Sliding mode control for networked systems with randomly varying nonlinearities and stochastic communication delays under uncertain occurrence probabilities

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Abstract

In this paper, we aim to propose the robust sliding mode control (SMC) scheme for discrete networked systems subject to randomly occurring uncertainty (ROU), randomly varying nonlinearities (RVNs) and multiple stochastic communication delays (MSCDs). Here, a series of mutually independent Bernoulli distributed random variables is introduced to model the phenomena of the ROU, RVNs and MSCDs, where the occurrence probabilities of above phenomena are allowed to be uncertain. For the addressed systems, an SMC strategy is given such that, for above network-induced phenomena, the stability of the resulted sliding motion can be guaranteed by presenting a new delay-dependent sufficient criterion via the delay-fractioning method. Moreover, the discrete sliding mode controller is synthesized such that the state trajectories of the system are driven onto a neighborhood of the specified sliding surface and remained thereafter, i.e., the reachability condition in discrete-time setting is verified. Finally, the usefulness of the proposed SMC method is illustrated by utilizing a numerical example.

Key words: Sliding mode control; Networked systems; Randomly varying nonlinearities; Multiple stochastic communication delays; Uncertain occurrence probabilities.

1 Introduction

Over the past several decades, as an effective control strategy, the sliding mode control (SMC) technique has been extensively applied in many practical domains due to its superiorities, such as the robustness against parametric uncertainties and unknown disturbances on the prescribed sliding surface. Accordingly, the application problems of SMC have gained certain research attention, such as in dynamical networks, robotics and spacecraft [1–4]. As such, many important SMC methods have been developed for various systems according to their structural characteristics, see e.g. [5,6]. For example, the SMC problem has been studied in [5] for uncertain delayed multi-agent systems with external disturbances. It is worthy of noting that, since most control algorithms have been implemented digitally in modern industrial fields, the discrete-time SMC design of control systems has increasingly become prevalent [7–11]. For example, the quasi-sliding mode idea has been proposed in [9] and the SMC problems for complex systems based on the quasi-sliding mode have been extensively addressed. For instance, based on the method in [9], the reaching law with the disturbance compensation scheme has been developed in [11] for a class of uncertain discrete-time systems.

As it is well known, the time-delays are often encountered in a large number of engineering systems probably because of the limited bandwidth and speed of the information processing. In fact, the existence of time-delays inevitably deteriorates the system performance [12, 13]. Therefore, the problems of analysis and synthesis for delayed systems have been investigated with hope to attenuate the negative effects of time-delays. Regarding the mixed time-delays, an efficient SMC method has been presented in [14] for discrete Markovian jump systems by employing the delay-fractioning approach. Actually, it is more common that the stochastic communication delays occur when the signals are transmitted by communication channels [15,16]. Recently, the robust control methods have been proposed in [17, 18] for nonlinear systems with MSCDs via the sliding mode conception. However, most of the existing results suppose that the occurrence probabilities of stochastic communication delays are deterministic, which would be a bit conservative in some cases, e.g., in the networked setting. Compared to the existing results, the MSCDs considered in this paper are allowed to have uncertain occurrence probabilities, which really reflects the case in practical applications. Besides, the delay-fractioning method is employed to handle such MSCDs with uncertain occurrence probabilities with hope to reduce the conservativeness. Hence, we aim to characterize the phenomenon of MSCDs with uncertain occurrence probabilities in a mathematical way firstly and then reveal the resulted effects onto the control systems. Hence, we aim to characterize the phenomena of MSCDs with uncertain occurrence probabilities in a mathematical way and reveal the resulted effects onto the control system design.

On the other hand, it is well recognized that both additive uncertainties and nonlinearities may lead to deteriorate the desired system performance if not appropriately addressed [19–23]. As such, a great number of techniques have been proposed for uncertain nonlinear systems, see e.g. [24–26]. For example, a neural-network-based SMC method has been developed in [27] for delayed systems with unknown nonlinearities. Recently, the so-called randomly occurring nonlinearities (RONs) have been modeled in [28] to depict the stochastic disturbances and the randomly occurring uncertainty (ROU) has been characterized in [29]. In [10,29], the robust SMC algorithms have been designed for networked systems with RONs based on the delay fractioning idea, where new stochastic stability criteria have been presented for resulted sliding motion. In addition, due to the unreliability of the networks or other reasons, the randomly varying nonlinearities (RVNs) have been modeled from a mathematical viewpoint. Accordingly, the effects of the RVNs have been discussed and some effective analysis methods have been given for various systems, such as the state estimation method [30], fault detection scheme and H_∞ SMC approach [31, 32]. To the best of our knowledge, the robust SMC problem has not been thoroughly studied for discrete networked systems with ROU, RVNs and MSCDs, not to mention the case that the occurrence probabilities are allowed to be uncertain. Consequently, we decide to further investigate the SMC problem for networked systems with RVNs under uncertain occurrence probabilities, and propose a new robust SMC strategy by resorting to the delay-fractioning idea.

In this paper, the robust SMC problem is addressed for networked systems subject to ROU, RVNs and MSCDs under uncertain occurrence probabilities. Firstly, a linear switching surface is designed. Next, a delay-dependent sufficient criterion is established by the Lyapunov stability theorem to ensure the robustly asymptotical mean-square stability of the resulted sliding mode dynamics. Besides, the original non-convex problem is transformed into an optimal one by introducing a computational algorithm. The novelties of the paper lie in that: (i) we examine the effects from the network-induced phenomena of the ROU, RVNs as well as MSCDs in a same framework; (ii) we make the first attempt to discuss the SMC problem for networked systems with MSCDs under uncertain occurrence probabilities, and the delay-fractioning approach is employed to better address such type of MSCDs with uncertain occurrence probabilities; and (iii) the information of addressed network-induced phenomena has been fully taken into account in order to achieve desired performance, and then a new SMC algorithm has been obtained to examine the effects from the ROU, RVNs and MSCDs under uncertain occurrence probabilities onto whole control system performance. At last, we propose numerical simulations to illustrate the usefulness of the presented control method.

Notations. The following notations will be used throughout this paper. $(\Omega, \mathcal{F}, \mathcal{P})$ denotes a probability space, where Ω is a sample space, $\mathcal{F}$ is the σ-algebra of subsets of Ω, and $\mathcal{P}$ is the probability measure on $\mathcal{F}$. $\mathbb{E}\{x\}$ is the mathematical expectation of random variable x. The superscript "T" denotes the matrix transposition. $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ represent, respectively, the n-dimensional Euclidean space and the set of all $n\times m$ real matrices. $P > 0$ ($P \geq 0$) means that P is a symmetric and positive definite (positive semi-definite) matrix. I and 0 represent the identity matrix and the zero matrix with proper dimensions. $\mathrm{diag}\{Y_1, Y_2, \ldots, Y_n\}$ stands for a block diagonal matrix where the diagonal blocks are matrices $Y_1, Y_2, \ldots, Y_n$. $\mathrm{col}\{\ldots\}$ represents a vector column with blocks given by the vectors in $\{\ldots\}$. $\|\cdot\|$ represents the Euclidean norm of a vector and its induced norm of a matrix. The star $(*)$ in a matrix denotes the term that can be induced by the symmetry. Matrices, if not explicitly stated, are assumed to be compatible for algebraic operations.

2 Problem Formulation and Preliminaries

In this paper, we consider the following class of discrete uncertain delayed nonlinear systems:

$$\begin{aligned} x_{k+1} &= (A + \alpha_k \Delta A)x_k + A_\tau \widetilde{x}_k + B(u_k + f(x_k)) \\ &\quad + \gamma_k D_1 g_1(x_k) + (1-\gamma_k)D_2 g_2(x_k) + E_1 x_k \omega_k, \\ \widetilde{x}_k &= \sum_{i=1}^{q} \beta_{k,i} x_{k-\tau_{k,i}}, \\ x_k &= \phi_k, \quad k = [-\tau_M, 0], \end{aligned} \tag{1}$$

where $x_k \in \mathbb{R}^n$ is the system state, $u_k \in \mathbb{R}^m$ stands for the control input, $\tau_{k,i} \in [\tau_m, \tau_M]$ $(i = 1, 2, \ldots, q)$ are time-varying delays, where τ_M and τ_m are known bounds, respectively. Besides, the lower upper τ_m can be expressed by $\tau_m = \tau p$ with τ and p being integers. $f(x_k)$ is a bounded nonlinearity in Euclidean norm. A, A_τ, B, D_1, D_2 and E_1 are known matrices, among which the matrix B is assumed to have full column rank. $\omega_k \in \mathbb{R}^1$ is a zero-mean Wiener process on a probability space $(\Omega, \mathcal{F}, \mathcal{P})$ with

$$\mathbb{E}\{\omega_k^2\} = 1, \quad \mathbb{E}\{\omega_k \omega_j\} = 0 \ (k \neq j).$$

The unknown matrix ΔA stands for the parameter uncertainty satisfying

$$\Delta A = HFE, \quad F^T F \leq I,$$

where H and E are known matrices of appropriate dimensions, and F is an unknown matrix.

The nonlinear function $g_i(x) : \mathbb{R}^n \longrightarrow \mathbb{R}^n$ $(i = 1, 2)$ represents the mismatched nonlinearity satisfying the following sector-bounded condition:

$$[g_i(x) - F_{1i}x]^T [g_i(x) - F_{2i}x] \leq 0, \quad \forall x \in \mathbb{R}^n, \tag{2}$$

where F_{1i} and F_{2i} are known matrices and $F_i = F_{1i} - F_{2i} > 0$.

The mutually independent random variables α_k, $\beta_{k,i}$ $(i = 1, 2, \ldots, q)$ and γ_k, which are employed to model the phenomena of ROU, RVNs and MSCDs, satisfy the Bernoulli distribution with

$$\begin{aligned} &\mathrm{Prob}\{\alpha_k = 1\} = \mathbb{E}\{\alpha_k\} = \bar{\alpha} + \Delta\alpha, \\ &\mathrm{Prob}\{\alpha_k = 0\} = 1 - \mathbb{E}\{\alpha_k\} = 1 - (\bar{\alpha} + \Delta\alpha), \\ &\mathrm{Prob}\{\beta_{k,i} = 1\} = \mathbb{E}\{\beta_{k,i}\} = \bar{\beta}_i + \Delta\beta_i, \\ &\mathrm{Prob}\{\beta_{k,i} = 0\} = 1 - \mathbb{E}\{\beta_{k,i}\} = 1 - (\bar{\beta}_i + \Delta\beta_i), \\ &\mathrm{Prob}\{\gamma_k = 1\} = \mathbb{E}\{\gamma_k\} = \bar{\gamma} + \Delta\gamma, \\ &\mathrm{Prob}\{\gamma_k = 0\} = 1 - \mathbb{E}\{\gamma_k\} = 1 - (\bar{\gamma} + \Delta\gamma), \end{aligned} \tag{3}$$

where $\bar{\alpha} + \Delta\alpha$, $\bar{\beta}_i + \Delta\beta_i$ and $\bar{\gamma} + \Delta\gamma \in [0, 1]$, $\bar{\alpha}$, $\bar{\beta}_i$ and $\bar{\gamma}$ are known scalars. Here, $\Delta\alpha$, $\Delta\beta_i$ and $\Delta\gamma$ depict the uncertain occurrence probabilities satisfying $|\Delta\alpha| \leq \epsilon_1$, $|\Delta\beta_i| \leq \epsilon_2$ and $|\Delta\gamma| \leq \epsilon_3$ with ϵ_j $(j = 1, 2, 3)$ being non-negative scalars. Obviously, we have $0 \leq \epsilon_1 \leq \min\{\bar{\alpha}, 1 - \bar{\alpha}\}$, $0 \leq \epsilon_2 \leq \min\{\bar{\beta}_i, 1 - \bar{\beta}_i\}$ and $0 \leq \epsilon_3 \leq \min\{\bar{\gamma}, 1 - \bar{\gamma}\}$.

Remark 1 *In (3), the Bernoulli distributed random variables α_k, $\beta_{k,i}$ $(i = 1, 2, \ldots, q)$ and γ_k are introduced to respectively characterize the so-called ROU, MSCD-s and RVNs. It should be mentioned that MSCDs are employed to describe the possible existence of different delays during the signals transmission via the various communication channels. Here, we make the attempt to* *characterize the case when the system in reality is unavoidably influenced by various types of network-induced phenomena and the occurrence probabilities could be uncertain due to some reasons [33, 34]. To be specific, the scalars $\Delta\alpha$, $\Delta\beta_i$ $(i = 1, 2, \dots, q)$ and $\Delta\gamma$ are used to depict the uncertain occurrence probabilities due probably to case that the exact probability information may not be easily obtained. In reality, by the repeated statistical tests, it is not technically difficult to obtain the corresponding information of RVNs with uncertain occurrence probabilities (i.e., the occurrence probabilities and the related upper bounds). Subsequently, the major effort is made to propose new robust SMC scheme for addressed networked systems against the ROU, RVNs and MSCDs under uncertain occurrence probabilities.*

Remark 2 *It is worthwhile to notice that the considered q different communication delays $\tau_{k,i}$ $(i = 1, 2, \dots, q)$ in (1) are time-varying and our aim is to propose new control method by attenuating the effects from the communication delays. In order to simplify the design idea of SMC, we let the upper/lower bounds of communication delays be same with the upper bound τ_M and lower bound τ_m. In fact, if we consider more general situations (e.g. $\tau_{k,i} \in [\tau_{m,i}, \tau_{M,i}]$), one possible common upper/lower bounds could be obtained easily by setting $\tau_m = \min\limits_{1\le i\le q}\{\tau_{m,i}\}$, $\tau_M = \max\limits_{1\le i\le q}\{\tau_{M,i}\}$.*

The objective of the paper is to propose a robust SMC strategy such that, for all ROU, MSCDs and RVNs, the robust mean-square asymptotical stability of the resulted sliding motion is ensured by proposing a delay-dependent sufficient criterion.

To proceed, we introduce the following necessary lemmas.

Lemma 1 *For a, b and $P > 0$ of compatible dimensions, we have*

$$a^T b + b^T a \le a^T P a + b^T P^{-1} b.$$

Lemma 2 *(S-procedure) Let $Q = Q^T$, H and E be real matrices of compatible dimensions, and uncertain matrix F satisfies $F^T F \le I$. Then $Q + HFE + E^T F^T H^T < 0$ holds, if and only if there exists a positive constant ε such that $Q + \varepsilon H H^T + \varepsilon^{-1} E^T E < 0$ or,*

$$\begin{bmatrix} Q & \varepsilon H & E^T \\ \varepsilon H^T & -\varepsilon I & 0 \\ E & 0 & -\varepsilon I \end{bmatrix} < 0.$$

Lemma 3 *(Schur complement) Given constant matrices $\mathcal{U}_1, \mathcal{U}_2, \mathcal{U}_3$ where $\mathcal{U}_1 = \mathcal{U}_1^T$ and $\mathcal{U}_2 = \mathcal{U}_2^T > 0$, then $\mathcal{U}_1 + \mathcal{U}_3^T \mathcal{U}_2^{-1} \mathcal{U}_3 < 0$ if and only if*

$$\begin{bmatrix} \mathcal{U}_1 & \mathcal{U}_3^T \\ * & -\mathcal{U}_2 \end{bmatrix} < 0 \quad \text{or} \quad \begin{bmatrix} -\mathcal{U}_2 & \mathcal{U}_3 \\ * & \mathcal{U}_1 \end{bmatrix} < 0.$$

Lemma 4 *[35] Let $M \in \mathbb{R}^{n\times n}$ be a positive semi-definite matrix. For $x_i \in \mathbb{R}^n$ and $a_i \ge 0$ $(i = 1, 2, \dots)$, if the series concerned are convergent, then*

$$\left(\sum_{i=1}^{+\infty} a_i x_i\right)^T M \left(\sum_{i=1}^{+\infty} a_i x_i\right) \le \left(\sum_{i=1}^{+\infty} a_i\right) \sum_{i=1}^{+\infty} a_i x_i^T M x_i.$$

3 Main Results

In this section, we aim to discuss the robust SMC synthesis problem for addressed networked systems with MSCDs, RVNs and ROU. Firstly, a linear switching surface is introduced. subsequently, the robust stability analysis problem of the corresponding sliding motion is discussed and a delay-dependent sufficient condition is given in terms of the delay fractioning method. In what follows, an LMIs-based minimization problem is given by introducing a computational algorithm. Moreover, the discrete-time reachability condition is tested by proposing a robust sliding mode controller.

3.1 Switching surface

In the paper, choose the following linear sliding surface:

$$s_k = G x_k - G A x_{k-1}, \tag{4}$$

where real matrix $G \in \mathbb{R}^{m\times n}$ is to be determined such that GB is non-singular and $G\widehat{D} = 0$ with $\widehat{D} := \begin{bmatrix} D_1 & D_2 & E_1 \end{bmatrix}$. In what follows, we select $G = B^T P$ with $P > 0$ to guarantee the non-singularity of GB.

It is worthy of noting that the ideal quasi-sliding mode satisfies the following condition

$$s_{k+1} = s_k = 0. \tag{5}$$

Then, it follows from (1), (4) and (5) that the equivalent controller can be given by:

$$u_{eq} = -(GB)^{-1}G\Big[(\bar{\alpha}+\Delta\alpha)\Delta A x_k + A_\tau \sum_{i=1}^{q}(\bar{\beta}_i+\Delta\beta_i) \times x_{k-\tau_{k,i}}\Big] - f(x_k). \tag{6}$$

Substituting (6) into (1) yields the following sliding mode dynamics equation:

$$\begin{aligned} x_{k+1} = {} & \mathcal{A}_k - B(GB)^{-1}G\mathcal{A}_k + B(GB)^{-1}GAx_k \\ & + A_\tau \sum_{i=1}^{q}[\beta_{k,i} - (\bar{\beta}_i+\Delta\beta_i)]x_{k-\tau_{k,i}} \\ & + [\alpha_k - (\bar{\alpha}+\Delta\alpha)]\Delta A x_k + \gamma_k D_1 g_1(x_k) \\ & + (1-\gamma_k)D_2 g_2(x_k) + E_1 x_k w_k, \end{aligned} \tag{7}$$

where

$$\mathcal{A}_k = [A + (\bar{\alpha}+\Delta\alpha)\Delta A]x_k + A_\tau \sum_{i=1}^{q}(\bar{\beta}_i+\Delta\beta_i)x_{k-\tau_{k,i}}.$$

Now, we analyze the robust asymptotic stability in mean square of the resulted closed-loop system (7) via the Lyapunov stability theorem.

Theorem 1 *Given a constant $\rho \in (0,1)$, if there exist matrices $P>0$, $Q_j>0$, $R>0$, $S>0$, real matrices $\mathcal{X}$, $\mathcal{Y}_j$ and $\mathcal{Z}_j$ $(j=1,2,\ldots,q)$, and scalar $\varepsilon>0$ satisfying*

$$\begin{bmatrix} \Lambda+\varepsilon\widehat{E}^T\widehat{E} & \sqrt{\tau_M}\mathcal{X} & d\mathcal{Y}_j & \widetilde{\Xi}_A^T PB & 0 & 0 \\ * & -\rho P & 0 & 0 & 0 & 0 \\ * & * & -\rho P & 0 & 0 & 0 \\ * & * & * & -B^TPB & 0 & 0 \\ * & * & * & * & -P & PH \\ * & * & * & * & * & -\varepsilon I \end{bmatrix} < 0, \tag{8}$$

$$\begin{bmatrix} \Lambda+\varepsilon\widehat{E}^T\widehat{E} & \sqrt{\tau_M}\mathcal{X} & d\mathcal{Z}_j & \widetilde{\Xi}_A^T PB & 0 & 0 \\ * & -\rho P & 0 & 0 & 0 & 0 \\ * & * & -\rho P & 0 & 0 & 0 \\ * & * & * & -B^TPB & 0 & 0 \\ * & * & * & * & -P & PH \\ * & * & * & * & * & -\varepsilon I \end{bmatrix} < 0, \tag{9}$$

$$B^TP\widehat{D}=0, \tag{10}$$

where

$$\begin{aligned} \Lambda = {} & 8\tilde{\Xi}_A^T P\tilde{\Xi}_A + \tilde{\Xi}_E^T P\tilde{\Xi}_E + \Xi_I^T[(2\rho\hbar-1)P+Q]\Xi_I \\ & + \Xi_\tau^T\tilde{\Phi}_\tau\Xi_\tau + W_R^T\mathcal{R}W_R + W_S^T\mathcal{S}W_S \\ & + \gamma_1\Xi_{g_1}^T D_1^T PD_1\Xi_{g_1} + \gamma_2\Xi_{g_2}^T D_2^T PD_2\Xi_{g_2} \\ & + \Pi_{g_1}^T\mathcal{F}_1\Pi_{g_1} + \Pi_{g_2}^T\mathcal{F}_2\Pi_{g_2}, \end{aligned}$$

$$Q = (\tau_M - \tau_m + 1)\sum_{j=1}^{q} Q_j,$$

$$\Xi_I = \begin{bmatrix} I_{n\times n} & 0_{n\times(m+q+3)n} \end{bmatrix},$$

$$\tilde{\Xi}_A = \begin{bmatrix} \hat{h}A_{n\times n} & 0_{n\times(m+q+3)n} \end{bmatrix},$$

$$\tilde{\Xi}_E = \begin{bmatrix} \hat{h}(E_1)_{n\times n} & 0_{n\times(m+q+3)n} \end{bmatrix},$$

$$\Xi_\tau = \begin{bmatrix} 0_{qn\times(m+1)n} & I_{qn\times qn} & 0_{qn\times 3n} \end{bmatrix},$$

$$W_S = \begin{bmatrix} I_{n\times n} \quad 0_{n\times(m+q+3)n} \\ \hline 0_{n\times(m+q+1)n} \quad I_{n\times n} \quad 0_{n\times 2n} \end{bmatrix},$$

$$W_R = \begin{bmatrix} I_{mn\times mn} \quad 0_{mn\times(q+4)n} \\ \hline 0_{mn\times n} \quad I_{mn\times mn} \quad 0_{mn\times(q+3)n} \end{bmatrix},$$

$$\Xi_{g_1} = \begin{bmatrix} 0_{n\times(m+q+2)n} & I_{n\times n} & 0_{n\times n} \end{bmatrix},$$

$$\Xi_{g_2} = \begin{bmatrix} 0_{n\times(m+q+3)n} & I_{n\times n} \end{bmatrix}, \quad \mathcal{R} = \mathrm{diag}\{R, -R\},$$

$$\Pi_{g_1} = \begin{bmatrix} I_{n\times n} \quad 0_{n\times(m+q+3)n} \\ \hline 0_{n\times(m+q+2)n} \quad I_{n\times n} \quad 0_{n\times n} \end{bmatrix}, \mathcal{S} = \mathrm{diag}\{S, -S\},$$

$$\Pi_{g_2} = \begin{bmatrix} I_{n\times n} \quad 0_{n\times(m+q+3)n} \\ \hline 0_{n\times(m+q+3)n} \quad I_{n\times n} \end{bmatrix}, \widehat{\beta} = 4\left[\sum_{i=1}^{q}(\bar{\beta}_i+\epsilon_2)\right],$$

$$\tilde{\Phi}_\tau = \mathrm{diag}\left\{\tilde{\Phi}_\tau^1, \tilde{\Phi}_\tau^2, \ldots, \tilde{\Phi}_\tau^q\right\}, \tilde{\Phi}_\tau^j = \beta_j A_\tau^T PA_\tau - Q_j,$$

$$\widehat{E}^T=\begin{bmatrix}\alpha_1 E^T\\ 0_{(m+q+3)n\times n_F}\end{bmatrix},\mathcal{F}_i=\begin{bmatrix}-\tilde{F}_i & \bar{F}_i\\ * & -2I\end{bmatrix}(i=1,2),$$
$$\tilde{F}_i=F_{1i}^TF_{2i}+F_{2i}^TF_{1i},\ \bar{F}_i=F_{1i}^T+F_{2i}^T,$$
$$\alpha_1=\sqrt{(2\rho\hbar+1)(\bar{\alpha}+\epsilon_1)(7\bar{\alpha}+9\epsilon_1+1)},$$
$$\beta_j=(2\rho\hbar+1)(\bar{\beta}_j+\epsilon_2)(\widehat{\beta}+1-\bar{\beta}_j+\epsilon_2),$$
$$\gamma_1=2(2\rho\hbar+1)(\bar{\gamma}+\epsilon_3),\quad \gamma_2=2(2\rho\hbar+1)(1-\bar{\gamma}+\epsilon_3),$$
$$\hat{h}=\sqrt{2\rho\hbar+1},\quad \hbar=2\tau_M-\tau_m,\quad d=\sqrt{\tau_M-\tau_m}, \tag{11}$$

then the sliding mode dynamics (7) is robustly asymptotically stable in mean square sense.

Proof: Firstly, we choose the following Lyapunov-Krasovskii functional:

$$V_k=\sum_{i=1}^{5}V_{ik}, \tag{12}$$

where

$$V_{1k}=x_k^TPx_k,$$
$$V_{2k}=\sum_{l=k-\tau}^{k-1}\Gamma_l^TR\Gamma_l,\quad V_{3k}=\sum_{l=k-\tau_M}^{k-1}x_l^TSx_l,$$
$$V_{4k}=\sum_{j=1}^{q}\sum_{i=k-\tau_{k,j}}^{k-1}x_i^TQ_jx_i+\sum_{j=1}^{q}\sum_{s=\hat{\tau}_M}^{-\tau_m}\sum_{i=k+s}^{k-1}x_i^TQ_jx_i,$$
$$V_{5k}=\rho\sum_{j=\hat{\tau}_M}^{0}\sum_{l=k-1+j}^{k-1}\eta_l^TP\eta_l+\rho\sum_{j=\hat{\tau}_M}^{-\tau p}\sum_{l=k-1+j}^{k-1}\eta_l^TP\eta_l,$$
$$\Gamma_l=\mathrm{col}\left\{x_l,x_{l-\tau},\ldots,x_{l-(m-1)\tau}\right\},$$
$$\eta_l=x_{l+1}-x_l,\quad \hat{\tau}_M=-\tau_M+1$$

with $P>0$, $Q_j>0$ $(j=1,2,\ldots,q)$, $R>0$ and $S>0$ being matrices to be designed. Then, let's calculate the difference of V_k along (7)

$$\mathbb{E}\{\Delta V_k\}=\sum_{i=1}^{5}\mathbb{E}\{\Delta V_{ik}\},$$

where

$$\begin{aligned}
&\mathbb{E}\left\{\Delta V_{1k}\right\}\\
=&\mathbb{E}\Big\{\mathcal{A}_k^TP\mathcal{A}_k-\mathcal{A}_k^TG^T(GB)^{-1}G\mathcal{A}_k+2(\bar{\gamma}+\Delta\gamma)\\
&\times\mathcal{A}_k^TPD_1g_1(x_k)+2[1-(\bar{\gamma}+\Delta\gamma)]\mathcal{A}_k^TPD_2g_2(x_k)\\
&+x_k^TA^TG^T(GB)^{-1}GAx_k+(\bar{\alpha}+\Delta\alpha)[1-(\bar{\alpha}+\Delta\alpha)]\\
&\times x_k^T\Delta A^TP\Delta Ax_k+(\bar{\gamma}+\Delta\gamma)g_1^T(x_k)D_1^TPD_1g_1(x_k)\\
&+[1-(\bar{\gamma}+\Delta\gamma)]g_2^T(x_k)D_2^TPD_2g_2(x_k)-x_k^TPx_k\\
&+x_k^TE_1^TPE_1x_k+\left(\sum_{i=1}^{q}[\beta_{k,i}-(\bar{\beta}_i+\Delta\beta_i)]x_{k-\tau_{k,i}}\right)^T\\
&\times A_\tau^TPA_\tau\left(\sum_{i=1}^{q}[\beta_{k,i}-(\bar{\beta}_i+\Delta\beta_i)]x_{k-\tau_{k,i}}\right)\Big\}
\end{aligned}\tag{13}$$

with $\mathcal{A}_k$ being defined below (7). By using Lemmas 1 and 4, it follows that

$$\begin{aligned}
&2(\bar{\gamma}+\Delta\gamma)\mathcal{A}_k^TPD_1g_1(x_k)\\
\le&(\bar{\gamma}+\Delta\gamma)\mathcal{A}_k^TP\mathcal{A}_k+(\bar{\gamma}+\Delta\gamma)g_1^T(x_k)D_1^TPD_1g_1(x_k)\\
\le&(\bar{\gamma}+\Delta\gamma)\mathcal{A}_k^TP\mathcal{A}_k+(\bar{\gamma}+\epsilon_3)g_1^T(x_k)D_1^TPD_1g_1(x_k),
\end{aligned}\tag{14}$$

$$\begin{aligned}
&2[1-(\bar{\gamma}+\Delta\gamma)]\mathcal{A}_k^TPD_2g_2(x_k)\\
\le&[1-(\bar{\gamma}+\Delta\gamma)]\mathcal{A}_k^TP\mathcal{A}_k+[1-(\bar{\gamma}+\Delta\gamma)]g_2^T(x_k)D_2^T\\
&\times PD_2g_2(x_k)\\
\le&[1-(\bar{\gamma}+\Delta\gamma)]\mathcal{A}_k^TP\mathcal{A}_k+(1-\bar{\gamma}+\epsilon_3)g_2^T(x_k)D_2^TP\\
&\times D_2g_2(x_k),
\end{aligned}\tag{15}$$

$$\begin{aligned}
&\mathcal{A}_k^TP\mathcal{A}_k\\
\le&2x_k^T[A+(\bar{\alpha}+\Delta\alpha)\Delta A]^TP[A+(\bar{\alpha}+\Delta\alpha)\Delta A]x_k\\
&+2\left[\sum_{i=1}^{q}(\bar{\beta}_i+\Delta\beta_i)x_{k-\tau_{k,i}}\right]^TA_\tau^TPA_\tau\Bigg[\sum_{i=1}^{q}(\bar{\beta}_i+\Delta\beta_i)\\
&\times x_{k-\tau_{k,i}}\Bigg]\\
\le&4x_k^TA^TPAx_k+4(\bar{\alpha}+\Delta\alpha)^2x_k^T\Delta A^TP\Delta Ax_k\\
&+2\left[\sum_{i=1}^{q}(\bar{\beta}_i+\Delta\beta_i)\right]\sum_{i=1}^{q}(\bar{\beta}_i+\Delta\beta_i)x_{k-\tau_{k,i}}^TA_\tau^TPA_\tau\\
&\times x_{k-\tau_{k,i}}\\
\le&4x_k^TA^TPAx_k+4(\bar{\alpha}+\epsilon_1)^2x_k^T\Delta A^TP\Delta Ax_k\\
&+\frac{\widehat{\beta}}{2}\sum_{i=1}^{q}(\bar{\beta}_i+\epsilon_2)x_{k-\tau_{k,i}}^TA_\tau^TPA_\tau x_{k-\tau_{k,i}}.
\end{aligned}\tag{16}$$

Notice that

$$\begin{aligned}
&\mathbb{E}\Big\{\Big(\sum_{i=1}^{q}[\beta_{k,i}-(\bar{\beta}_i+\Delta\beta_i)]x_{k-\tau_{k,i}}\Big)^TA_\tau^TPA_\tau\\
&\times\Big(\sum_{i=1}^{q}[\beta_{k,i}-(\bar{\beta}_i+\Delta\beta_i)]x_{k-\tau_{k,i}}\Big)\Big\}\\
=&\mathbb{E}\Big\{\sum_{i=1}^{q}[\beta_{k,i}-(\bar{\beta}_i+\Delta\beta_i)]^2x_{k-\tau_{k,i}}^TA_\tau^TPA_\tau x_{k-\tau_{k,i}}\Big\}\\
\le&\mathbb{E}\Big\{\sum_{i=1}^{q}(\bar{\beta}_i+\epsilon_2)(1-\bar{\beta}_i+\epsilon_2)x_{k-\tau_{k,i}}^TA_\tau^TPA_\tau x_{k-\tau_{k,i}}\Big\},
\end{aligned}\tag{17}$$

then it follows from (13)-(17) that

$$
\begin{aligned}
&\mathbb{E}\left\{\Delta V_{1k}\right\} \\
\leq &\,\mathbb{E}\Big\{8x_k^T A^T P A x_k + 8(\bar{\alpha}+\epsilon_1)^2 x_k^T \Delta A^T P \Delta A x_k \\
&+\widehat{\beta}\sum_{i=1}^{q}(\bar{\beta}_i+\epsilon_2)x_{k-\tau_{k,i}}^T A_\tau^T P A_\tau x_{k-\tau_{k,i}} + x_k^T A^T G^T \\
&\times (GB)^{-1} G A x_k + (\bar{\alpha}+\epsilon_1)(1-\bar{\alpha}+\epsilon_1) x_k^T \Delta A^T P \\
&\times \Delta A x_k + \sum_{i=1}^{q}(\bar{\beta}_i+\epsilon_2)(1-\bar{\beta}_i+\epsilon_2)x_{k-\tau_{k,i}}^T A_\tau^T P A_\tau \\
&\times x_{k-\tau_{k,i}} + 2(\bar{\gamma}+\epsilon_3) g_1^T(x_k) D_1^T P D_1 g_1(x_k) \\
&+2(1-\bar{\gamma}+\epsilon_3) g_2^T(x_k) D_2^T P D_2 g_2(x_k) + x_k^T E_1^T P E_1 \\
&\times x_k - x_k^T P x_k\Big\} \\
= &\,\mathbb{E}\Big\{8x_k^T A^T P A x_k + x_k^T A^T G^T (GB)^{-1} G A x_k + x_k^T E_1^T \\
&\times P E_1 x_k - x_k^T P x_k + \widetilde{\alpha}^2 x_k^T \Delta A^T P \Delta A x_k + \sum_{i=1}^{q}\widetilde{\beta}_i \\
&\times x_{k-\tau_{k,i}}^T A_\tau^T P A_\tau x_{k-\tau_{k,i}} + \widetilde{\gamma}_1 g_1^T(x_k) D_1^T P D_1 g_1(x_k) \\
&+\widetilde{\gamma}_2 g_2^T(x_k) D_2^T P D_2 g_2(x_k)\Big\} \\
= &\,\mathbb{E}\Big\{\xi_k^T\left[8\Xi_A^T P \Xi_A + \Xi_A^T G^T (GB)^{-1} G \Xi_A + \Xi_E^T P \Xi_E - \Xi_I^T\right. \\
&\left.\times P\Xi_I + \Xi_{\delta_A}^T P \Xi_{\delta_A} + \Xi_\tau^T \Phi_\tau \Xi_\tau\right]\xi_k + \widetilde{\gamma}_1 g_1^T(x_k) D_1^T P D_1 \\
&\times g_1(x_k) + \widetilde{\gamma}_2 g_2^T(x_k) D_2^T P D_2 g_2(x_k)\Big\}, \qquad (18)
\end{aligned}
$$

where

$$
\begin{aligned}
\xi_k &= \begin{bmatrix} \Gamma_k^T & \xi_{k,\tau}^T & g_1^T(x_k) & g_2^T(x_k) \end{bmatrix}^T, \\
\Xi_A &= \begin{bmatrix} A_{n\times n} & 0_{n\times(m+q+3)n} \end{bmatrix}, \\
\Xi_E &= \begin{bmatrix} (E_1)_{n\times n} & 0_{n\times(m+q+3)n} \end{bmatrix}, \\
\Xi_{\delta_A} &= \begin{bmatrix} \widetilde{\alpha}\Delta A_{n\times n} & 0_{n\times(m+q+3)n} \end{bmatrix}, \\
\xi_{k,\tau} &= \begin{bmatrix} x_{k-\tau_m}^T & x_{k-\tau_{k,1}}^T & \ldots & x_{k-\tau_{k,q}}^T & x_{k-\tau_M}^T \end{bmatrix}^T, \\
\Phi_\tau &= \mathrm{diag}\left\{\widetilde{\beta}_1 A_\tau^T P A_\tau, \widetilde{\beta}_2 A_\tau^T P A_\tau, \ldots, \widetilde{\beta}_q A_\tau^T P A_\tau\right\}, \\
\widetilde{\alpha} &= \sqrt{8(\bar{\alpha}+\epsilon_1)^2 + (\bar{\alpha}+\epsilon_1)(1-\bar{\alpha}+\epsilon_1)}, \\
\widetilde{\beta}_i &= (\bar{\beta}_i+\epsilon_2)(\widehat{\beta}+1-\bar{\beta}_i+\epsilon_2), \widetilde{\gamma}_1 = 2(\bar{\gamma}+\epsilon_3), \\
\widetilde{\gamma}_2 &= 2(1-\bar{\gamma}+\epsilon_3),
\end{aligned}
$$

and Ξ_τ, Ξ_I as well as $\widehat{\beta}$ are defined in (11).

Similarly, it can be derived that

$$
\begin{aligned}
&\mathbb{E}\left\{\Delta V_{2k}\right\} \\
=&\,\mathbb{E}\left\{\sum_{l=k+1-\tau}^{k}\Gamma_l^T R \Gamma_l - \sum_{l=k-\tau}^{k-1}\Gamma_l^T R \Gamma_l\right\} \\
=&\,\mathbb{E}\left\{\Gamma_k^T R \Gamma_k - \Gamma_{k-\tau}^T R \Gamma_{k-\tau}\right\}, \qquad (19) \\
&\mathbb{E}\left\{\Delta V_{3k}\right\} \\
=&\,\mathbb{E}\left\{\sum_{l=k+1-\tau_M}^{k} x_l^T S x_l - \sum_{l=k-\tau_M}^{k-1} x_l^T S x_l\right\} \\
=&\,\mathbb{E}\left\{x_k^T S x_k - x_{k-\tau_M}^T S x_{k-\tau_M}\right\}, \qquad (20) \\
&\mathbb{E}\left\{\Delta V_{4k}\right\} \\
=&\,\mathbb{E}\Big\{\sum_{j=1}^{q}\sum_{i=k+1-\tau_{k+1,j}}^{k} x_i^T Q_j x_i - \sum_{j=1}^{q}\sum_{i=k-\tau_{k,j}}^{k-1} x_i^T Q_j x_i \\
&+\sum_{j=1}^{q}\sum_{s=-\tau_M+1}^{-\tau_m}\left[x_k^T Q_j x_k - x_{k+s}^T Q_j x_{k+s}\right]\Big\} \\
\leq&\,\mathbb{E}\Big\{\sum_{j=1}^{q}\left[x_k^T Q_j x_k - x_{k-\tau_{k,j}}^T Q_j x_{k-\tau_{k,j}}\right] \\
&+\sum_{j=1}^{q}\sum_{i=k+1-\tau_M}^{k-\tau_m} x_i^T Q_j x_i + (\tau_M-\tau_m)\sum_{j=1}^{q} x_k^T Q_j x_k \\
&-\sum_{j=1}^{q}\sum_{i=k+1-\tau_M}^{k-\tau_m} x_i^T Q_j x_i\Big\} \\
=&\,\mathbb{E}\Big\{(\tau_M-\tau_m+1)\sum_{j=1}^{q} x_k^T Q_j x_k - \sum_{j=1}^{q}\big(x_{k-\tau_{k,j}}^T Q_j \\
&\times x_{k-\tau_{k,j}}\big)\Big\}, \qquad (21) \\
&\mathbb{E}\Big\{\Delta V_{5k}\Big\} \\
=&\,\mathbb{E}\Big\{\rho\sum_{j=-\tau_M+1}^{0}\sum_{l=k+j}^{k}\eta_l^T P \eta_l - \rho\sum_{j=-\tau_M+1}^{0}\sum_{l=k-1+j}^{k-1}\eta_l^T P \eta_l \\
&+\rho\sum_{j=-\tau_M+1}^{-\tau p}\sum_{l=k+j}^{k}\eta_l^T P \eta_l - \rho\sum_{j=-\tau_M+1}^{-\tau p}\sum_{l=k-1+j}^{k-1}\eta_l^T P \eta_l\Big\} \\
=&\,\mathbb{E}\Big\{\rho\tau_M\eta_k^T P \eta_k - \rho\sum_{l=k-\tau_M}^{k-1}\eta_l^T P \eta_l + \rho(\tau_M-\tau_m)\eta_k^T P \eta_k \\
&-\sum_{l=k-\tau_M}^{k-\tau_m-1}\eta_l^T \rho P \eta_l\Big\} \\
=&\,\mathbb{E}\Big\{\eta_k^T \rho\hbar P \eta_k - \sum_{l=k-\tau_M}^{k-1}\eta_l^T \rho P \eta_l \\
&-\frac{1}{q}\left(\sum_{l=k-\tau_{k,1}}^{k-\tau_m-1}\eta_l^T \rho P \eta_l + \sum_{l=k-\tau_M}^{k-\tau_{k,1}-1}\eta_l^T \rho P \eta_l\right)
\end{aligned}
$$

$$-\frac{1}{q}\left(\sum_{l=k-\tau_{k,2}}^{k-\tau_m-1}\eta_l^T\rho P\eta_l+\sum_{l=k-\tau_M}^{k-\tau_{k,2}-1}\eta_l^T\rho P\eta_l\right)$$
$$\cdots$$
$$-\frac{1}{q}\left(\sum_{l=k-\tau_{k,q}}^{k-\tau_m-1}\eta_l^T\rho P\eta_l+\sum_{l=k-\tau_M}^{k-\tau_{k,q}-1}\eta_l^T\rho P\eta_l\right)\Bigg\} \qquad (22)$$

with $\hbar = 2\tau_M - \tau_m$. Then, it follows from (19)-(22) that

$$\begin{aligned}
&\sum_{i=2}^{5}\mathbb{E}\{\Delta V_{ik}\}\\
&\le \mathbb{E}\Bigg\{2\rho\hbar x_{k+1}^T P x_{k+1} + 2\rho\hbar x_k^T P x_k + \Gamma_k^T R\Gamma_k - \Gamma_{k-\tau}^T R\\
&\times\Gamma_{k-\tau} + x_k^T S x_k - x_{k-\tau_M}^T S x_{k-\tau_M} + (\tau_M-\tau_m+1)\\
&\times\sum_{j=1}^{q} x_k^T Q_j x_k - \sum_{j=1}^{q} x_{k-\tau_{k,j}}^T Q_j x_{k-\tau_{k,j}} - \sum_{l=k-\tau_M}^{k-1}\eta_l^T\rho P\eta_l\\
&-\frac{1}{q}\left(\sum_{l=k-\tau_{k,1}}^{k-\tau_m-1}\eta_l^T\rho P\eta_l+\sum_{l=k-\tau_M}^{k-\tau_{k,1}-1}\eta_l^T\rho P\eta_l\right)\\
&-\frac{1}{q}\left(\sum_{l=k-\tau_{k,2}}^{k-\tau_m-1}\eta_l^T\rho P\eta_l+\sum_{l=k-\tau_M}^{k-\tau_{k,2}-1}\eta_l^T\rho P\eta_l\right)\\
&\cdots\\
&-\frac{1}{q}\left(\sum_{l=k-\tau_{k,q}}^{k-\tau_m-1}\eta_l^T\rho P\eta_l+\sum_{l=k-\tau_M}^{k-\tau_{k,q}-1}\eta_l^T\rho P\eta_l\right)\Bigg\}. \qquad (23)
\end{aligned}$$

In view of (18) and (23), we have

$$\begin{aligned}
&\mathbb{E}\{\Delta V_k\}\\
&\le \mathbb{E}\Bigg\{\xi_k^T\Big[8\tilde{\Xi}_A^T P\tilde{\Xi}_A + \tilde{\Xi}_E^T P\tilde{\Xi}_E + (2\rho\hbar-1)\Xi_I^T P\Xi_I\\
&+\Xi_I^T Q\Xi_I + \tilde{\Xi}_A^T G^T(GB)^{-1}G\tilde{\Xi}_A + \tilde{\Xi}_{\delta_A}^T P\tilde{\Xi}_{\delta_A}\\
&+\Xi_\tau^T\tilde{\Phi}_\tau\Xi_\tau + W_R^T\mathcal{R}W_R + W_S^T\mathcal{S}W_S + \gamma_1\Xi_{g_1}^T D_1^T P\\
&\times D_1\Xi_{g_1} + \gamma_2\Xi_{g_2}^T D_2^T P D_2\Xi_{g_2}\Big]\xi_k - \sum_{l=k-\tau_M}^{k-1}\eta_l^T\rho P\eta_l\\
&-\frac{1}{q}\left(\sum_{l=k-\tau_{k,1}}^{k-\tau_m-1}\eta_l^T\rho P\eta_l+\sum_{l=k-\tau_M}^{k-\tau_{k,1}-1}\eta_l^T\rho P\eta_l\right)\\
&-\frac{1}{q}\left(\sum_{l=k-\tau_{k,2}}^{k-\tau_m-1}\eta_l^T\rho P\eta_l+\sum_{l=k-\tau_M}^{k-\tau_{k,2}-1}\eta_l^T\rho P\eta_l\right)\\
&\cdots\\
&-\frac{1}{q}\left(\sum_{l=k-\tau_{k,q}}^{k-\tau_m-1}\eta_l^T\rho P\eta_l+\sum_{l=k-\tau_M}^{k-\tau_{k,q}-1}\eta_l^T\rho P\eta_l\right)\Bigg\}, \qquad (24)
\end{aligned}$$

where

$$\tilde{\Xi}_{\delta_A} = \begin{bmatrix}\alpha_1\Delta A_{n\times n} & 0_{n\times(m+q+3)n}\end{bmatrix},$$

and other matrices as well as parameters can be found in (11).

By considering (2), it is not difficult to obtain that

$$\begin{bmatrix} x_k \\ g_i(x_k)\end{bmatrix}^T \begin{bmatrix} -\tilde{F}_i & \bar{F}_i \\ * & -2I\end{bmatrix}\begin{bmatrix} x_k \\ g_i(x_k)\end{bmatrix} \ge 0 \ \ (i=1,2), \qquad (25)$$

where $\tilde{F}_i = F_{1i}^T F_{2i} + F_{2i}^T F_{1i}$ and $\bar{F}_i = F_{1i}^T + F_{2i}^T$. Furthermore, for any matrices $\mathcal{X}$, $\mathcal{Y}_j$, $\mathcal{Z}_j$ $(j = 1, 2, \ldots, q)$ with appropriate dimensions, we arrive at

$$\begin{aligned}
0 &= 2\xi_k^T\mathcal{X}\left[x_k - x_{k-\tau_M} - \sum_{l=k-\tau_M}^{k-1}\eta_l\right],\\
0 &= \frac{1}{q}\times 2\xi_k^T\mathcal{Y}_1\left[x_{k-\tau_m} - x_{k-\tau_{k,1}} - \sum_{l=k-\tau_{k,1}}^{k-\tau_m-1}\eta_l\right],\\
0 &= \frac{1}{q}\times 2\xi_k^T\mathcal{Z}_1\left[x_{k-\tau_{k,1}} - x_{k-\tau_M} - \sum_{l=k-\tau_M}^{k-\tau_{k,1}-1}\eta_l\right],\\
0 &= \frac{1}{q}\times 2\xi_k^T\mathcal{Y}_2\left[x_{k-\tau_m} - x_{k-\tau_{k,2}} - \sum_{l=k-\tau_{k,2}}^{k-\tau_m-1}\eta_l\right],\\
0 &= \frac{1}{q}\times 2\xi_k^T\mathcal{Z}_2\left[x_{k-\tau_{k,2}} - x_{k-\tau_M} - \sum_{l=k-\tau_M}^{k-\tau_{k,2}-1}\eta_l\right],\\
&\cdots\\
0 &= \frac{1}{q}\times 2\xi_k^T\mathcal{Y}_q\left[x_{k-\tau_m} - x_{k-\tau_{k,q}} - \sum_{l=k-\tau_{k,q}}^{k-\tau_m-1}\eta_l\right],\\
0 &= \frac{1}{q}\times 2\xi_k^T\mathcal{Z}_q\left[x_{k-\tau_{k,q}} - x_{k-\tau_M} - \sum_{l=k-\tau_M}^{k-\tau_{k,q}-1}\eta_l\right]. \qquad (26)
\end{aligned}$$

Then, substituting (25) and (26) into (24) yields

$$\begin{aligned}
&\mathbb{E}\{\Delta V_k\}\\
&\le\mathbb{E}\Bigg\{\xi_k^T\Big[8\tilde{\Xi}_A^T P\tilde{\Xi}_A + \tilde{\Xi}_E^T P\tilde{\Xi}_E + (2\rho\hbar-1)\Xi_I^T P\Xi_I + \Xi_I^T\\
&\times Q\Xi_I + \tilde{\Xi}_A^T G^T(GB)^{-1}G\tilde{\Xi}_A + \tilde{\Xi}_{\delta_A}^T P\tilde{\Xi}_{\delta_A} + \Xi_\tau^T\tilde{\Phi}_\tau\Xi_\tau\\
&+W_R^T\mathcal{R}W_R + W_S^T\mathcal{S}W_S + \gamma_1\Xi_{g_1}^T D_1^T P D_1\Xi_{g_1} + \gamma_2\\
&\times\Xi_{g_2}^T D_2^T P D_2\Xi_{g_2} + \Pi_{g_1}^T\mathcal{F}_1\Pi_{g_1} + \Pi_{g_2}^T\mathcal{F}_2\Pi_{g_2} + \Pi_1\\
&+\Pi_1^T + \tau_M\mathcal{X}(\rho P)^{-1}\mathcal{X}^T\Big]\xi_k
\end{aligned}$$

$$
\begin{aligned}
&+\frac{1}{q}(\tau_{k,1}-\tau_m)\xi_k^T\mathcal{Y}_1(\rho P)^{-1}\mathcal{Y}_1^T\xi_k\\
&+\frac{1}{q}(\tau_M-\tau_{k,1})\xi_k^T\mathcal{Z}_1(\rho P)^{-1}\mathcal{Z}_1^T\xi_k\\
&+\frac{1}{q}(\tau_{k,2}-\tau_m)\xi_k^T\mathcal{Y}_2(\rho P)^{-1}\mathcal{Y}_2^T\xi_k\\
&+\frac{1}{q}(\tau_M-\tau_{k,2})\xi_k^T\mathcal{Z}_2(\rho P)^{-1}\mathcal{Z}_2^T\xi_k\\
&\dots\\
&+\frac{1}{q}(\tau_{k,q}-\tau_m)\xi_k^T\mathcal{Y}_q(\rho P)^{-1}\mathcal{Y}_q^T\xi_k\\
&+\frac{1}{q}(\tau_M-\tau_{k,q})\xi_k^T\mathcal{Z}_q(\rho P)^{-1}\mathcal{Z}_q^T\xi_k\\
&-\Big(\Sigma_0+\Sigma_{11}+\Sigma_{12}+\Sigma_{21}+\Sigma_{22}+\cdots+\Sigma_{q,1}+\Sigma_{q,2}\Big)\Big\}\\
\le&\mathbb{E}\Big\{\xi_k^T\Big[\Pi+\tau_M\mathcal{X}(\rho P)^{-1}\mathcal{X}^T\Big]\xi_k\\
&+\frac{1}{q}(\tau_{k,1}-\tau_m)\xi_k^T\mathcal{Y}_1(\rho P)^{-1}\mathcal{Y}_1^T\xi_k\\
&+\frac{1}{q}(\tau_M-\tau_{k,1})\xi_k^T\mathcal{Z}_1(\rho P)^{-1}\mathcal{Z}_1^T\xi_k\\
&+\frac{1}{q}(\tau_{k,2}-\tau_m)\xi_k^T\mathcal{Y}_2(\rho P)^{-1}\mathcal{Y}_2^T\xi_k\\
&+\frac{1}{q}(\tau_M-\tau_{k,2})\xi_k^T\mathcal{Z}_2(\rho P)^{-1}\mathcal{Z}_2^T\xi_k\\
&\dots\\
&+\frac{1}{q}(\tau_{k,q}-\tau_m)\xi_k^T\mathcal{Y}_q(\rho P)^{-1}\mathcal{Y}_q^T\xi_k\\
&+\frac{1}{q}(\tau_M-\tau_{k,q})\xi_k^T\mathcal{Z}_q(\rho P)^{-1}\mathcal{Z}_q^T\xi_k\Big\}\\
=&\mathbb{E}\Big\{\xi_k^T\Big[\frac{1}{q}\times\frac{\tau_{k,1}-\tau_m}{\tau_M-\tau_m}\Big(\Pi+\tau_M\mathcal{X}(\rho P)^{-1}\mathcal{X}^T\\
&+(\tau_M-\tau_m)\mathcal{Y}_1(\rho P)^{-1}\mathcal{Y}_1^T\Big)+\frac{1}{q}\times\frac{\tau_M-\tau_{k,1}}{\tau_M-\tau_m}\\
&\times\Big(\Pi+\tau_M\mathcal{X}(\rho P)^{-1}\mathcal{X}^T+(\tau_M-\tau_m)\mathcal{Z}_1(\rho P)^{-1}\mathcal{Z}_1^T\Big)\\
&+\frac{1}{q}\times\frac{\tau_{k,2}-\tau_m}{\tau_M-\tau_m}\Big(\Pi+\tau_M\mathcal{X}\times(\rho P)^{-1}\mathcal{X}^T+(\tau_M-\tau_m)\\
&\times\mathcal{Y}_2(\rho P)^{-1}\mathcal{Y}_2^T\Big)+\frac{1}{q}\times\frac{\tau_M-\tau_{k,2}}{\tau_M-\tau_m}\times\Big(\Pi+\tau_M\mathcal{X}(\rho P)^{-1}\\
&\times\mathcal{X}^T+(\tau_M-\tau_m)\mathcal{Z}_2(\rho P)^{-1}\mathcal{Z}_2^T\Big)\\
&\dots\\
&+\frac{1}{q}\times\frac{\tau_{k,q}-\tau_m}{\tau_M-\tau_m}\Big(\Pi+\tau_M\mathcal{X}(\rho P)^{-1}\mathcal{X}^T+(\tau_M-\tau_m)\\
&\times\mathcal{Y}_q(\rho P)^{-1}\mathcal{Y}_q^T\Big)+\frac{1}{q}\times\frac{\tau_M-\tau_{k,q}}{\tau_M-\tau_m}\Big(\Pi+\tau_M\mathcal{X}(\rho P)^{-1}\\
&\times\mathcal{X}^T+(\tau_M-\tau_m)\mathcal{Z}_q(\rho P)^{-1}\mathcal{Z}_q^T\Big)\Big]\xi_k\Big\},
\end{aligned}
$$

where

$$
\begin{aligned}
\Pi\;=\;&8\tilde{\Xi}_A^TP\tilde{\Xi}_A+\tilde{\Xi}_E^TP\tilde{\Xi}_E+\Xi_I^T[(2\rho\hbar-1)P+Q]\Xi_I\\
&+\tilde{\Xi}_A^TG^T(GB)^{-1}G\tilde{\Xi}_A+\tilde{\Xi}_{\delta_A}^TP\tilde{\Xi}_{\delta_A}+\Xi_\tau^T\tilde{\Phi}_\tau\Xi_\tau\\
&+W_R^T\mathcal{R}W_R+W_S^T\mathcal{S}W_S+\gamma_1\Xi_{g_1}^TD_1^TPD_1\Xi_{g_1}\\
&+\gamma_2\Xi_{g_2}^TD_2^TPD_2\Xi_{g_2}+\Pi_{g_1}^T\mathcal{F}_1\Pi_{g_1}+\Pi_{g_2}^T\mathcal{F}_2\Pi_{g_2}\\
&+\Pi_1+\Pi_1^T,
\end{aligned}
$$

$$
\Pi_1=\frac{1}{q}\Big[\,\mathcal{X}\;\mathcal{Y}_1\;\mathcal{Z}_1\;\mathcal{Y}_2\;\mathcal{Z}_2\;\dots\;\mathcal{Y}_q\;\mathcal{Z}_q\,\Big]
\times\begin{bmatrix}
\underline{qI_{n\times n}\;\;0_{n\times(m+q)n}\;\;-qI_{n\times n}\;\;0_{n\times 2n}}\\
\underline{0_{n\times mn}\;\;I_{n\times n}\;\;-I_{n\times n}\;\;0_{n\times(q+2)n}}\\
\underline{0_{n\times(m+1)n}\;\;I_{n\times n}\;\;0_{n\times(q-1)n}\;\;-I_{n\times n}\;\;0_{n\times 2n}}\\
\underline{0_{n\times mn}\;\;I_{n\times n}\;\;0_{n\times n}\;\;-I_{n\times n}\;\;0_{n\times(q+1)n}}\\
\underline{0_{n\times(m+2)n}\;\;I_{n\times n}\;\;0_{n\times(q-2)n}\;\;-I_{n\times n}\;\;0_{n\times 2n}}\\
\dots\\
\underline{0_{n\times mn}\;\;I_{n\times n}\;\;0_{n\times(q-1)n}\;\;-I_{n\times n}\;\;0_{n\times 3n}}\\
\underline{0_{n\times(m+q)n}\;\;I_{n\times n}\;\;-I_{n\times n}\;\;0_{n\times 2n}}
\end{bmatrix},
$$

$$
\begin{aligned}
\Sigma_0\;=\;&\sum_{l=k-\tau_M}^{k-1}(\rho P\eta_l+\mathcal{X}^T\xi_k)^T(\rho P)^{-1}(\rho P\eta_l+\mathcal{X}^T\xi_k),\\
\Sigma_{11}\;=\;&\frac{1}{q}\sum_{l=k-\tau_{k,1}}^{k-\tau_m-1}(\rho P\eta_l+\mathcal{Y}_1^T\xi_k)^T(\rho P)^{-1}(\rho P\eta_l+\mathcal{Y}_1^T\xi_k),\\
\Sigma_{12}\;=\;&\frac{1}{q}\sum_{l=k-\tau_M}^{k-\tau_{k,1}-1}(\rho P\eta_l+\mathcal{Z}_1^T\xi_k)^T(\rho P)^{-1}(\rho P\eta_l+\mathcal{Z}_1^T\xi_k),\\
\Sigma_{21}\;=\;&\frac{1}{q}\sum_{l=k-\tau_{k,2}}^{k-\tau_m-1}(\rho P\eta_l+\mathcal{Y}_2^T\xi_k)^T(\rho P)^{-1}(\rho P\eta_l+\mathcal{Y}_2^T\xi_k),\\
\Sigma_{22}\;=\;&\frac{1}{q}\sum_{l=k-\tau_M}^{k-\tau_{k,2}-1}(\rho P\eta_l+\mathcal{Z}_2^T\xi_k)^T(\rho P)^{-1}(\rho P\eta_l+\mathcal{Z}_2^T\xi_k),\\
&\dots\\
\Sigma_{q,1}\;=\;&\frac{1}{q}\sum_{l=k-\tau_{k,q}}^{k-\tau_m-1}(\rho P\eta_l+\mathcal{Y}_q^T\xi_k)^T(\rho P)^{-1}(\rho P\eta_l+\mathcal{Y}_q^T\xi_k),\\
\Sigma_{q,2}\;=\;&\frac{1}{q}\sum_{l=k-\tau_M}^{k-\tau_{k,q}-1}(\rho P\eta_l+\mathcal{Z}_q^T\xi_k)^T(\rho P)^{-1}(\rho P\eta_l+\mathcal{Z}_q^T\xi_k),
\end{aligned}
$$

and Π_{g_1}, Π_{g_2}, $\mathcal{F}_i$ $(i=1,2)$ are defined in (11).

Finally, by utilizing the Lemmas 2-3, we can conclude that $\mathbb{E}\{\Delta V_k\}<0$ can be guaranteed by matrix inequalities (8) and (9). Hence, the sliding mode dynamics (7) is robustly asymptotically mean-square stable.

Remark 3 *During the proof of Theorem 1, we can see that more matrices should be found due to the utilization of the delay-fractioning approach. Accompanying with the rapid developments of the computation techniques, the resulted issue of the computation burden is not serious. Actually, there indeed exists a tradeoff between the computation burden and the conservatism. For this issue, we decide to adopt a compromise in an acceptable range, i.e., reduce the conservatism but increase certain computation burden.*

Remark 4 *In Theorem 1, a delay-dependent stability condition is presented for the sliding mode dynamics. It should be pointed out that, due to the existence of the equality constraint (10), the feasibility of newly proposed sufficient criterion cannot be easily checked. By utilizing the subsequent computational algorithm, the desired performance requirement can be ensured by solving a minimization problem.*

3.2 Computational Algorithm

It should be pointed out that the sufficient condition in Theorem 1 is non-convex due to the equality constraint (10). Based on the technique developed in [36], the condition (10) can be equivalently expressed by $\mathrm{tr}[(B^TP\widehat{D})^TB^TP\widehat{D}]=0$. Next, by introducing the matrix inequality $(B^TP\widehat{D})^TB^TP\widehat{D}\le\mu I$ with $\mu>0$ and utilizing the Lemma 3, it is easy to obtain

$$\begin{bmatrix} -\mu I & \widehat{D}^TPB \\ B^TP\widehat{D} & -I \end{bmatrix}\le 0. \tag{27}$$

Then, it is clear that the original non-convex problem can be converted into the minimization problem as follows:

$$\begin{aligned}&\min\ \mu\\&\text{subject to }\ (8),(9)\text{ and }(27).\end{aligned} \tag{28}$$

Remark 5 *It is worthwhile to mention that the minimization problem in (28) is an LMI-based one and its feasibility can be easily checked via the standard numerical software, then the stability problem of the resulted sliding motion is solved.*

3.3 Design of sliding mode controller

In this subsection, the reachability of the sliding surface (4) is discussed by designing the SMC law.

As in [9], we need to check the following inequalities with hope to achieve the desired performance:

$$\begin{cases}\Delta s_k=s_{k+1}-s_k\le-\kappa U\mathrm{sgn}[s_k]-\kappa Vs_k,\ \text{if } s_k>0,\\ \Delta s_k=s_{k+1}-s_k\ge-\kappa U\mathrm{sgn}[s_k]-\kappa Vs_k,\ \text{if } s_k<0,\end{cases}$$

where $U=\mathrm{diag}\{\mu_1,\mu_2,\dots,\mu_m\}\in\mathbb{R}^{m\times m}$, $V=\mathrm{diag}\{\nu_1,\nu_2,\dots,\nu_m\}\in\mathbb{R}^{m\times m}$, κ represents the sampling period, $\mu_i>0$ and $\nu_i>0$ $(i=1,2,\dots,m)$ are properly chosen constants satisfying $0<1-\kappa\nu_i<1$. To proceed, set

$$\begin{aligned}&\Delta_a(k):=\alpha_kG\Delta Ax_k,\Delta_f(k):=GBf(x_k),\\&\Delta_\tau(k):=GA_\tau\widetilde{x}_k=GA_\tau\sum_{i=1}^{q}\beta_{k,i}x_{k-\tau_{k,i}}.\end{aligned}$$

Then, suppose that $\Delta_a(k)$, $\Delta_\tau(k)$ and $\Delta_f(k)$ are bounded in Euclidean norm. This means that there exist known bounds $\underline{\delta}_a^i$, $\overline{\delta}_a^i$, $\underline{\delta}_\tau^i$, $\overline{\delta}_\tau^i$ $\underline{\delta}_f^i$ and $\overline{\delta}_f^i$ $(i=1,2,\dots,m)$ satisfying

$$\underline{\delta}_a^i\le\delta_a^i(k)\le\overline{\delta}_a^i,\ \ \underline{\delta}_\tau^i\le\delta_\tau^i(k)\le\overline{\delta}_\tau^i,\ \ \underline{\delta}_f^i\le\delta_f^i(k)\le\overline{\delta}_f^i,$$

where $\delta_a^i(k)$, $\delta_\tau^i(k)$ and $\delta_f^i(k)$ $(i=1,2,\dots,m)$ are the i-th elements of $\Delta_a(k)$, $\Delta_\tau(k)$ and $\Delta_f(k)$, respectively. In the subsequent, set

$$\begin{aligned}&\widehat{\Delta}_a=\begin{bmatrix}\hat{\delta}_a^1 & \hat{\delta}_a^2 & \dots & \hat{\delta}_a^m\end{bmatrix}^T,\ \ \hat{\delta}_a^i=\frac{\overline{\delta}_a^i+\underline{\delta}_a^i}{2},\\&\widetilde{\Delta}_a=\mathrm{diag}\left\{\tilde{\delta}_a^1,\tilde{\delta}_a^2,\dots,\tilde{\delta}_a^m\right\},\ \ \tilde{\delta}_a^i=\frac{\overline{\delta}_a^i-\underline{\delta}_a^i}{2},\\&\widehat{\Delta}_\tau=\begin{bmatrix}\hat{\delta}_\tau^1 & \hat{\delta}_\tau^2 & \dots & \hat{\delta}_\tau^m\end{bmatrix}^T,\ \ \hat{\delta}_\tau^i=\frac{\overline{\delta}_\tau^i+\underline{\delta}_\tau^i}{2},\\&\widetilde{\Delta}_\tau=\mathrm{diag}\left\{\tilde{\delta}_\tau^1,\tilde{\delta}_\tau^2,\dots,\tilde{\delta}_\tau^m\right\},\ \ \tilde{\delta}_\tau^i=\frac{\overline{\delta}_\tau^i-\underline{\delta}_\tau^i}{2},\\&\widehat{\Delta}_f=\begin{bmatrix}\hat{\delta}_f^1 & \hat{\delta}_f^2 & \dots & \hat{\delta}_f^m\end{bmatrix}^T,\ \ \hat{\delta}_f^i=\frac{\overline{\delta}_f^i+\underline{\delta}_f^i}{2},\\&\widetilde{\Delta}_f=\mathrm{diag}\left\{\tilde{\delta}_f^1,\tilde{\delta}_f^2,\dots,\tilde{\delta}_f^m\right\},\ \ \tilde{\delta}_f^i=\frac{\overline{\delta}_f^i-\underline{\delta}_f^i}{2},\end{aligned}$$

then we will design the desired robust sliding mode controller, which can ensure the reachability analysis.

Theorem 2 *For the sliding surface (4) with $G = B^T P$, if the minimization problem (28) is solvable and P is the solution to (28), then the sliding mode controller given by*

$$\begin{aligned} u_k = &-(GB)^{-1}\big[\kappa U\, sgn[s_k] + (\kappa V - I)s_k \\ &+(\widehat{\Delta}_a + \widetilde{\Delta}_a sgn[s_k]) + (\widehat{\Delta}_\tau + \widetilde{\Delta}_\tau sgn[s_k]) \\ &+(\widehat{\Delta}_f + \widetilde{\Delta}_f sgn[s_k])\big] \end{aligned} \tag{29}$$

can guarantee the discrete-time reaching condition.

Proof: Note that the proof of this theorem can be easily obtained and hence the proof is omitted for brevity.

Remark 6 *In this paper, we make great efforts to address robust SMC problem of addressed systems with network-induced phenomena, and several types of network-induced phenomena have been well discussed. The main difficulties during the derivation of new results can be summarized as follows: (i) how to fully consider the phenomenon of MSCDs and reflect the related information in main results? (ii) how to address the related sum terms induced by the MSCDs via the proper method? and (iii) how to integrate the SMC method with the delay-fractioning idea to attenuate the effects from MSCDs, ROU and RVNs subject to uncertain occurrence probabilities? To reply the above three issues, 1) we make an attempt to divide the sum term "$-\sum_{l=k-\tau_M}^{k-\tau_m-1}\eta_l^T\rho P\eta_l$" in (22) as*

$$\begin{aligned} &-\sum_{l=k-\tau_M}^{k-\tau_m-1}\eta_l^T\rho P\eta_l \\ =&-\frac{1}{q}\left(\sum_{l=k-\tau_{k,1}}^{k-\tau_m-1}\eta_l^T\rho P\eta_l + \sum_{l=k-\tau_M}^{k-\tau_{k,1}-1}\eta_l^T\rho P\eta_l\right) \\ &-\frac{1}{q}\left(\sum_{l=k-\tau_{k,2}}^{k-\tau_m-1}\eta_l^T\rho P\eta_l + \sum_{l=k-\tau_M}^{k-\tau_{k,2}-1}\eta_l^T\rho P\eta_l\right) \\ &\cdots \\ &-\frac{1}{q}\left(\sum_{l=k-\tau_{k,q}}^{k-\tau_m-1}\eta_l^T\rho P\eta_l + \sum_{l=k-\tau_M}^{k-\tau_{k,q}-1}\eta_l^T\rho P\eta_l\right); \end{aligned}$$

2) we use the free-weighting matrix technique to deal with the above sum terms induced by the MSCDs; and 3) we introduce the term V_{5k} below (12) with a parameter ρ to enhance the feasibility of the developed method. Moreover, the matrices F_{1i}, F_{2i} and scalars $\bar{\gamma}$, ϵ_3 are there for RVNs subject to uncertain occurrence probabilities, the scalars τ_m, τ_M, $\bar{\beta}_i$ and ϵ_2 reflect the time-varying MSCDs, and the matrices H, E and scalars $\bar{\alpha}$, ϵ_1 refer to the ROU. Therefore, the related information of the above mentioned factors has been clearly reflected in main results.

Remark 7 *Note that the addressed networked systems are influenced simultaneously by several stochastic phenomena, we have made great efforts to propose new robust control method. More specifically, the stability analysis problem of the sliding motion has been studied by proposing a new sufficient condition in terms of the delay fractioning method. In addition, the desired sliding mode controller has been designed, which can drive the system state trajectories onto the pre-defined sliding surface.*

4 An illustrative example

In this section, we provide numerical simulations to illustrate the usefulness of the obtained SMC method.

The system parameters in (1) are given by:

$$A=\begin{bmatrix} 0.15 & -0.25 & 0 \\ 0 & 0.13 & 0.01 \\ 0.03 & 0 & -0.05 \end{bmatrix},\quad B=\begin{bmatrix} 0.1817 & 0.4286 \\ 0.1597 & 0.793 \\ 0.1138 & 0.0581 \end{bmatrix},$$

$$A_\tau=\begin{bmatrix} -0.03 & 0 & -0.01 \\ 0.02 & 0.03 & 0 \\ 0.04 & 0.05 & -0.01 \end{bmatrix}, D_1=\begin{bmatrix} 0.025 & 0.1 & 0 \\ 0 & -0.03 & 0 \\ 0.04 & 0.035 & -0.01 \end{bmatrix},$$

$$D_2=\begin{bmatrix} -0.05 & 0.037 & -0.36 \\ 0 & 0.03 & 0 \\ 0.04 & 0.035 & 0.01 \end{bmatrix}, E_1=\begin{bmatrix} 0.015 & 0 & -0.01 \\ 0.01 & 0.015 & 0 \\ 0.02 & 0.025 & -0.01 \end{bmatrix},$$

$$H^T=\begin{bmatrix} 0.14 & 0.2 & 0.17 \end{bmatrix}, E=\begin{bmatrix} 0.2 & 0.1 & 0 \end{bmatrix}, F_k=\sin(0.6k).$$

Let the nonlinearities be given by:

$$\begin{aligned} f(x_k) &= \begin{bmatrix} 0.49\sin(x_{1,k}x_{3,k}) & 0.13\sin(x_{2,k}) \end{bmatrix}^T, \\ g_1(x_k) &= 0.5(F_{11}+F_{21})x_k + 0.5(F_{21}-F_{11})\sin(x_k)x_k, \\ g_2(x_k) &= 0.5(F_{12}+F_{22})x_k + 0.5(F_{22}-F_{12})\cos(x_k)x_k, \end{aligned}$$

where

$$\begin{aligned}
F_{11} &= F_{12} = \mathrm{diag}\{0.4, 0.5, 0.8\}, \\
F_{21} &= F_{22} = \mathrm{diag}\{0.3, 0.2, 0.6\}, \\
\sin(x_k) &:= \mathrm{diag}\{\sin(x_{1,k}), \sin(x_{2,k}), \sin(x_{3,k})\}, \\
\cos(x_k) &:= \mathrm{diag}\{\cos(x_{1,k}), \cos(x_{2,k}), \cos(x_{3,k})\},
\end{aligned}$$

and $x_{i,k}$ $(i = 1, 2, 3)$ is the i-th element of x_k. Let $\bar{\alpha} = 0.7$, $\epsilon_1 = 0.1$, $\bar{\beta}_1 = 0.6$, $\bar{\beta}_2 = 0.5$, $\epsilon_2 = 0.15$, $\bar{\gamma} = 0.82$, $\epsilon_3 = 0.07$. The bounds of the time-delay $\tau_{k,j}$ $(j = 1, 2)$ are set as $\tau_m = 3$ and $\tau_M = 6$. Choose the lower and upper bounds for $\delta_a^i(k)$, $\delta_\tau^i(k)$ and $\delta_f^i(k)$ $(i = 1, 2, \ldots, m)$ as

$$\begin{aligned}
&\underline{\delta}_a^i = - \parallel GH \parallel \parallel Ex_k \parallel, \quad \overline{\delta}_a^i = \parallel GH \parallel \parallel Ex_k \parallel, \\
&\underline{\delta}_\tau^i = -(\parallel GA_\tau \parallel \parallel x_{k-\tau_{k,1}} \parallel + \parallel GA_\tau \parallel \parallel x_{k-\tau_{k,2}} \parallel), \\
&\overline{\delta}_\tau^i = \parallel GA_\tau \parallel \parallel x_{k-\tau_{k,1}} \parallel + \parallel GA_\tau \parallel \parallel x_{k-\tau_{k,2}} \parallel, \\
&\underline{\delta}_f^i = - \parallel GB \parallel \parallel f(x_k) \parallel, \quad \overline{\delta}_f^i = \parallel GB \parallel \parallel f(x_k) \parallel .
\end{aligned}$$

Then, for prescribed scalars $p = 1$ and $\rho = 1.2 \times 10^{-4}$, solving the minimization problem (28) yields

$$\begin{aligned}
P &= \begin{bmatrix} 1.4008 & -0.7145 & 0.2304 \\ -0.7145 & 3.1500 & -0.4516 \\ 0.2304 & -0.4516 & 2.9226 \end{bmatrix}, \\
G &= \begin{bmatrix} 0.1666 & 0.3218 & 0.3023 \\ 0.0471 & 2.1655 & -0.0896 \end{bmatrix}, \\
\varepsilon &= 0.1952, \quad \mu = 0.0106.
\end{aligned}$$

In the simulation, let $\kappa = 0.06$ and $\mu_i = \nu_i = 0.1$ $(i = 1, 2)$. By applying newly synthesized sliding mode controller (29), the simulation results are obtained. Among them, Fig. 1 clearly shows that the system state trajectories $x_{1,k}$, $x_{2,k}$ and $x_{3,k}$ converge to a small neighborhood quickly in finite time. The control signal u_k is plotted in Fig. 5 and the communication delay $\tau_{k,i}$ $(i = 1, 2)$ is shown in Figs. 7-8. Besides, the sliding surface s_k and signal Δs_k are plotted respectively in Figs. 3 and 9. Besides, for the comparison purpose, we provide the related simulations by setting $\kappa = 0.6$ in order to illustrate the chattering effects, which have been shown in Figs. 2, 4 and 6. Overall, from the above simulation results, we can conclude that the proposed SMC method performs well.

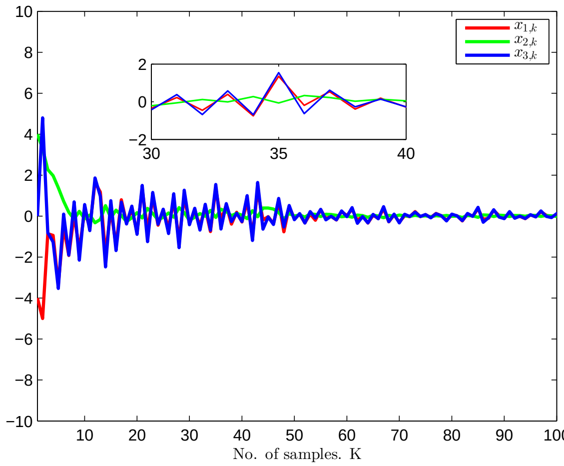

Fig. 1. The trajectory of state x_k(κ=0.06)

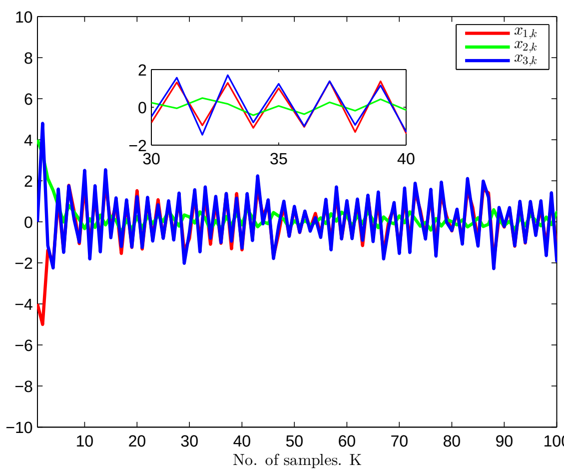

Fig. 2. The trajectory of state x_k(κ=0.6)

Remark 8 *It should be noted that the proposed stability conditions in Theorem 1 are delay-dependent, which are clearly affected by the delay interval. Accordingly, the actual discontinuous control will increase the gain and the chattering effects should be considered. From the simulations, we can see that smaller sampling period can reduce the chattering effects, which has been clearly shown in Figs. 1-6. Hence, during the implementation, we can increase the sampling frequency and then alleviate the chattering effects effectively.*

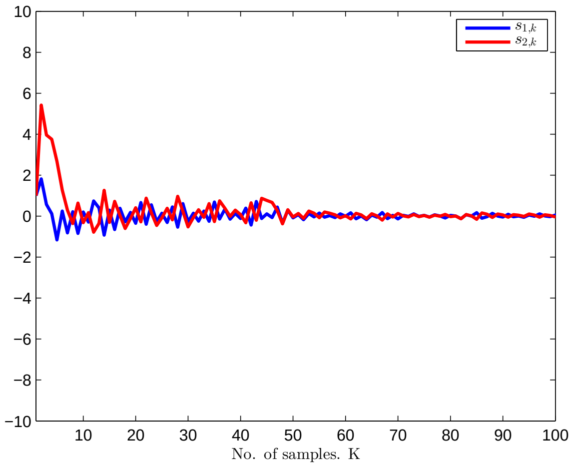

Fig. 3. The trajectory of sliding mode function $s_k(\kappa{=}0.06)$

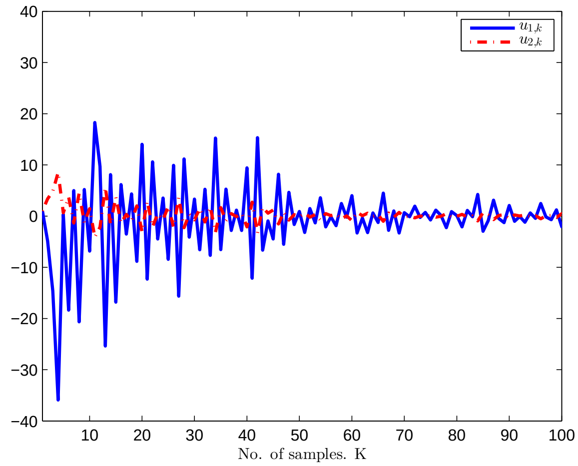

Fig. 5. The control signal $u_k(\kappa{=}0.06)$

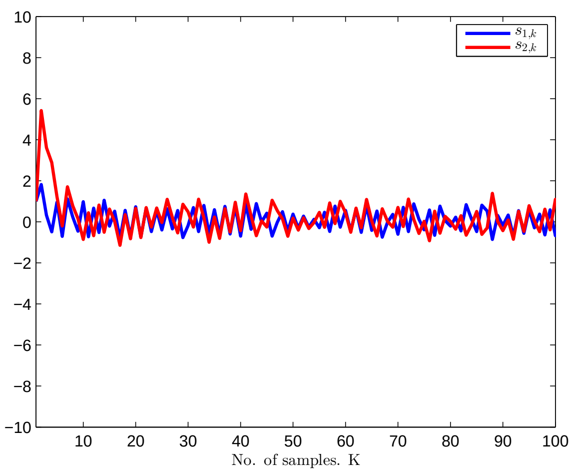

Fig. 4. The trajectory of sliding mode function $s_k(\kappa{=}0.6)$

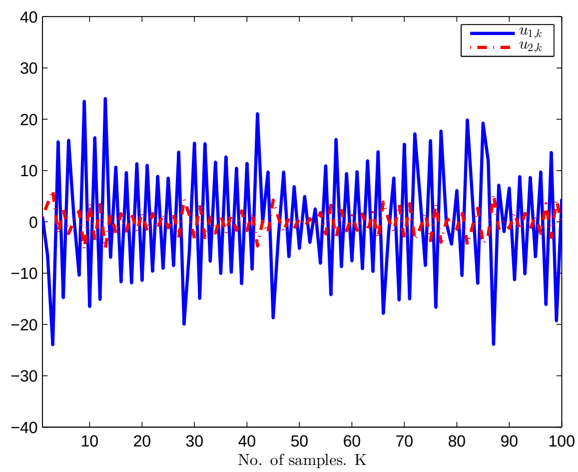

Fig. 6. The control signal $u_k(\kappa{=}0.6)$

5 Conclusion

In this paper, we have addressed the robust SMC problem for discrete networked systems subject to the ROU, RVNs and MSCDs under uncertain occurrence probabilities. The network-induced phenomena of ROU, RVNs and MSCDs have been characterized by a set of Bernoulli distributed random variables with uncertain occurrence probabilities. Firstly, we have introduced a linear switching surface. Secondly, a sufficient criterion based on the delay fractioning idea has been established to ensure the robust asymptotic stability of the sliding motion in the mean square. In the sequel, a minimization algorithm has been provided for convenience of examining the feasibility of the proposed method and an SMC law has been designed properly to guarantee the reachability analysis. It is worthwhile to note that the all parameter matrices in the sliding surface and SMC law can be easily obtained by solving an optimal problem with certain LMI constraints. Finally, a numerical example has been employed to show the feasibility of the new control method. It should be noted that it is of important significance to handle the problems of analysis and synthesis for networked systems with abruptly changed structures and incomplete measurements. Hence, further research topics include the extensions of the proposed SMC method to deal with the SMC problems for networked systems with Markovian jumping parameters and communication protocols.

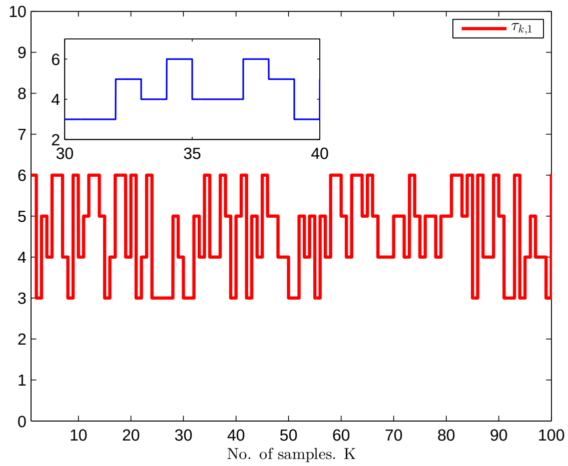

Fig. 7. The communication delay $\tau_{k,1}$(κ=0.06)

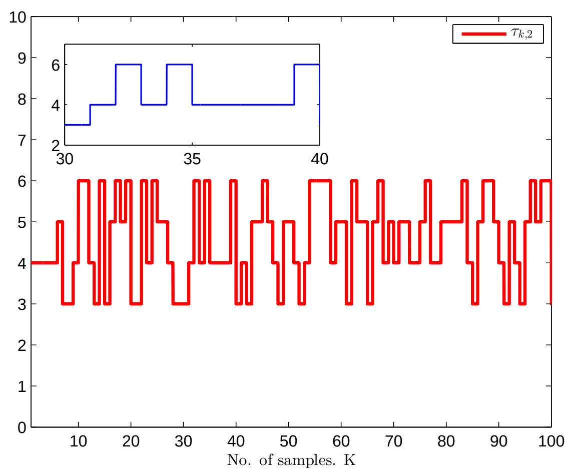

Fig. 8. The communication delay $\tau_{k,2}$(κ=0.06)

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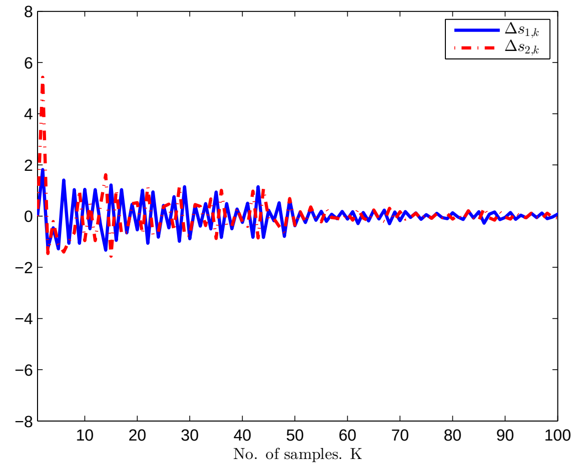

Fig. 9. The signal Δs_k(κ=0.06)

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