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**THE COGNITIVE AND AFFECTIVE DIMENSIONS OF
MATHEMATICAL DIFFICULTIES IN
SCHOOLCHILDREN**

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R11

Certificate of Research

This is to certify that, except where specific reference is made, the work described in this thesis is the result of the candidate's research. Neither this thesis, nor any part of it, has been presented, or is currently submitted, in candidature for any degree at any other University.

Signed *Morena Lebens*.....
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Date *10-9-08*.....

Abstract

Previous research (eg Barrouillet & Lepine, 2005; Cummings and Elkins, 1999) suggests that children with mathematical difficulties (MD) use inefficient problem-solving strategies and lack computational fluency. This thesis extends existing research by investigating the cognitive and affective dimensions of mathematical difficulties in schoolchildren using a variety of methodological approaches. The principal aim is to identify the characteristics of children with MD and compare them with children of average ability, to identify the factors relevant to the learning of mathematics.

In the existing literature (eg Baptist et. al., 2007), mathematical difficulties are primarily defined in terms of a cognitive deficit. However, learning may not only be a function of cognitive processes, and affective responses such as anxiety or attitude may play an important role in the learning of mathematics (McLeod, 1994). To investigate the relative salience of these factors, an instrument to measure affective responses towards mathematics in schoolchildren was developed. This instrument was then utilised to investigate differences in maths anxiety, self efficacy, perceptions of the learning environment and attitudes towards the teacher between children with MD and children of average mathematical ability. In order to investigate the cognitive aspects of MD, dual task experiments were used to examine the role of subvocal rehearsal in arithmetic problem solving. Mathematical problems were coupled with either a phonological or a neutral secondary task to find out how children with MD and average ability children use phonological working memory resources in arithmetic.

The cognitive aspects of MD for the learning process were addressed by evaluating two different types of mathematics instruction. A protocol analysis illustrated how the format of instruction affected children's use of arithmetic problem solving strategies and how strategy usage was influenced by individual differences in information processing. Overall, the results suggest that the cognitive processing deficits of children with MD seem to result from inefficient problem-solving strategy usage which occupies cognitive resources, however, problem solving was improved via a direct instruction intervention which provided worked-out examples and model strategies. A follow-up analysis found that the interventions for children with MD would need to go beyond the learning of specific abilities in order to produce sustainable long-term effects on school achievement.

CHAPTER 1

STATEMENT OF THE PROBLEM

CHAPTER 1: STATEMENT OF THE PROBLEM**1.1 BACKGROUND TO THE STUDY**

Basic mathematical abilities such as arithmetic are a pervasive requirement of everyday life and provide the essential foundation for dealing with various problem-solving situations. Basic mathematical abilities also constitute the essential means for dealing with more advanced skills that are central to many scientific disciplines and the majority of modern occupations (Geary, 1991). Thus, knowledge of basic mathematics is important for full occupational, social and economic participation in modern societies. However, international educational assessments from different countries provide compelling evidence that many pupils fail to acquire the mathematical abilities that are considered average for their age band and face difficulties in learning and problem - solving, despite having no identified learning disability (Unicef, 2002, Cumming, 2000, Cossey 1999, Dietz, 1998). Thus, difficulties in mathematics are not confined to children with learning disabilities such as dyscalculia, but are also common in mainstream education. In order to address the problem with learning and problem - solving, this thesis examines the affective and cognitive indicators of mathematical difficulties.

The domain of learning mathematics include a range of different aspects, such as algebra, geometry and statistics; however, much of the research on mathematical difficulties has focussed on basic mathematical abilities, such as arithmetic problem solving (Dowker, 2005), and this has contributed to the development of a systematic framework in this research area. This research focus is important, because arithmetic problem solving is one of the building blocks of mathematical ability and constitutes an essential foundation for learning more complex mathematical concepts.

This thesis aims to extend the previous research on learning arithmetic and arithmetic problem-solving in children with mathematical difficulties. It is argued that an understanding of the characteristics of children with mathematical difficulties is essential in order to provide a framework for identifying and ameliorating mathematical difficulties. The following paragraphs give a working definition of mathematical difficulties and establish the difference between mathematical difficulties and mathematical learning disability.

1.1.1 DIFFICULTIES IN LEARNING MATHEMATICS

According to Gersten, Jordan and Flojo (2005), children with difficulties in learning mathematics are those who perform either in the low/average or well below average range. More specifically, Passolungi and Siegel (2001) defined children as having mathematical difficulties (MD), if they fall below the 30th percentile of the overall mathematical performance band. Similarly, McLean and Hitch (1999) identified children with MD as those who scored below the 25th percentile in a standardized test. The definition of Gersten et al. (2005) has been adopted for the purpose of selecting the sample for this thesis. Based on the ability tracking system in Germany, it is possible to identify children with MD on the basis of the school track they attend.

As will be discussed in this thesis, the previous literature suggests that a number of different factors contribute to mathematical difficulties, including poor cognitive strategies (Kirschner et.al. 2006, Beishuizen, Van Putten, & Van Mulken 1997, Carnine, Jones & Dixon, 1994; Beishuizen, 1993, Vergnaud, 1982), maths-related anxiety, negative attitudes towards mathematics and instructional methods (Mayer, 2004, Baxter, Woodward & Olson, 2001, Bottge, 2001, Tuovinen & Sweller, 1999). However,

the attributes that characterise MD can be demarcated from mathematical disabilities, as illustrated below.

1.1.2 DIFFERENCES BETWEEN MATHEMATICAL DIFFICULTIES AND MATHEMATICAL DISABILITY

While low achievement is common to children with maths disabilities as well as those with maths difficulties, the concepts can be clearly demarcated from each other. Baptist, Minnie, Buksner, Kaye and Morgan (2007) summarised the similarities and differences between mathematical difficulties and mathematical disabilities, as illustrated in the table below.

Indicators of Mathematical difficulties	Indicators of Mathematical disabilities
<ul style="list-style-type: none"> ○ Working memory deficits – not specific to numeric processing ○ Procedural deficits: Immature counting strategies, more effortful and error-prone ○ Response to intervention – children with MD respond well to instructional interventions ○ No deficits in number representation but difficulties in computing/ retrieving basic facts from long-term memory 	<ul style="list-style-type: none"> ○ Working memory deficits – specific impairment of numeric processing ○ Conceptual and procedural deficits ○ Children do not respond well to instructional interventions ○ Deficits in number representation

Table 1.1: Indicators of mathematical difficulties

Table 1.1 shows that children with mathematical difficulties and those with mathematical disability differ along several dimensions. Most importantly, mathematics disabilities such as dyscalculia involve a “core deficit”, where the ability to represent numbers is impaired and the processing of numeric information is hampered (Chiappe, 2005). Children with mathematical difficulties, on the other hand, are able to process and manipulate numbers. They “have conceptual knowledge equivalent to that of normal students but do not develop strategies other than counting for the basic facts and

perform poorly on these tasks” (Cummings & Elkins, 1999, p. 152). This suggests that children with MD have no conceptual deficits, but rely on effortful and ineffective procedural strategies and have problems in retrieving procedural and factual knowledge from long-term memory. Further, Minnie et al. (2007) assume that children with dyscalculia have working memory deficits that are specific to numeric processes, whereas children with MD have a generalised deficit in working memory function. This has important consequences for the selection of participants for the present research.

Another important difference between the two groups outlined above concerns the contribution of innate cognitive deficits: while mathematical disabilities have a strong genetic component, mathematical difficulty refers to low achievement, which can be attributed to several non-genetic causes (Minnie et al., 2007). Indeed, another key difference between children with mathematical disability and children with mathematical difficulty is that the latter respond very well to educational interventions that provide individualised instruction. The classification framework for identifying children with MD provided by Minnie et al. (2007) can be criticised, because it is focussed on the cognitive markers of mathematical difficulties and does not account for the affective characteristics of children with MD. However, McLeod (1994) emphasizes that the learning of mathematics is not a purely cognitive endeavour, and that affective factors are central to the process of learning mathematics.

1.2 AIMS AND OBJECTIVES

As will be outlined below, several studies have investigated the characteristics of children with mathematical difficulties and there is evidence that cognitive as well as affective variables influence children’s mathematical abilities. Overall, the research so far has provided valuable insights into the nature of MD and provides the theoretical

and empirical foundation for the research in this thesis. It can be criticised that the level of analysis is limited in scope because a complex problem such as MD has to be investigated from different perspectives. However, the previous research within this area has tended to analyse particular aspects of mathematical difficulties in isolation. Based on the existing research, it is difficult to determine how the different aspects link together and to date there is no comprehensive framework for researching mathematical difficulties.

Therefore, the aim of this thesis is to extend the previous research by using different methodologies in order to investigate the various dimensions of mathematical difficulties from a range of perspectives, and to integrate the findings into a framework that can be applied by researchers and educators. Learning and problem-solving processes in children with MD will be examined in the thesis. The thesis can be separated in three main parts, as illustrated below.

Affective dimensions of mathematical difficulties

A key contribution of this thesis is an examination of the affective dimensions of mathematical difficulties, which have received little research coverage so far. In particular, little attention has been paid to children's attitudes towards the social environment in which the learning of mathematics occurs and the interpersonal relationships within this environment. The majority of attitude scales are designed to measure mathematical anxiety or self-efficacy, whereas attitudes towards the teacher or perceptions of the classroom environment reside at the periphery of research. Therefore, the first chapter deals with the psychometric development of an instrument to measure affective responses in schoolchildren. A scale to analyse affective responses towards

mathematics in children with MD is essential to identify negative patterns that might impede mathematical performance and to provide appropriate educational interventions. This instrument will be implemented to investigate differences between children of average mathematical ability and a control group. The comparison between children with MD and average ability children adds to an understanding of the relationship between cognitive and emotional factors in the learning of mathematics.

Cognitive dimensions of mathematical difficulties

Deficits in the area of arithmetic problem-solving processes have been described as typical characteristics of MD. According to Geary, Hoard and Hamson (1999, p. 223), a key characteristic of children with MD is the “error-prone use of developmentally immature problem-solving procedures.” It is argued that immature problem-solving algorithms go in line with an occupation of cognitive resources so that even simple arithmetic problems require extensive cognitive efforts (Tremblay & Lemoyne, 1986). Similarly, Barrouillet and Lepine (2005, p. 191) claim that children with MD “encounter a double difficulty when solving arithmetic problems. They retrieve the answers less frequently than do other children and, as a consequence, must rely more often on backup strategies – and they are also slower to perform those...the frequent recourse to slow arithmetic strategies increases the cognitive load involved in arithmetic problem solving in those children, who, in other respects have the lowest cognitive resources to cope with this extra cognitive load.”

Overall, the literature indicates that the usage of cognitive resources in is crucial for successful arithmetic problem-solving processes and children with MD seem to have deficits in this area. Previous research has used the working memory model to

conceptualise the cognitive architecture. It is assumed that working memory consists of a verbal and a visual processing device and a central executive for supervising and controlling cognitive processes. Thus, this concept is a prominent framework to study how visual and verbal resources are involved in arithmetic. However, the overall body of evidence regarding the cognitive mechanisms underlying arithmetic problem-solving in children with MD is not consistent. The majority of research on arithmetic problem-solving has been conducted with adults, mostly skilled university students, and it could be argued that these findings are of limited applicability in the context of MD in schoolchildren. A cognitive perspective will be employed to analyse the usage of working memory resources for arithmetic problem - solving in children with MD and average ability children. These findings can be used to derive principles for the type of instruction that is most appropriate for addressing particular mathematical difficulties.

Intervention and Evaluation

The previous sections outlined the fact that children with MD differ from children of normal ability in several cognitive and affective aspects. Thus, it could be argued that these children require a type of instruction that is different from the mainstream curriculum. The number of children with MD in many countries has heightened the need for instructional interventions to ameliorate MD. However, the present state of the literature offers contradictory findings about the effects of the type of instruction on the learning process in children with MD and only a few studies have attempted to trace the long-term effects of the direct instruction intervention. Therefore, this thesis adopts a diagnostic perspective and utilises different methods to evaluate a direct instruction intervention compared to minimally guided instruction for children with MD on a short-term and long-term basis.

It has been argued that direct instruction is beneficial for children with MD because it provides clearly structured and sequenced instruction and uses worked examples and modelled strategies. However, the possibility that direct instruction actually helps children with MD to adopt more efficient problem-solving strategies has not been examined adequately by previous research, although it is pertinent to investigate which specific aspects of an instructional intervention are beneficial for children with MD. Also, a major limitation of the existing literature on problem-solving strategy usage in children with MD is that there has been little discussion of the role of individual differences. There have been no controlled studies on the association between individual differences in representing and processing information and problem-solving. Therefore, the thesis aims to contribute to the literature by investigating how the format of instruction affects children's arithmetic problem-solving strategies, and to assess the effects of individual differences in cognitive processing through an analysis of children's verbal problem-solving protocols. The evaluation of different instructional approaches helps to determine the conditions under which the learning of mathematics can be facilitated in children with MD and this yields practical implications for educational practice and curriculum design.

Conclusion

To conclude, it has been argued that MD goes beyond low achievement in mathematics and includes several facets that have to be addressed by different perspectives and methodologies. The dimensions of MD range from endogenous, intra-individual factors such as working memory usage and anxiety to more exogenous factors such as instructional method and the learning environment. A coherent research framework which attempts to draw conceptual links between these factors has not yet

emerged, possibly due to the lack of dialogue between the different strands of research in this area. This assertion points to a necessity to bring cognitive and affective perspectives into a closer dialogue in research on children with MD.

CHAPTER 2

DEVELOPMENT OF THE AFFECTIVE RESPONSES TOWARDS MATHEMATICS SCALE

2.1 OVERVIEW

Affective responses are the least investigated aspect of human problem solving, yet it is probably the aspect most often mentioned as deserving further investigation. (Mandler, 1989, p. 3)

According to the definition of Minnie et. al. (2007), mathematical difficulties are defined in terms of cognitive factors; for example, the use of ineffective problem solving strategies, a lack of computational fluency and problems retrieving factual knowledge from long term memory. However, it is vital to point out that the learning of mathematics is not a purely cognitive endeavour. McLeod, (1994) emphasizes that affective factors are central to the process of learning mathematics. Similarly, Mandler (1989) highlights the urge to account for the role of affective factors when exploring cognitive processes such as the learning of mathematics. Therefore, the following two chapters are concerned with exploring the affective factors in the learning of mathematics in children with and without mathematical difficulties (MD). The present chapter illustrates and discusses the development and validation of the Affective Responses Towards Mathematics Scale (ARTMS). The subsequent chapter is concerned with the application of this instrument to measure affective responses towards mathematics in children with MD and average ability children from year five and year eight.

2.2 INTRODUCTION

The present section is concerned with the development of an instrument to measure children's affective responses towards mathematics. According to Hart Reyes (1984, p.558), the affective domain in the field of mathematics learning refers to "children's feeling about mathematics, aspects of the classroom such as teacher – student relationships or their perception of themselves as learners of mathematics." This definition seems to be useful, in that it includes cognitive aspects (perceptions) and emotional aspects (feelings). Moreover, the definition makes clear that learning is subject to influences from the children's environment such as the classroom and the interactions within this environment.

While the importance of affective factors for the learning of mathematics is an established tenet in the literature, the definition of concepts within the affective domain is less clear cut. A review of the literature indicates that research on affective responses is characterised by definitional ambiguity. Pajares (1992) criticises that terms such as beliefs, attitudes, perceptions and values are used interchangeably and lack a clear definition. According to Gopnik and Meltzoff (1997, p.13), perceptions can be understood as the equivalent to beliefs in that "perception and belief share many features...most significant, both perception and belief may be subject to misrepresentation with perceptual illusions." This definition implies that perceptions and beliefs are synonymous and can be clearly distinguished from attitudes. Indeed, Ajzen and Fishbein (1975) argue that, whereas attitudes refer to a person's favourable or unfavourable evaluation of an object, beliefs represent the information a person has about the object. Thus, it can be argued that affective responses consist of three concepts: 1.) beliefs/ perceptions 2.) attitudes 3.) anxiety.

The review of previous research indicated that the learning of mathematics in school is related to a variety of affective variables such as maths anxiety (Ashcraft & Kirk, 2001; Ma, 2004; Kellogg, Hopko, & Ashcraft, 1999), perceptions of the maths teacher, (Goh and Fraser, 1998; Wentzel, 1998), children's self-efficacy beliefs (Bandura, 1997; Marsh and Craven, 1997) and their perception of the classroom environment in maths lessons (Kellam et. al.,1998, Patrick, Kaplan and Ryan, 2007). However, there is no instrument to date which covers all of the affective domains listed above and which is suitable for children with respect to the number of items. In addition, a possible limitation of the existing scales is that they almost exclusively focussed on measuring mathematics anxiety (Wigfield & Meece, 1988) and self-efficacy and therefore these affective variables presumably received the most exhaustive research coverage, while other factors reside at the periphery of research.

A notable exception is the multidimensional scale developed by Fenemma and Sherman in 1976, which is one of the most frequently adopted scales to date. It consists of nine different subscales and the subscales were supposed to measure diverse aspects of mathematics such as attitude toward success in mathematics, the extent to which mathematics is perceived as a male domain (by children themselves as well by their parents), teacher attitudes, confidence in learning mathematics, mathematics anxiety, motivation for doing mathematics and the perceived usefulness of mathematics. Thus, the advantage of this scale over previous, predominantly unidimensional scales, is that it goes beyond a measurement of anxiety and self-efficacy. However, it could be tentatively argued that the Fenemma Sherman inventory is not free from methodological difficulties, given that subsequent studies did not confirm these nine subscales. Melancon, Thompson, and Becnel (1994) proposed only eight factors and the factor structure underlying the variables differed from the model assumed by Fennema and

Sherman. Similarly, Mulhern and Rae (1998) isolated only six factors and were unable to confirm the Fennema and Sherman model. Furthermore, the psychometric properties such as reliability and validity have been questioned (Suinn and Edwards, 1982). Another potential limitation of the Fenemma Sherman inventory is, that the scales consisted of 108 items and took children nearly 50 minutes to complete. It can be argued that this might have discouraged children from completing the scales to as high a standard as possible. A shorter inventory might therefore be more suitable for children.

A summary of the available literature on the development of psychometric tests to measure affective responses towards mathematics suggests that existing scales can be criticised for two main reasons: 1.) an overemphasis on factors such as anxiety and self-efficacy at the expense of other aspects of learning mathematics such as perceived conduct problems and the teacher–student relationship, 2.) methodological shortcomings resulting in inadequate psychometric properties. The present work attempts to address this issue through the development and implementation of a scale to measure affective responses of schoolchildren towards different aspects of learning mathematics.

2.3 VALIDATION OF THE INSTRUMENT

A preliminary pool of 45 items was included in the initial scale to capture children’s affective responses towards different aspects of learning mathematics such as maths anxiety, self–efficacy beliefs, perceptions of the maths teacher and children’s perceptions of the maths classroom environment. The items were developed on the basis of the existing literature. Overall, the review of the literature indicated that measures of affective responses towards maths need to go beyond the subject of mathematics but have to account for the social context in which the learning of mathematics takes place;

this is reflected in the item development for the purpose of the present scale. A detailed literature review for each of the affective variables will be provided in the second part of this chapter. The items were designed as a Likert-scale: 1 strongly disagree, 2 disagree, 3 neutral, 4 agree, and 5 strongly agree. Items which expressed negative responses were reverse scored. Thus, a mean score of less than 3 indicated a negative view of maths, while a mean score higher than 3 indicated a more positive view of maths. Because the sample consisted of German-speaking children, items were formulated in German.

2.3.1 PARTICIPANTS AND PROCEDURE

The sample ($n = 143$) consisted of a total of 77 boys and 66 girls from class five (age 10 – 11) and class eight (age 13 – 14). The mean age was 12.6 years. Children were classified as having mathematical difficulties if they attended a general secondary school as opposed to a comprehensive school. This decision is based on the ability tracking system in Germany, where schoolchildren are tracked on different secondary school types depending on their achievement in primary school. Primary school children with low achievement scores in core subjects such as mathematics are referred to general secondary schools, whereas children with average and above average grades are able to attend comprehensive school. 52 children were defined as children of average ability in terms of their performance, since they attended a general secondary school which requires children to perform at least on an average level. The sampling procedure is described in more detail on page 51 of this chapter. A total of 91 children were classified as children with MD, since they were tracked on a lower secondary school which indicates lower ability in terms of mathematical performance. The affective response scale was administered in a classroom, in groups of ten children. The

children were briefly introduced to the rationale of the survey. It was emphasized that their data would be handled confidentially and would not be shown to their mathematics teacher or anyone else. On average, the questionnaire took 30 – 35 minutes to complete.

2.3.2 DISCRIMINATORY ITEM ANALYSIS

All 45 items were subjected to an analysis in order to examine whether each item discriminates between those with positive attitudes and those with negative attitudes, rather than producing homogenous results across participants. Kline (1999) recommends that the scores of the top and lowest scoring quartiles should be used for the discrimination analysis. In order to do so, the scores for each respondent for each item of the scale were totalled to determine the top and the lowest performing quartile of children. The scores of the lowest performing quartile were subtracted from the scores of the top performing quartile. The difference was then divided by the number of participants in the quartile, which was 35. This calculation led to a discrimination index for each item. In order to increase the discriminative power of the scale, Kline (1999) recommends items below a threshold of 0.2 should be rejected because they do not discriminate between high and low respondents. After the discriminative item analysis, 37 items remained in the scale. The excluded items are listed in bold italics in table 2.1 below.

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Table 2.1: Discrimination index of the initial pool of items

Item	Discrimination Index
1. My grades will increase if I pay better attention in maths	.06
2. Maths is a subject I enjoy	.63
3. The teacher often doesn't notice if I put up my hand	.54
4. I am afraid to come up with a wrong answer	.83
5. The maths lesson is often disrupted by children	1.17
6. My teacher treats all children equally	.80
7. I am too afraid to ask if I don't understand	1.20
8. The more I learn for maths the better I get	.89
9. Only gifted students are able to succeed in maths	.89
10. I could achieve better maths grades, if I put in more effort	.66
11. My teacher chooses some children over others to answer questions	.83
12. I am afraid of maths examinations	1.03
13. Time flies in maths lessons	-.14
14. I like collaborative work in maths	.51
15. I enjoy the maths lessons with my teacher	.63
16. I am afraid to be picked in maths lessons	1.09
17. Our teacher explains a task until I have understood it	.69
18. I feel embarrassed when asked to come to the chalkboard	1.14
19. My teacher explains maths well	.71
20. My teacher often doesn't notice if I put my hand	.57
21. I prefer working alone in maths lessons	.80
22. My class fools around during maths lessons	.94
23. My teacher makes sure I really understand maths	.43

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24. It takes a long time until my teacher can really start teaching	.91
25. My teacher can't explain a new topic well	.57
26. My teacher asks if we understood a new topic	.94
27. I wonder what the others think if I come up with a wrong answer	.83
28. If my teacher explains something, it is difficult to understand	.97
29. I look forward to maths lessons	.80
30. When doing mathematics, I forget everything else around me	.17
31. It is quiet in our maths lessons	.94
32. Talent for maths is inborn	.63
33. I like groupwork in maths	.80
34. I am particularly nervous before maths lessons.	.17
35. Everyone who learns seriously can succeed in maths	.69
36. The maths lessons run without disturbances	.80
37. If you're not gifted, maths is very difficult	1.06
38. If I learn enough, I will have good grades in maths	.97
39. I will never be good at maths, no matter how much effort I put in it	1.00
40. If I learn enough, I will have good grades in maths	.11
41. I simply have no talent for maths and exercising doesn't help	1.49
42. I feel good if I am doing maths	.14
43. It is difficult for me to sit down and learn maths	.89
44. In maths lessons I am so nervous that I often don't understand anything	.23
45. Maths doesn't interest me	.09

2.3.3 MEASURES OF INTERNAL CONSISTENCY

The 37 items which remained after the discriminative item analysis were subjected to measures of internal consistency. In order to examine the internal consistency of the entire scale, Cronbach's alpha reliability coefficient was determined through a pairwise correlation test. Kline (1999) recommends 0.8 as an acceptability criterion for the reliability coefficient, whereas the British Psychological Society argues that a reliability coefficient of 0.7 is sufficient. Cronbach's alpha measurement of the overall scale resulted in a correlation coefficient of 0.88, which suggests consistency of the scale, despite the variety of items. However, it could be argued that Cronbach's alpha is too high. In order to confirm the internal consistency which has been suggested by the result of Cronbach's alpha coefficient, a split-half test was carried out to determine the strength of the relationship between two randomly divided parts of the scale (odd and even item numbers). The split half test resulted in a Spearman-Brown correlation coefficient of 0.83, which suggests that the instrument meets the criterion of internal consistency.

2.3.4 ITEM ANALYSIS

Another indicator of internal consistency is an analysis of the item-total correlation (Loewenthal, 2001). Kline (1986) proposes that the minimum criterion should be an item-total correlation of 0.2. An examination of the item-total correlation showed that the majority of items met this criterion, as laid out in table 2.2 below. While most items were beyond the 0.2 threshold, inter-item correlations are very moderate, if not low.

However, Cattell and Kline (1977) caution that a high degree of inter-item correlation might actually violate the validity of the instrument, because a high inter-item correlation means that the instrument is very narrow in its scope of measurement. According to Cattell and Kline (1977), internal consistency should be derived from high item-total correlations rather than high inter-item correlations. Despite the fact that this argument is theoretically valid, it is difficult to achieve satisfying item-total correlations without having inter-item correlation. Overall, the internal consistency of the present scale, as indicated by Cronbach's alpha reliability coefficient and the item-total correlations, can be regarded as acceptable.

2.3.5 FACTOR ANALYSIS

In order to reduce the number of interrelated items and detect the dimensions which underlie items with a high degree of correlation, a varimax orthogonal-rotation factor analysis was conducted on the scores in the affective responses to mathematics scale. This rotation method has been selected over the oblique rotation method because it is generally recommended when no a priori assumptions about the emerging factors are made (Kline, 1994). A set of eleven factors with an Eigenvalue greater than 1.00 was extracted. While it is generally accepted that factors with an Eigenvalue of 1.00 or more should be retained, the scree plot (figure 2.1) indicates that this is not useful in the present case because the plot fades out after the four factors. In conjunction with predictions in the literature on psychometric test development (cf. Loewenthal, 2001), the first one to four factors account for much of the variance. Therefore, the first four factors which are located just before the point where the scree plot drops off to plateau level were retained rather than relying solely on the Eigenvalue. The loading of items on each factor, as displayed in table 2.3, was then examined.

Scree Plot

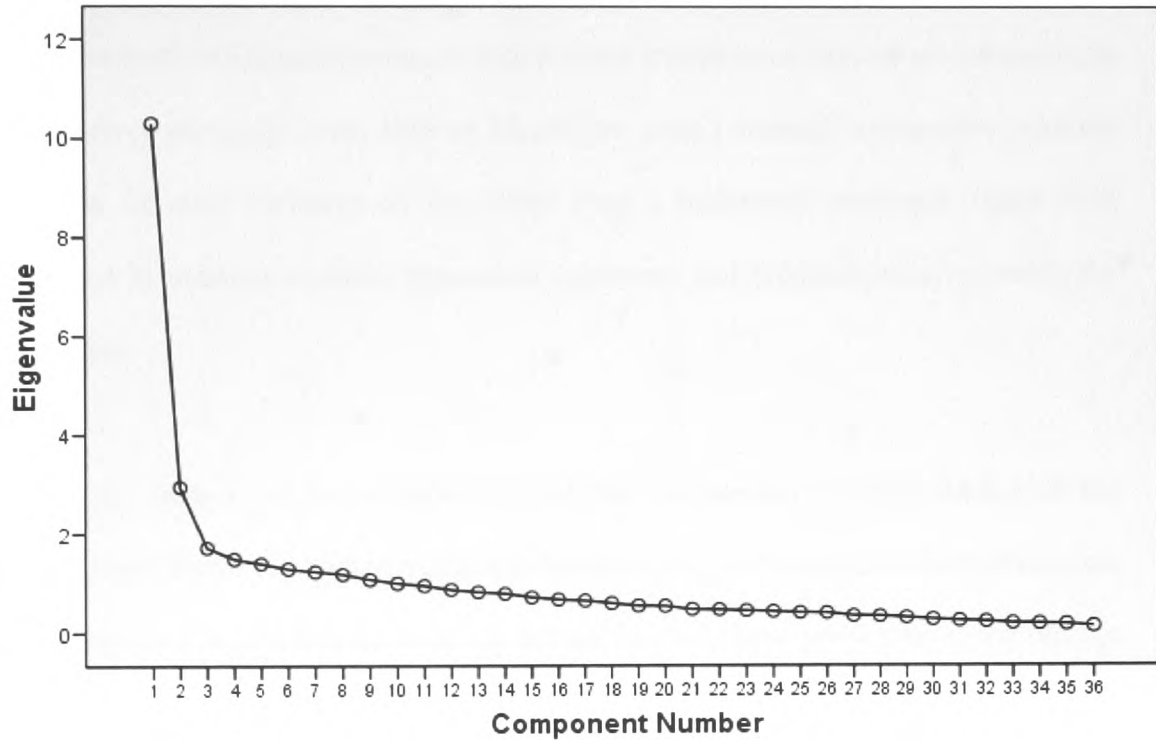


Figure 2.1 : Factor loadings displayed on a scree plot

There is disagreement about the criterion for including items. Kline (1994) proposes that a factor loading of 0.3 is acceptable. Similarly, Loewenthal (2001) recommends items with a loading of 0.4 or above should be included, while it might be acceptable to include items with a loading of 0.3 when there is a theoretical rationale and when it contributes to the labelling of factors (Loewenthal, 2001). In cases of crossloading, when an item had a high degree of loading from two factors, it was included in the factor where it was loaded most highly, because this strengthens the validity of the scale. When an item is loaded high on more than two factors, it is rejected from the scale

Having identified the items which share an underlying factor from an empirical point of view, the conceptual similarity of the variables within each factor was examined in accordance with the theoretical tenets in the literature and subscales were formed based on an examination of which items loaded most heavily on the factor. In cases where particular items with an Eigenvalue over 3 seemed incompatible with the majority of other variables on the factor from a theoretical viewpoint, items were excluded in order to establish theoretical coherence and unidimensionality within the subscales.

An analysis of the variables showed that the majority of items loading on the first factor referred to children's attitudes towards their mathematics teacher. Items such as "I enjoy the maths lessons with my teacher" go beyond a perception of the teacher and involve a favourable/ unfavourable evaluation of the teacher. This implies that the subscale does not only include perceptions of the teacher but also attitudes towards the teacher. In order to account for this, it seemed reasonable to label the subscale as "attitudes towards the teacher".

The second factor mainly included items which were indicative of strong tension and aversion towards mathematics, for example "I feel embarrassed when asked to come to the chalkboard" or "I am afraid to come up with a wrong answer". Each of the items in the subscale referred to negative feelings such as embarrassment, which go beyond a favourable/ unfavourable evaluation of mathematical activities; therefore the subscale was named "maths anxiety".

Items loading on the third factors mainly referred to children's perception of other children's behaviour in the maths classroom. Because the items such as "It takes a long time until my teacher can really start teaching" do not include a favourable or unfavourable evaluation of what happens in the classroom, it was decided to subsume the items in the scale as "perceptions" rather than "attitudes". Indeed, perceiving the classroom to be loud and distracting does not imply a negative evaluation. Some children, especially children who contribute to misconduct, might actually enjoy a loud classroom.

The fourth and final subscale includes only six items with loadings of 0.3 or above. Of these items, four referred to children's perception of their ability to succeed in mathematics in different situations. For example "If I learn well, I will have good grades in maths" and "I will never be good at maths." Given that the majority of items in this subscale referred to children's perceived capability to perform well in mathematics, the scale has been labelled as "self-efficacy beliefs". To sum up, the item loadings indicated that the four factors which emerged can be subsumed as: children's attitudes towards the teacher, maths anxiety, perceived classroom conduct and self-efficacy beliefs.

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Table 2.2: Results of the factor analysis

	Component			
	1	2	3	4
Rotation Sums of Squared Loadings	14,38 % of Variance	13,17 % of Variance	11,67 % of Variance	6,49 % of Variance
Item 2	-.12	.30	.23	.05
Item 3	.52	.28	.51	-.07
Item 4	.44	.44	.30	.15
Item 5	.19	.03	.84	.07
Item 6	.62	.31	.12	.19
Item 7	-.07	.32	.37	.15
Item 8	.26	.69	.09	.13
Item 9	.08	.37	.23	.13
Item 10	.23	.61	-.12	.24
Item 11	.64	.07	.40	-.18
Item 12	.30	.42	.32	.20
Item 14	.17	.15	-.03	.62
Item 15	.70	.05	.16	.18
Item 16	.09	.21	.13	.54
Item 17	.73	.10	.16	.10
Item 18	.46	.36	.15	.06
Item 19	.30	.43	.10	.16
Item 20	.17	.55	.03	-.03
Item 21	.04	.04	.48	.10
Item 22	.32	.12	.63	.11
Item 23	.71	.11	.19	.16
Item 24	.35	.05	.70	.01
Item 25	.67	.20	.03	.29
Item 27	.22	.38	.45	.04
Item 28	.61	.29	.43	.12
Item 29	.00	.65	-.10	.21
Item 31	.28	.26	.35	-.15
Item 32	.36	.56	.28	-.06
Item 33	.06	.28	.23	.23
Item 35	.25	.62	.08	-.03
Item 36	.26	-.10	.65	.26
Item 37	.37	.20	.40	-.08
Item 38	.27	.34	-.12	.49
Item 39	-.06	.52	.18	.42
Item 41	.31	.57	.15	.49
Item 43	.03	-.06	.28	.59

2.3.6 TESTS OF INTERNAL CONSISTENCY FOR THE SUBSCALES

Having derived the factors, the subscales were tested for internal consistency by conducting a reliability test and deriving the Cronbach's alpha correlation coefficient as an index of reliability. A Cronbach's alpha correlation coefficient between 0.6 and 0.7 was judged as low, a correlation coefficient above 0.8 was considered to be a good indicator for internal consistency (Kline, 1994). In line with the recommendations of Kline (1994) and Loewenthal (2001), it was decided to reject items with an item-total correlation coefficient lower than 0.3, because the 0.3 value means that the item would account for 10% of the variance within a scale, if squared. Items were rejected in cases where a removal of a particular item in a subscale would significantly improve the Cronbach's alpha reliability coefficient, but not if the removal would only lead to minor improvements. As can be seen in the tables below, the factors show a satisfactory degree of internal consistency, as indicated by the coefficient.

Table 2.3: Item analysis of attitudes towards the teacher subscale

Item	DESCRIPTION	Item - Total Correlation	Scale if item deleted
Item 3	The teacher often doesn't notice if I have difficulties	.61	.83
Item 6	My teacher approaches all children equally	.60	.83
Item 11	My teacher always selects the same children	.58	.83
Item 15	I enjoy the maths lessons with my teacher	.59	.83
Item 17	My teacher's explanations help me understand	.64	.83
Item 23	My teacher ensures that I understand maths	.67	.83
Item 25	My teacher can't explain a new topic well	.6	.82
Item 28	My teacher's explanations are difficult to understand	.72	.82

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Table 2.4: Item analysis of maths anxiety subscale

Item	DESCRIPTION	Item - Total Correlation	Scale if item deleted
ITEM 4	I am afraid to come up with a wrong answer	.47	.60
<i>Item 7</i>	<i>I am too afraid to ask if I don't understand</i>	.25	.70
Item 12	I am afraid of maths examinations	.53	.57
Item 18	I feel embarrassed when asked to come to the chalkboard.	.40	.63
Item 27	I wonder what the others think if I come up with a wrong answer	.50	.58

Table 2.5: Item analysis of perceived classroom conduct subscale

ITEM	Description	Item - Total Correlation	Scale if item deleted
Item 5	The maths lessons is often disrupted by pupils	.72	.74
Item 22	My class fools around during maths lessons	.65	.77
Item 24	It takes a long time until my teacher can really start teaching	.65	.76
Item 31	It is quiet during our mathematics lessons	.44	.81
Item 36	The maths lessons run without disturbances	.55	.78
<i>Item 37</i>	<i>If you're not gifted, maths is very difficult</i>	.43	.81

Table 2.6: Item analysis of the self-efficacy subscale

Item	Description	Item - Total Correlation	Scale if item deleted
ITEM 38	If I learn well, I will have good grades in maths	.43	.55
Item 39	I will never be good at maths	.43	.56
Item 41	I simply have no talent for maths	.58	.43
Item 43	It is difficult for me to sit down and learn maths	.25	.68

2.3.7 CRITERION VALIDITY

In order to measure criterion validity of each of the subscales, the difference between the affective response scores of secondary school children with MD (n=91) and children from an average ability control group (n=53) was determined. Table 2.7 shows a significant difference between the scores of the groups in all of the subscales: on the mathematics anxiety subscale $F(1, 41) = 30.61, p < .01$, the attitudes towards the teacher subscale $F(1, 41) = 29.85, p < .01$, on the self-efficacy subscale $F(1, 41) = 12.3, p < .01$ and the perceived classroom conduct subscale $F(1, 41) = 29.85, p < .01$.

Table 2.7: Mean affective response results for average achieving children and MD children

Factor	Ability	Mean	SD	N
Anxiety	Average	3.40	.57	52
	MD	2.82	.69	91
Attitudes towards the teacher	Average	3.95	.80	52
	MD	3.16	.86	91
Self-efficacy	Average	3.92	.56	52
	MD	3.50	.80	91
Perceived classroom conduct	Average	3.94	.58	52
	MD	2.80	.70	91

2.3.8 TEST – RETEST RELIABILITY

A Pearson correlation was used to establish test-retest reliability for each of the different subscales. The tests were conducted with a sample (n=24) of fourteen male and ten female secondary school children aged between ten and twelve (mean age eleven). In line with the recommendation of Kline (1986), the retest was administered three weeks after the first test, to ensure that children did not remember which answer

they chose in the first session. The result of the Pearson correlation test shows a significant test–retest reliability ($r=.86, p=0.01$).

Table 2.8: Mean affective responses of children with MD and average ability children in pretest and retest

	Mean	SD	N
Pre-test	156.67	21.40	24
Re-test	155.71	23.92	24

2.4 DISCUSSION

To recap, the ARTM scales were subjected to a range of tests of internal and external consistency, which indicated that the scale seems to be a sound psychometric instrument to measure affective responses towards different aspects of mathematics. The discriminatory analysis served to determine a discrimination index for each item and to exclude the items which did not adequately discriminate between respondents with positive attitudes and negative attitudes. The internal consistency of the entire scale was indicated by Cronbach’s alpha reliability coefficient, which was measured through a pair wise correlation test. The results of a split–half test and an examination of the item–total correlation confirmed the internal consistency indicated by the Cronbach’s alpha reliability coefficient. The factor analysis resulted in four factors: 1. maths anxiety, 2. self-efficacy beliefs, 3. children’s attitudes towards the teacher, 4. perceived classroom conduct.

A possible limitation of the scale development is that the study was conducted with children from class five and eight only and therefore the results cannot be accepted

as normative for other populations. Indeed, the fact that the majority of participants were derived from a population of children with MD might have had a serious effect on the results and future research might address this limitation by obtaining a heterogeneous population. Nevertheless, because the different subscales measure distinct aspects, it is hoped that useful information can be derived from the instrument. The scale reflects the assumption that research on children's affective responses towards mathematics should go beyond an exploration of anxiety and self-efficacy beliefs and the factors which emerged are in conjunction with previous literature.

CHAPTER 3

AFFECTIVE DIMENSIONS OF MATHEMATICAL DIFFICULTIES

CHAPTER 3 AFFECTIVE DIMENSIONS OF MATHEMATICAL DIFFICULTIES

3.1 AFFECTIVE RESPONSES IN CHILDREN WITH MATHEMATICAL DIFFICULTIES (MD)

The following chapter is concerned with the implementation of the ARTM scale in order to investigate differences in affective responses between children with MD and average ability children. The chapter starts with a review of the literature on the four aspects measured by the ARTM, namely self-efficacy, mathematics anxiety, attitudes towards the teacher and perceived classroom conduct

3.1.1 SELF-EFFICACY

Since Bandura's publication of the self-efficacy theory in 1977, the relationship between self-efficacy beliefs and academic achievement has attracted considerable research interest, particularly in the area of mathematical achievement. The concept of self-efficacy highlights the fundamental role of self-referential thought in mediating the relationship between cognitive processes, emotional reactions and behavioural actions. The self-efficacy concept is based on social cognitive theory, which assumes that achievement is based on interactions between people's behaviour, thoughts and beliefs and environmental circumstances (Bandura, 1986). In essence, the self-efficacy theory postulates that possession of the required skills is not sufficient to perform a given task successfully unless individuals perceive themselves as capable of implementing their skills and accomplishing the task. According to Bandura (1982, p.122), perceived self-efficacy is defined as "individuals' judgements of how well one can execute courses of action required to deal with prospective situations...people's judgements of their capabilities influence their thought patterns and emotional reactions during anticipatory

and actual transactions with the environment.” Bandura’s self-efficacy theory predicts that one’s perceived self-efficacy goes beyond a mere consideration of past achievement or an idiosyncratic judgement of future accomplishment.

Individual self-efficacy beliefs, whether correct or mistaken, constitute an important determinant of an individual’s choice of activities, environmental settings, task persistence in the face of obstacles and effort expended (Bandura, 1982). While individuals with low self-efficacy beliefs might reduce their effort or withdraw altogether when faced with difficulties and / or disappointment, individuals with strong self-efficacy beliefs are likely to expend greater effort and keep persisting. Persistence and expended effort, on the other hand, have considerable impact on achievement. Therefore, self-efficacy research is relevant to an understanding of the relationship between affective responses and learning processes in children with MD.

Self-efficacy and self-concept

Some scholars perceive self-efficacy and self-concept to be equivalent and use the terms interchangeably (Meece, Wigfield and Eccles, 1990). However, self-concept and self-efficacy can be distinguished with respect to their definition, measurement and interpretation. According to Marsh and Craven (1997, p.11), “self-concept refers to self perceptions formed through experience with the environment and in particular, through environmental reinforcements and the reflected appraisal of others.” Thus, a learner who is successful in mathematics and holds high self-efficacy beliefs might still have a low self-worth, because achievement in mathematics is not valued in the learner’s social environment.

Bong and Clark (1999) assume that self-concept is more likely to be affected by information from social comparisons than self-efficacy beliefs. Hanchon-Graham (2000) argues that a key difference between self-concept and self-efficacy lies in its measurement: whereas the former is concerned with measuring individuals' judgements of their capability to accomplish a specific task or attain a certain level of performance, the latter involves measurements of individuals' general self-worth with respect to performance. Overall, the literature indicates that self-concept and self-efficacy should be treated as separate constructs.

Self-efficacy and achievement

According to Bandura (1986), performance attainment is a fundamental source of self-efficacy beliefs. However, it is argued that self-efficacy judgements are not directly conveyed through performance attainment information. Pajares (1997) claims that it is not the objective information about one's achievement but individuals' interpretation of this achievement which is relevant with respect to self-efficacy. In other words, two children might achieve the same mathematics grade, but interpret it in entirely different ways, depending on different factors such as previous achievement, task difficulty and social comparison information from their social environment. For a low achieving child, a grade three might be interpreted as a success whereas, for an otherwise high achieving child, the same grade might give rise to disappointment and frustration. In cases where people cannot rely on their own previous experiences, for example if they learn a new skill or are confronted with novel tasks, they draw on the achievement of others in their social environment for their self-efficacy appraisals.

Collective efficacy and achievement

The fundamental role of the social context for individuals' self-efficacy beliefs has been translated into the concept of collective efficacy. According to Bandura (1997), collective efficacy can be defined as the perceived performance capability of an entire social system, such as a school. In essence, Bandura (1997) claims that if the collective efficacy is perceived as satisfactory, the individual maintains a positive self-efficacy belief and positive outcome expectations, even in light of occasional low achievement:

“It is the internal barriers created by perceptions of collective inefficacy that are... more demoralizing and behaviourally self-debilitating than are external impediments. People who have a sense of collective efficacy will mobilize their efforts and resources to cope with external obstacles to the changes they seek.” Bandura (198, p.24)

The concept of collective self-efficacy is relevant to an understanding of self-efficacy and achievement in school children because it highlights the fundamental role of peer influences in the classroom. Peer influences are particularly pronounced in peer networks such as the classroom environment because children in a classroom tend to be similar in age. Peer influences are mainly exerted through model similarity: the observation of peers who are similar to oneself and who are able to accomplish tasks successfully can raise children's own self-efficacy (Schunk, 1987). On the other hand, model similarity can have a negative impact on children's own self-efficacy beliefs. Children who witness continuous low achievement and repeated episodes of disappointment of similar peers can be led to believe that they are not capable of succeeding. Peer influences provide a source for self-efficacy appraisals for children

who are uncertain about their performance capabilities or who lack normative information to judge their capabilities.

To recap, observing the successful performance of others can facilitate one's own self-efficacy, whereas negative models can obstruct the formation of positive self-efficacy beliefs. This has considerable implications for the present research, which investigates self-efficacy beliefs of children who are tracked on different school types, based on their achievement. The segregation of children according to their achievement means that children are exposed to different social systems: children with mathematical difficulties (MD) are more likely to witness low performance and disappointment among their peers which, in turn, might have a negative effect on their own self-efficacy beliefs.

Overall, Bandura's concept of collective efficacy points out that judgements of one's capability to achieve a certain goal do not take place in a social vacuum. Rather, the performance of others is crucial for individual self-efficacy beliefs. Indeed, almost every achievement outcome is evaluated with respect to social criteria. Given that the present study seeks to determine the differences in self-efficacy between groups from different achievement levels and school systems, it appears that the concept of collective self-efficacy is highly relevant in this context.

Self-efficacy and attribution of success and failure

As has been mentioned above, self-efficacy appraisals are not conveyed through objective information on the achievement of one's performance as such, but rather through individual interpretation of this achievement. The interpretation of one's achievement is linked to people's causal ascriptions for their achievement outcomes

and, therefore, attribution style is an important source of self-efficacy (Schunk, 1983). According to Weiner (1992), individual achievement is most likely to be attributed to ability, effort, task complexity and luck. Each of these attributes can be considered along three different dimensions: *locus of control* (internal vs. external attributions), *stability* (perceiving a situation as fixed as opposed to open to change) and *controllability* (skill/efficacy vs. luck). According to Schunk (1982), attributional variables constitute an important source of self-efficacy, because attributes of academic success or failure are related to persistence, effort expended and choosing more challenging tasks.

An important outgrowth of the attribution theory, which is relevant to the self-efficacy concept, is the notion of learned helplessness (Seligman & Maier, 1967). In essence, learned helplessness means that students who attribute failure to internal, stable factors conceive themselves as unable to succeed in mathematics. They tend to view success as unattainable and outside their scope of action, are less motivated to approach more challenging tasks and are more inclined to withdraw when faced with disappointment (Dweck, 1986). Learned helplessness is reinforced in educational settings that place low value on effort but emphasize the importance of ability instead. Students who exhibit learned helplessness tend to attribute success in terms of external causes such as luck or ease of tasks, while failure is interpreted in terms of internal, stable causes. Learned helplessness is a considerable factor for self-efficacy beliefs because it influences the amount of effort that students are willing to invest (Kloosterman, 1983) and is thus related to persistence, effort and choosing more challenging tasks (cf. Hart Reyes, 1984).

Teacher behaviour can constitute an important source of self-efficacy beliefs, in that it influences learners' attributions of success and failure. This is the case, for example, if teachers want to make low achieving students feel safe through expressions of pity or excessive praise for relatively easy tasks. Indeed, it has been found that failure, which is perceived as being a result of low ability, evokes pity and empathetic helping behaviour in teachers (Weiner et al., 1982). Praise, in combination with pity, increased help and the absence of blame or anger may be interpreted by students as low ability cues (Graham, 1990). This was confirmed in Graham's 'induced failure' laboratory study (1984), where class six students were asked to solve a problem. The problem was designed in a way that made it difficult to solve and thus resulted in failure.

After the failure, the experimenter took on the role of a teacher and communicated either pity, anger or a neutral reaction. As predicted, the children who were exposed to the teacher's pity were most inclined to see their failure as a consequence of their low abilities. This suggests that praise does not inevitably have positive motivational implications. In fact, praise could be associated with high effort and high effort, in turn, might be perceived as compensation for low ability (Graham, 1990). This finding is relevant in the context of the present study because it is likely that children with MD elicit compassionate or angry responses from their teachers, which might lead them to develop learned helplessness and to attribute failure to stable, internal factors. According to Peterson's literature review (1990), such a negative attributional style is related to several affective and cognitive problems such as poor achievement, reduced help seeking behaviours, low levels of aspirations, absence of clearly defined goals and expectations and ineffective learning strategies.

To sum up, the individual interpretation of the causes of achievement outcomes play an important role for self-efficacy appraisals.

Self-efficacy and mathematical achievement

This section is specifically targeted reviewing the research on the relationship between mathematical achievement and self-efficacy beliefs. Maths self-efficacy has been defined as “individuals’ judgements of their capabilities to solve specific math problems, perform math-related tasks, or succeed in math-related courses” (Betz & Hackett, 1983, p.331). As will be explored in more detail, there are numerous studies which substantiate the predictive utility of self-efficacy beliefs for mathematical achievement. Indeed, there is evidence that self-efficacy exerts a greater influence on maths achievement than mental ability (Pajares and Kranzler, 1995). This supports Bandura’s assertion that self-efficacy beliefs influence effort expended and persistence and therefore mediate the effect of skill and ability on achievement outcomes.

One of the earliest research studies on self-efficacy beliefs and achievement is provided by Schunk (1982), who examined the effects of different attributional feedback on children’s self-efficacy beliefs and performance in subtraction. The sample consisted of 44 third-class children, who were learning how to carry out complex subtraction procedures at the time the experiment took place. The self-efficacy measure involved 25 pairs of subtraction problems, each accompanied by a ten point efficacy scale ranging from ten to 100. After each problem, children were asked to judge their capability to solve the problem correctly on the efficacy scale. After the efficacy assessment, children took part in a subtraction skill test, which consisted of 25 subtraction problems, ranging from two to six columns. Following the skill test, children were randomly assigned to different attributional feedback treatment, whereby the experimenter ascribed children’s achievement outcomes to one of four causes: ability attributional feedback (“you’re good at this”), effort attributional feedback

(“you’ve been working very hard on this”), ability and attributional feedback (“you’re good at this and you worked very hard”) or no attributional feedback. After three training sessions, the subtraction skill test was employed as a post-test to investigate any effects of the four attributional feedback treatments.

The results of an analysis of covariance showed that children who received ability attributional feedback exhibited significantly better subtraction skills than children who received other attributional feedback. The positive effect of ability attributional feedback might reflect the tendency in Western cultures to value ability more than effort (Graham, 1991). Hence, it could be that children perceived ability attributional feedback as social reinforcement, which might have promoted their self-efficacy beliefs more than effort attributional feedback. Effort attributional feedback, on the other hand, yielded significantly better subtraction skills than the no feedback condition. Children who were exposed to effort attributional feedback expended significantly more effort on the problems than children in the ability attributional and no feedback conditions. Thus, the provision of attributional feedback can facilitate self-efficacy and achievement and this challenges the assumption that self-efficacy is more than a mere reflection of past achievements.

Bandura (1986) predicted that self-efficacy beliefs are a more powerful predictor for achievement than prior experience. In addition, Bandura hypothesised that self-efficacy beliefs mediate the influences of self-concept and maths anxiety, which he describes as “common mechanisms” of human agency. Pajares and Miller (1994) employed the path analysis technique in order to investigate the predictive utility of self-efficacy for students’ problem solving performance and the role of self-efficacy beliefs in mediating self-concept and anxiety. Their sample consisted of 350

undergraduate students who completed an average of 10.3 college credits in maths courses. Self-efficacy beliefs were measured by the Mathematics Confidence Scale (MCS), which is specifically targeted towards college students with respect to the types of tasks. The scale asks participants to judge their ability to solve a given mathematical problem correctly. The problems differ with respect to the area of mathematics they cover (arithmetic, algebra, geometry), cognitive demands (computation, comprehension and application) and problem context (real and abstract).

Maths anxiety was measured by the Mathematics Anxiety Scale, consisting of ten items. The instrument that has been employed for measuring self-concept is an adapted version of the Self Description Questionnaires. Problem – solving performance was measured by an eighteen item, multiple choice instruments which is specifically targeted towards college students. The results of the path analysis indicated that self-efficacy was more predictive for performance than the self-concept measure, maths anxiety and previous experiences.

In a similar experiment, Pajares and Kranzler (1995) attempted to replicate the abovementioned results of Pajares and Miller (1994). Pajares and Kranzler (1995) extended the previous study by including the psychometric measure *g* as a measure of general mental ability in their path analysis. No previous study has addressed the issue of general mental ability and utilised the results of standardised aptitude tests, instead. However, Pajares and Kranzler (1995) point out that standardised aptitude tests performance is confounded by disparities in mathematical experiences as well as attitudinal factors and anxiety.

Therefore, the researchers recommend the usage of a nonverbal, untimed test of general mental ability in order to minimise the impact of confounding variables such as educational background. The sample consisted of 329 high school students from classes nine to twelve. In addition to the instruments in the Pajares and Miller study, Raven's Advanced Progressive Matrices, has been found to be a reliable indicator for mental ability. The results indicated that self-efficacy had a strong effect on both maths anxiety and problem-solving performance, even with a control in place for general mental ability. Also, self-efficacy was found to be partially responsible for the effects of *g* and maths background on maths anxiety and problem solving performance. Thus, the findings strengthen Bandura's assertion that self-efficacy is a fundamental variable for academic achievement outcomes.

The results of Pajares and Miller (1994) could also be replicated in a similar study by Kiamanesh, Hejazi and Esafahani (2004), who employed the same measures of maths anxiety as Pajares and Miller. The sample consisted of 400 class nine students. The researchers employed a twenty item scale to measure self-efficacy beliefs and devised a twenty item maths test to assess mathematical achievement. As in the previously described study, Kiamanesh et al. utilised the path analysis technique to examine direct and latent effects between variables. The results showed that maths self-efficacy was significantly more predictive for achievement than self-concept and anxiety. This finding is in line with those of Pajares and Miller (1994).

The finding adds clear support to Bandura's (1986) hypothesis that self-efficacy beliefs are a stronger predictor for achievement than other presumed factors. While the research studies of Pajares and Miller (1994), Pajares and Kranzler (1995) and Kiamanesh et al. were carefully designed and provide a valuable account of the

fundamental role of self–efficacy in achievement, it is important to point out that the research has been conducted with college students (Pajares and Miller study), high school students (Pajares and Kranzler study) or with class nine students (Kiamanesh et. al, 2004) and therefore the extent to which the findings apply to younger children might be limited. In addition, none of the studies reviewed so far addressed how self–efficacy beliefs in children differ as a function of differences in their environment such as the classroom. The present study attempts to account for this by investigating how children with and without mathematical difficulties (MD) from classes five and eight differ in their self–efficacy beliefs. Since children with and without mathematical difficulties are tracked on separate school types, it is hypothesised that these children are exposed to peer networks which differ with respect to their perceived collective efficacy. This, in turn, might result in different self–efficacy beliefs. In addition, it will be investigated whether the predictive utility of self–efficacy for individual differences in achievement differs across children with MD and average ability children from years five and eight.

3.1.2 MATHS ANXIETY

Maths anxiety has been identified as a key affective component which has attracted a considerable body of empirical scrutiny. According to Mc Leod (1992, p.36), maths anxiety “has probably received more attention than any other area that lies within the affective domain.” A selection of the available research will be reviewed in the following sections. Maths anxiety has been defined in different ways. According to Wood (1988, p.8), maths anxiety refers to “a general lack of comfort that someone might experience when required to perform mathematically.” The notion that maths anxiety refers to a general lack of comfort implies, that it is an attitudinal construct. However, Mc Leod (1992) criticises that defining maths anxiety in terms of a negative

attitude “does not seem adequate to describe some of the more intense feelings that students exhibit in [the] mathematics classroom.”

Hunt (1985) assumes that maths anxiety goes beyond a mere “lack of comfort” and defines it as a state of panic, paralysis and mental disorganisation which some people experience when asked to solve a mathematical problem. Correspondingly, Lewis (1970) proposes that maths anxiety is an emotional state which is characterised by fear and dread. Overall, the majority of the abovementioned definitions imply that maths anxiety is a construct which consists of attitudinal components (dislike), cognitive elements (negative thought) and strong emotional components (fear). Therefore, maths anxiety can be distinguished from attitudes towards mathematics and classified as an affective response instead.

Maths anxiety and achievement

A consistent finding in the literature on mathematics anxiety is the relationship between mathematics anxiety and achievement. One of the most compelling studies with respect to this issue is provided by Hembree (1990). Hembree conducted a meta-analysis on 151 studies on maths anxiety in order to examine the relationship between maths anxiety and achievement. The studies that were chosen for the meta-analysis were screened against several criteria; for example, studies which included measurements of maths anxiety had to be based on validated instruments. In addition, experimental studies were required to include a control group. In order to account for the rigour in the meta-analytic homogeneity test, each experimental group had to contain at least ten subjects. The screening resulted in a collection of 49 peer reviewed

articles, 23 articles from the Educational Resources Information Centre (ERIC), 75 doctoral dissertations and four reports from miscellaneous sources.

The range of participants' class levels varied from class three (primary school) up to post-secondary education. However, only seven of all the studies included participants from class six or below. Thus, the distribution of class levels in the 151 studies indicates that the vast majority of studies were concerned with older participants. Mathematics achievement for children from classes five to twelve was determined by their maths grade. In addition, measures of mathematical ability in different subtests, namely computation, concepts, problem solving, abstract reasoning and spatial ability were available for participants from class seven onwards. In essence, the results showed that maths anxiety was negatively related to all of these measures; there was a negative relationship between mathematics grade and anxiety, with a correlation coefficient of $-.36$.

The relationship between subtest-based achievement and anxiety was most pronounced in the subtest on abstract reasoning, with a correlation coefficient of $-.4$. The correlation coefficient for spatial ability and anxiety was $-.29$. The relationship between achievement in the problem solving subtest and anxiety was weaker, with a correlation coefficient of $-.27$, the same result was yielded for the relationship between achievement in the conceptual subtest and anxiety. The relationship between computational achievement and anxiety was $-.25$.

However, the overall utility of the findings on the relationship between subtest-based achievement and anxiety might be limited because these results were largely based on studies with samples from class seven onwards. In other words, it remains

open to what extent these findings apply to younger participants in the early secondary years. In addition, it could be criticised that no information was provided on the test instruments which were used to measure achievement. Consequently, it is not possible for the reader to determine the specific content of the different subtests such as the types of tasks and level of difficulty. The study might have been more persuasive if the author had adopted the criterion of choosing studies with standardised measurements of mathematical ability.

Another possible limitation of Hembree's study is that studies with participants from different age bands were merged in order to determine the relationship between achievement and school grade. However, it could be argued that academic demands, the classroom environment and the context in which achievement and anxiety were measured varies greatly between children from different age bands. The trajectories of anxiety and achievement over the years might not be adequately examined when the results of such a diverse population are merged together. Such an approach fails to account for the different factors which might influence the relationship between anxiety and achievement over time. Hence, it could be argued that the meta-analysis would have been more useful if it accounted for stability and a change in the relationship between anxiety and school grade across secondary years. Also, Hembree (1990, p.38) concludes that "there is no compelling evidence that poor performance causes mathematics anxiety." However, this claim can be criticised, given that a correlational research design cannot be used to infer a causal relationship. Despite these potential limitations, Hembree's study provides a valuable insight into the relationship between anxiety and achievement.

A more recent meta-analysis on the relationship between mathematics anxiety and achievement is provided by Ma (1999). The meta-analysis was based on a sample of 26 studies, which included eighteen peer reviewed articles, three unpublished articles and five dissertations. The analysis showed a correlation coefficient of -0.27 for the relationship between maths anxiety and achievements, which corroborates the results of Hembree (1990). In contrast to Hembree (1990), Ma investigated whether class level mediates the relationship between mathematics achievement and anxiety. The results led to the conclusion that, from class four onwards, there is a significant relationship between achievement and anxiety. There was no significant variance in the relationship between mathematics achievement and anxiety across classes four to nine.

However, Ma examined the development of maths anxiety over time by forming class-level pairs and by comparing maths anxiety and achievement from classes four to six, seven to nine, and ten to twelve, rather than examining the change and stability of the maths anxiety and achievement over time, on a year-by-year basis. It was found that there was no difference in the relationship between achievement and anxiety in classes four and six, classes seven and nine and classes ten and twelve. However, it remains open to doubt whether there is a difference when comparing classes four and ten, for example, which are characterised by different demands and classroom environments. The findings on the effects of class level on the relationship between maths anxiety and achievement might have been more informative if a different method of comparison across classes had been adopted.

Also, it should be noted that the majority of studies in Ma's meta-analysis used a standardised test to measure maths anxiety. It could be criticised that children who suffer a high level of anxiety are at a disadvantage in test situations because the

presence of mathematical stimuli elicits their anxiety, which in turn might constitute a serious impediment to children's test performance. In other words, individual differences in maths anxiety are likely to interfere with performance in a test situation where anxious individuals are exposed to mathematical stimuli which trigger their anxiety. Therefore, the test scores of anxious children might not be indicative of their achievement because they may be contaminated by anxiety (Kellogg, Hopko, & Ashcraft, 1999). A possible resolution would be to include studies which operationalise maths achievement in terms of teachers' mathematic grades rather than single test scores, since teachers' grades might provide a more realistic depiction of children's verbal and written achievement in the everyday classroom environment over a longer period of time.

Causal ordering of maths anxiety and achievement

To sum up the findings from the meta-analytic studies reviewed so far, there seems to be convincing evidence for a negative relationship between maths anxiety and achievement. However, the causal ordering of anxiety and achievement remains less clear. Newstead (1998, p. 54) argues that "this relationship...is essentially ambiguous with respect to the direction of causality." Several alternative models have been proposed to conceptualise the causal relationship between maths anxiety and achievement, which can be summarised as follows, according to Ma and Xu (2004):

- a) mathematics anxiety is an effect of low achievement
- b) mathematics anxiety causes low achievement
- c) there is a reciprocal, mutually reinforcing relationship between both factors

One limitation of this summary is, that it tends to simplify the complex relation between cognition and affect, especially because it fails to specify whether low achievement refers to achievement in single laboratory situations or to the school grade. In addition, the framework does not acknowledge the importance of mediator variables such as classroom environment, teacher supportiveness and other affective factors such as self-efficacy. These variables might have a considerable impact on children's perception of low achievement and, consequently, on the development or inhibition of maths anxiety. However, the framework proposed by Ma and Xu will be employed for the purpose of structuring the present literature review.

The deficit model

The hypothesis that maths anxiety is a function of low achievement has been incorporated within the deficit model, coined by Tobias (1985). According to the deficit model, "mathematics anxiety is the unpleasant remembrance of poor mathematics performance in the past." (Ma and Xu, 2004, p. 168). Low performance is conceived to be the result of poor learning habits and a lack of test-taking skills. However, one reservation that can be raised is that the deficit model lacks specification regarding the mechanisms by which the unpleasant remembrance of low achievement gives rise to maths anxiety. Indeed, the majority of children will have experienced occasional bad marks and disappointment and it would be pertinent for the deficit model to explain why some children develop maths anxiety in response whilst others do not.

One tentative explanatory model is provided by the self-efficacy concept, which emphasizes that it isn't achievement information that influences children's beliefs in their own capability per se, but rather their idiosyncratic interpretations of the

achievement within a particular context. Repeated episodes of failure and negative evaluation by significant others such as parents, peers and teachers are likely to have a detrimental effect on self-efficacy beliefs and might lead children to believe that their actions do not have any impact on the outcome. This in turn facilitates withdrawal and feelings of helplessness, which might be risk factors for maths anxiety. If this tentative explanatory model would be accepted, the deficit model is congruent with self-efficacy theory.

However, the prediction of the deficit model that maths anxiety is the product of low achievement has not gone unchallenged. Hembree's (1990) meta-analysis showed that cognitive and behavioural interventions were effective in reducing children's anxiety and this corresponded to an increase in achievement. Given that the interventions were targeted towards reducing anxiety rather than improving maths achievement, the findings can be taken as tentative evidence that maths anxiety is predictive for maths achievement, which contradicts the assumption of the deficit model. Indeed, Hembree (1990) concludes that "there is no compelling evidence that poor performance can cause mathematics anxiety."

The interference model

The hypothesis that mathematics anxiety is a cause for low achievement is advocated by the interference model, which was established by Mandler and Sarason (1952) and elaborated by Wine (1971). In essence, the interference model predicts that maths anxiety arises due to an inability to recall mathematical facts and procedures and therefore constitutes an impediment to the cognitive processes required for problem solving. Thus, the interference model is based on predictions which are opposite to the

deficit model that has been described above. The interference model receives empirical support when compared to more recent findings from research on the relationship between anxiety and working memory resources. In order to investigate this relationship, Kellogg, Hopko and Ashcraft (1999) have conducted a repeated measures experiment with 30 undergraduate students. Previous to the experiment, all participants completed the abbreviated Maths Anxiety Rating Scale, which contains 25 items (Mars, Alexander & Martray, 1989). Based on their results, participants were classified in one of three anxiety categories and were asked to solve a ten page arithmetic questionnaire which contained a series of easy and difficult mathematical tasks under both timed and not-timed conditions. Under timed conditions, participants were told that the amount of time for solving the tasks was limited and they should work as quickly and accurately as possible. Under not-timed conditions, participants were told that they could work through the tasks at their own pace. It was assumed that the timed conditions should create cognitive load in participants with a high level of anxiety, which would impede their performance more than participants with a normal or low level of anxiety.

However, it was found that individuals with a low, as well as high, level of anxiety were equally impeded in their arithmetic performance when tasks were performed under time pressured conditions. While there was no significant interaction effect between anxiety level and timing, the overall error rates of individuals with a high level of anxiety were significantly higher compared to individuals with a low level of anxiety. In addition, maths anxiety interacted with task difficulty, in that individuals with a high level of anxiety exhibited significantly higher error rates for difficult tasks than individuals with a low level of anxiety. This finding seems to suggest that anxiety interferes with cognitive processes during problem – solving resources and therefore

adds support to the prediction of the interference model. The findings were confirmed in another experiment by Ashcraft and Kirk (2001), as reviewed below.

In order to examine the relationship between working memory capacity and mathematics anxiety, Ashcraft and Kirk (2001) carried out a correlational study with 66 psychology undergraduate students. Participants were asked to complete the MARS inventory as an indicator for their anxiety. Based on their results on the MARS, participants were categorised in one of three anxiety categories: high, medium and low. Working memory capacity was measured by listening and computational span tasks, which require participants to memorise an increasing number of words or digits while carrying out simple verbal or arithmetic tasks. It was found that performance on both span tasks was negatively related to the measure of maths anxiety. However, a multiple regression analysis indicated that anxiety was a stronger predictor for computational span than for verbal span. Overall, the study shows that maths anxiety predicts working memory constraints and this result adds support to the inference model. Nevertheless, the study would have benefited from adopting a dual-task design because an occupation of working memory resources through a dual-task would provide a more straightforward examination of the effect of maths anxiety on performance. This issue has been addressed by a second experiment by Ashcraft and Kirk (2001).

Ashcraft and Kirk (2001) conducted another experiment with 45 psychology undergraduates to examine the role of anxiety in producing working memory constraints, which in turn inhibit achievement. As in their previous study, participants were categorised in one of three anxiety categories: high, medium and low. Participants were asked to solve 60 addition tasks, which involved different levels of difficulty: tasks with single digit operands (e.g., $4 + 5$), with a double and a single digit operand

(e.g., $13 + 4$), and with two double digit operands (e.g., $33 + 21$). Half of the problems in each set were no carry problems, and half involved a carry (e.g., $8 + 7$, $25 + 8$, $14 + 19$). The tasks were performed with and without a dual task. The dual task condition involved a letter recall task, where participants were asked to remember a set of either two or six randomly chosen consonants which were presented on a screen. If high mathematics anxiety was a function of general working memory deficits, then maths anxiety should impede performance irrespective of whether numerical or non numerical stimuli are presented. Hence, it could be hypothesised that individuals with a high level of anxiety perform significantly poorer in the letter recall dual tasks because this is associated with a heavy memory load. However, despite this heavy memory load, there was no significant variation in letter recall performance across anxiety levels. Ashcraft and Kirk (2001) conclude that “anxiety-related effects in the maths-only and dual-task conditions cannot plausibly be attributed to a general deficiency in working memory capacity for the higher anxiety groups (p. 236).”

While the interference between maths anxiety and working memory is unlikely to result from a generalised working memory deficit, Ashcraft and Kirk (2001) provide clear evidence that anxiety constitutes an impediment to achievement where more difficult arithmetic tasks are concerned. No significant differences between anxiety levels could be detected for the no-carry tasks, the variation span in error rate ranged from 0.2 for individuals with a low level of anxiety to 2.1 for individuals with a high level of anxiety. However, where carry-problems were concerned, there was a marked performance variation across anxiety levels. Participants with a low level of anxiety had an error rate of 5.2%, individuals of medium anxiety had an error rate of 7.7% and participants with a high level of anxiety had an error rate of 9.4% respectively. The finding that anxiety only interfered with the more difficult carry tasks, but not with the

easier no-carry tasks, clearly suggests that high anxiety interferes with tasks which rely extensively on working memory resources.

The strong link between anxiety and working memory capacity is also demonstrated by the finding that the performance of individuals with a high level of anxiety was more depressed when tasks were coupled with a dual task. The performance of individuals with a high level of anxiety in the medium memory load, two letter recall tasks was significantly lower than the performance of individuals with medium or low anxiety. The effect of the letter recall task was even more pronounced when carry tasks were solved. When carry problems had to be solved under heavy memory load conditions, the error rate in individuals with a high level of anxiety rose to 39%, which was significantly higher than the results of individuals with medium or low anxiety.

The results indicate that carrying procedures rely substantially on working memory resources, which are prone to interference in individuals with a high level of anxiety. However, the finding that error rates in these individuals varied substantially across difficulty levels and dual task demands suggests that the negative relationship between anxiety and working memory cannot be simplified to a consumption of working memory resources by anxiety. Rather, it seems that the relationship between working memory, achievement and anxiety is mediated by factors such as task difficulty and the associated working memory demands. Indeed, Kirk and Ashcraft (2001) argue that it is implausible to view working memory deficits as a precursor for anxiety, given that maths anxiety did not affect cognitive performance in the letter recall tasks.

Overall, the results of the reviewed studies indicate that the inability to recall prior knowledge in a given situation is likely to evoke feelings of pressure and tension, which occupy working memory resources and therefore interfere with cognitive processes required for the mathematical tasks which are specifically reliant on working memory resources (Ashcraft & Kirk, 2001). While anxiety can be beneficial for performance in certain cases, the general consensus in the literature is that feelings of anxiety impede performance (Ashcraft & Faust, 1994). To some extent, this conceptualisation of the anxiety–achievement relationship is congruent with the predictions of the cognitive load theory (CLT). To recap, this theory assumes that the restrictions of working memory (WM) mean that cognitive processing capacity is limited. According to Ashcraft & Kirk (2001), this should be particularly pronounced in individuals with a high level of anxiety, “who are already wasting working memory resources by attending their own anxiety.” (p. 236)

Conclusion

To recap, the evidence reviewed so far demonstrates that higher levels of anxiety are related to lower WM capacity, which impedes performance for complex numerical tasks. Ashcraft and Krause (2007) summarise the available evidence by arguing that “math anxiety seems to influence cognitive processing in a straightforward way – working memory resources” (p. 247). While the investigation of the impact of anxiety on cognitive processing has provided crucial insights into the relationship between anxiety and achievement, the origins and the development of anxiety is still unclear. One possible reason for why research on the development of mathematics anxiety is limited in scope is that the studies in this review have been mainly conducted with participants in upper secondary class or at university level and it is questionable to what

extent the results apply to younger samples. Indeed, no study to date has investigated maths anxiety and achievement in samples younger than class six (Hembree, 1990). Also, the studies have been carried out with an average achieving population. While individual achievement in standardised tests or teachers' grade was used in the studies in the meta-analyses reviewed above to identify low achieving children, none of the studies were targeted towards children with mathematical difficulties.

There is evidence that mathematical difficulties are related to the use of inefficient cognitive strategies, which in turn are linked to the misuse of working memory resources (Cumming and Elkin, 1999). In order to examine whether mathematical difficulties are linked to maths anxiety, an investigation of the relationship between maths anxiety and achievement in children with MD is justified. This target group has been identified as having manifest mathematical difficulties which go beyond an occasional bad grade and which involve a negative stereotype because children with MD and children of average ability are referred to different types of school. Thus, it can be hypothesised that children with MD are more likely to suffer negative experiences in the maths classroom than children of average ability, which might contribute to maths anxiety. Indeed, Ashcraft and Krause (2007) claim that the learning environment is a considerable risk factor for maths anxiety "we predict that math anxiety is learned in the classroom in front of the teacher and his or her peers. In short, lower than average math abilities and/ or working memory capacity, susceptibility to public embarrassment and an unsupportive teacher all may be risk factors for developing math anxiety" (p. 244).

3.1.3 ATTITUDES TOWARDS THE TEACHER

The present section is intended to provide a selective review of research on children's attitudes towards the teacher. It is important to emphasize that the present research is explicitly concerned with attitudes towards the teacher rather than with perceptions of the teacher. Pajares (1992) argues that terms such as perceptions, values and attitudes have been used interchangeably which leads to a definitional confusion. In line with the definition of Gopnik and Meltzoff (1997), perceptions and beliefs are synonymous in that both can be subject to misrepresentations. Attitudes, in contrast to beliefs or perceptions, involve a favourable or unfavourable evaluation of a person, event or object (Azjen and Fishbein, 1975). A typical belief or perception of the teacher would be "my teacher is confident" or "my teacher is strict." Such a belief does not automatically indicate whether a confident or strict teacher is perceived as positive or negative. Attitudes on the other hand would include more informative statements such as "I enjoy mathematics with my teacher." As will be documented in this literature review, children's individual differences in attitudes towards their teacher are linked to differences in achievement. More specifically, attitudes towards the teacher's teaching style, interpersonal behaviour and supportiveness influence children's achievement related beliefs and actual achievement. This issue has been addressed in a study by Goh and Fraser (1998).

Children's attitudes towards the teacher and cognitive outcomes

Using an adapted version of the Questionnaire on Teacher Interaction (QTI), Goh and Fraser (1998) examined children's perceptions of their teacher's interpersonal behaviour and its association with cognitive outcomes. Their sample consisted of a total of 1,512 children aged between ten and eleven. In its original format, the QTI consisted of 64 items. Several adaptations have been developed such as a shorter version which is targeted towards primary school children and which has been used by Goh and Fraser (1998). Cognitive performance was measured by a ten item mathematics achievement test. The results of a hierarchical linear model analysis suggest that successful mathematics achievement could be predicted by particular types of teacher behaviour such as leadership, understanding and helpfulness. As summarised by Goh and Fraser (1998) "better achievement was found in classes with an emphasis on more teacher Leadership, Helping/ Friendly and Understanding behaviours" (p.56).

Children's attitudes towards the teacher and motivational outcomes

Murdock and Miller (2003) examined the relationship between perceptions of the teacher and achievement motivation. Their sample consisted of 206 children from class eight. Motivation was assessed through three measures: self-reports of efficacy, self-reports of children's valuation of learning and teacher's report of children's effort. In addition, the researchers controlled for children's perceived motivational impact of parents and peers. The results of a regression analysis indicated that children's perception of the teacher accounted for a considerable amount of the variance in all of the three measures of motivation, when peer and parental influences were held constant.

This implies that a positive perception of the teacher can increase children's motivation and this, in turn, is related to an increase in achievement (Wentzel, 1998).

Midgley, Feldlaufer and Eccles (1989) investigated how children's perceptions of their teacher changed as a function of the transition from primary to secondary school and how these changes are related to changes in their perceived value of mathematics. A total of 2,501 children took part in the study. Based on performance in a standardised test of mathematical ability, children were grouped either as high achievers or children with MD. Children's valuation of mathematics was measured by two scales, each consisting of four items. Children's perception of the teacher was measured by a subscale, consisting of six items, from an instrument which assesses children's perception of the classroom environment. The data were collected before and after transition to secondary school, over a period of two years. A two-way analysis of variance showed that children who moved from supportive teachers to less supportive teachers after the transition to secondary school decline in their valuation of mathematics. Given that the perceived value of mathematics is an important factor for mathematical motivation and achievement, it can be concluded that children's perceptions of the teacher play a considerable role in the learning of mathematics.

Another interesting finding in the Midgley et al. (1998) study was that the decline in the perceived value of mathematics when moving to a less supportive teacher after transition from primary to secondary school was more pronounced for low achieving than for high achieving children. This implies that low achieving children are more affected by their perceptions of the mathematics teacher and the present study attempts to investigate this issue further. To date, there is no research on how the

relationship between individual achievement and children's perceptions of their teacher develop in children with MD and average ability children.

Conclusion

It is hypothesised that perceptions of the teacher are more predictive for individual achievement discrepancies in younger children, given that the teacher constitutes a significant person, a scaffold, for younger children. Older children, especially children of average ability who become more proficient and confident in mathematics, might be less dependent on the teacher and therefore the association between achievement and perceptions of the teacher decline over the years. Indeed, there is evidence that the impact of peer networks on children's affective responses towards mathematics increases, whereas the impact of the teacher declines. The rationale behind the present study is to examine this hypothesis and to charter stability and change in attitudes towards the teacher in children with MD and average ability children.

Perceptions of the classroom environment

During a mathematics lesson, time is not only spent on learning activities and instruction; maintaining classroom discipline and dealing with interruptions and misconduct can take up a significant proportion of the lesson time. This is important because the amount of time that children spend learning activities during their lessons is a key predictor for academic achievement (Walberg & Paik, 2000). In classrooms which are characterised by discipline problems and misconduct, the teacher is mainly concerned with non-curricular activities and children spend less time on their tasks. Thus, the classroom environment can be seen as a variable for academic achievement.

Peer influences in the classroom

The importance of the social context for understanding human cognition has been recognised from an early stage (1936) and has gained considerable interest since the publication of Bronfenbrenner's ecological systems theory. The interaction with others in a microsystem such as the classroom plays a critical role in children's cognitive development. Peers in the classroom, for example, serve as a model for behaviour and therefore influence children's own perception of classroom conduct and adaptive behaviour. As children move into adolescence, the influence of peers on children's academic achievement is more powerful than the influence of their parents (Steinberg, 1996) Indeed, Barth, Dunlap, Dane, Lochman and Wells (2004) argue that adaptive behaviours, for example task orientation and prosocial interactions, are facilitated in classrooms which consist mainly of children who exhibit these behaviours and provide role models for others. This has been confirmed in a longitudinal study conducted by Kellam, Xiange, Mersica, Brown, Ialongo (1998).

The researchers traced the development of children's behaviour from class one onwards, over a period of six years. The results indicated that aggressive boys who were exposed to aggressive classroom settings in class one maintained their level of aggressiveness until class six, compared to aggressive boys who were exposed to a non-aggressive classroom. This suggests that the classroom context is a significant variable for children's behaviour, which subsequently influences their learning outcomes. However, it could be argued that the emphasis on aggression at the expense of other factors such as task orientation or classroom conduct, in the Kellam et al. (1998) study limited the scope of their analyses.

Children's perception of the classroom, motivation and engagement

The study of Patrick, Kaplan and Ryan (2007) goes beyond an investigation of the relationship between classroom environment and aggression and examines the relationship between children's perception of the classroom and their engagement in the classroom. The researchers assumed that this relationship would be mediated by motivational beliefs (mastery goals, academic efficacy and social efficacy). Their sample consisted of 602 class five children from 31 classes. Children's perception of their classroom environment was measured using a standardised instrument consisting of a total of 24 items. Engagement was measured by two instruments. A six item scale measured children's ability to plan, monitor and regulate their cognition. Another five item scale measured perceived task related interaction.

The measure of children's motivation included a) a measure of mastery goals which were targeted towards children's desire to acquire academic competence, b) a measure of children's academic self-efficacy c) a measure of social motivational beliefs targeted towards children's perceived capability to interact effectively with their teacher and peers. The hypothesised relationship between perception of the classroom and engagement and the mediating role of motivational beliefs was tested using a structural equation modelling technique. The result of this analysis indicated that perceived task related interaction was significantly related to their maths achievement, as measured by children's report grades in maths. This relationship, in turn, appears to be mediated by children's perceived social efficacy and academic efficacy. Variance in perceived social efficacy and academic efficacy could be predicted from classroom environment variables. Overall, the researchers concluded that children's perceptions of their classroom environment were related to their motivation and academic engagement.

Given that motivation and engagement are crucial variables for mathematical achievement, the results seem to suggest that children's perception of their classroom environment is important for their learning outcomes in mathematics.

Conclusion

In summary, the studies reviewed so far clearly indicate that children's perception of their classroom environment is an important variable for mathematical achievement. Moreover, it appears to be an established tenet in the literature that children's behaviour in the classroom is influenced by the behaviour of their peers. Consequently, "classrooms which are mainly populated by children who exhibit deviant behaviour and misconduct are likely to perpetuate these maladaptive behaviours." (Barth et al., 2004, p. 127). However, the majority of studies have been conducted with average achieving children, and not on perceived classroom conduct in children with and without academic difficulties, who are tracked in different schools and therefore experience distinctive classroom environments.

The present study aims to address this issue by examining the relationship between academic achievement and perceived classroom conduct in children with MD and average ability children from classes five and eight. According to Kauffman (1997), "low achievement and behavioural problems go hand in hand" (p. 31). Therefore, it is hypothesised that children with MD are more likely to perceive their classroom as disruptive than children of average ability and that their individual achievement is more likely to vary as a function of individual differences in perceived classroom conduct because children with MD might be more responsive to the classroom environment and less focused on instructional activities. Also, it will be investigated whether

discrepancies in classroom perceptions of children with MD and average ability children are already present in class five, which is the first secondary school year, or whether they develop later, in class eight.

3.2 METHOD**3.2.1 DESIGN**

The intent of this quasi-experiment was to investigate the differences between children with and without mathematical difficulties in their affective responses towards mathematics, using the German version of the Affective Responses Towards Mathematics (ARTM) instrument.

3.2.2 PARTICIPANTS

The participants in this study were 52 children with MD and 91 children from an average achieving control group. The mean age was 12.6 years. Children from classes five (age 10 – 11) and class eight (age 13 – 14) took part in the study. Children from class five are in their first year of secondary school, children from class eight are reaching the end of their secondary school career, which goes up to year 10. The sampling of children with MD and average ability children and the characteristics of the sample will be described in more detail below.

Sampling procedure

Children were classified as having mathematical difficulties if they attended a general secondary school as opposed to a comprehensive school. This decision is based on the fact, that schoolchildren are tracked on different secondary school types depending on their ability in primary school, as determined by their report grades. Primary school children with low achievement scores in core subjects such as mathematics are referred to general secondary schools, whereas children with average and above average grades are able to attend comprehensive school. Although this

tracking system has been severely criticised in recent years, the tracking criterion to distinguish between children with MD and average ability children has been adopted for the present purpose to identify children with MD. Indeed, the ability tracking is based on teachers' assessment of children's mathematical ability, as reflected in children's report grades.

Report grades are not exclusively based on test performance. Apart from the results in assessments and tests, other factors are taken into account, for example children's behaviour in the classroom, their verbal contributions in the mathematics lesson and the quality of their homework. While report grades could be criticised for a lack of objectivity, they provide essential information to see if an intervention has the potential to improve children's ability to fulfil the requirements of everyday mathematics in a realistic context. Therefore, this criterion has a high degree of ecological validity in contrast to the standardised test of mathematical ability, which might not be indicative for children's ability to learn mathematics in the classroom setting.

Furthermore, children who have been referred to general secondary schools not only have difficulties in maths, but also in other core subjects such as German. This meets the definition of Minnie et al. (2007), who claim that mathematical difficulties are often part of a general lack of academic ability, whereas a mathematical disability means a specific impairment where only mathematical ability is affected. This justifies the sampling procedure that has been adopted for the purpose of this thesis.

This decision to adopt this criterion is supported by research outcomes from published performance benchmarks across secondary schools (Leutner, Wirth &

Fleischer, 2004). The performance benchmarks are the result of an empirical large scale assessment which was intended to examine the mathematical achievement of class nine students across different secondary school types. The results of the large-scale assessment indicate that general secondary school children lag behind their average achieving peers in mathematics, whereas children who attend comprehensive schools represent an average achieving population (Leutner, Wirth & Fleischer, 2004). This is illustrated in figures 3.1, 3.2 and 3.3 below. The tasks in the large scale assessment were targeted towards the abilities of class nine students. The tasks could be categorised in one of four levels of difficulty, ranging from P1 (basic) to P4 (complex) (www.learn-line.nrw.de/angebote/lernstand8/download/ergebn_05/kompetenzniveaus_05.pdf). These four different levels of achievement will be described briefly.

<p>P1 level:</p> <ul style="list-style-type: none">➤ Solving basic mathematical problems with obvious, routine strategies.➤ Extracting information from simple mathematical diagrams, charts or tables.➤ Students who do not achieve above P1 level do not fulfil the standards of class 9.	<p>P2 level:</p> <ul style="list-style-type: none">➤ Solving basic mathematical problems with known strategies.➤ Retrieving information from simple mathematical diagrams, charts or tables.➤ Bringing different pieces of information in relation to each other.
<p>P3 level:</p> <ul style="list-style-type: none">➤ Solving mathematical problems which require the selection and application of strategies.➤ Algorithms have to be inferred from similar strategies.	<p>P4 level:</p> <ul style="list-style-type: none">➤ Integrating information from different sources and solving complicated mathematical problems.

Figure 3.1: Different levels of mathematical achievement

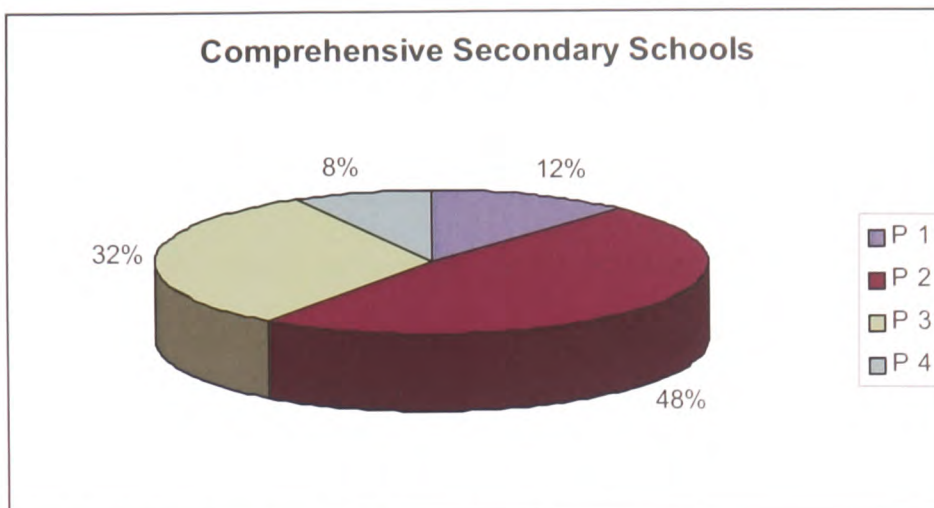


Figure 3.2: Achievement of children of average ability

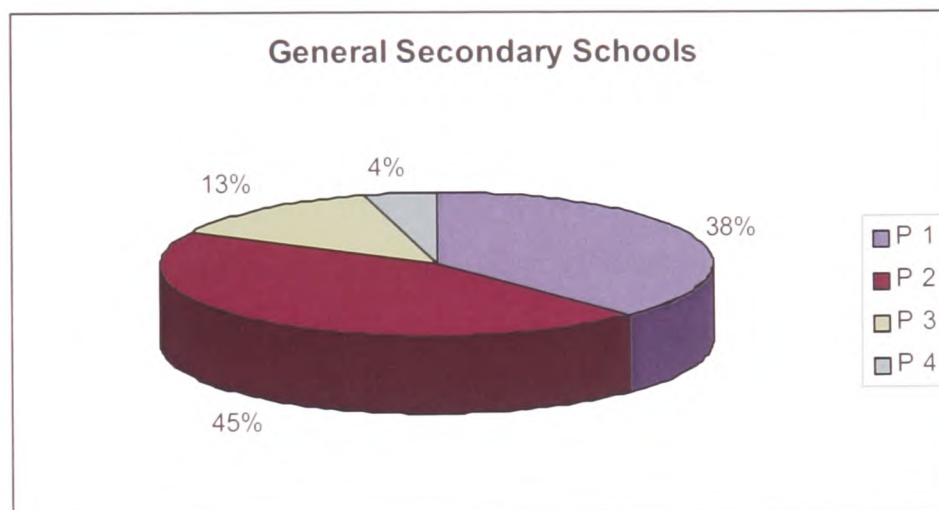


Figure 3.3: Achievement of children with MD

As can be seen from figures 3.2 and 3.3, the amount of children from general and comprehensive schools who achieve performance level P2 is comparable: 48% of children from comprehensive schools and 44.5 % of children from general secondary schools achieve level P2. While this implies that children from the two secondary school types are similar in performance, it is important to emphasize that there are considerable differences between children from general and comprehensive secondary schools where the higher performance levels, P3 and P4 are concerned. The figures indicate that at comprehensive secondary schools, 32% of students achieve performance level P3 but only 13% of students from general secondary school perform on this level.

The highest performance level, P4, is achieved by 8% of students from intermediate schools compared to 4% from general secondary schools. Students from general secondary schools, on the other hand, are over represented in the most basic performance level, P1. Nearly 40 % of the general secondary school students perform at this level. At comprehensive schools, only 12 % of the students fall within this category. The data described above support the view that, compared to their average ability peers from comprehensive schools, children from secondary schools generally seem to experience difficulties in mathematics.

The two schools selected for the research were chosen because their student population is comparable in many demographic variables. The information of the demographic composition of the student population has been derived from published data from a large scale educational assessment which included measures on the demographic characteristics of the catchment area. Based on this information, it was possible to categorise schools in one of three categories, as depicted in the table 3.1 below. The two schools where the present research took place fall into category one.

Table 3.1: Classification scheme for the demographic composition of the student population

	Category 1 general schools (36.4%)	Category 2 secondary schools (44.6%)	Category 3 secondary schools (less than 20%)
Student composition	At least 50% immigrant students with language deficits. At least 20% children from a low socio-economic background Almost no children from parents with higher education certificates.	25% – 50% immigrant students with language deficits. 10 – 20% children from low socio-economic background Up to 10% of children from families where at least one parent has a higher education certificate.	Less than 25% immigrant students with language deficits. Less than 10% of students come from families with a low social background More than 10% of children from families where at least one parent has a higher education certificate.
Catchment area	Children tend to live in urban high-density areas, which are characterised by a low real estate value, multi- storey buildings, large apartment blocks and the absence of garden or parks.	Children tend to live in suburban areas. The real estate value in these areas is middle to upper class.	Students live in rural suburban areas that are characterised by free-standing houses, green spaces and gardens.

Adapted from: (www.learn-line.nrw.de/angebote/lernstand8/download/ergeb_n_05/kompetenzniveaus_05.pdf.)

Materials/Apparatus

The ARTM was chosen because, unlike other instruments which are concerned with attitudes only, it is intended to measure affective responses. According to McLeod (1982), affective responses include not only attitudes but also beliefs and emotions such as anxiety. The intent behind implementation of the ARTM (Graff, Mayer, Lebens, 2007) was to assess the affective responses towards different aspects of mathematics of children with and without mathematical difficulties. The ARTM measures children’s affective responses in a Likert scale format, ranging from 1 (strongly disagree) to 5

(strongly agree). When items were phrased in a negative way, reversed coding was used to ensure that a mean score higher than 3 equals a positive affective response on the particular subscale and a mean score lower than 3 equals a negative affective response. The ARTM consists of four subscales which measure the following constructs: anxiety, affective responses towards the teacher, self-efficacy beliefs and affective responses towards the classroom environment. The tables below (3.2-3.5) provide a description of the subscales and the items which are included in each of the subscales.

As has been described earlier, the instrument meets the criteria of reliability and validity and measures children’s attitudes towards teacher–pupil interaction, classroom conduct during mathematics lessons, mathematics anxiety and self–efficacy.

Table 3.2: Items on the anxiety subscale

Item	Description
Item 4	I am afraid to come up with a wrong answer
Item 12	I am afraid of maths examinations
Item 18	I feel embarrassed when asked to come to the chalkboard.
Item 27	I wonder what the others think if I come up with a wrong answer

Table 3.3: Items on the attitudes towards the teacher subscale

Item	Description
ITEM 3	The teacher often doesn't notice if I have difficulties
Item 6	My teacher approaches all children equally
Item 11	My teacher always selects the same children
Item 15	I enjoy the maths lessons with my teacher
Item 17	My teacher's explanations help me understand
Item 23	My teacher ensures that I understand maths
Item 25	My teacher can't explain a new topic well
Item 28	My teacher's explanations are difficult to understand

Table 3.4: Items on the affective responses towards the classroom environment subscale

Item	DESCRIPTION
ITEM 5	The maths lessons is often disrupted by pupils
Item 22	My class fools around during maths lessons
Item 24	It takes a long time until we can really start the lesson
Item 31	It is quiet during our mathematics lessons
Item 36	The maths lessons run without disturbances
Item 37	If you're not gifted, maths is very difficult

Table 3.5: Items on the self-efficacy subscale

Item	Description
ITEM 38	If I learn well, I will have good grades in maths
Item 39	I will never be good at maths
Item 41	I simply have no talent for maths
Item 43	It is difficult for me to sit down and learn maths

The association between children's affective responses and their individual performance was measured using the individual mathematics grades of children, as indicated on their school reports. It is assumed that school grades provide a more realistic measure of children's mathematical ability than the results of a standardised test because children might fail in such a test as a result of test anxiety. The school grades, on the other hand, are composed not only of the average mark from three written mathematics tests, but also a variety of different assessments such as marked homework, participation in the classroom and the evaluation of children's overall achievement by the teacher. Therefore, it is assumed that children's school grades from one semester provide a comprehensive measure of their ability to perform mathematics in school. The grades range from one (very good) to six (fail), which means that a high grade indicates low achievement. Therefore, the coding of the grades is in direct opposition to the grading of children's affective responses.

3.2.3 PROCEDURE

The assessment took part towards the end of the first semester, meaning that children from year five had sufficient experience with learning mathematics at a secondary school level to express their affective responses towards the different aspects of learning mathematics in classroom settings. A teacher and the researcher were present to supervise the children; in order to achieve effective supervision, not more than fifteen children were tested at the same time. After students sat down, they were made to feel comfortable by approximately five minutes of informal talk. They were briefly introduced to the rationale of the attitude research. They were told that the researcher wanted to find out how children at their age think about learning mathematics. Because students voiced concern that their responses might be used

against them, it was emphasized that their data would be handled confidentially and would not be shown to their mathematics teacher or anyone else. A ruler was handed out to each child so that they could systematically go from item to item, making sure that they responded to each item and ticked only one option per row. After the students had finished the questionnaire, they were thanked and asked to go back to their class quietly. On average, the questionnaire took 30 – 35 minutes to complete.

3.3 RESULTS

The means in table 3.6 indicate that children with mathematical difficulties (MD) yielded lower scores on the anxiety scales than children of average ability. A low score indicates a negative affective response towards learning mathematics. It is important to emphasize that negative coding has been used for positively phrased items. Therefore, a low score indicates that children stated that they experienced feelings of anxiety and disagreed with items which stated that they experienced no or little anxiety. As can be seen in table 3.6, children with MD had a mean score of 2.82, whereas children of average ability had a mean score of 3.44. Thus, the low scores of the children with MD suggests that they are more anxious about learning mathematics than their average ability peers. The results of the two-between factor ANOVA show that the main effect of achievement is statistically significant $F_{(1, 139)} = 34.07, p < .05$.

Anxiety Subscale

Table 3.6: Mean scores and standard deviations for the results on the anxiety subscale

Group	Class	Mean	SD	N
<i>Children of average ability</i>	5	3.43	0.61	29
	8	3.46	0.53	23
	Total	3.44	0.57	52
<i>Children with MD</i>	5	3.10	0.60	46
	8	2.53	0.65	45
	Total	2.82	0.69	91

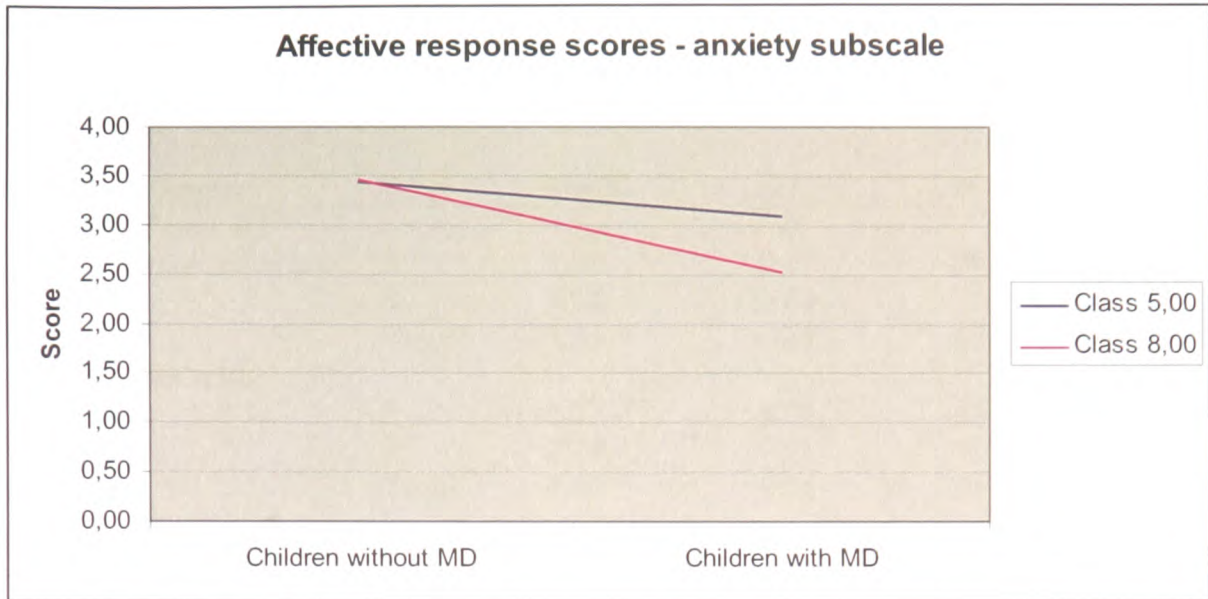


Figure 3.4 : Differences in mean anxiety scores between children with MD and average ability children from classes five and eight

As shown in figure 3.4, the anxiety score of children with MD and average ability children differs across classes five and eight. This reflects the fact that there is a significant interaction effect between the two variables achievement and class $F_{(1, 139)} = 7.92, p < .05$. This reflects the mean scores as displayed in table 3.6, which shows that children of average ability from class five yielded a score of 3.43 and children from class eight a score of 3.46.

*Attitudes towards the teacher***Table 3.7:** Mean scores and standard deviations for children's attitudes towards the teacher

Achievement	Class	Mean	SD	N
<i>Children of average ability</i>	5	3.98	0.75	29
	8	3.92	0.88	23
	Total	3.95	0.80	52
Children with MD	5	3.57	0.70	46
	8	2.74	0.81	45
	Total	3.16	0.86	91

Table 3.7 shows that the scores on the attitudes towards the teacher subscale differ between children of average ability and children with MD. Again, the Likert-Scale format of the instrument means that a score above 3 implies a positive affective response whereas a lower score implies a negative response. While both groups have a mean score above 3, table 3.7 indicates that children of average ability have a score of 3.95 compared to a score of 3.16 in the group of children with MD. A two-between factor ANOVA shows that the main effect of achievement is statistically significant $F_{(1, 139)} = 34.85, p < .01$. As with the anxiety subscale, the results of the ANOVA show that there is an interaction effect between achievement and class $F_{(1, 139)} = 8.24, p < .05$. This interaction effect is evident in figure 3.4.

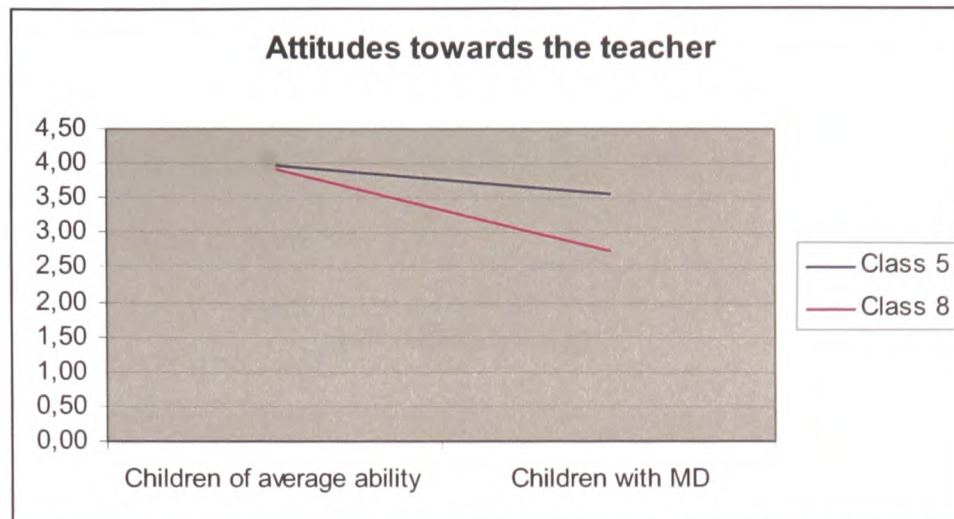


Figure 3.5: Differences in attitudes towards the teacher between children MD and average ability children from classes five and eight

Figure 3.5 indicates that the observed interaction effect can be attributed to the difference between children with MD from classes five and eight in their mean scores on the attitudes towards the teacher subscale. The mean scores of children of average ability, on the other hand, are similar across classes five and eight.

Self-efficacy beliefs

Table 3.8: Mean scores and standard deviations for children’s self – efficacy beliefs

Group	Class	Mean	SD	N
<i>Children of average ability</i>	5	3.86	0.61	29
	8	4.00	0.49	23
	Total	3.92	0.56	52
Children with MD	5	3.70	0.63	46
	8	3.31	0.90	45
	Total	3.50	0.80	91

In line with the findings from the analysis of the previous scales, table 3.8 shows that children with MD yielded lower scores on the **self-efficacy scale** than children of average ability. Children of average ability yielded a mean score of 3.92, compared to a mean score of 3.5 yielded by children with MD. The results of a two-between factor

ANOVA shows that the main effect of achievement is statistically significant $F_{(1, 139)} = 12.13, p < 0.1$. Also, the analysis indicates an interaction effect between achievement and class which is statistically significant $F_{(1, 139)} = 4.56, p < .05$. The interaction is graphically displayed in figure 3.6.

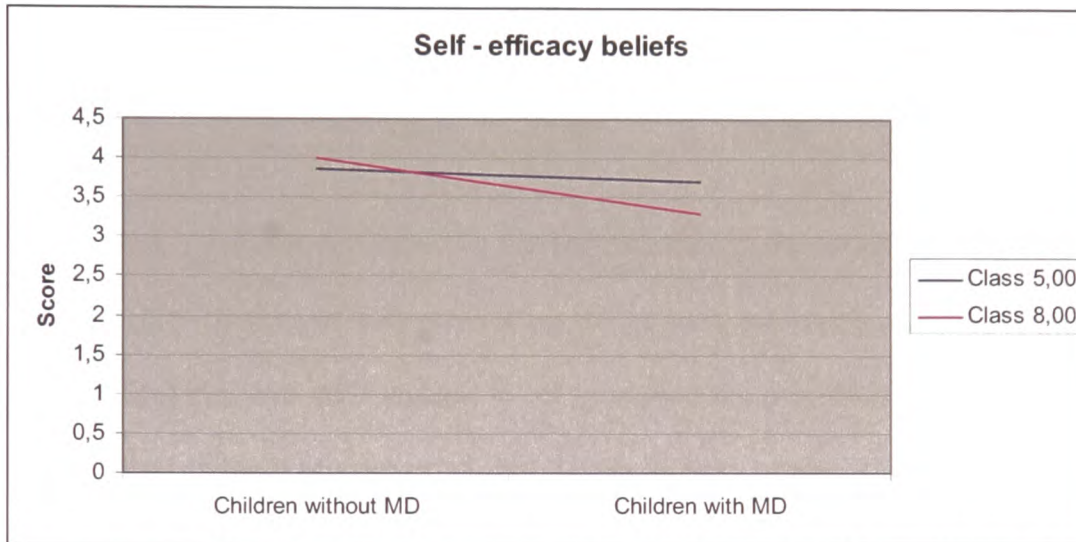


Figure 3.6: Differences in self–efficacy beliefs between children with and without MD from class five and eight

As shown in figure 3.6, children with MD from class eight had lower self–efficacy scores than children with MD from class five. This finding is also reflected in table 3.8, which shows that children with MD from class eight yielded a score of 3.31, whereas their peers from class five yielded a score of 3.7. Figure 3.6 shows that the finding is reversed in the group of children of average ability. As can be extracted from the means in table 3.8, children of average ability from class eight yielded a score of 4 compared to a lower score of 3.86 scored by children of average ability from class five.

Perceived classroom conduct

Table 3.9: Mean scores and standard deviations for children’s perceived classroom conduct

Group	Class	Mean	SD	N
Children of average ability	5	3.87	0.69	29
	8	4.02	0.41	23
	Total	3.94	0.58	52
Children with MD	5	2.89	0.72	46
	8	2.70	0.68	45
	Total	2.80	0.70	91

Table 3.9 shows that there is a marked difference between children with MD and average ability children in their mean score on the scale measuring perceived classroom conduct. Children with MD yielded lower scores on the perceived classroom conduct subscale than children of average ability. As can be seen in table 3.9, children with MD had a mean score of 2.8, whereas children of average ability had a mean score of 3.94. Thus, the low scores of the children with MD suggests that they reported more classroom disturbances than their average ability peers. The results of the two-between factor ANOVA show that the main effect of achievement is statistically significant $F(1, 139) = 99.05, p < .01$.

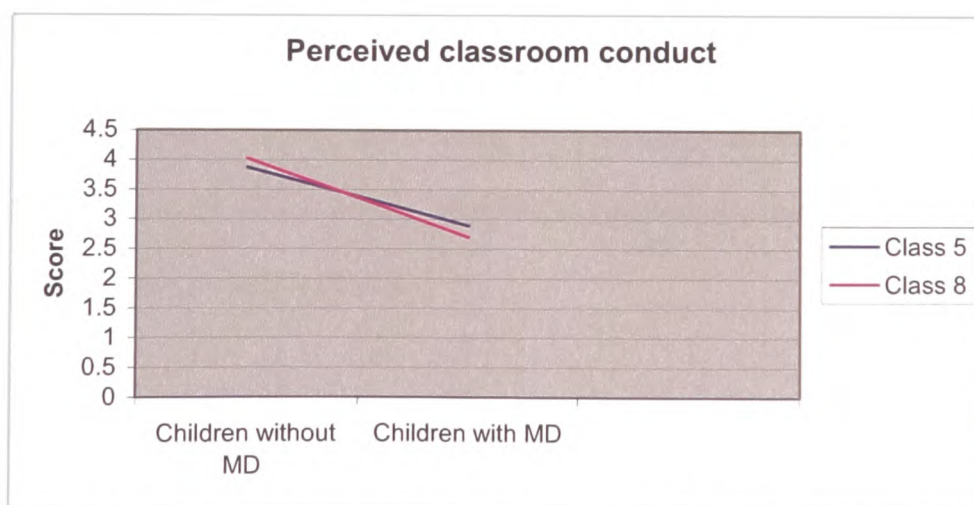


Figure 3.7: Differences in affective responses towards the classroom between children with MD and average ability children from class five and eight

3.4 DISCUSSION

The current study was intended to determine differences in affective responses towards mathematics between children with MD and children of average ability, and to determine the association between affective responses and mathematical achievement, as determined by school grades.

3.4.1 SELF-EFFICACY

A key finding which emerged from the current study is that children with MD from class eight hold significantly lower self-efficacy beliefs than their younger counterparts from class five. Hence, it seems that the self-efficacy beliefs of both younger and older children of average ability are similar, whereas the self-efficacy beliefs of children with MD seem to deteriorate over time. The finding that differences in self-efficacy beliefs between children only emerge in class eight seems to suggest that repeated experiences of satisfactory achievement helps children to maintain a positive judgement of their ability to accomplish their learning goals, whereas repeated experience of low achievement results in a negative perception of one's capability to attain learning goals. Overall, this finding unequivocally shows that self-efficacy is linked to achievement, which follows Bandura's argument that personal achievement is an important factor for self-efficacy.

Self- efficacy in children from class five

An interesting finding was that children with MD and average ability children from class five expressed similar self-efficacy beliefs. This finding was unanticipated, because the existing literature indicates that low achieving children are typically

associated with low self-efficacy beliefs. Such findings have previously been used to support the notion that low achievement precedes low self-efficacy beliefs. The finding that children with MD and average ability children do not significantly differ in their self-efficacy beliefs seems to contradict this assumption.

To recap, children were tracked on different secondary school types after four years of primary school, depending on their ability to perform maths during their primary school years. Thus, children with MD had already experienced frustration and low achievement in primary school before they were tracked with peers of the same ability in class five. In spite of these experiences, children with MD did not exhibit significantly lower self-efficacy beliefs than their average achieving peers. This implies that achievement itself constitutes a mediating factor rather than a causal factor for self-efficacy and corresponds to Schunk's (1983) assertion that while self-efficacy is affected by previous accomplishments it more than just a reflection of them."

Self- efficacy beliefs in children from class eight

Whereas children with MD and average ability children do not differ in their self-efficacy beliefs, there are significant disparities between reported self-efficacy beliefs in class eight. The findings show that children with MD express lower self-efficacy beliefs than their average achieving peers. Also, differences in self-efficacy were found to be a significant predictor for achievement differences among children from class eight with MD, but not for children of average ability.

However, if previous achievement is a determinant of self-efficacy, it could be argued that, by class eight, children with MD and average ability children have had

positive as well as negative results with respect to their achievements. In both groups, there are children who perform below average and children who perform above average. Therefore, it could be criticised that there should be no difference between the mean self-efficacy belief scores of children with MD and average ability children in class eight. As will be discussed below, Pajares (1997) provides a possible resolution to this.

Collective self-efficacy

While accepting that individual achievement is a key variable for self-efficacy beliefs, Pajares (1997) points out that achievement does not directly result in self-efficacy beliefs but that self-efficacy beliefs are based on an individual's interpretation of their learning outcomes. As has been indicated in the literature review, Bandura (1986) emphasizes that the achievement of others is an important source of self-efficacy. To recap, Bandura proposed the concept of "collective efficacy" to illustrate that judgments of one's capability to achieve a goal are always made with respect to social criteria, such as the performance of same-age peers. Collective efficacy refers to the perceived performance capability of an entire social system such as the classroom. According to Bandura (1982), individual difficulties often reflect group difficulties, which are considered to be more demoralising and behaviourally self-debilitating than external obstacles. However, people who foster a strong sense of collective efficacy are more likely to mobilise effort and persist in the light of obstructions, and this notion can be applied to the group of children with MD.

The collective efficacy concept can be utilised as an explanatory framework for present findings: children of average ability with a positive collective efficacy might be less affected by a negative test score or even failure than children with a history of MD, who receive more negative responses within their social surrounding and develop

negative collective efficacy beliefs over time. Collective efficacy is likely to be impoverished for children with MD. Also, children with MD are aware that their low ability is public knowledge. Because they are publicly identified as a group of children with MD due to attendance at a general secondary school, their situational circumstances are difficult. The school that children with MD attend indicates that these children do not meet the achievement standards of their average-achieving peers and this might create a social stigma for this group, resulting in a negative evaluation of their capabilities to attain a certain goal from both themselves and others. Thus, the experience of children with MD of being perceived, categorised and labelled as a low achiever gives rise to the decline in self-efficacy beliefs throughout their schooling career.

The results suggest, however, that negative collective self-efficacy beliefs are not automatically present in class five, at the start of secondary school. Rather, they seem to develop over time, as children with MD experience repeated episodes of frustration. Indeed, children with MD from class five experience their first year at a general secondary school and might therefore still reap the benefits from their previous experience at primary school, where children of different abilities were in mixed classes. It could be argued that in primary school, the collective self-efficacy beliefs were more positive.

Average-achieving peers might have provided a role model for children with MD and showed that effort has a positive effect on achievement. In such an environment, children with MD are able to maintain more positive self-efficacy beliefs. However, as time progresses, the collective self-efficacy beliefs of children with MD seem to deteriorate. Difficult situational circumstances and repeated negative

experiences of being perceived as a low achiever by others give rise to negative self labelling, which influences children's collective efficacy appraisals. The results indicate that, by class eight, the interaction of these factors contributed to a low collective efficacy which is predictive for children's achievement. Thus, the concept of collective self-efficacy indicates that, while individual achievement constitutes a mediating variable, the perception and interpretation of achievement within one's social reference framework is another important factor for efficacy appraisal.

Causal attributions and efficacy appraisal

As indicated in the literature review, the attributes of past outcomes constitute an important source of information for future efficacy expectations. According to self-efficacy researchers such as Bandura (1982) and Schunk (1983), the perception and interpretation of achievement is strongly linked to causal ascriptions of success or failure. There is evidence that achievement outcomes are mainly ascribed to either ability, effort or task difficulty. With respect to perceived ability, the present results indicate that children with MD are inclined to believe that the capability to achieve high marks in mathematics is inborn rather than a matter of learning and effort, a belief which might be sustained through repeated experience of failure. Such an attribution style implies that their actions have little impact on the outcome and this might lead them to believe that it is not worth putting in a great effort. Indeed, feeling the belief that achievement outcomes lie outside one's scope of action might give rise to learned helplessness and withdrawal behaviour. Children of average ability, on the other hand, are able to maintain a positive sense of self-efficacy because they attribute success or failure in mathematics to factors such as learning. Children with such an attribution

style are more likely to feel in charge of their learning process and put in more effort after experiencing failure, rather than withdrawing.

As a consequence of differences in causal ascriptions, children from an average achieving population and a low achieving population may respond differently to the same result. While a low achieving child might withdraw from further effort when confronted with a negative result, a child from an average achieving population might accept the negative result and possibly see it as an incentive to work even harder next time. This supports Bandura's (1982) argument that "those convinced of their inefficacy will cease trying, even though changes are attainable through concerted effort" (p.139). In summary, the literature on the importance of causal attributions for efficacy appraisal supports the argument that it is not the achievement outcome itself, but the individual's interpretation of the outcome that is a major source for future efficacy expectations.

Conclusion

To sum up, children with MD and average ability children do not differ significantly in their self-efficacy beliefs in their first year of secondary school. This seems to change over time, in that self-efficacy beliefs in children with MD seem to decrease between class five and class eight, whereas no differences were observed between children of average ability from classes five and eight. Thus, even if children from both groups are likely to have experienced failure at some point up to class eight, children of average ability from class eight displayed significantly more positive self-efficacy beliefs than children with MD. Therefore, the present results allow the tentative conclusion that low achievement in itself is not the source of low self-efficacy beliefs. This findings corroborate previous results, which show that self-efficacy beliefs go

beyond a reflection of past achievement and that the inferential process by which people derive those beliefs is based, not only on personal factors, but also on situational factors such as the social context which provides a reference framework for individual achievement (Bandura & Schunk, 1981).

Attendance at general secondary school is a synonym for low achievement and academic and behavioural difficulties, which might give rise to negative self-labelling processes. Being classified as children with MD is likely to be detrimental for children's perceived ability and might also decrease the effort that children devote to learning. Thus, it is safe to assume that, overall, children with MD foster more negative collective self-efficacy beliefs. In line with the literature discussed above, it seems plausible that the corrosion of self-efficacy in children with MD might be due to an interaction between environmental / situational efficacy appraisals, perceived collective efficacy and attribution style, which comprises perceived achievement and outcome expectations.

A key limitation of the present study is that no items were targeted towards collective efficacy. As has been discussed above, the interaction effect between perceived collective efficacy and situational / environmental responses towards children's achievement is an important source for efficacy expectations and future research on self-efficacy might therefore be more cognisant of this factor. In addition, future research might address the relationship between collective efficacy, situational / environmental experiences and causal ascriptions by investigating children's beliefs about the origins of their mathematical difficulties. Future research on these correlations of low self-efficacy is important to provide interventions which help to modify children's efficacy expectations.

3.4.2 MATHS ANXIETY

The results of maths anxiety in children with MD produced an outcome which corroborates the findings of a great deal of the previous work in this field: the findings show that children with mathematical difficulties (MD) are significantly more anxious than children of average ability. The findings are in conjunction with the results of Hembree (1990), who reported a significant negative correlation between maths anxiety and maths achievement in school children. Similarly, Ma (1999) found a significant correlation between achievement and anxiety, which was constant across gender, class, year of publication and the instruments that have been used to measure mathematics anxiety. However, as will be explored in the following discussion, the causal direction of mathematics achievement and anxiety remains ambiguous (Ma & Ho, 2000).

Developmental trajectories of maths anxiety

In addition to the overall difference in anxiety between children with MD and average ability children, the results suggest that anxiety seems to develop differently between classes five and eight in children with MD and average ability children. There is a significant difference between children with MD from classes five and eight, in that children with MD are significantly more anxious than average ability children from class five. This suggests that anxiety increases from class five to eight in children with MD. Contrary to expectations, this study did not find a significant difference in anxiety between children of average ability from classes five and eight. This finding is in opposition with the results of Hembree's meta-analysis (1990) of 151 studies on mathematics anxiety, which showed that mathematics anxiety increases up to class nine or ten and then starts to decrease. Hembree's findings have been utilised to support the

predictions of the deficit model, developed by Tobias (1985). In essence, this model assumes that mathematics anxiety is not the cause but the product of low achievement.

However, the prediction of the deficit model seems not to be supported by the present finding that anxiety seems to increase only in children with MD, whereas anxiety in children of average ability remains constant. It could be argued that, if the increase in anxiety can be attributed to the frequency of failure and low achievement, as predicted by the deficit model, then there should also be an increase in anxiety between average ability children from class five and eight because even average ability children are likely to have experienced low achievement. Indeed, individual grades are normally distributed in both groups and therefore there are children with low grades, even among the group of children of average ability. However, the results show that this is not the case. Therefore, it seems that increasing maths anxiety is not solely related to individual achievement per se, but might be mediated by a different factor, as will be discussed below.

Stressors for anxiety in children with MD

One possible explanation for the increase of anxiety in children with MD from class five to eight is that the repeated experience of being categorised and stigmatised as a low achiever leads to a negative self-evaluation, which might be more related to anxiety than individual achievement. Indeed, the association between individual mathematics grades and anxiety measurements differs markedly between children with MD and average ability children. According to the results of a multiple regression analysis, anxiety predicts nearly variance in achievement of children with MD whereas there is no significant association between anxiety level and mathematics grades for children of average ability.

One possible explanation for the pattern of findings is that children with MD from class eight with a history of repeated failure might be more aware that low mathematics grades have a serious effect on their future aspirations and, as a consequence, develop a fear of failure towards the end of secondary school. This is supported by the finding that there is no significant association between anxiety level and mathematics grades for children with MD from class five. Thus, the achievement of children with MD is not associated with anxiety in the beginning of secondary schooling but, over the years, anxiety appears to become a key predictor for achievement.

The finding that the effect of the role of anxiety on individual achievement decreases in children of average ability but increases in children with MD from class five to eight supports the hypothesis that anxiety is a function, not only of low achievement or failure, but of repeated negative experiences within a particular school environment. This follows the argument of Dossel (1993), who criticises that attempts to establish a causal relationship between anxiety and achievement have been unsuccessful in the past “perhaps anxiety and achievement are caused by a different factor – possibly the learning / teaching environment” (p.7). Indeed, Newstead (1998) emphasizes that environmental factors have considerable impact on maths anxiety. A similar notion has also been put forward by Stodolsky (1985), who perceives negative experiences in the classroom environment as one of the most important stressors for mathematics anxiety (Stodolsky, 1985).

The results seem to indicate that the longer children attend a general secondary school which is almost exclusively populated by low achieving children with learning difficulties, the greater the likelihood that they have experienced failure and frustration

which give rise to feelings of tension and anxiety and lead to a vicious circle. The negative experiences might be due to the fact that, irrespective of the individual grades, children with MD are considered to be “children with MD” because they attend a certain school type and have very limited occupational chances. Thus, children with MD are more likely to experience negative responses from parents, the teacher and even their peers which, in turn, lead to negative self-evaluation and anxiety. This might form a vicious circle, because there is evidence that anxiety interferes with the cognitive processes required for mathematics and this interference affects children’s performance (Kirk & Ashcraft, 2001). Such an explanation would be advocated by proponents of the interference model, which has been previously described.

Anxiety and achievement – the interference model

The interference model can be linked to more recent findings from information processing research on the relationship between anxiety and working memory resources (Ashcraft & Kirk, 2001). This would support evidence that children with MD have limited working memory resources and are therefore more at risk of cognitive overload than children of average ability. Cognitive overload might arise as a function of anxiety because limited working memory resources are occupied by intrusive thoughts and worry, which impedes recall of stored knowledge from long term memory. The inability to recall prior knowledge in a given situation is likely to reinforce feelings of pressure and tension, which then lead children with MD to respond more anxiously towards maths than children of average ability.

It could be argued that, if the relationship between achievement and anxiety in children with MD is due to the interference between feelings of anxiety and limited working memory capacities, then this should not only apply to children with MD from class eight but also to children with MD from class five, because children with MD have been identified as children with a risk of recall problems, irrespective of their class. However, the results not only showed that children with MD from class eight exhibited significantly more anxiety towards learning mathematics than children with MD from class five, but also that there was an association between anxiety in achievement for children with MD from class eight which was absent in class five. However, this should not be taken as evidence against the interference model.

Rather, it could be argued that the curriculum for children from class five at general secondary schools is not very complex and constitutes mainly a repetition of basic arithmetic which was taught in primary school, in order to remedy the deficits of children with MD who were tracked on the general secondary school because of their low achievement in primary school. Children with MD might feel relief because, for the first time, they are among peers of the same level of ability, which means they might feel less anxious about bad grades. It is likely that children with MD experienced this feeling of anxiety in primary school because children with MD and average ability children learned together in a mixed classroom, which meant children with MD were at a disadvantage compared to their average-achieving peers. Once children with MD were tracked onto the general secondary school, the feeling of being inferior and the fear of embarrassment might no longer exist because they are learning with peers who are similar in terms of achievement. Again, this would support Newstead's view (1998) that the classroom environment is a key factor in producing or preventing maths anxiety.

Therefore, it might be there is no interference between anxiety and cognitive processes in class five but the interference occurs in class eight, where more complex tasks are concerned which require more working memory resources. This supports the findings of Ashcraft and Faust (1994), who concluded that anxiety interferes mainly with more complex mathematical tasks which require more working memory space. Thus, the interference hypothesis seems to provide a valid explanatory model for the findings that a) there is a significant increase in anxiety among children with MD between classes five and eight b) there is an association between anxiety and individual achievement for children with MD from class eight, but not for children with MD from class five.

To recap, anxiety is predictive for the variance in achievement of children with MD from class eight, but not for children with MD from class five, which suggests that anxiety gradually increases over time. Contrary to expectations, this is the other way round for children of average ability, where anxiety is predictive for mathematics grades in class five but not in class eight. Despite this apparent inconsistent pattern of findings, these results add further support to the assumption of the interference hypothesis. According to Hunt (1985), everyday learning activities in the mathematics classroom such as going to the chalkboard or being asked by the teacher to answer questions verbally can cause anxiety in children because they fear revealing their inability in front of their peers.

The interference model predicts that these feelings of pressure and anxiety interfere with children's cognitive processes; this would explain the association between anxiety and achievement in children of average ability from class five. In contrast to their peers with MD, average ability children from class five might feel more pressure

and competitiveness due to a higher pace of instruction at comprehensive schools in the first year of the secondary school. In other words, the interference effect might be absent in children with MD because their curriculum is less complex and requires fewer cognitive processing resources than the curriculum for children of average ability.

Conclusion

The finding that there were no significant differences in anxiety between children from class 5 and 8 of an average ability cohort children whereas there is a significant increase in anxiety between children from class 5 and 8 points to a complex interplay between anxiety, achievement and the specific experiences children encounter in their particular school environment, which include the responses from significant others such as teachers, peers and parents. This notion corresponds to the argument forwarded by Ma and Xu (200), namely that anxiety is a function of the interaction of cognitive, personal and environmental factors. Indeed, the strength of the relationship between maths anxiety and achievement becomes insignificant or is substantially reduced when controlling variables such as attitudes towards the teacher, attitudes towards the mathematics classroom environment and children's perception of their abilities are introduced (Fennema & Sherman, 1977). An analysis of the measurement of self-efficacy and children's attitudes towards the teacher is therefore essential in order to discuss how these constructs are related to the measure of anxiety.

3.4.3 ATTITUDES TOWARDS THE TEACHER

To recap, “attitudes towards the teacher” are distinct from “perceptions of the teacher”. The rationale behind this decision is that the subscale contains items such as “I enjoy mathematics with my teacher,” which trap attitudes and therefore involve favourable or unfavourable evaluation, whereas other items such as “my teacher approaches all children equally” trap perceptions and beliefs, which merely represent an individual’s information about the object (Fishbein & Azjen, 1975).

Overall, the findings show that attitudes towards the teacher are significantly affected by children’s general achievement level: children with MD from classes five and eight respond more negatively towards their teacher than children of average ability. According to the ARTM scale, this finding indicates that children of average ability perceive their teacher as more helpful and supportive and they perceive the teaching style as more effective than their peers with MD. This corresponds to previous findings in the literature, which show that the perceived supportiveness of the teacher and the perceived quality of the teacher–student interaction emerged as reliable predictors of mathematical achievement (Dossey, 1992). In order to account for their difficulties, children with MD are tracked on general secondary schools, where behavioural peculiarities and a lack of classroom discipline are particularly prevalent. The finding that these children hold more negative attitudes towards the teacher imply that problematic behaviour and disobedience is related to lower levels of teacher support.

Developmental trajectories of attitudes towards the teacher

The previous literature shows that children's attitudes towards the teacher deteriorate as they proceed through the secondary school years (Midgley, Feldlaufer, Eccles, 1989). However, the present results provide only partial support for this notion; where the affective responses of children with MD actually appear to decrease over the years, the affective responses of children of average ability remain constant from class five to class eight. The finding that the decrease in affective responses over the years has only been observed for children with MD tentatively suggests that children's attitudes towards the teacher vary not only as a function of age, but also of achievement. Thus, it seems plausible that the attitudes towards the teacher develop differently across the secondary school years in children with MD and average ability children. It seems possible, however, that the teacher–student interaction differs between children with MD and average ability children and gives rise to different affective responses towards the teacher.

While there was no significant association between attitudes towards the teacher and individual grades for children with MD from class five, the result is different for children with MD from class eight where the response towards the teacher accounts for 41% of the variance in individual grade. One possible reason for this association might be that by class eight, children with MD might have developed their own explanatory model for academic success and failure and they might be aware that the support of the teacher is crucial for academic success. The idea that children from class eight developed their own theories for success or failure, in which the teacher plays an important role, can be linked to the concept of self–efficacy, which refers to children's evaluation of their ability to attain a certain level of achievement (Bandura, 1986).

Self-efficacy and attitudes towards the teacher

Based on the present results, it seems plausible to assume that children with MD from class eight believe that their achievement is largely determined by their teacher and his or her support and teaching style rather than the outcome of their own actions. Indeed, previous studies have demonstrated that average achievers tend to explain success in terms of internal, stable factors such as effective learning which is linked to a positive feeling of self-efficacy, whereas children with MD tend to attribute academic success, not to internal factors, but to external factors such as the teacher's support and they believe that success or failure is determined by external factors that lie outside their scope of action (Weisz, 1984). In other words, children with MD might simply withdraw when confronted with disappointing results and attribute their underachievement to the teacher whereas children of average ability might acknowledge that they should learn more.

This hypothesis might also explain why children with MD from class eight hold significantly more negative attitudes towards the teacher than children with MD from class five. To recap, it might be the case that children with MD from class eight attribute academic success to external factors such as teaching style or the level of support they receive, which results in: a.) attitudes towards the teacher which are more negative than the responses of class five children and b.) the association between their attitudes towards the teacher and their individual grades.

The abovementioned hypothesis receives further support from the finding that the association between attitudes towards the teacher of children with MD from class eight and their achievement is absent for children with MD from class five. Hence, it

seems possible that the association between children's affective responses and their achievement in class eight is linked to their theories of success and failure, as well as their self-efficacy beliefs, because children's belief systems develop over time. In class five, children might not perceive their teacher as an important factor for academic success, presumably because they believe that success is the outcome of their own actions. However, over the secondary school grades, children with MD form their own theories for the reasons behind their low achievement and this might result in the belief that academic success is largely determined by the teacher. This possibility is reversed for children of average ability, a group which is generally considered as average achieving, irrespective of their individual school grades. Whereas attitudes towards the teacher are predictive for individual grades in class five, such an association is absent in class eight, which suggests that children of average ability might begin secondary school with the assumption that their achievement can be attributed to the teacher, but then change this theory in class eight as a result of their experience of self-efficacy.

Teaching style and attitudes towards the teacher

An additional reason why there is a strong association between attitudes towards the teacher and the individual grades of children with MD from class eight but not from children of average ability from class eight is that the learning process of children with MD is more likely to be affected by the way in which mathematics is explained to them (Carnine, 1997). This supports a robust finding in the literature, namely that children with MD need highly structured and explicit instruction (Bottge, 2001; Kroesbergen & Van Luit, 2003; Kirschner, Sweller, Clark, 2006) while children of average ability are less dependent on the teacher because they are able to learn mathematical concepts and strategies through discovery and exploration. Children with MD need to have clear

explanations and worked out examples for how to use the appropriate strategies in order to solve a problem.

Therefore, teaching style and format of instruction are key factors for the individual achievement of generally low achieving children. The reason why there is no association between attitudes towards the teacher and individual grades of children with MD from class five might be that the curriculum for class eight is far more complex than in class five and the way in which mathematics is explained by the teacher might therefore be more important for older children with MD than for children from class five. Therefore it seems plausible that the supportiveness and teaching style of the teacher are predictive for the individual achievement of children with MD from class eight.

Conclusion

To sum up, attitudes towards the teacher are predictive for individual achievement in the early years, but not in class eight where children of average ability are concerned; however, this finding is reversed for children with MD, as attitudes towards the teacher only seem to have more impact in the later years. This implies that the role of the teacher in individual achievement gradually decreases in children of average ability who have established a positive self-concept and are able to perform at an average level, irrespective of the way in which mathematics is taught. The individual achievement of children with MD, on the other hand, appears to become more closely associated with their responses towards the teacher in the higher grades, presumably due to a lack of self-efficacy. This is related to both repeated episodes of disappointment

and the externalisation of attributes for success and failure. It is suggested that the association of these factors is investigated in future studies.

The most important limitation of the current study lies in the fact that each of the different samples had a different mathematics teacher, which means that differences in attitudes towards the teacher might be due to differences in the teachers' personalities rather than due to differences between children with MD and average ability children from classes five and eight. A longitudinal study of stability and change in children's affective responses towards one teacher over an extended period of time might address this issue. Given that a positive interaction between student and teacher is particularly important for at-risk children, future research might also be required to examine which interventions might help to improve children's affective responses towards their mathematics teacher.

3.4.4 PERCEIVED CLASSROOM CONDUCT

In contrast to previous studies which were concerned with children's perception of more general aspects of the classroom environment (cf. Goh and Fraser, 1998), the present study was specifically focussed on perceived classroom conduct as a specific domain of the classroom environment. The "perceived classroom conduct" subscale of the ARTM referred to the perceived behavioural conduct of children during mathematics lessons. The subscale referred to perceptions rather than attitudes because attitudes towards a certain object typically involve a favourable or unfavourable evaluation, whereas perceptions and beliefs merely represent an individual's information about the object (Fishbein & Azjen, 1975). The conceptual distinction between perceptions and attitudes is crucial because children who perceive their classroom environment as noisy and turbulent might not necessarily have a negative attitude towards misconduct.

Overall, the findings show that children with MD are more likely to perceive their classroom is subject to disturbances and misconduct than children of average ability. While the literature on differences between children with MD and average ability children with respect to their perceived classroom environment is not extensive, the present finding is consistent with other studies which showed that low achieving children tend to be associated with behavioural peculiarities, classroom disturbances and generally troubled classroom environments (Rösner, 1997). This supports Kauffman's (1997, p. 31) argument that "low achievement and behaviour problems go hand in hand." The absence of any interaction effects between the affective responses, ability and class suggests that the classroom environment remains relatively stable as children proceed across secondary school years.

Developmental trajectories of perceived classroom conduct

An interesting finding was that perceived classroom environment did not significantly differ between children from classes five and eight, neither among children with MD nor among children of average ability. It was anticipated that, at the start of adolescence, children in class eight in both groups would report more classroom disturbances. Children of this age, thirteen to fifteen, are typically assumed to be less conforming in the classroom and it was supposed that this affects the perception of their classroom environment. However, the results challenge this assumption. One potential explanation for these results is that while classroom conduct might actually decrease compared to that of class five, children in class eight might become habituated to their classroom environment. Indeed, perception does not arise within a social vacuum and children's perception of their classroom conduct might be biased because they are themselves part of their classroom environment.

Children of average ability from class eight perceive the classroom to be less disruptive than children with MD. Children with MD, on the other hand, seem to be exposed to a more difficult classroom environment and the results show that the perception of classroom difficulties is predictive for individual differences in achievement. Taken together, these findings are in conjunction with the findings of Goodenow (1992) and Hymel, Comfort, Schonert-Reichel and McDougall (1996), who found that children's perception of the classroom is an important predictor for academic achievement.

The association between children's perceptions of the classroom environment and achievement can be explained with reference to social cognitive concepts such as self-efficacy (Ames, 1992). As argued on page in the literature review, advocates of such an approach to classroom problems assume that "perceptions of the classroom influence students' beliefs about themselves...and these beliefs, in turn, influence the nature and extent of their engagement in academic tasks" (Patrick, Kaplan, Ryan, 2007, p. 83). Indeed, there is evidence that problematic classroom behaviour is indicative for children who are not able to follow the pace of instruction (Lopes, 2002) and it could be argued that children with learning difficulties are more likely to engage in misconduct because it distracts them from their academic limitations and helps them to gain attention from their peers and the teacher.

Trivial as it seems, being the centre of attention in the eyes of their teacher and peers might actually provide a source of self-worth to otherwise low achieving children. Classroom misconduct might therefore constitute an avoidance behaviour which helps children to protect their self-concept in the light of repeated experiences of learning difficulties and frustration. The present findings support the view that low achievement and classroom misbehaviour are factors which reinforce each other and lead to a vicious circle. In order to investigate the hypothesis that classroom disturbances are related to low levels of self-worth, it is important to discuss the present findings in relation to the results of the self-efficacy measure.

Another factor which might contribute to this vicious circle is the behaviour of the teacher. Teachers of low achieving children are likely to face more challenges and academic and behavioural difficulties than teachers of average achieving children and this might lead them to respond more aggressively and to adopt tactics such as

screaming, yelling and corporal punishment to manage the classroom. However, there is evidence that these tactics do not alleviate but increase incidences of misbehaviour (Lewis, 2001). In addition, children might engage in misbehaviour if they believe that they are unfairly treated by the teacher, that they receive insufficient support or if they are dissatisfied with the teaching style. Therefore, it is crucial to discuss the present results in relation to the findings of the “affective responses towards the teacher” subscale.

Conclusion

It is important to note that the cross-sectional nature of the data does not permit causal relationships to be inferred. Also, the association between perceived classroom conduct and the individual was investigated in the mathematics classroom only and it will be important to examine this association in other subjects. An investigation into how perceived classroom conduct relates to other aspects of the classroom environment such as perceived quality of peer relationships, perceived competitiveness and an integration of teachers' perception of the classroom constitute topics for further research. To recap, the direction of the relationship between low achievement, self-belief and behavioural problems in the classroom environment is not clear. It seems plausible that there is a reciprocal relationship between these factors and occurrence of one of these variables constitutes a "risk factor" which might trigger the occurrence of the other factor. Therefore, it is essential to triangulate the present results within the context of the findings of self-efficacy and the attitudes towards the teacher scales.

3.4.5 MULTIPLE REGRESSION RESULTS

Children with MD

The association between the dependent and independent variable is moderately strong (Multiple R = .51). The regression plane for these variables is statistically significant ($F(4, 86) = 7.6, p < .001$). Together, the variables math anxiety, attitudes towards the teacher, self efficacy and perceived classroom conduct accounted for 22 % of the variance in achievement (adjusted r^2). Taken separately, the variables attitudes towards the teacher ($t = -2.31, p < .05$) self efficacy ($t = -2.13, p < .05$) and perceived classroom conduct ($t = -2.49, p < .05$) are positively related to successful math achievement, as measured by children's school report grad in math ranging from 1 (very good) to 6 (fail). The relationship between math anxiety and math achievement is not statistically significant ($t = -1.15, p = .25$). The regression coefficient for attitudes towards the teacher was -.49. Thus, as attitudes towards the teacher increase by one unit, the math grade increases by .49. The regression coefficient for self efficacy was -.44, meaning that children's math grade increased by .44 as self efficacy increased by one unit.

Average ability children

When children of average ability are concerned, the association between the dependent and independent variable is moderately strong (Multiple R = .36). For perceived classroom conduct regression coefficient was -.54 which indicates that as perceived classroom conduct increases by one unit, math grade increases by .54. The standardized regression coefficients (beta) which has been adopted to assess the importance of the independent variables show that attitudes towards the teacher are a strongest predictor for math achievement (-.29). Perceived classroom conduct (-.22) is a stronger predictor

than self efficacy (-.21). However, the three variables attitudes towards the teacher, self efficacy and perceived classroom conduct are positively and significantly related to successful math achievement. The regression plane for these variables is not statistically significant ($F(4, 47) = 1.75, p = .154$). In other words the set of affective variables in this model do not help to predict math ability.

DISCUSSION

In essence, the results tentatively indicate that the math ability of average children is less affected by variation in affective responses towards maths whereas differences in math ability of children with MD can be partially explained by differences in the set of affective responses. It seems that maths ability in average children is stabilised to such an extent that their performance in maths is not a function of affective factors. Hence, it can be assumed that even negative affective responses do not impede the cognitive processing mechanisms involved in mathematical cognition to a significant extent. One possible interpretation for the finding that affective responses are no significant predictor for variance in math ability in average children is that the interference created from pessimistic and negative responses is below a particular threshold so that maths ability remains unaffected. To recap, Ashcraft and Kirk (2001) found that arithmetic problem-solving performance was a function of the interaction between working memory load and math anxiety. Hence, it might be that the impact of affective responses on math achievement in children with MD might be mediated by working memory capacity. As will be discussed in the following chapter, there is evidence that children with MD have working memory deficits and these deficits might increase the likelihood of intrusion induced by negative affective responses.

Besides the mediating role of cognitive factors such as working memory capacity, differences learning environment can be a possible explanation for the observed differences in the predictive value of affective responses when math ability is concerned. It has been mentioned that children with MD visit a particular school track and it is likely that the awareness of being labelled as a “low achiever” in combination with the open or latent responses and remarks from significant others, such as parents, siblings and peers, evoke negative attitudes, perceptions and self – beliefs among children and might increase anxiety. For children with MD, the continuing experience of being streamed in a “low ability” track is likely to create a feeling of tension and collective as well as individual inefficacy. In other words, the learning environment of children with MD is a mediating factor for the relationship between affective responses towards math and math ability.

To sum up the arguments that have been proposed so far in this discussion, affective variables seem to play a significant role in influencing math ability of children with MD, as measured by their school report grades. The finding that differences in anxiety, self-efficacy, attitudes and perceptions predict variance in math ability indicates that there is a strong linkage between this set of affective responses and learning outcomes in children with MD. Thus, it seems that children with MD can benefit from interventions which are concerned with increasing positive affective responses such as self-efficacy. Also, it has been suggested that positive affective responses can be promoted by keeping the learning environment heterogeneous as long as possible to enable children to observe children of average or above average ability in regulating the acquisition of learning mathematical concepts and procedures. Further research is essential to understand the mechanisms by which affective responses exert influence on math ability in children with MD.

3.5 SUMMARY OF RESULTS

The studies in this chapter were aimed at determining differences in affective responses towards mathematics between children with MD and average ability children and to investigate the association between affective responses and individual differences in achievement. Affective responses towards different aspects of mathematics were measured by the Affective Responses Towards Mathematics (ARTM) instrument, which comprises four subscales. Individual achievement was determined by children's term grades in mathematics, which are based on the maths teachers' assessment of children's written and verbal performance during maths lessons. The foregoing sections presented a description and discussion of the findings on children's affective responses towards different aspects of the mathematics classroom and provided an account of how the different measures of affective responses predict individual differences in classroom based achievement grades. The intent of the present section is to provide a summary of the key results and reiterate the implications that can be drawn from the different findings in order to synthesise and integrate the findings into an explanatory model for the overall pattern of results. The relative limitations of the current studies will be discussed towards the end of this section. Overall, the key findings were as follows:

3.5.1 ANXIETY

The results of the anxiety subscale of the ARTM indicated that:

- a) overall, children with MD are more anxious about learning mathematics than their average ability peers.

- b) reported anxiety seems to increase between classes five and eight in children with MD, but remains constant in children of average ability over the same period.

- c) the strength of association between anxiety and individual achievement seems to decrease in children of average ability over the years, but increases in children with MD between classes five and eight. It has been argued that this finding supports the hypothesis that anxiety is a function, not only of low achievement or failure, but of repeated negative experiences within a particular school environment. This corresponds with the argument forwarded by Drossel (1993, p. 7), who claims that studies on the causal ordering of anxiety and achievement which did not account for the environmental context are limited, since “perhaps anxiety and achievement are caused by a different factor – possibly the learning / teaching environment.”

- d) the hypothesis that environmental experiences contribute to the rise of anxiety in children with MD seems plausible, given that children with MD are more likely to experience negative responses from parents, their teacher and even their peers, which in turn leads to negative self-evaluation and anxiety. It has been argued that anxiety, environmental experiences and cognitive processes

reinforce each other in the form of a vicious circle, because anxiety interferes with the cognitive processes required for mathematics and this interference affects children's performance.

- e) anxiety is not predictive for individual achievement differences in children with MD from class five but predicts nearly 25% of the variance in the achievement of children with MD from class eight. Given that the curriculum is more difficult in class eight, it could be argued that there is no interference between anxiety and cognitive processes in class five, but only in class eight where more complex tasks are concerned which require more working memory resources.
- f) overall, the findings support the hypothesis that there is a reciprocal relationship between individual achievement, environmental experiences and anxiety, which in turn interferes with cognitive processes and impedes achievement, depending on the level of task difficulty. This explanatory model would explain the significant increase in anxiety among children with MD between classes five and eight and the association between anxiety and individual achievement for children with MD from class eight, but not for children with MD from class five.

Self-efficacy

In contrast to children of average ability, the self-efficacy beliefs of children with MD seem to deteriorate over time; children with MD from class eight hold significantly lower self-efficacy beliefs than their younger peers from class five. Given that attendance at a general secondary school is a synonym for low collective ability and academic and behavioural difficulties, a negative self-labelling process has been proposed as an explanation for the decrease of self-efficacy beliefs in children with

MD. It is argued that a low collective self–efficacy belief is not beneficial for children’s perceived self–ability and might induce feelings of helplessness and a perceived lack of agency which, in turn, might affect the effort that children devote to learning. In contrast, children of average ability are able to maintain a positive sense of self–efficacy because they are more likely to attribute success or failure in mathematics to factors such as learning.

3.5.2 PERCEIVED CLASSROOM CONDUCT

- a) children with MD are more likely to perceive their classroom is subject to disturbances and misconduct than children of average ability. This finding supports the argument that “low achievement and behaviour problems go hand in hand” (Kaufmann, 1997, p. 31). An unexpected finding was that no differences between children from class five and eight could be detected, neither among children with MD nor among average ability children.

3.5.3 ATTITUDES TOWARDS THE TEACHER

- a) Children of average ability perceive their teacher to be more helpful and supportive and the teaching style as more effective than their peers with MD. This corresponds to previous findings in the literature, which show that the perceived supportiveness of the teacher and the perceived quality of the teacher–student interaction emerged as reliable predictors of mathematical achievement (Dossey, 1992).

- b) attitudes towards the teacher in children with MD actually appear to decrease over the years, whereas the affective responses of children of average ability remain constant from class five to class eight. This implies that the perception of the teacher changes over the years. It has been proposed that class eight children with MD might have developed distinctive explanatory models for academic success and failure, in which the support and teaching style of the teacher plays a key role. In line with the tenets of attribution theory, it could be argued children with MD from class eight believe that their achievement is largely determined by their teacher rather than the outcome of their own actions. Such expectations might give rise to disappointment and negative affective responses towards the teacher, who is perceived as responsible for individual achievement.
- c) The hypothesis that children's causal ascriptions of success or failure influence their affective responses towards their teacher can be supported by the finding that, in contrast to children of average ability, affective response towards the teacher accounts for 41% of the variance in individual grade for children with MD from class eight. Thus, it appears that the abovementioned hypothesis serves as an explanatory model for the present findings.

3.5.4 CONCLUSION

Taken together, the present results suggest that children with MD tend to express more negative affective responses towards different aspects of mathematics than children of average ability. However, it seems that children with MD and average ability children differ with respect to stability and change in their affective responses. Apart from the results of the perceived classroom environment scale, the affective

responses of children with MD seem to decline between class five and class eight, whereas the affective responses of children of average ability from classes five and eight are similar. This implies that, in contrast to children with MD, the affective responses of children of average ability seem to be less affected by variances and remain stable. This is an unexpected finding, given that children from class eight are assumed to respond differently to their environment and experiences than children from class five. Indeed, it was expected that children with MD and average ability children should express a decline in self-efficacy, because it could be assumed that at the start of adolescence, children in class eight are more critical in their self-judgements.

However, the results showed that this is not the case for children of average ability. It has been argued that even children of average ability have experienced frustration and distress over the years. Nevertheless, the results indicate that children of average ability have different mechanisms to deal with negative experiences which enable them to maintain positive affective responses over time. In other words, it seems that children of average ability are able to balance out personal or interpersonal difficulties over the years, which results in stability in their affective responses. The assertion that children of average ability are less influenced by variance in affective responses than children with MD is supported by the results of the multiple regression analysis which showed that while affective responses were predictive for achievement variance in children with MD, affective responses did not explain the achievement variances in children of average ability.

In order to provide a more general explanatory framework for the present results, it has been proposed that the differences in stability and change of affective responses between children with MD and average ability children between classes five and eight

can be explained by a latent variable, namely collective self-efficacy. According to Bandura (1997), collective self-efficacy is concerned with the performance capability of a social system as a whole. Based on the present results, it has been suggested that collective efficacy constitutes a latent variable which mediates the affective responses of children with MD and average ability children, as well as the association between differences in affective responses and individual differences in achievement. However, it seems that children's perception of their collective social efficacy gradually develops as they progress through secondary school. Indeed, the present results show that there is no difference in self-efficacy belief between children with MD and average ability children, despite the fact that children with MD in class five already have a history of frustrating experiences and low achievement in primary school.

This finding substantiates the argument that differences in children's self judgements might not be due to individual low achievement as such, but are mediated by the experience of being exposed in a certain social system. Schools which are segregated with respect to ability and are exclusively populated by children with academic difficulties and low achievement might constitute a social system which is not beneficial for children's collective self-efficacy. It has been argued that negative collective self-efficacy beliefs are accompanied by withdrawal and negative self-labelling processes, which can occupy limited processing resources and therefore might constitute an impediment to cognitive processes. To sum up, collective efficacy seems to be an important variable where children's affective responses and their relationship to individual achievement are concerned.

However, a number of caveats have to be noted when considering the findings of the study. First of all, individual mathematics achievement refers to children's mathematics grade on their semester school report, which encompasses teacher-based judgment of the students' verbal and written performance in mathematics over the first half of the year. The predictive utility of children's grades from only one semester is limited because it is not possible to determine if a low grade is representative for the overall achievement of a particular child or whether it is just an occasional odd mark. This is especially true for children from class five, who are in their first year of secondary schooling after primary school, which is associated with a change of school environment.

Also, it is noteworthy that while there might be a normal distribution of the range of mathematics school grades in both groups, the grades do not refer to the same achievement. There are considerable differences in the level of difficulty of instruction and assessment, given that children with MD and average ability children attend different types of school. To recap, children with MD attend general secondary schools which make the lowest academic demands compared to other secondary school types in Germany and have a predominantly vocational orientation. Therefore, the curriculum for children of average ability is far more complex and the assessments are more demanding than for children with MD. Thus, a high grade in the group of children with MD does not refer to the same achievement standard as a high grade in the group of children of average ability. However, it could be argued that, despite these limitations, the results provide useful information.

Finally, it is important to re-emphasize that the design of the study does not permit conclusions to be drawn about the causal ordering of collective efficacy and the other affective constructs that have been measured. The results indicate tentatively that, while anxiety and attitudes towards the teacher constitute risk factors which are reciprocally related, their relative impact and their association with individual achievement is mediated by collective efficacy. Future research would be required to specify the role of collective efficacy, a construct which has so far only resided at the periphery of research on affective responses (Bandura, 1997).

Further research on the development and affective responses in children with MD would be important, because there is evidence that affective responses towards maths are linked to cognitive processing of mathematical information. As has been discussed in the literature review, anxiety has been described as acting “like a secondary task”, which seems to interfere with working memory processes and therefore increases cognitive load during problem-solving. Hence, it is pertinent to consider affective as well as cognitive aspects of MD in future research. In order to account for the importance of both aspects, the following study shifts the focus from the affective dimensions to the cognitive dimension of MD and examines working memory use in children with MD.

CHAPTER 4

COGNITIVE DIMENSIONS OF MATHEMATICAL DIFFICULTIES

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4.1 OVERVIEW

According to Geary, Hoard, Hamson (1999) the literature on the cognitive dimensions of difficulties in mathematics is not extensive, compared to the literature coverage of reading abilities. Therefore, many studies of children with MD have utilised models of normal arithmetic development in order to research the cognitive variables that contribute to the variation in achievement between children with MD and control group children. A key problem is that the domain of mathematics is extensive and covers different areas, such as algebra, geometry and arithmetic. Thus, it is difficult to provide a coherent and systematic framework for researching and identifying MD. However, an area where such a framework has been systematically developed over the past decades is that of arithmetic development in children, and this area is the focus of the present research. In the present chapter, the cognitive dimensions of MD will be investigated in terms of the working memory processes that are involved in arithmetic problem solving.

As will be shown, there is evidence that the usage of working memory resources in arithmetic problem-solving processes constitutes a fundamental factor for MD, because inefficient use of working memory resources impedes the direct retrieval of factual knowledge. Indeed, Adams and Hitch (1998) argue that working memory capacity accounts for much of the variance between average achieving children and children with MD. However, Penner-Wilger (2004) claims that the research on working memory usage in children with MD is limited in scope and the available evidence on

this question is contradictory. Therefore, the present chapter attempts to address this issue further.

The chapter consists of two studies. The first is concerned with the role of phonological working memory resources in the arithmetic performance of children with MD. Using a dual-task paradigm, the study investigates the extent to which children with MD use phonological resources when solving addition, subtraction, multiplication and division problems. To foreshadow the conclusion, the results indicate that children with MD use the phonological loop for multiplication and division but not for addition and subtraction, which implies that multiplication and division rely on different working memory mechanisms than addition and subtraction

In order to examine this finding further, the second study extends the first by comparing the usage of phonological resources for multiplication problems in children with MD and children from an average ability control group. The results suggest that average-ability children use phonological resources only for difficult multiplication problems, whereas children with MD also use phonological resources for simple problems, suggesting that supposedly simple problems constitute a difficulty for children with MD and require the same cognitive resources as difficult problems. It will be argued that the misuse of working memory resources might result in cognitive overload, which is reflected in less accurate retrievals with a high rate of intrusion errors. The findings will be synthesized in an overall discussion and the implications for the learning of mathematics in children with MD will be considered.

4.2 INDIVIDUAL DIFFERENCES IN ARITHMETIC ABILITY

One framework that has been frequently used in previous research to systematically examine individual differences in arithmetic ability is Baddeley and Hitch's (1974) multi-component model of working memory. The majority of studies on the usage of working memory components in arithmetic calculations have employed the dual-task paradigm, which is the dominant paradigm for isolating separate cognitive functions (Baddeley, 2000). Dual tasks are used to increase cognitive load during problem-solving processes. Cognitive load can be defined as the overall quantity of mental activity imposed on working memory at one certain point. Supposing that working memory resources are limited and that certain arithmetic processes are associated with a certain subsystem such as the phonological loop, arithmetic performance should be impeded when excessive cognitive load is induced through the administration of a secondary task that requires the processing resources of the given working memory (WM) subsystem. In other words, cognitive load should result in hampered performance if the processing resources required for the calculation are temporarily occupied through a secondary task. The performance decline would be an indicator of whether or not the particular WM component is involved in the calculation. If not, performance should not be impaired by the administration of the secondary tasks, because no cognitive load would arise if the arithmetic calculation and the secondary tasks were processed by different WM resources.

Different secondary tasks have been developed to increase cognitive load on different WM resources and to investigate their role during mental calculations. Visual memory has been loaded by asking participants to attend to irrelevant pictures while answering arithmetic tasks. Spatial memory load has been increased through secondary

tasks that involve hand movement (Logie, 1995). The phonological loop has been loaded through articulatory suppression tasks. Central executive load has been researched by using random letter generation tasks.

The possibility that differences in mathematical ability are linked to different cognitive processing profiles has been addressed in a study by Kroesbergen, Van Luit and Naglieri (2000). The Cognitive Assessment System (CAS) was employed to research the performance of children in the domains of Planning, Attention, Simultaneous processing and Successive processing (PASS). Although the CAS test is a test of general intellectual ability, it has been found to correlate strongly with achievement in mathematics. This suggests that the different components measured by the CAS can predict individual variation in achievement.

A total of 267 children with MD took part in the study. Children were identified as having MD if they performed in the lower 25th percentile in a standardized mathematics test. The results showed that children with MD and children of average ability have different profiles in the areas of planning, attention, simultaneous processing and successive processing. In all areas, children with MD scored significantly lower than their average-achieving peers.

In order to account for different types of MD, and to investigate whether these are related to specific cognitive deficits, the researchers distinguished between children with difficulties in learning multiplication facts on the one hand and children with difficulties in automatic fact retrieval of multiplication facts on the other hand. Children with difficulties in learning multiplication facts were defined as those who scored at least 1 SD below the average in a multiplication test. Children with difficulties in

automatic fact retrieval were defined as those who performed at an average level in a general multiplication test, but who scored at least 1 SD below average in a test of automaticity. The results showed that children with different types of difficulties have distinctive profiles with regards to their cognitive processing deficits: children with problems in learning multiplication facts had deficits in all the four areas of planning, attention, successive and simultaneous processing. Children with automaticity problems, on the other hand, had relatively high scores on the simultaneous processing scale. This implies that specific mathematical difficulties are linked to specific cognitive deficits.

Overall, the study of Kroesbergen et al. (2003) indicates that children with MD have deficits in the areas of attention and successive and simultaneous processing. The execution of these cognitive functions, in turn, is strongly related to working memory capacity. Therefore, the findings of Kroesbergen et al. (2003) suggest that children with MD have deficits in central executive functioning. To recap, the central executive is responsible for higher-order cognitive functions such as attention switching, inhibitory control and for the retrieval of information from long-term memory (Baddeley, 1996).

The possibility that children with MD have central executive impairments has been investigated by Passolungi and Siegel (2001), who researched the relationship between performance in WM tests and mathematics achievement in a total of 49 children from grade 4, some with average performance and others with mathematical difficulties. Children were defined as having MD if their score in a standardized national mathematics test fell below the 30th percentile. The children took part in different WM tests, which included non-numeric tasks such as a listening span test, a dual-task test and a listening span completion task as well as numerical tasks such as a

counting span test. The findings showed that children with MD showed significantly poorer performance in all WM tests than average-achieving children, even for tests which required no processing of numeric information. Therefore, the researchers concluded that children with MD have a general WM deficit. However, this conclusion has not gone unchallenged.

The possibility that specific WM components account for the variation between children with MD and average-achieving children has been addressed in a study by Andersson and Lyxell (2007). They investigated whether children with MD display WM deficits and whether these deficits are general WM deficits or are specific to a certain component. A total of 31 children with MD took part in their study, with an age-matched control group of average-achieving children. A pre-test of mathematical ability was conducted to confirm that children with MD can be distinguished from their average-achieving peers and the results showed that children with MD yielded significantly lower scores. Having established the performance difference between the groups, the children took part in a battery of tests that were designed to tap the different WM components. Tasks to tap central executive functioning included a verbal fluency task, a colour Stroop task, a crossing-out task to measure selective attention and a number-matching task to measure general processing speed. Tasks to tap the phonological loop included a digit span tasks, which required children to repeat sets of digits and to recall them in serial order, and a word span task, which required children to recall short words. Visuospatial sketchpad capacities were measured using a Corsi block span task.

The results show that children with MD performed significantly worse than their age-matched average achieving peers in the majority of central executive tasks and in the phonological loop tasks. However, no significant differences were found with respect to performance in tasks that tapped the visuospatial sketchpad. This finding is not consistent with the previously reviewed study of Passolunghi and Siegel (2001), where children with MD exhibited lower performance in verbal as well as visual WM tasks. Overall, the findings can be interpreted as evidence that children with MD have WM deficits, and that the major factors to explain these deficits are impaired central executive functions such as simultaneous processing and the storage of numerical information as well as the phonological loop. Nevertheless, Passolunghi, Vercelloni and Schadee (2007) argue that few studies have investigated the relationship between phonological working memory and arithmetic performance. The available evidence is contradictory, as will be reviewed below.

4.2.1 INDIVIDUAL DIFFERENCES IN PHONOLOGICAL LOOP USAGE

Geary, Hoard and Hamson (1999) investigated phonological loop usage in children with MD and children from an average ability control group. The sample consisted of 90 primary school children. The mathematical reasoning subtest of the Wechsler Individual Achievement Test was used to identify children with MD. Children who scored below the 30th percentile were classified as MD. Children took part in a regular digit span task and a backward digit span task in order to find out whether children with MD have a general deficit in maintaining temporary information in the PL or whether this deficit is specific to circumstances when children not only have to maintain information in the PL but also have to manipulate it. The backward digit span task requires children to listen to a sequence of single digit and to repeat the string of

digits in reversed order. The backward digit span task is a working memory test because children have to manipulate the remembered set of digits before reproducing them in opposite order.

Whereas no differences were found in the regular digit span task, children with MD performed significantly worse in the backward digit span task. This suggests that children with MD have specific problems with engaging in numeric processing when their PL is occupied. Thus, the pattern of findings indicates that the contribution of WM to arithmetic problem solving is reflected in the ability to temporarily encode information in the PL while processing and manipulating other information. However, a central limitation of Geary's study is that the phonological task involved only digits, rather than words. It could be hypothesized that the digits were temporarily retained via the VSSP rather than the PL. However, this assertion has not yet been subject to empirical investigations. While the studies reviewed so far provide evidence that performance in PL tasks accounts for variations in achievement between children with MD and children from an average ability control group, the overall evidence is not consistent.

Amico and Guarnera (2005) examined phonological, visuo-spatial and central executive working memory functions in children with MD compared to an age-matched control group. The sample consisted of 25 children with MD, who were identified by their teacher. Each child with MD was matched with an average-ability child of the same age and gender. Children took part in a series of WM tasks to tap the different WM components. The researchers took account of the shortcomings in the Geary study and used both numeric and linguistic items for testing phonological loop and central executive functioning. Children with MD performed significantly worse in all tests

involving the VSSP and the central executive, irrespective of whether numeric or linguistic material was used. There was no significant difference between children with MD and children from an average ability control group for the phonological loop tests, with the exception of the digit span task.

Therefore, Amico and Guarnera (2005) concluded that while the central executive and the VSSP are crucial for arithmetic problem solving, the role of the phonological loop has been overestimated in previous research. Thus, “the phonological loop is not the major factor in explaining arithmetical difficulties” (p. 201). However, it can be argued that the digit span task is not an exclusive measure of phonological loop functioning because it also involves the inhibition of irrelevant information and this implies an involvement of the central executive. The absence of any significant effects of the digit span task might therefore be explained in terms of a trade-off effect, meaning that the demands of the task were split between the central executive and the PL.

McLean and Hitch (1999) investigated the relationship between arithmetic difficulties and working memory impairments in 9-year-old children who had an average IQ and reading scores. Children with MD were compared with a group of children who were matched for age but were average achievers, and a group of younger children who also had significantly lower scores than the average achievers. A standardized arithmetic test was run to screen children with MD, who scored below the 25th percentile in the test. A standardized reading test showed no significant difference between the three groups. Children took part in a number of Working Memory tests, which were specifically targeted to capture specific WM components. Phonological loop functioning was measured using an auditory digit span and a non-word repetition task. The results showed no significant impairment of the phonological loop in children

with MD compared to the age-matched group and the ability-matched group. This replicates the findings of Amico and Guarnera (2005), who concluded that the phonological loop does not contribute to individual differences in mathematical ability. Similarly, a study conducted by Holme and Adams (2006) with 148 children aged 8 to 9 showed that while children's performance in visuospatial and central executive tasks was predictive of mathematics achievement, performance in phonological loop tasks was not. However, the criticism can be raised that the McLean and Hitch (1999) and Holme and Adams (2006) studies are based on separate measures of mathematical ability on the one hand and working memory capacity on the other hand, and do not actually measure the effects of occupying the phonological loop during arithmetic problem-solving. Both studies could be extended by employing a dual-task paradigm to examine individual differences in problem solving when the phonological loop is loaded through an articulatory suppression task.

To sum up, there is no consensus regarding the question of whether individual differences in mathematical ability are a function of phonological loop functioning. In order to examine this issue further, it is essential to review the research on the role of phonological resources in arithmetic problem solving.

4.2.2 PHONOLOGICAL LOOP USAGE AND ARITHMETIC PROBLEM SOLVING

According to Widaman (1989), arithmetic problem solving involves an inner speech process known as subvocal rehearsal. Subvocal rehearsal is mediated by the phonological loop. Competing models have been proposed to explain the role of the phonological loop in mental arithmetic. Despite having attracted considerable research,

the evidence on the role of the phonological loop (PL) in calculations is conflicting (Penner–Wilger, 2004). As will be reviewed below, the debate surrounding the circumstances under which the PL is recruited can be pinpointed to a central question, namely whether the PL is responsible for simple, retrieval-based calculations or for more complex problems. A related research question is thus whether the activation of the PL is tied to a specific arithmetic operation, or whether the recruitment of WM resources depends on other factors than arithmetic operation. The models of arithmetic cognition that will be introduced below adopt contradictory viewpoints on these questions.

Dehaene and Cohen (1995) advocate a triple code model of numerical cognition, which rests on the assumption that whether visual, phonological or abstract magnitude codes (number comparisons) are recruited depends on the particular arithmetic task to be performed. In other words, the model ties specific arithmetic tasks to specific working memory resources. Dehaene and Cohen (1997) propose that the acquisition of arithmetical procedures is governed by two kinds of strategies: procedural memory and rote verbal memorization. This has been confirmed in the literature (Zhou & Dong, 2003). Dehaene and Cohen (1995) maintain that the phonological loop is recruited for the rote memorization of number facts and for retrieval-based arithmetic tasks, presumably because rote memorization is a verbal-phonological process.

According to their triple code model, arithmetic facts are stored as verbal representations and accessed via the auditory–verbal code. Because multiplication is mainly taught via rote memorization, it follows that this operation requires more phonological resources than addition, subtraction or division. In fact, verbal rote memorisation is a common strategy to teach multiplication tables (Zhou & Dong, 2003).

In line with the assumption of Roussel et al. (2002), Cohen and Dehaene (2000) predict that direct retrieval is less plausible for addition and subtraction. In contrast to multiplication, subtraction requires the mental manipulation of numeric quantities, and more complex strategies, known as “semantic elaboration”, need to be employed instead of direct retrieval. Dehaene and Cohen (1997) do not exclude the possibility for verbal rote memorization and direct retrieval of basic addition facts. However, more complex addition problems are not solved in a verbal–phonological form and require abstract, quantity–based strategies instead. The verbal–phonological representational code is used for simple arithmetic problems, which can be solved via direct retrieval, whereas more difficult, multi-digit arithmetic tasks are addressed by the visual Arabic code, which is mediated by the visuospatial sketchpad (Campbell, 2004).

An opposing model of numeric cognition is the encoding complex model forwarded by Campbell (1994). While the triple–code model predicts that specific tasks are related to specific WM resources, Campbell maintains that the activation of visual or phonological codes depends on the format of presentation. Indeed, Campbell (1994) found that whether numbers were presented as digits (24) or written words (twenty–four) had a significant effect on arithmetic performance. Campbell agrees with the triple code model that information can be represented in visual or phonological form, but in contrast to the triple code model, the encoding complex model predicts that these codes do not work independently from each other. Rather, numeric information can activate competing responses in representational formats at the same time. The competition is based on the activation strength of the multiple responses and the response that receives the greatest excitatory input “wins” the competition. Excitatory and inhibitory potential are developed through task–specific practice. Thus, retrieval can be optimised through task–specific practice because this has a strengthening effect on the excitatory and

inhibitory networks. The following paragraphs provide a review of the evidence related to the abovementioned models.

Evidence for the triple code model

There is support for the prediction from Dehaene and Cohen's triple code model that the phonological code is used for basic arithmetic calculation, especially fact retrieval of verbal associations. Zhou, Chen, Dong, Zhang, Zhou, Zhao, Chen, Qiao, Jiang and Guo (2006) examined the event-related potentials elicited by simple addition, subtraction and multiplication problems using the EEG technique. The sample consisted of 18 Chinese undergraduate students. The researchers employed a delayed verification task, where participants were presented with an arithmetic problem in Chinese language and, after 1.5 seconds, were asked to state whether the answer that was presented was true or false. In essence, they found that the left anterior region of the brain, which is associated with phonological processing, is more involved in multiplication than in addition and subtraction. The researchers concluded that phonological processing is more salient for retrieval-based operations such as multiplication facts, but not for addition and subtraction, which are largely based on quantity manipulations. However, it could be argued that the sample consisted exclusively of Chinese students and arithmetic teaching in China might differ from the style of instruction found elsewhere. More precisely, it is argued that rote memorisation is greatly emphasized in Chinese mathematics instruction and this might result in different cognitive processing profiles. Indeed, linguistic differences in numeric representation might be a crucial factor which needs to be considered. Therefore, a cross-cultural comparison might provide more persuasive findings.

Support for Dehaene's notion that arithmetic cognition is modular with respect to the representational code is provided by Lee and Kang (2002). Lee and Kang employed a dual-task paradigm to research the role of the visuospatial sketchpad and the phonological loop in multiplication and subtraction. Their sample consisted of 10 undergraduate and graduate students, who solved 40 multiplication problems and 40 subtraction problems under three conditions: a condition where arithmetic problems were solved with no additional task (no dual task condition), a condition which involved the continuous repetition of a non-word to increase phonological load (articulatory suppression condition) and a condition where participants were asked to remember the shape and the location of a figure in order to match the figure later on (visuospatial load condition). In essence, they found that visuospatial secondary load produced the greatest interference effect on subtraction performance but had no disruptive effect on multiplication, whereas phonological load created through articulatory suppression impaired multiplication, but not subtraction.

They interpret their findings as showing that numeric representation in working memory is determined by the arithmetic function in a subsystem-specific manner, whereby retrieval-based multiplication tasks draw on phonological resources, whereas more complex tasks that rely on counting-based strategies are handled by visuospatial resources. This supports the prediction of the triple code model. However, Lee and Kang's findings (2002) can be criticised because the sample size is very limited in scope and restricted to skilled problem solvers (undergraduates and graduates). The stimuli consisted of easy arithmetic tasks, because both operands were single-digit numbers, while their sample consisted of highly skilled Korean university students. Therefore, it is safe to argue the participants in the Lee and Kang study relied mainly on direct retrieval to solve subtraction tasks, rather than strategies such as counting or

decomposition (Campbell & Xue, 2001). If the phonological loop was responsible for direct retrieval, then simple subtraction should be affected in the same way as simple multiplication by articulatory suppression.

Further evidence for the triple code model is provided by neuropsychological studies. Dehaene and Cohen (1997) report a case of selective impairment of multiplication relative to subtraction. Also, patients with damaged language areas typically have impaired multiplication abilities. This supports the concept of a dissociation between verbally mediated retrieval and operations that involve quantity manipulation (Dehaene and Cohen, 1997). Also, fMRI studies have revealed that subtraction is associated with increased activity in the superior parietal lobe, whereas multiplication draws mainly upon left-sided language areas (Chochon et al., 1999). This finding receives further support from lesion studies (Cohen et al., 2000). Nevertheless, the evidence for a role of the PL in retrieval-based arithmetic is not equivocal: Pesenti, Thioux, Seron and De Volder (2002) found no activation of cerebral language areas during fact retrieval. Similarly, Bull, Johnston and Roy (1999) concluded that no impairment of the phonological loop could be detected in children with MD. Indeed, a case study conducted by Butterworth, Cipolotti and Warrington (1996) showed that an impaired phonological loop did not affect arithmetic performance. As will be reviewed below, there is convincing evidence that indicates that the involvement of the phonological loop is linked to the level of task difficulty, in that the phonological loop is recruited for more complex rather than simple retrieval-based tasks.

Evidence for the network interference model

While the triple code model posits that the phonological loop mediates basic arithmetic facts, this model has been strongly challenged by a number of studies, as will be reviewed below. Many researchers found evidence that the phonological loop is recruited for more complex rather than simple, retrieval-based tasks and this contradicts the propositions of the triple code model. The role of the phonological loop in complex addition problems was addressed by Logie, Gilhooly and Wynn (1994). The sample of Logie et al. consisted of 24 participants with a mean age of 45 years. Participants were asked to listen to a cumulative sequence of two-digit numbers over a headphone, add them together and report the final total. Each participant solved a total of 40 arithmetic tasks, half of which were coupled with one of four secondary tasks (articulatory suppression, random generation of letter sequences, irrelevant pictures or hand movement).

Half of the tasks in the dual task and the non-task condition involved a maximum of two carries in the entire sequence. The secondary tasks were chosen to tap the specific WM components that are thought to be relevant for the task. The articulatory suppression task involved the continuous repetition of the word “the” while participants listened to the number sequence. The rationale behind this articulatory suppression task was to investigate the role of sub-vocal rehearsal in addition. The involvement of the central executive in mental addition was tested using a random letter generation task, which required participants to randomly articulate any letters from the alphabet in an arbitrary fashion. They were asked to avoid logical sequences (E F G H) or the spelling of words (D O G) in favour of random generations of letter strings.

The role of the visuospatial loop - i.e. the role of visual imagery - in mental addition was investigated by asking participants to attend to irrelevant pictures presented on a screen while keeping track of the running total of the number sequence. The hand movement task asked participants to press four buttons in a particular sequence. Only the articulatory suppression task and the random generation tasks impaired performance, which implies that both the phonological loop and the central executive play important roles in addition. However, it could be argued that the disruptive effect of the articulatory suppression task occurred because of the auditory presentation of the tasks, rather than because the phonological loop is involved in addition. Indeed, auditory presentation of the numbers requires the phonological loop to encode the numbers and it might be the case that articulatory suppression interferes with the initial encoding, whereas the addition process as such does not rely on the phonological loop.

In order to test this prediction, Logie et al. devised a follow-up experiment with a visual rather than auditory presentation of numbers. If phonological load caused by articulatory suppression - i.e. the disruptive effect of articulatory suppression - was a function of the auditory presentation, then it should be removed when numbers are presented visually. In essence, it was found that the disruptive effects of articulatory suppression that were found for auditory presentation also appeared with visual presentation. The researchers conclude that, "sub-vocal rehearsal assists retention of accurate running totals" (p.408). This runs counter to the predictions of the triple code model, which assumes that the process of sub-vocal rehearsal, as mediated by the PL, is responsible for the retrieval of basic arithmetic facts rather than keeping track of a running total of addition tasks. Rather, the findings support Campbell's network interference model, which posits that the recruitment of visual as opposed to

phonological resources depends on the difficulty and presentation of the task. However, a possible limitation of the Logie et al. (1994) study is that the arithmetic task consisted of a series of numbers and participants only stated the total. Thus, it is unclear whether the PL might also be involved in single tasks ($17 + 4 = ?$). Despite the possible methodological limitation of the Logie et al. (1994) study, the findings show that the argument forwarded by proponents of the triple code model, namely that retrieval-based simple arithmetic tasks make the greatest demands on the phonological loop (Zhou & Dong, 2003, Roussel et al., 2002), has not gone unchallenged. As will be illustrated below, there is evidence that the PL is not employed for fact retrieval in simple problem-solving processes but used for the temporary storage of intermediate results in more complex calculations.

Seyler, Kirk and Ashcraft (2002) investigated this possibility further through an analysis of participants' verbal protocols. It was found that for larger subtraction tasks, participants use more demanding, piecemeal strategies such as counting down or decomposing the large problem into smaller chunks. These strategies may require the learner to rehearse intermediate results by means of inner speech (sub – vocal rehearsal), which is disrupted through the concurrent articulatory suppression. Therefore, it seems plausible to argue that the PL is recruited for solving subtraction tasks, at least for more complex, multi-digit calculations, and this challenges the triple code model's assumption that fact retrieval is governed by the PL.

The notion that the phonological loop is required for complex arithmetic calculations rather than fact retrieval is also supported in a study by Fürst et al. (2000). They investigated whether different working memory components are responsible for solving complex mental addition tasks under different conditions. More precisely, the

researchers aimed to examine the separate functions of the central executive and the PL in arithmetic problem-solving processes. It was hypothesized that occupation of the phonological loop through secondary tasks would have a disruptive effect, especially for tasks that were presented only briefly and therefore increased the need for retention of information in the phonological loop. Their sample consisted of 30 university students, who solved a total of 36 three-digit addition problems, either under a brief-presentation condition or a continuous presentation condition. For participants in the brief-presentation condition, the arithmetic problems were shown for 4000 ms, whereas for participants in the continuous condition, the problem was visible throughout the experiment.

The arithmetic problems were coupled with two phonological dual tasks, which were intended to occupy the phonological loop. In the recitation task, the experimenter chose a random letter from the alphabet and asked participants to continue reciting the alphabet from that particular letter. In the articulatory suppression task, participants were asked to continuously repeat the word “the”. Corresponding to their hypothesis that the phonological loop is responsible for temporary storage of arithmetic information, it was found that articulatory suppression resulted in significantly lower results if the calculation task was only briefly presented, whereas there was no effect of articulatory suppression when the task was presented continuously. No differences were found between the two different phonological dual-task conditions (recitation vs. articulatory suppression). The researchers concluded that, “the phonological loop appears to be involved in maintaining problem information in mental arithmetic, but not in retrieving factual knowledge.” This challenges Dehaene and Cohen’s phonological storage hypothesis, which posits the phonological loop as a fact-retrieval device in simple tasks.

De Rammelaere, Stuyven and Vandierendonck (2001) investigated the proposition of the triple code model - namely that basic arithmetic facts are coded in a phonological format and that the phonological loop is a fact-retrieval device - by researching the effect of articulatory suppression on the verification of basic arithmetic problems. The sample consisted of 30 psychology students, with a mean age of 18.9 years. Participants were asked to verify arithmetic problems as correct or false. The problems involved one-digit numbers from 2 to 9. False problems were divided into small magnitude errors (± 1) and large magnitude errors (± 9). The verification of the sums was carried out under three conditions: without a dual task, with a phonological dual task or with a dual task to tap the central executive. Phonological load was induced through an articulatory suppression task (continuous repetition of the word "the"), which was intended to interrupt the sub-vocal rehearsal of arithmetic facts.

The central executive resources were constrained through a random generation task, which asked participants to tap a random, unpredictable rhythm while solving the problems. The results indicated that while the verification of sums was significantly impaired by central executive load, the articulatory suppression task had no significant effect, which implies that the phonological loop is not required for the verification of simple sums. In order to test whether these results are also valid for multiplication problems, De Rammelaere et al. replicated their experiment with multiplication rather than addition problems. As had been the case for addition, the occupation of the central executive obstructed performance, whereas occupation of the phonological loop had no significant effect on the verification of multiplication problems. Taken together, the results suggest that fact retrieval is mediated by the central executive rather than by the phonological loop, which presents a further challenge against the triple code model.

A similar result was yielded by Trbovich and LeFevre (2003), who investigated the role of the phonological loop and the visuospatial sketchpad in addition problems. The rationale was to find out whether the recruitment of the phonological loop (verbal code) as opposed to the visuo-spatial sketchpad (visual code) is a function of the format in which the arithmetic problem is presented. Their sample consisted of 96 participants with a mean age of 19 years. Participants solved a total of 72 arithmetic problems, which were either vertically aligned (one number written below the other) or horizontally aligned (one number written next to the other). Half of the participants in each format condition solved the arithmetic problems under a phonological dual task condition and half under a visual load dual task condition. A third of the problems were presented with no dual task (neutral condition).

The results show that the disruptive effect of phonological load through articulatory suppression was significantly more powerful than the effects of visual load if the problems were represented in a horizontal format, whereas the disruptive effect of articulatory suppression was less pronounced than the interference caused by visual load when the problems were represented in a vertical rather than a horizontal format. Trbovich and Lefevre concluded that participants tend to rely on phonological codes when processing horizontally aligned problems. The finding regarding the involvement of the phonological loop is interesting, because the phonological loop is used even when the material is presented in a visual format. To recap, the problems were continuously present in a visual format, so it could be assumed that participants solve the problems using the visual code and therefore do not need to draw on the phonological code.

One interpretation of this finding is that vertically aligned problems are solved via the VSSP, because the single digits are properly aligned and do not require a mental transformation of the problem because it becomes obvious which operands have to be added together column-wise. Indeed, vertical alignment of multi-digit problems is a common strategy for teaching written addition, and children should be more familiar with solving vertically aligned task than solving horizontally aligned tasks. Horizontal alignment of addition problems, on the other hand, requires the learner to keep track of units and tens and to add them separately, so participants might switch to the phonological code in order to maintain the operands and interim results via sub-vocal rehearsal. This adds support to the notion that the phonological loop is recruited for the maintenance of operands and interim results during complex tasks, and challenges the prediction of Dehaene and Cohen's triple code model, according to which the PL is involved in retrieval-based addition and multiplication problems. Rather, the findings go in line with Campbell's encoding complex model (1992), according to which the activation of various internal codes is tied to the format of presentation and secondary task.

Further evidence in favour of the encoding complex model is provided by Seitz and Schumann-Hengsteler (2002). A total of 24 German graduate students (mean age 24.5 years) took part in the study. The researchers used a dual-task methodology to investigate the role of the phonological loop and the central executive in solving addition and multiplication problems. Simple and difficult problems were used in order to examine the role of working memory resources in fact retrieval compared to more complex problem-solving procedures. The set of addition problems contained eight simple two-digit addition problems without a carrying operation (e.g. $12 + 76$) and eight difficult addition problems, which involved carrying ($45 + 18$). The set of

multiplication problems contained six simple problems, where both operands are 5 or smaller, and sixteen difficult multiplication problems, where one of the operands was a two-digit number. Participants were asked to solve the problems mentally and to speak the answers out loud.

All problems were performed under four conditions: a neutral condition, which involved no dual task, an articulatory suppression condition to engage the rehearsal system of the phonological loop, and a canonical number generation task and a random number generation task, which were intended to tap the central executive. The articulatory suppression task required participants to continuously repeat the syllable “bla”, while the canonical number generation task asked participants to count aloud “1 2 3 4 5 7 8”, which requires phonological resources as well as central executive resources to keep track of the counting sequence. The random number generation task involved the verbal production of an unpredictable number string, consisting of numbers 1 – 8. The results indicated that only the random number generation task disrupted performance for simple multiplication problems.

While the triple code model maintains that the phonological loop is recruited for simple, retrieval-based multiplication tasks, Seitz and Schumann–Hengsteler (2002) found no disruptive effects of articulatory suppression on the production or verification of single-digit multiplication facts. They concluded that, “temporary storage processes rely on phonological loop resources whereas fact retrieval relies on central executive resources” (p.301). Indeed, it seems plausible to hypothesize that complex tasks require participants to maintain temporary results and keep track of their mental calculations via sub-vocal rehearsal (De Rammelaere et al, 2001; Oberauer, Demmrich, Mayr & Kliegl,

2001) In other words, it appears that the PL is involved in more complex calculations, especially for the temporary retention of provisional results.

Similarly, Heathcote (1994) showed that the disruptive effect of articulatory suppression on the retention of intermediate results was significantly greater than the effects of visuospatial load. This provides further support for the assumption that the PL is involved in more complex calculations, especially for the temporary retention of provisional results. In essence, the above evidence implies that phonological resources play a role in complex calculations rather than simple fact retrieval. Moreover, the finding that the PL was involved in complex tasks across operations challenges the triple code model, which links specific WM resources to specific operations.

In essence, the research that has been reviewed so far provides no clear picture on the function of phonological representation in arithmetic problem-solving and this might be due to the methodological variety of the existing research. The studies in the review included dual-task experiments, correlational studies on the relationship between WM capacity and math ability, which employed different types of tasks (production vs. verification tasks) with different response requirements (verbal vs. written responses). Also, cross-cultural differences in learning maths and representing numbers have been considered as a potential explanation for the equivocal results. To recap, the available evidence regarding the role of the phonological loop in arithmetic problem solving is not yet consistent and especially the recruitment of the PL in all of the four basic arithmetic operations deserves further investigation to draw conclusions about whether the PL is responsible for fact retrieval or for complex calculations.

Given that no previous study addressed the question of how the PL is used in all four operations, this study is aimed at investigating the extent to which children with mathematical difficulties use subvocal rehearsal processes for solving addition, subtraction, multiplication and division problems. According to the triple code model, it could be hypothesized that the phonological loop is used for subvocal rehearsal in multiplication and division, but not addition and subtraction. The encoding complex model, on the other hand, assumes that the phonological loop is not tied to specific arithmetic operations as a mere fact retrieval device, but is used for the maintenance of temporary information during elaborate counting-based procedures via sub-vocal rehearsal. Based on the existing evidence which shows that children with MD experience problems in arithmetic fact retrieval and rely on counting-based procedure, it can be predicted that they use the phonological loop for subvocal rehearsal processes to solve basic arithmetic problems. Because children have more experience and practice in the domains of addition and subtraction, it can be hypothesized that the role of subvocal rehearsal is more salient in the domains of multiplication and division which should be reflected in a more extensive use of the phonological loop for these operations.

4.3 METHOD

4.3.1 DESIGN

A 4 x 3 within-participants design was employed to examine the interactions between arithmetic operation (addition, subtraction, multiplication and division) and dual-task condition (neutral condition, foot tapping condition, articulatory suppression condition). Articulatory suppression was chosen as a secondary task to load the phonological loop and to provoke interference with the subvocal rehearsal processes involved in arithmetic problem solving. The foot tapping task was chosen as a neutral distractor task which does not occupy working memory resources to the same extent as the articulation suppression task. Each operation was performed under each of the three conditions. The counterbalancing procedure will be explained in the “procedure” section.

4.3.2 PARTICIPANTS

A total of 67 children took part in the experiment. 27 children were from class 5 (16 males and 11 females) with a mean age of 11, 2 years, with 11 years being the minimum and 13 years the maximum age. 40 children (23 males and 17 females) from class 6, with a mean age of 12 years, with 11 years being the minimum and 14 years the maximum age. The sample was drawn from a school track which is specifically targeted towards children of lower ability, who do not have an identified learning disability but who performed consistently lower than average-ability peers during primary school. As has been mentioned, these children are characterised by severe academic underachievement and difficulties in the area of mathematics, and therefore the sample will be described as children with mathematical difficulties. It was decided to choose a younger sample, from the early secondary school classes, to minimize the potentially negative effects of exposure to the highly segregated school environment. Parents were informed about the experiment by the local educational authority and had the right to withdraw their children from participation. Participation was voluntary but an incentive (tickets for a premier – league soccer match) was offered for the ten best participants. The best participants were judged by adding the total amount of correctly solved arithmetic problems in each condition.

4.3.3 MATERIALS

The arithmetic test consisted of 24 arithmetic problems, six for each operation. The problems were presented on a sheet of paper. Given that the study involved three trials with a total of 72 problems, conducted with children with mathematical difficulties, it was decided to devise problems that were feasible for the children and

would keep them motivated. However, the question of what constitutes a simple as opposed to a difficult problem is contested. According to Seitz and Schumann-Hengsteler (2002), simple addition problems involve no carry, whereas difficult addition problems do. Thus, it was decided to use two-digit addition problems that did not involve a carry ($13 + 22$). Simple multiplication problems are described as problems where one operand is smaller than 6, while difficult multiplication problems are defined as problems where one operand is a two-digit number. Campbell (1997), on the other hand, argues that simple multiplication problems are problems with a product smaller than 25. However, it could be argued that such a cut-off criterion is arbitrary. For example, a problems such as $8 \times 4 = ?$ results in a product which is smaller than 25, but this problem is more demanding than a problem such as $10 \times 5 = ?$. Therefore, the proposal of Seitz and Schumann-Hengsteler (2002) was deemed more appropriate, and was used to devise a pool of simple and difficult multiplication and addition problems.

No such criteria were found with respect to the domains of subtraction and division. Therefore, it was decided to define simple subtraction problems as problems which do not involve a carry ($18 - 13 = ?$) and to define simple division problems as problems where the dividend is smaller than 6 ($42 : 6 = ?$). Problems which contained a 0, 1, or 2 as an operand (e.g., 5×0 , 1×4 , 2×3) and tie problems (e.g., $3 \times 3 = ?$) were rejected. A different set of problems was used in each of the three conditions. The problems were changed only marginally, in order to maintain a homogenous level of difficulty. Commuted pairs (e.g., $9 \times 4 = ?$ and $4 \times 9 = ?$) were considered as two different problems. The digits were aligned horizontally and presented on an A 5 sheet of paper. The reason for choosing horizontal rather than vertical alignment of problems was, that vertical alignment is typically associated with written arithmetic whereas the current experiment was targeted towards examining mental problem-solving.

4.3.4 PROCEDURE

Participants took part individually and were verbally instructed by the experimenter about the experimental procedure. It was emphasized that their performance would not affect their grades and that their results would remain confidential. Before the actual experiment started, participants were required to perform all the dual task conditions in order to make sure that they understood the requirements. In the foot tapping condition, participants had to tap with the foot in a continuous rhythm as exemplified by the experimenter. In the articulatory suppression condition, participants continuously repeated the non-word vowels “la-li-lu”. Participants were asked to start the particular secondary task a few seconds before they were allowed to turn over the problem sheets and to start solving them.

In order to avoid practice effects, the presentation of the three conditions was counterbalanced. However, it was assumed that children would feel more confident when starting with the neutral condition and in order to prevent distress, it was decided that this condition should come first. Therefore, incomplete counterbalancing rather than the traditional Latin square procedure was used, so that the neutral condition was always the one to start with. Only the order of the the two dual – tasks was permuted. The rationale behind the decision to have the neutral condition first is that a low performance under articulatory suppression or foot tapping might lead to frustration and distress among children of low ability which might have an inhibiting effect on subsequent performance. Arithmetic performance was measured in terms of the correctly answered problems in each domain. Each trial took between 4 and 5 minutes. After the procedure, children were thanked for their participation and received a small gift.

4.3.5 RESULTS

To recap, the aim of the present study was to examine the usage of the phonological loop for addition, subtraction, multiplication and division in children with and without mathematical difficulties (MD). A 4 x 3 analysis of variance was employed to analyse the effects of arithmetic operation and dual-task condition (neutral condition, foot tapping condition, articulatory suppression condition) on the problem solving ability of children with MD. The results of the mixed factorial repeated-measures ANOVA show that the effect of condition is statistically significant ($F_{(2, 66)} = 12.77, p < .01$). There was also a significant main effect of arithmetic operation ($F_{(3, 66)} = 13.28, p < .01$). As shown in table 4.1., the performance of children with MD differs across different conditions and across different operations. This reflects the fact that there is a significant interaction effect between the three variables ($F_{(6, 66)} = 12.98, p < .01$).

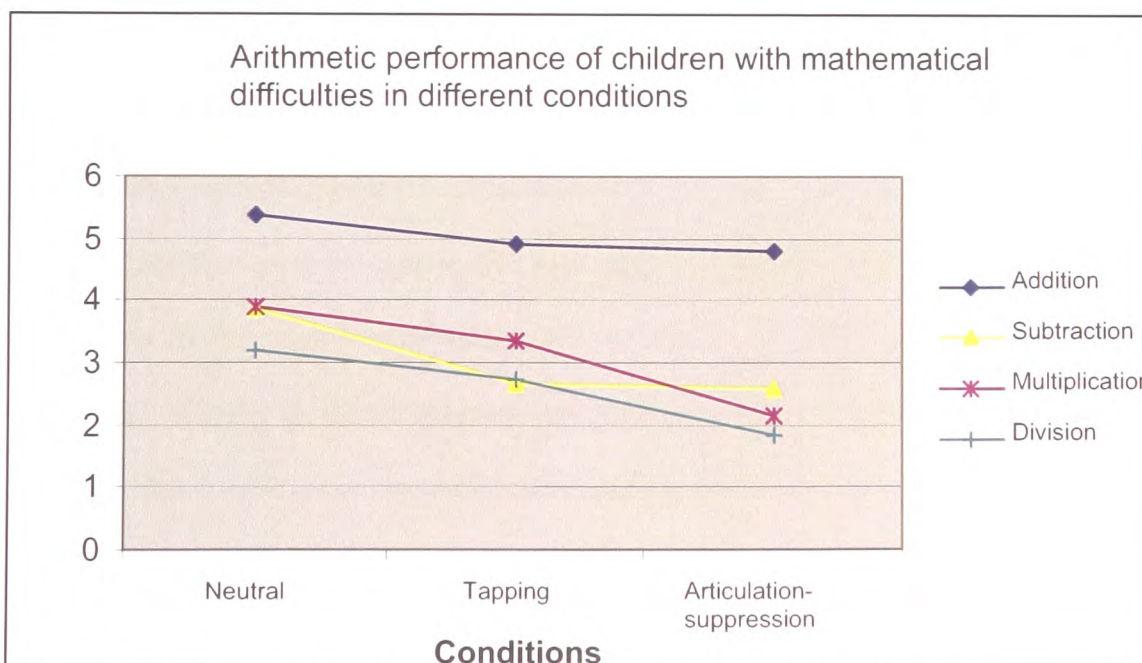


Figure 4.1: Interaction effect between arithmetic operation and condition

Table 4.1: Means and standard deviations for correctly solved arithmetic problems

Operation	Condition	Mean	Standard Deviation
Addition	Neutral	5.39	0.85
	Tapping	4.90	1.14
	Articulatory suppression	4.79	1.15
Subtraction	Neutral	3.84	2.01
	Tapping	2.63	1.87
	Articulatory suppression	2.57	1.59
Multiplication	Neutral	3.90	1.96
	Tapping	3.36	1.86
	Articulatory suppression	2.16	1.81
Division	Neutral	3.19	1.95
	Tapping	2.73	2.00
	Articulatory suppression	1.85	1.73

T-tests with Bonferonni's correction were used as post-hoc tests. The results show that addition performance in the foot tapping condition is lower than addition performance in the neutral condition, when no dual task is imposed. Also, addition performance is lower in the articulatory suppression condition compared to the neutral condition. These differences are statistically significant ($t_{(66)} = 2.8, p < .01$ and $t_{(66)} = 3.62, p < .01$, respectively). However, the difference between the foot tapping and the articulatory suppression condition is not statistically significant ($t_{(66)} = .63, p = .53$), indicating that foot tapping and articulatory suppression have effects of the same magnitude on addition performance.

For subtraction problems, performance in the foot tapping condition is lower than subtraction performance in the neutral condition. Also, subtraction performance is lower in the articulatory suppression condition compared to the neutral condition. The observed differences are statistically significant ($t_{(66)} = 4.78, p < .01$ and $t_{(66)} = 4.96, p < .01$, respectively). The results show a significant effect of foot tapping and a significant effect of articulatory suppression on subtraction performance. However, the difference between performance in the foot tapping and the articulatory suppression condition is not statistically significant ($t_{(66)} = .29, p = .77$). Therefore, the results for subtraction are similar to the results for addition, in that articulatory suppression did not add any extra distraction compared to foot tapping

For multiplication problems, there was no significant difference between performance in the foot tapping condition and performance in the neutral condition ($t_{(66)} = 2.08, p < .01$). However, multiplication performance was significantly lower in the articulatory suppression condition compared to the neutral condition ($t_{(66)} = 6.72, p < .01$). Also, the results show a significant difference between performance in the foot tapping and the articulatory suppression conditions, with a lower mean number of correct answers in the articulatory suppression condition ($t_{(66)} = 5.67, p < .01$).

For division problems, there was no significant difference between the foot tapping and neutral conditions ($t_{(66)} = 1.98, p < .05$). Division performance was significantly lower in the articulatory suppression condition compared to the neutral condition ($t_{(66)} = 5.29, p < .01$). Also, articulatory suppression resulted in a lower mean number of correctly answered division problems than the foot tapping condition ($t_{(66)} = 3.82, p < .01$). Thus, it seems that articulatory suppression has a significantly greater effect than foot tapping on performance in division problems. The results for division

are similar to the results for multiplication, in that the disruptive effect of articulatory suppression was greater than the effect of foot tapping. In summary, it seems that unlike addition and subtraction, multiplication and division involve a subvocal rehearsal process which is governed by the PL and which is interrupted if phonological load is increased through an articulatory suppression task.

4.3.6 DISCUSSION

The aim of the study was to address the question of how children with mathematical difficulties (MD) use the phonological loop (PL) in addition, subtraction, multiplication and division. Two theoretical models of arithmetic cognition bear centrally on this issue. To recap, the triple-code model proposes a phonological storage hypothesis, which maintains that basic arithmetic facts are stored in a learned lexicon of verbal associations through the process of subvocal rehearsal and simple multiplication and single-digit addition problems are usually solved by direct retrieval of facts from the long term memory (LTM), while more complex addition, subtraction and division problems require more elaborate cognitive processes, namely the semantic manipulations of numerical quantities, which are based on the visuospatial sketchpad (Cohen, 2000).

In its original prediction, the triple code model assumes that only single-digit addition and multiplication are coded in a verbal format. However, it is argued that basic division facts should also be coded in a verbal format, given that division problems are often recast as multiplication problems ($42: 6 = 6 \times ? = 42$) and therefore also draw on the learned lexicon of verbal associations (Mauro, 2003). The ecoding complex model on the other hand challenges the assumption that the different arithmetic operations are handled by specific working memory resources and maintains instead that factors such as problem difficulty, learners' ability and presentation format influence the usage of working memory resources for mental arithmetic problem-solving. However, there is no known research on how children with MD use the PL for solving addition, subtraction, multiplication and division. The current study addressed this issue as will be discussed in the following paragraphs.

4.4 PERFORMANCE OF CHILDREN WITH MD IN BASIC ARITHMETIC OPERATIONS

Overall, the small mean number of simple subtraction, multiplication and division problems that were solved correctly in the neutral condition seems to confirm that children with MD have severe deficits in basic arithmetic processing. The finding that foot tapping impaired addition and subtraction processes but not multiplication and division might be explained in terms of the problem-size effect, which would predict that whereas one-digit multiplication and division problems are mainly solved via direct retrieval, two-digit addition and subtraction problems can be solved via different alternative strategies, such as decomposition ($17 + 12 = 17 + 10 + 2$) or rounding ($17 + 12 = 20 + 12 - 3$) (LeFevre, Sadesky & Bisanz, 1996). While these strategies are not necessarily more difficult, these procedures take longer to perform than direct retrieval because they involve the temporary retention of information and/ or counting processes. Therefore it seems plausible that these operations are more fragile and more easily disrupted by foot tapping than multiplication and division. This would explain why a routine task such as foot tapping interferes with only with addition and subtraction.

Implications of the effect of articulatory suppression on different arithmetic operations

Turning now to the effects of the dual tasks, the results show that phonological load, induced through an articulatory suppression condition, had a detrimental effect on addition, subtraction, multiplication and division compared to the neutral condition, which suggests that sub-vocal rehearsal is involved performance in all of the four arithmetic operations. However, the results showed that the magnitude of the effect of phonological load caused by articulatory suppression differs across arithmetic

operations, which are differently affected by articulatory suppression. The results also showed that while foot tapping did not impair multiplication and division performance to a significant extent for, the occupation of the phonological loop through the articulatory suppression task did.

For addition and subtraction, on the other hand, there was no difference between the distraction caused by foot tapping and articulatory suppression. This implies that the effect of articulatory suppression on addition and subtraction performance did not go beyond the effect of a mere distracter task such as foot tapping. Overall, the results suggest that children with MD use different cognitive processing resources for addition and subtraction on the one hand and multiplication and division on the other; this goes in conjunction with the triple code model. The triple code model predicts that multiplication and division are more affected by articulatory suppression than addition and subtraction because multiplication is supposed to be learned via verbal rote memorization and based on verbal-phonological strategies: “multiplication tables are memorized as verbal associations between numbers represented as strings of words, and ... multi-digit operations are performed mentally using the visual Arabic code” (Dehaene, 1997, p. 46).

As has been illustrated in the literature review, a considerable number of studies show that the activation of the phonological loop is not exclusively tied to retrieval multiplication problems. Rather, there is evidence to suggest that the opposite might be the case, namely that the phonological loop is recruited for more complex rather than simple, retrieval-based problems. De Stefano & LeFevre (2004), for example, found that the PL plays a role in problems where intermediate results have to be retained in

memory, and concluded that problem difficulty has a considerable impact on the recruitment of WM components.

The finding that articulatory suppression interfered with the cognitive processes in complex multiplication problems is supported by the results of Seitz et al. (2000). In essence, they found no interference between articulatory suppression and multiplication problems when both operands were smaller than 6. Interference occurred only when larger operands were used. They concluded that there is no phonological-specific access to multiplication facts stored in long-term memory. This challenges the hypothesis that phonological resources are used for the retrieval of simple arithmetic facts and suggests instead that the phonological loop is used for the subvocal rehearsal of temporary results during counting processes. As Seitz et al. (2002, p.299) argue: “we infer that solving complex multiplication sums demands phonological loop and central executive processes, whereas retrieving numerical facts in solving simple multiplication sums requires only central executive processes.”

In support of Seitz et al. (2000), Logie et al. (1994) concluded that the role of the phonological loop is to maintain accuracy during mental arithmetic rather than retrieving number facts. Similarly, Hecht (2002) found evidence that the recruitment of the PL is influenced by the strategy that learners choose for solving arithmetic problems. It was found that the only strategy type to be affected by articulatory suppression were counting-based strategies, thus implying that this strategy type is partly based on the PL. This finding is at odds with the prediction of Cohen and Dehaene that the PL is not involved in counting-based procedures. Thus, instead of working memory components being linked to specific operations, as suggested by Dehaene and Cohen, other evidence suggests that the involvement of the phonological

loop is due to the problem difficulty, which requires participants to maintain temporary results and keep track of their mental calculations via sub-vocal rehearsal. Seitz et al. (2000) concluded, “We found no evidence of modality-specific access to numerical facts stored in long-term memory.” Instead, the findings of this study support the view that the PL is involved in complex problem solving processes, which involve the temporary retention of provisional results via sub-vocal rehearsal. In order to test this prediction, the following study investigates the effects of problem difficulty and mathematical ability on PL usage.

4.5 STUDY 2: WORKING MEMORY AND ARITHMETIC ABILITIES

Based on the results of the previously presented study, this study is intended to investigate how children with MD and children of average mathematical ability use the phonological loop for simple and difficult multiplication tasks. As has been mentioned previously, the literature on the differences between children with mathematical difficulties and children of average mathematical ability suggests that MD children have problems in retrieving facts from long-term memory and use an appropriate solution procedure. It is argued that individual differences in working memory usage account for these disparities in mathematical ability between children (Adams and Hitch, 1998). However, research findings with respect to phonological processing in children with MD are conflicting. The present experiment employs a dual-task paradigm to test the possibility that compared to children from an average ability control group, children with MD have specific problems with engaging in multiplication when their PL is occupied. In other words, it is hypothesized that for simple multiplication problems, an articulatory suppression task should cause greater interference for children with MD than for children from an average ability control group.

4.6 METHOD**4.6.1 DESIGN**

A 2 x 2 x 3 mixed factorial repeated measures design was employed to examine the interactions between ability level (mathematical difficulty vs. no mathematical difficulty), problem difficulty (simple vs. difficult) and dual-task condition (neutral condition, foot tapping condition, articulatory suppression condition). As in the previous study, the order of the tasks was counterbalanced. Arithmetic performance was

measured in terms of the correctly answered simple and difficult multiplication problems.

4.6.2 PARTICIPANTS

A total of 24 children with MD and 24 control group children of average mathematical ability from class 5 took part in the present study. The mean age was 10.4 years. The group of MD children consisted of 14 males and 10 females from the previous study. The average ability control group consisted of 13 males and 11 females. To reiterate, the educational system at secondary school level is based on ability tracking, which takes place in the final primary school year. Children with MD were defined as children who attended a general secondary school and are in a school track which has a vocational rather than an academic emphasis and which is specifically targeted towards children of lower ability, who do not have an identified learning disability but who performed consistently lower than average-achieving peers during primary school. Control group children were recruited from a comprehensive school, which is intended for average-achieving children without academic difficulties. At both schools, parents were informed about the experiment by the local educational authority and had the right to withdraw their children from participation. Participation was voluntary but an incentive (tickets for a premier – league soccer match) was offered for the ten best participants.

4.6.3 MATERIALS

The arithmetic test consisted of twenty visually presented multiplication problems, ten of which were classed as simple and ten as difficult problems. In line with the recommendations of Seitz and Schumann-Hengsteler (2002), simple multiplication problems are described as problems where one operand is 6 or smaller, whereas difficult multiplication problems are defined as problems where one operand is a 2-digit number. In contrast to the previous study, which involved a paper-and-pencil test, it was decided to use a computer-based presentation of the problems. The CLIC authoring software was used to design a test program that involved a randomised presentation of simple and difficult multiplication problems. As in the previous study, the problems were horizontally aligned and remained constantly visible. The number of correctly solved simple and difficult problems was presented at the end of the program.

4.6.4 PROCEDURE

The procedure was similar to the procedure in the previous study, with the exception that the children performed the test on a laptop. After children were familiarised with the secondary task, the test program started with three simple trial problems, which were not included in the analysis. Children were presented with simple and difficult multiplication problems and were asked to type the correct answer in a box via the keyboard. If they didn't know an answer, they were instructed to type in a "0". The problem was visible until children typed in an answer and pressed the "enter" bar on the keyboard. The result was available after the twenty problems have been solved. The procedure took approximately five minutes.

4.6.5 RESULTS

To recap, the aim of the present study was to examine the usage of the phonological loop for simple and difficult multiplication in children with MD and children from the control group. A 2 x 2 x 3 mixed factorial repeated measures analysis of variance was employed to analyse the effects of multiplication problem difficulty (simple vs. difficult), ability (mathematical difficulty vs. no mathematical difficulty) and dual-task condition (neutral condition, foot tapping condition, articulatory suppression condition).

Table 4.2: Means for correctly answered multiplication problems in children with MD and average ability children

Difficulty	Condition	Ability	Mean	Standard Deviation
Simple	Neutral	Control	9.46	0.66
		MD	8.88	1.36
Simple	Foot Tapping	Control	8.75	0.99
		MD	8.25	1.26
Simple	Articulatory suppression	Control	8.13	0.80
		MD	4.54	1.10
DIFFICULT NEUTRAL		Control	8.13	0.80
		MD	6.29	1.20
Difficult	Foot Tapping	Control	7.88	0.95
		MD	5.13	1.12
Difficult	Articulatory suppression	Control	4.00	0.88
		MD	4.04	1.12

Table 4.2 shows the means and standard deviations for the number of correctly solved simple and difficult multiplication problems in three different conditions. The results of the mixed factorial repeated measures ANOVA show statistically significant effect of ability ($F_{(1, 46)} = 85, 42, p < .01$), problem difficulty ($F_{(1, 46)} = 160, 40, p < .01$) and condition ($F_{(2, 92)} = 359, 65, p < .01$). The interaction of the three variables shows that articulatory suppression impairs complex but not simple problem-solving in children of average ability whereas both simple and complex problem-solving is impaired in children with MD.

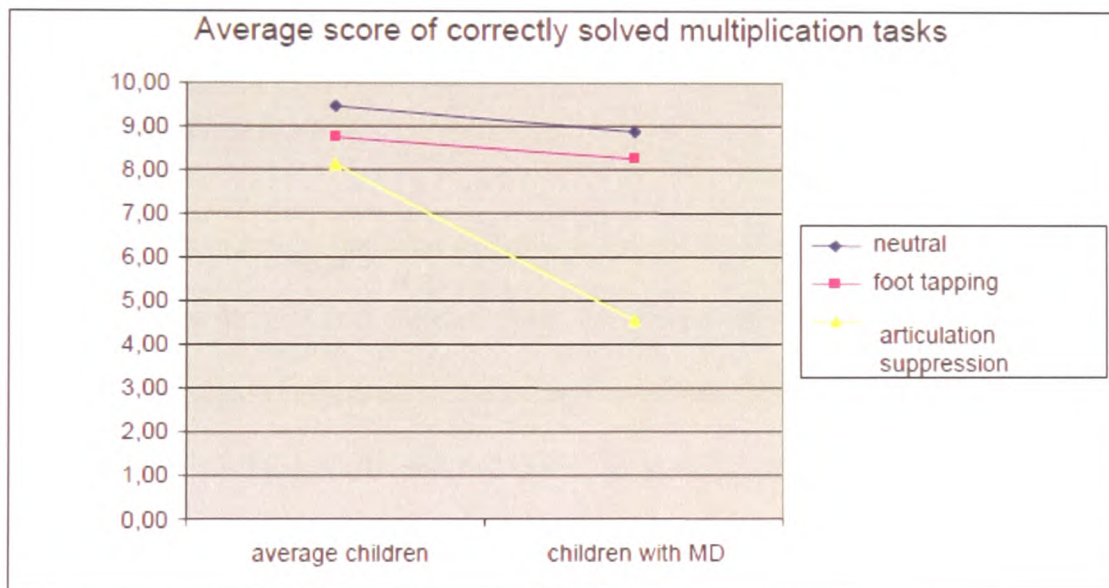


Figure 4.2: Performance of children with MD and average ability children in simple multiplication problems

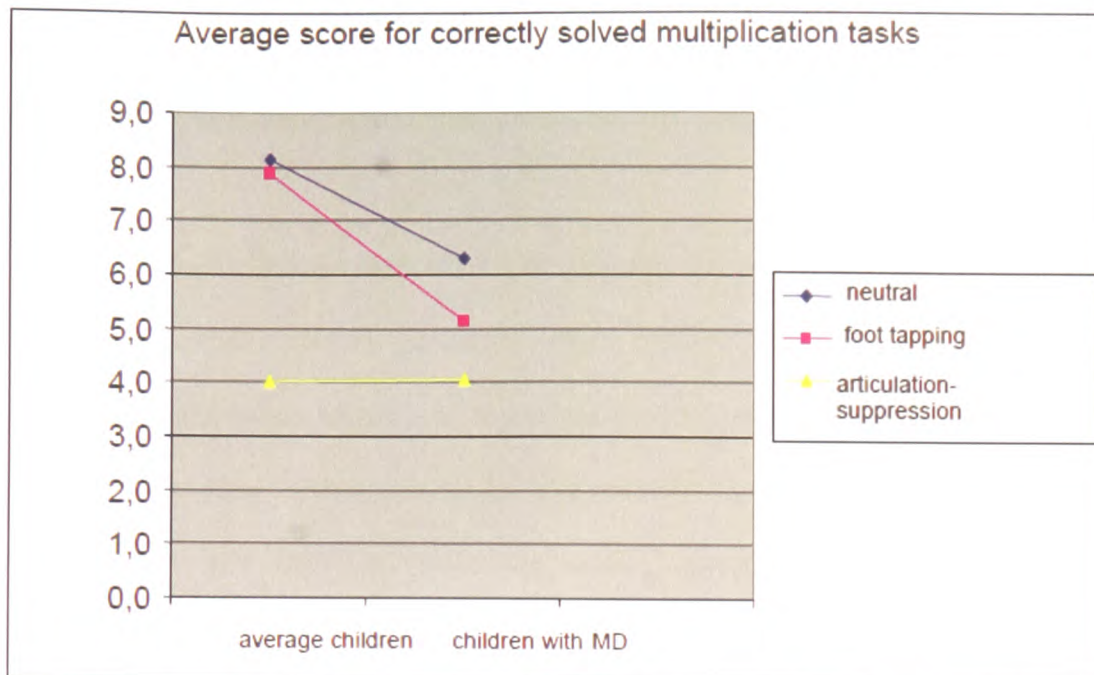


Figure 4.3: Performance of children with MD and average ability children in difficult multiplication problems

Simple multiplication

The descriptive results depicted in Table 4.2 show that for simple multiplication problems, children with MD and children from the control group performed worse in the foot tapping condition compared to the neutral condition. The effect of foot tapping is significant ($t_{(23)} = 3.21, p < .01$ and $t_{(23)} = 3.98, p < .01$, respectively). The mean number of correctly solved problems between children with MD and average achieving children is not statistically significant, either in the simple control or in the simple foot tapping condition ($t_{(46)} = 1.53, p = .13$).

While the two groups performed at similar levels in the simple foot tapping condition, there is a significant performance difference in the simple articulatory suppression condition ($t_{(46)} = 12.91, p < .01$). Indeed, the performance of the control group children for simple multiplication did not significantly differ between the foot tapping and the articulatory suppression condition ($t_{(23)} = 2.7, p < .01$). This pattern is

different for children with MD, who had fewer correctly solved problems in the articulatory suppression condition than in the tapping condition $t_{(23)} = 26.32, p < .01$.

The results indicate that while children with MD are disrupted in their ability to solve simple multiplication problems when articulatory suppression is induced, articulatory suppression merely had a distractor effect on control group children, who are nonetheless able to maintain an adequate level of performance. This implies that children with MD are less able than control group children to solve simple multiplication problems when the phonological loop is occupied.

Difficult multiplication

To recap, children with MD and children from the control group performed similarly when simple multiplication was performed in the control or in the foot tapping condition. However, when difficult multiplication was concerned, children with MD performed significantly worse than control group children in the control and in the foot tapping condition ($t(46) = 6.24$; $p < .01$ and $t(46) = 9.21$, $p < .01$, respectively).

As can be seen from Table 4.2, children from both groups performed worse under articulatory suppression than in the control or the foot tapping condition. For control group children, the mean score of correctly answered difficult multiplication problems is lower in the articulatory suppression than in the neutral and the foot tapping conditions. The results indicate that the differences are statistically significant ($t(23) = 14.84$, $p < .01$ and $t(23) = 14.65$, $p < .01$, respectively). The same finding applies to children with MD ($t(23) = 4.75$, $p < .01$ and $t(23) = 3.68$, $p < .01$, respectively).

Thus, the advantage of control group children seems to disappear under articulatory suppression: there is no longer a significant performance difference between children with MD and average-achieving children when difficult multiplication is coupled with articulatory suppression. Instead, for both groups of children, articulatory suppression creates a disruptive effect on difficult multiplication performance, which is distinct from the mere distraction caused by tapping. This indicates that articulatory suppression impedes the cognitive processes required for difficult multiplication. However, the effect of articulatory suppression is more profound for children with MD.

The results in table 4.2 indicate that in the articulatory suppression condition, children with MD do not perform better in simple than in difficult multiplication and the t-tests confirm that there is no significant performance difference ($t_{(23)} = 1.63, p = .12$). On the other hand, children of average mathematical ability performed significantly better for simple than for difficult problems in the articulatory suppression condition ($t_{(23)} = 16.47, p < .01$). Thus, it seems that in contrast to control group children, children with MD experience problems when multiplication has to be performed under articulatory suppression, irrespective of the level of difficulty.

4.7 DISCUSSION

The rationale behind the present study was to examine how children with and without mathematical difficulties (MD) use the phonological loop for simple and difficult multiplication problems in order to derive insights into the cognitive processing differences between children with MD and average achieving children. A mixed factorial repeated measures analysis of variance showed an interaction between ability (children with MD vs. control group children), problem difficulty (simple vs. difficult) and secondary condition (neutral condition, foot tapping and articulatory suppression).

4.7.1 SIMPLE MULTIPLICATION – NEUTRAL CONDITION

Overall, children with MD were able to solve nearly all simple multiplication problems correctly in the neutral condition. Thus, even in a test situation, children with MD demonstrated near-perfect performance in multiplication on the same level as control group children. However, despite similar performance, it is unclear to what extent children with MD and average children differed with respect to strategy usage.

According to Shrager and Siegler (1998), children use a variety of strategies for solving arithmetic problems. Individual differences in ability are an important variable when strategy usage is concerned. Lucangeli (2003), for example, found that more proficient children were more likely to solve multiplication problems via direct retrieval. Thus, it might be the case that average ability children were able to solve simple multiplication problems via direct retrieval whereas children with MD relied on less efficient strategies, such as decomposition of problems ($6 \times 4 = 6 \times 2 \times 2$). In other words, it seems that children with MD were able to compensate for retrieval deficits by using more effortful strategies and therefore managed to perform equally well as control group children. Such a hypothesis goes in line with findings that children with MD are more likely to use inefficient counting-based decomposition strategies rather than direct retrieval (Lemaire and Siegler, 1996; Lucangeli, 2003).

4.7.2 SIMPLE MULTIPLICATION – ARTICULATORY SUPPRESSION CONDITION

To recap, children with MD and average children performed equally well when simple multiplication was performed in the control or in the foot tapping condition. However, the results indicate that children with MD are more severely impaired by articulatory suppression in their ability to solve simple multiplication problems. Articulatory suppression caused a greater disruption of performance than foot tapping among children with MD. For control group children, on the other hand, the effects of articulatory suppression were not different from the effects of a mere distractor task such as foot tapping. Several implications can be derived from these findings.

With respect to the role of the PL, the finding that average-achieving children use fewer phonological resources than children with MD for simple problems goes in line with the assertion that the role of the phonological loop is to monitor the problem-solving procedure and to keep track of interim results in ongoing calculations. Secondly, the findings indicate that in contrast to children with MD, control group children are able to maintain an adequate level of performance irrespective of the occupation of the phonological loop. Further, it seems that only children with MD were severely affected by articulatory suppression: for control group children, articulatory suppression turned out to be much less demanding than had been anticipated and did not have a different effect from foot tapping.

One possible explanation for the findings is that children with MD do not automatically retrieve answers to simple multiplication problems but use laborious and ineffective counting-based processes, even when simple problems are concerned. This effect is even more pronounced for difficult problems: given that simple problems require considerable cognitive effort, it follows that complex problems exceed the problem-solving ability of children with MD. In sum, the marked disruption caused by articulatory suppression compared to foot tapping corresponds to the findings of Geary (1999) and Anderson and Lyxell (2007), who concluded that an occupation of the PL interrupts numeric processing in children with MD.

4.7.3 DIFFICULT MULTIPLICATION - NEUTRAL CONDITION

While children with MD and average children performed equally well when simple multiplication problems were performed in the neutral condition, there was a marked difference with regard to difficult multiplication problems. The finding that

children with MD performed worse than children without in complex multiplication problems is in agreement with the previously mentioned hypothesis, namely that children with MD used effortful decomposition strategies, rather than direct retrieval, to solve simple multiplication problems.

To recap, the results seem to indicate that children with MD were able to solve simple multiplication problems via alternative strategies such as counting or decomposition, which would explain why children with MD performed on the same level as control group children. However, these strategies seem not to work for complex multiplication problems. Thus, the present results suggest that deficits in working memory usage can be compensated in simple multiplication problems but not in difficult multiplication problems, which require the manipulation of numeric information and the maintenance of temporary information.

4.7.4 DIFFICULT MULTIPLICATION – ARTICULATORY SUPPRESSION CONDITION

While the findings showed that average-achieving children performed significantly better than children with MD when simple multiplication problems had to be solved under articulatory suppression, there was no significant difference between the two groups when difficult multiplication was performed under articulatory suppression, suggesting that the addition of phonological load through articulatory suppression impedes the cognitive processes required for difficult multiplication, irrespective of children's ability. Thus, the effect of a phonological working memory constraint such as articulatory suppression seems to be more profound in more difficult problems. However, children with MD performed equally badly in simple and difficult

multiplication tasks in the articulatory suppression condition, whereas control group children performed significantly better when simple rather than difficult multiplication problems had to be performed in the articulatory suppression condition. Thus, the interference effect of articulatory suppression is significantly less profound for simple than for difficult multiplication problems, suggesting that control group children use phonological memory resources more efficiently.

4.8 CONCLUSION

To sum up, differences in WM usage have been observed between children with MD and children of average ability. The relative effect of articulation suppression on simple and complex problem-solving in MD and average ability children seems to correspond to the general effect of ability. The interaction effect of articulation suppression, problem difficulty and math ability that has been observed supports the assumption that average ability children are competent in simple problem-solving but less competent in complex problem-solving whereas children with MD experience difficulties on both types of problems. It seems that average ability children need to use subvocal rehearsal for complex problems when simple problems are concerned whereas children with MD require subvocal rehearsal even for simple problems. This would explain why the phonological load, created through the articulatory suppression task, affected simple and complex problem-solving in MD children but only complex problem-solving in average ability children.

This difference in working memory usage might be a function of individual differences in strategy usage. This goes in line with previous research, which showed that children with MD have difficulties in the automatic retrieval of arithmetic facts.

The lack of automatization might result in the usage of inefficient strategies, which in turn occupy working memory resources: “inefficient processing of basic facts is expected to have high cognitive load...and a lack of automaticity on the basic facts contributes to failure on more complex tasks” (Cumming and Elkin, 1999, p. 160). Indeed, it has been shown in the present study that children with MD and children from an average ability control group were impaired in their ability to solve simple multiplication problems under dual task performance, whereas articulatory suppression yielded significantly poorer performance than did a foot-tapping task for children with MD. In other words, it seems that for children with MD, the effect of articulatory suppression is more severe than the effect of foot tapping.

For children with MD, this would suggest a marked interference between the phonological cognitive processes required for multiplication and articulatory suppression through an articulatory suppression task, so that even simple multiplication problems are not performed better than difficult problems. This supports the findings of Geary (1999), who concluded that children with MD have specific problems with engaging in numeric processing when their PL is occupied. It was proposed that these problems can be attributed to an inability to solve problems via direct retrieval because the associations of arithmetic facts in long-term memory are weak and difficult to access, so that even simple problems have to be solved using effortful decomposition strategies.

4.8.1 GENERAL DISCUSSION

The first study in this section showed that articulatory suppression had a more adverse effect on multiplication and division performance than on addition and subtraction performance in children with MD. The findings support the triple code model, which predicts that the usage of the phonological loop is tied to retrieval-based operations. However, previous evidence suggested that the recruitment of the phonological loop is not governed by the arithmetic operation but is influenced by factors such as problem difficulty. In order to investigate this question further, the second study compared phonological loop usage in children with MD and a control group for simple and difficult questions.

It was found that control group children used the phonological loop only for difficult problems, whereas children with MD used the phonological loop for simple as well as difficult problems. The results suggest that children of average ability do not use the phonological loop for simple problems, because they are able to retrieve the answers to simple problems directly from memory. However, the phonological loop becomes salient in difficult problems, which require the maintenance of interim results and which are solved via algorithmic back-up strategies rather than direct retrieval. In other words, phonological working memory constraints only appear in children of average ability as the demands of the problem increase. Children with MD, on the other hand, seem to use the PL in a different and less efficient way, so that even simple multiplication problems require the same cognitive effort as difficult problems.

Taken together, the findings support Campbell's encoding complex model, which assumes that the phonological loop is a function of variables such as problem difficulty rather than the type of operation. In line with the present findings, there is convincing evidence that phonological resources play a role in complex calculations rather than simple fact retrieval. This challenges the prediction of the triple code model that "the phonological loop is not available for complex operations such as $13 + 5$ or for subtraction or division problems that are not normally acquired by verbal rote learning." Instead, the results support the view that phonological resources will be used when children rely on strategies other than direct retrieval to solve multiplication problems. Children with MD seem to use phonological resources for simple problems, because they use non-retrieval, "back up" strategies even for solving simple multiplication problems, rather than retrieving the result automatically. On the other hand, no inference occurred for children from the control group, who were able to solve multiplication problems via direct retrieval.

Limitations and Conclusion

Several possible limitations of the present study have to be taken into consideration. Firstly, it has to be acknowledged that dual task performance was not measured and can therefore not be adequately analysed. The absence of a significant effect of foot tapping on multiplication and division might therefore be the result of a trade-off, whereby foot-tapping performance declined in order to perform the problems adequately. However, the impression of the experimenter was that the dual tasks were performed correctly. Another possible drawback of the present study is the relatively small number of problems from each operation. However, participants had to take part in all three conditions and it was decided to keep the number of problems minimal in

order to prevent distress among the students. The results indicated that despite the small number of problems, no floor or ceiling effects could be observed.

Also, it has been mentioned already that there is no agreement with respect to the question of what constitutes a simple as opposed to a difficult problem. However, the definition of problem difficulty has a considerable impact when examining the role of the phonological loop in arithmetic, particularly if comparisons are made across operations. Each operation possesses unique properties that influence cognitive processing and difficulty, and this makes it difficult to directly compare the operations. Nevertheless, given that the amount of previous research on subtraction and division was limited, it was necessary to find a starting point, and for the purpose of the present study, the criterion of Seitz and Schumann-Hengsteler (2002) was adopted and simple addition and multiplication problems were defined as two-digit addition problems which involve no carry, such as $13 + 22 = ?$ or $24 - 11 = ?$ Simple multiplication and division problems are described as single-digit problems where one operand is 6 or smaller.

However, Dehaene and Cohen (1997) argue that a seemingly simple problem such as " $13 + 5 = ?$ " constitutes a complex problem, which should therefore not be solved via the PL. If the criterion of Dehaene and Cohen was correct and if the problems in the present study were in fact complex problems, according to their prediction, then the finding that phonological resources were involved in these difficult problems strengthens the case against the triple code model, which assumes that phonological resources are used for rote learned facts whereas complex problems are based on semantic manipulation, as mediated via the VSSP.

4.8.2 FUTURE SUGGESTIONS

With respect to the effect of foot tapping, it seems possible that the foot-tapping task might not be merely a distractor task, but might involve CE resources in controlling the continuous tapping procedure. Further, it is suggested that the CE is required for simple problems to avoid associative confusion errors and inhibit inaccurate responses. This would explain why foot tapping caused interference in simple multiplication for both children with MD and control group children. Further research would be required to confirm that foot tapping involves central executive resources. Also, a major limitation of research in this area is that the evidence permits no consensus regarding the question of whether observed WM deficits are a precursor or a result of MD, and this question is awaiting research (McLean & Hitch, 1999).

Nevertheless, the finding that the fundamental misuse of working memory in children with MD elevates cognitive load to such an extent that even simple problem-solving processes require the same mental effort as difficult problems bears considerable implications for educational practice. The central question is, whether instructional intervention can help children with MD to overcome their working memory deficits, irrespective of whether or not these deficits are acquired or an innate characteristic of MD. It has been discussed in this chapter that the misuse of working memory use is potentially related to the use of inefficient problem-solving strategies. Deficits in procedural knowledge in LTM place unnecessary demands on limited WM capacity and therefore working out simple arithmetic problems requires considerable processing resources.

The possible linkage between working memory effort and strategy use leads to the question whether this misuse of working memory can be alleviated by teaching efficient problem-solving algorithms by means of explicit and structured instructional modelling of strategies and by facilitating the automatisisation of factual knowledge through repeated practice and feedback. These instructional features have been translated in a paradigm known as direct instruction (DI), which will be described and evaluated in the following chapter. The following chapter will address this issue by investigating the learning outcomes of children with MD when exposed to different instructions.

CHAPTER 5

INSTRUCTION FOR CHILDREN WITH MATHEMATICAL DIFFICULTIES

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5.1 OVERVIEW

The previous chapter examined the ways in which children with MD differ from average ability children with respect to their usage of working memory resources for mental arithmetic problem solving. To recap, the findings showed that children with MD are less able to solve simple problems under phonological load than average ability children. For children with MD, even simple problems require extensive phonological resources and this implies an increase of cognitive load during the problem-solving process. It has been suggested that differences in working memory usage can be attributed to differences in problem-solving strategies. In line with the previous literature, the findings seem to indicate that children with MD use inefficient algorithms for solving arithmetic problems and lack automatised basic facts whereas average ability children are able to solve simple problems via direct retrieval. Whether the misuse of WM resources and the extensive cognitive effort which follows from this misuse is a cause or an effect of inefficient strategy usage remains unclear. However, it seems pivotal to find out how the format of math instruction can be adapted to account for the cognitive processing deficits in children with MD. Therefore, this chapter examines how the type of instruction influences the learning of mathematics in children with mathematical difficulties (MD), by investigating the effects of minimally guided and direct instruction on arithmetic ability.

The term minimally guided instructions (MGI) was coined by Mayer (2004) as an umbrella term to cover constructivist-inspired learning activities such as discovery learning or cooperative learning. Direct instruction (DI) was concerned with the acquisition of efficient problem solving strategies. This approach is derived from a cognitive approach to learning which is particularly concerned with the reduction of cognitive load on memory resources so that these resources could be allocated to the learning processes, namely schema acquisition and automatisisation (Tuovinen, 1999).

In essence, a central difference between these approaches relates to the question, whether children learn most effectively through minimally guided inquiry (minimal guidance instruction) or by being exposed to highly directed and structured instructional activities (direct instruction) (Stein, Silbert and Douglas, 1997). There is evidence for both sides of the argument and the aim of this chapter is to extend the existing research. The first study is concerned with the effects of the different instructional intervention on the learning of mathematics, as determined by a pre-test post-test design. The second study is aimed at investigating the extent to which variation in learning gains can be explained by individual differences in affective responses towards mathematics.

The chapter starts with a review of the so called 'Follow-Through' study, a large scale longitudinal study from the United States on the learning in low achievers which involves an experimental investigation of the utility of different types of instruction for children with MD. While the results of the Follow-Through study imply that children with MD are better able to learn mathematics in DI settings, these results have to be treated with caution for a number of reasons which will be discussed. Also, more recent evidence shows that even children with MD can benefit from constructivist-inspired

MGI. The implications of this evidence will be considered in order to derive at specific research questions towards the end of this chapter.

5.1.1 LEARNING MATHEMATICS IN CHILDREN WITH MD – IMPLICATIONS OF THE FOLLOW THROUGH STUDY

The amount of literature on the effects of different forms of instruction on the learning in children with MD is limited because the majority of research centres on the identification and interventions for children with mathematical disabilities rather than difficulties (Baker, Gersten, Lee, 2002). A notable exception to this is the ‘Follow Through’ (FT) study. Initiated in the late 1960’s by the US Department of Education, the ‘Follow-Through’ study involved nearly 80,000 children from low-income backgrounds in the United States. It was intended to investigate the effectiveness of different types of instruction for children with MD (Adams and Engelmann, 1996). The comparison between different types of instruction was economically feasible because developers of the different educational intervention programs agreed to sponsor the implementation of their models at particular sites. The participating school districts chose a sponsor of an educational program that represented a certain educational approach. Each sponsor implemented its program at up to 8 sites, each of which had a non-treatment group as a control group. The different educational interventions that were evaluated in the Follow Through study can be categorised into three major models: the basic skills models, the conceptual skills model and the affective skills models. The first two instructional models are particularly relevant for the present purpose because they represent two opposing approaches to the learning of mathematics: direct instruction and the constructivist –inspired minimally guided instruction approaches.

The conceptual skill model rests on the constructivist approach to learning, which assumes children should construct their own meaningful understanding through self – directed discovery learning, cooperative learning or problem – based learning (Carpenter, 1999). It has been mentioned before that instructional models which can be attributed to constructivist influences such as the conceptual skill model can be summarised as minimal guidance instruction (MGI) (Mayer, 2004). Direct instruction (DI) on the other hand means “providing information that fully explains the concepts and procedures that children are required to learn as well as learning strategy support that is compatible with human cognitive architecture“ (Kirschner et. al., 2006, p. 76). Similarly, Engelmann and Carnine (1982, p. 24) define DI as “teaching rules, concepts, principles and problem – solving strategies in an explicit fashion.” In conjunction with cognitive approaches to learning, DI places great emphasis on automatising of procedural and factual knowledge and seeks to equip the learner with selected strategies which can be applied to a wide range of problems and which makes children aware that isomorphic problems require similar solutions. (Stein, Silbert and Carnine, 1997).

The outcomes of the different instructional models in the Follow Through study were measured in three different areas: basic skill outcomes, cognitive skill outcomes and affective outcomes. The data – analysis was undertaken by Stebbins, St. Pierre and Proper (1977). The researchers concluded that only direct instruction led to significant gains in all three areas. Interestingly, the direct instruction model led to a significant increase in cognitive skill scores, while the MGI models made no significant difference. Indeed, two of the three constructivist-inspired models had the lowest results in this measure. Bereiter and Kurland (1981) undertook a reanalysis of the results and this showed even stronger empirical evidence favouring for the direct instruction model. The

authors concluded that of all 9 instructional models, direct instruction led to the highest learning gains in all assessed areas.

The finding that DI is a more effective type of instruction than MGI for children with MD has been supported by Adams and Engelmann (1996). They carried out a meta-analysis of 37 studies which compared the effects of MGI and DI. It was found that 64% of the reviewed studies showed a significant difference in favour for DI on the 0.001 level. A total of 33 studies had a notable effect size of 1.11 or above. Thus, the critique directed against the DI approach, namely that an emphasis on basic skills is merely helpful for rote learning and does not result in conceptual change, is not supported by the data. Also, the results of Stebbins, St. Pierre and Proper (1977) challenge the frequently articulated concern that DI has an adverse effect on affective skills. Indeed, the affective skills models which were assumed to foster self-esteem were found to have adverse effects not only in the area of basic and cognitive skills but also in their distinctive domain of affective skills.

To recap, the MGI approach which has a considerable impact on mathematics curricula in the US, Australia, and Germany predicts that learning should be self-directed and problem – solving strategies should not be taught explicitly. However, the follow-through results showed that models within this tradition led to the weakest academic and affective gains. In sum, these models “produced children who were relatively poor in higher order thinking skills and models that emphasized improving children’ self-esteem produced children with the poorest self-esteem.” (Adams & Engelmann, 1996, p. 48). This evidence can be interpreted as undermining Battista’s (1994, p. 464) constructivist notion that “mathematical ideas must be personally constructed by children as they try to make sense of new situation.” The Follow-

Through study provides compelling empirical evidence that children with MD benefit from complete and explicit guidance rather than minimal guidance and this has been confirmed in different follow up studies, as will be discussed below.

5.1.2 LONG TERM EFFECTS OF DIRECT INSTRUCTION ON LEARNING MATHEMATICS

One frequently cited argument against direct instruction is that this intervention is a “pressure cooker” with little or no sustainable effect. However, follow-up studies of the DI implementation challenge this assumption. While the long – term effects of direct instruction will be discussed in more detail in chapter 7, the following paragraphs outline some of the follow – up studies on the long term outcomes of direct instruction. An influential analysis of the long – term effects of the direct instruction intervention that has been implemented as part of the Project Follow Through is provided Meyer (1984). Data were collected from over 80% of children who were exposed to DI and 76% of the control group children. Meyer researched how the performance of DI participants in grade 3rd correlated with their performance in 9th grade and how former DI participants compare with a control group when graduation rates, drop – out rates, college application rates and acceptance rates are concerned.

The data analysis indicated that 9th grade children who were exposed to the DI treatment in 3rd grade outperformed control group children in a standardized maths test. Also, children who took part in the DI intervention were significantly more likely to graduate on highschool and to be accepted on college than children from the control group.

Meyer's (1984) findings on the persisting effect of DI on later academic success did not replicate the results of Becker and Gersten (1982). Their follow – up sample consisted of 1097 former DI children and 970 children from a control group, who took part in different standardized tests of mathematical ability. It was found that although the former DI children performed significantly better than the control group, their overall performance declined after the DI treatment.

Despite the results of the Follow – Through study were successfully replicated and yielded significant long – term results, the design of the Follow – Through study and the methods used to analyse the results have not gone unchallenged. According to a paper of House, Glass, McLean and Walker (1978) the conclusions of the Follow Through study cannot be accepted as face value, because the study was defective in various aspects. Firstly, it is criticised that there is considerable variance between schools with respect to the effectiveness of the different models. According to House et. al. (1978) “the peculiarities of individual schools, neighborhoods and homes influence pupil's achievement far more than whatever is captured by labels such as “basic skills” or “affective education” (p. 152).

A second point concerns the measurement of the learning outcomes, which might be conceptualised too narrow. Advocates of the constructivist models claim that the selected instruments were biased towards models which produce measurable “mechanic” learning outcomes and these instruments could not capture the conceptual learning gains of the constructivist approach. Finally, it is criticised that the question “which model works best” was overemphasized at the expense of more important questions such as “what makes a model work and how can a model be improved.” Thus,

House et. al. (19789) concluded that the Follow – Through study did not demonstrate a superiority of the basic skill models to other models.

5.2 SUMMARY OF THE FOLLOW – THROUGH STUDY

To recap, the Follow-Through study showed that all models other than basic skills models had more adverse than positive effects in the area of basic skills: non-treatment children demonstrated better basic skills performance than their counterparts in the conceptual and the affective models. In other words, these approaches exacerbated the deficits that they claimed to ameliorate. Despite the great emphasis that the constructivist-inspired models place on conceptual skills, the data showed that no model had a positive impact in this area. The only model that increased conceptual skills was DI. This is interesting, because conceptual understanding is the key domain of constructivists and one frequently cited argument against DI is that it fails to account for conceptual understanding. However, the data fail to support this critique. Similarly, models which emphasized the role of affective factors failed to deliver their promises and led to more negative than positive outcomes in the area of affective skills. Basic skills models, on the other hand, significantly improved affective skills, suggesting that a positive self-concept is the result of rather than the prerequisite for successful basic skills acquisition.

In the light of the Follow-Through data, it can be concluded that children with MD are more likely to learn mathematics successfully when they receive explicit strategy instruction in form of modeled strategies and worked out examples as well as the opportunity to practice factual procedural and knowledge to the point of automatisisation. However, House et. al. (1978) noted several methodological limitations

of the Follow – Through study. A key limitation of the study is that it does not provide a more detailed analysis of which specific aspects of a given instructional model such as MGI or DI were effective or ineffective. In other words, it is not possible to specify whether the use of modeled strategies, the frequent provision of feedback or the highly structured pattern of each lesson contributed to the learning gains in the DI group. Also, the results fail to inform which particular instructional aspects made the MGI approach ineffective. However, the results are useful to the extent that they showed that low achievers from different student populations (rural, urban, ethnic minorities, economically deprived, non-native speakers) were able to acquire arithmetic skills successfully when learning with DI. Therefore, Adams (1996) concludes that DI possesses all the features necessary to produce outstanding academic gains for at-risk children and to increase their self-esteem.

5.2.1 FURTHER RESEARCH ON LEARNING WITH DI AND MGI

Further support for the DI approach is provided by Cardelle – Elawar (1995), who investigated the effects of DI on problem – solving strategies in terms of learning gains and attitudes towards mathematics. The sample in the study consisted of 489 children who were distributed in 18 classes, ranging from year 3 to 8. A total of 12 classes received the DI intervention, the control group consisted of 6 classes. The DI intervention was explicitly targeted towards improving metacognitive strategies for solving mathematical word problems among children with MD. Through the use of worked examples and extensive modeling, children learned how to extract the relevant information from a word problem. Under close supervision of a teacher, children learned were asked to apply the learned strategies to solve similar word problems in a systematic, step – by – step fashion. A control group received lecture – based

“traditional instruction” rather than individual modelling of example strategies. However, the paper provides little information on the instructional approach provided in the control group.

The intervention was implemented for one year. A pretest – posttest design was employed to determine the effectiveness of the DI intervention. Learning gains were measured by a specifically designed test. The test consisted of 20 problems which were developed in line with the curricular requirements for each grade. The results indicated that children made significant learning gains. Attitudes towards mathematics were measured by two scales, which addressed the extent to which children enjoy and value mathematics. Significant differences could be observed on both scales, in that children who took part in the DI intervention responded more positively.

Similar findings are provided by Moore and Carnine (1989), who researched the effects of DI implementation on the learning the concepts of ratios and proportions. The DI intervention involved modeling of each of the concepts through worked out examples and extensive practice of operations involving the concepts. As in the Cardelle – Elawar (1995) study, children were directly taught how to extract the relevant information from the problem and which problem – solving strategy to use. When working independently, children gradually moved from simpler to more complex problems and were closely monitored and received immediate feedback and correction of errors. Again, the learning gains in this study were statistically significant. Thus, the evidence reviewed so far suggests that children with MD respond better to DI than to MGI and this possibility has been experimentally tested in several studies which will be reviewed below.

The finding that learning process can be facilitated through the implementation of DI is in conjunction with the findings of a meta – analysis by Baker, Gersten and Lee (2002) on the effects of DI and MGI on the learning of mathematics in children with MD. Their research review was one of the first attempts to synthesize the existing research on the impact of teaching practices on learning processes in children with MD. The researchers selected only experimental or quasi – experimental studies which met the following criteria: studies had to be based on the actual implementation of a math instructional intervention which provided clear learning objectives and instructional activities. The interventions had to have a minimum time of at least 90 minutes. Also, the studies had to include a measure of mathematics achievement rather than attitudes. The researchers calculated the mean effect sizes for 15 studies from 1971 – 1999 which met the inclusion criteria. The studies were classified in two major categories, MGI and DI. The results showed that in general, DI had a positive effect with moderately string effect sizes whereas the effect size in the MGI studies was near to zero. The researchers concluded that the implementation of MGI is not effective for children with MD, an assertion which goes in conjunction with a research review of Mayer (2004).

Mayer (2004) carried out a research review on controlled studies from the mid 50's to the mid 80's, which provided empirical comparisons between DI and MGI. In essence, it was found that learners of all ages and abilities benefited more from DI than MGI. This is in conjunction with the tenets of a cognitivist perspective on learning as advocated by Kirschner et. al. (2006), who claim that the recent developments in the area of human cognition shows unequivocally that minimally guided instruction approaches are unlikely to facilitate the learning process because they fail recognise the structure and limitations of the human cognitive architecture, especially the limitations of working memory and the implications of these limitations for the learning process:

“Recommendations advocating minimal guidance during instruction proceed as though working memory does not exist or, if it does exist, that it has no relevant limitations when dealing with novel information” (Kirschner et. al. , 2006, p. 80). However, as will be illustrated below there is experimental evidence which does not support the above criticism and suggests that even children with MD can learn mathematics through a constructivist – inspired models of instruction.

Woodward and Baxter (1997a) investigated the learning of children with MD, who were exposed to a MGI intervention for one year. A total of 205 children from class 3 took part in the study, 104 children from the experimental group and 101 children from the control group. The Iowa test of basic skills was used to identify children with MD. The 34th percentile was the criterion for defining children with MD. According to this criterion, 17 children from the experimental group and 22 children from the control group had MD. The MGI intervention called “Everyday Mathematics” rests on the assumption that children already have informal conceptual understanding and heuristic problem solving strategies before they come to school and that they should be encouraged to draw on this knowledge to develop and construct problem solving strategies on their own. The problems used in the MGI intervention were explicitly chosen to prevent children from using modeled strategies and worked out examples as “cognitive shortcuts” and to encourage them to think and reason about the problems and to develop their own strategies by manipulating the numeric quantities. From the very beginning of the intervention, children were exposed to complex problems without modeled strategies or teacher intervention.

The rationale behind this approach was to facilitate genuine understanding of the underlying concepts. The intervention de-emphasized computations and explicit strategy instructions and focused on the use of number stories, mathematical word problems which are anchored in children's everyday experiences or subjects such as natural sciences or geography. It was anticipated that this sort of problems is meaningful to children, because it is related to children's everyday world. Children in the control group learned with a mathematics program which emphasized drill and practice of mathematical facts and algorithms. However, it is important to point out that this intervention involved mechanic rote learning of factual knowledge, which is not comparable to direct instruction.

The effectiveness of both interventions was determined by two dependent measures. The Iowa test of basic skills (ITBS) was used to measure computations, concepts and problem solving skills whereas the Informal Mathematics Assessment (IMA) test was used to test individual problem solving ability, especially the strategies which children adopted to derive the solution. The IMA involves a protocol analysis of children's problem solving strategies. According to the researchers, the analysis of the ITBS scores showed no significant differences between the groups. However, in the meta analysis of Baker, Gersten and Lee (2002) the study yielded a negative effect size of $- .024$, which shows that children from the comparison group outperformed the children who took part in the MGI intervention. This adds support to the notion that children with MD face difficulties when mathematics in MGI settings. While the meta-analysis reported a negative effect size, this result has to be treated with caution. A quantitative analysis of the IMA showed that results were significantly higher for children who took part in the MGI intervention "Everyday Mathematics". This challenges the conclusion of Baker et. al., that the implementation of MGI yields

negative learning outcomes for children with MD. Based on the the results of the IMA protocols, the authors conclude that MGI is also suitable for children with MD.

Nevertheless, this finding has to be treated with caution given that the sample consisted of middle class children whereas the sample in the Follow Through study consisted of economically deprived children. Indeed, Siegler (2003) notes that the majority of children with MD comes from parents who lack formal education and therefore it can be argued that children from middle class background are exposed more frequently to numeric quantities compared to children from lower socio - economic background. Therefore, middle class children might have benefited from this intervention because they already possessed some informal knowledge, which children from socio - economic background might lack.

Bottge and Hasselbring (1993) researched the effect of implementing MGI for children with MD. The sample included 83 children with MD from class 9. The intervention involved the presentation of authentic mathematic word problems via a videodisk. Word problems were “anchored” in a real – life context and therefore the intervention was described as contextualised instruction. In line with the MGI principles, no explicit guidance or worked out examples were provided. The intervention was implemented for 5 sessions of 40 minutes. A comparison group received traditional instruction in solving word problems. It was found that children in the intervention group yielded higher scores in a subsequent word problem solving test. However, in the review of Baker, Gersten and Lee (2002), this study yielded an effect size of .48. Thus, the effect size of the MGI intervention was smaller than the results of the DI interventions reviewed above.

Another possible limitation of the study is that it compared two different modes of delivering instruction, namely videodisk presented activities and traditional, paper pencil instruction. The different modes of delivery of instruction might have confounded the results, in that children from the intervention group were more excited about the new presentation of the learning material, an effect which is known as the “Hawthorne effect” (Rosenthal, 1966). In essence, Landsberger (1955) predicts that environmental changes such as changes in the work environment or educational interventions can have positive short – term effects, because participants see themselves as a member of something new. Children in the DI group were aware that the implementation of the DI intervention eFit was tested and this knowledge might have raised their motivation. Also, the “Hawthorne effect” can affect the teacher. The teacher as well as the children knew that they received special attention and this might have affected the short – term outcomes of the DI intervention.

A follow up study might have addressed this issue by investigating the long term effects of this intervention, once the initial excitement had diminished. Further research on the usefulness of MGI compared to DI for children with MD is necessitated. A closer examination of the role of the teacher in the DI learning outcomes would be an essential extension of the present research. Therefore, the study might have benefited from a research design in which children in the treatment and the non – treatment group receive their classroom based instruction from the same teacher.

5.2.2 SUMMARY

Given that the overall amount of evidence on the effects of MGI and DI on the learning of mathematics in children with MD is not clear cut (Baker, Gersten, Lee, 2002). Therefore, further evidence is necessary to derive inferences on the effects of the format of instruction on the learning processes in children with MD. In order to investigate this issue further, the present research study aims to address the question whether children with MD benefit from a web based DI intervention compared to children who are exposed to constructivist – inspired MGI. Given that the DI intervention is not regular, teacher – led instruction but web – based instruction, the following paragraphs will review some of the research on web – based instruction. A full appreciation of the available amount of research on CAI is beyond the scope of this thesis.

**5.3 COMPUTER ASSISTED INSTRUCTION FOR LEARNING
MATHEMATICS**

The implementation of computers for learning and teaching mathematics has a longstanding tradition. For convenience, the term CAI will be used to refer to all applications of computers in education which involve multimedia and hypermedia systems, such as drill and practice, tutorial, inquiry learning applications (Rosic, Glanivic & Stankov, 2006). An exploration of all CAI would exceed beyond the scope and purpose of this thesis. Therefore, the development of CAI will be illustrated by two examples, namely tutorial systems and intelligent tutoring systems.

The origins of educational technology can be traced back to the development of teaching machines. It was Skinner's dissatisfaction with the predominant educational practice that led him promote the use of teaching machines. He claimed that the increasing demand for education could not be addressed simply by providing more schools and more teachers. Instead, he called for a different educational approach: "in any other field a demand for increased production would have led at once to the invention of labor-saving capital equipment" (Skinner, 1958). The behaviourist approach advocated by Skinner criticised traditional education on the following grounds. First, it is argued that most children only learn in order to circumvent punishment, be it in terms of lower grades or negative feedback from the teacher. Further, it is criticised that the timing of teachers' feedback, which, according to Skinner, is often too late or absent. Further, it is argued that the teacher cannot account for the learning pace of individual children.

With the increasing availability of computers, advocates of the behaviourist approach proposed to implement them as universal learning machines. The first CAI programs were either pure presentations of textbook materials or followed the principles of programmed instruction (Klimsa, 1993). Later, CAI were designed as tutorial system, which are organized in a linear order. The order is determined by the designer and reflects his or her assumptions about an optimal sequence and received instant feedback. The learner then has merely to work through this fixed sequence. Whilst some systems provide some links to other topics, the majority are characterised by a strict and linear structure. This process can be described in terms of three major steps.

At first, the student is presented with the material and responds to this, for instance by completing missing spaces. If the program is well designed, the student gets prompt feedback in terms of whether the response was correct. Then the program displays the next frame (Jones & Mercer, 1993). Following Skinner's principle of operant conditioning, a correct response / desired behaviour is reinforced whilst incorrect responses are ignored (Jones & Mercer, 1993). One of the first examples of this kind stems from the work of Suppes (1969). In this project, each child was provided with a PC and was asked to work through a series of arithmetical tasks. The difficulty of the tasks was based on the child's previous performance. Children were found to improve significantly after taking part in the study.

Hasselbring et al. (1988) reported that drill – and – practice tutorial systems led to significant learning gains in children with learning handicaps. After a treatment period of 49 days, the number of fluently recalled mathematical facts increased by 73%. A comparison group of handicapped children who received regular mathematics education demonstrated no increase in fact recall. A similar comparison group of non-

handicapped children only learned 8 additional facts during the same period. Therefore, the researchers conclude that computer-mediated drill and practice is a helpful device in developing automaticity in basic fact recall in handicapped learners.

Based on the results from their meta-analysis of nearly 200 studies, Kulik and Kulik (1987) concluded that children demonstrated higher performance when computer usage was part of their mathematics education. Also, children learned more in less time. In 28 of all reviewed studies, the average reduction of instructional time was nearly 35%. According to Kulik and Kulik (1987), this is because computer-mediated instruction is well-designed instruction with unambiguous and explicit learning objectives as well as immediate feedback. Kulik and Kulik (1987) argue that instructional designers spend approximately 100 hours to design one hour of computer lessons, which is hardly ever possible for teachers. Another important factor is that computers are truly individualised in that they devote their sole attention to a single learner. Based on a meta-analysis of 254 evaluation studies, Kulik and Kulik (1991) concluded that computer-based instruction has positive outcomes for the learning process. Over 80% of the reviewed studies led to results that were higher than the average for traditional instruction.

5.3.1 INTELLIGENT TUTORING SYSTEMS

Intelligent tutoring systems (ITS) can be defined as “a generation of computer systems which provide children with learning and teaching environments adapted to their knowledge and learning capabilities. The goal of ITSs is to provide a learning experience for each student that approaches the standard of learning that he or she would receive in one – to – one tutoring from a human teacher.” (Rosic, Glavinic &

Stankov, 2006, p.4) According to Rosic et. al. (2006), ITS fall within the general category of web based instruction.

ITS are highly adaptive systems which are influenced by the methods of artificial intelligence. A characteristic feature of an ITS is that it consists of three parts: a domain model, a student model and a tutor model. The “knowledge domain” or “expert model” component incorporates a representation of declarative or procedural expert knowledge in a given domain (Blumstengel, 1998). Declarative knowledge defines concepts of the particular knowledge domain as well as relationships between them. Procedural knowledge incorporates rules that facilitate problem solving. This knowledge domain is structured in terms of diagrams or lists. The student model, or diagnosis model, is intended to provide a diagnosis of the learning process based on the learner’s input. In order to do so, the model charts and evaluates the course of learning and infers the learners’ current status of knowledge. Based on this evaluation, the model determines the learning path, the level of difficulty and the type and amount of feedback. The rationale behind the tutor model is to organize the presentation of learning material. Based on differences between the expert and student models, it simulates the decision-making process of a teacher, including the provision of appropriate pedagogical interventions. Similar to programmed instruction, ITS conceptualise learning in terms of behaviour. The domain model, for example, is based on certain behavioural objectives. The student model is concerned with the behavioural sequences of the student.

**5.3.2 DIFFERENCES BETWEEN ITS AND TRADITIONAL TUTORING
SYSTEMS**

In opposition to traditional tutoring systems, they provide "a dynamic and adaptive dimension to self-paced instruction" The main difference between TS and ITS systems is that the former are merely used for drill and practice whilst the latter add a diagnostic element (Yazdani,1987). Indeed, the importance of a diagnostic element is emphasized by other researchers such as Arroyo, Beck, Schultz and Park Woolf (1997). They argue that IT-based pre-tests that inform about individual differences will enhance the flexibility of ITS, to the benefit of the learner. In essence, ITS are intended to diagnose the current level of knowledge, to infer conclusions from this and to create instructions that reduce the differential between expert and novice.

ITS are termed intelligent because the communication model enables them to respond adaptively to the learning outcome. It is this feature which distinguishes ITS from TS. The rationale of fostering dialogue between teacher and learner accounts for the social aspects of the learning process: the learning process is not only based on the learner him/herself, but inevitably involves another person, be it the teacher, a peer, or even the author of the educational tool. According to Scrimshaw, children who work with a computer program are always in interaction with a "hidden teacher", namely the designer of the program (Scrimshaw, 1993) Similarly, if an individual learns word processing, he or she acquires knowledge that has been generated by other people such as the software designers. The sharing of knowledge is similar to the acquisition of cultural tools as emphasized by the social constructivist approach. ITS can be viewed as an extension rather than an opposition to behaviourist tutorial systems, because they recognize the active role of the child in the learning process. ITS are often based on web

– based learning platforms and can therefore be classified as web – based instruction (WBI).

Several definitions have been proposed for web- based instruction, some of which will be reviewed briefly in the present section. A frequently adopted definition has been forwarded by Khan (1997), according to which WBI refers to “...a hypermedia-based instructional program which utilizes the attributes and resources of the World Wide Web to create a meaningful learning environment where learning is fostered and supported” (p.47). Compared to other types of computer assisted learning, WBI possesses several distinctive advantages. Firstly, the non-linear and branched structure facilitates deeper information processing (Spiro & Jengh, 1990). The non –linear structure has been identified as one of the key benefits of WBI. Secondly, WBI provides different forms of instructional presentation modes which can be adjusted to meet the learning styles of the individual learner (Sitzmann, Kraiger, Stewart, & Wisner, 2006). According to Papanikolaou et. al. (2003), the hypermedia structure which underlies WBI enable to personalise the instruction processes and can address the specific needs of individual learners. This feature has also found to be beneficial for the learner by Welsh, Wanberg, Brown and Simmering (2003). However, the same researchers also point out potential limitations of WBI, which include a lack of access options such as low bandwidth and the absence of adequate IT facilities.

5.3.3 MEDIA AND METHODS DEBATE

To recap, the implementation of educational technology devices for mathematics instruction migrated from teaching machines to linear drill and practice tutorials and eventually ITSs. However, the notion that CAI possess unique advantages over

traditional instruction is subject of an intense and enduring debates between two camps of scholars, which will be outlined briefly. Scholars such as Kozma (1994) claim that the media determine the instructional methods that can be used. Kulik and Kulik (1987) for example argue that instructional designers spend approximately 100 hours to design one hour of computer lessons, which is hardly ever possible for teachers. Another important factor is that computers are truly individualised in that they devote their sole attention to a single learner. Thus, it appears that CAI incorporates some distinctive features such as immediate feedback and highly individualised instruction which cannot be provided in classroom settings. However, while Kozma assumes that the media make certain forms of instruction possible, it can be argued that the relationship between media and methods is the other way round, in that the instructional approach itself determines the instructional design of the media. This viewpoint is advocated by Clark (2001).

Several reviews of media effectiveness research led Clark (2001) to conclude that “learning is caused by the instructional methods embedded in the media presentation” (p.56). Based on a meta-analysis of experimental media effectiveness studies, he argued that the reviewed studies did not permit to determine causal relationships between the implementation of media and learning gains. He criticised that the majority of studies which reported significant improvements in learning through the implementation of educational media were flawed and failed to introduce sufficient experimental controls. Among the uncontrolled variables which are most likely to have obscured the results are differences in the instructional method, instructor effects, familiarity with the instructional method and a specification of the time that the learners actually spent on task in both conditions.

Evidence for Clark's view that the instructional paradigm is more important than the media which merely delivers it is provided by a meta-analysis of Sitzmann et. al. (2006). Sitzmann et. al. (2006), found that if WBI did not provide feedback to the learner and enough opportunity to practice, WBI was 20 % less effective than traditional, classroom based lessons. Although Clark's review includes several studies which showed that technology- delivered instruction was superior to classroom – based instruction, he maintains that most of these findings are artefacts resulting from inadequate empirical designs and the effects of CAI were actually determined by the instructional method embedded in the technology, which was not controlled for. Indeed, from a sample of 15 studies which accounted for instructional method, only two studies showed a significant effect of CAI.

Clark's line of argument would implicate that a comparison across media (traditional, classroom-based instruction vs. web – based instruction) and across methods (MGI vs. DI) is possible, because the DI approach underlying the eFit intervention is a more powerful variable for the learning process than the web – based delivery mode. Following the research on the outcomes of MGI and DI for children with MD that has been reviewed so far, the present study aims to investigate whether the web – based DI intervention eFit provides a more effective form of instruction for children with MD than MGI.

5.4 METHOD

5.4.1 PARTICIPANTS

A total of 194 children from year 5 and 6 completed the pre-test and the post-test. 58 children, 34 males and 24 females, were from a school where the DI intervention was implemented. The comparison school was selected because the student population was comparable with the student population at the intervention school with respect to ethnic and socio – economic background. The sampling procedure will be described in more detail in section 5.5.2 below. The mean age of the children in the experimental group was 11, 5 years, with 11 years being the minimum and 15 years the maximum age. The MGI group consisted of 5th and 6th graders from another general secondary school (70 females, 65 males). The mean age was 11, 3 years, with 11 years being the minimum and 15 years the maximum age. Of all participants, 118 children were from grade 6 and 76 were from grade 5. The increase of children in grade 6 is due to the fact that children who do not meet the standards at higher secondary school types are referred to general secondary schools. The parents of the participating children were informed about the experimental procedure by the local educational authority and they were ensured that children could withdraw at any point. Confidentiality of data was ensured.

5.4.2 SAMPLING

The sample from both groups was selected to ensure homogeneity with respect to the pupil composition. Therefore, ability level, ethnic and socio - economic background was controlled for in order to reduce the effect of these individual differences on the results. In order to match the participant composition of control group

to that of the experimental group, statistical figures from countywide educational assessments were used. Based on these data, 2 control schools were selected which have a similar catchment area and pupil composition. The chosen control schools met the following criteria: 1. they were situated in low income areas, 2. at least 20 % of the pupil population consists of children with immigrant background who are not proficient in the German language, 3. at least 20 % of the population comes from a family background which has been identified as “low socioeconomic status” (low SES).

5.5 MATERIAL

5.5.1 MINIMALLY GUIDED INSTRUCTION

The traditional mathematics lessons are based on the mandatory regulations of state curriculum, which determines the content and skills to be learned during one schoolyear. According to this curriculum, MGI is the dominant approach to teaching mathematics. The mathematics curriculum in NRW is strongly influenced by the principles of the National Committee of Teachers of Mathematics (NCTM), which defines mathematics as an intellectual and creative field and emphasises self – directed learning and collaborative work as the cornerstones of mathematics learning (Winter, 1995). In line with the NCTM principles, the steering group for the mathematics curriculum argues that a “small – step approach” with explicit explanation should be avoided in maths instruction. Guided practice sessions should be as brief as possible and more time should be allocated to independent strategy choice, in order to facilitate children’ use of heuristic, “rule of thumb”, strategies. (Bruder, 2002).

According to the curriculum, 6th class children for example should be able to develop their own mathematical examples and strategies and to test them through trial

and error procedures. Indeed, trial and error and discovering own examples and pathways are deemed to be the most important problem strategies for classes 5 and 6. The rationale behind this approach is the uncontested assumption that developing own heuristic strategies improves children' problem solving ability and is superior to strategy instruction and guided practice (Tietze, Klika & Wolpers, 1997). The impact of the constructivist – inspired NCTM principles on the mathematics curriculum in NRW is evident in the table below.

Table 5.1: Comparison between the statewide, the countrywide and the NCTM curricula

NRW core curriculum	National educational standards (2003)	NCTM Standards (2000)
Process-related standards	General competencies	Process standards
Reasoning and communicating	Mathematical reasoning	Reasoning and proof
	Communicating	Communication
Problem solving (investigating an solving problems)	Mathematical problem solving	Problem solving
Modelling (finding and using models)	Mathematical modelling Using mathematical representations	Representations, Connections
	Dealing with symbolic, formal and technical elements of mathematics	
Using media and tools		
Content-related standards	Competencies related to overarching ideas	Content standards
Arithmetic / Algebra (dealing with numbers and symbols)	Number	Number & Operations Algebra
Functions (describing and investigating dependence and change)	Functional connection	
Geometry (grasping plane and spatial structures by measure and shape)	Space and Form Measurement	Geometry Measurement
Stochastic (working with data and chance)	Data and Chance	Data analysis and probability

Corresponding to the NCTM principles, the curriculum for mathematics at general secondary schools recommends the presentation of “open – ended tasks” which encourage children to discover appropriate strategies and to present and justify it. Tasks should not be phrased as questions because this is assumed to impede the child's

creativity. The curriculum makes clear that mathematics education should be based on active – explorative and social learning and emphasizes skills such as verbalising, communicating, reflecting, inventing and developing. This is illustrated in the following tasks, taken from the standard schoolbook for mathematics in class 5.

Example problems from a schoolbook:

“From all 75 children of class 5, 37 children play soccer, 28 children play table tennis and 15 children play both. How many children play neither soccer nor table tennis? Think and reason. “

“Think of a “mathematical story” and create your own exercises. “

“There are 3 houses, each with 8 storeys. Each storey consists of three flats. How many flats are there? Use different solution pathways and explain them. “

Figure 5.1: Problems taken from a mathematic book for 5th graders

As evident in the above, the tasks in the mathematics schoolbooks require children not only to solve a mathematical problem but to discover the problem at first. Apart from general hints such as ‘think and reason’, no clear task is provided to the children. Indeed, the curriculum is based on the assumption that children learn most effectively when given the chance to approach new tasks independently, even if their current state of knowledge is deficient. According to this view, the teacher cannot facilitate children’s mental development but can merely offer indirect help so that children can finally help themselves (Grundkonzeption des Zahlenbuchs Klasse 1 NRW, Lehrerband, Verlagsabdruck aus dem Internet). Discovery learning is seen as the legitimate and most effective form of learning (Begleitband Zahlenbuch, Klett Balmer Verlag). Jahnke (2001) argues that effective tasks are tasks which make children wondering and which lead them to search, discover, errate and invent. It is up to the children to discover and find out. The heavy emphasis on the development and

application of heuristic strategies corresponds to the constructivist – inspired MGI approach.

5.5.2 DIRECT INSTRUCTION INTERVENTION “eFit”

Whereas children from the control group are solely exposed to the MGI curriculum as described above, children from the intervention group receive direct instruction which is mediated by the web-based intelligent tutoring system eFit. In order to enable the children to access the programme, the teacher has to set up an account for each student. Once the teacher has set up an account for each of the children, they are able to log in via a web browser. The first step for the children in the eFit intervention is to take part in the eFit test, a standardized mathematical test to diagnose children’s individual level of ability.

The eFit intervention is based on the principles of direct instruction which aims to improve procedural knowledge in terms of improved problem solving behaviour and automatization of factual knowledge. As has been mentioned, the usage of efficient problem solving strategies and their application to novel situations yields frequent difficulties for children with MD (Bransford & Schwartz, 1999). It is important to address this shortcoming, because evidence shows that performance is based on individuals’ ability to retrieve previously learned patterns and to apply them to new situations. Ericsson and Polson (1988), for example, found that although individuals had the same amount of practice in performing a task, their performance differed depending on whether or not they used efficient strategies to perform the task. When choosing a strategy, children tend not to choose the most effective one but the one they are most familiar with (Lucangelo, 2003). Therefore, the emphasis should not be on teaching

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many different strategies but the most effective ones so that children become familiar with them and make more use of it. Instructional recommendations to improve strategy instruction include using advanced organisers and worked examples.

The screenshot shows a web browser window with the URL <http://www.efit-nrw.de> and the page title "Klett". The page is titled "Mit Größen rechnen Grundlagen" and features a shopping list table. The table has three columns: item description, unit price, and total price. The items and their calculations are as follows:

Item	Unit Price	Total Price
15 Packungen Servietten zu je 0,90 €	15 · 90 Cent	15 · 1 €
5 Packungen Haushaltsrollen zu je 2,15 €	5 · 2 €	5 · 3 €
120 Beutel Plastikbesteck zu je 19 Cent	150 · 20 Cent	25 €
3 Packungen Partyteller zu je 18,50 €	3 · 18 €	3 · 20 €
9 Notizblöcke zu je 1,15 €	2 · 5 €	10 · 1 €
2 Packungen Papiertaschentücher zu je 4,35 €	120 · 20 Cent	2 · 4 €
Spül- und Putzmittel für 21,40 €	9 · 2 €	20 €

Navigation buttons on the right include "Prüfen", "Tipp", "Lösung", "Noch mal", and "OK". The browser's taskbar shows the Start button and several open applications.

Figure 5.2: Explicit demonstration of isomorphic problems

The screenshot shows a web browser window with the URL <http://www.efit-nrw.de> and the page title "Klett". The page is titled "Addieren und Subtrahieren Fortgeschrittene" and features a worked example for written addition. The example shows a grid with the numbers 4, 7, 7, 8, 8, 6, 5, 1, 1, 1, 1, 4, 3, 6, 2. Arrows point to specific digits and the text "Umkehrrechnung" is used to explain the process. The calculations shown are:

Umkehrrechnung: $4 - 1$ (Übertrag) =

Umkehrrechnung: $16 - 8 - 1$ (Übertrag) =

Rechne: $7 + 6 + 1$ (Übertrag) =

Navigation buttons on the right include "Prüfen", "Tipp", "Lösung", "Noch mal", and "OK". The browser's taskbar shows the Start button and several open applications.

Figure 5.3: Detailed and explicit strategy instruction for written addition

5.5.3 WORKED OUT EXAMPLES

An important instructional technique for helping children with MD to become successful problem solvers is the explicit teaching of strategy use, through the use of worked out examples. Worked examples help children to recognise a problem which require similar strategies. This is an important step towards higher levels of abstraction. Abstraction involves the search for commonalities between problems within a category while ignoring redundant, case-specific details of certain exemplars so that problem solving strategies can be transferred from worked examples to other problems. Ausubel (1978) points out that worked out examples can facilitate the learning process. As illustrated in the below screenshot, these principles can be found in the eFit intervention.

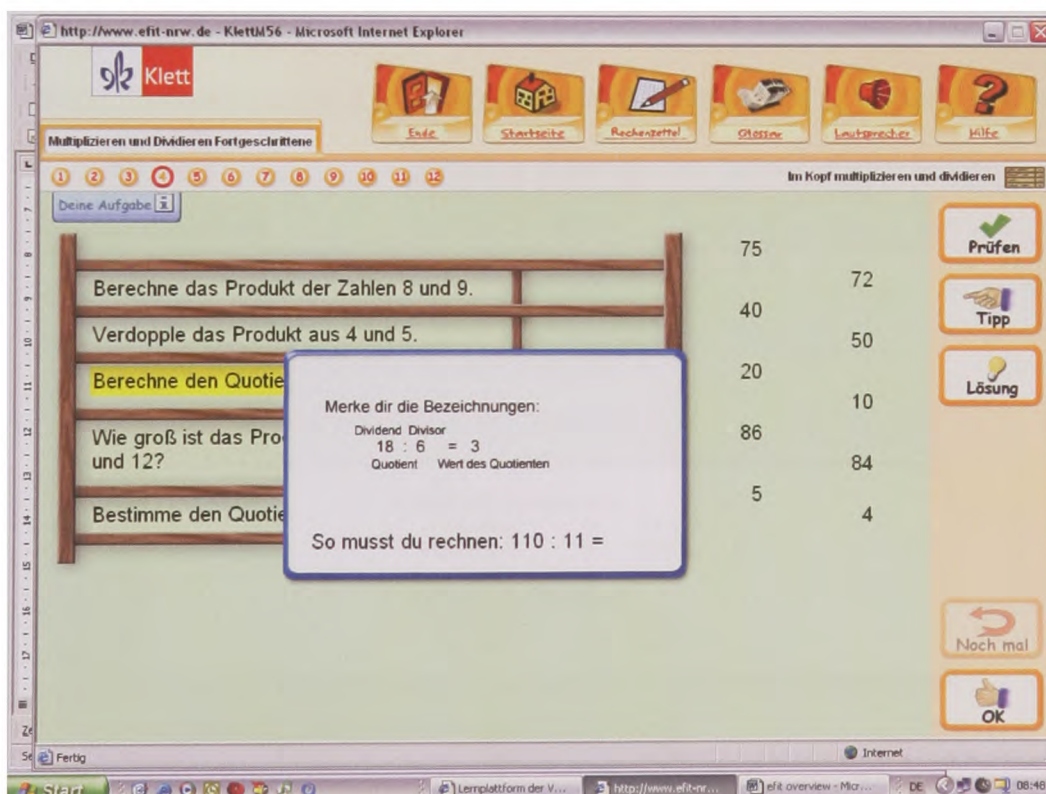


Figure 5.4: Worked out examples and detailed explanations for division

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The screenshot shows a web browser window with the URL <http://www.efit-nrw.de>. The page title is "Klett" and the main heading is "Multiplizieren und Dividieren Fortgeschrittene". A navigation bar includes icons for "Ende", "Startseite", "Rechenzettel", "Glossar", "Lehrsprecher", and "Hilfe". A progress bar shows steps 1 through 12, with step 7 highlighted. The main content area displays a grid for the task "Deine Aufgabe" and a large pop-up box with the following text:

1. Bei der schriftlichen Multiplikation wird der zweite Faktor in Einer, Zehner, Hunderter usw. zerlegt.
 2. Beginne mit der höchsten Stelle und berechne die Teilprodukte.
 3. Zuletzt werden die Teilprodukte addiert.

Aufgabe: $357 \cdot 245$

The pop-up box also shows three columns of handwritten multiplication work:

$\begin{array}{r} 357 \cdot 200 \\ \hline 71400 \\ 357 \cdot 40 \\ \hline 14280 \\ 357 \cdot 5 \\ \hline 1785 \\ \hline 87465 \end{array}$	$\begin{array}{r} 71400 \\ + 14280 \\ + 1785 \\ \hline 87465 \end{array}$ Kurzform	$\begin{array}{r} 357 \cdot 245 \\ \hline 71400 \\ 14280 \\ 1785 \\ \hline 87465 \end{array}$
--	--	---

On the right side of the interface, there are several interactive buttons: "Prüfen", "Tipp", "Lösung", a right arrow, a left arrow, "Noch mal", and "OK".

Figure 5.5: Explicit strategies and explanations for written multiplication

The screenshot shows a web browser window with the URL <http://www.efit-nrw.de>. The page title is "Klett" and the main heading is "Mit Größen rechnen Grundlagen". A navigation bar includes icons for "Ende", "Startseite", "Rechenzettel", "Glossar", "Lehrsprecher", and "Hilfe". A progress bar shows steps 1 through 22, with step 18 highlighted. The main content area displays a shopping list table with a large pop-up box providing examples for rounding numbers.

	20 €	
Apfelschorle und Wasser	256,80 €	30 €
Bonbons	27,40 €	60 €
Banane		
Gurke		
Orangensaft		
Schokolade		
Kakao und Milch	229,80 €	160 €
Äpfel	58,70 €	240 €
Obstsalat	107,80 €	260 €

The pop-up box contains the following text:

Beispiele für Abrunden:
 $74,50 \text{ €} \rightarrow 70 \text{ €}$
 $161,30 \text{ €} \rightarrow 160 \text{ €}$

Beispiele für Aufrunden:
 $267,89 \text{ €} \rightarrow 270 \text{ €}$
 $148,30 \text{ €} \rightarrow 150 \text{ €}$

On the right side of the interface, there are several interactive buttons: "Prüfen", "Tipp", "Lösung", a right arrow, a left arrow, "Noch mal", and "OK".

Figure 5.6: Worked out examples for how to round numbers

5.5.4 ADVANCE ORGANISERS

Ausubel's (1978) advance organiser theory informs how instruction can be designed in way which facilitates the correct application of strategies. Ausubel emphasizes that the external structuring of material has a significant impact on the learning process. The linkage of new concepts with prior experiences at the beginning of a new learning unit occurs through pre-structured guidance and hints: the so-called advance organisers. Advance organisers provide an overview of the topic and the learning goal of a unit and therefore provide a 'guided transfer' of prior knowledge to new problems. More importantly, they help to establish the connection between existing strategies and new content, by pointing out similarities as well as differences. Advanced organisers facilitate the efficient application of known strategies to novel problems with an isomorphic structure through instructional design which draws out the similarities between different types of problems. In fact, Ausubel points out that it is important to draw connections between the concepts within a cognitive structure and to show up similarities as well as distinguishing features in order to make them meaningful and more memorable to the learner. Perceiving the conceptual category to which a given problem belongs is one of the key aspects of arithmetic problem-solving according to Sandrini et al. (2003). In other words, one factor for the positive learning outcome of eFit is that children were explicitly shown how apparently different problems belong in fact to the same category and therefore require similar strategies. Ausubel recommends the use of organisers with an emphasis on explanation when new content is to be introduced, and also on contrasting and comparing for content that is known but has to be recalled into memory again. This is evident in the efit screenshots below

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The screenshot shows a web browser window displaying a math learning interface. The page title is "Multiplizieren und Dividieren Fortgeschrittene". The interface includes a navigation bar with icons for "Karte", "Startseite", "Rechenzettel", "Glossar", "Lehrplan", and "Hilfe". A progress indicator shows steps 1 through 12, with step 4 highlighted. The main content area contains a list of math problems:

- Berechne das Produkt der Zahlen 8 und 9.
- Verdopple das Produkt aus 4 und 5.
- Berechne den Quotienten aus 72 und 4.
- Wie groß ist das Produkt aus 5 und 12?
- Bestimme den Quotienten aus 86 und 2.

A pop-up box provides a conceptual explanation for multiplication:

Merke dir die Bezeichnungen:
Faktor Faktor
 $6 \cdot 5 = 30$
Produkt Produktwert

So musst du rechnen: $8 \cdot 9 =$

On the right side, there is a table of numbers and a vertical list of buttons: "Prüfen", "Tipp", "Lösung", "Noch mal", and "OK".

Figure 5.7: Explicit strategy instruction, with conceptual explanations

5.5.5 MEASURE OF ARITHMETIC ABILITY

The initial measure of arithmetic ability which was used to determine pre-test and re-test performance is an essential part of the eFit intervention. The test is a subscale of the Heidelberger Rechentest (HRT), which is an established instrument to measure the arithmetic problem solving abilities of children from grades 4 – 6. It measures abilities in the areas of addition, subtraction, multiplication and division. Children are required to select the appropriate strategy for solving multi-digit problems such as $546 - ? = 342$ or problems such as $23 \times 4 - 38$, which require conceptual knowledge. Other tasks require children to apply the numeric place value system or to complete rows of numbers in a logical manner by applying the correct arithmetic operation.

The HRT has been validated with a large sample ($N = 3.075$) of primary school children across Germany. Test – retest reliability of the HRT scales was established with a sample of 246 primary school children, which led to a reliability coefficient of .87 for the arithmetic subscale, which has been adopted for the eFit test. The results of the Heidelberger Rechentest correlate with children's school grades in mathematics (-.67), meaning that the criterion validity of the Heidelberger Rechentest is satisfactory. Based on the percentage correct children achieve in the test, eFit classifies children in one of three categories: no additional needs, mediocre additional needs and severe additional needs in the areas of addition and subtraction and / or multiplication and division.

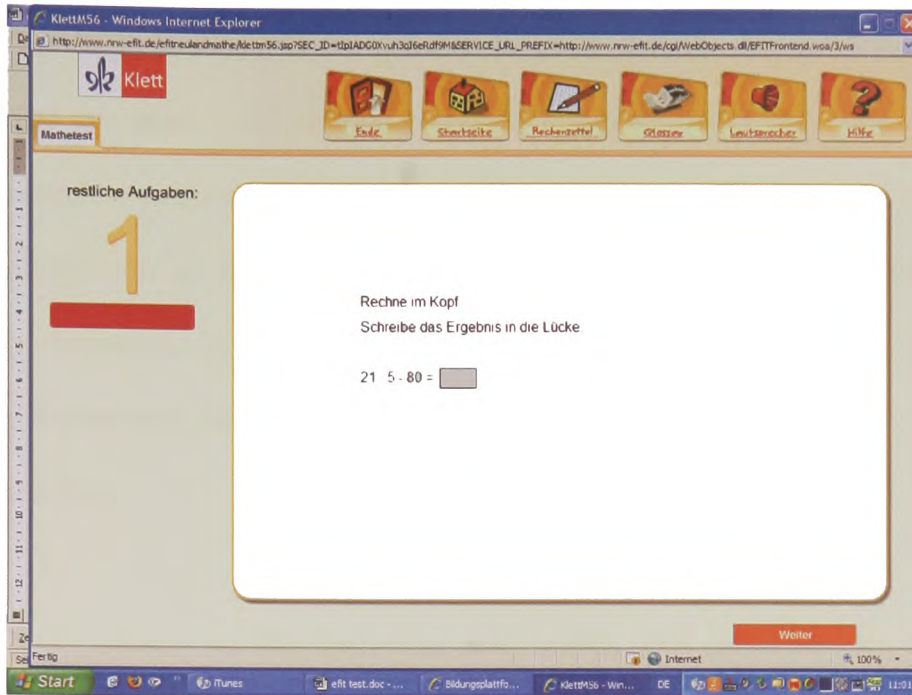


Figure 5.8: Example of a task from the test of arithmetic skills



Figure 5.9: Example of a test problem: “the antecedent of 7499 is....?”

The test takes approximately 40 minutes and starts with a short trial test to familiarise the children with the test format. The actual test consists of 61 basic mathematical problems which correspond to the mathematics curriculum at the start of class 5. Each task is presented on a separate slide. Once the participant has activated the

“forward” button, it is not possible to go backwards. The tasks increase in difficulty. The test starts with simple, two-digit arithmetic tasks and then moves on to more complex, multi-digit tasks. The children do not receive feedback for each task. After completion of the test, a screen display provided a brief indication of children’s performance. A laughing or a sad smiley shows the areas where the student demonstrated problems in the test. A detailed diagnosis is only accessible for the eFit teacher.

Based on the individual test results, eFit generates an individual learning schedule for each student, which covers the aspects that appear to be difficult for the student. Thus, each student receives a tailored learning schedule with learning activities that meets his or her individual requirements. The individual learning schedule consists of lessons which cover different domains of mathematics, in line with the curricular demands for class 5 children. The level of difficulty and the extent of repetition depends on the children’s test performance.

5.6 DESIGN / PROCEDURE

5.6.1 PRE-TEST AND POST-TEST

The pre-test was conducted in September/ October 2005 and the retest data collection took place from May to June 2006. Before any assessment was conducted, the participating schools were asked to send a list with the names of the participants and to indicate whether all children would be eligible to participate in the research. The key criterion was that the participants were able to conduct a computer-based assessment. Children were then allocated a number to identify them so that all information collected from children remained confidential and anonymous.

For the experimental group, the pre-test constituted an essential part of the eFit treatment. The control classes of the participating schools were invited to take the pre-test at the experimental school. Arrangements were made with the school principal and the teachers of the control group classes. The test took place in the computer room of the experimental school, where 18 computers with Internet access were available. The teachers of the control classes were asked to inform the children and parents about the rationale of the test in advance. Therefore, the participants were aware of the procedure on their arrival.

When the participants arrived in the computer room, they were asked to sit down and to listen to the instructions. In order to keep the children's attention, the instructions were kept minimal. It was emphasized that the test has no relevance for their grades or school certificates and that the results would be treated confidentially. A short explanation of how to run the test and the kinds of exercises that they would experience was given. After the briefing, children received their login details on a sheet. The eFit teacher helped to make sure that each participant logged in successfully. Before the actual test started, children were asked to run a short trial test to ensure that they were all able to carry out the test on the PCs. The short trial served to evaluate participants' eligibility to participate in the research. The children had no problems with carrying out the trial test and none of the participants who arrived for experimental sessions had to be excluded. However, the computer-presented test did not need explicit instruction. In cases where children were insecure, they could run the trial again. After the trial, the test was started.

The test was administered in groups of approximately 15 children. Children who finished the test were asked to remain silent. Two teachers were present to supervise the test but no additional help or explanations were given to the children during the test. The responses made by participants were automatically recorded. Although eFit did not measure the actual amount of time that children needed to solve a single question, it was decided to administer the tests within a time span of 30 minutes. After a trial with two average children, this was deemed to be an appropriate measure of arithmetic fluency. Every student managed to solve the tasks within the time span, although some had to hurry toward the end because of misallocation of time. Once all children had finished the test, they were thanked and given a little present for their participation. The same procedure was carried out from May 2006 onwards. For the post-test, all participants had to attend the experimental school to run the test again. In Germany, this is towards the end of the school year, and after nearly a full school year, children' achievement is supposed to reflect the outcome of the particular educational treatment they have received.

5.6.2 RESULTS

An analysis of covariance was employed to research whether eFit leads to significant learning gains for children with MD from class 5 and 6 at lower secondary schools. The data are based on the results from the pre-test and retest and analysed in terms of percentage correct.

Table 5.2: Means and Standard deviations for Addition and Subtraction

Treatment	Mean	SD	N
no treatment	52.02	21.24	136
treatment	73.34	16.69	58
Total	57.69	22.421	194

After adjusting the pre-test scores using analysis of covariance (ANCOVA), there was a significant effect of treatment ($F(1, 191) = 54.89; p < .01$). Adjusted mean recognition scores suggest that children who took part in the eFit treatment made significantly greater improvement in addition and subtraction than children from the MGI group. Children from the eFit group had a mean percentage correct of 73.34 compared to a mean percentage correct of 51.02 in the control group. The value of Cohen's d is 1.17 with an effect size correlation of $r^2 = .25$, meaning that 25% of the variance in addition and subtraction performance can be explained by treatment.

Table 5.3: Means and Standard deviations for Multiplication and Division

Treatment	Mean	SD	N
no treatment	52.15	21.56	135
treatment	78.16	18.75	58
Total	59.97	23.91	193

Table 5.3 shows that children in the eFit treatment group yielded a mean score of 78.16, compared to a mean score of 52.15. A one-way analysis of covariance, using pre-test scores as a covariate, showed that this difference was statistically significant ($F(1, 190) = 91.47; p < .01$). The value of Cohen's d is 1.29, which represents an

effect size correlation of $r^2 = .29$. This means that once pre-test scores were held constant, 29 % of the variance could be explained by treatment.

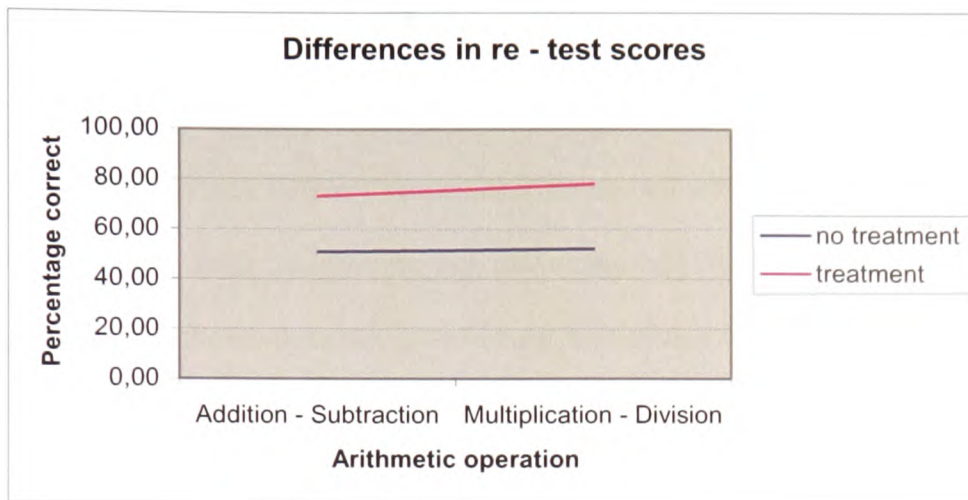


Figure 5.10: Differences in re – test scores between children from the DI and MGI group

5.6.3 DISCUSSION

The results of the analysis of covariance showed, that children from the DI group performed significantly better in the re – test of addition and subtraction as well as multiplication and division than children from the MGI group. One potential explanation for this finding is, that children who were exposed to the eFit program experienced highly individualised instruction in addition to the group instruction which takes place in their traditional math lessons. Such an explanation would support the argument that ITS are particularly suitable for the provision of individualised instruction (Rosic, Glavinic & Stankoc, 2006), because they measure and diagnose children knowledge, determine the difference between actual and anticipated level of knowledge and provide learning activities which are adapted to this difference. Indeed, a growing body of evidence points to the effectiveness of ITS (Fletcher, 2003; Murray, 1999).

Rosic, Glavinic & Stankoc (2006) conclude that „ITSs still represent the best way to enable one to one instruction”.

However, the possibility that the success of eFit can be attributed to the computer – assisted delivery mode has to be treated with caution. Despite individualised instruction is being cited as the key advantage of ITS, the overall amount of research suggests that the implementation of computer assisted instruction has not met the initial expectations and the notion that educational technology possess unique advantages over traditional instruction is subject of intense debate. Clark (1985) for example cautions that „learning is caused by the instructional methods embedded in the media presentation.” Clark (2001) criticised that the majority of studies which reported significant improvements in learning through the implementation of educational media did not control for variables such as differences in the instructional method, instructor effects, familiarity with the instructional method and a specification of the time that the learners actually spent on task in both conditions. Evidence for Clark’s viewpoint is provided by a meta –analysis of Sitzmann et. al. (1996). Sitzmann et. al. (2006), found that if WBI did not provide feedback to the learner and enough opportunity to practice, WBI was 20 % less effective than traditional, classroom based lessons. This supports the assertion of Clark the effects of CAI were actually determined by the instructional method embedded in the technology, which was not controlled for. According to this line of argument, it seems that the direct instruction approach that underlies eFit rather than the computer assisted delivery mode is likely to be a major source for the performance differential between treatment and non – treatment school.

The finding that the eFit group outperformed the minimally guided instruction group goes in line with the overall literature that has been reviewed so far, which

suggests that children with MD respond better to direct, teacher-led instruction, where the learning goals and the path to achieve these goals are stated and taught explicitly, rather than minimally guided - instruction. The results add support to Kroesbergen, Van Luit & Maas' (2004) conclusion that with MD benefit more from clearly structured explicit instruction. Hogan-Gancarz (1999) found that children with learning difficulties who received explicit instruction for strategy use in addition to their maths program outperformed children who did not receive this instruction. This goes in line with the present finding. As illustrated in the figure, eFit provided explicit and unambiguous strategies for addition problems. These highly structured explanations and the extensive amount of practice are likely to have contributed significantly to the learning gains of children in the eFit treatment. Overall, the present results support the view that MGI, with core components such as discovery learning, does not provide an effective format of instruction for children with MD.

To recap, the rationale behind the concept of discovery learning is that children's' active engagement with problems facilitates the construction of meaningful knowledge. Kirschner et. al. (2006) cast doubt on the notion that discovery learning activities are required to maximise the process of knowledge construction. Instead they argue that the majority of learners of all ages is able to construct knowledge when provided with appropriate information and structured guidance and there is no reason to assume that providing only fragmentary evidence leads to qualitatively better representations than providing the information fully. Kirschner et. al. (2006, p. 77) claim that „The goal of instruction is rarely simply to search for or discover information. The goal is to give learners specific guidance about how to cognitively manipulate information in ways that are consistent with a learning goal, and store the result in long-term memory.”

5.6.4 CONCLUSION AND FURTHER RESEARCH

A number of caveats need to be noted with respect to the design of the present result. Firstly, the present might have benefitted from controlling for variables such as differences in the instructional method, instructor effects, familiarity with the instructional method and a specification of the time that the learners actually spent on task in both conditions. Indeed, while children from the eFit group performed significantly better than children from the MGI group, the small effect sizes suggest that variance re- test results might be attributed to factors other than the implementation of the eFit treatment. To recap, the eFit program does not only constitute a different instructional medium but by focussing on arithmetic problem solving ability, it might also provide an instructional method which is different to the way mathematics is taught in the classroom.

It is important to point out that children from the eFit group had an advantage because they took the test in a familiar surrounding, whereas children from the non – eFit group were neither used to the surrounding nor used to solve problems on a computer. Because e-Fit used the same medium for the teaching as for the testing, it could be argued that eFit was "teaching to the test". Therefore, the study could have been modified by administerin a paper – pencil test in a neutral environment which is not familiar to both parties.

Although eFit was designed in line with the curricular requirements, it was explicitly focussed on children's deficits, as indicated by their results in the pre – test. This was not the case in the comparison group, where the teacher was concerned with progressing through the curriculum. Thus, children from the eFit group had excessive

opportunity to ameliorate their deficits whereas their peers from the non – eFit group had to progress to more complex areas of maths, irrespective of minor or major deficits. While the emphasis on remedying children’s deficits in the eFit program is likely to be successful in terms of the outcomes of the re-test, this might be to the expense of other parts of the curriculum. In other words, the eFit program “trained for the test” and the instructional familiarity that children had with the eFit program might have obscured the results.

While the present study accounted for the key factors such as length of treatment, age and general ability level, further research might examine other potential factors which might have contributed to the learning gains, such as the effects of the teacher’s behaviour, children’s motivation and the time spent on task. Joy and Garcia (2000) recommend researchers to depart from the media vs. method debate and to address the question which combination of instructional and media will confer the greatest learning benefits for the particular target group. By conclusion, the learning gains give good reason to believe that DI can provide the structure and guidance needed for children with MD to succeed when it comes to arithmetic problem – solving.

The results do not imply that DI is an educational panacea for mathematics instruction. As has been emphasized, the DI intervention was targeted towards arithmetic problem - solving. While fluency in arithmetic operations is essential for more complex mathematical tasks, further research would be required to investigate if the successful implementation of the DI intervention applies to other domains of mathematics such as geometry or algebra.

5.7 STUDY 2: AFFECTIVE RESPONSES TOWARDS MATHEMATICS AND LEARNING GAINS

The results of the previous study suggested that children with mathematical difficulties (MD) benefit from direct instruction and it was assumed that this instructional approach serves the cognitive requirements of children with MD better than minimally guided instruction. However, this doesn't implicate that the learning gains can be solely explained in terms of the cognitive mechanisms involved in learning mathematics. It has been previously mentioned that affective factors have a profound impact on learning mathematics (McLeod, 1994) and this has been confirmed in the foregoing study in section 2, which showed that MD are not only a function of cognitive deficit but can also be defined in terms of cognitive dimensions.

To recap, it was found that children with MD tend to forward negative affective responses towards mathematics, compared to average ability children. Also, the study demonstrated how individual differences in affective responses predicted variance in achievement. This underpins the importance of investigating the affective correlates involved in the learning of mathematics. Therefore, the examination of the affective factors which are related to the learning outcomes constitutes an essential component of the evaluation of the direct instruction intervention.

5.7.1 METHOD

Participants

Participants were the same as in the sample from the DI intervention group (N = 58, mean age 11, 5 years). Data were only collected from all children in the DI intervention group.

Material and Procedure

The ARTMS was used to measure affective responses towards maths in children from the DI intervention group. The instrument was administered in groups of 10 – 15 children. The procedure is the same as described in study 2, where the ARTMS was implemented.

Results

A multiple regression, using backwards selection, was carried out to examine the extent to which affective responses predict the individual learning gains in the domains of addition and subtraction and multiplication and division, of children who took part in the DI intervention, as determined by the difference between pretest and posttest. The scores on the affective responses towards mathematics scale, which measured anxiety, attitudes towards the teacher, self efficacy and perceived classroom conduct, were entered as predictor variables in the initial model.

Table 5.4: Affective responses and learning gains in multiplication and division

	R - value	R²	Adjusted r²	F - value	Significance
Model 1	.25	.06	-.01	.9	.47
Model 2	.25	.06	.01	1.21	.32
Model 3	.25	.06	.03	1.82	.17
Model 4	.18	.03	.02	1.89	.18

As can be seen in table 5.4., none of the regression models predicted the variance in multiplication and division learning gains to a significant extent. Table 5.5 shows that this is different when addition and subtraction are concerned.

Table 5.5: Affective responses and learning gains in addition and subtraction

	R - value	R²	Adjusted r²	F - value	Significance
Model 1	.43	.19	.13	3.03	.025
Model 2	.43	.19	.14	4.12	.011
Model 3	.42	.18	.15	6.02	.004
Model 4	.38	.15	.13	9.63	.003

As can be extracted from table 5.5, the first model explains a significant amount of variance in the dependent variable ($F_{(1, 56)} = 3.03; p < .05$). There is a moderate multiple correlation between the predictor variables and learning gains ($r = .43$). The value of the adjusted r shows that taken together, the predictor variables account for 13 % (adjusted r^2) of the variance in the learning gains. However, the standardized B value indicates that none of the individual predictor variables contributes significantly to the variance in learning gains, only scores on the “attitudes towards the teacher” scale approached significance. Anxiety was the weakest predictor variable in the first model ($\beta = .01$) and therefore excluded in the second model.

The remaining predictor variables in the second model explain a significant amount of variance in learning gains ($F_{(1, 56)} = 4.12; p < .05$). There is a moderate multiple correlation between self efficacy, attitudes towards the teacher and perceived classroom conduct and learning gains in addition and subtraction ($r = .43$). The value of the adjusted r shows that taken together, the predictor variables account for 14 % (adjusted r^2) of the variance in the learning gains. However, the standardized B value show that none of the individual predictor variables reached significance in the second model.

In the third model, self efficacy was removed due to a small standardized B value ($\beta = .86$). The third model yielded a moderate correlation between perceived classroom conduct and attitudes towards the teacher ($r = .42$) and explains 15 % (adjusted r^2) of the variance in learning gains. This regression model is statistically significant ($F_{(1, 56)} = 6.02 p < .05$). However, only the predictive value of attitudes towards the teacher reached significance ($F_{(1, 56)} = 9.63; p < .05$). Attitudes towards the teacher emerged as the strongest predictor variables in the final model, when perceived classroom was eliminated.

There is a small (multiple $r = .46$) but significant correlation between attitudes towards the teacher and learning gains in addition and subtraction. If attitudes towards the teacher dropped by one unit, learning gains in addition and subtraction increased by .38 standard deviation units. In other words, negative attitudes towards were related to positive learning gains in addition and subtraction. Overall, attitudes towards the teacher accounted for 13 % (adjusted r^2) of variance in learning gains in addition and subtraction among children from the DI intervention group.

5.7.2 DISCUSSION

The present study was intended to examine how differences in affective responses towards mathematics, as measured by the affective responses towards mathematics scale, predict variance in learning gains among children who took part in the direct instruction intervention eFit. The results of a multiple regression with backwards selection showed that children's score in the attitudes towards their teacher subscale was predictive for variance in learning gains in addition and subtraction. This goes in conjunction with the previous literature. Goh and Fraser (1998) for example reported that attitudes towards the teacher were positively related to achievement. Similarly, Murdock and Miller (2003) found that attitudes towards the teacher correlated positively with motivation. However, the present results documented an inverse relationship in that negative attitudes towards the teacher were related to higher learning gains. In other words, children who are dissatisfied with their teacher tended to outperform their peers with more positive attitudes towards the teacher.

One possible explanation is, that children with negative attitudes towards the teacher benefited more because their teacher was absent during the eFit sessions. Children whose cognitive processes are otherwise impeded by their negative attitudes are advantaged in a setting where instruction is mediated by a computer. It is unclear if negative attitudes towards the teacher are a causal factor for low achievement, or the other way round. However, the present results implicate that children with negative attitudes significantly improve their learning in settings where their teacher is not present.

Nevertheless, a problem with this interpretation is that it is not possible to separate the teacher and his or her teaching style from the format of instruction. Children who disagreed with the item “my teacher can explain maths very well” might attribute the inability to explain maths to the teacher, even though it is the underlying instructional curriculum which determines how mathematics is taught in secondary schools. In other words, the negative attitudes might be partly directed against the format of instruction rather than the teacher, who delivers the instruction. This would implicate, that children with negative attitudes improved not because the teacher was absent, but because a different instructional approach was adopted in eFit. However, a more controlled research design would be required to address this issue in more detail.

An unexpected finding was that affective responses in general and attitudes towards the teacher in particular were not predictive for variance in learning gains in multiplication and division. However, this finding provides some support for the notion that addition and subtraction at the one hand and multiplication and division on the other hand are operated by different cognitive processing resources. While multiplication and division are mainly learned via rote memorisation, addition and subtraction require the usage of problem solving strategies, an ability which was explicitly trained in the eFit intervention. Indeed, the direct instruction approach places considerable emphasis on the acquisition and usage of efficient algorithmic problem solving strategies. It can be argued that the format of instruction and also the role of the teacher is more salient for addition and subtraction, because the available problem solving strategies for multiplication and division are limited compared to the strategies available for addition and subtraction. Therefore, learning multiplication and division might not be influenced by attitudes towards the teacher. However, the present set of data is not sufficient to draw valid conclusions on this question. Again, a controlled

study on the role of attitudes towards the teacher in different arithmetic domains would be required.

5.7.3 CONCLUSION AND FUTURE RESEARCH

The study might have benefited from employing a pretest – posttest design, in order to chart how the role of affective responses changes throughout the intervention period. However, it was assumed that the learning gains constitute a valid indicator of the progress over the intervention period. Also, the results would have been more convincing if it included data from the MGI group as well. Indeed, it has been discussed that attitudes towards the teacher are predictive for learning gains in addition and subtraction because the teacher and more specifically, the type of instruction plays a central role in DI and it would be have been useful to investigate whether this is different in the child - centred MGI group, where the role of the teacher in learning mathematics is less emphasized. However, the present results present a starting point for conducting further research on the predicitive value of affective responses on the learning of mathematics under different instructional conditions.

To sum up, the chapter demonstrated that the format of instruction has considerable impact on the learning outcomes in children with MD in that the web-based DI intervention improved children's ability to solve arithmetic problems successfully. It has been mentined that the significant learning gains of children in the DI group can be attributed to the strong emphasis on modelling selected procedural strategies , presenting worked-out examples and clearly structured explanations of mathematical concepts such as place value in the DI intervention, which can promote the acquisition of procedural schemata in LTM and usage of efficient problem-solving

strategies in children with MD. To recap, chapter 4 reviewed evidence that strategy use is strongly related to working memory use: immature procedural strategies such as counting require a considerable amount of the limited working memory capacity in order to monitor and supervise the non-automatic problem-solving process and this increases the cognitive load in children with MD. In children with MD, the use of inefficient strategies exacerbates and the misuse of working memory resources for solving arithmetic problems seems to go hand in hand. Thus, investigating the potential of an instructional intervention to improve strategy use is pivotal to derive implications for educational practice and for improving the cognitive processing mechanisms in children with MD. In order to address this issue, the following chapter is aimed at investigating the possibility that children from the DI group employ more efficient problem-solving strategies than children who were exposed to MGI.

CHAPTER 6

ARITHMETIC PROBLEM-SOLVING STRATEGIES IN CHILDREN WITH MATHEMATICAL DIFFICULTIES

CHAPTER 6 ARITHMETIC PROBLEM-SOLVING STRATEGIES IN CHILDREN WITH MATHEMATICAL DIFFICULTIES

6.1 OVERVIEW

The previous study showed that children who were exposed to the web-based direct instruction intervention outperformed a control group of children who received traditional, minimally guided instruction (MGI) in the domain of basic arithmetic. It was proposed that one potential explanation for the significant learning gains in the eFit treatment group is that in contrast to the MGI approach, the direct instruction approach facilitated the application of procedural knowledge in children with mathematical difficulties (MD), specifically the use of effective mental arithmetic strategies. Indeed, according to Geary, Hoard and Hamson (1999), improvements in arithmetic ability are reflected in the distribution of problem-solving strategies. In order to investigate this issue further, the present study utilises a protocol analysis of children's arithmetic strategies to address the question of whether direct modelling of arithmetic procedures and concepts through worked examples and model strategies improves problem-solving ability in children with MD.

To recap, the DI approach is based on explicit strategy instruction and direct modelling of strategies and worked examples, whereas the MGI approach maintains that children should discover effective procedures and concepts in order to construct a meaningful understanding of mathematics. In the present study, children's verbal reports of how they derived the solutions to addition, subtraction, multiplication and division problems will be used to analyse whether children from the DI treatment group use strategies which differ in terms of efficiency from the strategies employed in a

control group of similar ability. The chapter starts with a consultation of the existing literature about the cognitive development of effective arithmetic strategy usage.

6.2 LITERATURE REVIEW

6.2.1 COGNITIVE DEVELOPMENT OF ARITHMETIC PROBLEM-SOLVING STRATEGIES

One of the most efficient ways to solve an arithmetic problem is the memory-based retrieval of arithmetic facts. The automatization of basic facts is an essential requirement for fluent arithmetic problem solving because it minimises the demands on working memory (Ashcraft, 1992). Cumming and Elkin (1999), for example, argue that a lack of automaticity on simple problems is associated with high cognitive load, which contributes to failure on more difficult problems. However, despite the importance of automaticity as a foundation for efficient problem solving, there is a general agreement that it would be impossible to derive the solution to unknown or unrecallable facts without procedural, non-retrieval strategies, which serve as “back up” strategies. These strategies are also known as algorithmic strategies. The answer to the problem 15×6 , for example, might not be directly retrieved from memory but can be computed via the decomposition strategy ($15 \times 3 = 45$; $45 \times 2 = 90$). This suggests that factual and procedural knowledge are functionally dependent and that fact retrieval alone is not sufficient for successful arithmetic problem solving. Rather, procedural, non-retrieval strategies constitute an important complement to retrieval-based strategies.

According to Lemaire and Siegler (1995), the usage of procedural strategies becomes more frequent as problems become more complex. Also, the use of efficient procedural strategies is crucial with respect to cognitive efficiency, because compacted

strategies eliminate the necessity of storing each basic arithmetic fact individually. Knowing that $18 + 18$ can be solved by the same strategy as 2×18 eliminates the cognitive effort of representing these facts separately in LTM. In order to keep the demands on working memory minimal and to avoid cognitive overload, it is vital for learners to use efficient non-retrieval strategies. For a problem such as $19 + 17$, immature strategies such as finger counting interfere with the actual problem-solving procedure and create cognitive load, whereas a decomposition strategy such as $20 + 17 - 1$ helps the learner to solve the problem in manageable chunks. However, it has been mentioned already that the usage of efficient strategies constitutes a key problem for children with mathematical difficulties (MD).

Compared to children of average mathematical ability, children with MD face fundamental impediments in the context of arithmetic problem solving: firstly, they are less likely to solve arithmetic problems via direct retrieval than average ability children. Instead, they tend to rely on immature and inefficient counting-based strategies, which impede problem-solving procedures (Barrouillet & Lepine, 2005). Similarly, Geary, Hoard and Hamson (1999, p. 223) argue that children with MD have problems with shifting from algorithmic, procedural-based problem-solving strategies to retrieval-based problem solving and are characterised by “error-prone use of developmentally immature problem-solving procedures.” Secondly, they experience difficulties in executing procedural strategies fast and accurately. Thus, even simple problems require the same cognitive processing effort as complex problems (Tremblay & Lemoyne, 1986).

Before delving into an examination of the strategic difficulties in children with MD, the previous sections provide a review of the literature on arithmetic strategy

development. It is assumed that models of normal arithmetic strategy development provide a framework from which to derive implications for arithmetic strategy development in children with MD.

Different theories have been posited to explain how children develop arithmetic strategies. In essence, theories of arithmetic development fall into three categories:

- a.) Retrieval theories in which the speed and accuracy of retrieval are based on the semantic structure of the network of stored associations. Accurate and fast retrieval of an answer depends on the semantic features of the problem, such as physical similarity and magnitude similarity. More specifically, it is argued that a problem such as 5×6 might activate related problems such as 5×7 , leading to interference and in turn to slow latency and high error rates (Campbell, 1995).
- b.) Retrieval theories in which the speed and accuracy of retrieval are based on exposure to and frequency of arithmetic problems. In contrast to the previously reviewed semantic account, this model predicts that accurate and fast retrieval of an answer depends on the associative strength between problem and answer, which is a function of cumulated practice (Ashcraft, 1992).
- c.) Multiple-procedure theories, which propose that direct retrieval of arithmetic facts is just one of many different strategies and supplemented by procedural, non-retrieval backup strategies (Siegler, 1988).

Both retrieval theories of arithmetic development share the assumption of a developmental pathway or evolutionary progression from simple and effortful counting-based strategies to the use of retrieval-based strategies and fluent mastery of facts, with ineffective strategies being replaced as newer, more effective strategies are learned (Ashcraft, 1982; Siegler, 1988). This notion has been challenged by Siegler's overlapping waves theory, a multiple-procedure theories, which is concerned with the development of arithmetic strategies. The overlapping waves theory is based on evidence that children use multiple strategies over long periods of time for different types of tasks and continue to use older strategies alongside more effective ones, although they are able to explain the advantages of the more recent strategies (Shrager & Siegler, 1998). In other words, older and less efficient strategies persist alongside recently learned strategies. The majority of studies on the development of arithmetic strategies are based on the protocol analysis paradigm, whereby children's verbal explanations are tape recorded, transcribed and analysed. As will be evident in the studies reviewed below, the terminology is not consistent because researchers have developed different labels in order to code the variety of different strategies that children employ.

6.2.2 ADDITION AND SUBTRACTION

An early but seminal study on the development and change in children's addition strategies is provided by Siegler and Jenkins (1989). The rationale behind the experiment was to investigate the transition from the inefficient "sum" strategy (adding two numbers by counting on from the smaller addend: $3 + 5 = 4; 5; 6; 7; 8$) to the faster "min" strategy (adding two numbers by counting on from the larger addend: $3 + 5 = 6; 7; 8$). Both the "min" and the "sum" strategy should gradually replace the immature

“counting all” strategy, one of children’s earliest strategies, whereby two numbers are added by simply counting one unit after another until the result is attained (Siegler & Jenkins, 1989). This strategy is the simplest and least efficient arithmetic strategy, because it involves the counting of all numbers.

Siegler and Jenkins (1989) adopted the microgenetic method, which was originally formulated and advanced by Siegler (1984). The microgenetic method enables researchers to chart children’s cognitive development in fixed intervals over a certain amount of time. The Siegler and Jenkins study consisted of a pretest and a practice period of 11 weeks. The pretest was intended to identify children who had mastered counting to a sufficient extent but who were not proficient in the “min” strategy. In the pretest, children were presented with addition problems, with addends ranging from 1 to 5, and were asked to explain verbally how they solved the problems. In another part of the pretest, children were asked to consider alternative strategies. Based on their pretest results, a total of 10 children, aged 4 to 5, were chosen to take part in the microgenetic study. These children solved 78% of the addition problems correctly and used the “sum” method most frequently (43% of the time).

During the 11-week practice period, the children who were selected from the pre-test were individually tested in three sessions per week, each of which involved the presentation of approximately seven addition problems. Children’s strategies were examined using a combination of video recordings of overt problem-solving behaviour and verbal self-reports of how they derived their solutions. In cases when overt problem-solving behaviour could not be observed or was ambiguous, the verbal self-reports were used to analyse and classify the strategy. Up to week 7, the problems consisted of addends ranging from 1 to 5. The researchers noticed that while the

majority of children used the sum strategy at this point, they only used this strategy occasionally. This suggests that the transition to a more efficient strategy proceeds slowly. In order to provide an incentive for children to move to the sum strategy as the dominant strategy, the researchers decided to use “challenge problems” with one small addend and one large addend ($2 + 21$) during week 8. It was hypothesized that by using these problems, children are forced to use the sum strategy and that the advantages of the sum strategy over the min strategy become obvious. From week 9 to 11, a mixture of easy and challenging problems was employed.

In the final session, children used a total of 8 strategies. However, the frequency of usage differed considerably across strategies: the inefficient counting-from-first strategy (counting from 1 until the result is attained) was used only 1% of the time, with a moderate percentage correct rate of 40%. Children guessed the result 2% of the time, with a percentage correct rate of only 20%. The sum strategy was used 34% of the time, with a percentage correct of 89%. The min strategy was used by 9 children, with a percentage correct rate of 86%. The retrieval strategy was used 22% of the time, again with a high proportion of correct answers: 89%. Thus, the majority of children were able to use the two most efficient strategies for addition accurately. The results support the assertion of Shrager and Siegler (1998), who claim that despite the fact that a variety of strategies coexist at a given time, there is a general tendency to move towards more advanced, retrieval-based strategies. Taken together, the results indicate that regular practice facilitated children’s transition from the inefficient sum strategy to the accurate mastery of the min strategy and direct retrieval.

The previous study has demonstrated that counting-based strategies such as the counting all and counting up strategies are slow and inefficient. In contrast to immature counting-based procedures, decomposition strategies involve the mental manipulation of numeric information rather than overt or sub-vocal counting and can therefore be considered as more efficient. Beiszhuisen (1993) distinguishes two different decomposition strategies: the “1010” strategy ($27 + 34 = 20 + 30 + 7 + 4$), which involves the decomposition of both addends into tens and units, and the “N 10” strategy, a complete number strategy whereby only one addend is decomposed ($27 + 34 = 27 + 30 + 4$). The 1010 strategy and the N10 strategy are also known as the Decomposition method and the Jump method (Thompson, 1999). As will be illustrated later, Foxman and Beiszhuisen (2002) found that the Jump strategy and rounding strategy were preferred by more proficient pupils, while the Decomposition method was favoured by less able pupils. The N10 C strategy is a rounding strategy. Children do not decompose all numbers but round one of the addends to the nearest unit of tens, add it to the first addend and subsequently eliminate the difference.

Canobi (2005) investigated strategy usage in primary school children, aged 5 – 7. The sample included 20 children from grade 1 (mean age 5.8 years), 20 children from grade 2 (mean age 6.7 years) and 20 children from grade 3 (mean age 7.9 years). Children were asked to solve single-digit addition and subtraction problems and to explain which strategy they had used to solve the problem. The strategies were coded as retrieval, decomposition, counting all (when children started counting from 1: $5 + 2 = 1; 2; 3; \dots 7$), counting on (when children counted from one addend: $5 + 2 = 6; 7$), counting up (when children solved subtraction by counting forward from the smaller number: $5 - 2 = 3; 4; 5$), counting down (traditional subtraction: $5 - 2 = 4; 5; 3$), or finger counting. Based on a protocol analysis of children’s verbal self-reports, the results

indicate that the most frequently used strategies were “counting on” for addition and “counting down” for subtraction: 38% of the time, the addition tasks were solved using counting up and 34% of the time the subtraction tasks were solved using the counting down strategy. Thus, counting appears to be the dominant strategy in the early stages of development. Another study by Canobi (2004) lends support to this notion.

In a similar study, Canobi (2004) examined the strategies used to solve addition and subtraction problems. The sample included a total of 60 children, 30 from grade 2 (mean age 7.3 years) and 30 from grade 3 (mean age 8.3 years). Children were asked to solve a total of 48 addition and subtraction problems and to provide verbal self-reports of how they derived their solutions. It was observed that counting-on was the dominant strategy for addition, with a frequency of 55%, while counting-up appeared to be the dominant strategy for subtraction, with a frequency of 43%. The high frequency of the counting-up strategy indicates that addition is one of the earliest strategies for solving subtraction tasks, and this suggests that mastery of addition is an important step in strategy development. This confirms earlier evidence that young children rely on counting-up strategies first before they move on to more elaborate strategies (Fuson & Willis, 1988).

With respect to the development of subtraction strategies, there is a conflict as to whether instruction should emphasize conceptual understanding or procedural knowledge as the starting point for learning subtraction. Bryant (1999), for example, argues that an understanding of the inverse relationship between addition and subtraction is essential for a genuine understanding of these operations. The counting up strategy for a problem such as $7 - 4$ means that children state the subtrahend (4) first and then count up to seven while monitoring the number of forward counts: “4 – 5, 6, 7”.

Another strategy involves counting down. For a problem such as $7 - 4$, this involves stating the minuend (7) first, counting back 4 and stating the last number counted.

Selter (2001) investigated the difficulties that children encounter when solving three-digit subtraction problems. The sample consisted of 300 primary school children who were in the transition from grade 3 to 4, who were asked to solve six written three-digit subtraction tasks which were presented in the form of a class test. Selter found that students experienced the greatest difficulties in written subtraction when the minuend contained a 0 ($601 - 374$) and borrowing from 0 was required to solve the problem. The results showed that less than 50% of all children were able to solve such a problem correctly. A similar result was found by Fiori and Zuccheri (2005), who examined children's written subtraction errors. Their sample, consisting of 732 children aged 9 – 12, was asked to solve nineteen written subtraction problems, presented as a class test. In line with the finding of Selters, their results showed that subtraction tasks where the minuend contained a 0 caused the greatest difficulties to the majority of children. One possible explanation for why children have problems in multi-digit subtraction tasks containing a zero is the failure to fully understand the positional numerical system, i.e. place values (Fiori & Zuccheri, 2005). This would suggest that a lack of conceptual understanding causes procedural difficulties, in that children are unable to use arithmetic strategies correctly if they do not understand the underlying conceptual features of the problem.

However, the notion that conceptual understanding is a prerequisite for the efficient usage of problem solving strategies has not gone unchallenged. Indeed, there is evidence that even if children are challenged to think about the underlying concept, they are not able to execute the task correctly by applying this conceptual understanding.

This finding was reported by Davis and McKnight (1980). They investigated the subtraction strategies of 3rd and 4th graders who were able to solve subtraction problems correctly but had problems when the minuend contained a zero.

The children were asked to solve a subtraction problem that required them to borrow across zeros, such as $8002 - 48$. After presenting the problem, children were involved in different tests that were targeted to facilitate their conceptual understanding of subtracting with zeros. These tests included word problems involving subtraction and number estimations. If the children were able to solve one of the conceptual tests correctly, they were asked to apply this concept to the subtraction problem at hand. However, none of the children in the study was able to carry out the subtraction problem correctly and instead adopted the previous strategies, which were incorrect. This implies that children's conceptual understanding was not sufficient to enable them to execute the subtraction procedure correctly and that an understanding of the procedure might be more important than an understanding of the concept behind the procedure. In other words, to acquire competence in arithmetic, children need to learn the strategic procedures; this goes in line with the assumptions of the direct instruction (DI) approach (Kirschner, 2006)

However, a study conducted by Thornton (1990) provides evidence that children's strategy development can be significantly enhanced through instructional interventions, which emphasize the acquisition and correct usage of efficient problem-solving strategies, such as direct instruction. The sample in the study consisted of a total of 70 1st grade children (38 in the experimental group and 22 in the control group), who were either exposed to a strategy-based intervention or drill and practice intervention. Thornton (1990) argued that conventional subtraction instruction overemphasizes both

conceptual understanding and drill and practice to the expense of another important phase, namely explicit strategy instruction. In other words, it is argued that the transition phase, which leads to an automatic retrieval of subtraction facts, has not been addressed in current instructional practices, according to Thornton (1990).

Children in the strategy instruction group received direct instruction in how to use non-counting strategies to solve unknown subtraction facts, whereas children in the control group received traditional drill and practice to memorise subtraction facts to the point of automatic recall. The overall rationale behind the strategy-based intervention was that “when concept understanding for subtraction has been demonstrated and prior to drill for speed, teachers should intervene and actively teach children solution strategies for working out answers to unknown subtraction facts” (Thornton, 1990, p. 36). The active role of the teacher in guiding children’s conceptual and procedural understanding and the explicit teaching and modelling of solution strategies captures the essence of the direct instruction approach that was discussed in the previous study.

Apart from the different instructional approaches that were used to teach subtraction, variables such as teacher’s experience, amount of time spent on maths and the textbook were identical. Two different methods were used to examine changes in children’s strategy development. Based on a microgenetic research design, a total of five verbal protocol analyses were carried out at three-week intervals over a period of five months to investigate mental strategy development. Children’s verbal responses to nine subtraction problems were coded as correct and quick (latencies faster than two seconds), correct and slow (latencies slower than two seconds), incorrect or not attempted. The solution strategies were coded as a) no attempt, b) show all, c) mental counting back, d) mental counting up, e) addition, f) memorization or g) other. Each

child's preferred strategy was determined for each protocol analysis interview. Written subtraction tests, which were carried out at the end of the intervention and approximately three months later, as a measure of retention, were employed to examine to differences in written strategy usage between the two groups. The test consisted of 54 subtraction problems.

The protocol analysis indicated that after one year of strategy-based treatment, over 50% of children moved from inefficient counting strategies to more elaborate and efficient non-counting strategies. Only 19% of students in the control group who received drill and practice treatment managed to move from counting strategies to more elaborate strategies. Those children from the strategy-based treatment group who still relied on counting strategies were significantly more likely to use the shortest counting strategy, "mental counting up", than children from the control group: children who were exposed to the strategy-based intervention chose the most efficient counting strategy 82% of the time, compared to control group students, who did so 58% of the time. Indeed, immature and laborious counting strategies were more prevalent among students from the control group.

Also, students from the drill and practice control group were more inclined to give up when faced with a problem. In the first protocol analysis, 36% of the children from the control group made no attempt to answer the problem, compared to 9% of the children from the strategy intervention group. In the final protocol analysis, 18% of the children from the control group but only 1% of the children from the strategy intervention group made no attempt to answer the problem. Further examination of the data indicated that children from the strategy group were not only more likely to choose

more effective strategies for subtraction but also solved the problems with greater speed and accuracy than children from the control group.

An analysis of the percentage of subtraction problems that could be correctly solved within two seconds showed that in the first protocol analysis, 16% of the children in the control group and 36% of the children in the strategy intervention group solved the problems quickly and correctly. In the final protocol analysis session, only 30% of the children in the control group but 79% of the children in the strategy intervention group did so. Thus, children from the strategy–intervention group made significant improvements in speed and accuracy over a five-month period, compared to children from the control group. The overall trend in the protocol analysis results in favour for the strategy–intervention group was supported by significant differences on the written subtraction tests. When the test was administered at the end of the intervention, the mean number of correctly solved problems was 39.6 out of a total of 54 problems for the strategy–intervention group and 18.3 for the control group. When the test was repeated after three months, strategy–intervention children still outperformed children from the control group (with a mean score of 42.9 compared to 18.1). This suggests that the positive outcome of the strategy intervention goes beyond a short–term effect and continues even after the intervention has finished.

One drawback of the Thornton study is that children had already been exposed to their instructional intervention for two to three months by the time the first protocol analyses took place. Thus, the results of the first protocol analysis cannot be treated as pre-test data, because the results might already be contaminated by the effects of the intervention. Also, the written subtraction task was only administered as a post-test, at the end of the intervention, and three months later as a measure of retention. However,

the study could have been improved by conducting a written test as well as the protocol analysis prior to the intervention. A more careful pretest – post-test design would provide a more coherent view on the effects of strategy-based instruction intervention. Overall, the research that has been reviewed so far indicates that direct (or automatic) retrieval constitutes an efficient alternative to procedural algorithms. This supports Canobi's (2005, p. 222) argument that "in children's addition and subtraction, skilled problem solving is characterized by high levels of speed and accuracy and relatively frequent use of retrieval-based strategies." As will be reviewed below, the study of Imbo and Vandierendonck (2007) suggests that the retrieval strategy is more efficient compared to non-retrieval, procedural strategies, because direct retrieval reduces working memory requirements.

The study conducted by Imbo and Vandierendonck (2007) included a total of 63 children. Of the total participants, 21 were from grade 4 (mean age = 10 years), 21 were from grade 5 (mean age = 11.1 years) and 21 were from grade 6 (mean age = 12.2 years). Children took part in a choice and a no-choice condition. Each condition comprised 32 simple addition problems, ranging from $2 + 9$ to $9 + 8$. In the choice condition, children could choose among four different arithmetic strategies. In the no-choice condition, children were forced to use one particular strategy to solve all problems. The no choice/ retrieval condition required the children to retrieve the result automatically, the no choice / transformation condition required children to make an intermediate step by decomposing one of the numbers and adding it together in sequence, and the no choice / counting condition required children to count sub-vocally until they reached the correct total. In each condition, half of the problems were coupled with executive working memory load. Executive load was induced through a random reaction time task, which required children to press "4" on a numerical keyboard when

they heard a high tone and a “1” if they heard a low tone. Overall, the accuracy rate for the central executive task was very high, at 95% or above in all conditions.

An analysis of the reaction time in the no-choice condition in terms of strategy execution speed showed a significant effect of executive working memory load on reaction time: strategies were executed faster in the no-load compared to the load condition. The disruptive interference effect of working memory load was largest when children were forced to use the transformation strategy. This effect was larger than the effects on retrieval and counting. To sum up, the results show that executive resources are involved in all of the strategies that children have used. The non-retrieval strategies, however, seem to require more working memory resources compared to retrieval. According to Imbo and Vandierendonck (2007), “non-retrieval strategies are composed of multiple retrievals from long-term memory, these strategies also contain several processes that might require extra executive resources such as performing calculations, manipulating interim results and monitoring counting processes” (p. 286).

There was also an interaction effect between grade, working memory load and strategy: the effects of load on retrieval speed declined linearly from grade 4 to grade 6, whereas the reaction times for the non-retrieval strategy did not change across grades. In other words, children execute the retrieval strategy significantly faster and therefore more efficiently as they grow older and the increasing efficiency in terms of strategy speed reduces the requirement for working memory resources. Indeed, Imbo and Vandierendonck (2007) conclude that, “more frequent retrieval leaves more working memory resources free for other uses.” Thus, arithmetic strategic competence seems to develop towards increasing efficiency of retrieval strategies, which goes in line with decreasing working memory demands. However, one possible drawback of the choice /

no-choice method that has been used in this study is that researchers cannot determine whether children actually used one of the four strategies or whether they used a different strategy. Also, it is impossible to determine whether the use of retrieval strategies increases the available working memory capacities or whether children with high working memory capacities are more likely to use retrieval strategies. This question has been addressed by Barrouillet and Lepine (2005).

Barrouillet and Lepine (2005) investigated the possibility that WM resources affect the use of arithmetic problem-solving strategies and examined whether individual differences in WM capacity determine the frequency of retrieval strategy use for single-digit addition. They hypothesized that children with low WM resources use direct retrieval less frequently, because poor WM resources result in weaker and less accessible associations in long-term memory. Their sample included 91 children, of whom 47 were from grade 5 (mean age = 8.11 years) and 44 from grade 4 (mean age = 9.11 years). In order to discriminate between children with high and low working memory capacity, working memory capacity was tested through the administration of a counting span task and a reading letter span task. Children whose test results deviated by $-.67$ units or more from the mean were considered as having low WM capacities. The arithmetic test consisted of 80 simple addition tasks, with combinations from $1 + 1$ to $9 + 9$. After solving a problem, the child was asked to explain how he or she had derived the solution. The researchers discriminated between procedural, non-retrieval strategies and retrieval strategies. For both 4th and 5th graders, there was a positive correlation between working memory capacity and frequency of retrieval strategy. High working memory capacity was related to more frequent retrieval usage, a trend that was most pronounced for smaller problems where retrieval is the predominant strategy. Also, it was found that children with low working memory capacities were faster in retrieving

the solution than children with high working memory capacities. Thus, Barrouillet and Lepine (2005) conclude that, “children’s working memory resources affect both the frequency and the efficiency of the retrieval strategy in simple arithmetic problem solving” (p.204).

However, the study might benefit by including a sample of children with average working memory capacity rather than comparing children with high and low capacity. Indeed, this comparison might have led to floor and ceiling effects. Furthermore, Barrouillet and Lepine treat working memory resources as an innate capacity, which is either intact or weak. At the same time, they do not address the question of why children have poor memory resources and how these resources develop over time. Given that the children had had 4 or 5 years of learning experience at school, their scores in the counting and reading letter span tasks might be a result of their learning mechanisms, in that children who used efficient reading and counting strategies from an early age might develop better associations in long-term memory and therefore be more skilled in counting and reading span tasks than children who relied on weak strategies. Thus, these measures of working memory might be a function of inefficient strategies rather than the other way round. The relationship between working memory capacity and strategy usage is not clear. Therefore, the results of Barrouillet and Lepine’s (2005) study are not conclusive and further research would be needed in order to substantiate their findings.

6.2.3 MULTIPLICATION AND DIVISION

A meta-analysis conducted by Sherin and Fuson (2005) showed that children’s earliest multiplication strategy is to transform the calculation into an addition problem

and derive the solution by simply adding the multiplicand several times as determined by the multiplier ($21 \times 4 = 21 + 21 + 21 + 21$). However, the meta-analysis indicates that the developmental trend in multiplication is similarly to the development of addition and subtraction strategies, in that children tend to move from immature counting strategies, as described above, to direct retrieval of multiplication facts. This assumption goes in line with the findings of Lemaire and Siegler (1996).

A valuable account of children's strategy usage is provided by a longitudinal study by Lemaire and Siegler (1995), who investigated which strategies were used for multiplication, how often they were used and how effectively they were applied. The sample consisted of 22 children from grade 2 (mean age 8.1 years). In the experimental sessions, children were asked to solve a set of 40 orally presented multiplication problems, which were combinations of multiplicands (1 to 9) and multipliers (1 to 9). Unlike the majority of studies, which rely on the protocol analysis method to determine the strategies that children use to solve arithmetic problems, Lemaire and Siegler (1995) did not ask the children to explain how they solved the problem. Instead, children were videotaped and their strategy usage was inferred from their problem-solving behaviour. When children displayed no overt problem-solving behaviour such as finger counting or sub-vocal counting, it was inferred that they had solved the problem via direct retrieval.

In essence, it was found that children used multiple strategies for solving multiplication tasks, which included direct retrieval, repeated addition, writing up the problem or saying "I don't know". In the first session, the retrieval strategy was used 38% of the time, repeated addition was employed 30% of the time and children answered, "I don't know" 32% of the time. In the second session, the percentage of retrieval use increased to 62%, whereas the frequency of repeated addition and

“unknown solution” declined to 22% and 15%, respectively. The trend towards using retrieval for multiplication problems increased in the third session, when retrieval was used 92% of the time, while repeated addition decreased to 6%. In the third session, the answer “I don’t know” was given only 2% of the time. Thus, the usage of repeated addition and “I don’t know” decreased continually, whereas the usage of direct retrieval increased. Also, direct retrieval came to be executed with greater speed and accuracy. Errors in retrieval execution decreased significantly, from 27% in the first session to 12% in the second session and 9% in the third session. Taken together, the findings suggest that while children use multiple strategies for multiplication, there is a strong tendency for direct retrieval to become the dominant strategy for multiplication, and to be executed faster and more accurately over time.

One possible drawback of the study is that strategy usage was inferred from the presence or absence of overt problem-solving behaviour. To recap, children were classified as having retrieved the answer if no overt problem-solving behaviour was evident. This operationalisation rests on the assumption that any response that did not entail overt figuring is based on direct retrieval. However, the emphasis on overt behaviour rather than cognitive processes suggests that Lemaire and Siegler’s classification scheme might be over-simplified. The absence of children’s verbal protocols makes it difficult to determine which strategies had been actually used.

To recap, the previous study provided evidence that children use different strategies for solving multiplication problems and this challenges the assumption that multiplication is synonymous to the direct retrieval of facts from a network of stored knowledge representations (Ashcraft, 1992). However, the sample in the previous study consists of young children, who might use non-retrieval strategies merely because the

multiplication facts are not yet readily available due to a lack of experience and practice. Despite the variety of strategies that children used, a clear trend towards increasing use of direct retrieval was observed and it seems that with increasing experience and practice, the associations between stored facts in the mental network are strengthened to an extent that non-retrieval strategies might ultimately be replaced by direct retrieval. If the use of retrieval as opposed to non-retrieval backup strategies for multiplication is a function of the frequency of exposure to multiplication facts, then it would follow that given the extensive amount of evidence, adults solve simple multiplication problems by direct retrieval because there is no necessity for them to rely on backup strategies. This question has been addressed by LeFevre, Bisanz, Buffone and Sadesky (1996).

In essence, the researchers aimed to investigate whether adults continue to use different strategies for solving multiplication problems or whether they uniformly retrieve the answers from memory. Their sample consisted of 16 undergraduate students (mean age 22 years). Participants were asked to solve a total of 100 single-digit multiplication problems. After solving each problem, participants were asked to report their solution procedure. The experimenter coded the strategies immediately into predefined categories: responses were coded as retrieval in cases when participants indicated that they “just knew” the answer from memory. Responses were classified as “derived fact strategy” if participants drew on a known fact to derive the solution for a related problem. An example for the derived fact strategy is the problem “7 x 12”. In this case the related fact, which is retrieved automatically, would be “7 x 11”. Based on the result of the retrieved fact, the problem can be solved by adding another “7”. The repeated addition strategy entailed the addition of an operand until the result is reached. Responses were coded as rules if participants indicated that they used a common rule,

for example “ $0 \times N$ is always 0”. Responses were classified as “number series” if participants solved the problem by drawing on memorized number strings such as “5, 10, 15, 20...”

The results showed that direct retrieval constitutes the most frequently used strategy: it was used for 87.9% of all problems and 100% of all participants used direct retrieval at least once. However, it is important to note that only 28% of the participants solved all of the problems via direct retrieval. Instead, participants seem to have selected strategies depending on the nature of the problem. A total of 56% of the participants used the derived fact strategy at least once. While this strategy was used for only 5.6% of the problems, it was mainly used to solve problems with operands larger than 6. The derived fact strategy was significantly slower and more error-prone than the retrieval strategy. The repeated addition strategy was used at least once by 38% of all participants, for a total of 2.2% of all problems. Repeated addition was primarily used for problems where one of the operands was a 2 (e.g. 2×7 solved as $7 + 7$). Analysis of latencies indicated that this strategy was executed efficiently, in that repeated addition was not significantly slower than retrieval. The number series strategy was employed at least once by 19% of all participants, but this strategy was used infrequently, for only 2.5% of all problems. This strategy was mainly adopted for problems containing operands of 3 or 5. The frequency of using rules for solving the problems was negligible. Overall, the findings provide evidence that even for single-digit multiplications, adults continue to use multiple routes to the solution. Non-retrieval strategies were not used in a random way, but were employed for particular types of problem, which suggests that adults use strategies adaptively. While the use of non-retrieval strategies persists in adults, the results suggest that retrieval is the most

frequently used strategy, and is accompanied by fast latencies and low error rates. However, the study might benefit from a larger sample.

To recap, the previous study documented that despite the fact that adults continue to use different strategies for multiplication, the majority of participants used retrieval as a means to solve the majority of problems quickly and accurately. As will be demonstrated in the following studies, retrieval is also a frequently used method for division. Division is not introduced before class 3 and this means that it is the operation that children have practiced least. While children experience little direct experience of practicing division facts, an understanding of the reciprocal relationship between multiplication and division enables them to solve division problems by drawing on related multiplication facts (Mauro, LeFevre & Morris, 2003). Not surprisingly, then, division problems have been identified as the most difficult operations to learn (Siegler & Shipley, 1995).

Of all four basic arithmetic operations, division is introduced last because the other operations are often used as a foundation for teaching division (Parmar, 2003). In particular, the fluent mastery of multiplication is seen as an essential prerequisite for learning the concept of division. Indeed, there is evidence that there is a strong relationship between the cognitive processes and memory representations involved in multiplication and division (Mauro, LeFevre & Morris, 2003). More specifically, it has been proposed that besides direct retrieval, another important strategy to solve division problems is “mediated retrieval”. This hypothesis has been addressed in the study reviewed below.

Mauro, LeFevre and Morris (2003) investigated the possibility that individuals rely on one of two retrieval-based strategies for solving division problems, namely direct retrieval or mediated retrieval. Mediated retrieval refers to an intermediary problem-solving process whereby the division problem is first converted into a multiplication-based format. The multiplication fact is accessed and retrieved as the solution for the division problem. The sample for the Mauro et al. (2003) study included 44 undergraduate students (mean age 22.5 years). Participants were asked to solve a total of 36 division problems in five representational formats, presented on a computer screen. The formats differed in two ways, namely with respect to their operation symbol (multiplication vs. division) and with respect to the location of the missing element that had to be solved by the participants ($72 : 8 = ?$, $72 : ? = 9$, $8 \times ? = 72$). Problems were classified as large problems and small problems, with dividends of 25 or less.

The results showed that participants were significantly faster when the division problems were converted into corresponding multiplication problems than they were with problems in division format. However, the advantage of the multiplication format was confined to large problems and the format had no effect on solution latencies for small problems. One possible explanation for this finding is that while small problems can be solved via direct retrieval, large division problems are less practiced and the division facts are less accessible. Latencies for small problems, on the other hand, varied as a function of the position of the missing element. Participants were significantly faster at solving a small problem such as $12 : 4 = ?$ than a problem in an unconventional format such as $12 : ? = 4$. Thus, an unusual division format has a negative effect on solution latencies for small problems. Mauro et al. (2003) conclude that, “these findings are consistent with the notion that multiplication is the primary mode of representation for both multiplication and division problems and that strategic

recasting of division problems is most likely to occur on the largest and therefore, most difficult problems” (p. 169).

Robinson et al. (2005) investigated developmental changes in children’s division strategies. A total of 122 children from grade 4 to 7 were asked to solve 32 simple division problems. The protocol analysis method was used to investigate differences in strategy use. Children were instructed to solve the task, give the answer verbally and then explain how they had derived the solution. This is different from other studies, where children are encouraged to “think aloud”, in other words to verbalise their mental strategic processes while solving the computation, rather than solving the computation first and giving an explanation afterwards. The time the children needed from reading the task to stating the answer was recorded in order to measure solution latencies.

In essence, it was found that three strategies were consistently used across grades, namely addition, multiplication and direct retrieval. Younger children tended to use addition strategies to solve division problems by adding the divisor until the dividend was reached ($49 : 7 = 7+7+7+7+7+7+7$). Older students, on the other hand, recast the division as a multiplication problem ($49 : 7 = 7 \times 7$), which then became the dominant strategy even in adults (Robinson et. al., 2002). This has been confirmed by Lucangeli et al. (2003), who argue that the transformation of a division problem into a multiplication problem “served to transform the complexity of division into a more manageable and familiar multiplication” (p. 519)

Overall, there was no increase in the usage of the retrieval strategy and the frequency of retrieval usage was stable across grades. Children in grade 4 used the retrieval strategy for 16.4% of the tasks, while children in grade 5 used retrieval for 17.

6% of the tasks, children in grade 6 used retrieval in 14.7% of the tasks, and children in grade 7 used retrieval for 16.4% of the tasks. Interestingly, the accuracy and solution of the retrieval strategy remained consistent. Contrary to the intuitive assumption that older children from higher grades were faster and more accurate when deriving the answer via direct retrieval, it was found that there were no marked differences in the effectiveness of direct retrieval between older and younger children. In all grades, the multiplication strategy yielded significantly longer solution latencies than direct retrieval. Thus, the tendency for children to move from the addition strategy to the multiplication strategy across grades 4 to 7 points to a cognitive progression towards a more competent strategy.

Table 6.1: Frequency, accuracy and latency rates of children's use of different division strategies

Strategy	Use ^a (%)	Once ^b (%)	Half ^c (%)	Accuracy ^d (%)	Latency ^e (S)
GRADE 4					
Retrieval	16.4	62.5	12.5	92.7	1.8
Multiplication	15.2	46.9	12.5	88.5	3.5
Addition	53.0	81.3	56.3	73.7	8.4
GRADE 5					
Retrieval	17.6	66.7	10.0	93.5	2.3
Multiplication	48.8	93.3	50.0	92.5	3.3
Addition	21.8	70.0	20.0	87.7	5.2
GRADE 6					
Retrieval	14.7	71.4	8.6	93.0	1.9
Multiplication	53.7	88.6	45.7	96.2	2.4
Addition	16.7	65.7	8.6	92.9	3.7
GRADE 7					
Retrieval	16.4	64.0	12.0	93.9	1.6
Multiplication	71.0	100.0	88.0	95.0	2.6
Addition	4.8	11.1	0.0	94.7	5.1

^a Percentage of problems on which the strategy was used.

^b Percentage of students who used the strategy at least once.

^c Percentage of students who used the strategy on at least half of the problems.

^d Percentage correct strategy use.

Adapted from: Robinson, K. M., Arbuthnott, K. D., Rose, D., McCarron, M. C., Globa, C. A. & Phonexay, S. D.(2006). Stability and change in children's division strategies. *Journal of Experimental Child Psychology*, 93, 3, 224-238.

To recap, younger children transform division problems into addition problems, whereas children from higher grades recast division problems into multiplication problems. This may be explained in terms of the increasing automatisisation of multiplication and division facts in older children. Younger children who have not yet automatised the multiplication tables thus rely on addition strategies as a safe but slow backup strategy to derive the results. Robinson et al.'s findings are highly relevant, because they clearly indicate that unlike other basic arithmetic operations, there is no significant change across age-bands and grades towards increasing usage of retrieval strategies for division problems.

Thus, the arithmetic strategic development for division seems to be an exception to the pattern found for other operations. On the other hand, it can be argued that no other operation than division enables students to select from two strategies that are equally effective. For addition, subtraction and multiplication, retrieval becomes the dominant strategy. In division problems, there is thus a competition between two strategies, namely retrieval and multiplication. However, given that the multiplication strategy has been found to be equally efficient for solving division problems, there is no need for children to move from multiplication strategies to retrieval strategies.

6.2.4 SUMMARY OF PROBLEM-SOLVING STRATEGIES

The studies on children's use of arithmetic strategies as reviewed above illustrate that researchers use different classification devices for arithmetic strategies. While the table 6.2 below is not exhaustive, it exemplifies that certain problem-solving strategies have been repeatedly found across different studies. Researchers have used different labels which refer to the same strategies and table 6.2. provides an overview of the strategy labels from previous studies.

CHAPTER 6 ARITHMETIC PROBLEM-SOLVING STRATEGIES IN CHILDREN WITH MATHEMATICAL DIFFICULTIES

Table 6.2: Summary of arithmetic problem solving strategy labels found in previous research

	Lucangeli	Thompson	Beiszhuizen Foxman	Example
Mental addition $38 + 26$	N 10	Jump Method	N 10	$38 + 20 = 58$ $58 + 6 = 64$
	C 10 / Formation of units of tens	Compensation	N10 (Rounding) C	$38 + 30 = 68$ $68 - 4 = 64$
	Decomposition strategy	Split method	1010	$30 + 20 = 50$ $8 + 6 = 14$ $50 + 14 = 64$
	Mental algorithm (MA)			$\begin{array}{r} 38 \\ + 26 \\ \hline 64 \end{array}$
	Automatic Calculation (AUTO)	Automatic Calculation (AUTO)	Automatic Calculation (AUTO)	
Mental subtraction $64 - 26$		Jump Method	N 10	$64 - 20 = 44$ $44 - 6 = 38$
	C 10 / Formation of units of tens	Compensation	N10 C (Rounding)	$64 - 30 = 34$ $34 + 4 = 38$
	Decomposition strategy	Split method	1010	$60 - 20 = 40$ $4 - 6 = ?$ Often invalid
	Mental algorithm (MA)			$\begin{array}{r} 64 \\ - 26 \\ \hline 38 \end{array}$
	Automatic Calculation (AUTO)	Automatic Calculation (AUTO)	Automatic Calculation (AUTO)	
Mental multiplication 21×3	Different operation (DO)			$21 + 21 + 21$
	Mental algorithm (MA)			$\begin{array}{r} 21 \times 3 \\ \hline 3 \\ 20 \\ \hline 23 \end{array}$
	Automatic Calculation (AUTO)	Automatic Calculation (AUTO)	Automatic Calculation (AUTO)	
Mental division $42 : 6$	Different operation (DO)			$6 \times 6 \times 6 \times 6 \dots$
	Mental algorithm (MA)			$\begin{array}{r} 42 : 6 \\ \hline 7 \end{array}$
	Automatic Calculation (AUTO)	Automatic Calculation (AUTO)	Automatic Calculation (AUTO)	

To sum up the reviewed literature on children's arithmetic strategies, children use non-retrieval strategies more frequently for addition and subtraction than for multiplication and division. While children and even adults use multiple strategies for solving arithmetic problems, it seems that proficient problem solvers frequently rely on direct retrieval, despite the fact that non-retrieval procedures can be executed equally efficiently. For instance, division problems can be solved efficiently by transforming them into multiplication problems. This strategy seems to be just as effective as automatic, direct retrieval. The overall body of research lends support to Siegler's notion that direct retrieval from long-term memory is the result of a developmental process, by which the frequent use of algorithmic, non-retrieval strategies strengthens the associations between problems and answers in the interconnected associative network in long-term memory. In fact, "skilled problem solving is characterised by high levels of speed and accuracy and relatively frequent use of retrieval-based strategies" (Canobi, 2005, p. 222).

As has been mentioned, the reason why procedural, non-retrieval strategies are less effective than direct, automatic retrieval is that they involve the temporary maintenance and manipulation of interim results and thus make heavier demands on working memory capacities. Counting-based strategies are least effective. The formation of a mental algorithm, for example, requires extensive working memory resources and is therefore less efficient compared to other strategies such as the jump strategy and the rounding strategy. To recap, children use different strategies for a given arithmetic operation and therefore strategic competence goes beyond the application of one standard strategy, but requires the learner to choose adaptively between different available strategies to yield the most effective combination of cognitive efficiency and accuracy (Lemaire & Siegler, 1995).

6.2.5 STRATEGY USAGE IN CHILDREN WITH MD

A key limitation of the research that has been reviewed so far is that it has been conducted with average-achieving children, who are free from mathematical difficulties. While research on normally developing children provides valuable insights into the development of arithmetic strategic competence, it is important to detect the difference in strategy usage between children with MD and average ability children. One of the key problems for children with mathematical difficulties (MD) is the effective application of procedural knowledge, namely the correct, flexible and effective usage of arithmetic strategies (Lucangeli, 2003). According to Cummings and Elkins (1999), children with MD “have conceptual knowledge equivalent to that of normal students but do not develop strategies other than counting for the basic facts and perform poorly on these tasks.” Kroesbergen (2004) argues that the acquisition, selection and application of strategies (strategy usage) constitute a major problem for low achievers. Similarly, Pellegrino and Goldman (1987) showed that low achievers did not possess the same repertoire of strategies as their normal achieving peers and therefore relied on inefficient counting strategies. When strategy recall is hampered, it follows that students may actually perceive routine problems as complex, non-routine problems (Tremblay & Lemoyne, 1986, p. 2): “students experience difficulties are unable to recognize isomorphic problems, so that each problem is considered as a new problem...and therefore requiring an original solution...the recall of known strategies is probably based on this judgment of isomorphism and may therefore be greatly influenced by the subject’s previous experience.”

A second characteristic of low-achieving children, which is related to the difficulties in strategy usage, is that children have deficits in storing and accessing

numeric facts in and from long-term memory: “children with low achievement on arithmetical computation may not only have poor strategic development but also poor memory development” (Cummings & Elkins, 1999). Because of this memory deficit, arithmetic facts are not sufficiently automatised, so that low-achieving children rely on counting-based strategies to solve computations. Laborious and ineffective strategies such as counting rely extensively on limited working memory resources, which results in cognitive overload and constitutes an impediment to fluent arithmetic problem solving. Indeed, children with MD make more errors and have slower reaction times when retrieving arithmetic facts from memory compared to children with MD (Geary, Hoard & Hamson, 1999).

To recap, the studies of Imbo and Vandierendonck (2007) and Barrouillet and Lepine (2004) lend support to the notion that individual differences in WM capacity influence strategy use. More specifically, lower WM capacities are associated with less frequent use of retrieval strategies. However, the overall body of evidence that has been reviewed suggests that direct retrieval is more effective compared to algorithmic, non-retrieval strategies. Given that working memory deficits are a frequently cited characteristic of mathematical difficulties, it might be that children with MD use less effective strategies than average ability children. This issue has been addressed in a study by Foxman and Beiszhuisen (2002).

Foxman and Beiszhuisen (2002) investigated the arithmetic strategies used by a sample of 247 eleven-year-old children. Based on a standardized written test of mathematics ability, which measured children’s ability in different categories such as numeracy, geometry, algebra, probability and statistics, the children were classified into one of three performance bands in order to compare the strategy usage of children of

different abilities. Children were provided with a booklet containing problems from each arithmetic operation and were asked to read out each problem and explain how they would work out the answer. In essence, the researchers found that the low-ability children generally used decomposition methods: they split both numbers into tens and units and calculated the components separately (the 1010 strategy). This strategy was the least chosen by children from the higher ability bands. Children from the higher ability bands, on the other hand, were more likely to use complete number strategies, such as the Jump (N 10) or Rounding strategy (N 10C), where one number in the calculation is left complete, rather than to decompose both numbers.

For a two-digit subtraction task, thirty-one children from the high ability band and sixteen children from the middle attainment band chose the N10 strategy, which means that one number is left complete and the other number is subtracted in a sequence of units and tens ($64 - 27 = 64 - 20 = 44; 44 - 7 = 37$). In contrast, only 8% of the children from the low ability band chose this strategy and 27% of these children chose the 1010 method, whereby both numbers are split into units and subtracted separately. However, this method is difficult to apply and prone to conceptual errors, especially if the unit of the first number is smaller than the unit of the number to be subtracted: $64 - 27 = 60 - 20 = 40; 4 - 7 = ?$. Indeed, of all 27% of the children who used this method, only 13% derived the correct result, and it seems that these children used manipulatives such as fingers or overt counting to derive the solution. A similar finding emerged for multiplication, as illustrated below.

For the multiplication problem “ 4×1.99 ”, 57% of the children from the high ability band used the rounding strategy to derive the solution ($4 \times 2 = 8; 8 - 0.4 = 7.96$), and 87% of these children calculated the result correctly. Only 22% of the children from

the low ability band used this strategy, 74% of these children solved the problem successfully. The majority of children from the low ability band, 35%, used a decomposition number strategy ($4 \times 1 + 4 \times 0.90 + 4 \times 0.09$) but only 7% of these children derived the correct result. This ineffective strategy was used infrequently by children from the middle ability band (23%) and children from the high ability band (14%). However, 45% of children from the high ability band were able to get the correct result using this strategy, compared to only 22% of children from the middle attainment band. Thus, the decomposition strategy appears to be an ineffective and error prone strategy for multi-digit multiplication.

A similar trend was found for complex multiplication problems. 20% of the children from the high ability band, 11% of the children from the middle ability band but only 7% of the lowest attainment band used a complete number strategy, meaning that only one number was split into units and tens ($16 \times 25 = 6 \times 25 = 150; 10 \times 25 = 250; 250 + 150 = 400$). However, 100% of the children who used this strategy solved the problem correctly. The majority of children from the low ability band used the separate digits strategy, whereby both numbers are split in units and tens, multiplied separately and then added together ($16 \times 25 = 5 \times 6 = 30; 20 \times 10 = 200; 30 + 200 = 230$). However, this strategy is incorrect and none of the children who used this strategy derived the correct result. It can be argued that Decomposition methods are less efficient and rely more heavily on cognitive resources than Jump methods, because the Decomposition method involves a long calculation pathway, whereby interim results have to be maintained in the working memory (Adams and Hitch, 1998). Thus, the Foxman and Beiszhuisen study suggests that children of lower mathematical ability use less effective strategies than children from the high ability band.

To recap, difficulties in strategy usage have been cited as a key factor in mathematical difficulties (Foxman & Beiszhuisen, 2003). There is evidence to suggest that strategy usage is a function of ability-related differences. The influence of mathematical ability on the efficient usage of strategies in the domains of addition and subtraction has also been addressed by Torbeyns, Verschaffel and Ghesquiere (2004). The sample consisted of twenty-six second-graders with strong mathematical abilities and twenty-five second-graders with mathematical difficulties. Mathematical ability was determined by the results of standardized ability tests. Children were asked to solve twenty addition problems and twenty subtraction problems.

Problems were devised in a choice and a no-choice condition. In the choice condition, children were asked to choose among three different strategies, namely direct retrieval, decomposition and counting on, by means of pictures which provided a visual representation of the strategies. After choosing a strategy, children were asked to write their answers in the corresponding picture. Children who chose the retrieval strategy were asked to just write down the result, children who chose the decomposition strategy were asked to write down the entire solution strategy in sequence and children who chose the counting strategy were told to actually count sub-vocally or with fingers. In contrast to the conventional protocol analysis method, which relied on children's verbalised solution strategies, no verbal explanation was required from the children. In the no-choice condition, children were asked to use one particular strategy to solve the problems.

In the choice condition, considerable differences were observed in strategy usage between children with strong mathematical abilities and children with MD. A total of 68.68% of children with strong mathematical abilities solved addition and subtraction

problems via direct retrieval. Only 31.77% of children with MD used this strategy. The frequency of use of the decomposition strategy was similar for children with strong mathematical abilities and children with MD: 20.78% of the strong group and 25.67% of the MD group solved addition and subtraction problems via the decomposition strategy. However, the least efficient strategy, counting on, was chosen by the majority of children with MD, 42.56%, but only 10.54% of children with strong mathematical abilities used this strategy.

Also, significant differences were observed in the speed and accuracy of strategy execution. The proportion of correctly solved addition and subtraction problems was 0.24% in the strong group but only 0.12% in the group of children with MD, when the retrieval strategy was used. Children with strong mathematical abilities applied the decomposition strategy more accurately and faster than children with MD, with an accuracy proportion of 0.99 and a mean solution time of 07.56 s compared to an accuracy proportion of 0.87 and a mean solution time of 14.63, respectively. Overall, the results indicate that children with strong mathematical abilities use the retrieval strategy more often, more accurately and more quickly, and the inefficient counting strategy less frequently when solving addition and subtraction problems, than children with MD, who relied mainly on the immature counting strategy. This supports the assumption of Canobi (2005, p. 222), that skilled arithmetic problem solving is characterised by the frequent use of direct retrieval.

While the Torbeyns et al. (2004) study increased the knowledge about strategy development, it is important to point out that children with MD were not compared with a group of average achieving children but with a group of children who were particularly strong in mathematics, a fact which might have obscured the results. The

study might have benefited from including an additional group of children of average mathematical ability, to determine how children with MD deviate in arithmetic strategic competence from an average population.

The studies that have been reviewed so far lend support to the argument put forward by Barrouillet and Lepine (2005), who argue that children with MD “encounter a double difficulty when solving arithmetic problems. They retrieve the answers less frequently than do other children and, as a consequence, must rely more often on backup strategies – and they are also slower to perform those...the frequent recourse to slow arithmetic strategies increases the cognitive load involved in arithmetic problem solving in those children, who, in other respects, have the lowest cognitive resources to cope with this extra cognitive load.”

The present research question asks whether pupils who were exposed to the direct instruction intervention eFit are more competent in using strategies for different arithmetic tasks than children from a control group, who were exposed to constructivist-inspired instruction, which emphasizes the discovery rather than the direct instruction of strategies. In contrast to constructivist-inspired instruction, the direct instruction approach places great emphasis on explicit strategy instruction through the employment of worked examples and the usage of advanced organisers to structure material into meaningful units (Stein, Silbert and Carnine, 1997).

To recap, direct instruction means, “providing information that fully explains the concepts and procedures that students are required to learn as well as learning strategy support that is compatible with human cognitive architecture” (Kirschner et al.,2006, p. 76). Engelmann and Carnine (1982, p. 24) define DI as “teaching rules, concepts,

principles and problem – solving strategies in an explicit fashion.” The MGI approach, on the other hand, assumes that strategies should not be formally taught in an explicit fashion. Rather, children should construct their own meaningful understanding through self-directed discovery learning, cooperative learning or problem-based learning (Carpenter, 1999).

Therefore, it is worth investigating whether children in the direct instruction group actually demonstrate enhanced strategic competence compared to children who were exposed to MGI. According to Lemaire and Siegler (1995), strategic competence can be defined in terms of an improvement in four dimensions, namely 1) the repertoire of strategies an individual can draw on, 2) how frequently the different strategies are used, 3) how efficiently the strategies can be executed in terms of accuracy and speed, and 4) the efficient and adaptive choice of strategy, i.e., the strategy that leads to a fast and accurate answer. The aim of the current study is twofold: firstly, the study aims to examine the two aspects in order to determine whether children who were exposed to the DI treatment employ more efficient strategies than children from the MGI group. Secondly, it will be investigated whether strategy use is associated with preferences in the representation and processing of information.

With regards to the first aim of the study, which is concerned with the exploration of differences in strategy use between DI and MGI children, it is difficult to define a strategy as efficient and there is no consensus in the literature regarding the features of “efficient strategies.” For the purpose of the present study, efficiency will be defined with reference to the working memory theory. Hence, a strategy will be judged as efficient (1) if it is generalisable to other problems of the same difficulty, i.e. the strategy is likely to lead to the correct result when applied to similar problems of the

same operation and (2) if demands on working memory are kept minimal. Hence, a strategy which involves several steps, the maintenance of interim results and/ or counting processes is less efficient than strategies such as direct retrieval, the jump method and the compensation strategy, which involve fewer steps

6.2.6 COGNITIVE STYLE AND PROBLEM-SOLVING STRATEGIES

As has been discussed in chapter 4, working memory plays a fundamental role in arithmetic problem-solving. To recap, the model of Baddeley and Hitch assumes that WM consists of a visual and a verbal sub-system which are monitored by the central executive. Hence, information can be represented and processed in a verbal-phonological as well as in a visuo-spatial format. While competing theories exist about the specific function of the visuo-spatial sketchpad and the phonological loop in the mental arithmetic problem-solving process, there is a general consensus that verbal and visual WM components are used in a systematic way, depending on the specific characteristic of the problem. Problem difficulty, presentation duration, presentation format and arithmetic ability have been found to influence the recruitment of visual or verbal resources. The question whether the use of WM resources is related to distinct preferences in representing and processing information resides at the periphery of research and there is no known empirical evidence that visual and verbal WM capacity are related to preferences in representing numeric information. As will be discussed in the following paragraphs, individual preferences in representing and processing information have been summarised as cognitive style. The current study is intended to examine whether cognitive style is associated with the use of specific problem-solving strategies.

The concept of cognitive style was originally coined by Allport (1937), who defines cognitive style as a constant way of perceiving, remembering, thinking and problem solving. A similar definition is adopted by Goldstein and Blackman (1978), who assume that cognitive style mediates the mental processes that occur between stimuli and responses. The present work utilises a more precise definition of cognitive style, as proposed by Riding (1991). According to their definition, cognitive style refers to an individual's preferred and habitual approach to organizing and representing information.

Riding and Cheema (1991) came up with more than 30 cognitive/ learning styles. Based on several reviews, they concluded that these could be mapped onto two basic cognitive style dimensions. The two-dimensional model provides a helpful device to summarise the large number of cognitive styles that have been identified so far. According to the two-dimensional model, individual differences in cognitive style can be mapped on two dimensions, the wholist – analytic and the verbaliser – imager dimension. The wholist – analytic dimension (WA dimension) is concerned with individual differences in the cognitive processes of structuring and organising information. The verbaliser – imager dimension (VI dimension) addresses individual differences in the mental representation of information. A relatively straightforward measure of those dimensions is provided by Riding's cognitive styles analysis. This measure assesses both dimensions by means of basic tasks, which are assumed to be indicative of learners' underlying cognitive processing and of the ways in which the learner usually structures and represents information during thinking.

Corresponding to the fundamental assumption of Gestalt psychology, which predicts that “a whole is more than the sum of its parts”, the cognitive processing strategies of wholists are holistic, in that information are treated as an integral entity. The environment is experienced in a more global fashion, since their perception passively conforms to the impact of the dominant field or context. This means that they are likely to learn material in the way in which it is presented, rather than making attempts to restructure it. However, a drawback of this cognitive style is that wholists tend to have difficulties in delineating the boundaries between the different entities, which might lead to a “blurred” view of the information. As a consequence, wholists tend to experience problems when asked to break down information and to pay sufficient attention to the separate elements.

Analytics, on the other hand, tend to perceive information as an organized array of individual elements and therefore focus on the single elements of which the information is comprised rather than the overall information as such. They experience their environment in an analytical manner, in that objects are experienced separately from their backgrounds. This allows field independent learners to impose their own organization on unstructured material and to extract relevant information from a surrounding context. One advantage of this cognitive style is that analytics can figure out the essence of a problem relatively quickly, because their attention is drawn to potentially relevant aspects. However, this often occurs at the expense of ignoring other information, such as the relationship between single elements that contribute to the characteristic of the overall problem. Because some parts attract more attention than others, it might be difficult for analytics to maintain a balanced view of the entire situation.

The verbal – imagery dimension includes three types: verbalisers, imagers and bimodals. In essence, verbalisers think about the information they receive in terms of words or verbal associations, whereas imagers consider it in terms of mental pictures. Bimodal individuals are able to switch between the two modes of representation. The assumption that some individuals have a mainly verbal way of representing information while the others have a visual way of representing information was articulated from the 1880s onwards (Galton, 1883; James, 1890). This notion formed the basis for research on style representations. The verbal–imagery dimension was coined in Paivio’s dual coding theory. Thus, before reviewing this dimension, a brief description of Paivio’s work is needed.

For Paivio (1986), the distinct feature of human cognition is the ability to cope simultaneously with language and with non-verbal objects and events. Moreover, Paivio claims that, “the language system is peculiar in that it deals directly with linguistic input and output (in the form of speech or writing) while at the same time serving a symbolic function with respect to nonverbal objects, events, and behaviours. Any representational theory must accommodate this dual functionality” (p 53). In essence, this theory predicts that there are two cognitive subsystems, for verbal and non-verbal information, which are distinctive but interconnected.

Following Paivio, visualisers are individuals whose thoughts refer to mental images, whilst verbalisers are more likely to think in words. Whilst Riding (1991) acknowledges that some individuals have a preference for both representational modes, he assumes that others have a preference for either the visual or the verbal mode. Assessment of learners’ preferred mode was initially based on the imagery code test. However, this assessment merely equated verbal coding preferences to a

slower performance in imagery tasks (Riding, 1991). In order to account for this limitation, Riding (1991) created a computer-based assessment of the verbaliser imagery style, which involved the presentation of word pairs. Riding (1991) confers the intuition that that imagers favour pictorial material, in contrast to verbalisers, who learn best through text material.

6.2.7 COGNITIVE STYLE AND INFORMATION PROCESSING STRATEGIES

As indicated above, individual differences in cognitive style constitute a variable for individuals' cognitive processing and mental representations of information. Thus, it can be hypothesized that cognitive style affects individuals' mental representation of arithmetic problems and is manifested in their preferred problem-solving strategies. While there is no known study on the role of cognitive style on arithmetic processing, Graff (2003) provides convincing evidence that individual differences in cognitive style are manifested in different strategies of processing and extracting information from a web-based instructional system.

The sample in the study consisted of fifty undergraduate students (mean age 23.2 years). Participants took part in the Cognitive Style Analysis and were allocated to one of two conditions. In the long page condition, the information was presented on eleven pages. In the short page condition, the same information was more segmented and was distributed over twenty-three web pages. Thus, the long page condition contained more words per page in contrast to the short-page condition, in which the material was more clearly arranged due to its segmentation. Based on the information provided in the text, participants were asked to produce an essay, which was marked with respect to the level of detail provided. In essence, the results showed that

wholists outperformed analytics in the segmented, short page condition. Graff (2003) hypothesized that the pre-structured representational format in the short page condition interfered with the tendency of analytics to impose their own structure for organising information. Overall, the study confirmed that individual differences in cognitive styles constitute a significant factor in information processing strategies. While the Graff (2003) study has been carried out in a non-mathematical domain, it seems very informative with respect to the current study.

To recap, no previous study to date has examined the role of cognitive style in arithmetic strategy usage and the rationale behind this study is to address this shortcoming. Based on the evidence that has been reviewed so far, it can be hypothesized that individual differences in cognitive style go in line with individual differences in arithmetic strategy usage. More specifically, it can be assumed that imagers use the mental algorithm more frequently than verbalisers.

6.3 METHOD

DESIGN

A chi-square design was employed to examine the association between cognitive style (verbaliser, imager, bimodal) and arithmetic problem-solving strategy use.

6.3.1 PARTICIPANTS

The sample consisted of children with MD who were recruited from two secondary schools in similar socioeconomic catchment areas. The schools gave their consent to the children's participation. The children were from grade 6, aged 11 – 14 and had no physical or psychological conditions. A total of 22 children took part in the

web-based, direct instruction intervention eFit (mean age 12 years, 13 males, 9 females) and 20 children came from a comparison school, where the eFit intervention was not implemented (mean age 12.2 years; 13 males, 7 females). Only the children from the eFit intervention group took part in the cognitive style analysis.

6.3.2 MATERIAL

Eight arithmetic problems were presented to the children. Each problem was presented separately on a sheet of paper. The problems were developed in collaboration with a mathematics teacher. Based on a small pilot study with four children from class 6, all but one of the problems seemed appropriate. The problem which caused difficulties in the pilot study was the problem “20 x 16”, which was initially chosen as the difficult multiplication problem. However, all of the children in the pilot study experienced difficulties with it, and therefore this problem was replaced in order to avoid distress.

For each arithmetic operation, there was a simple and a difficult problem. For addition and subtraction, simple problems were not defined in terms of the number of digits they entailed. Rather, problems were defined as simple if they did not require carrying (carry problem: $15 + 19$; non-carry problem: $15 + 13$) irrespective of the number of digits. The simple addition problem was $138 + 51$; the difficult addition problem was $82 + 79$. The simple subtraction problem was $46 - 24$; the difficult subtraction problem was $91 - 37$. Simple multiplication problems were defined as problems with one-digit numbers; simple division problems were defined as problems whose solution was a one-digit number.

While previous research has defined simple problems as problems with no operand larger than 6, it was decided to adopt a different definition for the purpose of the present study: for the simple multiplication problem, the problem “7 x 8” was chosen in order to provide an incentive for children to consider using a non-retrieval strategy. The problem 6 x 14 was chosen as a difficult multiplication problem, because it is assumed that this problem is not too taxing but can be solved using a range of different strategies. For simple division, the problem “42 : 7” was chosen because it is unlikely that children will use non-retrieval strategies for smaller problems. For difficult division, the problem “121 : 11” was selected. Again, it was assumed that children would be able to solve this problem by means of different strategies, such as fact transfer, repeated addition or direct retrieval.

Cognitive Style Assessment

The CSA is a computer-based assessment. On the basis of three sub-tests, it determines a participant's position on the wholist-analyst (WA) and the verbaliser-imager (VI dimension). The first part assesses the VI dimension. It consists of written statements and the subject is asked to judge whether or not an object falls within a certain category or, alternatively, has the same colour as another object. The second sub-test asks the participant to differentiate between two similar geometrical shapes. The rationale behind this is to determine the individual's position on the wholist style. The third sub-test is related to the analytic style and requires participants to decide whether or not one shape is contained in another one. The participant's task is merely to respond to the stimuli by pressing one of two keys. Speed and accuracy of responses are then fed into a calculation of individual style.

The CSA was conducted with groups of approximately ten subjects. In line with the recommendation of Riding, instruction was kept minimal. Participants were told that they had to carefully read the computer-presented instructions and that they should use the red and the blue keys to respond to the stimuli. Apart from that, the CSA ran without interference. The responses made by participants were recorded automatically.

6.3.3 PROCEDURE

Participants were tested individually. After their arrival, children were told that they had to solve some mental arithmetic tasks and that their answers would be tape-recorded. It was emphasized that their answers would not be given any marks and that the data would be handled confidentially. After the briefing, children received a sheet of paper with basic arithmetic problems and were asked to explain how they would solve each of the problems. The pilot study showed that most of the children were reluctant to articulate their thoughts. In order to account for this, they received the following verbal instruction: “imagine your schoolmate is ill and has to stay at home. He has missed a lot of maths lessons and the teacher has asked you to help him to catch up. Now you phone him to explain these tasks.” The overall majority of children responded very well to this instruction and did not need any further prompting.

Each session took 15 – 20 minutes. The interviews were tape recorded and transcribed. The categorisation of children’s strategies was checked using a double blind test. An independent judge (teacher in training) was asked to read through the transcripts and indicate which of the categories would apply.

Data Analysis

Children's strategy usage was determined by an analysis of the transcribed protocols. A total of 336 protocols were analysed and coded independently by the experimenter and a mathematics teacher in order to decide in which category a problem – solving strategy belongs. The strategies that have been identified in previous studies served as a coding device to categorise children's problem-solving strategies in the present study. Table 6.2. exemplifies the problem-solving strategies that have been found in previous research. These It is evident that the same strategies have consistently been found across different studies. However, table 6.2 shows different researchers have used different labels for the same strategy, so it was ensured that the labelling of strategies in this studies was consisten to prevent definitional ambiguity and overlap. As shown in the table, the different problem – solving strategies can be clearly distinguished from each other. The strategies that children actually used in the present study are represented in the tables below.

Inter-rater reliability approached 95 %, with 15 cases of disagreement. If there was a case of disagreement, for example whether the strategy $16 \times 4 = 10 \times 4 + 6 \times 4$ constitutes fact transfer or decomposition, consensus was reached through a close re-examination of the problem-solving strategy in table 6.2. In analysing the data, the focus was on strategy usage rather than accuracy. Therefore, accuracy of the answers was not subject to investigation.

Table 6.3: Addition strategies identified in the protocol analysis

Addition strategies	Example
Jump method N 10	$138 + 50 = 158$ $158 + 1 = 159$
Decomposition/ Split method / 1010	$130 + 50 = 180$ $8 + 1 = 9$ $180 + 9 = 189$
Mental algorithm	$\begin{array}{r} 138 \\ + 51 \\ \hline 189 \end{array}$
C 10/ Formation of units of tens	$138 + 60 = 198$ $198 - 9 = 159$
Direct retrieval	“Just knew it” / immediate answer
No answer	

Table 6.4: Subtraction strategies identified in the protocol analysis

Subtraction strategies	Example
Jump method N 10	$46 - 20 = 26$ $26 - 4 = 22$
Decomposition/ Split method / 1010	$40 - 20 = 20$ $6 - 4 = 2$ Often invalid if second unit is larger than 1 st unit
Mental algorithm	$\begin{array}{r} 46 \\ -24 \\ \hline 22 \end{array}$
C 10/ Formation of units of tens	$91 - 40 = 51$
Direct retrieval	“Just knew it” / immediate answer
No answer	

CHAPTER 6 ARITHMETIC PROBLEM-SOLVING STRATEGIES IN CHILDREN WITH MATHEMATICAL DIFFICULTIES

Table 6.5: Multiplication strategies identified in the protocol analysis

Multiplication strategies	Example
Fact transfer including different operation	$7 \times 7 = 49$ $49 + 7 = 56$
Decomposition/ Split method / 1010	6×10 6×4
Direct retrieval	“Just knew it” / immediate answer
No answer	

Table 6.6: division strategies identified in the protocol analysis

Division strategies	Example
Fact transfer including different operation	$121 : 11 = 11 \times ? = 121$
Direct retrieval	“Just knew it” / immediate answer
No answer	

6.3.4 RESULTS

Chi-square tests were carried out on the results of the protocol analysis, in order to address the questions of whether children from the direct instruction (DI) and minimally guided instruction (MGI) groups differed in strategy use across different types of tasks. Categories of strategies were devised on the basis of an analysis of children's verbal protocols, and the numbers of children from each group who fell into certain categories were statistically examined. The number of categories used varied across operations, reflecting the fact that different arithmetic operations are associated with certain cognitive strategies. No significant differences could be detected between children from the eFit and the control group when easy problems were administered. Therefore, it was decided to focus on difficult problems where differences could be observed. This seems justified, given that the use of procedural, non-retrieval strategies is more prevalent in relation to difficult problems

Difficult Subtraction

Table 6.7: Strategies used for difficult subtraction

Treatment	Retrieval	Jump	Mental algorithm	Decomposition	No answer
DI	3	13	5	0	1
MGI	9	8	0	2	1

As can be seen from table 6.7, a total of thirteen children who were exposed to direct instruction used the jump strategy for difficult subtraction, compared to eight children from the minimally guided instruction group. There was also a difference in usage of the mental algorithm strategy, in that five children from the DI but no child from the MGI group used this strategy. A total of nine children from the MGI group retrieved the result to the difficult subtraction task from memory, compared to only

three children from the direct instruction group. The results of a 2 x 2 chi-square showed that this pattern of association between the type of instruction that children received and the usage of the jump strategy and the mental algorithm strategy for solving difficult subtraction is a statistically significant interaction ($\chi^2 (1) = 10.73, p < .01$). Fisher's exact test was used because the sample size is small and some of the cells contain a zero. The finding seems to represent the fact that based on the odds ratio, children who received DI were 2.17 times more likely to use the jump method for difficult subtraction than children who received MGI.

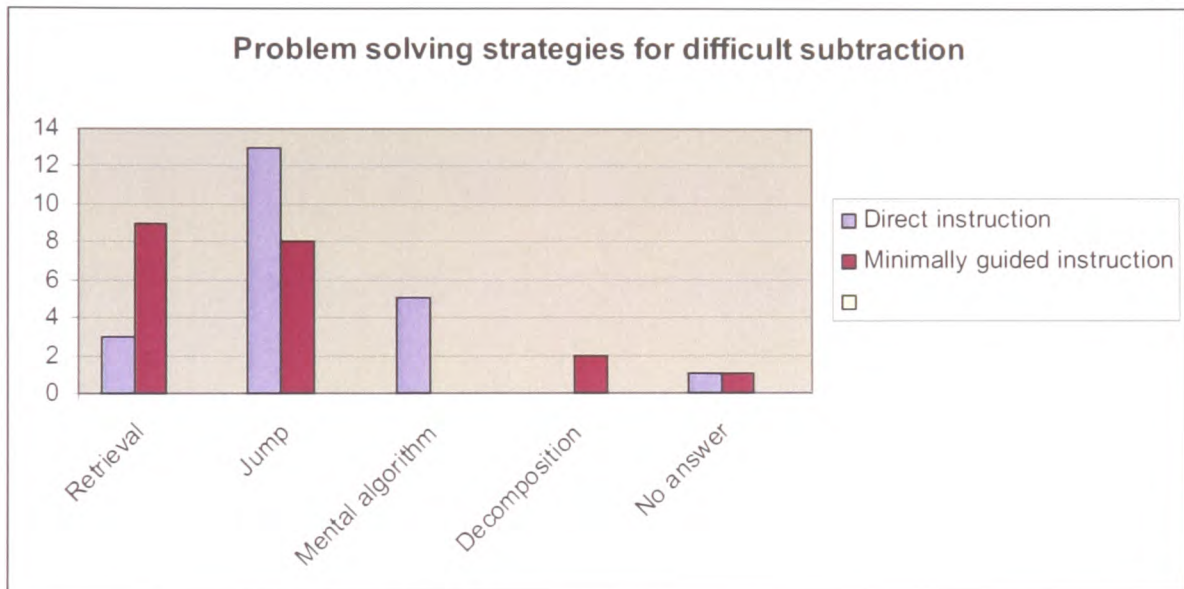


Figure 6.1: Strategies used for difficult subtraction

Difficult Multiplication

Table 6.8: Problem-solving strategies for difficult multiplication

Treatment	Direct retrieval	Decomposition	Fact transfer	No answer
DI	13	9	0	0
MGI	8	6	5	1

Table 6.8 indicates that for difficult multiplication, children from the DI group used the decomposition strategy more frequently than children from the MGI group. A total of nine children from the direct instruction group used the decomposition strategy compared to six children in the minimally guided instruction group. An even more pronounced difference can be detected when the frequency of use of direct retrieval is concerned, a strategy which was employed by thirteen children from the direct instruction group but only eight children from the minimally guided instruction group.

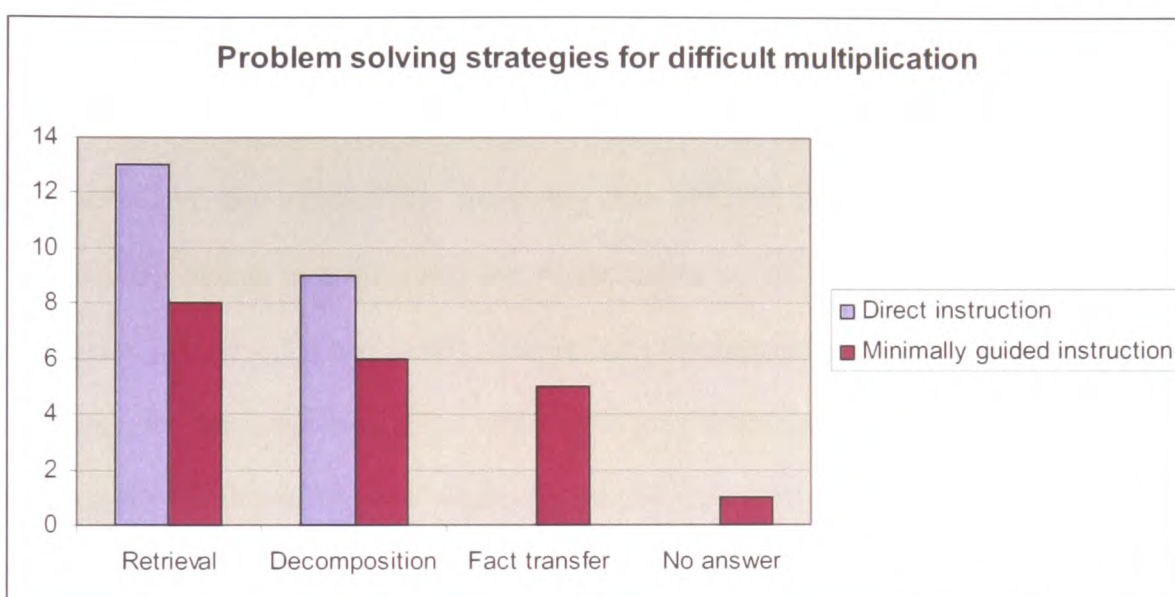


Figure 6.2: Problem-solving strategies for difficult multiplication

No child from the DI group had to derive the result of the difficult multiplication problem via the derived fact transfer strategy, whereas five children from the MGI group adopted this method. The results of the chi-square test showed that this pattern of association between the treatment children received and the strategies they used for difficult multiplication is significant ($\chi^2 (3) = 7.46, p < .05$). This represents the fact that based on the odds ratio, children who were exposed to direct instruction were 1.73 more likely to adopt the decomposition method and 2.16 times more likely to employ the automatic retrieval method than children exposed to minimally guided instruction. Also, children in the MGI group were significantly more likely than children from the DI group to solve difficult multiplication tasks via fact retrieval. It was not possible to

compute the odds ratio for the fact transfer category, because the value in one of the cells was zero.

Difficult Division

Table 6.9: Problem-solving strategies for difficult division

Treatment	Direct retrieval	Fact transfer	no answer
DI	18	3	1
MGI	8	10	2

Table 6.9 documents that children from the DI group used direct retrieval more frequently and fact transfer less frequently than children from the MGI group. The association between treatment and the problem-solving strategies used is statistically significant ($\chi^2 (2) = 7.81, p < .05$). This reflects the fact that based on the odds ratio, children from the direct instruction group were 6.75 times more likely to use automatic retrieval than children from the minimally guided instruction group. Children from the MGI group, on the other hand, were 0.16 times more likely to solve difficult division problems via fact transfer and 0.43 times more likely to come up with no answer. In essence, the results show that for difficult division, children from the DI group were more likely to be able to recall answers automatically, whereas children from the MGI group tended to derive their results via fact transfer.

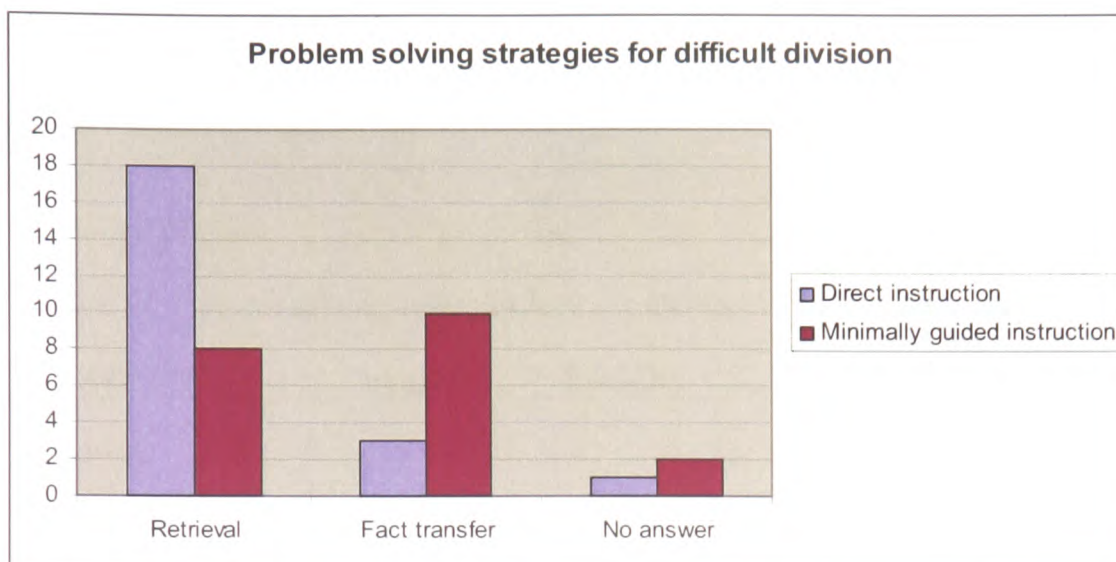


Figure 6.3: Strategies for difficult division

6.3.5 RESULTS – COGNITIVE STYLE AND STRATEGY USE

A chi-square test was carried out on the results of the protocol analysis, in order to address the question of whether cognitive style is associated with strategy use across different types of tasks. Overall, six different types of arithmetic strategies were derived on the basis of children’s interviews: the jump strategy, the split or decomposition strategy, the formation of units of tens strategy, the mental algorithm strategy and the answer “I don’t know”, meaning that a child was not able to answer the question.

A total of twenty-two children took part in the protocol analysis. Based on the results of the cognitive style analysis, seven of them were identified as imagers, nine as bimodals and six as wholists. Categories of strategies were devised on the basis of an analysis of children’s verbal protocols and the numbers of children from each group who fell into certain categories were statistically examined. The number of categories used varied across operations, in that some strategies were been used for particular

operations, reflecting the fact that different arithmetic operations are associated with certain cognitive strategies.

Easy Addition

Table 6.10: Problem-solving strategies for easy addition

STRATEGY	Imager	Bimodal	Verbaliser
DIRECT RETRIEVAL	1	1	4
Jump	4	6	2
Decomposition	2	2	0

Table 6.10 shows that three different strategies were used for easy addition tasks, namely the jump strategy, the decomposition strategy and the direct retrieval strategy. The group of children identified as imagers chose each of the strategies equally often. All of the children with a bimodal cognitive style chose the jump strategy, whereas the majority of verbalisers recalled the result automatically. The results of a 2 x 4 chi square showed that the observed association pattern between children's cognitive style and the usage certain strategies for solving difficult multiplication tasks approached statistical significance ($\chi^2(6) = 11.82, p = .052$).

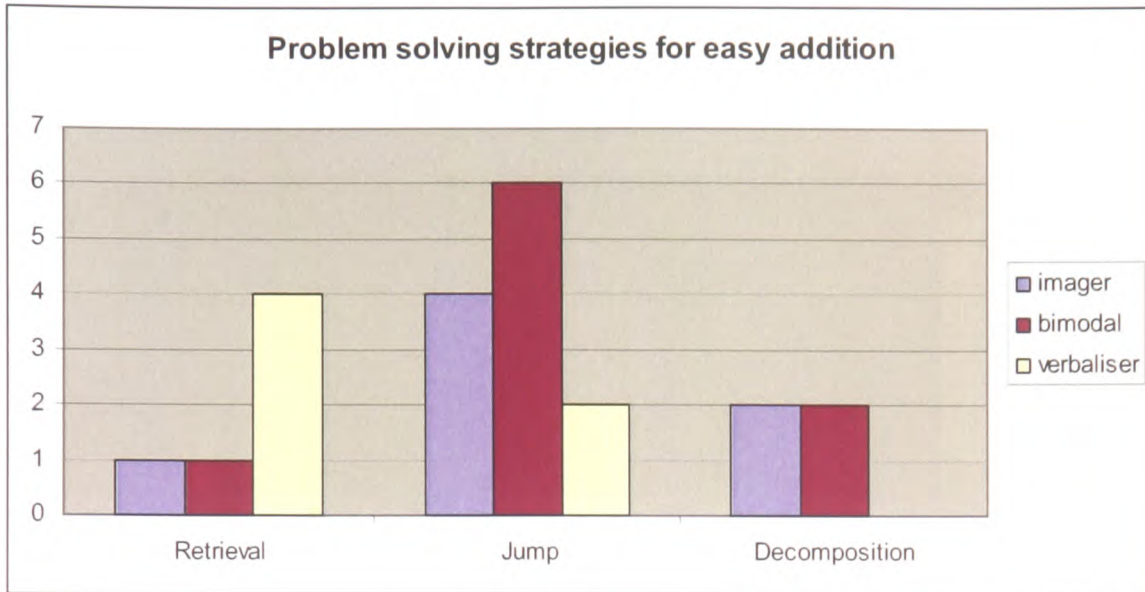


Figure 6.4: Problem-solving strategies for easy addition

Difficult Addition

Table 6.11: Problem-solving strategies for difficult addition

STRATEGY	Imager	Bimodal	Verbaliser
Direct retrieval	0	0	1
Jump	1	2	0
Decomposition strategy	0	1	1
Mental algorithm strategy	4	2	0
C 10	2	4	4

Table 6.11 shows that for difficult addition, imagers predominantly used the mental algorithm, whereas verbalisers used the formation of units of tens strategy. For bimodals, the usage of strategies was more dispersed, in that there was no marked difference between the usage of the different strategies. The results of a 2 x 2 chi-square showed that the observed association pattern for solving difficult addition tasks was statistically significant ($\chi^2 (6) = 26.75, p < .01$).

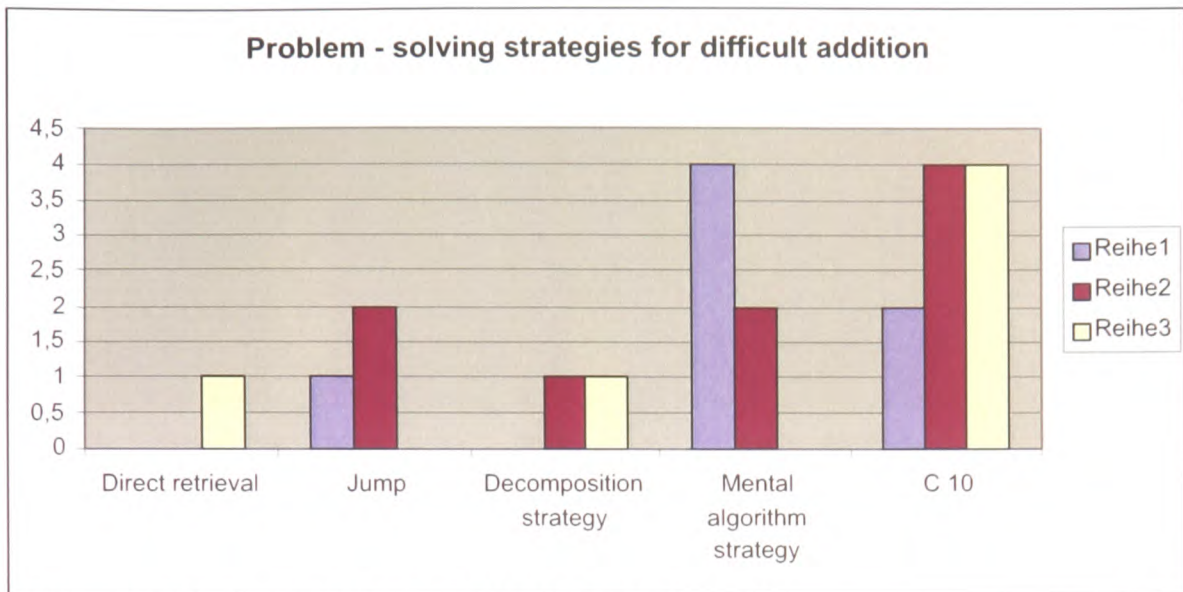


Figure 6.5: Problem-solving strategies for difficult addition

Easy Subtraction

Table 6.12: Problem solving strategies for easy subtraction

STRATEGY	Imager	Bimodal	Verbaliser
Direct retrieval	2	2	4
Jump	3	6	2
Mental algorithm	2	1	0

As table 6.12 shows, the pattern of strategy usage for easy subtraction is similar to the pattern of strategy usage for easy addition, in that imagers tended to choose different strategies whereas bimodals predominantly selected the jump strategy. As was the case for easy addition, verbalisers were more likely than imagers and bimodals to solve the easy tasks via automatic retrieval. The results of a 3 x 3 chi-square showed that the observed association pattern was statistically significant ($\chi^2(4) = 10.13, p < .05$).

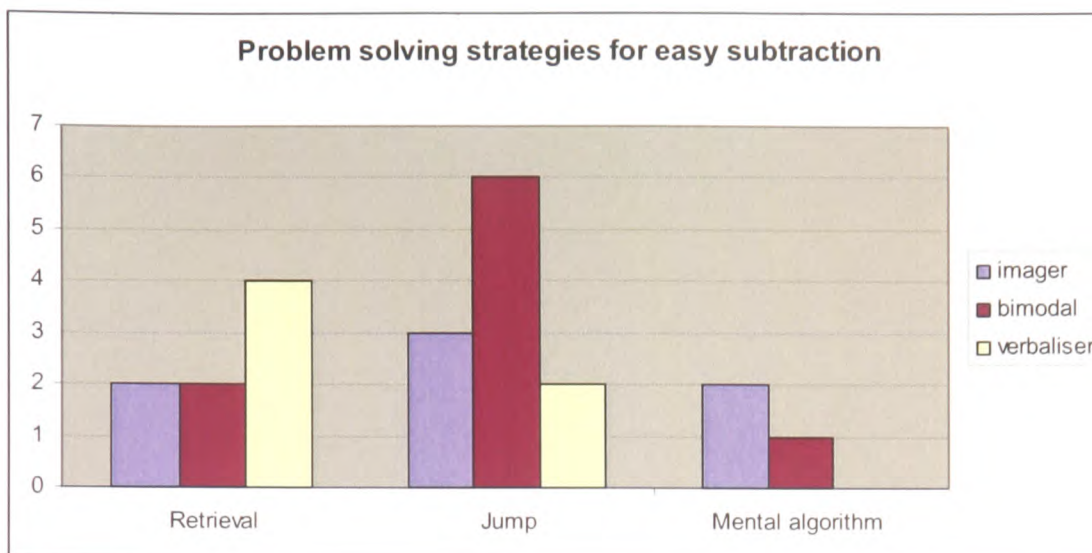


Figure 6.6: Problem-solving strategies for easy subtraction

Difficult Subtraction

Table 6.13: Problem solving strategies for difficult subtraction

STRATEGY	Imager	Bimodal	Verbaliser
Direct retrieval	1	0	2
Jump	2	7	4
Mental algorithm	4	1	0
No answer	0	1	0

As can be seen from table 6.13, imagers were more likely than bimodals or verbalisers to select the mental algorithm strategy, whereas verbalisers almost exclusively used the jump strategy. The results of a 3 x 4 chi-square showed that this pattern of association was statistically significant ($\chi^2(6) = 16.72, p < .05$).

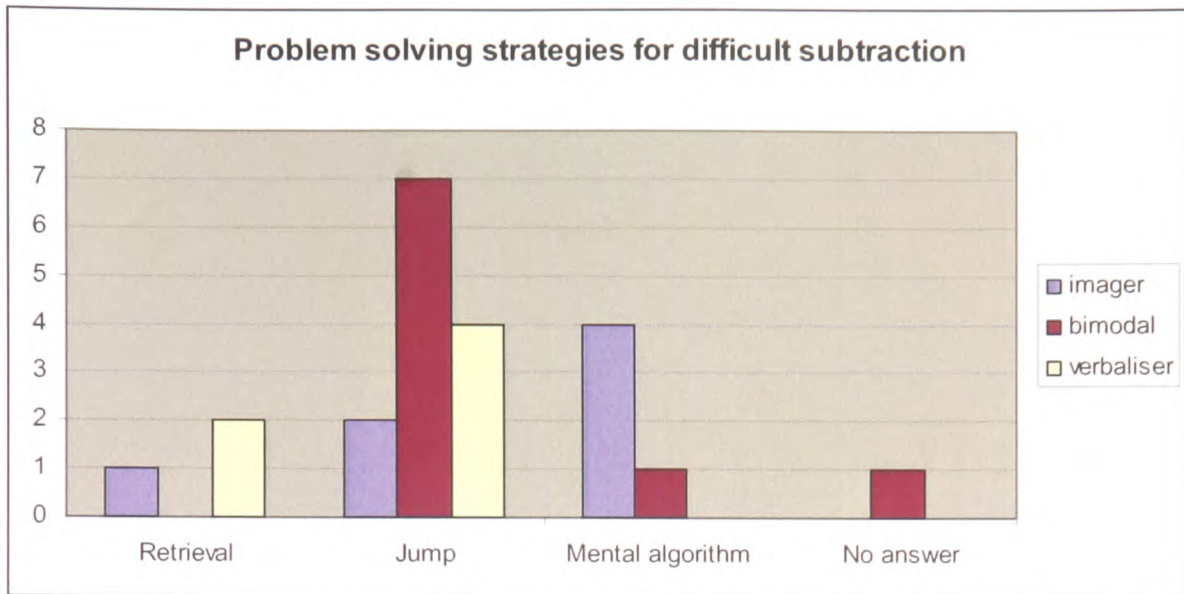


Figure 6.7: Problem-solving strategies for difficult subtraction

Easy Division

Table 6.14: Problem-solving strategies for easy division

STRATEGY	Imager	Bimodal	Verbaliser
Direct retrieval	7	6	5
Fact transfer	0	3	1

Table 6.14 shows that whereas imagers and verbalisers almost exclusively relied on automatic retrieval when solving easy division tasks, bimodals used the direct retrieval and the fact transfer strategy equally. The results of a 2 x 2 chi-square showed that this observed association pattern approached statistical significance ($\chi^2 (2) = 5.94, p = .068$).

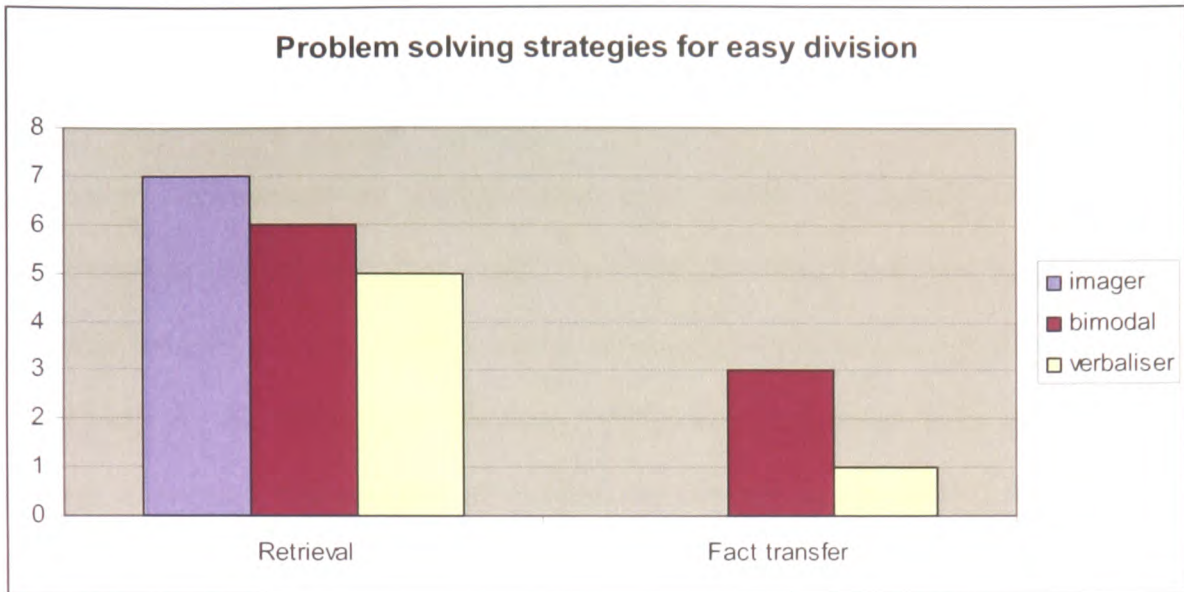


Figure 6.8: Problem-solving strategies for easy multiplication

6.4 DISCUSSION

The purpose of this research was to explore the strategy use of children who were exposed to a web-based direct instruction treatment compared to the strategy use of children from a control group. As will be illustrated below, the results indicate that children from the eFit treatment group and children from the control group did not differ in strategy usage for easy arithmetic problems. However, the eFit children were able to use strategies in a more adaptive way than children from the control group when difficult problems were concerned.

Children from the MGI group used direct retrieval more often than children from the DI group for difficult subtraction. According to Canobi (2005) and Imbo and Vandierendonck (2007), frequent use of retrieval-based strategies indicates skilled problem solving in addition and subtraction. However, the research has been largely conducted in the area of simple, one-digit addition and subtraction problems. While

direct retrieval constitutes an efficient strategy in the context of easy addition and subtraction problems, it could be argued that it might be a risky strategy for difficult subtraction. In contrast to multiplication facts, which are mainly learned via memorisation and are therefore readily available for direct retrieval, retrieval is normally not used as the primary means for solving difficult subtraction problems. This corresponds to the findings of Thornton (1990), who found that even after direct strategy instruction, the majority of children did not use direct retrieval for solving subtraction problem but improved their procedural, non-retrieval strategy usage and used more efficient non-retrieval strategies instead. Therefore, it is surprising that children from the MGI group used this strategy most frequently.

Unlike children from the control group, who used direct retrieval for solving difficult subtraction problems, children from the eFit group used the jump strategy and the mental algorithm strategy. The jump strategy has been described as efficient by Foxman and Beiszhuisen (2002), who found evidence that more proficient children tend to use the jump strategy in solving difficult addition and subtraction problems. In the light of this evidence, the present findings suggest that children from the DI group were more likely to adopt an efficient non-retrieval strategy for solving difficult subtraction problems than their peers from the MGI group.

The frequent use of the mental algorithm strategy among DI children can be explained in terms of the features of the DI treatment. As has been mentioned in chapter 6, the DI intervention placed great emphasis on the acquisition and use of procedural, non-retrieval strategies for solving written addition and subtraction problems and it is possible that children used these strategies as “safe” strategies to solve mental problems rather than trying to retrieve unknown facts from memory. Given that the eFit

intervention was entirely based on visual–textual explanations of strategies, it is likely that children who were exposed to this intervention became used to this representational format and therefore used the mental algorithm strategy more frequently than children from the MGI group. This exemplifies how children with MD who took part in the direct instruction intervention were able to make adaptive choices, meaning that they did not simply use a standard strategy for addition but rather chose the strategy that yielded the most effective combination of efficiency and accuracy.

In both groups, the most frequently used strategy for solving multiplication and division problems was direct retrieval. This confirms previous evidence that has documented convincingly that although several strategies exist for multiplication and division, retrieval is the major strategy for these problems because multiplication is mainly taught via automatisisation (Lee and Kang, 2002). However, there seems to be a strategic advantage favouring children who were exposed to the DI treatment. For difficult multiplication and division, more children from the DI group than from the MGI group were able to solve the task via automatic retrieval. Twice as many children from the eFit group than from the control group solved the difficult division task via automatic retrieval. Children from the control group relied on fact transfer, which is less efficient than direct retrieval. Thus, it seems that children from the DI group used problem-solving strategies that are less likely to overload phonological resources. This finding links back to the proposition, that direct instruction is beneficial for children with MD because it addresses one of their main problems, namely the usage of efficient strategies. The results indicate that the direct and explicit modelling of problem solving strategies and the usage of worked out examples facilitates the problem solving ability of children with MD better than open – ended, minimally guided instruction which

might increase cognitive load in children who have difficulties in using efficient strategies.

The finding that children with MD were able to use direct retrieval to solve difficult multiplication and division problems is notable, especially with respect to an earlier finding in this thesis on phonological working memory usage in children with MD. To recap, it has been proposed that the misuse of phonological resources in children with MD is a function of using inefficient algorithmic problem-solving strategies, which might increase cognitive load on the phonological loop during problem solving. However, the impact of working memory load on problem-solving activities is diminished if children are able to use efficient strategies such as retrieval (Imbo & Vandierendonck, 2007).

The relationship between strategy use and working memory use has considerable implications when the impact of instructional interventions is concerned. Provided that certain strategies such as direct retrieval are more efficient with respect to working memory load, the finding that the DI has the potential to improve strategy usage in children with MD suggests that working memory misuse in children with MD can be alleviated through the format of instruction. Although it is not clear whether working memory misuse in children with MD reflects an innate cognitive deficit or merely a developmental delay of strategy development, it seems that working memory use can be improved indirectly by teaching children efficient strategies which require less working memory resources and which reduce cognitive load during arithmetic problem-solving processes.

In sum, children from the eFit group were more likely to select strategies that are most effective for the particular task and to solve difficult multiplication and division problems via direct retrieval. These results support the assumption that the web-based direct instruction intervention, which emphasized the teaching of explicit strategies in a clearly structured fashion, improved the arithmetic strategic competence of children with MD. This finding corresponds to previous research, which shows that children with MD benefit more from direct instruction compared to MGI. Mercer et al. (1994) question the usefulness of a constructivist approach for low achievers by arguing that, "to expect students who have a history of problems with automaticity, metacognitive strategies, memory, attention, generalization, proactive learning and motivation to engage in efficient self-discovery learning is not plausible." Teaching explicit

To recap, the literature provided evidence that children with MD have specific problems with finding the relevant information in a problem and selecting the appropriate strategy (Kirschner et. al., 2006). Consequently, the search for information during the process of discovery learning and the exploration of several possible solution strategies puts heavy demands on the limited working memory resources, especially for children with MD, who do not possess sufficient prior knowledge in LTM to integrate the new information into existing schemata. In other words, it can be argued that children with MD lack the slots necessary to categorise the new information and recognise similarities to existing mental structures. This means that the discovery of information is meaningless if learners lack the prior skills and knowledge necessary to make sense of the novel information. The free discovery of information impedes the learning process because the limited WM resources of low achievers are occupied with information searching and can therefore not be employed for processing information

into LTM. Consequently, learners can search for a long time with no or little change in cognitive structure taking place.

This in turn implies that children with MD who were exposed to a direct instruction intervention that emphasized explicit modelling of procedures and concepts improved in terms of both procedural and conceptual knowledge compared to children with MD who were exposed to minimal guidance instruction, which emphasizes the construction of conceptual understanding and the discovery and usage of heuristic strategies. Indeed, the acquisition of effective strategies is a major aim of the direct instruction approach. To recap, direct instruction explicitly emphasizes the acquisition of effective strategies and the automatization of arithmetic facts. Given that children who took part in the direct instruction eFit intervention demonstrated superior performance in strategy use and frequently used direct retrieval to derive the results, it seems that the learning gains in the eFit treatment group can be attributed to the effects of direct instruction.

Taken together, the findings indicate that children from both groups used a variety of different strategies and it seems that they made adaptive choices with regard to the particular demands of different types of problems. This is not consistent with retrieval-based models of arithmetic cognition, which assume that with increasing practice, procedural back-up strategies are ultimately replaced by automatic retrieval. Instead, the results seem to lend further empirical support to Siegler's overlapping waves model, a multiple-procedure model that proposes that direct retrieval of arithmetic facts is just one of many different strategies and is supplemented by procedural, non-retrieval backup strategies

The overlapping waves model predicts that although children acquire more efficient arithmetic strategies as part of their learning process, they continue to use less effective strategies or switch between equally effective strategies to yield an effective combination of speed and accuracy. In other words, there is no arbitrary point at which a strategy can be defined as efficient or inefficient. Instead, the efficiency of a strategy can be understood only in the context of the problem. Thus, the present findings challenge the assumption of Rabinowitz and Woolley (1995) that there is a developmental progression towards increasing usage of retrieval in that “younger children gradually move from procedural strategies towards the more automatic and efficient fact retrieval characteristic of older children and adults.” Rather, the results show that children use different strategies, which are more or less efficient with respect to the nature of the problem at hand. However, children from the eFit group were more likely to use strategies in an adaptive and efficient way. Thus, it seems that the format of instruction is a significant variable in the development of children’s mental arithmetic strategies.

6.4.1 COGNITIVE STYLE AND PROBLEM-SOLVING STRATEGIES

To reiterate, the study aimed to investigate the association between children’s cognitive style, as determined by the cognitive style analysis, and their use of different arithmetic strategies for solving easy and difficult addition, subtraction, multiplication and division problems, which were analysed and classified by means of children’s verbal self reports. Recent evidence has suggested that adults with different cognitive styles show considerable variability in the selection of information processing strategies. Such evidence, however, has had little impact on models of children’s

arithmetic strategy usage so far. No previous study has examined the relationship between cognitive style and arithmetic strategy usage.

Based on a consultation of the related literature on cognitive style and individual cognitive processing differences, this study has been driven by the assumption that individual differences in cognitive style lead children to develop preferences for particular problem-solving strategies. More specifically, the goal of this research was to test the hypothesis that children who were identified as visualisers use the mental algorithm strategy, a visualization strategy, more often than children who were identified as verbalisers

The hypothesis was supported to some extent. For easy addition, there is a tendency for verbalisers to use direct retrieval more frequently compared to imagers and bimodals. The trend for verbalisers to rely primarily on direct retrieval is even more pronounced for easy subtraction problems. The finding that verbalisers use retrieval strategies more frequently than imagers or bimodals was not anticipated. As has been mentioned in the literature review, verbalisers tend to represent information in a verbal-phonological format. Thus, the present results seem to confirm that verbal-phonological resources are an essential component of arithmetic problem solving, as predicted by different models of mental arithmetic. To recap, there is evidence that arithmetic facts are stored in a phonological code and retrieval is mediated by the phonological loop. The present findings that verbalisers use retrieval as the primary strategy for solving addition and subtraction support this model.

When difficult addition or subtraction problems were concerned, imagers used the mental algorithm strategy more frequently than verbalisers or bimodals. This

confirms the intuitive assumption that imagers are more likely to use strategies which involve a visualization of the arithmetic problem and which therefore correspond to the individual's habitual way of representing and processing information. It is safe to argue that the visualization process involved in the mental algorithm strategy is mediated by the visuo-spatial sketchpad. The usage of this working memory component for mental arithmetic might be more advanced in visualisers.

A somewhat unexpected finding was that verbalisers were more likely to use the jump strategy for difficult subtraction. One possible explanation is that the jump strategy requires children to temporarily store an interim result in working memory while keeping track of the ongoing calculation. There is evidence that the phonological loop is the key agent for the temporary storage of interim results during ongoing computations. Therefore, it seems that verbalisers, whose preferential format of representing information is the verbal-phonological code, are more proficient in executing the jump strategy. For bimodals, there was no significant difference between the usage of the different strategies, which supports the assumption that bimodals have no preferred representational code.

The majority of children solved easy and difficult multiplication and division problems via direct retrieval, irrespective of individual differences on the verbal-imager dimension. Overall, it seems that the impact of individual differences in cognitive style on strategy use is more powerful for addition and subtraction, where different alternative, non-retrieval strategies are available, compared to multiplication and division, which are solved more frequently via direct retrieval. The present results add further support to the notion that phonological working memory resources are a key

agent for arithmetic problem solving. However, the present findings are not unequivocal.

As has been mentioned in a previous chapter, there is no consensus with respect to the question of whether the phonological loop is involved in fact retrieval or used for the temporary maintenance of information during complex calculations. The results add support for both propositions: on the one hand, verbalisers, who are assumed to be particularly proficient in using the verbal–phonological format, have been found to use direct retrieval as their preferred strategy more frequently than imagers or bimodals. This finding suggests that phonological resources are mainly responsible for fact retrieval and substantiates the results of Noel, Seron and Trovarelli (2004), who found that phonological loop capacity was correlated with the use of the retrieval strategy. However, on the other hand, the present results showed that for difficult subtraction, verbalisers were more inclined to use the jump method, which is supposed to involve the temporary rehearsal of interim results in a phonological format. Thus, the question of whether phonological resources are used for retrieval–based or more complex problems cannot be clearly answered. Rather, the results point to an alternative conclusion, namely that while arithmetic facts are retrieved via phonological code, phonological resources are also required for maintaining these facts during complex calculations. Further research would be needed to address this issue further. Despite methodological shortcomings, the study has illuminated the link between cognitive styles and strategy usage, which has not been investigated previously.

6.4.2 LIMITATIONS

The research design of the present experiment could be criticised because children were asked to solve the problems and provide concurrent verbal reports of their solution strategies. Previous studies used a stepwise approach, where the solution of the problem and the problem solving strategy were reported in sequence. Mauro et al. (1996), for example, asked participants to solve the problem first and to explain the strategy only after the solution had been computed. However, the rationale for asking children to provide on-line reports of their solution strategies was that concurrent verbal reports during the problem-solving process would provide a more valid account of their strategy usage than asking children to compute the solution and then provide a retrospective report of the solution strategy.

A major limitation of the present study is that it measured a skill - strategy usage - that was explicitly trained and emphasized by the direct instruction intervention. While strategy usage is also a core component of the MGI approach, it might have been emphasized to a lesser extent because the teacher was concerned with progressing through the curriculum. Thus, children from the eFit group had extensive opportunities to improve strategy usage, whereas their peers from the non-eFit group had to cover a variety of other areas of maths. As already mentioned in the discussion of the previous study, the eFit intervention “trained for the test” and this might be one factor that contributed to the superiority of the children who took part in the eFit intervention. However, the implication that can be drawn from this hypothesis is that a strong focus on explicit strategy instruction helps children with MD to improve their arithmetic strategic competence. Hence, this aspect seems to deserve greater emphasis in current mathematics curricula.

While the DI intervention seemed to improve strategy use in children with MD, which resulted in significant learning gains in the domain of arithmetic, it is important to investigate to which this effect can be sustained over an extended period of time. In chapter 5, it has been cautioned that the significant learning gains of children in the DI group might be explained in terms of the “Hawthorne effect”, which means that the novelty of the web- based DI intervention rather than the instructional features of the intervention itself promoted the learning gains. In order to address this issue, the following chapter is devoted to researching the long-term effects of the DI intervention.

Chapter 7

Exploring the Long-Term Effects Of Direct Instruction

CHAPTER 7 EXPLORING THE LONG-TERM EFFECTS OF
DIRECT INSTRUCTION

7.1 OVERVIEW

The previous study showed that the format of instruction constitutes an important variable for the learning of mathematics in children with mathematical difficulties (MD). Children with MD who took part in the web-based direct instruction intervention made significant improvements in arithmetic ability compared to children with MD who were exposed to minimally guided instruction. Also, a protocol analysis study indicated that children in the direct instruction group used more efficient problem-solving strategies than those in the MGI group. The finding that children with MD who were exposed to the DI treatment used more advanced problem-solving strategies indicates that DI has the potential to facilitate cognitive change in children with MD. Taken together, these findings substantiate the previous evidence that direct instruction is a more effective instructional approach for children with MD than minimally guided instruction. However, it is pertinent to enquire whether the DI intervention has enduring and sustainable effects and whether the learning gains can be maintained even after the end of the intervention.

The evidence with respect to the long-term effects is not extensive. Although previous studies suggest that DI has positive long-term effects (Tarver & Jung, 1995), it has been criticised as a “pressure cooker” in the media and it has been argued that any observed effects are merely short-term gains (Hechinger, 1986). Therefore, it is important to investigate whether the advantage shown by children from the DI group endures beyond the implementation period.

7.1.1 THE LONG-TERM EFFECTS OF INSTRUCTIONAL INTERVENTIONS

Hurry and Sylva (2007) investigated the short-term, medium-term and long-term outcomes of two different reading interventions that were targeted towards children with reading difficulties. The sample included 400 children at the age of 6. A total of 95 children were assigned to an instructional intervention called “Reading Recovery”, while the remaining 97 children were assigned to the “Phonological Training” intervention. The remaining children were in the control group and received no additional intervention. While both interventions had a positive and significant short and medium-term impact on children’s reading abilities, the significance ceased after 3 1/2 years. According to the authors, the results suggest that the instructional interventions alone are not sufficient to alleviate learning difficulties.

Hasselbring and Goin (2005) researched the long-term effects of the FASTT program (Fluency and Automaticity through Systematic Teaching with Technology) for children with mathematical difficulties. The development of the FASTT program was based on previous research that showed that traditional drill-and-practice computer programs did not facilitate automaticity and fluency but merely reduced procedural counting time. In other words, children did not move from counting-based procedures to direct retrieval as a result to exposure to drill and practice programs. Instead, immature counting-based procedures were executed faster. Thus, the programs were not successful in helping children to become more efficient problem solvers. The sample in the Hasselbring and Goin study consisted of two groups of children with mathematical difficulties and one group of children with average mathematical abilities. The groups were matched for age, gender and ethnicity. One of the groups of children with MD was exposed to the computer-based FASTT intervention, while the other groups received

drill-and-practice fluency instruction delivered by the teacher. The FASTT intervention consisted of 54 sessions of 10 minutes each.

The results of the post-test showed that children in the experimental group acquired 24 new arithmetic facts, while children with MD gained no new facts. Interestingly, even children of average ability only gained 8 facts during the period. However, this might be because children of average ability knew more facts than their peers with MD prior to the intervention, and because of this advantage, their potential for improvement might have been smaller. A follow-up study after 4 months indicated that children who were exposed to the FASTT intervention were able to recall an average of 20 facts, which suggests that the intervention yielded positive long-term effects. However, reservations could be raised because the follow-up study was only carried out with children from the experimental group. Thus, no information is available on the performance of children in the control groups. The study might have benefited from investigating the long-term effects of the teacher-based fluency treatment in the control groups also.

One of the largest educational research studies is the Head Start Project. This ongoing project started in 1965 in the US and included 561,000 children, most of whom had Afro-American backgrounds and were from economically deprived households. Children can take part in the project at the age of 3 or 4. Apart from a facilitating learning environment, Head Start includes several social aspects such as access to preventive medical care and a healthy diet. Garces, Thomas and Currie (2000) investigated the long-term socio-economic outcomes of the Head Start project. Their sample consisted of approximately 4000 adults, born between 1964 and 1977, who took part in the Head Start project or in any other preschool or day-care program. The results

indicate that children who participated in the Head Start project are 28% more likely to enter college than siblings who did not attend pre-school and 20% more likely to enter college than siblings who attended any other preschool.

A more detailed analysis of the long-term effects of the Head Start project was undertaken by the US Department of Health and Human services. Their sample included a total of 4667 children, distributed across 23 states, who were randomly assigned to the Head Start group or a control group at the age of 3 or 4 years. The Head Start group consisted of 2783 children, while the control group consisted of 2783 children. However, it is important to note that the control group is not a no-treatment group. Rather, parents of children in the control group had the opportunity to enrol their children in other forms of childcare and preschool arrangements. Baseline data were established in 2002, when children entered the program. From 2002 onwards, data were collected annually until 2006. The effect of the Head Start project on the development of children's cognitive abilities was assessed in 5 areas: (1) pre-reading skills, such as letter recognition, (2) pre-writing skills, such as writing letters, (3) vocabulary knowledge, (4) oral comprehension and phonological awareness, which includes the ability to recognise spoken words and discrete phonemes, and (5) early numeracy skills.

The results showed that exposure to the Head Start arrangement had a significant effect on pre-reading skills: after one year, children in the Head Start group performed significantly better than the control group in standardized tests of letter and word identification and letter naming. These abilities are considered to be important requirements for the development of literacy. In addition, the results indicate an advantage of Head Start children in the area of pre-writing skills. The group of three-year-old Head Start children performed significantly better than the control group in the

“draw a design” test, a standardized test of perceptual motor skills which involves copying geometrical shapes. Among children in the 3-year-old group, a significant difference in average scores was found between Head Start and control group children in the vocabulary test. The results for phonological awareness and early numeracy skills showed no significant differences between Head Start and control group children.

Apart from the cognitive gains among children in the Head Start group, the project was found to have a significant impact on children’s social and behavioural development. Three-year-old children in the Head Start group scored significantly better on both the Total Problem Behaviour measure and the hyperactive behaviour measure. Also, positive effects on the Aggressive Behaviour measure were reported among 4-year-old Head Start children from English-speaking families. Overall, the findings seem to imply that the Head Start project yields positive cognitive and social outcomes for children from economically deprived family backgrounds.

An empirical investigation of the long-term effects of the DI treatment was undertaken by Meyer (1984), who managed to locate over 80% of former DI children and 76% of the particular control group. At the end of grade 9, the DI group was 7 months ahead of the control group. Meyer researched the following two questions:

1. How the performance of DI participants in 3rd grade correlates with their performance in 9th grade.
2. How high school graduates who were DI participants in 3rd grade compare with a control group with respect to graduation rates, drop-out rates, college application rates and acceptance rates.

The groups were matched for ethnic composition and socio-economic status. In both groups, the proportion of children from low socio-economic status households was over 70%, as indicated by the reception of welfare aid for dependent children. The results showed that 9th grade children who were exposed to the DI treatment in 3rd grade performed significantly better on a standardized maths test, with a score of 8.59 compared to a score of 7.95 in the control group. High school graduates who received DI treatment in 3rd grade were more successful in all three areas than graduates from the control group. 63% of former DI treatment children had graduated from high school, compared to 38% of the control group. 37% of the treatment group applied to college, compared to 22 % of the control group. A total of 34% of the participants from the treatment group applied successfully for college, compared to an acceptance rate of 17% in the control group.

Similar findings were reported by Gersten, Keating and Becker (1988). While the researchers found that the advantage of former DI children decreased, it never disappeared. One possible explanation for the decreasing advantage is that after the treatment, the children received different learning experiences than during DI lessons. However, the positive long-term effects of DI persisted beyond school education: Meyer, Gersten & Gutkin (1983) documented significantly higher rates of graduation, college application, college acceptance and college retention and lower drop-out rates for former DI children.

Similarly, Becker and Gersten (1982) conducted a further follow-up study in order to determine the long-term effects of learning with DI in a large sample of 5th and 6th graders. 1097 former FT children and 970 non-FT children were tested in different domains of mathematics, such as computational abilities, conceptual knowledge and

problem solving. While the former FT children outperformed the control group, their overall performance declined after the DI treatment, which suggests that DI treatment needs to be implemented for a longer period of time in order to have sustainable effects on the learning of mathematics. This assumption is supported by the findings of Tarver and Jung (1995), who compared the mathematical achievement scores and attitudes towards mathematics of 1st and 2nd graders who were exposed to DI or MGI. The achievement tests involved different subtests such as computational abilities, conceptual knowledge and strategy application.

The attitude test was designed by the experimenter to measure a range of different aspects, such as children's perceived value of mathematics and their confidence in their mathematics ability. After the first year of the intervention, children in the DI group performed significantly better in computations, but not in concepts and applications, but at the end of the second year, the DI children performed significantly better in all areas and also held more favourable attitudes towards mathematics than did children from the control group. This challenges the assumption that DI is a 'pressure cooker' and suggests that DI needs to be implemented for a longer period of time in order to ensure positive long-term effects on the learning of mathematics.

Overall, the literature that has been reviewed so far indicates that DI interventions can have considerable long-term effects on academic achievement. The aim of the current follow-up analysis is to investigate whether these findings also apply to the web-based DI intervention that has been evaluated in chapter 5 and 6. While the previous chapters dealt with the effects of DI on specific mathematical ability, the current study is concerned with the long-term effects on children's school grades, which have been collected 17 months after the DI intervention.

METHOD

7.1.2 DESIGN AND PARTICIPANTS

A longitudinal design was employed to investigate the long-term effects of DI on maths ability in children with MD. In March 2008, the winter report grades in mathematics were collected from 52 out of a total of 58 children from the former DI intervention cohort and 44 out of a total of 47 children from one of the former minimally guided instruction groups. Some children had moved away or had to repeat a year and therefore their report grades were not available. Children who had attended class 5 and 6 at the time of the intervention were in class 7 and 8 by the time the report grades were collected.

7.1.3 MATERIAL / PROCEDURE

As has been mentioned in chapter 5, children's learning outcome in the intervention study was based on a standardized test, which provided a valuable account of mathematical abilities in the domain of arithmetic. However, it was decided to examine long-term effects on the basis of school report grades rather than using a standardized test, because it was deemed to be important to investigate whether the intervention has positive long-term outcomes for children's ability to learn mathematics in the everyday classroom setting. On request, the schools provided lists of the latest report grades. The report grades in Germany range from 1 (very good) to 6 (fail). Report cards are handed out in winter and in summer. In contrast to performance in standardised tests, the school grades are based on continuous assessments, which include homework, written assessments, verbal contributions and classroom behaviour.

7.1.4 RESULTS

An independent t-test was performed to examine the differences in report grades of children from the former DI group and the MGI group. As can be seen in Table 7.1 below, children from the former DI cohort have slightly better grades than children from the MGI group. However, the results of the t-tests show that these results are not significant ($t_{(94)} = -1.02, p = .31$).

Table 7.1: Differences in school report grade between children from the DI and MGI group

Group	Mean Grade	SD	N
Direct instruction	3	1.28	52
Minimally guided instruction	3.25	1.08	44

7.2 DISCUSSION

This follow-up study was intended to examine the long-term effects of the direct instruction intervention eFit, based on school report grades that were collected 17 months after the end of the intervention. The reason for using school grades rather than the scores from a standardised test to determine the long-term outcomes of the instructional intervention was that school grades constitute an ecologically valid indicator of the long-term outcomes of instructional intervention. While standardised tests are administered in an experimental context, school grades reflect children's ability to perform mathematics in the everyday classroom setting over a longer period of time. While the post-test study in chapter 5 showed a significant advantage favouring the DI group in terms of scores on the standardised test, the present study addressed the question of whether the DI intervention also had a positive long-term effect on school

grades. However, the long-term data showed no significant difference between the school grades of children from the DI and the MGI group.

Hence, the current results do not support the findings of previous studies on the long-term effects of DI, which have reported advantages that persisted even to the level of college education (Meyer, 1984). Instead, it might be the case that the improvement of the children in the DI group that has been reported in the post-test, which followed immediately after the treatment, can be explained in terms of the novelty of the DI treatment. Hence, it is possible that the change of instruction, rather than the quality of DI instruction as such, influenced children's performance and resulted in a significant improvement. In other words, the mere exposure to a new format of instruction, especially the use of an ICT device for learning mathematics, elevated children's performance to a significant extent. Such an effect is known as the "Hawthorne effect" (Landsberger, 1955).

In essence, the Hawthorne effect predicts that environmental changes, such as changes in the work environment or educational interventions, can have positive short-term effects, because participants see themselves as members of something new. Children as well as the mathematics teacher in the DI group were aware that the implementation of the DI intervention eFit was being tested, and this knowledge might have raised their motivation. The results suggest that the novelty effect diminished after a certain period and this would explain why the DI intervention showed no long-term effects on children's school grades. This problem could have been addressed by implementing the DI intervention for a longer period. Indeed, the DI intervention in the Meyer study was implemented for a longer time span than the intervention evaluated in the present research.

To recap, the DI intervention examined by Meyer was part of the large-scale educational study known as “Follow Through”, which was intended to investigate the effects of different educational paradigms, such as DI and MGI. Schools could decide which approach to adopt and teachers received extensive training in addition to learning materials and equipment. Therefore, children in schools where DI was implemented received DI exclusively, whereas children in the present research study only received two hours of DI per week. In sum, the scope of the DI implementation in the Meyer (1984) study, which yielded considerable long-term outcomes with respect to the academic success and educational careers of the former participants, is not comparable to the present DI implementation.

Thus, the finding that children from the DI group did not outperform children from the MGI group in terms of school grades does not imply that the DI intervention has no positive long term-effects at all. Rather, a possible explanation for the present results is that the DI intervention was too short and not intense enough to produce effective long-term advantages in children with MD. After children have been exposed to the intervention for nine months, for just two hours per week, all children went on to exclusively receive the minimally guided instruction as recommended in the curriculum guidelines, and this might explain why the advantage of the former DI cohort diminished.

Indeed, it might be that although DI resulted in positive short-term gains, children with MD need longer exposure to the DI intervention in order to benefit on a long-term basis. It could be that these skills have no significant long-term effect unless they are required and practiced in the classroom as part of the mathematics curriculum, and this might explain why the results did not show a significant long-term advantage

favouring the DI intervention group. Thus, it seems that the DI intervention served as a scaffold, helping children to learn arithmetic concepts and procedures more effectively. Once the scaffold is removed, the children are no longer able to maintain their learning advantage when the curriculum moves to increasingly complex contents and requires the acquisition of new skills. This implies that instructional interventions such as the DI program eFit might need to be implemented for a longer period of time in order to assess the long-term outcomes. Further research would need to address the question of whether a continuous long-term implementation of DI, which can be adjusted to different curricular requirements, has the potential to yield effective long-term outcomes even if more difficult mathematical contents are addressed.

It remains unknown why children from the DI were not able to maintain their advantage by transferring their previously learned abilities to new learning situations. The DI intervention that was evaluated in the current thesis was concerned with arithmetic abilities and therefore promoted specific mathematical skills more than others. A possible explanation is that the concern with arithmetic problem-solving procedures and strategies was emphasized at the expense of a wider conceptual understanding of how the acquired knowledge could be applied to other mathematical domains. Indeed, it is worth noting that classes 5 and 6 are designed as an “orientation stage”, whereas the mathematics curriculum for classes 7 and 8 goes beyond the area of arithmetic computations and includes more complex and abstract algebraic concepts such as equations and rational numbers.

Thus, the content of mathematics lessons changed considerably during the 17 months between the final post-test and the collection of long-term data. This implies that children have to deal with different mathematical contents during their maths

lessons. While the arithmetic skills that were emphasized in the DI instruction constitute a prerequisite for more complex problem-solving activities, the abstract nature of these mathematical concepts requires children to manipulate and apply existing skills and to adopt different modes of numeric representation. Hence, it seems that children made significant learning gains in a specific mathematical domain, which required the application of previously learned skills. However, it seems that the skills were domain-specific and could not be transferred to new domains.

According to Piaget's theory of cognitive development, the application of previously learned skills to new situations requires abstract thinking to recognise how previously learned strategies that have been acquired in a particular domain can be transferred to new domains. The current findings indicate that the children in the DI group succeeded in performing arithmetic operations in the context of concrete problem-solving situations, which were embedded in a specific learning environment, namely the web-based DI intervention. While this constituted a significant improvement, it seems that the children did not progress to the stage of formal operations, which enables them to mentally manipulate their knowledge and to recognise the conceptual relationship between the new problems and the previous situation. It seems that children were able to improve their arithmetic abilities in a specific learning context and to apply their skills to similar problem-solving situations. Thus, children did not adopt a qualitatively different mode of thinking and numeric representation because the DI intervention improved domain-specific rather than general problem-solving abilities. This would explain why the DI intervention had no long-term effect on their school grades.

Overall, the findings on the long-term effects of DI are not clear-cut, especially because no standardised instrument has been used to investigate long-term effects. It is important to note that school grades are not comparable to the standardized measure of mathematical ability that was used in chapter 5. School grades usually have a “normal distribution”, meaning that the majority of children perform around the mean, whereas fewer children perform in the upper or lower bound. In practice, teachers tend to avoid using grade 6 on school reports, because this indicates failure and might seriously impede children’s future academic and occupational careers. Hence, the range of grades is small and it is difficult for children to make significant improvements.

While standardised tests of mathematical ability measure a specific cognitive ability, school grades are determined by various non-cognitive factors such as motivation, social adjustment and communication skills, as well as endogenous factors such as parental support. The correlation between mathematical ability and school grades are typically between 0.6 and 0.7 (Tent, 2001). Consequently, specific mathematical ability is just one of several components that are relevant to children’s school grades. School grades are not exclusive measures of mathematical ability as such, but rather of the ability to learn mathematics in the classroom setting, which includes children’s classroom behaviour, their ability to participate successfully in cooperative work, their verbal contributions in mathematics lessons and the quality of their homework. Hence, children’s learning gains in a specific area of mathematics such as arithmetic are not necessarily reflected in better school grades. Therefore, the current results cannot be taken as evidence that the DI intervention had no long-term effects on children’s mathematical ability. Instead, the results suggest that instructional interventions which help children to improve specific mathematical abilities are not

sufficient to improve general academic performance in a real-life setting such as the classroom.

7.3 CONCLUSION AND FUTURE RESEARCH

As mentioned above, a possible limitation of the current study is that it measured the long-term effects of DI on school achievement rather than the effect on specific mathematical abilities. To recap, the finding that children from the DI group performed significantly better after the treatment is based on a standardized instrument to measure mathematical abilities. The correlations between school achievement in terms of grades and mathematical abilities are moderate (Tent, 2001), which implies that school grades are subject to a variety of influences. Future research might investigate the long-term effects of web-based DI on mathematical abilities by utilising a standardised instrument rather than using school grades. Also, further research would be required to determine the conditions under which an instructional intervention such as DI can yield sustainable long-term effects not only on specific mathematical abilities but on general mathematical achievement in terms of school grades. For this purpose, follow-up studies would be necessary to investigate the long-term effects when the DI intervention has been implemented for a longer period of time

The overall evaluation of the DI intervention provides justification for such an attempt and suggests that DI is a suitable type of instruction to address the cognitive and affective dimensions of mathematical difficulties. The Head Start study showed that educational interventions that are implemented for a longer period of time have the potential to improve children's cognitive and social development considerably and can

lead to social and economic gains in later life. Hence, the positive short-term outcomes of the DI intervention eFit warrant further investigation.

Chapter 8

General Discussion

CHAPTER 8 GENERAL DISCUSSION

This thesis examined the cognitive and affective characteristics of schoolchildren with mathematical difficulties, and the role of instruction in ameliorating such difficulties. Children with MD have no learning disabilities such as dyscalculia, but have impoverished mathematical abilities and do not achieve the curricular performance standards for children of their age group. In the literature, children with MD are defined as those whose performance falls in the lowest 25th – 30th percentile. The present thesis used a similar criterion. Children with MD were identified on the basis of the secondary school track they attend. This is supposed to be a reliable indicator of MD, because children are tracked into different secondary school types on the basis of their academic ability. The results of large-scale educational assessments indicate that the majority of children from general secondary schools perform in the lowest attainment band, compared to children from comprehensive schools or grammar schools.

It has been argued that previous accounts of mathematical difficulties are limited in that they have emphasized the cognitive aspects of MD, whereas the role of affective factors has received less research coverage. However, such a one-dimensional perspective fails to account for the complex and multifaceted nature of MD. The present thesis has aimed to address this issue in order to develop a more comprehensive model of mathematical difficulties, which goes beyond an examination of the cognitive dimensions of MD and which can be applied by researchers and educators. The thesis was divided into six major sections and brings together different methodological approaches, such as dual-task studies, protocol analysis and questionnaires to investigate MD from different perspectives and on different levels of analysis, because

it is assumed that the different facets of MD cannot be captured by a single research method.

The thesis outlined the psychometric development of an affective response scale, which can be implemented to identify children with MD. The affective response scale covered four aspects, namely mathematics anxiety, self-efficacy, attitudes towards the teacher and perceived classroom conduct. These four factors have been derived on the basis of a factor analysis. As has been shown, children with MD differed from average achieving children in that they were more negative in their affective responses towards mathematics on each of the four subscales. Further, the findings showed that individual differences in affective responses towards mathematics predict individual differences in mathematics performance.

Negative affective responses were most pronounced in older children, from class 8. In general, it was observed that affective responses in children with MD decreased from grade 5 to grade 8, whereas affective responses in children of average ability remained constant. Anxiety, for example, is not predictive of individual achievement differences at the start of secondary schooling, but predicts nearly 25% of the variance in achievement of children with MD from class 8. Therefore, the finding implies that negative affective responses in children with MD are not present from the outset but develop over time. This challenges the assumption of the interference model, which predicts that math anxiety is a stressor for low achievement. The relationship between math anxiety and achievement seems to be more complex than assumed by either the interference or the deficit model and the results tend to support the argument of Newstead (1998) and Dossel (1993), who maintain that the anxiety – achievement

relationship is mediated by another factor, namely children's learning environment including the student – teacher relationship.

In order to investigate this possible explanatory model further, future research would be required to identify the mechanisms by which anxiety increases in children with MD. This might involve a more detailed analysis of the aspects which are likely to increase anxiety in children with MD, such as possible anxiety - evoking situations and teacher – student interaction. In order to prevent a vicious circle of mathematical difficulties and negative affective responses, an early identification of these affective indicators is essential for educators.

With respect to the cognitive aspects of MD, the findings indicate that there are considerable differences in the use of phonological working memory for arithmetic problem solving between children of average ability and children with MD. As has been discussed, children of average ability used phonological resources only for difficult problems, whereas children with MD used phonological resources even for simple multiplication problems. The finding that simple and difficult problems require the same cognitive effort in children with MD suggests that these children treat simple problems in the same way as they treat difficult problems and therefore use phonological resources less economically than children of average ability. It can be argued that the misuse of phonological resources for simple problems constitutes an impediment to successful problem solving, especially in instructional arrangements such as minimally guided instruction, which impose a high level of external cognitive load in that children are asked to discover and use heuristic strategies independently.

One possible explanation for why children with MD misuse phonological resources might be that they rely on inefficient algorithmic problem-solving strategies, which require excessive working memory resources. This corresponds to the findings of Imbo and Vandierendonck (2007) and substantiates the argument of Barrouillet and Lepine (2004) that “the frequent recourse to slow arithmetic strategies increases the cognitive load involved in arithmetic problem solving in those children, who, in other respects, have the lowest cognitive resources to cope with this extra cognitive load.” Future research might employ neuroimaging techniques to specify the cognitive load associated with the usage of different problem solving strategies in children with MD. Measuring the cognitive efficiency of different strategies would provide valuable insights for educational practice and would help educators to adapt instruction to the cognitive requirements of children with MD.

The implications of the findings on working memory usage in children with MD for instruction have been addressed by evaluating the effects of a direct instructional intervention compared to minimally guided instruction, which is the dominant paradigm in mathematics education. The results of a standardized test of arithmetic ability showed that children in the direct instruction groups had a significant advantage over children in the minimally guided instruction group. It is argued that direct instruction is more beneficial for children with MD, because the direct teaching of explicit model strategies and worked examples reduces cognitive load and helps children to develop more efficient problem-solving strategies, which links back to the finding that children with MD use working memory in an inefficient way, even when solving simple problems.

The Affective Responses towards Mathematics scale was implemented to account for the role of affective factors in the learning process. Of all affective dimensions, children's attitudes towards the teacher predicted the greatest variance in the learning outcome. The relationship between learning outcome and attitudes towards the teacher was inverse, meaning that children with lower attitudes towards their teacher made greater learning gains in the direct instruction intervention. The intervention was run without the regular classroom teacher and therefore it has been suggested that these children benefited from the intervention because their negative attitudes towards the teacher might have impeded their ability to learn mathematics in the regular classroom setting. Also, the results might imply that children's negative attitudes are not directed towards the teacher but towards the type of instruction, instead. This assumption is supported by the finding that the type of instruction had a significant effect on children's arithmetic performance. The development of a scale to investigate children's attitudes towards the type of instruction would be important to examine how these attitudes predict individual differences in achievement. Therefore, this topic deserves further investigation.

An analysis of children's protocols indicates that children from the direct instruction group were more likely to use efficient algorithmic and retrieval-based strategies for difficult problems than children from the minimally guided instruction group. It has been argued that the differences in strategy usage can be attributed to the direct instruction intervention, because the explicit modelling of example strategies is a focal point in direct instruction, whereas minimally guided instruction is based on the assumption that children should discover and invent heuristic rather than algorithmic strategies. The finding that children with MD require considerable phonological working memory resources supports the view that the direct modelling of selected

strategies is more effective than discovery learning, a process which in itself increases cognitive load and therefore constitutes an impediment to the learning processes of children with MD.

It has been proposed that the usage of problem-solving strategies might be a function of individual differences in the representation and organisation of knowledge. However, to date no known study addressed the relationship between individual differences in cognitive style and arithmetic strategy usage. Therefore, it was decided to examine the association between individual differences in cognitive style and problem-solving usage, an issue that has not been addressed by previous research. The results showed that children with a verbal cognitive style were more likely to use the direct retrieval strategy, whereas imagers used the mental algorithm strategy. The finding that children with a verbal cognitive style are more likely to retrieve answers directly goes in line with the finding that the phonological loop plays an important role in retrieval-based operations. Children with a verbal cognitive style might be more adept in using phonological working memory resources, which confers an advantage when it comes to the direct retrieval of answers. The linkage between verbal cognitive style and phonological loop functioning in arithmetic awaits further research. More specifically, the usage of neuroimaging techniques such as fMRI might provide further insights into the role of individual cognitive processing differences in strategy usage. Findings on the effect of cognitive style differences on problem solving behaviour would have considerable practical implications for the design of instruction.

Overall, the findings on the effects of DI indicate that the DI intervention improved arithmetic abilities and problem-solving strategies in children with MD. Based on the results, it was assumed that DI resulted in a cognitive change in terms of a

restructuring of procedural knowledge. Hence, although the causal relationship between working memory functioning and strategy usage remains unclear, it seems that children with MD can improve their problem – solving strategies when provided with instruction which accounts for the limitations of working memory. However, as will be elaborated below, it would be pertinent to inquire the origins of working memory deficits in children with MD.

In order to investigate whether the DI intervention improved mathematical abilities on a long-term basis, a follow-up study was carried out 17 months after the end of the intervention. The results showed that no significant difference between the groups could be detected, which suggests that the advantage of the DI group over the MGI group ceased over time. This seems to run counter to the assumption that DI led to a sustainable cognitive reorganisation of procedural knowledge in long term memory. However, the long-term study is not free from methodological limitations and further research is awaited to examine the long-term effects of the web-based DI intervention. For example, it was emphasized that while the DI intervention was implemented for only 17 months, MGI is the dominant instructional paradigm for teaching maths at secondary schools. The continuous exposure to MGI might have conferred an advantage to this approach compared to DI. In the light of the positive evaluation of the DI approach, it was suggested that the intervention might need to be implemented for a longer period of time in order to produce sustainable long-term effects. Future research would be required to address the learning of mathematics in children with MD who are constantly exposed to a direct instruction treatment, rather than switching back to minimally guided instruction. The overall model of mathematical difficulties that emerges from the findings will be outlined below.

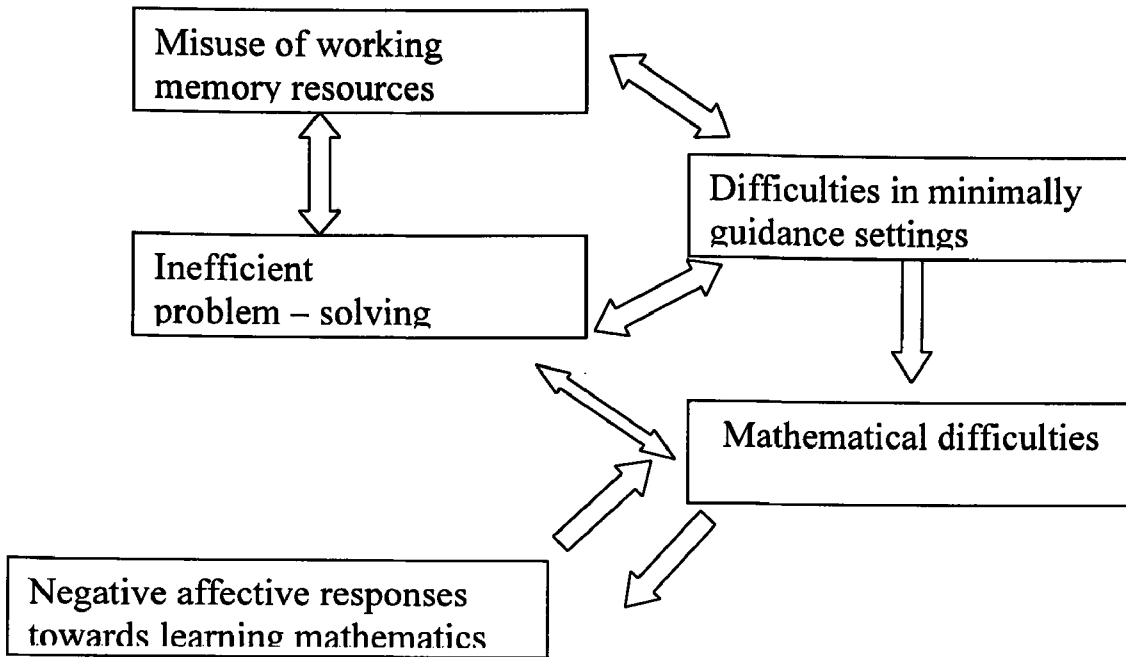


Figure 8.1: A model of mathematical difficulties

Taken together, the model of MD as displayed in figure 8.1, which emerges from the current findings suggests that the misuse of working memory resources and inefficient cognitive processing strategies are the main determinants of MD. While the causal ordering of these two factors was not subject to investigation, the finding that children with MD have problems in the area of simple arithmetic suggests that the cognitive processing difficulties are present from an early stage. Hence, it is possible that the misuse of working memory is in fact indicative for a working memory deficit, as argued by Schuchardt et. Al. (2008). To reiterate, children with MD require considerable working memory resources to compute simple problems, presumably because they rely on inefficient and error-prone algorithmic strategies. Future work is required to investigate why children use inefficient strategies. Simple arithmetic problems are one of the first mathematical concepts that children learn in school and it seems that mathematical difficulties become manifested at this stage.

The notion that working memory deficits are the key predictor of mathematical difficulties and are a reason for rather than an outcome of inefficient strategy usage receives empirical support from Schuchardt et. Al. (2008). Similarly, Barrouillet and Lepine (2005) perceive cognitive constraints as the key aspects of MD. According to their argument, cognitive deficits are not a function of inefficient strategy usage. Instead, these deficits are further perpetuated by inefficient strategies and this interaction reinforces MD. In order to address this possibility, further research would be required to investigate the origins of individual differences in working memory usage for simple arithmetic in young children. Neuro-imaging techniques might be employed to examine working memory functioning in a cohort of young primary school children and the results could be used to research how working memory functioning predicts children's arithmetic ability in follow up studies. This research would permit researchers to identify risk factors with respect to working memory functioning and to establish a causal link between working memory deficits and inefficient strategy usage.

Without efficient strategies at hand and with a lack of automatisisation, even simple problems become difficult to solve and require extensive cognitive effort. As a consequence, the level of intrinsic cognitive load that arises due to the lack of efficient strategy does not permit children to discover and use heuristic problem-solving strategies independently. This argument goes in line with the predictions of the cognitive load theory, which assumes that whereas automatised processes reduce demands on working memory resources, a lack of automatisisation on basic facts increases the intrinsic load during problem – solving. This in turn puts further constraints on the amount of WM resources which can be devoted to schema acquisition and therefore impedes more complex learning processes (Chandler and Sweller, 1991). To recap, the cognitive load theory predicts that learning process will be maximized if

sufficient WM resources can be devoted to encoding of information into LTM. However, this learning process will be impeded if limited WM resources are occupied by the conscious processing of basic facts.

In essence, the model of MD assumes that because of their deficits in strategy usage, children with MD are severely disadvantaged in the context of minimally guided instruction compared to children of average mathematical ability and this contributes to the manifestation of MD. Similarly, Mercer (1994) claims that "to expect students who have a history of problems with automaticity, metacognitive strategies, memory, attention, generalization, proactive learning and motivation to engage in efficient self-discovery learning is not plausible." As shown in the model, the systematic cognitive disadvantage of children with MD who are exposed to minimally guidance setting is likely to result in negative self-efficacy appraisals and increased anxiety. Towards the end of primary school, the difficulties are manifest and children with MD are tracked into a lower secondary school type. The segregation seems to evoke a labelling process because children are categorised as "low achievers" by peers, teachers and parents. The segregation into a homogenous pool of low achieving children and the stigmatisation of these children is likely to lead to debilitating self-efficacy appraisals which result in a negative collective self-efficacy appraisal. Hence, the segregation process can depress self-efficacy beliefs on an individual and a collective level. Negative self-efficacy, in turn, is a significant predictor for low achievement as has been shown in chapter 3. Hence, the model proposes that exposure to minimally guidance settings results in a vicious cycle between MD and negative affective responses in children with MD.

According to Bandura, it could be predicted that collective self-efficacy, which develops over time, is the key stressor for other negative affective responses and this

concept should therefore receive increased attention in further research. Hence, further research might adopt a longitudinal design to identify the stage at which the decline in self – efficacy occurs. It is hypothesized that the secondary school segregation goes in line with a stigmatisation, because children are tracked on different school types depending on their academic abilities. Future work is needed to explore the mechanisms by which the school transition influences children’s affective and socio – cognitive development. More precisely, the development of self – efficacy beliefs could be investigated by administering the ARTMS to children before and after transition to different secondary schools. In addition, the implementation of an instrument to measure collective self – efficacy would substantiate the argument that school segregation has a negative effect on the development of affective responses towards maths. account for the responses that children with MD receive in their social environment. Children defined as children with MD visit the lowest school track and might therefore be the target of discouraging remarks from parents and peers.

However, the model indicates that the use of efficient problem-solving strategies, which can be facilitated through the type of instruction, is a key factor in ameliorating mathematical difficulties. In line with the tenets of the cognitive load theory, it is assumed that the type of instructional activities and the presentation of instructional material determine whether sufficient WM resources can be allocated to schema acquisition and automatisisation, which are seen as the major mechanisms for learning. However, it remains unclear if the use of efficient strategies also has an effect on working memory functioning. In other words, future research might address the question whether strategy instruction helps children with MD to deal with limited WM resources or whether it is possible to overcome the hypothesized deficit by improving WM functioning. The latter proposition goes in line with findings which show that

children with WM deficits who were exposed to a WM training program were able to increase their WM capacities to a significant extent. Hence, WM capacity is not a fixed characteristic but can be expanded through appropriate intervention. Again, this finding yields positive implications when children with MD are concerned.

The finding that the type of instruction can improve problem-solving strategies and performance holds important implications for educators: while future work in the area of cognitive psychology is needed to examine whether or not the misuse of working memory resources reflects an underlying cognitive deficit, it seems that mathematical difficulties are not inevitable but can be challenged through instructional interventions which address the specific characteristics of children with MD. Nevertheless, the overall pattern of results as summarised in the model of MD indicates that these interventions have to be implemented at an early stage and for a sufficient period, in order to prevent a cycle of inefficient problem strategy usage, working memory constraints, low performance and negative affect. Further, the results indicate that the learning environment plays an important role in developing and reproducing affective responses. More precisely, the ability tracking after primary school is likely to evoke detrimental self – stereotyping processes due to the perceived stigma which arises from the school segregation.

Children with MD would benefit from remaining within a heterogenous group of learners as long as possible. Especially for children from deprived social backgrounds it would be helpful to learn together with children from average socio-economic backgrounds, because the latter are more likely possess the socio-cultural capital required to meet the demands of the school environment, which has been described as a “middle class institution” (Bourdieu & Wacant, 1982). Indeed, the role of the home

environment in the generation, another factor which deserves attention in future research. Roscigno (1998) for example demonstrated the strong effect of cultural capital resources on maths achievement: students from households with more than 50 books scored higher in maths assessments than students from households without these resources.

The role of the home environment seems to play a considerable role for mathematical development. Luo, Jose, Huntsinger and Pigott (2007) carried out a large scale study to investigate possible factors which explain the outstanding mathematical ability in Asian Children, compared to American children. They conclude that these considerable ability disparities between the ethnic group can be largely explained in terms of home environment and parental support: “we believe that the most likely explanation is cultural differences in family environment and upbringing. East Asian people...traditionally place greater emphasis on memorization and practice, effort, concentration and persistence” (p. 610). If children are encouraged from early on to practice arithmetic skills with a high degree of persistence and if they engage in activities which require them to concentrate, it is reasonable to assume working memory development will be facilitated over time and confers these children an advantage in school. It would be important for children with MD to have the opportunity to exchange with more able children who can serve as role models. Keeping the learning environment heterogeneous as long as possible would help children with MD to improve their maths skills by observing children of average or above average ability learning mathematical concepts and procedures.

For educational policy makers, this assertion would imply that the current educational system which segregates children into different ability tracks after four

years of primary school, might need to be revised in order to enable children of different abilities to share the same learning environment for a longer period of time.

The popularity of MGI approach in current curricula, with its emphasis on self-directed discovery learning bears considerable societal implications. If children are perceived as responsible for their learning, it follows that learning difficulties can be attributed to the children themselves rather than to factors such as the learning environment, the type of instruction or even the educational system. Consequently, a phenomenon such as MD which is the result of a complex interplay between cognitive, affective and social – developmental factors is reduced to the level of the individual learner. According to such a reductionist approach, it is the child itself rather than the teacher who is in charge of ameliorating MD.

It is important to consider the moral implications of educational approaches and the ideological agenda behind the MGI approach seems to take away the responsibility for children's learning from teachers and educational policy makers. The assumption that children are responsible for structuring their own learning process and this might reflect a wider societal trend towards an emphasis on individual advancement and personal responsibility, as proposed by neoliberal rhetoric. By conclusion, the model proposed in the current thesis suggests that educational policy makers would need to reconsider the abovementioned MGI recommendations and adapt the curriculum with respect to the cognitive and affective characteristics of children with MD.

REFERENCES

REFERENCES

- Adams, G.L., & Englemann, S. (1996). *Research on Direct Instruction: 25 years beyond DISTAR*. Seattle, WA: Educational Achievement Systems.
- Adams, J. W., & Hitch, G. J. (1998). Children's arithmetic and working memory. In C.Donlan (Ed.) *The development of mathematical skills*. Hove: Psychology Press.
- Alexander, L. and Martray, C. (1989) The development of an abbreviated version of the Mathematics Anxiety Rating Scale. *Measurement and Evaluation in Counseling and Development*, 22, pp. 143 - 150.
- Allport, G.W. (1937). *Cognitive style: five approaches and relevant research*. Wiley, New York
- Arroyo, I., Beck, J., Schultz, K., Woolf, B.P., (1997) Piagetian Psychology in Intelligent Tutoring Systems, *Proceedings of The Ninth World Conference on Artificial Intelligence and Education*. France.
- Ashcraft, M. H. (1992). Cognitive arithmetic: A review of data and theory. *Cognition*, 44, 75-106.
- Ashcraft, M.H. and Faust, M.W. (1994) Mathematics anxiety and mental arithmetic performance: An exploratory investigation. *Cognition and Emotion*, 8, pp. 97 – 125.
- Ashcraft, M.H., and Kirk, E.P. (2001) The relationships among working memory, math anxiety, and performance. *Journal of Experimental Psychology – General*, Vol. 130 (2): 224 – 237.
- Ashcraft, M. H., & Krause, J. A. (2007). Working memory, math performance, and math anxiety. *Psychonomic Bulletin & Review*, 14, pp. 243-248.
- Baker, S., Gersten, R., & Lee, D.S. (2002). A synthesis of empirical research on teaching mathematics to low-achieving students. *Elementary School Journal*, 10, 51-73.

REFERENCES

Bandura, A., & Schunk, D. H. (1981). Cultivating competence, self-efficacy, and intrinsic interest through proximal self-motivation. *Journal of Personality and Social Psychology*, 41, pp. 586-598.

Bandura, A. (1982). Self-efficacy mechanism in human agency. *American Psychologist*, 37, pp. 122-147.

Bandura, A. (1986). *Social foundations of thought and action: A social cognitive theory*. Englewood Cliffs, NJ: Prentice-Hall.

Bandura, A. (1997). *Self-efficacy: The exercise of control*. New York: Freeman.

Bandura, A. (1977). Self-efficacy: Toward a unifying theory of behavioral change. *Psychological Review*, 84, pp. 191-215.

Baptist, J.; Minnie, L. Buksner, S. Kaye, R., Morgan, J. (2007) Screening in the early years for mathematics difficulties and disabilities: identifying red flags to support early learners at risk. *Orbit*. Vol 37, No.1.

Barrouillet, P.; Lepine, R. (2005) Working memory and children's use of retrieval to solve addition problems. *Journal of Experimental Child Psychology*, 91 (3). pp. 183-204

Barth, J. M., Dunlap, S. T., Dane, H., Lochman, J. E., & Wells, K. C. (2004). Classroom environment influences on aggression, peer relations, and academic focus. *Journal of School Psychology*, 42, pp. 115-133.

Battista, M.T. (1994) Teacher Beliefs and the Reform Movement in Mathematics Education. *Phi Delta Kappan*, pp. 462-70.

Becker, W. C., & Gersten, R. (1982) A follow-up of follow through: Meta-analysis of the later effects of the Direct Instruction Model. *American Educational Research Journal*, 19, 75-93.

REFERENCES

- Beishuizen, M. (1993). Mental strategies and materials or models for addition and subtraction up to 100 in Dutch second grades. *Journal for Research in Mathematics Education*, 24, 294- 323.
- Beishuizen, M., Van Putten, C. & Van Mulken, F. (1997). Mental arithmetic and strategy use with indirect number problems up to one hundred. *Learning and Instruction*, 7, 87-106
- Bereiter, C., & Kurland, M. (1981). A constructive look at Follow Through results. *Interchange*, 12(1), 1-22.
- Betz, N. E., Hackett , G. (1983). The Relationship of Mathematics Self-Efficacy Expectations to the Selection of Science-Based College Majors. *Journal of Vocational Behavior*, 23, pp. 329-345.
- Blumstengel, A. (1998). Entwicklung hypermedialer Lernsysteme, Berlin: Wissenschaftlicher Verlag.
- Bong, M. & Clark (1999). Comparison between self-concept and self-efficacy in academic motivation research. *Educational Psychologist*, 34 (3), pp.139-153.
- Bottge, B.A., & Hasselbring, T.S. (1993). Taking word problems off the page. *Educational Leadership*, 50(7), 36-38.
- Bransford, J.D.; Schwartz, D.L. (1999): Rethinking Transfer: A Simple Proposal With Multiple Implications. In: Review of research in Education 24. Washington D.C.: American Educational Research Association, S. 61-100.
- Briant, P., Christie, C., Rendu, A.(1999) Children's understanding of the relation between addition and subtraction: inversion, identity, and decomposition, *Journal of Experimental Child Psychology*, 74(3), pp.194-212.
- Bronfenbrenner, U. (1979). *The Ecology of Human Development: Experiments by Nature and Design*. Cambridge, MA

REFERENCES

- Bruder, R. (2003): Konstruieren – Auswählen – Begleiten. Über den Umgang mit Aufgaben. In *Friedrich Jahresheft XXI*, Seelze, 12-15.
- Bull, R., Johnston, J.S. & Roy, J.A. (1999). Exploring the roles of the visual-spatial sketch pad and central executive in children's arithmetic skills: Views from cognition and developmental neuropsychology. *Developmental Neuropsychology*, 15(3), 421-442.
- Campbell, J. I. D., & Clark, J. M. (1992). Numerical cognition: An encoding-complex perspective. In J. I. D. Campbell (Ed.). *The nature and origins of mathematical skills* (pp. 457-491). Amsterdam: Elsevier Science.
- Campbell, J. I. D. (1994). Architectures for numerical cognition. *Cognition*, 53, pp. 1-44.
- Campbell, J. I. D. (1995). Mechanisms of simple addition and multiplication: A modified network-interference theory and simulation. *Mathematical Cognition*, 1, pp. 121-164.
- Campbell, J.I.D., & Xue, Q. (2001). Cognitive arithmetic across cultures. *Journal of Experimental Psychology: General*, 130, pp. 299-315.
- Canobi, K.H. (2004). Individual differences in children's addition and subtraction knowledge, *Cognitive Development*, 19, 81 – 93.
- Canobi, K. H. (2005). Children's understanding of addition and subtraction. *Journal of Experimental Child Psychology*, 92, 220-246.
- Cardelle-Elawar, M. (1995). Effects of Metaocognitive Instruction on Low Achievers in Mathematics Problems. *Teaching & Teacher Education*, 11(1), 81-95.
- Cattell, R.B. and Kline, P. (1977). *The Scientific Analysis of Personality and Motivation*. London: Academic Press.

REFERENCES

Chochon, F., Cohen, L., Van de Moortele, P. And Dehaene, S. (1999). Differential contributions of the left and right inferior parietal lobes to number processing. *Journal of Cognitive Neuroscience*, 11, 617 – 630.

Cipolotti, L, Warrington, E. K., & Butterworth, B. (1995). Selective impairment in manipulating Arabic numerals. *Cortex*, 31, 73-86.

Clark RE. (1994). Media will never influence learning. *Educational Technology Research and Development*, 42, pp. 21–29.

Clark, R. E. (2001). *Learning from Media: Arguments, Analysis and Evidence*. Greenwich, Conn: Information Age Publishers.

Cumming, J., & Elkins, J. (1999). Lack of automaticity in the basic addition facts as a characteristic of arithmetic learning problems and instructional needs. *Mathematical Cognition*, 5(2), pp. 149-180.

D'Amico, A., & Guarnera, M. (2005). Exploring working memory in children with low arithmetical achievement. *Learning and Individual Differences*, 15(3), pp. 189–202.

Davis, R., McKnight, C. (1980). The Influence of Semantic Content on Algorithmic Behavior. *Journal of Mathematical Behavior*, 1, pp. 39-87.

Dehaene, S. (1997). *The number sense*. New York: Oxford University Press.

Dehaene, S., Cohen, L. (1997). Cerebral pathways for calculation: double dissociation between rote verbal and quantitative knowledge of arithmetic. *Cortex*, 33, pp. 219-250.

Dehaene, S., Cohen, L. (1995). Towards an anatomical and functional model of number processing. *Mathematical Cognition*, 1, pp.83-120.

REFERENCES

- De Rammelaere, S., Stuyven, E., & Vandierendonck, A. (2001). Verifying simple arithmetic sums and products: Are the phonological loop and the central executive involved? *Memory & Cognition*, 29, pp. 267-273.
- DeStefano, D. & LeFevre, J.-A. (2004). The role of working memory in mental arithmetic. *European Journal of Cognitive Psychology*, 16(3), 353-386.
- Dossel, S. (1993). Maths anxiety. *Australian Mathematics Teacher*, 49 (1), pp. 4-8.
- Dweck, C. (1986). Motivational processes affecting learning. *American Psychologist*, 41, pp.1040 - 1048.
- Engelmann, S., & Carnine, D. (1982). *Theory of instruction: Principles and applications*. New York: Irvington.
- Ericsson, K. A., & Polson, P. G. (1988). An experimental analysis of the mechanisms of a memory skill. *Journal of Experimental Psychology: Learning, Memory and Cognition*, 14, 305-316.
- Fennema, E.H. & Sherman, J.A. (1978). Sex – related differences in mathematics achievement and related factors: a further study. *Journal for Research in Mathematics Education*, 7, pp. 189 – 203.
- Fishbein, M., & Ajzen, I. (1975). *Belief, Attitude, Intention, and Behavior: An Introduction to Theory and Research*. Reading, MA: Addison-Wesley.
- Fiori C., Zuccheri L., (2005). An Experimental Research on Error Patterns in written subtraction, *Educational Studies in Mathematics*, 60 (3), pp.323-331.
- Fleischer, J., Pallack, A., Wirth, J. & Leutner, D. (2005). *Vergleichende Schulrückmeldungen im Rahmen der Lernstandserhebungen in Nordrhein-Westfalen*. Poster präsentiert auf der 67. Tagung der Arbeitsgruppe Empirisch-Pädagogischer Forschung (AEPF), Salzburg, Österreich.

REFERENCES

- Fletcher, J.D. (2003) Evidence for learning from technology – assisted instruction. In H.F. O' Neill, Jr. & R.S. Perez (Eds.) *Technology Applications in education : a learning view*. NJ: Lawrence Erlbaum Associates Publishers, Mahwab.
- Foxman, D., Beishuizen, M (2002). Mental calculation methods used by 11-year-olds in different attainment bands: a reanalyse of data from the 1987 APU survey in the UK - In: *Educational Studies in Mathematics*, 51, S. 41-69.
- Fraser, B.J. (1986). *Classroom Environment*. London: Croom Helm.
- Geary, D. C., Hoard, M. K., & Hamson, C. O. (1999). Numerical and Arithmetical Cognition: Patterns of Functions and Deficits in Children at Risk for a Mathematical Disability. *Journal of Experimental Child Psychology*, 74(3), 213–239.
- Goh, S.C. & Fraser, B.J. (1998). Teacher Interpersonal Behaviour, Classroom Environment and Student Outcomes in Primary Mathematics in Singapore Instruments, in B.J. Fraser and K.G. Tobins (Eds). *International Handbook of Science Education*, Kluwer Academic Publishers, UK.
- Goodenow, C. (1992). Strengthening the links between educational psychology and the study of social contexts. *Educational Psychologist*, 27, pp. 177-196.
- Gopnik, A. & A. N. Meltzoff (1997). *Words, thoughts, and theories*. Cambridge, Mass.: Bradford, MIT Press.
- Graff, M. G. (2003). Learning from web-based instructional systems and cognitive style' *British Journal of Educational Technology*, 34, (4) 407-418.
- Graham, S. (1984). Communicating sympathy and anger to black and white children: The cognitive (attributional) consequences of affective cues. *Journal of Personality and Social Psychology*. 47(1), pp. 40-54.
- Graham, S. (1991). A review of attribution theory in achievement contexts. *Educational Psychology Review*. 3(1), pp. 5-39.

REFERENCES

Hanchon Graham, L. (2000). *Self – efficacy, motivation constructs and math performance of middle school students: a three year longitudinal study*. Unpublished PhD thesis. Emory University. Emory.

Hart Reyes, L. (1984). Affective variables and mathematics education. *The Elementary School Journal*, Vol. 84, Nr. 5, pp. 558 – 581.

Hasselbring, T.S., Goin, L., & Bransford, J.D. (2006). Developing math automaticity in learning handicapped children: The role of computerized drill and practice. *Focus on Exceptional Children* pp. 1-7.

Hasselbring, T. S., & Goin, L. (2005). *FASTT Math*. [Computer Software]. Boston, MA: Tom Snyder Productions.

Hechinger, F. M. (1986, April 22). Preschool programs. *New York Times*, 17.

Hecht, S.A. (1999). Individual solution processes while solving addition and multiplication facts in adults. *Memory & Cognition*, 27, pp. 1097-1107.

Hembree, R. (1990). The nature, effects, and relief of mathematics anxiety. *Journal for Research in Mathematics Education*, 21, pp. 33-46.

Hogan - Gancarz, C. R. (1999) *Working memory and mathematics: Cognitive learning strategies use with students with learning disabilities*. Dissertation Abstracts International Section A: Humanities and Social Sciences Feb 1999 Vol 59(8-A).

House, E. R., Glass, G. V., McLean, L. D., & Walker, D. F. (1978). No simple answer: Critique of the Follow Through evaluation. *Harvard Educational Review*, 48(2), 128-159.

Hunt, G. E. (1985). Math anxiety—Where do we go from here? *Focus on Learning Problems in Mathematics*, 7, pp. 29-40.

REFERENCES

Hurry, J. & Sylva, K. (2007). Long term outcomes of reading interventions. *Journal of Research in Reading*, 30, (2), 1-22.

Hymel, S., Comfort, C., Schonert-Reichel, K., & McDougall, P. (1996). Academic failure and school dropout: The influence on peers. In J. Juvonen & K.R. Wentzel (Eds.), *Social Motivation: Understanding children's school adjustment*. New York: Cambridge University Press.

Imbo, I., Vandierendonck, A. (2007). The development of strategy use in elementary school children: Working memory and individual differences. *Journal of Experimental Child Psychology*, 96 (4), pp. 284–309.

Jahnke, S. (2001) *Sinnstiftender Mathematikunterricht für Mädchen und Jungen*. Schneider-Verlag Hohengehren

Jones, A. and Mercer, N. (1993). Theories of Learning and Information Technology. In Scrimshaw, P. (Ed.) *Language, Classroom and Computers*. London: Routledge.

Kauffman, J.M. (2005). *Characteristics of children's behavior disorders (7th ed.)*. Columbus, OH: Merrill.

Kellam, S. G., Ling, X., Merisca, R., Brown, C. H., & Ialongo, N. (1998). The effect of the level of aggression in the first grade classroom on the course and malleability of aggressive behavior into middle school. *Development and Psychopathology*, 10, pp. 165-185.

Kellam, S.G., Brown, C.H., Poduska, J., Ialongo, N., Wang, W., Toyinbo, P., Petras, H., Ford, C., Windham, A., Wilcox, H.C., (2008). Effects of a universal classroom behavior management program in first and second grades on young adult behavioral, psychiatric, and social outcomes. *Drug Alcohol Dependency*, 95, pp. 5 – pp. 28.

Kellogg, Jeffrey S.; Hopko, Derek R.; Ashcraft, Mark H. (1999). The Effects of Time Pressure on Arithmetic Performance. *Journal of Anxiety Disorders*, 13 (6), pp. 591–600.

REFERENCES

Khan, B. H. (Ed.). (1997). *Web-based instruction*. Englewood Cliffs, N.J.: Educational Technology Publications.

Kiamanesh, A.R., Hejazi, E. and Esfahani, Z.N.(2004). The Role of Math Self-Efficacy, Math Self-Concept, Perceived Usefulness of Mathematics and Math Anxiety in Math Achievement. *Proceedings of the 3rd International Biennial SELF Research Conference, Self-Concept, Motivation and Identity: Where to go from here?* Berlin, 4 - 7 July, 2004.

Kirschner, P.A., Sweller, J. & Clark, R.E. (2006). Why minimal guidance during instruction does not work: An analysis of the failure of constructivist, discovery, problem-based, experiential, and inquiry based teaching. *Educational Psychologist*, 41, 75-86.

Kline, P. (1994). *An easy guide to factor analysis*. London: Routledge.

Klimsa, P.(1993). *Neue Medien und Weiterbildung: Anwendung und Nutzung in Lernprozessen der Weiterbildung*. Deutscher Studien-Verlag, Weinheim. pp. 164.

Kline, P. (1999). *The handbook of psychological testing*. 2. London: Routledge.

Kloosterman, P. (1983). *Causal attribution theory and sex-related differences in mathematics*. Paper presented at the research pre-session of the annual meeting of the National Council of Teachers of Mathematics, Detroit.

Kozma RB. (1994). A reply: Media and methods. *Educational Technology Research and Development*, 42, 11–14.

Kroesbergen, E.H., Van Luit, J.E.H., & Maas, C.J. (2004). Effectiveness of explicit and constructivist mathematics instruction for low-achieving students in the Netherlands. *The Elementary School Journal*, 104 (3), pp. 233-251.

Kulik, J.A, Kulik, C.C (1987). Review of recent literature on computer-based instruction, *Contemporary Educational Psychology*, 12, pp.222-30.

REFERENCES

- Lee, K.; Kang, S. (2002). Arithmetic operation and working memory: differential suppression in dual tasks. *Cognition*, 83,(3), pp. 63-68.
- LeFevre, J., Sadesky, G. S., & Bisanz, J. (1996). Selection of procedures in mental addition: Reassessing the problemsize effect in adults. *Journal of Experimental Psychology: Learning, Memory, and Cognition*, 22, pp. 216-230.
- Lemaire, P., & Siegler, R. S. (1995). Four aspects of strategic change: Contributions to children's learning of multiplication. *Journal of Experimental Psychology: General*, 124, 83-97.
- Lewis, A. (1970). The ambiguous word "anxiety." *International Journal of Psychiatry*, 9, pp. 62-79.
- Lewis, K. G. (2001). Using midsemester student feedback and responding to it. *New Directions for Teaching and Learning*, 87, pp. 33-44.
- Loewenthal, K.M. (2001). *An Introduction to Psychological Tests and Scales (2nd ed)*, Psychology Press, Hove.
- Logie, R. H., Gilhooly, K.J., & Wynn, V. (1994). Counting on working memory in arithmetic problem solving. *Memory & Cognition*, 22 (4), pp. 395-410.
- Lucangeli D., Tressoldi P.E., Bendotti M., Bonanomi M., Siegel L.S. (2003). Effective Strategies for Mental and Written Arithmetic Calculation from the Third to the Fifth Grade. *Educational Psychology*, 23, pp. 507-520.
- Luo, Z., Jose, P. E., Huntsinger C. S., & Pigott T. D. (2007). Fine motor skills and mathematics achievement in East Asian American and European American kindergartners and first graders. *British Journal of Developmental Psychology*, 595–614.

REFERENCES

Ma, X. (1999). A meta-analysis of the relationship between anxiety toward mathematics and achievement in mathematics. *Journal for research in mathematics education*, 30(5), pp. 520-540

Ma, X. & Xu, J. (2004). The Causal Ordering of Mathematics Anxiety and Mathematics Achievement: A Longitudinal Panel Analysis. *Journal of Adolescence*, 27 (2), pp. 165-179.

Mandler, G. & Sarason S. B. (1952). A study of anxiety and learning. *Journal of Abnormal & Social Psychology*. 47, pp.166-73.

Marsh, H.W. (1987). The Big-Fish-Little-Pond effect on academic self concept. *Journal of Educational Psychology*, 79, pp. 280–295.

Marsh, H.W. & Craven, R. (1997). Academic Self-concept: Beyond the dustbowl. In G.D. Phye (Ed.), *Handbook of classroom assessment*. San CA: Academic Press.

Mauro, D. G., LeFevre, J., & Morris, J. (2003). Effects of problem format on division and multiplication performance: Evidence for mediation of division by multiplication. *Journal of Experimental Psychology: Learning, Memory, & Cognition*, 29, 163-170.

Mayer, R. (2004). Should there be a three-strikes rule against pure discovery learning? The case for guided methods of instruction. *American Psychologist*, 59, pp.14–19.

McLeod, D.B. (1992). Research on affect in mathematics education: A reconceptualization. In D.A. Grouws (Ed.) *Handbook of research on mathematics teaching and learning*. New York: Macmillan.

Meece, J. L., Wigfield, A., & Eccles, J. S. (1990). Predictors of math anxiety and its consequences for young adolescents' course enrollment intentions and performances in mathematics. *Journal of Educational Psychology*, 82, pp. 60-70.

REFERENCES

- Melancon, J. G., Thompson, B., & Becnel, S. (1994). Measurement integrity of scores from the Fenemma-Sherman Mathematics Attitudes Scales: The attitudes of public school teachers. *Educational and Psychological Measurement*, 54 (1), pp. 187-192.
- Meyer, L. (1984). Long-term academic effects of the Direct Instruction Project Follow Through. *Elementary School Journal*, 84, 380-394.
- Moore, L. & Carnine, D. (1989). Evaluating curriculum design in the context of active teaching. *Remedial and Special Education*, 10, 28-37.
- Mulhern, F. & Rae, G. (1998). Development of a shortened form of the Fennema-Sherman Mathematics Attitudes Scales. *Educational and Psychological Measurement*, 58(2), pp. 295-306.
- Murdock, T. B., & Miller, A. D. (2003). Teachers as sources of middle school students' motivational identity: Variable-centered and person-centered analytic approaches. *Elementary School Journal*, 103, pp. 383-399.
- Murray, T. (1999). Authoring intelligent tutoring systems: an analysis of the state of the art. *International Journal of Artificial intelligence in education*, 10, pp. 98 – 129.
- National Council of Teachers of Mathematics. (2000). *Standards for School Mathematics*. Retrieved March 1, 2006, from <http://www.nctm.org/standards/standards.htm>
- Newstead, K. (1998) Aspects of children's mathematics anxiety. *Educational Studies in Mathematics* 36 (1), pp. 53-71.
- Paivio, A. (1986). *Mental representations: a dual coding approach*. Oxford, England: Oxford University Press.
- Pajares, M. F. (1992). Teacher's beliefs and educational research: Cleaning up a messy construct. *Review of Educational Research*, 62, pp.307-322.

REFERENCES

Pajares, F., & Miller, M. D. (1994). The role of self-efficacy and self-concept beliefs in mathematical problem solving: A path analysis. *Journal of Educational Psychology*, 86, pp.193-203.

Pajares, F. and Kranzler, J.H. (1995). Self-efficacy beliefs and general mental ability in mathematical problem-solving. *Contemporary Educational Psychology*, 20, pp.426-443.

Pajares, F. (1997). Current directions in self-efficacy research. In M. Maehr & P. R. Pintrich (Eds.). *Advances in motivation and achievement*. Greenwich, CT: JAI Press.

Papanikolaou, K., Grigoriadou, M., Kornilakis, H., & Magoulas G. (2003) Personalising the Interaction in a Web-based Educational Hypermedia System: the case of INSPIRE. *International Journal of User Modeling and User-Adapted Interaction*, 13 (3), 213-267.

Parmar, R.S. (2003). Understanding the concept of division: Assessment considerations. *Exceptionality*, 11, pp. 177 – 189.

Patrick, H., Ryan, A.M. and Kaplan, A. (2007). Early adolescents' perception of the classroom social environment, motivational beliefs and engagement. *Journal of Educational Psychology*, 99 (1), pp. 83 – 98.

Pesenti, M., Thioux, M., Seron, X., & De Volder, A. (2000). Neuroanatomical substrate of Arabic number processing, numerical comparison and simple addition: A PET study. *Journal of Cognitive Neuroscience*, 12 (3), 461-479.

Peterson, C. (1990). Explanatory style in the classroom and on the playing field. In S. Graham & V.S. Folkes (Eds.) *Attribution theory: Applications to achievement, mental health, and interpersonal conflict*. Hillside, New Jersey: Lawrence Erlbaum Associates.

Riding, R., & Cheema, I. (1991). Cognitive styles - an overview and integration. *Educational Psychology*, 11 (3-4), pp. 193-215.

Riding, R. (1991). *Cognitive Styles Analysis*. Birmingham: Learning and Training Technology.

REFERENCES

- Robinson, K. M., Arbuthnott, K. D., Rose, D., McCarron, M. C., Globa, C. A. & Phonexay, S. D. (2006). Stability and change in children's division strategies. *Journal of Experimental Child Psychology*, 93, 3, pp. 224-238.
- Rosic, M.; Glavinic, V.; Stankov, S. (2006). Intelligent tutoring systems for the new learning infrastructure. *AIS SIGSEMIS Bulletin*, 2 (4).
- Roussel, J., Fayol, M., & Barrouillet, P. (2002). Procedural vs. direct retrieval strategies in arithmetic: A comparison between additive and multiplicative problem solving. *European Journal of Cognitive Psychology*, 14, pp. 61–104.
- Sandrini, M., Miozzo, A., Cotelli, M. and Cappa, S.F. (2003) The residual calculation abilities of a patient with severe apraxia: evidence for a selective deficit of subtraction procedures, *Cortex*, 39, pp. 85 – 96
- Seitz, K., & Schumann-Hengsteler, R. (2002). Phonological loop and central executive processes in mental addition and multiplication. *Psychologische Beiträge*, 44, pp. 275-302.
- Seyler, D.J., Kirk, E.P, & Ashcraft, M.H. (2003). Elementary subtraction. *Journal of Experimental Psychology: Learning, Memory, and Cognition*, 29, pp. 1339-1352.
- Schunk, D. H. (1987). Peer models and children's behavioral change. *Review of Educational Research*, 57, pp. 149 - 174.
- Schunk, D. H. (1982a). Effects of effort attributional feedback on children's perceived self-efficacy and achievement. *Journal of Educational Psychology*, 74, pp. 548-556.
- Scrimshaw, P., (1993). *Language, classrooms and computers*, Routledge, London.
- Selter, Ch. (2001). Addition and Subtraction of Three-digit Numbers: German Elementary Children's Methods, Success and Strategies. *Educational Studies in Mathematics* 47/2, pp. 45-173.

REFERENCES

- Sherin, B., & Fuson, K. (2005). Multiplication strategies and the appropriation of computational resources. *Journal for Research in Mathematics Education* 36(4), pp. 163-171.
- Shrager, J. & Siegler, R. S. (1998). SCADS: A model of children's strategy choices and strategy discoveries. *Psychological Science*, 9, pp. 405-410
- Siegler, R.S. (1988). Strategy choice procedures and the development of multiplication skills. *Journal of Experimental Psychology: General*, 116, pp. 258 – 275.
- Siegler, R.S. and Shipley, C. (1995). Variation, selection and cognitive change. In T.J. Simon & G.S. Halford (Eds.) *Developing cognitive competence: New approaches to process modelling*. Hillsdale, NJ: Lawrence Erlbaum.
- Siegler, R. S. (2003.) Implications of cognitive science research for mathematics education. In Kilpatrick, J., Martin, W. G., & Schifter, D. E. (Eds.), *A research companion to principles and standards for school mathematics* (pp. 119-233). Reston, VA: National Council of Teachers of Mathematics.
- Siegler, R., and Jenkins, E. (1998). Strategy Discovery and Strategy Generalization. In: *How Children Discover New Strategies*. Lawrence Erlbaum Associates, Hillsdale, NJ, pp. 1-20.
- Sitzmann, T., Kraiger, K., Stewart, D., & Wisher, R. (2006). The comparative effectiveness of Web-based and classroom instruction: A meta-analysis. *Personnel Psychology*, 59, pp. 623-664.
- Spiro, R.J., Jengh, J. (1990). Cognitive Flexibility and Hypertext: Theory and Technology for the Non-linear and Multidimensional Traversal of Complex Subject Matter. D. Nix & R. Spiro (eds.), *Cognition, Education and Multimedia*. Erlbaum, Hillsdale, NJ.

REFERENCES

Stebbins, L. B., St. Pierre, R. G., Proper, E. C., Anderson, R. B., & Cerva, T. R. (1977). *Education as experimentation: A planned variation model: Vol 4-A. An evaluation of Follow Through*. Cambridge, MA: Abt Associates.

Stein, M., Silbert, J., & Carnine, D. (1997). *Designing effective mathematics instruction: A direct instruction approach*. Upper Saddle River, New Jersey: Merrill.

Steinberg, Laurence. (1996). *Beyond the classroom: Why school reform has failed and what parents need to do*. New York: Simon & Schuster.

Stodolsky, S. (1985). Telling math: origins of math aversion and anxiety. *Educational Psychologist*, 20, (3), pp. 125 – 133.

Suppes Patrick (1969). *Sets and Numbers (Books 3-6)*. Revised edition. New York: Random House.

Tarver, S. G. & Jung, J. S. (1995). A Comparison of Mathematics Achievement and Mathematics Attitudes of First and Second Graders Instructed With Either a Discovery-Learning Mathematics Curriculum or a Direct Instruction Curriculum. *Effective School Practices*, 14(1), 49-56.

Tietze U.P., Klika M, Wolpers H., (1997). *Mathematikunterricht in der Sekundarstufe II. Band 1: Didaktische Grundfragen - Didaktik der Analysis*. Verlag Vieweg, Braunschweig.

Thompson, I. (1999). Mental calculation strategies for addition and subtraction. *Mathematics in School*. London: The Mathematical Association.

Thornton, C. (1990). Strategies for the basic facts. In J. Payne (Ed.), *Mathematics for the young child (pp. 133-51)*. Reston, VA: National Council of Teachers of Mathematics.

Tobias, S. (1985). Math anxiety and physics: The problem of teaching 'difficult' subjects. *Physics Today*, 38 (6), pp. 61-68.

REFERENCES

- Torbeyns, Joke; Verschaffel, Lieven; Ghesquiere, Pol (2004). Strategic aspects of simple addition and subtraction: the influence of mathematical ability. In: *Learning and Instruction*, 14, (2), pp. 177–195.
- Trbovich, P. L., & LeFevre, J.-A. (2003). Phonological and visual working memory in mental addition. *Memory & Cognition*, 31(5), pp. 738-745.
- Tremblay, C., Lemoyne, G. (1986). Addition and multiplication: Problem – Solving and Interpretation of relevant data. *Educational Studies in Mathematics*, 17, (2).
- Tuovinen, J. E. (1999). *The cognitive load of discovery learning*. Unpublished PhD, University of New South Wales, Sydney.
- Tuovinen, J. E., & Sweller, J. (1999). A comparison of cognitive load associated with discovery learning and worked examples. *Journal of Educational Psychology*, 91, pp.334–341.
- U.S. Department of Health and Human Services, Administration for Children and Families (2005). *Head Start Impact Study: First Year Findings*. Washington, DC.
- Walberg, H. J., and S. J. Paik. (1997). Home Environments for Learning. In Herbert J. Walberg and G. D. Haertel, eds. *Psychology and Educational Practice*. Berkeley, Calif.: McCutchan Publishing.
- Weiner, B. (1992). *Human Motivation: Metaphors, Theories and Research*. Newbury Park, CA: Sage Publications.
- Weiner, B., Graham, S., & Chandler, C. C. (1982). Pity, anger, and guilt: An attributional analysis. *Personality and Social Psychology Bulletin*, 8, pp. 226-232.
- Wentzel, K. R. (1998). Social support and adjustment in middle school: The role of parents, teachers, and peers. *Journal of Educational Psychology*, 90, pp. 202-209.

REFERENCES

Widaman, K.F., Geary, D.C., Cormier, O. & Little, T.D. (1989). A componential model of mental addition. *Journal of Experimental Psychology: Learning, Memory & Cognition*, 15, pp. 898–919.

Wigfield, A. & Meece, J.L. (1988). Math anxiety in elementary and secondary school students. *Journal of Educational Psychology*, 80, pp. 210- 216.

Wine, J. D. (1971). Test anxiety and direction of attention. *Psychological Bulletin*, 76, pp. 92-104.

Winter, H. (1995): Mathematikunterricht und Allgemeinbildung. In *Mitteilungen der Gesellschaft für Didaktik der Mathematik*, 61, pp. 37-46

Wood, E. (1988). Mathematics anxiety and elementary teachers: What the research tells us. *For the Learning of Mathematics*, 8 (8).

Woodward, J., & Baxter, J. (1997a). The effects of an innovative approach to mathematics on academically low achieving students in mainstreamed settings. *Exceptional Children*, 63(3), pp. 373-388.

Yazdani, M. (1987). Artificial Intelligence, Powerful Ideas and Children's learning in Rutkowska & Crook (eds.) *Computers, Cognition and Development*. Chichester: Wiley.

Zhou, X. Chen, C., Dong, Q., Zhang, H. Zhou, R., Zhao, H., Chen, C., Qiao, S., Jiang, T. and Guo, Y. (2006). Event-related potentials of single-digit addition, subtraction, and multiplication, *Neuropsychologia*, 44, pp. 2500–2507.

Zhou, X., & Dong, Q. (2003). The representation of addition and multiplication. *Acta Psychologica Sinica*, 35, pp. 345–351.

REFERENCES

Table: Inter-item correlations

Item	Corrected Item-Total Correlation	Cronbach's Alpha if Item Deleted
Item 2	.17	.83
Item 3	.17	.83
Item 4	.25	.83
Item 5	.39	.82
Item 6	.22	.83
Item 7	.31	.83
Item 8	.30	.83
Item 9	.29	.83
Item 10	.21	.83
Item 11	.37	.82
Item 12	.23	.83
Item 14	.22	.83
Item 15	.26	.83
Item 16	.29	.83
Item 17	.32	.83
Item 18	.40	.82
Item 19	.42	.82
Item 20	.24	.83
Item 21	.22	.83
Item 22	.43	.82
Item 23	.26	.83
Item 24	.44	.82
Item 25	.33	.83
Item 26	.44	.82
Item 27	.40	.82
Item 28	.42	.82
Item 29	.19	.83
Item 31	.34	.83
Item 32	.24	.83
Item 33	.24	.83
Item 35	.44	.82
Item 36	.31	.83
Item 37	.43	.82
Item 38	.37	.82
Item 39	.46	.82
Item 41	.43	.82
Item 43	.21	.83