Multi-Hop Consensus for High-Order Integrator Multi-Agent Systems

Chang-Jiang Li[1] and Guo-Ping Liu[2]

***Abstract*— This paper investigates consensus problems for high-order integrator multi-agent systems subjected to constant network communication delay and fixed topology. A new protocol is proposed based on multi-hop relaying process. For the case without delay, the consensus protocol only requires the network connectivity condition, yet does not require the final consensus value to be constant. For the case with delay, a necessary and sufficient condition to reach consensus for the delay and connectivity is derived. Numerical examples are given to demonstrate the effectiveness of the theoretical results.**

I. Introduction

Consensus seeking has been studied for decades as a central problem of cooperative control. Consensus problem, can be summarized to be finding some communication rules that synchronize the agents to a common behavior. Study on consensus has been extensive in the recent decades. Since the study by Ofati Saber [1], consensus problem for multi-agent system has been studied for mainly two kinds of agent dynamics: integrator dynamics [1], [2] and LTI dynamics [3], [4]. Studies also involves topology, fixed topology, switching topology [2], uncertainty [5], [6], network delay [6], [7]. Roughly speaking, the achieving of consensus requires the connectivity of all agents, fixed or periodic [1], [2]. Still most research on consensus [6], [8], [9] assumes the consensus value to be constant, which may not be the case in the sense that the information state of each agent may be dynamically evolving in time. This happens in some formation control problems where the formation is moving.

In the designing of consensus protocols, the tunning of protocol parameters is important, especially when the agent is with high-order integrator dynamics. Most of the existing consensus protocols for integrator multi-agent systems depends on the selection of parameters. The protocols proposed in [10] requires a stable polynomial concerning the eigenvalues of the Laplacian matrix and the coefficients. The coefficients of the protocol proposed in [11] for second-order integrator multi-agent system also has to satisfy certain requirements. In the works above, the parameter tuning is a troubling problem. For large scale networks, finding the eigenvalues of Laplacian matrix can be a problem. Moreover, these parameters has to be designed within the consensus stable region and may not satisfy robust requirements in some cases. As is known in [1], the consensus protocol for first-order integrator multi-agent system relies only on the connectivity of the work. So the problem is, is there some simpler protocols for high-order integrator multi-agent system that is free of or relies less on the parameter designing?

Since consensus problem is concerned with a network of a autonomous agents, it is natural to introduce into consensus seeking problem some network communication rules, which is also termed as routing protocols that specify how a pair of nodes communicate and share information with each other. In particular, the paper is interested in a routing protocol called distance-vector routing protocols which introduce the concept of communication distance and expand the network connection [12], which attempted to boost convergence for large scale networks. In this paper, based on the concept of communication distance, multi-hop consensus protocols for continuous multi-agent system with high-order integrator dynamics are proposed, and consensus analysis are provided. Compared to previous work [10], the results only requires the connection condition, saving the trouble of tunning the protocol parameters, moreover, the final consensus value is not tuned to be constant. For the case with time delay, necessary and sufficient conditions to guarantee consensus are derived.

This paper is organized as follows. The first section gives literature review on consensus seeking and multi-hop consensus. Some preliminaries of graph theory are reviewed in the second section. Section III gives the main results. Numerical examples are presented in the next section. Section V concludes the paper.

II. Preliminaries

In this section, preliminaries on graph are provided.

To model a multi-agent system, a directed graph $\mathcal{G} = \{\mathcal{V}, \mathcal{E}, \mathcal{A}\}$ is introduced with a node set $\mathcal{V}$ indexed by $i = 1, \cdots, n$, and an edge set $\mathcal{E}$ and a nonnegative weighted matrix $\mathcal{A} = [a_{ij}]$. $a_{ji} > 0$ is meant that there exists communication from node i to node j and no communication otherwise. Node j is said to be a neighbor of node i if $a_{ij} > 0$, and the set of agent i's neighbors is denoted by N_i. $\mathcal{G}$ is said to be connected if there exists an sequence of ordered edges of form $(s_i, s_{l_1}), (s_{l_1}, s_{l_2}), \cdots, (s_{l_q}, s_j)$, where $s_{l_k} \in \mathcal{V}, k = 1, 2, \cdots, q$ for all $s_i, s_j \in \mathcal{V}$.

For a directed graph $\mathcal{G} = \{\mathcal{V}, \mathcal{E}, \mathcal{A}\}$, multiple two-hop paths may exist between a pair of vertices, thus following the work of [13], the adjacency matrix of a two-hop graph

*This work was supported in part by the National Natural Science Foundation of China under grant 61333003. Corresponding Author: Chang-Jiang Li.

[1]Chang-Jiang Li is with Center for Control Theory and Guidance Technology, Harbin Institute of Technology, Harbin 150001, China albert.author@papercept.net

[2]Guo-Ping Liu is with School of Engineering, University of South Wales, Pontypridd CF37 1DL, UK and also with Center for Control Theory and Guidance Technology, Harbin Institute of Technology, Harbin 150001, China guoping.liu@ia.ac.cn

$\mathcal{A}^{(2)} = a_{ij}^{(2)}$ is defined to be

$$a_{ij}^{(2)} = \sum_{j} a_{ik} a_{kj}$$

or equivalently

$$\mathcal{A}^{(2)} = \mathcal{A}^2$$

Furthermore, the m-hop adjacency matrix can be given by

$$\mathcal{A}^{(m)} = \mathcal{A}^m.$$

The Laplacian matrix $\mathcal{L}$ of graph $\mathcal{G}$ is defined by $\mathcal{L} = \mathcal{D} - \mathcal{A}$, where $\mathcal{A}$ is the graph's weighted matrix, and $\mathcal{D}$ is diagonal matrix with its i-th diagonal element equal to $\sum_{j \in N_i} a_{ij}$. $\mathcal{L}$ is always semi-definite.

This kind of definition, using the nearest neighbor rule, implies that agent i has a weight a_{ij} for its neighbor j. To precede, define weight diffusion rule $\mathcal{L}^{(l)} = [l_{ij}^{(l)}]$ as follow

$$l_{ij}^{(1)} = l_{ij} = \begin{cases} -a_{ij} & i \neq j \\ \sum_{j=1, j\neq i}^{n} a_{ij} & \text{otherwise} \end{cases} \tag{1}$$

$$l_{ij}^{(l+1)} = \sum_{k=1}^{n} a_{ik}(l_{ij}^{(l)} - l_{kj}^{(l)}) = \sum_{k=1}^{n} l_{ik} l_{kj}^{(l)}$$

Let $\mathcal{C}$ be a unit ball of $\mathbb{R}^n$, i.e. $\mathcal{C} = \{x \in \mathbb{R}^n | \|x\| \leq 1\}$.

III. Main Results

Consider a multi-agent system with high-order integrator dynamics. To describe the communication of the system, consider graph $\mathcal{G} = \{\mathcal{V}, \mathcal{E}, \mathcal{A}\}$, where $\mathcal{V}$ is the node set, $\mathcal{E}$ is the edge set, $\mathcal{A} = [a_{ij}]$ is the adjacency matrix.

The aim of this paper, is to

Problem 1: design some consensus protocol based on multi-hop neighboring information such that

$$\lim_{t\to\infty} |x_{li}(t) - x_{lj}(t)| = 0, \forall\, l, \forall\, j, \forall\, i. \tag{2}$$

This paper utilizes multi-hop protocol to synchronize the states of the agents. The multi-hop protocol uses the information of multi-hop neighbors. To implement the weight diffusion rule, first, agent i has to produce a weight a_{ij} for each of its neighbor j, then using rule to attain the multi-hop weight. Note that every hop of weight diffusion still uses the nearest neighbor rule. Based on the weight diffusion rule, one has the following result

$$\mathcal{L}^{(n)} = \mathcal{L}^n \tag{3}$$

For notational convenience, impose the following notation. Define

$$x^1(t) = \begin{bmatrix} x_{11}(t) \\ x_{12}(t) \\ \vdots \\ x_{1n}(t) \end{bmatrix}, x(t) = \begin{bmatrix} x^1(t) \\ x^2(t) \\ \vdots \\ x^d(t) \end{bmatrix}$$

A. Continuous-Time Case

Consider a multi-agent system with integrator dynamics as follows

$$\begin{aligned} \dot{x}_{1i}(t) &= x_{2i}(t) \\ \dot{x}_{2i}(t) &= x_{3i}(t) \\ &\vdots \\ \dot{x}_{di}(t) &= u_i(t) \end{aligned} \tag{4}$$

where $x_{ij}(t) \in R$ is the j-th state of agent i, $u_i(t)$ is the control input of agent i.

Then the compact form of the system dynamics is the following

$$\begin{aligned} \dot{x}^1(t) &= x^2(t) \\ \dot{x}^2(t) &= x^3(t) \\ &\vdots \\ \dot{x}^d(t) &= u(t) \end{aligned}$$

First the consensus protocol is given as follows

$$u_i(t) = -\sum_{l=1}^{d} \sum_{j=1}^{n} b_l l_{ij}^{(d+1-l)} x_{lj}(t)$$

then

$$\begin{aligned} u(t) = &-b_1 \mathcal{L}^{(1)} x^d(t) - b_2 \mathcal{L}^{(2)} x^{d-1}(t) \\ &- \cdots - b_d \mathcal{L}^{(d)} x^1(t) \end{aligned}$$

where $u(t) = \begin{bmatrix} u_1(t) & u_2(t) & \cdots u_n(t) \end{bmatrix}^T$.

Now one can obtain the compact closed-loop system dynamics as follows:

$$\dot{x}(t) = \Omega x(t) \tag{5}$$

where

$$\Omega = \begin{bmatrix} & I_n & & \\ & & \ddots & \\ & & & I_n \\ -b_d \mathcal{L}^{(d)} & -b_{d-1}\mathcal{L}^{(d-1)} & \cdots & -b_1 \mathcal{L} \end{bmatrix}$$

Theorem 2: Let $b_i = C_d^i q^i$, where q is a positive constant. Then protocol (III-A) solves Problem 1 for system (4) if system (4) contains a spanning tree.

Proof: In light of the weight diffusion rule, it can be obtained that

$$\Omega = \begin{bmatrix} & I_n & & \\ & & \ddots & \\ & & & I_n \\ -b_1 \mathcal{L} & -b_2 \mathcal{L}^2 & \cdots & -b_1 \mathcal{L}^d \end{bmatrix}$$

Using the Schur Complement Theorem, one has

$$\det(sI_n - \Omega) = \det(s^d I + b_1 \mathcal{L}^1 + \cdots + b_d \mathcal{L}^d)$$

Consider $b_i = C_d^i q^i$,

$$\begin{aligned} \det(sI_{nd} - \Omega) &= (\det(sI_n + q\mathcal{L})^d) \\ &= (\det(sI_n + q\mathcal{L}))^d \end{aligned} \tag{6}$$

Since the graph has a spanning tree, 0 is a simple eigenvalue of $\mathcal{L}$, the rest eigenvalues of $\mathcal{L}$ are all with positive real parts. Together with (6), it can be obtained that

- 0 is eigenvalue of Ω with algebraic multiplication d.
- The rest eigenvalues of Ω are all with negative real parts, since $q > 0$

To this point, one can obtain that

$$\lim_{t\to\infty} e^{\Omega t} = P \lim_{t\to\infty} \begin{bmatrix} S(t) & 0_{dn-d,d} \\ 0_{d,dn-d} & e^{\bar{J}t} \end{bmatrix} P^{-1} \tag{7}$$

where,

$$\begin{aligned} P &= \begin{bmatrix} w_1 & w_2 & \cdots & w_{dn} \end{bmatrix} \\ P^{-1} &= \begin{bmatrix} \nu_1^T & \nu_2^T & \cdots^T & \nu_{dn}^T \end{bmatrix} \\ S(t) &= \begin{bmatrix} 1 & t & \frac{t^2}{2} & \frac{t^3}{6} & \cdots \\ & 1 & t & \frac{t^2}{2} & \cdots \\ & & \ddots & t & \cdots \\ & & & 1 & t \\ & & & & 1 \end{bmatrix} \in \mathbb{R}^{d\times d} \end{aligned}$$

Since there exists a spanning tree,

$$\mathcal{L}1_n = 0_n, p^T\mathcal{L} = 0_n$$

where p^T is some left eigenvector of $\mathcal{L}$.

Now let $p^T 1_n = 1$, and take

$$w_1 = \begin{bmatrix} 1_n \\ 0_n \\ 0_n \\ \vdots \\ 0_n \end{bmatrix}, w_2 = \begin{bmatrix} 0_n \\ 1_n \\ 0_n \\ \vdots \\ 0_n \end{bmatrix}, \cdots, w_d = \begin{bmatrix} 0_n \\ 0_n \\ 0_n \\ \vdots \\ 1_n \end{bmatrix}$$

and

$$\begin{aligned} \nu_1 &= \begin{bmatrix} p^T & 0_n^T & 0_n^T & \cdots & 0_n^T \end{bmatrix} \\ \nu_2 &= \begin{bmatrix} 0_n^T & p^T & 0_n^T & \cdots & 0_n^T \end{bmatrix} \\ &\vdots \\ \nu_d &= \begin{bmatrix} 0_n^T & 0_n^T & 0_n^T & \cdots & p^T \end{bmatrix} \end{aligned}$$

then

$$\begin{aligned} &\lim_{t\to\infty} e^{\Omega t} \\ &= Pe^{Jt}P^{-1} = P \lim_{t\to\infty} \begin{bmatrix} S(t) & 0_{dn-d,d} \\ 0_{d,dn-d} & 0_{dn-d,dn-d} \end{bmatrix} P^{-1} \\ &= \lim_{t\to\infty} \sum_{i=1}^{n}\sum_{j=1}^{n} w_i S(t)_{ij}\nu_j \\ &= \lim_{t\to\infty} \sum_{i=1}^{d}\sum_{j=i}^{d} w_i\nu_j \frac{t^{j-i}}{(j-i)!} \\ &= \lim_{t\to\infty} \begin{bmatrix} 1_n p^T & t1_n p^T & \frac{t^2}{2}1_n p^T & \cdots & \frac{t^{d-1}}{(d-1)!}1_n p^T \\ & 1_n p^T & t1_n p^T & \cdots & \frac{t^{d-2}}{(d-2)!}1_n p^T \\ & & \ddots & \vdots & \vdots \\ & & & 1_n p^T & t1_n p^T \\ & & & & 1_n p^T \end{bmatrix} \end{aligned}$$

where the second equality is because the eigenvalues of Ω except 0 are with negative real parts.

This means, when t goes to ∞,

$$\begin{aligned} x^d(t) &= 1_n p^T x^d(0) \\ x^{d-1}(t) &= 1_n p^T x^{d-1}(0) + t1_n p^T x^d(0) \\ &\vdots \\ x^1(t) &= \sum_{i=0}^{d-1} \frac{t^i}{i!} 1_n p^T x^{d-i}(0) \end{aligned} \tag{8}$$

Now we know that $|x_{ij}(t) - x_{ik}(t)| \to 0$. That is, consensus is reached finally. ■

Remark 3: Since Theorem 2 introduces the multi-hop topology, utilizing more information than single-hop consensus protocol, the consensus condition only requires that the topology has a spanning tree. Compared to [14], the proposed consensus protocol is free of trouble of determining parameters for the topology.

B. Continuous-Time with Delay

This subsection focuses on the simplest possible case where the each one-hop delay of a pair of agents equals τ. The control input is given by

$$\begin{aligned} u(t) = &-b_1\mathcal{L}^{(1)}x^d(t-\tau) - b_2\mathcal{L}^{(2)}x^{d-1}(t-2\tau) \\ &- \cdots - b_d\mathcal{L}^{(d)}x^1(t-d\tau) \end{aligned} \tag{9}$$

Before providing the main result, the following result is needed.

Lemma 4: Consider time-delay system given as follows

$$\dot{x}(t) = -qx(t-\tau) + \delta(t), x(t) = \phi(t), t \in [-\tau, 0] \tag{10}$$

where $x(t) \in \mathcal{R}$, $\delta(t)$ is a small signal that converges to zero. Suppose the nominal system asymptotically stable with respect to $x = 0$. Then system (10) is asymptotically stable with respect to $x = 0$.

Proof: By Theorem 1.5.2 in [15], one has the fundamental solution $X(t)$ of the nominal system is with $|X(t)| \leq ke^{-ct}$, where k, c are some positive constants since the nominal system is asymptotically stable.

Let $y(t)$ be solution of the nominal system, by variation of constants formula, one has that the solution of (10) $x(t)$ can be given as follows

$$x(t) = y(t) + \int_0^t X(t-s)\delta(s)ds \tag{11}$$

Note that

$$\begin{aligned}\lim_{t\to\infty} |\int_0^t X(t-s)\delta(s)ds| &\leq k \lim_{t\to\infty} \int_0^t e^{-c(t-s)}|\delta(s)|ds \\ &= k\delta(\infty) \lim_{s\to 0} \frac{1}{s+c} = 0,\end{aligned}$$

where the last equality is based on final theorem of Laplacian Transform. Take limit of (11), one has $x(\infty) = 0$, completing the proof. ■

Now we are in position to provide our main result.

Theorem 5: Consider a network of integrator agents with equal communication time-delay in all links. Assume the network topology is fixed, undirected, and connected. Then, protocol (9) with $\tau_{ij} = \tau$ globally asymptotically solves the average-consensus problem if either of the following equivalent conditions is satisfied.

1) $\tau \in (0, \tau^\star)$, where $\tau^\star = \pi/2\lambda_n, \lambda_n = \lambda_{max}(\mathcal{L})$.
ii) The Nyquist plot $N(s) = e^{-\tau s}/s$ has a zero encirclement around $-1/(q\lambda_k(\mathcal{L})), k > 1$.

Proof: Let $z(t) = x^1(t)$, then one has

$$\begin{aligned} z^{(d)}(t) = &-b_1\mathcal{L}^{(1)}z^{(d-1)}(t-\tau) \\ &- b_2\mathcal{L}^{(2)}z^{(d-2)}(t-2\tau) \\ &- \cdots - b_d\mathcal{L}^{(d)}z(t-d\tau)\end{aligned} \tag{12}$$

Consider $b_i = C_d^i q^i$, let $h_m(t) = \sum_{i=0}^m C_m^i \mathcal{L}^i z^{(m-i)}(t-i\tau)$. From (12), one can get the following equation

$$\dot{h}_d(t) = -q\mathcal{L}h_d(t-\tau) \tag{13}$$

Define consensus error $q(s) = P's$, where

$$P' = \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & & & \\ \frac{1}{\sqrt{2}} & & -\frac{1}{\sqrt{2}} & & \\ \frac{1}{\sqrt{2}} & & & \ddots & \\ \frac{1}{\sqrt{2}} & & & & \frac{1}{\sqrt{2}} \end{bmatrix}$$

It is clear that $P = \begin{bmatrix} \frac{1}{\sqrt{n}} 1_n^T \\ P' \end{bmatrix}$ is with $P^T P = I_n$. Next let $p(s) = Ps$. Now one has

$$\begin{aligned} -\frac{d}{dt}p(h_d(t)) &= qP^T\mathcal{L}Pp(h_d(t-\tau)) \\ &= q\begin{bmatrix} 0 & 1^T\mathcal{L}P' \\ 0 & (P')^T\mathcal{L}P' \end{bmatrix} p(h_d(t-\tau)) \end{aligned} \tag{14}$$

therefore,

$$\frac{d}{dt}q(h_d(t)) = -Sq(h_d(t-\tau)), \tag{15}$$

where $S = q(P')^T\mathcal{L}P'$.

Now the consensus problem of (13) boils down to the stability of (15). Take Laplacian Transform for the equation above,

$$sI + e^{-\tau s}S = 0 \tag{16}$$

From the structure of S it can be obtained that S is positive definite if the graph has a spanning tree. Based on Theorem 1.4.1 in [15], (15) is asymptotically stable if and only if all roots of (16) has negative real parts. In other words, (15) is asymptotically stable if and only if all roots of the following equation

$$s + qe^{-\tau s}\lambda_k(\mathcal{L}) = 0, k = 2, \cdots, n \tag{17}$$

has negative real parts.

Define $Z_{\tau,s} = s + e^{-\tau s}$. It is clear that λ_k satisfies the following equation

$$\frac{1}{q\lambda_k(\mathcal{L})} + \frac{e^{-\tau s}}{s} = 0$$

Now it is clear that if all the Nyquist plot of $N(s) = e^{-\tau s}/s$ around $-1/(q\lambda_k\mathcal{L})$ for $k > 1$, then all the zeros of $Z_{\tau,s}$ are stable.

To find the bound of τ, suppose there $s = jw$ is one of the roots of equation (12), then there must be

$$\begin{aligned} jw + e^{-jw\tau}\lambda_k = 0 \\ -jw + e^{jw\tau}\lambda_k = 0 \end{aligned} \tag{18}$$

By some simple calculation, one would have

$$\lambda_k^2 = w^2 \text{ and } \sin(\tau w) = 1 \tag{19}$$

To this point, one can conclude if and only if $\tau \in [0, \frac{\pi}{2\lambda_{max}})$ can be consensus of (15) be reached.

Without loss of generality, let $h_d(t) = 1_n * f_d(t) + \delta(t)$, where $f_d(t)$ is a scalar function, and δ is a signal that asymptotically converges to zero.

Since $h_d(t) = \dot{h}_{d-1}(t) + \mathcal{L}h_{d-1}(t-\tau)$, now one has

$$\dot{h}_{d-1}(t) = -q\mathcal{L}h_{d-1}(t-\tau) + 1_n * f_d(t) + \delta(t) \tag{20}$$

then

$$\frac{d}{dt}p(h_{d-1}(t)) = -Sp(h_{d-1}(t-\tau)) + P'\delta(t)$$

which is a perturbed delayed linear differential equation.

Based on Lemma (4), one would have $\lim_{s\to 0} p(h_{d-1}(t) = 0$.

Using the same procedure, one would get $p(h_m(t), m = d, d-1, d-2, \cdots$ converges to zero and finally get $p(h_0(t)) = p(z(t))$ converges to zero, implying the first state of each agent reaches consensus. Naturally, based on the dynamics of the agents, all the states of the agents reach consensus.

■

C. Algorithms

The protocol proposed in this paper, is also originated from practical network routing protocols such as AODV protocol.

The AODV protocol is one of the common routing algorithms used in ad-hoc networks and is based on the principle of discovering routes as needed. In a wireless network each node can only communicate with the nodes next to it. AODV allows nodes to relay messages through their neighbors to nodes with which they cannot directly communicate by using multi-hop communication. The proposed protocol also uses this procedure, and to make use of the protocol, a messages relayed must be attached with a stamp indicating the count of hops (CoH) it has been relayed.

When one node needs to send a message to the destination node that is not its neighbor, a path discovery is initiated by broadcasting a Route REQuest (RREQ) packet to its neighbors. The RREQ message contains several fields such as the source, the destination, count of hops so far (CoH), a Sequence Number (SN1) which serves as a unique ID, and a lifespan (TTL). Each node receiving a RREQ will rebroadcast it to a destination node. The destination node receives the RREQ and sends the RREP to the intermediate nodes in the reverse paths.

In some cases, a messages relayed in the network may be relayed to original senders, creating loops of network. This phenomenon, termed as flood of RREP, may cause the drop of network performance and even cause network breakdown. To avoid this, TTL is introduced. TTL is an integer equal to the order of integrator. Whenever the message is relayed, TTL will be reduced by one. A message with negative TTL will be dropped.

Due to the flood of RREP messages, an intermediate node can receive multiple RREPs. But the fresher RREP route, referred to as the shortest hop count (i.e. the minimum numbers of hops in the path to the destination, SHC), will be used. To find SHC, SNs serve as time stamps allowing nodes to determine the timeliness of each packet and to prevent the creation of loops. A higher Sequence Number refers to a fresher route. SN helps to find the path with the minimum transmission time, though, TTL helps to find the path with the minimum transmission hop. Moreover, each node needs to maintain to route table.

This implementation of the proposed consensus protocol is an analogy of network routing protocol like AODV. Yet it still needs test since most network runs in discrete-time, which is our future job.

IV. Numerical Example

This section provides numerical examples to demonstrate the effectiveness of our protocol. Consider a multi-agent system with three agents indexed by $1, 2, 3$, each agent is with third-order integrator dynamics, and the connectivity of the system is given by the adjacency matrix as follows

$$\mathcal{A} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

In the following, two examples are given, both set $q = 1$.

A. Fixed Case

The case takes no delay into consideration.

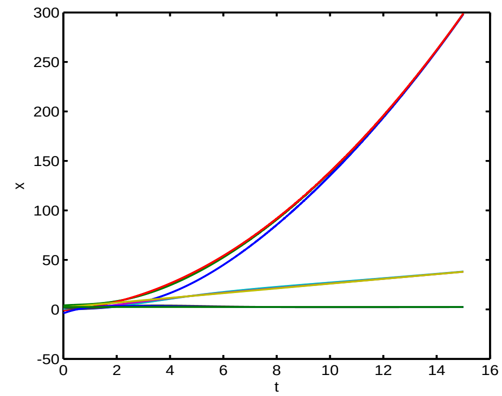

Fig. 1. State trajectory without delay

B. Case with delay

In this case, the delay is set to be $\tau = 0.15$.

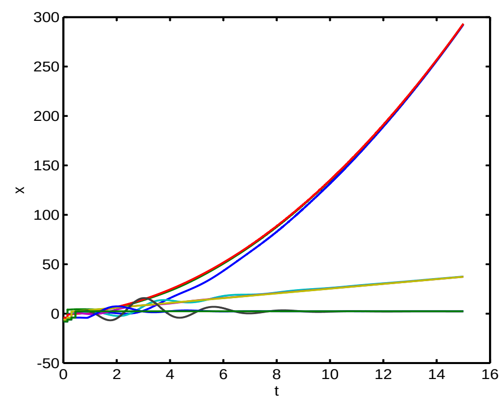

Fig. 2. State trajectory with delay

V. Conclusion

Consensus problems lie in the core of distributed control and optimization problem. Existing consensus protocols for high-order integrator multi-agent system has difficulty in tunning protocol parameters. Using multi-hop consensus protocols, high-order consensus protocol saves the trouble of tunning parameters and simplifies the procedures for protocol designing. Both theoretical and numerical examples have proved the effectiveness of multi-hop consensus protocol.

Still, the protocol is limited to fixed topology case, topology switching and network delay have not been taken into consideration, which are part of our future work.

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