SEARCH FOR A MOVING TARGET

GEORGINA MARY WOODWARD

Submitted to the C.N.A.A. in partial fulfilment of the requirements for the degree of Doctor of Philosophy.

Sponsoring Establishment
The Polytechnic of Wales

March 1989

To
Annette and Peter

CONTENTS

ACKNOWLEDGEMENTS

ABSTRACT

1 THE SEARCH PROBLEM AND ITS HISTORY 1

1.0 Introduction 1

1.1 The Origin of Search Theory 1

1.2 The Scope of Application of Modern Search Theory 5

1.3 Models of the Search Process 13

1.3.1 Target Wherabouts 14

1.3.2 Detection Capability 16

1.1.3 Updating Target Information 24

1.4 Optimisation 27

1.4.1 Infinitely Divisible Search Effort 29

1.4.2 Discrete Sequential Search 31

1.5 The Need for a Revised Search Model 33

2 A MODEL OF THE SEARCH PROCESS 36

2.0 Introduction 36

2.1 The Search Process 37

2.2 Derivation of the Updating Procedure 39

2.2.1 An Alternative Approach 43

2.3 The Search Area 45

2.4 Specification of Target Motion 52

2.4.1 Speed 52

2.4.2 Direction 56

2.5 Transition Matrices 57

2.5.1 The Time Interval Between Looks 57

2.5.2 Calculation of Transition Probabilities 60

2.6 Determination of Search Path 65

2.7 An Example Search Strategy 68

3 OPTIMISATION 76

3.0 Introduction 76

3.1 Criteria for Optimisation 77

3.2 Optimal Probability of Detection 78

3.2.1 Dynamic Programming 80

3.2.2 Exhaustive Evaluation of Strategies 82

3.2.3 Elimination of Strategies 85

3.3 Detection Probability Per Unit Cost 93

3.3.1 Optimisation 95

3.4 Sub-Optimal Solutions 101

4 ANALYSIS OF THE SEARCH MODEL 109

4.0 Introduction 109

4.1 Elements of the Search Model 110

4.2 Initial Distribution and Detection Capability 110

4.3 Target Movement 113

4.3.1 Target Speed 114

4.3.2 Target Direction 120

4.4 Conclusion 123

5 REVISED MODEL OF TARGET MOVEMENT 124

5.0 Introduction 124

5.1 Double Grid System 125

5.2 Updating at Unit Time Intervals 126

5.2.1 Transition Probabilities 128

5.3 Analysis of the New Model of Target Movement 132

6 RESULTING SEARCH STRATEGIES 136

6.0 Introduction 136

6.1 Optimising Probability of Detection 137

6.2 The Standard Case 142

6.2.1 The Best Strategy for Twenty Looks 151

6.2.1 Detection Probability in Restricted Strategies 160

6.3 Variations on the Standard Case 165

6.3.1 Target Direction 166

6.3.2 Target and Searcher Speed 169

6.3.3 Detection Capability 172

6.3.4 Initial Conditions 175

6.4 Optimising Detection Probability Per Unit Cost 178

6.5 Concluding Remarks 183

7 CONCLUSION

REFERENCES

APPENDIX

ACKNOWLEDGEMENTS

I wish to express my sincere thanks to my supervisors; Dr. R.J.Wiltshire, Dr. D.M.Jones, and Prof. J.D.Griffiths for their advice and guidance in the course of this work, with my particular gratitude to Dr. Wiltshire for his encouragement during the preparation of this thesis.

I would also like to thank my daughter Annette for her assistance in typing the text, and Dr. H. Zand for his helpful comments.

ABSTRACT

SEARCH FOR A MOVING TARGET

GEORGINA WOODWARD

A mathematical model of a discrete sequential search for a target moving in discrete space is given. The model is based on a Bayesian updating algorithm giving successive probability distributions of target position at intervals throughout the search. Updating allows for target movement and for negative information gained from unsuccessful search.

The search is conducted by taking a sequence of discrete, instantaneous looks at chosen points, or nodes, of the search area. The sequence of chosen nodes is termed a strategy. The successive target position distributions allow the probability of detecting the target to be found for any strategy.

The model is an improvement over previous discrete sequential search models with respect to the following points. Target movement between nodes of the search area is formulated in terms of statistical information of target speed and direction, which are likely to be known. The time interval between looks, and target movement during this time, are related to the distance travelled by the searcher between search nodes. Also, with each look, the searcher has a view of surrounding nodes as well as the chosen search node. Implementation of these refinements is aided by considering the search area to consist of a finite, isometric pattern of nodes.

Optimisation of strategies with respect to both detection probability and detection probability per unit cost is considered, and a criterion given in each case to assist optimisation. However, in practice, these criteria are of limited use, and full optimisation can only be carried out in a limited range of cases. Restricting both the planning horizon of the optimisation process, and searcher travel distance, allows sub-optimal strategies to be found in a wider range of cases. Results suggest that the detection probability of strategies found under these restrictions is normally close to optimal.

CHAPTER ONE

THE SEARCH PROBLEM AND ITS HISTORY

1.0 INTRODUCTION

The process of search is a fundamental part of human behaviour, but it is only in comparatively recent times that a mathematical theory of search has been developed. In this chapter the early history of search theory is outlined, and some of the current applications of the theory are introduced. Ways in which search processes may be modelled mathematically are also discussed and the extent to which optimal solutions have been obtained for these models is examined. The inadequacy of present models in providing practical guidance for single searcher, moving target searches is discussed, and an approach is proposed that would provide a strategy for such a search.

1.1 THE ORIGIN OF SEARCH THEORY

The short history of search theory, as part of the general development of Operations Research, can be traced to its beginning during World War II.

Operations Research did not emerge as a coherent professional field until the early years of the war. Its development, chronicled by Larnder (1984), can be traced to the need to answer a specific military threat: the defence of Britain against air attack. The pre-war build up of German air power posed for Britain a serious problem of early warning against airborne attack. The development of radar to counter this threat led to the establishment of the first experimental radar station in 1937, followed by four more by the summer of 1938.

Early trials had seemed encouraging, but a major air defence exercise, carried out in July 1938, revealed that the additional stations did not improve effectiveness as expected. It became clear that there was a need to coordinate and correlate the often conflicting information received from these stations. A proposal was therefore made that research should be carried out into the operational, as opposed to the technical, aspects of the radar system. The term 'Operational Research' (O.R.) was coined as a suitable description of this new branch of applied science.

The Operational Research Group thus formed played a key role in the analysis and evaluation of the radar system, its impact on air tactics and its effectiveness against enemy raids. The subsequent extension of O.R. analysis to the prediction of the outcome of future operations was even more important in its consequent effect on policy decisions.

The entry of the United States into World War II brought further defence problems. A major threat was that of U-boat attack on U.S. shipping. In answer to this the Antisubmarine Warfare Unit (A.S.W.) was set up to study and coordinate defence against German submarines. Collaboration with the British had demonstrated the value of employing civilian operational analysts on defence projects. Hence a non-military scientific task force was recruited, under the leadership of Philip Morse, to assist the A.S.W. unit in analysing the U.S. antisubmarine effort.

The primary danger from a submarine lay in its ability to remain undetected, so the process of finding it was an important part of the counteraction. Morse and his colleagues identified a set of important quantities involved in the search process, and derived equations relating them, enabling the prediction of search efficiencies and effective search patterns. In this way the early theory of search uniting physical and operational attributes of target and searcher through mathematical concepts was developed. The recommendations made on the basis of this work measurably improved the tactics of convoy protection and submarine search.

The A.S.W. Operations Research Group, as they were called, eventually grew to about forty members, including Bernard Koopman and George Kimball who are recognised as being among the founders of modern search theory. An account of the development of the group and its contribution to the early days of O.R. in the United States is given by Morse (1986).

At the end of the war, the work of the group was consolidated into a series of reports, one of which was 'Search and Screening' written by Koopman (1946). This remained classified for many years, but a series of papers on search, Koopman (1956a), (1956b) and (1957) were published. This work laid the foundation for modern search theory. An expanded and updated version of Search and Screening, Koopman (1980) has since been produced.

Applications of search theory can today be found in fields much wider than this military origin, although the military influence still persists, particularly in the common use of the word 'target' for the object of the search.

1.2 THE SCOPE OF APPLICATION OF MODERN SEARCH THEORY

Search theory today is a broad field of applied science encompassing many disciplines and having application to a wide range of practical problems. Some of the more common areas of application of search theory are given below.

(a) Military

Search theory has today, as at its origin, important military and defence application. Areas of interest range from the traditional topics of efficient search patterns for enemy units and effective barrier patrol, to tracking and guidance systems of modern missiles. Much of the research in the military field is necessarily classified, but Sutcliffe (1985) indicates that most of current British military research centres on the mechanism of detection, in particular with respect to target characteristics and behaviour, sensor characteristics and the environment.

Detectability is an important consideration to both searcher and target. For the searcher, it is important firstly, to have detection equipment which is as efficient as possible. Considerable research effort is devoted to the continuing development of sophisticated radar, sonar and satellite surveillance hardware, aided by the signal processing and image enhancement capabilities of modern computers. Secondly it is important to know the true operational capability of such equipment, in order to accurately estimate the efficiency of any search. The performance of equipment in the far from ideal conditions of combat, under the supervision of operators under stress, may be very different from that measured under laboratory conditions. Gathering reliable data in these conditions is often problematic.

Conversely a target wishing to remain unseen must aim to present an image, or behave in a way that will minimise the probability of recognition by enemy sensors. One of the design requirements of a modern battleship, for example, is a profile which will give a radar image that is as confusing as possible. Also such tactics as the ejection of metallic chaff from ships in order to confuse enemy rocket guidance systems, and low flying of aircraft to avoid radar detection are commonly practiced.

(b) Search and Rescue

Rescue services are confronted daily with the problem of locating missing persons lost in hostile environments. Such examples as air-sea rescue operations and mountain searches are seen frequently in news reports. Planning searches of this type has traditionally relied heavily on the intuition and experience of the leader of the search team. Increasingly, however, search theory methodology is being employed to assist in the efficient deployment of search resources.

The missing person (the search target) is often at risk from the environment, or from injuries that may have been sustained. For this reason the overriding concern of a search and rescue (S.A.R) operation is normally the recovery of the target in the shortest possible time. Optimal utilisation of the manpower and equipment available to the search is therefore of paramount importance.

The most extensive application of computer aided search planning in S.A.R. is the United States Coast Guard Computer Aided Search Planning System (C.A.S.P.) described by Richardson and Discenza (1980). This system has been used by the U.S. Coast Guard, in planning open water searches, since 1974. The principal output from the C.A.S.P. system is a sequence of probability 'maps' which display the current target location probability distribution throughout the search period.

The initial probability map, of target position at the time of the incident, is produced by Monte Carlo methods from a number of scenarios of the events leading to the incident, weighted according to credibility. Monte Carlo simulation is also used to update this probability map to account for subsequent drift, due to currents and winds, in the time period until the search is started. C.A.S.P. then gives guidance on the allocation of search effort based on optimal search theory. If the day's search is unsuccessful, Bayesian updating is used to reflect this negative information and the steps of updating for movement and of search planning are repeated. C.A.S.P. has proved to be a very useful aid and has been credited with saving many lives.

Many of the problems associated with planning S.A.R. operations are described in Haley and Stone (1980), pages 45-71. Particular difficulty is frequently found in producing accurate initial probability distributions from the vague and conflicting information that is often presented to search planners. Similar problems are discussed by Hypher (1980) and Mattson (1980).

(c) Recovery and Clearance

The problems associated with clearing debris and recovering equipment for analysis, following a disaster, are similar in nature to those of a S.A.R. operation. The principal difference, however, is that the priority is not normally speed of detection, but completion of the operation with minimum cost.

Richardson and Stone (1971) describe the operations analysis of the deep water search for the remains of the submarine Scorpion. The search, which took place over a period of five months, was conducted using a towed platform carrying cameras, magnetometers, and sonar equipment. The platform was submerged to depths of up to two miles. The search area covered approximately 150 square miles of ocean floor. Search effort was allocated within this area on the basis of an a priori distribution of Scorpion's wherabouts (produced in a similar manner to that used in the C.A.S.P. system) and information gained during the search.

One of the main problems in allocating search effort was in estimating search effectiveness when knowledge of the capabilities of the sensors against the target were uncertain. Analysis carried out after Scorpion had been found, showed that sensor capabilities had been overestimated, indicating that the search plan had not been as efficient as anticipated. Other problems included navigational uncertainties making it difficult to execute the search as planned, and difficulty in deciding how much effort shold be expended in close investigation of some contacts. This occured when it was uncertain whether an object detected was in fact Scorpion, or a false target such as a magnetic rock.

Grasty (1980) describes a clearance operation of a different nature following the disintegration of the nuclear powered Russian satellite COSMOS 954 on re-entry into earth atmosphere. Radioactive debris, which was scattered over a large area of Canada's Northwest Territories, had to be located and retrieved. Gamma ray detection equipment was used in airborne reconnaissance flights. Here again some difficulty was encountered in distinguishing true radioactive contacts from naturally occuring sources of radiation, and in pinpointing the exact ground location of contacts detected from the air, particularly in densely wooded areas.

(d) Medical

There is currently substantial medical interest in increasing the efficiency of procedures for screening and monitoring large populations for the early indications of certain diseases. Diseases such as cancer, hypertension and glaucoma can be treated much more effectively if detected in their early stages, before symptoms become apparent to the patient. If screening can be made more efficient, larger groups of people can be tested.

Kolesar (1980) describes an application of search theory to the location of retina blind spots characteristic of the disease glaucoma. The eye is tested for blind spots by determining the patient's response to a small light stimulus directed at points on the retina. Kolesar has shown that, by positioning test points optimally on the retina, the number of points tested can be reduced from the seventy two commonly used to only ten, while maintaining a 95% probability of detecting a blind spot if present.

(e) Other Applications

Additional areas to which search theory is currently applied include surveillance, mining and exploration, and industry.

The purpose of a surveillance operation is not to find a specific target, but to monitor the state of a given system by information gathering. Search theory can assist in planning a surveillance operation by determining an optimal allocation of the available surveillance resources in order to maximise information gain. Pollock (1980) analyses the surveillance problem and indicates the role of search theory within this larger context.

Areas of application of this approach include allocation of police patrol routes, pollution control, and monitoring livestock and fish populations in order to determine stock levels, and enforce hunting and fishing regulations. These and other applications are discussed in Haley and Stone (1980), pages 99-112.

Exploration for coal, oil and mineral deposits often involves drilling test bores in areas where geological survey data indicates deposits may be found. Optimal placement of test bores, to increase the probability of a find, can be assisted by utilisation of both positive and negative results (from successful and unsuccessful bores respectively) to update current geological information. Surkan (1975) developed a computer program to assist in exploration for oil deposits. This quantifies prior information from geological surveys into a density map for oil deposits, which is then modified in a Bayesian manner for information gained during exploration.

Kadane (1980) describes several industrial applications including, minimising the cost of quality control testing and fault detection, the management of research and development projects, and maintainance scheduling. All of these may be viewed as task sequencing problems within the constraint of minimising costs.

The application of search theory in many of the areas described above has only been made possible by the use of high speed computer processing. This has enabled the construction of a variety of mathematical models, many of which would have been impossible to attempt by other means. Some common features of current search models are examined in the next section.

1.3 MODELS OF THE SEARCH PROCESS

A number of different approaches to modelling search processes may be found in current literature, reflecting the diversity of situations to which they apply. A comprehensive index of papers on the topic can be found in Strumpfer (1980).

Several features can, however, be identified as common to many models. These features, which may briefly be summarised as:

1) Target wherabouts

2) Detection capability

3) Updating target information

are discussed in the following sections.

1.3.1 TARGET WHERABOUTS

The exact position of the target is clearly unknown at the start of the search, otherwise no search would be necessary. However, an a priori probability distribution of the possible wherabouts of the target can frequently be calculated from known information. This information may consist, for example, of a last reported position, or a re-entry trajectory as in the COSMOS 954 search, or geological data as in oil exploration.

The a priori distribution of a target located in an open area can, in pricipal, be described mathematically by a continuous probability density function over two (or three) dimensional Euclidean space. Search effort may then be allocated to any point in the search area. Stone (1975) page 20, gives an example of a bivariate normal distribution representing the position of a ship in distress. The distribution, centered on the ship's reported position, describes the error in the navigational system used.

In practical searches it is frequently found to be more convenient to divide the search area into rectangular cells, where a cell represents the smallest region to which search effort can be allocated. A discrete a priori distribution over the set of cells of the search area may then be determined. The search area is divided in this way in the C.A.S.P. system and also in the search for Scorpion. As in these examples, described in section 1.2(b), Monte Carlo simulation is often used to produce the a priori distribution when this approach is taken.

A discrete distribution is also appropriate when the target may be located in one of a number of disjoint locations, for example an object hidden in one of a number of boxes. Pollock (1970), Dobbie (1974) and Wegener (1982), among others, formulate search models of this type. Here it is often assumed that the probability of the target being in any box is known in advance.

Other applications where a discrete distribution is appropriate, suggest a conceptual, rather than physical search space. In the case of quality control testing the target is a fault (if any) in a manufactured article. The search space consists of the set of all possible faults and an initial distribution may be determined from the frequency of occurance of each fault.

In cases where an a priori distribution is required, but no information is available, a uniform distribution over the search space may be used.

1.3.2 DETECTION CAPABILITY

In order to plan a search it is necessary to have a measure of the effectiveness of the search effort applied. This requires knowledge of the ability of the sensor to detect the target. At any instant a sensor will not detect a target with total certainty, there is normally some probability, dependent on range and operational conditions, that the target will be overlooked. Search effectiveness relates the amount of effort spent looking in an area to the probability of detecting the target given that it is in that area.

Mathematically search effort is regarded in one of two ways: as discrete units of a fixed quantity applied sequentially such as the light stimulus used in the glaucoma test, or as continuously applied amounts available in any required quantity such as the towed sensors used in the search for Scorpion. Detection probability in each of these cases is discussed below.

a) Discrete Effort

Search where effort is available in discrete units is called discrete search, not to be confused with the description of the search space or target motion. A unit of search effort is termed a look or glimpse and is assumed to be instantaneous. The quantity of search effort expended can be measured by the number of looks taken.

The instantaneous probability of detection in one look, conditional on the target being present, may vary with time, position and range. Koopman (1980) page 54, however, considers the simplest case where detection probability is constant. Assuming that the probability of detection, p, in each look is independent of other looks, the probability of detecting the target in n looks, denoted P_n is

$$P_n = 1 - (1 - p)^n \qquad (1.1)$$

by the usual rules of probability.

Pollock (1970) and Kan (1977) consider searches for an object hidden in one of a number of boxes, where the detection probability is independent of time but dependent on the box searched, while Wegener (1982) allows detection probability to be dependent not only on the box searched, but on the number of looks already taken in that box, thus removing the need for the condition of independence.

Koopman, Chapter 3.2, discusses the dependence of instantaneous detection probability on range for a look taken over an open area, and extends the application of discrete effort to the case of a searcher moving over open water, taking a series of discrete looks as it travels. The approach taken here is somewhat different from the usual discrete effort search as the glimpses are assumed to occur rapidly, leading to a formulation similar in nature to the continuous case discussed next.

b) Continuous Effort

Detection capability of a continuous sensor can be expressed in terms of a detection function relating the amount of effort expended in an area to the probability of detecting a target in that area.

Koopman (1956a) and (1956b), characterised the detection ability of a continuous sensor in terms of its lateral range function and sweep width. The lateral range of a target from a sensor when both are travelling with fixed speed and course is the distance of their closest approach, denoted by x in diagram 1.1.

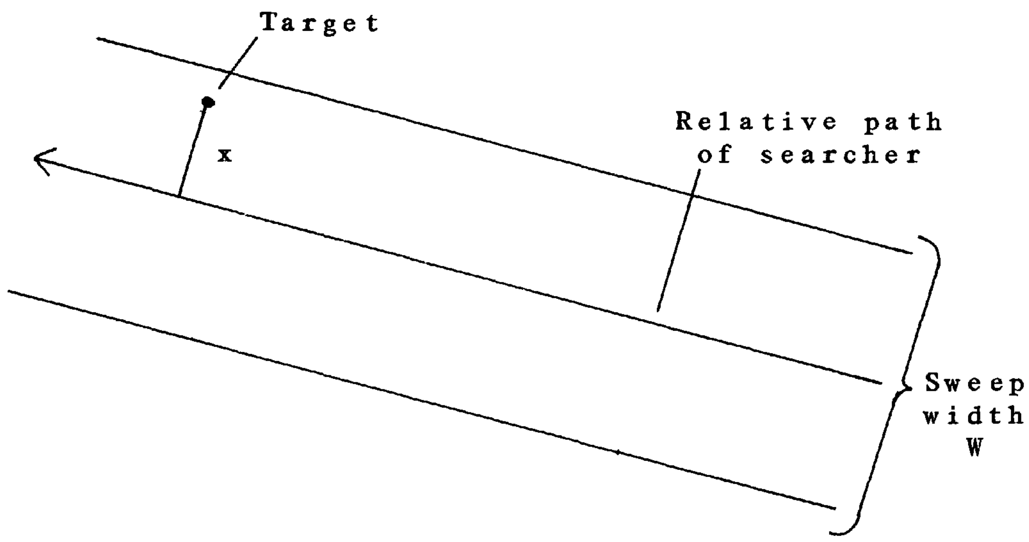
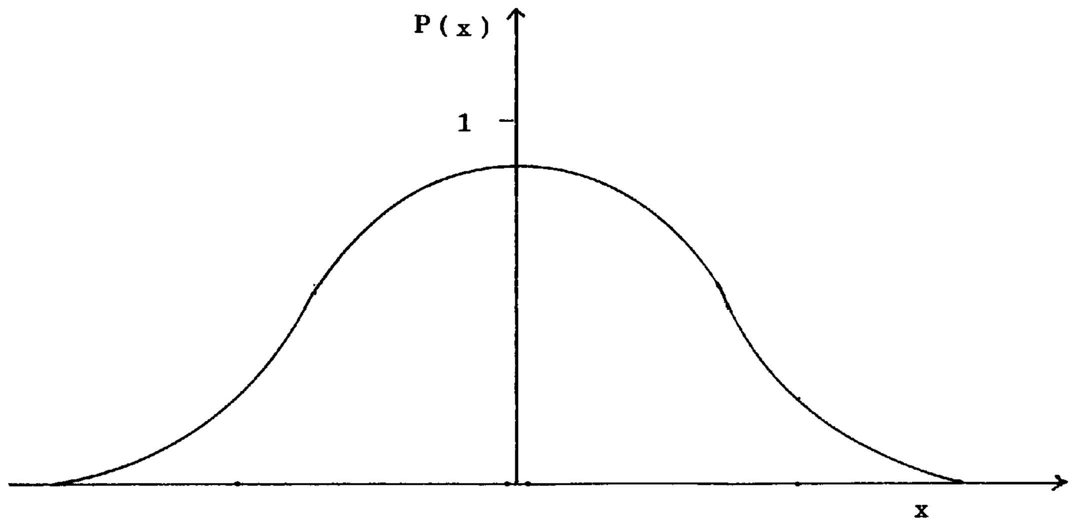


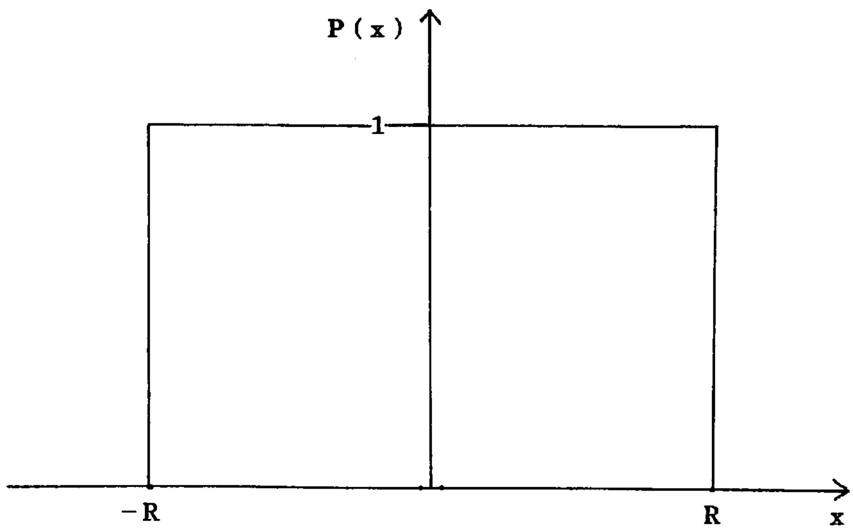
Diagram 1.1 The Path Swept by a Sensor

The lateral range function is a measure of the detection ability of the sensor in terms of the lateral range of the target. It gives the cumulative probability, P(x), that the sensor, as it travels along its course, will detect a target having lateral range x.

In general the lateral range function will be dependent on the relative velocity of target and searcher as well as on operational factors. A typical lateral range function is shown in diagram 1.2a, while diagram 1.2b shows the lateral range function of an idealised sensor having a definite range law of detection. This idealised sensor will detect with total certainty any target within range R, any target outside this range will not be detected.



1.2a



1.2b

Diagram 1.2 Lateral Range Functions

The sweep width of the sensor, denoted by W, is given by the integral of the lateral range function, thus

$$W = \int_{-\infty}^{\infty} P(x)\, dx \qquad (1.2).$$

Sweep width is a commonly used measure of the capability of a continuous sensor.

A sensor with effective sweep width W has a probability of detecting a target equivalent to that of a sensor with a definite range law of detection, of range $R = W/2$. Thus the product (path length travelled) x (effective sweep width) gives an area equivalent to that 'swept clean' by such a sensor on the same path, as illustrated in diagram 1.1. This product is used as a measure of the search effort applied, from which the detection probability can be calculated.

The classical exponential detection function formulated by Koopman as the 'law of random search', for a target located in an area A, is based on the following assumptions:

1) The target distribution is uniform in A.
2) The sensor has a definite range law of detection of width W/2.

3) The observer's path is random in A, in the sense that different small portions of the path are placed independently of one another in A.

This leads to a detection probability p given by

$$p = 1 - e^{-WL/A} \tag{1.3}$$

where L is the total path length in A.

Assumption 3 is, in practice, impossible to satisfy in most applications. However the law of random search is frequently assumed to apply as it gives a useful lower bound on the detection probability, even when the search path is far from random.

In the C.A.S.P. system the search effort applied to any cell is assumed to be uniformly distributed over the cell area. This is approximated in practice by performing equally spaced parallel sweeps covering the cell. Allowing for the uncertainties in navigational accuracy leads to a detection function equivalent to the law of random search.

c) Further Considerations

The above discussions on discrete and continuous application of search effort assume that the detection capability of the sensor against the target is known.

In practice this is frequently not the case, either because of the difficulty in assessing operational values, or because the state of the target is not known. Richardson and Belkin (1972) consider a search model where the sweep width of the sensor is fixed, but only known as a prior probability distribution, while Discenza and Stone (1981) look at a survivor search where the target may change state during the search, each state having a different detectability.

A further assumption is that the sensor has perfect discrimination, that is it will not mistakenly detect a false target such as the magnetic rock found in the search for Scorpion. Where false targets may be detected a distinction must be made between detecting a object and conclusively identifying it to be the target after closer investigation. Dobbie (1973) and Stone et al (1972) consider searches in the presence of false targets.

The models so far considered also assume that the search is passive, that is that the search conditions are in no way changed by the presence of the searcher. In particular it is assumed that the target is not aware of the search, so unable to change its behaviour in order to avoid or assist detection. Dobbie (1975) models a search where the target attempts to avoid detection by moving away from the vicinity of the searcher when it becomes aware of the searcher's presence.

1.3.3 UPDATING TARGET INFORMATION

As the search progresses, the current estimate of the position distribution of the target may need to be updated. There are two reasons for updating. Firstly, if the target can move, the position distribution will change with time as the search is conducted. Secondly, when search effort is applied to a region and the target is not detected, an a posteriori probability that the target is in that region can be calculated.

Considering firstly target motion. This may be described in a variety of ways, but is frequently modelled as a stochastic process which may occur in either discrete or continuous time.

Pollock (1970), Kan (1977) and Washburn (1980) consider searches for a target moving between a set of discrete cells or boxes. In each case, the motion is described by a discrete time Markov process determined by a known set of constant transition probabilities $\{p_{ij}\}$, which give the probability that the target is located in box j at time t_{k+1} given that it is in box i at time t_k. Dobbie (1974) also considers Markov target motion between discrete cells but here motion is in continuous time described by known transition rates between cells.

Pursiheimo (1978) investigates a case where target motion between boxes is conditionally deterministic. The motion is uniquely determined by a route function partly dependent on the initial box. In contrast the most general type of motion is modelled by Stone (1979). Here movement is described by a stochastic process $\{X_t, t \geq 0\}$ representing virtually any reasonable type of target motion in Euclidean n space.

On a practical level the C.A.S.P. system allows for target motion by computing a set of drift vector probability distributions. These represent the effect on the target of winds and ocean currents at various geographical locations. Updating is performed daily by Monte Carlo methods, the target being moved along in short time intervals until updating is complete.

Updating the target distribution for additional information gained as the search progresses may be carried out using Bayes' rule. In its simplest form the posterior probability that the target is located in area A, given an unsuccessful search, can be expressed as

$$P(A|S) \quad = \quad S)$$

where A is the event 'ta ev event 'search fails'. Tfai updated in a separate se system, or in conjunca target movement as perfct a

Pollock provides an algœs target location distribm d a target moving with Ming between two boxes. It oxe located in box 1 with pr 1 2 with probability $1-\pi$. lit or box 2 and detects theete q_2 respectively, condily, the box in which the lcich moves from box i to box i p_{ij}. The updated probabied 1 after an unsuccessfulsuc is denoted by π'. π'.

If a look is taken in bcake equation (1.4), ,

$$P(A) = P(1) = \pi = P$$

$$\text{and} \quad P(A|S) = P(A$$

Thus, with allowance also for target movement, the updated probability is given by

$$\pi' = \frac{\pi(1-q_1)p_{11} + (1-\pi)p_{21}}{1-q_1\pi} \qquad (1.5a).$$

Similarly, if a look is taken in box 2,

$$\pi' = \frac{\pi p_{11} + (1-\pi)(1-q_2)p_{21}}{1-q_2(1-\pi)} \qquad (1.5b).$$

Explicit updating in this manner is only performed when effort is available in discrete units, however implicit allowance for the effect of previous search is made in any search plan that proceeds in time.

1.4 OPTIMISATION

A search operation normally has only a limited amount of search effort or resources availible. The basic problem of search is the optimal distribution of these resources subject to the constraints and requirements of the individual application.

A rule specifying the way search effort is distributed in space is called an allocation. When the rule also dictates how the search should be carried out in time it is called a search plan or strategy. In the case of discrete effort, a search plan consists of a sequence of locations $X_1, X_2, \ldots. X_n$ at which a look is to be taken at time $t_1, t_2, \ldots. t_n$.

In the continuous case, effort applied over a discrete or continuous search space is often assumed to be infinitely divisible. That is, the effort available at any time can be distributed as finely as desired over the search space. Where a fixed amount of effort is available an allocation of this effort takes the form of a real valued function of the search space.

Constraints on a search plan may take the form of limits to the total time or cost available for the search, and to the rate at which search effort may be applied. An optimal search plan is one that gives a distribution of effort within these constraints that is optimal subject to a given criterion.

The applications discussed in section 1.2 illustrate the more commonly applied optimisation criteria. These include maximising the probability of detection within a given cost or time, maximising the probability of detection per unit cost and minimising the expected time to detection.

Optimisation of stationary target searches has been thoroughly investigated. However optimal plans for moving target searches are considerably more difficult to find. Progress that has been made in finding optimal plans for discrete sequential searches, and searches with infinitely divisible effort is discussed below.

1.4.1 INFINITELY DIVISIBLE SEARCH EFFORT

a) Stationary Target

Koopman (1946) first investigated the optimal allocation of search effort for a stationary target, finding a solution for an exponential detection function and circular normal target distribution. A method of finding allocations for more general target distributions is given in Koopman (1957). Stone (1975) provided necessary and sufficient conditions for optimal allocation within a given cost, giving a method of solution using Lagrange multipliers.

Where more search effort becomes available with time, a search plan which gives an optimal allocation by time t for all $t \geq 0$ is called uniformly optimal. Stone (1975) showed the existence of uniformly optimal plans under very general conditions and again used Lagrange multiplier techniques to obtain solutions.

b) Moving Targets

Dobbie (1974) considered a search with exponential detection function and continuous time Markov target motion between two discrete cells. He gave distributions of search effort maximising the probability of detection by a fixed time T, and minimising the expected time to detection.

Results for continuous space moving target models were initially restricted to exponential detection function and special types of target motion, such as special Markov processes (Hellman (1972) and Saretsalo (1973)), or conditionally deterministic motion (Stone and Richardson (1974) and Persiheimo (1978)). However, Stone (1979) gave necessary and sufficient conditions for optimal search plans for a wide class of stochastic target motion in discrete or continuous time and discrete or continuous space.

In the case of discrete time target motion and exponential detection function, Stone's conditions have a simple interpretation, as follows. If at some instant t, the target position distribution (given the failure of previous search) is g_t, then the optimal allocation of effort at that instant is the same as that for a stationary target search with target distribution g_t. The previously known results constitute special cases of Stone's result.

Practical methods of solution, (other than the simple two cell model considered by Dobbie), are restricted to the case of discrete time motion with exponential detection function. Brown (1980) gave an iterative algorithm for this case that produces a sequence of search plans which converges to an optimal plan. Conditions under which the algorithm may be applied were further investigated by Washburn (1983).

Brown compared the results obtained with those found by an incrementally optimal or myopic plan. This allocates effort such that each increment of effort applied yields the maximum increase in detection probability considering the previous increments. The comparison showed that, unlike the stationary target case, moving target optimal plans are not normally uniformly optimal. That is for $n < m$ an optimal plan for m time intervals need not be an extension of the optimal plan for n time intervals.

1.4.2 DISCRETE SEQUENTIAL SEARCH

a) Stationary target

Pollock (1960) first considered the case of a single searcher taking a sequence of discrete looks for a stationary target. He produced optimal strategies minimising the expected time to detection in the two region case using a dynamic programming approach. Black (1965) showed by a simple geometric argument that the expected cost of the search is minimised by taking the next look in the region for which the probability of finding the target, (given the failure of previous looks), divided by the cost of that look, is greatest.

More recently, Wegener (1980) minimised the expected search cost, considering only those strategies which are ultimately certain to find the target. Lössner and Wegener (1982) consider a variation on the basic search model introducing a cost penalty for the searcher switching from one cell to another.

b) Moving Target

Results for moving target searches with discrete effort have proved difficult to obtain. Pollock (1970) gave expressions for the minimum expected number of looks and the maximum probability of detection with a given number of looks when the search space consists of two discrete boxes. Using a dynamic programming technique not easily extended to a larger search space he obtained solutions in two special cases. These are the case of perfect detection and the 'no learning' case, where transition prbabilities $p_{ij} = v_j$ for all i, resulting in no information being gained from previous searches.

Kan (1977) extended Pollock's model to an arbitrary number of cells and obtained solutions in the no learning case and when the transition probabilities are represented by a Jordan matrix. Washburn (1980) applied an algorithm similar to that used by Brown (1980) but showed that when effort is not infinitely divisible the strategies obtained may not be optimal.

1.5 THE NEED FOR A REVISED SEARCH MODEL

The presently available models of moving target searches do not allow the calculation of an optimal path for a single searcher to follow over an open search area.

Search plans found under the assumption of infinite divisibility of effort are in practice impossible to implement, although they are sometimes used as a guide for the allocation of effort, as is the case in the C.A.S.P. system. However, approximating a search plan found under this assumption is only appropriate where a large number of search vehicles are available or target speed is very slow in relation to the time required to perform the search.

Some attempts have been made to express probability of detection for a single searcher travelling a specific path. Mangel (1982) for example gives an expression for the detection probability in the special case of target motion being a diffusion process. This has however, only been sucessful for very limited types of target motion, and the expressions found have proved very complicated.

Ideally it would be desirable to develope a model that would allow the specification of a continuous optimal path over the search area. However, in view of the difficulty in obtaining expressions for detection probability in this case, a more feasible approach would appear to to be to approximate the continuous path by a sequence of discrete searches.

Current models of discrete sequential search processes are not applicable to search over a large, two dimensional search area, for two reasons. Firstly, they are normally formulated as 'box type' searches, where the target can only be detected if it is in the box, or cell, in which a look is taken. This does not allow a situation where the cell size and spacing may be small in relation to the detection range of the searcher, giving the possibility of detection of a target in a nearby cell. Secondly target movement is normally expressed as a known, fixed set of transition probabilities between cells, implying a fixed time interval between looks. More realistically, the velocity of the target would be known, and the time interval between looks determined by the time taken by the searcher to travel across the search area.

This thesis attempts to extend the discrete sequential search model in these areas, in order to produce practical search strategies applicable to a search for a target moving in two dimensional space. In the following chapters a search model is developed, encompassing these features, that allows the calculation of detection probability for any search strategy. The problem of determining optimal strategies is also addressed, and a selection of the resulting search strategies obtained are presented and discussed.

CHAPTER TWO

A MODEL OF THE SEARCH PROCESS

2.0 INTRODUCTION

In this chapter an algorithm is developed for the determination of discrete search strategies for a moving target. The basis of the algorithm is a Bayesian updating procedure allowing probability distributions of the target position to be found prior to each look being taken. From these distributions the probability of detection for any strategy can be found.

Use of the updating procedure requires that the detection capability of the sensor and an initial probability distribution of target location are known. Further statistical knowledge is required concerning the speed and direction of motion of the target, from which a set of transition matrices (also required in the procedure) are determined. Calculation of these matrices is greatly simplified by restricting searcher and target movement to an isometric grid such as those illustrated in diagram 2.3.

2.1 THE SEARCH PROCESS

For the purpose of modelling the search process it is supposed that the search is conducted in the following manner.

Assume that the search area consists of a finite set of discrete locations which will be termed nodes. For simplicity, assume also that the search area is known to contain the target (although the model may also be used in the case of a defective initial distribution). The searcher takes a sequence of discrete looks at intervals throughout the search. Between looks, target and searcher are constrained to move from node to node within the search area.

At each stage of the procedure the searcher chooses a node (called a search node) at which the next look is to be taken. The look is assumed to occur instantaneously, and at that moment the searcher has a view of the surrounding nodes as well as the search node. Denote the set of nodes of the search area by X. If node $J \varepsilon X$ is the chosen search node, the target can be detected with known probability $\emptyset_{Jh} \varepsilon [0,1]$ given it is at node $h \varepsilon X$ at this time. A look can only be taken at a node.

The time interval between looks is not assumed to be constant. During this interval the target moves between nodes with Markov motion described by a set of known transition probabilities which are dependent on the length of the interval.

A discrete collection of nodes is not the most natural way in which to model a two dimensional search area. However this, together with the assumption of discrete effort gives a search model that is amenable to numerical calculation, as discussed in section 1.5.

The simplifications of discrete time Markov target motion and instantaneous detection are made in order to minimise the amount of computation involved in updating the target position distribution. This computation will be considerable because of the large number of nodes required to adequately represent a two dimensional search area.

Also to minimise computation, added assumptions are made that neither target motion, nor the detection ability of the searcher change with time, or as a result of previous search (i.e. it is a passive search). It is also assumed that the search is not complicated by the presence of false targets, and that if a detection is made, no further investigation is needed so the procedure will terminate. In practice this is often not the case, as indicated in section 1.2, however these simplifications are frequently applied, for example by Pollock (1970), Kan (1977), and Eagle (1984).

In the following section an algorithm is derived which allows an updated probability distribution of target position to be calculated. Although position may not be the only target statistic in error, (the speed or direction of motion may also have been incorrectly estimated), updating target position is the most natural approach as it is the target's correct location that is being sought. The model could, in theory, be extended to also allow target velocity, or even detectability, to be updated. However this approach was not taken because of the complexity of the resulting updating algorithm.

2.2 DERIVATION OF THE UPDATING PROCEDURE

A procedure for finding an updated probability distribution of target position following an unsuccessful look and and subsequent target movement can be found as follows.

Assume that at time t_k the probability distribution of target position is given by $\{\pi_i, i\varepsilon X\}$ with $\sum_i \pi_i = 1$.

This is implicitly conditional on the target being in the search area and being undetected at any time $t < t_k$ (otherwise the search would have terminated). Suppose a look is made at node $J \varepsilon X$ at time t_k and denote the updated distribution at some later time t_{k+1} by $\{\pi'_i, i \varepsilon X\}$.

Denote the following events:-

i' = target at node i at time t_{k+1},

$\bar{D}_J$ = target not detected from J at time t_k,

h = target at node h at time t_k,

S_{hi} = target moves from node h to node i in the time interval $t_{k+1} - t_k$.

The updated probability π'_i, that the target is at node i at time t_{k+1}, given that the look at time t_k is unsuccessful, can be expressed as:

$$\pi'_i = P(i' | \bar{D}_J)$$

$$= \frac{P(i' \cap \bar{D}_J)}{P(\bar{D}_J)}$$

by the usual rules of probability.

The target can only occupy one node at any time, so as mutually exclusive events this can be written

$$\pi'_i = \frac{\sum_h P(h \cap \bar{D}_J \cap S_{hi})}{\sum_h P(h \cap \bar{D}_J)}$$

$$= \frac{\sum_h P(h)\ P(\bar{D}_J | h)\ P(S_{hi} | (h \cap \bar{D}_J))}{\sum_h P(h)\ P(\bar{D}_J | h)} \qquad (2.1).$$

again by the usual rules of probability.

To simplify this a little the following notation is introduced:-

Let

$$\eta_{Jh} = P(\bar{D}_J | h) \qquad J, h \varepsilon X$$

Thus η_{Jh} is the probability that a target at node h is not detected by a look from node J. (i.e. $\eta_{Jh} = 1 - \emptyset_{Jh}$)

Also let $\Delta t = t_{k+1} - t_k$, and

$$\rho_{hi}(\Delta t) = P(S_{hi} | (h \cap \bar{D}_J)) \qquad h, i \varepsilon X \text{ with } \sum_i \rho_{hi} = 1$$

That is $\rho_{hi}(\Delta t)$ is the transition probability from node h to node i for the time interval $t_{k+1} - t_k$, of length Δt. It is conditional on the target being at node h at time t_k, and not detected by the look made at node J at this time.

And, by definition, $\pi_h = P(h)$.

Substitution in equation (2.1) gives

$$\pi'_i = \frac{\sum_h \pi_h \eta_{Jh} \rho_{hi}(\Delta t)}{\sum_h \pi_h \eta_{Jh}} \qquad (2.2).$$

The updating expressions given by Pollock (1970), equations (1.5a) and (1.5b), are a particular case of equation (2.2). If the search space consists of just two nodes, and the possibility of detection restricted to a target at the node at which the look is taken, equation (2.2) reduces to equations (1.5a) and (1.5b).

Use of this algorithm to produce successive target position distributions requires three components. These are:

1) an initial probability distribution of target position,

2) a set of non-detection probabilities $\{\eta_{Jh}\}$ for searcher and target at any positions J and h respectively in the search area, and

3) transition probabilities $\rho_{hi}(\Delta t)$ between any pair of nodes h and i for a variety of time intervals Δt.

Estimation of initial target position and of the detection capability of the searcher can present complex problems, as indicated in Chapter 1. However, for the purpose of the present model, it is assumed that an initial target distribution and the non-detection probabilities are known. Ways in which they may be specified are discussed in section 2.7.

In search models of this type, it is normally also assumed that transition probabilities governing target motion are known. Pollock (1970), Kan (1977), Washburn (1980) and Eagle (1984) all take this approach, allowing only a fixed time interval between looks. It is however, more realistic that target motion in a two dimensional area would be known in terms of speed and direction of movement. Determination of transition probabilities from this information for time intervals of varying length will be examined in detail in section 2.4.

2.2.1 AN ALTERNATIVE APPROACH

An alternative derivation of equation (2.2) can be obtained by considering the two steps of

a) updating following an unsuccessful look, and

b) updating to allow for target movement,

as separate operation as performed in the C.A.S.P. system. This can be achieved as follows.

Let $\{\pi_i^s, i \varepsilon X\}$ be the updated position distribution following an unsuccessful look at node J, (with no subsequent movement). By Bayes rule, the posterior probability π_i^s that the target is at node i is

$$\pi_i^s = \frac{\pi_i \ \eta_{Ji}}{\sum\limits_h \pi_h \ \eta_{Jh}} \qquad (2.3).$$

Also, let $\{\pi_i^m, i \varepsilon X\}$ be the position distribution updated for target movement only in time interval Δt. The probability π_i^m that the target is at node i after this interval is

$$\pi_i^m = \sum_h \pi_h \ \rho_{hi}(\Delta t) \qquad (2.4).$$

The probability that the target is at node i at time t_{k+1}, following an unsuccessful look and subsequent movement, denoted by $(\pi_i^s)^m$ is given by the composition of expressions (2.3) and (2.4):

$$(\pi_i^s)^m = \sum_h \left[\frac{\pi_h \ \eta_{Jh}}{\sum\limits_h \pi_h \ \eta_{Jh}} \right] \rho_{hi}(\Delta t)$$

$$= \frac{\sum\limits_h \pi_h \ \eta_{Jh} \ \rho_{hi}(\Delta t)}{\sum\limits_h \pi_h \ \eta_{Jh}} \qquad \text{(which is equation 2.2).}$$

This approach, of updating in two separate steps, will be considered further in Chapter 5, where a revised method of determining target transition probabilities is discussed.

This two-stage method of updating can also lead to an alternative algorithm. By performing the updating steps in reverse order, first allowing for movement, and then for an unsuccessful look, a sequence of distributions of target position immediatly following each look can be obtained.

Denote the updated probability that the target is at node i, following movement during time Δt and a subsequent unsuccessful look at node J, by $(\pi_i{}^m)^s$. The composition of expressions (2.3) and (2.4) in this order gives

$$(\pi^m{}_i)^s = \frac{\sum_h \pi_h \; \rho_{hi}(\Delta t) \; \eta_{Ji}}{\sum_h \left(\sum_k \pi_k \; \rho_{kh}(\Delta t) \right) \eta_{Jh}} \qquad (2.5).$$

The double sum in the denominator makes this expression a little more awkward to use than equation (2.2). Also, it is more useful to have information about target position immediatly before the decision of where to place each look is made, so equation (2.2) was chosen as the basis of the updating algorithm in preference to equation (2.5).

2.3 THE SEARCH AREA

Before target transition probabilities for any given time interval can be determined, consideration must be given to the specification of the search area.

It was initially thought that a continuous two dimensional search area could be used, divided into discrete rectangular cells, as in the C.A.S.P. system. However it was found that direct calculation of transition probabilities between cells from information of target speed and direction was too complicated to be of practical use. This is because the probability of movement from one cell to another depends not only on velocity, but also on position within a cell as illustrated by diagram 2.1. These quantities are only known probabilistically and evaluation of the resulting trigonometric expressions would have been too complex to incorporate in the updating algorithm. Monte Carlo methods, used in the C.A.S.P. system, would also be unsuitable in this case as updating in this way between successive looks would be far too time consuming.

Other cell shapes, such as triangular or hexagonal divisions, were also considered but these offered no advantage over rectangular cells. Approximating the search area by a collection of discrete points, as illustrated in diagram 2.2, was therefore found to be the most suitable approach.

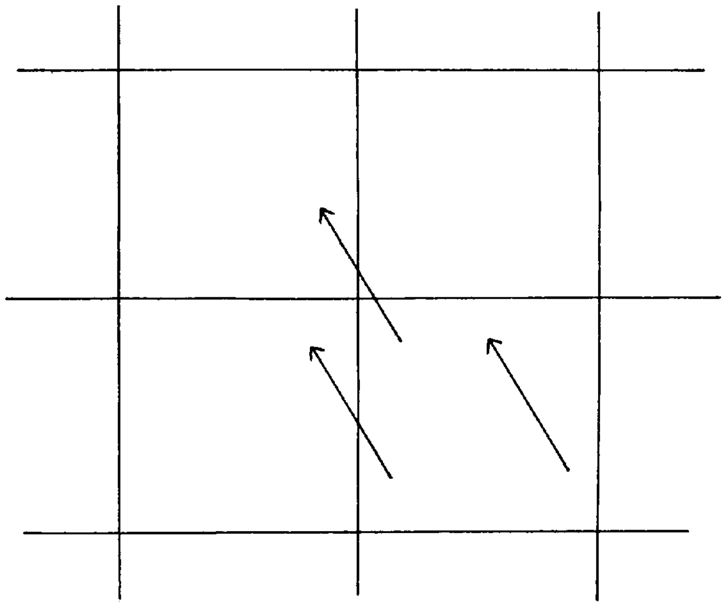


Diagram 2.1 Target Movement Between Cells

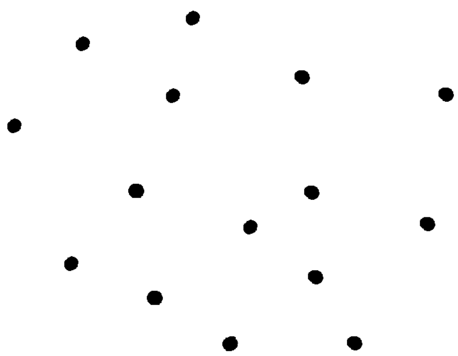
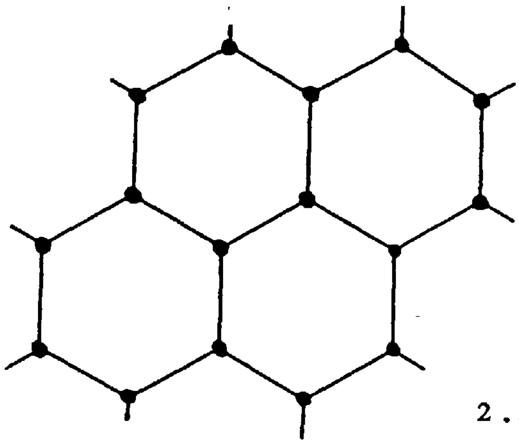


Diagram 2.2 A Collection of Discrete Nodes

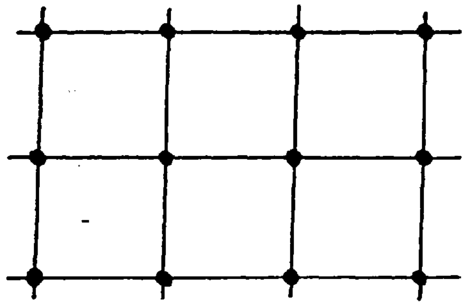
The target transition probability between any two nodes is dependent on the relative position and spacing of the nodes. Consideration must therefore be given to the arrangement of the nodes of the search area. An isometric arrangement, where neighbouring nodes are equidistant from each other is computationally the most convenient. The three possible such arrangements in two dimensions are illustrated in diagrams 2.3a, 2.3b and 2.3c which show nodes having three, four and six neighbours respectively.

The principal advantage of such an arrangement is that, if the target is constrained to move along the grid lines indicated in the diagrams, the distance of travel between any two nodes is always an integer multiple of the node spacing. This greatly simplifies the formulation of transition probabilities. A further advantage is obtained in determining the time interval between looks if searcher movement is also constrained in this way. This point is discussed in section 2.5.

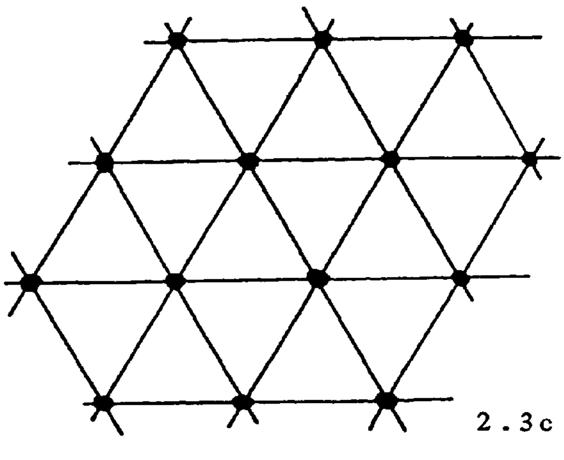
Constraining target and searcher movement in this way clearly restricts the direction of motion, however the computational advantages of this simplification are substantial. To minimise the restriction on direction, the node arrangement shown in diagram 2.3c was chosen, allowing six directions of travel from each node.



2.3a



2.3b



2.3c

Diagram 2.3 Isometric Node Arrangements

For computational convenience the nodes are considered to have unit spacing and are given coordinates by means of a pair of non-orthogonal axes as shown in diagram 2.4. With this coordinate system, the minimum distance N, along the grid lines, between two nodes having coordinates (x_1,y_1), (x_2,y_2) respectively is given by the relation

$$N = \begin{cases} |x_1-x_2| + |y_1-y_2| & \text{if } (x_1-x_2)(y_1-y_2)>0 \quad (2.6a) \\ \max\{ |x_1-x_2|,|y_1-y_2| \} & \text{if } (x_1-x_2)(y_1-y_2)\leq 0 \quad (2.6b). \end{cases}$$

Thus in diagram 2.4, the distance from the node labeled A, of any node in the shaded regions is given by expression (2.6a), while the distance of any node in the non-shaded regions is given by (2.6b). (In the case of nodes on the boundary of the two regions, the two expressions are equivalent.)

For example, the distance between points A(1,-1) and B(2,1), shown in diagram 2.4, is

$$N = |1-2|+|(-1)-1| = 1+2 = 3 \text{ units},$$

and the distance between A(1,-1) and C(-3,2) is

$$N = \max \{ |1-(-3)|,|(-1)-2| \} = \max \{ 4,3 \} = 4 \text{ units}.$$

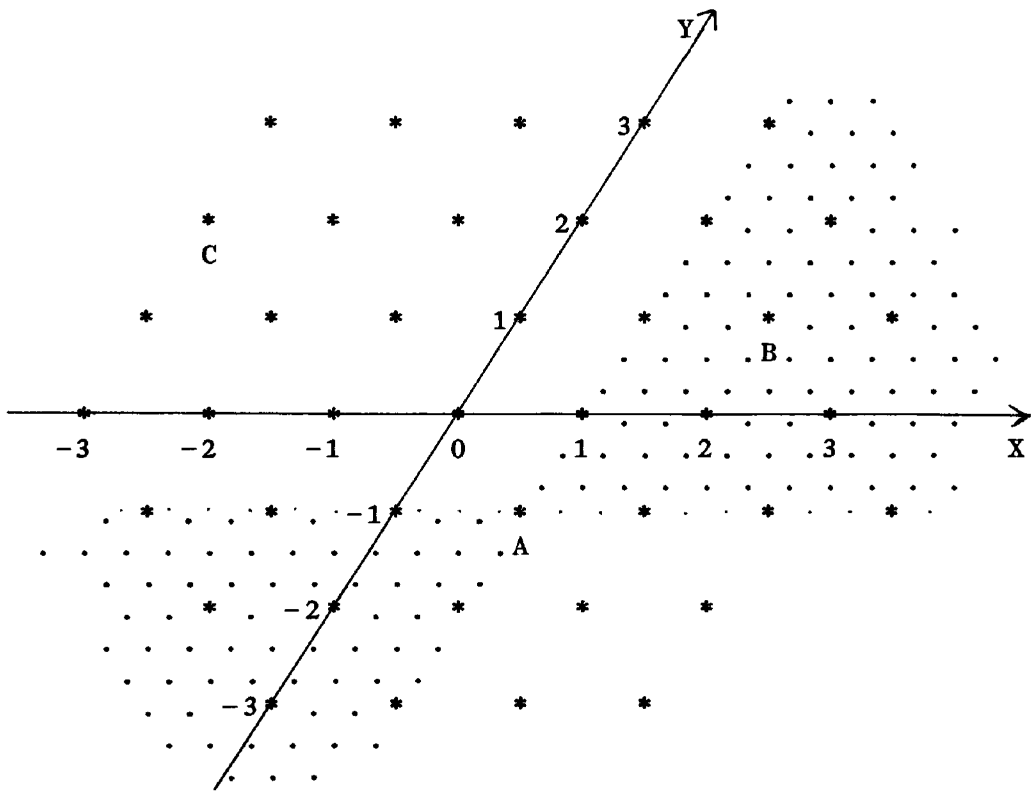


Diagram 2.4 Illustrating the Distance Function

2.4 SPECIFICATION OF TARGET MOTION

It is assumed that the motion of the target may be specified as independent probability density functions of speed and direction of movement. Such information might be available from historical data of similar targets, or estimated from prevailing search conditions.

The assumption of independence may not be appropriate to every search, strong winds for instance may make travel in one direction slower than another. However, this assumption simplifies the calculation of transition probabilities. Also to facilitate this calculation, target motion is assumed to be independent of position. Again this would not be appropriate in every situation, as geographical features might, for example, impede movement in some locations. The two components of motion are discussed in the following sub-sections.

2.4.1 SPEED

The distribution of target speed may be specified by any probability density function appropriate to the search under consideration.

For computational investigation of the search model it is assumed that the speed is given by a Beta distribution. This has the form:

$$B(x) = A\ x^{\alpha-1}\ (1-x)^{\beta-1} \qquad x\varepsilon[0,1] \qquad (2.7)$$

with

$$A = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\ \Gamma(\beta)}$$

The Beta distribution was used because it is of a suitable shape and has the necessary flexibility to model a range of target behaviour.

It is assumed that three statistics of target speed are known, allowing specification of a particular Beta distribution. These are:

1) The maximum target speed, denoted by V_{tmax}
   This allows the speed distribution to be specified over the interval [0,1]. Denoting the speed distribution by s(v), and target speed by V_t this can be achieved by setting $v = V_t/V_{tmax}$.

2) The normal running speed of the target, denoted by V_{tn}.
   This is assumed to determine the mode of the distribution, from which a relation between the parameters α and β can be found.

The mode, M, of the Beta distribution with $\alpha, \beta > 1$ is given by

$$M = \frac{\alpha-1}{\alpha+\beta-2} \qquad (2.8).$$

Rearranging and setting $M=V_{tn}/V_{tmax}$ gives the relationship between the parameters,

$$\alpha-1 = \frac{M\ (\beta-1)}{1-M} \qquad (2.9).$$

3) The probability that the target is moving at a speed greater than $\frac{1}{2}(V_{tn}+V_{tmax})$, denoted by λ. This statistic determines the top tail of the Beta distribution from $\frac{1}{2}(M+1)$ to 1. Substituting expression (2.9) for $(\alpha-1)$ in equation (2.7) (written in terms of v), allows β to be found from the expression

$$\int_{\frac{1}{2}(M+1)}^{1} A\ v^{c(\beta-1)}\ (1-v)^{\beta-1}\ dv = \lambda \qquad (2.10)$$

with

$$c = \frac{M}{1-M}$$

These three statistics were chosen because they constitute information that might reasonably be known about target behaviour, and allow the parameters α and β to easily be calculated. The mode was chosen as statistic two in preference to the mean, because it was felt that it would be more likely that the normal running speed would be known, rather than the average running speed, particularly in the case of skew distributions.

It might also seem more natural to give the value of the integral from M to 1 as statistic three , but in the case of a symmetric distribution this is always equal to 0.5 whatever the value of $\alpha = \beta$. To avoid being unable to determine the parameters in this situation the specified statistic was chosen, although the integral over another range could equally well be used.

The parameters α and β were found by evaluating the integral given in equation (2.10), using a N.A.G. computer routine. A range of values of β in intervals of 0.1 was taken, and that giving the closest approximation to λ chosen. This degree of accuracy was felt to be sufficient for the purpose of investigating the model. In practice, more accurate interpolation could be used if the accuracy of target information was sufficient to warrant this.

The restriction was made that α, $\beta \geq 1$. This was imposed to exclude Beta distributions having infinite values at $v = 0$ and $v = 1$ as these were not felt to be suitable to represent target speed. (Although equation (2.8) is not strictly defined at $\alpha = \beta = 1$, equation (2.10) may still be used to specify a uniform distribution.)

A further restriction was imposed that α, $\beta <= 10$, for two reasons. Firstly it was thought that this range gave a sufficient degree of flexibility in specifying target movement. Secondly, large changes in α and β above this range are required to produce small changes in the value of the integral. The degree of accuracy to which λ is likely to be known was not thought to warrant taking extreme values of α and β, although again in practice this restriction could be lifted if desired.

2.4.2 DIRECTION

Clearly on the grid described there are six possible directions of travel from any node. In order to specify the distribution $d(\theta)$ a convention is adopted for numbering the directions as shown in diagram 2.5.

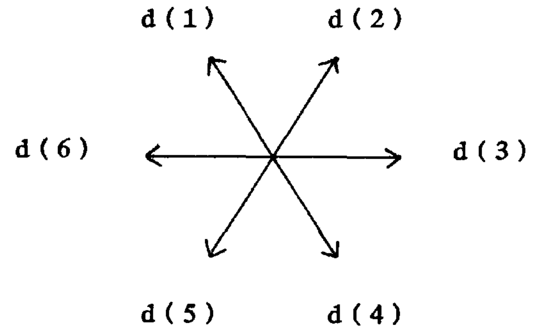


Diagram 2.5 Numbering System for Direction

Any suitable discrete distribution may be used to give the direction of motion of the target, ranging from the case where $d(\theta)$ is uniform, expressing an unknown direction, to the case $d(\theta) = 1$ for $\theta = a$, $d(\theta) = 0$ for $\theta \neq a$, $\theta, a \varepsilon \{1, 2, ..,6\}$ giving an exact direction.

2.5 TRANSITION MATRICES

Target movement, specified by the distributions of speed and direction discussed in section 2.4, must be translated into transition probabilities between nodes for use in the updating procedure given by equation (2.2). The transition probabilities are also dependent on the length of time between successive looks, which is discussed in the next sub-section.

2.5.1 THE TIME INTERVAL BETWEEN LOOKS

To minimise the computation involved in updating the target distribution it is essential that the transition probabilities be known before the updating procedure begins. Clearly a set of transition probabilities can be calculated for each of only a limited number of time intervals. A suitable method of determining a finite set of time intervals, realistic to a physical search, is to consider how long it would take the searcher to travel between consecutive search nodes.

The simplification is made that the searcher moves between nodes by the shortest path, along the grid lines described in section 2.3, travelling at a constant speed V_s. If consecutive search nodes are distance N units apart, the corresponding time interval Δt between looks is given by

$$\Delta t = \frac{N}{V_s} \qquad (2.11).$$

Diagram 2.6 illustrates the set of nodes at distance three units from a search node labelled J. The choice of any one of these as the next search node would result in a time interval of $3/V_s$ between the looks.

With this convention a set of transition probabilities are required for each time interval N/V_s for $N = 0,1,\ldots N_{max}$, where N_{max} is the maximum distance, over the grid, between any two nodes.

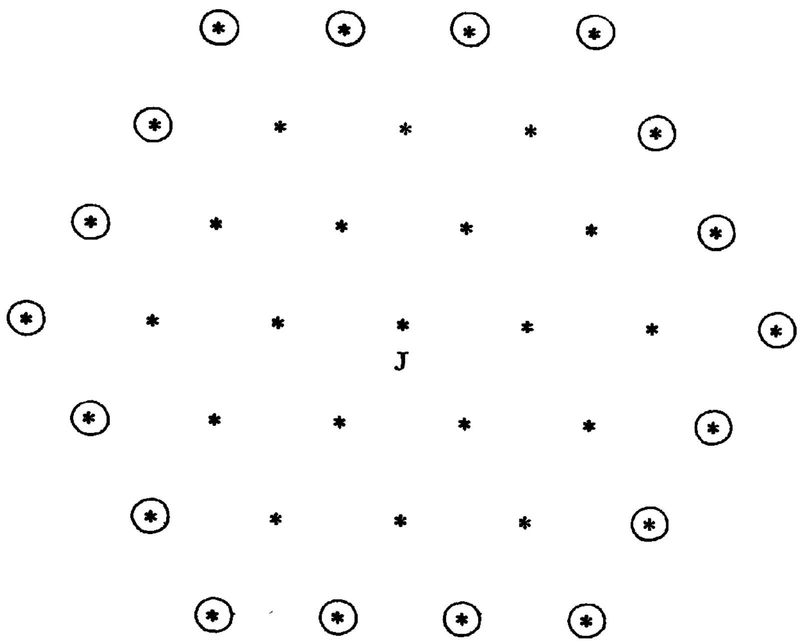


Diagram 2.6 Nodes at Distance Three Units from Search Node J

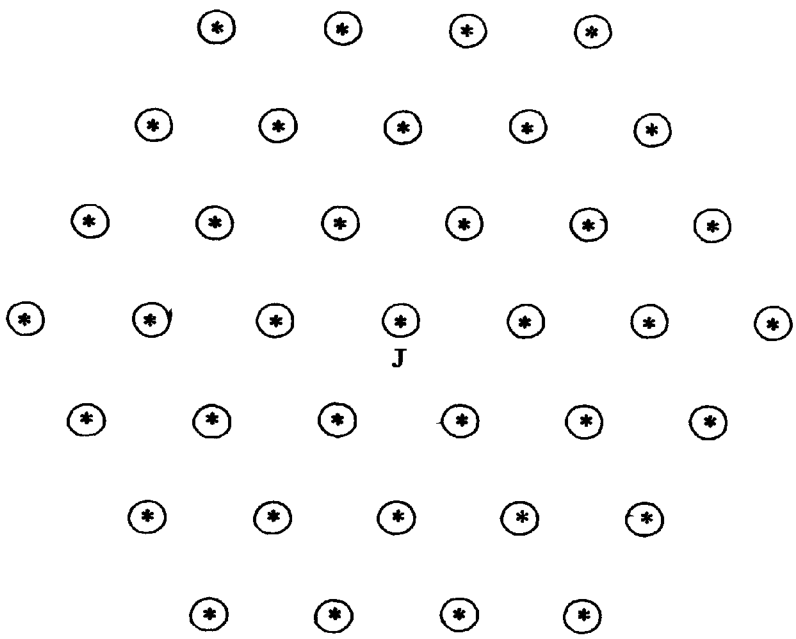


Diagram 2.7 Nodes Within Three Units of Search Node J

An alternative approach would be to specify a maximum searcher speed, V_{smax} and calculate a set of transition probabilities for each time interval N/V_{smax} for $N = 0,1,...N_t$, where $N_t \geq N_{max}$ is the maximum allowable time interval between looks. At time N the searcher might choose either, to take a look at any node K such that the distance between K and the previous search node is less than or equal to N units, or wait until the next time. The set of nodes from which a search node may be chosen at time N = 3 are illustrated in diagram 2.7.

Although this approach would avoid limiting the searcher to a constant speed, it would greatly increase the computation involved in the decision process used to determine search strategies, so was not adopted.

2.5.2 CALCULATION OF TRANSITION PROBABILITIES

Suppose the searcher travels a distance N units between consecutive search nodes, so that, by equation (2.11), $\Delta t = N/V_s$. Assume that at the begining of the interval Δt the target is at node h, and consider the probability that at the end of this interval it is at node i, where $h,i \varepsilon X$, the set of all nodes. This may be expressed in terms of distance travelled and path taken.

To achieve this the following events are defined:

$R_{\Delta t}$ = target moves a distance R in time Δt,

S_{hi} = target path starting at node h ends at node i.

As the nodes have unit spacing the target must move an integral number of units to reach another node. Therefore $R_{\Delta t}$ takes the discrete values 0,1,2,..... . These distances are exhaustive and mutually exclusive, so the transition probability from node h to node i, $\rho_{hi}(\Delta t)$, defined in section 2.2, may be written as:

$$\rho_{hi}(\Delta t) = \sum_{R=0}^{\infty} P(R_{\Delta t} \cap S_{hi})$$

$$= \sum_{R=0}^{\infty} P(R_{\Delta t}).P(S_{hi}|R) \qquad (2.12)$$

by the usual rules of probability.

Thus the two components, distance and path taken, may be considered separately.

a) Distance Travelled

The distance R moved by the target at speed V_t in time Δt is

$$R = V_t \Delta t$$

$$= V_t \frac{N}{V_s} \qquad \text{by equation (2.11).}$$

Thus the probability that the target moves distance R units in Δt is equivalent to the probability that the target speed is

$$V_t = \frac{R.V_s}{N} \qquad \text{for } N \neq 0.$$

However, R can only take integer values, so to convert the continuous distribution of target speed into discrete probabilities the assumption is made that

$$P(R_{\Delta t}) = P\left(\frac{(R-\frac{1}{2})V_s}{N} \leq V_t \leq \frac{(R+\frac{1}{2})V_s}{N}\right)$$

As this is dependent on the distance N between consecutive search nodes, $P(R_{\Delta t})$ is more clearly written as $P(R|N)$. Dividing by V_{tmax} to scale to [0,1] allows $P(R_{\Delta t})$ to be expressed in terms of the speed distribution s(v) by the integral

$$P(R_{\Delta t}) = P(R|N) = \int_{K(R-\frac{1}{2})}^{K(R+\frac{1}{2})} s(v)\, dv \qquad (2.13).$$

$$\text{where } K = \frac{V_s}{N\ V_{tmax}}$$

To avoid difficulties when the range of integration falls partially outside the interval [0,1], over which the function s(v) is defined, the additional definition is made that $s(v) = 0$ for $v < 0$ and $v > 1$.

Discretising target speed in this way is not suitable in the case of a slow moving target, as it can lead to target motion being artificially frozen. This can be seen by considering the case $V_s > 2.V_{tmax}$ with $N = 1$, giving a value of $K > 2$. Evaluating equation (2.13) in this case, with any speed distribution, gives $P(R|N) = 0$ for all $R > 0$. Thus if the searcher moves only one unit between looks, as is frequently the case when optimal strategies are considered, the target is stationary.

Even in cases where $V_s < 2.V_{tmax}$ this straightforeward discretisation of target speed, which was initially employed, proved to be far from satisfactory. This is because it results in uncceptable distortions of the speed distribution, as illustrated by the results shown in Chapter 4. Ways in which this might be overcome are investigated in Chapter 5, and an alternative approach proposed.

In addition to the description of target speed by the function s(v), the target may also have a known probability of being stationary. Denoting this probability by q, the probabilities $P(R|N)$ defined by equation (2.13) may be adjusted in this case by writing

$$P'(R|N) = \begin{cases} q + (1-q)\, P(R|N) & R = 0 \\ (1-q)\, P(R|N) & R > 0 \end{cases} \qquad (2.14).$$

b) Path Taken

Consider the probability $P(S_{hi}|1)$ that the target moves from node h to node i given R = 1, and denote this by

$$P(S_{hi}|1) = m_{hi}.$$

The probability m_{hi} is easily expressed in terms of the probability distribution of target direction $d(\theta)$, since for neighbouring nodes $m_{hi} = d(\theta) \quad \theta \varepsilon \{1,2,..6\}$ where θ is the direction of travel from node h to node i, and for non-neighbouring nodes $m_{hi} = 0$. Let m_{hi} be element (h,i) of matrix M. It can then be seen by applying the well known result of Markov chains that, for any $R \geq 0$,

$$P(S_{hi}|R) = \text{element } (h,i) \text{ of } M^R \qquad (2.15)$$

(with M^0 = the identity matrix).

Thus the probability that the target moves from node h to node i, given that it moves R units, is contained in the appropriate power of matrix M.

c) Matrix Form of Transition Probabilities

With $P(R_{\Delta t})$ and $P(S_{hi}|R)$ defined in terms of the distributions of speed and direction by equations (2.13) and (2.15) respectively, transition probabilities for each of the range of time intervals may be found by equation (2.12).

A limit may be placed on the range of summation of equation (2.12) as follows. Any value of R such that

$$R \geq \frac{1}{K} + \frac{1}{2} \qquad (2.16)$$

(where K is given in equation (2.13)), gives a range of integration of expression (2.13) outside the interval [0,1], giving $P(R_{\Delta t}) = 0$. Denoting by L the smallest value of R for which equation (2.16) holds, equation (2.12) may be written

$$\rho_{hi}(\Delta t) = \sum_{R=0}^{L} P(R_{\Delta t}).P(S_{hi}|R)$$

$$= \sum_{R=0}^{L} P(R|N).P(S_{hi}|R) \qquad (2.17).$$

The set of transition probabilities for each time interval is more conveniently expressed in matrix form. Writing

$$P_N = \sum_{R=0}^{L} P(R|N)\ M^R \qquad (2.18),$$

the transition probability $\rho_{hi}(\Delta t)$ corresponding to Δt defined by a distance N between consecutive search nodes, is given by element (h,i) of matrix P_N.

2.6 DETERMINATION OF SEARCH PATH

A sequence of looks forming a search strategy may be planned by considering the probability of detection for that strategy, calculated from the successive distributions of target position produced by equation (2.2). The simplest form of planning is that of an incremental, or myopic strategy, which places each look to give the highest probability of detection at that stage.

Use of the updating procedure to plan a strategy, with the transition matrices described in section 2.5, is not as straightforeward as with a single transition matrix such as those used by Pollock (1970) and Kan (1977). This is because the target position distribution at the time of each look is dependent, through the transition probabilities, on the distance travelled by the searcher between search nodes.

Consider, for example, the situation illustrated in diagram 2.8, with the searcher positioned at node J. If the next look is to be taken at one of the nodes labelled i, at distance 1 unit from J, then the target distribution immediatly before that look may be found by using elements of the transition matrix P_1 (defined in equation (2.18)) in the updating algorithm. If, however, one of the nodes labeled h is to be the next search node, a different target distribution formed using elements of P_2 will be required, and so on.

```
*   *   *   *   *
    h   h   h

  *   *   *   *   *   *
  h   i   i   h

    *   *   *   *   *   *
    i   J   i   h

      *   *   *   *
      i   i   h
```

Diagram 2.8 Nodes at Various Distances from Search Node J

Therefore, to determine the next search node of an incremental strategy, the distance to each node of the search area, from the current search node, must be found. The corresponding target distribution may then be generated in order to calculate the probability of detecting the target with a look taken at that node. In this way the node giving the highest detection probability can be chosen.

In practice, this is most economically performed by considering the nodes in order of ascending distance from the current search node. This allows the target distribution corresponding to each distance to be calculated only once, and then over-written after the detection probability for every node at that distance has been found. An example of a search strategy planned in this way is given in the following section.

2.7 AN EXAMPLE SEARCH STRATEGY

The following example illustrates a simple incremental search strategy, planned using the algorithm developed in this chapter. The algorithm was implemented in a FORTRAN computer program, which allows the input of the parameters of initial distribution, detection capability, and target movement.

The search area consists of a hexagonal array of nodes, of a size chosen by the user, up to a maximum size of 91 nodes. For ease of reference the nodes are numbered in a spiral manner from the centre, the numbering of the largest array being illustrated in diagram 2.9. Once the size of search area is chosen the parameters are specified in the following way.

a) Initial Disribution

For simplicity the program allows only a limited range of distributions, which may be specified in one of two ways. The first method, which is used in the following example, gives a symmetric distribution on the central seven nodes. It is specified by giving the probability π_1 that the target is at the central node; the remaining probability $1 - \pi_1$ is then uniformly distributed over the six nodes surrounding the central node. Alternatively a uniform distribution, (again symmetric about the central node), may be specified,

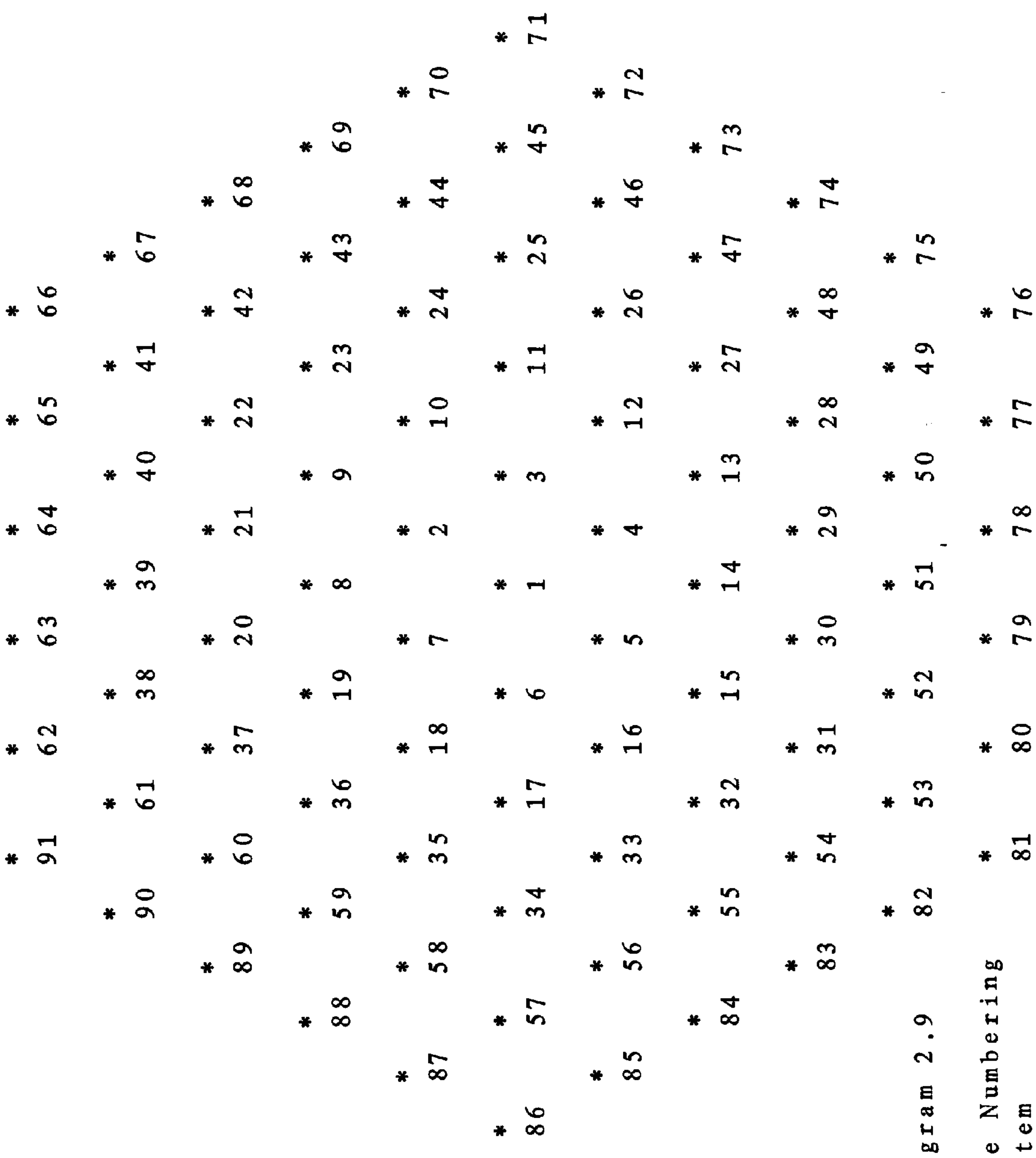


Diagram 2.9

Node Numbering System

the range of the distribution being chosen by the user.

The program could easily be adapted to enable more detailed specification of initial position. This could be done by allowing the position probability at each node to be input individually, or by discretising a continuous distribution in a suitable way.

b) Target Movement

Target movement is specified by giving values of the parameters of target and searcher speed and target direction discussed in sections 2.4 and 2.5.

The program calculates the appropriate Beta distribution from the target speed parameters V_{tmax}, V_{tn} and λ defined in section 2.4.1. Specifying the searcher speed V_s enables the distribution to be discretised using equation (2.13). It is then combined with the matrices constructed from the six direction probabilities to form the final transition matrices, as described in section 2.5.2.

To enable the the model to be used on the search area shown in diagram 2.9, the direction probabilities have to be adapted at edge and corner nodes in the way illustrated in diagram 2.10. This is so that the requirement $\sum_i \rho_{hi} = 1$ is satisfied for all $h \in X$. (An alternative approach might be to introduce an absorbing state, the target being 'lost' on entering this state.)

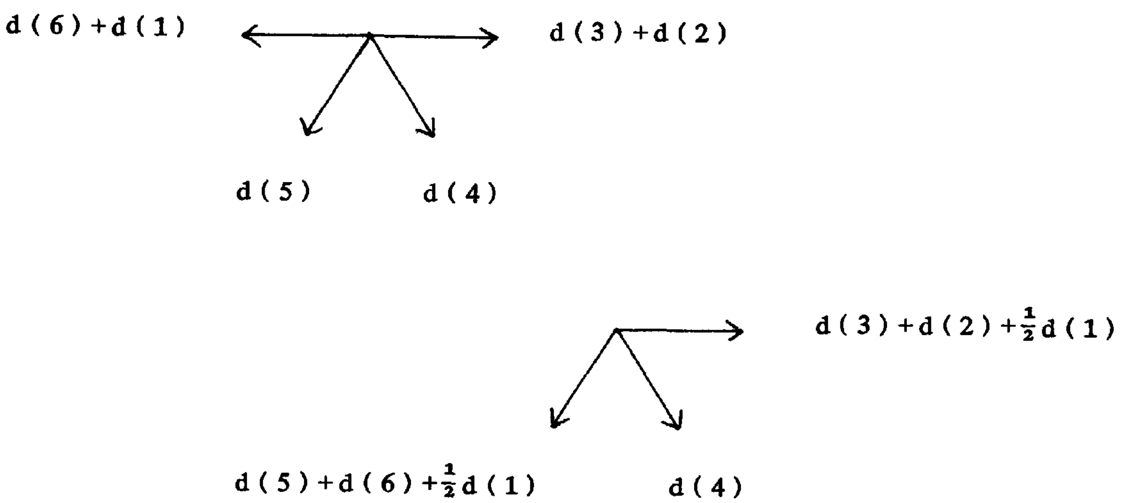


Diagram 2.10 Adaptation of Direction Probabilities at Edge and Corner Nodes

c) Detection Capability

Again for the purpose of simplifying the input to the program, it is assumed that detection capability is independent of position, and dependent only on target range from the current search node. It is specified by giving the six probabilities that a target, at range 0, 1, 2,...5 units from the search node, will not be detected. The non-detection probability at range greater than five units is assumed to be equal to one. From these values the matrix of non-detection probabilities required in the algorithm is constructed.

Again the facility to provide a more detailed description of detection capability could easily be incorporated in the program.

The above information enables the matrices required in the updating algorithm to be set up. The user then specifies the initial position of the searcher and the number of looks (following the initial look) that are required.

In the following example of an incremental strategy, each search node is chosen to give the highest probability of detection at that look. This incremental method of planning does not necessarily give optimal probability of detection for the search as a whole. The production of optimal strategies is investigated in Chapter 3.

The values of the parameters used in the example are as follows.

Size of search area = 91 nodes.

Initial distribution:

$$\pi_i = \begin{cases} 0.4 & i = 1 \\ 0.1 & i \in \{2,3,...7\} \\ 0 & i \in \{8,9,...91\}. \end{cases}$$

Target and searcher speed:

$V_{tmax} = 20$ $\qquad$ $V_s = 30$

$V_{tn} = 12$ $\qquad$ $\lambda = 0.15$

giving

$\alpha = 2.20$ $\qquad$ $\beta = 1.80$

(actual value of integral = 0.152).

Target direction:

$$d(\theta) = \begin{cases} 0.1 & \theta \in \{1,2\} \\ 0.2 & \theta \in \{3,4,5,6\}. \end{cases}$$

Non-detection probabilities:

range	0	1	2	3	4	5
probability	0.1	0.5	0.9	1.0	1.0	1.0

Initial position of searcher = node 37.

Number of looks = 15.

The resulting search strategy is shown in Table 2.1. The table gives the node at which each look is to be taken (numbered as in diagram 2.9), the resulting probability of detecting the target with that look, and the cumulative probability of detection for the search so far. The cumulative probability does not include the probability of detection at the initial node as this is not part of the decision process. The path of the strategy is illustrated in diagram 2.11.

Table 2.1 The Search Strategy

Look Number	Search Node	Detection Probability	Cumulative Probability
1	1	0.4231	0.4231
2	1	0.3049	0.5990
3	4	0.2610	0.7036
4	5	0.2141	0.7671
5	6	0.1855	0.8103
6	3	0.1884	0.8460
7	2	0.1264	0.8655
8	14	0.1207	0.8817
9	16	0.1000	0.8936
10	12	0.0931	0.9035
11	50	0.0791	0.9111
12	53	0.0783	0.9181
13	48	0.0771	0.9244
14	32	0.0783	0.9303
15	50	0.0805	0.9359

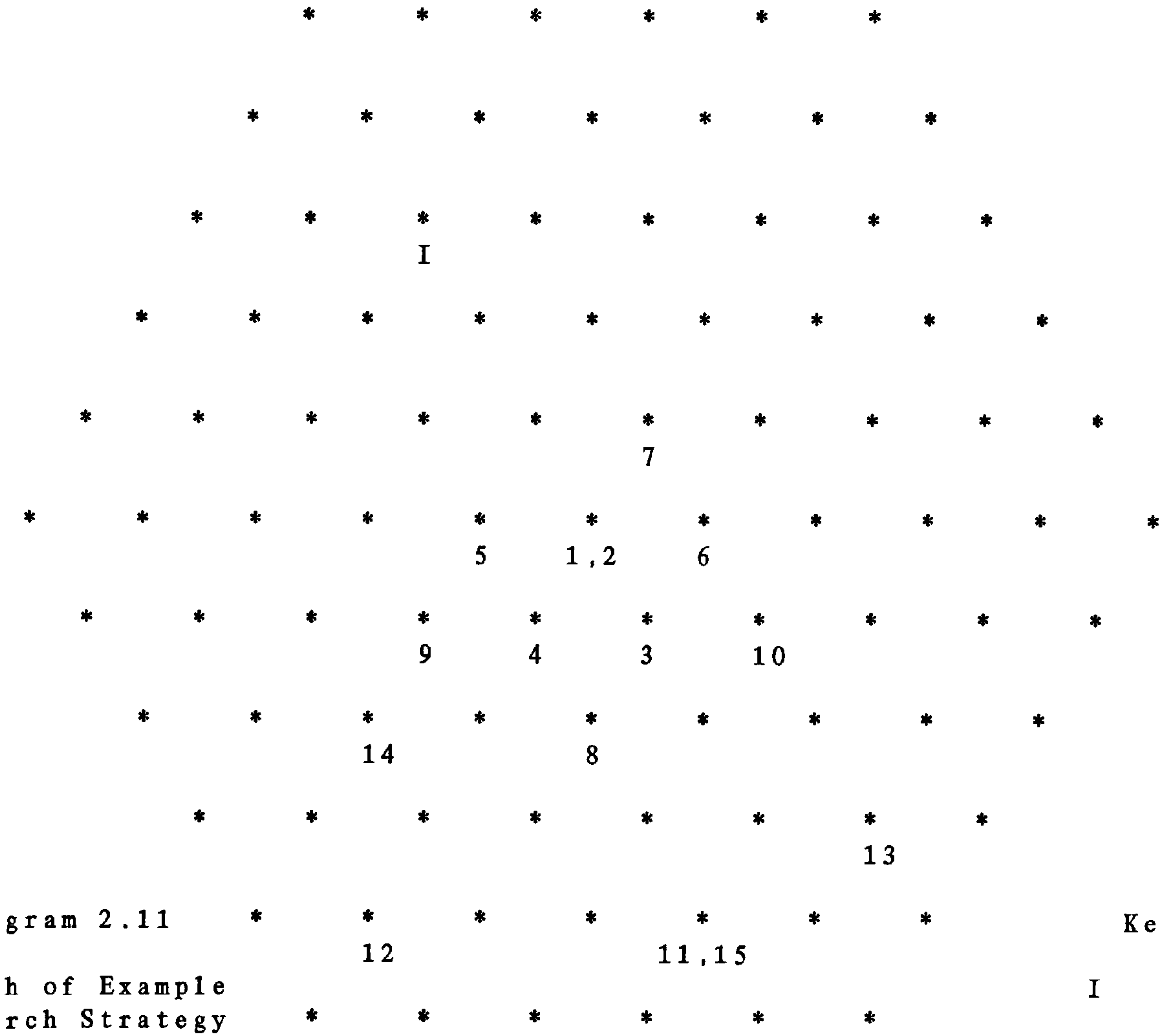


Diagram 2.11

Path of Example
Search Strategy

Key

I	Initial Search Node
1,2,..	Position of Search Node 1,2,..etc.

CHAPTER THREE

OPTIMISATION

3.0 INTRODUCTION

An incremental, or myopic strategy for a moving target search, as illustrated by the example given in section 2.7, does not necessarily give the maximum probability of detection. This was shown by Brown (1980) in the case of infinitely divisible effort, and is illustrated in the present case by the results given in Chapter 6.

This chapter examines ways in which optimal strategies may be obtained using the search model developed in Chapter 2. Two optimisation policies are considered in section 3.1, and ways in which the optimisation may be carried out for each case are discussed in sections 3.2 and 3.3. For cases where optimisation cannot be achieved alternative methods of solution leading to sub-optimal strategies are considered in section 3.4.

3.1 CRITERIA FOR OPTIMISATION

The applications described in section 1.2 illustrate some of the more common objectives of search planning, which may be used as criteria under which optimal search paths are sought. A common requirement is to maximise the probability of detecting the target, subject to some limit on the total available search resources. Alternatively, the return on the cost invested in the search may be of importance, with with detection probability in relation to cost being the relevant criterion.

Optimisation with respect to each of these requirements is investigated in this chapter. In the first case, total search resources are measured by the number of looks available, and maximisation of detection probability in a given number of looks is considered. In the second case detection probability per unit cost is optimised, again within the constraint of a specified number of looks. The quantities that may be involved in the measurement of search cost are discussed in section 3.3.

These two represent the most common optimisation requirements, but are not the only possible criteria. Pollock (1970), for example, considers the minimum expected number of looks to detection in the two cell case. This approach, however, assumes unlimited search resources are avaliable if needed, and may lead to infinite solutions. Since, in practice, the available resources are limited, this approach was not adopted.

3.2 OPTIMAL PROBABILTY OF DETECTION

An expression giving the probability of detecting the target in a given number of looks can be obtained as follows.

Suppose that at some time t_k the searcher is at a known node I, where $I \varepsilon X$, the set of all nodes of the search area. Assume that the target position distribution at time t_k, denoted by $\{\pi_i(t_k), i \varepsilon X\}$, is known, and consider a sequence of M+1 looks taken at nodes $I, J_1, \ldots. J_M$ at times $t_k, t_{k+1}, \ldots. t_{k+M}$. Such a sequence of nodes will be termed a strategy.

Denote the probability that target is not detected by the look at time t, given that it is undetected at any time $t' < t$, by $\bar{P}(t)$. The probability that the target remains undetected after M+1 looks is given by the product of these conditional probabilities for times $t_k, t_{k+1}, \ldots. t_{k+M}$. Thus the total probability of detection in M+1 looks, denoted by D_{M+1} is

$$D_{M+1} = 1 - \prod_{\alpha=0}^{M} \bar{P}(t_{k+\alpha}) \qquad (3.1).$$

Evaluation of D_{M+1} for any known strategy is straightforward. Let η_{Jh} denote the probability that a look taken at node J will not detect the target, given that it is at node h (as previously defined in section 2.2), and denote the target position distribution at time $t_{k+\alpha}$ by $\{\pi_i(t_{k+\alpha}), i\varepsilon X\}$. Then, if node J_α is the chosen search node at time $t_{k+\alpha}$, the corresponding probability $\bar{P}(t_{k+\alpha})$ is given, by the usual rule for conditional probability, by

$$\bar{P}(t_{k+\alpha}) \quad = \quad \sum_h \pi_h(t_{k+\alpha}) \; \eta_{J_\alpha h} \qquad (3.2).$$

The distribution $\{\pi_i(t_{k+\alpha})\}$ may be obtained from $\{\pi_i(t_{k+\alpha-1})\}$ by use of equation (2.2). Thus, knowing $\{\pi_i(t_k)\}$ and the sequence of search nodes, D_{M+1} can be found.

Obtaining an optimal strategy however, presents problems. Denoting the sequence of search nodes $J_1, J_2, \ldots . J_M$ by S, the maximum obtainable probability of detection in M+1 looks starting at a fixed node I is

$$\text{Max}[\ D_{M+1}\ ] \quad = \quad \underset{S}{\text{Max}}[\ 1 - \prod_{\alpha=0}^{M} \bar{P}(t_{k+\alpha})\ ] \qquad (3.3).$$

This maximisation cannot easily be performed as will be seen in the following sub-sections, where methods of finding optimal strategies are examined.

3.2.1 DYNAMIC PROGRAMMING

Dynamic programming has been applied in the optimisation of discrete stationary target search problems, and some simple moving target problems. Pollock (1970), for example, gave analytic solutions by this method for some special cases of the two cell moving target problem.

Although equation (3.3) can be expressed as a dynamic programming recursion, the application of this method is of little practical use in this case, as can be seen from the following analysis.

Let $V_{M+1}(\{\pi_i(t_k)\},I)$ denote the maximum obtainable probability of detection in M+1 looks, first looking at node I at time t_k with prior distribution $\{\pi_i(t_k), i \varepsilon X\}$.

By assumption, I and $\{\pi_i(t_k)\}$ are known, so

$$V_1(\{\pi_i(t_k)\},I) = 1 - \bar{P}(t_k) \quad \text{is known.}$$

From equation (3.3)

$$V_{M+1}(\{\pi_i(t_k)\},I) = \underset{S}{\mathrm{Max}}[1 - \prod_{\alpha=0}^{M} \bar{P}(t_{k+\alpha})]$$

$$= \underset{S}{\mathrm{Max}}[1-\bar{P}(t_k)+\bar{P}(t_k)(1 - \prod_{\alpha=1}^{M} \bar{P}(t_{k+\alpha}))]$$

$$= 1-\bar{P}(t_k)+\bar{P}(t_k)\underset{S}{Max}[1 - \prod_{\alpha=1}^{M} \bar{P}(t_{k+\alpha})]$$

$$= 1-\bar{P}(t_k)+\bar{P}(t_k)\underset{J\varepsilon X}{Max}\ V_M(\{\pi_i(t_{k+1})\},J)$$

$$(3.4).$$

Unfortunatly, equation (3.4) is not readily solvable because of the dependence of the distribution $\{\pi_i(t)\}$ at any time $t > t_k$ on the chosen search path. This can be seen by inspection of equation (2.2). Writing $\pi_i(t_{k+\alpha+1})$ for π' and $\pi_i(t_{k+\alpha})$ for π to make explicit the time dependence, and denoting the search node at time $t_{k+\alpha}$ by J_α (all other notation being as defined in section 2.2), equation (2.2) becomes

$$\pi_i(t_{k+\alpha+1}) = \frac{\sum_h \pi_h(t_{k+\alpha})\ \eta_{J_\alpha h}\ \rho_{hi}(\Delta t)}{\sum_h \pi_h(t_{k+\alpha})\ \eta_{J_\alpha h}} \qquad (3.5)$$

Thus at each time $t_{k+\alpha+1}$ the target position distribution is dependent explicitly on the search node J_α, and implicitly on the previous path $I,J_1,\ldots.J_{\alpha-1}$ (through the distribution $\{\pi_i(t_{k+\alpha})\}$). It is also dependent on the search node $J_{\alpha+1}$ chosen at time $t_{k+\alpha+1}$ due to the dependence of the transition probability ρ on the travel time Δt between nodes J_α and $J_{\alpha+1}$.

The only restriction that can be placed on $\{\pi_i(t)\}$ at any time $t > t_k$, when the path is unknown is

$$\pi_i(t) \geq 0, \qquad \sum_i \pi_i(t) = 1.$$

Thus the dynamic programming state space for a search area of N nodes is $X \times R^N$ where $X = \{1,2,..N\}$, and R is the non-negative real numbers (with the additional constraints given above).

The complexity of finding optimal feasible solutions to this problem is prohibitive for all but simple cases. Eagle (1984) presents a method of removing unnecessary vectors from the state space in a less complex moving target model. The method however requires the solution of a potentially large linear programming problem, which would make its application to the present model unsuitable.

3.2.2 EXHAUSTIVE EVALUATION OF STRATEGIES

Optimal strategies can in principal be found by evaluation of detection probability for every possible strategy, enabling the strategy or strategies giving the highest detection probability to be found.

For this purpose equation (3.3) may be re-written in the following way:

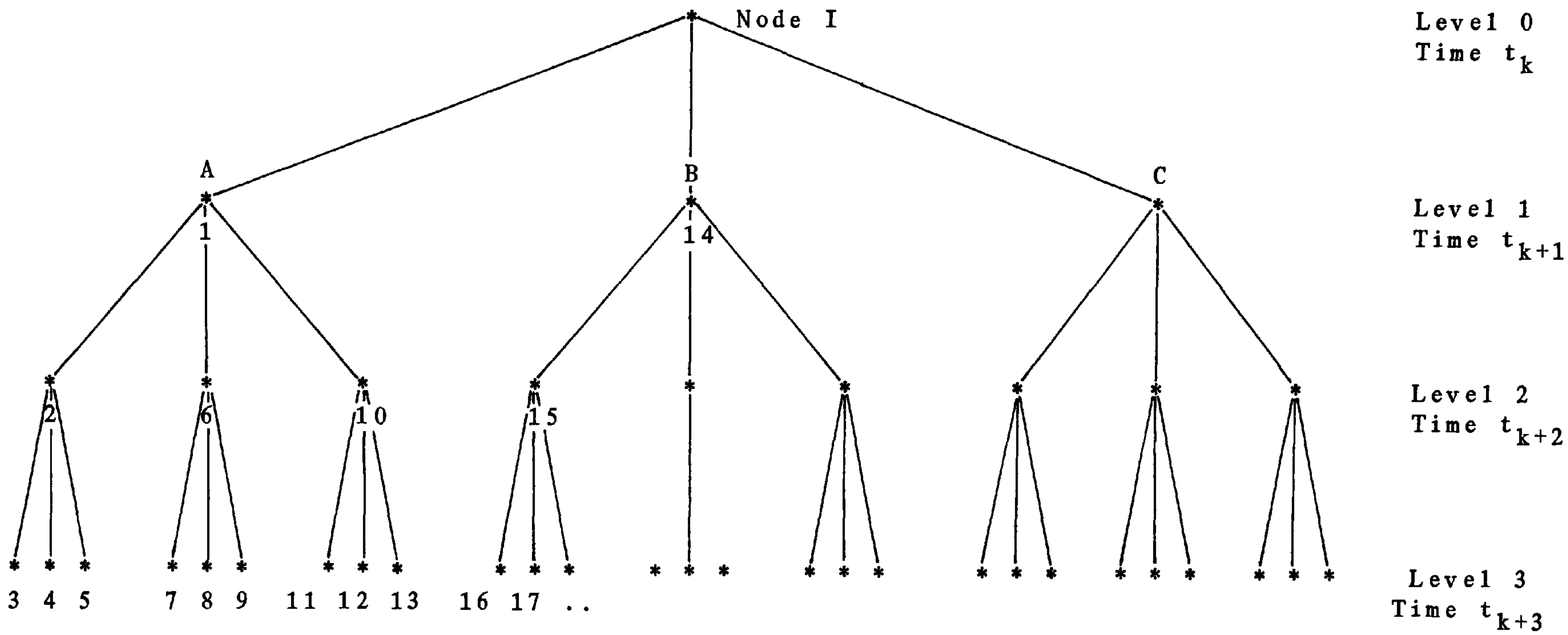
$$\operatorname{Max}[\, D_{M+1}] = 1 - \operatorname*{Min}_{S}[\, \prod_{\alpha=0}^{M} \bar{P}(t_{k+\alpha}) \,]$$

$$= 1 - \bar{P}(t_k) \operatorname*{Min}_{S}[\, \prod_{\alpha=1}^{M} \bar{P}(t_{k+\alpha}) \,]$$

$$(\text{since } \bar{P}(t_k) \text{ is known}) \qquad (3.6).$$

An optimal strategy is therefore one which minimises the overall probability that the target is undetected in the M looks taken at times $t_{k+1}, t_{k+2}, \ldots . t_{k+M}$. For every possible sequence S, the corresponding values $\bar{P}(t_{k+\alpha})$, $\alpha = 1, \ldots, M$, may be found in the way discussed in section 3.2, and hence an optimal strategy can be chosen.

As the target position distribution at each stage is dependent on the sequence of search nodes to that point, the evaluation and minimisation is most economically performed as a depth first tree search, backtracking through the tree until all possible strategies have been considered. The order of evaluation of non-detection probabilities is illustrated in diagram 3.1 in the case of four looks taken in a search area of three nodes. As explained in section 2.6, further economy can be made by considering the nodes at each step in ascending order of distance from the previous search node. (This refinement is not shown in the diagram.)



Key

A B C Nodes
1 2 3 ... Order of evaluation

Diagram 3.1 Order of Evaluation of Non-Detection Probabilities

In practice, the size of problem that can be optimised in this way is severely limited. This is because a sequence of M+1 looks in a search area of N nodes generates N^M different strategies, (as can be seen from diagram 3.1). The volume of computation therefore grows exponentially with the number of looks. Although ever-increasing computing speeds may make this approach more feasible in the future, at present only problems of a limited size can be optimised in this way if all possible strategies have to be evaluated. However, in the following sub-section a condition is found under which some strategies may, theoretically, be eliminated from evaluation.

3.2.3 ELIMINATION OF STRATEGIES

Unfortunatly, since equation (3.6) requires the minimisation of the product of non-detection probabilities $\bar{P}(t_{k+\alpha})$, $\alpha = 1,...,M$, each of which is in the range $0 \leq \bar{P} \leq 1$, no strategy can be chosen or eliminated on the basis of the partial product $\prod_{\alpha=1}^{U} \bar{P}(t_{k+\alpha})$, $U < M$ (except where this is zero, giving certain detection). A criterion has however been found which allows a node J_α to be eliminated from consideration as search node at time $t_{k+\alpha}$. The criterion is an extension of the result given by Kan (1977).

Kan considers a search for a target moving from box to box in a search area of N boxes. The search model is a particular case of the model considered here, with the following restrictions:

1) Searches take place at unit time intervals with target transition probability from box to box between these times described by a single Markov transition matrix, (denoted by $\{\rho_{ih}\}$ in the following equation (3.7)).
2) A target can only be detected if it is in the box in which the look is taken, i.e. $\eta_{ij} = 1$ whenever $i \neq j$. (For simplicity the probability of detection $(1-\eta_{ii})$ is denoted by $\emptyset_i$ in equation (3.7)).

Theorem 1 of Kan may be written in the following way. If at time t there exists a box J such that:

$$\pi_J(t)\emptyset_J\rho_{Jh} \geq \pi_i(t)\emptyset_i\rho_{ih} \quad \text{for all } h, i\varepsilon X \qquad (3.7)$$

then to maximise the probability of detecting the target an optimal strategy looks in box J at time t.

A comparable result for the present search model may be obtained as follows.

Let S' denote the final M looks of a strategy S of length A+1+M looks, A > 0, and assume that the initial A search nodes of S have already been decided. In order to choose the next node, J, of the strategy in an optimal way, J must be chosen so that the probability of detection in the strategy JS' is a maximum.

Define the following notation.

Let $F_{M+1}(\{\pi_i\}, JS')$ = Probability of detection in M+1 looks with prior distribution $\{\pi_i\}$, first looking at node J, then following strategy S'.

Also, let $G_M(i, S')$ = Probability of detection in M looks following strategy S' <u>given</u> target at node i.

(As the first A nodes of S are known, $\{\pi_i\}$ is known.)

To emphasise the dependence on strategy, denote the probability that the target is undetected by the look taken at node J by $\bar{P}_J$, and the target position distribution following this unsuccessful look and subsequent movement by $\{\pi_i^J\}$. Then, by equation (3.2), omitting the time dependence for simplicity,

$$\bar{P}_J = \sum_h \pi_h \eta_{Jh} \qquad (3.8)$$

and, by equation (3.2)

$$\pi_i^J = \frac{\sum_h \pi_h \eta_{Jh} \rho_{hi}}{\sum_h \pi_h \eta_{Jh}}$$

$$= \frac{\sum_h \pi_h \eta_{Jh} \rho_{hi}}{\bar{P}_J} \qquad (3.9)$$

By comparison with equation (3.4), again omitting the time dependence, the probability of detection for the sequence of looks JS′ may be expressed as follows.

$$F_{M+1}(\{\pi_i\}, JS') = 1 - \bar{P}_J + \bar{P}_J F_M(\{\pi_i^J, S'\})$$

$$= 1 - \bar{P}_J + \bar{P}_J \sum_i \pi_i^J G_M(i, S')$$

(by the usual rule for conditional probability),

$$= 1 - \bar{P}_J + \bar{P}_J \sum_i \left[\frac{\sum_h \pi_h \eta_{Jh} \rho_{hi}}{\bar{P}_J} \right] G_M(i, S')$$

(by equation (3.9)),

$$= 1 - \bar{P}_J + \sum_i \sum_h \pi_h \eta_{Jh} \rho_{hi} G_M(i, S'). \qquad (3.10)$$

Now since $\sum_i \rho_{hi} = 1$ for all $h \varepsilon X$, equation (3.8) may be expressed as

$$\bar{P}_J = \sum_i \rho_{hi} \sum_h \pi_h \eta_{Jh}$$

$$= \sum_i \sum_h \pi_h \eta_{Jh} \rho_{hi}$$

Thus from equation (3.10),

$$F_{M+1}(\{\pi_i\},JS') = 1 - \sum_{i}\sum_{h}\pi_h \eta_{Jh} \rho_{hi}(1 - G_M(i,S')) \quad (3.11)$$

The conditional probability $G_M(i,S')$ is independent of previous search strategy, thus if for some $J\varepsilon X$

$$\sum_{h}\pi_h \eta_{Jh} \rho_{hi}(\Delta_1 t) \quad \leq \quad \sum_{h}\pi'_h \eta_{Lh} \rho_{hi}(\Delta_2 t) \quad (3.12)$$

for all $L\varepsilon X$, $i\varepsilon X$, $\Delta_1 t, \Delta_2 t \ \varepsilon \ \{0,1,2,\ldots\}$,

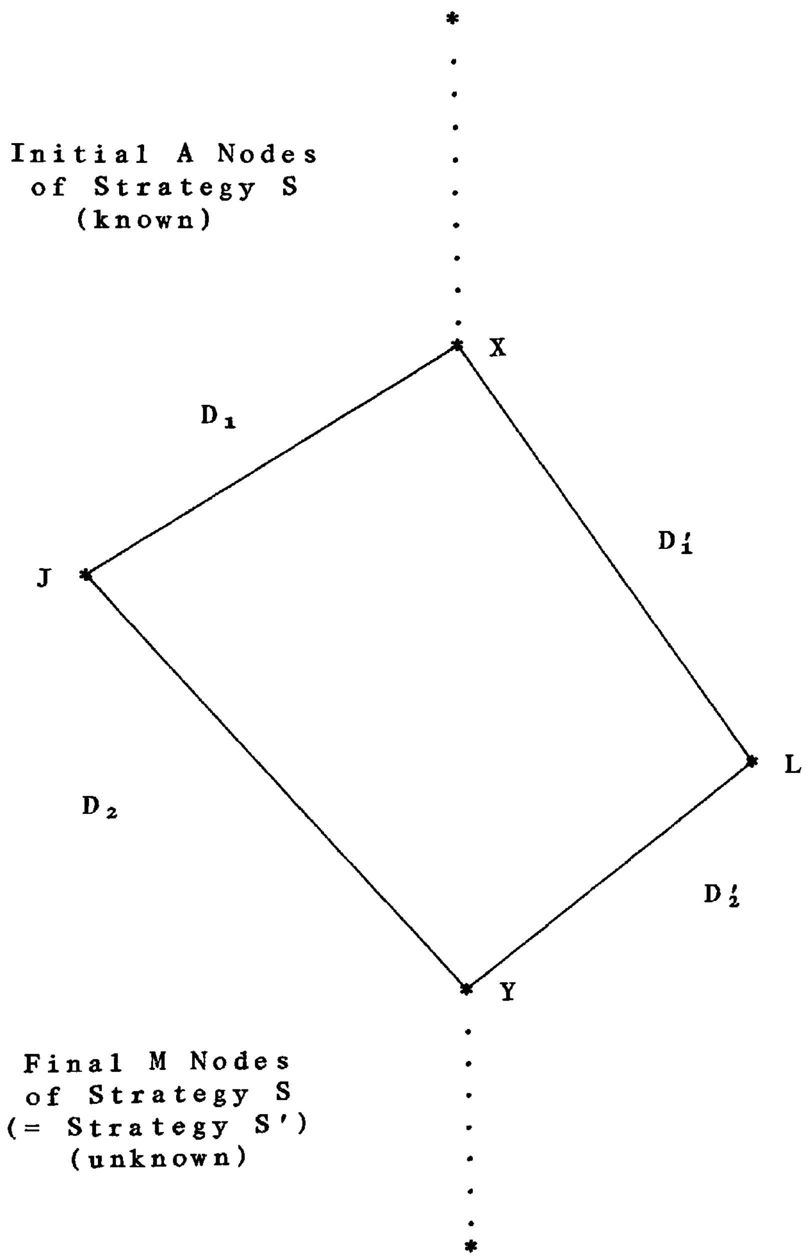
where $\{\pi_i\}$ and $\{\pi'_i\}$ are possibly different target position distributions, as explained below, then

$$F_{M+1}(\{\pi_i\},JS') \geq F_{M+1}(\{\pi'_i\},LS')$$

for any $L\varepsilon X$ and any sequence S'.

Hence, if there exists a node J for which condition (3.12) holds, then J must be chosen as the next search node in order to give optimal probability of detection in the final M+1 looks of strategy S.

In equation (3.12), the distribution $\{\pi'_i\}$ may differ from $\{\pi_i\}$ if nodes L and J are at different distances from the previous search node. These distances are shown as D_1 and D'_1 in diagram 3.2, which illustrates the choice between sequences JS' and LS'. Also shown are the distances D_2 and D'_2 to the first node of sequence S' from J and L respectively. These determine the time intervals $\Delta_1 t$ and $\Delta_2 t$ which in general will not be known.



Key

X	A^{th} node of strategy S
J, L	alternative $(A+1)^{th}$ nodes of strategy S
Y	first node of strategy S'
D_1, D_1'	known distances
D_2, D_2'	unknown distances

Diagram 3.2 Showing the Choice Between Two Strategies

Condition (3.12) has a simple physical interpretation. The quantity $\sum_h \pi_h \eta_{Jh} \rho_{hi}(\Delta_1 t)$ is the probability that the target will be undetected by the look taken at node J <u>and</u> be situated at node i at time $\Delta_1 t$ after that look is taken.

In practice it is unlikely that any node will satisfy condition (3.12), however the converse of this condition may be used to eliminate nodes from consideration at each level of the tree search in the following way.

Suppose, given the first A nodes of S, there exists a node K for which

$$\sum_h \pi_h \eta_{Kh} \rho_{hi}(\Delta_1 t) \quad > \quad \sum_h \pi'_h \eta_{Lh} \rho_{hi}(\Delta_2 t) \qquad (3.13)$$

for some $L \varepsilon X$ and all $i \varepsilon X$, $\Delta_1 t, \Delta_2 t \ \varepsilon \ \{0,1,2...\}$, then K cannot be the A+1th node of S if S is to be optimal.

To illustrate the use of conditions (3.12) and (3.13), consider the nodes labeled A, B and C at level 1 of the search tree shown in diagram 3.1. If, say node A satisfied condition (3.12), then the branches containing nodes B and C could not contain optimal strategies so could be ignored, thus eliminating two thirds of the possible strategies. Alternatively if condition (3.12) could not be satisfied, but say node C fulfilled condition (3.13), then the branch containing node C could be ignored, and only strategies in branches A and B need be compared for optimality.

It is clear that if one of these conditions could be found to hold, particularly at the early decisions of the search, a significant reduction could be achieved in the number of strategies to be investigated. However, the application of these conditions involves a substantial amount of computation. This is particularly wasteful if unsuccessful, because the comparisons have to be made across the tree at each level, so losing the economy of the depth first approach.

Some preliminary investigation of the usefulness of these criteria indicated that in a few cases up to 10% of possible strategies could be eliminated, but in the majority of cases little or no reduction in the number of paths could be made. In many cases full implementation of these criteria would increase, rather than decrease the computation time. For this reason the criteria were not incorporated in the optimisation programs used to determine the strategies presented in Chapter 6.

It is clear from the above discussion that alternative methods of reducing the volume of computation must be considered, in order to make the calculation of search paths feasible. Some alternative approaches, leading to strategies that may not necessarily be optimal, are discussed in section 3.4.

3.3 DETECTION PROBABILITY PER UNIT COST

The costs involved in a search operation vary greatly with the application considered. Mathematically search cost is frequently defined as a function of the node or area searched. Black (1965) defines search cost in this way in a stationary target model as does Kan (1977) in a moving target case.

In a practical search operation over a two dimensional area, for example an airborne search, costs will be in terms of manpower, equipment and fuel. In general these costs depend mainly on the duration of the search, thus for simplicity it is assumed that search cost is proportional to the time taken to complete the search operation.

As only instantaneous search is being considered, and the searcher is assumed to move at constant speed, the search time is determined by the distance travelled by the searcher. Thus in order to maximise the probability of detection per unit cost, the equivalent problem of maximising probability of detection per unit distance, in a given number of looks, is considered.

Detection probability per unit cost in a given number of looks may be defined in the following way. Consider again a sequence of M+1 looks and let R_k be the distance between search nodes J_{k-1} and J_k (with $J_0 = I$). Denote the detection probability per unit cost in the M searches following the initial search of node I by C(M), with all other notation as previously defined.

Then C(M) may be defined by

$$C(M) = \frac{1 - \prod_{k=1}^{M} \bar{P}(t_k)}{\sum_{k=1}^{M} R_k} \qquad (3.14)$$

Alternatively, detection probability per unit cost could be defined over the M+1 looks as

$$C(M+1) = \frac{1 - \prod_{k=0}^{M} \bar{P}(t_k)}{\sum_{k=0}^{M} R_k} \qquad \text{with } R_0 = 0. \qquad (3.15)$$

However, the initial node I is pre-determined, and there is no associated travel distance to that node, so definition (3.14) was chosen in preference.

Difficulties would arise in the calculation of C(M) from equation (3.14) if $\sum_k R_k = 0$. To avoid this the restriction is made that the searcher is not allowed to take consecutive looks at the same node, thus imposing a minimum travel distance of 1 unit between consecutive search nodes.

An alternative approach might be to introduce a nominal cost of looking at each node, the total cost of each look being composed of this nominal cost plus the distance cost. For simplicity, however, the first alternative was chosen.

3.3.1 OPTIMISATION

The maximum probability of detection per unit cost can be expressed, using definition (3.14) as

$$\operatorname{Max} C(M) = \operatorname*{Max}_{S} \left[\frac{1 - \prod_{k=1}^{M} \bar{P}(t_k)}{\sum_{k=1}^{M} R_k} \right] \qquad (3.16).$$

Several approaches were taken to this maximisation problem, but as in the previous case, the optimisation is not, in general, easily achieved. Dynamic programming again proved to be unsuitable. In this case equation (3.16) appears not to be expressable as a dynamic programming relation, as the sum in the denominator prevents the expression from being separated in a suitable way. A sequence of nodes maximising C(M) can be found by exhaustive evaluation of strategies in a similar way to that described in section 3.2.2, but as previously discussed, this is only feasible for a limited range of cases.

A condition comparable with (3.12) does not appear to be obtainable in this case, again because of the inseparability of equation (3.16). However, a simple alternative criterion can be given, which can be of use in some cases in reducing the number of strategies to be investigated in the exhaustive evaluation.

The condition stems from observing from equation (3.14) that, for any strategy,

$$C(M) \leq \frac{1}{\sum_{k=1}^{M} R_k} \qquad (3.17).$$

Now, suppose for some strategy S with path length $\sum_k R_k = L$, it is known that the total probability of not detecting the target is H_L. Then for strategy S, equation (3.14) gives

$$C(M) = \frac{1 - H_L}{L} \qquad (3.18).$$

It then follows from equation (3.17) that any strategy with path length L+W such that

$$\frac{1}{L + W} \leq \frac{1 - H_L}{L} \qquad (3.19)$$

cannot give a higher probability of detection per unit distance than strategy S, whatever the detection probability for that path.

Rearranging equation (3.19) gives

$$W \geq \frac{LH_L}{1 - H_L} \qquad (3.20).$$

Denoting the smallest value of W for which equation 3.20 holds by α,

$$\alpha = \frac{LH_L}{1 - H_L} \qquad (3.21).$$

Thus any strategy of path length L+W with $W > \alpha$ can be ignored in the optimisation process.

This condition can be very conveniently incorporated into the exhaustive optimisation process to provide an upper limit to the path length of those strategies that need to be evaluated. For a search consisting of

M looks (following an initial look at node I) this can be done in the following way. The application of the restriction discussed in section 3.3, disallowing successive looks at the same node, gives a minimum path length of M units. If the optimisation is carried out using the depth first tree search, with nodes taken in order of ascending distance from the previous search node, as previously discussed, a value of H_M for this minimum distance is the first to be calculated. This can be used to find an initial value for α. As the optimisation proceeds, this value can be updated whenever a strategy giving a higher probability of detection per unit distance is found.

To illustrate the use of condition (3.21) suppose that, in a search of 5 looks (following the initial look), the probability of detecting the target with a strategy S of minimum path length is 0.6. This gives H_L = 0.4, and L = 5, so from equation (3.21)

$$\alpha = \frac{5 \times 0.4}{0.6} = 3.3$$

As path length can only take integer values, any strategy with path length greater than 8 units will therefore give a lower probability of detection per unit distance than strategy S.

It can be seen from the results given in Chapter 6 that the values quoted here are not untypical of those that can be expected in some cases. Since, on the 91 node grid shown in diagram 2.6, paths of up to 50 units are possible for a strategy of 5 looks, significant savings in computation can be made in cases such as this.

However, where detection probability is lower, the saving is less significant. This can be seen from diagram 3.3 which shows a graph of α against detection probability $(1 - H_L)$ for various values of L. It can be seen from the graph that condition (3.21) is of most use when high detection probability is obtained in minimum path length, giving a low value of α.

Considering the case where $L = 10$, with a detection probability of 0.4, a value of $\alpha = 15$ is obtained from the graph, giving a maximum total path length of $L + W = 25$ units. While this still represents a 50% reduction in maximum path length for 5 looks, the saving in computation will be insufficient to allow optimisation.

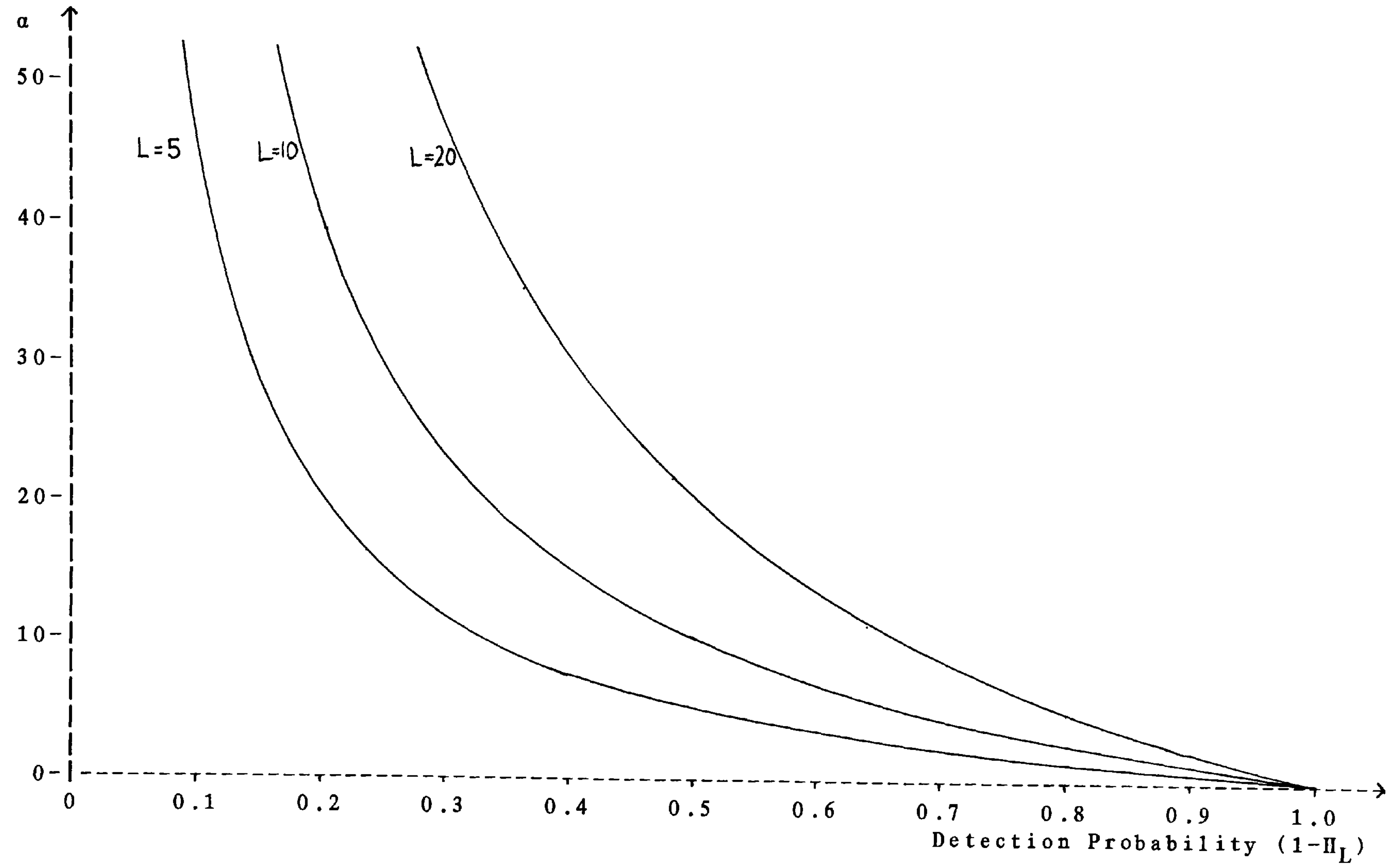


Diagram 3.3 Graph of α Against Detection Probability for Various Values of Path Length L

In such cases it may be possible to put an upper bound $B < 1$ on the maximum achievable detection probability, allowing equation (3.19) to be replaced by

$$\frac{B}{L + W} < \frac{1 - H_L}{L}$$

leading to a lower value of α. This might be possible, for example, if the search sensor had poor detection capability.

In general however, in cases where optimisation is not possible with the aid of condition (3.21), it will be necessary to employ the alternative methods of solution discussed in the following section.

3.4 SUB-OPTIMAL SOLUTIONS

This section considers further ways of reducing the amount of computation in the exhaustive optimisation process, in cases where the application of conditions (3.13) and (3.21) are insufficient. The methods produce solutions which may not necessarily be optimal but, as demonstrated by the results given in Chapter 6, are acceptable approximations to optimal solutions.

Three methods of solution are considered. These are:-

1) Planning the search path to a limited horizon.
2) Limiting the distance travelled by the searcher between consecutive search nodes.
3) The introduction of a moving grid system.

The first two methods have been used extensively in obtaining the results shown in Chapter 6, the third is introduced to enable the model to be applied to a search for a target moving through a large area.

The methods are discussed below in relation to the optimisation of detection probability, but are equally applicable to optimisation with respect to cost.

1) Limited Horizon.

As stated in section 3.2.2, the number of possible strategies for a sequence of M + 1 looks in a search area of N nodes is N^M (assuming that the first node is pre-determined). Thus the computation time increases exponentially with the number of looks over which the optimisation is carried out. One method of reducing the computation time is to plan the strategy in a step by step manner, optimising over only a limited number of future looks before each decision is made.

In a sequence of M+1 looks starting at node I_1 this may be achieved by finding an optimal strategy $(I_1, J_1, J_2, \ldots J_K)$ for the first K+1 of these looks. The node J_1 is taken as the next search node of the sub-optimal path, it then becomes the initial node I_2 of the optimisation procedure over the next K+1 looks and the process is repeated. The procedure is carried out M-K+1 times, the last giving a path $(I_{M-K+1}, J_1, J_2, \ldots J_K)$ which is optimal for the last K+1 looks.

The number of possible strategies is therefore reduced from N^M of length M+1 nodes to $(M-K+1) \times N^K$ of length K+1 nodes. Thus for say, 11 looks over 91 nodes, optimising over 3 looks ahead, the reduction will be from 91^{10} strategies of 11 nodes to 8×91^3 of 4 nodes with a corresponding reduction in computation time.

Optimal strategies cannot always be found for comparison, but results obtained by restricting planning horizon in this way suggest that this method of solution gives strategies with detection probability that is close to optimal. In the cases considered in Chapter 6, it is frequently found that the detection probability obtained from a myopic strategy (which is optimised over just one look ahead) differs only minimally from an optimal value, the difference being typically of the order of 2%.

2) Limited Distance.

An alternative method of reducing the number of strategies considered is to restrict the maximum distance that the searcher is permitted to travel between consecutive search nodes. Again taking the example of M + 1 looks over 91 nodes. If the searcher is restricted to move at most two units between looks, it can be seen from the grid illustrated in diagram 2.9 that at each decision there is a choice of at most 19 nodes. This restricts the number of strategies from 91^M to at most 19^M, again giving a substantial saving in computation.

From a practical point of view, restricting searcher movement in this way is often more realistic than allowing a free choice of search node. For example, a searcher physically travelling over a two dimensional search area is unlikely to pass over a search node without investigating it.

The results discussed in Chapter 6 indicate that, in the majority of cases, the searcher will choose to move only a limited distance between looks (typically no more than two). Thus restricting searcher movement in this way is unlikely to greatly affect the overall detection probability. Some exceptions to this are discussed further in Chapter 6.

3) Moving Grid.

The procedures described above allow sub-optimal solutions to be found for more prolonged searches over larger search areas. There is still however a comparitively small limit to the size of search area that can be used. This is because updating target position and optimising over many hundreds of nodes would be prohibitive in terms of computer time and available memory. In order to apply the model to searches over a larger area the concept of a search grid moving with the searcher is introduced here.

One arrangement for such a grid is illustrated in diagram 3.4. The diagram shows a hexagonal grid of nodes T_1 termed the target grid over which a target position distribution $\{\pi_i \ , \ i \varepsilon \ T_1\}$ is known, with $\sum_i \pi_i = 1$. The searcher is assumed to be situated at the central node (I). Surrounding node I is the grid of nodes S_1 from which the next search node may be chosen.

Suppose node J is the next search node. A new target grid T_2 centred on J is defined as follows. Nodes belonging to both T_1 and T_2 retain the same target position probability, all other nodes in T_1 are ignored. Nodes now in T_2 but not in T_1 are given probability 0. The target position distribution on T_2

is updated and normalised following the look at node I, and the process is repeated.

At each stage the true probability that the target is in area T must be maintained in order to calculate the probability of detection at each look. The procedure is carried out until either the required number of looks has been made, or the probability that the target is in area T falls below a specified level.

In order to minimise the error in computed detection probability caused by the truncation of the target distribution, the size if grids S and T are chosen so that

$$\text{radius of } T \geq \text{radius of } S + R + 1$$

where R is the maximum range of detection of the searcher. This ensures that all nodes 'visible' from grid S are contained within grid T, and that the edge nodes of T, which are likely to be the most in error, are outside the range of detection.

It is clear that this approach is most suitable in cases where the direction of motion of the target is well defined as this will result in the least 'loss of probability' when the target grid is moved.

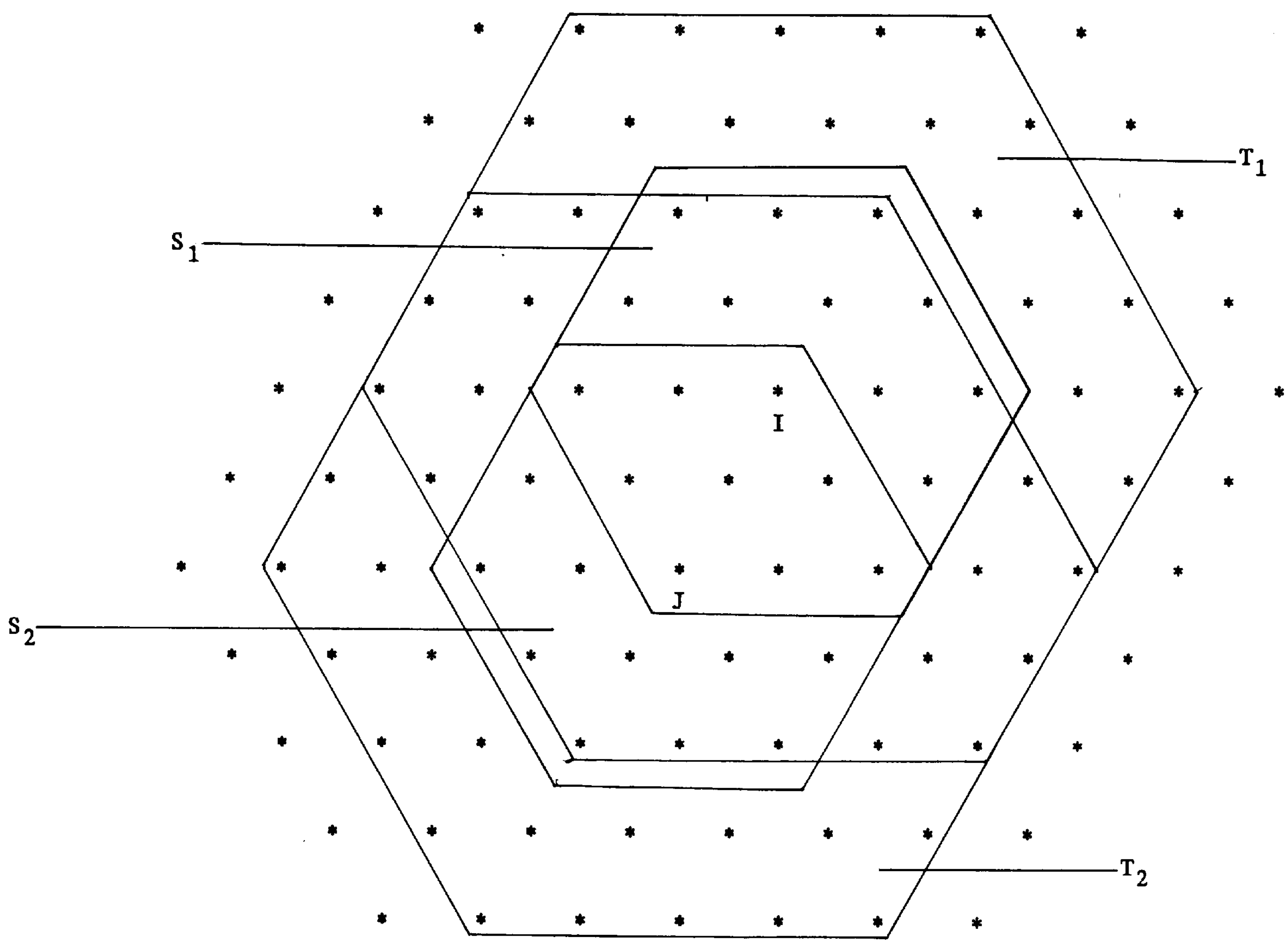


Diagram 3.4 Illustrating the Moving Grid System

The application of the restrictions discussed above allow search strategies to be calculated for grids that might realistically represent a physical search area. It is often necessary to apply more than one restriction, i.e. restricting movement and optimising over a limited horizon, but in cases where comparisons can be made, the results obtained compare favourably with optimal solutions. An analysis of the methods of producing search strategies discussed in this chapter is given in Chapter 6.

CHAPTER FOUR

ANALYSIS OF THE SEARCH MODEL

4.0 INTRODUCTION

Investigation of the search model developed in Chapter 2 can be considered from two aspects. These are, firstly verifying that the algorithm given by equation (2.2) produces an acceptable updated target distribution, and secondly that search strategies generated from this distibution are, by some measure, sensible. The first of these aspects is investigated in this chapter, while verification of search strategies is considered in Chapter 6.

The updated distribution is considered with respect to the individual components of the search model. It is shown to be consistent with predicted values except in relation to target speed. The model of target speed is shown in section 4.3.1 to produce anomalies in the target distribution which might unacceptably influence the choice of search strategy. An alternative model of target speed is given in Chapter 5.

4.1 ELEMENTS OF THE SEARCH MODEL

The updating algorithm, equation (2.2), takes three components, initial target position, searcher detection capability and target movement, to calculate a revised estimate of target wherabouts following each look. Equation (2.2) is reproduced here as equation (4.1):

$$\pi'_i = \frac{\sum_h \pi_h \eta_{jh} \rho_{hi}(\Delta t)}{\sum_h \pi_h \eta_{jh}} \qquad (4.1),$$

where π_h, π'_i denote target positon probability,

η_{jh} denotes non-detection probability,

and $\rho_{hi}(\Delta t)$ denotes transition probability for the time interval Δt,

as defined in section 2.2.

In order to verify that the algorithm gives sensible information about target position, the updated distribution is considered, in the following sections, with respect to each of the three components.

4.2 INITIAL DISTRIBUTION AND DETECTION CAPABILITY

The initial target distribution $\{\pi_h,\ h\varepsilon X\}$, and the set of non-detection probabilities $\{\eta_{jh},\ j,h\varepsilon X\}$ (giving the probability that a target situated at node h will not be detected by a look taken at node j), are assumed to be known parameters of the algorithm, and are thus not considered with respect to modelling. However, the effect on the updated distribution of varying both initial target position and detection capability has been investigated.

Target distributions obtained from a computer program implementing the algorithm, have been found in all cases to be in accordance with expected values in relation to these parameters. (This is illustrated by the results discussed in sections 6.2 and 6.3.) This observation is supported by the following analytic investigation of two special cases.

Consider first the case where the target is stationary, and the searcher has zero probability of detection at any node. As no information is gained from the search, and there is no target movement, the target position distribution should remain unchanged. This may be verified by setting

$$\eta_{jh} = 1 \quad \text{for all } j,h \varepsilon X,$$

and the transition probabilities

$$\rho_{hi} = \begin{cases} 1 & \text{if } h=i \\ 0 & \text{if } h \neq i. \end{cases}$$

Equation (4.1) then gives

$$\pi'_i = \frac{\pi_i}{\sum_h \pi_h}$$

$$= \pi_i \qquad \text{since } \sum \pi_h = 1.$$

Thus the distribution is unchanged, as expected.

Next consider the case where, again the target is stationary, but where the searcher has certain detection at the search node j, and zero probability of detection at all other nodes. In this case the updated target position probability should be reduced to zero at the search node. Again this may be verified by setting

$$\eta_{jh} = \begin{cases} 0 & \text{if } j=h \\ 1 & \text{if } j\neq h \end{cases}$$

and

$$\rho_{hi} = \begin{cases} 1 & \text{if } h=i \\ 0 & \text{if } h\neq i. \end{cases}$$

Then, by equation (4.1)

$$\pi'_i = \frac{\pi_i \ \eta_{ji} \ \rho_{ii}}{\sum_h \pi_h \ \eta_{jh}}$$

$$= \begin{cases} 0 & \text{if } i = j \\ \dfrac{\pi_i}{\sum_{h\neq j} \pi_h} & \text{if } i \neq j. \end{cases}$$

Thus the probability density function becomes zero at the search node as expected, and is increased proportionally at the remaining nodes to normalise the distribution.

This argument may be extended to other simple cases, such as where there is certain detection at nodes other than the search node, or where detection probability at the search node takes some value other than 1, where a corresponding reduction in position probability will be seen from equation (4.1).

4.3 TARGET MOVEMENT

Target motion, discussed in sections 2.4 and 2.5, is specified by independent distributions of speed and direction of movement, which are converted into transition probabilities for use in the updating algorithm. The set of transition probabilities $\{\rho_{hi}(\Delta t),\ h,i \varepsilon X\}$, giving the probability that a target situated at node h will move to node i in the time interval Δt, is formed from the two distributions by use of equation (2.17). In order to validate the target movement component of the algorithm it is necessary to check that these transition probabilities adequately represent the underlying speed and direction distributions when applied in equation (4.1).

4.3.1 TARGET SPEED

Target speed is given by a probability density function on the interval $[0,V_{tmax}]$, scaled to $[0,1]$. This is discretised by use of equation (2.13) to give the probability that the target moves an integral number of units in a given time interval. As the discussion in section 2.5.2 indicates, this discretisation is unsuitable where target speed is slow in relation to searcher speed. However the following results show that, even where the ratio of target speed to searcher speed is higher, this approach can lead to unacceptable distortions of the target speed distribution.

Some preliminary analysis was carried out by imposing various restrictions on searcher movement to force different time intervals between looks. The results showed some anomalies in target motion which were investigated further. The effect is most clearly shown by considering the case where the target is initially situated at a given node, and moves in a known direction, so that the resulting target distribution is spread along a line of nodes as illustrated in diagram 4.1. Setting $\eta_{jh} = 1$ for all $j,h \in X$ gives zero probability of detection, so any change in the target distribution will be due only to target movement, and should be independent of the number of looks taken.

$$\underset{I}{*} \longrightarrow * \longrightarrow * \longrightarrow * \longrightarrow * \longrightarrow *$$

Diagram 4.1 Target Movement in a Known Direction from Initial Node I

This case was examined using various values of searcher to target speed, both skew and symmetric Beta distributions and various time intervals between looks. Examples of the resulting target distributions are shown in diagrams 4.2 to 4.5. The diagrams, which show target distribution after a period of five time intervals (i.e. the time required for the searcher to move five units), were produced by placing the following restrictions on searcher movement. Firstly the searcher was constrained to move exactly one unit between consecutive looks. Diagrams 4.2a to 4.5a show the resulting target distribution after five looks had been taken. Next the searcher was forced to move five units between looks, diagrams 4.2b to 4.5b show the resulting target distribution after just one look.

Both sets of distributions represent target movement after five time units, so they should correspond, however significant differences can be seen between the two sets. The difference is caused by the way in which the speed distribution is discretised to give the distance travelled in different time intervals.

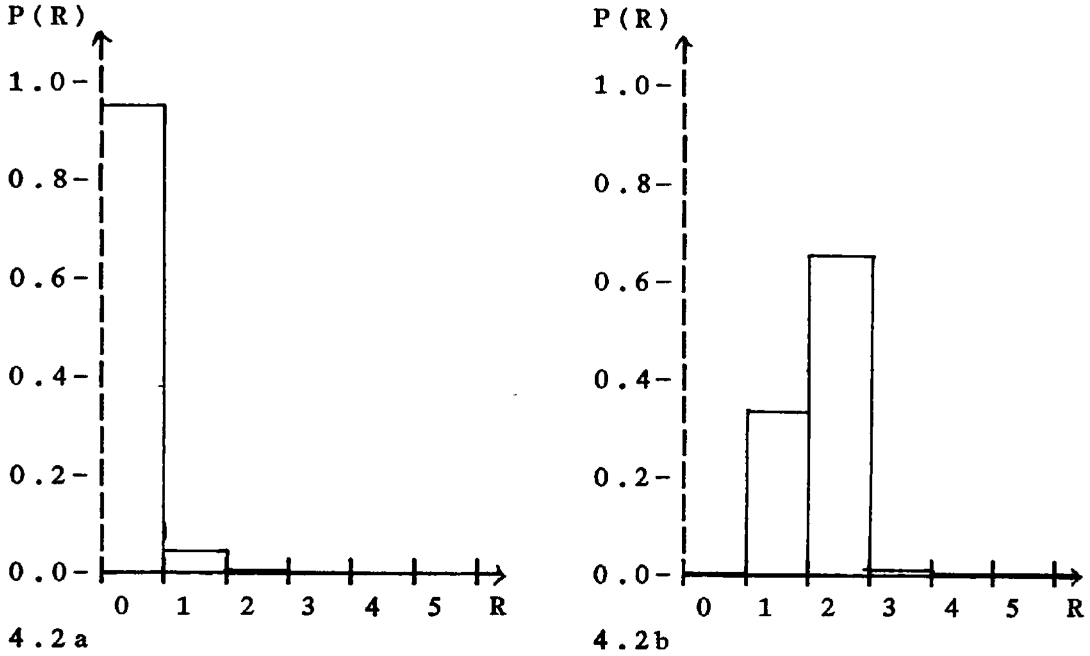


Diagram 4.2
Distribution of Target Position in the Case
V_s=30 V_{tmax}=20 V_{tmode}=10 α=10 β=10

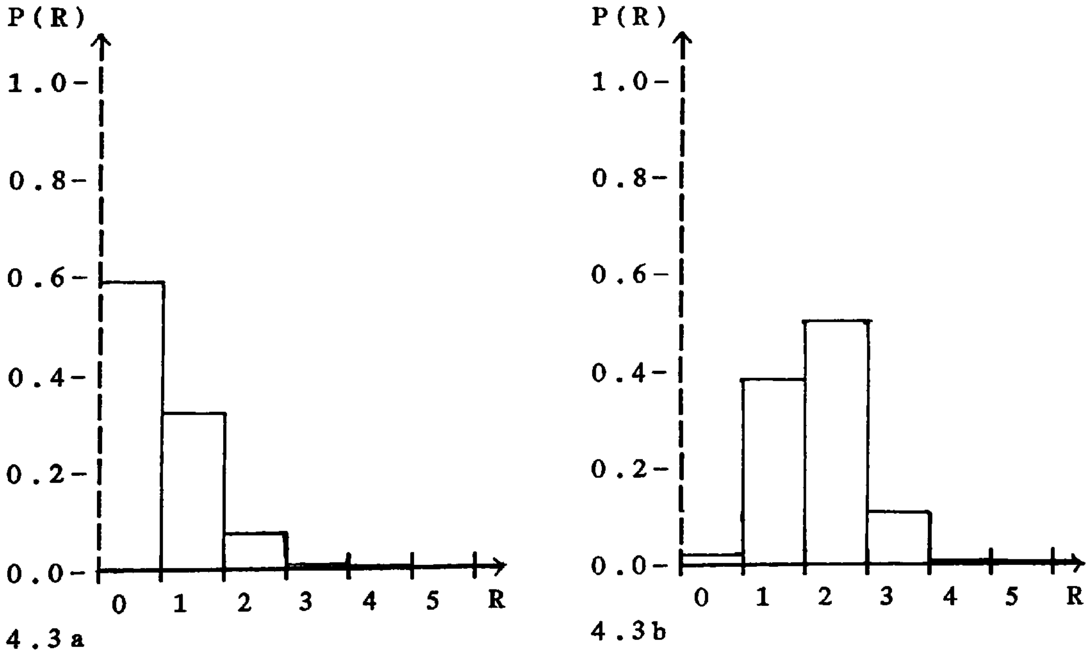


Diagram 4.3
Distribution of Target Position in the Case
V_s=30 V_{tmax}=20 V_{tmode}=10 α=3.1 β=3.1

Key R = distance moved by target

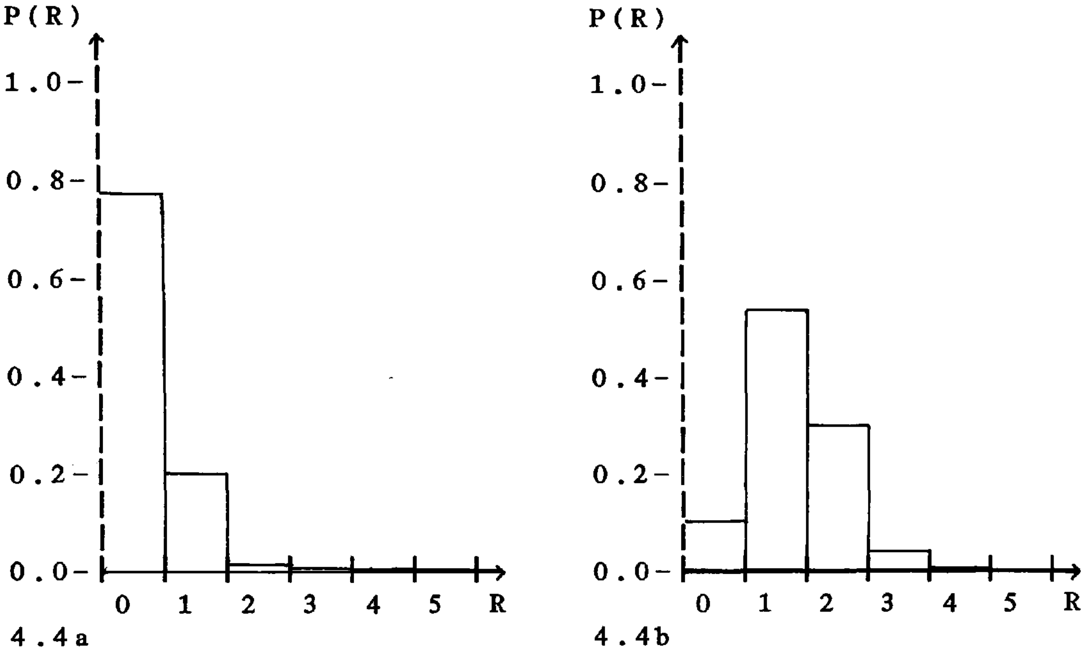


Diagram 4.4
Distribution of Target Position in the Case
$V_s=25$ $V_{tmax}=20$ $V_{tmode}=5$ $\alpha=2.23$ $\beta=4.7$

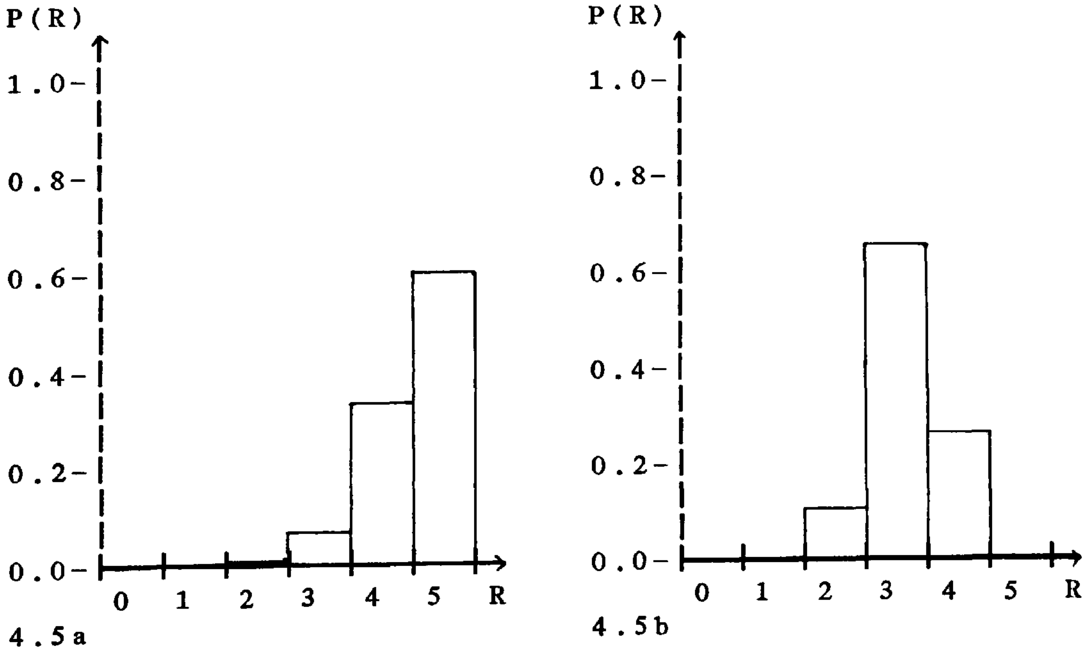


Diagram 4.5
Distribution of Target Position in the Case
$V_s=25$ $V_{tmax}=20$ $V_{tmode}=17$ $\alpha=8.93$ $\beta=2.4$

Key R = distance moved by target

Consider the case illustrated by diagrams 4.2a and 4.2b. These distributions result from a symmetric Beta distributions where $\alpha = \beta = 10$, and the ratio of searcher speed to maximum target speed is 3:2. The probability that the distance travelled by the target is R units, when the searcher moves N units between looks, is given by equation (2.13) as

$$P(R|N) = \int_{K(R-\frac{1}{2})}^{K(R+\frac{1}{2})} A\, x^9 \,(1 - x)^9 \, dx$$

for $0 \leq x \leq 1$ with $K = \dfrac{3}{2N}$, and A constant.

The resulting discretisation for the cases N = 1 and N = 5 respectively are shown in diagrams 4.6a and 4.6b. It can be seen from the diagrams that the discretisation is much coarser in the first case than in the second.

The target position distribution shown in diagram 4.2a results from five applications of equation (4.1) using transition probabilities formed from the coarse discretisation shown in diagram 4.6a. Since in one time unit the target has only a small probability of moving, the repeated application of this results in a target position distribution heavily biased towards the starting node.

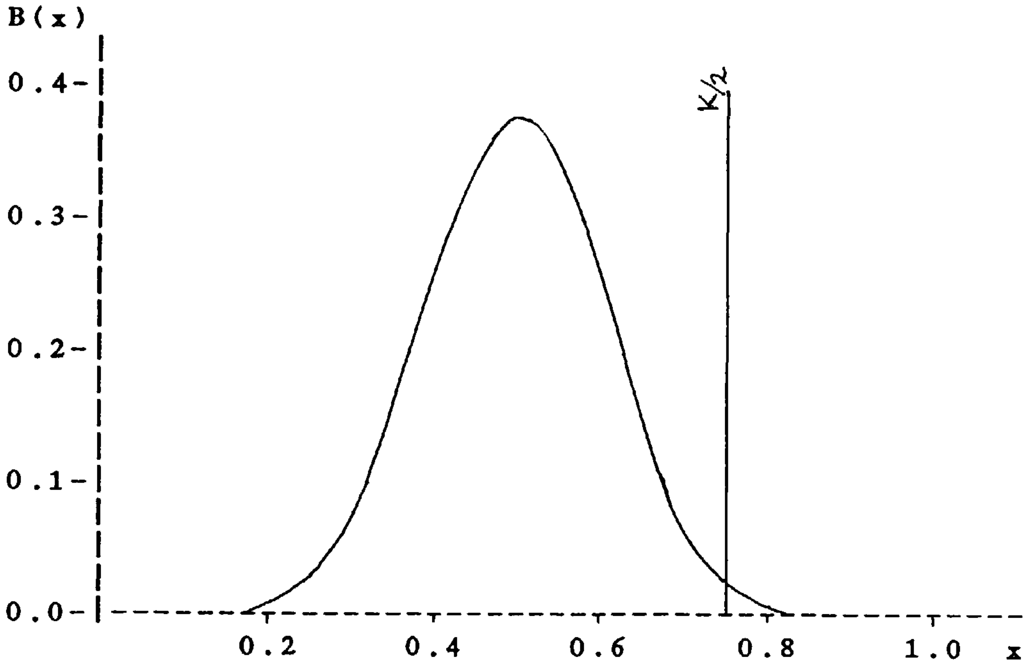


Diagram 4.6a Showing the Discretisation of $B(x)=Ax^9(1-x)^9$ in the Case N=1

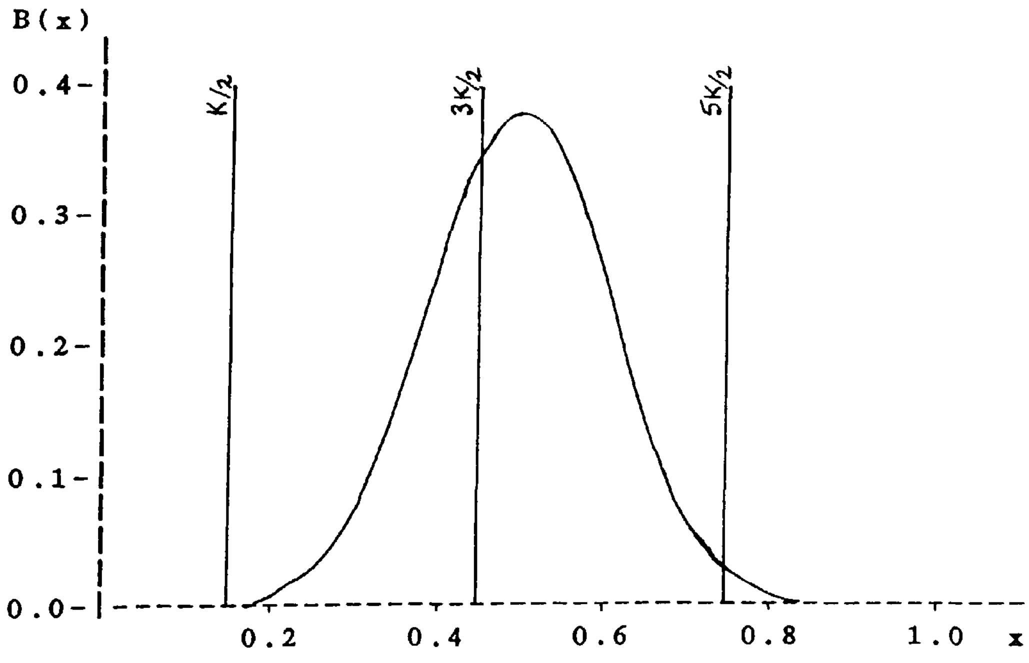


Diagram 4.6b Showing the Discretisation of $B(x)=Ax^9(1-x)^9$ in the Case N=5

(Vertical scale = $A.10^{-5}$)

In contrast the position distribution in diagram 4.2b results from one application using the discretisation show in diagram 4.6b, and can therefore be considered to be the true representation of target movement after five time intervals.

The effect is clearly most pronounced in cases where the discretisation for one time unit results in a high probability of the target remaining stationary, as in the case just considered, or in a high probability of it moving, as in the case illustrated in diagram 4.5.

However, even in cases where the problem is less marked, this model of target speed is far from satisfactory because the differences in resulting target position distribution depend on searcher strategy. Hence, when optimising, the search strategy will be artificially influenced by the differences in target distribution. Alternative approaches to modelling target speed are examined in Chapter 5, where a revised model is presented.

4.3.2 TARGET DIRECTION

The direction component of target movement was also investigated. This was done in conjunction with target speed, as the example given in section 4.3.1 illustrates, and also independently by setting the probability that the target moves one unit in each time interval equal to one. The latter allows the distance moved by the target to be known exactly, enabling the direction component of the motion to be clearly identified.

The results obtained were, in all cases, found to be consistent with anticipated target movement. It must however be noted that, in cases where the direction of motion is uncertain, the assumed Markov property of the motion, dicussed in section 2.1, results in the spread of the target distribution being rather slower than might at first sight be expected.

This may be illustrated by considering the case where the target is initially at the central node, and has unknown direction of movement. With notation as defined in section 2.4, this is given by setting the direction distribution $d(\theta) = 1/6$ for $\theta = 1,2,..6$, where θ denotes direction of movement. Setting $\eta_{ij} = 1$ for all $i,j \varepsilon X$, and the probability that the target moves one unit in each time interval equal to 1, as previously discussed, the target position distribution obtained after three time intervals is shown in diagram 4.7.

* * * *
.0046 .0139 .0139 .0046

* * * * *
.0139 .0278 .0278 .0278 .0139

* * * * * *
.0139 .0278 .0694 .0694 .0278 .0139

* * * * * * *
.0046 .0278 .0694 .0556 .0694 .0278 .0046

* * * * * *
.0139 .0278 .0694 .0694 .0278 .0139

* * * * *
.0139 .0278 .0278 .0278 .0139

* * * *
.0046 .0139 .0139 .0046

Diagram 4.7 Target Position Distribution After Three Time Intervals

It can be seen that the target still has a high probability of being at, or around, the central node after moving three units. This is because at each node the target has equal probability of moving in any direction, and thus has a high probability of returning to nodes previously visited. This is a feature of the Markov assumption of target movement, and might be consistent with the movement of a crash survivor wandering aimlessly around, or with a military target taking evasive action by changing direction frequently to avoid detection. It would not however be a suitable model in a case where the direction of movement was initially unknown, but once chosen was assumed to be maintained.

4.4 CONCLUSION

The discussion in the preceding sections indicates that the updating algorithm, developed in Chapter 2, produces sensible target position distributions in relation to all aspects of the search model, with the exception of target speed. An alternative, more acceptable, approach to the problem of discretising the speed component of target motion is developed and discussed in Chapter 5. Further analysis of the search model with regard to detection probability, search strategy and optimisation is given in Chapter 6.

CHAPTER FIVE

REVISED MODEL OF TARGET MOVEMENT

5.0 INTRODUCTION

It was shown in section 4.3 that the discretisation of target speed introduced in section 2.5.2, leads to unsatisfactory anomalies in target movement when applied in the updating algorithm. Two alternative methods of overcoming the problem of target movement are discussed in this chapter. The first method, involving a separate grid for target movement, was found to give some improvement in target motion. However the motion is still unsatisfactory, and the method is computationally unsuitable. The second method, updating the target distribution at unit time intervals, results in a target position distribution which is somewhat flatter than the true distribution. However, the distribution is a significant improvement over that produced by the initial model, and is sufficiently close to the true distribution to be acceptable.

5.1 DOUBLE GRID SYSTEM

The anomalies in target movement identified in section 4.3 are due to the very course discretisation of the target speed distribution that results when the searcher moves only short distances between looks. An initial attempt to reduce this effect was made by introducing a second, more closely spaced grid for target movement, the spacing of the target grid being chosen so that the nodes of the two grids coincide.

An example of this double grid system is illustrated in diagram 5.1. The diagram shows a target grid superimposed on the search grid, the target grid having node spacing one third of that of the search grid. With this arrangement, the searcher is still restricted to travel integer distances, while the distance travelled by the target can take the values 0, 1/3, 2/3, ... etc.

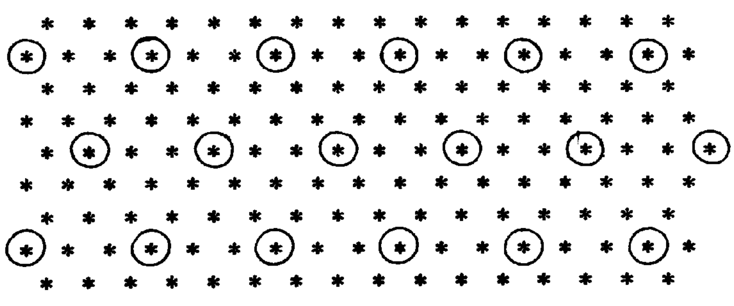


Diagram 5.1 Double Grid

Key * nodes belonging to target grid
* nodes belonging to both target and searcher grid

It was found that this approach gave some improvement in target motion, however the effect shown in diagrams 4.2 to 4.5 was still present. This is because, although the discretisation of the speed distribution is less coarse, it is still different for different time intervals. Thus, even with a very fine spacing for the target grid, it is likely that this problem would still be apparent.

In addition, this method has the following disadvantages. The introduction of extra nodes for target movement significantly increases the computation time required for updating, thus magnifying the problem of optimisation. Also the use of separate grids places a greater restriction on the movement of the searcher than of the target, which is undesirable.

Because of the problems outlined above, an alternative approach, not presenting these disadvantages, is discussed in the following section.

5.2 UPDATING AT UNIT TIME INTERVALS

An alternative approach to the problem of modelling target movement is to update the target position distribution at equal time intervals, irrespective of whether a look is to be taken at that time.

This may be performed by considering the updating algorithm in the form discussed in section 2.2.1. There it was shown that updating using the algorithm given by equation (2.2) is equivalent to updating in two stages, firstly applying equation (2.3) to allow for information gained from a look, and then applying equation (2.4) to give target movement prior to the next look. Equations (2.3) and (2.4) are reproduced below as (5.1) and (5.2), where π_i^s and π_i^m are the position probabilities at node i updated for search and for movement respectively, (all notation being as defined in section 2.2).

$$\pi_i^s = \frac{\pi_i \eta_{ji}}{\sum_h \pi_h \eta_{jh}} \tag{5.1}$$

$$\pi_i^m = \sum_h \pi_h \rho_{hi}(\Delta t) \tag{5.2}$$

Assume for convenience that the searcher takes unit time to travel between adjacent nodes, hence the time interval between looks is always an integral number of time units. The target position distribution can therefore be updated at unit time intervals, applying equation (5.2) to give only target movement at those times when no look is taken, and equation (2.2) (i.e. equations (5.1) and (5.2) combined) to allow also for information gained at times when a look is taken.

Updating in this way ensures that the modelling of target movement is independent of searcher activity.

5.2.1 TRANSITION PROBABILITIES

In order to update at unit time intervals, transition probabilities must be found to represent the known distribution of target speed. Clearly it is not sufficient to simply use the discretisation of the speed distribution for one time interval given by equation (2.13) because, as shown in section 4.3.1, repeated application of this results in distorted representation of target movement. As an alternative to this the following approach was taken.

Assume that the initial target position distribution is known at time t_0, and that updating is to be carried out at unit intervals t_1, t_2,... following t_0. Let the time interval between t_{N-1} and t_N be interval N, and define the following events:

I,N = target moves I units in interval N.

R,t_N = target moves R units in the time interval t_N-t_0, (of length N units).

Also, the additional assumption is made that the maximum speed of the target is no greater than the speed of the searcher. This restriction is not unrealistic because, if the searcher was unable to move at least as fast as the target, the target could outrun it. With this restriction the target is not able to move more than one distance unit in each time interval. That is, it can either move 0 units or 1 unit in any interval N, hence for all N,

$$P(0,N) = 1 - P(1,N) \qquad (5.3).$$

For $N > 1$ the probability that the target does not move in interval N can be expressed as a weighted sum of probabilities conditional on its movement in the previous N-1 time units. Thus

$$P(0,N) = \sum_{R=0}^{N-1} P(0,N \mid R,t_{N-1}).P(R,t_{N-1}) \qquad (5.4).$$

In order to find P(0,N) for any $N > 1$ the components of the sum may be evaluated in the following way.

If the target moves R units in N time intervals, this can occur in one of two ways: by moving R units in the first N-1 intervals and 0 units in interval N, or moving R-1 units in the first N-1 intervals and 1 unit in interval N. These events are exhaustive and mutually exclusive, so the elements of the sum are given, for $R > 0$, by

$$P(0,N \mid R,t_{N-1}).P(R,t_{N-1}) = $$
$$P(R,t_N) - P(1,N \mid R-1,t_{N-1}).P(R-1,t_{N-1}) \qquad (5.5)$$

and in the case $R = 0$,

$$P(0,N \mid 0,t_{N-1}) . P(0,t_{N-1}) = P(0,t_N) \qquad (5.6).$$

Also, for any $R > 0$ and $N > 1$

$$P(1,N \mid R-1,t_{N-1}) = 1 - P(0,N \mid R-1,t_{N-1}) \qquad (5.7).$$

Now for any R, $P(R,t_N) \equiv P(R|N)$, where $P(R|N)$ is the probability that the target moves R units given that the searcher moves N units, as defined in section 2.5.2. Hence $P(R,t_N)$ may be evaluated by use of equation (2.13):

$$P(R,t_N) = \int_{K(R-\frac{1}{2})}^{K(R+\frac{1}{2})} s(v)\, dv \qquad (5.8)$$

$$\text{with } K = \frac{V_s}{N\, V_{tmax}} .$$

The probability that the target does not move in the first time interval, $P(0,1)$ may be obtained directly from equation (5.8). Then for any $N > 1$, the elements required in the evaluation of $P(0,N)$ by equation (5.4) may be found by use of equation (5.6) for the case $R = 0$, and then equations (5.7) and (5.5) for successive values of R.

Having found $P(0,N)$, $P(1,N)$ may be found from equation (5.3). A set of transition probabilities for use in the updating algorithm, can then be constructed for each time interval as follows. Let $\rho_{hi}(N)$ denote the probability that the target moves from node h to node i in time interval N, given that it is at node h at time t_{N-1}, then

$$\rho_{hi}(N) = \begin{cases} P(0,N) & \text{for } h = i, \\ P(1,N).d(\theta_{hi}) & \text{for } h,i \text{ 1 unit apart,} \\ 0 & \text{for } h,i > 1 \text{ unit apart,} \end{cases} \tag{5.9}$$

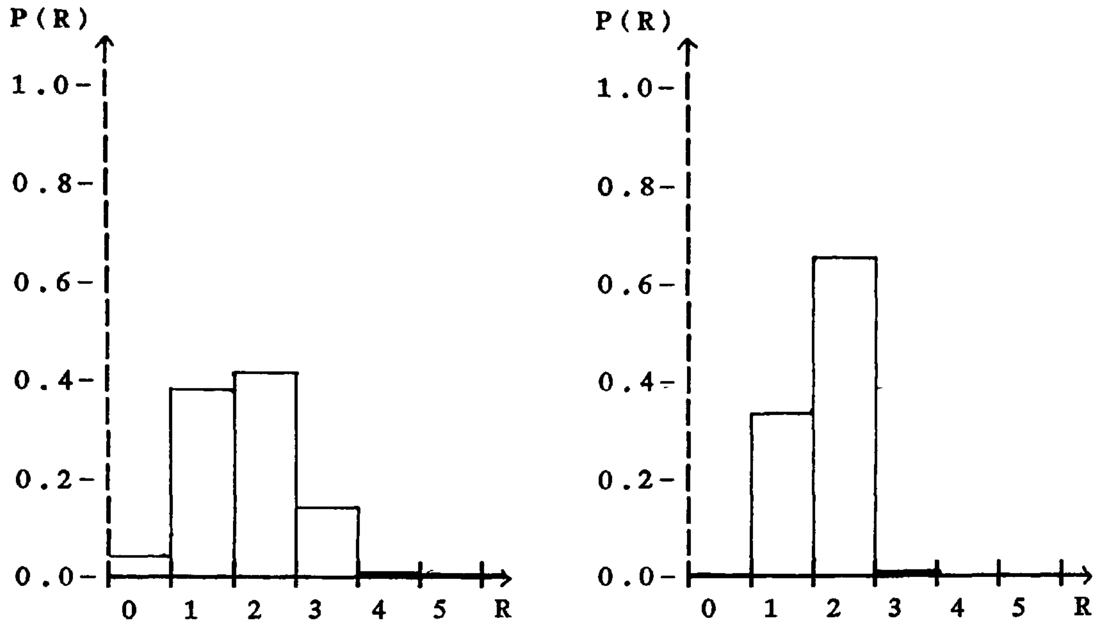
where θ_{hi} is the direction of travel from node h to node i, and $d(\theta_{hi})$ is the direction probability as defined in section 2.4.2.

Although this approach requires a different set of transition probabilities for each time interval, there will only be at most seven non-zero values associated with each node. As the assumption was made in section 2.4 that target motion is independent of position, these values will be the same for all nodes of the search area. Hence it is only necessary to store the set of seven values for each time interval instead of a full transition matrix.

The maximum number of time intervals that need be considered can be calculated from the number of looks taken multiplied by the maximum distance between any two nodes of the search area. For example, a search consisting of 10 looks over an area with maximum node separation of 10 units will require no more than 100 time units to complete. Hence the transition probabilities can be calculated in advance and stored in a file ready to be accessed by the search program.

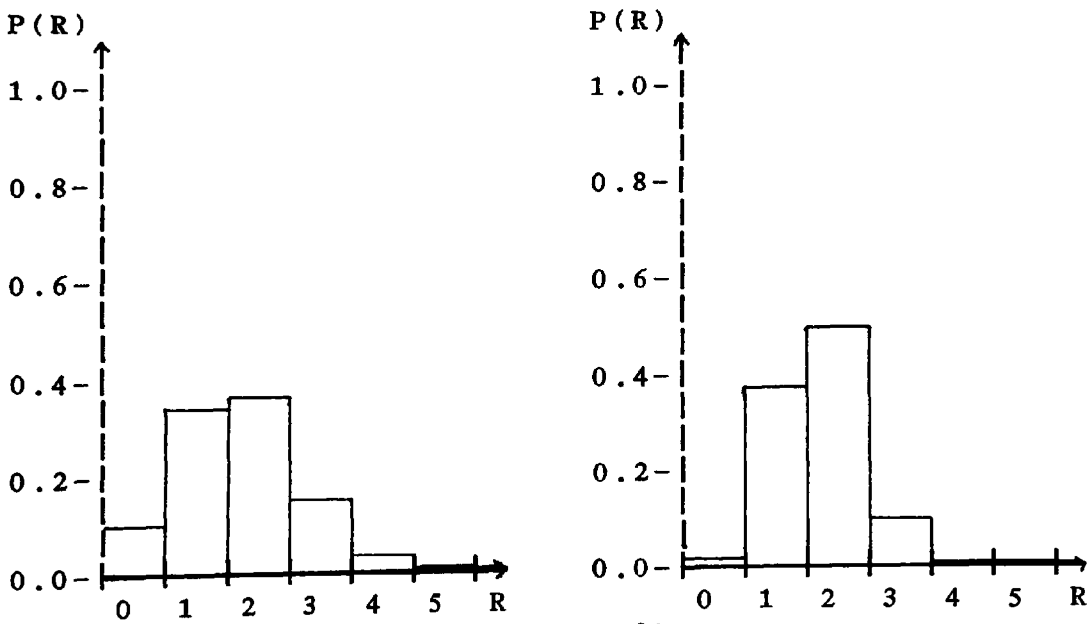
5.3 ANALYSIS OF THE NEW MODEL OF TARGET MOVEMENT

The target speed distributions resulting from this new model of target movement were investigated in a similar way to that described in section 4.3.1. Unidirectional target movement and zero detection capability were applied with various speed distributions. In each case the resulting distribution of target position was compared after various time intervals with that obtained by applying equation (2.13). The position distributions for the cases considered in section 4.3.1 are illustrated in diagrams 5.2a to 5.5a. The corresponding 'true' distributions given by equation (2.13) (diagrams 4.2b to 4.5b), are reproduced here for comparison as diagrams 5.2b to 5.5b.



5.2a 5.2b

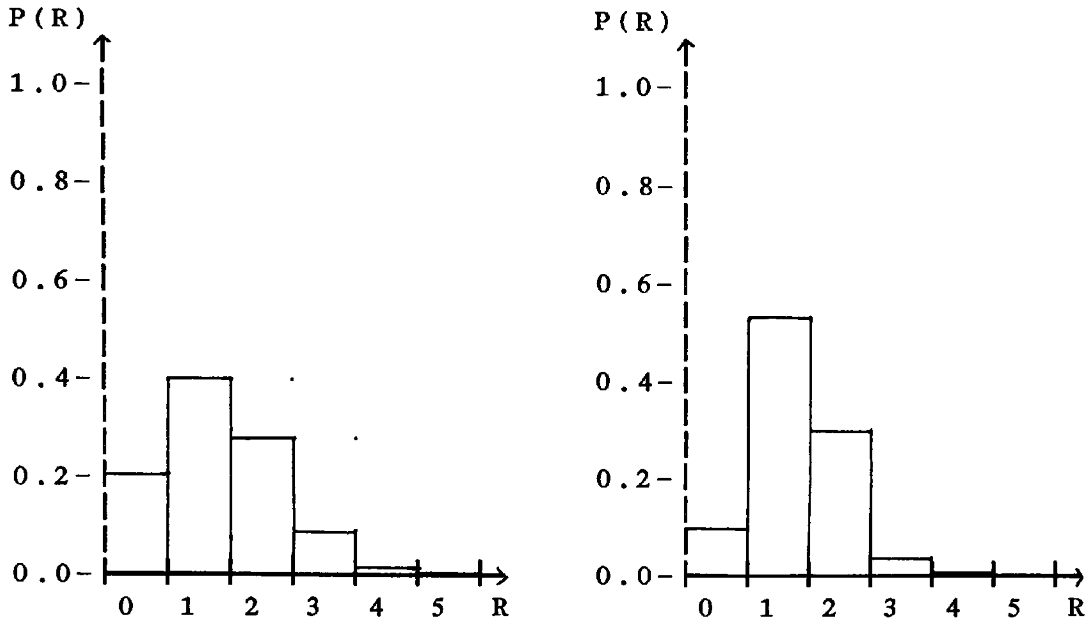
Diagram 5.2
Distribution of Target Position in the Case
$V_s = 30$ $V_{tmax} = 20$ $V_{tmode} = 10$ $\alpha = 10$ $\beta = 10$



5.3a 5.3b

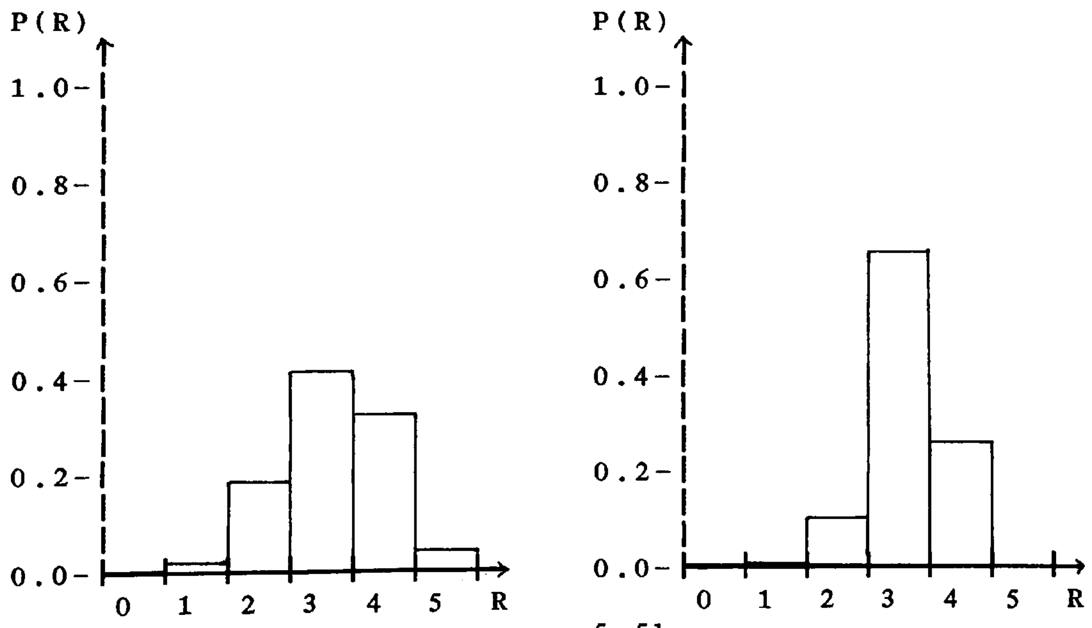
Diagram 5.3
Distribution of Target Position in the Case
$V_s = 30$ $V_{tmax} = 20$ $V_{tmode} = 10$ $\alpha = 3.1$ $\beta = 3.1$

Key R = distance moved by target



5.4a 5.4b

Diagram 5.4
Distribution of Target Position in the Case
$V_s=25$ $V_{tmax}=20$ $V_{tmode}=5$ $\alpha=2.23$ $\beta=4.7$



5.5a 5.5b

Diagram 5.5
Distribution of Target Position in the Case
$V_s=25$ $V_{tmax}=20$ $V_{tmode}=17$ $\alpha=8.93$ $\beta=2.4$

Key R = distance moved by target

It can be seen that the distribution of target position given by the new model is somewhat flatter than the true distribution. However, in all cases, both the range and mode of the position distribution correspond with those of the 'true' distribution.

This method clearly gives a significant improvement over the target movement produced by the previous model. It is also computationally convenient and ensures that the movement of the target is unaffected by the decision strategy of the searcher. For these reasons this model was chosen as the most satisfactory way found to model target movement.

CHAPTER SIX

RESULTING SEARCH STRATEGIES

6.0 INTRODUCTION

In this chapter, search strategies obtained using the search model and optimisation procedures developed in the previous chapters are discussed, and a selection of the results presented.

The strategies were obtained by the process of exhaustive evaluation, described in section 3.2.2, with extensive use being made of limiting planning horizon and travel distance (as discussed in section 3.4) in order to make this approach feasible. In the case of optimising detection probability per unit cost, condition (3.21), given in section 3.3.1, was also used to reduce the amount of computation. Optimisation with respect to detection probability is discussed in section 6.1, and with respect to detection probability per unit cost in section 6.4.

Comparison of results with strategies obtained by other methods has not been possible because of the differences in approach. Most current models are concerned with allocation of available effort over the entire search area at any instant, and are thus not comparable with a discrete sequential approach. As discussed in Chapter 1, the discrete models that exist are presented in very simple cases which cannot usefully be compared with the present model. An approach was made to the Department of Operational Analysis Establishment for any relevant data that might be of use in verification of the model, however none was available.

In view of this lack of supporting evidence to verify the model, extensive evaluation of the resulting search strategies and associated detection probabilities has been carried out. To study the behaviour of the model a typical case was chosen as a standard, to which variations were made in one set of parameters at a time. The resulting strategies are discussed in sections 6.2 and 6.3 in relation to optimising probability of detection, and the differences in results obtained when optimising with respect to unit cost are discussed in section 6.4.

6.1 OPTIMISING PROBABILITY OF DETECTION

The procedure for optimising detection probability by exhaustive evaluation described in section 3.2.2 was coded in a FORTRAN computer program SEAR14. The program is designed to be used with the transition probabilities, described in section 5.2.1, which are created by a second FORTRAN program, TRANSP.

The information required in the decision process is given in the following way. The parameters of target and searcher speed and target direction, specified in the same way as in the example in section 2.7, are given as input to TRANSP. The program calculates the appropriate Beta distribution and creates a file of transition probabilities which may subsequently be read by SEAR14. The remaining search parameters of initial target position and searcher detection capability, again specified in the way discussed in section 2.7, are given as input to SEAR14. As before, an hexagonal search area is used, the size of which must be specified, with a maximum of 91 nodes. The initial position of the searcher, and the number of looks required must also be given as input.

SEAR14 allows the restrictions on searcher range and planning horizon, discussed in section 3.4, to be given. Planning horizon is specified as the number of future looks over which the optimisation is to be performed, ranging from one look, giving an incremental strategy, to the total number of looks, giving an optimal strategy. An option is also included allowing an additional restriction to be imposed, preventing the searcher from taking consecutive looks at the same node. (Thus giving a minimum distance of one unit between search nodes.) This is to enable the strategies to be compared with those found when optimising with repect to unit cost, where the same restriction is used for reasons discussed in section 3.3.

The decision process is carried out as a tree search as illustrated in diagram 3.1, the depth to which the tree is searched before a decision is made being determined by the restriction on planning horizon. The search strategy is chosen by calculating the total non-detection probability for each possible strategy (or, in the case of limited planning horizon, part strategy). The values are compared to an accuracy of 0.5×10^{-6}, (to minimise the effect of computer rounding errors on the decision process), and the strategy with the lowest non-detection probability is selected. Where two or more strategies give the same value within this tolerance, only one is chosen, this being the strategy with the shortest path length. If the path length is also the same the first of these strategies to be found is selected. No look is taken at the initial node, as it is not part of the decision process, so it is considered merely as a starting point for the searcher with no information being gained. This aids comparison with results obtained when optimising with respect to unit cost.

The output from SEAR14 is as follows. Firstly a map showing the initial target position distribution over the hexagonal search area is shown, and the searcher starting point is indicated. In cases where there is no limit on planning horizon, the chosen strategy is listed (by node number as shown in diagram 2.9), with the cumulative probability of detection using that strategy. The program then outputs more detailed information about the strategy, giving for each look, the probability of detection with that look, the cumulative detection probability for the search to that point, and a map of the target position distribution immediately before the look is taken.

In cases where planning horizon is limited, the projected strategy for the required number of future looks is given. The cumulative detection probability for the search, up to and including the projected strategy is also output. The detailed information described above is given about the first node only of the projected strategy, (as this becomes the next search node of the final strategy,) before the decision process is repeated and the next projected strategy output. Finally a summary showing the complete search strategy and overall cumulative detection probability is given.

The behaviour of the decision process when varying the length of search, and under differing restrictions on planning horizon and searcher range, was investigated by producing strategies for the cases shown in table 6.1. The table shows the maximum length (i.e. number of looks) of strategies found under the restrictions indicated. In all cases a search grid of 91 nodes was used.

Table 6.1 Maximum Search Length

Planning Horizon	Searcher Range 1	2	3	4	5	6	7	8	9	10
1	20	20	20	20	20	10	10	10	10	10
2	20	20	20	20	10	10	10	10	10	10
3	20	10	10	5	4	3				
4	20	7	4							
5	10	5								
6	7									
7	7									

The results are limited to those given in the table because the combinatorial possibilities made the computer time required for optimisation of cases outside this range very high. It was felt that a better understanding of the model could be gained by investigating prolonged searches over the maximum size search area, than by complete optimisation of cases of a small number of looks and a small search area.

The effect of using different values for the various parameters of the search model was investigated by choosing a 'standard' case to which changes could be made systematically in one set of parameters at a time. The results obtained in the standard case are given in detail in section 6.2, and a summary of the differences found in variations on the standard case is given in section 6.3.

6.2 THE STANDARD CASE

The set of values shown below was chosen as a 'standard' basis for investigation of the search model because it was felt to be a fairly typical case that could easily be interpreted. The initial target distribution, non-detection probabilities and a symmetric speed distribution were chosen for simplicity, while the distributions of target direction and speed were selected as being non-uniform, but not too deterministic. The searcher to target speed ratio was thought to be representative of a typical case. (However in practice, what might be typical would depend on the particular application.)

The parameters chosen for the standard case are given here with all notation as defined in sections 2.2 and 2.4. The initial distribution is specified according to the node numbering in diagram 2.9, and target direction according to diagram 2.5.

Size of search area = 91 nodes.

Initial ditribution:

$$\pi_i = \begin{cases} 1 & i = 1 \\ 0 & i \in \{2,3,\ldots 91\}. \end{cases}$$

Target and searcher speed:

$V_{tmax} = 20$ $\qquad$ $V_s = 25$

$V_{tn} = 10$ $\qquad$ $\lambda = 0.1$

giving

$\alpha = 3.1$ $\qquad$ $\beta = 3.1$

(actual value of integral = 0.0995).

Target direction:

$$d(\theta) = \begin{cases} 0.1 & \theta \in \{1,2\} \\ 0.2 & \theta \in \{3,4,5,6\}. \end{cases}$$

Non-detection probabilities:

range	0	1	2	3	4	5
probability	0	1	1	1	1	1

Initial position of searcher = node 19.

Minimum distance between search nodes = 1 unit.

The standard case was investigated with the range of restrictions shown in table 6.1. The results obtained are shown in tables 6.2 to 6.8. Each table shows, for a particular restriction on planning horizon, the resulting strategy and associated cumulative detection probability, for each of the restrictions on maximum searcher range. Where the tables are left blank, the strategy and detection probability are the same as those for the next lower searcher range.

The strategy is specified by giving the sequence of search nodes, numbered according to diagram 2.9, at which successive looks are to be taken. A fold-out copy of the node numbering system, for use with tables 6.2 to 6.9, is located in the appendix.

The following sub-sections contain further examination of the standard case. In section 6.2.1 one strategy is chosen for closer analysis, and in section 6.2.2 the effect of changing the optimisation restrictions is investigated.

Table 6.2

Strategies with Planning Horizon of 1 Look

Look No.	Max. Searcher Range 1		Max. Searcher Range 2		Max. Searcher Range 3		Max. Searcher Range 4		Max. Searcher Range 5	
	Sch. Node	Cum. Det. Prob.	Sch. Node	Cum. Det. Prob.	Sch. Node	Cum. Det. Prob.	Sch. Node	Cum. Det. Prob.	Sch. Node	Cum. Det. Prob.
1	7	.0272	1	.3486						
2	1	.3727	4	.4398						
3	4	.4639	5	.5078						
4	5	.5315	6	.5547						
5	6	.5757	1	.5907						
6	1	.6097	3	.6198						
7	3	.6382	4	.6385						
8	4	.6564	14	.6527						
9	14	.6705	5	.6651						
10	5	.6824	6	.6767						
11	1	.6930	1	.6868						
12	2	.7021	3	.6959						
13	3	.7097	12	.7040						
14	12	.7175	13	.7112						
15	13	.7245	15	.7180						
16	14	.7308	16	.7238						
17	15	.7372	6	.7291			79	.7301		
18	16	.7434	1	.7340	29	.7341	77	.7362		
19	6	.7487	13	.7386	79	.7408	81	.7432		
20	1	.7530	51	.7438	76	.7475	77	.7496	76	.7504

Searcher Range 6 - 10 (10 looks only), strategy the same as the first 10 looks of case with maximum range 2.

Table 6.3

Strategies with Planning Horizon of 2 Looks

	Max. Searcher Range 1		Max. Searcher Range 2		Max. Searcher Range 3		Max. Searcher Range 4	
Look No.	Sch. Node	Cum. Det. Prob.	Sch. Node	Cum. Det. Prob.	Sch. Node	Cum. Det. Prob.	Sch. Node	Cum. Det. Prob.
1	7	.0272	1	.3486				
2	1	.3727	4	.4398				
3	5	.4637	5	.5078				
4	4	.5316	6	.5547				
5	3	.5778	1	.5907				
6	1	.6117	3	.6198				
7	6	.6388	2	.6378				
8	5	.6568	4	.6530				
9	14	.6708	14	.6661				
10	4	.6828	5	.6775				
11	12	.6934	16	.6873				
12	3	.7032	6	.6960				
13	1	.7121	1	.7034				
14	6	.7200	13	.7108				
15	16	.7273	30	.7171	31	.7165	12	.7174
16	15	.7340	79	.7234	78	.7234	78	.7241
17	30	.7398	78	.7294	79	.7300	80	.7308
18	29	.7453	28	.7345	76	.7370	76	.7380
19	13	.7506	77	.7404	79	.7424	79	.7449
20	4	.7553	76	.7468	81	.7503	81	.7526

Searcher Range 5 - 10 (10 looks only), strategy the same as the first 10 looks of case with maximum range 2.

Table 6.4

Strategies with Planning Horizon of 3 Looks

	Max. Searcher Range 1		Max. Searcher Range 2		
Look No.	Sch. Node	Cum. Det. Prob.	Sch. Node	Cum. Det. Prob.	
1	7	.0272	1	.3486	Maximum searcher range 3, 10 looks only:
2	1	.3727	4	.4398	Strategy as First 10 Looks of Range 2
3	5	.4637	5	.5078	
4	4	.5316	6	.5547	Maximum searcher range 4, 5 looks only:
5	3	.5778	1	.5907	Strategy as First 5 Looks of Range 2
6	1	.6117	2	.6133	
7	6	.6388	3	.6358	Maximum searcher range 5, 4 looks only:
8	16	.6552	12	.6520	Strategy as First 4 Looks of Range 2
9	5	.6699	4	.6664	
10	14	.6832	14	.6796	Maximum searcher range 6, 3 looks only:
11	13	.6944	---------------		Strategy as First 3 Looks of Range 2
12	12	.7050	(10 looks only)		
13	3	.7149			
14	1	.7237			
15	6	.7308			
16	16	.7369			
17	15	.7427			
18	30	.7481			
19	29	.7531			
20	13	.7581			

Table 6.5

Strategies with Planning Horizon of 4 Looks

Look No.	Max. Searcher Range 1 Sch. Node	Max. Searcher Range 1 Cum. Det. Prob.	Max. Searcher Range 2 Sch. Node	Max. Searcher Range 2 Cum. Det. Prob.	Max. Searcher Range 3 Sch. Node	Max. Searcher Range 3 Cum. Det. Prob.
1	7	.0272	1	.3486	1	.3486
2	1	.3727	3	.4273	4	.4398
3	3	.4512	4	.4963	5	.5078
4	4	.5201	5	.5543	6	.5547
5	5	.5773	6	.5937	(4 looks only)	
6	16	.6029	1	.6211		
7	6	.6325	7	.6387		
8	7	.6497	(7 looks only)			
9	1	.6682				
10	3	.6816				
11	4	.6943				
12	13	.7054				
13	29	.7142				
14	30	.7227				
15	15	.7316				
16	16	.7386				
17	6	.7444				
18	1	.7498				
19	4	.7552				
20	13	.7602				

Table 6.6

Strategies with Planning Horizon of 5 Looks

	Max. Searcher Range 1		Max. Searcher Range 2	
Look No.	Sch. Node	Cum. Det. Prob.	Sch. Node	Cum. Det. Prob.
1	7	.0272	1	.3486
2	1	.3727	3	.4273
3	3	.4512	4	.4963
4	4	.5201	5	.5543
5	5	.5773	6	.5937
6	16	.6029	(5 looks only)	
7	6	.6325		
8	1	.6556		
9	4	.6713		
10	14	.6852		

Table 6.7

Strategy with Planning Horizon of 6 Looks

Look No.	Max. Searcher Range 1: Sch. Node	Max. Searcher Range 1: Cum. Det. Prob.
1	7	.0272
2	1	.3727
3	5	.4637
4	4	.5316
5	3	.5778
6	1	.6117
7	6	.6388
8	5	.6568

Table 6.8

Strategy with Planning Horizon of 7 Looks

Look No.	Max. Searcher Range 1: Sch. Node	Max. Searcher Range 1: Cum. Det. Prob.
1	7	.0272
2	1	.3727
3	3	.4512
4	4	.5201
5	5	.5773
6	6	.6144
7	1	.6400

6.2.1 THE BEST STRATEGY FOR TWENTY LOOKS

In order to examine one strategy more closely, the case giving the highest probability of detection in twenty looks is considered. This is found in the case with a planning horizon of four looks and maximum searcher range one unit. The path of the strategy is illustrated in diagram 6.1, and more detailed information about the decision process for this case is shown in table 6.9.

In addition to the information already given in table 6.5, table 6.9 shows the projected search path at each stage of the decision process, and the cumulative detection probability for a search pursuing that projected strategy. The projected path should be read from the table diagonally, for example at the first decision, the projected path is 7,1,5,4 with cumulative probability 0.5316, while the final strategy looks at nodes 7,1,3,4 with cumulative probability 0.5201. Note that the cumulative detection probability is higher for the projected strategy than for the first four looks of the final strategy. Wherever the two differ this will normally be the case, since the projected strategy is optimal for the four looks (with this planning horizon) following the decision. Any deviation from this strategy made by the following decisions causes a reduction in detection probability for these looks, in order to gain higher detection probability with subsequent looks (unless a different path having the same detection probability is chosen).

Also shown in table 6.9 is the probability of detecting the target with each look (given the failure of previous looks). It can be seen that, as the search progresses, the probability of detection with each look tends to decrease as the target distribution spreads. However, detection probability is not strictly decreasing as would be expected in the case of a stationary target, as discussed by Black (1965) for example. This is firstly because the target distribution is changing with time, so the probability that the target is at a particular node when a look is taken will vary with the timing of of that look (in addition to the effect of previous looks). Secondly, as discussed above, the decision process may choose to look at a node with relatively low detection probability in order to increase detection probability later in the search. A third factor is the restriction on searcher range, which in some cases may prevent a look being taken at a node with higher detection probability.

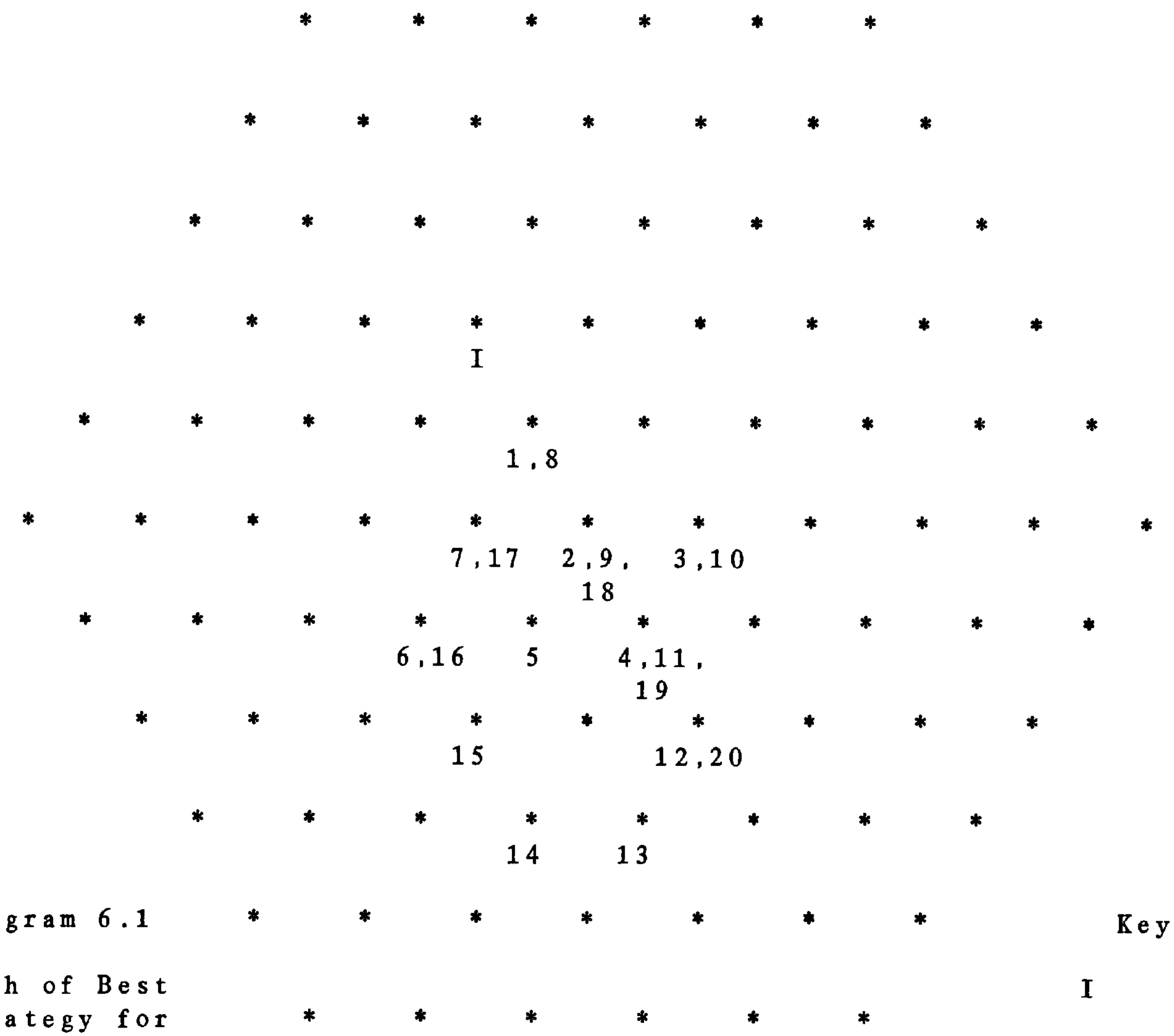


Diagram 6.1

Path of Best
Strategy for
Standard Case.

Key

I — Initial Node

1,2,.. — Position of Search Node 1,2,..etc.

Table 6.9

Best Strategy for the Standard Case:

Planning Horizon 4 Looks, Maximum Searcher Range 1 Unit.

Look No.	Actual Path	Detection Probability	Cumulative Probability	Projected Path				Projected Cum. Prob.
1	7	.0272	.0272	7				
2	1	.3552	.3727	1	1			
3	3	.1251	.4512	3	5	5		
4	4	.1255	.5201	4	4	4	4	.5316
5	5	.1192	.5773	5	5	5	3	.5778
6	16	.0605	.6029	16	1	6	6	.6144
7	6	.0746	.6325	6	6	6	1	.6400
8	7	.0468	.6497	7	1	1	16	.6556
9	1	.0527	.6682	1	1	4	4	.6713
10	3	.0404	.6816	3	3	4	14	.6852
11	4	.0400	.6943	4	4	4	14	.6966
12	13	.0361	.7054	13	13	14	14	.7067
13	29	.0300	.7142	29	14	14	15	.7163
14	30	.0299	.7227	30	30	15	15	.7246
15	15	.0318	.7316	15	15	15	16	.7320
16	16	.0263	.7386	16	16	16	5	.7388
17	6	.0223	.7444	6	5	5	5	.7449
18	1	.0209	.7498		1	4	4	.7505
19	4	.0217	.7552			4	13	.7555
20	13	.0203	.7602				13	.7602

As diagram 6.1 illustrates, the search is concentrated around the lower central area of the search grid, reflecting the initial central position of the target and the subsequent downward bias of target motion. In order to relate the strategy to target position, the target distributions after 10 and 20 time units are shown in diagrams 6.2 and 6.3 respectively. Diagrams 6.2a and 6.3a show the distribution at these times if no search is made, whilst diagrams 6.2b and 6.3b show the target distribution obtained from the updating procedure when the strategy in diagram 6.1 is pursued.

Comparison of the distributions with and without the search shows how knowledge of the wherabouts of the target is modified by the information gained from unsuccessful looks. In particular it can be seen that the concentration of looks in the central area results in the target position probability being reduced in this area. This reduction in probability can also be seen in nodes below the area searched, since the predominantly downward target movement results in reduced probability that the target is subsequently in this area following the unsuccessful looks. Corresponding to this is an increase in probability in the upper and edge regions of the grid where no looks are taken.

```
         *     *     *     *     *     *
       .0000 .0000 .0001 .0001 .0000 .0000

       *     *     *     *     *     *     *
     .0001 .0002 .0004 .0005 .0004 .0002 .0001

     *     *     *     *     *     *     *     *
   .0002 .0007 .0018 .0028 .0028 .0018 .0007 .0002

   *     *     *     *     *     *     *     *     *
 .0004 .0016 .0050 .0100 .0126 .0100 .0050 .0016 .0004

 *     *     *     *     *     *     *     *     *     *
.0004 .0022 .0083 .0208 .0335 .0335 .0208 .0083 .0022 .0004

*     *     *     *     *     *     *     *     *     *     *
.0003 .0017 .0080 .0249 .0504 .0643 .0504 .0249 .0080 .0017 .0003

 *     *     *     *     *     *     *     *     *     *
.0009 .0044 .0166 .0417 .0669 .0669 .0417 .0166 .0044 .0009

   *     *     *     *     *     *     *     *     *
 .0016 .0065 .0200 .0399 .0503 .0399 .0200 .0065 .0016

     *     *     *     *     *     *     *     *
   .0018 .0059 .0144 .0227 .0227 .0144 .0059 .0018

       *     *     *     *     *     *     *
     .0013 .0033 .0064 .0080 .0064 .0033 .0013

         *     *     *     *     *     *
       .0006 .0014 .0021 .0021 .0014 .0006
```

Diagram 6.2a

Target Position Distribution
After Ten Time Intervals
With No Search

```
                 *     *     *     *     *     *
               .0000 .0001 .0001 .0001 .0001 .0000

              *     *     *     *     *     *     *
            .0001 .0004 .0008 .0010 .0008 .0005 .0002

           *     *     *     *     *     *     *     *
         .0004 .0014 .0033 .0049 .0051 .0034 .0015 .0005

        *     *     *     *     *     *     *     *     *
      .0009 .0032 .0085 .0132 .0166 .0153 .0082 .0030 .0008

     *     *     *     *     *     *     *     *     *     *
   .0010 .0045 .0137 .0221 .0175 .0332 .0262 .0118 .0037 .0009

  *     *     *     *     *     *     *     *     *     *     *
.0008 .0038 .0135 .0264 .0252 .0148 .0000 .0271 .0108 .0029 .0006

     *     *     *     *     *     *     *     *     *     *
   .0022 .0081 .0202 .0298 .0409 .0480 .0394 .0201 .0068 .0018

        *     *     *     *     *     *     *     *     *
      .0038 .0111 .0239 .0393 .0484 .0420 .0254 .0103 .0033

           *     *     *     *     *     *     *     *
         .0043 .0110 .0228 .0325 .0320 .0219 .0104 .0039

              *     *     *     *     *     *     *
            .0032 .0073 .0129 .0153 .0123 .0068 .0030

                 *     *     *     *     *     *
               .0017 .0035 .0052 .0051 .0033 .0016
```

Diagram 6.2b

Target Position Distribution
After Ten Time Intervals
With Best Search Path

```
          *     *     *     *     *     *
       .0002 .0003 .0004 .0004 .0003 .0002

       *     *     *     *     *     *     *
    .0005 .0008 .0011 .0012 .0011 .0008 .0005

    *     *     *     *     *     *     *     *
 .0011 .0018 .0027 .0034 .0034 .0027 .0018 .0011

   *     *     *     *     *     *     *     *     *
.0018 .0032 .0055 .0077 .0087 .0077 .0055 .0032 .0018

   *     *     *     *     *     *     *     *     *     *
.0023 .0044 .0084 .0133 .0168 .0168 .0133 .0084 .0044 .0023

   *     *     *     *     *     *     *     *     *     *     *
.0026 .0048 .0098 .0172 .0243 .0273 .0243 .0172 .0098 .0048 .0026

   *     *     *     *     *     *     *     *     *     *
.0052 .0090 .0169 .0266 .0335 .0335 .0266 .0169 .0090 .0052

   *     *     *     *     *     *     *     *     *
.0085 .0132 .0221 .0310 .0348 .0310 .0221 .0132 .0085

    *     *     *     *     *     *     *     *
 .0106 .0148 .0221 .0275 .0275 .0221 .0148 .0106

       *     *     *     *     *     *     *
    .0102 .0131 .0176 .0196 .0176 .0131 .0102

          *     *     *     *     *     *
       .0095 .0126 .0155 .0155 .0126 .0095
```

Diagram 6.3a

Target Position Distribution
After Twenty Time Intervals
With No Search

```
      *     *     *     *     *     *
   .0005 .0006 .0007 .0008 .0006 .0005

     *     *     *     *     *     *     *
  .0011 .0015 .0019 .0021 .0020 .0015 .0011

    *     *     *     *     *     *     *     *
 .0022 .0031 .0043 .0052 .0053 .0045 .0032 .0022

   *     *     *     *     *     *     *     *     *
.0035 .0053 .0077 .0097 .0107 .0101 .0079 .0052 .0033

  *     *     *     *     *     *     *     *     *     *
.0044 .0071 .0107 .0132 .0143 .0155 .0146 .0108 .0067 .0040

 *     *     *     *     *     *     *     *     *     *     *
.0053 .0080 .0123 .0143 .0117 .0101 .0174 .0172 .0121 .0073 .0047

  *     *     *     *     *     *     *     *     *     *
.0096 .0128 .0159 .0148 .0153 .0073 .0198 .0176 .0122 .0087

   *     *     *     *     *     *     *     *     *
.0144 .0166 .0178 .0182 .0199 .0000 .0204 .0163 .0133

    *     *     *     *     *     *     *     *
 .0176 .0183 .0188 .0189 .0201 .0198 .0177 .0164

     *     *     *     *     *     *     *
  .0179 .0181 .0189 .0190 .0186 .0171 .0166

      *     *     *     *     *     *
   .0188 .0216 .0237 .0232 .0205 .0175
```

Diagram 6.3b

Target Position Distribution
After Twenty Time Intervals
With Best Search Path

Information about target wherabouts gained by the search is however, dispersed relatively quickly due to target movement, as transition between nodes re-distributes the position distribution. An example of this can be seen in diagram 6.2b, where the position probability at the central node (which is the ninth search node of this strategy) has already risen from zero, following the ninth look, to 0.0148 just one time unit later. Comparison of the two sets of diagrams also clearly shows how the search destroys the symmetry of the target distribution.

6.2.2 DETECTION PROBABILITY IN RESTRICTED STRATEGIES

To illustrate the effect of changing the restrictions on planning horizon and searcher range, tables 6.10 to 6.14 show the detection probability for searches of 4, 5, 7, 10, and 20 looks repectively under the range of restrictions shown in table 6.1. It must be noted that the probabilities quoted differ in some cases from those shown in tables 6.2 to 6.7. This is because, where the number of looks is less than that shown in tables 6.2 to 6.7, the detection probability for the projected path up to that number of looks is given.

Table 6.10 Four Looks

Planning Horizon	Searcher Range 1	2	
1	.5315	.5547	Ranges 3 - 10 as Range 2
2	.5316	.5547	Ranges 3 - 10 as Range 2
3	.5316	.5547	Ranges 3 - 5 as Range 2
4	.5316	.5547	Range 3 as Range 2

Table 6.11 Five Looks

Planning Horizon	Searcher Range 1	2	
1	.5757	.5907	Ranges 3 - 10 as Range 2
2	.5778	.5907	Ranges 3 - 10 as Range 2
3	.5778	.5907	Ranges 3 - 4 as Range 2
4	.5778	.5937	
5	.5778	.5937	

Table 6.12 Seven Looks

Planning Horizon	Searcher Range 1	2	
1	.6382	.6385	Ranges 3 - 10 as Range 2
2	.6388	.6385	Ranges 3 - 10 as Range 2
3	.6388	.6385	Range 3 as Range 2
4	.6400	.6387	
5	.6400		
6	.6400		
7	.6400		

Table 6.13 Ten Looks

Planning Horizon	Searcher Range 1	2	
1	.6824	.6767	Ranges 3 - 10 as Range 2
2	.6828	.6775	Ranges 3 - 10 as Range 2
3	.6832	.6796	Range 3 as Range 2
4	.6852		
5	.6852		

Table 6.14 Twenty Looks

Planning Horizon	Searcher Range 1	2	3	4	5
1	.7530	.7438	.7475	.7496	.7504
2	.7553	.7468	.7503	.7526	
3	.7581				
4	.7602				

Considering firstly the range restriction. It can be seen from the tables that, with 10 or fewer looks, the detection probability changes when the searcher range is increased from 1 to 2, but that no further change is found when increasing the range beyond 2.

Inspection of the corresponding paths (shown in tables 6.2 to 6.7) shows that for ranges of 2 or more the initial look is taken at the central node, requiring a move of two units from the starting position. After this the strategy usually moves only one unit between looks, and never more than two even when the restriction is lifted. In the case of a small number of looks (4 or 5), this change in strategy results in an overall increase in detection probability for the search as a whole. However, in the case of a longer search (more than 7 looks), increasing searcher range leads to a decrease in overall detection probability.

The reason for this is that, when the range is restricted to 1 unit, the first look is taken at node 7, yielding a low detection probability for that look. When this is one of only a small number of looks it results in a lower detection probability than can be achieved if the searcher can move a greater distance. However, when taken as part of a longer search, the information gained at this early stage, before the target distribution has time to spread, leads to greater certainty about target position in later looks, outweighing the initial low detection probability. The dependence of strategy on search length is discussed further in section 6.5.

The searcher normally chooses to move only one unit distance at each look because taking longer time intervals between looks allows the target distribution time to spread, reducing the certainty of target position. An exception to this is seen in the case of 20 looks, where the searcher travels the maximum allowable distance between the later looks of the strategy. It can be seen from diagram 6.3b that, by this stage of the search, target position probability starts to accumulate at the lower edge of the search grid, due to the downward movement of the target and flattening of the distribution by earlier looks. Taking large time intervals between looks allows this accumulation to take place.

This behavior can be seen in many cases where target movement has a directional bias, and is particularly marked where target to searcher speed ratio is high. However this may not be considered to be a representative feature of the search, but an effect produced by the limitation of a finite search area.

The maximum change in detection probability with searcher range is an increase of about 4% found in the case of a strategy of 4 looks. However the behaviour of the decision process with respect to searcher range depends heavily on the case under consideration, as will be discussed in sections 6.3 and 6.5.

It can be seen from tables 6.10 to 6.14 that increasing the planning horizon results either in no change in strategy, or to a small increase in detection probability. The maximum increase is seen in the case of twenty looks where, with searcher range 1, increasing the planning horizon from 1 to 4 looks results in an increase in detection probability of just under 1%. It will be seen in the following section, that increasing the planning horizon may in some cases lead to a drop in detection probability, although as in this case, any change is normally small. The effect of changing planning horizon is discussed more fully in section 6.5.

6.3 VARIATIONS ON THE STANDARD CASE

The effect of using various values of the search parameters was investigated systematically by making changes to the standard case. The resulting strategies are discussed below, with particular regard to any unexpected behaviour, under the following headings:

a) Target direction.

b) Target and searcher speed.

c) Detection capability.

d) Initial conditions.

6.3.1 TARGET DIRECTION

Five representative direction distributions were investigated, as summarised in table 6.15, (the directions being as specified in diagram 2.5).

Table 6.15 Direction Distributions Investigated

Case	d(1)	d(2)	d(3)	d(4)	d(5)	d(6)
1	1/6	1/6	1/6	1/6	1/6	1/6
2	0	0	0.2	0.3	0.3	0.2
3	0	0	0	0.2	0.6	0.2
4	0	0	0	0	1	0
5	0	0.5	0	0	0.5	0

In all cases the resulting search paths closely reflect the direction distribution of the target, and as would be expected, the probability of detection in any given number of looks increases with increasing certainty about target direction.

In the uniform case the search path is concentrated mainly around the central seven nodes. This reflects the uncertainty in direction coupled with the high probability of the target returning to the central node, caused by the Markov transition probability, as discussed in Section 4.3.2. In the remaining cases the path strongly follows the movement of the target except for instances where the searcher 'wastes time' in order to increase detection probability at a later look.

An extreme case of time-wasting can for example, be seen in the second case listed in table 6.15. Here, at one point in the strategy having a planning horizon of two looks and searcher range ten, the path jumps from the lower part of the search area to a node at the top edge of the grid. This area has zero target position probability, but gives the maximum time interval before the next look, at a node at the bottom edge of the grid, allowing the target position probability to accumulate in this region. As previously discussed, this anomaly is a result of the finite search grid.

An interesting example illustrating the effect of the iter-relationship between planning horizon and searcher range on detection probability can be seen in the fourth case listed in table 6.15. Here target direction is known and, where searcher range is two (or less), the searcher follows the target systematically down the diagonal line of movement. The strategy in the case of range two, illustrated in diagram 6.4, gives certain detection in six looks. When searcher range is increased, but planning horizon is limited to one look, the path jumps nodes as it travels down the diagonal and then back-tracks allowing the possibility of target and searcher passing each other between looks. Diagram 6.5 illustrates the path generated with searcher range three, which gives a detection probability of a little over 0.9. When planning horizon is increased, the strategy returns to the path giving certain detection in six looks. This is another demonstration that increased searcher range can lead to lower detection probability when planning horizon is limited.

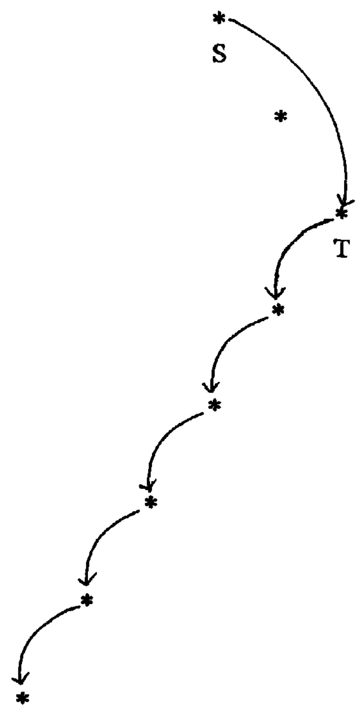


Diagram 6.4
Search Path With
Range 2.

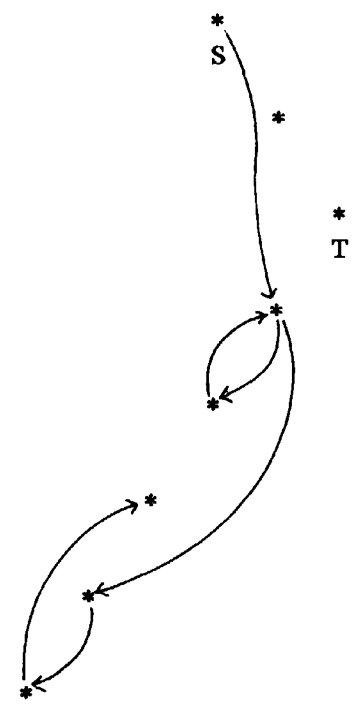


Diagram 6.5
Search Path With
Range 3.

Key: S = Searcher start node. T = Target start node.

6.3.2 TARGET AND SEARCHER SPEED

A number of cases were investigated to assess the effect of changing the target speed distribution, and the ratio of target to searcher speed. In general it was found that the search strategies and detection probabilities were in accordance with expected behaviour. When searcher speed is high in relation to normal target speed, detection probability is generally higher than in the standard case, and the path is concentrated more around the initial position of the target. This is because, when the searcher can travel faster there is a smaller time interval between looks, allowing the target distribution less time to spread. When searcher speed is low in relation to normal target speed the converse is true.

Three cases were considered in which the target speed distribution remained symmetric with mode 10, but changed in degree of certainty by setting $\lambda = 0.25$, 0.1, and 0.005, giving $\alpha = \beta = 1$, $\alpha = \beta = 3.1$ and $\alpha = \beta = 10$ respectively. These three distributions were combined with searcher speeds of $V_s = 21$, $V_s = 25$ and $V_s = 28$. (The case $\alpha = \beta = 3.1$, $V_s = 25$ is the standard case.) Table 6.16 summarises the maximum detection probability obtained in ten looks for these cases.

Table 6.16 Detection Probability in Ten Looks with Symmetric Target Speed Distributions

α, β	$V_s=21$	$V_s=25$	$V_s=28$
1.0	0.6411	0.7181	0.7576
3.1	0.6383	0.6852	0.7107
10.0	0.6258	0.6585	0.7253

It can be seen from the table that, for each distribution, the probability of detection increases as searcher speed increases, as expected. However, the table also shows that for any particular searcher speed, the detection probability normally tends to decrease as the target speed distribution becomes more certain (i.e. as α and β increase in value). This appears to be counter to expectation as increased knowledge of target behaviour should increase the probability of detection. Inspection of the early transition probabilities for these cases reveals that this apparent contradiction is a result of the discretisation of target motion coupled with the initial starting position of the searcher.

Examination of the detection probabilities for each look of the respective paths shows that the increased probability of detection in the case of the less certain distributions occurs only in the first looks of each strategy. The reason for this can be seen from table 6.17 which, taking the example where $V_s = 28$, shows the probability that the target moves 1 unit in each of the first two time intervals.

Table 6.17 Transition Probability in First Two Time Intervals for Searcher Speed 28.

t	$\alpha,\beta=1$	$\alpha,\beta=3.1$	$\alpha,\beta=10$
1	0.3000	0.1588	0.0326
2	0.3500	0.6100	0.8800

The table shows that the target has an increasingly smaller probability of moving in the first time interval, and an increasingly larger probability of moving in the second interval, as the value of α and β increases. Since the searcher is initially two units away from the target, this results in a lower detection probability in the first two looks in the case of a more definite speed distribution. This is due to a smaller probability of detecting the target after the first time interval, and a higher probability that the target distribution will have spread to the surrounding nodes after the second time interval.

Thus the effect results from the initial position of the searcher relative to the target. This has been verified by investigating these cases with the searcher initially one unit away from the target, at node 7. With this starting position the effect shown in table 6.16 is not present. This behaviour is a consequence of the discrete search space and is likely to be apparent in any model which discretises target movement in this way.

6.3.3 DETECTION CAPABILITY

The effect of changing both the range and capability of detection was investigated by considering the range of cases summarised in table 6.18.

Table 6.18 Cases of Non-Detection Probability Considered

Case	Range 0	1	2	3	4	5
1	0.3	1	1	1	1	1
2	0.3	0.5	1	1	1	1
3	0.3	0.5	0.7	1	1	1
4	0.3	0.5	0.7	0.9	1	1
5	0.5	0.7	0.9	1	1	1
6	0.7	0.9	1	1	1	1
7	0.9	1	1	1	1	1

The table shows values of non-detection probability for a target at various ranges. These values were chosen to give a systematically increasing range of detection (cases 1 to 4), and a systematically decreasing detection capability (cases 4 to 7).

In all cases the resulting detection probabilty and search strategy are much as expected. As the range of detection is increased, detection probability increases accordingly. However, this increase is dramatic when changing detection range from case 1 to case 2 of table 6.18. For example, the highest probability of detection in ten looks in case 1 is 0.5431, while the best strategy in case 2 gives a detection probability of 0.9424 in the same number of looks. The increase is gained mainly in the early looks of the search. This is because the greater range of detection enables the searcher to 'see' much of the area in which the target distribution lies during the first stages, before the distribution has time to spread.

As the range of detection is increased further, the increase in detection probability is less marked. This is partly because the target distribution is initially compact, so a wider detection range gives little improvement in the important early looks. Also the detection capability at wider ranges is lower (as might realistically be the case), so the overall increase in detection capability is less significant.

When the range of detection is wide, the search strategy is initially concentrated around the central node, with the searcher repeatedly returning to node 1, because much of the target distribution can be seen from this point. Later, as the target distribution spreads, the searcher moves around the lower part of the grid, consistently moving more than one unit at a time (where this is permitted) because of the ability to 'see' the nodes adjacent to each search node.

As the detection capability is reduced, detection probability in any given number of looks drops accordingly. The search path again tends to remain around the central node when detection capability is low, in this case because little information is gained from each look, so the target distribution does not flatten as quickly as in the standard case.

One interesting point that can be seen in case 1 of table 6.18 is that, as in many of the other cases considered, increasing planning horizon sometimes leads to a slight drop in overall detection probability. (This clearly only occurs when the higher planning horizon is still lower than the total search length.) Here, in a search of 10 looks with searcher range 1, an increase in planning horizon from 4 to 5 looks results in a drop in detection probability from

0.5493 to 0.5466. This is because the projected strategy for the first seven looks with planning horizon 4, gives a probability of detection of 0.4945. With planning horizon 5, a different strategy is chosen giving a marginally increased probability of 0.4946 in seven looks. However, the change in path results in a reduction in the maximum obtainable detection probability in the remaining looks, thus giving a drop in the overall value for 10 looks. Any such drop in detection probability is normally small, this being the largest change observed in the cases considered.

6.3.4 INITIAL CONDITIONS

The changes in initial conditions were made in the initial distribution of target position, and in the starting position of the searcher.

The effect of changing the initial distribution was investigated by gradually increasing the spread of the distribution, culminating in a uniform distribution over all 91 nodes of the search area. The resulting detection probabilities were again as expected; the greater the spread of the initial distribution, the lower the probabilty of detection in any given number of looks.

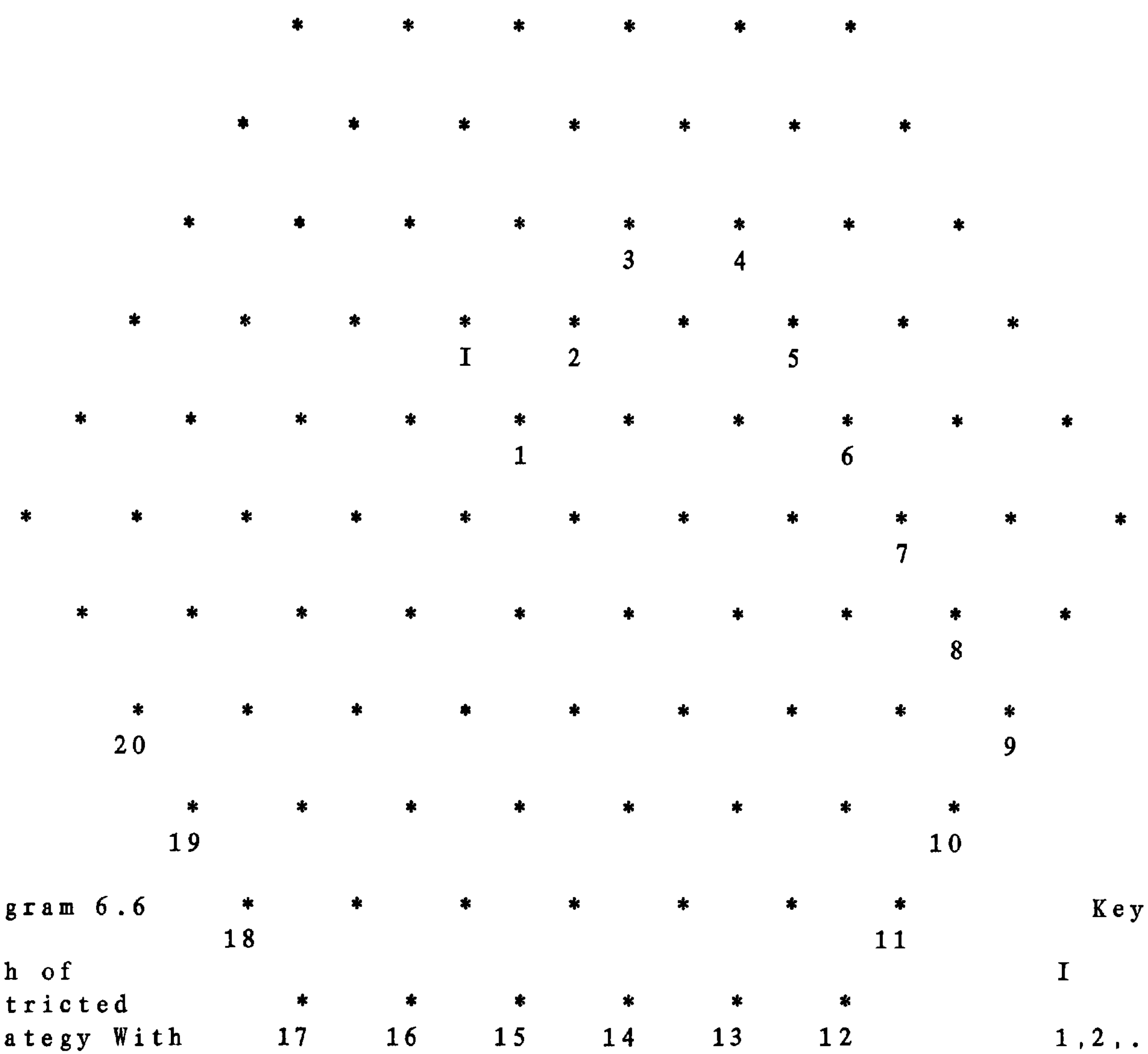


Diagram 6.6

Path of
Restricted
Strategy With
Uniform Target
Distribution.

Key

I Initial Node

1,2,... Position of Search Node 1,2,...etc.

Generally the search paths generated were also much as expected, with the pattern of looks being more spread around the search grid in the case of less certain initial distributions. However, in the case of a uniform distribution over the entire search area, the strategies exhibit some interesting features. Where searcher range is unlimited, the edge effects, due to target probability accumulating at the edge nodes of the grid, dominated the search strategy. This is because initially all nodes have equal probability of housing the target, but after the first time interval the downward target movement results in target probability immediatly beginning to accumulate at the lower edge of the grid. The searcher therefore moves directly to this area, and the entire search is conducted along the lower edge of the search area.

If however searcher range is restricted, a different behaviour emerges. The path of the strategy in the case with planning horizon 1 and searcher range 1 is illustrated in diagram 6.6. Here it can be seen that the searcher initially moves against the main direction of movement of the target. This is because, following the look at the first search node (labeled 1 in the diagram), the reduction in position probability due to this look influences position probability more at nodes below the search node than above. This is due to the prevailing direction of movement of the target as previously discussed. The searcher therefore keeps moving upward until the target probability in the upper region of the grid begins to drop (again due to the downward movement of the target). The path then turns downwards and again edge effects begin to dominate.

The effect of changing searcher starting position has already been discussed in relation to the target speed distribution. However a number of additional cases have been investigated, all giving results in accordance with expected behaviour. Generally, the further away the searcher is initially from the target distribution, the lower the detection probability due to the spreading of the position distribution prior to the first look.

6.4 OPTIMISING DETECTION PROBABILITY PER UNIT DISTANCE

The second optimisation criterion, introduced in section 3.3, was also investigated. The procedure was coded as a FORTRAN program, SEAR15, with input parameters the same as those described for SEAR14, with the exception of the minimum distance between search nodes. This is constrained to be one unit, for reasons discussed in section 3.3.

Again options allowing the restriction of planning horizon and maximum searcher range were included, and the criterion presented in section 3.3.1, giving a limit to the optimal total distance, was also implemented. Where a sub-optimal strategy is found by limiting planning horizon, this criterion is applied only within each projected strategy, not to the final strategy as a whole.

In many cases the results that would be obtained from SEAR15 could be predicted from the results generated by SEAR14, so only a limited number of cases were considered. The standard case was again investigated in detail. The detection probabilities in this case, with respect to planning horizon and searcher range, are shown in tables 6.19 and 6.20 for 5 and 10 looks respectively.

Table 6.19 Five Looks

Planning Horizon	Searcher Range 1	Searcher Range 2	
1	.5757	.5907	Ranges 3 - 10 as Range 2
2	.5778	.5778	Ranges 3 - 10 as Range 2
3	.5778	.5778	Ranges 3 - 4 as Range 2
4	.5778	.5778	
5	.5778	.5778	

Table 6.20 Ten Looks

Planning Horizon	Searcher Range 1	2	
1	.6824	.6767	Ranges 3 - 10 as Range 2
2	.6828	.6828	Ranges 3 - 10 as Range 2
3	.6832	.6832	Range 3 as Range 2
4	.6852		
5	.6852		

Here detection probability is given, rather than probability per unit distance, in order to aid comparison with tables 6.11 and 6.12. From this comparison it can be seen that, in most cases, results obtained when optimising with respect to distance are the same as those found when optimising detection probability with searcher range limited to one unit. Similar results can be seen in any given search length. (In all cases where detection probability is the same, the corresponding strategies are identical.)

The only exception to this is in the myopic case with maximum searcher range greater than one. Here the detection probability is the same as that when optimising probability of detection with the same planning horizon, and maximum searcher range two. When planning horizon is increased, the strategy reverts to that of the minimum path length.

The reason for this is that, if the first look is made at a node at minimum distance, the resulting detection probability is very low. When this is optimised over a planning horizon of one look, a higher detection probability per unit distance can be achieved by taking the first look at the central node, two units from the starting point. However, when optimised over more than one look, the increase in distance outweighs the increase in cumulative detection probability for two or more looks, so the minimum path length is chosen.

To illustrate the form of detection probability per unit distance that can be expected, table 6.21 shows the search strategy, detection probability per unit distance for each look, and detection probability per unit distance for the strategy as a whole, in one case. The standard case with planning horizon 3 looks and maximum searcher range 3 units was chosen so that a prolonged search with reasonable searcher range could be examined.

It can be seen from comparison with table 6.4 that the strategy is the same as that found when optimising detection probability with the same planning horizon and searcher range 1. Since the strategy has minimum path length (and hence only one distance unit between search nodes) the detection probabilities and probabilities per unit distance have the same values. Apart from the first two values, the detection probability per unit distance for the search as a whole, decreases with the length of the search. This is because of the decreasing return in detection probability discussed in section 3.3.

Table 6.21 Strategy Found by Optimising Probability of Detection Per Unit Distance

Planning Horizon: Three Looks
Maximum Searcher: Range One Unit

Look No.	Search Node	Prob. Per Unit Dist. for Look	Prob. per Unit Dist. for Path
1	7	0.0272	0.0272
2	1	0.3727	0.1864
3	5	0.4637	0.1546
4	4	0.5316	0.1329
5	3	0.5778	0.1156
6	1	0.6117	0.1019
7	6	0.6388	0.0913
8	16	0.6552	0.0819
9	5	0.6699	0.0744
10	14	0.6832	0.0683
11	13	0.6944	0.0631
12	12	0.7050	0.0587
13	3	0.7149	0.0550
14	1	0.7237	0.0517
15	6	0.7308	0.0487
16	16	0.7369	0.0461
17	15	0.7427	0.0437
18	30	0.7481	0.0416
19	29	0.7531	0.0396
20	13	0.7581	0.0379

In general the results obtained when optimising detection probability per unit distance show that, in the majority of cases, the optimal path is that with minimum path length, thus the strategies are the same as those when optimising with respect to detection probability with searcher range 1. In particular the time-wasting behaviour present in the previous case, and loss of detection probability due to search paths jumping as in the cases illustrated in diagram 6.5, is not seen.

6.5 CONCLUDING REMARKS

From the foregoing results, it appears that the search model normally produces search paths that are sensible in relation to the given search parameters. Also, predicted detection probabilities usually accord with the level of information about target location and movement, and with searcher detection capability. Two exceptions to this are the anomalies in detection probability in relation to the target speed distribution, discussed in section 6.3.3, and the time-wasting behaviour observed in several cases where target position probability accumulates at edge nodes. Strategies found under restrictions on planning horizon and searcher range appear, in general, to be reasonable approximations to an optimal path.

The change in detection probability observed when planning horizon is increased is normally small, suggesting that any increase obtained when full optimisation is performed, will not be major. Often any increase is gained by re-ordering the search nodes of a strategy found with a lower planning horizon rather than by major changes in path. This suggests that fully optimal paths will probably also not differ greatly from the sub-optimal cases (with the exception of instances where time-wasting is a feature).

Where optimisation is not possible, the results suggest that a planning horizon of two or three looks is sufficient to produce a reasonably good strategy. Raising planning horizon much above this level involves substantial computation time for all but the most restricted cases. Also, as illustrated by the example given in section 6.3.3, increased planning horizon may result in a drop, rather than an increase in detection probability.

The situation with regard to searcher range is less easy to generalise. Normally little increase in detection probability is found when extending searcher range beyond two units, as the loss in information due to target movement is normally a deterrent to large time intervals between looks. There are however some exceptions to this, which are:

a) where the searcher is initially some distance from the target distribution,

b) where the searcher has a significant range of detection,

c) where the searcher wastes time in order to allow target position probability to accumulate in one area.

In the first case it may be most economical in terms of computation time, to allow the searcher unrestricted range for the first look only. However, the effectiveness of this in terms of gain in detection probability depends heavily on the initial conditions and the total length of the search. If, in a prolonged search with limited planning horizon, a low probability of detection can be obtained at an intermediary node, then higher overall probability of detection is obtained when searcher range is restricted (as discussed in section 6.2.2 in the standard case). Alternatively, if there is zero detection probability at intervening nodes, (for example where target movement is away from the searcher), or the search is short, then increased range leads to increased detection probability.

In cases where time-wasting is evident, substantially higher detection probability can in extreme cases, be gained with unrestricted searcher range. However, the associated strategies often bear little relation to the parameters of the search. In these cases restricted searcher range produces strategies much more representative of target behaviour, and less influenced by the artificial limitation of a finite search area. For this reason it is felt that restricting searcher range is benefical in these cases, even if the associated detection probability is not optimal.

From the above discussion, it appears that normally little is lost in terms of detection probability, when planning horizon and searcher range are restricted, and that restricting searcher range can produce more sensible strategies in some cases where the limitations of the model cause unwelcome searcher behaviour. The results indicate that if complete optimisation could be carried out over a prolonged search, the resulting strategies may well be dominated by the edge effects of the finite grid, thus the inability to perform complete optimisation is not a serious drawback.

CHAPTER SEVEN

CONCLUSION

The aim of this work was to produce a mathematical model of a moving target search which would give a reasonable approximation to a physical search in two dimensional space, and provide an optimal search strategy that could readily be interpreted in a physical, rather than mathematical form.

The model presented in the preceding chapters does provide such a strategy which accords with intuitively expected behaviour in relation to target movement and searcher capability. Although optimisation can only be carried out in the simplest cases, acceptable sub-optimal solutions can be found by limiting the planning horizon of the decision process, and range of movement of the searcher. Imposing these limitations would allow the use of a much larger search grid than that considered here.

The maximum size of 91 nodes considered in the evaluation presented in Chapter 6, was chosen in order to investigate the optimisation process as fully as possible. (Initially a further consideration was the amount of storeage required for the transition matrices formulated in section 2.5, however the revised approach taken in section 5.2 does not present such problems.) However, strategies obtained under limitations on optimisation appear to be good approximations to optimal strategies, so full optimisation is not necessary.

When planning horizon and maximum searcher range are fixed, the increase in computation time required when the number of nodes is increased, depends primarily on the time required to update the target position distribution. Thus if planning horizon is limited to two looks, and searcher range to two or three units, a search area in the region of 1000 nodes would be feasible. (Parallel processing of the updating procedure could greatly increase this figure.) This increased search space would give a more acceptable approximation to a two dimensional search space.

Some problems associated with the model still remain. The approximation of continuous target movement over the discrete search space, although much improved by the revised model introduced in Chapter 5, is still not totally satisfactory. There are two aspects to this problem, firstly the use of a discrete search space with discrete instantaneous search can produce anomalies in detection probability as discussed in section 6.3.2, and secondly discretisation leads to a flattening of the target speed distribution, as illustrated in section 5.3.

The first of these problems might be reduced by relaxing the constraint of constant searcher speed. This possibility, discussed in section 2.5.1, would be feasible with the restrictions on optimisation discussed above. An alternative approach, relevant to both problems, might be to consider a search model with continuous transition rates between nodes, and non-instantaneous detection.

A further problem is the accumulation of target position probability at edge nodes of the grid, as described in section 6.3.4. The use of an extended search area, together with the restriction on searcher range, would miminise the effect of this accumulation on search strategy. Use of the moving grid system introduced in section 3.4 would also reduce this effect, and would be particularly appropriate in the case of target movement having a strong directional bias, where the effect is most marked. An alternative might be to model the edge nodes of the grid as absorbing states of the Markov process, the target being 'lost' on entering one of these nodes.

REFERENCES

Black W.L.	1965	Discrete Sequential Search Information and Control Vol.8, p159-162
Brown S.S.	1980	Optimal Search for a Moving Target in Discrete Time and Space Operations Res. Vol.28, p1275-1289
Discenza J.H. and Stone L.D.	1981	Optimal Survivor Search with Multiple States Operations Res. Vol.29, p309-323
Dobbie J.M.	1973	Some Search Problems with False Contacts Operations Res. Vol.21, p907-925
Dobbie J.M.	1974	A Two-Cell Model of Search for a Moving Target Operations Res. Vol.22, p79-92
Dobbie J.M.	1975	Search for an Avoiding Target SIAM Jnl. Applied Mathematics Vol.28, p72-86
Eagle J.N.	1984	The Optimal Search for a Moving Target When the Search Path is Constrained Operations Res. Vol.32, p1107-1115
Grasty R.L.	1980	The Search for COSMOS-954 Search Theory and Applications p211-220
Haley K.B. and Stone L.D. (editors)	1980	Search Theory and Applications Plenum Press
Hellman O.	1972	On The Optimal Search for a Randomly Moving Target SIAM Jnl.Applied Mathematics Vol.22, p545-552
Hypher R.P.	1980	Search and Rescue in Canada Search Theory and Applications p73-85
Kadane J.B.	1980	Industrial Applications of Search Theory and Applications p205-210

Kan Y.C. 1977 Optimal Search of a Moving Target
Operations Res. Vol.25, p864-870

Kolesar P. 1980 An Application of Search Theory in Medicine: The Detection of Glaucoma
Search Theory and Applications
p199-204

Koopman B.O. 1946 Search and Screening
Office of Chief of Naval Operations
Washington,D.C.
O.E.G. Report No.56.

Koopman B.O. 1956a The Theory of Search
I. Kinematic Bases
Operations Res. Vol.4, p324-346

Koopman B.O. 1956b The Theory of Search
II. Target Detection
Operations Res. Vol.4, p503-531

Koopman B.O. 1957 The Theory of Search
III. The Optimum Distribution of Searching Effort
Operations Res. Vol.5, p613-626

Koopman B.O. 1980 Search and Screening
Pergamon Press

Larnder H. 1984 The Origin of Operational Research
Operations Res.
Vol.32, No.2, p465-475

Lössner U. and Wegener I. 1982 Discrete Sequential Search with Positive Switch Cost
Math.of Operations Res.
Vol.7, p426-440

Mangel M. 1982 Probability of Success in the Search for a Moving Target
Operations Res. Vol.30, p216-222

Mattson R.J. 1980 Overland Search for Missing Aircraft and Persons
Search Theory and Applications
p87-92

Morse P.M. 1986 The Beginings of Operations Research in The United States
Operations Res. Vol.34, p10-17

Pollock S.M.	1960	Optimal Sequential Strategies for Two Region Search when Effort is Quantized Op. Res. Center, MIT, Cambridge, Mass. Interim Tech. Rept. No. 14, (AD 238 662)
Pollock S.M.	1970	A Simple Model of Search for a Moving Target Operations Res. Vol.18, p883-903
Pollock S.M.	1980	Surveillance and Search Theory Search Theory and Applications p113-126
Pursiheimo U.	1978	On the Optimal Search for a Moving Target in Discrete Space Proceedings 8th IFIP Conference on Optimisation Techniques, p177-185
Richardson H.R. and Belkin B.	1972	Optimal Search with Uncertain Sweep Width Operations Res. Vol.20, p764-784
Richardson H.R. and Discenza J.H.	1980	The United States Coast Guard Computer Assisted Search Planning System (CASP) Naval Research Logistics Quarterly Vol.27, p659-680
Richardson H.R. and Stone L.D.	1971	Operations Analysis During the Underwater Search for Scorpion Naval Research Logistics Quarterly Vol.18, p141-157
Saretsalo L.	1973	On the Optimal Search for a Target Whose Motion is a Markov Process Jnl.Applied Probability Vol.10, p847-856
Stone L.D., Stanshine J.A. and Persinger C.A.	1972	Optimal Search in the Presence of Poisson-Distributed False Targets SIAM Jnl. Applied Mathematics Vol.23, p6-27
Stone L.D. and Richardson H.R.	1974	Search for Targets with Conditionally Deterministic Motion SIAM Jnl.Applied Probability Vol.27, p239-255
Stone L.D.	1975	Theory of Optimal Search Academic Press, New York

Stone L.D.	1979	Necessary and Sufficient Conditions for Optimal Search Plans for Moving Targets Math. of Operations Res. Vol.4, p431-440
Strumpfer J.	1980	Search Theory Index Institute for Maritime Technology Simonstown, South Africa Technical Note TN-017-80
Surkan A.J.	1975	Sequential Search by Computer in Exploration Notes of American Ass. of Petroleum Geologists Res. Symp. Stanford Univ
Sutcliffe P.M.	1985	Personal communication M.O.D. D.A.O.E. Weybridge, Surrey
Washburn A.R.	1980	On a Search for a Moving Target Naval Research Logistics Quarterly Vol.27, p315-322
Washburn A.R.	1983	Search for a Moving Target: The FAB Algorithm Operations Res. Vol.31, p739-751
Wegener I.	1980	The Discrete Sequential Search Problem with Nonrandom Cost and Overlook Probabilities Math.of Operations Res. Vol.5, p373-380
Wegener I.	1982	The Discrete Search Problem and the Construction of Optimal Allocations Naval Research Logistics Quarterly Vol.29, p203-212

APPENDIX

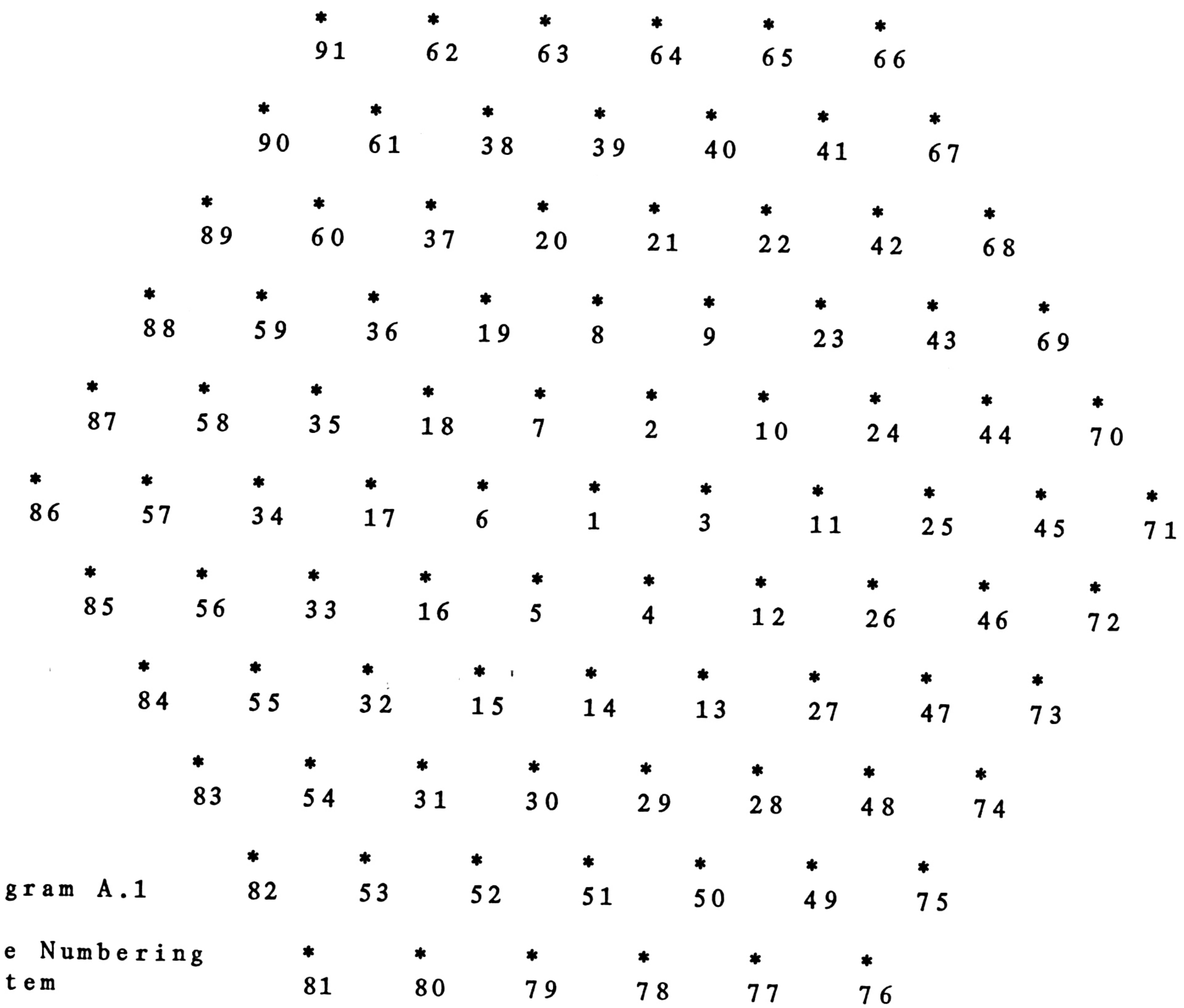


Diagram A.1

Node Numbering System