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Geraint James Davies

Numbers, Dimensions, Perceptions: Resonances from Dürer's *MELENCOLIA I*

VOLUME I

Geraint James Davies, BA (Hons)

Numbers, Dimensions, Perceptions: Resonances from Dürer's *MELENCOLIA I*

Submitted for PhD

University of Wales, Newport
School of Art, Media and Design

2012

This thesis was submitted in partial fulfilment of the requirements of the University of Wales for the degree of Master/Doctor of Philosophy.

DECLARATION

This work has not previously been accepted in substance for any degree and is not being concurrently submitted in candidature for any degree.

Signed .... `(candidate)

Date 04.04.2012

STATEMENT 1

This thesis is the result of my own investigations, except where otherwise stated. Where correction services have been used, the extent and nature of the correction is clearly marked in a footnote(s).

Other sources are acknowledged by footnotes giving explicit references. A bibliography is appended.

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I hereby give consent for my thesis, if accepted, to be available for photocopying and for inter-library loan, and for the title and summary to be made available to outside organisations.

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Abstract

This PhD combines theory and practice in order to explore how art can express fundamental scientific truths, principally through an understanding of numerical harmony and its expression in and through geometric form. Consequently, in making accommodation and finding concomitance between Art and Science, this thesis intends to address, on the one hand, some quite broad and difficult philosophical questions within the truth paradigm. A number of dual relationships are investigated, this principally through a central and more clearly defined dichotomous relationship between 'the literary' and 'the mathematical', where the one is imbued with the uncertainty and difficulties in interpretation and understanding of 'word', the other being the more rationally dependable and more consistent language of 'number', principally expressed through and within examples of proportional and geometric harmonies. All this encompasses an important 'human' condition, that of the 'believer'/'rationalist' duality, that then indicates the biologically based problem of psychological accommodation.

On the other hand, in order to further elucidate the ontological/phenomenological divide as a precursor to understanding (the most) difficult issues within the truth paradigm, and ultimately the relationship between perception and consciousness, an in-depth investigation is made into the mathematical properties of perfect and imperfect harmonic relationships, taking the enigmatic polyhedron in Dürer's *MELENCOLIA I* as a central focus of a wider locus that encompasses also some related properties of Islamic two-dimensional patterning as well as important numerical relationships within three-dimensional geometric space. This is a practice based thesis and, therefore, the parallel practical and artistic nature of this investigation is seen as important in enabling the acquisition of further knowledge and in helping to underpin the theoretical, as well as in corroborating a number of innovative mathematical propositions.

Lastly, the bilingual nature of this document and the difficulties encountered in attempting to obtain as closely similar nuances and meaning within each language as it is possible to do through word and syntax – these being culturally different and contextually, experientially acquired – only serves to highlight and accentuate one of the central issues concerned; in that although thinking (as a cognitive process) may well happen in and through an internalized verbal language, the potential presence of ontological realities that are outside our ability to perceive and to comprehend may best be indicated by a further understanding of mathematical and essentially numerical truths. These may be indicated visually through geometry and, it is contended, can therefore best be disseminated culturally as included within a form of artistic representation, either overtly or covertly.

Contents

VOLUME I

Thesis

Chapter 2
The Power of Word: the Beauty of Number

A. Language

B. Mysticism

C. Mathematics

D. Development

Chapter 3
Perceiving Form and Number

A. The numerical and two-dimensional structure

B. Connecting Two- and Three-dimensional Structure through Number

C. Historical Resonances

Exhibition

Codicil

VOLUME II

Thesis Plates

Exhibition Plates

List of Plates

Introduction

Chapter 2

Plate 1 DŰRER, Albrecht. 1513. *Knight, Death and the Devil.*
[engraving, 247 x 190 mm] URL

Plate 2a DŰRER, Albrecht. 1514. *Melencolia I.*
[engraving, 241 x 192 mm] London: British Museum,
Dept. of Prints & Drawings, AN 550243001©
[© Trustees of the British Museum] and URL

Plate 2b Relating the polyhedron to the magic square.

Plate 3 DŰRER, Albrecht. 1514. *St. Jerome in his Study.*
[engraving, 249 x 192 mm] URL

Plate 4 Questionable attribution to DŰRER, Albrecht. 1504. Title plate to
Messahalah de Scientia Motus Orbis. URL

Plate 5 RAPHAEL. 1509-10. *School of Athens.*
[fresco, 500cm x 770cm] Vatican City: Apostolic Palace, Rome. URL

Plate 6 Loadstone/magnetite, mainly octahedral.
Archived Huañaquino, Potosí Department, Bolivia. URL

Plate 7 (upper) Fused octahedral crystals. URL
(lower) Iron pyrite dodecahedral forms. URL

Plate 8 (upper) Some rhombic faces within magnetite sample from the Tazheran massif, Pribaikal. URL
(lower) Martite Pseudomorph after Magnetite. Harts Range, NT Australia. URL

Plate 9a ANON. 1514-1550. [copy of DŰRER engraving] London: British Museum,
Dept. of Prints & Drawings, AN 550244001©. URL
[© Trustees of the British Museum]

Plate 9b (upper) Detail from previous image.
(middle) Detail from Wölflin/Waetzoldt/Steck sources. [as for Plate 2a] URL
(lower) Detail from British Museum print, AN 30294001© – Panofsky/Bartrum.
[© Trustees of the British Museum] URL

Plate 10 DŰRER, Albrecht. 1493. *Self-portrait.*
[oil on linen, transferred from vellum, 57 x 45 cm] Musee du Louvre, Paris. URL

Plate 11a Extract from: YATES, Frances A. 1960. 'Ramon Lull and John Scotus Erigena',
Journal of the Warburg and Cortauld Institutes, Vol. 23, No. 1 / 2, pp. 1-44.

Plate 11b Detail from previous image.

Plate 12a DŰRER. *Dresden Sketchbook.* Sketch of his polyhedron.
[202 x 193 mm; watermark: waterwheel with letter *p*.]

In STRAUSS, Walter L. 1972. *Albrecht Dürer: The Human Figure; the Complete 'Dresden Sketchbook'.* New York: Dover Publications, Inc, p. 283.

Plate 12b An enlargement of the detail.

Plate 13a Photograph of *Μελεγχολία Α, Β, Γ, Δ, Ε* and *Ζ.* Glynn Vivian Art Gallery, Swansea, 31st July – 11th Sept, 2009.

Plate 13b ditto. Glynn Vivian Art Gallery, Swansea, 31st July – 11th Sept, 2009.

Plate 14 Photograph of *Μελεγχολία Γ, Δ* and *Ε.*

Plate 15 DŰRER, Albrecht. 1525. *Underweysung der Messung.*
Perspective drawing apparatus.
In DÜRER, Albrecht. 1972. *Underweysung der Messung* (a facsimile reprint of unrevised 1525 edition). Portland, Oregon: Collegium Graphicum.

Plate 16 DŰRER, Albrecht. Sketch dated 1514. *Dresden Sketchbook.*
[228 x 187 mm; Watermark: anchor in circle]
In STRAUSS, Walter L. 1972. *Albrecht Dürer: The Human Figure; the Complete 'Dresden Sketchbook'.* New York: Dover Publications, Inc, p. 293.

Plate 17a Eberhard Schröder diagram.
In SCHRÖDER, Eberhard. 1980. *Dürer – Kunst und Geometrie.* Stuttgart: Birkhäuser, Pl. 59.

Plate 17b Eberhard Schröder diagram.
In SCHRÖDER, Eberhard. 1980. *Dürer – Kunst und Geometrie.* Stuttgart: Birkhäuser, Pl. 60.

Plate 17c Eberhard Schröder diagram.
In SCHRÖDER, Eberhard. 1980. *Dürer – Kunst und Geometrie.* Stuttgart: Birkhäuser, Pl. 61.

Plate 18a Terence Lynch's Fig. 3.
In LYNCH, Terence. 1982. The Geometric Body in Dürer's Engraving Melencolia I. *Journal of the Warburg and Courtauld Institutes, University of London, Volume Forty-Five.* [Also now available online from JSTOR]

Plate 18b Terence Lynch diagrams and notes.
In LYNCH, Terence. 1982. The Geometric Body in Dürer's Engraving Melencolia I. *Journal of the Warburg and Courtauld Institutes, University of London, Volume Forty-Five*, p. 228.

Plate 18c Terence Lynch diagrams and notes.
In LYNCH, Terence. 1982. The Geometric Body in Dürer's Engraving Melencolia I. *Journal of the Warburg and Courtauld Institutes, University of London, Volume Forty-Five*, p. 231.

Plate 19a Sketches from Dürer's Nürnberg-Codex.
In WEITZEL, Hans. 2007. Zum Polyeder auf *A. Dürers* Stich Melencolia I -

ein Nürnberger Skizzenblatt mit Darstellungen archimedischer Körper. *Sudhoffs Archiv, Band 91, Heft 2. (British Library 8509.150000)*, p. 156.

Plate 19b Sketches from Dürer's Nürnberg-Codex.
In WEITZEL, Hans. 2007. Zum Polyeder auf *A. Dürers* Stich Melencolia I – ein Nürnberger Skizzenblatt mit Darstellungen archimedischer Körper. *Sudhoffs Archiv, Band 91, Heft 2. (British Library 8509.150000)*, p. 157.

Plate 20 HANS HOLBEIN the Younger. 1533. *The Ambassadors.*
London: National Gallery. URL

Plate 21 FLUDD, Robert. 1617. Illustration of monochord.
In FLUDD, Robert. 1973. *Utriusque Cosmi Maioris Scilicet et Minoris Metaphysica, Physica Atqve Technica Historia.* [Facsimile - a complete photographic record of the original 1617-19 edition in the Bancroft Library.] Berkeley: University of California.

Plate 22 Fig.15 from DŰRER. 1525 edition. *Underweysung der Messung.*
In DÜRER, Albrecht. 1972. *Underweysung der Messung* (a facsimile reprint of unrevised 1525 edition). Portland, Oregon: Collegium Graphicum.

Plate 23 Attributed to BARBARI, Jacopo de'. 1495. *Painting of Luca Pacioli.*
[oil on wood. 99 x 120 cm]
Naples: Museo Nazionale di Capodimonte. URL

Plate 24 DŰRER. Albrecht. 1532 Latin edition. Pages from *De Symmetria Patrium in Rectis Formis Humanorum Corporum.*
In DÜRER, Albrecht. 2003. *De Symmetria Patrium in Rectis Formis Humanorum Corporum* (1532 Latin edition) and *Underweysung der Messung* (1538 German edition) – full photographic images of single bound edition, together with English translation. Octavo Editions/The Warnock Library (www.octavo.com). Also view un-translated only at www.rarebookroom.org

Plate 25 DŰRER, Albrecht. Drawing from *Dresden Sketchbook.*
[152 x 142 mm; no watermark]
In STRAUSS, Walter L. 1972. *Albrecht Dürer: The Human Figure; the Complete 'Dresden Sketchbook'.* New York: Dover Publications, Inc, p. 289.

Plate 26 PACIOLI, Luca. 1509. *Divina Proportione.* Leonardo da Vinci's drawing of an Archimedean solid.
In PACIOLI/PAGIOLI, Luca. 1509. *Divina Proportione* and *Euclidis Megarensis Philosophi Acutissimi Mathematicorum* [microfilm]. The Institute of Chartered Accountants in England and Wales.

Plate 27 PACIOLI, Luca. 1509. *Divina Proportione.* Leonardo da Vinci's drawing of a Platonic solid.
In PACIOLI/PAGIOLI, Luca. 1509. *Divina Proportione* and *Euclidis Megarensis Philosophi Acutissimi Mathematicorum* [microfilm]. The Institute of Chartered Accountants in England and Wales.

Plate 28	PACILI, Luca. Fig.13 within a page of *Divina Proportione*. In PACIOLI/PAGIOLI, Luca. 1509. *Divina Proportione* and *Euclidis Megarensis Philosophi Acutissimi Mathematicorum* [microfilm]. The Institute of Chartered Accountants in England and Wales.
Plate 29	Fig.13 isolated.
Plate 30	DÜRER, Albrecht. Dodecahedron net: Fig. 33 from *Underweysung der Messung*. In DÜRER, Albrecht. 1972. *Underweysung der Messung* (a facsimile reprint of unrevised 1525 edition). Portland, Oregon: Collegium Graphicum.
Plate 31	DÜRER, Albrecht. Preparatory drawing. London: British Library. [MS Sloane 22758.]
Plate 32	DÜRER, Albrecht. Fig. 24 from *Underweysung der Messung*. In DÜRER, Albrecht. 1972. *Underweysung der Messung* (a facsimile reprint of unrevised 1525 edition). Portland, Oregon: Collegium Graphicum.
Plate 33	DÜRER, Albrecht. *Geometric Designs*. London: British Library. [MS Sloane 5229, fol.142r] In KEMP, Martin. 1990. *The Science of Art: Optical Themes in Western Art from Brunelleschi to Seurat.* London & New Haven: Yale University Press, p. 57.
Plate 34	Full-sized version of *Μελεγχολία B.* **cre8ive** Exhibition, Oriel Lliw Gallery, Pontardawe, 26th Aug – 19th Sept, 2009.
Plate 35	Another view of Full-sized version of *Μελεγχολία B.* **cre8ive** Exhibition.
Plate 36	Closer view to show detail of 'magic triangle'. **cre8ive** Exhibition.
Plate 37	*The Emergence of Phi*, acrylic on canvas, 2009. **cre8ive** Exhibition.
Plate 38	Golden section patterns.
Plate 39	*The Consolidation of Phi*, acrylic on canvas, 2009. **cre8ive** Exhibition.
Plate 40a	Preparatory outline drawing.
Plate 40b	Initial colouring stages.
Plate 40c	Further colouring stages.
Plate 40d	Developmental sequence and final realisation.
Plate 41a	Indication of the proposed 5:4 truncation integrated within Dürer's polyhedron.
Plate 41b	A two-dimensional indication of (i) the 80° angle for Dürer's three-dimensional polyhedron and (ii) a 63½° angle as reference to the three-dimensional golden rhombic angle.

Chapter 3

Plate 42	Illustration within: REISCH, Gregor. 1503. *Margarita Philosophica.* Freiburg: Johann. Schott. URL
Plate 43	"A Lady with Flower and Fly Whisk" India, Mughal; c.1630 [miniature: 25 × 14 cm] (David Collection). In FOLSACH, Kjeld von. 2007. *For the Privileged Few: Islamic Miniature Painting from the David Collection.* Louisiana: Louisiana Museum of Modern Art, p.192.
Plate 44	Miniature from Hafiz-i Abru's ***Majma al-tawarikh***, "Noah's Ark", Iran (Afghanistan), Herat; c. 1425. [Leaf: 42.3 × 32.6 cm] (David Collection). URL
Plate 45	*Mourning of the Death of Muhammed*, Siyer-I Nebi: The Life of the Prophet, Istanbul: 1595 in Topkapi collection. URL
Plate 46	Mosaic within the Great Mosque of Damascus, 715. URL
Plate 47	Mosaic tile-work within Sirçali Madrassa, Konya, 1243. In RICE, David Talbot. 1975. *Islamic Art.* London: Thames & Hudson, Plate 174.
Plate 48	Darb-i Imam shrine in Isfahan, Iran, built in the 1600s. URL
Plate 48a	Developmental drawing.
Plate 48b	Developmental drawing.
Plate 48c	Developmental drawing.
Plate 48d	Developmental drawing.
Plate 48e	Developmental drawing.
Plate 48f	Developmental drawing.
Plate 48g	Detail from Darb-i Imam shrine.
Plate 48h	Developmental drawing.
Plate 48i	Developmental drawing
Plate 48j	*Before 5, Beyond 4.*
Plate 48k	*Beyond 4*, isolated.
Plate 49	Gunbad-i Kabud tomb tower in Maragha, Iran, c.1197. URL
Plate 50	Arabesque from the Alcázar of Seville, once a Moorish palace. URL
Plate 51	Fragmentary bowl. Benaki Museum, Athens.

In PHILON, Helen. 1980. *Early Islamic Ceramics (Ninth to Late Twelfth Centuries) – Vol.1 from the Catalogue of Islamic Art of the Benaki Museum, Athens.* London: Sotheby Parke Bernet Publications/Islamic Art Publications, Fig. 515.

Plate 52 Mosaic tile pattern in the courtyard of the Madrasa al-'Attarin, Fez, 14th century. In CRITCHLOW, Keith. 1976. *Islamic Patterns: An Analytical and Cosmological Approach.* London: Thames & Hudson, cover photograph.

Plate 53 ***Jali*** screen (one of a pair), second half of 16th century; Mughal, India. [carved red sandstone. H. 73 ¼ in. (186 cm), W. 51 3/16 in. (130 cm), Th. 3 9/16 in. (9 cm)]. Rogers Fund, 1993 (1993.67.2); The Metropolitan Museum of Art. URL

Plate 54 Octagonal pattern, from the Alhambra, Granada. URL

Plate 55 Harmonious patterning, from the Alhambra. URL

Plate 56 Tile detail and plaster work from the Alhambra. URL

Plate 57a Plaster work within the Alhambra. URL

Plate 57b Plaster work within the Alhambra. URL

Plate 58 Nasrid plasterwork pannel from the Dar al-Manjara l-kubra (Royal Chamber of Santo Domingo, Granada). [Photograph taken from *Journal of Cultural Heritage* 5, 2004, p 85.] In BUENO, Ana García & FLÓREZ, Víctor J. Medina. 2003. The Nasrid plasterwork at "qubba Dar al-Manjara l-kubra" in Granada: characterisation of materials and techniques. *Journal of Cultural Heritage, 5(2004) 75-89, p.85.* Also URL

Plate 59 Upper section of the eastern wall, El Transito Synagogue, Toledo. Model - Beit Hatfutsot, the Core Exhibition. URL

Plate 60 Dado panel, first half of 15th century; Mamluk, Egypt. [polychrome mosaic. H. 45 ¼ in. (114.9 cm), W. 23 in. (58.4 cm)] Gift of the Hagop Kevorkian Fund, 1970 (1970.327.8). The Metropolitan Museum of Art. URL

Plate 60a Fusion of two replicated panels.

Plate 60b Developmental drawing.

Plate 60c Developmental drawing.

Plate 60d Developmental drawing.

Plate 61 Analysis of the geometry within Critchlow's example of Islamic design. In CRITCHLOW, Keith. 1976. *Islamic Patterns: An Analytical and Cosmological Approach.* London: Thames & Hudson, p. 97.

Plate 62a	Geometric Seljuk carving on wooden door from Konya. In RICE, David Talbot. 1975. *Islamic Art.* London: Thames & Hudson, Plate 176.
Plate 62b	Door wing from Bey Hekim, small mosque in Konya [same as above]. In the Ince Minare Medrese Museum, Konya. Seljuk period, 13th century AD. URL
Plate 62c	Bottom of left door panel.
Plate 62d	Top of left door panel.
Plate 62e	Whole pattern conjoined.
Plate 62f	Developmental drawing.
Plate 62g	Developmental drawing.
Plate 62h	Developmental drawing.
Plate 62i	Developmental drawing.
Plate 62j	Developmental drawing.
Plate 63	Enlargement to show detail from Mosaic tile-work within Sirçali Madrassa, Konya, 1243. In RICE, David Talbot. 1975. *Islamic Art.* London: Thames & Hudson, Plate 174.
Plate 64a	Developmental drawing.
Plate 64b	Detail of developmental drawing.
Plate 64c	Detail of developmental drawing.
Plate 65a	Developmental drawing.
Plate 65b	Developmental drawing.
Plate 65c	Developmental drawing.
Plate 65d	Developmental drawing.
Plate 65e	Developmental drawing.
Plate 66	Karatay Medrese, Konya, CE 1260 (AH 649). Ref TUR 0319: Pattern in Islamic Art. URL
Plate 67	Fig. 4 (Lu & Steinhardt). [No workable *girih* surface pattern can be manufactured for the Penrose 'dart' B, thus any thesis for pattern superimposition becomes redundant and comparability becomes incomplete.] In LU, Peter J. and STEINHARDT, Paul J. 2007. Decagonal and Quasi-Crystalline Tilings in Medieval Islamic Architecture. *Science,*

23 February 2007, Vol 315, pp.1106-1110. Also available: URL.

Plate 68 Fig. 2 (Lu & Steinhardt). **A** – Periodic *girih* pattern from the Seljuk Mama Hatun Mausoleum in Tercan, Turkey (~ 1200 C.E.), but with – **B, C & D** in analysis of the Maragha tower.
In LU, Peter J. and STEINHARDT, Paul J. 2007. Decagonal and Quasi-Crystalline Tilings in Medieval Islamic Architecture. *Science, 23 February 2007, Vol 315, pp.1106-1110.* Also available: URL.

Plate 69 Fig. 1 (Lu & Steinhardt), where – **E** shows a periodic pattern at the Timurid shrine of Khwaja Abdullah Ansari at Gazargah in Herat, Afghanistan (1425 to 1429 C.E.) and – **G** shows the pattern outlines for Panel 28 of the Topkapi Scroll, with coloured *girih* tile shapes superimposed from **F.**
In LU, Peter J. and STEINHARDT, Paul J. 2007. Decagonal and Quasi-Crystalline Tilings in Medieval Islamic Architecture. *Science, 23 February 2007, Vol 315, pp.1106-1110.* Also available: URL.

Plate 70 Separation of the linear surface patterns (in blue) from the outline shapes of the *girih* templates (in red).

Plate 71 Aperiodic surface pattern from Penrose rhombi.
In LIVIO, Mario. 2003. *The Golden Ratio: the story of phi, the extraordinary number of nature, art & beauty.* London: Hodder Headline Book Publishing, p. 205.

Plate 72 'Fig.1 The Blue Tomb, Gunbad-i-Quabud (built A.D. 1196-1197) in Marãgha, Western Iran' (as described by Emil Makovicky).
In HARGITTAI, István, ed. 1994. *Fivefold Symmetry.* Singapore: World Scientific Publishing Co. Pte. Ltd., p. 68.
[compare with **Plates 49** and **68**]

Plate 73a (upper) 'Fig. 2a Line reconstruction of principal ornamentation net from one side of the tomb tower and from the adjacent corner pilasters (shaded) of the tomb' (as drawn by Emil Makovicky).
In HARGITTAI, István, ed. 1994. *Fivefold Symmetry.* Singapore: World Scientific Publishing Co. Pte. Ltd., p. 70.
(lower) *girih* templates superimposed (by this author).

Plate 73b The anomaly of the surface pattern upon template 'e' within the standard *girih* theoretical model.

Plate 74 Developmental drawing.

Plate 75 Developmental drawing.

Plate 76 Developmental drawing.

Plate 77 Integration of the Penrose 'kite' shape within a decagon, in relation to the rhombus within a star pentagon.
In HARGITTAI, István, ed. 1994. *Fivefold Symmetry.* Singapore:

World Scientific Publishing Co. Pte. Ltd., p. 285.

Plate 78a Detail of *girih* patterning applied to central section of the Darb-i Imam shrine.

Plate 78b Comparison of scale within the Darb-i Imam shrine mosaic.

Plate 79a (above) Panel 28 of the Topkapi Scroll. Image taken from Sebastian R. Prange's article, The Tiles of Infinity, *Saudi Aramco World, pp. 24-31, Vol.60, No.5 (Sept/Oct, 2009)*
(below) superimposition of larger scale *girih* template (in blue).

Plate 79b Further pattern indications from developing area 'in-fill'.

Plate 79c Further expansion of design.

Plate 79d Indications of *girih* templates.

Plate 79e Comparison of scale.

Plate 79f Panel 28 of the Topkapi Scroll.
In NECIPOĞLU, Gülru. 1995. *The Topkapi Scroll – Geometry and Ornamentation in Islamic Architecture*. Santa Monica: The Getty Center for the History of Art and the Humanities.

Plate 79g Panels 53 and 54 from the Topkapi Scroll. [Ref. as for **79f**]

Plate 79h Panels 55 and 56 from the Topkapi Scroll. [Ref. as for **79f**]

Plate 79i Panel 63 from the Topkapi Scroll. [Ref. as for **79f**]

Plate 79j Panels 89ab and 90ab from the Topkapi Scroll. [Ref. as for **79f**]

Plate 80a Developmental drawing.

Plate 80b Developmental drawing.

Plate 80c Developmental drawing.

Plate 80d Developmental drawing.

Plate 81 ESCHER, M. C. 1958. *Man with cuboid.*
[wood engraving, 2½ x 2½" (64 x 64 mm)]
In THÉ, Erik; VELDHUYSEN, Mark. 2000. *The Magic of Escher.* London: Thames & Hudson, p.143.

Plate 82 ESCHER, M. C. 1958. *Belvedere.*
[lithograph, 18¼ x 11⅝" (462 x 295 mm)]
In THÉ, Erik; VELDHUYSEN, Mark. 2000. *The Magic of Escher.* London: Thames & Hudson, p.142.

Plate 83 St Catherine's monastery in Sinai. URL

Plate 84 Tower at St Catherine, Sinai. URL

Plate 85 The Kaaba at Mecca which incorporates the 'black stone'. URL

Plate 86 ESCHER, M. C. 1950. *Order and Chaos (Contrast).*
[Lithograph, 11 x 11" (280 x 280 mm)]
In LOCHER, Dr. J. L.; LANDSHOFF, Andreas; ELFFERS, Joost; THÉ, Erik; VELDHUYSEN, Mark. 2000. *The Magic of Escher.* London: Thames & Hudson, p.103.

Plate 87 ESCHER, M. C. 1952. *Gravitation.*
[lithograph and watercolour, 11¾ x 11¾" (297 x 297 mm)]
In LOCHER, Dr. J. L.; LANDSHOFF, Andreas; ELFFERS, Joost; THÉ, Erik; VELDHUYSEN, Mark. 2000. *The Magic of Escher.* London: Thames & Hudson, p.101.

Plate 88 (above) Selected feature from the floor of St. Mark's Basilica, Venice.
(below) Similarly constructed mosaic tessera at St. Mark's Basilica. URL

Plate 89 ESCHER, M. C. 1943. *Reptiles.*
[lithograph, 13⅛ x 15⅛" (334 x 385 mm)]
In LOCHER, Dr. J. L.; LANDSHOFF, Andreas; ELFFERS, Joost; THÉ, Erik; VELDHUYSEN, Mark. 2000. *The Magic of Escher.* London: Thames & Hudson, p.175.

Plate 90 Outer 'skeletal' dodecahedron and a 'solid' inner star dodecahedron (*great stellated dodecahedron*).
In GHYKA, Matila. 1977. *The Geometry of Art & Life.* New York: Dover Publications, p. 48.

Plate 91 A 'solid' *small stellated dodecahedron* enclosed within a 'skeletal' icosahedron.
In GHYKA, Matila. 1977. *The Geometry of Art & Life.* New York: Dover Publications, p. 49.

Plate 92 Illustration from JAMNITZER. 1568. *Perspectiva Corporum Regularium.* URL

Plate 93 Fra GIOVANNI da Verona. c. 1520. Two of many intarsia at the Church of Santa Maria, Verona. URL

Plate 94 Platonic solids.

Plate 95 Duals.

Plate 96 Numerical breakdown.

Plate 97 From *first* dual, to truncation, to a dual of the truncation.

Plate 98 From *second* dual, to truncation, to a dual of the truncation.

Plate 99 From *third* dual, to truncation, to a dual of the truncation.

Plate 100	Dual Platonic solids, their truncations and their further duals.
Plate 101	Numerical breakdown, re. column II.
Plate 102	Numerical breakdown, re. column III.
Plate 103	Rhombic triacontahedron and rhombic hexecontahedron.
Plate 104	Numerical breakdown of **Plate 103**.
Plate 105	(above) *acute* golden rhombus; (below) *obtuse* golden rhombus.
Plate 106	(left column) rhombi in two-dimensional space filling; (right column) rhombi in three-dimensional space filling.
Plate 107	5/4 logarithmic spiral.
Plate 108	$\sqrt{\varphi}$ spiral.
Plate 109	φ spiral.
Plate 110a	(above) ammonite fossil. URL (below) Nautilus pompilius. URL
Plate 110b	Abalone shell. URL
Plate 111	Pythagorean Lambda and Tetractys.
Plate 112	GAFFURIO, Franchino. 1492. Plate from *Theorica Musicae*. URL
Plate 113	GIKATILLA, Abraham Joseph. 1516. *Portae Lucis (Shaarei Orah)* or 'Gates of Light'. Title page to the Latin translation by Paulus Ricius. Augsberg: Johann Miller. Now in Biblioteca Marucelliana, Florence. [The Center for Online Judaic Studies] URL
Plate 114	Figured numbers.
Plate 115a	Stained glass panel depicting Dr Lorenz Tucher, provost of the Lorenzkirche in Nuremberg, dated 1487. Germanisches Nationalmuseum, Nürnberg. In BROWN, Sarah. 1994. *Stained Glass: An Illustrated History*. London: Bracken Books, p. 101.
Plate 115b	WOLGEMUT, Michael. 1478. *Portrait of Ursula Tucher*. [Oil on panel] Museumslandschaft of Hesse, Kassel. URL
Plate 116	DÜRER, Albrecht. 1499. *Portrait of Elsbeth Tucher*. [Oil on panel, 29 x 23 cm.] Kassel, Staatlische Kunstsammlungen. URL
Plate 117a	Design for stained glass window. Niedersächsisches Landermuseum, Landesgalerie, Hannover.

In BROWN, Sarah. 1994. *Stained Glass: An Illustrated History.* London: Bracken Books, p. 97. URL

Plate 117b Detail from previous image.

Plate 118 A companion panel (stained glass) to one depicting Dr. Sixtus Tucher, 1502. In BROWN, Sarah. 1994. *Stained Glass: An Illustrated History.* London: Bracken Books, p. 97.

Plate 119 (left) Design by WOLGEMUT, Michael for: (right) the frontispiece to the *Nuremberg Chronicle*. In BARTRUM, Giulia. 2003. *Albrecht Dürer and his Legacy: The Graphic Works of a Renaissance Artist.* London: The British Museum Press, p. 95.

Plate 120 WOLGEMUT, Michael. 1486. *Portrait of a 'Young Man'.* [Oil on linden panel, 16 x 12½ x 1⅝ " (40.6 x 31.1 x 4.1 cm)] Detroit Institute of Arts. URL

Plate 121a DŰRER, Albrecht. 1516. *Portrait of Michael Wolgemut*, originally painted in 1516 and later annotated in (or after) 1519. Nuremberg: Germanisches Nationalmuseum. URL

Plate 121b Detail from above image.

Plate 121c Ditto.

Plate 122a DŰRER, Albrecht. 1490. Coat of Arms, painted on the reverse of **122b**. In NORBERT, Wolf. 2006. *Albrecht Dürer, 1471-1528: The genius of the German Renaissance.* Köln: Taschen, p. 93. URL

Plate 122b DŰRER, Albrecht. 1490. *Albrecht Dürer the Elder with Rosary.* [mixed media on panel, 47.5 x 39.5 cm] Florence, Galeria degli Uffizi. In NORBERT, Wolf. 2006. *Albrecht Dürer, 1471-1528: The genius of the German Renaissance.* Köln: Taschen, p. 7. URL

Plate 122c Attributed to DŰRER, Albrecht. c.1490. *Portrait of Barbara Dürer.* [oil on panel, 47 x 36 cm] Germanisches Nationalmuseum, Nuremberg. In NORBERT, Wolf. 2006. *Albrecht Dürer, 1471-1528: The genius of the German Renaissance.* Köln: Taschen, p. 10. URL

Plate 122d DŰRER, Albrecht. 1498. *Self Portrait.* [oil on panel, 52 x 41 cm] Madrid: Museo Nacional del Prado. URL

Plate 122e DŰRER, Albrecht. 1500. *Self Portrait.* [oil on panel, 67 x 49 cm] Munich: Arte Pinakothek. URL

Plate 122f DŰRER, Albrecht. 1500. *Portrait of a Young Man.*

[mixed media on limewood, 29 x 21 cm]
Munich: Bayerische Staatsgemaldesammlungen, Alte Pinakothek. URL

Plate 123a Attributed to MEMLING, Hans. 1496. *Portrait of Jacob Obrecht.*
[oil on panel, 50.8 x 36.1 cm (with frame); 44 x 28 cm (painting only)]
Kimball Art Museum, Fort Worth. URL

Plate 123b Detail from **123a** and **123c**.

Plate 123c CHRISTUS, Petrus. 1449. *A Goldsmith in His Shop, Possibly Saint Eligius.*
[oil on wood; overall: 39 $^3/_8$ x 34 ¾", painted surface: 38 $^3/_8$ x 33 ½"]
New York: Metropolitan Museum of Art (Robert Lehman Collection). URL

Plate 123d BOUTS, Dirk. 1462. *Portrait of a Man (Jan van Winckle?).*
[oil with egg tempera on oak, 31.6 x 20.5 cm]
London, The National Gallery. URL

Plate 123e(i) Chart copied from Isaac Taylor's "The Origin of the Alphabet".
In WOODRUFF. Charles E. 1909. The Evolution of Modern Numerals from Ancient Tally Marks. *The American Mathematical Monthly, Vol. 16, No. 8/9, pp. 125-133, p. 128.* URL

Plate 123e(ii) Title plate to Gregor Reisch's *Margarita Philosophica*, 1503, Friburgi ed., indicating the subjects that are discussed. URL

Plate 123e(iii) Page from the 'Arithmetica' section of Gregor Reisch's *Margarita Philosophica*, 1503, Friburgi ed., indicating differences between printed and hand written numbers within the text. URL

Plate 123e(iv) The continuing page from the 'Arithmetica' section of Gregor Reisch's *Margarita Philosophica*, 1503, Friburgi ed., indicating further the differences between printed and hand written numbers within the text. URL

Plate 123e(v) Plate from Gregor Reisch's *Margarita Philosophica*, 1515, Argentine ed., introducing the 'Musica' section. URL

Plate 123e(vi) Page from Gregor Reisch's *Margarita Philosophica*, 1515, Argentine ed., to show written music of the time. URL

Plate 123e(vii) Plate from Gregor Reisch's *Margarita Philosophica*, 1515, Argentine ed., introducing the section on 'Geometria'. URL

Plate 124a Attributed to SCHONGAUER, Martin. C*hrist as Teacher.*
[pen and ink, 20.7 x 12.4 cm];
inscribed by Dürer: *Das hat hubsch martin gemacht jm 1469 jor.*
In ROWLANDS, John & BARTRUM, Giulia. 1993.
Drawings by German Artists in the Department of Prints and Drawings in the British Library: The Fifteenth Century, and the Sixteenth Century by Artists born before 1530 (Vols I & II). London: The British Museum, p.9 & PLATE 12.

Plate 124b SCHONGAUER, Martin. (1469). *Young woman fanning a fire with a bird's wing.*
[pen and ink with touches of pink and red wash, 18.3 x 14.2 cm]
In ROWLANDS, John & BARTRUM, Giulia. 1993.
Drawings by German Artists in the Department of Prints and Drawings in the British Library: The Fifteenth Century, and the Sixteenth Century by Artists born before 1530 (Vols I & II). London: The British Museum, p.10 & PLATE 13.

Plate 124c SCHONGAUER, Martin. 1470. *Head of a high priest.*
[pen and ink over traces of black chalk, 17.9 x 14.2 cm]
In ROWLANDS, John & BARTRUM, Giulia. 1993.
Drawings by German Artists in the Department of Prints and Drawings in the British Library: The Fifteenth Century, and the Sixteenth Century by Artists born before 1530 (Vols I & II). London: The British Museum, p.10 & PLATE 14.

Plate 125a DŰRER, Albrecht. 1526. *Portrait of Johannes Kleberger.*
[Oil on linden panel, 36.7 x 36.6 cm]
Kunsthistorisches Museum, Vienna. URL

Plate 125b Detail from the above.

Plate 126a 'Geomantic Figures with Related Planets from *Henry Cornelius Agrippa, His Fourth Book of Occult Philosophy* (London, 1655)'.
In TYSON, Donald, ed. 2007. *The Three Books of Occult Philosophy written* (1533) *by Henry Cornelius Agrippa of Nettesheim, translated out of Latin by James Freake* (1651). Woodbury, MN: Llewellyn Publications, p. 774.

Plate 126b Mathematical patterning of the 16 'figures' of geomancy.

Conclusion

Plate 127 DŰRER, Albrecht. 1506. *Christ among the Doctors.*
[oil on panel, 65 x 80cm] Museo Thyssen-Bornemisza, Madrid. URL

Plate 128 COLLIER, John. 1883. *Portrait of Charles Darwin (copy of 1881 portrait).*
[oil on canvas, 49½ x 38 in] National Portrait Gallery, London. URL

Introduction

(That) the knowledge at which geometry aims is knowledge of the eternal,
and not of aught perishing and transient.

Plato
Book VII of *The Republic*
(Botton & Jowett, 1999: 283)

(i) Preamble

It was originally intended that this thesis should be expressed in the Welsh language alone. It soon became apparent to me that a bilingual document would be preferable. I wanted to reach as wide an audience as possible, to subject my research to potentially more robust critical appraisal and to engage more effectively within a broader academic framework. A bilingual document would also better serve to elevate the Welsh language within academia. Indeed, it is apposite that the fruits of my research is expressed in two distinctly different and separate languages as one of the themes that pervades this thesis is the uncertainty of 'understanding' as a result of the inculcation of different and competing belief structures that are developed through exposure to evolving cultural and essentially linguistic influences.

Notwithstanding the importance of language, not least as a vehicle of expressing and disseminating 'thought', it is the philosophical concept of mathematical enquiry that is central to this thesis. But before I outline further the structure and themes pursued, I think it important to place the central tenet of my enquiry in the context of my own precursor research and practice, and then to explain how it was that my enquiries inevitably gravitated towards and now revolves around one particular image, viz. Dürer's *Melencolia I* of 1514.

Developing an individual philosophical understanding of universal matters in relation to the behavioural and essentially biological experience of being existent requires some understanding of science. Although some scientific 'discoveries' lay claim to chance, classical science is based on the empirical evaluation of *a priori* ideas. Crucially, accuracy in quantitative measurement, as well as in devising and setting the strict parameters for testing and confirming, is essential. This strict adherence to scientific methodology, it would seem, is in contradistinction to the more 'interpretive' view and practice adopted by most artistic practitioners. And this is what has always interested me, being both an inquisitive 'artist' and 'scientist', these terms to be viewed in their broadest sense.

(ii) Background

As a former musician, artisan and now fine artist, I cannot avoid the understanding of the very personal and interpretive nature of artistic engagement, even to the extent of negative realisation. It is interesting that many deeply engaged practitioners have taken the scientific route to 'balance' what may be an inevitable personal proclivity with a more 'rational' evaluation of contingent matters, to perhaps arrive at a more benign, universal conclusion. As a result, some of my output attempts to demonstrate that 'cold' but quite beautiful mathematical basis to life – this quite separate from the emotions and chaos of human affairs – as in the ordered structure of inorganic atomic and subatomic activity. See **Appendix A** for a brief explanation of my construct, *Element 88* **(Plate A1)**, completed in 2007 and which acts as a template for the electron configuration of the elements as well as an expression of the beauty I see in mathematical form. [The copious research material that would normally support such extensive investigations is not referenced here or included in my bibliography as I am only making fleeting reference here to previous research activity as an example of engagement with science.] This clear-cut numeracy is in contrast with the mathematical harmonies that may be envisioned within micro-structures and arrangements of more infinitely complex organic function and that demonstrate more evolving structured determination. With regard to the latter, **Appendix B** addresses a question that I have not seen posed in academic texts, where I ponder whether any form of mathematical patterning pertains – other that of course the most fundamental triplet-base genetic code itself – in the attraction through tRNA of any one of 20 amino acids in the production of protein chains, this from the potentiality of the 64 triplet-based codons, and as pre-determined by DNA sequencing. *RNA's Degenerate Code*, 2008 **(Plate B4)**, shows the culmination of my investigations in the form of a two-dimensional leaded window, and also three-dimensionally in cubic form as in **Plate B5**. [Again, supporting reference material with regard to biological investigations is too vast to include in this document.]

As I have this proclivity to analyse the natural world scientifically, and to express findings through visual codes, I was curious to know whether, and understand how, other polymath art practitioners and theorists might have expressed their own view of a world otherwise heavily influenced by cultural, religious and political factors. An image that comes immediately to mind with regard to DNA, of course, is Salvador Dalí's *Galacidalacidesoxyribonucleicacid*, 1963 **(Plate C1)**. Salvador Dalí (1904-1989) produced many works of art where geometric symbolism is integral to or somehow subsumed within them. He even used a pentagonal geometric template in his preparatory drawing for *Leda Atomica*, 1947 (Descharnes & Néret, 2001: 422), and annotated it with the formula $[(R/2) \sqrt{(10 - 2\sqrt{5})}]$ (Ghyka, 1977: 17) for calculating the length of the sides of the pentagon circumscribed within a circle; such geometry he learned directly from the mathematician, Prince Matila Ghyka (Descharnes & Néret, 2001: 551). Dalí was fascinated by mysticism and religion, producing *Corpus Hypercubus (Crucifixion)* **(Plate C2)** in 1954, where he created a three-dimensional cross composed of eight cubes; and then, in 1955 he produced *The Last Supper* **(Plate C3)**, with its clear pentagonal reference. There are, of course, many other examples of mathematical and religious symbolism within his extensive oeuvre.

Another and more recent artist who is interested in matters to do with perception and religious belief is Anselm Kiefer (b.1945). In one particular series of work, his *Merkaba* of 2002, Kiefer takes Jewish Kabbalah as his subject, a topic to which I also refer – quite crucially – in Chapter 3, Part 3.16 The Kabbalah. One particular image **(Plate D1)** from this body of work, i.e. *Sephiroth* (Kiefer, 2002: 49), is particularly interesting, where he integrates a three-dimensional Platonic solid, the dodecahedron, within the two dimensional geometry of the structure of the mystical *sephiroth.*

There are many other examples in past and more recent art history where this interest in relating mathematical substrates to a particularly 'human' view of understanding and belief is evident. For example, Graham Sutherland's underlying geometrical but concurrently 'earthy' spiritual expressions would also have been worth considering, as well as the contrastingly more 'neutral' and clean forms of Barbara Hepworth, such sculptures being quite distinct from the expressive subject of the human form, as exemplified by Auguste Rodin.

However, it is not the purpose of this thesis to analyse and gather information about such individual artists and how they may have interpreted their own particular views but to gain a better understanding of a particular 'believer/rationalist' dichotomy and, possibly, to determine what it is that, fundamentally, underlies perception and, ultimately, consciousness. With this in mind, I return to Kiefer, for in his *Melencholia*, 1990-91 **(Plate D2)**, he has appropriated that particularly interesting geometric object which originates from Dürer's print, *Melencolia I* of 1514, and then represented it within an historical context of his own. Kiefer has 'used' this object – best described as a truncated rhombohedron – a number of times and in a number of different contexts. It is this particular three-dimensional form that will provide a pivotal focus for my thesis.

(iii) <u>Structure</u>

Chapter 1.

The 'believer/rationalist' dichotomy is introduced through looking at the assertions of some major figures in the history of the development of thought and where such philosophers have not, in my view, been 'rational' in presenting their particular understanding of what they 'believe' in relation to a concept of deity, which, for them, is clearly and most unequivocally synonymous with the monotheism of *Christianity*. The separate **(A)** metaphysical and **(B)** empirical approaches employed by these philosophers to justify a 'belief' in the biblical God – this promoted by invoking the logic and certainty of mathematics – I find most puzzling, as I, personally, experience the clarity and detached truth that is to be found through numeracy and geometry to be totally distinct from the deeply psychological problem of conformity to sociologically induced 'beliefs'. This first chapter is therefore intended to introduce the very 'human' problem of a common inability by *biological* man to separate himself from his function as a member of a powerfully determinate 'flock', that separateness involving an understanding of wholly universal matters where the evolution of human behaviour has no impact whatsoever, and where such behaviour, in its most combative form, serves as evidence of the ultimate distinction between dichotomy and unity and also the void between the *concept* of universality and its human *perception* through contingency.

Chapter 2.

This next chapter continues the religious theme with regard to the power of 'word' to persuade, then to further establish the distinction between **(A)** the verbal and **(B)** the numerate, particularly through an understanding of proportionality through geometric structure. This is where Dürer's *Melencolia I* is introduced as a vehicle to accentuate not only the persistent problem of visual perception per se but also to highlight a further problem with scientific investigation and the importance of accuracy. In this section I also first propose the possibility that Dürer might have seen some mystical properties to the numbers 4, 5 and 9 and has subsumed this 'insight' into his polyhedron. From a detailed analysis of the mathematical literature with regard to the exact constitution of the polyhedron that Dürer has represented, it is clear that some investigators seem to display an inability to question most thoroughly what may be termed their own 'individual scientific beliefs' for, within the published literature, there are many competing and inconsistent views expressed, mostly under the guise of attention to mathematical rigour. That they cannot agree as academic scientists should raise some very serious questions, not only with regard to their 'thinking', but – more importantly – with regard to their assertiveness as scientists, and most certainly in consideration of their non-empirical methodology. I find this as troubling as the assertiveness of the fundamentally religious.

Chapter 3.

Both themes of numerical harmony and the significance of the rhombus as a geometric shape are systematically developed in this last chapter through an investigation into the relationship between two-dimensional and three-dimensional symmetries. Properties of **(A)** such symmetries within two-dimensional patterning, with particular reference to Islamic designs, forms the first part of this enquiry with drawings and paintings created concurrently in the development of ideas and as additional expressions of artistic engagement. Many theoretical and empirical aspects became increasingly evident through my enquiries, and in my resultant analyses I have detailed some mathematical inconsistencies that were found in a number of practical and historic instances. The interpretive views of scientific theorists, expressed within academic journals and other publications, again raises questions with regard to individual assertions in the field of mathematics when interfacing with human expressions in the form of artistic representation.

The second part of this chapter deals with **(B)** three-dimensional harmonies, investigated both theoretically and practically. Indeed it was essential for me to engage practically with this most difficult aspect of my enquiry in order that I should gain a better understanding of how, traditionally, this subject would have been investigated and understood, especially with regard to any conclusions that might be able to be drawn as to the possible constitution of Dürer's polyhedron. I was intent upon a continuation of my empirical approach, that which supports my otherwise 'academic' investigative methodology. M.C. Escher is cited here, particularly, as a convenient link and valuable reference point. The three-dimensional constructs that result from these activities form a major part of the exhibition material that accompanies this thesis. This is where the possibility that Dürer might have seen some mystical properties to the numbers 4, 5 and 9 is further related to certain harmonic proportions common to both two- and three-dimensional symmetries.

The last part of this final chapter brings my investigations full circle as I complete some **(C)** historical connections in forging links with the Greeks, forward through Jewish Kabbalah, and finally on and back to the focal point of Dürer the polymath, the complex religious mathematician and artist. The final proposal that I make here is that the portrait of a 'Young Man' ('by Wolgemut'), dated 1489 **(Plate 120)**, may actually be a portrait of Dürer, possibly even a self-portrait by Dürer himself, but any provenance and attribution has yet to be corroborated by Detroit Institute of Arts, Michigan, where I also mention my theory regarding Dürer's occasionally unusual configurations of the numbers 4, 5 and 9, viz. my original e-mail, from geraintdavies37@hotmail.com to libraryadmin@dia.org, 26/02/2011, with a reply from Jennifer Meekh of 28/02/2011, stating her forwarding my request to curators Salvador Salort-Pons (SSalortPons@dia.org) and Nancy Sojka (nsojka@dia.org). Also, my other enquiry for corroboration of what I propose is a spurious attribution (Brown, 1992: 97) and where I also propose my theory as to Dürer's unusual calligraphy of the numbers 4, 5 and 9 in relation to two particular examples of stained glass windows illustrated in her book, *Stained Glass: An Illustrated History*, has not been responded to by the author, viz. my e-mail, geraintdavies37@hotmail.com to sarah.brown@york.ac.uk, 25/02/2011.

It has to be finally explained that the structure of this thesis is perhaps rather unusual, where Dürer's polyhedron and Dürer the man are used as an occasional returning reference point, but also, principally, as an example of a particular form of artistry that contains a complex reference to mathematical substrates as well as, in this particular case, also important references to issues of fundamental 'belief'. There is, therefore, within this thesis, a weaving in and out as connections are made between the various dichotomous relationships investigated and where I have taken the convenience of a specific work of art as a tying-in mechanism. Clearly, the wider philosophical questions asked within this thesis are as important as the specific issues addressed, where particulars such as absolute accuracy are seen as a bench mark for mathematical truth, exemplified principally through numeracy and geometric analysis, then translated as and through various forms of artistic expression. It has of course to be appreciated that many of the figures and working drawings (together with some photographs of pencil drawings) are included as exactly what they are and as they were produced in the investigative process, much as Bruce Nauman included most effectively within his *Raw Materials*, where he has also, incidentally, hidden some quite basic algorithms which may appear, at first sight/hearing, as randomly designed and confused text (Nauman, 2004).

Chapter 1

Certitude and belief

Infini - rien. - Notre âme est jetée dans le corps, où elle trouve nombre, temps, dimensions. Elle raisonne là-dessus, et appelle cela nature, nécessité, et ne peut croire autre chose.

Blaise Pascal
Pensées
(Brunschvicg, 1922: 434)

Infinity – nothingness. The soul is cast into the body, where it finds number, time, dimensions. It argues on that basis, and all these it calls nature, necessity, and it can't believe anything else.

Blaise Pascal
Pensées
(Bishop, 1966: 218)

A. Metaphysics

1.1 Viewing Perception and Consciousness

What is it that I, as an artist, wish to express? Most practitioners would probably be able to answer this question quite easily, especially if they were traditional landscape artists or portraitists. However, my entry into the world of art practice has come later in life, and what I bring forward for others to consider is not perhaps the norm. It would certainly be difficult to classify my work within the field of contemporary art practice, for that would seem to be predominantly concerned with the subjective. My work tends to veer towards the objective. Why is this?

Now there are two questions to be answered. However, both are very closely linked, and when the latter is explained, the former will become self-evident. When I was posed with a particularly difficult philosophical problem that I felt needed to be solved, it became clear to me that to be able to command any true understanding of my situation required a certain degree of detachment. This was in response to experiences that had impinged upon, and totally altered my personal world. Any indications of clarity or subsequent control are of course expressed in hindsight – quite different from the experience. Eventually, as logic prevailed, it became clear that any attempt to view this personal predicament that did not contain an objective element was bound to place any form of perception at its extreme pole, either ignoring or not accepting the rationality of what may best be described as the principle of linear, dichotomous polarity. This polarity can subsequently be viewed from outside its own dimension. Such a psychological approach requires a form of mathematical thinking that then becomes more philosophical in nature, and may best be explained through the following geometric analogy.

Take a straight line AC with its midpoint B. Viewed from A, B represents an indeterminate point between the extremes of A and C. Psychologically, C would be viewed more extremely from A rather than B, such that each point on a diametric but graduated polarity is related to another purely in accordance to its linear position. However, when positioned along the line AC, only the view relative to one's own position can be appreciated. If one were now to extend to D a line from B, orthogonal to AC, thus constructing the triangle ACD, various positions can be adopted along the line BD. Previous relationships along the line AC will be viewed differently from each point along the line BD, i.e. it is easier and better to visually assess the distance and mathematical relationship between AB from D rather than along the same axis, from C. This sort of approach, where a linear dichotomous polarity is introduced, is akin to that promoted by the American psychologist, George Kelly, and outlined in his book, *A Theory of Personality: the Psychology of Personal Constructs*, but where he develops this much further within a range of other corollaries.

It is difficult to encompass advanced philosophical ideas in terms also of mathematics, psychology, or indeed, any or all other forms of scientific enquiry and analysis. But what I would contend is that the wider the net is cast, the greater and deeper also is the knowledge and understanding. Ultimately, what my art practice is intended to be is an expression of my sincere attempts at understanding principles within a particularly difficult area, what I refer to as 'mathematical substrates' – those that are consistent and universal, those that underpin the science of life – and then to express them through the aesthetics of visual form, but without losing the integrity of truth to science. I am passionate about the fundamental scientific principles that underlie our common physical and psychological world, particularly our extraordinary faculties of perception, whose anomalies indicate that 'other' ontological world, completely distinct from the phenomenological. It may be that our perception of consciousness may purely be a false construct formed by independently predetermined neuronal activity, as Benjamin Libet's revealing experiments seem to indicate. Libet conducted a series of controversial experiments, involving invasive neural surgery, inserting probes that detect neuronal activity deep within the brain. In his wide-ranging book, *Consciousness Explained,* Daniel Dennett makes particular reference to these experiments (Dennett, 199:153-166) and objections that have been raised against their interpretation. [Tor Nørretranders, in his book *The User Illusion*, provides a profound insight into the science of consciousness, and devotes a whole chapter to the implications of Libet's work (Nørretranders, 1999: 213-250), giving the background to, and an excellent detailed analysis of, the complex science involved for anyone wishing to follow this fascinating topic further.]

In general then, the results of these groundbreaking experiments would seem to suggest that there was up to an approximate half-second delay between (i) the evidence of neuronal firing that marked precisely the moment of a consciously driven motor/tactile act, and (ii) the exact moment when the conscious thought occurred that set into operation that particular act. If we consciously pre-determine action by thought, clearly we would expect that the thought should precede the neuronal evidence of action generated by that thought, but it would seem, from this experimental evidence, that the reverse is true. I only touch upon this anomaly, for to understand fully the detailed science of the series of experiments that were conducted (Libet, 1981) requires an advanced knowledge of neurology, an expert and esoteric science: and not all agree (Churchland, 1981) with Libet's interpretation that leads to such a controversial conclusion. Clearly, how scientists view the results of Libet's series of experiments has huge implications as to whether or not we have any 'free will' at all. Could it really be then that, in the last analysis, we are no more than biologically reactive organisms (albeit highly evolved and complex) whose differing qualities of perceptions are governed by relevant endogenous and exogenous factors, and that consciousness is purely an after effect to a predetermination that is completely outside our control, much as we would like to think otherwise? This difficult issue of 'free will' will tend to rise again in this and other chapters, also in relation to its historical interpretation within the domain of theology, perhaps a different concept entirely from what scientists may mean by biological 'free will'. [The Catholic view holds on to tradition and is still heavily influenced by the biblical and personal interpretation expounded through the writings of St. Thomas Aquinas (1225-1274) where the view is that man certainly has free will. An extract, on this subject, is taken from his *Summa Theologica* (1265-1274) and is given as **Appendix 1**.]

1.2 Philosophy and mathematical foundations

What then, as non-specialist scientists, are we to make of consciousness, and that which generates its possibility, i.e. perception? René Descartes (1596-1650), arguably the founder of modern philosophy, famously coined the phrase *I think, therefore I am* (Chávez-Arvizo, 1997: 92). Perhaps it should be introduced as *je pense, donc je suis,* as it was expressed in the original French publication of his *Discourse on the Method*, 1637, but it is more often quoted as *cognito ergo sum* as it would also have been known from his *Principles of Philosophy*, 1644, then published in Latin. I can only work from further translations into English and interpret meaning, firstly in relation to our modern scientific understanding of the perceptive process, and then as this relates to belief. Notwithstanding the problems with nuance in translation, Descartes, in para. 32 (Part IV) of his *Discourse on the Method*, preambles with this statement:

> … it is sometimes requisite in common life to follow opinions which one knows to be most uncertain, exactly as though they were indisputable …in this case I wished to give myself entirely to the search after truth …
>
> (Chávez-Arvizo, 1997: 91)

The first part of the statement, I would contend, has to do with the same psychology involved in what is, for many, the (linguistic) formation of fundamental religious *belief*. Then there is the more challenging, and most difficult search to attempt – the search for *truth*. Here, it is necessary to unpick, inductively, all the layers and connections of understanding, despite there being in-built false perceptive processes to factor, arriving finally at a core conception and (mathematical) foundation that is quantitatively sound. Only then is it possible to make the reverse journey to fully comprehend a process that is indicated by the complexity of the on-going creation and evolution of our own biological world. Deduction will therefore always be a questionable process, as compounding effects increase and epistemology expands. Collectively, this encompasses a whole teleological process of mind and thought.

In his *Rules for the Direction of the Mind*, Descartes makes clear his own view of the fundamentality of mathematics as a demonstrator of certitude:

> ... in our search for the direct road towards truth we should busy ourselves with no object about which we cannot attain a certitude equal to that of the demonstrations of arithmetic and geometry.
>
> (Chávez-Arvizo, 1997: 7)

Incidentally, with regard to certitude in the modern world, I am mindful here of the many misinterpretations of the 'uncertainty principle', a term appropriated from the field of quantum physics (Hawking, 1988: 53-61), when applied outside its proper domain. By the same token, elliptic and hyperbolic geometry are both theoretically and practically valid when applied to the surface planes to which they apply. But when this translates to a belief that Euclidian or planar geometry is false, this indicates a lack of understanding as to the reality that elliptic, hyperbolic and indeed even more complex geometries within 'Hilbert space' are quite impossible to comprehend or to define without the very basic concepts of Euclidian geometry itself. Without the contentious and axiomatic parallel or fifth postulate, all geometry is incomplete. [See **Appendix 2** for Euclid's parallel postulate.]

Similarly, axiomatic principles apply also to numeracy. Clearly, we can have no conception or understanding of 'rational' and 'irrational' numbers without the certainty of the concept of 'integers'. Numbers are the bedrock of all arithmetic and geometry. Only because of the existence of that absolute *whole* quantitative quality embedded in our concept of numbers is it possible for us to comprehend, for example, the magic of the Golden Section, whose historical symbol is φ (*phi* / *fi*) but sometimes also given now by the symbol τ (*taf*). Its quantitative formula is $(\sqrt{5} + 1)/2$, which translates to the irrational number 1.61803398875... It exists in the latter state as numerically infinite: at the same time it is a completely contained and tangible entity, a perfect geometric form. Furthermore, our concept of 'real numbers' requires our ability to conceive of measurable points on a line. [See **Appendix 3** for a definition of mathematical terms used here.] Even though, perhaps, nothing in the measurable, physical world is exact, we can only know that because of exactitude itself, the absolute given.

Ultimately, pure mathematics has to do with the signification and application of absolutely precise quantitative thought, which can then be applied within the field of the qualitative. How precise and absolute the process is, defines the nature of the language used, and thus mathematics becomes a language of accuracy and of thought. Importantly, it has to be certain and reliable for us to know and appreciate its obverse. Marcus Giaquinto states in the conclusion to his book, *The Search for Certainty: A Philosophical Account of Foundations of Mathematics*:

> The final balance, then, is positive. Though we cannot be *certain* of the reliability of *all* of classical mathematics, we can be certain of the reliability of a significant part of it, and we can be confident in the reliability of all of it.
>
> (Giaquinto, 2006: 229)

For myself, I am clear in my own mind as to the mathematical properties of Euclidian geometry, as to what I see is certain and exact. When it comes to its practical application, I am most mindful of that which is theoretically correct and accurate and that which the uncertainties of visual perception might wish us to believe. It is easy to be tricked into considering certain geometric relationships or proportions as valid, which, after careful calculation, turn out only to be approximations, albeit their sometimes being extremely close.

1.3 A mathematical proof of God?

Returning to Descartes, it is interesting that, when he has considered what it is possible for him to believe is true from the point of view of biological consciousness – that is, *I think, therefore I am* – he is then able to arrive at, through what amounts to a series of Socratic, syllogistic arguments:

> ... that all the things that we very clearly and very distinctly conceive of are true, is certain because God exists, and that He is a perfect Being, and that all that is in us issues from Him.
>
> (Chávez-Arvizo, 1997: 96)

As A. K. Stout puts it in his essay, *The Basis of Knowledge in Descartes*:

> Thus Descartes has in fact invoked God to prove the truth of the same general rule with which he started – that all that is clearly and distinctly perceived is true.
>
> (Doney, 1970: 176)

Descartes' God, therefore, I would take to be a deity born of some sort of reason, of deduction, the creation of a supreme mathematical mind. Can this be the same God of the Israelites, or God taken forward by the Christians, then to be appropriated by Islam? Descartes clearly does not provide any validation of God in terms of the product of any particular creed or culture, as an anthropological legacy. On the contrary, Descartes' conception of God is that which is validated through logic alone. By referencing the axiomatic truths of geometry as entities at the metaphysical level, or as 'forms' – distinct from their demonstration in the physical world – Descartes invokes a similar 'act of faith' to validate God, as expressed in his *Discourse on the Method*:

> … I saw very well that if we suppose a triangle to be given, the three angles must certainly be equal to two right angles; but for all that I saw no reason to be assured that there was any such triangle in existence, while on the contrary, on reverting to the examination of the idea which I had of a Perfect Being, I found that in this case existence was implied in it in the same manner in which the equality of its three angles to two right angles is implied in the idea of a triangle; or in the idea of a sphere, that all the points on its surface are equidistant from its centre, or even more evidently still. Consequently it is at least as certain that God, who is a Being so perfect, is, or exists, as any demonstration of geometry can possibly be.
>
> (Chávez-Arvizo, 1997: 94)

Are we, however, to take it that Descartes is synonymously referring to a cultural deity, or is he wishing only to validate the concept of a generic god? We shall have this same problem of reference when others similarly wish to validate God through reason, notably Baruch Spinoza (1632-77). Spinoza's God, however, was a far more pantheistic conception, his having rejected the strict Judaism of his cultural background. In his *Ethics*, published in1674, which is full of syllogistic development formed from questionably axiomatic statements, he attempted a proof of the existence and the attributes of God by mimicking the structure of the geometric proofs of Euclid, and mixing in the type of linguistic arguments that Descartes adopted. He soon produces a similar circular argument:

Proposition 11

God – in other words a substance consisting of infinite attributes, each of which expresses eternal and infinite essence – necessarily exists.

Demonstration. If you deny this, conceive (if this can be done) that God does not exist. Therefore (by Axiom 7), his essence does not involve existence. But this (by Proposition 7) is absurd; therefore, God necessarily exists. Q. E. D.

(Parkinson, 2000: 82)

[Axiom 7 states that 'the essence of whatever can be conceived as not existing does not involve existence' (Parkinson, 2000: 76), and Proposition 7 states that 'it belongs to the nature of substance to exist' (Parkinson, 2000: 78).]

1.4 <u>The Dialectic</u>

A century later, George Wilhelm Friedrich Hegel (1770-1831) develops a complex system of philosophy that clearly separates a God of logic from the biblical God. The idea of a God (an Absolute), infinite in magnitude and nature, develops from a dawning of consciousness. This is dealt with in his *Logic*, published in 1812-16. Here, he also makes reference to mathematics in the development of his ideas, but the language he uses – perhaps it would be clearer in German – does not necessarily promote a greater understanding of the difficult concepts that are inherent in any enquiry into metaphysics:

> Number is a thought, but thought in its complete self-externalisation. Because it is a thought, it does not belong to perception: but it is a thought which is characterised by the externality of perception. – Not only therefore *may* the quantum be increased or diminished without end: the very notion of quantum is thus to push out and beyond itself.
>
> (Wallace, 1892: 195)

The notion of number as a concept, and its development in number theory, is clear to me, especially as it is applied: but 'thought' and 'thought in its complete externalisation'? I am not clear as to a precise meaning here. 'Thought': would we understand its true meaning as it might apply to modern psychology? And what does he mean by perception: the purely biological and phenomenological, or some sort of other more nebulous cerebral process? This is where the semiotic element of language and 'thinking' become troublesome, and, for me, the clarity of mathematics becomes helpful.

Language can be a powerful tool in expressing the qualitative. Where the clinical, quantitative element of mathematics formalises and makes accurate, language 'softens' and makes more pliable. Language can seduce, as does perception – perception in the sense of the faculty of our 'senses' – our sight, hearing, touch, taste, smell. The visceral is surely qualitative, but when the scientist analyses the complex processes involved (the physics, the chemistry, the biology), proper understanding is not possible without the basic tenet of the measurable. This is an argument that will run quite consistently throughout this thesis, and will be seen, as themes are developed, to be crucial in the thinking process and the formation of my constructs.

Returning to Hegel; his *Logic* was preceded in 1807 by his *Phenomenology of Spirit,* where he deals with the spirituality of *mind*. Here, when deliberating what is meant by such concepts as soul, consciousness, mind, morality and ethics, he makes an initial fleeting connection with theology, although he makes no particular biblical references:

> The word 'Mind' (Spirit) – and some glimpse of its meaning – was found at an early period: and the spirituality of God is the lesson of Christianity.
>
> (Wallace, 1894: 7, as Part 384 of the *Encyclopaedia*)

Hegel's *Philosophy of Nature* was to be published last, and first disseminated in published form in 1816, as the second of the trilogy, collectively titled *Encyclopaedia.* Here, he outlines what he knows and understands of the natural world as science, a very basic attempt at an epistemology. But before he delves into what could be quantifiable science, Hegel explains his notion of Nature in relation to his system of philosophy, as in the *Zusatz* for Section 247:

> If God is all-sufficient and lacks nothing, why does He disclose Himself in a sheer Other of Himself? The divine Idea is just this: to disclose itself, to posit this Other outside itself and to take it back again into itself, in order to be subjectivity and Spirit. The Philosophy of Nature itself belongs to this path of return; for it is that which overcomes the division between Nature and Spirit and assures to Spirit the knowledge of its essence in Nature.
>
> (Miller, 1970: 14)

This again, perhaps of necessity, is a 'linguistic' expression of a very difficult philosophical concept, and its interpretation is dependent upon an understanding of his language, as opposed to its being expressed in a 'universal language of understanding'. Is there a meeting ground between the more conceptual and qualitative (expressed linguistically), and the more concrete and quantitative (expressed mathematically)? Between Section 254 to 261, within his analysis of Space and Time, Hegel develops his views on the purpose and place of mathematics within his scheme. It would appear that Hegel views mathematics to be a separate discipline, that of measure, and not a particularly useful tool for thought:

> The simple elementary figures and numbers, on account of their simplicity, can be used for *symbols* without fear of misunderstanding; but even so, these symbols are too heterogeneous and cumbersome to express thought. The first efforts of pure thought had recourse to such aids, and the Pythagorean system of numbers is the famous example of this.
>
> (Miller, 1970: 38)

Hegel then amplifies upon what he understands of what are really very complex and mathematical sciences, as is the case for light (Sections 275 to 277). Unfortunately, he tends again towards the metaphysical, and is quite critical of any opposing understanding of a more physical nature. He is quite dismissive of Newton:

> The conception of discrete, simple *rays* of light, and of *particles* and *bundles* of them, which are supposed to constitute a limited expansion of light, is of a piece with those other barbarous categories for whose prevalence in physics Newton is chiefly responsible.
>
> (Miller, 1970: 92)

Newton was a brilliant theoretical scientist and sincere in his attempts at understanding. It is a brave man who, from limited empirical evidence, attempts to theorise *a priori*. Indeed, Newton's contention that there was a mathematical connection between a particular Pythagorean musical scale and the seven 'colours' that he would explain through the prismatic refraction of light, was not quite correct. In his defence, he had no empirical method to test and possibly prove this. However, the 'idea' of some sort of Pythagorean relationship, I would contend, was valid; only he took (as if by faith) one particular scale as his template, that of the Dorian mode (Newton, 1952: 154-5 & 212). It is interesting that Hegel, in his section on Sound (Sections 300 to 302), accepts and shows his understanding of Pythagorean mathematics and its relevance in understanding how pitch acuity is possible. However, when it comes to his writing and understanding about colour, the influence of Goethe is very apparent (Miller, 1970: 194-217, Part 320 of the *Encyclopaedia*). Goethe, of course, was also noted for his opposition to

Newton's approach towards a Theory of Colour, and to mathematicians in particular.

I have made a particular study of the mathematical relationships that may be manifest between differing and various wavelengths that, when in combination, produce the perception within the brain of individual colours. This I have compared with the chromatic ($\sqrt[12]{2}$) musical scale, somewhat differently from Newton, arriving at some interesting $\sqrt[n]{2}$ proportions (with their approximate Pythagorean equivalents) that are common to (i) various relationships within the colour spectrum and (ii) harmonic intervals in sound. This template explains the relative proportions within the Helmholtz-Maxwell-Young tricolour theory, as it does also for the quite different but related proportions within Hering's opponent/opposing colour theory. In addition, this visual and numerical template indicates the more complex but integrated relationship between the complementary colours that may well indicate how the contiguous colour effect is produced. Suffice it to say that I still find as somewhat enigmatic the total scientific understanding of a series of processes whereby the initial light impinging upon an external object is perceived as a subject of colour and form within the brain.

1.5 The phenomenological/ontological divide

In my comments on Hegel's dialectic, and elsewhere, I have used the term 'metaphysics'. I have been using the term non-specifically, and in its loosest meaning, as dealing with the fundamental concepts of reality, existence, substance, causality, etc. Generally speaking then, the dictionary definition indicates to us that metaphysics has to do with

> ... an understanding of the ultimate reality which lies behind that which we confront in sensory experience. This understanding is not itself based on sensory experience, but on rational analysis or insight.
>
> (Mautner, 2000: 351)

Indeed, prior to Hegel, Immanuel Kant (1724-1804) had attempted to clarify this problem of the duality that we are aware of, between that which we know that is contingent and phenomenological, and that mysterious 'other' that has to do with 'mind'. There is the undoubted knowledge of some unfathomable intelligence outside and beyond our own contingency, some sort of ontology totally separate from our own necessities. This ontology would seem to alight within us a consciousness that makes *a priori* judgements possible; but for this 'insight' to hold true, it must be tested with our limited 'other' capacities, dictated by the uncertainty of 'mind' being itself subject to imperfect information formed from sensory input. These are Kant's well-known expressions on the subject:

> ... all the "ideas" that come to us involuntarily (as those of the senses) do not enable us to know objects otherwise than as they affect us; and consequently that as regards "ideas" of this kind even with the closest attention and clearness that the understanding can apply to them, we can by them only attain to the knowledge of *appearances*, never to that of *things in themselves*.
>
> (Abbott, 1923: 70)

This we might apply purely with reference to understanding consciousness but, for Kant, this principle applies also to morals, for the above quotation comes from his *Fundamental Principles of the Metaphysic of Morals*, first published in 1787, where he addresses the subject of the immutability of 'free will'. But the subject of cognition had previously been deliberated by him at considerable length within the context of a transcendental deduction, derived from the relationship between the *a priori* and the *a posteriori*, this in *The Analytic of Pure Reason*, first published in 1781. Here, he includes a mathematical analogy:

> ... if it were to be supposed that space and time are in themselves objective and conditions of the possibility of things in themselves, then it would be shown, first, that there is a large number of *a priori* apodictic and synthetic propositions about both, but especially about space, which we will therefore here investigate as our primary example. Since the propositions of geometry are cognized synthetically *a priori* and with apodictic certainty, I ask: Whence you take such propositions, and on what does our understanding rely in attaining to such absolutely necessary and universally valid truths? There is no other way than through concepts or through intuitions, both of which, however, are given, as such, either *a priori* or *a posteriori*. The latter, namely empirical concepts, together with that on which they are grounded, empirical intuition, cannot yield any synthetic proposition except one that is also merely empirical, i.e., a proposition of experience; thus it can never contain necessity and absolute universality of the sort that is nevertheless characteristic of propositions of geometry.
>
> (Guyer, 1998: 187)

However, by the end of a long and complicated thematic development within this book, Kant concludes that 'the belief in a God and another world is so interwoven with my moral disposition that I am in as little danger of ever surrendering the former as I am worried that the latter can ever be torn away from me' (Guyer, 1998: 689). [It will be interesting to compare this view with what Pascal also has to say upon this very point, and which is deliberated at the end of the next section of this chapter, Part 1.6 The theological God and metaphysics.] The metaphysical stance in relation to the subject of 'free will' is taken further in his *Dialectic of Pure Practical Reason,* first published in 1788, a year after *Fundamental Principles of the Metaphysic of Morals*. Here he further develops his moral argument, and indicates that the belief in the benevolence of God should be in relation to what he sees as a higher duty towards ownership of one's own contribution to that end:

> Therefore those who placed the end of creation in the glory of God (provided that this is not conceived anthropomorphically as a desire to be praised) have perhaps hit upon the best expression. For nothing glorifies God more than that which is the most estimable thing in the world, respect for His command, the observance of the holy duty that His law imposes on us, when there is added thereto His glorious plan of crowning such beautiful order of things with corresponding happiness.
>
> (Abbott, 1923: 228)

Kant then places a condition on this:

> That in the order of ends, man (and with him every rational being) is *an end in himself,* that is, that he can never be used merely by any (not even God) without being at the same time an end also himself, that therefore *humanity* in our person must be *holy* to ourselves, this follows now of itself because he is the *subject of the moral law,* in other words, of that which is holy in itself, and on account of which and in agreement with which alone can anything be termed holy. For this moral law is founded on the autonomy of his will, as a free will which by its universal laws must necessarily be able to agree with that to which it is to submit itself.
>
> (Abbott, 1923: 229)

He is clearly not letting individual man off the hook. Then, further theological issues of morality, such as 'evil' and 'original sin', are taken up by Kant in *Religion within the Limits of Reason Alone*, first published in 1793. [Augustine of Hippo (354-430), a contemporary of St Jerome (347-420), is attributed with originally promoting the importance of the Biblical concept of 'original sin'.] Although the title might suggest otherwise, Kant is now happy to accept the biblical teaching whereby 'all agree that the world began in a good estate' and that 'this happiness vanished like a dream and that a Fall into evil' will end 'with the Last Day and the destruction of the world at hand' (Greene, 1960: 15). However, he is clear that such faith in historical fatality does not abrogate man from individual responsibility, for 'man *himself* must make or have made himself into whatever, in a moral sense, whether good or evil, he is or is not to become' (Greene, 1960: 40). Clearly, Kant has finally taken 'reason' to mean something other than that which is not dictated by 'faith', indicating an inconsistent relationship between his earlier analysis (through *a priori* logic) of a perceptive process that leads to cognition, and the theological stance of acceptance (through religious faith) of that which he readily accepts as being born of a cultural inheritance.

1.6 The theological God and metaphysics

Of course, this now begs the question, what do we mean when we talk of God? It is clear that there is a problem here when, as with Kant and so many others, the biblical God is sometimes confusedly synonymous with a conceptual 'god of mind'. The further back we go in the history of philosophical enquiry, the more they seem to merge. Gottfried Wilhelm Leibniz (1646-1716), for example, although a profound mathematician and thinker, could not separate his belief in a theological God from that which he explained from logical argument, the former a product of cultural contingency, and the latter developed from a cerebral enquiry into the metaphysical. In the opening to his *Discourse on Metaphysics* he states:

> The greatest knowledge and omnipotence contain no impossibility. Consequently power and knowledge do admit of perfection, and in so far as they pertain to God they have no limits. Whence it follows that God who possesses supreme and infinite wisdom acts in the most perfect manner not only metaphysically, but also from the moral standpoint.
>
> (Montgomery, 1902: 3)

Leibniz gains validation for his continued reasoning from the scriptures and even allows God the high moral ground in any and all questionable circumstances, and he states:

> For God foresees from all time that there will be a certain Judas, and in the concept or idea of him which God has, is contained this future free act... God has found it good that he should exist notwithstanding that sin which he foresaw.
>
> (Montgomery, 1902: 50)

This positioning of morality vis-à-vis the metaphysical is significant, and is taken up further in the many letters between himself and Monseur Antoine Arnauld, an eminent theologian, writer and mathematician of his time. Then, in his *Monadology*, Leibniz chooses to further employ arguments within the domain of logic to arrive at his continued conviction. This duality of argument – the employment of logic to underscore belief – I find most intriguing, and I now include (as **Appendix 4**) a particular passage from the *Monadology* as another example of a reference to the relationship between Reason, Mathematics and Truth that many philosophers have been happy to use for their own purposes, and in pursuit of the validation of their own particular beliefs. It is a fascinating insight into a particular thought mechanism and argument that arrives at a metaphysical concept (elsewhere defined by Leibniz in terms of a theory of organisation, formed of relationships within the substance of *monad*) that necessitates the momentum of a supreme and all-encompassing entity, but which I cannot see should, therefore, necessarily be synonymous with the particular anthropological form of Judeo-Christianity in the guise and person of biblical God.

Finally, through the common link of M. Arnauld, who was a Jansenist and a vociferous opponent of the Jesuits, we come to the brilliant Blaise Pascal (1623-62), himself also a Jansenist. Pascal, the great mathematician and observer of man and Nature, is known to have been seized by what must have been an intense revelation, for he carried a memorial to that experience, dated 23rd November 1654, which was discovered upon his death in the form of a piece of paper and parchment, sewn into his coat linings, wherein he expresses:

> God of Abraham, God of Isaac, God of Jacob, not of the philosophers and scholars.
> Certitude, certitude, feeling, joy, peace.
> *God of Jesus Christ.*
>
> (Bishop, 1966: 54)

And this separation of the logical, mathematical part of his mind from that formed of his revelation, is distinct and clear. Furthermore, in Pascal's *Pensées,* which are his thoughts that predominate on religion, he comments (No. 214, Lafuma – No. 282, Brunschvicg):

> We know truth not only by reason but also by the heart. By this latter means we know the first principles, and it is in vain that reasoning, which has no connection with them, tries to combat them.
>
> (Bishop, 1966: 198)

And he reiterates, singularly, in No. 225-278:

> It is the heart that feels God, not reason. That is what faith is, God felt in the heart, not by reason.
>
> (Bishop, 1966: 200)

Then, in No. 343-233 (the French original introduces this chapter), he begins with a philosopher's statement:

> *Infinity – nothingness.* The soul is cast into the body, where it finds number, time, dimensions. It argues on that basis, and all these it calls nature, necessity, and it can't believe anything else.
>
> (Bishop, 1966: 218)

However, this statement leads him, through more argument, to an opposing view, and to the passage for which he is perhaps best known:

> ... either God is or He is not. Which side shall we lean to? Reason can't determine anything about it. There is an infinite chaos separating us. A game is going on in the infinite reaches of this infinite distance, where heads or tails will turn up. What will you bet?
>
> (Bishop, 1966: 219)

I leave the metaphysics as it might apply to religion – and the problems as to whether religious conviction relates at all to logic – with Pascal, whose position is, I find, disarmingly honest and very human. [It might be interesting at this point to refer back to Part 1.5 <u>The phenomenological/ontological divide</u> and reflect upon what Kant seems also to be implying, possibly having been influenced by Pascal's thoughts, as to the impossibility of equating *faith* with *reason*.] Next, I shall be looking at how a more empirical approach to questioning consciousness may elucidate matters further.

B. Physics

1.7 The School of Empiricism

Thus far I have been looking at how philosophers have dealt with the difficulty of explaining the ontological in terms of their approach to connecting the metaphysical to logic, and by their invoking the power of mathematics, often quoting the certainty of geometric concepts and axioms as proof of the reality of their own beliefs. This sort of logic (in their terms) within the realms of metaphysical enquiry, based principally upon a belief versus disbelief dichotomy, has been utilised by many philosophers to prove the existence not only of god as a metaphysical concept, an immutable intelligence, but perversely (in my view) to validate the existence of the monotheistic God, believed in as that deity whose attributes and laws are unquestioned, being explained and transmitted through an ancient text that is the Torah.

For me, however, there is no *logic* that I could accept as valid whose base is an acceptance of the purely apocryphal. Judaism, Christianity and Islam are *faiths* based upon an historical account, promoted and propagated by transmission through language, and it is interesting that this problem of the persistence of belief in a biblical God as a provable and – it would appear from metaphysical accounts thus far analysed – a proved God, is found also in the literature of the empiricists. This I find surprising, for empiricism, as I understand it, has fundamentally to do with that which is provable not ontologically, but phenomenally, in the experimental and lived world; that is the world of the physical, mediated by the senses.

1.8 Mind as *tabula rasa*

With this in mind, I now turn to the British School of Empiricism, and look at some of the thoughts and beliefs of its main interlocutors, namely Locke, Berkley and Hume. John Locke (1632–1704) is immediately problematic for, although his thoughts and writings on the subject of how we might acquire knowledge of our own particular world may be based fundamentally upon the overriding influence of experience – with linguistic input playing an important role – he does not seem to equate the latter with how this very behavioural phenomenon may itself be responsible for religious belief, that which is not supported by empirical proof. In his early writings, about thirty years before publishing *An Essay concerning Human Understanding*, Locke indicates an acceptance of biblical teaching, as evidenced by his *Essays on the Law of Nature,* written in Latin around 1660, or shortly thereafter:

> Since God shows Himself to us as present everywhere and, as it were, forces Himself upon the eyes of men as much in the fixed course of nature now as by the frequent evidence of miracles in time past, I assume there will be no one to deny the existence of God, provided he recognizes either the necessity for some rational account of our life, or that there is a thing that deserves to be called virtue or vice.
>
> (Leyden, 1954: 109)

Locke invites us to accept the presence of God from the evidence of past miracles. This presupposes a belief in miracles per se in the same way that one might suppose that we would believe that man could live to be upwards of nine centuries old, as testified for Methuselah and some other ancient biblical characters. Locke's proviso is also that we recognise 'some rational account of our life'. This is rather puzzling, given that such belief in questionable miracles (biblical) is viewed as rational; or otherwise that there is a moral paradigm in terms of virtue or vice. This either/or codicil seems for Locke to be relevant to his thesis for an acceptance of the presence of God, where (to me) there is no necessity to introduce the relationship between the irrational and the rational, other than to point out their contradictory position as necessity, and that virtue or vice separately exist as concepts within the moral paradigm. Is Locke inviting us to choose between a rational, intellectual approach, and that of an emotive, spiritual alternative? In 1690, Locke published *An Essay concerning Human Understanding*, where, in Book II, he introduces the idea that the mind may be viewed as a blank slate, a *tabula rasa,* upon which impressions from life are imprinted. Locke expresses it thus:

> Let us then suppose the Mind to be, as we say, white Paper, void of all Characters, without any *Ideas*; How comes it to be furnished ? Whence comes it by that vast store, which the busy and boundless Fancy of Man has painted on it, with an almost endless variety? Whence has it all the materials of Reason and Knowledge? To this I answer, in one word, From *Experience*: In that, all our Knowledge is founded; and from that it ultimately derives it self. Our Observation employ'd either about *external, sensible Objects; or about the internal Operations of our Minds, perceived and reflected on by our selves, is that, which supplies our Understandings with all the materials of thinking.* These two are the Fountains of Knowledge, from whence all the *Ideas* we have, or can naturally have, do spring.
>
> (Nidditch, 1991: 564)

Locke's belief is that knowledge and understanding comes from experience. Now would this not explain the interesting phenomenon of particular religious beliefs, an acquisition of lore as external source? Moreover, in the case of teaching within the three main monotheistic religions, this would explain that experience of unquestioned acceptance upon the specific threat of perdition. What experience could be more frightening and more psychologically powerful to a developing psyche than this? Neither is such an experience particularly comforting in terms of the developmental need to satiate enquiry and understanding as a purely biological function. As for Locke, what is perverse about the position of such psychological acceptance of religious belief is that the very necessary element of empirical evidence is clearly lacking within his scheme. How can historical narrative be taken as factually true, and how can this be taken as reliable in terms of experience? Surely, subjection to such apocryphal scriptural evidence cannot be taken as a valid experience upon which one can base a credible philosophy. It is surprising that Locke does not see or admit to this possibility when devoting a chapter in Book IV to the *Knowledge of the Existence of a God*, wherein he lays down the ground rules – like his contemporary, Spinoza – with the assertion as axiomatic that there *is* a God, and that this can be demonstrated:

> … we are capable of *knowing*, i.e. *being certain that there is a* GOD, and how we may come by this certainty, I think we need go no farther than our selves, and that undoubted Knowledge we have of our own Existence.
>
> (Nidditch, 1991: 619)

However, Locke had already stated in his previous chapter that knowledge of oneself is gleaned through 'intuition', which is not commensurate with 'demonstration'. This is what he has to say:

> Let us proceed now to enquire concerning our Knowledge of the *Existence* of Things, and how we come by it. I say then, that we have the Knowledge of *our own Existence* by Intuition; of the *Existence of* GOD by Demonstration; and of other Things by Sensation.
>
> (Nidditch, 1991: 618)

Here then is another example of an assertion of *belief*, but not the *demonstration* that is implied by Locke and which might be supported by robust logic outside the complexity of linguistic thought and interpretation. This is important, and in Locke's case it is very interesting, for Locke devoted Book III to the significance of language and words, but this mostly in terms of semiotics. He avoids, or perhaps he was unable to see or admit to, the implications that might arise from the power to inculcate religious thoughts and beliefs that language has, irrespective of any truth in other terms; or might this be what he is implying in Chapter XVIII of Book IV on *Faith and Reason*, where he states:

> For our simple *Ideas* then, which are the Foundation, and sole Matter of all our Notions, and Knowledge, we must depend wholly on our Reason, I mean, our natural Faculties; and can by no means receive them, or any of them, from *Traditional Revelation.* I say, *Traditional Revelation,* in distinction to *Original Revelation.* By the one, I mean that first Impression, which is made immediately by GOD, on the Mind of any Man, to which we cannot set any Bounds; and by the other, those Impressions delivered over to others in Words, and the ordinary ways of conveying our Conceptions one to another.
>
> (Nidditch, 1991: 690)

This example of the merging of paradigms is in contradistinction to Pascal's understanding of the impossibility of making such an accommodation. I include now, as **Appendix 5**, the continuation of his argument which (interestingly) cites both mathematical and biblical references but where he appears to be giving precedence to the importance of what might seem to be a greater truth of the senses. I cannot overstate how important this allegiance to the empirical that is evident in Locke's statements. However, a little later, he crucially contradicts himself by saying, in §8:

> But since GOD in giving us the light of *Reason* has not there-by tied his own Hands from affording us, when he thinks fit, the light of *Revelation* in any of those Matters, wherein our natural Faculties are able to give a probable Determination, *Revelation,* where God has been pleased to give, *must carry it, against the probable Conjectures of Reason*
>
> (Nidditch, 1991: 694)

Now Locke cannot have it both ways – but he has – by indicating that any abilities that we might have to conjecture through empiricism (and Locke states that empiricism is paramount), God has given us that faculty, although that faculty itself cannot support the validity of a God in the terms as proclaimed in the Torah. Now this is very significant, for Locke, like the philosophers of metaphysics, has created a false circular argument. So, although Locke proclaims that he places his allegiance with empiricism, he has, at the same time, invoked the very mechanisms that have been shown to be popular with the continental metaphysical philosophers, i.e. that concomitant *belief* in an intangible deity, and (and this is crucial) in terms of biblical teaching, as if this were indisputable knowledge as truth.

1.9 Objects of the mind

The next of the foremost empiricists is Bishop George Berkeley (1685-1753). If Locke was convinced that knowledge was acquired through experience, it will be seen that Berkeley was certainly not convinced by the evidence of the phenomenal world. His contention, it will be seen, is that objects exist in the mind only as perceptions but that objects have no reality in themselves in that they exist only for the observer. In section CXXV of his *Essay towards a New Theory of Vision* of 1709, Berkeley makes reference to Locke:

> After reiterating endeavours to apprehend the general idea of a triangle, I have found it altogether incomprehensible. And surely if any one were able to introduce that idea into my mind, it must be the author of the Essay concerning Human Understanding;
>
> (Lindsay, 1963: 70)

However, he makes this reference only to indicate that, in his opinion, the source of Locke's understanding is erroneous. In his preceding section CXXIV, Berkeley had himself concluded that:

> It is commonly said, that the object of geometry is abstract extension; but geometry contemplates figures: now figure is the termination of magnitude, but we have shown that extension in abstract hath no finite terminate magnitude, whence it clearly follows that it can have no figure, and consequently is not the object of geometry.
>
> (Lindsay, 1963: 69)

Berkeley is making the important distinction between (a) objects that he classifies as *figure*, that we see as contingent and therefore within our experience (having sensible properties such as magnitude or colour), and (b) that of *extension*, the 'idea' or concept of such an object without its physical properties, metaphysically, as it were. Berkeley is expressing an opinion here that geometry has to do with that which is understandable by means of the tangible properties of the senses, manifest as *figure*. He then goes on to say:

> It is indeed a tenet as well of the modern as of the ancient philosophers, that all general truths are concerning universal abstract ideas; without which, we are told, there could

> be no science, no demonstration of any general proposition in geometry. But it were no hard matter, did I think it necessary to my present purpose, to show that propositions and demonstrations in geometry might be universal, though they who make them never think of abstract general ideas of triangles or circles.
>
> (Lindsay, 1963: 70)

As I understand it, Berkeley is of the opinion that we make universal application in geometry only from the particularly observed, not from any abstract concepts, at the same time denying the existence of observed objects. Such a theory is clearly problematic in that it contains a fundamental contradiction. This is interesting, for, as a mathematician (as well as a theologian), he would have been well aware of the concept of infinitesimal calculus, this having been introduced into the domain of mathematics in the previous century by both Newton and Leibniz. If we apply the concept of infinitesimal reduction to the thickness, for example, of the lines that form a visually perceived triangle, we would arrive at an imagined visual concept of that same triangle in its measure and magnitude exactly, but more precisely, because it would lack the problematic thickness of the lines of its cruder initial physical manifestation, a particular encumbrance to perfect accuracy. The triangle would, therefore, become unseen, and move into the realms of the imagined, as particular concept, and which would remain equally abstract with regard to its geometric properties, but absolutely perfect. This could equally be applied to other potential magnitudes and measure, in exactly the same manner as any observed triangle, or, indeed, any other shape. Anyone who practices geometry to absolute precision, in the two or three dimensions, and particularly in the domain of practical construction, knows this property of geometry only too well. This is absolutely crucial to understanding the relationship between the observed and the theoretical elements of geometry, the *figure* and the *extension.*

I turn now to Berkley's *A Treatise concerning the Principles of Human Knowledge,* published in 1710, where he states:

> IV. *The vulgar opinion involves a contradiction.*- It is indeed an opinion *strangely* prevailing amongst men, that houses, mountains, rivers, and in a word all sensible objects have an existence natural or real, distinct from their being perceived by the understanding. But with how great an assurance and acquiescence soever this principle may be entertained in the world; yet whoever shall find in his heart to call it in question, may, if I mistake not, perceive it to involve a manifest contradiction. [For what are the forementioned objects but the things we *Perceive* by sense, and what do

> we perceive *besides our own ideas or sensations;* and is it not plainly repugnant that any one of these or any combination of them should exist unperceived?]
>
> (Lindsay, 1963: 114)

Here we have Berkeley contradicting what he had been proposing with regard to his view that *extension,* if it does at all exist in geometry, can only be understood in terms of the *figure*. However, when it comes to another sensible world, Berkeley now proposes that whatever we see is pure perception, and that things exist only as such. Here, Berkeley seems now to be abandoning the very idea of *figure*. Things appear as such only because we perceive them, but have no reality outside our perception. Berkeley here is simply describing the process of psychology, that which happens in the mind of an individual. Yes, of course, the perception is not the object. But perception cannot happen unless it is related to the object of comprehension, in the same way that the knowledge of the properties of a triangle cannot be known conceptually without the *experience* (as Locke had already proposed) of its viewing. Then, in section XCI, Berkeley reiterates:

> All the difference is, that according to us the unthinking beings perceived by sense have no existence distinct from being perceived, and cannot therefore exist in any other substance, than those *unextended, indivisible substances,* or *spirits, which act, and think, and perceive them*: whereas philosophers vulgarly hold, that the sensible qualities exist in an *inert, extended, unperceiving substance,* which they call *matter,* to which they attribute a natural subsistence, exterior to all thinking beings, or distinct from being perceived by any mind whatsoever, even the eternal mind of the Creator, wherein they suppose only ideas of the corporeal substances created by him: if indeed they allow them to be at all *created.*
>
> (Lindsay, 1963: 159)

Clearly, for Berkeley, *matter* – separate from its perception – does not exist. For him, God is in this same matter-less realm and so his argument forms the perfect backdrop to a criticism of that which is material and which might give support to scepticism of the metaphysical in religious terms:

> XCIV. *Of Idolaters.* - The existence of matter, or bodies unperceived, has not only been the main support of atheists and fatalists, but [on the same principle doth *idolatry* likewise in all its various forms depend.] Did men but consider that the sun, moon, and stars, and every other object of the senses, are only so many sensations in their minds, which have no other existence but barely being perceived, doubtless they would never fall down and worship their own *ideas*; but rather address their homage to that eternal

invisible Mind which produces and sustains all things.

(Lindsay, 1963: 160)

Berkeley then has managed to make a case for accepting the world as sensation, evoked from that which is somehow not material, a very particular and peculiar sort of empiricism, more to do with the psychology of perception than it has to do with anything else, but somehow dragging into this realm elements of the metaphysical when he relates it to religion.

1.10 Human nature

The Scottish philosopher, David Hume (1711-76), was to look at the problems that Berkeley raised as to how we think about what we commonly perceive as objects, as well as the other issues that have been consistently raised as to our perception of geometric principles and how we may view faith in terms of its psychology and also its anthropology, religion acting as a power in political terms, as well as having its role in functioning as an individual and societal psychological crux. Let us look first at what Hume has to say about objects. In his monumental work, *A Treatise of Human Nature* of 1739/40, which he largely re-vamps in his *Enquiry concerning the Human Understanding* of 1748, he states in Book I (Of the Understanding):

> 'Tis certain, that almost all mankind, and even philosophers themselves, for the greatest part of their lives, take their perceptions to be their only objects, and suppose, that the very being, which is intimately present to the mind, is the real body or material existence. 'Tis also certain, that this very perception or object is suppos'd to have a continu'd uninterrupted being, and neither to be annihilated by our absence, nor to be brought into existence by our presence.
>
> (Selby-Bigge & Nidditch, 1990: 206-7)

Now this view is one to which I would subscribe, as opposed to Berkeley's somewhat more conceptual view of the phenomenal world. As an artist, one only has to consider the contiguous colour effect, and other visual anomalies, to understand that it is the perception of an object as a biological process which is the problem, not that there is no object to be perceived in terms of physical science, and that an object should cease to exist when we do not observe it. It would be ridiculous to assume that because we may not be aware (through sense) of an object at any time, that such awareness has any bearing upon that object, when it is the presence or the lack of the perception that is the factor that affects the apparatus of the mind, not that the object needs our perception to exist.

The distinction that Hume makes is between the perception of observed objects and the memory thereafter of the impression made by the object in that the imagining of the continuation of existence of that body cannot be taken as anything other than that imagination of a continuance. Empirically this has to be true, and only repeated observations of objects leads to the view that a belief in the continuance of an object, whether in an altered or unaltered state, may become valid. Hume finally makes the same point as his contemporary, Kant, as to the distinction between the process of observation and that of the observed, in that, as Hume puts it:

> We never can conceive any thing but perceptions, and therefore must make every thing resemble them.... As we suppose our objects in general to resemble our perceptions, so we take it for granted, that every particular object resembles that perception, which it causes.
>
> (Selby-Bigge & Nidditch, 1990: 216-7)

I shall not dwell further on this opinion that seems to be accepted and supported by a modern understanding of the same distinction between *thought*, as psychological process, and *matter*, in terms of physical science. Unfortunately, when it comes to matter, pinpointing the physical position of what is ultimately the dual nature of energy at the quantum level is proving somewhat problematic for the scientist, in the same way that we might try to find the ultimate perfect physical geometry outside conception. Again, this highlights the distinction between how we perceive and understand through thought, and any form of ontology outside that domain. For ultimate comprehension, we would have to deconstruct our biology and be outside thought. Thus man may perversely conceive or aspire to deity. And on that subject of deity, Hume also had a dual distinction to make. But before we see finally what he has to say about religion, I want to make brief reference to what he has to say about geometry. In Book I, again from *A treatise of Human Nature*, Hume says:

> There have been many objections drawn from the *mathematics* against the indivisibility of the parts of extension; tho' at first sight that science seems rather favourable to the present doctrine; and if it be contrary in its *demonstrations*, 'tis perfectly conformable in its *definitions*.
>
> (Selby-Bigge & Nidditch, 1990: 42)

Hume, surprisingly, cannot conceive of the principle of physical reduction to perfect concept, and therefore perfect exactitude in measure, as already discussed in relation to infinitesimal calculus in my deliberations on Berkeley. As Berkeley was unable to perceive a perfect geometrical concept as potentially manifest in physical terms, neither was Hume able to either:

> But I go farther, and maintain, that none of these demonstrations can have sufficient weight to establish such a principle, as this infinite divisibility; and because with regard to such minute objects, they are not properly demonstrations, being built on ideas, which are not exact, and maxims, which are not precisely true. When geometry decides any thing concerning the proportions of quantity, we ought not to look for the utmost *precision* and exactness. None of its proofs extend so far. It takes the dimensions and proportions of figures justly; but roughly, and with some liberty. Its errors are never considerable; nor wou'd it err at all, did it not aspire to such an absolute perfection.
>
> (Selby-Bigge & Nidditch, 1990: 44-5)

Clearly, both Berkeley and Hume are unable to accept an ontology that is distinct from their empirical world, an ontology that is there to support the theoretical exactitude that provides an ability to function in practice, albeit only approximately, when, for example, two-dimensional fields interface with the delineation of lines, or where three-dimensional objects physically touch. As a craftsman in a physical world, yes, I am in agreement with Hume that I cannot aspire to absolute perfection, but it is the understanding of infinite divisibility, and therefore the ability to utilise perfect numerical concepts, that makes approximation possible out of *absolutely perfect* geometry.

And now to Hume's take on religion. In the next chapter, I shall be referring to the relevance and the effect of religious persecution upon the formation (and especially the expression) of thought in an age previous to the Enlightenment. The powers of the Inquisition were, however, in continuance. Indeed, the holding or expression of heretical thought was potentially dangerous and still punishable by death. And so, as with all other expressions of philosophical thought already discussed in relation to religion, this has always to be considered as highly relevant in all cases so far discussed. This was particularly the case for Hume as he was to outline his dual approach to the question of religious faith. He was to publish *Natural History of Religion* in 1757, followed by his *Dialogues concerning Natural Religion,* completed in 1751-5, with revisions in 1761 and 1776. In his *Natural History of Religion,* Hume outlines and traces the history of religion – not in any great depth or detail – from the origin of polytheism to the formation of the monotheistic faiths, and about which he has this to say:

> The intolerance of almost all religions, which have maintained the unity of God, is as remarkable as the contrary principle of polytheists. The implacable narrow spirit of the JEWS is well known. MOHAMETANISM set out with still more bloody principles; and even to this day, deals out damnation, though not fire and faggot, to all other sects. And if, among CHRISTIANS, the ENGLISH and DUTCH have embraced the principles of toleration, this singularly has proceeded from the steady resolution of the civil magistrate, in opposition to the continued efforts of priests and bigots.
>
> (Root, 1957: 50)

Hume, with his Calvinistic background, certainly had the courage of his convictions in making such a bold statement of criticism. It is no small wonder that he was, therefore, concerned about potential reactions, given such outspoken expressions. Hume, in his *Natural History of Religion*, where he briefly outlines the historical and anthropological, clearly separates this from the philosophy: this he addresses in his *Dialogues concerning Natural Religion*, where the three main characters, Cleanthes, Philo and Demea put forward their arguments in the form of a debate, perhaps drawing upon the influence of the Socratic model of Plato. This is a complex work, and Philo's central viewpoint stands somewhere uncomfortably between the two opposing stances of Cleanthes and Demea, but showing eventually more sympathy with the empirical view. The term 'natural religion' in the title refers to a belief in a deistic (not a theistic) God who can be defined according to our experience of some sort of order in the natural world as viewed by thinking man, and compared to man. Man is all too aware of a teleology that is involved in the progress of his affairs and that of the rest of Nature, an understanding born of analysis and experience. This view is presented by the character of Cleanthes, where, in putting forward his argument from the evidence of design, he describes, through the inductive thought process:

> Look round the world: Contemplate the whole and every part of it: You will find it to be nothing but one great machine, subdivided into infinite number of lesser machines, which again admit of subdivisions, to a degree beyond what human senses and faculties can trace and explain. All these various machines, and even their most minute parts, are adjusted to each other with accuracy, which ravishes into admiration all men, who have ever contemplated them.
>
> (Smith, 1947: 143)

He then further states:

> The curious adapting of means to ends, throughout all nature, resembles exactly, though it much exceeds, the productions of human contrivance; of human design, thought, wisdom, and intelligence. Since therefore the effects resemble each other, we are led to infer, by all the rules of analogy, that the causes also resemble; and that the Author of nature is somewhat similar to the mind of man; though possessed of much larger faculties, proportioned to the grandeur of the work which he has executed. By this argument *a posteriori*, and by this argument alone, we do prove at once the existence of a Deity, and his similarity to human mind and intelligence.
>
> (Smith, 1947: 143)

Demea holds the opposing theistic view, in that God can only be known through *a priori* reasoning or through revelation, a *fideism* or blind faith that is indeed independent of reason.

1.11 Towards a conclusion

This thesis has been looking in quite some detail at this apparent contradiction within the theistic stance as compared with the more rational and humanistic approach based upon man's perceptions through experience, and so it is perhaps appropriate to conclude this chapter with reference again to Hume's *Dialogues* where Philo seems finally to side with a form of deism; this, it is believed by most commentators, is probably the voicing of Hume's own opinions. I do not intend to analyse Hume's text further, for this is a difficult work that takes many turns of argument, but, as a lead into my next chapter on Dürer's *Melencolia I*, I shall take one more reference from the *Dialogues*, when Philo remarks:

> It is true; both fear and hope enter into religion; because both these passions, at different times, agitate the human mind, and each of them forms a species of divinity, suitable to itself.... When melancholy, and dejected, he has nothing to do but brood upon the terrors of the invisible world, and to plunge himself still deeper in affliction. It may, indeed, happen, that after he has, in this manner, engraved the religious opinions deep into his thought and imagination, there may arrive a change of health or circumstances, which may restore his good humour, and raising cheerful prospects of futurity, make him run into the other extreme of joy and triumph. But still it must be acknowledged, that, as terror is the primary principle of religion, it is the passion which always predominates in it, and admits but of short intervals of pleasure.
>
> (Smith, 1947: 225-6)

It is clear then that there is a profoundly important element that comes into play with regard to faith, and that is how emotions may severely hamper the individual's ability to rationalise. The most powerful element contained in all three major monotheistic religions is that of fear, that most primeval and dreadful fear of a threatening unknown or an imaging of what may otherwise be perceived as a known world. Indeed, fear is a most powerful and elemental emotion, connected with the mechanism of survival and as thus has a clear biological function. In the next chapter, we shall see how this element of fear and imagination has been used as a tool by both church and state, and begin now to look and see whether (and in what way) any artists have responded to the big questions surrounding belief, and what may or may not be perceived as reality.

[Chronology

René Descartes (1596-1650)
Blaise Pascal (1623-62)
John Locke (1632-1704)
Baruch Spinoza (1632-77)
Sir Isaac Newton (1642-1727)
Gottfried Wilhelm Leibniz (1646-1716)
Bishop George Berkeley (1685-1753)
David Hume (1711-76)
Immanuel Kant (1724-1804)
George Wilhelm Friedrich Hegel (1770-1831)]

Chapter 2

The Power of Word: the Beauty of Number

> Refusing to abandon art to the whim of the moment, Dürer insisted on certainty, not only in shape and proportion but in faith. Whether tirelessly pursuing research into proportion or immersed in Luther's writings, truth was his invariable preoccupation.
>
> (Steck, 1964: 119)

A. Language

2.1 Historical context

I have deliberated at some length as to how some of the major philosophers of the Enlightenment seemed to have had considerable difficulty in separating belief, principally in the form of *a priori* faith, from any mechanisms by which we may otherwise perceive our world, and then understand what constitutes consciousness. Whether their stance was influenced by fear of retribution from the religious authorities and from organised society, or whether it was the effect of subjection to a religious linguistic culture that could not be psychologically shaken, is a mute point. I suspect that it may equally have been either or both. Indeed, it has always been dangerous to question the status quo, and the power of language seems to have dictated the political and personal affairs of man throughout history. But as evolving linguistic tradition seems to be distinct from the inheritance and knowledge of universal mathematical concepts, it is the latter that has always been at the heart of the building of all the great organised civilisations, as the legacy of their terrestrial architecture, and the thirst for a better knowledge and understanding of the celestial, would indicate. It is that mathematical and structural element that I shall now be pursuing and developing to quite some detail in the unfolding of this chapter.

2.2 The language of fear

It is important, however, to see where the linguistic element within the context of fear is totally distinct from the functioning of a more logical and rational part of the psyche that clearly has more to do with mathematical structure. The Torah begins with *Genesis*:

BERE'SHIT
1. When God began to create heaven and earth – the earth being unformed and void, with darkness over the surface of the deep and a wind from God sweeping over the water – God said, "Let there be light"; and there was light. God saw that the light was good, and God separated the light from the darkness. God called the light Day, and the darkness He called Night. And there was evening and there was morning, a first day.

(Berlin & Brettler, 1999: 12)

God, at the beginning, gives names to His creations. For the monotheistic faiths, we can see, therefore, how important words are. The Torah is disseminated through word, as *the word.* Without words, there is no language, no expression; no means by which the mind can create. And for Christians, truth is also seen in terms of the word. Indeed, light and dark is seen in the same (and then additional) terms, for in *The New Testament*, with the opening of *The Gospel according to John,* 1.1, the word now becomes synonymous with God, in a reiteration of creation:

In the beginning was the Word, and the Word was with God, and the Word was God. He was in the beginning with God; all things were made through him, and without him was not anything made that was made. In him was life, and the life was the light of men. The light shines in the darkness, and the darkness has not overcome it.

(Division, 1973: 87)

The further theatrical power that words have is shown in *Deuteronomy*, 4:

The mountain was ablaze with flames to the very skies, dark with densest clouds. The LORD spoke to you out of the fire; you heard the sound of the words but perceived no shape – nothing but a voice.

(Berlin & Brettler, 1999: 371)

The text continues, and what was an historical instruction for the survival of the Jewish people – transmitted through word as oral instruction and scripture – becomes a central mantra also for Christians and Mohammedans:

He declared to you the covenant that He commanded you to observe, the Ten Commandments; and He inscribed them on two tablets of stone.

(Berlin & Brettler, 1999: 371)

What is most revealing is how the modern Jewish version of the same text differs linguistically from the modern Christian version. The *Torah* states in *Deuteronomy*, 6:

And this is the Instruction – the laws and the rules – that the LORD your God has

> commanded [me] to impart to you to be observed in the land that you are about to cross into and occupy, so that you, your children, and your children's children may revere the LORD your God and follow, as long as you live, all His laws and commandments that I enjoin upon you, to the end that you may long endure.
>
> (Berlin & Brettler, 1999: 371)

However, the recent *Revised Standard Version of The Holy Bible* retains a somewhat different Christian interpretation as to how we should engage with God when the word 'revere' is translated as 'fear' for the very same biblical text:

> Now this is the commandment, the statutes and the ordinances which the LORD your God commanded me to teach you, that you may do them in the land to which you are going over, to possess it; that you may fear the LORD your God, you and your son's son ...
>
> (Division, 1973: 160)

Whether out of fear or not, The Welsh language version of Deuteronomium VI, 5 within the British and Foreign Bible Society's *Y Bibl Cyssegr-lan* of 1896, exalts us to love (câr di) God:

> PENNOD VI, 5. Câr di gan hynny yr ARGLWYDD dy DDUW â'th holl gallon,
> ac â'th holl enaid, ac â'th holl nerth.

[There is, of course, an interesting history as to how whatever may be regarded as original text has come to us. Translated initially into Greek, predominantly for Diaspora Jewish consumption, then into Latin by St. Jerome for a more exclusive use by the Catholic Church (Dürer of course made a print of St. Jerome in his study to accompany *Melencolia I*), it was not until the advent of the mechanical printing press, in the 15th century, that it was possible for translations to be made available to the public in their native tongue, with Martin Luther's translations and outspoken, moralistic teaching being most pertinent when we relate this again to Dürer.]

2.3 The climate of fear

A compounding element to the problem of the interpretation of that which is literary, notwithstanding the additional danger of a quite *literal* interpretation of text, is the *enforcement* of any interpretation. It is clearly problematic when a faith is based on either love or fear, but when the same text can be interpreted as either or both, such differences give rise to competing factional interests, especially when appropriated and massaged by other cultures for whom the

Torah was not intended. And this has been the tragic history of the legacy of a literature that has been so important to the survival of, and a defining of the inheritance of the Jewish people. There can be no doubt as to the stark teaching of the Torah and to whom its laws apply. *Deuteronomy* 13 states:

> If your brother, your own mother's son or daughter, or wife of your bosom, or your closest friend entices you in secret, saying, "Come let us worship other gods" .... do not assent or give heed to him. Show him no pity or compassion, and do not shield him; but take his life. Let your hand be the first against him to put him to death, and the hand of the rest of the people thereafter. Stone him to death, for he sought to make you stray from the LORD your God, who brought you out of the land of Egypt, out of the house of bondage. Thus all Israel will hear and be afraid, and such evil things will not be done again in your midst.
>
> (Berlin & Brettler, 1999: 395-6)

The Gospel tells us that Christ taught a completely different morality, one of tolerance and non-violence. However, whilst accepting the Christian Gospel, the Qur'ân is directed towards the retention of a parallel legacy to Judaism, with descent down to Muhammad claimed from the line of Ishmael, Israel (Isaac)'s half brother and Abraham's first son by his second wife, thereby separating themselves from Judaism, at the same time retaining a perverse distinction in claiming allegiance (this also upon threat of punishment) to the singular same monotheistic deity, but in the name of Allâh:

> Sûrah 42. Ash-Shûra Part 25
>
> 15. So unto this (religion of Islâm alone and this Qur'ân) then invite (people) (O Muhammad), and stand firm [on Islâmic Monotheism by performing all that is ordained by Allâh (good deeds), and by abstaining from all that is forbidden by Allâh (sins and evil deeds), as you are commanded, and follow not their desires but say: "I believe in whatsoever Allâh has sent down of the Book [all the holy Books, - this Qur'ân and the Books of the old from the Taurât (Torah), or the Injeel (Gospel) or the Pages of Ibrâhîm (Abraham)] and I am commanded to do justice among you.
>
> (Al-Hilâlî, 1420 A.H.: 653-654)

And it is this use of language, particularly by invoking religious text, that it is dangerous to challenge (to gain and to justify 'political' power) and which now becomes a very powerful tool. There is now an additional fear, not just the fear instilled by the text, but the fear of an even more immediate threat of physical torture, as was the practice of the Catholic Inquisition, when it was deemed heretical to live by or teach by a different moral code derived from an interpretation contrary to that which was decreed by the powers that be.

It is necessary to make an important distinction here between a singular Jewish faith and other faiths based upon a history that is not theirs. The Torah, otherwise interpreted as the Christian 'Pentateuch' or the Muslim 'Tawrat' (Genesis through to Deuteronomy), the Tanakh (Hebrew Bible) and the Targum (Aramaic translation of the Hebrew Tanakh) are texts that relate to a particular people. Their Talmud and Midrash are rabbinic texts that further lay down interpretation and expansion upon their own culture and laws. These are texts that have come out of, and are based upon, an account of the unfolding of the history of an oppressed people, a long and convoluted journey that has its origins in Babylonian Ur and Abraham's consequent journey to Canaan (it is thought in around 1,800 B.C.). Any poetic invention upon that history might be viewed as an attempt to bring rationality into what may seem to be an irrational world. In grappling with tragedy, and trying to relate to a hostile world, any element of self-punishment is clearly a matter of accommodation between a particular people (within their own developed society) and their God. Unfortunately, the Qur'ân then appropriates this same biblical history, citing justification through an alleged lineage from Ishmael, and perpetuates the tragic theme in a continuation of judgement, turning the self-judgment of a particular people into an additional external judgment upon them:

> Sûrah 5. Al-Mâ'idah Part 6
> 78. Those among the Children of Israel who disbelieved were cursed by the tongue of Dâwûd (David) and 'Isâ (Jesus), son of Maryam (Mary). That was because they disobeyed (Allâh and the Messengers) and were transgressing beyond bounds.
>
> (Al-Hilâlî, 1420 A.H.: 159-160)

And for all additional others, who do not abide by the teaching of the Qur'ân, there is also a littering and perpetuation of threats of perdition, exemplified by:

> Sûrah 15. Al-Hijr Part 14
> 2. How much would those who disbelieved wish that they had been Muslims [those have submitted themselves to Allâh's Will in Islâm i.e. Islâmic Monotheism – this will be on the Day of Resurrection when they will see the disbelievers going to Hell and the Muslims going to Paradise].
>
> (Al-Hilâlî, 1420 A.H.: 338)

and most emphatically again in:

Sûrah 22. Al-Hajj Part 17
57. And those who disbelieved and belied Our Verses (of this Qur'ân), for them will be a humiliating torment (in Hell).
(Al-Hilâlî, 1420 A.H.: 451)

The last book of the Bible, *Revelation*, perhaps contains the starkest of all warnings with regard to a belief in a judgement and punishment in 'the hereafter'. [Interestingly, there are many instances where there are indications as to there being some mystical significance to the numerical, with the example in *Revelation 13* giving rise to much fanciful conjecture:

This calls for wisdom: let him who has understanding reckon the number of the beast, for it is a human number, its number is six hundred and sixty-six.
(Division, 1973: 235)]

[Please see the significance of 666 in occult and mathematical terms at the end of Part 2.10 The magic square.]

Clearly, we now see the power of words – words used in judgment – and how they can be used to inculcate fear. Within Christianity, the Inquisition stamped a perverse moral authority upon all, through fear and torture. Its powers could be used for personal or political reasons, or enacted in a purely arbitrary manner to gain the same punitive effect. This was not just the inculcation of a fear of 'the hereafter', but it was the enacting of persecution in the lived and *real world*. So, was it this same fear that influenced the philosophers of the Enlightenment to create some sort of *a priori* rationale, based upon language, in validating the biblical God, at the same time dualistically referring to an overriding benevolence – a love out of fear? It was Leibniz who said in his *Discourse on Metaphysics*:

IV. That love for God demands on our part complete satisfaction with and acquiescence in that which he has done.
The general knowledge of this great truth that God acts always in the most perfect and most desirable manner possible, is in my opinion the basis of the love which we owe God in all things; for he who loves seeks his satisfaction in the felicity or perfection of the object loved and in the perfection of his actions.
(Montgomery, 1902: 7)

Is the strength of such an ambivalent emotion dominant, and can a psychological divorce from the influences of the un-emotive and logically rational part of the psyche be so easily induced, thus creating 'conformity' through a compound duality that can function happily within the same person?

2.4 The artist's contribution

It is at this point that I now wish to examine whether there have been any artists, at any stage in our history, who may have taken up the difficult theme as to whether logic, perhaps defined in terms of *mathematical* structure, relates in any way to a different understanding from belief based upon *linguistic* inheritance, and included any pertinent symbolism in their work, covert or otherwise. And one particular work stands out as highly significant, that is Dürer's print, *Melencolia 1,* dated 1514 **(Plate 2a)**. It is interesting that Dürer included his title within the very print itself: a linguistic message that must surely be intentional for this inclusion is quite novel in terms of art history. [It should be noted that the title of this print, when referred to in publications, is often spelt differently, as exemplified by this translation of Dürer's journal, and this issue will be dealt with later in Part 2.9 The further significance of melancholy:

> Then I sent to his secretary, Erasmus, who drew out the Request for me, a *Sitting Jerome*, the *Melancholy*, the *Anthony*, and the two new *Marys*.
>
> (Conway, 1958: 103)]

Dürer's print has intrigued analysts over the years. For the first time in art history, that we are aware of, we have a singular work of art that includes an unmistakable and sophisticated form of mathematical symbolism where it is the polyhedron that clearly stands out as being the most incongruous (and therefore most significant) object in this print, seeming also to be related to the equally intriguing magic square **(Plate 2b)**.

The particular significance, in terms of (a) what this strange polyhedron may symbolise, is one aspect that is relevant to this enquiry, but, for me, what is equally important is (b) what its actual and exact constitution as a geometric object may conceivably be. Let us now look at each aspect in turn and deal initially with the symbolic and conjectural.

2.5 Durer's polyhedron as symbol

So far in this thesis, I have been deliberating two seemingly related issues: (i) the necessary dichotomous relationship between the phenomenally perceived (as a psychological function) and what *may actually be* in terms of some sort of ontology (totally unrelated to our biological function as homo sapiens sapiens) and (ii) how an acceptance and accommodation to a coercive environment may affect our rational judgment. My question now is, does this geometric object that Dürer has invented, relate in any way to this debate? To me, it clearly does, although obviously at a purely personal level. It symbolises, for me, exactly that delineation between the two psychological fields, and highlights that very difficulty in understanding the relationship between a clearly defined geometric exact truth (as a symbol of that ontological necessity) and that of our culturally complex interface at a viscerally social level.

It is significant that Dürer, in addition to his being highly artistic, possessed an advanced mathematical mind. But did this strange object have a uniquely different symbolic meaning for Dürer? When taken in its historical context, and at the personal level, I suppose it must have. Dürer would have been conditioned, to a greater or lesser degree, by geographic location and consequently influenced by both the cultural strictures and the turbulence brought about by the vicissitudes of his age. Historically, and in terms of this enquiry, this was an extremely important period in the development of man's concurrent relationship with science and theology, as medieval beliefs were being influenced either by the dissemination of knowledge from earlier major cultures, particularly Greek and Roman, or by new scientific understanding promoted by an emerging and powerful Arabic culture. It was the dawning of the Renaissance, a challenge to medievalism and the occult traditions, as humanism and radicalism challenged the traditional powers of theologically and politically based structures. Let us, therefore, first of all look at the polyhedron in terms of the occult.

B. Mysticism

2.6 Cornelius Agrippa

Henry Cornelius Agrippa of Nettesheim (1486-1535) was a close contemporary of Dürer, at one time serving Emperor Maximilian I who was, coincidentally, also Dürer's patron. Although associated initially with the town of Cologne, then Metz and Lyon, he travelled widely, taking on a number of different high-level posts throughout a constantly changing career, as a sometime soldier, occultist, physician and diplomat. What is of particular interest here is his writings on the occult. Although not published fully until 1531, his *Three Books of Occult Philosophy* had already been prepared in first draft form by 1510, four years before the production of Dürer's *Melencolia I.* It is quite possible therefore that Dürer may well have been aware of the details of the magical arts that Agrippa was intent on disseminating through this polemic work. Indeed, in about 1509, a certain friend writes to Agrippa:

> "The bearer of these letters is a German, native of Nuremberg, but dwelling at Lyons; and he is a curious inquirer after hidden mysteries, a free man, restrained by no fetters, who, impelled by I know not what rumour concerning you, desires to sound your depths."
>
> (Tyson, 2007: xvii)

This would seem to indicate that rumours were rife and that interest had spread widely, with Dürer's home town of Nuremberg being mentioned in connection with this spreading of information.

Agrippa's knowledge of the occult had been harvested from many sources; from knowledge of the Pythagorean Greeks to the teaching of the Platonists and Aristotelians; from the historicism of the Torah and longstanding Hebrew traditions to the medieval mysticism of the Kabbalah. It is thought that much information about the latter would have been gleaned from the works of Johannes Reuchlin, viz. *De verbo mirifico,* published in Germany, in 1494. It is said (Tyson, 2007: xviii) that Reuchlin had greatly influenced the thoughts of Erasmus and Luther. Dürer, of course, was also in contact with Erasmus, indeed made a portrait of him, and followed avidly the works of Luther. It is feasible, although not certain, that Dürer may well have been influenced to include references to the occult in this one particular print, *Melencolia I,* possibly as a counterpoint to the biblical references that are integral to so many of his other works, this at a particularly vulnerable phase of his life, in mourning the death of his mother that very same year of 1514. [We will see later how Dürer's title, *Melencolia I,* may also be related to Agrippa's work, as a possible reference to one of the four humours.]

Agrippa's *Three Books of Occult Philosophy* is a huge and vast compilation of mystical information, infused with many mathematical references, but he is very careful not to exclude the divine:

> From one man, *Adam*, all men proceed, from that one all become mortal, from that one *Jesus* Christ they are regenerated: and as saith *Paul*, one Lord, one faith, one baptism, one God, and father of all, one mediator betwixt God and man, one most high Creator, who is over all, and in us all.
>
> (Tyson, 2007: 241)

Despite many other references that indicate a biblical belief within his *Three Books of Occult Philosophy*, the material is clearly mystical. It is interesting that Agrippa was to publish *De vanitate et incertitudine scientiarum et atrium / The Vanity and Uncertainty of the Sciences and the Arts* in 1530, just a year before the first edition of his *Three Books of Occult Philosophy*. In *De vanitate* Agrippa espouses a contrary view to the gaining of wisdom from enquiries into what can only be described as the occult, wherein he expresses scepticism of any knowledge that is not gained from divine revelation. This indicates further the ambivalence that exists, for whatever reason, in the possession of opposing views within the same person.

2.7 Melancholy in context

Reference is often made in the standard literature to the supposed connection between melancholia or melancholy – as one of the four humours – and the planet Saturn, and therefore in terms of its astrological significance. There is also a connection made with one of the four elements (earth), a clear legacy of the Classical Greek view. Other bodily and pseudo-medical elements are also related within an overall scheme. All the connections and relationships are commonly tabulated thus:

HUMOUR	TEMPER	ORGAN	NATURE	ELEMENT
Yellow bile	Choleric	Gall bladder	Warm Dry	Fire
Blood	Sanguine	Head	Warm Wet	Air
Black bile	Melancholic	Spleen	Cold Dry	Earth
Phlegm	Phlegmatic	Lungs	Cold Wet	Water

[I have placed the rows in this particular order such that the elements of fire, air, earth and water are seen also to represent the tetrahedron, the octahedron, the cube and the icosahedron, respectively, when the dodecahedron would then further represent the cosmos in Plato's cosmological system (Fauvel & Gray, 1987: 76-79).]

Frances Yates, in her book *The Occult Philosophy in the Elizabethan Age*, devotes a whole chapter (Yates, 1979: 49-59) to the subject of occult philosophy and melancholy, making a special reference to Dürer's *Melencolia I.* She makes the same supposed connection with Agrippa that I have already mentioned, as do many others. Such research draws from the complex history of the development of the inventive beliefs that are integral to the occult, with resonances from the Platonic age, continuing right through to medievalism. [However, I do have one critical comment to make upon a particular assumption that Yates makes in her analysis of Dürer's *Melencolia I*, and this is included in Part 2.9 The further significance of melancholy.]

Not only would Agrippa's work have been of interest at this time, but also the writings attributed to Hermes Trismegistus, entitled *Hermetica*. See extract as **Appendix 6** which, drawing upon an Egyptian and Greek heritage, connects the 'four elements' – in rather a peculiar way – to a particular concept of 'reality', a topic (as perception) that was discussed within Chapter 1. Elsewhere, Trismegistus (as *Hermes*) has a lot to say about God as Good, greatly emphasising this benevolent aspect of God. As with Agrippa, there is an attempt made by this author to make connection with, and an accommodation between, monotheism and Classical beliefs. In taking a quote from the end of this particular extract, and knowing that Dürer would have thought deeply about the perceptive process, this view might well apply to *Melencolia 1* in general, and to his polyhedron in particular:

> ... all this is false, and deceives the eyes of those who look at it; they think that what they see is real, but it is really an illusion.
>
> (Scott, 1993: 149-150)

It is possible that Dürer would also have had knowledge of certain terra-centric astrological beliefs from another source, viz. that of the Jewish Persian known as Messa Halah, or Masha'allah ibn Athari (c.740-815). Indeed, it is stated on the internet (Wikipedia, 2009-2010, and repeated elsewhere) that it was Dürer who provided the title plate (see **Plate 4**) for the 1504 translation (from the Arabic to Latin) of his book, *De Scientia Motus Orbis*. This is doubtful as no reference to this can be found in the standard academic literature and there is no obvious tell-tale monogram that I can see subsumed within this plate that would corroborate. Dürer did, however, in 1502, provide a woodcut for the title plate for a book entitled *Philosophy* where the four medieval temperaments of choleric, sanguine, phlegmatic and melancholy are depicted allegorically as the four winds (Bartrum, 2003: 64-65).

2.8 Standard research

The writings of Erwin Panofsky (Panofsky, 1971: 157-171), Heinrich Wölfflin (Wölfflin, 1971: 200-208) and many others, are commonly cited in connection with the interpretation of such possible mystical symbolism within Dürer's print, as well as interpretations of the symbolic meaning of the other objects in addition to the polyhedron and magic square. Such information is valuable in as much as it contextualises this particular print in terms of all his other works, and also in comparison with other artworks of the period. Further historical information about his contacts and other influences of his age are to be found in varying detail in many publications, the most interesting of which probably include those of Wilhelm Waetzoldt and Max Steck. Waetzoldt, for example, refers to the melancholic character being written about by Marsilius Ficinus in his German translation (dated 1505) of his book on the triple life, *De Vita Triplici* (Waetzoldt, 1950: 80). Ficinus writes about melancholia being a characteristic temperament of all the great artists. Waetzoldt makes further comment to drive home his point:

> The prevalence of the opinion that melancholy was a typical spiritual phenomenon in the creative man is also illustrated by the remark of the Farnese legate Paulucci (1519) that Raphael was a man with a propensity to melancholy, 'like all men of such outstanding significance'.
>
> (Waetzoldt, 1950: 80)

[It is known that Raphael – the creator of the celebrated *School of Athens*, 1509-10 **(Plate 5)** where the theme is clearly humanistic – was held in high esteem by Dürer and that they once exchanged works of art as personal gifts to each other (Wood, 2009).]

2.9 The further significance of melancholy

In Book I, Chapter LX of his *Three Books of Occult Philosophy*, Agrippa makes particular reference to the melancholy humour, and, with reference to Aristotle's book of *Problems*, he states:

> By melancholy, saith he, some men are made as it were divine, foretelling things to come, and some men are made poets. He saith also, that all men that were excellent in any science, were for the most part melancholy.
>
> (Tyson, 2007: 188)

In his further commentary on the melancholy humour, Agrippa continues to explain the nature of this mental state:

> So great also they say the power of melancholy is of, that by its force, celestial spirits also are sometimes drawn into men's bodies, by whose presence, and instinct, antiquity testifies men have been made drunk, and spake most wonderful things. And that they think happens under a threefold difference, according to a threefold apprehension of the soul, viz. imaginative, rational, and mental.
>
> (Tyson, 2007: 188)

His further explanation of these three related states are given as **Appendix 7**, and it is significant, in my opinion, that this view (by Agrippa) of these particular and graduating mental conditions, may quite possibly be the very themes that Dürer is alluding to in his triptych, that is (i) *Knight, Death and the Devil* (1513), (ii) *Melencolia I* (1514) and (iii) *St. Jerome in his Study* (1514). See **Plates 1, 2a** and **3** respectively. Certainly the presence of a dog and also the hourglass in each print suggests a possible link between the three prints. This, to me, is significant. However, I see the way in which Agrippa formulates his view of the melancholic state and how this may relate to Dürer's treatment of this subject somewhat differently from Frances Yates, whose book I referred to earlier. Yates is of the opinion that *Melencolia I* relates to the first state that Agrippa describes (see again **Appendix 7**) and questions whether this particular print was intended as the first of a series (Yates, 1979: 54), with two remaining unaccounted for, presumably entitled *Melencolia II and III*. However, although *Melencolia I*'s title may initially indicate the contrary, I would contend that the first state that is described by Agrippa could actually be referred to within *Knight, Death and the Devil*, not *Melencolia I*, with the apocalyptic and military element that Agrippa includes in his text being quite evident in this print. [By taking and then abridging a section (mentioned as a translation of the manuscript version of 1510) from Klibansky, Panofsky and Saxl's 1964 *Saturn and Melancholy*, it is not clear whether Yates has excluded this most important element herself (Yates, 1979: 53) – or whether it was not included by Agrippa in 1510 – and, if by doing so, she then ignores its implications.]

I am of the view that *Melencolia I* would then continue as a reference to the second 'rational' state, with *St. Jerome in his Study* finally referring to the third state, that of divine revelation. [Dürer produced o number of different versions of *St. Jerome* prior to the 1514 print, therefore St. Jerome, as the first translator of the Bible into Latin (in c.390-405), must have been important to him.] There would therefore not be a need to consider a *Melencolia II and III* as such. Indeed, it is possible that the '*I*' in *Melencolia I* may not actually be a numerical reference at all. [I deal with this issue in Part 2.11 The magic square.]

I would contend that linking the three states of melancholy as described by Agrippa to the extant and evident triptych is far more plausible than the idea that there may be additional prints simply because of the numerical possibility that the title *Melencolia I* might suggest. Later in her chapter on this subject, Yates amplifies upon her initial view, and expresses her disagreement with Panofsky's interpretation of *St. Jerome* – that of a mathematical order of the room, a quite clear use of perspective, and a further and obvious indication of holy contemplation. Panofsky's view is that this is intended to contrast with the disorder and incertitude of *Melencolia I.* This, Yates asserts, takes no account of Agrippa's 'three stages of inspired vision' (Yates, 1979: 58). She puts forward her own concluding theory thus:

> In fact I would suggest that the *St Jerome* may actually be *Melencolia III.* At this stage the *mens* sees into all truth, including the divine truth of geometry, obscurely and confusedly distinguished by *imaginatio* in *Melencolia I.*
>
> (Yates, 1979: 58)

Under this scheme, where then is the rational (*ratio*), and an inferred *Melencolia II*? Has Yates fallen into the same trap as Panofsky in linking *Melencolia I* only with *St Jerome*? Certainly Dürer's own written accounts (Conway, 1958: 92-126) indicate that he sometimes sold them together but not exclusively so, often selling them with others, therefore not necessarily, or consistently, as a pair. Indeed, *Melencolia I* was sometimes sold with his other and earlier print of St. Jerome, the *Sitting Jerome*, dated 1512.

It is also significant that both William B. Scott and W. M. Conway (in his translation of Moritz Thausing's particular research on Dürer's writings) have Dürer himself, in the journal of his journey to the Netherlands, July 1520-July 1521, referring to *Melencolia I* consistently as simply *Melancholy* (again Conway, 1958: 92-126). [The original book in which Dürer made these records is lost but an old copy of his journeys and accounts remains.] The 'I' is always excluded and the English spelling is probably purely a conventional translation of what, in modern German, is 'melancholie'. To add to the confusion, many modern translations of the title have it as *Melancholia I* or simply *Melancholia,* as with Bartrum, whereas Panofsky and Yates remain true to the title within the print. Is the significance of the 'I' in the print itself therefore perhaps given too much prominence by commentators and theorists when Dürer himself excludes the 'I' in his own records? It is assumed that Thausing would not have deviated from the extant copy and would not have made alterations that would not have been true to what had originally been recorded (presumably) as Dürer's own titling and conventional spelling. Conway's own translation into English is therefore (probably) from closely original text; otherwise, both authors will have crucially altered and misrepresented an important element of extant documentation. Additionally, the print that is otherwise commonly known as *St Jerome in his Study*, is translated from these accounts more directly by Conway as *Jerome in the Cell*, and therefore has a slightly different connotation to the former. Then, in another place it is translated simply as *Jerome*, and in yet another case it is given more fully as *St Jerome in the Cell.* As there are these significant differences in the titling, it is probable that Conway/ Thausing were quite accurately replicating those slight and human inconsistencies that often occur in personal writing. However, we cannot be certain, for Conway uses a quite different titling and spelling in the commentary, which could equally indicate either the noted differentiation or a contrary inconsistency:

> Memorable are the two following years, 1513 and 1514, for the appearance of Dürer's three central engravings – the 'Knight and Death', the 'S. Jerome in his Study', and the 'Melancolia'.
>
> (Conway, 1958: 77)

[The latter spelling ('Melancolia') could be a misprint, for in another place (Conway, 1958: 183) it is spelt as 'Melencolia'.]

Returning now to Yates, and her linking *Melencolia I* to occult philosophy, I would agree with her that *St Jerome* may be referring to the third stage of the mental (*mens*), i.e. that of learning 'the secrets of divine things, as the Law of God' (Tyson, 2007: 189) as the final stage of revelation and salvation, and that this is a quite obvious interpretation; I would still contend that *Melencolia I* more logically may refer to the second stage of the rational (*ratio*) with all its mathematical and more humanistic symbolisms. This leaves *Knight, Death and the Devil* acting as the first print in a series of three (clearly predating the other two by one year) and as an initial reference to the imagination (*imaginatio*) within the occult philosophy, indicating that realisation of the stark nature of the contingent world and the apocalyptic inferences that can be drawn from this knowledge from scriptural teaching.

When we look at other research material, away from the specific issue of how the melancholy reference may relate to Dürer's own treatment of the subject, there is also Joseph Leo Koerner's interesting contextualising of Dürer's self-portraiture, which looks at another aspect of his work and personal world. However, I do not propose to comment upon Dürer too generally and make an extensive trawl of the more standard literature and evaluate whether their interpretations may or may not be valid, for this is the common route of so many researches, resulting in the repetition and recycling of the same kind of historical material, ad nauseam. This does not help us get any closer to any truth about Dürer's print that is not conjectural. However, with regard to possible occult influences, we may take it on *faith* that there may have been a connection with mystical symbolism being made by Dürer, and that we may refer to Agrippa's tome on this subject as being his potential or major source of reference. However, this is in no way certain.

2.10 Dürer's polyhedron as stone

In Book I, Chapter XXV: *What things are saturnine, or under the power of Saturn*, Agrippa writes:

> Saturnine things, amongst elements, are Earth, and also Water: amongst humours, black choler that is moist, as well natural, as adventitious, adjust[1] choler is excepted. Amongst tastes, sour, tart, and dead. Amongst metals, lead, and gold, by reason of its weight, and the golden marcasite[2]. Amongst stones, the onyx,[3] the ziazaa,[4] the camonius,[5] the sapphire, the brown jasper, the chalcedon,[6] the loadstone, and all dark,

weighty, earthly things. [Notes given in **Appendix 8.1**]

(Tyson, 2007: 83)

It is, therefore, the 'accepted wisdom' that Dürer's polyhedron, as stone, and as a mystical object, probably represents Saturn, for 'black choler' would indicate 'melancholy'. Which of these stones might Dürer have been referring to, if he was at all? Now, Agrippa tells us in Book 111, Chapter 11: *Of concealing of those things which are secret in Religion*:

> ... and divine *Plato* commanded,[2] that holy and secret mysteries should not be divulged to the people; *Pythagoras* also,[3] and *Porphyrius* consecrated their followers to a religious silence; ... [Notes given in **Appendix 8.2** provide further insight.]

(Tyson, 2007: 443)

Might this very concealment be the reason for Dürer's enigmatic object? Was he acting out some sort of personal Pythagorean ritual in withholding whatever information or message he was guarding covertly within the object itself? Is he hiding some sort of mysticism, or just the mathematics within what is to us an uncertain geometry? [This particular issue is dealt with in Chapter 3, Part 3.15 Figured numbers.]

Now there are many questions and much to ponder. I shall attempt to address their relationship and any implications that might arise in turn; but firstly, what might the stone represent in terms of its geology? If it indeed refers to Saturn, then a study of Agrippa's tome indicates a number of options. The most likely candidate is probably the loadstone that Agrippa mentions, some crystalline structures of which might well relate to the shapes and angles that may be constituted within Dürer's polyhedron. These I shall be proposing later in my technical analysis.

Loadstone or lodestone (Fe_3O_4), also known as magnetite, is magnetic and commonly forms as an octahedron (see **Plate 6** and **Plate 7**, upper image) but some variations can show different configurations. For the Greeks, the octahedron, as one of the Platonic solids, represented Air (gas).

Loadstone can also form rhombic faces, examples of which are shown in **Plate 8**, upper image. The significance of the rhombic faces and their relationship within solid geometry to the Platonic forms is hugely important. This will be shown in the development of my own constructs as a direct reflection of such important properties, a major consideration within this thesis.

The other possibility is that Dürer might have been making reference to golden marcasite or iron pyrite (FeS_2) that Agrippa also refers to, commonly known as fool's gold. The particular iron pyrite sample in **Plate 7**, lower image, is clearly in the form of an isometric crystal – pyritohedron (pentagonal dodecahedron). Cubic forms are also common. For the Greeks, the cube represented Earth (solid), and the dodecahedron, Ether (quintessence).

The fact therefore that loadstone and iron pyrite are to be found – one might even say are 'created' – in these beautiful geometric forms, and whether Dürer was aware of this may be significant, in that Dürer may have been making a link between a presumed occult reference and what he knew about the fascinating mathematical resonances within and between the Platonic and other regular solids. From the viewpoint shown in **Plate 8**, lower image, this sample certainly exhibits certain geometric features that are similar to those that constitute Dürer's polyhedron, although the acute angle would probably be 60° for loadstone.

2.11 <u>The magic square</u>

Next, I wish to explore any connection that might be implied between Dürer's polyhedron and his magic square, firstly in terms of any possible connection with Saturn. The print of *Melencolia I* that is published by Panofsky in his book *The Life and Art of Albrecht Dürer (181, fig. 209)* and by Guilia Bartrum in *Albrecht Dürer and his Legacy (fig. 128)* which is in the British Museum, shows the 9 differently (lower image of **Plate 9b)** from that which is in the print commonly shown on the Internet **(Plate 2a**, and middle image of **9b)** and shown by Wölflin in *The Art of Albrecht Dürer (Plate 87)*, by Waetzoldt in *Dürer and his Times (Plate 7)* and by Steck in *Dürer and his World* [p.99 – reference given as 'a copperplate engraving made by Dürer in the Year of his mother's death. (Reproduction: F. Bruckmann KG, Munich)']. In both versions, the 9 is represented un-conventionally and Dürer must have had a reason for this. Why did he go to the extent of altering his plate for the second version? Did the number 9 hold some sort of symbolic significance for him? [The British Museum also has a copy, among others, that was made of Dürer's engraving that very closely resembles the original, with the 9 here somewhat similar to that within the original print referred to by Wölflin, Waetzoldt and Steck. See **Plate 9a** and detail in upper image of **Plate 9b**. Dürer, of course, was very much copied by others, with this copy being quite closely faithful to the original in most of its detail and form.]

This brings me back to the conjectures that surround the inclusion of the title *Melencolia I* within the print. Could it be that Dürer intended this mystery through the unconventional spelling, and what is *assumed* to be a number 1, thus purposely making us question further? Might he actually be creating a link and a reference to the number 9 within the magic square; for it might be that the 'I' in the title is actually the *letter* I, not the *number* 1, with 'I' (being the ninth letter of the alphabet) intended to represent the *number* 9? This would act to highlight its significance and may also be intended as indicating that further bridge between (a) the uncertainty that is linguistic interpretation and thought (through the unconventional spelling and what might be assumed to be a number, within that context of lettering) and (b) that which is more clear and certain, i.e. the actual numerical and mathematical.

In *The Three Books of Occult Philosophy*, Agrippa mentions the Greek's use of ordered sequencing in assigning a letter to each number (Tyson, 2007: 308). In their first sequence, the number 9 is synonymous with the letter I:

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
Α	Β	Γ	Δ	Ε	Ζ	Η	Θ	Ι	Κ	Λ	Μ	Ν	Ξ	Ο	Π	Ρ	Σ	Τ	Υ	Φ	Χ	Ψ	Ω

It was also known that, prior to Agrippa, those who followed the principles of Ramon Lull (1232 - c. 1316) assigned 'letter-notation to notions so exalted and abstract as the names, attributes, or dignities of God' (Yates, 1979: 11). In addition to the primary set of letters (linking the first four letters of the alphabet as an astrological reference) there was also a series of nine dignities, each designated in Lullian 'Art' by one letter out of the series BCDEFGHIK. The letter A was excluded, for this letter stood for God. The letter 'I', in this one major scheme, stood for VERITAS, translated as 'truth' (Yates, 1960: 1). See **Plate 11a** and detail in **Plate 11b**. It must be noted, however, there were other slightly different letter schemes where this did not pertain.

[This theme is further developed, but with particular reference to the *Sephiroth*, in Chapter 3, Part 3.16 The Kabbalah.]

It is also significant that Dürer has included what one presumes to be the letter S in front of the dating of his print *Knight, Death and the Devil* of 1513. See **Plate 1** again. Could this possibly again be a reference to the number 9, for one could conjecture that S might stand for September, the ninth month of the calendar; or might it stand for Saturn and might all this then indicate a possible link between this print and *Melencolia I*? [Panofsky believes the S to be the abbreviation for Salus (Panofsky, 1971: 151), i.e. the Latin for health, wellbeing or salvation.]

Looking now at the number 5 within the magic square of *Melencolia I*, this also seems to be a variation from the norm, with what we know as the number 5 appearing to having been rotated by 180°. A study of Dürer's other works reveals rather an interesting pattern. In his dating, he occasionally adopts a different calligraphy from his norm, this for any one of only three particular numbers, and it is always one of these three numbers that is given this unconventional treatment, none other. The numbers are 4, 5 and 9. One relationship is immediately self-evident, that of the addition of the first two numbers to produce the third.

Many artists of the Renaissance included themselves covertly in their paintings, but it was Dürer who was to champion self-portraiture in terms of the overt and as an exercise in the representation purely of the self. And if you look at the dating of the first of his painted self portraits, dated 1493 **(Plate 10)**, you will see how he has represented the number 4. [This quirky 4 can be also found in some of his sketches of this period.]

Now if we go back to Agrippa, we may be able to see some possible motive in Dürer having chosen to alter his nine, this being some covert indication as to its significance. The 9 is of course included in the magic square of 4 x 4. Agrippa, in Book II, Chapter XXII (Tyson, 2007: 318-328) connects this square with Jupiter. Agrippa's magic square is shown below in **Fig. 1**:

4	14	15	1
9	7	6	12
5	11	10	8
16	2	3	13

Fig. 1

Dürer may have chosen to use this particular square so that he could alter the sequences (please refer to Dürer's version) to show the print's dating (in the bottom row of his version) and possibly to show his initials, in reverse (indicated then by the 4 [D] and the 1 [A] either side). But if he is indicating that the number 9 is to be regarded as significant, it may be that he is making reference here to the very first of the magic squares, the 3 x 3, containing 9 cells whose sum is 45 (see **Fig. 2**). Note also the incidental numerical reverberation within the number 45 as 9 x 5 = 45. This magic square is clearly indicated in Agrippa's book as being connected to the planet Saturn (Tyson, 2007: 321), or Saturnus in Latin: and thus we come back full circle to the possible reference to 'melancholy'.

4	9	2
3	5	7
8	1	6

Fig. 2

Note that 5 is in a commanding and central position, and it is worth seeing what mystical and religious significance Agrippa holds for this number:

> Also this number hath great power in expiations: for in holy things it drives away devils. In natural things, it expels poisons. It is also called the number of fortunateness, and of favour, and it is the seal of the Holy Ghost, and a bond that binds all things, and the number of the cross, yea eminent with the principal wounds of Christ, whereof he vouchsafed to keep the scars in his glorified body. The heathen philosophers did dedicate it to be so much more excellent than the number four, by how much a living thing is more excellent than a thing without life.
>
> (Tyson, 2007: 262)

He then continues to give this number further biblical significance:

> For in this number the father *Noah* found favour with God, and was preserved in the flood of waters. In virtue of this number *Abraham*, being an hundred years old, begat a son of *Sarah*, being ninety years old, and a barren woman, and past child bearing, and grew up to be a great people.
>
> (Tyson, 2007: 262)

For the number 4, although he finds also religious significance in this number, Agrippa makes allusion to the Greeks:

> The Pythagoreans call the number of four tetractys, and prefer it before all the virtues of numbers, because it is the foundation, and the root of all other numbers; whence also all foundations, as well in artificial things, as natural, and divine, are four square ... and it signifies solidity ...
>
> (Tyson, 2007: 254)

[The significance of the tetractys is investigated in Chapter 3, Part 3.15 Figured numbers.]

For the number 9, Agrippa includes a dedication to the nine Muses, the nine blessed orders of angels and includes a biblical reference in 'the ninth hour our Lord *Jesus* Christ breathed out his spirit' (Tyson, 2007: 284), one of many in his book.

How much of the above was known to Dürer and whether any such mysticism is being referred to in *Melencolia I* can only be a matter of conjecture, but we do know he had associations within humanist circles. He is, therefore, more than likely to have been well informed about such matters through, for example, his life-long friend Wilibald Pirkheimer, or even through his connection with the eminent Desiderius Erasmus (1466/7?-1536).

Additionally, as an interesting adjunct (not necessarily relevant to Dürer's particular magic square) I wish briefly to refer back to the biblical passage within *Revelation 13* that was mentioned towards the end of Part 2.3 The climate of fear. This is in reference to the significance that is given to the number 666. Agrippa states, when referring to 'the planets, their virtues, forms, and what divine names, intelligences, and spirits are set over them':

> The fourth table is of the Sun, and is made of a square of six, and contains thirty-six Numbers, whereof six in every side, and diameter produce 111, and the sum of all is 666.
>
> (Tyson, 2007: 319)

If we look at Agrippa's magic square (Tyson, 2007: 319) we can see clearly that any row or column adds up to the same number (in this case 111) as do both diagonals, and that this is a property of any magic square:

6	32	3	34	35	1
7	11	27	28	8	30
19	14	16	15	23	24
18	20	22	21	17	13
25	29	10	9	26	12
36	5	33	4	2	31

Agrippa's magic square

The number six certainly has a particular significance in biblical terms. The Torah, as part of the Tanakh, and as the foundation stone of both Christianity and Islam, claims that the world was created by God in six days and that He rested on the seventh. However, this biblical play on the number 666 is clearly mystical, taking the properties of a magic square and placing it outside its own domain and ascribing to it the power of religious significance. No small wonder that many academics question whether *Revelation* as the last book of the New Testament should have a place within the Christian canon.

Next, and moving now away from any occult references, I wish to investigate whether the numbers 4, 5 and 9 now have any relevance purely with regard to their mathematical properties as they apply to the possible geometry within the polyhedron.

C. Mathematics

2.12 The polyhedron

It is inevitable that my enquiries into the possible geometric structure of the polyhedron were conducted initially at the more empirical level, for I am a practical two and three dimensional artist/artisan. Much as I am able to conjecture and theorise *a priori*, I am of the opinion that it is *empiricism* that informs that crucial faculty as we function within our phenomenal world, and it is this *empiricism* which finally clarifies or corrects any mistaken theorising, provided we also understand that distinct relationship between the phenomenal and the ontological, discussed at length in Chapter 1. My approach to the problem at hand has therefore been predominantly practical, using the mathematical knowledge gleaned through geometric analysis as a further informant to the development of my own constructs. It is this investigative, constructive practice that has developed a better understanding and ability to 'see' better – to visualise in three dimensions – through to the ontological 'ideal'; to then appreciate how this complex geometry relates back crucially to the two-dimensional and, finally, to further appreciate what are pure, numerical concepts, which is right at the very core of understanding.

In researching the literature upon the subject of Dürer's polyhedron, I have found, with one notable exception, that the approach of researchers has been predominantly theoretical, lacking (to me) the necessary empirical evaluation that one might have expected as final proof of their theories. Their starting point is *a priori,* their conclusions are *a priori*. I, however, intend to evaluate both *a priori* and *a posteriori*, as I try to place in context all the possibilities of the numerical relationships that are to be found in the competing standard theories. I shall be developing my own reasoning and experimental approach – much as did Terence Lynch – and will be proposing two additional possibilities that seem to me to be equally (if not more) plausible, and which are clearly not in the literature. It is now necessary to deal with the more difficult technical and mathematical aspects of this enquiry.

2.13 A truncated hexahedron

What then, in general, is the geometric structure of Dürer's polyhedron? Fortunately, there is no need for conjecture here, for we have the evidence from a preparatory sketch **(Plate 12a)** that Dürer made for this particular print (Strauss, 1972: 282). His sketch is included in a compilation of a number of various preparatory works, known as the *Dresdner Skizzenbuch (Dresden Sketchbook).* Both Schröder and Weitzel refer specifically to this source in their deliberations. [I have re-drawn this sketch to make his numbering and lettering clearer and more understandable. This is taken from an enlarged and more detailed view **(Plate 12b)** and shown below as **Fig. 3**. The numbers are not to be confused with different numbering systems that have been used by other researchers.]

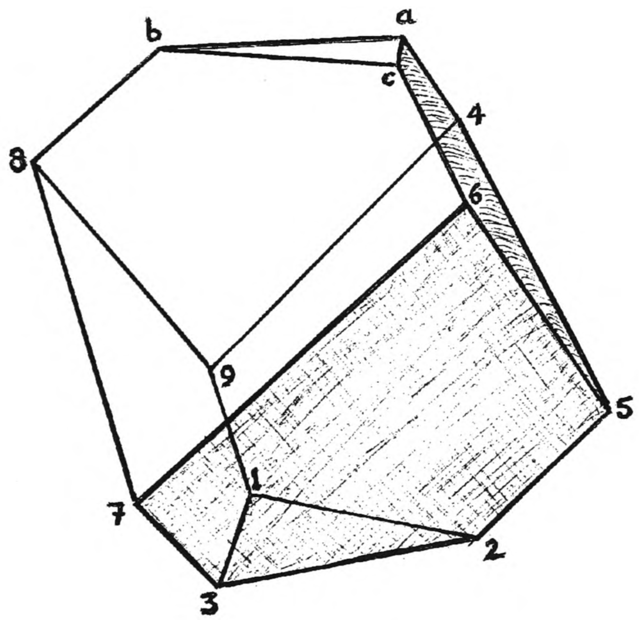
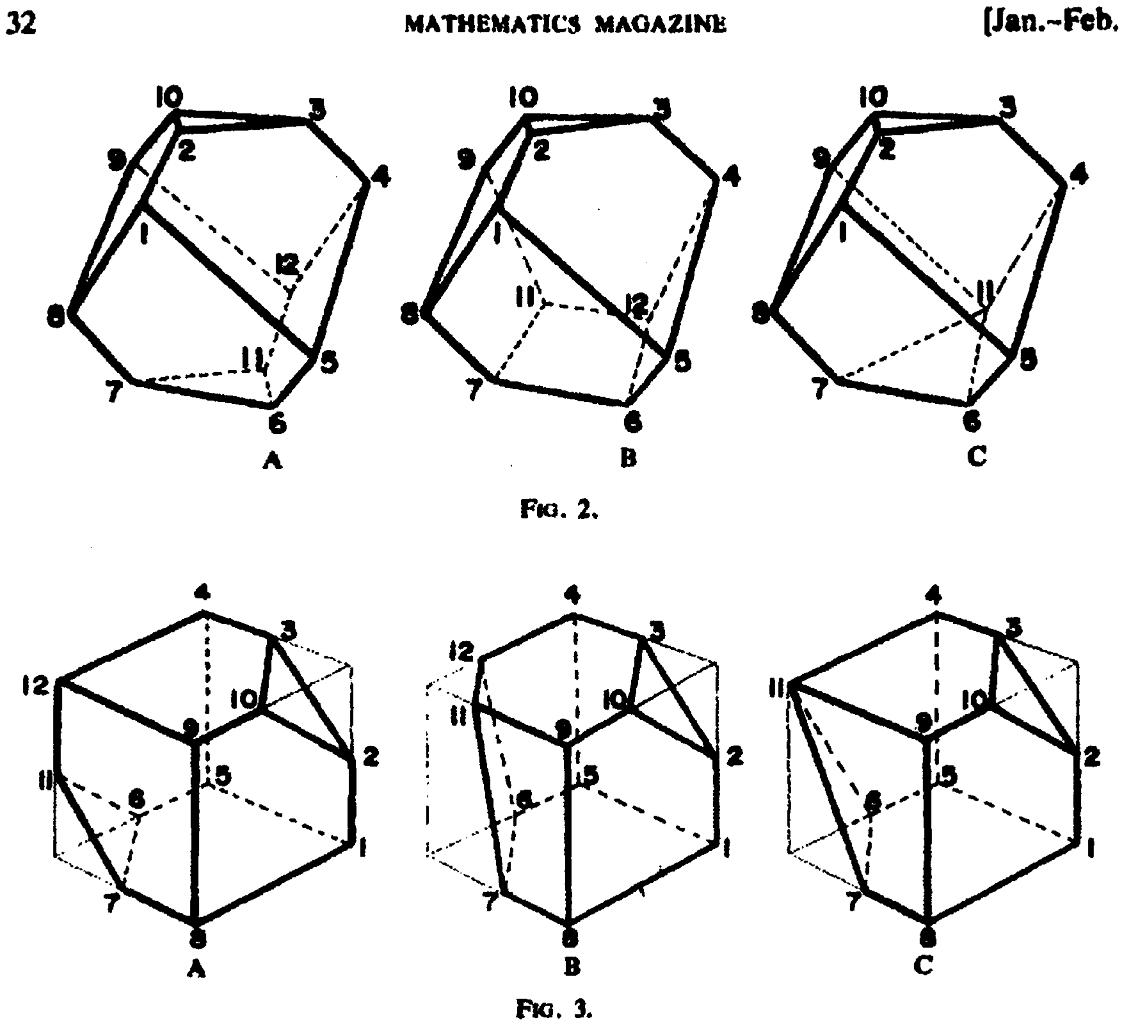


Fig. 3

We can, therefore, discount any contrary conjecture such as those configurations that are muted by P. J. Federico in his paper, 'The Melancholy Octahedron', 1972, specifically the configurations B & C in his FIG. 2, and B & C in his FIG. 3, shown below:



[Federico's FIG. 2 & 3]

To be fair, Federico does not indicate that we should take his proposals, necessarily, as serious propositions, but had we not the evidence of Dürer's sketch they would have to be regarded as valid initial considerations. In his paper, the representation given as A, in his FIG. 2, is exactly what Dürer's sketch shows (expressed in mirror image) and generally described as a truncated hexahedron. The representation A, now in his FIG. 3, gives a somewhat different view of another proposal that is often muted, i.e. a truncated cube, itself being a specific form of a truncated hexahedron. With regard to the latter, and in an otherwise quite informative and interesting book, *Squaring the Circle*, Paul Calter says of Dürer's polyhedron:

> People who have analyzed the polyhedron in this engraving have decided that it is actually a cube with opposite corners cut off.
>
> (Calter, 2008: 308)

This, unfortunately, is to rather simplify and completely misrepresent the case. As soon as such a hypothesis is tested empirically, it is found not to be at all plausible. The necessary practical approach to testing had already been taken by Terence Lynch, back in 1982 (Lynch, 1982: 226-70). In this paper, 'The Geometric Body in Dürer's Engraving *Melencolia I*' he states:

> Models were made with pentagons that had angles at the apex of 90°, 75° and 60°, but none of these was successful (Fig. 2). No better result was achieved when pentagons were used that were based on the Golden Section, (a ratio of 1.618 to 1), a method used in the past to construct regular pentagons. As the sketches [shown separately below] in Fig. 2 show, the model with 90° at the apex, that is to say a cube, was far too squat; and that with 60° was far too tall; and although the model with 75° was closer to the shape in the engraving, it was still too high. From these experiments one fact emerged that agreed with the findings of Niemann, namely that the apex angles of the pentagons seemed to be between 80° and 83°, and the side angles to be between 97° and 100°.

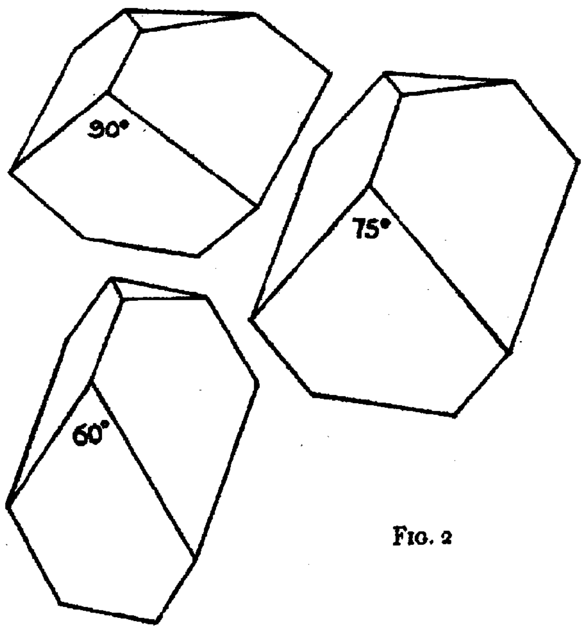


FIG. 2

This concurs with the results of tests conducted using my own accurately constructed models, some of which are displayed in **Plates 13a, b & 14**. If we were to accept, for the moment, as the first of my two original propositions (see **Fig. 4** below), an apex angle of 80° and a side angle of 100°, this gives an indication of the geometry that is muted for each of the six polygonal faces of Dürer's polyhedron (the top and bottom faces being equilateral triangles):

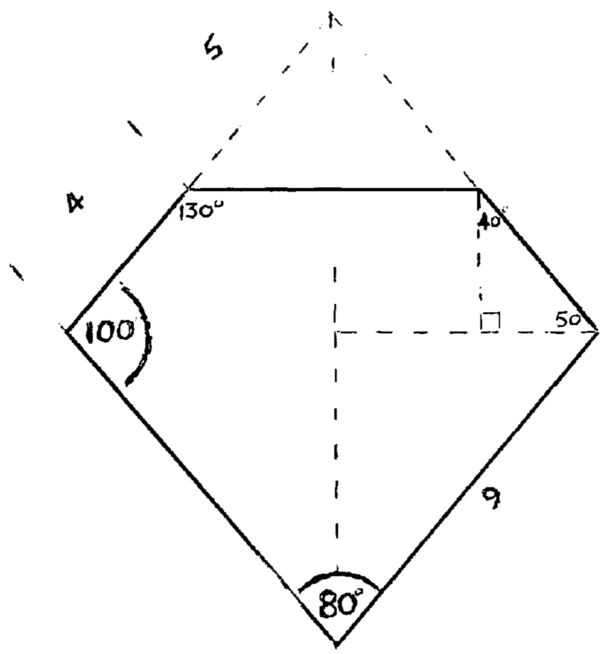


Fig. 4

From the above figure, it can be seen that this face can be regarded as a semi-truncation of a rhombus. Note the numerical harmony (4: 5: 9) in both linear and angular proportions that are integral to it, and how this might relate to what was discussed previously in Part 2.11 The magic square. How much of a truncation that we should assign to it is a matter of considerable debate. Some researchers are of the opinion that it should be regarded as a half way truncation such that a side of 9 units of measure should have its opposite shorter parallel side measuring 4½ units. A greater truncation is deemed to be an aberration, and assumed to be an error made by Dürer.

This is what Lynch contends. That, of course, presupposes that Dürer had not been at all accurate in the preparation and execution of his printing. This has to be considered as a possibility, and Lynch shows the overlay of the supposed discrepancy upon his theoretical half way truncated model. However, Lynch fails to point out that there is also a considerable discrepancy between the outlines indicated in the sketch from the *Dresden Sketchbook* (**Plate 12a**) with that of his final print. I assume that Lynch would have known of this sketch, although he makes no reference to it. In **Fig. 5** below, I have overlaid Dürer's final printed version upon the reverse image of his sketch (enlargement shown in **Plate 12b**) to show the margin of error, where the outline of the sketch (thinner line) is shown to be that much wider that his final print (shown in broader line). This mismatch might well be explained by his viewing point for the sketch being slightly more 'face-on', i.e. slightly to the right, compared with the viewing point for his final print. He could equally have placed and oriented the model of the polyhedron he was using slightly differently in each case. The latter is highly likely as the model was more than probably moved slightly or moved entirely and replaced between any preparatory sketch and the final version.

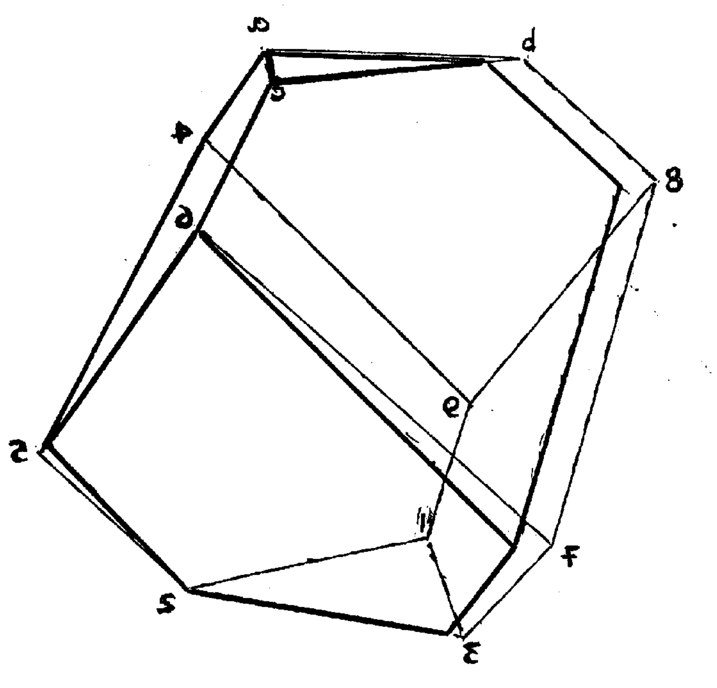


Fig. 5

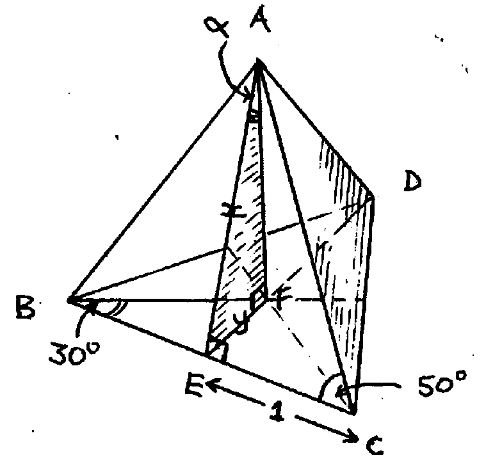
2.14 Perspective drawing

It should also be noted that whereas lines b-8 and 5-2 (see **Fig. 3** and **5**) are drawn in parallel, lines 4-9 and 6-7 are drawn in convergent mode towards an imaginary point, very far off bottom left. Although the points 4 and 6 are forward of their respective conjoining points (9 and 7), such perspective correction is probably excessive and not congruent with the same perspective that should therefore have been given to lines b-8 and 5-2 that are parallel to them, but not drawn to perspective. Lines 7-8, 1-9, 6-c and 5-4 are clearly drawn in perspective, indicating a sharp depth of field. What is important, however, is that *both* sketch *and* final version include a truncation greater than a half. It is unlikely that Dürer would have made the same error twice if he was working from the exact half-way truncated model that is proposed by Lynch and others. Indeed, for the sketch (with its greater width), the truncation is in excess of that which is displayed by his print, a further and clearer indication that the model from which Dürer was working had a truncation greater that a half. I would contend, therefore, that Dürer's working model would have had to have integrated within it a truncation greater that a half and that this truncation was clearly a physical attribute of the model.

Dürer, of course, had made a special study of perspective drawing. He published his findings in *Underweysung der Messung*, and the 1525 edition shows the first of his inventions for perspective drawing (**Plate 15**). However, sketches made in 1514 (Strauss, 1972: 292) and a later one in 1515 (Bartrum, 2003: 219) – the former (**Plate 16**) being of the same date as *Melencolia I* – would indicate that he might already have made, and was, therefore, using this apparatus for achieving accurate perspective drawings: and when we look at Eberhard Schröder's analysis of Dürer's polyhedron, it is clear that he (the researcher) had exactly this in mind – indeed he refers to it – and he provides us with his own detailed perspective drawings (**see Plates 17a, b** and **c**) to propose the numerical relationship 2: $\sqrt{3}$ for the diagonals of a non-truncated rhombohedron (Schröder, 1980: 72), giving an angle of 81.8°. The angle itself he does not actually state, only the 2: $\sqrt{3}$ relationship. [This 2: $\sqrt{3}$ proportion is a very important one, and we shall be meeting it again in my further deliberations.]

Lynch uses the same configurations as does Schröder, basing his theory upon a relationship between the square grid and a right-angled semi-truncated rhombus (in this case, a semi-truncated square) that can be superimposed upon it. Compare Lynch (**Plate 18a**) with Schröder (**Plate 17a**). Both rely upon the 30° angle of tilt that the faces display in their (side) plan view. Lynch is then of the opinion that when the face is viewed head on and tilted back by 30°, it would take on the appearance of the semi-truncated square that he visualises within the magic square (see Schröder, **Plate 17b** and Lynch, **Plate 18b**), and that the angle would therefore have had to originally been 81.8°. For the trigonometric calculations, see Lynch, **Plate 18c**. This, of course, presupposes an angle of inclination of 30°, which (itself) is based strictly upon plan drawing.

If, however, we detach ourselves from what appears initially to be an *a priori* hypothesis, and see how Lynch originally came to formulate his theory, it was initially through *empiricism*. He had taken the side view of his model that represented the 80°- 83° form and noticed that the angle of inclination was approximately 30°. The word *approximately* is very important here, for I could equally use his logic by also working back from my *a priori* theory and claim that my 80° proposal could be just as valid, as a tilt of 28.97675° is also *approximately* 30°. The trigonometry is given below in a 3D representation (as **Fig. 6**) where three isosceles triangles, Δ ABC, Δ ADC and Δ ADB, sit upon the equilateral triangle, Δ BDC, such that angle BAC for the front face of this irregular tetrahedron is 80°.



$$x = \tan 50^\circ \quad (AE = \tan \angle ACE)$$
$$y = \tan 30^\circ \quad (EF = \tan \angle EBF)$$
$$\sin\alpha = \frac{y}{x} = \frac{\tan 30^\circ}{\tan 50^\circ} = 0.4844544$$
$$\alpha = 28.97675^\circ$$
$$\simeq 29^\circ \quad (\angle EAF)$$

Fig. 6

Both his theory and mine, therefore, have to be regarded as somewhat speculative as they are based upon initial approximations and the assumption that there *may be* a relationship between the polyhedron and the magic square. Schröder, however discounts any mystical connections, and bases his 2: √3 proportion purely upon plan drawing from a cubic base, but whose mathematical considerations would tend to lend support to Lynch's theory. The fact that Durer's sketch and print display quite clearly a greater truncation than half way must, unfortunately, bring such theories into serious question.

Notwithstanding such disputed positions, the outcome of most research is that the apex angle is of the order of 80°, with variation either side of only a few degrees. This is born out by Weitzel's tabulation of some of the research material (Weitzel, 2007: 152). In addition to giving us his extensive and very comprehensive overview of so much research material, which includes some complex mathematical formulae, Weitzel also draws our attention to sketches that are to be found in the Nürnberg-Codex of Dürer's manuscript. Certainly, there is one sketch (Weitzel, 2007: 157; **Plate 19b**) that appears to resemble Dürer's polyhedron, but this is actually a visual aberration, as it seems that he is attempting to devise a novel configuration out of pentagonal and triangular faces. [It is possible that he was working towards an Archimedean snub dodecahedron, which possesses many more faces than Dürer has here, and that his own unique polyhedron might have come about from these experimental drawings.]

The same conjecture applies to a previous sketch (Weitzel, 2007: 156; **Plate 19a**) which might indicate the individual shape of the faces of his disputed polyhedron, this from an initial plan view. The acute angle of this pentagon approximates 80°. The latter appears to be the shape we are looking for, but only as a two-dimensional play upon the theme of what is actually depicting a quite different solid object. In fact, a number of these sketches appear to be experimental drawings of how we may transform a Platonic tetrahedron into its related Archimedean form (the truncated tetrahedron) by truncating each of the four corners to create four new triangular faces, thus forming four new hexahedrons out of the original triangles of the tetrahedron, whereas others seem to form some sort of pentagonal faces, forms that are not altogether clear. As such, these experimental sketches actually now serve to draw our attention to such visual anomalies and show how it is possible to be tricked into imaginary visualisations of both two- and three-dimensional images. Such anomalies certainly highlight the highly interpretive nature of visual perception. [I shall be dealing with the significance and importance of this later, and with particular reference to a revealing and perceptually intriguing drawing that I have noticed within Pacioli's *Divina Proportione* that shows the configuration of a dodecahedron, set perfectly within its related Platonic solid, the icosahedron.]

To Weitzel, the magic square is also significant. The summary to his paper states:

> The two free parameters of a truncated rhombohedron have to be chosen in a way that (i) its front orthogonal elevation is nearly quadratical and with the form of the magic square, and that (ii) it possesses approximately a circumscribed sphere. Both conditions result in a value of 79.2° for the angle of the rhombohedron.
>
> (Weitzel, 2007: 173)

The calculation by which Weitzel arrives at the angle 79.2° is complex, whereby the relationship of the diagonals are given by the formula: $3n\ [1 - 2\ /\ (\sqrt{}\ (9n^2\ /\ m^2 - 3))]\ /\ 4$. He does not address the vexed question as to the amount of truncation that would be integral to his model. It has also to be noted that his 2007 paper (published in German) is a considerable expansion and further hypothesising based upon his initial paper in 2004 (published in English) where he first refers to the sketches within the Stadtbibliothek Nürnberg. Here he proposes a 79.5 ± 0.5° for a truncated rhombohedron that would be inscribed perfectly within a circumsphere, as is the case for all Platonic and Archimedean solids. Here he cites Schreiber:

> This result should confirm Schreiber's [1999] hypothesis of a circumsphere of the solid.
>
> (Weitzel, 2004: 14)

However, this first attempt at confirmation is more than questionable, as Schreiber's own hypothesis is based upon a completely different angle – that is 72° (Schreiber, 1999: 376). Schreiber's own hypothesis for the 72° golden angle we can dismiss on the grounds of empiricism, as already stated when considering Lynch's paper. Indeed, Schreiber provides no proof or theoretical grounds for his hypothesis, other than the obvious significance of this angle as being related to φ, the golden section, $(\sqrt{5}+1)/2$. In contrast, and by utilising some advanced mathematical formulae (drawing upon her knowledge as a crystallographer), Caroline MacGillavry cautiously proposes a number of configurations whose conclusions might then be averaged out, and expressed commonly, as 79 ± 1°. One piece of research that she alludes to, however, is significant, for it touches (possibly) upon a model that I had already developed independently. This is what she has to say about Grigoriev and Schafranovsky's research:

> They compare the Melencolia polyhedron with two different fluorite crystals, one an octahedral body with two small and six larger faces ($\alpha = 60°$), the other a combination of the cube ($\alpha = 90°$) with two small octahedral faces. They suggest Dürer constructed a sort of hybrid model from these two shapes, with $\alpha = 72°$ because of his known

preference for the golden rule.

(MacGillavry, 1981: 294)

The theoretical model that I propose can be seen in **Plates 13a & 13b**, fourth from the left, and integrates both 90° and 72° angles. But before I next discuss the implications of the mathematical coding that I have integrated into this and my other maquettes, I wish to refer back to one more theory, that which involves anamorphosis and which relates to an 80° hypothesis, at the same time conceding the likelihood of 90° for the actual model.

2.15 Relating an 80° acute angle to a 79% anamorphism

The artistic interpretation of anamorphosis is perhaps best known through Hans Holbein's *The Ambassadors,* completed in 1533 **(Plate 20)**, not long after the production of Dürer's print. There are, of course, other elements that may be equally important when considering the significance of the various technical instruments and other objects that are also displayed within this painting. However, it has been suggested that there may be a relationship between the skull in Holbein's painting and that which might be implied within the prominent face of Dürer's polyhedron. Hideko Ishizu, in a very recent and interesting paper (Ishizu, 2009), makes this very suggestion but further proposes that Dürer might have been making reference to what has become known as the Delian problem (where the early Greeks tried to devise a formula so that they could convert the linear dimensions of a cube for a doubling of its volume), and that this is reflected in the 79% formula that he adopts for the amount of anamorphosis that he proposes that Dürer's polyhedron represents. [The percentage can be stated more accurately as 79.37%, in the conversion of $(\sqrt[3]{2})^{-1}$.]

In Book 4 of his *Underweysung der Messung,* Dürer does indeed show a practical method for drawing the increased square with rule and compass (Dürer's Fig. 44) and whose cube would lead to a doubling of the volume. **Appendix 9** shows this method. The expressions are not easy to follow in translation, but the important factor to appreciate is the difficulty in gaining perfect accuracy when attempting to ensure (in practice) that lines *gh* and *hi* are equal in length and where their scale is relatively small. The second stage in this method could also lead to possible inaccuracies. However, when this method was assessed through trigonometric calculation, it was found to be correct to within eight decimal places, with the equal lines both at 0.57968159, this when working back with a scientific calculator from hk = 1.25992104989/90. It is fascinating to me that the process is at all correct, and why it actually should be. However, the theoretical background to any proof of the accuracy of this method is beyond the scope of this enquiry.

The following diagram (**Fig. 7**) and accompanying calculations show a different but more direct method of arriving at $\sqrt[3]{2}$, equally consistent in testing and in practice.

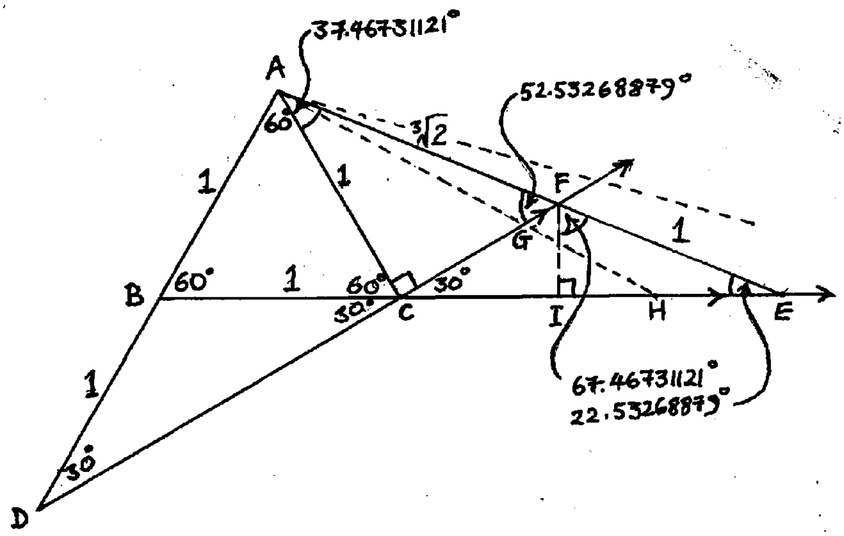


Fig. 7

To draw a line to equal $\sqrt[3]{2}$ as an extension of 1, move the point G on straight line AH in the direction of F so that H also extends to E. Keeping the line AE straight and when the line FE equals 1, then AF = 1.2599210499 (or $\sqrt[3]{2}$) and where CE = 1.58740105196. This can be shown to be correct by the following calculations:

By Pythagoras, assuming AF to be $\sqrt[3]{2}$ (1.2599210499) when FE is 1,

then $\angle$CAF = 37.46731121°.

Therefore $\angle$AFC = 52.53268879°.

If we subtend a line FI to meet line CE at right angles to it at I,

then as $\angle$IFE = 67.46731121°, line FI = 0.38321046827.

Also cos $\angle$IFE = 0.38321046827

Therefore FE must = 1.

Returning to Ishizu's theory, he suggests that this value of $^3\sqrt{2}$ is the same amount by which a truncated cube could be expanded within a vertical plane and thus converted to an 80° polyhedron. The same formula can be applied in a reverse process by which an 80° polyhedron could equally be converted back to a truncated cube. It is proposed that Dürer could therefore have been working from a 90° model, which can be made to appear – through anamorphosis – as if it were an 80° polyhedron. Ishizu states that there is evidence from a manuscript dated 1513 that Dürer clearly had the technical knowledge for conducting anamorphism (Ishizu, 2009: 183), this before he published sketches in 1528 within his work *Vier Bücher von Menschlicher Proportion* to indicate the elongation of the proportions of a head within a grid in both front and side views. Ishizu, therefore, concludes that

> This proves that a cube with an inscribed human head was one of Dürer's basic concepts then, and so was a cube within an inscribed skull.
>
> (Ishizu, 2009: 183).

This proposal has some novel attraction. However, I have one fundamental problem with this theory: it lacks the robustness of necessary practical accuracy. Ishizu gives the ratio of vertical enlargement to produce an 80° replication from a truncated cube (90° model) at 'about 1.277' (Ishizu, 2009: 188). [I would concur that he is perfectly close to the mark, with my calculation for the increase in height AF from a 90° model to the 80° shown in my **Fig. 6** providing 1.2768770 from $\sqrt{}$ (tan 50°- ⅓) / ($\sqrt{}$⅔).] However, for the Delian reference to hold true, the proportion would have to be exactly $^3\sqrt{2}$, i.e. 1.25992105. Moreover, and quite crucially, what Ishizu has actually done is not strictly an anamorphism in the artistic sense (which would be a tilting of the *replicating surface* to produce an elongated effect upon that same surface) but a quite different distortion induced by tilting the surface plane of the *object itself*, which is quite a different matter. However, by this method, one can only tilt an 80°

polyhedron to look like a 90° cube, not the other way round, for tilting a 90° cube would create a 'squashed' effect with an angle in excess of 90°. A more practical approach that would provide an actual anamorphic drawing (as opposed to Ishizu's Fig.13, a digitally manipulated image distortion) is much simpler and more direct, and thus my calculation for the Delian anamorphism differs considerably. The eventual appearance of an acute angle to the polyhedron would have to be approx. 76.9° (where $\tan \alpha = 2 \times (\sqrt[3]{2})^{-1}$), not the ≈ 80° that would be produced by Ishizu model, with an angle of tilt to the viewing and drawing face at approx. 37.5° (where $\cos \alpha = (\sqrt[3]{2})^{-1}$).

My calculations for the direct practical approach are given below as **Fig. 8** where the replicating plane AC indicates the necessary angle of tilt (α) that would result in the $\sqrt[3]{2}$ elongation to the plane of the object face, represented by AB.

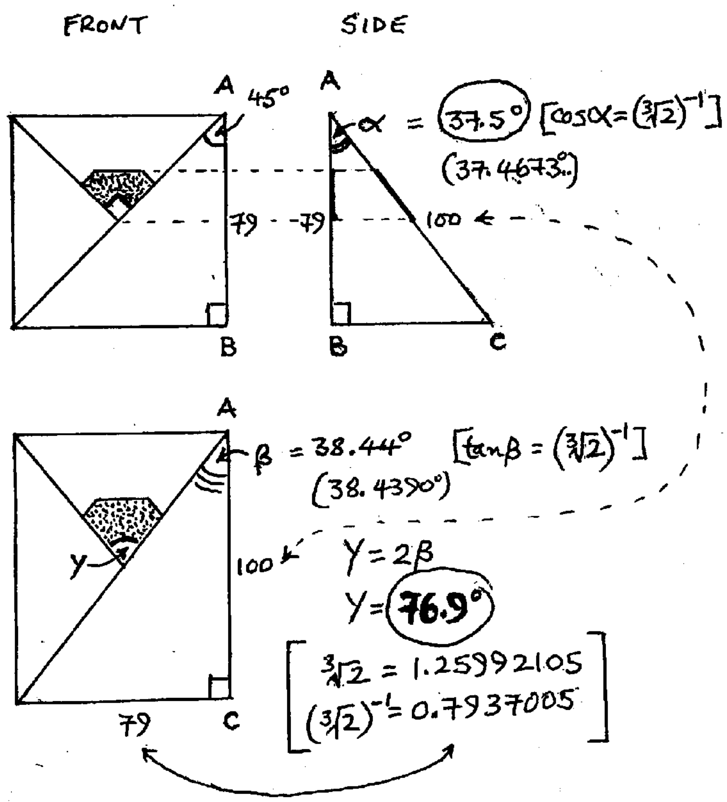


Fig. 8

[For an angle of 80° for γ, the anamorphic proportion would have to be 1.19175, i.e. (tan β @ 40°)$^{-1}$, clearly not close enough to the $^3\sqrt{2}$ proportion of 1.25992105 and certainly nothing to do with the 1.277 produced by Ishizu's quite different mathematical model.]

I cannot overemphasize the importance of the distinction between two different views of a surface or object depending upon (a) *its* tilt – not of its replication – away from a vertical plane, thus causing a truncated effect and (b) the distorting effect of true anamorphosis/anamorphism which is an elongation of an image that is applied upon a replicating surface, and then the different and more 'natural' appearance that is produced by tilting that same *replicating surface* away from its natural viewing plane. The process, mathematics and optics involved in each case are quite different and distinct.

I am quite clear in my mind that, whatever principle Dürer was engaged with, he would have been working from an actual scale model (whatever that might have been) and that he would have been basing his drawings and final plates upon their two-dimensional replication through the use of that apparatus of his own devising. Were it to be feasible, a version of Ishizu's hypothesis could be tested using a replica of Dürer's perspective drawing device by tilting the glass face by approx. 37.5° from the vertical for an eventual appearance to the truncated cube model of an acute angle of 76.9° for the Delian proportion, but as this process is clearly not what Ishizu describes, there would not be any profit in taking this approach. Another clear weakness to this theory is that the problem of how much truncation there is within the model has also been totally ignored.

2.16 The musical analogy

What is of further interest to me, of course, are the further numerical resonances that arise out of an appreciation of the mathematics involved in what would be a Pythagorean approach to the Delian problem. Below, as **Fig. 9**, are represented three cubes to show the relationship between their linear and volumetric dimensions.

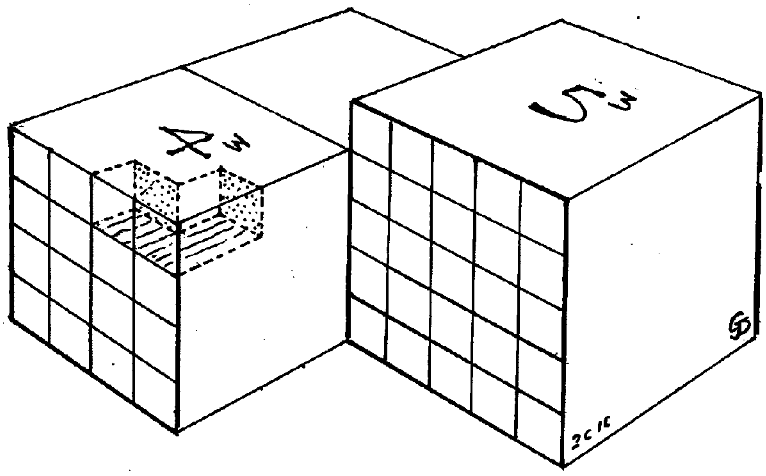


Fig. 9

Note that if we were to take the three small cubic units that are indicated in the figure away from the two 4^3 cubes, whose combined total is 128 cubic units (i.e. 2 x 64), this would equal the volume (125 cubic units) of the larger 5^3 cube. In precise terms, the amount by which the linear proportions of a cube exceed those of another twice its volume, is given as $\sqrt[3]{2}$. This factor translates numerically, as we have already seen, to 1.25992105, which is fairly close to 1.25, i.e. 1¼ or 5/4. Immediately, we see a resonance with the 5:4 truncation that I propose in my initial theoretical model (**Fig. 3**) but as there is only an approximation here, I do not think it significant.

This 5:4 proportion also approximates the Pythagorean proportion 81:64, and represents the relationship between the wavelengths of two musical notes, a diatonic major third interval apart, in the same way that its reciprocal would be represented in terms of their frequency. To illustrate this, we could set a monochord so that the dividing bridge is placed to give a measured ratio of 5:4 and thus, at a given tension, to produce two precise notes a major third interval apart. This 5:4 ratio is indicated by the first diagrammatic representation within **Fig. 10**, and then by its further compound effect within the longer more complex model. [Incidentally, the link between Pythagorean relationships and the celestial order that is seen in Robert Fludd's representation (see **Plate 21**) is also relevant in the context of the occult, elements of which were discussed previously.]

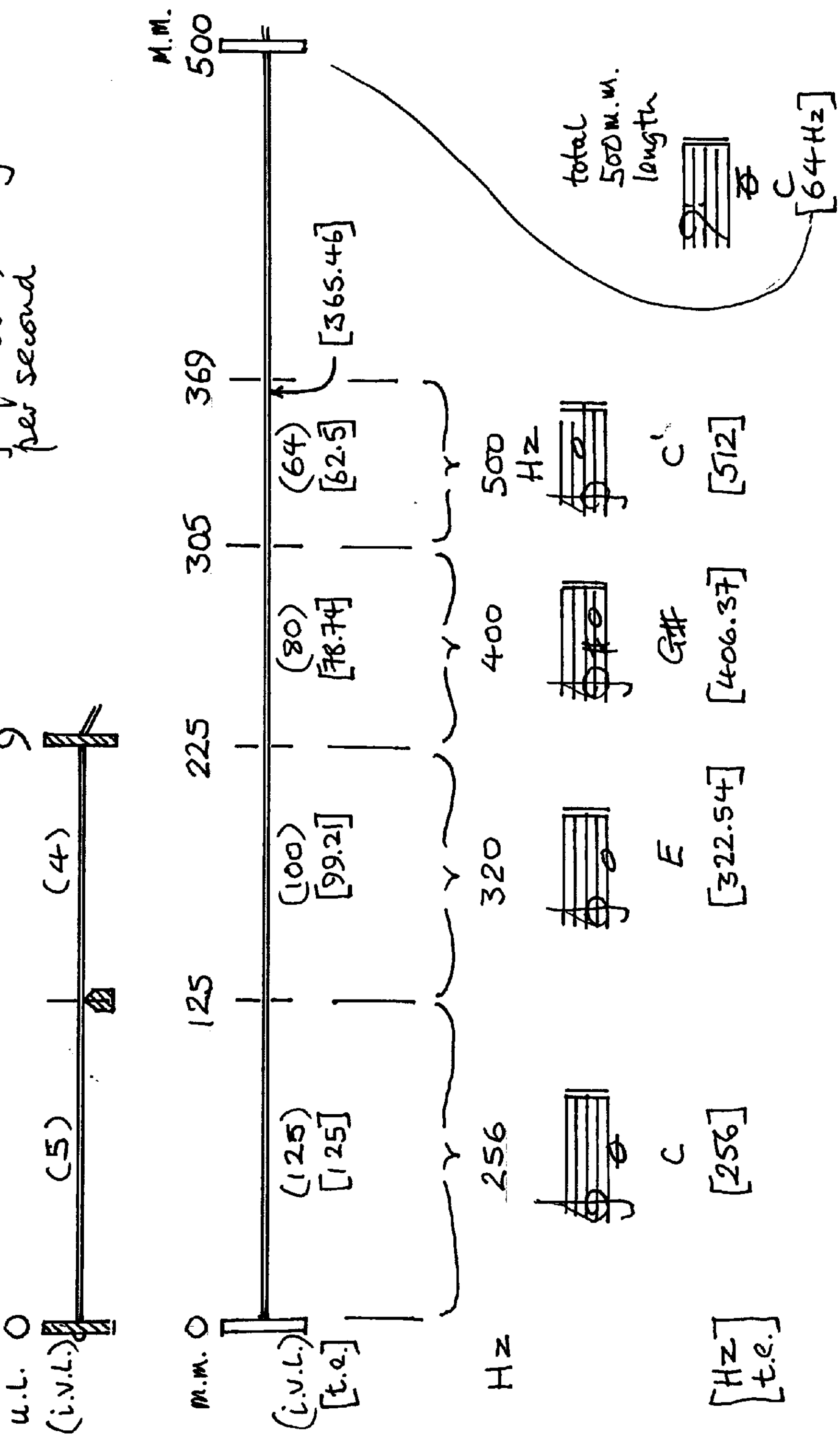

u.l. unit lengths
(i.v.l.) individual vibrating lengths
[t.e.] tempered equivalents
Hz frequencies, as cycles per second
u.l. 0
(i.v.l.)
(5)
(4)
9
m.m. 0
(i.v.l.)
[t.e.]
125
225
305
369
M.M. 500
(125)
[125]
(100)
[99.21]
(80)
[78.74]
(64)
[62.5]
[365.46]
Hz
256
320
400
500 Hz
C
E
G#
C'
[Hz]
[t.e.]
[256]
[322.54]
[406.37]
[512]
total 500 m.m. length
C
[64 Hz]

Fig. 10

Drawing a bow across the string length of 5 units would give, let's say, the note middle C, which can be given a value between 256 and 264 Hz (cycles/sec.). [For numerical convenience, I shall be using the lower value, 256 Hz.] The string length of value 4 units would produce a higher note, a major third above, viz. E (256 x 5/4 = 320 Hz). Remember that any decrease in vibrating length means an increase in frequency, such that the vibrating length is related as a reciprocal of the frequency. Interestingly, three major thirds are contained within an octave, e.g. C → E → G# → C^1, where C → C^1 represents a theoretical doubling of the frequency. We would therefore expect the multiplication, 5/4 x 5/4 x 5/4, to result in 2. It does not. The calculation (viz. 5/4 x 5/4 x 5/4 = 125/64 = 1.953125) produces a very close approximation to 2 for this combination of three intervals of a diatonic third. The resultant note would, therefore, be marginally under an octave, i.e. middle C (256 Hz) would produce an octave (C^1) under the expected 512 Hz (i.e. 256 x 125/64 = 500 Hz). Only the *tempered* major thirds, as exact consecutive $^3\surd 2$ multiples, would produce a perfect octave. Tempered values are indicated by square [ ] brackets within **Fig. 10**.

Furthermore, there are four minor third intervals within an octave, e.g. C → Eb → Gb → A → C^1, where the ratio of a minor third interval is 6:5. [This is not shown in my figure.] The minor third interval is, of course, the very fundamental interval that gives the minor key its more sombre qualities, compared to the major key. The minor key could be regarded as more 'melancholic'. I therefore see a possible resonance for the 6:5 proportion within Richter (1957) and Sixel (1990)'s theories, referred to by Weitzel (Weitzel, 2004: 146). They both propose a 79.6° for the apex angle as a reflection of the 5:6 relationship between the two different lengths of the diagonals within a rhombus possessing such an angle. This is very close to 80°.

The calculation, 6/5 x 6/5 x 6/5 x 6/5, for the minor third interval, gives us 1296/625 = 2.0736 and the resultant octave above middle C would, therefore, be slightly sharp at 530.84 Hz, not the desired 512 Hz. These aberrations exemplify the problems encountered by all serious executant musicians in making accommodations within the mismatch between the more strictly defined (a) Pythagorean and (b) Diatonic relationships within 'just' (or pure) intonation, which we know and feel at the visceral and auditory level, and the mathematical reality that, for each of the twelve semitones within a chromatic octave, the desired 16:15 (i.e. 1.0666 recurring) interval for the diatonic semitone does not exactly equate with the very exact and smaller ratio of $^{12}\sqrt{2}$ (i.e.1.0594631...) that is used in 'tempered' tuning to achieve perfectly even semitones across the whole harmonic spectrum (Helmholtz, 1885). See **Appendix 10** for a comparison between the two unresolved 'just'(or pure) systems.

What may seem to be simple solutions to the problem of justifying intervals within a harmonic structure belie a somewhat complex field. Strictly speaking, what is referred to as the Just Pythagorean scale actually utilises only the *one* whole tone interval, that of 9:8, so that the major third (9/8 x 9/8 = 1.265625) is slightly larger than the diatonic interval of a third (5/4 = 1.25) and tends to indicate more of a resolution towards the perfect fourth. Likewise, the major seventh (as leading note) also melodically resolves towards the octave through the same smaller semitone interval of 256:243 (i.e. $4^4/3^5$, which converts to 1.0534979). However, the Just Diatonic scale that I have been referring to utilises the larger 16:15 semitone interval (which converts to 1.0666 recurring) and, therefore, *two distinct and different* whole tones are required, viz. the same '*major* 9:8 tone' utilised in the Pythagorean scale, and an additional '*minor* tone of 10:9', so that the major and minor thirds (5:4 and 6:5 respectively) are then made possible within the octave. [The 'tempered' whole tone ($^6\sqrt{2}$ or 1.1224622) lays between the major Pythagorean 9:8 whole tone (at 1.125) and the minor 10:9 whole tone (at 1.1111111), closer to the 9:8 than the 10:9.]

Interestingly, Robert Lawlor, in his book, *Sacred Geometry: Philosiphy and Practice*, proposes another ratio for the semitone, that of 19:18, which he gleans, in a novel way, from the number 1.05147 as his ratio of the diagonal of a pentagon to its height (Lawlor, 1982: 51). The ratio 19:18 actually gives us 1.0555 recurring, somewhere between the 256/243 or $4^4:3^5$ (converted then to 1.0534979) and 16:15 (converted to 1.0666 recurring) of the just semitones, whereas a more accurately calculated pentagonal ratio of 1.0514622 (by either $2 / \sqrt{(1+1/\varphi^2)}$ or $\sqrt{(1+1/\varphi^2)/(1+\varphi/2)}$) is actually smaller than, and much closer to, the Pythagorean $4^4:3^5$ (or 256/243). Lowler's approximate 19:18 ratio for the pentagonal relationship can therefore be discounted in favour of the closer Pythagorean 256:243 (or $4^4:3^5$) interval.

The relevant ratios are tabulated below:

Ratios (accurate to 7 decimal places)		Difference
(Lawlor's incorrect approx.	= 1.05147)	
Pentagonal ratio	**= 1.0514622**	
	}	0.0020357
Pythagorean 256/243	**= 1.0534979**	
	}	0.0020577
(Lawlor's theoretical 19/18	= 1.0555556)	
	}	0.0039075
Tempered $^{12}\sqrt{2}$	= 1.0594631	
	}	0.0072036
Diatonic 16/15	= 1.0666667	

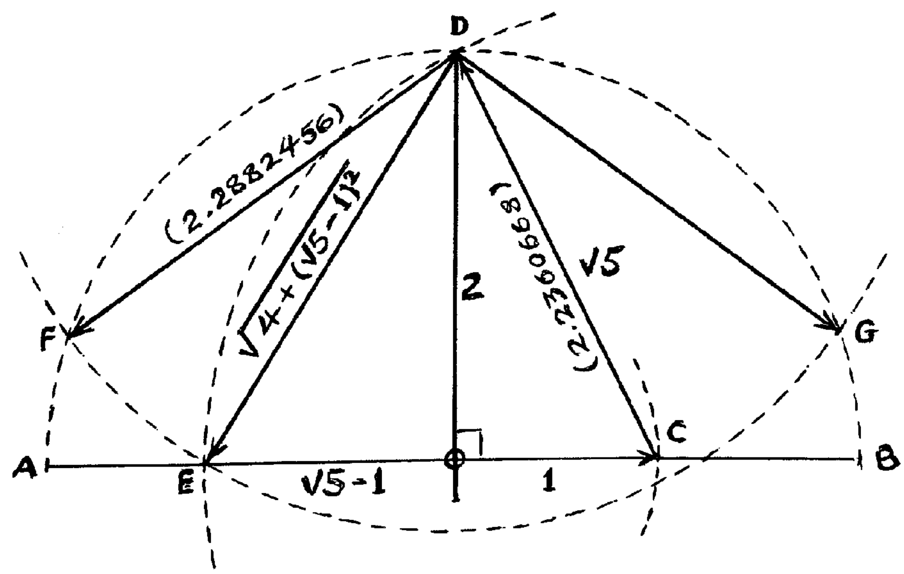
2.17 Practical implications

What might then at first glance appear to be quite a simple set of proportions, the now 'just' 5:4 and 6:5 may be seen as highly complex in terms of their relationships beyond themselves. As a mathematician, specialising in proportional relationships, Dürer would have been only too well aware of the problems (in the mathematical world) of knowing what is perfectly exact, this through a knowledge that is ontological truth, and relating that to what we must grapple with as approximations in the world of biological endeavour. Perhaps there is an element of this that we might think is implied in Dürer's polyhedron, for we cannot achieve perfection. So how do we 'represent' in the practical world? And this, partially, is what Dürer was attempting to address in his *Underweysung der Messung*. Dürer's intention seems to have been two-fold: (i) to make his readers appreciate the pure nature of geometry but, on the other hand, and probably more importantly, (ii) to show how that knowledge can be translated, in a practical way, for craftsmen to be able to draw with rule and compass, turning theory into practice. For anyone who has attempted to draw a perfect pentagon from first principle, they will understand, and Dürer attempts to provide a purely practical solution. In that this is not possible (to the finest degree of accuracy) is not his failing. The problem lies within the complexity of both theoretical understanding and the impossibility of perfectly realising (in practical terms) that which is only perfect in ontological terms, a problem that was discussed in Chapter 1.

In a paper, *Albrecht Dürer and the Regular Pentagon,* Donald W. Crowe (Hargittai, 1994: 465-487) deals in great detail with this issue of precision, for the geometry is not simple or straightforward. He states that Dürer had considerable assistance with the purely mathematical complexities from both his friend Pirckheimer and Johannes Werner, a prominent mathematician, also living in Nuremberg. Dürer gives two methods for the construction of the pentagon and I include, as **Appendix 11**, the drawings and a translation of Dürer's text for both these methods.

The first method, Crowe tells us, is based on Ptolemy's direct proof. This method is theoretically correct but its accurate realisation is not easy to execute in practice. The method begins with the radius (let's say R) of a circle, represented by Dürer as *b-c* in the first circle within his fig.15, (see again **Appendix 11** and **Plate 22** for a copy of the original). He correctly goes through the process of creating a second arc, *e-d* (this measuring R/2 x $\sqrt{5}$). A further adjustment is then made in increasing the span still further to *f-d,* so that it can then proceed to sequentially divide the circumference of the circle into five equal measures. The formula for this final measurement is given by Matila Ghyka in his book, *The Geometry of Art and Life*, as R/2 x $\sqrt{[10 - 2\sqrt{5}]}$ (Ghyka, 1977: 17). I prefer to view this length through the adoption of a different formula (and equally correct) whose direct trigonometric arrangement is shown in **Fig. 11** below whereby $\sqrt{[10 - 2\sqrt{5}]}$ is replaced by $\sqrt{[4 + (\sqrt{5}-1)^2]}$. The line E-D is the hypotenuse of the right-angled triangle OED and whose other sides measure 2 for O-D and $\sqrt{5}$-1 for E-O.

Fig. 11



By this method, the act of increasing the span of the compass twice initiates two stages of potential inaccuracy. This is further compounded by repeated augmentation of any such inaccuracy through the repositioning and drawing of five separate arcs along the circumference as the third and last phase of the process. Theoretically, one should arrive back at the exact initial point after proceeding around the circumference. In practice this is very difficult to accomplish and the fine-tuning is achieved by trial and error. Perversely, Dürer's second method should theoretically produce an angle (for lines meeting at the circumference) of 108° 21' 58" (Hargittai, 1994: 477), slightly over the desired exact 108°. But in practice, the results of this method show no significantly higher degree of practical inaccuracy because it is procedurally much simpler, without any alteration of radii involved.

2.18 Pacioli's *Divina Proportione*

It is thought also that Dürer would have gained a lot of instruction from Luca Pacioli's *Divina Proportione,* 1509, and *Euclidis Megarensis Philosophi Acutissimi Mathematicorum*, the latter his Latin translation of Euclid's *Elements*. A painting exists **(Plate 23)** that is supposed to be that of Luca Pacioli and attributed commonly to Jacopo de' Barbari, an important contemporary and acquaintance of Dürer. Indeed, it is further thought that Barbari was also fascinated by the proportions of the human body, which may have prompted Dürer, not being able (again apocryphally) to get much out of Barabari, to make such a comprehensive study himself, publishing his detailed mathematical analysis of a large number of body types in his *De Symmetria Patrium in Rectis Formis Humanorum Corporum* (1532 Latin edition). This massive body of work, containing page upon page of detailed mathematical proportions, represents that compulsion Dürer must have had, to find and express mathematical substrates, even within the diversity of the human body **(Plate 24)**. The *Dresden Sketchbook*, also, is made up predominantly of proportional studies of the human form, and one particular sketch **(Plate 25)** perhaps indicates that dual aspect that appears to be central to Dürer's thinking: a deep humanity on the one hand, and that purity of mathematical thought on the other. Whereas the mystical female figure in *Melencolia I* (whilst looking outwards with dividers held in her hand) may indicate the contemplation of some geometric profundity at an unknown level, the contemplation indicated by this sketch (perhaps of the three-dimensional world in some 'upper realm') is somewhat more graphically indicative.

I have already mentioned that it is often regarded that *Melencolia I* represents a seminal moment in the history of mathematical referencing within works of art. This, of course, may not be strictly true, for we could view the portrait of Luca Pacioli (**Plate 23**), which pre-dates *Melencolia I*, as indicating the contemplation of the mathematical world as much as it may have been intended purely to be (and probably commissioned as) a straight-forward portrait. However, the representation of the Archimedean rhombicuboctahedron – as an apparent transparrent object, with its uncertain mode of suspension and its possible function here as a form for the containment of a liquid – certainly indicates mystery. Did Dürer see and take inspiration from this portrait, and who is the other person that is included? It has been suggested that the other character in the painting might be Dürer himself. These are imponderable questions, and their resolution will have to remain outside the scope of this present enquiry.

2.19 Further to Pacioli's *Divina Proportione*

At this point, I wish to refer back to Weitzel, and the reference to the sketches to be found in the Nürnberg-Codex of Dürer's manuscripts. It is clear that the process of investigating the three-dimensional through the two dimensions of geometric drawing, and how this also relates to perspective, soon introduces (and engages us in) further perceptual difficulties; and in looking through Pacioli's *Divina Proportione*, I also noticed an interesting play upon the visual perceptive process that relates to further resonances within the Platonic world. In Pacioli's book, Leonardo da Vinci's drawings of the Archimedean (**Plate 26**) and Platonic (**Plate 27**) solids are representations from real life. However, the further drawings within the text are equally interesting (**Plate 28**). 'Figure 13' (**Plate 29**) from this particular page of *Divina Proportione* indicates how the icosahedron is related to the dodecahedron, and how the twenty vertices of the internal dodecahedron coincide with the same number of centres of the faces of the icosahedron. When we look closer at this figure, we become aware of further geometries arising out of this representation of the three-dimensional. My **Fig. 12** isolates the shape of the icosahedron and compares it to Pacioli's 'figure 13'. [Note the slightly different geometries from slightly different views. Mine is exactly topological.]

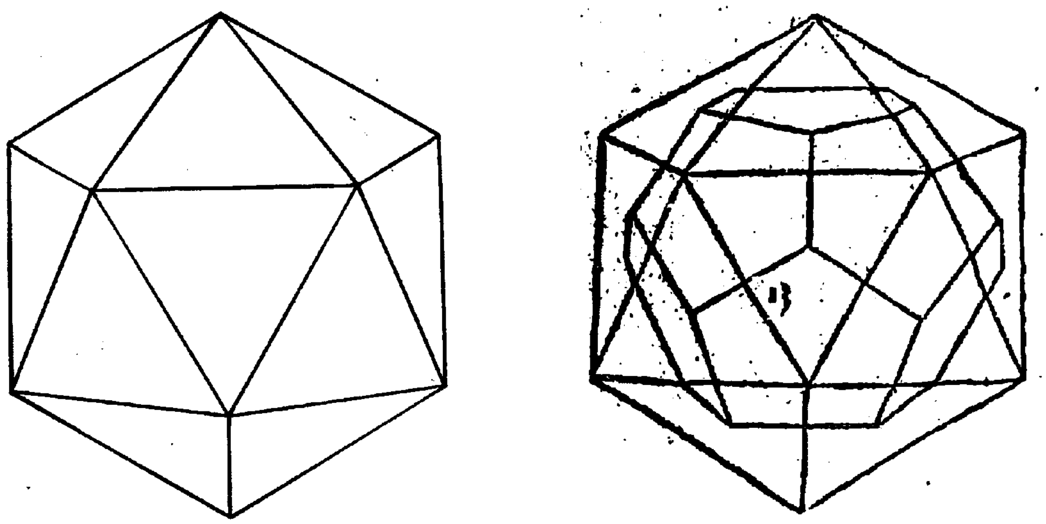


Fig. 12

A central equilateral triangle conjoined within an external hexagon can also be isolated as another pattern within Pacioli's Fig. 13. See my **Fig. 13** below.

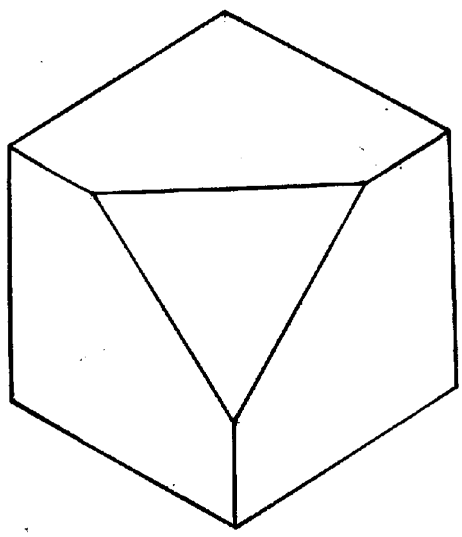


Fig. 13

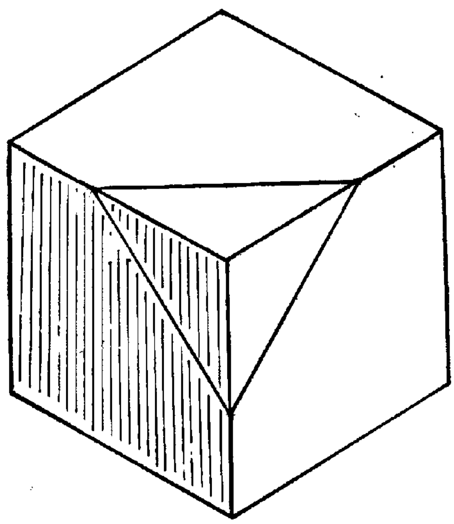


Fig. 14

This could be viewed as representing a three-dimensional truncated cube, as indicated in **Fig. 14**. The interesting two-dimensional proportions that come out of this view are then seen in **Fig. 15**. These relate to some exact (and some very close approximations to) Pythagorean proportions, including the 4:5 proportion that may be the truncation that Dürer intended to represent in his polyhedron and which then may be seen in terms of acting as a further allusion to the important properties of the very basic cube.

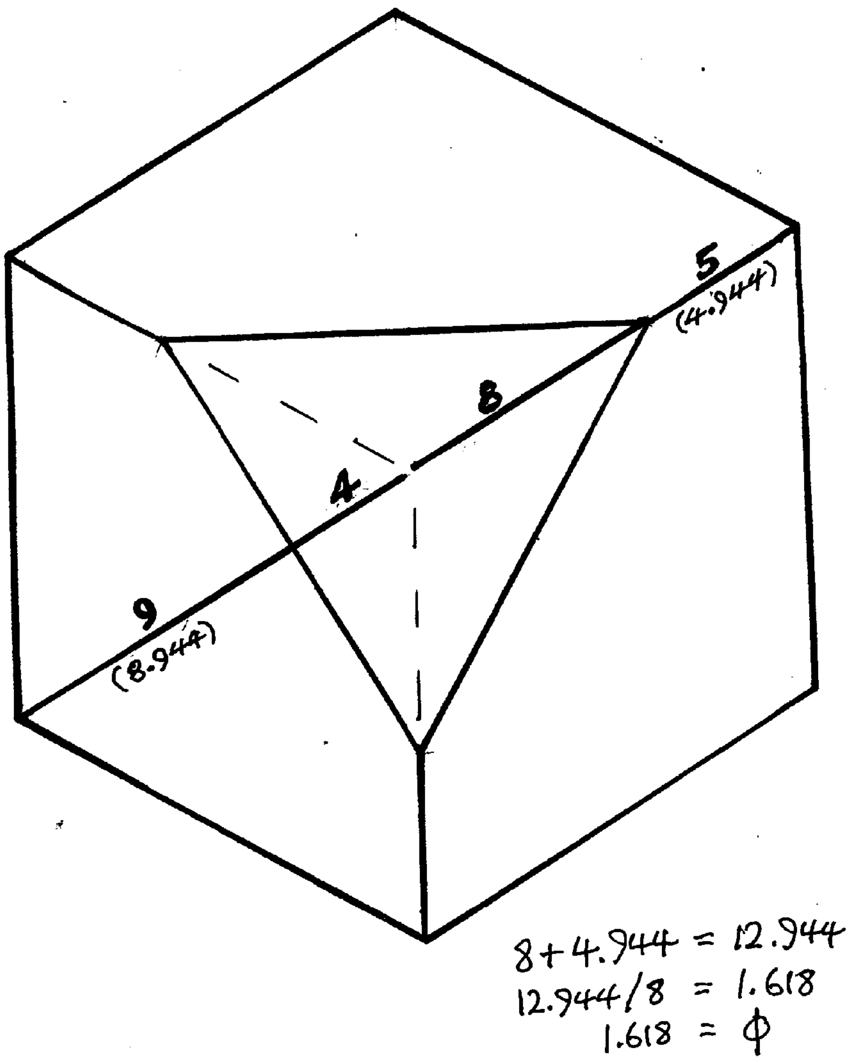


Fig. 15

These proportions arise out of a more complex analysis that is now shown in **Fig. 16**.

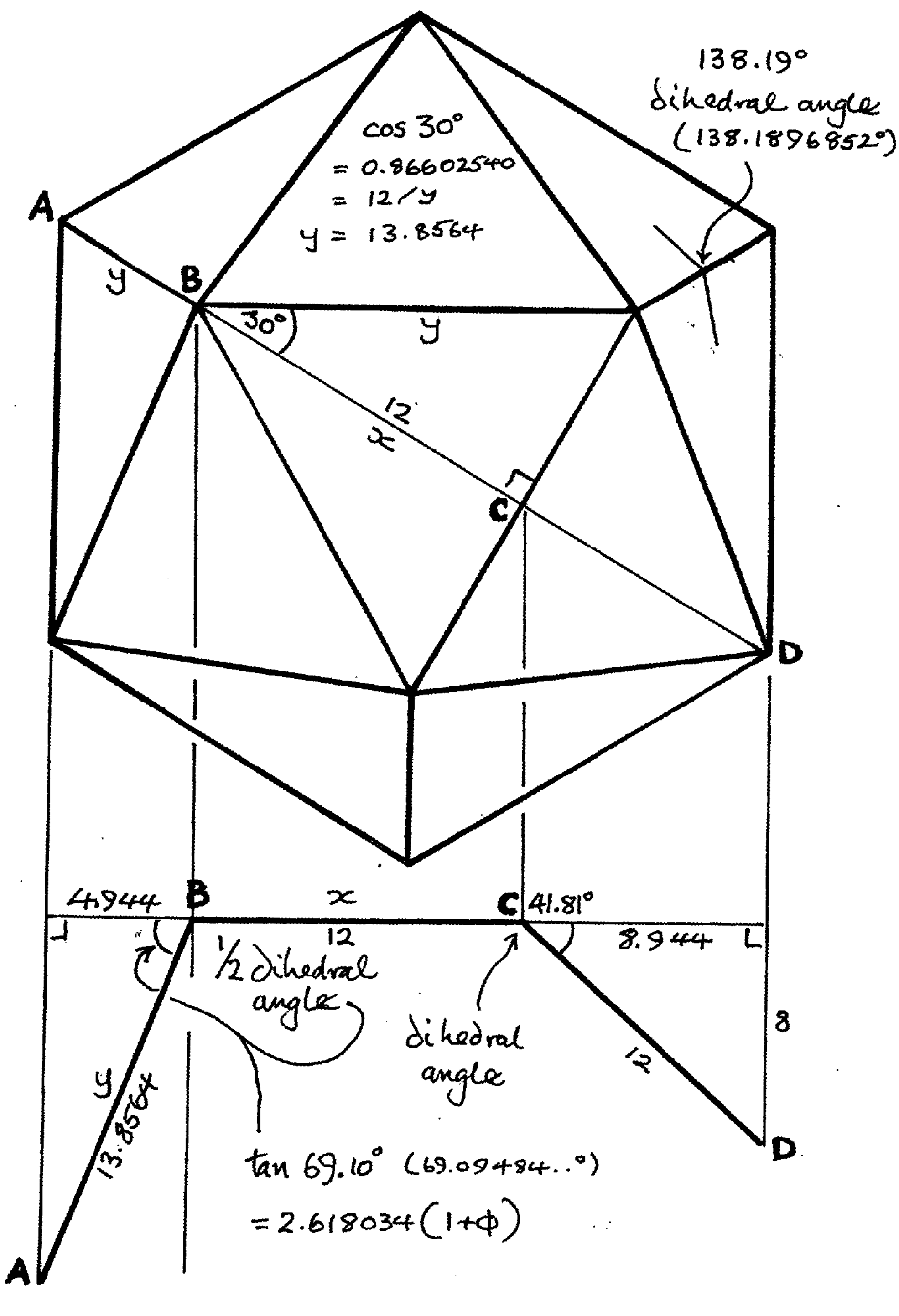

138.19°
dihedral angle
(138.1896852°)
cos 30°
= 0.86602540
= 12/y
y = 13.8564
A
B
y
30°
y
12
x
C
D
4.944
B
x
C
41.81°
12
8.944
1/2 dihedral angle
dihedral angle
8
12
y
13.8564
tan 69.10° (69.09484..°)
= 2.618034 (1+ϕ)
A
D

Fig. 16

Although line A-B represents a side of actual length 'y' (13.8564 units), its two dimensional representation is indicated by a line that has the measure of 4.94 units (very close to 5). Furthermore, as the two-dimensional representation of line C-D is 8.944 units (very close to 9) but whose actual length in the three dimensional object is the same as B-C (i.e. 12 units of measure), the geometry then becomes complete. The further incidental calculation that was shown in the previous figure, **Fig. 15**, also indicates the exactitude of what is actually an integrated golden proportion within the same two-dimensional geometry. What is now crucially important is that this geometry indicates not only Pythagorean relationships (through very close approximation) but also the intrinsic and reflective nature of the exact golden proportions that are integral to the design, and are manifested in both two- and three-dimensional relationships within the complex geometries of all the Platonic solids.

D. Development

2.20 Construction and testing of maquettes

I now wish to look a little more into practical considerations. I have mentioned how Dürer, in his *Underweysung der Messung*, provided detailed instructions as to how two-dimensional geometric forms may be constructed and the theoretical considerations that underlie them. Additionally, Dürer was to show the connections between the edges of three-dimensional objects in the novel form of 'nets' (see **Plate 30** and preparatory drawing, **Plate 31)** to further assist in, and better the process of their construction and assembly. Within Book 4 of his *Underweysung der Messung* – immediately after his explanation of the solution to the Delian problem – Dürer proceeds to explain the method of construction of the five Platonic solids and ten out of the remaining regular (Archimedean) solids through this very process of constructing 'nets' (Dürer's figures 29 through to 43a). Clearly, this draws upon knowledge gained from the practice of constructing Platonic and Archimedean solids that must have been exercised by other investigators also, including Leonardo da Vinci. To be able to provide such accurate two dimensional representations of such solids for inclusion in Pacioli's *Divina Proportione*, Leonardo would have had to been working from actual models (See again **Plates 26 & 27**). It is known that Leonardo and Dürer were as much highly skilled practicing artisans as they were supreme artists, and this is the approach that now underpins the furthering of my own investigations. We can now similarly represent Dürer's polyhedron in the form of a 'net'. **Fig. 17,** below, represents my experimental model (see fifth from left as *Μελεγχολια Ε* in **Plates 13a & 13b)** that possesses a 72° apex angle and a truncation that reflects also the golden proportion.

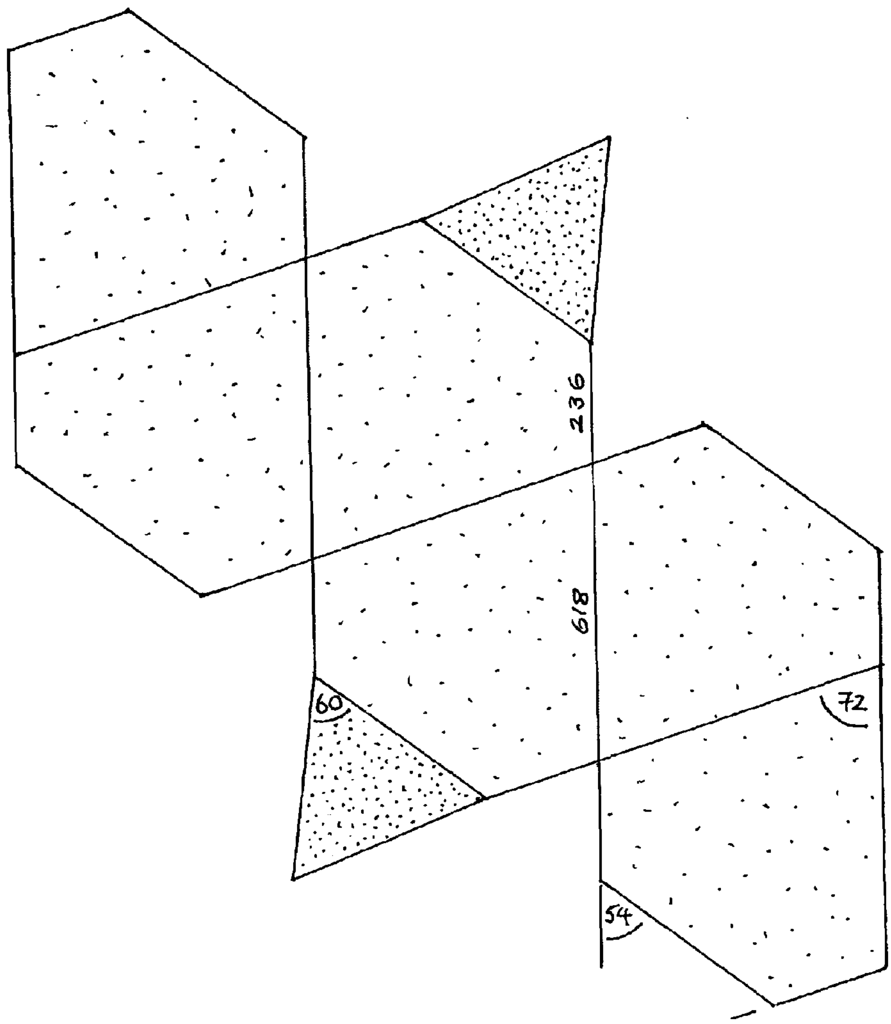


Fig. 17

2.21 Two and three-dimensional patterns arising

I now present an analysis of my own practical constructions and theoretical rationale with regard to Dürer's polyhedron and indicate the mathematical relationships that form the additional patterning that I have inferred as additional resonance. Consider the six maquettes displayed in **Plates 13a** and **13b**. In order, from left to right, the first three show:

(α) a half way semi-truncation to each 72° face,

(β) a half way semi-truncation to each 80° face and

(γ) a half way semi-truncation to each 90° face.

Invoking the Greek reference, I shall call the first maquette *Μελεγχολία Α*. Its face is adorned to indicate how we can display a Penrose tile pattern (**Fig. 18**) to show the implications of the golden angle, 72°, and how the kite and dart shapes relate to the larger 72° and smaller 36° rhombi:

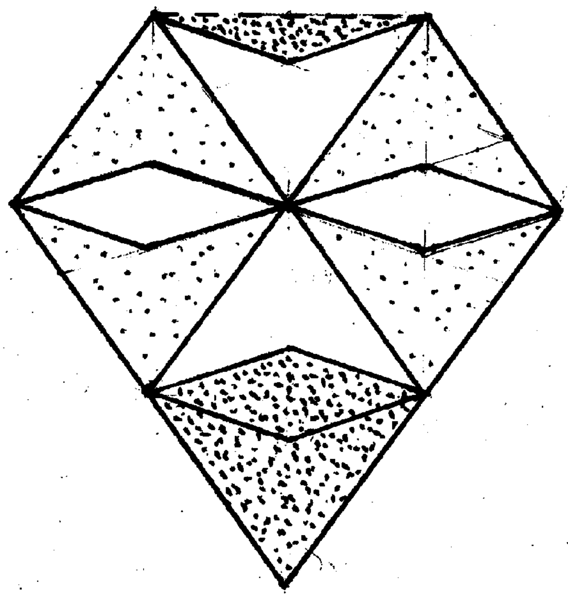


Fig. 18

The knowledge that non-periodic patterns – as well as the periodic pattern that I have shown – can be made from the 72° and 36° rhombi is usually attributed to Roger Penrose, these rhombi thereby often being described as 'Penrose tiles'. However, their patterning properties would have been known from much earlier times, as the profound complexity of Islamic patterns would indicate, a topic that I shall be returning to again and will be analysed in some considerable detail in Chapter 3. Indeed, figure 24 **(Plate 32)** within his *Underweysung der Messung* shows that Dürer had the knowledge of the relationship of both these rhombi to the pentagon itself, and thus their potential for periodic patterning in conjunction with the pentagon itself, with multiple arrangements therefore being implicit. [Some of his sketches **(Plate 33)** show also how three-dimensional effects can be created not only from 60° and 30° rhombi but also with the additional inclusion of the square.] The other paired forms that are derived from the division of the 72° rhombus that Penrose formulated are also included in the basic introductory pattern to the face of this first of my developmental maquettes. Penrose, of course, modified these forms by adding small directional peaks and including corresponding indents to the edges so that their orientations would thus make non-periodic patterns imperative (Penrose, 1990: 178), hence their basic forms being described as *aperiodic*, that is non-periodic only.

The second maquette, *Μελεγχολία B*, indicates how we might develop a symmetrical repeat pattern **(Fig. 19** below) from the {40° 50° 90°} triangle, at the same time implying the geometry of the half way semi-truncated 80° rhombus by combining it with the smaller {50° 80° 50°} isosceles triangle. This was derived from the geometry and implied resonances already introduced within **Fig. 4**.

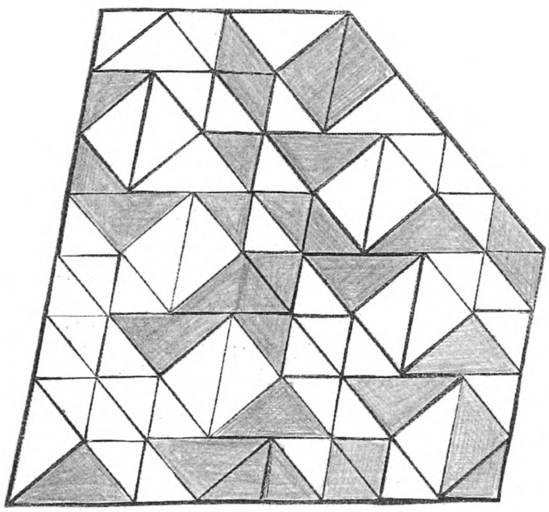


Fig. 19

This patterning I have also added to and placed upon a 'full-size' model as an imitation of Durer's polyhedron (**Plates 34 & 35**) but with, of course, the incorrect half way truncation. However, I do not think that such an object was ever produced to that 'life-size scale' as a model for Dürer's print. As a mathematical exercise, and as an aesthetic that is intended to add further resonance, I have also re-configured Dürer's magic square into a novel magic triangle of my own devising and incorporated it as an additional layer upon the structure. See **Plate 36**. The four central triangles (with their numbers 8, 2, 11 and 13) add up to the same number (34) as for the magic square. The other groups of four numbered triangles, at each of the three main corners, exhibit a similar function, providing the same sum, 34.

The third maquette, *Μελεγχολια Γ*, which is shown as an enlargement that is additional to **Plates 13a &b**, indicates how we can develop
(i) the golden proportion (φ) from a {1 2 √5} triangle on the one front face, to create
(ii) another golden linear proportion on the leading edge of the third face (through taking a √5+1 arc through the bottom adjoining face) and to further develop
(iii) a reduction spiral pattern in 3D of the golden proportion on that third face.
[The other geometry that can be found within the front face, of course, is the classic Pythagorean {3 4 5} triangle, when we subtend a line to create this right-angled triangle from the 36.87° isosceles triangle, this itself created through the half-way semi-truncation of the square.]

Fig. 20, below, provides a geometric explanation of the development of φ from a basic square, and **Fig. 21** shows the 3D form that is indicated on maquette *Μελεγχολια Γ* and which was developed from the 2D logarithmic spiral configuration that emanates from φ.

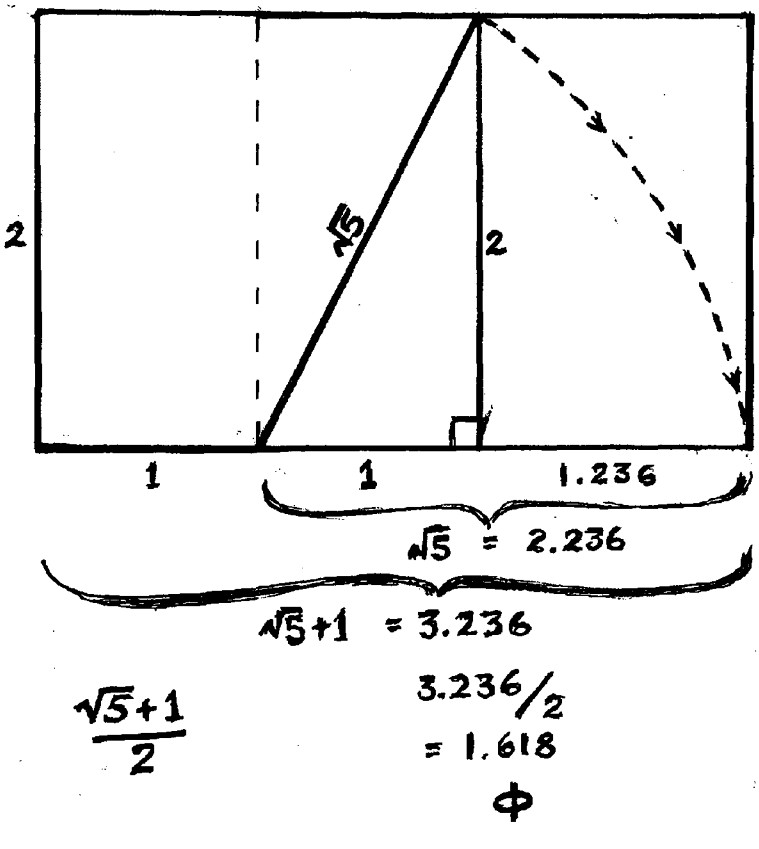


Fig. 20

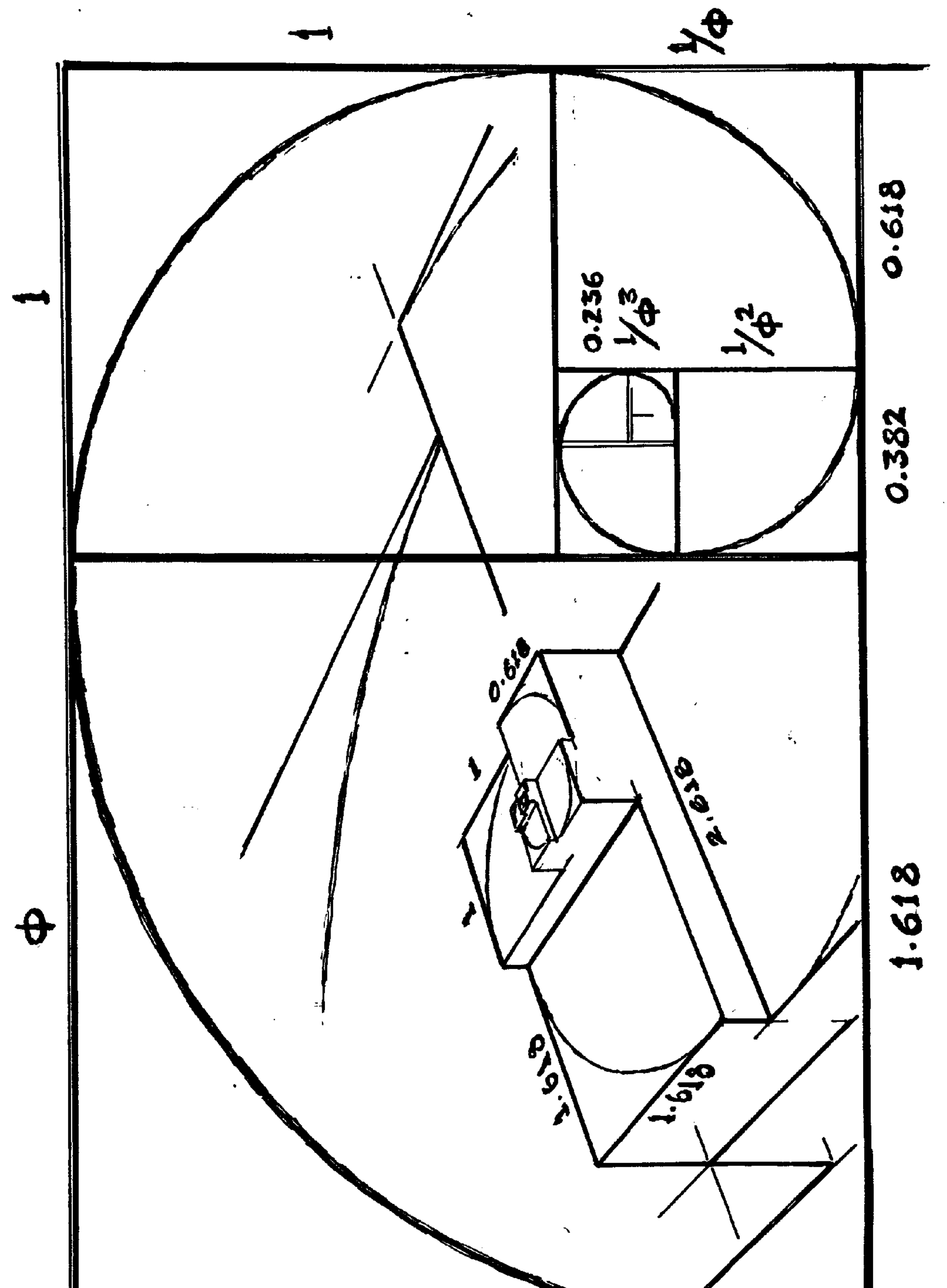


Fig. 21

The method of continuing the development of φ from its initial linear proportion to the construction of the pentagon and its related pentangle is represented by a further work of art, *The Emergence of Phi* (see **Plate 37**). [This work is clearly related to the resonances that I have integrated into the truncated cube, maquette *Μελεγχολία Γ*.] In addition, this painting is designed as a play on the visual perceptive process whereby we are continually questioning the plane at which any particular shape may be posited. The painting of a green border was purposely devised. This reduces an outer physical 6 x 7 framing to an inner visual 4 x 5 frame, this to reflect the Pythagorean 4:5 proportion, whereby 20 imaginary 1 x 1 squares would exactly cover what may be viewed as the window through which one could contemplate further any indications within what appears to be rather an uncertain background. This background could be considered in terms of a reference to the golden section with its property of infinite continuance.

2.22 The median truncation

We have seen how the first three maquettes have been judiciously placed in series to emphasize the significance of the median 80° face (β), lying between the two discounted extremes of 72° (α) and 90° (γ), each one displaying the half-way truncation. Next, I wish to pursue the implications of Grigoriev and Schafranovsky's research that were mentioned by MacGillavry in her paper where it was muted that Dürer's polyhedron might be a hybrid of a 90°and 60° polyhedron, resulting in a 72° median angle. That each angle might be 72° of course has already been discounted. This is the form that is probably being suggested by MacGillavry. My contention is, however, that Dürer's polyhedron might be a hybrid, not of a 90°and 60° polyhedron but of a 90°and 72° polyhedron. Additionally, this model – and this will be my second theoretical model – does not produce the common median angle of (approx.) 80° polyhedron but a more complex model that physically retains *both* the 90°and 72° as reference within its structure, and a truncation that is a median between the halfway and the golden section. This complexity I shall now explain.

Consider the maquettes that are third, fourth and fifth in my series, viz. *Μελεγχολία Γ*, *Μελεγχολία* Δ and *Μελεγχολία Ε*. [This particular part of the series is shown separately as **Plate**

14.] The 90° *Μελεγχολία Γ* has, of course, been previously analysed but what does the 72° *Μελεγχολία* Δ display that is different from the 72° *Μελεγχολία A*? In essence, very little: the only difference is that *Μελεγχολία* Δ posses a golden section truncation (see **Fig. 17** again for its net diagram) instead of the half-way truncation. The surface pattern that I have integrated into this model reflects the relationship of the golden section to the three shapes that are shown, viz. the pentagon, the pentagram and the golden {72° 36° 72°} isosceles triangle itself. **Fig. 22**, below, details these relationships:

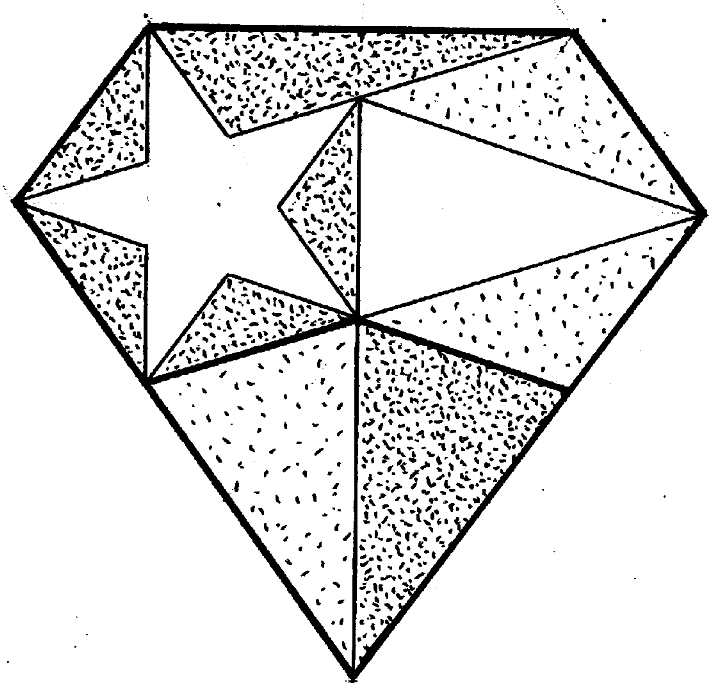


Fig. 22

There are many ways in which we could view the infinite variety of internal relationships within this shape. We can clearly see the proportional progress of the golden triangle in relation to diminution of scale as we isolate this feature in **Fig. 23** below:

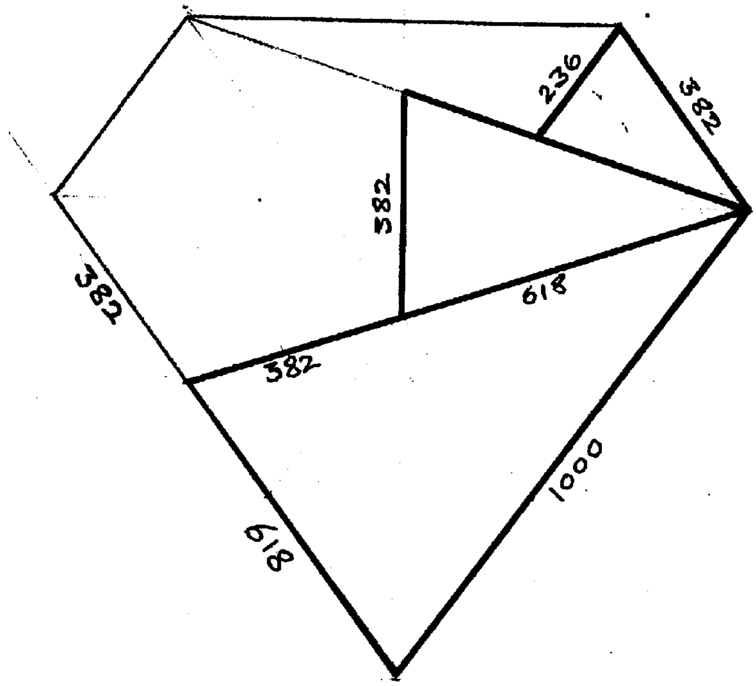


Fig. 23

As we start then to develop the conjoining relationships by altering the scale, thereby adding more internal linear detail, interesting conflicts of visualisation and patterning emerge. This can be seen in **Plate 38**, as the 36° pentangle and the 108° pentagon compete with the 72° pentangle at the centre. Within this partially obstructed 72° pentangle can be visualised the Penrose 'kites'. Also, within its centre, the decagon – made up of ten golden triangles – can equally be visualised. [The decagon is a shape that will feature importantly later on in my analysis of Islamic *girih* patterning in Chapter 3.]

In continuance of this theme, I have developed a painting, *The Consolidation of Phi*, 2009 **(Plate 39)**, to reflect further the internal relationships of *phi* (φ) within this shape. The way I have patterned the Penrose tiles and used the properties of visual perception through designed colour balances to create a particular effect is, of course, relevant to the many ongoing themes that are being discussed within this thesis. The first painting processes are shown as **Plates 40 b, c and d**. However, the painting actually developed out of an initial exercise in developing geometric relationships and the three-dimensional cubic effects that are created (see **Plate 40a**), dealing with purely graphic and mathematical considerations. The manner by which the contiguous colour effect dominated was to dictate the development of this particular painting and there were many other different interpretations that could have been harvested from this colouring exercise. I eventually chose, with this particular version, to highlight the formation of the 72° pentangles and the many subtle golden section arrangements seen within the overall patterning. This meant the loss of the many three-dimensional 'step-like' perceptions invoked by other stronger colour balances.

2.23 <u>Hybrid model</u>

We have seen, in quite some detail, the many ways in which the golden section is infused into both *Μελεγχολία Γ* and *Μελεγχολία Ε*. Is it now at all possible to construct a model that incorporates all of these significations? I believe it is, and my model, seen as *Μελεγχολία Δ*, does this and more: it also incorporates the 4:5 proportion within its truncation. Consider the net of this model below.

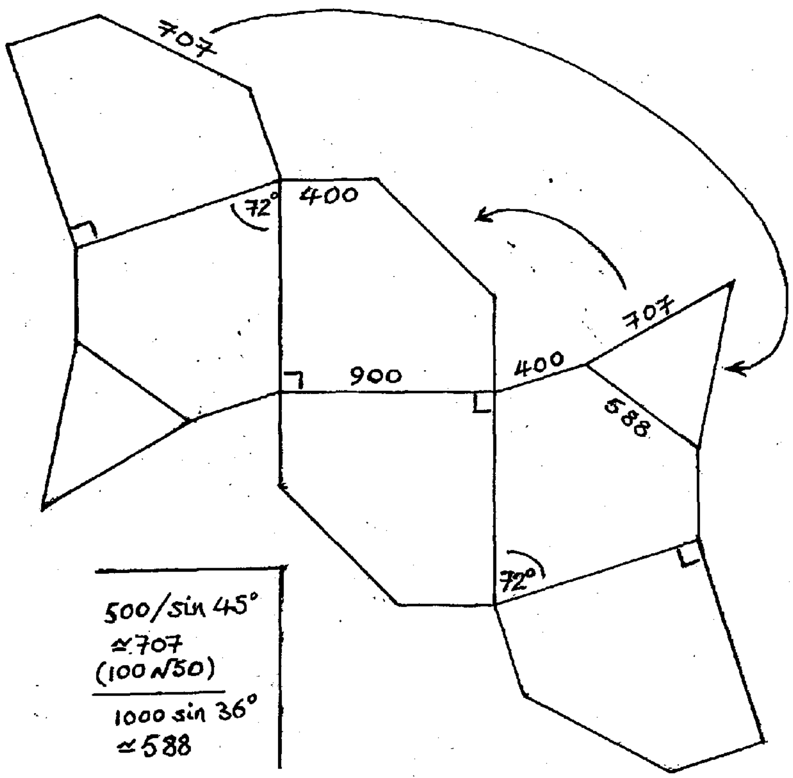


Fig. 24

Four of the irregular pentagonal faces display 90° apex angles and the remaining two opposing faces with 72° apex angles, such that apex angle 6-7-8 (see my **Figs. 3 & 5**) of Dürer's polyhedron may be regarded as having the 72° angle to its front upward facing polygon, with corner 5-6-7 then having the contrary 90° larger apex angle, facing downwards. As a construct that incorporates mathematical resonances from both *Μελεγχολία Γ* and *Μελεγχολία Ε*, *Μελεγχολία* Δ then must also make an accommodation between the two parent models with regard to its truncation. The arithmetic mean between the truncation of 0.5 (x) for *Μελεγχολία Γ* and 0.618 (y) for *Μελεγχολία Ε* would produce a truncation of 0.559, by the formula $\frac{1}{2}(x + y)$. The geometric mean, arrived at through the formula $\sqrt{x\,y}$, would produce a truncation of 0.556188. The latter is very close to the truncation of 0.555556 which is the truncation that would be obtained from a proportion of 5:4, i.e. 5/9. The harmonic mean, the reciprocal of the arithmetic mean of the reciprocals, i.e. $[\frac{1}{2}(x^{-1} + y^{-1})]^{-1}$, gives 0.552784 and is a little further away in the opposite direction. I shall, therefore, adopt the closest, simplest and more resonant proportion, the 5:4.

A truncation of 5:4 for *Μελεγχολία* Δ now suggests some interesting geometries for the 90° faces. Consider **Fig. 25** below.

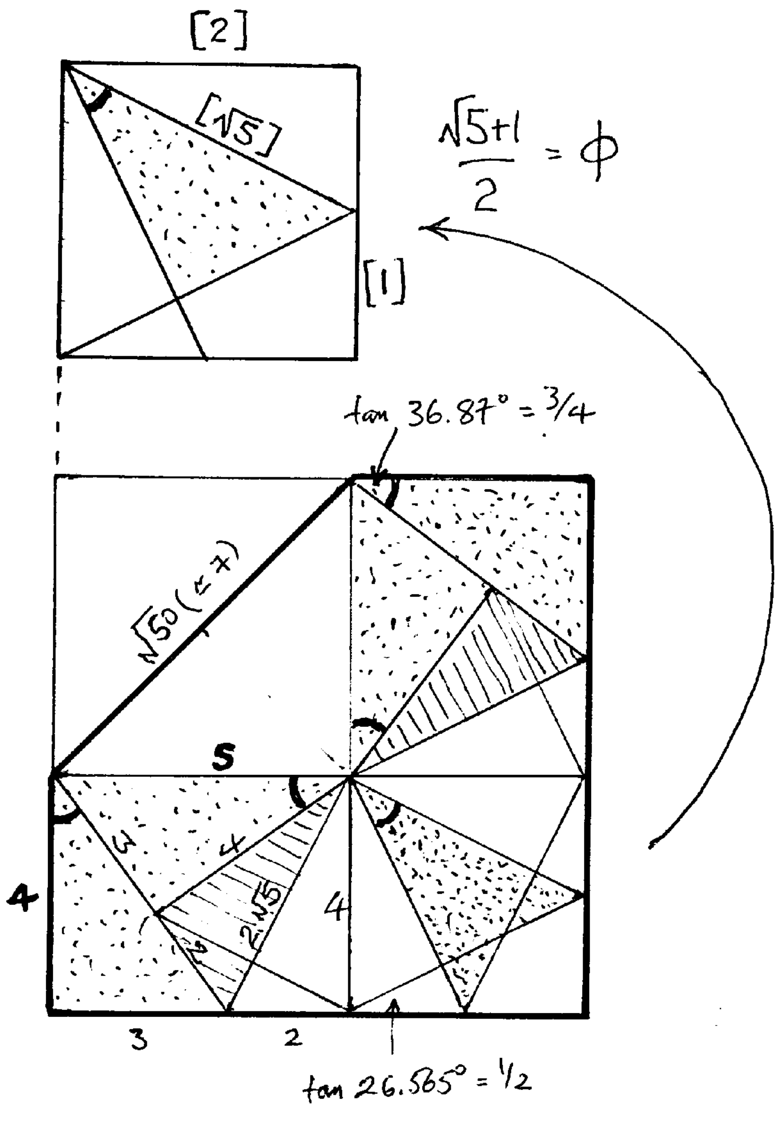


Fig. 25

Interesting configurations can be seen within both 5 x 4 rectangles that accommodate perfectly two {3 4 5} triangles and two other triangles with the configuration {1 2 √5 but of magnitude {2 4 2√5}. We can also see a relationship, albeit perhaps a rather tenuous one, between the {3 4 5} configuration triangle and the {1 2 √5} configuration triangle, formed within the ([2] x [2]) square. This is referred to by Jay Kappraff in his paper, *Mathematics and*

Mysticism of the Golden Mean (Hargittai, 1994: 38) and by Robert Lawlor in his *Sacred Geometry* (Lawlor, 2007: 38), using the identical colour indicated figures. [Who was working from whom?] I prefer to view the relationship that both see, more simply as the addition of the two sides, √5 and 1, and the subsequent division of this sum by the remaining side (i.e. by 2), numerically giving us the golden section. I would say that the formation of a {3 4 5} triangle is purely incidental to this arrangement within the square and not strictly related to the golden section. Furthermore, this relationship within the {1 2 √5} triangle should not be confused with the more traditional golden relationship seen within the {72° 36° 72°} isosceles triangle. However, Lawlor sees some mystical properties in what he refers to as the Diaphantine or Sacred Triangle, that is, the {3 4 5} triangle:

> … the irrational roots symbolise the constant, creative process of acting and reacting energy. This immeasurable gestating force emanates from the incomprehensible Unity. That which is comprehensible is no other than a momentary limitation of this One, indefinable Being into a definable moment: 'Necessary, then, all that is definable arises out of an Indefinable All'.
>
> (Lawlor, 2007: 38)

Lawlor does not cite his included reference but does go on to find some further 'esoteric significance' (Lawlor's words) in Theon's geometric explanations. Although it is possible to see all these mathematical relationships, I would not go as far as Lawlor in assigning to them properties other than geometric.

However, moving away now from the truncated square, when we come to analyse the arrangements that I see within the other two 72°gons, we are engaged again with the issue that Dürer faced with his two methods of constructing a regular pentagon, that of the distinction between what we might consider as theoretically accurate and what are acceptable practical approximations. **Fig. 26a**, below, indicates some basic geometric arrangements that can be developed within the 4:5 truncated model that reflect the golden ratio.

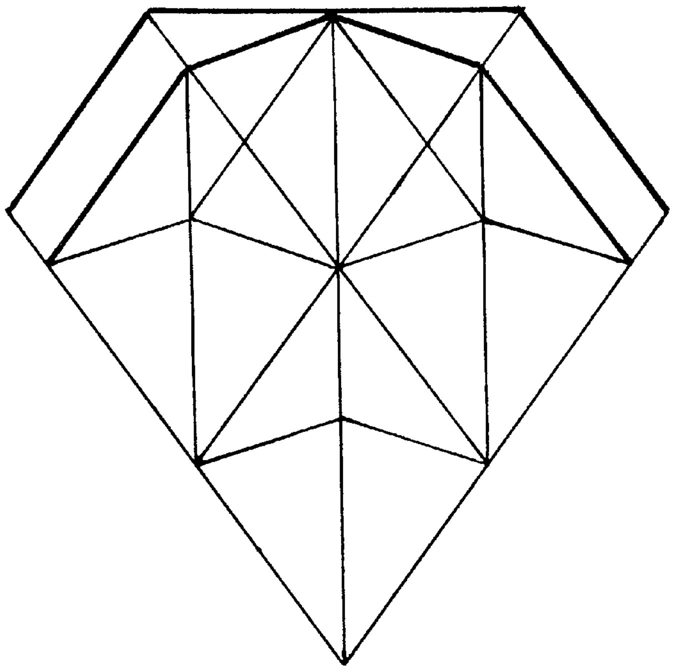


Fig. 26a

A pattern that might indicate the botanical, in the form of a maple leaf, could be envisaged, as well as the arrangement that I formed on the maquette itself (see *Μελεγχολία Δ*, **Plate 13a**):

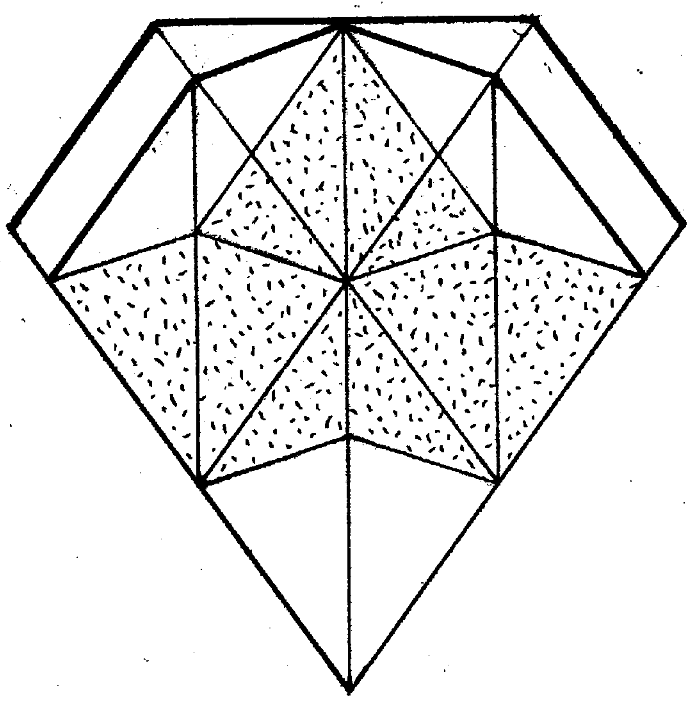


Fig. 26b

However, when we do the maths, we find that the height of the geometric arrangement that I have invented (1047.214 units) is marginally shorter than the actual height of the gon itself (1051.722 units). The degree of error is actually well under ½ %, and might therefore be dismissed in practical terms. Nevertheless, this mismatch – within what is otherwise a visually very close match – is theoretically significant. See **Fig. 27** below.

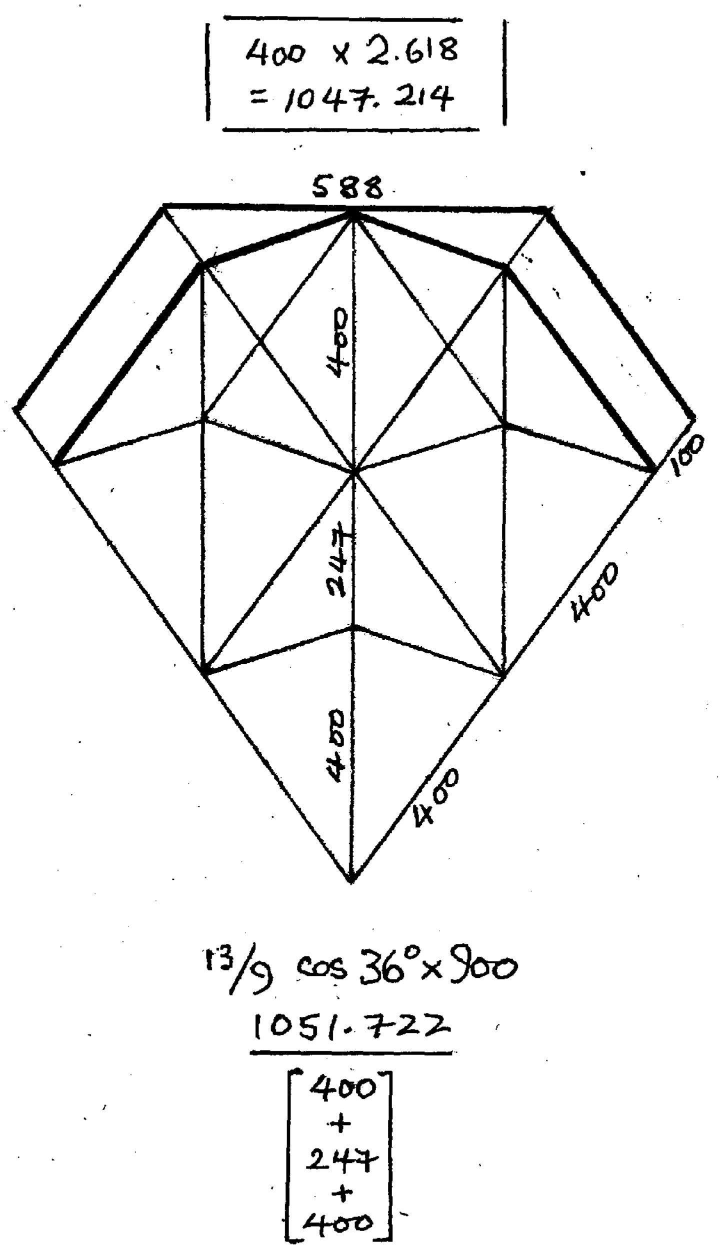


Fig. 27

2.24 Concluding remarks

It is clear then that (i) *Μελεγχολία Δ*, with its 5:4 truncation and the mathematical references that might be implied by the geometries that can be 'read' into the faces, along with (ii) the far simpler *Μελεγχολία Ζ*, with its similar 5:4 truncation and the repeated 4,5,9 reference inferred by the {40° 50° 90°} triangular arrangements, are the two most likely possibilities and are empirically far closer to Dürer's model than any of the other theoretical models that have been put forward by other researchers. However, being true to the scientific method, all models are to be tested, using the same practical method, already discussed, that Dürer would probably have employed in preparing his image and subsequent plate, and whose results are discussed in the thesis conclusion. A linear indication of the proposed 5:4 truncation is now superimposed upon Dürer's print to further represent and make clearer this proportion as **Plate 41a**.

Before moving on to the next chapter, where I relate the two- to the three-dimensional in much greater depth, I wish finally to consider a particular two-dimensional aspect of Dürer's polyhedron that seems to have escaped the attention of all investigators, and that is the significance of the singular orientation that Dürer has chosen in both preparatory sketch and final version. If we were to consider that this orientation, and its slight integral inclination, is not accidental, as indeed we would conceive that neither is the constitution of his three-dimensional construct accidental, then the two-dimensional angles produced by the print for this particular orientation may hold some significance also. **Plate 41b** indicates the two most important angles by which we might initially judge this three-dimensional form. What is immediately clear is that, if we consider that Dürer's polyhedron might be constituted as per my model *Μελεγχολία Ζ*, as I suspect is probably the case, then this particular view of the object conveniently retains the 80° angle within its two-dimensional replication. Might Dürer therefore have chosen his viewpoint to suggest this possibility to the serious investigator? But why, then, the approximate 63½° angle to the other corner? This had initially intrigued me, but if we might consider that Dürer – and we cannot corroborate this sort of accuracy from the print – was creating a reference to the golden rhombic angle of ~ 63.436° (whose tangent is exactly 2, as we have already seen from the {1 2 √5} triangle) then this would indicate that my further investigations into the relationship between the two- and three-dimensional have been proven to be more than relevant to what is was that Dürer might have been suggesting. [The huge importance of this angle to an appreciation of the golden section in three-dimensionality is dealt with comprehensively in the unfolding of Chapter 3, with its culmination in Part 3.12 Total space filling solids.]

Chapter 3

Perceiving Form and Number

> ... the beauty and the order of regular bodies are overwhelming. There's nothing you can do, for they're there. If you then insist upon speaking about god: they have something divine, at least nothing human.
>
> M. C. Escher
> On *Möbius Strip,* 1961
> (Locher, 2000: 184)

A. The numerical and two-dimensional structure

In the previous chapter, I investigated a particular three-dimensional geometric object (as a subject of art) principally in terms of its mathematical properties but also for any possible philosophical or religious signification that it may otherwise possess. If it is that one is looking for absolute truth and certainty in any of these domains, then this enigmatic object, when investigated thoroughly, has much to offer. For me, personally, one might say that it singularly signifies the struggle for a particular form of understanding, an understanding that is totally separate from, and not encumbered by, any cultural influences; that which does not have to rely upon a conjectural 'truth', needing either those mystical linguistic expressions or the quasi-mathematical signifiers of occult tradition to give it validation. Interestingly, many unforeseen mathematical elements have arisen from this investigation, not least the artistic and mathematical properties of patterning. Therefore, this investigation would be incomplete without referring to Islam and its art.

3.1 Islamic art

The subject of belief – in terms of how we understand the relationship between the phenomenological and the ontological – ties in with a separation between the mathematically logical and the biologically psychological. I have already touched briefly upon how Islam, as a religion, stands in relation to other faiths with regard to its historical and cultural significance, and in relation to the fundamental and literal teaching of both Christianity and Judaism. But how does it relate to its art?

The first element that is striking about Islamic art is its relative lack of iconography (seemingly) and the impression of a preponderance towards intricate two-dimensional patterning. It must be appreciated, however, that this impression is somewhat misguided and that, although geometric patterning is a common visual and adorning element within its religious architecture, this does not necessarily play a dominant roll in relation to the force of its literary teaching, and neither (incidentally) is the element of patterning exclusive to Islam. When it comes to private tastes, and for the rich and powerful, there are countless examples of the human and personal being portrayed within medieval Islamic art. This less highlighted element of Islamic art can clearly be seen in the more covert miniature paintings from The David Collection (Folsach, 2007) and in various other major collections. The subject matter for such painted images range from the depiction of the politically powerful – as well as those chronicling events associated with their exploits – to images depicting events from the Torah and the Koran, such as that of Noah's ark in **Plate 44** and the Prophet Mohammed shown in **Plate 45**. More 'liberal' subject matter included scenes from harems, with other such 'personal' images being evidenced by **Plate 43**. But for more normal religious and architectural purposes, adorning, either through the use of formal geometric pattern or the more organic beauty of fine calligraphy, would both seem to be an intrinsically important and a common factor within Islamic art. However, although there are some examples of more freely organic mosaic patterns, the majority of Islamic architecturally artistic expressions, that which is commonly known to Western culture, seem to be (on balance) principally geometric, showing either/both a repetition of pattern or some internal visual/mathematical resonance – sometimes simple, sometimes quite complex.

Perhaps one of the best examples of organic forms is to be found in the Great Mosque of Damascus, 715 (see **Plate 46**). This, of course, dates from quite early Islam. In contrast, the Sirçali Madrassa in Konya, 1243, displays a variety of what seem to be quite simple mosaic patterns (**Plate 47**). [Later on, in Part 3.5 <u>Wooden Seljuk craftsmanship</u>, I shall be referring to an example of wooden Seljuk craftsmanship and patterning, this again from Konya.]

However, if we look more closely at the patterning that is shown top right of this black and white photograph from Rice Talbot's most informative book, *Islamic Art*, it becomes apparent that some quite complex patterning occurs here. [This I shall also be analysing in some depth later, in Part 3.6 <u>The geometry of contrast</u>.] Then, a particularly good example of quite detailed and complex mosaic patterning is found in the façade to the Darb-i Imam shrine in Isfahan, Iran (see **Plate 48**). The Gunbad-i Kabud tomb tower in Maragha, also in Iran (see **Plate 49**), then shows what, at first glance, appears to be a plain linear and repetitive form of geometric patterning to its lower external façade, now somewhat degraded. [These latter two examples will also be analysed in more detail in later sections.]

3.2 <u>Basic symmetry</u>

Through looking more closely at some other examples, in addition to those given above, I shall now introduce certain basic patterning properties that relate to the symmetrical properties of the equilateral triangle, the square and the regular pentagon. The further significance of the 'stand alone' properties of these three geometric shapes will then be discussed in Part 3.8 <u>Prelude to 3D harmonies</u> and how these mathematical properties also further reverberate in three-dimensional geometry, culminating finally in an evaluation of the relationship of these geometries to certain rhombi whose particular constitution are fundamental to either patterning properties or to total space-filling.

Patterning around a single point for the triangle and the square is obviously very straight forward and needs no explanation. Much Islamic patterning makes use of this. The primary shape of the equilateral triangle gives rise, through successive sub-division, to hexagonal and further to dodecagonal (twelve-fold) designs. This can clearly be seen in **Plate 50** where twelve strap-like forms radiate from the centre of each hexagon, this type of work consequently often being described as 'strap work'. Note the many six pointed stars created, sometimes referred to as the Star of David, or the Seal of Solomon, symbolic within Judaism. Interestingly, Islam may not have been averse to making use of this geometric symbol, as is evident from one particular artwork (see **Plate 51**). It is significant that Islamic conquerors often retained and incorporated the architectural and artistic features of religious buildings when they converted them into mosques and we therefore have to be careful as to what may or may not have been appropriated, adapted or subsumed within (or as) their own expressions.

What interests me particularly is how many Islamic designs contrive to create the five pointed stars (pentangles) from what is clearly not a five-fold pentagonal design. These pentangles are clearly irregular and do not conform to the necessary golden section proportion for them to be aesthetically pleasing. This feature can be seen in **Plate 52**, another example of 'strap work', where the external framing is hexagonal, with quite perfect eight pointed stars integral to each side, but which also contrives to create twelve five-pointed stars around the central motif. There are instances of this contrivance within octagonal designs also. **Plate 53** shows a clear intent to create pentangles, secreted within groups of *four* 'roughly pentagonal forms' around a central point (an obvious contradiction). The design is clearly four-fold with the competing, and more correct, octagonal geometries clearly evident. **Plate 54**, from the Alhambra in Granada, is another example of this geometric discomfort, with a preponderance of golden lopsided pentagonal stars around and in between the octagonal motifs being evident. In **Plate 55**, however, the marriage works between the four-fold symmetry of the square motifs and with what could otherwise – in isolation – be viewed as a three-fold division within the point radiation of the other integrated motifs, as twelve is divisible by both four and three.

Indeed, many styles of ornamentation and geometries can be seen in the Moorish Alhambra at Granada and **Plate 56** gives an indication of this, for it's evident that the Alhambra not only displays geometric designs through its coloured mosaics **(Plate 56)** but also more calligraphic and ornately detailed decoration in its plasterwork (see **Plate 57a**). There are, however, clear indications of geometric design even within such ornate plaster motifs and this is evidenced by **Plate 57b**. The property of such repeated rhombic design is perhaps clearer and more evident in its Nasrid precursor at Dar al-Manjara l-kubra **(Plate 58)**, also at Granada. [The significance of the repetition of the rhombus will be seen to be integral to some of the more complex five-fold Islamic designs, dealt with in my more detailed analyses within proceeding parts.]

It is worth noting here that such intricate decorative ornamentation and geometric patterning is not exclusive to Islam. **Plate 59** shows both organic decoration as well as octagonal geometries either side of the central hexagonal patterned window in a Jewish synagogue. Jewish architecture and decoration (Folberg, 2000) uses many forms of patterning other than that formed of, or making reference to, the Star of David. Patterning, within the religious context, is therefore not exclusively Islamic, as has already been indicated.

3.3 <u>True five-fold symmetry</u>

Perfect five-fold symmetry is complex. I return now to the example given earlier in Part 3.1 <u>Islamic art</u>, viz. the Darb-i Imam shrine in Isfahan, **Plate 48**. To initiate the analysis, **Plate 48a** shows the overall grid upon which the intricate mosaic fill is based. The five-fold symmetry now becomes clearer as we can see some perfect pentagonal, as well as Penrose 'kite' forms, quite distinctly. This design can be extended beyond its architectural enclosure, as shown in **Plate 48b**. This extension has the advantage of showing from where the four centres radiate, i.e. from points (marked 'x') at the very edges of this particular architectural frame, and how some of these intricate pentagonal designs, when extended beyond their individual scales, can be viewed as being part of a larger scaled pattern, and could conceivably be the basis of a simple lateral series of golden section lozenges or rhombi, which will be seen to be the pattern element in some other Islamic designs, such as the Dado panel in Mamluk (**Plate 60**). If an actual reference beyond an individual scale were to be given or implied, this rhombic shape could equally be part of a more complex and larger *girih* design. [The many interesting properties of *girih* patterning, which includes a further and important reference to the Darb-i Imam shrine as a more likely *girih* scaled extension design, is looked at separately and discussed in further detail in Part 3.7 <u>*Girih* patterning</u>.]

To further this particular analysis, **Plate 48c** concentrates the mind more upon the implied larger rhombus, whereas **Plate 48d** shows where the extreme points of the Penrose 'kites' meet but do not intersect, giving shape to the pentagonal forms between them, with the exception of the void rhombus at the centre, but which itself has two Penrose 'kites' integral to it. The Penrose 'kites' are set outside and at the limits of each star decagon (with obtuse angled points) and whose centres of radiation define the magnitude of the rhombus. This rhombus then has the potential to enable the creation of an infinite repeat pattern if required. The star decagon is, indeed, crucial to this arrangement, and will later be seen (also) as the related surface pattern upon one of the five essential geometric template shapes within *girih* patterning.

Next, I wish to consider a certain feature that is integral to my developmental image (**Plate 48e**) that introduces another element that is fundamentally integral to this design and which will feature in the linking, eventually, of 2D geometric and numerical interrelationships with 3D symmetrical structures and to indicate further the balances that may be implied by Dürer's polyhedron. What seems never to be highlighted in the literature about these Islamic designs, complex that they are, is the significance of the different numerical proportions, implied within each different design, that define that space between the outer limits of the star decagons and whose centres are posited at the conjoining points of the overall rhombus pattern. This space, between the defined limits of each of the star decagons, is responsible for the creation of the variously different geometric patterns in that difficult-to-resolve 'strip' or 'space between' rhombic repeat designs. What then is the mathematics involved here and does it have any significance? Let me then analyse this particular proportional element firstly within the mosaic of the Darb-i Imam shrine in Isfahan.

In **Plate 48e** I have superimposed (in red) a golden section triangle upon the pentagon to show the mathematical relationship between (i) the one 'strip' (partially dot-shaded in both **Plate 48c** and **48e**) that defines the inner diameter of the star decagon, where there are two either side of the central strip that is a lateral extension of the smaller central rhombus (shaded grey in **Plate 48d**) and (ii) that central un-shaded strip itself. [The golden triangle that is indicated in **Plate 48e** is also shown (to the left) in **Plate 48f** where it is additionally demonstrated that this triangle (to the right) can also be internally divided to obtain a different configuration where the pentagon is placed centrally to create a φ: 1: φ proportional division of the longer side.]

If we consider the broken green line AB, within **Plate 48e**, that joins two corners of the pentagon, to have the value of ~1.618 (where we might consider the side of the pentagon, CB, to be one unit in length) then, as CB equals BE, BF equals 2. The proportion AB: BF is therefore

~1.618 / 2 and whose value at 0.809 is extremely close to 4:5 (or 0.800). This discrepancy of 1.125% towards the larger value would seem to indicate that this closeness, not being acute enough, is not significant enough for there to be any conclusions to be drawn, other than this closeness to the 4:5 proportion may well invoke a possible Gestalt perceptual attraction. Mathematically, however, there is here only the direct relationship of ~ 0.809, being half the golden section, but this *halving* of a complex proportion may also, in itself, have some more subtle Gestalt significance. [It may be that I am here attempting to fit modern theory to that which it does not relate, just as Makovicky (considered later in Part 3.7 *Girih* patterning) saw a 'parallel' between the patterns in Penrose tiling and somewhat 'similar' arrangements that he placed as overlay to – and therefore somehow integral to – *girih* patterning, whereas I see them as quite separate but similarly integrated pattern systems.]

There is also another interesting element to the design of the Darb-i Imam shrine mosaic; that is the smaller scale geometric arrangement within each of the different shapes already delineated by this analysis. If we now zoom in and look particularly at the individual pentagonal shapes, we see within them the arrangement shown in **Plate 48g**. Many overlapping pentagonal geometries compete with each other as our visual perceptive processes create different arrangements as an active kaleidoscope. This is where we may again be able to appreciate the properties of *girih* patterning. Some arrangements that recur for me are shown as **Plates 48h & 48i** where it is the rhombus that is overlaid in overlap, indicating again the integral property of the rhombus to these five-fold patterns. From this exercise there developed a dual painting (see **Plate 48j**) entitled *Before 5, Beyond 4* which is indicative of the visual and mathematical complexities of the golden section that invokes – and is evocative of – the number 5, compared with the purity and simplicity of that which is its immediate precursor and, by negative extension, indicates further important properties of what can be mistakenly viewed as quite simple numbers, viz. 3, 2 and 1, not forgetting the fundamental concept and properties of 0. What I mean here will become clearer (and might be held in mind from here-on) when, in my concluding section to this chapter on three-dimensional geometry, the comparative relationship between these most primary of numbers then relate to each other. This is further related to

Dürer's possible symbolic allusion to not only a 4:5 harmony, but also the 3:3, 3:4 and 3:5 harmony, wherein the triple use of the 3 (as a comparative base) might possibly be that which is indicated by Dürer's enigmatic 9. With regard to *Before 5, Beyond 4,* the order in which the two paintings are placed is significant and not accidental, whereby the expansive and inductive process of mind might conjecture a continuance in either/both directions; towards infinity as a concept of magnitude (and also complexity) in the one direction, and towards more finite but perhaps more difficult philosophical concepts in the opposite, as a stepped numerical approach towards simplicity terminates quite fundamentally but monumentally with (and in) 0 itself (as compared with 1), with the relationship between the latter two symbols (as concept) possibly being the most difficult to resolve and where (here) reciprocals might relate to opposite infinite entities.

In more strict mathematical terms and for number theory to work, there has to be an initial concept of **1** as a basis for ∞ (infinity), with an extension (as a concept of increasing magnitude) towards ∞ by successive additions of **1** to **n** (number); thereafter the principle of reciprocation by inversion is necessary for the obverse process to indicate the equally infinite nature and quantification of **0**, this through successively smaller fractions, such that

$$n < [(n+1) / 1] \rightarrow \infty$$
$$1 > [1 / (n+1)] \rightarrow 0$$

This principle of reciprocation will be seen to be absolutely fundamental to an understanding of the mathematical properties of such harmonies that are secreted within the symmetry of 3D basic structuring that are to be discussed more fully in **Section B. Three-dimensional Structure**.

It may be worth noting here also that this particular quantitative concept of zero, as opposed to the concept of simple 'absence', would not have been possible without the Hindu-Arabic system to which the Catholic Church in the Europe of the Middle Ages was opposed. Historians are of the opinion that:

> The Catholic Church wanted to keep control of education by maintaining its hold on numbers, and in addition opposed the system from the Islamic world on religious grounds. Mathematicians who practiced the arcane systems of mathematics using an

> abacus were protected by the Church. So strong was the opposition to the popularization of Hindu-Arab numerals that, it is said, some poor souls were even burned at the stake as heretics for using them.
>
> (Rooney, 2008: 56-57)

Now, with regard to Dürer, the emergence of the printing process – shortly before he was using numerical proportional concepts in his copious calculations and published texts – meant that the dissemination of this highly contentious knowledge was eventually impossible to be controlled by the Catholic Church. Whether it is that Dürer might have been less certain of criticism of a more serious nature that he gave to us his enigmatic polyhedron to ponder over is a mute point, if it indeed was that he might have harboured more 'dangerous' humanist thoughts that would have challenged more fundamentally the Catholic teaching of the time. Religious principles espoused within his personal correspondences certainly indicate that he had a strong religious sympathy for Lutheran principles. That humanist teaching was a discursive topic in Dürer's own time is evidenced by the Latin publication of Reisch's *Margarita Philosophica*, where, in one of the woodcuts, the allegorical 'Arithmetica' is depicted indicating a preference towards the whole number system which is clearly emblazoned upon her dress. It is believed to have been first published in 1503 (Cunningham & Kusukawa, 2010: ix) and **Plate 42** shows a coloured illustration of this edition. Gregorius Reisch (c.1467-1525) was confessor to Emperor Maximilian 1, Dürer's patron and, in his exalted and somewhat protected position, would likely to have been in contact with most of the important humanists of his time, including Erasmus. [Of course, for the Greeks, as therefore for the humanists, the problem as to the nature of 'one', as a concept, had long been pondered, and in Plato's *Parmenides*, where Parmenides initially asks of Aristoteles: 'If one is, …, the one cannot be many?', he concludes his dialogue, after much convoluted Socratic development of the query, that 'whether one is or is not, one and the others in relation to themselves and one another, all of them, in every way, are and are not, and appear to be and appear not to be.' (Jowett, 2006: 41; 78). Clearly, a numerical, quantitative approach would result in a better understanding, that which imprecise sophist language cannot, such language ending up contorted within itself; the philosophical problem is the exact nature of singularity, oneness, wholeness, totality, *monad*, the concept of singularity within singularity, which then gives rise to further numeracy; any such concept can only be clearly understood numerically, for it is, ultimately, a *mathematical* problem, becoming problematic linguistically only without reference to the practice and knowledge of pure mathematics. If *one* (being *one* of many other such *ones*) was/were simply to view *one* as being part of a 'whole', that 'whole' itself being *one*, it is soon seen that the problem is with the language and with linguistic thinking, not with what is really a very straightforward 'concept' or 'idea', that is *one*. I don't think *one* has a problem with itself; only *singular* and *collective* man has a problem with his own thinking!]

3.4 <u>More complex golden section relationships</u>

Returning now to more contingent and less philosophical matters, the Mamluk dado panel **(Plate 60)** shows another classical Islamic pattern, a good and clear example of decagonal motifs arranged repetitively on an equally repetitive rhombic grid design, and where six larger size 'kite' shapes are employed to help join the points of the star decagons around one central star decagon, with the other four points overlapping naturally. Note that it is the ubiquitous rhombus that is created through this overlap. The small isolated and conjoined pentagons that are then formed from this design, together with their internal pentangles, are clearly not a contrivance and conform to exact regular pentagonal geometry. **Plate 60b** further indicates and differentiates between the four smaller overlapping Penrose 'kites' (shaded in green) – those that relate to the periphery of other star decagons – and the other small Penrose 'kites' (outlined in red) that relate to the central star decagon and which are, in turn, united to those of the other star decagons by the larger Penrose 'kites', their having been created to unify the pattern.

What remains to be evaluated now is whether or not there may be a simple harmonic proportion integral to the space that lies between the limits of the star decagons for the rhombic template, similar to my analysis for the Darb-i Imam shrine mosaic. **Plate 60a** shows quite clearly the repeat pattern and the channels between the star decagons. What then is the mathematical relationship here?

Plate 60c indicates firstly the mathematical relationship between the larger and smaller kite shapes employed, which is obviously proportionally 3:2, with **Plate 60d** then showing the further geometry involved that clearly indicates a formula [$2\varphi / (2 + \varphi^{-1})$] for the relationship between the centre to edge of the star decagon and the strip between, which is proportionally the same as the 3.236068 / 2.381966 measurements indicated within this working drawing. This formula [$2\varphi / (2 + \varphi^{-1})$], which translates numerically to approx. 1.35857, indicates the very subtle reverberation of the golden section within these designs, this formed crucially from the tripartite division of overlapping shapes, the pentagons in the case of the smaller scale design of the Darb-i Imam shrine mosaic, and the larger Penrose 'kites' in the Mamluk dado panel design. [Perhaps the significance of this subtle tripartite division within the context of the golden section might be noted here, for this subtle and somewhat hidden 3:5 harmonic relationship will become more importantly evident within the later context of 3D geometries.] **Plate 60d** serves also to highlight the creation of the rhombus through these overlaps.

What my working drawing does additionally clearly indicate is that the two pentangles contained within the fusion of what must be two irregular pentagons cannot be truly regular, where lines HI and FG would have to be of one unit length for regularity to be true. Instead, they measure ~ 0.854102 (from [$(3/\varphi)-1$]) with only the lines indicated in green being the true measurement of φ (at ~1.618034) for the conjoining lines of the pentangles, and where the lines HE and EF are of unit length for the enclosing pentagons. The contrivance of creating two touching pentangles is visually subtle but, nonetheless, a contrivance, for they do not conform to theoretical accuracy.

Why then this complexity of design? Well, although certain five-fold patterns, through the 72° golden angle, can be created that expand outwards infinitely from one point, as through the 72° / 108° rhombus, once a repeat or conjoining pattern is to be created, either a 36° angle of the golden triangle or a further pentagonal division of the 108° angle is required which then necessitates a displacement or an overlap, hence the creation of irregular shapes other than uniformly pentagonal. [It will later be seen, in Part 3.7 *Girih* patterning, that a 72° solution – external and integral to the decagon – is possible, this through and within background *girih* patterning, but where the actual overlay patterns still create compromise shapes.] This property is what makes patterning of the 'golden angles' so fascinating and complex and which has been much celebrated in Islamic decoration. The more basic elements of this subject (but certainly not all) are covered in the analyses of both Lawlor and Critchlow's well-known publications, *Sacred Geometry* and *Islamic Patterns*, respectively. However, there are geometric complexities within Islamic designs that are well beyond these analyses.

I have already referred to Lawlor's preponderance towards the mystical, and his misplaced specific reference to the musically harmonic in my previous chapter. It might, therefore, be apposite to mention here an important element of subjectivity that also clearly colours Critchlow's perspective upon this subject, and which is anathema to me as an objective investigator. His *belief*, as distinct from his geometric analysis, is stated formally before the pages of his book begin to be numbered:

> To the one God
> and his messenger, the Prophet Muhammad
> to whom we owe Islam,
> and to all those master-craftsmen,
> known and unknown, who dedicated
> their lives to beauty through the
> arts of Islam.
>
> (Critchlow, 2008)

I mention Critchlow's publication here as well because he gives us on his page 97 a clear line drawing of what on first viewing indicates almost the exact same geometric design as that which is manifest in the Mamluk dado panel. However, I have taken a quite different analytical approach to Critchlow in outlining the particular geometric overlaps that are involved and have shown in my **Plate 61** how it is six perfect pentagons that surround the star decagon here in Critchlow's example (as distinct from eight in the Mamluk dado panel) and that if we were to attempt to construct conjoining pentangles we would find here also an irregularity but of a greater magnitude than is the case with the Mamluk dado panel. In Critchlow's design, the larger Penrose 'kite' is divided differently such that it would appear that AB equals BC (at $\frac{1}{2}\varphi$ or 0.809...) and the longer sides (at 2φ or 2.168...) are therefore not sub-divided equally, with the single unit of measure at centre and which defines the measurement of the sides of all the regular pentagons.

3.5 Wooden Seljuk craftsmanship

In the furtherance of attempting to gain a deeper understand of what constitutes accurate proportional geometry (as opposed to what is contrivance to gain a particular visual effect) I now wish to put forward my analysis of an interesting patterning within a particular early Islamic wooden door that was mentioned in my preamble to Islamic patterning (see **Plates 62a & b**) that attempts – but fails – to resolve the difficulties of overlay that have been highlighted in previous examples. This impressive object is of particular interest to me not only because of the material of its manufacture, but also because of its elaborate, technically challenging construction, where the quality of craftsmanship is (from the photographic evidence) likely to be quite superb. David Talbot Rice, in his already mentioned book, *Islamic Art*, describes this object as being a pair of twelfth century doors at Konya, which is in Turkey, and points out that 'doors of this type once existed in nearly all of the more important mosques, madrassas and hans' (Rice, 1975: 174-175). This type of work is typical of Seljuk craftsmen. The Seljuks were a conquering tribe of the Middle Ages and defended the Islamic world against the Crusaders. They conquered large parts of the Byzantine Empire, especially what was known as Anatolia, or Asia Minor.

We have already seen ornate and curvilinear detailing within strictly geometrical patterning, both in the examples of the Alhambra (**Plate 57a & b**), and within Nasrid plasterwork (**Plate 58**). This type of ornate detailing is also evident within the smaller geometric panels that collectively make up the otherwise straight-line geometries of this Seljuk wooden door (see detail in **Plates 62c & d**). I say 'straight-line geometries' quite loosely here for, on closer inspection and further analysis of both doors, it soon becomes apparent that there is something about this complex patterning that is rather questionable. Clearly, each door panel is a mirror image of the other. At the bottom of each panel can be seen a regular pentagon (see again **Plate 62c**) and this appears, on first inspection, to be indicative of five-fold symmetry. If we conjoin and match the panels more closely, as in **Plate 62e**, it becomes apparent that each door represents a small portion of a much larger and quite elaborate golden section design.

Plate 62f indicates the central pentagon (highlighted in black) that conjoins the upper motif, centred round a star decagon (highlighted in red), and the lower motif, itself a pentagon (highlighted with green border) centred round a smaller pentangle (highlighted also in red). The geometric shapes that expand out of the pentangle are then shown as **Plate 62g**. The problem that now arises out of this arrangement is how to conjoin both geometries (mathematically correctly) through the shapes highlighted in **Plate 62h**. This same shape is now shown in its correct delineation and scale in my line drawing, **Plate 62i**, where it is labelled 'A'. It can be seen, from the detailed breakdown of the measurements that conform to the golden section, that for this shape 'A' to fit within the upper decagonal motif, it has to be distorted and lengthened to accommodate what should be a smaller scale for the above decagonal geometries and which are shown separately within **Plate 62j**. When we consider the different shapes and magnitude of 'B' (especially) – together with 'C' and 'D' in both of my theoretical models – that could correctly relate to 'A', with the one making the shape 'B' and the other making the shape 'D' to closely resemble the Seljuk model, and then look along the lines of geometry within the door panels, it is clear that a huge amount of license has been taken to create a workable geometry that somehow unifies these differences of scale; to bend and skew, thus to create the effect that the geometries within the panels might be credible, as if conforming to some legitimate mathematical pattern. Patently, they do not.

By contrast, what can also be seen in Konya is an example of some interesting and subtle patterning that *does* now conform to perfectly clear and correct geometry. This is analysed next.

3.6 <u>The geometry of contrast</u>

The mosaic tile-work at the Sirçali Madrassa in Konya (**Plate 47**) was introduced within Part 3.1 <u>Islamic art</u>, and **Plate 63** now isolates, as a section from the black-and-white photograph by Talbot Rice, a particularly interesting pattern that displays an unusual form of three-fold symmetry. I have drawn, as **Plate 64a**, that which can be worked out from such minimal photographic evidence (with all the problems of distortion through parallax being evident), and the amount of pattern that is displayed here is shown in its entirety, as a front view, with selected sections given as **Plates 64b & c**. **Plate 65a** then shows my rough working drawing, created to understand, and to indicate, how the geometry and mathematics works out; how the subtle off-setting is achieved to make the design possible, through what might be described as positive and negative channelling (my terminology). Here, the base and sides of the equilateral triangle has 15 divisions such that there are 225 (i.e.15^2) smaller triangles created within. [There is a further connection here, where 'squared numbers' are dealt with, later, in conjunction with the 'Tetractys' within 3.15 Figured numbers.] I have circled (O) and also marked (with✓) three points (•) in this drawing where the conjoining lines meet within the grid structure that is indicated in my completed design (see again **Plates 65a, b** and **c**). An appreciation is then gained as to the angular displacement of these points of conjunction, where the centres of both the black (hexagonal) and the white (triangular) design elements meet.

Next, there was a necessity to scale this up to form the grid for the overall design and to separate the centres of the black star motifs from those of the white tri-star channelling. **Plate 65b** now shows the overall base 30 triangle (double the previous) and where the hexagonal centres of the star motifs can now be seen as actual hexagons at their displaced locations, now clearly placed (and this is the important consideration) in the same location upon each individual overlapping 19-base triangle. Only an odd number base would create a dual black and white channel grid system like this, with the smaller 4-base end triangles delineated creating this particular angular displacement. This system is very subtle (if not complex) and quite beautiful to traverse in its drawing and then its final realisation as **Plate 64a**. The intervening phases are shown, as the build-up of patterning is achieved within **Plates 65c** and **65d**.

Talbot Rice mentions that the tile colours used by the Seljuk craftsmen were limited to blue, black and white. It is assumed that the light channels would therefore have been white, with the darker tiles then being black. A mid to light tone to the tiles can be seen elsewhere in some of the other patterns. This tone must then indicate the (lighter) blue. I have taken some considerable license with my own sketch (**Plate 65e**), and separated the different three and six fold dark patterns by the use of contrasting colours, red and green; this colour scheme is clearly not as per original and is intended to be experimentally indicative only. [Subsequent to my having made this analysis, another example (colour **Plate 66**) has come to light, this now in black and blue, that shows what appears at first sight to be a similar pattern but where the geometry of the black hexagonal star has been 'fudged' and where the proportional geometry of the rest therefore differs in an unsuccessful attempt to make the darker strips align with the darker, and the lighter strips with the lighter.]

3.7 *Girih* patterning

I now wish to look at the important properties of *girih* patterning, initially through considering an interesting and in-depth paper on this subject published in 2007 by Lu and Steinhardt, entitled 'Decagonal and Quasi-Crystalline Tilings in Medieval Islamic Architecture', where the Darb-i Imam shrine mosaic and the Gunbad-i Kabud tomb tower in Maragha are central to their analysis. Their particular way of looking at patterning – where a comparison between *girih* tile pattern formation and the way that Penrose formulated his two more obvious dual patterns is also discussed – is perfectly valid as an intellectual exercise in establishing possible connections and is, of course, theoretically very interesting. However, it would be arrogant to think that both these ways of looking at pattern would not have been known to Islamic practitioners, a point that is not lost to Lu and Steinhardt. There are, clearly, many ways of viewing such patterned arrangements; and this is the intriguing element within sequential, overlapping and expanding Islamic golden section designs, such that their viewing becomes an active and intellectual activity within what would otherwise be a purely aesthetic or artistic engagement. However, it is clear that, although it is my contention that a separate Penrose 'kite' itself may well feature in *girih* patterning as a tile shape (that is, apart from and to the exclusion of the 'dart' shape), the type of linear patterns on these tiles is impossible to arrange upon the smaller Penrose 'dart' shape in the same way that they are with a the *quite different* curvilinear Penrose overlay patterning, which is why the smaller 'dart' shape was never utilised in *girih* patterning.

This particular anomaly is actually highlighted by the comparative patterning of Lu and Steinhardt's own Fig.4 (my **Plate 67**) that shows the relationship between the two different surface pattern arrangements and where I will be able to show that at least two distinct *girih* surface patterns are able to be applied to the Penrose 'kite' shape but not the smaller and more confined 'dart' shape. What it is that Lu and Steinhardt are pointing out is that, whereas it is perfectly possible to superimpose a Penrose configuration upon a *girih* template, its scale would have to be smaller (reduced by a factor of φ^{-1} or ~ 0.618) than the scale of the corresponding *girih* template, i.e. the scale shown for the right hand examples of both Penrose template **A** and **B** of their Fig.4 (my **Plate 67**), and that such a pattern could then only be aperiodic. I would contend, however, that the simpler dual Penrose tiling system is not really relevant within the context of more complex Islamic *girih* patterning because of the totally different surface pattern and edge-matching proportional arrangements that are employed by two quite different and separate systems *but* that the separate Penrose 'kite' shape may well have been employed by *girih* designers as individual templates at a larger scale (increased by a factor of φ or ~1.618) compared with the left hand example of **A** within Fig. 4. Such similarly proportioned and sized *girih* templates – but with different surface patterns – are proposed by me as **E** and **F** in my **Plate 76**. [Note that Lu and Steinhardt's templates **A** and **B** have not been drawn accurately to the exact same scale as their **C**, **D**, and **E**, adding further confusion to an understanding of the issues involved.]

It is now necessary to simplify in order to explain and determine the nature and the complexity of *girih* patterning, and to further explain how I see the possible (if not necessary) inclusion of the Penrose 'kite' tile shape as a template, which (it is proposed) would be able to form and replace the standard model, single decagon. **Plate 70** shows the separation between the *girih* tile shapes themselves (templates) and the linear surface patterns placed upon them. The lengths of all the edges of the templates are equal, with many computations of alignment then possible, whereby the judicious choice of angles from the possible 72°, 108°, 144° and 216° being always able to be combined to form the necessary total of 360° at every apex and without the necessity of the inclusion of a 36° angled point that applies (at some stage) within the Penrose aperiodic system for total space fill. The exact same angles (72°, 108°, 144° and 216°) are also formed by the lines that constitute the surface patterns themselves so that perfect alignment and/or extension of linear patterns is made possible at the conjunction of the half-way point of each edge of template. The exact same rules and conditions with regard to angles therefore apply to both surface pattern and template. Such a linear surface design that would conform to the conditions just described is simply not possible upon a Penrose 'dart' shape and where not all the edges of the templates are of equal length. In attempting to compare and find conjunctions or similarities between the two systems, this crucial point should not be ignored. [What is possible to generate, interestingly, is a non-periodic, straight-line geometric surface pattern (see Fig.104 of **Plate 71**) from the 72°/108° and 36°/144° Penrose *rhombic* templates (Livio, 2003: 204-205) but which then again does not conform to the geometries generated by the *girih* templates. Clearly, it would be possible to design others.]

Lu and Steinhardt's paper of February 2007 is actually predated by an equally interesting and related paper by Emil Makovicky. This paper was first published in 1992 as '800 -Year-Old Pentagonal Tiling From Marāgha, Iran, and the New Varieties of Aperiodic Tilings it Inspired', and is included (pp.67-86) within a most interesting and informative collation of papers edited by István Hargittai, published as *Fivefold Symmetry*. Here, Makovicky analyses the *girih* pattern from the tomb tower in Maragha and in particular relation to Penrose tile patterning where he creatively attempts to find a correlation between *girih* and Penrose templates at a 1:1 comparative scale, a scale that is just as inappropriate as that of Lu and Steinhardt's more correct 1: φ^{-1} comparative scale for aperiodic overlay. This mismatch by Makovicky is further commented upon by Lu and Steinhardt in their response paper of November 2007. What is of interest to me is how, in their concentration upon attempting to apply modern theory to old practices, Lu and Steinhardt on the one hand and Makovicky on the other – through their initial, convoluted and opposing positions, and then within their subsequent comment and response papers – both miss the opportunity to see the significance of a very crucial design feature that is actually visually included (albeit only incidentally) within both their initial analyses, but which is not seen as being significant to a fuller understanding of *girih* patterning, although it is referred to in passing by Makovicky, who states:

> Eye-attracting rosettes of this kind are common in Islamic wall ornaments, but those used here (only once per each side of the building) are completely foreign to the rest of the pattern, and will therefore be omitted from further consideration.
>
> (Hargittai, 1994: 73)

The design feature to which Makovicky incidentally and dismissively refers is that which can be seen quite clearly in his Fig. 2a (my **Plate 73a**) and to which Lu and Steinhardt make no reference at all, other than its necessary and unavoidable visual inclusion as being integral to image **C** within their Fig. 2 (my **Plate 68**). Note the distinct surface pattern upon the decagonal *girih* template, shaded light blue. This is not the same surface pattern that is seen upon the same *girih* decagon template of image **E** in Lu and Steinhardt's Fig. 4 (my **Plate 67**), shown also in **F** within their Fig.1 (my **Plate 69**).

Makovicky had taken, as evidence for his thesis, a different photograph of the architecture in question, see his Fig.1 (my **Plate 72**), reproduced from a 1936 publication by A. Godard (Hargittai, 1994: 86). He then reconstructs and represents the external plaster *girih* pattern that is applied to what he refers to as 'The Blue Tomb', as already shown (my **Plate 73a**). At the bottom of this plate, I have taken and re-worked this image and included my own development upon the design, to indicate the interplay between the perception of both overlapping and stand-alone decagonal shapes formed from the judicious placement of the *girih* shapes **A**, **B** and **C**, which are shown in **Plate 76**. Within this development, I have highlighted the decagon in question differently and given it a green outline. This is the same decagon that Lu and Steinhardt highlighted (blue shaded) within illustration **C** of their Fig. 2 (my **Plate 68**). **Plate 73b** then indicates the differentiation between the implied decagons (**a**, **b**, **c** and **d**) and the complementary five pointed stars formed from the contiguous placement of rhombic *girih* templates **C** around the one point. [Incidentally, see how similar to those patterns of previous **Plates 48h** & **i** are the effects of overlapping arrangements that are shown in **Plate 74** when colour fields are applied to the overlap of **a** and **b** within **Plate 73b**.]

The particular aspect of early Islamic understanding of the relationship between (i) the star shape formed from five 72° rhombi and (ii) their complimentary interlocking decagons, is discussed very comprehensively in another paper by Wasma'a K. Chorbachi and Arthur L. Loeb (Hargittai, 1994: 283-305); and their Fig.1 (my **Plate 77**) further indicates, quite clearly, the evidence that the division of the decagon into five Penrose 'kite' shapes was a fundamental element within their understanding of the geometric intricacies of five-fold symmetry and its further utilisation in design. This understanding is additional to the knowledge that the decagon can be formed, but with less surface pattern symmetry, from *girih* templates **A** and **B**. This is hugely important, and may give weight to my theory as to how it may be that the individual surface pattern design was formed within the decagonal shape that I have outlined in green and labelled **e** in **Plate 73**. I would contend that – other than there being another quite separate and full decagonal template with a different surface pattern from that indicated in **Plate 70** – the surface pattern upon decagon **e** can only be formed by the repeat use, around a centre point, of the proposed *girih* template **F** (see **Plate 76** again). The other (conventional) single decagonal pattern indicated in **Plate 70** could equally have been formed from the repeat use of the smaller template **E,** and the theory then becomes complete, with templates **A**, **B**, **C**, **D**, **E** and the additional **F** making a more comprehensive application possible. The upper image within **Plate 75** provides further supporting visual evidence through the repeat use of template **F** to form the surface pattern that is evident upon *girih* shape **e**. The inclusion of the lower pattern design within **Plate 75** is incidentally provided to show the simple expansive periodic properties of template **B** (through its repeated placement) that is additional to its more complex relationship with other templates: such infinitely expanding and perfect periodic patterns can also be created with repeated individual use of both templates **A** and **C**.

Before concluding finally by defining the mathematical and visual properties of one particular expanding overlay of *girih* patterns found within the Topkapi Scroll, I wish to make one important comment upon what seems to be a somewhat common and exclusively mathematical and theoretical approach to analysing what are, ultimately, *practical* realisations created in a very distant era, with all the technical problems inherent in their construction, and which should (in my view) not be divorced from their very distinct and cultural context. Lu and Steinhardt's contention that, compared to the direct strap work method, greater accuracy is gained through the repeated positioning of these *girih* tiles to create a larger area of pattern with (allegedly) less accumulated distortion of angle and line (Lu & Steinhardt, 2007: 1106/7), could perhaps have been indicated more clearly and definitively by them. If they are referring here to the macro scale rather than to any infill at a micro scale that would be working outwards towards a larger scale, then I would agree with them. Although a pattern may be planned and confirmed more easily through the placing and manipulation of small *girih* tiles, they must then conform to, and be judiciously placed within, an overall mathematically calculated and measured area whose external limits are exact and finite for its final practical realisation. If this is ignored, and the pattern is laid out without regard to this principle, then the increased accumulation of slight and inevitable errors in the placement of individual tiles will eventually lead to ever greater distortions as the design expands in physical terms. Additionally, even if it were possible to make initially 'perfect' unfired clay forms from 'perfect' moulds, the manufacture of 'perfectly' proportioned individual tiles would have been almost impossible to produce, given the distortion and shrinking properties of ceramics in the firing process.

The problem of addition and increase of scale is a difficulty that all experienced and fine artisans committed to excellence must grapple with, which is why the more accurate practitioner has always had to know the maths *at all scales*, and very accurately. For *girih* designs, this is no simple matter, thus the staged scaling of the *girih* pattern must conform to the architectural design to which it is applied, or vice versa. It is possible that *girih* templates, at the larger scale, may have been used for this purpose. If this was the case, their considerably larger size would have necessitated their having been made from a different and lighter material, such as a wooden frame, and at a scale that would have considerably exceed the size of the smaller ceramic templates. In some cases the factor of increased scale would have been enormous, as is evidenced by a further analysis of the patterning properties of the Darb-i Imam shrine's complex and detailed mosaic. [See again **Plate 48**.] This pattern is now investigated in terms of its different scales of *girih* patterning.

The blue-grey lines of the design that were highlighted by me in black in **Plate 48a** constitute the larger-scale surface pattern applied to the larger-scale *girih* template design that is not evident at all at this stage. The central rhombus that is formed by this surface pattern arrangement that was highlighted in my previous analysis by its grey shading within **Plate 48d** is now seen, in **Plate 78a**, as being integral to a *girih* pattern template arrangement whose central 'bowtie' shape is outlined in red and the surface pattern lines indicated in blue. The much smaller scale *girih* templates that form the detailed mosaic design are then highlighted by green outline. What is important to note is the comparative accuracy of the large scale design in contradistinction to conciderable inaccuracies in the tile placements and the proportions of some motifs within the more detailed design at this smaller scale. In its practical relisation, the large scale overall design would have been established and mapped out first, and the smaller scale ceramic tiles would then have been placed *within* this arrangement such that the decagonal 'rosettes', indicated at the conjunction points by (O) and elsewhere as (•) in **Plate 78b**, would 'take up the slack' (as it were) to enable a more accurate placement of the close-knit detailed design. This is evidenced by the fact that these separately delineated decagonal motifs are slightly larger and less regularly shaped than the other more accurate, integrated decagonal shapes. How might the artisans then have made their calculations, and what is the difference in scale between the smaller detailed *girih* ceramic templates and the larger overall template design?

Plate 78b provides an indication as to how we might view this comparison, where the single units of length of a selection of the smaller *girih* templates are shown in black, and which can be related to the larger scale by their arrangement along the one red line of the larger 'bow-tie' template. This indicates an expansion of scale by a factor of $2(\varphi^2 + \varphi)$ or, in numeric terms, ~ 8.472. This is a huge difference in scale. How might this have been managed and calculated in practical terms? As the mosaic tiles are clearly marginally smaller than they would have been for a totally (and impossibly) accurate fill, the large scale templates would have to have been marginally larger than the theoretical ~ 8.472 increase of scale. As additionally shown in **Plate 78b**, the larger scale could easily have been measured and achieved in actual practice by placing, at right angles to each other, two measures of equal size, their being six times the unit measure of the smaller tiles. The hypotenuse of a right-angled triangle thus formed would measure ~ 8.485 (from $\sqrt{72}$, or $6\sqrt{2}$). This would then provide the larger measure required but would clearly not be significantly larger than the theoretical ~ 8.472 (at 0.15% difference) to account for the irregularities observed. The necessary 'allowance' could easily have been achieved otherwise, as in the natural contraction of the tiles in their firing, with such potentially greater difference in the smaller templates having a more significant and accumulative effect than a small accommodation at the larger scale.

I now conclude this section on the relationship between *girih* template and surface pattern geometries with a brief analysis, initially, of one of the many pattern designs included in what has become known as the Topkapi Scroll, being included with many other precious and important artefacts that are housed within the Topkapi Palace, Istanbul. **Plate 79a** first of all shows Panel 28 (as catalogued by Gürlu Necipoğlu) of this scroll, and then my superimposition of the larger scale template in blue to complement the original red outlines of the surface pattern. In **Plate 79b**, I further show the potential effects of in-filling that was seen in the mosaic tiling of the Darb-i Imam shrine and the different geometric shapes thus highlighted. In **Plate 79c**, I have extended the design beyond the limits of the original, with **Plate 79d** then highlighting the template lay-out to indicate more clearly the relationship of scale between the two interrelated sets of *girih* pattern/template designs. The difference in scale between the two designs can also now be calculated by comparing rhombus ABCD with larger scale rhombus AFGH within **Plate 79e**.

Take line AB to be of unit length. Since BE is equal to CE at ~ 1.618 (i.e. φ) then AF, being twice AE, is calculated as ~ 5.236 (the numerical form of $2(1+ \varphi)$, or $2\varphi^2$). Such a difference of scale is not as great as was evidenced within the Darb-i Imam shrine design, so how might the artisan have measured *this* difference of scale? Fortunately, this scaled measurement can equally be easily produced, for $\mathbf{2\varphi^2}$ can also be expressed as $\mathbf{\sqrt{5}+1+2}$ (i.e. ~ 5.236 = ~ 2.236 +1+2). This particular increase of measure from what could be *any unit length* – and this is the practical beauty of both my proposed methods – could be produced easily by opening out a simple triangle, where two sides (meeting at right angles) would measure a single and double unit of length respectively, with the resultant hypotenuse automatically providing what would otherwise be the more difficult numerical measure of $\sqrt{5}$. [This is the exact same basic triangle that is integral to the square shown previously in **Fig. 19** of Chapter 2 and also in my painting, *The Emergence of Phi* (**Plate 37**) which shows its use in the production and measurement of φ as $(\sqrt{5}+1)/2$.] What could be simpler? Any necessary adjustments to this comparative scaling, in order to facilitate ease of tile placement, could then have been made as indicated for the Darb-i Imam shrine.

Since conducting the above analysis (all of which remains relevant) I have been able to access Gürlu Necipoğlu's book on the Topkapi Scroll (Necipoğlu, 1995), which is much in demand, and I can now put my previous analysis in further and better context. Indeed, it is now clear that many researchers have either not been aware of, ignored, or omitted to mention further elements that would fundamentally question their own theories. **Plate 79f** shows Panel 28 now in its entirety, but what is of particular interest is that many panels, including Panels 53, 54, 55, 56, 63 and 90a (shown within **Plates 79g, h, i & j**), exhibit surface patterns that do not arise from the sole use of what has been limitedly described in the literature as *girih* patterning. It would appear that commentators have been highly selective and exclusive in their choice of patterns in support of their arguments. Clearly, the reason for the different patterns now found within Panels 53, 54, 55, 56 and indeed some others that also exhibit five-fold symmetry, is that there were a number of other templates used by these Islamic designers in addition to those classified by Lu & Steinhardt, et al. They are to be clearly seen in the aforementioned panels. For example, in Panel 56, there are two differently identified templates, in addition to the standard rhombic *girih* template that is also integral to this particular pattern design. Not all the sides of these newly identified templates are equal and, as such, they do not conform to the neat theories that have been proposed with respect to a more limited selection of templates. There are also a number of subtle angular variations from 'the norm' to the surface patterns applied to the standard decagonal *girih* template, as in Panels 56 and 90a, with yet another totally different and very intricate pattern indicated in Panel 54. This, of course, now gives added weight to my contention that there might indeed have been a *girih* template such as that shown as **F** in my **Plate 76**, or perhaps its decagonal equivalent. There are also two variations to the surface patterns of what are novel tetragons, the one seen in Panels 53 & 56, and the other in Panel 63. The latter tetragons are then fused together to form the one novel hexagonal *girih* template, found in Panel 90a, where there is also an additional and novel irregular pentagonal *girih* template, a fusion of three golden section triangles, and with the regular pentagonal *girih* template containing the 36° star pentagon instead of the 72° star pentagon surface pattern normally placed within the regular pentagonal *girih* template. Panel 90a exhibits perfect pentagonal symmetry. Yet another different surface pattern is seen upon the regular pentagonal *girih* template in Panel 63. It is my view, therefore, that these variations should not be ignored or excluded when considering patterns for critical analysis, and one should not select only from what may be regarded perhaps as the more obvious and commoner patterns that seem to have been adopted by Islamic practitioners. [It is thought that the Topkapi Scroll was compiled in the late fifteenth or sixteenth century, somewhere in western or central Iran (Necipoğlu, 1995: 38/39) and measures between 33 and 34cm high.]

Before moving on to analysing three-dimensional structures, I would like to end this topic with a most subtle and profound observation made by the Sunni scholar Muhammad al-Dawwānī (1424-1503), possibly adapted from an earlier Shi'i text by the Mongol-Ilkhanid mathematician and astronomer Nasir al-Din al-Tusi (1210-1274):

> Now nature is superior to art, for it is derived from the highest of sources, without the intervention of human judgment; whereas art proceeds solely from such intervention. Nature then is the pedagogue and preceptor of art; and as the perfection of things secondary lies in their resemblance to originals, the perfection of art must lie in its resemblance to nature which resemblance it may attain by anticipating or postponing means, and arranging them generally in their appropriate course: so that that perfection which, under Providence, is effected by the agency of nature, may be accomplished by art under the guidance of human will; and this with a peculiar virtue belonging only to art.
>
> (Necipoğlu, 1995: 117)

B. Connecting Two- and Three-dimensional Structure through Number

3.8 Prelude to 3D harmonies

Clearly, symmetry is important and integral to patterning. I now introduce some developmental drawings whose numeric content will later be seen to relate directly to certain numerically significant proportions and symmetries also found in Platonic, Archimedean and Catalan solids, with the latter shapes displaying rhombic faces. These drawings show how the different numerical series (they become increasingly more evident within this staged breakdown) relate – quite fundamentally – whole number sequences to their reciprocals, just as the magnitude of the angles of the different polygons mirror inversely the number of their sides. This is akin to wavelength reciprocating frequency in sound, and this analogy is very pertinent with regard to *arithmetic* series mirroring its reciprocal as *harmonic*.

In sound, the *physics* of basic harmonic rules dictate that, for **Fig. 28** below, the relationship of the frequencies of each note within – let's say – the second inversion of the C major triad (where the fifth note of the scale is in 'root' position) is, in ascending order, **3**: **4**: **5** (or **12/4**: **16/4**: **20/4**) for the notes G (96 Hz), C (128 Hz) and E (160 Hz).

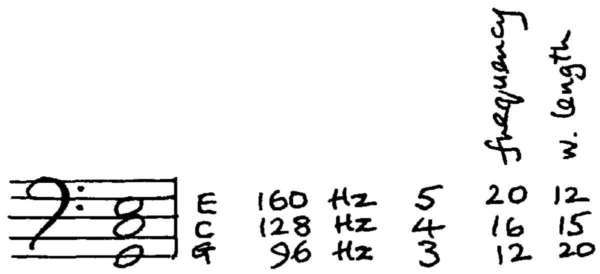


Fig. 28

The reciprocal, for wavelength, would be the equivalent of **1/3**: **1/4**: **1/5** (or **20/60**: **15/60**: **12/60**). It can then be seen that, in now comparing frequency with wavelength, the numbers **12** and **20** are interchangeable either side of both arithmetic and harmonic sequences, with the middle (mean) numbers, 15 and 16, differing by a single unit. [This difference is interesting and is also explained by the semitone between B (480 Hz) and C (512 Hz) being in the proportion 15/16. The frequency 480 Hz for B can also be corroborated as its being the major third above G whose important ratio at **5/4** provides the same result.] The numbers **12** and **20** will later be seen to be highly significant in my analysis of three-dimensional geometry and will be seen to be integral to the physical structures of the second and third Platonic duals, and in some other further related solids also. [The Greeks were aware of more complex numerical sequences where the 'mean' of two numbers could be calculated; in fact, they formulated seven beyond the *arithmetic*, *geometric* and *harmonic* means. Their algebraic expressions are described and explained by Sir Thomas Heath in his *History of Greek Mathematics, Vol.I* (Heath, 1981: 85-89).]

Plate 80a now introduces three basic triangles, viz. the equilateral, the right-angled isosceles and the golden section 36° isosceles triangle which, in turn, are seen to relate directly (internally) to the equilateral triangle itself again, the square, and the regular pentagon, respectively. When we divide 180° (this being the sum of the angles of any triangle) by **3**, **4** and then **5** – to obtain 60°, 45° and 36° respectively – it is seen that **three** 60° angles are replicated in the first (equilateral) triangle, **two** 45° angles in the second (isosceles) triangle and **one** 36°angle (only) in the third (golden section) triangle. This may seem to be rather simple and obvious, but here we can see the very beginnings of a core numerical harmony within geometric structure. These are the most basic and most fundamental elements of two-dimensional geometry, that which underpins the increasingly more complex properties of three, four and five-fold symmetries in the third dimension, where a [2: 3: **3**], [2: 3: **4**] and [2: 3: 5] relationship will be proposed, and where this will (eventually) be seen also to generate, through reciprocation, a particular and significant mathematical matrix (see **Plate 96**). As it is the initiating thrust of this thesis, we must bear in mind here also the most crucial significance of Dürer's own possible numerical reference, where it is proposed that the relationship 4:5 (also the major third interval proportion that was previously referred to) is perhaps the most fundamentally important factor here, being at the very core of what I see as the cusp between simplicity and complexity that was alluded to in my explanation of the underlying philosophical concept that produced my dual painting, *Before 5, After 4*. [See again Part 3.3 True fivefold symmetry.] **Plate 80b** then shows how (i) the equilateral triangle finally relates to the 60°/120° rhombus, (ii) the square relates to itself as a 90° rhombus and, finally, (iii) the regular pentagon relates to a 72°/108° rhombus which itself is then related to the 36°/144° rhombus. Remaining plates within this series then show (a) the simplicity of 3-fold symmetrical arrangements, (b) the only possible configuration for simple 4-fold symmetry, with the necessity for (c) more complex arrangements for 5-fold symmetry. [**Plate 80d** is included to show how actually it might be possible to create a regular arrangement of pentagons where all their sides are equal *but* where it is necessary that *not all* the angles are equal, such that there are two 90° angles, two 114.295° angles and one 131.41° angle.]

3.9 The Esher connection

It may be said that Dürer was fascinated by the mathematics of form, particularly the expression of the three-dimensional through the complexity of two-dimensional perspective representation. M. C. Escher (1898-1972), however, was to take the problem of visual perception and the artistic representation of the perceived world a step further. Through the manipulation of what is essentially a cerebral process (a function laid down developmentally by necessary contingent experience), he plays with our natural and compelling instinct to interpret and make sense of what we see. Escher, of course, produced many prints that challenge this natural, psychological process and thus to induce a surreal perceptive experience.

As a convenient link to my investigations, I make brief reference to one of these prints, viz. *Man with cuboid*, 1958 **(Plate 81)**, which shows a person contemplating what seems to be a strangely formed cube. This curious object is the classic Necker Cube, an impossible form. Now, in the second half of this chapter, I shall be developing an understanding of the geometry of three-dimensional solids and investigating further numerical relationships gained through the experience of their construction, those that are far more complex than the cube. Any investigation into geometry necessitates the use of number, if only as a function of measure, and what I wish to emphasize again is the relationship between number and thought and, ultimately, that understanding which leads to the formation of a particular belief, not reliant upon or influenced by the teachings of any culturally or politically motivated ideology. Upon this numeric connection and principle this whole thesis depends, and around which it revolves.

Where then does Escher (and his cube) come in here? Well, there are many connecting elements here. Firstly, Escher – and I suspect Dürer also – constantly questioned himself as much as he did the world around him:

My picture **[Belvedere]** is no higher math. That I cannot state this more precisely is

> caused by my lack in mathematical training, I suppose. That is actually what is absorbing about my position as regards mathematics: our realms touch each other but do not overlap. I regret that!
>
> M. C. Escher
> (Locher, 2000: 145)

This is, of course, somewhat of an understatement, for Escher's knowledge, understanding and application of mathematical form was clearly profound; but here (see **Plate 82)** he is referring to the print from which he has extracted his figure and subject of contemplation for *Man with cuboid*. Escher's utilisation of illusory techniques and geometric patterning might seem all-pervasive in his works but there are many examples that also suggest to me a deep understanding of the connection between harmonic systems and the profoundly visceral nature of the biological world, particularly the ultimate feature of duality; that which is manifest in two forms, in complement and in opposition, in black and in white. To me, these *biological* inclusions, with all their stark implications, are a feature of his work that is as important as the mathematical and illusory elements. In consideration of this, and potential 'melancholic' implications, I wonder whether *Belvedere*, 1958, might have been inspired by enquiries that he might have conducted on a more personal level into theological issues, as another facet of what could be seen as the psychological duality of man. This, of course, is a fanciful notion but, when I compare certain architectural features within his *Belvedere* with photographic evidence of certain manifestations at St Catherine's Monastery in Sinai **(Plate 83)**, I am struck by some similarities. Could it be that it is the domed roof and curved arches of the monastery wall that are being mimicked in *Belvedere* and whose internal ladders can also just be seen in the photograph of the tower **(Plate 84)**?

Certainly the site of St Catherine's Monastery is historically important, for this is where the Codex Sinaiticus had been kept until it emerged in the mid-nineteenth century. The Codex Sinaiticus (http://codexsinaiticus.org, 2011) provides a valuable insight into how history is 'written', for this early hand written version of the Greek biblical text, which is thought to date from about the mid-fourth century, includes the New Testament in original vernacular language (koine) as well as the Old Testament, otherwise known as the Septuagint. Both texts show the signs of heavy annotation, with the inclusion of books not found in the Hebrew Bible but then also with some parts missing of what we would now regard as the New Testament, as in the absence here of some of the traditional accounts of Christ's resurrection.

St Catherine's Monastery is also thought to be the location of, and thus synonymous with, the apocryphal and biblical 'burning bush', its inhabitants still being allowed their Christian independence and freedom although being located within long established Arab territory. Similar arches can be seen, of course, within the architecture that surrounds the Kaaba at Mecca **(Plate 85)**, also referred to as 'The Cube' (although not, strictly, geometrically, a perfect cube), and which externally incorporates the all-important 'Black Stone' within one of its corners. Would there be any benefit in considering a thematic link here with Dürer's polyhedron? Probably not.

Clearly, these links are very tenuous and purely conjectural, indeed one might say 'inventive', as might the thought of connecting any imagined symbolic element of significance with regard to Dürer's polyhedron (as stone) to a similarly important site, such as that which is housed within the visually and politically imposing Islamic mosque in Jerusalem, and which is fittingly referred to as the 'Rock of the Dome'. Both Arab and Jew, of course, each claim their own historic connection and rights to this significant and 'holy' site, also known as the Temple Mount or Haram al Sharif. However, it is more likely, and this according to his own admission, that it was the Norman, Romanesque, Saracen and Moorish architectural features of southern Italy which influenced Escher to include such features in his work (Locher, 2000: 54). Incidentally, regarding the Moorish influence, Escher's beautiful coloured crayon and chalk sketches of 'majolica' tile work that he copied from the Alhambra is testimony to his knowledge and interest in their two-dimensional geometric patterning (Locher, 2000: 57).

As has been previously mentioned, geometric patterning is not exclusive to Islamic architecture; and as I make further connection with Escher and his use of a certain geometric form in both *Order and Chaos (Contrast)*, 1950 **(Plate 86)** and *Gravitation*, 1952 **(Plate 87)**, we clearly see some very fine Byzantine *pietre dura* work (composed of marble or other such selected stone intarsia such as lapis lazuli, agate and porphyry) within the St. Mark's Basilica in

Venice, c.1430 (upper image of **Plate 88**) where the knowledge of both two-dimensional and three-dimensional geometry has been cleverly utilised in the depiction of this very same form. [This, of course, is contemporary with Dürer.] The judicious use of contrast in colour and shade creates the impression of three-dimensionality, and some sources attribute such work to the celebrated Florentine painter, craftsman and mosaicist, Paolo Uccello (1397-1475). In the first example, it is what may be termed the *small stellated dodecahedron* that is indicated, and in the second instance it is the three-dimensionality of the implied repeated cubes that is central to the design. However, in Escher's work, as already mentioned, the connection between the mathematical and the biological is never far away, where 'man' or 'beast' is often indicated as integral to a possibly more Darwinian view of reality, and which is perhaps the deeper indication intended within *Reptiles*, 1943 **(Plate 89)** but where the solid form of the dodecahedron is also included.

The relationship between the outer 'skeletal' dodecahedron and what may be termed a 'solid' inner star dodecahedron, or *great stellated dodecahedron*, is now shown in **Plate 90**. Conversely, **Plate 91** indicates a 'solid' *small stellated dodecahedron* enclosed within a 'skeletal' icosahedron. There are differences of opinion within the literature regarding nomenclature for these 'stellated' forms and this can therefore be a cause for some confusion; they are sometimes referred to as Kepler's star polyhedra (Cromwell, 1997: 168-173). Johannes Kepler (1571-1630), a contemporary of Galileo Galilei (1564-1642) – Galileo being well known for his having been tried and subsequently detained by the Inquisition – is of course also well known for his mathematical and astronomical investigations and resultant theories, where he was to further develop the heliocentric ideas of Nicolaus Copernicus (1473-1543), and who was himself clearly heavily influenced by humanist ideas that harp back to Greek thought and principles. [It is worth noting here, in connection not only with mathematical matters but astronomical issues also, that Copernicus was contemporary with Dürer, with less than two years separating their ages, and where, therefore, the implication of possible influences upon Dürer cannot be disregarded, this in contradistinction to the Ptolemaic, geocentric teaching espoused by Reich in his *Margarita Philosophica*, a book already referred to towards the end of

Part 3.3 True five-fold symmetry in connection with **Plate 42**.]

Kepler was clearly much influenced by Greek ideas, so much so that he famously attempted to associate the planetary orbits with the five Platonic solids being inscribed within an integrated system of spheres (Hoyle, 1962: 92-120); but prior to Kepler's mathematical analyses of these and other polyhedra, Wenzel Jamnitzer (1507/8-1585), in his *Perspectiva Corporum Regularium* of 1568, had also provided perspective drawings of these and various other related polyhedra as a development of the very basic descriptions of the Platonic solids and their supposed association with the elements of fire, earth, air, and water that had been written about by Plato in his *Timaeus* and other texts, and also the more purely geometric and more detailed considerations found within Book XIII of Euclid's *Elements* (Heath, 1956: 440-511). Although not born there, Jamnitzer, as a resident of Nürnberg, had proceeded after Dürer as a celebrated and leading goldsmith, also himself gaining the patronage of royalty and nobility. **Plate 92** shows Jamnitzer's own illustrations of (i) to the right, the *great stellated dodecahedron* within a freely suspended icosahedron while (ii) to the left, his version of the rhombic triacontahedron accentuates more a configuration that appears as a conglomeration of a number of three pointed stars. However, it is the importance of the concomitant *rhombic* element of this latter polyhedron that I shall be accentuating in the development of my particular theme.

Clearly, there are many common and related areas of interest that apply to this thesis. The history of the development of three-dimensional representation and perspective drawing, for example, will now have to be viewed as too extensive a topic to be followed in depth, where Uccello would have to be included among many other practitioners and theorists, along with Piero della Francesca, Brunelleschi and Alberti (Cromwell, 1997: 114-121), and who were to provide a more credible alternative to earlier Byzantine negative perspective interpretations. My concern has been throughout, as it was with analysing Dürer's polyhedron, in the *actual* geometry, not necessarily its two-dimensional depiction with all the inherent technical problems and issues of psychological interpretation. However, before returning to Escher, another important practitioner and theorist of Dürer's era who should perhaps also be mentioned is Fra Giovanni da Verona (c.1457-1525), the author of a number of publications on the subject of geometry and perspective, specialising in wood intarsia, some of which indeed depict three-dimensional geometric structures exemplified here – in **Plate 93** – by one of the many intarsia within the Church of Santa Maria in Organo, Verona.

I now return to the 'impossible' cube within Escher's *Man with cuboid*, with all its integral implications as to the uncertainty of perception. What I shall now be attempting to do is not to question visual perception per se but to question whether there is some significant numeric and harmonic connection between what is the 'real' cube and other more complex but clearly related geometric forms. To do this through reference to Escher is, of course, very relevant, for not only did he use such previously referred to techniques to induce the perception of three-dimensional visual anomalies upon a two-dimensional surface, but he also made a special study of three-dimensional form in its own right:

> If you were to ask, why do you do such silly things, such absolute objective things that have nothing personal any more? Then all I can answer is: I cannot stop it. This particular case has never been satisfactorily resolved, as far as I know, by that group of folks, around 1500 and 1600, Dürer, Pacioli, Barbaro, and even Leonardo. Undoubtedly they were originally interested in pure form in the same way as I am: the beauty and the order of regular bodies are overwhelming. There's nothing you can do, for they're there. If you then insist upon speaking about god: they have something divine, at least nothing human. But we cannot do anything about the idiosyncrasy of wanting to bear witness to what touches us either. We cannot stop it. My personal result, of course, is a wash out, a proof of impotence; I may say that, just so long as nobody else says it. But I have given it my best effort and more than that I cannot do. Who knows, perhaps I'll do it better some time later on.
>
> M. C. Escher
> On *Möbius Strip*, 1961
> (Locher, 2000: 184)

[An excerpt from this quotation was used at the head of this chapter.]

To understand what it is that lies behind this compulsion, it was necessary for me also to go through the same steps of actual construction that Escher and his predecessors would have experienced; to better understand the mind-set of the polymath, but – more importantly – to gain for myself a better understanding of the mathematics that underlies structure at its most basic level. It could then indeed be said that the intended 'ultimate' symbolic function of Dürer's polyhedron might have been to indicate to the enquirer – and thus to induce the necessary response towards that conclusion – that true understanding can only be gained through committed, personal endeavour. As the result of such exacting and necessarily time-consuming activities, and also from knowledge gained through additional academic investigations (Coxeter, 1973; Cromwell, 1997; Cundy & Rollet, 2007; Ghyka, 1997; Hargittai, 1994; Heath, 1956 & 1981; Wenninger, 1975), I now introduce certain numeric relationships that were implied earlier in my deliberations, and show how the cube continues to take centre stage, literally, as one of the five Platonic solids in the development of my theme towards the final significance of the rhombus within the third dimension.

3.10 Solid geometry and number

Plate 94 shows the cube in a central position. Although the cube may seem to be separated more from the others by its perhaps greater familiarity, with each right-angled face aligned in direct relationship to the three planes of dimension, and each at right angles to each other, it is in fact partnered by, and most directly related to, the octahedron. Such pairs are known as 'duals', where corresponding faces and vertices (points where the corners of shapes meet) are interchangeable. This relationship is shown in simpler visual terms in **Plate 95**.

The octahedron constitutes *eight* triangular faces (each obviously having **3** sides) where it is **4** of these triangles that must meet at any one of its *six* vertices. Conversely, the cube has *six* faces (each having **4** sides) with **3** meeting at any one of its *eight* corners (vertices), hence the common numeric ratio **3:4** for both octahedron and cube. Note that the number of faces and number of vertices are also in the ratio of **3:4**, simplified from {*six:eight*}, with the number of edges (at twelve) being common to both. [The Schäfli symbol system denotes the octahedron as {3,4} and the cube as {4,3}. In this system, the fist number of each pair is the number of sides to each hedron, with the second number indicating how many hedrons meet at each vertex.]

Consider now the next set of duals, the icosahedron and the dodecahedron. The icosahedron constitutes *twenty* triangular faces (again the numerical significance of **3** related to the triangle) where it is **5** of these triangles that now meet at any one of *twelve* vertices. Conversely, the dodecahedron has *twelve* pentagons (each having **5** sides) with **3** meeting at any one of *twenty* vertices. The common ratio here is now **3:5**, as it is for the ratio of number of faces to vertices, simplified from {*twelve*: *twenty*}. The number of edges is common to both (at thirty). [The

Schäfli symbol for the icosahedron is therefore {3,5}, and for the dodecahedron is {5,3}.]

The tetrahedron is the simplest of the Platonic solids and, as a dual, is a mirror and duplication of itself, with *four* triangular faces and *four* vertices, where **3** triangular faces (with **3** sides) meet at any one vertex, hence the concomitance **3:3**. Although introduced last, it is the first and most primary of the Platonic solids, and from which the others can be developed sequentially, as we shall see later. [The Schäfli symbol for the tetrahedron is simply {3,3}.]

Note that the ratio 2:3 is common to all the Platonic solids, in that it is to be found for each one as a simplified ratio somewhere within the numbers of faces, vertices and sides. This is significant, for if we tabulate further (see **Plate 96**) we can see that in taking the core ratios **[2: 3: 3]**, **[2: 3: 4]** and **[2: 3: 5]** and inverting them as we did for sound, their reciprocals become [1/2: 1/3: 1/3], [1/2: 1/3: 1/4] and [1/2: 1/3: 1/5] respectively. These three groups, when now expressed as [**6**/12: **4**/12: **4**/12], [**12**/24: **8**/24: **6**/24] and [**30**/60: **20**/60: **12**/60], can then be simplified and further expressed as **[6: 4: 4]**, **[12: 8: 6]** and **[30: 20: 12]**; these are the very same ratios for the numbers of [edges: vertices: faces] seen in the tabulation for the Platonic solids (see again **Plate 94**). As the ratio 2: 3 (simplified) is seen to be common to the three sets of duals **[2: 3: 3]**, **[2: 3: 4]** and [**2: 3: 5**], it is the simpler **[3: 3] [3: 4] [3: 5]** sequence indicated by **Plate 95** that becomes increasingly significant within the context of greater complexity and in trying to understand what it is that Dürer *may* have been signifying in *Melencolia 1*.

What may be worth noting here, with regard to the Platonic solids, and also in further connection with Dürer, is the commonality of the equilateral triangle within each of the three groups of duals. The triangle is, of course, the first and simplest straight line geometric shape possible and of such fundamental importance that Regiomontanus made it the sole topic of one of his many studies. Significantly, this person, Johannes Müller von Königsberg (1436-1476), but otherwise known as Regiomontanus, moved to Nuremberg in 1471, the same year that Dürer was born, there building an astronomical observatory as well as establishing a printing press. He prepared, but never published, several mathematical works, including one entitled *De triangles.* Unfortunately, he died before this and all his other mathematical works could be produced by the press that he had himself established. As a native of Nuremberg, and although not exactly his contemporary, Dürer could not have failed to have known of Regiomontanus' important work on mathematics and astronomy; indeed, it is alleged that:

> In 1523 the famous painter Albrecht Dürer acquired Regiomontanus' copy of the [Cardinal] Bessarion manuscript f.a.332. This copy is now lost.
>
> (Ragep, 1996: 94)

The manuscript that is being referred to was a redaction by Alfred the Great (848-c.900) of Euclid's *Elements* by Boethius. [Because of inconsistencies within the standard literature, dates for Boethius are given loosely here as c.480-c.525.] Regiomontanus also holds another particular place within the history of the development of knowledge about geometry, for it is claimed that it was 'Regiomontanus who introduced trigonometry into the form that we would recognise it today' (Sozio, 2005: 8).

3.11 Connecting to Archimedean and Catalan solids

The numeric and geometric relationship between the three sets of Platonic duals can be developed further. Consider now **Plate 97**. If we truncate the tetrahedron by deleting each corner as far as the half way point of every edge, the resultant shape is the octahedron. Conversely, we might regard the tetrahedron as constituting a core octahedron with four half-scale tetrahedra added to this central structure. From **Plate 95** we already know that the dual of the octahedron is the cube. The cube is a total three-dimensional space filler.

Consider then **Plate 98**. If we truncate further either one of the second classification of duals (see again **Plate 95**), i.e. the octahedron or the cube itself, again to the halfway point of each one of the edges, we arrive at the cuboctahedron (an Archimedean solid), whose dual is the rhombic dodecahedron (a Catalan solid) with twelve faces. Similar to the cube, the rhombic dodecahedron is also a total three-dimensional space filler, whose convenient form is utilised by 'nature' in the construction of the waxed bee hive and its cells.

Lastly, consider **Plate 99**. Taking the third classification of duals, the resultant truncation of either the icosahedron or the dodecahedron is the icosidodecahedron (another Archimedean solid) whose dual is the rhombic triacontahedron (another Catalan solid) this time with thirty faces. The whole staged process and classification can now be represented as **Plate 100**.

If we now consider column II from **Plate 100** and represent it in isolation as **Plate 101**, and then do likewise for column III, representing it anew as **Plate 102**, it can be seen that the same **[1: 2: 5]** relationship between the three distinct 'duals' classification A, B and C, identified in **Plate 94** (synonymous with column I of **Plate 100**), is being carried through for each new column, i.e. columns II and III, respectively. This relationship is most significant and can be represented in musical terms by, arguably, the most beautiful of all triads, shown as **Fig. 29** below, not necessarily, of course, at the particular pitch that is exemplified here. It is the **[1: 2: 5]** relationship that is critical. Note also how the C in the middle of the triad is represented by **4** for frequency but then by **5** for wave length, as the reciprocal of [**2**: **4**: **10**] becomes [**10**/20: **5**/20: **2**/20].

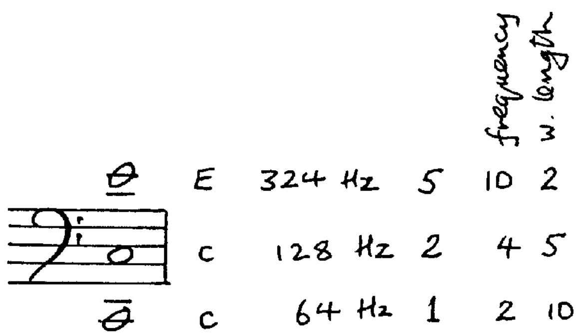


Fig. 29

Personally, I would contend that the **2: 5** ratio, represented by the interval of a tenth between C (128 Hz) and E (324 Hz) above, supported then by the lower octave C (64 Hz) as **1**, is perhaps the most evocative of all resonances, when such intervals are executed exactly, perfectly in balance and in continuum. Anyone with any musical sensitivity cannot but be affected as some most fundamental connection is made, somewhere deep in the psyche it seems to me, with perhaps the most powerful concept of all, that of outward infinity. If it is *this* connection with the absolute nature of number is what the Pythagoreans were moved by, then I can perhaps understand how it is possible to believe that the two are one; the one being the *psychological response* to the other; the other being the physical manifestation of *mathematical essence* through its aural sensing as pure vibration.

3.12 Total space filling solids

Let us return to the geometry, and consider further the three solids arrived at in column III of **Plate 100**, now in terms of their space filling properties. The cube can, of course, with its edges meeting at right angles, i.e. with a dihedral angle of 90°, be used repetitively to fill space totally. The rhombic dodecahedron, with its dihedral angle of 120°, also fills space totally, as a honeycomb, with either three or four vertices meeting at one point (see **Plates E3 & E5**). It can also be further viewed as being configured with twelve solids meeting around a single central solid. The rhombic triacontahedron, however, is a little more interesting. It has a dihedral angle of 144°, which is $^2/_5$ of 360°. Any complement, as $^3/_5$ of 360° (i.e. 216°), requires either a doubling of the 108° angle, or a tripling of 72°. The object that fulfils both of these criteria is the rhombic hexecontahedron (see **Plates 103 & 104**) in that it possesses both 216° and 72° dihedral angles. Part of the single rhombic triacontahedron fits perfectly into the twelve star-like pentangular recesses that are integral to, and form the external surface of, the rhombic hexecontahedron, as indicated by **Plate 103**, but no converse configuration is possible that would then correctly surround a single rhombic triacontahedron through the very same star pentangular recesses of the rhombic hexecontahedron. Total space filling is therefore not possible with the rhombic tria- and hexe-contahedra.

The rhombic hexecontahedron actually constitutes 20 fused *acute* golden rhombohedron/hexahedron, with each hexahedron possessing both dihedral angles 72° and 108° (see upper form in **Plate 105**). It is the one acute end of each of the 20 hexahedra that meets the others at the centre of the rhombic hexecontahedron, with the other end of each hexahedron forming the 20 outer vertices. However, there is another golden rhombohedron, the *obtuse* golden rhombohedron/hexahedron, which possesses what are the complementary dihedral angles 36° and 144° (see lower form in **Plate 105**). Both types of golden rhombohedra/hexahedra constitute six golden ~ 63.435° rhombic faces. For the *acute* or 'stretched' version, three ~ 63.435° angles meet at opposite poles; and for the *obtuse* or 'flattened' version, it is the three ~116.565° angles that meet at opposite poles. One might be tricked by two-dimensional representation into thinking (e.g. Livio, 2003: 206) that the faces differ in the two versions, but *both* most certainly are composed *only* of ~ 63.435°/ ~116.565° golden rhombi. It is actually the *dihedral* angles that are reminiscent of the *two-dimensional* 72° and 108° golden angles seen in the one Penrose rhombus, with the 36° and 144° angles in the other (see again **Plate 71**). In combination, the *acute* and *obtuse* golden rhombo/hexahedra can be used to make three-dimensional space-filling patterns, but only aperiodically. Indeed, it is from these two golden rhombohedra that a solid triacontahedron can actually be formed:

> A model can be built up from twenty rhombohedra, ten acute and ten obtuse, bounded by such rhombs. (The thirty faces of the triacontahedron are accounted for as follows. Seven of the obtuse rhombohedra possess three each, and nine of the acute rhombohedra possess one each. The remaining four rhombohedra are entirely hidden in the interior.)
>
> (Coxeter, 1973: 26)

As a final clarification of the different angles involved in two- and three-dimensional patterning, **Plate 106** indicates (left column) the rhombi involved in the classification of two-dimensional space filling that I analysed through Islamic patterning (the 36° angle shown here as integral within the 72° model), these to be contrasted with (right column) the three rhombi that were then evident in total space filling three-dimensional solids, where 63.43495° is the rhombic angle for both forms of rhombic triacontahedron, 70.52878° being the rhombic angle for the 'honeycomb' rhombic dodecahedron, and the obvious 90° rhombus (as square) for the cube. Note that the square is common to both two and three-dimensional patterning. Note also, most particularly, the significance of the most primary of trigonometric whole number ratios to the rhombic acute angle for, in their order, (i) the cube, (ii) the rhombic triacontahedron and both forms of the rhombic hexahedron and (iii) the rhombic dodecahedron:

$$\sin 90° = \mathbf{1{:}1}$$
$$\tan 63.43495° = \mathbf{2{:}1}$$
$$\sec (1/\cos)\ 70.52878° = \mathbf{3{:}1}$$

[Note:
For a clearer and additional understanding of matters raised in the last three sections, viz. 3.10 Solid geometry and number, 3.11 Connecting to Archimedean and Catalan solids and 3.12 Total space filling solids, it would perhaps be beneficial, at this point, to cross-refer with the geometric elements that are included in the Exhibition section at the end of this thesis and where the accompanying photographic evidence – included at the end of Volume II – makes for better visual clarification.]

3.13 Spirals

Much has been made, by some commentators, of the visual resemblance of the two-dimensional 'golden spiral' to similar spirals in nature, especially those of the nautilus and abalone shells. However, when referring to the specific logarithmic spiral of the golden section, it is not the nautilus but only certain species of the abalone that occasionally exhibit a near perfect 'golden spiral' in the appearance of their shells. [Covers of both Mario Livio's *The Golden Ratio* and Matila Ghyka's *The Geometry of Life* feature the nautilus shell spiral, with the former unfortunately displaying an obvious inconsistency with its theme.] Some of the different gnomonic spirals possible are shown quite clearly by examples in Chapter VI of Ghyka's book, and others by me in samples of my detailed drawing analyses of certain proportions that are related to this thesis, viz. **Plates 107, 108 & 109**. The 5:4 and $\sqrt{\varphi}$ spirals of **Plates 107 & 108** form the generic basis of the violin scroll and which closely resemble the spirals of ammonite fossils and certain nautilus shells **(Plate 110a)** whereas the φ spiral of **Plate 109** more closely resembles the abalone **(Plate 110b)** although, again, there are many variations.

C. Historical Resonances

3.14 A Greek view of mathematics

Fleeting reference has previously been made to Plato in relation to the nature of 'one', in the conclusion to Part 3.3 True five-fold symmetry, from which the concept of proportions can be developed. As some of the aforementioned solids are named after him, it might be relevant at this point to make proper reference to the Greek philosopher, Plato (c.427-347 BC). Although he was obviously otherwise steeped in knowledge with regard to harmony and form, in terms of both number and geometry, in his *Timaeus* Plato provides us with the most minimal of mathematical descriptions (Jowett, 2009: 135-137), indeed nothing for the dodecahedron, for his interest here seems to be in relating the first four geometric solids to the 'four elements', developing his metaphysical ideas from the cube as 'earth'. This view of 'matter' is preceded by his relating the nature of number, with its generation as initiated by God, and its computation through a process of initially subtracting 'part' units – as single, individual units of equal value – from the entity of a larger *universal* 'whole' (as distinct from 'whole' *numbers*), and then creating numerical proportions from distinct/different 'whole' numbers that were formed by the utilisation of these initial, single, 'part' units, this through a convoluted, developmental process of addition, subtraction, multiplication and division, and then to further this principle of mathematical harmony to its ultimate transformation and expression as the three-dimensional nature of a larger planetary 'cosmos' (Jowett, 2009: 115-117). The proportions developed and described here by Plato are the very same as those that can be formed from the numbers that constitute the *Lambda* **(Plate 111)**, also referred to again by Plato, but this in relation to a concept of motion as 'sensation' (Jowett, 2009: 124). From these numbers, Plato indicates how one might generate the Pythagorean ratio **256: 243** (4^4: 3^5). A perfect musical analogy that reflects his description of sequences, and which ends with the important 256: 243 ratio, is how the smaller semitone (256/243) that is found between the major third (81/64) and perfect fourth (4/3) intervals of the Pythagorean scale is the interval that is derived from combining two **equal** intervals (of 9/8) to form the *Pythagorean major third* (9/8 x 9/8 = 81/64), and then compared with the perfect fourth (4/3). This semitone interval (256/243) is distinct from the marginally larger semitone (16/15) of the diatonic scale, referred to in Part 3.8 Prelude to 3D harmonies, when it is the combination of two **unequal** whole tones (9/8 and 10/9) that are involved in producing the smaller *diatonic major third* interval (5/4). In summary:

(i) the *Pythagorean semitone* derives from:

[8/9 x **8/9]** x **4/3** = (8 x 8 x 4): (9 x 9 x 3) = (64 x 4): (81 x 3) = **256: 243**, and

(ii) the *diatonic semitone* derives from:

[8/9 x **9/10]** x **4/3** = (8 x 9 x 4): (9 x 10 x 3) = 4/5 x 4/3 = **16: 15**

That which it is that Plato explains stems from an earlier tradition and particular life-style that was practiced by the followers of the mathematician and philosopher, Pythagoras of Samos (estimated at roughly 570-490 BC). In the same way that we have to rely upon the writings of Plato to have an interpretation of the thoughts and teachings of Socrates, information about Pythagoras relies upon the historical writings of others, so being definitive as to what he really thought is impossible; there is nothing that might have been recorded by Pythagoras himself that is extant. This is perhaps not surprising if it is supposed that he was intent upon 'in-house' secrecy, much as it is for the Freemasons – another fascinating subject somewhat beyond the scope of this enquiry – where, for them, it is the *Biblical* that co-exists in ritual secrecy with the mathematical. However, what is relevant here, from the evidence of repeated historical writing, is the Pythagorean's belief that numbers possess attributes or powers further than that which we might ascribe to them computationally. The apparent form that such thoughts or beliefs took are described and included in the *Life of Pythagoras* by Iamblichus (c.245-c.325 AD) (Guthrie, 2010: 1-62), a contemporary of another commentator, Porphory (234-c.305 AD); but what is of particular interest and relevance here are the details that are recorded in the extant fragments of Pythagoras's not quite contemporary, Philolaus (c.470-c.385 BC) – although **Plate 112** includes a late 15th century depiction of him at work with Pythagoras – where he explains in a little further detail (Guthrie, 2010: 95-102; from Boeckh) that which Plato also was to later describe in the *Timaeus*, including some further information about the Tetractys **(Plate 111)**. I shall be explaining the significance and importance of the Tetractys in the next section.

[These fragments of Philolaus were known to Boethius who was mentioned previously in connection with Euclid's *Elements*. An(i)cius Manlius Severinus Boethius is an historically important figure. His dates are variously given as 475 and 480 - 524 or 525. It would appear that he was affected by, and embroiled within, the constant power struggles as a result of the uncomfortable relationship between the strict doctrine and internal, divisive politics of Roman and Byzantine Catholicism on the one hand, and the more liberal and educated views of an emerging humanist tradition (later to flourish in the middle ages) on the other, although he is claimed to have died a Christian martyr. However, in relation to the topic of numerical proportions and music, his *De Institutione arithmetica* comprises a translation into Latin of both Nicomachus's lost *De musica* and Ptolemy's *Harmonics* (Sadie, 1980: Vol. 2, pp. 844-5).]

Clearly, what is important and relevant to this thesis are these constant references to *numerical* harmony, that which the Greeks certainly knew applied to *musical* harmony most particularly but which they also thought could be applied more *universally*, as evidenced from another fragment of Philolaus:

> The world's nature is a harmonious compound of infinite and finite elements; similar is the totality of the world itself, and of all it contains... All things, at least those we know contain number; for it is evident that nothing whatever can either be thought or known, without number.
>
> (Guthrie, 2010: 95)

But it is the element of *absolute mathematical purity*, found in the harmonies of both music and geometry, is that which I wish to distinguish – and indeed isolate – from any concomitant alliance with less universally benign forms of belief that seem to be manifest in the religions of all otherwise mathematically literate cultures and civilisations, as if there were some sort of psychological necessity in the balancing of order with chaos, or the left cerebral hemisphere with the right, as if gripped or necessitated by some convoluted and complex form of binary 'conjoined separation'.

3.15 Figured numbers

The Tetractys is now to be considered, for it might be that Dürer had been influenced by the numerical reverberations that are implicit within it. In the sequence 1, 2, 3 & 4, indicated by the dots in **Plate 111**, all the numbers add up to 10. This simple fact seems to have held some particular mystical significance for the Pythagoreans (Heath, 1921: 77). This same mystical significance given to the number ten pertains also in the Jewish *Qabalah* or Kabbalah, where a hierarchical and interrelated arrangement of divine essences is shown in the form of the Sephiroth **(Plate 113)**. [See also next Part 3.16 The Kabbalah.]

However, if we take the sequence 1, 3 & 5, derived from the number of triangles that the dots form, the addition of the first two numbers equals **4**, which, when added to the third number **(5)**, gives us **9**. Might it have been the knowledge of the mystical practices of the Pythagoreans that initially influenced Dürer to perhaps see some hidden significance, more in *these* particular numbers? As already mentioned in my previous chapter, alterations to, or variations of, the numbers **4**, **5** and **9** are to be seen within the dating of a significant number of his works, in addition to *Melencolia I*: it is only these three particular numbers that Dürer alters.

In considering number sequences further, the subject of what may otherwise be termed Greek 'triangular', 'square' and even 'oblong' numbers (Heath, 1921: 76-84, Vol. I) becomes somewhat more involved and where the geometry is presented rather differently from my particular representation within **Plate 111** which is intended to integrate the first two categories as Dürer might have seen them, but which may then not be aligned so conveniently with the nomenclature within the standard literature. To avoid any confusion, and to highlight the significance of the early Greek mathematician's use of geometric form, as an additional interpretation and explanation of number grouping and sequencing, I therefore now present the subject a little more clearly.

Consider now **Plate 114**. Firstly, there are the 'triangular' numbers which, as an extension of the Tetractys, can continue indefinitely, each number increasing in magnitude according to the 'n^{th} term' algebraic expression [2 (n + 1) / 2]. The placing, within a geometric frame, of the dots or counters, as individual units, is intended to aid numeric understanding through visual representation, much as a child learns to count manually with counters. It is important to understand that these numbers do not represent measurement or spatial quantification in the same way that numeracy is otherwise applied in geometry. Here, the geometry arises purely out of a unified placement of individual 'points'. These points are to be regarded as representing a *unit of quantity*, not as significant spatial points with relative or specific measurable distances separating them. Geometric form arises here as an incidental, secondary construct. The right-angled frames that include and contain each number grouping, were referred to by the Greeks as *gnomons*, although this word was also used by them in relation to a different application.

[I cannot overemphasise the distinction that is being made here, between (i) the numeracy of geometric structure, where concepts of both number and space are *absolutely integral* and related to each other (indeed incorruptible) and (ii) a geometry that is *applied to* numeric structure. It is the latter that is now being discussed in this particular section, in much the same way that both constructs of mine, *Element 88* **(Plate A1)** and *RNA's degenerate code* **(Plates B4 & B5)**, were designed to express separate but quite complex numerical configurations, these in the form of *idealised* but didactically correct geometric structures (see **Introduction**).]

Secondly, there are the 'square' numbers. Whereas the numbers in the 'triangular' number sequence were separated by the arithmetic series 2, 3, 4, 5..., the numbers in the 'square' number sequence are separated by the arithmetic series 3, 5, 7, 9..., according to the algebraic expression n^2. This is where Dürer might have been further intrigued by 5 being the difference between the 'square' numbers 4 and 9, as 2^2 and 3^2.

Then there are the 'oblong' numbers, so called because the numbers **6, 12, 20, 30**..., derived from [n (n+1)], can be conveniently contained within an expanding oblong grid, generated by the addition of each successive *gnomon* that creates the arithmetic sequence 4, 6, 8, 10... (these numbers derived from 3+1, 4+2, 5+3, 6+4). The 'oblong' numbers reverberate also as (**2 x 3** = 6), (**3 x 4** = 12), (**4 x 5** = 20), (**5 x 6** = 30)... Intriguingly, the numbers **6, 12, 20, 30** might now also be considered as being included within a classification of the Platonic solids (see again **Plate 94**), variously as either numbers of faces, vertices or edges. It would be tempting to assign some sort of significance to this, but it is only a quirk. How easily we might ascribe some *mystical quality* to a number or a sequence, when it is simply *freely disengaged property* that numbers possess within a purely mathematical context; not that mathematics is simple; only the numbers are. [Early reference and basic descriptions of the 'triangular', 'square' and 'oblong' numbers with their *gnomons* are to be found in the fragments of Philolaus (Guthrie, 2010: 98/99).]

3.16 The Kabbalah

The image that is most associated with The Kabbalah is the *Sephiroth*, an example of which is given as **Plate 113**, already introduced at the beginning of the previous section, and also mentioned in my **Introduction** in relation to a more recent visual expression by Anselm Kiefer. There would now seem to be certain parallels between (a) the way the Pythagoreans may have connected numbers to elements outside their pure computational property and (b) the adoption of such connections and resultant 'merging' by Jewish Kabbalah tradition, whose roots are thought by some (Mathers, 2005: 21) to predate the Pythagoreans; this is exemplified by a somewhat more complex association between numbers and now letters, and even other visual or imaginary phenomena. It is unfortunate that much of the literature related to Kabbalah is often confused and contradictory, but it is clear that, working from published early text, various relationships were being made by early commentators and practitioners between (i) Hebrew holy letters, (ii) holy names formed of these letters and (iii) numerical attributions and values to these letters, i.e. *gematria*, as well as (iv) the physical manifestations of deity as described by these linguistic and numeric constructs.

Consider now this passage from Chapter III of a particular Kabbalist book, *SPRA DTzNIOPVTHA* or *The Book of Concealed Mystery*, translated from the Latin in 1887 by Samuel Liddel MacGregor Mathers, this being the first of what he considers to be the three most important books of what he calls the "Zohar":

14. Those nine (paths of Microprosopus) are evolved from the perfect name (that is, from the understanding or mother, in whom they were conceived; for unto her pertaineth the name IHVH, which is Tetragrammaton expressed and Elohim hidden, which form the nine in power). And thence are they planted into the perfect name, like as it is written, Gen. ii. 8: "And IHVH ALHIM planted" (that is, these nine

> letters of the perfect masculine and feminine name, so that they may be a garden – that is, Microprosopus in action).
>
> (Mathers, 2005: 82)

[Microprosopus is the collective term for the nine lesser Sephira below the first. The Macroprosopus constitutes all ten Sephira as the *Sephiroth* or SPIRVTH, with KETHER, the Crown as first Sephira; the other nine are named, in descending order: CHOKHMAH, BINAH, CHESED, GEBURAH, TIPHERETH, NETZACH, HOD, YESOD MALKHUTH.]

In addition to also giving facial imaging some special esoteric significance throughout all three translated books, especially the feature of the beard, there is here, as elsewhere in the text, an attempt to give significance to some sort of mystical relationship between Divinity and the numerical values given to Hebrew letters and to their formations as words. In the quoted case, significance is given to the division of nine into four and five. We came across a similar example of this *gematria* in Chapter 2, Part 2.11 The magic square (**Plates 11a & 11b**), where, in that case, association was made between the first ten letters of the alphabet (as 'divine' letters) and, again, the number nine itself when the first is excluded as symbolic of the singular deity.

In contrast to the above publication by MacGregor Mathers, Gershom Scholem's very truncated précis form of the so-called *Zohar*, its origin thought by him to date back to the Spanish Kabbalists of the 1300s, and entitled *Zohar: The Book of Splendor (Basic Readings from the Kabbalah)*, is short and simple, very prosaic, with only one brief reference being made to divine names, and this in relation to the ten Sephira:

> But when He had created the shape of supernal man, it was to him for a chariot, and on it he descended, to be known by the appellation YHVH, so as to be apprehended by his attributes and in each particular one, to be perceived. Hence it was he caused himself to be named *El, Elohim, Shaddai, Zevaot* and YHVH, of which each was a symbol among men of his several divine attributes, making manifest that the world is upheld by mercy and justice, in accordance with man's deeds.
>
> (Scholem, 1949: 52)

The above reference to the 'chariot' is in keeping with the subject of German Jewish mysticism that Kiefer used as his theme for *Merkaba*, fleetingly referred to in my **Introduction**. However, Scholem's 'basic readings' could mislead the selective reader as to the full contents of the *Zohar* for Charles Poncé in his *Kabbalah: An Introduction and Illumination for the World Today* states that 'the published editions of the *Zohar* occupy some 2,500 pages' (Poncé, 2004: 47) and indicates further the type of themes considered, those that would have included the Kabbalistic subjects detailed by both Mathers, and also Gikatilla who is next to be considered.

The particular image of the *Sephiroth* that I have shown (**Plate 113**), is the title page to the Latin translation of 1516, by Paulus Ricci, of a late medieval Kabbalist treatise, this by Abraham Joseph Gikatilla (1248-after 1304), entitled *Portae Lucis*, or *Gates of Light*, wherein he makes many and various connections between Hebrew words with their apparently significant letter formations, as found within the *Torah*, and the Jewish oral tradition. Gikatilla's treatise is voluminous and here he is intent upon pressing home the usual Kabbalist's understanding of the significance of 'words' within the written *Torah*, this in relation to the affairs of man, but also in relation to an ordered structure. This is exemplified by the following two extracts:

[1.]

> And it is said:
>
> My eyes are fixed on You, O *YHVH ADoNAY*, I seek refuge in you...
>
> (Psalm 141:8)
>
> You also must know that when these two Names are joined together as one, *YHVH* is altered in the way it is vocalised and is pronounced *ELoHIM*, which alludes to the attribute of *BINaH* and that unites with the attribute *MaLCHUT*, which is known as *ADoNAY*.
>
> (Gikatilla, 1994: 153)

[2.]

> For the Tree of Knowledge's essence is the Tree of the Sphere of Knowledge, which is the Middle Line. Just as the Tree of Life is the tree of the Sphere of *BINaH*, so *MaLCHUT* is the Tree of Knowledge. This is encoded in the letters *hay, vav, hay*: first *hay* – *ChaYYiM*; *vav* – the Tree of Life; last *hay* – the Tree of Knowledge. The essence of this is reflected in the communing of those who know the secrets of the heavenly emanations.
>
> (Gikatilla, 1994: 219-220)

In consideration of the above, it is worth looking further at the physical manifestations of the visual *Sephiroth* now in terms of its geometric implications and the numerical proportions contained within, in addition to the significance indicated previously that was given through *Qabalah* to letters and their sequential ordering, being associated also firmly with number quantitatively. [*Qabalah* is synonymous with Kabbalah, the former being more consistent with the Hebrew QBLH (Mathers, 2005: 2).] It is quite possible that Dürer would have been aware of this very particular image – if not this actual one, then some similar other – and of the copious symbolic text that accompanied it, this a number of years before the book was finally published. Considering the printer of this book, Johann Miller of Ausburg, was located not that far from Nuremberg, Dürer may even have been initially approached with a commission for this print, or may have known of its imminent publication, given the circles in which he moved. This being a possible scenario, might it be that Dürer was influenced by such an image and text to codify something of significance within his own title, *Melencolia I,* and actually set it within his print, similar to that which was muted in Chapter 2, Part 2.11 The magic square? However, in this particular case, it might be that he was assigning numerical values to each letter sequentially within the German alphabet, such that the sum of the numbers synonymous with the first five letters of *Melencolia I*, viz. M(13), E(5), L(12), E(5) and N(14), equal exactly the sum of the next five letters plus the added I, as C(3), O(15), L(12), I(9), A(1) and I(9). The sum, for each equal group, is 49 – a number that displays two of Dürer's 'magic' numbers, 4 and 9 – and which itself, coincidentally, but significantly, is also a 'square' number (7^2). There is also another balance within the lettering of MELEN: COLIA I where the numbers of letters used for each side are indicative of the ratio 5:6, which is, significantly, the 'melancholic' musical interval of a minor third. Again, all this is pure conjecture, even invention, but as such beliefs and practices seem to have been rife within non-Catholic circles and probably known to most enquiring humanists, it may be more than an outside possibility.

It is now possible that this print – where Gikatilla is depicted holding and contemplating the *Sephiroth* – may have also inspired Dürer to invent the geometry that underlies his polyhedron. Consider again **Plate 113**. The first and last Sephira may be viewed as representing masculine and feminine polarities. Excluding these two, i.e. Kether and Malkhuth, an interesting geometry arises whereby a *truncated rhombus*, delineated by the now five remaining upper Sephira (Chokhmah, Binah, Chesed, Geburah and Tiphereth), is joined – with the central Sephira (Tiphereth) as the link – to the lower four Sephira (Tiphereth, Netzach, Hod and Yesod) which form as a *perfect rhombus*, with Tiphereth (the central Sephira) being common to both forms. However, in the case of some other images of the *Sephiroth*, where the individual Sephira are not so conveniently aligned, this does not pertain. Consequently, such a theory would have to be viewed as being most tenuous and only holds as a possibility if it was this particular image that Dürer might have seen and consequently been inspired by. It is also possible that, as there are 16 joining pathways, with a bifurcation of the second and third Sephira, this is also another numeric reverberation that might have been carried through intentionally, and subsumed within the 4^2 magic square of *Melencolia I.*

3.17 Further symbolic possibilities

From further evidence of other interesting dating practices that may be strongly related to, or even attributable to Dürer, it is now to be considered whether it may have been the initial influence of one particular person who may indeed have pointed out the significance of the number *four* – as a possible allusion to the Tetractus – and perhaps other related numeric patterns, and who may have been responsible for initiating Dürer's developing interest in the nature and 'possibilities' of numbers and letters.

Before opening his own workshop in Nuremberg in 1495, this immediately upon his return from his first Italian trip, Dürer had been apprenticed to arguably the most important workshop in Nuremberg at that time, that of Michael Wolgemut (1434-1519). [This was a three years apprentideship, starting from 30th November 1486 (Bartrum 2003: 92-93), and given this start date, Dürer's particular association with Wolgemut might conceivably have extended a little beyond 1489 and into 1490.] Here, major commissions were undertaken to provide plates for publications such as the extremely important *Nuremberg Chronicle*, finally completed and published by Anton Koberger in 1493; but there were also designs for stained glass projects undertaken, even through (it would appear) to actual completion in glass (Brown, 1992: 100-101). An example of such a commission is a panel **(Plate 115a)**, dated 1487, that depicts – in very fine detail – Dr Lorenz Tucher, one of a number of individuals portrayed from an important and influential Nuremberg family. [Note that the 1478 portrait of Ursula Tucher **(Plate 115b)** is attributed to Wolgemut and that the embellishment between the names is similar to that which

Dürer used between the word MELENCOLIA and the I in *Melencolia I*.] However, what is important to note about the stained glass panel is the configuration of the number *four*. It therefore has to be strongly considered that this might be the work of Dürer; he was in training with Wolgemut at this very time. If it is not Dürer's work, then probably Wolgemut himself would have been responsible. However, this suggestion may not conform exactly to what is elsewhere stated:

> The changing relationship between designers and craftsmen in the later Middle Ages is revealed by a comparison of the working practices in the Nuremberg workshops of Michael Wolgemut and Veit Hirschvogel the Elder. In the studio of Wolgemut, where Dürer trained as a young man, members of the atelier were involved in every stage of the execution of the workshop's stained-glass commissions. Dürer and his pupils, on the other hand, produced numerous designs for stained glass, but were not involved in their technical execution.
>
> (Brown, 1992: 97)

[In 1499, Dürer was to paint portraits of Hans and Felicitas Tucher, as well as Niclas and Elsbeth Tucher, the compositions reminiscent of portraits that had already been done of family members that are attributed to Wolgemut. In Dürer's portrait of Elsbeth Tucher (**Plate 116**), the meaning of what might be the letters MHI MNSK on the band of her headscarf is most intriguing and where only the letters ?H? MNSK are clearly discernable. Similarly, there is mystery in the strange way her name is written above the date, top right, as ELSPETNICLA TVCHERNZ67L, but where the last two characters are not certain; perhaps it is an embellishment of T at the end: it's difficult to tell.]

What has to be considered next is a later preparatory design, dated 1502 (**Plate 117a**), for another stained glass window (**Plate 118**), being the companion panel to one depicting Dr Sixtus Tucher, the final product executed by the workshop of Viet Hirschvogel the Elder. I would question the attribution of the production of the original design to 'the so-called Benedict Master, an artist in the circle of Albrecht Dürer' (Brown, 1992: 97) as Dürer's tell-tale monogram (lower image, **Plate 117b**) is just to be seen above the EM that ends COLLOCEM. This is also born out by the more recent statement, with reference to the same stained glass window, '... a trefoil window after a design by Albrecht Dürer, attributed to the workshop of

Veit Hirsgovel the Elder, showing Death on Horseback Taking Aim at Provost Dr Sixtus Tucher, dated to 1502' in Issue 39 (April 2010) of Vidimus (http://www.vidimus.org, 15.11.11). Tellingly, 1502 is the date of Dürer the Elder's death, and we clearly see in this drawing the unusual configuration of the *five*; the date does not appear in the glass panel. However, what concerns me, and still brings into question who might actually have been responsible for the drawing, is the fact that, to the best of my knowledge, Dürer does not seem to have utilised, in any of the works that have come to my attention, this particular variant of the *five*, nor, indeed, the Z version of 2 (upper image, **Plate 117b**). [These variants will be discussed towards the end of Part 3.19 <u>Flemish and other possible connections</u> with reference to their written inclusion within various editions of Gregor Reisch's *Margarita Philosophica.*] But we need to go back to Dürer's apprenticeship with Wolgemut to try to piece together some credible theory as to why there are so many anomalies.

There exists a preparatory drawing for the frontispiece to the *Nuremberg Chronicle* **(Plate 119)** that was made at Wolgemut's workshop. The importance of this major work – published by the influential Anton Koberger – has already been indicated, and the initial commission in 1487-8 (Bartrum, 2003: 96) to provide the illustrations to the text for this massive work coincides exactly with Dürer's apprenticeship at Wolgemut's workshop, and so the preparatory work for the frontispiece woodcut may well have been in the planning stage before the 1490 date that is integrated within the preparatory drawing. It is the unusual configuration of the *four* that is interesting. The date is not transferred to the blank tablet within the final copy and realisation of the printed book, this in keeping with an evolving code for the replication of original work and where certain details within the original or preparatory drawings were not included, in recognition of the artist's original work (Wood, 2009: 11-13 [English translation]). This same code, therefore, may account for the previous case of the missing date to the Tucher stained glass window.

3.18 <u>Wolgemut and the *Portrait of a 'Young Man'*</u>

Predating this, there is a portrait of a 'Young Man' **(Plate 120)**, dated 1486, which is reputed to have been painted by Wolgemut, with the *four* again configured in the idiosyncratic way that Dürer himself was to include in the dating of his own self-portrait of 1493 (see again **Plate 10**). Might this 1486 portrait of a 'young man' be of Dürer the young apprentice, and might this then indicate further the possible initial influence and importance of Wolgemut? Indeed it might conceivably be a self-portrait by Dürer himself. [It is noted in both my **Introduction** and **Conclusion** that Detroit Institute of Arts, Michigan, the owners of this portrait, have been unable to give an opinion as to this suggested possibility or indeed to provide corroboration of their own contrary attribution.] There are striking similarities within both portraits, in the composition, and in the placing of the date. Furthermore, Dürer himself was to make a portrait of Wolgemut (**Plate 121a**), and he later annotated it to include the date of his former master's death in 1519, indicating that the portrait was a likeness of Wolgemut in 1516, aged eighty-two. In this annotation (see **Plate 121b**), the *nine* is configured idiosyncratically. Intriguingly, the original date may have been subtly altered, with the 1 between the 5 and 6 within the date having the appearance of a lower case ϋ (see **Plate 121c**). [Unfortunately, this is not obvious, for many images provided by the Internet appear to have been somehow manipulated and enhanced such that the date on many appear as the original 1516, so sight of the actual portrait would be required to corroborate.] If it is the case that the date was altered by Dürer, but it is not clear that it is, might this be an additional hidden symbolism, as a lasting private tribute from former 'pupil' to 'master', for it is possible that the ϋ – the second letter in Dürer's name – was meant to stand for the number *two* within a possible re-dating to1526, and that he might also have considered himself significantly indebt to, and therefore 'second' to, the master and mentor who may have originally introduced him to the esoteric secrets of numeric thinking? This is hugely conjectural, but not beyond the bounds of possibility. [In consideration of the date 1526, it is interesting that Dürer was to include some interesting symbolism within his portrait of Johannes Kleberger of that same date (**Plate 125a**). The symbolism within this portrait will be considered in Part 3.20 Further possible influences.]

3.19 Flemish and other possible connections

At this point, I wish to isolate and concentrate briefly upon the likely source of the unusual configuration of the number 4 within Dürer's possible scheme and where Wolgemut, if indeed it was Wolgemut, might also have been influenced to adopt this feature from the evidence of its having been occasionally used by some earlier Flemish artists. Indeed, it is thought by some commentators that it was Wolgemut, or perhaps Hans Pleydenwurff, who may have been instrumental in the introduction of certain Flemish influences into German art of the period. Be that as it may, there is clear evidence that Dürer used the unusual configuration of the 4 a little before its obvious inclusion in his self-portrait of 1493 **(Plate 10)**. [This portrait has already been mentioned in the previous chapter, and in relation to the number 4.] For example, this same use of the unconventional 4 (this time in its more angular form) can be seen within the 1490 dating of a painting by him of the Dürer family coat of arms **(Plate 122a)** that he placed upon the reverse side of the portrait of his father **(Plate 122b)**. This portrait is thought to partner another that he also made of his mother, and **Plate 122c** is attributed accordingly. Note that the portrait of his mother is not dated, nor does it have his monogram, and that the actual face side of the portrait of his father includes the conventional 4 within the dating, which is placed above his monogram. Perhaps his two other self-portraits should now also be included, if only to note evidence within these also of the continuation of numeric symbolism, although these examples are somewhat far less obvious and pronounced. His self-portrait of 1498 **(Plate 122d)** displays a 9 with a hooked tail, but his 1500 self- portrait **(Plate 122e)** has its 5 only slightly in the form of an 's', much less obviously idiosyncratic than he sometimes forms his 5, as for example in his striking *Portrait of a Young Man* **(Plate 122f)** of that same year.

Away now from Dürer, I wish to consider the portrait of Jacob (H)olbrecht **(Plate 123a)**, a musician who was much celebrated in his time, which is attributed to Hans Memling (c.1430/40-1494), an important Flemish artist, and which has a date upon its frame that is two years after the artist's death. If it is that Memling had not completed this portrait before his death, then it is possible that the later date of 1496 could have been added by the artist who may have completed it. Were this to be the case, might this artist have been Dürer, possibly commissioned through the auspices of Maximilian I, the Holy Roman Emperor, for both artists were connected with him, especially Dürer, who was later to gain his patronage and, importantly, was to paint a most noted portrait of him? This is, of course, highly conjectural and probably unlikely, for it is at a somewhat later date that Maximilian's patronage can be ascribed more definitely to Dürer. However, what is particularly to be considered is the configuration of the 4. This is detailed in the upper image of **Plate 123b**. Again, this same unusual configuration can be seen within a much earlier painting, this by another important Flemish artist, Petrus Christus (1425-1476), that is dated 1449, viz. *A Goldsmith in His Shop, Possibly Saint Eligius* (**Plate 123c**). This date is detailed in the lower image of **Plate 123b**. Here, however, the 4 is far less cursive, as is also the case in Dürer's example of 1490 (**Plate 122a**).

Yet another Flemish example of the cursive style of the variant 4 can most clearly and unambiguously be seen within Bouts' *Portrait of a Man (Jan van Winckle?)* (**Plate 123d**), dated 1462. Dirk Bouts (1415-1475) was another important painter who was also influenced by the much-celebrated Jan van Eyck and Rogier van der Weyden. [Interestingly, a similar 4 can clearly be seen within the date that appears – together with an inscription – upon the frame to van Eyck's *The Madonna and Child* of 1439 (Whinney, 1968: Plate 23a), but it is not known by me whether this frame is contemporary with the painting, nor who might be responsible for the date and inscription.] It is, however, most certainly clear from Dirk Bouts' example, and a little less so from some others, that unusual configurations of the number 4 had, at least occasionally, been used by Flemish or, perhaps, also even other associated artists, this before the evidence of any use by Dürer, or possibly also Wolgemut. However, to be more detailed and definitive about all this would require more protracted research, somewhat outside the scope of this current enquiry. Moreover, the research would need to be extended beyond any Dutch /Flemish influence, for who might have been the artist responsible for the illustration within Gregor Reich's *Margarita Philosophica*, first published in 1503 by Johann Schott (see again **Plate 42**)? Was it Alban Graf, as it is supposed, from the prior commissioning by Johannes Amerbach (Amerbach & Halporn, 2000: 46-50)? The same 'unusual' *four* – here integral within the Lambda numbers placed upon the dress of allegorical Arithmetica – is seen to differ (this within the same illustration) from the 'conventional' *four*, seen upon Boethius's table. [It would now seem that both versions of the *four* were in use in Europe as far back as the 14th century, with the 'unusual' form in use in 13th century France, and even earlier in 12th century England (Woodruff, 1909: 128). See **Plate 123e(i)**.]

Also, most importantly – and in relation to previous comments about the preparatory drawing (**Plate 117**) for its related stained glass window – it has now been discovered that the distinctive Z version of the 2 and the same 'inverted' configuration for the 5 (without its straight line), can be found quite consistently, in written form, within the printed text of the earlier editions of Gregor Reisch's *Margarita Philosophica*, with the written *five* looking somewhat similar to a straight lined 9 at times, together with a written inverted V, visually quite distinct from, but representing the printed 7 (http://www.digital-collections.de/index.html, 08.11.11). Examples of the differences between the written and the printed numbers are given as **Plates 123e(iii)** and **(iv)** from the Friburgi (Fribourg) first edition of 1503. Certain differences between both the printed numbers and hand written inclusions are to be found in further editions, but both the Strasbourg edition of 1504 and the Basileae (Basel) version of 1508 display a written *five* similar to that of the original 1503 edition. Then, in the Argentoratum (Strasbourg) edition of 1508, the Argentin[a]e of 1512, and the Argentine of 1515, the hand written *five* tends now towards something appearing somewhat more similar to a conventional 5 but with the straight line, that would have been above, being missing. The written *five* takes on a very strange angular form in the Basilea of 1517, and by the Basileae of 1535 there are very few inclusions of numerals that are now hand written, with the written *five* appearing more similar to an S but with the 2, when written, still retaining the form of a Z, with the written *seven* appearing now more like the printed 7. [I include also, as **Plates 123e(v)**, **(vi)** and **(vii)**, two interesting woodcut prints from the 1515 edition, these being sub-title plates that relate to subjects within this thesis, with a sample also of how differently music was written then.]

3.20 Further possible influences

Lastly, what has to be considered is the possible influence of Martin Schongauer (c.1450-1491). After completing his apprenticeship with Wolgemut, Dürer went in search of him at Colmar but by the time he got there, Schongauer had died. It would appear that Dürer had admired his work, and it is thought that it may have been Dürer who was responsible for the annotation of some of his drawings and that he may have collected them through Schongauer's brother after the artist's death in the February of 1491 (Rowlands, 1993: 9-11). What is important about these annotations and minor additions is the calligraphic nature of the numbers within the dating. Consider the annotation to Schongauer'ds sketch, **Plate 124a**, presumably in Dürer's own hand. Here, the *nine* has its curved tail. Then in **Plate 124b**, the 1469 date has been added to Schongauer's own monogram, with the tell-tale unusual configuration of both *four* and *nine*. Interestingly, there is a drawing **(Plate 124c)** that may also have been executed by Schongauer (Rowlands, 1993: 10); it has his monogram and is dated 1470. What is significant, when thinking back to Dürer's 1499 portrait of Elsbeth Tucher **(Plate 116)** is Schongauer's inscription on the high priest's hat, the letters ANΓOH SEION. The first word contains the Greek letter Γ, but the word itself is not understood. Considering that one might associate a Greek spelling originally with the translation from Hebrew of religious texts for a Diaspora Jewish audience, and any Latin spelling then associated more with a later and more Catholic audience, might this perhaps be another example of a hidden reference to Kabbalah? The second word corresponds with Zion, but spelt the Greek way, with letters SE replacing the letter Z in Zion, the latter spelling as we would know it in English, but then spelt Seion in Welsh.

As a final example of the possibility that Dürer may indeed have been much influenced by Kabbalah and other occult traditions – particularly the *mystique* of their symbolism – as much as he was knowledgeable about the pure mathematical properties of number and their importance as touchstones in attempting to get closer to the possibility of *absolute truth*, I wish to cite Dürer's portrait of Johannes Kleberger of 1526 **(Plate 125a)**, mentioned only a little earlier. **Plate 125b** isolates and enlarges two elements of symbolism evident within this particular portrait. Consider firstly the symbol that follows the Roman numeral XXXX, a number that indicates the age of the portrayed. In Chapter XXXIII (Of the seals and characters of natural things) within BOOK I of his *Three Books of Occult Philosophy*, Agrippa shows the letters or characters of the *Sun* (Tyson, 2007: 103), and the character shown by Dürer in this portrait of Kleberger may well be a geometric imitation of the very end character shown by Agrippa, albeit

Agrippa's form is far more cursive. What is not conjectural is the clear symbolism of the sign for Leo, which Dürer has highlighted in gold and surrounded by what appear to be stars (see again **Plate 125b**, upper image). [Incidentally, might the symbol for the constellation Leo be what influenced Dürer to portray his number 9 in the manner that he occasionally did, and might this therefore be his own personal reference and tribute to St Jerome – inextricably associated of course with the lion – and a particular subject matter that was to occupy him repeatedly?]

As regards the portrait of Kleberger, the similarity of form here – between the sign for Leo and Dürer's 9 – is surely significant. In occult philosophy, Leo is strongly associated with the sun, or *solis*, and we can see from **Plate 126a** (Tyson, 2007: 774), taken from a 1655 translation of Agrippa's *Of Geomancy* or his *Fourth Book of Occult Philosophy*, the possibility that Dürer may have been making some sort of link here between three astrological elements, viz. the configuration of (a)'*The lesser Fortune*', clearly linked in geomancy to (b) the sign for *solis*, and then (c) that further association with the constellation Leo. That which we may regarded for the moment as the six geomantic points of '*The lesser Fortune*', appear in this portrait as 'stars'. In geomancy, of course, the location of these geometrically configured points are to do with an occult association with *earth*. However, this link will have to be considered as questionable, for although the points * are not perfectly placed in their ideal geometrical locations by Agrippa for what could have been a more uniform grid, Dürer's 'stars' are even less in conformity with a more accurate placement, a greater accuracy that is indicated in my **Plate 126b**. There is a * * * straight alignment to three of the 'stars' for the configuration indicated within the portrait and – as it will be seen in the concluding part on geomancy – this contradicts the *mathematical* principles that underlies the particular patterning in geomancy.

But before I look further into the purely mathematical elements that are implied within and by the 16 geomantic configurations, I wish to point out another interesting reverberation that now, resultantly, applies to the now debated *Portrait of a 'Young Man'* by Wolgemut / (Dürer?). Consider again the form to the dating of this portrait (**Plate 120**). What now becomes clearly noticeable and recognisable is the sign for *solis*, placed between the idiosyncratic configuration of the 4 (the number *four* here is crucial) and the conventional 8. Significantly, in his important table of the planets, as Chapter XXII in Book II of his *Three books of Occult Philosophy*, where here he associates the planets with magic squares and their Hebrew equivalents in letter form, together with their various and varying 'Seals or Characters', Agrippa links the sun with the *fourth* magic square, that of 6^2 (Tyson, 2007: 324). Coincidence? Perhaps. This now makes one wonder whether that elaborate calligraphy, either side of the date within the portrait, also holds some hidden symbolism.

3.21 <u>Geomancy and associated elements</u>

Now to finally consider the more purely mathematical properties of the 16 symbolic 'figures' of geomancy; **Plate 126a** indicates the sixteen 'figures' as being within a pattern or order that satisfies the classification requirement of geomancy's rather complex matrix of *astrological processing* (Tyson, 2007: 773-784), a subject and process that is beyond this current enquiry and where there are systems other that that outlined by Agrippa to be considered also. However, if we look at the actual geometric pattern that pertains for each 'figure' within this set of symbols, the *mathematical order* becomes clear. See now my **Plate 126b**. There is nothing random or divinatory here. Each 'figure' constitutes 4 'rows', each triple cell 'row' conforming to one of two mathematical patterns, viz. |X|O|X| or |O|X|O|, where X represents the star-like * of geomancy and O represents its absence. It can now be seen that each of the sixteen 'figures' are vertically symmetrical, whereas additional lateral symmetry pertains only in the top four 'figures' of my novel lay-out. As with Greek 'triangular', 'square' and 'oblong' number sequences that were discussed previously, the geometry is necessary only to illustrate the mathematical configurations. The linear geometry is not quantitatively significant, as it would be in Euclidian geometry; what is relevant is the significance of the symmetrical arrangement of 'units' within their triple base sets as 'rows' – 'units' expressed by the presence or absence of *, i.e. X or O respectively in my plan – and how these triple base sets (or 'rows') relate in vertical sequence to each other within their individual geometric forms, or 'figures'. Note the six pairings, pairs that indicate a reversal of pattern. Only 16 configurations are possible, i.e. 2^4, from the formula 2^n, where 'n' denotes the number of 'rows', with the *two* denoting the number of the possible configurations within each 'row', viz. |X|O|X| or |O|X|O|. Were there to be six 'rows', there would be 64 possible configurations or 'figures', again from the same formula 2^n, in this case equalling 2^6, or 64.

With this in mind, there are echoes here with the mathematical patterning of the hexagrams within the *I Ching*, although the inclusion here of other and various mathematical stages within its operation towards the selection of one out of 64 possible outcomes differs considerably. In the *I Ching*, each outcome has its accompanying text (Wilhelm, 1950), which may or may not have a secondary stage. This is dependant upon the primary selection process in working towards a binary outcome for their individual triple cells, this being a little more mathematically complex, where there are 8 possible initial outcomes here because there is no restriction to the *chance order* within the combination, such that each random triple combination is then simplified to the visual binary form of either – – or – (synonymous with *yin* and *yang* respectively) and whose initial triplet combination may be additionally indicated by either X (*yin*) or O (*yang*) in going on to the second stage for three 'heads' and three 'tails' in a row when using the simpler method of flipping a coin.

Ignoring then this initial process in getting to the binary form, there are 64^2 (i.e. 4096) possible secondary outcomes to these 64 hexagrams. This does not correlate with the initial limit of 16 patterns or 'figures' in geomancy whose subsequent strictly mathematically processed ranking arrangement of new 'figures' is created out of the initial selection, by chance, of only four 'figures' by assigning to each row the differentiation of either * or ** (these symbolising even and odd numbers of randomly created marks in the ground) but where there is a greater complexity of process and cross referencing inherent thereafter, involving the twelve 'houses' and seven 'planets' of astrology, all of which is then associated with accompanying astrological texts. As there are16 different possible computations to each of the first four 'figures' of chance, the number of possible outcomes, ready for the next stage, has therefore to be 16^4, i.e. 65536, as a new set of four 'figures' is created out of the first group, this next process not through chance, but by mathematical design. I shall not dwell upon the next mathematical and astrological stages of geomancy as the complexity of outcome becomes more unmanageable. None of these complexities is present in the *I Ching*.

Be this as it may, it is the clean and subtle mathematical reverberations, inherent within the various stages of the *I Ching*, which is the subject of my final 3D construct. However, in the final exhibition, this construct is displayed in conjunction with its text, indicating a direct and integral link here between the mathematical and the literary, that which is pure *ratio* with what is *freely interpretive* and which is clearly contingent upon chance. This indication is in contradistinction to the otherwise unresolved relationship between, on the one hand, the pure detachment of *ratio* and, on the other, the internal drive of a personal morality, which would seem to be born of the interaction between pre-determined biological programming and the influence of external contingency, largely in the form of other people's *beliefs* or behaviour, a major theme that runs throughout this thesis. Photographs of my novel 3D representation of the *I Ching* arrangement, entitled *Newidiadau / Changes*, is included with other photographs of my exhibition and is attached to Volume II. [See **Plates E24** and **E25**.]

Conclusion

This thesis set out to attempt to answer some quite broad philosophical questions within the truth paradigm and also to answer some very particular, more directly focused questions of a more numeric and geometric nature, this in relation to a number of specific examples of mathematical expressions within the domain of artistic representation. Indeed, the many mathematical analyses within this investigation have been central to an understanding of how some polymath artists may have attempted to subsume within their work a similar indication of their understanding of matters that are outside the normal compass of artistic expression.

I shall deal firstly with the more specific issues before coming to a broader philosophical conclusion.

One of the main intents of this investigative thesis has been to unravel the mystery surrounding the exact mathematical constitution and symbolic meaning of the polyhedron within Dürer's *Melencolia I*, this acting also as a connecting bridge to other related themes. I have made a number of detailed proposals to this end, having investigated the standard and then more technically specific theoretical models that have been proposed by others, this in relation to both the wider cultural and historical background to Dürer's print, and also with regard to any indications of the possible influence that thinkers and practitioners of his age with whom he would have had contact might have had upon his individual thinking and beliefs. However, a definitive and undisputed answer to the question as to what the exact mathematical constitution of the polyhedron that Dürer may have modeled and subsequently drawn will have to remain largely unresolved for, although I have been able to point out flaws within the non-empirical approach of other investigations that have led to competing theoretical models, I do not lay claim to a definitive answer to this question myself.

Given the nature of the subject, my own theoretical models are based upon a combination of strictly mathematical and loosely mystical theory, the latter based upon Dürer's possible mindset, accompanied and corroborated by experimental modeling. Irrespective of any potential merits there may be in the background thinking to my two innovative models, they are strictly theoretical and therefore there cannot be any certainty, and where the one relies upon a personal reading of an assumed harmonic resonance (the 5:4 within 9), possibly supported by historical narrative and the unusual calligraphy within the dating of certain works by Dürer. Indeed, in using the method of observation through and drawing upon glass that Dürer would probably have used, such a final empirical test – as would be desirable – introduces an additional element of uncertainty, through potential distortion and approximation before and in transfer of the image to plate, for we do not know how perfect the glass was that he might have used, or the angle of the polyhedron base plane to glass. In addition, two of his inscribed lines are not perfectly straight, thus an absolutely definitive answer is not possible, but – and assuming a 90° angle of base to glass – the viewing of each of my three-dimensional models through the superimposition on to glass of a clear tracing taken from Dürer's final print, does indicate a tantalizingly close approximation to Dürer's model in outline, but the match is not perfect enough for me to lay claim to an undisputed exact replication. Indeed, this is in keeping with my general view that too many claims to accuracy and truth are made – and this (to me) is not acceptable in science – that are based upon conjecture from *a priori* belief and stem from a certain confidence in an individual's own abilities to convince and be convinced, claims that are then furthered through their becoming generally accepted as 'facts' within particular disciplines, as if it were possible to rely upon our uncertain faculties of perception and intellectual function to lay down such claims, especially when unsupported by empirical evidence, itself – by definition – questionable as being outside the domain of absolute ontological truth. This is particularly the case in theoretical physics where, for example, such sweeping statements as 'the *big bang* happened 13.7 billion years ago' are made.

However, for me, much has been gained through this enquiry, not least the possibility that I may have discovered some important misattributions made by other art academics and, in particular, that the 1486 *Portrait of a 'Young Man'* (**Plate 120**), purported to be by Michael Wolgemut, may well be of or by Dürer or even (more importantly) a hitherto missed and therefore unknown first self portrait in oil by the young apprenticed Dürer. Had I not been pursuing the possible significance of certain numerical relationships that Dürer may have been influenced by, this would not have come to light. Unfortunately, Detroit Institute of Arts, who own this painting, remain non-committal, not offering any opinion whatsoever further to my follow up e-mail (geraintdavies37@hotmail.com to jmeekhof@dia.org, 19.07.2011; their reply 22.07.2011). A follow up e-mail to Sarah Brown, on the subject of other attributions and my theories regarding Dürer's unusual configurations to the numbers in his datings, again received no reply (geraintdavies37@hotmail.com to sarah.brown@york.ac.uk, 19.07.2011). Original correspondences are referred to in my Introduction.

In the furtherance of gaining a better understanding of the numerical relationships associated with patterning and total space filling properties within the plane of the two-dimensional, and then of three-dimensional space, the information gathered from an initial study of Islamic patterning has been unexpectedly revealing, both mathematically and culturally. From the point of view of the duality within Islam, the expressions of a different truth knowledge (mathematical and consistent) through its geometric art that is in direct contradiction to its liturgical teachings (historical and linguistic), only goes to highlight that distinction between that which is completely unchangeable, explicable only through quantifiable logic, and that which is biologically complex and less well defined, being changeable and variably reactive to the random elements of chance, but which still has at its core that duality of function and organization whose most basic and absolute foundations must lie at the very opposite pole to the evolution of organic complexity. Dürer's compelling *Christ among the Doctors* of 1506 **(Plate 127)** has been included as an expression of the human condition that needs no further explanation, where the innocence and goodness of Christ stands in opposition to the power of biological determinism and cynicism, exemplified here by the scribes in taking historical teaching as their tools of justification, and then later by the cruelty of the members of the Sanhedrin in sealing his fate. In the final analysis, it is that underlying adversarial simplicity, together with the complex mechanisms of biological evolution, that hold the answers to man's behaviour. No small wonder that – on the one hand – man's DNA holds at its core such a simple mathematical code, but that – on the other – the teleological drive of the trans generational syndrome of individual behaviour cannot be accounted for by this, only by a complexity that is mirrored in the behaviour of the collective flock. As a final reference to this, I include the impressive 1883 portrait by John Collier of Charles Darwin (**Plate 128**) whose individual contribution to a further understanding of biological determinism is pivotal and historically so important.

Exhibition Photographs

Note:

The following list is essentially descriptive, where these photographs are provided primarily as a brief and selective record of – acting also as a visual accompaniment to my report upon – the Exhibition; it is not intended as an academic reference. However, comprehensive and full referencing of individual books and documents that are incidentally cited here (as being 'within' the photographs) is to be found within the report itself. Professional photographs taken by Paul Duerinckx are acknowledged and indicated by **Plate***. Those which were taken by the author, and on one occasion by Joanna Porter, are otherwise indicated by **[Plate]** and **{Plate}**, respectively.

Plate* E1	Table 1 (introductory), with a Welsh Family Bible, upon which is placed a mathematical construct comprising a single two-tone coloured rhombic dodecahedron upon six conjoined plain dodecahedra. A quantity of bilingual background information sheets about each assemblage is also to be found here.
[Plate] E2	A closer view of the above features.
Plate* E3	Isolation of the combined feature that is a single, two-tone coloured rhombic dodecahedron placed upon six conjoined plain dodecahedra.
Plate* E4	Two different views of the same rhombic dodecahedron (with its integral tetrahedron) to indicate the phenomenon of visual perceptive creativity, i.e. where a cubic shape may be implied but where it is not present in actuality.
Plate* E5	The visual impression produced by this view of the six conjoined rhombic dodecahedra is indicative of (and could be viewed as a 2D reference to) Dürer's magic square.
Plate* E6	Table 2, incorporating two thematic titles, viz. 1. *Genesis* (on the left) and 2. *Development* (middle and right).
[Plate] E7	The Hebrew Bible, within theme 1. *Genesis.*

Plate* E8 The tetrahedron and elements of truncation.

Plate* E9 A view that shows the 'English Bible' within theme 2. *Development*; also showing the tetrahedra of theme 1. *Genesis* in the background.

Plate* E10 Placed upon individual cubes – and in order from right to left – are the octahedron, the cuboctahedron and the rhombic dodecahedron.

Plate* E11 Showing a different orientation at the centre – and now in order from left to right – are the octahedron, the cuboctahedron and the rhombic dodecahedron again, similarly placed upon their related cubes.

Plate* E12 'Net' of the rhombic dodecahedron.

Plate* E13 Table and title 3. *Appropriation*, where the Qur'an is surrounded by the mathematical theme of five-fold symmetry. The coloured construct that is included within this assemblage (and also shown separately in **[Plate] E15)** constitutes a single *obtuse* golden rhombic hexahedron (in its skeletal form), in combination with two *acute* golden rhombic hexahedra (in solid form), to show how the complementary dihedral angles make possible the build up of a complete rhombic triacontahedron or, indeed, other more random aperiodic 3D shapes.

Plate* E14a From left to right – the icosahedron, dodecahedron, icosidodecahedron and rhombic triacontahedron.

Plate* E14b Again, as above.

[Plate] E15 (above) – the combination of one skeletal *obtuse* rhombic hexahedron with two solid *acute* rhombic hexahedra as a demonstration of total space filling and aperiodic patterning in 3D, and in relation to:
(below) – the rhombic triacontahedron (on the left) which can be formed from a combination of the above. [The golden proportion rectangle, painted green upon the 2D face of the rhobohedron, is subsumed within and integral to this geometry, as it is also subsumed within other 3D pentagonal geometries. See next photograph.]

Plate* E16 Showing (on the left) how the 12 corners of three golden section rectangles, set orthogonally, coencide with the 12 apices of the icosahedron (on the right).

Plate* E17 Table and title 4. *Mystery.* Various objects and literature associated with Dürer and his enigmatic polyhedron.

Plate* E18 The geometric theme, accentuated by my own two theoretical models, set on grey bases.

[Plate] E19 The red outline upon the acetate sheet, which can be used for testing both models, can just be seen here; seen here also is the facsimile of a page from Erasmus's Greek and Latin New Testament (bottom right).

[Plate] E20 My 80° theoretical model.

[Plate] E21 My combined 72°/90° theoretical model.

Plate* E22 A template that indicates the coincidental 63½° 2D angle within Dürer's print (refer back to **Plate 41b**) and which may have been intended as a possible reference to the very same angle for the rhombic faces that constitute the 3D golden rhombic hexahedron (*acute* version shown here).

Plate* E23 The 80° angle for the 3D polyhedron itself.

Plate* E24 Table and title 5. *Eastern Vision.* The *I Ching* theme, with my novel 3D construct, and a violin scroll in relation to the original block of wood from which it is being formed.

[Plate] E25 Novel 3D construct showing a random example of a hexagram arrangement.

[Plate] E26 The violin scroll and peg box in the process of their formation.

Plate* E27 A photograph by me of the above in relation to the logarithmic 5/4 spiral, together with a photograph of the beginning stages of assembling an eventual rhombic hexecontahedron (see **Plates 103 & 104** again) by use of 30 golden rhombic hexahedra (*acute*).

Plate* E28 My natural photographs (reproduced courtesy of Peter Finnemore) of the developmental stages in the construction of some of my mathematical models.

Plate* E29 A selected view of the above.

Plate E29x	An original photograph of mine that was taken in the developmental process, and selected here from the display.
Plate* E30	A selected view of some of my developmental work, pinned onto two wall mounted boards.
Plate* E31	Table and title 6. *Transparency.*
Plate* E32	A closer view of the above.
Plate* E33	Eight stellations attached to an internal icosahedron, whose points correspond with the eight corners of the golden section related cube.
Plate* E34	The dodecahedron in golden section relation to the three scales of cube within the assemblage (see again **Plate* E32)** but with only two arranged here, all the various sized cubes being related to each other in golden section proportion.
[Plate] E35	Another selected and partial view, placing the assemblage in a different context of the exhibition space and to include also the related icosahedron.
[Plate] E36	Another contextual view of the above.
[Plate] E37	Table and title 6. *Transparency*, in relation to the exhibition space and to Table1 (to be seen in the background) as the introduction to the exhibition.
[Plate] E38	A sample of the templates used in the construction process and placed here at random upon the lower shelf of the draftsman's table.
{Plate} E39	A view of the exhibition space, looking towards the photographic section.
{Plate} E40	A view towards the back wall.
Plate* E41	A view from the photographic section.
Plate* E42	A view of the paintings on the back wall.
Plate* E43	*The Emergence of Phi.* [Acrylic on canvas, 70 x 60 cm.]
Plate* E44	*The Consolidation of Phi.* [Acrylic on canvas, 70 x 60 cm.]
Plate* E45	*Fundamental Resonances.* [Oil on canvas, 20 x 30 ins.]
[Plate] E46	A mock-up of a proposed 3D version on the same theme as the latter, viz. *Fundamental Resonances.*

Codicil

Plate F1	'Net' of the solid golden rhombic hexahedron (*acute*).
Plate F2	'Net' of skeletal rhombic dodecahedron, with the two relevant templates.
Plate F3	First stages of assembling the rhombic triacontahedron (skeletal version).
Plate F4	Staged assembly of the solid icosidodecahedron and its 'dual', the rhombic triacontahedron (the latter already shown in **Plate F3**).
Plate F5	Additional assembly to **Plate F3**.
Plate F6	Side view to the same as **Plate F5**.
Plate F7	Further addition to the solid icosidodecahedron, shown in relation to the completed solid icosahedron and dodecahedron and its partially completed dual, the skeletal rhombic triacontahedron.
Plate F8	Completed skeletal rhombic triacontahedron, in relation to one of the first stages in the development and construction of the rhombic hexecontahedron, formed of the fusion of 60 acute golden rhombic hexahedra.

Exhibition Report

PhD Exhibition

Swansea Grand Theatre, Arts Wing, Level 2

10^{th} – 27^{th} January 2012

This exhibition was targeted at the more thoughtful; it was carefully designed and set up as a comprehensive display, to include the many and various artefacts produced in the development of this thesis and as a further visual and 'actual' three-dimensional realisation of those particulars which (by nature of the medium) had otherwise to be represented two-dimensionally within this document. That which is a two-dimensional representation of an *idea* in its own right (as distinct from an *idea* subsumed within, or represented by, the three-dimensional) is presented through my paintings. [See **Plates E40** through to **E45**.] These paintings may be considered at many intellectual levels, dependent upon interest in, and knowledge of, mathematics and structure – where it may then be possible to be lead to certain philosophical realisations – and which also incorporate essential elements of perceptive 'shifts'. Contrastingly, my natural photographs (see **Plates E27**, **E28** and **E29**, reproduced for this exhibition courtesy of Peter Finnemore) of the developmental stages in the construction of some of my mathematical models (photographs taken in my domestic and essential working environment as evidence and record of process and direct contingency) were displayed and positioned on the opposite wall to the framing of a facsimile of Dürer's print, the latter being a very high resolution digital print (© Trusties of the British Museum) and reproduced (courtesy again of Peter Finnemore) to the actual size of the original, to create further resonance within the exhibition, and as a further direct appreciation of the difficulty in understanding or imagining the exact nature of unknown or unfamiliar and complex three-dimensional forms (displayed actuarially on the tables for direct comparison) from their two-dimensional representation.

[There are further interesting optical anomalies that could also have been discussed in my thesis, but which are beyond its present scope, such as the difference in function of the eye's pliable but fixed position lens compared with the variable positioning of the 'hard' camera lens/lenses and whereby the latter inevitably leads to different and differing distortional factors. Most crucially, there is also another important matter which has to be considered, that of the relationship between the viewing distance in relation to the size of the object which is to be replicated. It is therefore impossible to gain empirical *photographic* proof of any theoretical model of Dürer's polyhedron that could be regarded as evidentially unequivocal, as is the clear case with **Plates E20** and **E21**.]

In addition, this exhibition was intended to function as dual-purpose: (i) as an interactive engagement with the general public – and as such it was extremely successful, especially in consideration of the in-depth discussions that I had with a number of very interested and well informed scientists – and (ii) to provide an opportunity where examiners of this thesis could enquire about and discuss any particular issues they may have had with understanding the geometry, etc. and (most importantly, and essentially) where I could further provide quite comprehensive evidence (but where it was impossible to include all) of *the practice element and its relevance within the thesis development*, thus to enable a necessarily robust evaluation for the award of a *practice-based* PhD.

In essence then, there were two elements that ran concurrently through and within the exhibition, as a reflection of – and reference to – the dual theme within the thesis: that of (A.) the literary, and (B.) the mathematical.

[Clearly, such a large and expansively themed exposition could not effectively be displayed in a more minimal, perhaps more contemporary mode; but then this would not have been appropriate as a 'minimalist' statement was not being made here. Indeed, a subsequent re-setting in contemporary mode was temporarily made to tests such efficacy, but this only corroborated its undesirability and to confirm its lesser impact with no comparable inquisitiveness being shown and clearly no potential being able to be offered for the viewer to engage further.]

Thus, for this particular exhibition, the two themes – the literary and the mathematical, that also involve the important element of historicism and visceral engagement with contingency – were given further symbolism, and referenced by their being displayed upon a variety of different period tables as a very particular and personal engagement. This further contrasted with the open architecture of the building, a modern adaptation to an Art Deco and somewhat industrial structure. A brief background information sheet was provided and I, as the artist, was present at all times should there be the indication of a desire to enter into dialogue. The exhibition was intended as much more than a visual exploration. There was here a tacit invitation to journey further. This element of voluntary, mutual engagement was an important interactive element of the exhibition and it is of particular delight to me that so many members of the viewing public did engaged with me, some at great length, and many towards a gratifyingly deep level of understanding, a clear vindication of the unique style and content of the comprehensive display that was provided.

Here now is a short summary of that which was provided for the exhibition, where further and some additional commentary is also provided within the list of photographic plates that apply to this section:

Table 1 [See **Plates E1** and **E2**.]

An introduction to the exhibition

A. Literary – a Welsh Family Bible, viz. *Bibl yr Addoliad Teuluaidd,* 1875, Y Parch Peter Williams translation.

B. Mathematical – a single, two-tone coloured rhombic dodecahedron upon six conjoined plain dodecahedra. [**Plate E4** indicates the many and various perceptive anomalies involved with the viewing of the singular rhombic dodecahedron, and also in multiple combination for this construct in **Plates E3 & E5**.]

This table introduces succinctly these two contrasting themes, in that the setting of the Family Bible, placed upon the tablecloth, references the historical and the contingent; a domestic setting that is evocative of Welsh Non-conformism where, as I remember from childhood, the gentleness of Christ's morality and teaching that was promoted for the children contrasted vividly with the 'hell and damnation' judgment and warning separately issued – and most dramatically – to the adults from the pulpit on high. In contrast, the geometric structure invites the viewer to contemplate it as a pure mathematical structure. However, it could also be viewed as a reference to elements within the first chapter of the thesis, that which deals with that attempt by various past philosophers to prove the existence of a Biblical God through citing geometric 'truths'.

Table 2 (incorporates two thematic titles) [See **Plate E6**.]

Title 1: *Genesis* [See **Plate E7**.]

A. Literary – Hebrew Bible, 1931, printed by Preussische Druckerel-und Verlags-A.-G., Berlin.

B. Mathematical – tetrahedron and elements of truncation. [See **Plate E8**.]

The reference here is very clear, with the TaNaKH in Hebrew indicating the historical beginnings of the development of the major Western religious movements and their promotion through language. Similarly, the root of three-dimensional Euclidian geometry is indicated by the unique duality of the tetrahedron, with its 'dual' being itself.

Title 2: *Development* [See **Plate E9**.]

A. Literary – *The Holy Bible: Revised Standard Version* (Division, 1973).

B. Mathematical – development from the 'duals' that are the cube and octahedron, through their truncation, to the rhombic dodecahedron as a 'dual' of the cuboctahedron. [See **Plates E10** and **E11**.]

Here is indicated, as the title suggests, the historical development of Christian religion as a direct lineage from its development out of essentially Judaic history, religion and politics, to its present-day linguistic version. The mathematical indicates the development towards the total space-filling rhombic dodecahedron, such properties already having been introduced in **Table 1**. The assemblage also includes an example of a two-dimensional 'net' arrangement [see **Plate E12**] that constitutes the rhombic dodecahedron (solid version).

Table 3 [See **Plate E13**.]

<u>Title 3</u>: *Appropriation*

A. Literary – *The Noble Qur'an*. (Al-Hilâlî, 1420 A.H.)

B. Mathematical – (i) the furtherance of five-fold symmetry (3D), from the 'duals' dodecahedron and icosahedron, via the icosidodecahedron, through to the rhombic triacontahedron [see **Plates E14a** and **E14b**]; (ii) a development of further complex geometries through the fusion of the core *acute* rhombic hexahedron and also its truncation [see **Plate E29x**, as photographed by me in its developmental stage], and then its relation to its complement, the *obtuse* rhombic hexahedron which, in combination, can make up the form of the rhombic triacontahedron or, indeed any other aperiodic 3D pentagonal patterning [see **Plate E15**]; (iii) the 2D basic templates of Islamic *girih* patterning. [Examples of complex *girih* patterning were displayed amongst other drawings on wooden wall-panels. See background in **Plate E30**.] Additionally, when golden section rectangles are set orthogonally to form a three-dimensional construct, their corners can clearly be seen to correspond with the apices of the icosahedron. [See **Plate E16**.]

The title here suggests how it is that Islam has appropriated and assimilated into their religion the historicism of Judaism and Christianity, at the same time indicating 'hell and damnation' as a punishment to those who do not then adhere to the singular teaching of the Qur'ân. Interestingly, whilst adhering strictly to an unaltered and therefore un-evolved historical literature (but then interpreting that literature variously), Islam then seems to celebrate a beauty and a universal truth that is not part of their religious literature but which can be found in geometric and particularly pentagonal (or five-fold) symmetry, such properties being manifest in the many decorations of their holy buildings. Such complementary notions within the same culture should, however, not necessarily lead to unquestioned credence of their literary legacy.

Table 4 [Plate E17]

Title 4: *Mystery*

A. Literary – (i) facsimile of a page from Erasmus's Greek and Latin New Testament [fol. a 1^r, Gospel of Matthew; Novvm instrument[m] omre. Edited, translated and annotated by Erasmus of Rotterdam., 2 pts. Basel: Johan Froben, March 1516. URL]; (ii) facsimile of a page from Luther's translation of the above New Testament. [2nd edition (1529) of his 1522 translation. URL]

A/B. Literary/Mathematical – (i) facsimiles of Dürer's publications, viz. *De Symmetria Partium in Rectis Humanorium Corporum* and *Underweysung der Messung* (Dürer, 2003); (ii) *Chambers's Six-figure Mathematical Tables* (Comrie, 1963) and (iii) *Euclid's Elements* (Simson, 1827).

B. The Purely Mathematical – (i) six different experimental models of Dürer's enigmatic polyhedron; (ii) two novel theoretical models of mine on grey bases. [See **Plates E18, E20** and **E21**.] These can be tested empirically by looking through the acetate sheet – having folded the latter away from the replication of Dürer's print – and comparing the red outline with the configuration of the objects. [See **Plate E19**.]

As the title suggests, this table – being directly in front of the print of *MELENCOLIA I* – references the central focus of the thesis, the enigmatic polyhedron, and where the significance of all the above is self-explanatory. Included also among the geometrical objects is a plywood template to replicate the 80° and 63½° two-dimensional angles of the polyhedron within the print, with the golden rhombic hexahedron (with its 63½° acute angle) so placed as to indicate the significance of this coincidence (or possible indicatory design). [See **Plates E22 & E23**.]

Table 5 [Plate E24]

Title 5: *Eastern Vision*

A. Literary – *The I Ching or Book of Changes* (Wilhelm, 1950)

B. Mathematical – a novel 3D interpretation of the different possible configurations. [See **Plate E25**.]

C. Sculptural – a violin scroll and peg box in the process of their formation. [See **Plate E26**.] This functions as a visual tie to the photographic display [see **Plate E27**] and further references the complexity of logarithmic/gnomic spirals; drawings of specific spirals and details of their mathematical properties are exhibited elsewhere in the exhibition, together with actual samples of close resemblances from Nature, in the form of shells and fossils.

Table 6 [Plate E31, and then through to **E38**.]

Title: *Transparency*

B. Mathematical (only) – a linear series of constructs to show the Golden Section relationship between various related solids, in terms of both scale and form, and particularly in relation to the cube.

Clearly, the placing of this assemblage upon a draftsman's table, without any reference to the literary or liturgical, is indicative of the separation that I make between (i) the pure order and clarity that is apparent and *transparent* within geometry and its underlying numeracy, and (ii) the chaos and discord of human affairs, played out so tragically and fatally through his confrontational politics and 'beliefs'. **Plate E37** and **E39** show this assemblage in the context of the exhibition space, with **Plate E38** showing some of the templates used in the construction of forms within the exhibition, a topic that is touched upon, briefly, in my Codicil. The icosahedron seen within **Plates E35** to **E37**, with 12 of its open faces covered by a variety of colour filters, was to be developed (and will now be made at a larger scale as a future project) as a 3D symbolic version of the numerical balances to be found within my 2D *Fundamental Resonances* **(Plate E45)**, relating the resonances between the tetrahedron, the octahedron and the icosahederon in an integral ratio of (4:**4**):(6:**8**):(12:**20**). [See scale mock-up as **Plate E46**.]

Codicil

1. Clarification

A question that I am sometimes asked by those who may initially be unfamiliar with my work, my thoughts and (particularly) my writings, is whether or not my visual and practical expressions are intended to be – or indeed are – Minimalist and/or Constructivist; a hugely open question and a subject that encompasses a quite vast and separate field of individual and collective practice and theory, somewhat beyond the central focus and wider locus of the stated *subject* – and declared *object* – of this practice based thesis. Be that as it may, although it may otherwise be natural that my own visual expressions should be compared by others with – and therefore thought somehow to relate to – the work of such noted schools or movements, from *my* perspective and stand point, I would clearly have to answer in the negative, in that I do not conform with or adhere to any of their various manifestos, and therefore do not consciously work in any recognized manner other than that which is my own. Clearly, some sort of subliminal influence may be unavoidable and it is possible that some elements in my work may be a reflection of this inevitability. Certainly a number of individual artists within these various loosely-termed movements, many of which are related by influence, or have certain cross-over elements, have interested me greatly; but then so have many other artists and movements whose methods, expressions and subject matter do not compare in any recognizable manner whatsoever with Minimalism, Constructivism or other related movements.

Joseph Albers, for example, would be a case in point, for one has only to look at the paintings in my exhibition, and immediately the unavoidable phenomenon of the contiguous colour effect manifests itself and where this, in combination with other perceptive effects, becomes an integral and important part of the viewing experience. But then Albers' actual art practice and intent differs completely from mine, although the influence of his work on colour theory, disseminated through his *Interaction of Color*

(Albers, 2006), remains profound. My uncompromising geometric patterns have an underlying purpose towards the expression of a pure mathematical/philosophical understanding and may therefore be regarded as having an additional didactic potential. The aesthetic then, although carefully and judiciously engineered, is a by-product of the perceptive process as it engages with the subject matter – which remains totally independent of all such effects – as pure concept, and therefore not to be confused with some of those visually less complex and more random elements that were then taken forward by, for example, Ellsworth Kelly. The influence may be there, but the work is totally different. Therefore, much as I admire much that came out of the Bauhaus and other related movements, the purpose behind my work is totally different, although perhaps certain principles remain common, such as (for me) the distillation through the process of induction (that may be manifest also in others) and the greater clarity of visual organization, sometimes regarded, erroneously, as simplicity.

The same can be said in relation to the *De Stijl* movement in that here, again, the visual aspects are important. For me, it is the pure *numerical* truth that is manifest in the mathematics of geometric function – but which is also expressive within Nature – that is the subject, not necessarily the organization of geometric shapes to produce an artistic effect although, clearly, that is most desirable and then associated with visual perception as part of the aesthetic. As an example of this distinction, Mondrian would be a case in point, where his development towards geometric expression from the organic subject of nature can be traced from his *Red Tree* of 1908, through his *Apple Tree in Bloom* of 1912, then to his *Composition* of 1913 (Fuchs, 1978: 175-186), before he settled on his purely geometric 'compositions' but with this being quite distinct from any expression of purely analytical geometry per se.

Again, the promotion of the geometrical by the Supremacists would be relevant to this discourse, particularly the symbolic and emblematic importance that Malevich placed upon the square; but *I* seek to divorce such forms from 'the personal' – a total separation (as objectivity) from the expression of feelings, contrary to that which it was declared by many as an intent within the Supremacist movement, as it was also for Kandinsky (but differently expressed in his manifesto, *Concerning the Spiritual in Art* (Kandinsky, 1994)) – to make only expressions of adherence to analytical truth by allowing geometry its detachment from such references although, granted, my geometry was placed vis-à-vis the literary and liturgical in my exhibition, but this only as an expression of *distinction from* such contingent matters, and solely in the context of this particular thesis. The square *I* would now view as the first of three particular rhombi, whereby the sine of the angle constituting the first is 1, the tangent of the second is 2 and the secant of the third is 3, all these in relation to the three-dimensional. El Lissitzky, however, was to harvest the didactic force and potential of the geometric form and place it in a context that was state political. My purpose is totally apolitical. Tatlin, again was highly political, and in his *Thesis of 1919* he states:

> 6) Invention is always the working out of impulses and desires of the collective* and not of the individual. (* By collective Tatlin means a society which first and foremost is new and building socialism.)
>
> (Zhadova, 1984: 238)

There is also the historical influence of those such as Naum Gabo, whose *Realist Manifesto* of 1920 (in conjunction with Antoine Pevsner) again contained a political intent, but where his advanced mathematical input – as it was also for Barbara Hepworth – has more to do with truly profound artistic expression (as through the representation of three-dimensional structure that is wholly inspirational) than it has to do with the mathematics of more fundamental proportional numerical substrates, which is the somewhat less mathematically complex (but perhaps philosophically more accessible) subject of my own expressions. This is what Naum Gabo then had to say in 1937 – note particularly what he has to say about philosophy – within his *The Constructive Idea in Art*, and where I sometimes interject within the text, using square brackets [ ]:

> The force of Science lies in its authoritative reason. The force of Art lies in its immediate influence on human psychology and in its active contagiousness. Being a creation of Man it re-creates Man [but can also express that which is purely objective and outside the domain of Man]. Art has no need of philosophical arguments [clearly not all Art], it does not follow the signposts of

> philosophical systems [again, not all Art does]; Art, like life, dictates systems to philosophy. It is not concerned with the meditation about what is and how it came to be [but these are the very investigations of my art]. That is a task for Knowledge. Knowledge is born of the desire to know, Art derives from the necessity to communicate and to announce [or it can be a pure expression of understanding or realization, and not *necessarily* produced for an external audience]. The stimulus of Science is the deficiency of our knowledge. The stimulus of Art is the abundance of our emotions and our latent desires. Science is the vehicle of facts – it is indifferent, or at best tolerant, to the ideas that lie behind facts. Art is the vehicle of ideas and its attitude to facts is strictly partial. Science looks and observes, Art sees and foresees. Every great scientist has experienced a moment when the artist in him saved the scientist. "We are poets," said Pythagoras, and in the sense that a mathematician is a creator he was right [Gabo is certainly being 'poetical' here].
>
> (Bann, 1974: 213)

Clearly all this is (and has to be) an over-simplification of matters, for there was much more to the individual intent of the various practitioners and promoters of 'minimalism' and 'constructivism' than can be known and expressed here, and which is, as already indicated, a massive, massive subject in its own right, where the enquiry would have to extend comprehensively beyond Europe and Russia, and forward to include the American influences of those such as Sol LeWitt with his large-scale graphic 'minimalism' seen in his wall drawings (LeWitt, 1984), and then his equally large geometric expressions in the three-dimensional, as exemplified in his treatment of the visualization of the cube in its many computationally incomplete stages of form and formation, viz. his *Incomplete Open Cubes* of 1974 (Baume, 2001), and in relation to the less clearly defined (to me) treatment of 'minimalist' intent in the presentations of Donald Judd. As for the latter, I quote:

> When asked in 1990 whether his viewer should 'understand something' or 'just look', Judd replied:
> "That's the division between thought and feeling. You have to do it [think and feel] all at once. You have to look and understand, both. In looking you understand; it's more than you can describe. You look and think, and look and think, until it makes sense, becomes interesting."
>
> (Serota, 2004: 40)

Perhaps such expressions in explanation of his contribution to the latter stages of 'minimalism' within art history are best summed up in:

> The literalist aesthetic most associated with minimalism held that a work should reveal nothing other than its constitutive materials and manner of construction.
>
> (Meyer, 2001: 7)

[Clearly, such non-disclosure is totally opposite to the deeply didactic content subsumed within my work, there to be investigated or which can be clearly explained.]

2. Construction of models

Now as for materials used and methods of construction, that's another question that has been asked of me and which has been of quite some interest to the viewing public. Again, the answer has to be general, for to explain the techniques that were applied in what is a technically complex, extremely time-consuming and exacting craft – requiring skill and precision – is clearly impossible without demonstration, but I shall now give an outline and overview to indicate some of the stages and general practical elements involved. At the outset, it has to be realized that nothing was involved here other than my own personal skills as an artisan; no machinery whatsoever, no modern technology, no automation, and no laser cutting or any such modern technical interventions. Accurate calculation and measurement, however, was of paramount importance at all stages, as any slight inaccuracies would otherwise be further compounded at each additional stage as constructs were built up. Thus, high-grade (precision) traditional engineering measuring tools had to be utilized. The material used throughout was a high grade of Finnish, birch three-ply (1.5 mm).

The first and most important stage in the creation of these forms is the production of the templates as a standard reference for the making of each individual 'face'. These have to be fashioned as accurately as it is possible to do when cutting with a sharp blade and then finishing with fine files and a 'true', emery-faced surface. Each 'face' is then individually cut – again accurately – to the template that is appropriate to the structure being developed. The gluing edges to each 'face' are further beveled to the necessary angle in accordance with the dihedral angle that is required, using variously either a paring chisel; a Stanley No. 9½ low-angled plane; or an even lower angled No. 60½; no mean task, given the small scale of many of the individual pieces and the pliable properties of such thin ply. The gluing process is less complex but still requires some accuracy in the placement of the pieces as they are temporarily held in place by adhesive tape at this stage. PVA glue is used (somewhat less traditionally) in this final process of construction because of its relative convenience, as compared with Scotch glue, and for its quick drying property.

Clearly, since each and every stage in the above process is executed by hand, it is inevitable that perfect accuracy is not possible but, as it is with violin-making, that is the charm of the end product; although not truly perfect, the object appears to the untrained eye as being quite beautifully – even perfectly – formed, as the maker's very individual input and artistry is somehow felt; clear evidence of that particular contingency and visceral contact that is made with the 'perfect' ontological as an intent to somehow replicate such impossible perfection. This is important to me, as a very individual engagement with, and as a vehicle for intellectual development and a further understanding of, a subject that is wholly impersonal in itself but which is clearly personal to me (perhaps this is what Gabo was referring to as 'stimulus' and 'emotion'), and somewhat different from that which motivated the artisans of the Arts and Crafts movement, should a further comparison be suggested here.

Plates F1 through to **F8** now indicate some of the constructional stages that were fleetingly captured by me as a partial record, and as being illustrative also, of *process*.

Appendices

Appendix A

The Periodic Table of Elements, **Plate A2**, indicates the seven different periods, in their vertical order, where the sum of the individual members of each lateral groupings are 2, 8, 8, 18, 18, 32 and 32 respectively. **Plate A3** then indicates the detailed *electron configuration* of each element in conformation with the formula $2n^2$ to give the series: 2, 8, 18, 32, to be followed by its mirror image 32, 18, 8, [2], providing for 120 possible configurations but, in accordance with generally accepted scientific theory, with the last 2 electrons shown as deleted from the series to provide for 118 theoretical elements. The detailed breakdown of the number sequences for each individual element seen in **Plate A3** indicates a further subdivision of electron shell arrangements such that 2, 8, 18, 32, 32, 18, 8 can be represented in more detail by the sequence:

2, (2+6), (2+6+10), (2+6+10+14), (2+6+10+**14**), (2+6+**10**), (2+**6**).

Plate A4 is my own template and visual system of configuration, designed to reflect the mathematical patterning and further complex sequencing that the data shown in **Plate A3** would seem to indicate. This forms the basis to my sculpture, ***Element 88*** **(Plate A1)** which has the structure 2, 8, 18, 32, 18, 8, 2 and, as a palindrome, is intended to represents the electron configuration of the element *Radium*, discovered in 1898 by Curie & Curie, whereby the dark scrim 'blank' squares indicate the 'missing' electrons from the overall representative template, those that are highlighted in **bold** in the above sequence, but where the graduated use of colour filters is intended to indicate what is a further complexity in the sometimes dislocated 'staged' build-up of the sequence, another somewhat separate matter.

The base, although primarily functional, echoes the numerical theme of the sculpture itself. Its construction is designed so that there are 88 squares [(10 x 9) – 2] with 54 contained in the main platform and 34 within the surrounding lower tier. This reflects the number of electrons for *Radium* at 88.

Appendix B

Plate B1 shows a table of the 64 triplet based codons for the Genetic Code, and alongside each codon is indicated the amino acid that is attracted for ribosomal attachment. By colour coding for groups, as in **Plate B2**, we can then list the groupings as below:

5 groups (yellows) comprising 4 different codons [for amino acids *Ala, Gly, Pro, Thr & Val*]
9 groups (blues) comprising 2 different codons [for *Asn, Asp, Cys, Gln, Glu, His, Lys, Phe & Tyr*]
3 groups (reds) comprising 6 different codons [*Arg, Leu & Ser*]
1 group (greens) comprising 3 codons [for *Ile*]
2 *individual* codons (violet) [for *Trp* and *Met* (acting as 'Start' code AUG)]
3 *individual* codons (not coloured) [for 'End' codes UAA, UAG & UGA]

This analysis can then be further represented as **Plate B3** where a certain mathematical pattern emerges to indicate a proportional sequence that begins with a simple and perfect 1:1 concomitance for the first (totally in yellow, as 4:4) of the eight overall groupings (each constituting 8 individual codons), only to be balanced at its opposite apex by a complex but interesting arrangement that includes the three 'Stop' codes. This last grouping is then represented by somewhat different hues within my artistic realisation of the overall numerical balance as a leaded glass, hexagonal arrangement, entitled *RNA's Degenerate Code*, 2008 **(Plate B4)**:

1 group (3 slightly different greens for the 3 codons) [coding for *Ile*]
2 *individual* codons (turquoise & deep violet) [for *Trp* and *Met* – the 'Start' code AUG]
3 *individual* codons (deep ambers) [for 'End' codes UAA, UAG & UGA]

Plate B5 then represents *RNA's Degenerate Code*, 2008, as a painted wooden construct. Here there are 4 x 4 x 4 = 64 cubes, referencing – again by colour code – the various groupings for the 64 individual codons, and then set on an icosahedron, constituting 20 hedrons (*three-sided*), to further reference both the *triplet*-based code and the 20 amino acids. This is a complex piece of work that can also be partially dismantled to show further the visual harmony of the mathematical balances contained within.

Chapter 1

Appendix 1

Extract from: St. Thomas Aquinas' *Summa Theologica.*

Obj. 5. Further, the Philosopher says (*Ethic.iii.*5): *According as each one is, such does the end seem to him.* But it is not in our power to be of one quality or another; for this comes to us from nature. Therefore it is natural to us to follow some particular end, and therefore we are not free in so doing.

On the contrary, It is written (Ecclus. Xv. 14): *God made man from the beginning, and left him in the hands of his own counsel;* and the gloss adds: *That is of his free-will.*

I answer that, Man has free-will: otherwise counsels, exhortations, commands, prohibitions, rewards and punishments would be in vain. In order to make this evident, we must observe that some things act without judgment; as a stone moves downwards; and in like manner all things which lack knowledge. And some act from judgment, but not a free judgment; as brute animals. For the sheep, seeing the wolf, judges it a thing to be shunned, from a natural and not a free judgment, because it judges, not from reason, but from natural instinct. And the same thing is to be said of any judgment of brute animals. But man acts from judgment, because by his apprehensive power he judges that something should be avoided of sought. But because the judgment, in the case of some particular act, is not from a natural instinct, but from some act of comparison in the reason, therefore he acts from free judgment and retains the power of being inclined to various things. For reason in contingent matters may follow opposite course, as we see in dialectic syllogism and rhetorical arguments. Now particular operations are contingent, and therefore in such matters the judgment of reason may follow opposite courses, and is not determinate to one. And forasmuch as man is rational is it necessary that man have a free-will.

(Aquinas, 1947: 418)

Appendix 2

Extract from: *The Concise Oxford Dictionary of Mathematics.*

parallel postulate The axiom of Euclidean geometry which says that, if two straight lines are cut by a *traversal* and the interior angles on one side add up to less than two right angles, then the two lines meet on that side. It is equivalent to Playfair's axiom, which says that, given a point not on a given line, there is precisely one line through the point parallel to the line. The parallel postulate was shown to be independent of the other axioms of Euclidean geometry in the nineteenth century, when *non-Euclidean geometries* were discovered in which the other axioms hold but the parallel postulate does not.

(Clapham, 1996)

[Notes, critical and geometrical, on Definition XXXV, Axiom XII and Proposition XXIX in Euclid's Book I, otherwise collectively referred to as the 'parallel postulate', are to be found in Simson's *The Elements of Euclid* (Simson, 1827: 269-270)].

Appendix 3

Extract from: *The Concise Oxford Dictionary of Mathematics.*

integer One of the 'whole' numbers: ..., -3, -2, -1, 0, 1, 2, 3, ... The set of all integers is often denoted by **Z**. With the normal addition and multiplication, **Z** forms an *integral domain*. Kronecker said, 'God made the integers; everything else is the work of man'.

real number The numbers generally used in mathematics, in scientific work and in everyday life are the **real numbers.** They can be pictured as points of a line, with the integers equally spaced along the line and a real number b to the right of the real number a if $a < b$. The set of real numbers is usually denoted by **R**. It contains such numbers as 0, ½, -2, 4.75, $\sqrt{2}$ and π. Indeed, **R** contains all the rational numbers but also numbers such as $\sqrt{2}$ and π that are irrational. Every real number has an expression as an infinite *decimal fraction*.

rational number A number that can be written in the form a/b, where a and b are integers, with $b \neq 0$. The set of all rational numbers is usually denoted by **Q**. A real number is rational if and only if, when expressed as a decimal, it has a finite or recurring expansion.
For example,

$$5/4 = 1.25,\ 2/3 = 0.666...,\ 20/7 = 2.857142\ 857142\ ......$$

A famous proof, attributed to Pythagoras, shows that $\sqrt{2}$ is not rational, and e and π are also known to be irrational.

irrational number A real number that is not *rational.* A famous proof, sometimes attributed to Pythagoras, shows that $\sqrt{2}$ is irrational; the method can also be used to show that numbers such as $\sqrt{3}$ and $\sqrt{7}$ are also irrational. It follows that numbers like $1+\sqrt{2}$ and $1/(1+\sqrt{2})$ are irrational. The proof that e is irrational is reasonably easy, and in 1761 Lambert showed that π is irrational.

(Clapham, 1996)

Appendix 4

Extract from: Leibniz's *Monadology*.

31. Our reasoning is based upon two great principles: first, that of Contradiction, by means of which we decide that to be false which involves contradiction and that to be true which contradicts or opposes to the false.
32. And second, the principle of Sufficient Reason, in virtue of which we believe that no fact can be real or existing and no statement true unless it has a sufficient reason why it should be thus and not otherwise. Most frequently, however, these reasons cannot be known to us.
33. There are also two kinds of Truths: those of Reasoning and those of Fact. The Truths of Reasoning are necessary, and their opposite is impossible. Those of Fact, however, are contingent, and their opposite is possible. When a truth is necessary, the reason can be found by analysis in resolving it into simpler ideas and into simpler truths until we reach those which are primary.
34. It is thus that with mathematicians the Speculative Theorems and the practical Canons are reduced by analysis to Definitions, Axioms, and Postulates.
35. There are finally simple ideas which no definition can be given. There are also the Axioms and Postulates or, in a word, the primary principles which cannot be proved and, indeed, have no need of proof. These are identical propositions whose opposites involve express contradictions.
36. But there must be also a sufficient reason for contingent truths or truths of fact; that is to say, for the sequence of the things which extend through out the universe of created beings, where the analysis into more particular reasons can be continued into greater detail without limit because of the immense variety of the things in nature and because of the infinite division of bodies. There is an infinity of figures and of movements, present and past, which enter into the efficient cause of my present writing, and in its final cause there are an infinity of slight tendencies and dispositions of my soul, present and past.
37. And as all this detail again involves other and more detailed contingencies, each of which again need of a similar analysis in order to find its explanation, no real advance has been made. Therefore, the sufficient or ultimate reason must needs be outside of the sequence or series of these details of contingencies, however infinite they may be.
38. It is thus that the ultimate reason for things must be a necessary substance, in which the detail of the changes shall be present merely potentially, as in the fountain-head, and this substance we call God.
39. Now, since this substance is a sufficient reason for all the above mentioned details, which are linked together throughout, *there is but one God, and this God is sufficient.*

(Montgomery, 1902: 258-9)

Appendix 5

Extract from: John Locke's *An Essay concerning Human Understanding.*

Book IV, Ch. XVIII

§ 4. *Secondly,* I say, that *the same Truths may be discovered, and conveyed down from Revelation, which are discoverable to us by Reason,* and by those *Ideas* we naturally may have. So GOD might, by Revelation, discover the Truth of any Proposition in *Euclid;* as well as Men, by the natural use of their Faculties, come to make the discovery themselves. In all Things of this Kind, there is little need or use of *Revelation,* GOD having furnished us with natural, and surer means to arrive at the Knowledge of them. For whatsoever Truth we come to of our own *Ideas,* will always be certainer to us, than those which are conveyed to us by *Traditional Revelation.* For the Knowledge, we have, that this *Revelation* came at first from GOD, can never be sure, as the Knowledge we have from the clear and distinct Perception of the Agreement, or Disagreement of our own *Ideas,* v.g. If it were revealed some Ages since, That the three Angles of a Triangle were equal to two right ones, I might assent to the Truth of that Proposition, upon the Credit of Tradition, that it was revealed: But that would never amount to so great a Certainty, as the Knowledge of it, upon the comparing and measuring my own *Ides* of two right Angles, and the three Angles of a Triangle. The like holds in matter of Fact, knowable by our Senses, v.g. the History of the Deluge is conveyed to us by Writings, which had their Original from Revelation: And yet no Body, I think, will say, he has as certain and clear Knowledge of the Flood, as *Noah* that saw it; or that he himself would have had, had he then been alive, and seen it. For he has no greater an assurance than that of his Senses, that it is writ in the Book supposed writ by *Moses* inspired: But he has not so great an assurance, that *Moses* writ that Book, as if he had seen *Moses* write it. So that the assurance of its being a Revelation, is less still than the assurance of his Senses.

(Nidditch, 1991: 690-1)

Chapter 2

Appendix 6

Extract from: *Hermetica,* attributed to Hermes Trismegistus.

... reality exists only in things everlasting – . The everlasting bodies, as they are in themselves,- that is very fire, earth that is very earth, air that is very air, and water that is very water, - these indeed are real. But our bodies are made up of all these elements together; they have in them something of fire, but also something of earth and water and air; and there is in them neither real fire nor real earth nor real water nor real air, nor anything that is real. And if our composite fabric has not got reality in it to begin with, how can it see reality, or tell reality?

All things on earth then, my son, are unreal; but some of them, – not all, but some few only, – are copies of reality. The rest are illusion and deceit, my son; for they consist of mere appearance. When the appearance flows in from above, it becomes an imitation of reality. But apart from the working of power from above, it remains an illusion; just as a painted portrait presents to us in appearance the body of the man we see in it, but is not itself a human body. It is seen to have eyes, and yet it sees nothing; it is seen to have ears, and yet it hears nothing at all. The picture has all else too that a living man has, but all this is false, and deceives the eyes of those who look at it; they think that what they see is real, but it is really an illusion.

(Scott, 1993: 149-150)

Appendix 7

Extract from: Agrippa's *Three Books of Occult Philosophy*, Book I, Chapter LX.

They say therefore, when the mind is forced with a melancholy humour, nothing moderating the power of the body, and passing beyond the bounds of the members, is wholly carried into the imagination, and doth suddenly become a seat for inferior spirits, by whom it oftentimes receives wonderful ways, and forms of manual arts. So we see that any most ignorant man doth presently become an excellent painter, or contriver of buildings, and to become a master in any such art. But when these kinds of spirits portend to us future things, then they show those things which belong to the disturbing of the elements, and changes of times, as rain, tempests, inundations, earthquakes, great mortality, famine, slaughter, and the like. As we read in *Aulus Gelius,* that *Cornelius Patarus* his priest did at that time, when *Caesar,* and *Pompey* were to fight in Thessalia, being taken with a madness, foretell the time, order, and issue of the battle.

But when the mind is turned wholly into reason, it becomes a receptacle for middle spirits. Hence it obtains the knowledge, and understanding of natural, and human things. So we see that a man sometimes doth on a sudden become a philosopher, physician, or an excellent orator, and foretells mutations of kingdoms, and restitutions of ages, and such things as belong to them, as the sibyl did to the Romans.

But when the mind is wholly elevated into the understanding, then it becomes a receptacle of sublime spirits, and learns of them the secrets of divine things, as the Law of God, the orders of the angels, and such things as belong to the knowledge of things eternal, and salvation of souls. It foretells things which are appointed by God's special predestination, as future prodigies, or miracles, the prophet to come, the changing of the Law. So the sibyls prophesied of Christ a long time before his coming. So *Virgil* understanding that Christ was at hand, and remembering what the sibyl *Cumea* had said, sang thus to *Pollio*:

> Last times are come, Cumaea's prophecy
> Now from high heaven springs a new progeny,
> And time's great order now again is born,
> The Maid returns, Saturnian realms return.

(Tyson, 2007: 189)

Appendix 8.1

Notes

1. *adjust* – Dry, burning.
2. *golden marcasite* – Iron pyrites, or fool's gold.
3. *onyx* – Black onyx, a form of chalcedony, an opaque black or dark brown stone, usually with a white line running across it. Sometimes the line forms a circle, and the stone is then called lynx-eye onyx. Connected astrologically with Capricorn and Saturn, it was used in rosaries and to avert the evil eye.
4. *ziazza* – "A black and white stone; it renders its possessor litigious, and causes terrible visions" (Spence [1920] 1968, 439).
5. *camonius* – In the Latin *Opera*, camonius.
6. *chalcedon* – Chalcedony, a form of silica. "Take the stone which is called *Chalcedonius*, and it is pale, brown of colour, and somewhat dark (*Book of Secrets* 2.22 [Best and Brightman, 36]).

(Tyson, 2007: 83)

Appendix 8.2

Notes

2. *Plato commanded* –
But the best way would be to bury them [the Mysteries] in silence, and if there were some necessity for relating them, only a very small audience should be admitted under pledge of secrecy and after sacrificing, not a pig, but some huge and unprocurable victim, to the end that as few as possible should have heard these tales. (Plato *Republic* 2,378a [Hamilton and Cairs, 624-5])

3. *Pythogoras also* – On this subject Clemens Alexandrius writes: "They say that Hipparchus, the Pythagorean, being guilty of writing the tenets of Pythagoras in plain language, was expelled from the school, and a pillar raised for him as if he had been dead" (*Stromateis* 5.9. In *Ante-Nicene Christian Library*, vol.12).

Pythagoras enforced not only a silence concerning the secrets of his fraternity but a period of general silence to be endured by all disciples: "Pythagoras enjoyed young men five years' silence, which he called *echemychia*, abstinence from all speech, or holding of the tongue" (Plutarch *On Curiosity* 9, trans. Philemon Holland. In *Plutarch's Moralia; Twenty Essays* [London: J.M.Dent and Sons, n.d.], 143).

(Tyson, 2007: 443)

Appendix 9

Extract from:

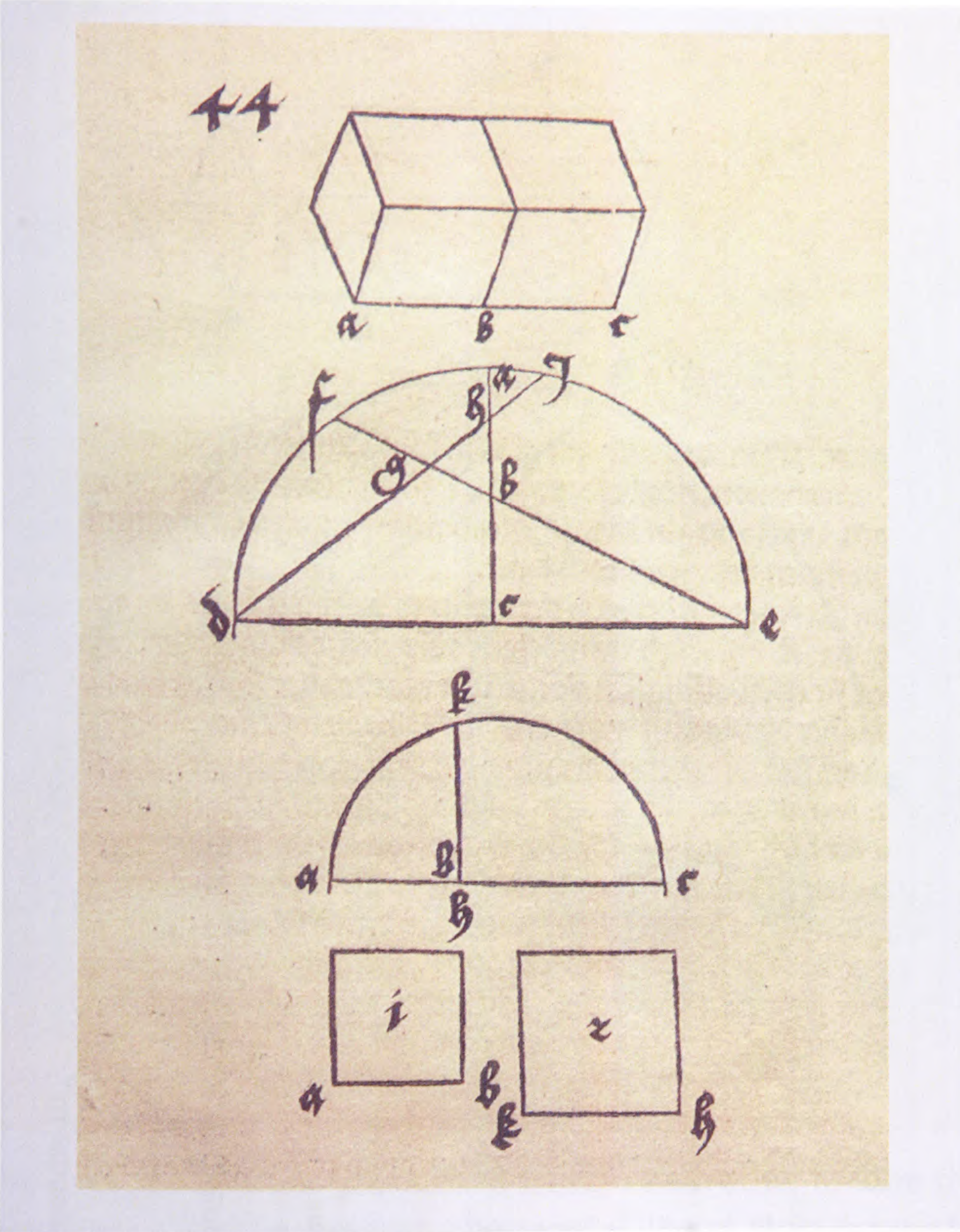
Walter L. Strauss translation of *Underweysung der Messung*, originally published as *The Painter's Manual* (New York: Abaris Books, Inc., 1977) pp. 174-175 and the image overleaf (Dürer's ffig. 44) from www.octavo.com as Octavo Edition, 2003 [VIEW 162: page 01v].

First place two equal cubes or dice next to each other [and mark them as below with three points, to form line *abc*.] Then erect a line of the same length as *ac* at right angles on a horizontal line *de*. Draw a semicircle from the center *c* through points *d, a* and *e*. Then draw a straight line from *e* through *b* to the periphery of the semicircle and mark it point *f*. Then mark a small ruler with a center point and divide both sides evenly with the same scale of numbers beginning from the center with ı [and continuing in either direction]. By moving the ruler you will find the first line and then the second for the doubled cube. To do this, place one end of this ruler on point *d* and keep it anchored on this point, whether you move the ruler up or down. And if you move the other part of the ruler, its center must always remain on line *abc*. Now move the ruler until you find the middle between line *ef* and the periphery of the semicircle, and where the movable member of the ruler crosses line *ef* mark point *g*. Where it crosses line *abc* mark point *i*. In this way, *gh* and *hi* will become equal in length, and *hc* will denote the first side of the doubled cube.

{**Note here by author of this thesis**:

'and *hc* will denote the first side of the doubled cube' is an unfortunate phrase in translation as the line *kh* (**not** this initial *hc*) eventually becomes the actual 'side of the doubled cube' in the next stage of the process.}

Next place the line *hc* horizontally adjacent to line *ab*, representing the side of the simple cube, to form line *agc*. Put one leg of a compass on the midpoint of line *ac* and draw a semicircle from *a* to *c*. Then erect a vertical line from *h* to the periphery and mark point *k*. This line *kh* represents the side of the doubled cube, as shown in the diagram below.


44
a
b
c
f
a
d
h
g
b
d
c
e
k
a
b
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z
a
b
k
h

Appendix 10

Pythagorean (P)

Interval (P)		**9/8** ↓		**9/8** ↓		256/243 ↓		[9/8] ↓		**9/8** ↓		**9/8** ↓		256/243 ↓	
Ratio (P)	1/1		9/8		81/64		4/3		3/2		27/16		243/128		2/1
Note	**C**		**D**		**E**		**F**		**G**		**A**		**B**		**C**
Ratio (maj)	1/1		9/8		5/4		4/3		3/2		5/3		15/8		2/1
Interval (maj)		↑ **9/8**		↑ (10/9)		↑ 16/15		↑ [9/8]		↑ (10/9)		↑ **9/8**		↑ 16/15	

Diatonic major (maj)

Diatonic minor (min)

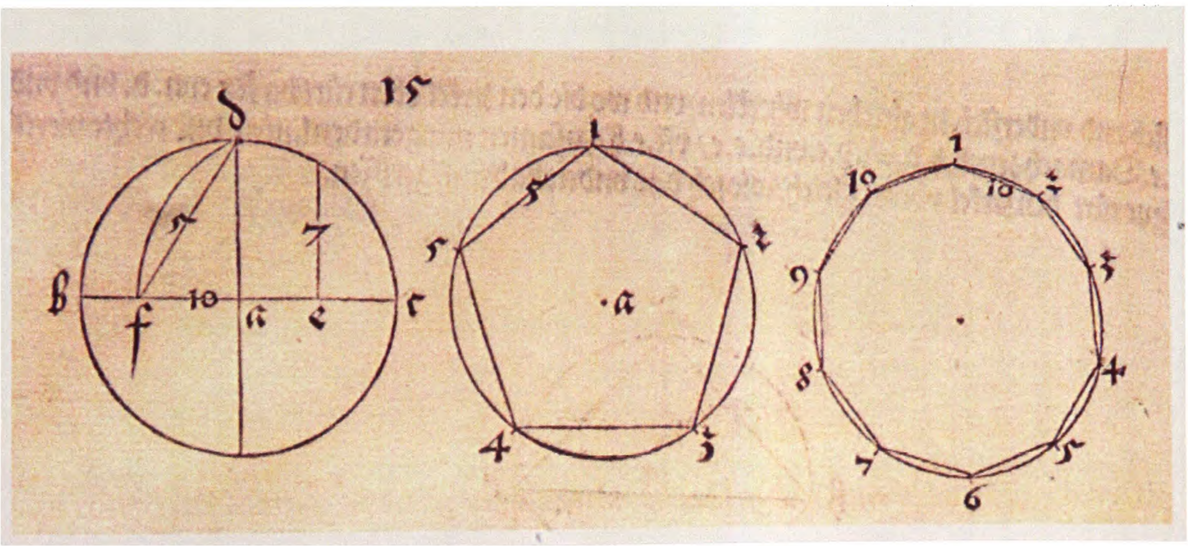
Interval (min)		**9/8** ↓		16/15 ↓		(10/9) ↓		[9/8] ↓		16/15 ↓		**9/8** ↓		(10/9) ↓	
Ratio (min)	1/1		9/8		6/5		4/3		3/2		8/5		9/5		2/1
Note	**A**		**B**		**C**		**D**		**E**		**F**		**G**		**A**

Appendix 11

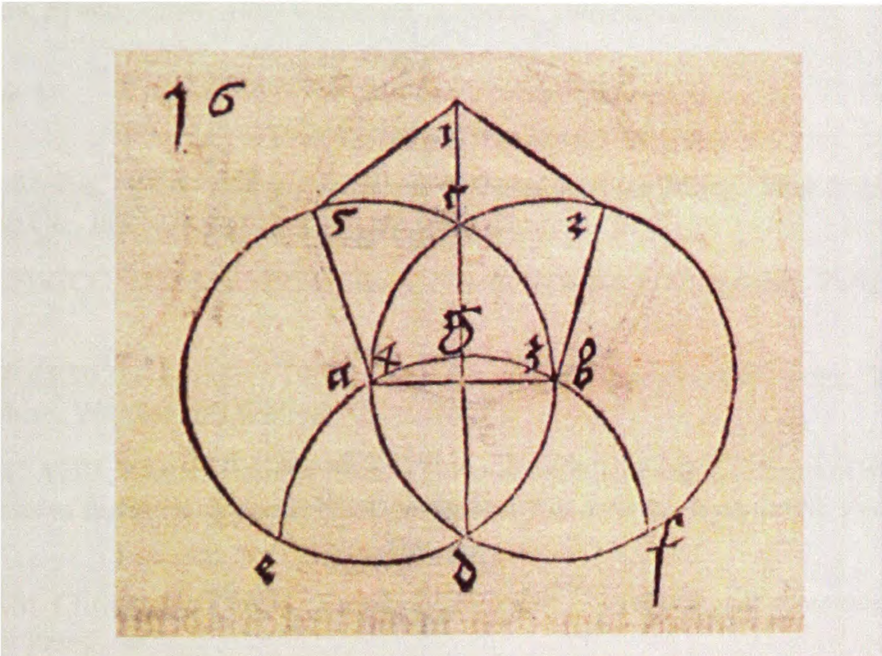
Extract from:

Walter L. Strauss translation of *Underweysung der Messung*, originally published as *The Painter's Manual* (New York: Abaris Books, Inc., 1977) pp. 58-59, and these images from www.octavo.com as Octavo Edition, 2003 [VIEW 111: page E4v].

Now it is necessary to construct a pentagon within a circle. To do this, first draw a circle with centre *a* and draw a horizontal line through this centre *a*. Where this horizontal line crosses the periphery of the circle on both sides, mark points *b* and *c*. Then draw a vertical line through the centre *a* at right angles to the horizontal line. Where it crosses the periphery on top, mark point *d*. [Then divide line *ac* in two halves and mark the midpoint *e*.] Now draw a straight line *ed* and place one leg of a compass on point *e* and the other on point *d* and draw an arc down to the horizontal line *bc*. Where the arc crosses this line, mark point *f*. Then connect *f* and *d*. This line *fd* represents one side of the pentagon, whereas line *fa* is equal to one side of a ten-sided figure. Then divide line *ac* into two halves and erect a vertical line up to the periphery. Its length is equal to approximately one-seventh of the circle. This is shown in the following diagram.



Construction of a pentagon without changing the opening of the compass is accomplished as follows. Draw two circles which overlap so that the periphery of each touches the other's centre. Then connect the two centre points *a* and *b* with a straight line. The length of this line is equal to one side of the pentagon. Where the two circles cross, mark *c* on top and *d* on the bottom and draw line *cd*. Then take the compass without changing its opening and place one leg on *d* and with the other draw an arc through the two circles and their centres *a* and *b*. And where the periphery is crossed by this arc, mark point *e* and *f*. Where the vertical line *cd* is crossed, mark the point *g*. Then draw the line *eg* and extend it to the periphery of the circle. Mark this point *h*. Then draw another straight line *fg* and extend it to the periphery of the circle. Mark that point *i*. Then connect *i* with *a* and *h* with *b* and it will give you three sides of the pentagon. Then erect two inclined lines of equal length to *ih* until they meet on top. You will then have constructed a pentagon, as shown below.



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facsimile of a page from Erasmus's Greek and Latin New Testament
http://smu.edu/bridwell_tools/publications/ryriecatalog/5_2ab.jpg
20th Aug 2011.

facsimile of a page from Luther's translation of the above
http://www.for-martin-luther.com/leaf_luther1529_200.jpg
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Plate A2
http://www.click4chill.com/wp-content/uploads/2010/07/periodic_table.gif
20th March 2011.

Plate A3
http://www.avon-chemistry.com/electron_config_2.jpg
20th March 2011.

Plate C1
http://files.myopera.com/marcjoel/albums/670975/Dali%20DNA.jpg
21st March 2011.

Plate C2
http://www.theartistsalvadoridali.com/dalihypercube.jpg
21st March 2011.

Plate C3
http://metrobibleblog.files.wordpress.com/2010/03/dali-last-supper.jpg
21st March 2011.

Plate D2
http://farm2.static.flickr.com/1071/4729028956_1a4c8e888e_z.jpg
21st March 2011.

Chapter 2

Plate 1
http://upload.wikimedia.org/wikipedia/commons/2/23/Knight-Death-and-the-Devil.jpg
[within] http://blogomata.wordpress.com/category/durer/
9th March 2011.

Plate 2a
http://pavlopous.files.wordpress.com/2009/08/durer_melancholia_i.jpg
[within] http://blogomata.wordpress.com/category/durer/
9th March 2011.

Plate 3
http://mappingthemarvellous.files.wordpress.com/2009/04/durer-hieronymus-im-gehaus.jpg [within] (http://blogomata.wordpress.com/category/durer/
9th March 2011.

Plate 4
www.deliciarum.info/21/07/2009/mashallah/
9th March 2011.

Plate 5
http://www.mezzo-mondo.com/arts/mm/raphael/RAF001.html
9th March 2011.

Plate 6
http://www.irocks.com/db_pics/mdpics/MD-24380a.jpg
12th March 2011.

Plate 7
(upper) http://www.thunderhealing.org/rock/magnetite.jpg
12th March 2011.
(lower) http://math.ucr.edu/home/baez/week247.html
12th March 2011.

Plate 8
(upper) http://maurice.strahlen.org/minerals/pics/magnetite-tazh.jpg
12th March 2011.
(lower) http://farm5.static.flickr.com/4061/4447112896_181487ae63.jpg
12th March 2011.

Plate 9a
http://www.britishmuseum.org (research)
9th March 2011.

Plate 9b
(upper) as for Plate 9a
(middle) www.georgehart.com/virtual-polyhedra/durer.html
9th March 2011.
(lower) http://www.britishmuseum.org (research) [AN00030294_001.jpg]
8th December 2011.

Plate 10
http://www.ibiblio.org/wm/paint/auth/durer/self/
9th March 2011.

Plate 20
http://nationalgallery.org.uk/paintings/hans-holbein-the-younger-the-ambassadors
9th March 2011.

Plate 23
http://www.spoonfiles.konstvet.uu.se/image/900/pacioli.jpg
9th March 2011.

Chapter 3

Plate 42
http://datapeak.net/images/MargaritaPhilosophica.jpg
9th March 2011.

Plate 44
http://www.davidmus.dk/assets/2204/16_2-8-2005-Noah's-ark-Hafis-Abru-2.jpg
14th March 2011.

Plate 45
http://www.ee.bilkent.edu.tr/~history/Pictures1/im50.jpg
14th March 2011.

Plate 46
http://www.holidayinisrael.com/UserFiles/Islamic_Art_Museum_004.jpg
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Plate 48
http://math.ucr.edu/home/baez/week247.html
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Plate 49
www.panoramio.com/photo/10767971
9th March 2011.

Plate 50
www.monmouth.edu/academics/mathematics/faculty/bodner.asp
9th March 2011.

Plate 53
http://www.metmuseum.org/toah/works-of-art/1993.67.2
9th March 2011.

Plate 54
http://farm2.static.flickr.com/1094/532795488_e48f87f3fc.jpg
9th March 2011.

Plate 55
http://riv.zcache.com/moorish_tile_the_alhambra_spain_poster_p228467793907682062trma_400.jpg
9th March 2011.

Plate 56
www.5cense.com/Granada_Seville.htm
9th March 2011.

Plate 57a
http://www.eso-garden.com/index.php?/weblog/the_alhambra_tour/
9th March 2011.

Plate 57b
http://www.spain-holiday.com/blog/wp-content/uploads/2010/09/Court-of-the-Lion-Alhambra.jpg
9th March 2011.

Plate 58
http://www.elsevier.com/locate/culher
9th March 2011.

Plate 59
http://www.bh.org.il/database-article.aspx?48725
9thMarch 2011.

Plate 60
http://www.metmuseum.org/toah/images/hb/hb_1970.8.jpb
9th March 2011.

Plate 62b
http://k43.pbase.com/06/28/410728/1/104013255.6imE2zor.Konyasept20084039.jpg
14th March 2011.

Plate 66
www.patterninislamicart.com/ia/l/tur_0318.jpg
9th March 2011.

Plate 67
http://www.physics.princeton.edu/~steinh/LuSteinhardt2007.pdf
9th March 2011.

Plate 83
http://farm6.ststic.flickr.com/5008/5254957760_71768406be.jpg 9th
9th March 2011.

Plate 84
www.shutterstock.com
9th March 2011.

Plate 85
http://muslimvoices.org/files/2008/10/kaaba-400-250.jpg
9th March 2011.

Plate 88
(above) http://images-mediawiki-sites.thefullwiki.org/11/1/5/9/53348271005621129.jpg
9th March 2011.
(below) www.callumjames.blogspot.com/2009_09_01_archive.html
9th March 2011.

Plate 92
www.georgehart.com/virtual-polyherdra/jamnitzer.html
9th March 2011.

Plate 93
http://www.ac-noumea.nc/maths/polyhedr/img_art/imtarsia.jpg
9th March 2011.

Plate 110a
(above) http://s4.hubimg.com/u/851567_f260.jpg
9th March 2011.
(below) http://stoneplus.cst.cmich.edu/zoogems/shell-nautilusWikipedia.jpg
9th March 2011.

Plate 110b
http://www.magicalagora.com/images/IB552.jpg
14th March 2011.

Plate 112
www.amigodaalma.com.br/up-contents/uploads/Pitágoras3.TheoricaMusicae.1492.Gaffurius-1451-1522.0.3.jpg
9th March 2011.

Plate 113
http://cojs.org/cojswiki/images/5/5c/Portae_Lucis.jpg
9th March 2011.

Plate 115b
http://commons.wikimedia.org/wiki/File:Michael_Wolgemut_-_Portrait_of_Ursula_Tucher_-_WGA25863.jpg
27th October 2011.

Plate 116
http://www.albrecht-durer.org/72554/Portrait-Of-Elsbeth-Tucher-large.jpg
9th March 2011.

Plate 117a&b
http://uploads2.wikipaintings.org/images/albrecht-durer/horse-final-death.jpg
9th November 2011

Plate 120
http://www.dia.org/user_area/comping/41.1-S1.jpg
9th March 2011.

Plate 121a,b&c
http.//irea.wordpress.com
9th March 2011.

Plate 122a
http://hoocher.com/Albrecht_Durer/Alliance_Coat_of_Arms_of_the_Durer_and_the_Holper_Families_1490.jpg
27th October 2011.

Plate 122b
http://hoocher.com/Albrecht_Durer/Portrait_of_Durer_s_%20Father_1490.jpg
27th October 2011.

Plate 122c
http://durerart.com/Images?Paintings/Barbara-Durer.jpg
27th October 2011.

Plate 122d
http://www.ibiblio.org/wm/paint/auth/durer/self-26.jpg
27th October 2011.

Plate 122e
http://www.ibiblio.org/wm/paint/auth/durer/self-28.jpg
27th October 2011.

Plate 122f
http://media.artfinder.com/works/r/akg/5/6/3/97365_full_570x702.jpg
27th October 2011.

Plate 123a
http://lovinglifebeingabitch.files.wordpress.com/2011/07/portrait-of-jacob-obrecht.jpg
27th October 2011.

Plate 123c
www.willwhitakerbooks.com/up-content/uploads/2011/09/St-Eligius-Petrus_Christus_003.jpg
27th October 2011.

Plate 123d
http://museodelprado.es/uploads/pics/Dirk_Bouts_04.jpg
27th October 2011.

Plate 123e(i)
http://www.jstor.org/stable/2970818
5th November 2011.

Plate 123e(ii)
http://daten.digitale-sammlungen.de/bsb00012346/image_5
8th November 2011.

Plate 123e(iii)
http://daten.digitale-sammlungen.de/bsb00012346/image_178
8th November 2011.

Plate 123e(iv)
http://daten.digitale-sammlungen.de/bsb00012346/image_179
8th November 2011.

Plate 123e(v)
http://daten.digitale-sammlungen.de/bsb00006315/image_180
8th November 2011.

Plate 123e(vi)
http://daten.digitale-sammlungen.de/bsb00006315/image_195
8th November 2011.

Plate 123e(v)
http://daten.digitale-sammlungen.de/bsb00006315/image_207
8th November 2011.

Plate 125a
http://upload.wikimedia.org/wikipedia/commons/f/f5/Albrecht_Durer_081.jpg
9th Sept 2011.

Plate 144
http://freechristimages.org/images_Christ_life/Christ_Among_The-Doctors_Albrecht_Durer_1506.jpg
12th Apr 2011.

Plate 145
http://blogs.ft.com/scienceblog/files/2009/06/27-collier_portrait-of-charles-darwin.jpg
12th Apr 2011.

Exhibition

facsimile of a page from Erasmus's Greek and Latin New Testament
http://smu.edu/bridwell_tools/publications/ryriecatalog/5_2ab.jpg
20th Aug 2011.

facsimile of a page from Luther's translation of the above
http://www.for-martin-luther.com/leaf_luther1529_200.jpg
20th Aug 2011.

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