External Consensus of Networked Multi-agent Systems with Nonlinear Dynamics and Random Network Delay*

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***Abstract*— In this paper, the external consensus problem of networked multi-agent systems with fixed undirected topology, random network delay and nonlinear dynamics is considered. The unknown nonlinear dynamics existing in the systems can be described as RBF-ARX model. Besides, the random network delay can be compensated by introducing the prediction strategy. The RBF-ARX model is used as prediction model to design the output predictor, which is to generate the prediction sequence of agent's output and control input. Then, the designed selector chooses the proper value from the available sequences corresponding to the value of the network delay. Based on the mentioned above, a distributed external consensus control algorithm is proposed to make the considered systems with external reference input achieve consensus. Finally, an example is given to illustrate the validity of the proposed algorithm.**

I. Introduction

In the past few years, the cooperative control problems for multi-agent systems have attracted extensive attention in automatical control field [1]. With the rapid development of network technology, all agents communicate with each other via their local controller over the communication network, which comes out to the networked multi-agent systems(NMAS). Compared to the conventional control system, NMAS can achieve complex goals by exchanging information among agents and cooperating with each other. Therefore, NMAS has the advantages of flexibility, reliability and parallelism. NMAS has a wide range of potential applications in uncertain environments, such as, unmanned aerial vehicle(UAV) formation [2], intelligent transportation [3], multi-robot systems [4], etc. In distributed coordinated control of NMAS, a significant problem is to design distributed coordinated control algorithm such that consensus can be achieved on a decision value.

Consensus can be achieved by the implementation of the designed control algorithm. Different consensus values create different types of tasks for all agents. The most common consensus problem considered in the literatures is average consensus [5]-[6], which consensus value is calculated based on the initial value of every agents in NMAS. Besides, min-max consensus [7]-[8] and consensus function [9]-[10] are also depending on the initial value form agents. Therefore, these types of consensus problems can only be applied and limited to the NMAS with non-zero initial value. Another type of consensus problem that does not have any constraints on the initial value is called external consensus [11]-[12]. This type of consensus value is determined by the external reference input given to one or more agents in the NMAS.

Various literatures have researched the external consensus problem of NMAS. In [11], a consensus framework of consensus algorithm design with external input for a general linear NMAS is proposed. With the designed control algorithm, the consensus value is independent of the initial value of agents. In [12], an improvement work of [11] on the whole NMAS stability has been discussed and proved by demonstrating the proposed control algorithm with two continuous inhomogeneous systems using PI controller. However, these studies do not take the network induced constraints into account. The utilization of a communication network in the NMAS inevitably introduces network constraints, especially network delay. The presence of network delay will lead to significant deteriorations of stability and control performance of systems. In order to solve the problem, paper [13] considers the external consensus problem of linear NMAS with constant network delay and adopts a prediction strategy to compensate the effects of constant network delay. It then motivates us to adopt prediction strategy in the context of NMAS application. In the real problem of the NMAS application, the network delay is often times random or time varying and the system structure is always nonlinear. Thus, to investigate the NMAS with random network delay and nonlinear dynamics is helpful.

It is clear that the successful adopt of prediction strategy in nonlinear model is highly dependent on having a reliable system model. Thus, to look for a model that effectively describe the nonlinear dynamics of the system, and should also be easily usable in designing a prediction strategy are of importance. In model based real-time control strategies for nonlinear system, radial basis function (RBF) networks offer a framework for the modeling of the system, because of its simple topological structure, precision in nonlinear dynamics approximation, and fast learning. However, a great number of centers may be needed in order to reach the required accuracy, which leads to difficulties in computing. Time series model is another model in modeling nonlinear system, which is a mothed of curve fitting and parameter estimation. And the fast and accurate on-line estimation of

*This work is supported by the National Natural Science Foundation of China under Grant 61573379 and the Hubei Provincial Natural Science Foundation of China under Grant 2015CFA010 and the 111 project under Grant B17040.

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the time-varying parameters is difficult as well. Then, Vesin [14] proposes a method that uses a set of RBF networks to approximate the coefficients of a state-dependent AR model, yield RBF-AR model. It has the advantages of both the state-dependent AR model in description of nonlinear dynamics and the RBF networks in function approximation. What's more, Peng *et al* [15] extends the RBF-AR model to the RBF-ARX model, and proposes a fast converging structured nonlinear parameter optimization algorithm which is an off-line parameter optimization method. It can avoid difficulty of fast and accurate on-line estimation, and easily obtain the local model at each sampling interval. The RBF-ARX model is successfully apply in many field so far [16]-[17].

With this background, the external consensus problem of NMAS with fixed undirected topology, random network delay and nonlinear dynamics is considered. The nonlinear dynamics of agents can be described as RBF-ARX model. It is used as prediction model to design the output predictor, which generates the prediction sequence of agent's output. Then, the designed selector chooses the proper value from the available sequences corresponding to the network delay. Based on the description above, a distributed external consensus control algorithm is proposed to make the considered system achieve consensus. The rest of this paper is organized as follow. In Section II, some basic concepts for algebraic graph theory and the problems of considered system are introduced. In Section III, the main methods are proposed to solve the considered problems. In Section IV, simulation results are presented to demonstrate the validity of the proposed methods. Finally, the conclusion is given in SectionV.

II. Preliminaries and Problem Formation

Graph theory is an indispensable tool for the study of NMAS. Assumed that NMAS consist of N agents, then the network topology of agents can be represented using a graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$, where the set of nodes $\mathcal{V} = \{1, 2, ..., N\}$ and the set of edges $\mathcal{E} = \{(i, j) : i, j \in \mathcal{V}\} \subseteq \mathcal{V} \times \mathcal{V}$. Label $\mathcal{A} = [a_{ij}]_{N\times N}$ refers to an adjacency matrix with elements satisfying $a_{ii} = 0$ and $a_{ij} > 0$, which provides a numerical representation of agents' relationship. If there is an edge between i and j, then its elements describe as $a_{ij} > 0$. Besides, the graph $\mathcal{G}$ is said to be unweighted if all $a_{ij} = 1$. The set of neighbors of agent i is denoted by $N_i = \{j \in \mathcal{V} : (i, j) \in \mathcal{E}, i \neq j\}$. The Laplacian matrix $\mathcal{L}$ of graph $\mathcal{G}$ can be obtained as $\mathcal{L} = [l_{ij}]_{N\times N}$, where

$$l_{ij} = \begin{cases} |N_i|, & i = j. \\ -1, & i \neq j \text{ , } i \ is \ adjacent \ to \ j. \\ 0, & otherwise. \end{cases}$$

The structure of NMAS to be studied here is shown in Fig. 1. Consider a group of N nonlinear SISO agents whose dynamics features depend on time-varying working-points. The dynamics of the agent i is described by

$$\begin{aligned} y_i(t) = f_i(&y_i(t-1), ..., y_i(t-n_{y,i}), u_i(t-1), ..., \\ &u_i(t-n_{u,i})), \quad i = 1, 2, ..., N \end{aligned} \tag{2}$$

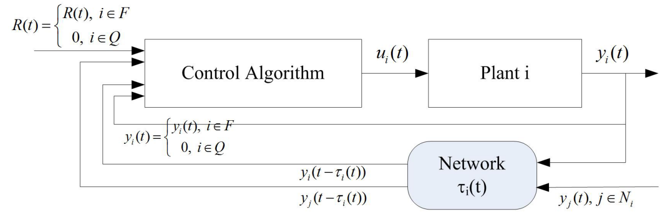

Fig. 1. Block diagram of agent i

where $y_i(t)$ and $u_i(t)$ are the output and control input of agent i; and $f_i(\cdot)$ is the unknown nonlinear dynamics.

Defining the vector

$$\mathbf{X}_i(t-1) = [y_i(t-1), ..., y_i(t-n_{y,i}), u_i(t-1), ..., u_i(t-n_{u,i})]^T$$

then model (2) can be rewritten as

$$y_i(t) = f_i(\mathbf{X}_i(t-1)) \tag{3}$$

Every agent has its own local controller and it can be designed as the transfer function type

$$G_i(q^{-1}) = \frac{D_i(q^{-1})}{C_i(q^{-1})}, \quad i = 1, 2, ..., N \tag{4}$$

where

$$C_i(q^{-1}) = 1 + c_{i1}q^{-1} + ... + c_{in_{c,i}}q^{-n_{c,i}}$$
$$D_i(q^{-1}) = d_{i0} + d_{i1}q^{-1} + ... + d_{in_{d,i}}q^{-n_{d,i}}$$

$n_{c,i}$ and $n_{d,i}$ are the polynomial orders of $C_i(q^{-1})$ and $D_i(q^{-1})$, $n_{c,i} \geq n_{d,i}$. The controller in (4) is designed without considering the network delay to achieve the desired performance of NMAS. Classical or advanced control methods can be used for this purpose.

Then, the common external consensus algorithm for NMAS with random network delay, but without network delay compensation can be described as follow

$$u_i(t) = \begin{cases} G_i(q^{-1})\Big(R(t) - (y_i(t) \\ \quad + K_i \sum_{j\in N_i} (y_i(t-\tau_i(t)) - y_j(t-\tau_i(t)))\Big), i \in F \\ -G_i(q^{-1})\Big(K_i \sum_{j\in N_i} (y_i(t-\tau_i(t)) - y_j(t-\tau_i(t)))\Big), \\ \quad i \in Q \end{cases} \tag{5}$$

where F is the set of agents with external reference input; Q is the set without external reference input; $\tau_i(t)$ is the random network delay; K_i is the coupling gain with a positive constant [18] and it is used as a tunable parameter that strengthens or loosens the coupling between the individual agents. It is quite intuitive for tuning these gains. Higher or lower values will apply when it is important to keep the tracking firmly or when we want to loosen or set free of the connection.

In this work, we consider the case that the network topology of NMAS is fixed and undirected with random network delay and nonlinear dynamics (3). The NMAS is then said to achieve consensus if it satisfies

$$\lim_{t\to\infty} \|y_i(t) - y_j(t)\| = 0, \quad i,j \in \mathcal{V}, \quad i \neq j \tag{6}$$

III. Main Methods

In this section, the main methods to solve the considered problems are discussed. First, the RBF-ARX model is introduced to approximate the nonlinear dynamics of the system. Then, the external consensus algorithm based on prediction strategy is proposed. At the end, the prediction strategy is presented to make the proposed algorithm work.

A. RBF-ARX model

RBF-ARX model is a global model which combines with RBF networks and ARX model. It uses fast converging structured nonlinear parameter optimization algorithm to identify the parameters. This is an off-line nonlinear model parameter optimization method, depending partly on the Levenberg-Marquardt method (LMM) for nonlinear parameter optimization and partly on the Least-Squares method (LSM) for linear parameter estimation. In general, RBF-ARX model uses far fewer centers when compared with a single RBF model and shows a better performance than the ARX model.

RBF-ARX model has the common structure as follow

$$\begin{aligned} y(t) =& \varphi_0(\mathbf{W}(t-1)) + \sum_{i=1}^{n_y} \varphi_{y,i}(\mathbf{W}(t-1))y(t-i) \\ &+ \sum_{i=1}^{n_u}(\mathbf{W}(t-1))u(t-i) + e(t) \end{aligned} \tag{7}$$

$$\begin{aligned} &\varphi_0(\mathbf{W}(t-1)) = c_0^0 + \sum_{k=1}^{m} c_k^0 exp\{-\lambda_k^y \|\mathbf{W}(t-1) - \mathbf{Z}_k^y\|_2^2 \\ &\varphi_{h,i}(\mathbf{W}(t-1)) = c_{0,i}^h + \sum_{k=1}^{m} c_{k,i}^h exp\{-\lambda_k^h \|\mathbf{W}(t-1) - \mathbf{Z}_k^h\|_2^2 \\ &\mathbf{W}(t-1) = [w(t-1), w(t-2), ..., w(t-n_w)]^T \\ &\mathbf{Z}_k^h = [z_{k,1}^h, z_{k,2}^h, ..., z_{k,n_w}^h]^T, \quad h = y, u \end{aligned}$$

where y and u are the output and the input; n_y, n_u, m and n_w are the output order, input order, RBF networks order and the number of selected state information; $z_k^h(k = 1, 2, ..., m; h = y, u)$ are the network centers; $\mathbf{W}(t-1)$ are the vector of state information, which can be selected as input, output or the combination of input and output; λ_k^h are the scaling parameters; c_k^0 and $c_{k,i}^h(k = 0, 1, ..., m)$ are the scalar weighting coefficients; $e(t)$ is white noisy. More detail about the building and parameter estimation of RBF-ARX model can be found in paper [16]-[17].

Then, by using the RBF-ARX model, nonlinear agents (3) can be described as

$$A_{t,i}(q^{-1})y_i(t) = A_{0,t,i} + B_{t,i}(q^{-1})u_i(t-1) \tag{8}$$

$$\begin{aligned} A_{0,t,i} &= c_{0,i}^0 + \sum_{k=1}^{m_i} c_{k,i}^0 exp\{-\lambda_{k,i}^y \|\mathbf{W}_i(t-1) - \mathbf{Z}_{k,i}^y\|_2^2\} \\ A_{t,i}(q^{-1}) &= 1 + \sum_{l=1}^{n_{y,i}}\{c_{l,0,i}^y + \sum_{k=1}^{m_i} c_{l,k,i}^y \\ &\quad exp[-\lambda_{k,i}^y \|\mathbf{W}_i(t-1) - \mathbf{Z}_{k,i}^y\|_2^2]\}q^{-l} \\ B_{t,i}(q^{-1}) &= \sum_{l=1}^{n_{u,i}}\{c_{l,0,i}^u + \sum_{k=1}^{m_i} c_{l,k,i}^u \\ &\quad exp[-\lambda_{k,i}^u \|\mathbf{W}_i(t-1) - \mathbf{Z}_{k,i}^u\|_2^2]\}q^{-l} \\ \mathbf{W}_i(t-1) &= [w_i(t-1), w_i(t-2), ..., w_i(t-n_w)]^T \\ \mathbf{Z}_{k,i}^h &= [z_{k,1,i}^h, z_{k,2,i}^h, ..., z_{k,n_w,i}^h]^T, \quad h = y, u \end{aligned}$$

There, $A_{0,t,i}$ is represented the local mean; q^{-1} is the unit delay operator.

Noticed that, if all the coefficients of $A_{t,i}(q^{-1})$, $B_{t,i}(q^{-1})$ and $A_{0,t,i}$ are fixed at instant t, model (8) at time t is a local linearized ARX model with a constant term.

B. External Consensus Algorithm

Obviously, due to the existence of network delay in (5), the NMAS is unable to achieve external consensus and produces oscillating outputs. For that reason, the prediction strategy is proposed to predict the agent's output in order to solve the external consensus problem for NMAS with network delay.

Thus, to compensate the effects caused by the network delay, the external consensus algorithm based on prediction strategy is designed as

$$u_i(t) = \begin{cases} G_i(q^{-1})\Big(R(t) - (y_i(t) \\ \quad + K_i \sum_{j\in N_i}(\overline{y}_i(t) - \overline{y}_j(t))\Big), i \in F \\ -G_i(q^{-1})\Big(K_i \sum_{j\in N_i}(\overline{y}_i(t) - \overline{y}_j(t))\Big), i \in Q \end{cases} \tag{9}$$

where $\overline{y}_i(t)$ and $\overline{y}_j(t)$ are the predicted output value of agent i and j.

C. Prediction Strategy

The proposed external consensus algorithm (8) requires predicted output $\overline{y}_j(t)$. Let

$$y_{ei}(t) = \begin{cases} y_i(t) + K_i \sum_{j\in N_i}(\overline{y}_i(t) - \overline{y}_j(t)), i \in F \\ K_i \sum_{j\in N_i}(\overline{y}_i(t) - \overline{y}_j(t)), i \in Q \end{cases} \tag{10}$$

and model (8) also can be described as

$$\begin{aligned} A_{t,i}(q^{-1}) &= 1 + a_{t,i1}q^{-1} + ... + a_{t,in_{y,i}}q^{-n_{y,i}} \\ B_{t,i}(q^{-1}) &= b_{t,i0} + b_{t,i1}q^{-1} + ... + b_{t,in_{u,i}}q^{-n_{u,i}} \end{aligned} \tag{11}$$

Then the prediction strategy based on model (4) and (11) is used to generate the output sequence recursively from time $t - \tau_{max} + 1$ to t for all agents, where τ_{max} is the upper bound of the random network delay and $\widetilde{t} = t - \tau_{max}$. For $p \in [1, \tau_{max}]$, the prediction strategy can be described as

$$
\begin{aligned}
y_i(\widetilde{t}+p|\widetilde{t}) =& \\
&- \sum_{f=1}^{\min\{n_{y,i},p-1\}} a_{\widetilde{t}+p|\widetilde{t},if} y_i(\widetilde{t}-f+p|\widetilde{t}) \\
&- \sum_{f=p}^{n_{y,i}} a_{\widetilde{t}+p|\widetilde{t},if} y_i(\widetilde{t}-f+p) \\
&+ \sum_{f=0}^{\min\{n_{u,i},p-2\}} b_{\widetilde{t}+p|\widetilde{t},if} u_i(\widetilde{t}-f-1+p|\widetilde{t}) \\
&+ \sum_{f=p-1}^{n_{y,i}} b_{\widetilde{t}+p|\widetilde{t},if} u_i(\widetilde{t}-f-1+p) + A_{0,\widetilde{t}+p|\widetilde{t},i}
\end{aligned} \tag{12}
$$

$$
u_i(\widetilde{t}+p|\widetilde{t}) = \begin{cases}
- \sum_{f=1}^{\min\{n_{c,i},p-1\}} c_{if} u_i(\widetilde{t}-f+p|\widetilde{t}) \\
- \sum_{f=p}^{n_{c,i}} c_{if} u_i(\widetilde{t}-f+p) + D_i(q^{-1}) R(\widetilde{t}+p) \\
- \sum_{f=0}^{\min\{n_{d,i},p-1\}} d_{if} y_{ei}(\widetilde{t}-f+p|\widetilde{t}) \\
- \sum_{f=p}^{n_{d,i}} d_{if} y_{ei}(\widetilde{t}-f+p), \qquad i \in F \\
- \sum_{f=1}^{\min\{n_{c,i},p-1\}} c_{if} u_i(\widetilde{t}-f+p|\widetilde{t}) \\
- \sum_{f=p}^{n_{c,i}} c_{if} u_i(\widetilde{t}-f+p) \\
- \sum_{f=0}^{\min\{n_{d,i},p-1\}} d_{if} y_{ei}(\widetilde{t}-f+p|\widetilde{t}) \\
- \sum_{f=p}^{n_{d,i}} d_{if} y_{ei}(\widetilde{t}-f+p), \qquad i \in Q
\end{cases} \tag{13}
$$

From (12) and (13), it can be seen that the accuracy of the prediction strategy depends on the accuracy of model (4) and (11). Besides, the output and control input sequence have two separated part. The first part is the prediction sequence while the second part is the current signal sequence. This recursive prediction algorithm can reduce the complexity of computing. Based on the above equations, the output prediction of agent i at time t with random network delay $\tau_i(t)$ can be simplified as $\overline{y}_i(t) = y_i(t - \tau_{max} + \tau_i(t)|t - \tau_{max})$, which is applied into the consensus algorithm (9). Thus, a selector must be designed to select the proper value from the prediction output sequences corresponding to the network delay. It can be designed as

$$
\mathbf{S} = [\mathbf{0}_{1\times(\tau_i(t)-1)} \quad 1 \quad \mathbf{0}_{1\times(\tau_{max}-\tau_i(t))}] \tag{14}
$$

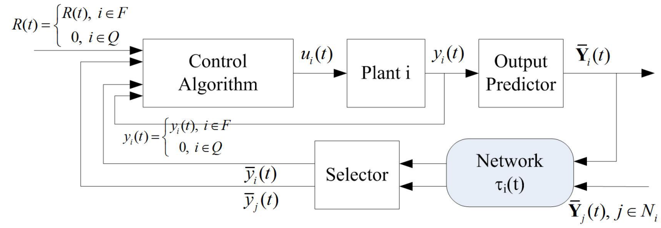

Fig. 2. Block diagram of agent i with output predictor

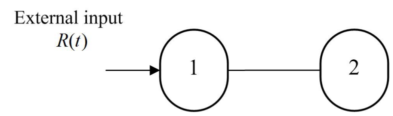

Fig. 3. The network topology of the NMAS

so

$$
y_i(t - \tau_{max} + \tau_i(t)|t - \tau_{max}) = \mathbf{S}\overline{\mathbf{Y}}_i \tag{15}
$$

$$
\overline{\mathbf{Y}}_i = [y_i(t - \tau_{max} + 1|t - \tau_{max}), ..., y_i(t|t - \tau_{max})]^T
$$

The structure of agent i with output predictor is shown in Fig. 2.

IV. Simulations

In this section, we provide an example for the considered NMAS under consensus algorithm (5) and (9). Considered two types of systems with nonlinear dynamics as follow.
Agent1

$$
\begin{aligned}
y(t+1) =& \frac{2.5y(t)y(t-1)}{1 + y^2(t) + y^2(t-1)} \\
&+ 0.7sin(0.5(y(t) + y(t-1))) \\
&+ 1.2u(t) + 1.4u(t-1)
\end{aligned} \tag{16}
$$

Agent2

$$
y(t+1) = \frac{y(t)}{1 + y^2(t)} + 0.2y(t-1) + u(t) \tag{17}
$$

The local controllers have the structures as

$$
G_1(q^{-1}) = \frac{0.2q + 0.3}{q - 1} \qquad G_2(q^{-1}) = \frac{0.1q - 0.09}{q - 1} \tag{18}
$$

The NMAS consists of two agents and the network topology is fixed and undirected shown in Fig. 3. Only agent 1 receives the external reference input. Here, for simplicity, select $(K_1, K_2) = (0.5, 3.8)$, sample time $T = 1s$. Under these values, the maximum allowable constant network delay can be tested as 8-step. To evaluate the performance of the designed consensus algorithm in the NMAS, a random network delay varying between 10-step to 12-step delay is induced in the system.

Based on Fig. 4 and Fig. 5, it is observed that the NMAS is unable to achieve consensus with random network delay under consensus algorithm (5). Because the random network delay is over the maximum allowable constant network delay and the system does not receive any compensation. And the NMAS with consensus algorithm (9) has produced a

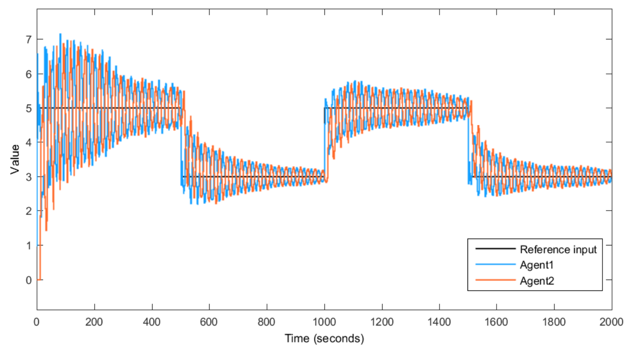

Fig. 4. Agent's output with random network delay under consensus algorithm (5)

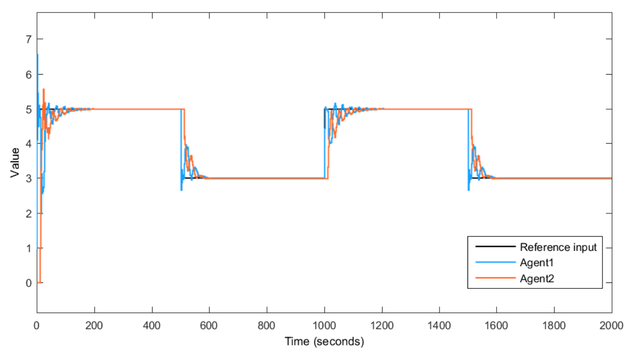

Fig. 5. Agent's output with random network delay under consensus algorithm (8)

stable output due to the insertion of the delay compensation. These results indicate the potential of the proposed prediction strategy to compensate for bounded random network delay effectively and solve the NMAS external consensus problem.

V. CONCLUSIONS

In this paper, the external consensus problem of NMAS with fixed undirected topology, random network delay and nonlinear dynamics has been investigated. Because of the nonlinear dynamics of agents, the system is uncertain and hard to analyze. Thus, we introduce the RBF-ARX model to approximate the nonlinear dynamics. Moveover, the existence of network delay effects the control performance of the system. Then, the prediction strategy is adopted to compensate the network delay. According to the simulation, the external consensus problem of the considered NMAS is ensured to be solved by the proposed methods mentioned above. In the future works, we will focus on the mathematical analysis of the stability of the system.

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