Tracking control of wheeled mobile robots with communication delay and data loss

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Abstract This paper considers the tracking control problem of a wheeled mobile robot under situation of communication delay and consecutive data packet dropouts in the feedback channel. A tracking controller in discrete-time domain for the case of ideal network condition is first derived, and then the networked predictive controller as well as two algorithms for dealing with communication delay and consecutive data packet dropouts are proposed. Simulation and experimental results verify the realizability and effectiveness of the proposed algorithms.

Keywords Communication delay, consecutive data loss, networked predictive control, trajectory tracking, wheeled mobile robot

1 Introduction

In recent years, with the rapid development of network technology and the widespread use of the Internet, the use of networks in control systems has dramatically increased [1], [2], [3], [4] and the discussions of controller design for networked control systems (NCSs) have been developed in depth [5], [6], which has allow the control of large distributed systems. When a communication network is introduced into a traditional control system, the design process of control scheme and stability analysis of the system are changed, and the traditional control system is transformed into a networked control system. Compared to the traditional control systems, the spatial distribution of sensors, controllers and actuators of a NCS offers many benefits such as cost reduction, increased structure flexibility, and easy maintenance.

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However, the communication delays and data packet dropouts in a networked control system are inevitable, and which will affect the performance of the system and may even make the system unstable. To compensate the network delay and data loss, methods of networked predictive control, for linear [7], [8], [9], [10] and nonlinear systems [11], have been proposed, which consider the stability of closed-loop NCSs and can actively compensate for network delays and data packet drops. These methods have been successfully applied in linear systems, such as in DC servo [12], [13] and ball-and-beam systems [14]. Nevertheless, only few applications of nonlinear systems have been implemented based on above approach.

To solve the tracking control problem of wheeled mobile robots, authors in [15] and [16] used the output feedback and state feedback methods. Rao et al. [17] studied navigation problem of Non-holonomic mobile robot using neuro-fuzzy logic approach. Huang et al. [18] considered the problem of global tracking and stabilization control of internally damped mobile robots with unknown parameters subject to input torque saturation and external disturbances. In [19], [20], [21], tracking control laws were designed at the torque level for driving mobile robots. In [22], an adaptive moving-target tracking control scheme was constructed for a vision-based mobile robot.

Nevertheless, with the communication network introduced into the closed-loop of wheeled mobile robots, the tracking control methods proposed above seem to be ineffective, as they were designed where the network condictions are ideal. Fortunately, several researchers have made efforts to solve these problems.

In [23], the formation tracking problem of nonlinear multi-agent systems with communication delays was investigated by using a novel stochastic analysis approach. The authors in [24] derived the exact discrete-time model of a mobile robot with transport delay, and designed a path-tracking control law based on cascaded systems theory. To compensate the performance degradation due to delays, the authors in [25], [26] proposed a predictor-based control strategy. Moreover, the experiment in [26] used an interconnected robotic platform which was located partly in Netherlands and Japan, but the case of data loss was not addressed.

To the best of our knowledge, this is the first time a tracking control problem is solved using networked predictive control scheme under conditions of communication delays and data packet dropouts. Considering the continuous time domain tracking controller proposed in [26], [27], [28], this paper obtained similar results by extending this method into discrete-time domain. Following that, a networked predictive control scheme was designed to compensate the negative effects caused by communication delays and data losses.

The contribution of the current paper is twofold. First, a networked predictive control strategy and two algorithms are proposed, which are capable of compensating for communication delays and data packet losses. Second, the algorithms are validated using an experimental platform specifically designed for the purposes of this paper, where a wireless network is used in the feedback channel. It is worth noting that the results obtained from simulation and experiments are consistent.

The remainder of this paper is organized as follows. Section 2 provides some mathematical tools. Section 3 proposes tracking controller for a wheeled mobile robot without and with adverse network conditions in the feedback channel. In Section 4, the experimental platform designed for this paper is described. Simulation and experimental results are presented in Section 5. Finally, this paper concludes with a discussion in Section 6.

2 Preliminaries

In this section we present some mathematical tools that are employed in further parts of this article. Consider the parameterized discrete-time cascaded systems of the form

$$x(k+1) = f_T(k, x(k), z(k)) \tag{1}$$

$$z(k+1) = g_T(k, z(k)) \tag{2}$$

where $x, z \in R^n$ and $T > 0$ is a parameter. There also exists the unperturbed auxiliary system

$$x(k+1) = f_T(k, x(k), 0) \tag{3}$$

A function $\alpha : R_{\geq 0} \to R_{\geq 0}$ is said to be of class $\mathcal{K}(\alpha \in \mathcal{K})$ if it is continuous, $\alpha(0) = 0$, and strictly increasing; A function $\alpha : R_{\geq 0} \to R_{\geq 0}$ is said to be of class $\mathcal{K}_\infty$ if $\alpha \in \mathcal{K}$ and, in addition, $\lim_{s\to\infty} \alpha(s) \to \infty$. A function $\gamma : R_{\geq 0} \to R_{\geq 0}$ is said to be of class $\mathcal{N}$ if $\gamma(\cdot)$ is continuous and nondecreasing.

Assumption 2.1 *[29] There exist $\gamma_2 \in \mathcal{N}$, $\gamma_1, \gamma_3 \in \mathcal{K}_\infty$ and $T^* > 0$ such that for all $\xi \in R^n$, $k \geq 0$ and $T \in (0, T^*)$, we have $|f_T(k, x, z)| \leq \gamma_1(|\xi|)$ and $|f_T(k, x, z) - f_T(k, x, 0)| \leq T\gamma_2(|x|)\gamma_3(|z|)$.*

Lemma 2.2 *[29] Suppose that f_T of the system (1) satisfies Assumption 2.1. Then, the overall system (1) and (2) is Uniformly Globally Asymptotically Stable (UGAS) if the following conditions hold:*

1) system (3) is Lyapunov UGAS;

2) system (2) is UGAS;

3) system (1) and (2) satisfies the property of globally uniformly bounded (UGB).

Proposition 2.3 *[29] Given positive numbers c_1, c_2, c_3, c_4, p_M, $\widehat{T}$, a persistently exciting function $p_T(\cdot)$, positive definite, radially unbounded, and continuous functions $\psi : R^n \to R_{\geq 0}$ and $V_T : R \times R^n \to R_{\geq 0}$, suppose that the following conditions hold for all $x \in R^n$, $T \in (0, \widehat{\widehat{T}})$ and all $k \geq 0$:*

$$c_1\psi(x) \leq V_T(k, x) \leq c_2\psi(x) \tag{4}$$

$$\frac{V_T(k+1, F_T(k, x)) - V_T(k, x)}{T} \leq -c_3 p_T(k)\psi(x) + Tc_4\psi(x) \tag{5}$$

$$p_T(k) \leq p_M \tag{6}$$

Then, there exists $\varepsilon^* > 0$ and for each $\varepsilon \in (0, \varepsilon^*)$ there exist positive numbers d_1, d_2, d_3, T^* such that the function

$$U_T(k, x) := V_T(k, x) + \varepsilon W_T(k, x) \tag{7}$$

with $W_T(k,x) := -T\sum_{i=k}^{\infty} e^{(k-i)T} p_T(i) V_T(k,x)$ *satisfies for all* $x \in R^n$, $T \in (0, T^*)$ *and* $k \geq 0$

$$d_1\psi(x) \leq U_T(k,x) \leq d_2\psi(x) \tag{8}$$

$$\frac{U_T(k+1, F_T(k,x)) - U_T(k,x)}{T} \leq -d_3\psi(x) \tag{9}$$

Proposition 2.4 *[29] Consider the system (1) with input* z*. Suppose that there exist* $\widetilde{\alpha}_1$, $\widetilde{\alpha}_2$, $\varphi \in \mathcal{K}_\infty$, $\widetilde{\gamma}_1$, $\widetilde{\gamma}_2 \in \mathcal{N}$, $T^* > 0$, $c \geq 0$ *and for each* $T \in (0, T^*)$ *there exists* $V_T : R_{\geq 0} \times R^{n_x} \to R_{\geq 0}$ *such that for all* $x \in R^{n_x}$, $z \in R^{n_z}$, $k \geq 0$ *and* $T \in (0, T^*)$ *we have that*

$$\widetilde{\alpha}_1(|x|) \leq V_T(k,x) \leq \widetilde{\alpha}_2(|x|) + c \tag{10}$$

$$V_T(k+1, f_T(k,x,z)) - V_T(k,x) \leq T\widetilde{\gamma}_1(|z|)\varphi(V_T(k,x)) + T\widetilde{\gamma}_2(|z|) \tag{11}$$

$$\int_1^\infty \frac{ds}{\varphi(s)} = \infty \tag{12}$$

If, furthermore, the solutions of (2) satisfy the summability condition

$$T\sum_{k=k_0}^{\infty} \mu(|\phi_T^z(k,k_0,z_0)|) \leq \rho(|z_0|) \tag{13}$$

with some $\rho \in \mathcal{K}_\infty$ *and* $\mu(s) := \widetilde{\gamma}_1(s) + \widetilde{\gamma}_2(s)/\varphi(1)$ *then, the system (1), (2) are UGB.*

3 Design of tracking controller

In this section, a local tracking controller under ideal network conditions is provided firstly, then networked predictive tracking controller which solve time-delays and consecutive data dropouts in the feedback channel is proposed.

3.1 Local controller

Consider the discrete-time kinematics model of a wheeled mobile robot as follows

$$\begin{aligned} x(k+1) &= x(k) + T\upsilon(k)cos\theta(k) \\ y(k+1) &= y(k) + T\upsilon(k)sin\theta(k) \\ \theta(k+1) &= \theta(k) + T\omega(k) \end{aligned} \tag{14}$$

where $q(k) \triangleq [x(k), y(k), \theta(k)]^T$ is state of mobile robot, and $x(k)$, $y(k)$ represent position information of mobile robot in global coordinate frame, $\theta(k)$ denotes the angle between heading direction of mobile robot and x-axis of global coordinate frame at stepping time k, respectively. $u(k) = [\upsilon(k)\ \omega(k)]^T$ is control input of the system, $\upsilon(k)$ and $\omega(k)$ are also linear and angular velocities of mobile robot. T is sampling period with $0 < T < 1s$.

Supposing that there is a virtual mobile robot, whose moving trajectory is considered to be the reference trajectory of real mobile robot, and the kinematics characteristics of this virtual mobile robot satisfy (14), then the positional relationship between virtual and real mobile robots can be described in Figure. 1.

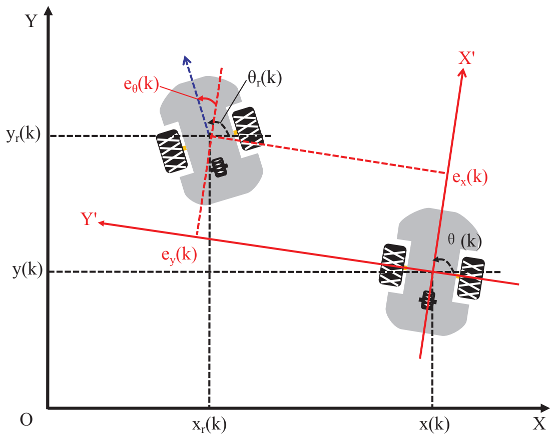


Figure 1: Positional relationship between virtual and real mobile robot.

The state of virtual mobile robot is defined as $q_r(k) \triangleq [x_r(k), y_r(k), \theta_r(k)]^T$. With positional relationship as shown in Figure. 1, the state deviation can be described as

$$E(k) = \begin{bmatrix} e_x(k) \\ e_y(k) \\ e_\theta(k) \end{bmatrix} = \begin{bmatrix} cos\theta(k) & sin\theta(k) & 0 \\ -sin\theta(k) & cos\theta(k) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_r(k) - x(k) \\ y_r(k) - y(k) \\ \theta_r(k) - \theta(k) \end{bmatrix} \tag{15}$$

where $E(k) \triangleq [e_x(k), e_y(k), e_\theta(k)]^T$ is state error. The detailed expressions of the state error's components are

$$\begin{aligned} e_x(k) =& x_r(k)cos\theta(k) - x(k)cos\theta(k) + y_r(k)sin\theta(k) - y(k)sin\theta(k) \\ e_y(k) =& - x_r(k)sin\theta(k) + x(k)sin\theta(k) + y_r(k)cos\theta(k) - y(k)cos\theta(k) \\ e_\theta(k) =& \theta_r(k) - \theta(k) \end{aligned} \tag{16}$$

Using (16) and the system model of mobile robot (14), as well as the fact that

$$\begin{aligned} cos\theta(k+1) &= cos\theta(k) - Tsin\theta(k)\omega(k) \\ sin\theta(k+1) &= sin\theta(k) + Tcos\theta(k)\omega(k) \end{aligned}$$

it gives

$$\begin{aligned} e_x(k+1) =& e_x(k) + T\omega(k)e_y(k) + T\upsilon_r(k)cose_\theta(k) - T\upsilon(k) \\ e_y(k+1) =& e_y(k) - T\omega(k)e_x(k) + T\upsilon_r(k)sine_\theta(k) \\ e_\theta(k+1) =& e_\theta(k) + T[\omega_r(k) - \omega(k)] \end{aligned} \tag{17}$$

where $\upsilon_r(k)$ and $\omega_r(k)$ are linear and angular velocities of virtual mobile robot, respectively. The next step is to design tracking controller $u(k) \triangleq [\upsilon(k), \omega(k)]^T$ that eliminate the state deviations.

In this paper, a well-known tracking controller is chosen, which was proposed and examined in [26]-[28]. The discrete-time form of this tracking controller is

$$\begin{aligned} \upsilon(k) &= \upsilon_r(k) + k_x e_x(k) - k_y \omega_r(k) e_y(k) \\ \omega(k) &= \omega_r(k) + k_\theta e_\theta(k) \end{aligned} \tag{18}$$

where k_x, k_y and k_θ are positive numbers with properly chosen values, and $\upsilon_r(k)$, $\omega_r(k)$ are reference inputs. To compact the notation, in the sequel it will use $\psi_k/\psi_{*,k} = \psi(k)/\psi_*(k)$ for short. Substituting (18) into (17), the state error dynamics of mobile robot could be derived as

$$\begin{cases} \begin{bmatrix} e_{x,k+1} \\ e_{y,k+1} \end{bmatrix} = A_k \begin{bmatrix} e_{x,k} \\ e_{y,k} \end{bmatrix} + G_k e_{\theta,k} & \text{(19a)} \\ e_{\theta,k+1} = (1 - T k_\theta) e_{\theta,k} & \text{(19b)} \end{cases}$$

with

$$A_k = \begin{bmatrix} 1 - Tk_x & T\omega_{r,k}(1+k_y) \\ -T\omega_{r,k} & 1 \end{bmatrix}, \; G_k = T \begin{bmatrix} k_\theta e_{y,k} + \upsilon_{r,k} \frac{cose_{\theta,k} - 1}{e_{\theta,k}} \\ -k_\theta e_{x,k} + \upsilon_{r,k} \frac{sine_{\theta,k}}{e_{\theta,k}} \end{bmatrix}$$

Lemma 3.1 *Consider the discrete-time kinematic model of wheeled mobile robot (14); if tracking controller (18) is used, then state error system (19) is UGAS.*

Proof It is obvious that the state error system (19) satisfies the discrete-time cascaded systems (1) and (2), and tracking controller (18) is a special case of section IV in [29] with $T\eta = -k_y\omega_r(k)e_y(k)$. It can be proved that

$$\begin{aligned} \eta &= -\frac{k_y \omega_r(k) e_y(k)}{T} \\ \Rightarrow |\eta| &= \left| \frac{k_y \omega_r(k)}{T} \right| |e_y(k)| \\ \Rightarrow |\eta| &\leq \left| \frac{k_y \omega_r(k)}{T} \right| \sqrt{e_x^2(k) + e_y^2(k)} = K|X| \end{aligned}$$

with $K = |k_y\omega_r(k)/T|$ and $|X| = \sqrt{e_x^2(k) + e_y^2(k)}$, which satisfies the assumptions of Proposition 4 in [29], then the state error system (19) could be proved to be UGAS, and the proof process of which is omitted due to its similarity.

Remark 3.2 The discrete-time tracking controller (18) is a direct extension of the controller proposed in [26]-[28] under continuous-time. The main value of Lemma 3.1 lies in that it demonstrates that this control scheme ensures stability, and this result is vital for designing the networked predictive controller in the following sections.

In above results, communication problems are not taken into consideration. When a communication network is introduced into a traditional control system, the design process of control scheme and stability analysis of the system are changed, which transforms the traditional control system into a networked control system. The communication delays and data packet dropouts in a networked control system will affect the performance of control system and may even make the system unstable. To this end, new tracking controllers should be utilized to overcome these undesirable effects.

3.2 Networked predictive tracking controller for constant delay

In this section, the case that a constant communication delay τ exists in the feedback channel (from sensor to controller) is considered (see Figure. 2), where $0 < \tau \leq \tau_{max}$ and τ_{max} is assumed to be upper bound of the delay. At stepping time k, the delayed state $q_{k-\tau}$ is received. When using tracking controller (18), the linear and angular velocities of mobile robot become oscillatory (see Figure. 7 in section 5.2), and the actual moving trajectory of mobile robot cannot converge to the reference due to that the control goal is $q_{k-\tau} \to q_{r,k}$.

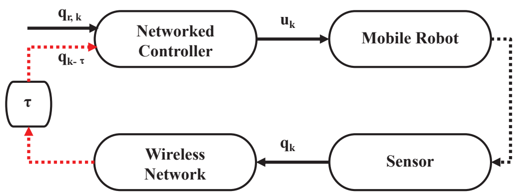


Figure 2: A constant time-delay exists in the feedback channel of the mobile robot.

To cope with this problem and to compensate the effect of time-delay, a network-based predictive control scheme is proposed in Algorithm 1, and the following predictive tracking controller is designed.

$$\begin{aligned} \widehat{\upsilon}_{k|k-\tau} &= \upsilon_{r,k} + k_x \widehat{e}_{x,k|k-\tau} - k_y \omega_{r,k} \widehat{e}_{y,k|k-\tau} \\ \widehat{\omega}_{k|k-\tau} &= \omega_{r,k} + k_\theta \widehat{e}_{\theta,k|k-\tau} \end{aligned} \tag{20}$$

By setting $\widehat{q}_{k-\tau} = q_{k-\tau}$ and $\widehat{E}_{k-\tau} = E_{k-\tau}$, based on predictive tracking controller (20), system model of mobile robot (14) and Algorithm 1, the estimate state error system is given as

$$\begin{cases} \begin{bmatrix} E_{x,k+1} \\ E_{y,k+1} \end{bmatrix} = \begin{bmatrix} C_x & TC_y\Omega_{r,k} \\ -T\Omega_{r,k} & I \end{bmatrix} \begin{bmatrix} E_{x,k} \\ E_{y,k} \end{bmatrix} \\ + \begin{bmatrix} TC_\theta \overline{E}_{y,k} + TV_{r,k}\Theta_{cos} \\ -TC_\theta \overline{E}_{x,k} + TV_{r,k}\Theta_{sin} \end{bmatrix} E_{\theta,k} & \text{(21a)} \\ E_{\theta,k+1} = (I - TC_\theta)E_{\theta,k} & \text{(21b)} \end{cases}$$

with $E_{\gamma,k} = [\widehat{e}_{\gamma,k}, \widehat{e}_{\gamma,k-1}, \cdots, \widehat{e}_{\gamma,k-\tau}]^T$, $\gamma \in (x, y, \theta)$. $\overline{E}_{\delta,k} = diag\{\widehat{e}_{\delta,k}, \widehat{e}_{\delta,k-1}, \cdots, \widehat{e}_{\delta,k-\tau}\}$, $\delta \in (x, y)$. $V_{r,k} = diag\{\upsilon_{r,k}, \upsilon_{r,k-1}, \cdots, \upsilon_{r,k-\tau}\}$, $\Omega_{r,k} = diag\{\omega_{r,k}, \omega_{r,k-1}, \cdots, \omega_{r,k-\tau}\}$, $C_x = diag\{1-$

Algorithm 1 Networked predictive control scheme for constant communication delay in the feedback channel

1: Set up a buffer to store $q_{r,k}$ at each sampling step.
2: At stepping time k, for $k \in (0, \infty)$. If $0 \leq k < \tau$, return to step 1 (store $q_{r,k}$). If $k \geq \tau$, the delayed state information $q_{k-\tau}$ is received; Calculate state error $E_{k-\tau}$ using (16), then $u_{k-\tau}$ is derived.
3: Obviously, it has $\widehat{u}_{k-\tau|k-\tau} = u_{k-\tau}$, then the predictive controller $\widehat{u}_{k|k-\tau}$ can be derived using following steps.
4: **for** $i = 1$; $i \leq \tau$; $i++$ **do**
5: Substitute $\widehat{u}_{k-\tau+i-1|k-\tau}$ into system model (14), which gives $\widehat{q}_{k-\tau+i|k-\tau}$.
6: Calculate state error $\widehat{E}_{k-\tau+i|k-\tau}$ using (16) and predictive controller $\widehat{u}_{k-\tau+i|k-\tau}$ through (20) with $\widehat{q}_{k-\tau+i|k-\tau}$ and $q_{r,k-\tau+i}$.
7: **end for**
8: Finally, let $u_k = \widehat{u}_{k|k-\tau}$.

$Tk_x\}$, $C_y = diag\{1 + k_y\}$, $C_\theta = diag\{k_\theta\}$, $\Theta_{cos} = diag\{\frac{cos\widehat{e}_{\theta,k}-1}{\widehat{e}_{\theta,k}}, \cdots, \frac{cos\widehat{e}_{\theta,k-\tau}-1}{\widehat{e}_{\theta,k-\tau}}\}$ and $\Theta_{sin} = diag\{\frac{sin\widehat{e}_{\theta,k}}{\widehat{e}_{\theta,k}}, \cdots, \frac{sin\widehat{e}_{\theta,k-\tau}}{\widehat{e}_{\theta,k-\tau}}\}$.

Next, our main result is given and which will prove that the closed-loop estimate state error system (21) is UGAS.

Theorem 3.3 *Consider the discrete-time kinematics model of wheeled mobile robot (14) with constant communication delay τ in the feedback channel, where $0 < \tau \leq \tau_{max}$. There exist positive numbers ω_m and ω_M, with $0 < \omega_m < \omega_M$. If the predictive tracking controller (20) is used, and the following conditions hold*

$$k_x \in (0, \frac{1}{T}),\ k_y > 0,\ k_\theta \in (0, \frac{2}{T}) \tag{22}$$

$$max\{|\upsilon_{r,k}|,\ |\omega_{r,k}|, \frac{|\omega_{r,k} - \omega_{r,k-1}|}{T}\} \leq \omega_M \tag{23}$$

$$min\{|\upsilon_{r,k}|,\ |\omega_{r,k}|, \frac{|\omega_{r,k} - \omega_{r,k-1}|}{T}\} \geq \omega_m \tag{24}$$

then the closed-loop estimate state error system (21) is UGAS, and the constant time-delay exists in the feedback channel can be compensated actively.

Proof The Lemma 2.2 is used to prove this result, the state x in cascaded system (1) corresponds here to $X = [E_{x,k}\ E_{y,k}]^T$ and the input z in (1) and (2) corresponds here to $z = E_{\theta,k}$. Assumption 2.1 hold trivially with $\xi = \sqrt{|X|^2 + \omega_M^2/(k_\theta^2 + 2k_\theta\omega_M)}$, $\gamma_1(|\xi|) = (|A_k| + T|z| * \sqrt{k_\theta^2 + 2k_\theta\omega_M})|\xi|$, $\gamma_2(|X|) = \sqrt{(k_\theta^2 + 2k_\theta\omega_M)|X|^2 + \omega_M^2}$ and $\gamma_3(|z|) = |z|$, where

$$A_k = \begin{bmatrix} C_x & TC_y\Omega_{r,k} \\ -T\Omega_{r,k} & I \end{bmatrix}$$

and $\gamma_1(\cdot),\ \gamma_3(\cdot) \in \mathcal{K}_\infty$, $\gamma_2(\cdot) \in \mathcal{N}$.

It is obviously that the subsystem (21b) is uniformly globally exponentially stable with $k_\theta \in (0, \frac{2}{T})$. Then, it is going to prove that the unperturbed dynamics $X(k+1) = A_k X(k)$ in (21a) is UGAS and that (21a), (21b) are UGB.

Proposition 2.3 is adopted to prove UGAS of dynamics $X(k+1) = A_k X(k)$. To that end, consider the Lyapunov function $V_T(k, X) := |X|^2 - E^T_{x,k}\Omega_{r,k-1}E_{y,k}$, where $|X|^2 = E^T_{x,k}E_{x,k} + E^T_{y,k}E_{y,k}$ and $\Omega_{r,k-1} = diag\{\omega_{r,k-1}, \cdots, \omega_{r,k-\tau-1}\}$. Observer that this function is positive definite and radially unbounded for sufficiently small $\Omega_{r,k-1}$, it have that

$$c_1|X|^2 \le V_T(k, X) \le c_2|X|^2 \tag{25}$$

with $c_1 = 1 - 0.5\omega_M$ and $c_2 = 1 + 0.5\omega_M$. Thus (4) hold with $\psi(X) = |X|^2$. Next, it gives

$$\begin{aligned}
&\Delta V_T(k, X)\\
=&V_T(k+1, X+1) - V_T(k, X)\\
=&-E^T_{x,k}E_{x,k} - E^T_{y,k}TC_y\Omega^2_{r,k}E_{y,k} + E^T_{y,k}T^2C_y^2\Omega^2_{r,k}E_{y,k} + E^T_{x,k}(C_x^2 + T^2\Omega^2_{r,k} + TC_x\Omega^2_{r,k})E_{x,k}\\
&+E^T_{x,k}(2TC_xC_y\Omega_{r,k} - 2T\Omega_{r,k} - C_x\Omega_{r,k} + \Omega_{r,k-1} + T^2C_y\Omega^3_{r,k})E_{y,k}\\
\le&-E^T_{x,k}E_{x,k} - T(1+k_y)\omega_m^2E^T_{y,k}E_{y,k} + [(1-Tk_x)^2 + T^2\omega_M^2 + T(1-Tk_x)\omega_M^2]E^T_{x,k}E_{x,k}\\
&+T^2(1+k_y)^2\omega_M^2E^T_{y,k}E_{y,k} + [2T(1-Tk_x)(1+k_y)\omega_M + 2T\omega_M + (1-Tk_x)\omega_M + \omega_M\\
&+T^2(1+k_y)\omega_M^3]E^T_{x,k}E_{y,k}\\
\le&-T\Lambda_1E^T_{x,k}E_{x,k} - T\Lambda_2E^T_{y,k}E_{y,k} + T^2\Lambda_3E^T_{x,k}E_{x,k} + T^2\Lambda_4E^T_{y,k}E_{y,k}\\
&+T^2\Lambda_5(E^T_{x,k}E_{x,k} + E^T_{y,k}E_{y,k})
\end{aligned}$$

with $\Lambda_1 = \frac{1}{T}$, $\Lambda_2 = (1+k_y)\omega_m^2$, $\Lambda_3 = \frac{(1-Tk_x)^2}{T^2} + \omega_M^2 + \frac{\omega_M^2}{T}(1-Tk_x)$, $\Lambda_4 = (1+k_y)^2\omega_M^2$, $\Lambda_5 = \omega_M\{\frac{1}{T}[(1-Tk_x)(1+k_y)+1] + \frac{1}{2T^2}(2-Tk_x) + \frac{1}{2}(1+k_y)\omega_M^2\}$. There exists $K_1 > 0$ such that

$$\frac{\Delta V_T}{T} \le -p_T(k)|X|^2 + TK_1|X|^2 \tag{26}$$

Hence (5) holds with $c_4 = K_1$, $c_3 = 1$ and $p_T(k) = min\{\Lambda_1, \Lambda_2\}$, $K_1 = max\{\Lambda_3, \Lambda_4, \Lambda_5\}$. Notice that the condition (6) also holds in view of (22) and (24). There exists a function $U_T : Z_{\ge 0 \times R^2 \to R}$ such that (8) and (9) hold, thus it is concluded that the unperturbed dynamics $X(k+1) = A_kX(k)$ in (21a) is UGAS.

Furthermore, subsystems (21a) and (21b) should be proven to be UGB. We proceed to verify the conditions of Proposition 2.4 with $f_T(k, X, z) := F_{1T}(k, X) + G_T(k, X, z)$ as defined in (21a) and with $V_T(k, X) = U_T(k, X)$. In view of (8) and (9), bounds (10) holds trivially with $\widetilde{\alpha}_1(|X|) = d_1|X|^2$, $\widetilde{\alpha}_2(|X|) = d_2|X|^2$ and any $c \ge 0$, we can also find k_1, $k_2 > 0$ with $\varphi(s) = s$, $\widetilde{\gamma}_1(s) := k_1s$ and $\widetilde{\gamma}_2 := k_2s$, then (11) and (12) hold. Select a linear function $\rho(s) := \rho s$, which is a nondecreasing function of linear growth, and due to the fact that $\phi_T^{e_\theta,k}$ decays uniformly exponentially to zero, then (13) holds finally. The prove process is complete and the overall system (21a) and (21b) is UGAS.

Remark 3.4 In this paper, the networked predictive control scheme is based on the assumption of an exact knowledge of the communication delay. However, if the communication delays are not known exactly (time-varying delays), the time-varying delays should be first tested, and then an acceptable upper bound (τ_{max}) can be selected. If the actual delay $\tau(k)$ is smaller than τ_{max}, $\tau(k)$ is increased to τ_{max} (through technical means), then the time-varying delays are transformed into a known constant delay. If the actual delay $\tau(k)$ is larger than τ_{max}, this data packet is considered to be lost. In this manner, the robustness of the networked predictive control scheme is increased.

3.3 Networked predictive controller with data loss

The state information of mobile robot in the feedback channel is transmitted through a wireless network, e.g. WiFi. Consequently, some data packet losses are inevitable due to the nature of wireless networks. Once a data packet is lost in the feedback channel, the system becomes open-loop. A simple approach to cope with this problem is to use historical state information as current state information, but for this the number of consecutive packet drops should be small. To this end, a new algorithm, which is able to compensate the effect of consecutive packet losses in the feedback channel is proposed in this paper.

Algorithm 2 Networked predictive control scheme for data loss in the feedback channel

```
1: Set up a variable delay_ step. At sampling step k, data loss occurs in the feedback channel;
2: if delay_ step = 0 then
3:     Estimate q̂_k using q_{k−1} and Algorithm 1;
4: else
5:     Estimate q̂_k using q̂_{k−1|k−τ−1} and Algorithm 1;
6: end if
```

Remark 3.5 The value of variable *delay_ step* in Algorithm 2 is obtained through subtraction of the time stamp between sensor and networked controller, whose running clocks are synchronized, and the network delay is assumed to be a constant value.

Theorem 3.6 *Consider the discrete-time kinematics model of wheeled mobile robot (14) with consecutive packet drops in the feedback channel. If Algorithm 2 is used, the closed-loop predictive control system is UGAS, and the consecutive packet drops in the feedback channel can be compensated actively.*

The prove process of Theorem 3.6 is omitted due to its similarity to Theorem 3.3.

4 Experimental platform

In this section, the experimental platform for trajectory tracking control of a wheeled mobile robot is described. The experimental platform, shown in Figure. 3, consists of three parts: The wheeled mobile robot, a NetCon-STM32 and a Vicon system.

The Vicon system is an optical motion capture system, which is produced by Oxford Metrics

Limited (OML) in U.K., and it is the world's first product for an optical motion capture system. It consists of a group of network connected Vicon MX motion capture cameras and other equipment, and it establishes a complete three-dimensional motion capture system to provide real-time optical data. These data can be used for real-time online or offline motion capture and analysis.

In the experimental platform designed in this paper, four markers were placed on the mobile robot, which composed a quadrangle. These markers could be captured by Vicon cameras, which were placed on the ceiling of laboratory room. Vicon server obtained the coordinates of four markers and transformed which into state information (including position and angle) of mobile robot. Finally, the state information was sent to networked controller, which was located on mobile robot, through a wireless network. Thus, the closed-loop tracking control system was realized.

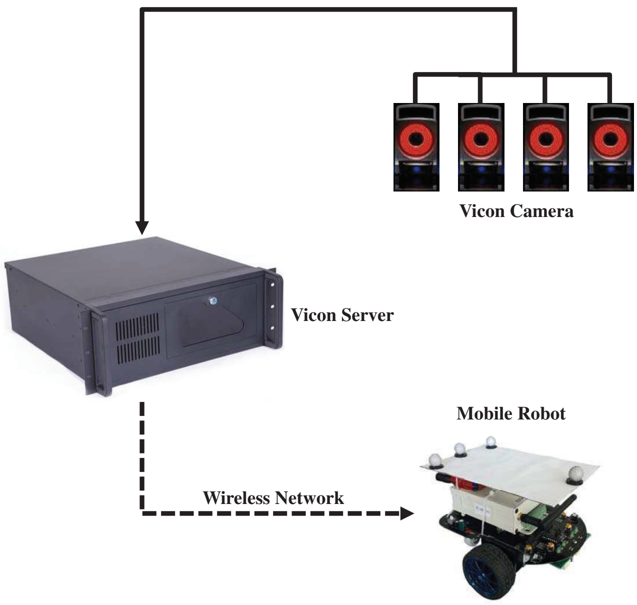


Figure 3: Experimental platform for tracking control of wheeled mobile robot.

4.1 Wheeled mobile robot

The wheeled mobile robot, which was controlled by the networked controller, had two driven wheels individually actuated by means of stepping motors. An omni-directional wheel was placed at the back to keep balance. Details of the mobile robot designed for the purposes of this paper are shown in Figure. 4.

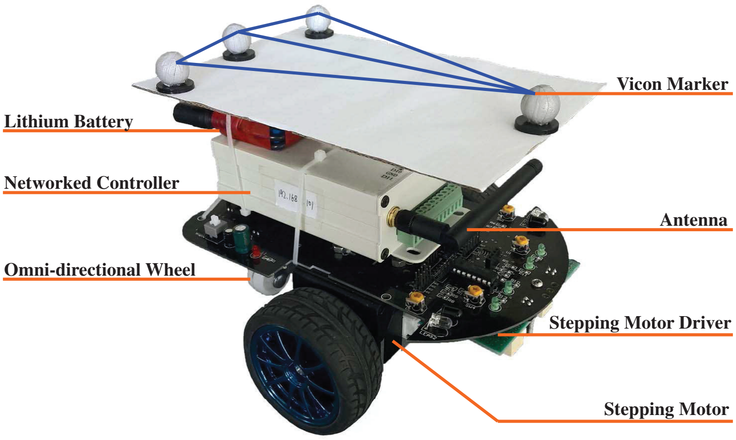


Figure 4: Details of wheeled mobile robot.

4.2 NetCon-STM32

The NetCon-STM32 was designed and developmented by authors of this paper, and it was composed of two parts: the NetconLink-STM32 and Networked Controller. Overall structure of the NetCon-STM32 is shown in Figure. 5.

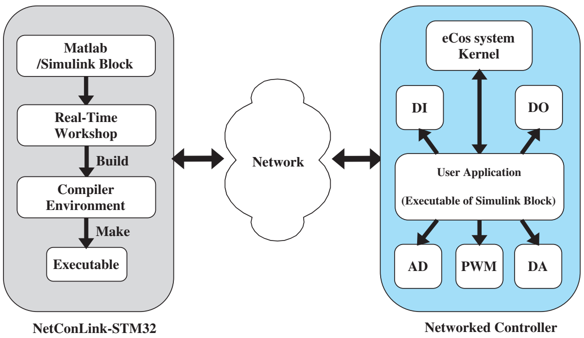


Figure 5: Overall structure of NetCon-STM32.

In the PC side, a script program, named as NetconLink-STM32, was integrated in the Real-Time Workshop of Matlab, which played an important role in compiling, linking and downloading. The detailed workflow of the NetconLink-STM32 will be described in following steps.

Firstly, a Simulink Block designed by the user in Matlab/Simulink was built and exported into C files. Secondly, the C files were compiled and linked under a Linux environment, provided by Cygwin, and combined into an executable file. Finally, the executable file was sent to Networked Controller by a socket script file through wireless network.

In the Networked Controller side, the STM32F207 was used as a micro-controller and the eCos system was chosen as its operating system. There were two high priority threads running in the Networked Controller: one was used for network listening and the other for user applications.

The user application obtained through the Simulink Block was the interface between the control algorithm and the hardware resources in the Networked Controller. The hardware resources of the controller included an analog to digital converter (AD), a digital to analog converter (DA), a digital input (DI), a digital output (DO) and a Pulse Width Modulation (PWM) function. Moreover, the Networked Controller could communicate with PC and/or other Networked Controllers using network send and network receive modules through UDP protocol.

5 Simulation and Experiment

In this section, simulation and experimental results of wheeled mobile robot's tracking control, obtained using the same parameters, are provided. The reference trajectory is a circular line which satisfies the kinematics model of wheeled mobile robot (14) and having the following form

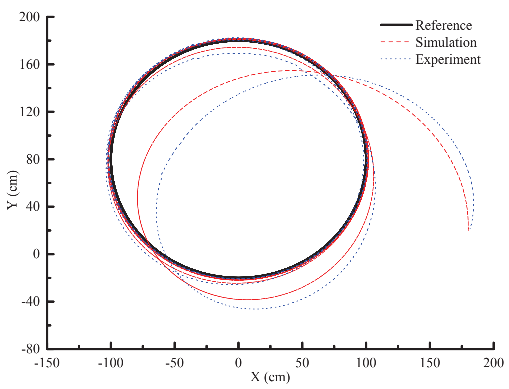
$$\begin{aligned} x(k) &= x_{rc} + rsin\theta(k) \\ y(k) &= y_{rc} - rcos\theta(k) \end{aligned} \tag{27}$$

where $(x_{rc}, y_{rc}) = (0, 80)[cm]$ was the center of the reference circular trajectory, and $r = 100cm$ was the radius. For tracking controllers (18) and (20), $\upsilon_r(k) = 39.27cm/s$, $\omega_r(k) = 0.393rad/s$, with $k_x = 0.24$, $k_y = 0.36$ and $k_\theta = 0.2$. The sampling time was set to 100ms.

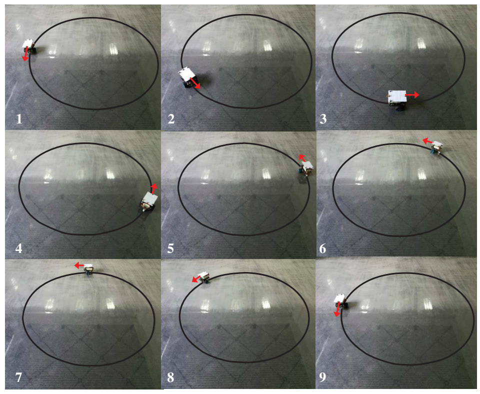
Remark 5.1 At the beginning of the test, some comparisons were made using different sampling periods. Under sampling periods of 20ms, 50ms and 100ms, the difference between test results was negligible. However, when communication delay exist in the feedback channel, of e.g. 1000ms, tracking controller (20) had to perform 50 iterations for a sampling period of 20ms, but only 10 iterations for a period of 100ms. Considering that the kinematics model of wheeled mobile robot was inaccurate, the iteration steps should be decreased to diminish the accumulative error. Thus, sampling period was selected as 100ms.

5.1 Tracking control without communication delay

For the general case, without any communication delay in the feedback channel, tracking controller (18) was used. The simulation and experimental results of the trajectory are shown in Figure. 6(a) and 6(b).



(a) Reference, simulation and experimental results.



(b) Experimental result of wheeled mobile robot.

Figure 6: Simulation and experimental results without communication delay in the feedback channel

In Figure. 6(a), solid line denotes the reference circular trajectory described by (27), while dashed and dotted lines represent the simulation and experimental results of moving trajectory of mobile robot, respectively. It can be seen that the actual trajectory finally converges to the reference trajectory.

5.2 Constant communication delay

In the laboratory environment, the data packets were transmitted using a Wireless Local Area Network by TCP/IP protocol. The communication delay using this method was small (less than 10ms), which was less than one sampling period (100ms in this paper). When considering communication delay values larger than one sampling period, an artificial delay function is implemented. Here the delay function block in Simulink Library Browse is selected, which can be used in simulation research and can be downloaded into networked controller.

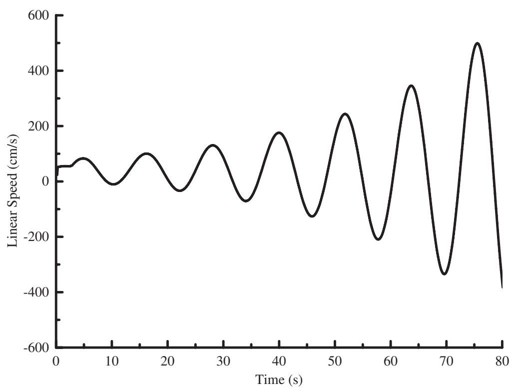
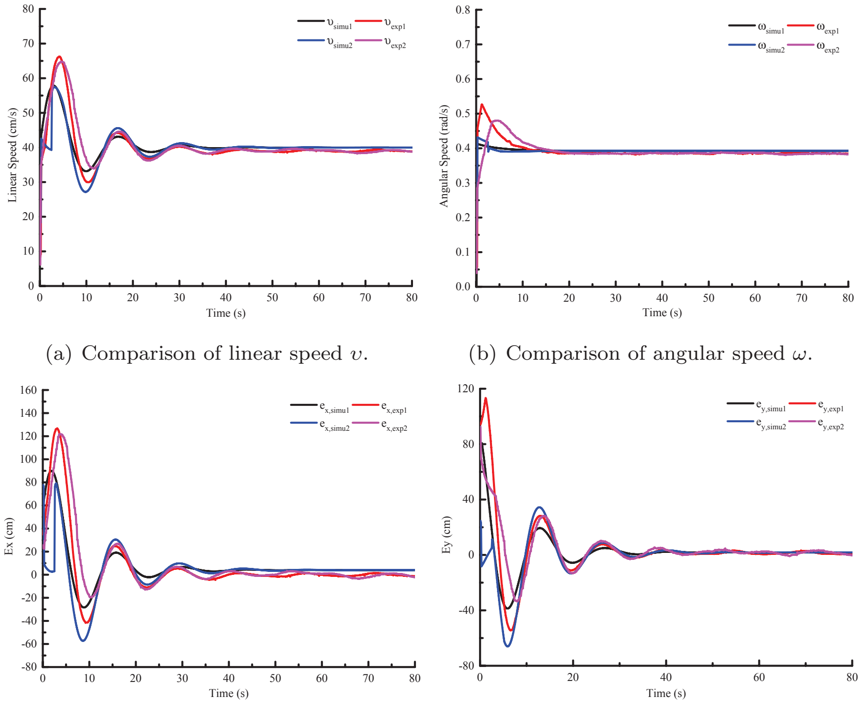


Figure 7: Linear speed of mobile robot in simulation using tracking controller (18) under a 25-step delay.



(a) Comparison of linear speed υ.

(b) Comparison of angular speed ω.

(c) Comparison of state error e_x.

(d) Comparison of state error e_y.

Figure 8: Simulation and experimental results with constant communication delay in the feedback channel.

Moreover, during the simulation study, since the Simulink block used PC clock, the execution time of each individual block was synchronized, whereas for the experimental part, the execution time of networked controller and sensor were assumed to be synchronized (due to the delay between them being smaller than one sampling period). In other words, the communication delay in the feedback channel was artificially introduced on the controller side, thus allowing us to realize a time-delay both in simulation and experiment.

When a constant communication delay τ exists in the feedback channel, tracking controller (18) cannot satisfy the control performance requirements anymore, as shown in Figure. 7. Consequently, networked predictive controller (20) should be applied.

In this subsection, a constant communication delay (2500ms) was implemented in the feedback channel of wheeled mobile robot. The networked predictive controller was used to compensate the effect of time-delay, simulation and experimental results are shown in Figure. 8.

In Figure. 8, ϕ_{simu1} and ϕ_{exp1} denote simulation and experimental results without communication delay, ϕ_{simu2} and ϕ_{exp2} represent for simulation and experimental results with constant communication delay in the feedback channel, respectively, for $\phi \in (\upsilon, \omega, e_x, e_y)$. The trajectory of ϕ_{simu1} and ϕ_{simu2} in simulation (ϕ_{exp1} and ϕ_{exp2} in experiment) are almost consistent.

Finally, the linear and angular velocities (υ and ω) converge to the desired velocities (shown in Figure. 8(a) and Figure. 8(b)), and state errors e_x and e_y(see Figure. 8(c) and Figure. 8(d)) converge to zero.

Consequently, the constant communication delay in the feedback channel can be compensated actively. Moreover, the simulation and experimental results are almost consistent.

5.3 Data packet drops

With the introduction of a communication network into the feedback channel of wheeled mobile robot, problems of communication delay and data packet drops are inevitable. The problem of constant communication delay was analyzed in previous subsection, so data packet drops will be processed in this subsection.

Generally speaking, in a real application of a wheeled mobile robot, data packet drops will happen in two cases. Case 1: Since the kinematics model of mobile robot is inaccurate, if communication delay in the feedback channel exceeds the upper bound τ_{max}, when iterating the future states based on this delayed state, the accumulated error will increase rapidly. As a result, the data packet is regarded as lost. Case 2: If the sensor and/or wireless router fails for a finite time period, the state of mobile robot will never be transmitted to the controller through feedback channel, in which case it is also considered that the data packet is lost.

In both the above cases, the state of mobile robot is considered as lost on controller side, in which case Algorithm 2 should be adopted to compensate for the data loss.

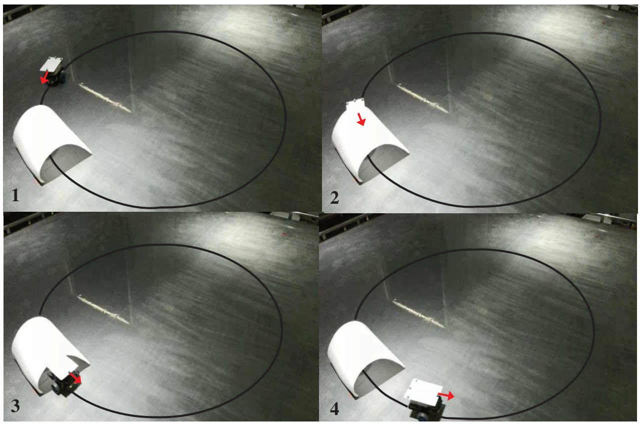
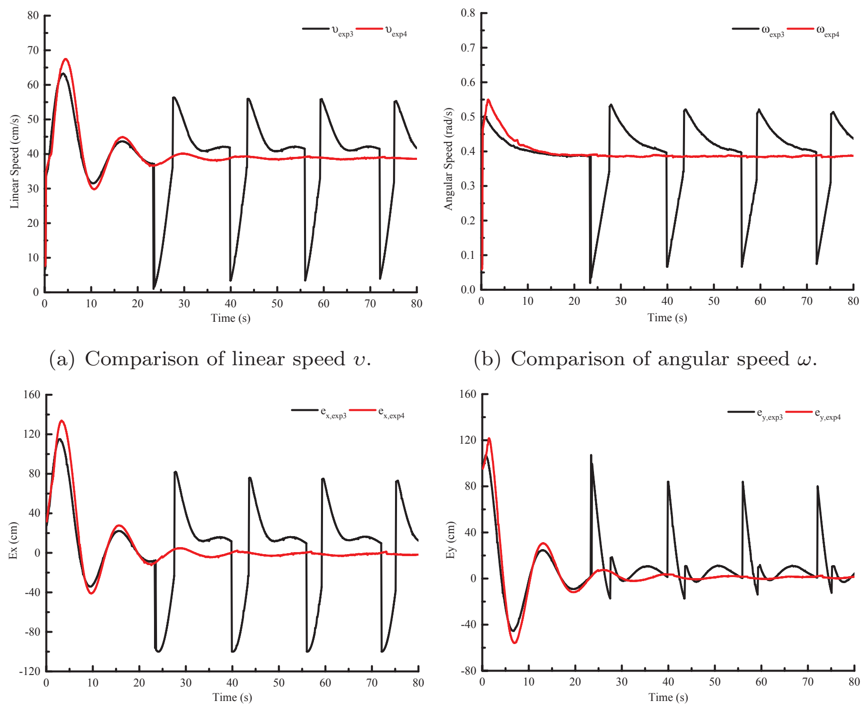


Figure 10: Experimental results while traveling through a tunnel.

To simulate data loss in the feedback channel and make the experiment more interesting, the solution of traveling through a tunnel is adopted. When mobile robot travels through the tunnel, the Vicon markers placed on mobile robot cannot be detected by Vicon cameras, in which case the Vicon server will send a NAN response to networked controller. Hence, the data packets are considered to be lost in the feedback channel (case 2).

The experimental results are shown in Figure. 9 and Figure. 10. In Figure. 9, φ_{exp3} and φ_{exp4} are experimental results without and with compensation, respectively, when data packets



(a) Comparison of linear speed υ.

(b) Comparison of angular speed ω.

(c) Comparison of state error e_x.

(d) Comparison of state error e_y.

Figure 9: Experimental results with data packet drops in the feedback channel.

are lost periodically in the feedback channel, for $\varphi \in (\upsilon, \omega, e_x, e_y)$. In Figure. 10, a tunnel is placed on the moving trajectory of mobile robot. When the mobile robot traveling through this tunnel, the Vicon markers placed on mobile robot are covered by tunnel. Then Vicon server cannot detect the state information of mobile robot and will send an NAN signal to mobile robot, which means that the state information of mobile robot is lost in the feedback channel. Whereas, it is obvious that, the negative effects under data losses are compensated by Algorithm 2 both in Figure. 9 and Figure. 10.

6 Conclusion and Discussion

In this paper, the tracking control problem of a wheeled mobile robot under communication delay and consecutive data packet drops in the feedback channel was considered. A local tracking controller under discrete-time was derived firstly. To solve the communication delay and consecutive data loss problems, a networked predictive tracking controller and two algorithms were proposed. The simulation results were clearly verified by experiments, which demonstrates the effectiveness of the proposed networked predictive tracking controller.

Objectively speaking, the kinematics model did not describe the dynamics characteristics of wheeled mobile robot fully, especially when there were uncertainties in system model and sliding friction in experimental environment, as well as other factors. Hence, a more precise dynamics model of wheeled mobile robot will be identified in our future work.

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