Scaling Direct Drive Robots *

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Abstract

Recent experimental and analytical evidence indicates that direct drive robots become very practical and economical at miniature and microscopic scales, so it is interesting to understand quantitatively the properties of direct drive robots under scaling transformations. This leads to a study of how screws and their dual co-screws behave under the group of similarity transforms. This group is the group of isometries together with dilations. Several different representations are found on the space of screws and complementary representations are found on the dual space of co-screws. From the electromagnetic theory of the force and torque on a magnet in a magnetic field, we derive the scaling properties of the electromagnetic wrench. Hence, these results can be directly applied to the scaling of direct drive motors [1]. We conclude by proposing a scale-invariant measure for direct drive actuator performance.

1 Introduction

Understanding scaling properties of direct drive actuators is extremely important for robot miniaturization. As we design robots at increasingly smaller scales, we are repeatedly confronted with problems such as, how much power would a given robot require, if it were built at a smaller scale? Or, given the choice between two actuator designs, say two already built and their properties known at one scale, can we decide which would be superior at another scale? In general, scaling properties depend on assumptions about what factors we hold constant across scales. One approach considers the *specific torque* [3][6] determined when we fix an operating temperature, obtain a maximum operating torque, and divide by actuator mass. That approach is sensible because one of the primary design constraints is heat dissipation, but this paper reports a complimentary view of actuator scaling, in which assume that scaling a physical object like a motor is a pure dilation that preserves all the dimensional proportions of the object and its constituent parts but leaves the material density properties, such as magnetic remenance and resistivity intact; such a scaling would amount to building the same device out of the same materials, but at a different scale. [1]

The existence of multi-axis direct drive actuators like the Spherical Pointing Motor [2], Lorentz Force Levitation devices [7], and Sawyer and other Linear Motors [9][10], requires that we utilize more general concepts than the scalar variables such as torque and motor constant that are conventionally applied to single-axis motors, so we analyze the scaling properties in terms of the group of proper rigid body motions $SE(3)$. For us a screw will be an element of the Lie algebra $se(3)$, in some other work these elements are called motors or twists the name screw being reserved for elements of the projective space formed from the Lie algebra. The importance of the Lie algebra elements is that they represent generalized velocities. They are six component vectors, the first three components of which are the angular velocity of the rigid motion and the last three are a linear velocity characteristic of the motion. We call the dual of the Lie algebra $se^*(3)$, the space of co-screws. These co-screws are linear functionals on the screws. Generalized momentum and generalized forces are co-screws. The generalized force vectors are combinations of forces and

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[1] Thinking about what to leave invariant across scales often leads to confusion, for example it is easy to fall into an infinite set of choices by reasoning from Ohm's Law. Consider that doubling the dimensions doubles the length and radius of wire, which halves its resistance, so the voltage is half if the current is the same, but shouldn't the current be quadrupled because the cross-section of wire is four times larger? Then, should we change the voltage or change the resistance? We can put down this apparent paradox by considering only the effects of a pure dilation, because the properties of interest such as the motor constant K_M and mass m can be shown, up to a first order, to be independent of wire diameter.

torques usually called wrenches. In defiance of the more common notation we will write wrenches with the first three components giving the torque about the origin and the second three the force on the body. The reason for this change is that we can now write the evaluation of a co-screw on a screw as a simple matrix multiplication. For example, if the wrench:

$$\mathcal{W} = \begin{pmatrix} \boldsymbol{\tau} \\ \mathbf{F} \end{pmatrix}$$

acts on a rigid body which is moving with velocity screw:

$$\mathbf{s} = \begin{pmatrix} \boldsymbol{\omega} \\ \mathbf{v} \end{pmatrix}$$

then the power exerted is given by:

$$\mathcal{W}^T \mathbf{s} = \boldsymbol{\tau} \cdot \boldsymbol{\omega} + \mathbf{F} \cdot \mathbf{v}$$

See [11] for a more detailed account.

The scaling properties of screws and other representations of $SE(3)$ have hardly been considered. The notable exception is Donelan and Gibson's work [4] on the classification of screw systems up to scale invariance. We follow their treatment closely introducing the group of similarity transformations of $\mathbb{R}^3$, denoted $\text{Sim}(3)$. These transformations preserve the scalar product of a pair of vectors in $\mathbb{R}^3$ but only up to multiplication by a positive scale factor. Although not a symmetry group of any physical device, a knowledge of the action of this group is important when considering design issues.

2 The Group of Similarity Transforms

The group of similarities of $\mathbb{R}^3$ can be generated by translations, rotations and dilations about the origin. The action of the group $\text{Sim}(3)$ on a point $\mathbf{x}$ in $\mathbb{R}^3$ can be written as:

$$M : \mathbf{x} \longmapsto R(s\mathbf{x}) + \mathbf{t}$$

where R is a rotation matrix, $\mathbf{t}$ a translation vector and s the scale factor, that is a real number greater that zero.

An element M of the group can thus be written as a 4×4 matrix:

$$M = \begin{pmatrix} sR & \mathbf{t} \\ 0 & 1 \end{pmatrix}$$

The Lie algebra of this group is seven dimensional, elements can be written as 4×4 matrices, this time of the form:-

$$X = \begin{pmatrix} \sigma I_3 + \Omega & \mathbf{v} \\ 0 & 0 \end{pmatrix}$$

where Ω is an anti-symmetric 3×3 matrix, $\mathbf{v}$ a 3 vector and σ a real number. If we think of the group element as the exponential of the Lie algebra element; $M = e^X$, a pure dilation about the origin would have, $s = e^\sigma$.

Now the adjoint action of the group on its Lie algebra can be written as the conjugation:

$$Ad(M)X = MXM^{-1}$$

Using the block matrix form above this becomes:

$$Ad(M)X = \begin{pmatrix} \sigma I_3 + R\Omega R^T & \sigma\mathbf{t} - R\Omega R^T\mathbf{t} + sR\mathbf{v} \\ 0 & 0 \end{pmatrix}$$

If we write the Lie algebra element as a 7×1 vector:

$$X = \begin{pmatrix} \boldsymbol{\omega} \\ \mathbf{v} \\ \sigma \end{pmatrix}$$

the operator $Ad(M)$ can be written as a 7×7 matrix:

$$Ad(M) = \begin{pmatrix} R & 0 & 0 \\ TR & sR & -\mathbf{t} \\ 0 & 0 & 1 \end{pmatrix}$$

where T is the 3×3 anti-symmetric matrix which satisfies; $T\mathbf{v} = \mathbf{t} \times \mathbf{v}$ for any vector $\mathbf{v}$. We can think of the screws as forming a six dimensional subspace in this Lie algebra, that is we identify the screws with Lie algebra elements of the form:

$$\mathbf{s} = \begin{pmatrix} \boldsymbol{\omega} \\ \mathbf{v} \\ 0 \end{pmatrix}$$

It is easy to see that this subspace is invariant under the action of the similarity group. We obtain a six dimensional representation of the similarity group on the space of screws:

$$\mathbf{s}' = \begin{pmatrix} R & 0 \\ TR & sR \end{pmatrix} \mathbf{s}$$

So the effect of a dilation about the origin on a screw $\mathbf{s}^T = (\boldsymbol{\omega}^T, \mathbf{v}^T)$ will be to change it to; $(\mathbf{s}')^T = (\boldsymbol{\omega}^T, s\mathbf{v}^T)$, where s is the scale factor of the dilation.

This representation is exactly the representation of the group on the six dimensional angular velocity–velocity vectors. To see this recall that the linear velocity of a point $\mathbf{x}$, in $\mathbb{R}^3$ attached to a rigid body moving with instantaneous screw; $\mathbf{s}^T = (\boldsymbol{\omega}^T, \mathbf{v}^T)$ is given by:

$$\dot{\mathbf{x}} = \boldsymbol{\omega} \times \mathbf{x} + \mathbf{v}$$

see for example [12]. Under an arbitrary similarity:

$$\mathbf{x} \longmapsto R(s\mathbf{x}) + \mathbf{t}$$

we expect the velocity to change to; $R(s\dot{\mathbf{x}})$, the translation part disappears because the velocity is essentially given by the difference of successive position vectors. Now if the angular and linear velocity changes according to:

$$\boldsymbol{\omega} \longmapsto R\boldsymbol{\omega}, \qquad \text{and} \qquad \mathbf{v} \longmapsto sR\mathbf{v} + \mathbf{t} \times R\boldsymbol{\omega}$$

as in the adjoint representation, then we get the correct transformation properties for the velocities of point in $\mathbb{R}^3$.

This is not the only possible representation of the group of similarities on the six dimensional space of screws. For example, screws are also used to represent lines in space. The Plücker coordinates of a line through two points $\mathbf{x}$ and $\mathbf{y}$ are given by:

$$\mathbf{s} = \begin{pmatrix} \mathbf{x} - \mathbf{y} \\ \mathbf{x} \times \mathbf{y} \end{pmatrix}$$

The first three components here give a vector in the direction of the line while the second triple give the moment of the line about the origin. The relevant representation here is the antisymmetric square of the 4×4 representation; $\wedge^2$. Using the 4×4 representation to find the action on points we see that:

$$\wedge^2 M : \mathbf{x} - \mathbf{y} \longmapsto sR(\mathbf{x} - \mathbf{y})$$

and

$$\wedge^2 M : \mathbf{x} \times \mathbf{y} \longmapsto s^2 R(\mathbf{x} \times \mathbf{y}) + sR(\mathbf{x} - \mathbf{y}) \times \mathbf{t}$$

Hence we can write the 6×6 matrix for $\wedge^2 M$ as:

$$\wedge^2 M = \begin{pmatrix} sR & 0 \\ sTR & s^2 R \end{pmatrix}$$

Recall that, for $SE(3)$ the adjoint representation and the antisymmetric square of the 4×4 representation are the same. In fact there are lot of inequivalent six dimensional representations of the similarity group. It is straightforward to see that for any n the following matrix gives a representation of Sim(3)

$$\Gamma_n(M) = \begin{pmatrix} s^n R & 0 \\ s^n TR & s^{n+1} R \end{pmatrix}$$

So we have that:

$$Ad(M)|_{\sigma=0} = \Gamma_0(M), \qquad \text{and} \qquad \wedge^2 M = \Gamma_1(M)$$

Notice that, although all these are different six dimensional representations they give the same action on the projective space $\mathbb{P}\mathbb{R}^5$. Since multiplying the homogeneous coordinates by an overall factor of s^n has no effect. This means that the results of Donelan and Gibson apply to all of them.

Now we find the action of the similarity group on the dual space of co-screws. That is the wrenches. The pairing of a velocity screw with a wrench is not necessarily invariant with respect to scaling. The pairing of a co-screw $\mathcal{M}^T = (\mathbf{j}^T, \mathbf{p}^T)$ with a screw is given by:

$$\mathcal{M}^T \mathbf{s} = \mathbf{j} \cdot \boldsymbol{\omega} + \mathbf{p} \cdot \mathbf{v}$$

Although we no longer expect this to be invariant as it is for rigid transformation, we do expect it to scale as some power of of the scale factor s. This is because this pairing gives physical quantities like energy and power, depending on what co-screw we use, and we expect these physical quantities to scale as s^n. Note that, we can think of this as defining a number of one dimensional representations of Sim(3). We can write these representations as:

$$\alpha_n(M) = s^n$$

So for example, we expect the mass of an object to transform according to the α_3 representation.

This means that we expect the co-screws to transform according to representations Γ_n^*, which satisfy:

$$\Gamma_n^{*T} \Gamma_m = \Gamma_m^T \Gamma_n^* = I_6 \otimes \alpha_{m+n} \tag{1}$$

As a result, we get a sequence of inequivalent representations of the similarity group on the co-screws, given by the matrices:

$$\Gamma_{n+1}^*(M) = \begin{pmatrix} s^{n+1} R & s^n TR \\ 0 & s^n R \end{pmatrix}$$

For example, take a wrench $\mathcal{W}^T = (\boldsymbol{\tau}^T, \mathbf{F}^T)$, where $\mathbf{F}$ is the total force acting on a rigid body and $\boldsymbol{\tau}$ the torque about the origin. The pairing of this with a velocity screw gives the power being expended. Now, for dimensional reasons we expect the power to scale as s^5, since the units of power are [mass][length]^2[time]$^{-3}$ and we expect the mass to scale as s^3 and the length as s. Hence, the representation obeyed by the wrenches is:

$$\mathcal{W}' = \begin{pmatrix} s^5 R & s^4 TR \\ 0 & s^4 R \end{pmatrix} \mathcal{W}$$

that is Γ_5^*. The effect of a pure dilation about the origin is given by the matrix:

$$\Gamma_5^*(s) = \begin{pmatrix} s^5 I_3 & 0 \\ 0 & s^4 I_3 \end{pmatrix}$$

so that; $\mathcal{W}_s^T = (s^5\boldsymbol{\tau}^T, s^4\mathbf{F}^T)$. In future we will use subscript s to denote a quantity subject to a pure dilation.

This however, takes no account of how the wrench is produced. Suppose the wrench were produced by the action of gravity. Such a weight wrench would have the form:

$$\mathcal{W} = \begin{pmatrix} -mg\mathbf{r}\times\mathbf{k} \\ -mg\mathbf{k} \end{pmatrix}$$

where m is the mass of the rigid body, g acceleration due to gravity, $\mathbf{r}$ the position vector of the center of mass and $\mathbf{k}$ the unit vector in the upward direction. Now if we scale the size of the body by s this will change to:

$$\mathcal{W}_s = \begin{pmatrix} -s^4mg\mathbf{r}\times\mathbf{k} \\ -s^3mg\mathbf{k} \end{pmatrix}$$

That is, the weight wrench transforms according to the Γ_4^* representation. The discrepancy in the exponent of s is accounted for by the fact that that we have implicitly assumed that size of the planet earth was not scaled, that is we assumed g was constant. This was reasonable since the point of these scaling relations is usually to see the effect of building a device at a different scale here on earth. If we were to insist that the size of the planet should be scaled along with the rigid body under consideration then we would use the relation:

$$F = G\frac{mM_\oplus}{R^2}$$

for the magnitude of the force. We would then recover the the scaling relation:

$$\mathcal{W}' = \begin{pmatrix} s^5R & s^4TR \\ 0 & s^4R \end{pmatrix}\mathcal{W}$$

There is another geometrical six dimensional representation of Sim(3), this is the fourfold antisymmetric power of the standard representation, however a small calculation shows that:

$$\wedge^4 M = \Gamma_{-1}^*(M)$$

Finally here, we note the following relations among the tensor products of the representations we have found:

$$\alpha_m\otimes\Gamma_n = \Gamma_{m+n}, \qquad \text{and} \qquad \alpha_m\otimes\Gamma_n^* = \Gamma_{m+n}^*$$

In the following we look at the scaling relation for the wrench caused by the magnetic field of current flowing in a coil of wire acting on a permanent magnet. That is a direct dive motor. In this case it is not easy to see what implicit assumptions are being made.

3 Electro-Magnetism

Suppose we have a permanent magnet sitting in an external magnetic field with flux density $\mathbf{B}$. The field produces a force an torque on the magnet, the total wrench acting on the magnet due to the field can be written:

$$\mathcal{W} = \begin{pmatrix} \boldsymbol{\tau} \\ \mathbf{F} \end{pmatrix} = \begin{pmatrix} \int \mathbf{r}\times(\mathbf{J}\times\mathbf{B}) \\ \int \mathbf{J}\times\mathbf{B} \end{pmatrix}$$

where the integrations are over a virtual current distribution $\mathbf{J}$, representing the magnet (see [8]). Now, in trying to determine the scaling properties of these quantities we don't need to use very sophisticated model of the interaction between magnet and the current in the wires. A first approximation will do since we expect the higher approximations to have the same scaling properties. Hence, we may represent the torque on the magnet by the simple formula:

$$\boldsymbol{\tau} = \boldsymbol{\mu}\times\mathbf{B}$$

where $\boldsymbol{\mu}$ is the magnetic moment of the magnet and $\mathbf{B}$ is the flux due to the coil. We can assume that the magnetic moment of the magnet is proportional to the volume of the magnet. Hence, if the scaling does not affect the material properties of the magnet only it's size, then $\boldsymbol{\mu}$ scales as s^3.

It is important that we model the wire as having some thickness. However to first order we can ignore this thickness assuming the diameter of the wire is small compared to the size of the motor. The finite thickness of the wire will give rise to second order effects. So we approximate the magnetic flux due to the coils as the integral:

$$\mathbf{B} = \frac{\mu_0}{4\pi}I\oint\frac{\mathbf{dc}\times\mathbf{a}}{|\mathbf{a}|^3}$$

where $\mathbf{a}$ is a vector directed from the source point to the field point. The integration is over all source points that is along all the wires. Scaling the coils give the new flux:

$$\mathbf{B}_s = I_s\frac{\mu_0}{4\pi}\oint\frac{s\mathbf{dc}\times s\mathbf{a}}{s^3|\mathbf{a}|^3} = s^{-1}\mathbf{B}I_s/I$$

here I_s, is the new current in the wire.

Combining these relations we arrive at an expression for the torque at the new scale:

$$\boldsymbol{\tau}_s = \boldsymbol{\mu}_s\times\mathbf{B}_s = s^3\boldsymbol{\mu}\times s^{-1}\mathbf{B}I_s/I = s^2\boldsymbol{\tau}I/I_s$$

The results of the last section tell us that the force must scale as s if the torque scales as s^2, that is, this wrench transforms according to the Γ_2^* representation. For a pure dilation we have:

$$\mathcal{W}_s = \begin{pmatrix} \boldsymbol{\tau}_s \\ \mathbf{F}_s \end{pmatrix} = \begin{pmatrix} s^2\boldsymbol{\tau} \\ s\mathbf{F} \end{pmatrix} I_s/I = \begin{pmatrix} s^2 I_3 & 0 \\ 0 & sI_3 \end{pmatrix} \mathcal{W} I_s/I \quad (2)$$

4 Conclusion

Suppose we have a direct drive robot which can generate just enough force and torque to lift its links with no payload. If we scale down this machine, that is dilate with a scale factor $s < 1$, then the force and torque required to lift the links reduces rapidly. The torque, for example scales as s^4, this is because the wrench required to lift the links is a weight wrench obeying the representation Γ_4^*. The wrenches generated by the direct drive motors obey the representation Γ_2^* and hence reduce less rapidly. The result is that scaling down the robot will yield more useful torque and force.

A common way to compare motors is the torque constant K_T, which is the amount of torque developed per unit current. Even for a single axis motor this is not really a constant but depends on the position of the rotor, see [1]. For the multi-axis machines we want to consider we would have a torque constant for each axis which we can combine with a force constant K_F for each axis. The force constant being the force developed per unit current. The combination is a wrench:

$$\mathcal{K} = \begin{pmatrix} \mathbf{K}_T \\ \mathbf{K}_F \end{pmatrix}$$

which we could call the wrench constant of the device. From equation (2) we see that the wrench constant scales as Γ_2^*, since for a pure dilation we have:

$$\mathcal{K}_s = \begin{pmatrix} s^2\mathbf{K}_T \\ s\mathbf{K}_F \end{pmatrix}$$

Another commonly quoted characteristic of motors is the motor constant. The power consumed by the motor is proportional to the square of the torque. The motor constant is the constant of proportionality. Notice that the motor constant can be written as; $K_M = K_T/\sqrt{Z}$ where Z is the resistance of the motor windings. Now when we scale the motor the length of the wire in the windings increases as s and the area of the wire scales as s^2. So if the resistivity of the wire remains the same, the resistance of the windings will scale as s^{-1}. We can introduce the solenoid constant $K_S = K_F/\sqrt{Z}$, and combine the two to produce a wrench:

$$\mathcal{M} = \begin{pmatrix} \mathbf{K}_M \\ \mathbf{K}_S \end{pmatrix}$$

This wrench scales according to the $\Gamma_{5/2}^*$ representation, a pure dilation gives:

$$\mathcal{M}_s = \begin{pmatrix} s^{5/2}\mathbf{K}_M \\ s^{3/2}\mathbf{K}_S \end{pmatrix}$$

Next we look again at the power, as mentioned in section 1 the power is given by the matrix product $\mathcal{W}^T\mathbf{s}$. When we look at the power exerted by the motor the wrench we must use is the electromagnetic one obeying the Γ_2^* representation, the screw is a velocity screw and hence transforms according to the Γ_0 representation. So using formula (1) we can see that the output power of the motor scales as s^2. Using the same argument we can see that the power required to lift the weight of the robot's links scales as s^4, since we must use a weight wrench here to compute the power. The input power is given by I^2Z, where I is the current and Z the resistance. We cannot make much progress with this relation without some model of the electrical circuit for the motor. However, if we revert to the one axis case we can write the current in terms of the torque and torque constant:

$$P_{in} = \left(\frac{\tau}{K_T}\right)^2 Z$$

Hence, we see that the input power scales as s^3. The efficiency of the motor, that is the output power divided by the input power then scales as s^{-1}.

Notice that none of the above constants is invariant across scales (see Figure 2). Such an invariant would be useful when comparing devices of widely differing sizes.

For a single axis motor we can produce a scale invariant measure by dividing the motor constant by a power of the mass:

$$Q = K_M/m^n, \qquad \text{so that} \qquad Q_s = s^{5/2}s^{-3n}Q$$

Choosing $n = 5/6$ ensures that $Q_s = Q$, that is, Q is scale invariant.

The scatter plot in Figure 1 compares direct drive actuators across scales using a modified version of our Q variable that takes into account the difference in actuator workspace. Our design task is to build a robot finger or leg with 90 degrees of motion per joint. Some actuators have less than 90 degree workspaces, so we normalize their mass by a factor of s_m Here

$$Q = \frac{K_M}{(s_m m)^{5/6}}$$

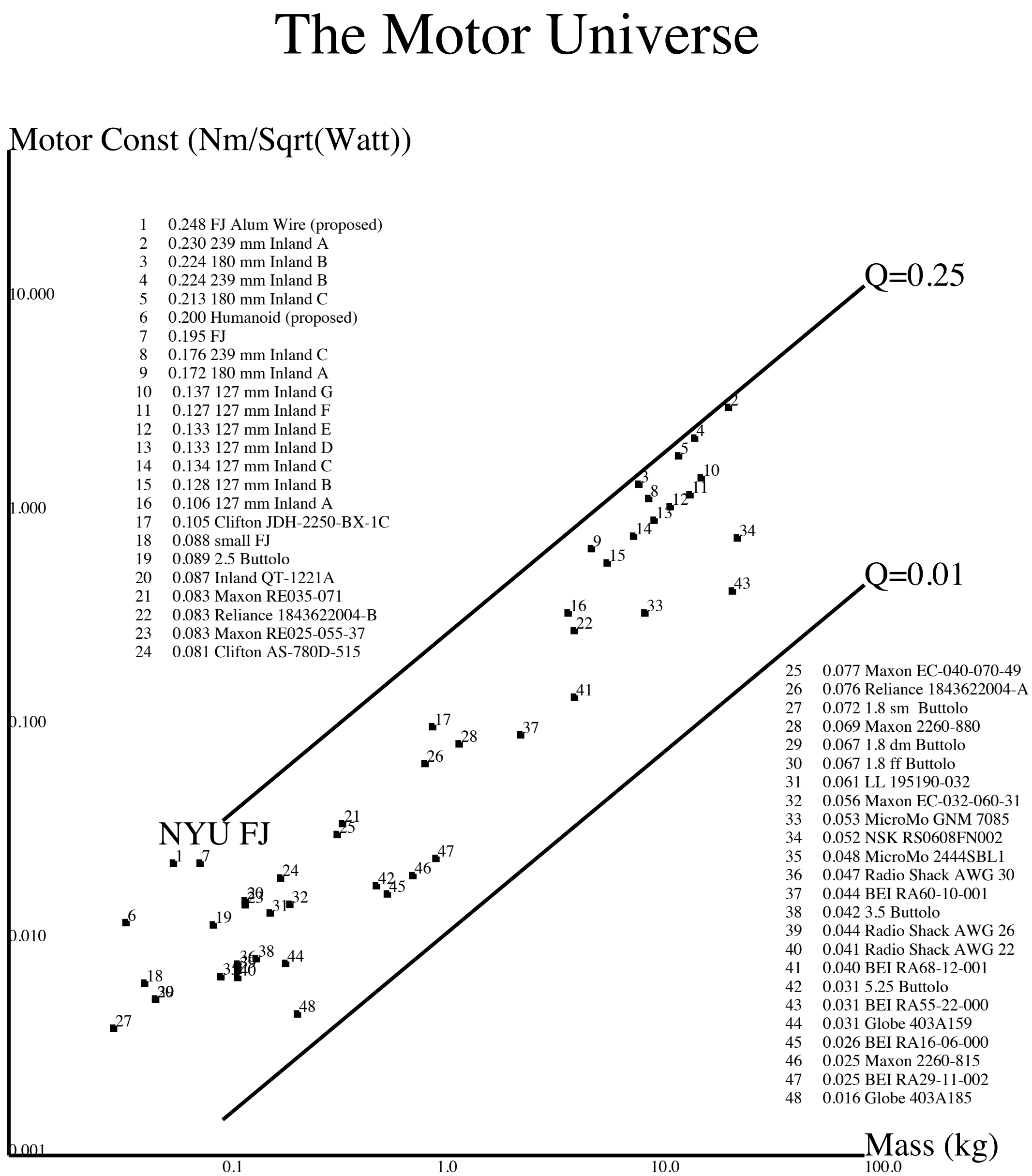

Figure 1: The graph plots 48 commercial and experimental direct drive actuators across a range of sizes from a few grams up to $26Kg$ on a log-log scale. The vertical axis denotes motor constant and the graph shows the well-known property that larger motors have larger motor constants. The diagonal lines plot constant contours of our proposed quality measure Q (see text), by which the actuators are ordered.

where

$$s_m = \max(1, 2\omega/\pi)$$

and ω is the actuator's workspace in radians.

Some manufacturers (e.g. BEI and Inland) offer only "unframed" motors, i.e. devices that do not include structural housing and bearings necessary in any given application. We estimate that in general one must scale the mass of an unframed motor by a factor of 2 in order to build the actuator into a realistic application. Our experimental actuators include all the mass of the robot link needed to connect one joint to another.

The devices listed are all commercial actuators except the ones with the names "FJ" and "Humanoid" which are based on measurements from experimental devices built in our lab. The "FJ" is built and tested and the "FJ Alum Wire" is what we project to achieve once we obtain aluminum wire. Strictly speaking, the "Radio Shack" actuators were actually torque measurements using commercial prefabricated air-core inductors and a $3/8'' \times 3/4''$ cylinder of NdFeB material. The table shows the well known fact that motor constant K_M increases with increased mass, i.e. bigger motors have bigger motor constants. But suppose we want to know, given an actuator technology, what would happen if we built the same device smaller or larger. The scale-independent parameter Q allows us to compare, for example, the large 26 *Kg* NSK motor with with 30 *gm* disk drive motor analyzed by Buttolo et al. [3]. Figure 1 illustrates this Q function in comparison with a variety of commercial actuators, including the Inland motors analyzed in [6] and our experimental actuators [13][14].

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Quantity	Symbol	Units	Scaling Property	Mass Relation
Resistance	Z	Ω	$Z_s = Z/s$	$O(m^{-1/3})$
Coil current	I	*Amps*	I_s independent of I	
Magnet current	I_m	*Amps*	$I_{ms} = sI_m$	$O(m^{1/3})$
Magnetic torque	$\boldsymbol{\tau}$	*Nm*	$\boldsymbol{\tau}_s = s^2\boldsymbol{\tau} I_s/I$	$O(m^{2/3}I_s/I)$
Torque Constant	K_T	*Nm/Amp*	$K_{Ts} = s^2 K_T$	$O(m^{2/3})$
Motor Constant	K_M	$Nm/\sqrt{Watt}$	$K_{Ms} = s^{\frac{5}{2}} K_M$	$O(m^{5/6})$
Magnetic force	F	*N*	$F_s = sFI_s/I$	$O(m^{1/3}I_s/I)$
Force Constant	K_F	*N/Amp*	$K_{Fs} = sK_F$	$O(m^{1/3})$
Solenoid Constant	K_S	$N/\sqrt{Watt}$	$K_{Ss} = s^{\frac{3}{2}} K_S$	$O(m^{1/2})$
Required Power	W	*Watts*	$W_s = s^3 W$	$O(m)$
Efficiency	E	*dimensionless*	$E_s = E/s$	$O(m^{-1/3})$
Motor quality	Q	$Nm/\sqrt{Watt}/(Kg)^{5/6}$	$Q_s = Q$	$O(1)$

Figure 2: This table summarizes some important scaling properties of familiar scalar properties of direct drive actuators that follow from the main results in this paper.