Data-Driven Leader-follower Output Synchronization for Networked Non-linear Multi-agent Systems with Switching Topology and Time-varying Delays*

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Abstract This paper studies the output synchronization problem for a class of networked non-linear multi-agent systems with switching topology and time-varying delays under the leader-follower settings. To synchronize the outputs, a leader is introduced whose connectivity to the followers varies with time, and a novel data-driven consensus protocol is proposed, where the reference input is designed to be the time-varying average of the agents' outputs. Both cases when the reference input is known to the leader and the case otherwise are considered. The protocol does not use explicit or implicit information of its mathematical model. Sufficient conditions are derived to guarantee the closed-loop stability and consensus convergence. Numerical simulations are conducted to demonstrate the effectiveness of the proposed method.

Keywords Non-linear system, data-driven, output synchronization, switching topology, time-varying delay, networked multi-agent system.

1 Introduction

Networked multi-agent systems (NMASs), as a significant part of networked control systems, have been finding wide applications in various control systems in recent years [1, 9, 10, 13, 15, 16, 18]. Consensus problem, or synchronization problem, is a key problem in networked multi-agent systems. Roughly speaking, consensus problem attempts to give rules that synchronize the states/outputs of the agents to a common behaviour. There are extensive results concerning consensus problem in the literature, and the most important issues that impact consensus problem include: delay and the switching of network topology. There also exists much research considering the two issues. To name a few, [13] claimed that it requires the graph to be (jointly) connected (with spanning tree rather than strongly connected) to synchronize the states of the agents, [18] extended Ren's results and proposed that such requirement is still

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needed to synchronize networked multi-agent systems with time-varying delay. Later literature mainly considered the dynamics like second-order integrator [8], high-order integrator [14], LTI dynamics [12]. To sum up, consensus achieving requires the connectivity of all agents.

Despite extensive results concerning consensus problem, yet one obvious problem for these results is that they require the dynamics of the agent to be known. These results, based on accurate mathematical model, is often called model based control (MBC) approach. However, it is difficult to establish the accurate mathematical model, let alone most current plants or agents contain more or less non-linearity. In practical application, first hand principles and system identification may contain un-modelled non-linearities or even may not be available. Therefore, it is necessary to take non-linearity into consideration when studying consensus problem, moreover, it is necessary to study consensus problem without accurate model. It is also worth mentioning that traditional adaptive control approach to solve consensus problem may not work to satisfy this kind of requirement, since, some adaptive protocols are based on the assumptions that the parameter varies slowly or the structure is known [7]. It is well known that neural networks (NNs) have excellent functionality for non-linear approximation. Using this property, adaptive NN consensus control for multi-agent systems with unknown nonlinear dynamics has been studied in [2, 19].

Like PID control, model free adaptive control (MFAC) proves an effective approach to implement data-driven control (DDC) for discrete-time non-linear systems with unknown dynamics [3, 4, 6]. This approach, is independent of mathematically model since it is based on system identification of un-known dynamics. The kind of approach, by estimation of the so-called pseudo partial derivative of the non-linear system, constructs the I/O relationship of the plant/agent, based on which the control law is designed. The paper solves the leader-follower consensus problem based on MFAC approach for a class of networked non-linear multi-agent systems with switching topology and time-varying delay. Both case when the reference input is known and the case otherwise are considered, a novel consensus protocol is proposed, the theoretical analysis is provided, with results illustrated by numerical examples.

The paper is organized as follows. The first section gives literature review on consensus seeking and data-driven control. Some preliminaries of graph theory are reviewed in the second section. Section III and IV give the problem formulation and the main results, respectively. Numerical examples are presented in the next section. Section VII concludes the paper.

2 Preliminaries

Generically, a directed graph can be represented by $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$, where $\mathcal{V}$ is the node set and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$. A graph can also be represented by a $0-1$ matrix $\mathcal{A} = [a_{ij}]$ in the sense that $a_{ij} = 1 \Leftrightarrow (j, i) \in \mathcal{E}$, and $\mathcal{A}$ is termed as adjacency matrix. Let $P = [p_{ij}]$ be non-negative matrix can also be used to represent a graph, denoted by $\mathcal{G}_P$ in the sense that $p_{ij} > 0 \Leftrightarrow a_{ij} = 1$.

A graph has a spanning tree if there exists a path from certain node (referred to as the root) to each of the rest nodes. Let $\mathcal{G}_c$ be a set of graphs with $\mathcal{G}_c = \{\mathcal{G}_i = \{\mathcal{V}, \mathcal{E}_i\} | i = 1, 2, \cdots, d\}$, the joint graph of $\mathcal{G}_c$ is defined to be $\{\mathcal{V}, \cup_{i=1}^{d} \mathcal{E}_i\}$. $\mathcal{G}_i, i = 1, 2, \cdots, d$ is said to have a joint spanning tree if their joint graph has a spanning tree.

Hereafter, Δ denotes the backward difference operator defined by $\Delta x(k) = x(k) - x(k-1)$.

3 Problem Formulation

Consider a networked non-linear multi-agent system, where there are a group of n agents with dynamics described by (1), referred to as the followers, plus one additional agent with label $n+1$, which is referred to as the leader and is also with dynamics (1):

$$y_i(k+1) = f_i(y_i(k), \cdots, y_i(k-n_{yi}), u_i(k), \cdots, u_i(k-n_{ui})) \tag{1}$$

where $y_i(k) \in \mathbb{R}, u_i(k) \in \mathbb{R}$ are the output and input of agent i, respectively; $f_i(\cdot)$ is an unknown non-linear function; and n_{yi} and n_{ui} are the unknown orders of the output and input, respectively.

The objective of this paper, is to design some $u_i(k)$ such that the outputs of the agents can be synchronized, or mathematically

$$\lim_{k\to\infty} |y_i(k) - y_j(k)| = 0, i, j \tag{2}$$

The assumptions of [6] below are adopted and imposed for agent i.

Condition 3.1 The partial derivative of $f_i(\cdots)$ with respect to $u_i(k)$ is continuous.

Condition 3.2 System (1) is generalized Lipschitz, i.e., $|\Delta y_i(k+1)| \leq \overline{\phi}|\Delta u_i(k)|$ for any k and $\Delta u_i(k) \neq 0$, where $\overline{\phi}$ is a positive constant.

Under Assumptions 3.1 and 3.2, non-linear system (1) can be transformed into the following PPD data model:

$$\Delta y_i(k+1) = f_{ci}(k)\Delta u_i(k) \tag{3}$$

For stability analysis the following assumption is also adopted from [6].

Condition 3.3 The PPD $f_{ci}(k)$ satisfies $f_{ci}(k) > 0$ or $f_{ci}(k) < 0$ for all time instant k around a certain operating point.

For assumptions above, the following remarks are made

Remark 3.4 For a completely unknown non-linear system, the verification of Assumption 3.1, 3.2 and 3.3 is a priori. From a practical perspective, Assumptions 3.1, 3.2 and 3.3 are reasonable and acceptable and there are practical examples showing this point [4, 5].

Since the dynamics of system (1) is unknown, the data-driven control framework is first proposed as follows

$$\begin{aligned}\widehat{f}_{ci}(k) = \widehat{f}_{ci}(k-1) + \frac{h_i \Delta u_i(k-1)}{m_i + \Delta u_i^2(k-1)}(\Delta y_i(k) \\ - \widehat{f}_{ci}(k-1)\Delta u_i(k-1))\end{aligned} \tag{4a}$$

$$\widehat{f}_{ci}(k) = \widehat{f}_{ci}(1) \text{ if } |\widehat{f}_{ci}(k)| < e_i \\ \text{or } \Delta u_i(k-1) < e_i \\ \text{or } \mathrm{sign}(\widehat{f}_{ci}(k)) \neq \mathrm{sign}(\widehat{f}_{ci}(1)) \tag{4b}$$

$$u_i(k) = u_i(k-1) + \frac{r_i \widehat{f}_{ci}(k)}{\mu_i + \widehat{f}_{ci}^2(k)}(r_i(k+1) - y_i(k)) \tag{4c}$$

where $\widehat{f}_{ci}(k)$ is the estimation of $f_{ci}(k)$ with the initial value $\widehat{f}_{ci}(1)$, h_i, r_i, μ_i, m_i are positive weighting factors, e_i is a positive constant, $r_i(k+1)$ is the reference input to be designed later for agent i.

Remark 3.5 Framework (4) are in purely distributed manner since each agent has independent set of constants referred to in (4), which are specified respectively to each agent. Moreover, the protocol involves only the input and output data of the agent.

The next job is clear, to synchronize the outputs or design protocol $u_i(k)$, one need to design the reference input $r_i(k+1)$ for each agent.

4 Consensus Protocol

From last section, to synchronize the outputs of the agents one needs to design the reference input for each agent. In the section, a novel protocol is proposed under the data-driven control scheme (4), the details about the scheme can be find in [13, 18].

4.1 Leader Dynamics

In this paper, two cases are considered, the case with unknown reference input, and the case when the leader is with a known reference input y_d. It is assumed that the followers can obtains information from the neighbouring agent including the leader, depending on the displace/range to the other agents, i.e., the connectivity to the leader is time-varying.

For the case with unknown reference input, the reference input is designed to be the following

$$r_{n+1}(t+1) = \frac{1}{2} y_{n+1}(t-1) + \frac{1}{2} y_{n+1}(t) \tag{5}$$

The first result shows that the systems of the leader in both cases are BIBO stable.

Lemma 4.1 *Consider a networked multi-agent system of n agents with non-linear dynamics described by (1). Let Assumption 3.1, 3.2, 3.3, then agent $n+1$ with reference input (5) is stable Moreover, output and input of agnet $n+1$ is bounded.*

Proof The proof is performed in three steps. First, we show that $\widehat{f}_{ci}(k)$ is bounded. This step is similar with that of [6, Chapter 4].

Let condition of (4b) hold, then, it is clear that $\widehat{f}_{ci}(k)$ is bounded. Otherwise, define $\widetilde{f}_{ci}(k) = \widehat{f}_{ci}(k) - f_{ci}(k)$. From the (4a) it can be given that

$$\widetilde{f}_{ci}(k) = \left(1 - \frac{h_i \Delta u_i^2(k-1)}{m_i + \Delta u_i^2(k-1)}\right) \widetilde{f}_{ci}(k-1) + f_{ci}(k-1) - f_{ci}(k) \tag{6}$$

Take absolute value of (6), one has

$$|\widetilde{f}_{ci}(k)| \leq |\left(1 - \frac{h_i \Delta u_i^2(k-1)}{m_i + \Delta u_i^2(k-1)}\right)||\widetilde{f}_{ci}(k-1)| + |f_{ci}(k-1) - f_{ci}(k)| \tag{7}$$

Without loss of generality, based on (4b), it is obvious that

$$|\left(1 - \frac{h_i \Delta u_i^2(k-1)}{m_i + \Delta u_i^2(k-1)}\right)| \leq d_1 < 1 \tag{8}$$

where d_1 is some positive constant.

This leads to that $|\widetilde{f}_{ci}(k)|$ is bounded since $|f_{ci}(k-1) - f_{ci}(k)|$ is. Subsequently, $|\widehat{f}_{ci}(k)|$ is bounded.

Next, we show that the tracking error asymptotically converges to zero.

Given (4c) it can be obtained that

$$\Delta u_i(k) = \frac{r_i \widehat{f}_{ci}(k)}{2(\mu_i + \widehat{f}_{ci}^2(k))}((y_i(k-1) - y_i(k)) \tag{9}$$

Next, define

$$s_i(k) = r_i \frac{f_{ci}(k)\widehat{f}_{ci}(k)}{\mu_i + \widehat{f}_{ci}^2(k)}.$$

By (3),

$$y_{n+1}(k+1) = (1 - \frac{1}{2} r_{n+1} s_{n+1}(k)) y_{n+1}(k) + \frac{1}{2} r_{n+1} s_{n+1}(k) y_{n+1}(k-1) \tag{10}$$

or compactly

$$\begin{bmatrix} y_{n+1}(k+1) \\ y_{n+1}(k) \end{bmatrix} = \begin{bmatrix} 1 - \frac{1}{2} r_{n+1} s_{n+1}(k) & \frac{1}{2} r_{n+1} s_{n+1}(k) \\ 1 & 0 \end{bmatrix} \begin{bmatrix} y_{n+1}(k) \\ y_{n+1}(k-1) \end{bmatrix} \tag{11}$$

The resetting algorithm (4b) makes sure that $s_i(k) \geq 0, \ \forall\, i$. Moreover, since f_{ci} is bounded from both above and below and the sign is unchanged, and the fact that $\widehat{f}_{ci} \geq e_i$, it is clear that there exist some constants d_2, d_3 with

$$0 < r_{n+1} d_2 \leq s_i(k) \leq r_{n+1} d_3 \tag{12}$$

Note that $\begin{bmatrix} 1 - \frac{1}{2} r_{n+1} s_{n+1}(k) & \frac{1}{2} r_{n+1} s_{n+1}(k) \\ 1 & 0 \end{bmatrix}$ is SIA [17], $\begin{bmatrix} y_{n+1}(k) \\ y_{n+1}(k-1) \end{bmatrix}$ converges to $\begin{bmatrix} 1 \\ 1 \end{bmatrix} c$, where c is some constant. This implies that $y_{n+1}(k)$ converges to some constant c.

Finally, by [4, 6] it can be given that $u_{n+1}(k)$ is also bounded.

Remark 4.2 For agent $n+1$, it is not necessary that $r_{n+1}(k+1) = \frac{1}{2} y_{n+1}(t-1) + \frac{1}{2} y_{n+1}(t)$, in fact, it can be any convex combination of $y_{n+1}(k-s), s = 1, 2, \cdots, N$, where N is some constant integer.

4.2 Case with Switching Topology and Time-Varying Delay

Without loss of generality, the time-varying delay induced by network at k, is denoted by $\tau_{ij}(k)$. Next, the following assumption is imposed

Condition 4.3 There exists some upper bound τ with $\tau_{ij}(k) \leq \tau$.

Condition 4.4 There exists some positive integer B and a strictly increasing sequence of integers t_k with $t_{k+1} - t_k \leq B$, satisfying that for all k, $\mathcal{G}_{\mathcal{A}(t)}, t = t_k, t_k + 1, \cdots, t_{k+1} - 1$ have a joint spanning tree rooted at the leader.

Condition (4.4), or periodic switching connectivity condition to be specific, is similar with the condition in [13]. It is clear that the time-varying of delays and topology switching are correlated. For their relationship details the readers are referred to reference [18].

Represent the connectivity topology of the followers by matrix $\mathcal{A}(k) = [a_{ij}(k)]$.

Note that for agent i, its own output $y_i(k)$ is still available, thus (4a) and (4b) is still valid. The key of this problem is to design the reference input $r_i(k+1)$ such that the outputs of all agents can be synchronized. In this paper, the tracking reference input is proposed as follows

$$\begin{aligned} r_i(k+1) = & \frac{1}{\sum_{j \in N_i(k)} a_{ij}(k) + b_i(k) + k_i} \Big(\sum_{j \in N_i(k)} a_{ij}(k) y_j(k - \tau_{ij}(k)) \\ & + b_i(k) y_0(k - \tau_{i0}(k)) + k_i y_i(k-1) \Big), i < n+1 \\ r_{n+1}(k+1) = & \begin{cases} \frac{1}{2}(y_{n+1}(k) + y_{n+1}(k-1)) & r_{n+1}(\infty) \text{ is unknown} \\ r_{n+1} & \text{otherwise} \end{cases} \end{aligned} \tag{13}$$

where r_{n+1} is the known tracking reference input, $N_i(k)$ is the set of agent is neighbours from the original group of followers. $b_i(k) > 0$ if agent $n+1$'s output can be obtained by agent i, otherwise $b_i(k) = 0$, $k_i \in (0, 1)$ is weighting factor for agent's past information.

Theorem 4.5 *Consider a networked multi-agent system of n agents with non-linear dynamics described by (1). Let Assumption 3.1, 3.2, 3.3, 4.4 and 4.3 hold. Protocol given by (4) and (13) asymptotically solves the output consensus problem for system (1). Moreover, output and input of the system is bounded.*

Proof The first step is to prove that $\widehat{f}_{ci}$ is bounded and the proof is the same with that of Lemma 4.1.

For the second step,

Case I:

From last Lemma 4.1 it is clear that $y_0(\infty) = c$, where c is some constant. Next,

$$\begin{aligned}\Delta u_i(k) &= \frac{r_i\widehat{f}_{ci}(k)}{\mu_i+\widehat{f}^2_{ci}(k)}\Big(\frac{1}{\sum_{j\in N_i(k)} a_{ij}(k)+b_i(k)+k_i}\Big(\sum_{j\in N_i(k)} a_{ij}(k)y_j(k-\tau_{ij}(k)) \\ &\quad + b_i(k)y_{n+1}(k-\tau_{i(n+1)}(k)) + k_i y_i(k-1)\Big) - y_i(k)\Big) \\ &= \frac{r_i\widehat{f}_{ci}(k)}{\mu_i+\widehat{f}^2_{ci}(k)}\Big(\sum_{i=1}^{n+1} m_{ij}(k)y_j(k-\tau_{ij}(k)) - y_i(k)\Big), i<n+1 \\ \Delta u_i(k) &= \frac{r_i\widehat{f}_{ci}(k)}{2\mu_i+2\widehat{f}^2_{ci}(k)}(y_i(k-1)-y_i(k)), i=n+1\end{aligned} \tag{14}$$

where

$$m_{ij}(k) = \begin{cases} \frac{a_{ij}(k)}{\sum_{j\in N_i(k)} a_{ij}(k)+b_i(k)+k_i} & j<n+1, j\neq i \\ \frac{k_i}{\sum_{j\in N_i(k)} a_{ij}(k)+b_i(k)+k_i} & j=i \\ \frac{b_j(k)}{\sum_{j\in N_i(k)} a_{ij}(k)+b_i(k)+k_i} & j=n+1 \end{cases}$$

then by (4c),

$$\Delta y_i(k+1) = r_i\widetilde{s}_i(k)\sum_{j=1}^{n+1} p_{ij}(k)(y_j(k-\tau_{ij}(k)) - y_i(k)), \tag{15}$$

where $\tau_{ii}(k)=1, \widetilde{s}_i(k)=s_i(k)$ if $i<n+1$, $\widetilde{s}_i(k)=s_i(k)/2$ if $i=n+1$.

Next, define $P(k)=[p_{ij}(k)]=\begin{bmatrix}M^T(k) & m^T(k)\end{bmatrix}^T$, $m(k)=\begin{bmatrix}0 & 0 & \cdots & 0 & 1\end{bmatrix}\in\mathbb{R}^{n+1}$. Note that $P(k)$ is stochastic with positive diagonals. Furthermore, $G_{P(k)}$ has a joint spanning tree. In order to facilitate our analysis, we introduce some notations. Define

$$\Lambda(P(k)) = \{B=[b_{ij}] | b_{ij}=p_{ij}(k) \text{ or } b_{ij}=0\}.$$

Let $P_i(k)\in\Lambda(P(k)), i=0,1,\cdots,\tau$ such that

$$\text{diag}(P(k)) + \sum_{j=0}^{\tau} P_j(k) = P(k)$$

Define

$$\begin{aligned} S(k) &= \text{diag}\{\widetilde{s}_1(k), \widetilde{s}_2(k), \cdots, \widetilde{s}_{n+1}(k)\}, \\ R &= \text{diag}\{r_1, r_2, \cdots, r_{n+1}\}, \end{aligned}$$

let $Q(k)=RS(k)$.

Now let

$$\begin{aligned} y(k) &= \begin{bmatrix} y_1(k) & y_2(k) & \cdots & y_{n+1}(k)\end{bmatrix}^T \\ Y(k) &= \begin{bmatrix} y^T(k) & y^T(k-1) & \cdots & y^T(k-\tau).\end{bmatrix}^T \end{aligned} \tag{16}$$

Based on (15),

$$Y(k+1) = \Omega(k)Y(k), \tag{17}$$

where

$$\Omega(k) = \begin{bmatrix} \begin{matrix} I_{n+1} - Q(k) \\ +Q(k)P_0(k) \end{matrix} & \begin{matrix} \mathbf{diag}(Q(k)P(k)) \\ +Q(k)P_1(k) \end{matrix} & \cdots & Q(k)P_{\tau-1}(k) & Q(k)P_\tau(k) \\ I_{n+1} & 0 & \cdots & 0 & 0 \\ 0 & I_{n+1} & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & I_{n+1} & 0 \end{bmatrix}$$

It is easy to show $\Omega(k)$ has all row sums equal to one. As is shown in the last proof, $s_i(k)$ is bounded from both below and above by some positive constant, thus it is clear that there exists some sufficiently small r_i such that $\Omega(k)$ is non-negative. It is clear that $\Omega(k)$ can also be non-negative if r_i is small enough. Based on Theorem 2 of [18], one has $y(k)$ asymptotically reaches consensus if $P(k)$ has a periodically joint spanning tree. Since the overall system has a periodically joint spanning tree rooted at the leader, consensus can be reached.

Finally, we show that $u_i(k)$ is bounded. As Lemma 4.1 shows, the output of the leader asymptotically converges to some constant, since consensus can be reached, by [4, Chapter 4], one can conclude that the reference input is bounded.

Case II:

This case can viewed as a special case of the first case. Introduce a virtual leader labelled by $n+2$, whose dynamics is described as follows

$$y_{n+2}(k+1) = \frac{1}{2}y_{n+2}(k) + \frac{1}{2}u_{n+2}(k) \tag{18}$$

It can be verified that the dynamics of this virtual leader satisfies Assumption 3.2, 3.1, 3.3. Assume that $\Delta y_{n+2}(1) = 0, y_{n+2}(1) = r_{n+1}, y_{n+2}(2) = r_{n+1}, \Delta u_{n+2}(1) = 0$. Using framework (4) to track r_{n+1}, it is easy to obtain that $u_{n+2}(k) = r_{n+1}$ for all k. Thus $y_{n+2}(k) = r_{n+1}$ for all k.

To this point, the consensus problem in this case can be viewed as all $n+1$ agents try to synchronize with the virtual agent $n+2$. Using the result of Case I one can get that consensus can be obtained.

Remark 4.6 The protocol in this paper is proposed based on MFAC approach. The main difference between MFAC based consensus protocol and the model-based adaptive consensus protocol are: 1) Traditional adaptive consensus protocol is proposed for a time-invariant or slowly time-varying system with a known system structure and system orders, whereas MFAC is proposed for unknown non-linear systems; 2) The traditional protocol design depends on the mathematical model, whereas the MFAC design depends merely on the I/O data.

5 Numerical simulations

The non-linear plant in [11] is considered in our example, with little parameter variation. We consider a group of 4 agents with dynamics given as follows

$$\begin{aligned} x_i(k) &= 1.5u_i(k) - 1.5u_i(k)^2 + 0.5u_i(k)^3 \\ y_i(k+1) &= 0.6y_i(k) - 0.1y_i(k-1) + 1.2x_i(k) - 0.1x_i(k-1) \end{aligned} \tag{19}$$

where agent 1 is set to be the leader.

The topology switching is given as follows

$$\mathcal{A}(t) = \begin{cases} A_1 & \text{mod } (t,3) \in [0, 1.2) \\ A_2 & \text{mod } (t,3) \in [1.2, 2.4) \\ A_3 & \text{mod } (t,3) \in [2.4, 3), \end{cases} \tag{20}$$

where

$$A_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, A_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0.5 & 0.5 \\ 0 & 0.5 & 0 & 0.5 \end{bmatrix}, A_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0.5 & 0.5 \\ 0 & 0 & 0.5 & 0.5 \end{bmatrix}$$

It is clear that the topology, though switches, has a periodically joint spanning tree. In the following, two numerical examples are given to demonstrate the effectiveness of the proposed protocols, respectively. Both examples are with a step-size $h = 0.02$.

5.1 The Case without reference input

In this example, the network delay is time-varying, with a upper bound $\tau = 2$. The communication delay between agents is randomly chosen. The initial outputs of each agent are randomly chosen from $[-5, 2.5)$. Using the proposed protocol is synchronized the outputs of this system, the trajectory is given by Fig 1.

As is show by the Fig. 1, the outputs are the agents are synchronized, and the stability of the system is also guaranteed.

5.2 The Case with reference input

In this example, the network delay is time-varying, with a upper bound $\tau = 2$. The communication delay between agents is randomly chosen. The initial values of each agent are randomly chosen from $[-5, 2.5)$, the reference input is set to be $r_{n+1} = 2$. Using the proposed protocol is synchronized the outputs of this system, the trajectory is given by Fig 2.

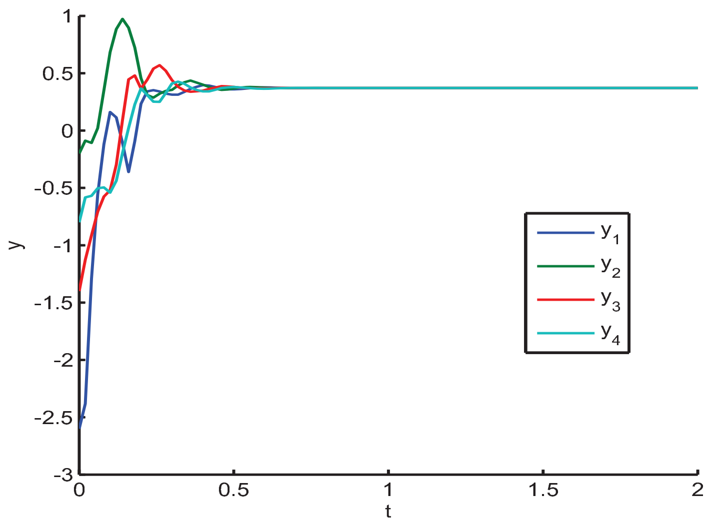

Figure 1: Output trajectory without reference input

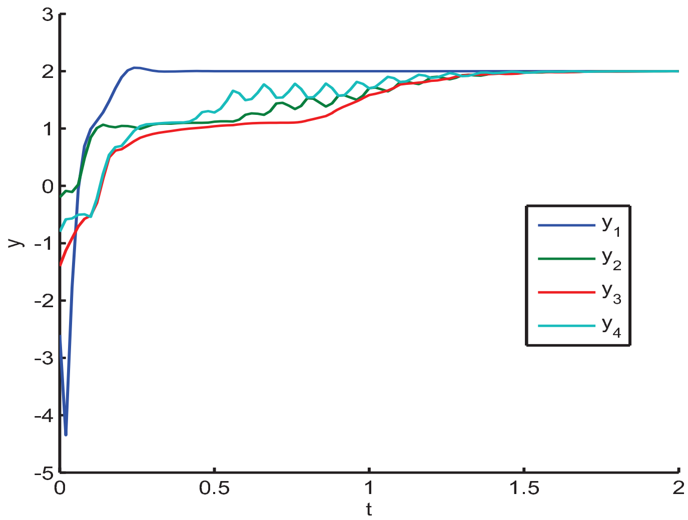

Figure 2: output trajectory with reference input

From Fig. 2 it is clear that the outputs of the agents are stable, the consensus of the system is also reached. Also note that neither multi-agent system in the two examples is stable with respect to the origin.

6 Conclusion

Consensus problem is fundamental problem for distributed control and optimization. In this paper, we have presented novel consensus protocol under the leader-follower setting based on data-driven control for a class of networked non-linear networked multi-agent system, where periodic topology and time-varying network delay exist simultaneously. The protocol has been used to synchronize the outputs of the agents, proved to be effective both theoretically and numerically. Contributions of this paper includes: 1) Leader-follower consensus problem for networked multi-agent system with switching topology, time-varying delay and a kind of non-linearity has been considered, and then novel consensus protocols based on data-driven networked control scheme have been designed. 2) The proposed protocols are in purely distributed manner, which is easy to implement in practice. 3) Consensus can be guaranteed along with system stability by the proposed consensus protocol.

Future research efforts will be devoted to extending the proposed method to more general non-linear multi-agent systems and also some other network-induced constraints such as data quantization, time-varying sampling.

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