# Output Consensus of Networked Multi-Agent Systems with Time-Delay Compensation Scheme

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## Abstract

The output consensus problem of linear discrete-time multi-agent systems (DTMASs) with heterogeneous dynamics and a communication delay is investigated in this paper. In order to remove the negative effects of the communication delay, the networked predictive control scheme is introduced to compensate for the network delay actively. A novel distributed protocol is proposed with the predictions of agents' outputs at current time, instead of available outdated data. For DTMASs with heterogeneous agents and a constant communication delay, the necessary and sufficient conditions of output consensus are obtained while agents' states are not measurable. Simulation result is further presented to demonstrate the effectiveness of the theoretical results.

*Keywords:* output consensus, multi-agent systems, networked predictive control, discrete-time systems

## 1. Introduction

In recent years, the coordination control of multi-agent systems (MASs) is a hot topic and has received spreading attention from various fields of science,

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including control engineering, mathematics, physics, robots and artificial intelligence. As a basic and important problem, the consensus of multi-agent systems has been investigated extensively [1–3].

With the development of network technology, more and more networks (e.g., the Internet and the Ethernet) have been applied to distributed control systems (See Figure 1). Although the networks make it convenient to control large distributed systems, there are many control issues that occur in conventional control systems, such as network delay and data dropout[4, 5]. However, time delay can degrade the performance of control systems and even destabilize the system. Hence, a few research works on consensus problems of MASs with communication time delays have emerged recently[6, 7]. The idea of tackling communication delays is mainly categorized into passive acceptance approach and active compensation approach. The passive acceptance approach means that delayed information is used directly. In the active compensation approach, the prediction intelligence of each individual is considered.

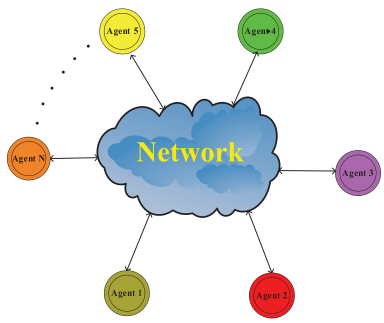


Figure 1: Networked multi-agent systems of $N$ agents.

When each agent lies in a closed convex constraint set, a novel approach is proposed to tackle the consensus of MASs with unbalanced graphs and bounded communication delays in [8]. The consensusability problem of continuous-time MASs with time-varying communication delay was discussed in [9], and a consensus protocol was designed based on the low gain solution of a parametric algebraic Riccati equation. In [10], consensus problem for MASs with second-order dynamics under delayed and intermittent communication was studied, by using Lyapunov approach and graph theory. Under the condition that the union graph is strongly connected and balanced, the consensus problem of discrete-time second-order MASs with bounded time-varying delays and time-varying topology was studied in [11]. Inspired by numerous results on the predictive intelligence of natural bio-groups, [12] and [13] proposed centralized and decentralized model predictive control protocols for linear dynamic networks without leaders, which show that the predictive protocols can accelerate consensus speeds and reduce sampling frequencies. Also the work in [14] considered the input saturation constraints case and proposed a decentralized predictive mechanism and predictive pinning control to achieve the consensus and improve consensus performance. Besides, the consensus problem for MASs with communication delays is considered by using the predictive control approach in [15–18]. For continuous-time first-order and second-order multi-agent systems with an uniform constant communication delay, a weighted average prediction was introduced into the existing consensus protocol to simultaneously improve the maximum tolerant delay and consensus convergence speed in [15] and [16]. For DTMASs with general linear dynamical nodes and a common constant network delay, a distributed protocol was proposed to compensate for communication delay actively, based on the networked predictive control scheme (NPCS) in [17], and necessary and sufficient conditions of the consensus have been obtained. And on this basis, the problem of consensus for DTMASs with non-uniform linear dynamical nodes and a common constant network delay has been discussed in [18]. By using the NPCS and dynamic output feedback, sufficient conditions of the consensus have been provided under mild assumptions in [18]. Therefore, it is a promising topic how to improve the performance of MASs by fully utilizing the prediction intelligence of each individual.

In this paper, the output consensus problem of linear DTMASs with heterogeneous dynamics and a communication delay is considered, where the dynamics systems are non-identical for different agents. By exploiting the NPCS proposed by [19, 20], agents' outputs at current time are predicted. A novel distributed protocol is derived based on predictions of outputs, instead of available outdated information, which can compensate for communication delay actively. For DTMASs with heterogeneous dynamics and a constant communication delay, the necessary and sufficient conditions are obtained to ensure agents' outputs to reach a common value. The proposed consensus conditions are independent of the network delay and only dominated by agents' dynamics and communication topology.

The paper is organized as follows. Section 2 briefly reviews some preliminaries of graph theory and formulates the problem to be investigated. For DTMASs with a constant communication delay, the protocol design and consensus analysis are presented in Section 3. To illustrate the proposed theoretical results, a numerical example is provided in Section 4. Finally, Section 5 makes concluding remarks.

## 2. Preliminaries and Problem Statement

Some basic concepts and properties in graph theory are briefly introduced, which are very important and helpful in the analysis of multi-agent systems. Let $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ be a weighted digraph of order $N$, where the set of nodes $\mathcal{V} = \{1, 2, \cdots, N\}$, set of edges $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$, and a nonnegative weighted adjacency matrix $\mathcal{A} = [a_{ij}] \in M_{_N}(\mathbb{R})$. An edge from $i$ to $j$ is denoted by $e_{ij} = (i, j)$ and the adjacency element $a_{ji}$ associated with the edge $e_{ij}$ is positive, i.e., $e_{ij} \in \mathcal{E} \Leftrightarrow a_{ji} > 0$. The set of neighbors of the node $i$ is denoted by $N_i = \{j \in \mathcal{V} : (j, i) \in \mathcal{E}\}$. A directed path is a sequence of edges in a digraph of the form $(i_1, i_2)$, $(i_2, i_3)$, $\cdots$, $(i_{f-1}, i_f)$, where $j = 1, \cdots, f \in \mathbb{Z}^+$, $i_j \in \mathcal{V}$ and $(i_j, i_k) \in \mathcal{E}$. If there exists a directed path from node $i$ to node $j$, then node $j$ is said to be reachable from node $i$. The set of all reachable nodes to node $i$ is denoted by $N_i^*$. The Laplacian matrix $\mathcal{L} = [l_{ij}] \in M_{_N}(\mathbb{R})$ of the weighted digraph $\mathcal{G}$ is defined as $\mathcal{L} = \mathcal{D} - \mathcal{A}$, where $\mathcal{D} = \mathrm{diag}(d_{in}(1), d_{in}(2), \cdots, d_{in}(N))$ and $d_{in}(i) = \sum_{j=1, j\neq i}^{N} a_{ij}$, $i = 1, 2, \cdots, N$. Obviously, all the row-sums of $\mathcal{L}$ are zero, which implies that $\mathcal{L}$ has always a zero eigenvalue corresponding the right eigenvector $\mathbf{1}_N$. For a comprehensive restatement of the graph theory, the reader is referred to [21].

Let $\mathbb{R}$ and $\mathbb{C}$ be the real and complex number filed, respectively. $M_{m,n}(\mathbb{F})$ denotes the set of all $m$-by-$n$ matrices over a field $\mathbb{F}$, and $M_{n,n}(\mathbb{F})$ is abbreviated to $M_n(\mathbb{F})$. The set of nonnegative integers is denoted by $\mathbb{Z}^+$. A vector valued function $\mathrm{vec}(\cdot)$ of a matrix is defined as $\mathrm{vec}(A) = [\ A_1^{\mathrm{T}} \ \ \cdots \ \ A_n^{\mathrm{T}}\ ]^{\mathrm{T}} \in M_{mn,1}(\mathbb{F})$, where $A_k$ is the k-th column of $A$, $k = 1, 2, \cdots, n$. A matrix $V \in M_n(\mathbb{C})$ is said to be Schur if $\sigma(V) \subseteq U_0$, where $\sigma(V)$ represents the spectrum of matrix $V$, and $U_0$ denotes an open circle of radius 1 centered at 0. The Kronecker product of $A = [a_{ij}] \in M_{m,n}(\mathbb{F})$ and $B = [b_{ij}] \in M_{p,q}(\mathbb{F})$ is denoted by $A \otimes B$ and is defined to be the block matrix $A \otimes B = \begin{bmatrix} a_{11}B & \cdots & a_{1n}B \\ \vdots & \ddots & \vdots \\ a_{m1}B & \cdots & a_{mn}B \end{bmatrix} \in M_{mp,np}(\mathbb{F})$. 0 represents zero matrix with an appropriate dimension. $\mathbf{1}_N$ denotes a $N$-dimension column vector with all entries equal to one. $\mathrm{diag}(\cdot)$ represents a block-diagonal matrix.

Consider a multi-agent system composed of heterogeneous $N$ agents, where dynamics of agent $i$ are described by a linear discrete-time system:

$$\begin{aligned} x_i(t+1) = & \ A_i x_i(t) + B_i u_i(t), \\ y_i(t) = & \ C_i x_i(t), \ t \in \mathbb{Z}^+, \\ x_i(t) = & \ \varphi_{xi}(t), \ -2\tau \leq t \leq 0, \\ u_i(t) = & \ \varphi_{ui}(t), \ -2\tau \leq t \leq 0, \\ y_i(t) = & \ \varphi_{yi}(t), \ -2\tau \leq t \leq 0, \\ i = & \ 1, 2, \cdots, N, \end{aligned} \tag{1}$$

where $x_i \in M_{n_i,1}(\mathbb{R})$, $u_i \in M_{m_i,1}(\mathbb{R})$ and $y_i \in M_{l,1}(\mathbb{R})$ are the state, control input and measured output of agent $i$, respectively; $A_i$, $B_i$, $C_i$ are constant matrices. $\tau$ is a transmission delay of the network, which implies that agents are compelled to receive data with $\tau$-step lag. And $\varphi_{xi}(\cdot)$, $\varphi_{ui}(\cdot)$ and $\varphi_{yi}(\cdot)$ represent the initial state, initial control input and initial output, respectively.

A weighted digraph described the information exchange between agents is usually expressed as $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$, where $\mathcal{V} = \{1, 2, \cdots, N\}$ is a nonempty set of nodes representing $N$ agents. The directed edge $e_{ij} \in \mathcal{E}$ means that agent $j$ can receive the information from agent $i$. It is assumed that information exchanged among all agents is achieved by the network with a constant delay $\tau$, and states of all agents are not available but their outputs can be measured. Due to the heterogeneity, the state consensus (i.e. $\lim_{t\to\infty} \|x_i(t) - x_j(t)\| = 0, \ \forall \ i, j \in \mathcal{V}$) is generally impossible. Hence, the output consensus of DTMAS (1) with a constant communication delay is discussed. A distributed protocol is designed and the conditions of consensus are provided, based on the networked predictive control scheme.

## 3. Protocol Design and Consensus Analysis with Compensation Scheme

In this section, it is considered that the information exchanged among all agents is achieved by the network with a constant delay $\tau$, where $\tau$ is a known positive integer. Due to the network delay, it can not be achieved that each agent receives current information from other agents at time $t$, yet it only obtains their information at time $t-\tau$. Hence, the predictive control method is adopt to overcome actively the effect of network delay in this section.

When the states of the plant are not measurable, it is often efficient to estimate them using an observer[22, 23]. In order to guarantee the existence of controllers for DTMAS (1), the following assumption can reasonably be made:

**Assumption 1.** *Each agent can receive information from itself and all reachable nodes to it, i.e., for any $j \in \{i\} \cup N_i^*$, agent $i$ can receive information from agent $j$, $i \in \mathcal{V}$.*

Because agent $i$ receives information from agent $j$ ($j \in \{i\} \cup N_i^*$) with time delay $\tau$, in order to overcome the effect of the network delay, based on the output data of agent $j$ up to time $t-\tau$, the state predictions of agent $j$ from time $t-\tau+1$ to $t$ are constructed as:

$$\begin{aligned} \hat{x}_j(t-\tau+1|t-\tau) = \ & A_j\hat{x}_j(t-\tau|t-\tau-1) + B_j u_j(t-\tau) \\ & +L_j[y_j(t-\tau) - C_j\hat{x}_j(t-\tau|t-\tau-1)], \end{aligned} \tag{2a}$$

$$\begin{aligned} \hat{x}_j(t-\tau+d|t-\tau) = \ & A_j\hat{x}_j(t-\tau+d-1|t-\tau) \\ & +B_j u_j(t-\tau+d-1), \\ d = \ & 2,3,\cdots,\tau,\ j \in \{i\} \cup N_i^*, \end{aligned} \tag{2b}$$

where $\hat{x}_j(t-\tau+1|t-\tau) \in M_{n_j,1}(\mathbb{R})$ and $u_j(t-\tau) \in M_{m_j,1}(\mathbb{R})$ are the one-step ahead state prediction and the input of the observer at time $t-\tau$, and matrix $L_j \in M_{n_j,l}(\mathbb{R})$ can be designed using observer design approaches, $\hat{x}_j(t-\tau+d|t-\tau) \in M_{n_j,1}(\mathbb{R})$ is state prediction of agent $j$ at time $t-\tau+d$ on the basis of the information up to time $t-\tau$, and $u_j(t-\tau+d-1) \in M_{m_j,1}(\mathbb{R})$ is the input at time $t-\tau+d-1$, $d = 2,3,\cdots,\tau$, $j \in \{i\} \cup N_i^*$.

It is seen from (2) that the procedure of state predictions of agent $j$ can be summarized as two steps. Firstly, the observer provides an one-step ahead state prediction using the output at time $t-\tau$. Secondly, based on the input information available, the state predictions of agent $j$ from time $t-\tau+2$ to $t$ are constructed. It follows from (2) that, based on the data up to time $t-\tau$, the output prediction of agent $j$ at time $t$ can be constructed as

$$\hat{y}_j(t|t-\tau) = C_j\hat{x}_j(t|t-\tau),\ j \in \{i\} \cup N_i.$$

Thus, the following protocol based on the output feedback is designed.

For agent $i$ of DTMAS (1) with a constant network delay $\tau$, the protocol based on the NPCS is designed as:

$$\begin{aligned} u_i(t) = \ & u_i(t|t-\tau) \\ = \ & F_i\hat{y}_i(t|t-\tau) + K_i\hat{\zeta}_i(t|t-\tau), \\ i = \ & 1,2,\cdots,N, \end{aligned} \tag{3}$$

where $\hat{\zeta}_i(t|t-\tau) = \sum_{j\in N_i} a_{ij}(\hat{y}_j(t|t-\tau) - \hat{y}_i(t|t-\tau))$ is output prediction difference between agent $i$ and agent $j$, and $F_i, K_i \in M_{m_i,l}(\mathbb{R})$ are matrices to be designed, $i = 1, 2, \cdots, N$.

**Definition 1.** *For DTMAS (1), protocol (3) is said to solve output consensus problem if* $\lim\limits_{t\to\infty} \|y_i(t) - y_j(t)\| = 0,\ \forall\ i, j = 1, 2, \cdots, N.$

Let

$$\begin{aligned}
&\delta_i(t) = y_i(t) - y_1(t),\ i = 1, 2, \cdots, N,\\
&\delta(t) = \left[\begin{array}{cccc} \delta_2^{\mathrm{T}}(t) & \delta_3^{\mathrm{T}}(t) & \cdots & \delta_{N}^{\mathrm{T}}(t) \end{array}\right]^{\mathrm{T}},\\
&x(t) = \left[\begin{array}{cccc} x_1^{\mathrm{T}}(t) & x_2^{\mathrm{T}}(t) & \cdots & x_{N}^{\mathrm{T}}(t) \end{array}\right]^{\mathrm{T}},\\
&y(t) = \left[\begin{array}{cccc} y_1^{\mathrm{T}}(t) & y_2^{\mathrm{T}}(t) & \cdots & y_{N}^{\mathrm{T}}(t) \end{array}\right]^{\mathrm{T}},\\
&e(t) = \left[\begin{array}{cccc} e_1^{\mathrm{T}}(t) & e_2^{\mathrm{T}}(t) & \cdots & e_{N}^{\mathrm{T}}(t) \end{array}\right]^{\mathrm{T}}.
\end{aligned}$$

From Definition 1, protocol (3) solves the output consensus problem if and only if $\lim\limits_{t\to\infty} \|e(t)\| = 0$ and $\lim\limits_{t\to\infty} \|\delta(t)\| = 0$ simultaneously hold. A lemma needs to be presented before the main results of this paper.

**Lemma 1.** *[24] Let $A \in M_{m,n}(\mathbb{F})$, $B \in M_{p,q}(\mathbb{F})$ and $C \in M_{m,q}(\mathbb{F})$ be given and let and $X \in M_{n,p}(\mathbb{F})$ be unknown. The matrix equation*

$$AXB = C$$

*is equivalent to the system of $qm$ equations in $np$ unknowns given by*

$$(B^{\mathrm{T}} \otimes A)\mathrm{vec}X = \mathrm{vec}C.$$

Using the NPCS, the necessary and sufficient conditions of DTMAS (1) achieving output consensus are proposed under protocol (3).

**Theorem 1.** *Consider DTMAS (1) with a directed topology $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ and constant communication delay $\tau > 0$. Protocol (3) solves the output consensus problem if and only if the following conditions hold:*

($\mathrm{a}_1$) *$(A_i, C_i)$ is detectable, $i = 1, 2, \cdots, N$.*

($\mathrm{a}_2$) $\mathrm{rank}(C_i^{\mathrm{T}} \otimes C_iB_i) = \mathrm{rank}([C_i^{\mathrm{T}} \otimes C_iB_i \quad \mathrm{vec}(C_iA_i)])$.

(a$_3$) *There exists* $K_i \in M_{m_i,l}(\mathbb{R})$, $i = 1, 2, \cdots, N$, *such that* $-RC_{_D}B_{_D}K_{_D}(\mathcal{L}_2 \otimes I_l)$ *is Schur, where* $\mathcal{L}$ *is the Laplacian matrix of digraph* $\mathcal{G}$, *and*

$$\begin{aligned} B_{_D} &= \mathrm{diag}(B_1, B_2, \cdots, B_N), \\ C_{_D} &= \mathrm{diag}(C_1, C_2, \cdots, C_N), \\ K_{_D} &= \mathrm{diag}(K_1, K_2, \cdots, K_N), \\ R &= \left[\begin{array}{cc} -\mathbf{1}_{_{N-1}} & I_{_{N-1}} \end{array}\right] \otimes I_l, \\ \mathcal{L}_2 &= \mathcal{L}[\begin{array}{cc} 0 & I_{_{N-1}} \end{array}]^{\mathrm{T}}. \end{aligned}$$

*Proof.* Because $(A_i, C_i)$ is detectable, there exists $L_i \in M_{n_i,l}(\mathbb{R})$ such that $A_i - L_iC_i$ is Schur, $i = 1, 2, \cdots, N$. Then, we take $L_i$ as a gain matrix of observer (2a), $i = 1, 2, \cdots, N$. For agent $i$, it follows from (2) that the predictive state of agent $j$ at time $t$ is

$$\begin{aligned} \hat{x}_j(t|t-\tau) = \; & A_j^{\tau-1}(A_j - L_jC_j)\hat{x}_j(t-\tau|t-\tau-1) \\ & + \sum_{s=1}^{\tau} A_j^{\tau-s}B_ju_j(t-\tau+s-1) \\ & + A_j^{\tau-1}L_jy_j(t-\tau), \;\; j \in \{i\} \cup N_i^*. \end{aligned} \tag{4}$$

By the way of iteration, the state of system (1) can be expressed by

$$\begin{aligned} x_i(t) = \; & A_i^{\tau}x_i(t-\tau) + \sum_{s=1}^{\tau} A_i^{\tau-s}B_iu_i(t-\tau+s-1), \\ i = \; & 1, 2, \cdots, N. \end{aligned} \tag{5}$$

Combining (4) and (5) yields

$$\begin{aligned} \hat{x}_j(t|t-\tau) = \; & x_j(t) + A_j^{\tau-1}e_j(t-\tau+1), \\ \hat{y}_j(t|t-\tau) = \; & y_j(t) + C_jA_j^{\tau-1}e_j(t-\tau+1), \;\; j \in \{i\} \cup N_i^*. \end{aligned} \tag{6}$$

Substituting (6) into (3) derives

$$\begin{aligned} u_i(t) = \; & F_iy_i(t) + F_iC_iA_i^{\tau-1}e_i(t-\tau+1) \\ & - K_i \sum_{j=1}^{N} l_{ij}\left(\delta_j(t) + C_jA_j^{\tau-1}e_j(t-\tau+1)\right), \\ i = \; & 1, 2, \cdots, N. \end{aligned}$$

So the closed-loop system of system (1) subjected to distributed control (3) can be described as

$$\begin{aligned} x_i(t+1) = \; & A_ix_i(t) + B_iF_iy_i(t) + B_iF_iC_iA_i^{\tau-1}e_i(t-\tau+1) \\ & - B_iK_i \sum_{j=1}^{N} l_{ij}\delta_j(t) - B_iK_i(l_i \otimes I_l)C_DA_D^{\tau-1}e(t-\tau+1), \end{aligned} \tag{7}$$

$$
\begin{aligned}
y_i(t+1) = &\ C_i(A_i + B_iF_iC_i)x_i(t) - C_iB_iK_i(\tilde{l}_i \otimes I_l)\delta(t) \\
&+C_iB_iF_iC_iA_i^{\tau-1}e_i(t-\tau+1) \\
&-C_iB_iK_i(l_i \otimes I_l)C_DA_D^{\tau-1}e(t-\tau+1) \\
i = &\ 1, 2, \cdots, N,
\end{aligned} \tag{8}
$$

where $\tilde{l}_i = l_i[\ 0 \quad I_{_{N-1}}\ ]^{\mathrm{T}}$, and $l_i$ is the $i$-th row of Laplacian matrix $\mathcal{L}$.

From Lemma 1 and

$$
\operatorname{rank}(C_i^{\mathrm{T}} \otimes C_iB_i) = \operatorname{rank}([C_i^{\mathrm{T}} \otimes C_iB_i \quad \operatorname{vec}(C_iA_i)]),
$$

there exists $F_i \in M_{m_i,l}(\mathbb{R})$, such that

$$
C_i(A_i + B_iF_iC_i) = 0,\ i = 1, 2, \cdots, N. \tag{9}
$$

So (8) is reduced to

$$
\begin{aligned}
y_i(t+1) = &\ C_iB_iF_iC_iA_i^{\tau-1}e_i(t-\tau+1) - C_iB_iK_i(\tilde{l}_i \otimes I_l)\delta(t) \\
&-C_iB_iK_i(l_i \otimes I_l)C_DA_D^{\tau-1}e(t-\tau+1) \\
i = &\ 1, 2, \cdots, N.
\end{aligned} \tag{10}
$$

Then the following compact form can be consulted:

$$
\begin{aligned}
y(t+1) = &\ -C_DB_DK_D(\mathcal{L}_2 \otimes I_l)\delta(t) \\
&+C_DB_D(F_D - K_D(\mathcal{L} \otimes I_l))C_DA_D^{\tau-1}e(t-\tau+1),
\end{aligned}
$$

where $F_{_D} = \operatorname{diag}(F_1, F_2, \cdots, F_N)$,

The error system can be represented as

$$
\begin{aligned}
\delta(t+1) = &\ Ry(t+1) \\
= &\ \Omega_1\delta(t) + \Omega_2e(t-\tau+1),
\end{aligned}
$$

where

$$
\begin{aligned}
R &= \begin{bmatrix} -\mathbf{1}_{_{N-1}} & I_{_{N-1}} \end{bmatrix} \otimes I_l, \\
\Omega_1 &= -RC_DB_DK_D(\mathcal{L}_2 \otimes I_l), \\
\Omega_2 &= RC_DB_D(F_D - K_D(\mathcal{L} \otimes I_l))C_DA_D^{\tau-1}.
\end{aligned}
$$

From (2a),

$$
e_i(t+1) = (A_i - L_iC_i)e_i(t),\ i = 1, 2, \cdots, N. \tag{11}
$$

Therefore, the error system can be described as

$$
\begin{bmatrix} \delta(t+1) \\ e(t-\tau+2) \end{bmatrix} = \begin{bmatrix} \Omega_1 & \Omega_2 \\ 0 & \Omega_3 \end{bmatrix} \begin{bmatrix} \delta(t) \\ e(t-\tau+1) \end{bmatrix},
$$

where $\Omega_3 = A_D - L_D C_D$ and $L_{_D} = \mathrm{diag}(L_1, L_2, \cdots, L_N)$.

From Definition 1, protocol (3) solves the output consensus problem if and only if $\Omega_1$ and $A_i - L_i C_i$ are Schur, $i = 1, 2, \cdots, N$. The proof is completed. $\square$

When agents exchange information with a constant network delay, Theorem 1 indicates that, under protocol (3) based on the NPCS, the output consensus of DTMAS (1) is independent of the network delay and only dominated by agents' dynamics and communication topology.

**Theorem 2.** *Consider DTMAS (1) with a directed topology $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ and constant communication delay $\tau > 0$. Protocol (3) solves the output consensus problem if and only if*

$$\mathrm{rank}(C_i^{\mathrm{T}} \otimes B_i) = \mathrm{rank}([C_i^{\mathrm{T}} \otimes B_i \ \ \mathrm{vec}(A_i)]),$$

*and* $(\mathrm{a}_1)$ *and* $(\mathrm{a}_3)$ *in Theorem 1 hold.*

*Proof.* From Lemma 1 and

$$\mathrm{rank}(C_i^{\mathrm{T}} \otimes B_i) = \mathrm{rank}([C_i^{\mathrm{T}} \otimes B_i \ \ \mathrm{vec}(A_i)]),$$

there exists $F_i \in M_{m_i,l}(\mathbb{R})$, such that

$$A_i + B_i F_i C_i = 0, \ i = 1, 2, \cdots, N.$$

So (8) remains to be reduced to (10). Similar to derivations in Theorem 1, protocol (3) can solve the output consensus problem. The rest of proof is omitted. $\square$

## 4. Simulation

In this section, a numerical example is presented to illustrate the effectiveness of the proposed theoretical results.

**Example 1.** *Consider DTMAS (1) with three agents indexed by* 1, 2 *and* 3, *respectively. The dynamics of agent $i$ ($i = 1, 2, 3$) are described by (1), where*

$$A_1 = \begin{bmatrix} 0.6 & 1 & 2 \\ 0 & 0.5 & 0 \\ 0 & 0 & 0.5 \end{bmatrix}, \ B_1 = \begin{bmatrix} 1 & 3 \\ -1 & 6 \\ 5 & 0 \end{bmatrix}, \ C_1 = \begin{bmatrix} 0 & -1 & 2 \end{bmatrix}; \qquad (12)$$

$$A_2 = \begin{bmatrix} 0.4 & 0 & 0 \\ 1 & 0.5 & 1 \\ 0 & 0 & 0.4 \end{bmatrix}, \; B_2 = \begin{bmatrix} 4 & 1 \\ 0 & 0.5 \\ 1 & 1 \end{bmatrix}, \; C_2 = \begin{bmatrix} 1 & 0 & -2 \end{bmatrix}; \tag{13}$$

$$A_3 = \begin{bmatrix} 0.6 & 0 & 0 \\ 0 & 0.6 & 0 \\ 1 & 2 & 0.8 \end{bmatrix}, \; B_3 = \begin{bmatrix} 1 & 3 \\ 4 & 2 \\ 1 & 1 \end{bmatrix}, \; C_3 = \begin{bmatrix} 1 & -2 & 0 \end{bmatrix}. \tag{14}$$

*The interconnection among the three agents is described by* $\mathcal{G}$ *in Fig. 2 with the adjacent matrix*

$$\mathcal{A} = \begin{bmatrix} 0 & 0 & 1 \\ 2 & 0 & 0 \\ 0 & 3 & 0 \end{bmatrix},$$

*and Laplacian matrix*

$$\mathcal{L} = \begin{bmatrix} 1 & 0 & -1 \\ -2 & 2 & 0 \\ 0 & -3 & 3 \end{bmatrix}.$$

*It is assumed that the communication delay* $\tau$ *is equal to* 3.

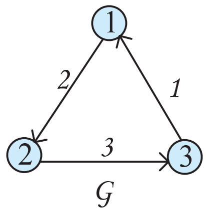


Figure 2: Fixed topology.

*Next, one possible way is presented to obtain the observer gain matrices* $L_i$ *in (2a) and controller gain matrices* $F_i$ *and* $K_i$ *in (3),* $i = 1, 2, 3$*. It is obvious that* $(A_i, C_i)$ *is detectable,* $i = 1, 2, 3$*. Hence, for an arbitrary matrix* $Q_i > 0$*, discrete-time algebraic Riccati equation*

$$A_i P_i A_i^{\mathrm{T}} - P_i - A_i P_i C_i^{\mathrm{T}} (I + C_i P_i C_i^{\mathrm{T}})^{-1} C_i P_i A_i^{\mathrm{T}} + Q_i = 0 \tag{15}$$

*has an unique solution* $P_i > 0$ *satisfying that* $A_i - L_i C_i$ *is Schur, where* $L_i = A_i P_i C_i^{\mathrm{T}} (I + C_i P_i C_i^{\mathrm{T}})^{-1}$*,* $i = 1, 2, 3$*. By choosing* $Q_1 = Q_2 = Q_3 = 2I_3$ *and using Matlab, solutions of Riccati equation (15) and gain matrices are* *obtained as:*

$$P_1 = \begin{bmatrix} 28.4235 & 2.9915 & 1.6361 \\ 2.9915 & 2.5424 & 0.2484 \\ 1.6361 & 0.2484 & 2.1698 \end{bmatrix},$$

$$P_2 = \begin{bmatrix} 2.3106 & 1.4137 & 0.1407 \\ 1.4137 & 11.2883 & 0.7439 \\ 0.1407 & 0.7439 & 2.0995 \end{bmatrix},$$

$$P_3 = \begin{bmatrix} 2.9131 & 0.4237 & 5.7007 \\ 0.4237 & 2.2775 & 3.0217 \\ 5.7007 & 3.0217 & 85.5894 \end{bmatrix},$$

*and*

$$L_1 = \begin{bmatrix} 0.5616 \\ -0.0911 \\ 0.1822 \end{bmatrix}, \; L_2 = \begin{bmatrix} 0.0728 \\ -0.1854 \\ -0.1456 \end{bmatrix}, \; L_3 = \begin{bmatrix} 0.1094 \\ -0.2188 \\ -0.5712 \end{bmatrix}.$$

*On the other hand,* $\mathrm{rank}(C_i^{\mathrm{T}} \otimes C_i B_i) = \mathrm{rank}([C_i^{\mathrm{T}} \otimes C_i B_i \quad \mathrm{vec}(C_i A_i)])$*. So there exist matrices*

$$F_1 = \begin{bmatrix} 0.1943 \\ 0.4395 \end{bmatrix}, \; F_2 = \begin{bmatrix} -1.3600 \\ -2.3200 \end{bmatrix}, \; F_3 = \begin{bmatrix} -0.0360 \\ 0.8520 \end{bmatrix}$$

*satisfying* $C_i(A_i + B_i F_i C_i) = 0, \; i = 1, 2, 3$*. Then, feedback gain matrices can be obtained by*

$$K_1 = \begin{bmatrix} 162.5188 \\ 297.9512 \end{bmatrix}, \; K_2 = \begin{bmatrix} 162.5188 \\ 325.0376 \end{bmatrix}, \; K_3 = \begin{bmatrix} -0.0774 \\ 0.5415 \end{bmatrix}.$$

*It is easy to verify that* $-RC_{_D}B_{_D}K_{_D}(\mathcal{L}_2 \otimes I_l)$ *is Schur. Hence, protocol (3) solves the consensus problem by Theorem 1.*

*When* $\tau = 3$*, initial conditions of DTMAS (1) is chosen as*

$$\varphi_{x1}(0) = \begin{bmatrix} 1 \\ 3 \\ -2 \end{bmatrix}, \; \varphi_{x2}(0) = \begin{bmatrix} -3 \\ -6 \\ 1 \end{bmatrix}, \; \varphi_{x3}(0) = \begin{bmatrix} 4 \\ -2 \\ 2 \end{bmatrix},$$

$\varphi_{xi}(t) = 0$ *and* $\varphi_{yi}(t) = 0, \; -6 \leq t \leq -1, \; i = 1, 2, 3$*. The output trajectories of three agents and estimate error trajectories are shown in Fig. 3 and Fig. 4–Fig. 6, respectively.*

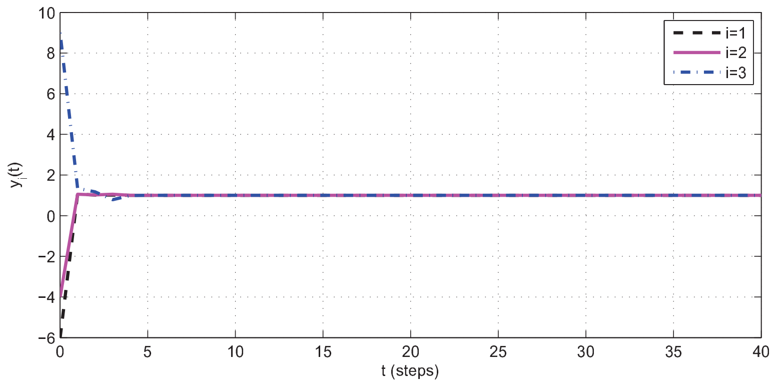


Figure 3: The output trajectories $y_i(t)$ $(\tau = 3)$.

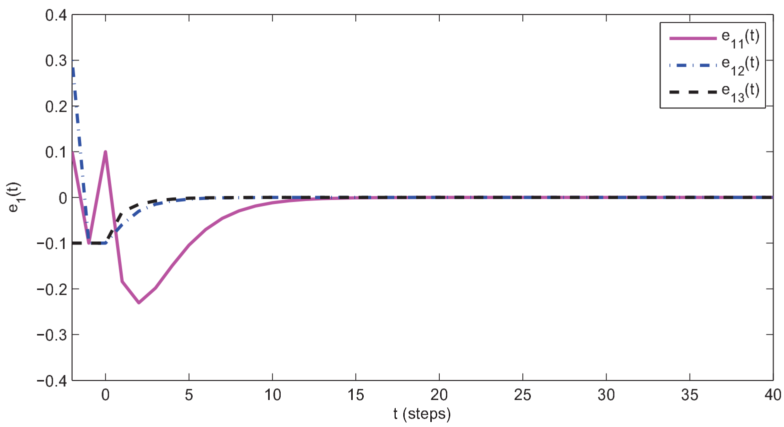


Figure 4: The error trajectories $e_1(t)$ $(\tau = 3)$.

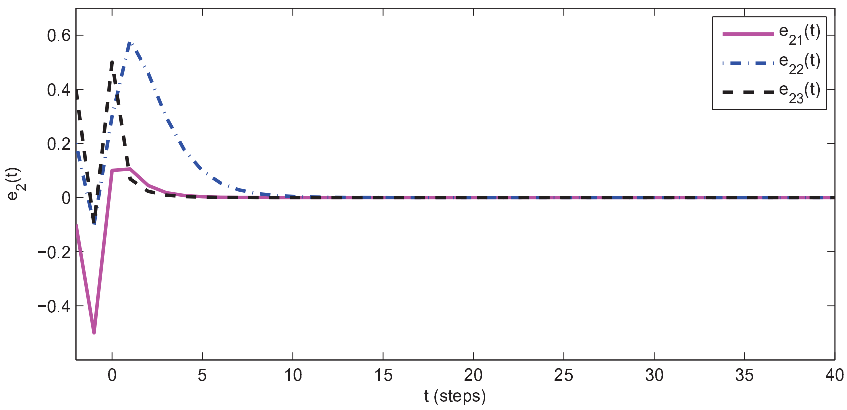


Figure 5: The error trajectories $e_2(t)$ $(\tau = 3)$.

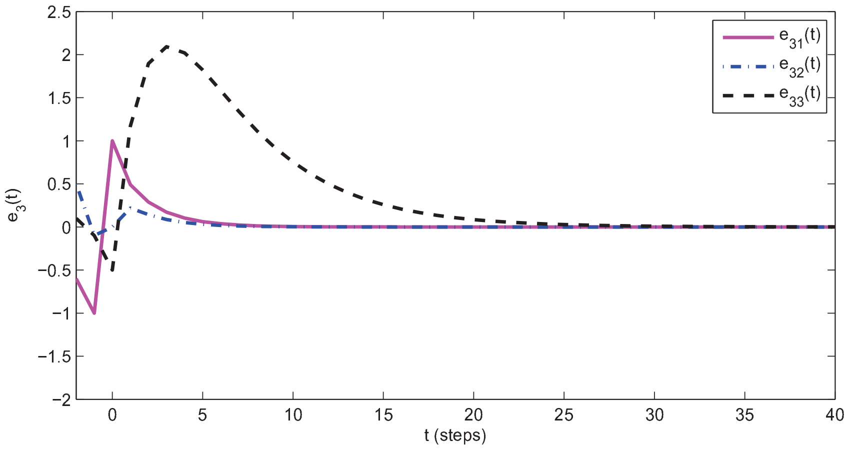


Figure 6: The error trajectories $e_3(t)$ $(\tau = 3)$.

## 5. Conclusions

The output consensus problem of DTMASs with heterogeneous dynamical agents and a communication delay has been discussed in this paper, where agents are described by linear discrete-time time-invariant systems. It is assumed that states of all agents are not available but their outputs can be measured. Under the fixed and direct topology, the disturbed control protocol for every agent is designed to compensate for the network delay actively, based on the networked predictive control scheme. The delay-independent necessary and sufficient conditions have been obtained to ensure that the outputs of all agents reach an agreement. A numerical example has demonstrated the effectiveness of the obtained theoretical results.

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