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Admissible consensus for networked singular multi-agent systems with communication delays

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This study concerns the admissible consensus problem for networked singular multi-agent systems with communication delays and agents described by general singular systems. Only the information of outputs is available through the network. An observer-based networked predictive control scheme (NPCS) is employed to compensate for the communication delays actively. Based on NPCS and dynamic compensator (dynamic output feedback), a novel protocol is proposed. Based on graph, algebra and singular system theory, the necessary and sufficient conditions are given to guarantee existence of the proposed protocol. The conditions depend on the topologies of singular multi-agent systems and the structure properties of each agent dynamics. Moreover, a consensus algorithm is provided to design the predictive protocol. A numerical example demonstrates the effectiveness of compensation for networked delays.

Keywords: Admissible consensus, networked singular multi-agent systems, communication delays, networked predictive control, dynamic compensators

1. Introduction

Consensus control problems of multi-agent systems are the foundation of distributed cooperative control theory, which have attracted much attention of researchers from control fields (Fax & Murray, 2004; Seo et al., 2009; Wang & Yu, 2016). Consensus control problems are applied in many problems including flocking problem, formation problem, distributed sensor networks and congestion control in communication networks, and so on (Guo et al., 2014; Shen et al., 2010; Shi & Shen, 2015; Tang et al., 2016; Yu et al., 2013; Zhu et al., 2013).

It is worth mentioning that singular systems have a natural representation of dynamic systems. And singular systems describe a larger class of systems than normal linear system models(Li et al., 2014; Xia et al., 2009). Hence, comparing with normal linear systems, singular systems have a more comprehensive background, such as power systems (Hill & Maareels, 1990), social economic systems (Luenberger & Arbel, 1977), circuit systems (Sastry & Desoer, 1981), and so on. In addition, since the theory of singular systems was put forward, it has provided a extensive application background and gradually developed into the important branch of control theory (Li & Zhang, 2012; Yang et al., 2013; Yang & Liu, 2012). Moreover, the concept of singular multi-agent systems has been introduced in Yang & Liu (2012). In the last two decades, many results of state space systems have been extended to singular systems (Alma & Darouach, 2014; Yang & Liu, 2014; Yang et al., 2010). However, rare works have been published to deal with consensus of singular multi-agent systems. Hence this paper puts an intensive study on admissible consensus problems with networked multi-

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agent systems composed of singular systems as the research object.

The necessary and sufficient consensus conditions with respect to a set of admissible consensus protocols have been given for singular multi-agent systems with fixed topologies (Yang & Liu, 2012). For singular multi-agent systems with agents described by homogenous or heterogenous singular systems, Yang & Liu (2011) has provided consensus conditions based on algebra, graph and singular system theory. Xi et al. (2012) has solved consensus problem for linear time-invariant singular swarm system. For singular high-order multi-agent systems with switching topologies, Xi et al. (2014) has dealt with guaranteed-cost consensus problems. The admissible consensus problem for heterogeneous descriptor multi-agent systems has been studied in Yang & Liu (2015). However, it is regrettable that Yang & Liu (2012) and Xi et al. (2012, 2014); Yang & Liu (2015, 2011) do not consider the communication delays. When agents exchange information through a wired or wireless network, due to the limitations of network bandwidth and transmission speed, network induced time delays will appear inevitably (Yang et al., 2014). Moreover, time delays usually affect the performance of networked multi-agents and even lead to system instability (Wu et al., 2011). Thus, it is essential to eliminate or reduce the negative effect of networked delays. Based on NPCS, static output feedback and observer, Yang & Liu (2014) has proposed protocols to guarantee that the studied singular system achieve consensus.

In this paper, the admissible consensus problem is considered for networked singular multi-agent systems with communication delays and agents described by general singular systems. An observer-based NPCS is employed to compensate for the communication delays effectively and actively. Based on the dynamic compensator and NPCS, a novel protocol is proposed to solve the admissible consensus problem for singular multi-agent systems. The provided numerical example demonstrates the effectiveness of compensation for communication delays. It is worth mentioning that the theoretical results obtained in Yang & Liu (2014) are one particular case of this paper, which implies that the given conclusions in this paper are generalizations of the results obtained in Yang & Liu (2014).

The rest of this paper is organized as follows. Some preliminaries on graph theory and singular system theory, and the problem formulation are described in Section 2. In Section 3, NPCS is employed to compensate for the communication delays actively. Admissible consensus analysis is discussed for the studied system with communication delays in Section 4. Moreover, the proposed consensus protocol is designed based on NPCS and the dynamic compensator. In Section 5, a numerical example will be given to verify the feasibility of the theoretical results. Conclusion remarks are stated in Section 6.

Throughout the paper, $\mathbb{R}$, $\mathbb{C}$ and $\mathbb{R}^{m\times n}$ represent the real plane, the complex plane and a set of all real matrices of dimension $m \times n$, respectively. Let $\sigma(A)$ be the set of all eigenvalues of the square matrix A. For the given vector x, $\|x\|$ represents the Euclidean norm of x. A matrix $H \in \mathbb{R}^{n\times n}$ is said to be Schur stable if $\sigma(H) \subseteq D(0,1)$, where $D(0,1)$ expresses the interior of an identity circle whose center is the origin.

2. Preliminaries and problem formulation

2.1 *Preliminaries*

In general, information exchanges among agents are achieved through a wired or wireless network for (singular) multi-agent systems. The networked communication topology of multi-agent systems can be modeled by directed or undirected graphs (Skelon et al., 1998). Let $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ be a directed digraph with a set of nodes $\mathcal{V} = \{1, 2, \cdots, N\}$ denoting the agents, a set of directed edges $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ and a nonnegative weighted adjacency matrix $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{N\times N}$. In $\mathcal{G}$, a directed edge from i to j is denoted as an ordered pair $(i, j) \in \mathcal{E}$, where the vertex j is called the child vertex and the vertex i is called the parent vertex. The set of the i-th agent's neighbors is denoted by $N_i = \{j \in \mathcal{V} | (j, i) \in \mathcal{E}\}$. The weighted adjacency matrix $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{N\times N}$ with nonnegative elements, where $a_{ii} = 0, a_{ij} > 0 \Leftrightarrow j \in N_i$, otherwise $a_{ij} = 0$. The Laplacian matrix $\mathcal{L}_{\mathcal{G}} = [l_{ij}] \in \mathbb{R}^{N \times N}$ of the digraph $\mathcal{G}$ is defined as

$$l_{ij} = \begin{cases} -a_{ij}, & i \neq j, \\ \sum\limits_{k=1, k \neq i}^{N} a_{ik}, & i = j. \end{cases} \tag{1}$$

Lemma 1: *(Ren & Beard, 2005) The Laplacian matrix $\mathcal{L}_{\mathcal{G}}$ of a directed graph $\mathcal{G}$ has at least one zero eigenvalue and all non-zero eigenvalues are in the open right-half plane. Furthermore, $\mathcal{L}_{\mathcal{G}}$ has exactly one zero eigenvalue if and only if $\mathcal{G}$ has a directed spanning tree.*

Before moving on, the following definitions and lemmas are introduced.

Definition 1: (Yang et al., 2004) Let $E, A \in \mathbb{R}^{n \times n}$.

(i) The pair (E, A) is said to be regular if $\det(sE - A) \not\equiv 0$ for some $s \in \mathbb{C}$;
(ii) The pair (E, A) is said to be causal if (E, A) is regular and $\mathrm{deg}\det(sE - A) = \mathrm{rank}E$ for $\forall s \in \mathbb{C}$, where $\det(\cdot)$ and $\deg(\cdot)$ represent determinant of a matrix and degree of a polynomial, respectively;
(iii) Singular discrete-time system

$$Ex(k+1) = Ax(k)$$

is said to be regular and causal, if the pair (E, A) is regular and causal.

Definition 2: (Yang et al., 2004) Singular discrete-time system

$$Ex(k+1) = Ax(k) + Bu(k), \tag{2a}$$

$$y(k) = Cx(k), \tag{2b}$$

is said to be Y-controllable, if there exists a state feedback

$$u(k) = Fx(k) + v(k)$$

such that the closed-loop system

$$Ex(k+1) = (A + BF)x(k) + Bv(k) \tag{3}$$

is causal, where $v(k)$ is a new input.

Definition 3: (Yang et al., 2004) System (2) is said to be Y-observable, if at arbitrary time k, $x(k)$ is uniquely determined by the initial condition and $\{u(i),\ y(i),\ i = 0, 1, \cdots, k\}$.

Lemma 2: *(Yang et al., 2004) System (2) is Y-controllable if and only if*

$$\mathrm{rank} \begin{bmatrix} E & 0 & 0 \\ A & E & B \end{bmatrix} = \mathrm{rank}(E) + n;$$

System (2) is Y-observable if and only if

$$\text{rank}\begin{bmatrix} E & A \\ 0 & E \\ 0 & C \end{bmatrix} = \text{rank}(E) + n.$$

Lemma 3: *(Yang et al., 2004) For system (2), there exists an output feedback*

$$u(k) = Fy(k) + v(k)$$

such that the closed-loop system (3) is causal if and only if system (2) is Y-controllable and Y-observable.

2.2 *Problem formulation*

Consider a networked singular multi-agent system composed of N agents indexed by $1, 2, \cdots, N$, respectively. The dynamics of the i-th agent are described by a singular discrete-time system:

$$Ex_i(k+1) = Ax_i(k) + Bu_i(k), \tag{4a}$$

$$y_i(k) = Cx_i(k), \tag{4b}$$

where $x_i(k)$ is the state, $u_i(k)$ is the control input, $y_i(k)$ is the measured output, $E, A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times q}$, $C \in \mathbb{R}^{m\times n}$ and $\text{rank}E = r \leq n$. The communication topology is described by a directed digraph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$. Assume that the communication delay d is a known and constant positive integer.

Remark 1: In the above singular multi-agent system, if matrix E is nonsingular, this singular multi-agent system becomes a normal linear multi-agent system. Therefore, from taking account of the wide rang of elements of matrix E, it is obtained that singular multi-agent systems is a generalization of normal linear multi-agent systems.

Remark 2: Usually, when information exchanges among agents through the shared network, the communication delays change in a certain range, that it is $0 \leq d(t) \leq d_M$, where d_M is the admissible upper bound of delays. If $d(t) \leq d_M$, set $d = d_M$ which implies that at some time, data obtained from the shared network does not immediately used, but is forced to wait for the delay time reaching d. By this way, the time-varying delay problem into the constant delay one. Hence it is reasonable to assume that the communication delay d is a known and constant positive integer. Although this method has a certain conservative, directly dealing with the time-varying delay problem is sometimes very difficult. This method can be as an effective method to solving time-varying delay problem indirectly.

Adopt the following protocol:

$$u_i(k) = Fy_i(k) + v_i(k), \ i \in \mathcal{V}, \tag{5}$$

where $F \in \mathbb{R}^{n\times q}$ and $v_i(k)$ will be designed as follows.

Definition 4: For networked singular multi-agent system (4), protocol $u_i(k)$, $i \in \mathcal{V}$ is said to solve the admissible consensus problem (or networked singular multi-agent system (4) achieves admissible consensus via protocol $u_i(k)$) if the closed-loop system via $u_i(k)$ is causal, and the following condition holds:

$$\lim_{k\to\infty} \|x_j(k) - x_i(k)\| = 0, \ \forall i, j \in \mathcal{V}. \tag{6}$$

The aim of this paper is to solve the following consensus problem.

Problem 1: For networked singular multi-agent system (4) with the directed topology $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ and the communication delay d, design protocol (5) to solve the admissible consensus problem, where $v_i(k)$ can be dependrd on values of the output y_i up to time step $k-d$.

3. Compensation for communication delays

If there exists $F \in \mathbb{R}^{n\times q}$ in protocol (5) such that the closed-loop system

$$Ex_i(k+1) = (A + BFC)x_i(k) + Bv_i(k) \tag{7}$$

is causal, then there exist two nonsingular matrices P and Q such that

$$PEQ = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}, \ P(A+BFC)Q = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix},$$

$$PB = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}, \ CQ = \begin{bmatrix} C_1 & C_2 \end{bmatrix}, \tag{8}$$

$$Q^{-1}x_i(k) = \begin{bmatrix} x_{i1}^T(k) & x_{i2}^T(k) \end{bmatrix}^T,$$

where $A_{11} \in \mathbb{R}^{r\times r}$, $A_{12} \in \mathbb{R}^{r\times(n-r)}$, $A_{21} \in \mathbb{R}^{(n-r)\times r}$ and $A_{22} \in \mathbb{R}^{(n-r)\times(n-r)}$ such that $\det(A_{22}) \neq 0$ which implies system (7) is causal (Yang et al., 2004). Then the restricted equivalent form of the system (7) is obtained:

$$x_{i1}(k+1) = A_{11}x_{i1}(k) + A_{12}x_{i2}(k) + B_1v_i(k), \tag{9a}$$

$$0 = A_{21}x_{i1}(k) + A_{22}x_{i2}(k) + B_2v_i(k), \tag{9b}$$

$$\begin{aligned} y_{i1}(k) &= C_1x_{i1}(k), \ y_{i2}(k) = C_2x_{i2}(k), \\ y_i(k) &= y_{i1}(k) + y_{i2}(k), \end{aligned} \tag{9c}$$

Hence (6) holds if and only if

$$\lim_{k\to\infty} \|x_{j1}(k) - x_{i1}(k)\| = 0 \tag{10}$$

and

$$\lim_{k\to\infty} \|x_{j2}(k) - x_{i2}(k)\| = 0 \tag{11}$$

hold, simultaneously.

It can be obtained from (9b) that

$$x_{i2}(k) = -A_{22}^{-1}[A_{21}x_{i1}(k) + B_2v_i(k)]. \tag{12}$$

Substituting (12) into (9a) derives

$$x_{i1}(k+1) = \hat{A}x_{i1}(k) + \hat{B}v_i(k), \tag{13a}$$

$$y_{i1}(k) = C_1x_{i1}(k), \tag{13b}$$

where

$$\hat{A} = A_{11} - A_{12}A_{22}^{-1}A_{21},\ \hat{B} = B_1 - A_{12}A_{22}^{-1}B_2. \tag{14}$$

Since information exchanges with the communication delay d, at time k, the i-th agent only receives information from the j-th agent that was released at time $k-d$, $j \in N_i$. In order to compensate for the networked communication delay actively and effectively, NPCS proposed in Liu et al. (2007) is employed to solve the studied consensus problem. Based on the output data from the j-th agent up to time $k-d$, construct state and output predictions of the j-th agent from time $k-d$ to time k as follows:

$$\hat{x}_{j1}(k-d+1|k-d) = \hat{A}\hat{x}_{j1}(k-d) + \hat{B}v_j(k-d) + L[y_{j1}(k-d) - C_1\hat{x}_{j1}(k-d)] \tag{15a}$$

$$\hat{y}_{j1}(k-d+1|k-d) = C_1\hat{x}_{j1}(k-d+1|k-d) \tag{15b}$$

$$\hat{x}_{j1}(k-d+2|k-d) = \hat{A}\hat{x}_{j1}(k-d+1|k-d) + \hat{B}v_j(k-d+1), \tag{16a}$$

$$\hat{y}_{j1}(k-d+2|k-d) = C_1\hat{x}_{j1}(k-d+2|k-d) \tag{16b}$$

$$\vdots$$

$$\hat{x}_{j1}(k|k-d) = \hat{A}\hat{x}_{j1}(k-1|k-d) + \hat{B}v_j(k-1) \tag{17a}$$

$$\hat{y}_{j1}(k|k-d) = C_1\hat{x}_{j1}(k|k-d) \tag{17b}$$

where $\hat{x}_{i1}(k)$ and $v_i(k)$ are the state and control input of the predictor (15), respectively. $\hat{x}_{j1}(k-d+s|k-d)$ and $\hat{y}_{j1}(k-d+s|k-d)$, $s = 1, 2, \cdots, d$, are the state and output predictions of the j-th agent from time $k-d$ to time k, respectively. $L \in$ is a $\mathbb{R}^{r\times m}$ matrix which will be suitably designed. According to observer design approaches (Yang et al., 2004), design $L \in \mathbb{R}^{r\times m}$ to guarantee that $\lim\limits_{k\to\infty} \|\hat{x}_{i1}(k) - x_{i1}(k)\| = 0,\ i \in \mathcal{V}$. Obviously, using the above method is required to construct input predictions $v_j(k-d+s|k-d)$ of the j-th agent to replace with $v_j(k-d+s),\ s = 1, 2, \cdots, d$. Thus, for NMAS (13) with the communication delay d, the protocol for the i-th agent based on NPCS and the dynamic compensators is adopted as

$$z_i(k+1) = \bar{A}z_i(k) + \bar{B}(\sum_{j=1}^{N} a_{ij}[\hat{y}_{j1}(k|k-d) - \hat{y}_{i1}(k|k-d)]), \tag{18a}$$

$$v_i(k) = \bar{C}z_i(k) + \bar{D}(\sum_{j=1}^{N} a_{ij}[\hat{y}_{j1}(k|k-d) - \hat{y}_{i1}(k|k-d)]), \tag{18b}$$

where $z_i(k) \in \mathbb{R}^{n_c}$ is the state of the dynamic compensator, $\bar{A} \in \mathbb{R}^{n_c \times n_c}$, $\bar{B} \in \mathbb{R}^{n_c \times m}$, $\bar{C} \in \mathbb{R}^{q \times n_c}$ and $\bar{D} \in \mathbb{R}^{q \times m}$ are constant matrices which will be designed.

The predictive processes is explained in detail as follows. At time k, the j-th agent can only obtain data $\{u_j(k-d),\ y_j(k-d),\ z_j(k-d)\}$ from the j-th agent. Based on $\{u_j(k-d),\ y_j(k-d),\ z_j(k-d)\}$, using the following steps to construct input predictions of the j-th agent and states of dynamic compensator (18) from time $k-d+1$ to time $k-1$:

Step 1: Based on data $\{v_j(k-d),\ y_{j1}(k-d)\}$, one obtains $\hat{x}_{j1}(k-d+1|k-d)$ and $\hat{y}_{j1}(k-d+1|k-d)$ from (15);
Step 2: Using data $\{z_j(k-d),\ y_{j1}(k-d)\}$ and (18a), one obtains $z_j(k-d+1)$;
Step 3: Based on data $\{z_j(k-d+1),\ y_{j1}(k-d+1|k-d)\}$, one obtains $v_j(k-d+1|k-d)$ from (18b);
Step 4: Based on data $\{x_{j1}(k-d+1|k-d),\ v_j(k-d+1|k-d)\}$, one obtains $\hat{x}_{j1}(k-d+2|k-d)$ and $\hat{y}_{j1}(k-d+2|k-d)$ from (16);
Step 5: Repeat the above steps, it can be obtained:

$$\begin{gathered}\{v_j(k-d+2|k-d),\ z_j(k-d+2)\}\\ \{v_j(k-d+3|k-d),\ z_j(k-d+3)\}\\ \vdots \\ \{v_j(k-1|k-d),\ z_j(k-1)\}\end{gathered}$$

Remark 3: Comparing the dynamic compensator with the state feedback, the former one would make more sense in consensus protocols. Due to constraints on measurement or economic costs in practice, it is sometimes hard to measure information of all states directly (Yang & Liu, 2011). However, only the relative information of all outputs is available. When $n_c = 0$, protocol (18) is simplified into a static output feedback in Yang & Liu (2014):

$$v_i(k) = \bar{D}(\sum_{j=1}^{N} a_{ij}[\hat{y}_{j1}(k|k-d) - \hat{y}_{i1}(k|k-d)]). \tag{19}$$

Since the static output feedback can not fully reach the function of the state feedback, protocol (18) can overcome this limitation when $n_c > 0$. Moreover, protocol (18) can also increase design degrees of freedom, which can guarantee that the obtained closed-loop system has better performance. Hence it is easy to see that protocol (19) is one particular case of protocol (18).

Definition 5: For NMAS (13) with $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ and the communication delay d, protocol (18) is said to solve the consensus problem (or NMAS (13) achieves consensus via protocol (18)), if the following conditions hold:

$$\lim_{k\to\infty}\|x_{j1}(k)-x_{i1}(k)\|=0,\ \lim_{k\to\infty}\|z_i(k)\|=0,\ \lim_{k\to\infty}\|e_i(k)\|=0,\ \forall i,j\in\mathcal{V},$$

where $e_i(k)=\hat{x}_{i1}(k)-x_{i1}(k)$.

Remark 4: When protocol (18) solves consensus problem of NMAS (13), one has

$$\lim_{k\to\infty}\|z_i(k)\|=0,\ \lim_{k\to\infty}\|\delta_i(k)\|=0,\ \lim_{k\to\infty}\|e_i(k)\|=0,\ i\in\mathcal{V},$$

where $\delta_i(k)=x_{11}(k)-x_{i1}(k),\ i\in\mathcal{V}\setminus\{1\}$, According to (18b), one has

$$\lim_{k\to\infty}v_i(k)=0, i\in\mathcal{V}. \tag{20}$$

Using (12), one obtains

$$x_{i2}(k)-x_{12}(k)=-A_{22}^{-1}A_{21}\delta_i(k)-A_{22}^{-1}B_2[v_i(k)-v_1(k)].$$

Combining $\lim\limits_{k\to\infty}\|\delta_i(k)\|=0$ with (20) yields

$$\lim_{k\to\infty}\|x_{j2}(k)-x_{i2}(k)\|=0,\ \forall i,\ j\in\mathcal{V}.$$

Based on the previous preparation, solving Problem 1 has been converted to solving the following Problem.

Problem 2: Design a matrix F and protocol (18) to guarantee that the closed-loop system (7) is causal and NMAS (13) achieves consensus via protocol (18), simultaneously.

4. Analysis of admissible consensus for networked singular multi-agent systems

Theorem 1: *For networked singular multi-agent system (4) with $\mathcal{G}=(\mathcal{V},\mathcal{E},\mathcal{A})$ and the communication delay d, protocol (5) solves the admissible consensus problem if and only if the following conditions hold:*

(i) *Each agent of system (4) is Y-controllable and Y-observable;*
(ii) *There exist matrices F, $\bar{A}$, $\bar{B}$, $\bar{C}$, $\bar{D}$ and L such that*

$$\Delta=\begin{bmatrix}\Delta_{11} & S[I_N\otimes(\hat{B}\bar{C})]\\ \bar{\mathcal{L}}\otimes(\bar{B}C_1) & I_N\otimes\bar{A}\end{bmatrix}$$

and $\hat{A}-LC_1$ are Schur stable, where $\otimes$ is the Kronecker product of matrices, $\Delta_{11}=I_{N-1}\otimes\hat{A}-(\mathcal{L}_{22}-\mathbf{1}_{N-1}\mathcal{L}_{12})\otimes(\hat{B}\bar{D}C_1)$, $\bar{\mathcal{L}}=\mathcal{L}_{\mathcal{G}}\begin{bmatrix}0\ I_{N-1}\end{bmatrix}^T$, $\mathbf{1}_{N-1}=[1\ 1\ \cdots\ 1]^T\in\mathbb{R}^{N-1}$, $\begin{bmatrix}l_{11} & \mathcal{L}_{12}\\ \mathcal{L}_{21} & \mathcal{L}_{22}\end{bmatrix}=\mathcal{L}_{\mathcal{G}}$, $\hat{A}$ and C_1 are defined by (8) and (14), respectively.

Proof. According to Lemma 3, there exists an output feedback (5) such that the closed-loop system (7) is causal if and only if each agent of system (4) is Y-controllable and Y-observable. It will be shown that NMAS (13) achieves consensus via protocol (18) if and only if condition (ii) holds as follows. Denote

$$\begin{aligned}x(k) &= [\, x_{11}^T(k)\; x_{21}^T(k) \cdots\; x_{N1}^T(k) \,]^T,\\ z(k) &= [\, z_1^T(k)\; z_2^T(k) \cdots\; z_N^T(k) \,]^T,\\ e(k) &= [\, e_1^T(k)\; e_2^T(k) \cdots\; e_N^T(k) \,]^T,\\ \delta(k) &= [\, \delta_2^T(k)\; \delta_3^T(k) \cdots\; \delta_N^T(k) \,]^T.\end{aligned}$$

It is easy to see that $x(k)$ and $\delta(k)$ such that

$$\delta(k) = Sx(k), \tag{21}$$

where $S = [\, \mathbf{1}_{N-1}\; -I_{N-1} \,] \otimes I_n$.

It can be concluded from Definition 5 that protocol (18) solves the consensus problem for system (13) if and only if $\lim_{k\to\infty} \|\delta(k)\| = 0$, $\lim_{k\to\infty} \|z(k)\| = 0$ and $\lim_{k\to\infty} \|e(k)\| = 0$ hold.

By the iteration of system (13a), the state $x_{j1}(k)$ can also be written as

$$x_{j1}(k) = \hat{A}^d x_{j1}(k-d) + \sum_{s=1}^{d} \hat{A}^{d-s}\hat{B}v_j(k-d+s-1). \tag{22}$$

Using (13a) and (15a), one obtains

$$e_i(k+1) = (\hat{A} - LC_1)^{d-1} e_i(k-d+1). \tag{23}$$

From (15a), (16a) and (17a) by the way of iteration, one obtains

$$\hat{x}_{j1}(k|k-d) = \hat{A}^{d-1}(\hat{A} - LC_1)\hat{x}_{j1}(k-d) + \sum_{s=1}^{d} \hat{A}^{d-s}\hat{B}v_j(k-d+s-1) + \hat{A}^{d-1}Ly_{j1}(k-d),\; j \in \mathcal{V}, \tag{24}$$

Combining with (22), (23) and (24) drives

$$\begin{aligned}\hat{x}_{j1}(k|k-d) =& \hat{A}^{d-1}(\hat{A} - LC_1)\hat{x}_{j1}(k-d) + z_j(k) - \hat{A}^d x_{j1}(k-d) + \hat{A}^{d-1}LC_1 z_j(k-d)\\ =& x_{j1}(k) + \hat{A}^{d-1}e_j(k-d+1),\end{aligned}$$

Using the definition of $\mathcal{L}_\mathcal{G}$ and the above equation, one has

$$\sum_{j=1}^{N} a_{ij}[\hat{y}_{j1}(k|k-d) - \hat{y}_{i1}(k|k-d)] = C_1(\bar{\mathcal{L}}_i \otimes I_n)\delta(k) - C_1\hat{A}^{d-1}(\mathcal{L}_i \otimes I_n)e(k-d+1),$$

where $\mathcal{L}_i = \begin{bmatrix} l_{i1} & \bar{\mathcal{L}}_i \end{bmatrix}$, $\bar{\mathcal{L}}_i = \begin{bmatrix} l_{i2} & l_{i3} & \cdots & l_{iN} \end{bmatrix}$. Substituting the above equation into (18) yields the closed-loop system which is made of system (13) and protocol (18)

$$x_{i1}(k+1) = \hat{A}x_{i1}(k) + \hat{B}\bar{C}z_i(k) + \hat{B}\bar{D}C_1(\bar{\mathcal{L}}_i \otimes I_n)\delta(k) - \hat{B}\bar{D}C_1\hat{A}^{d-1}(\mathcal{L}_i \otimes I_n)e(k-d+1),$$

$$z_i(k+1) = \bar{A}z_i(k) + \bar{B}C_1(\bar{\mathcal{L}}_i \otimes I_n)\delta(k) - \hat{B}C_1\hat{A}^{d-1}(\mathcal{L}_i \otimes I_n)e(k-d+1).$$

Hence,

$$x(k+1) = (I_N \otimes \hat{A})x(k) + [I_N \otimes (\hat{B}\bar{C})]z(k)[\mathcal{L}_{\mathcal{G}} \otimes (\hat{B}\bar{D}C_1\hat{A}^{d-1})]e(k-d+1) + [\bar{\mathcal{L}} \otimes (\hat{B}\bar{D}C_1)]\delta(k),$$

$$z(k+1) = (I_N \otimes \bar{A})z(k) + [\bar{\mathcal{L}} \otimes (\bar{B}C_1)]\delta(k) - [\mathcal{L}_{\mathcal{G}} \otimes (\bar{B}C_1\hat{A}^{d-1})]e(k-d+1),$$

where $\bar{\mathcal{L}} = \mathcal{L}_{\mathcal{G}} \begin{bmatrix} 0 \ I_{N-1} \end{bmatrix}^T$. Using (21) and

$$S[\bar{\mathcal{L}} \otimes (\hat{B}\bar{D}C_1)] = -(\mathcal{L}_{22} - \mathbf{1}_{N-1}\mathcal{L}_{12}) \otimes (\hat{B}\bar{D}C_1),$$

one obtains

$$\eta(k+1) = \Sigma\eta(k),$$

where

$$\eta(k) = \begin{bmatrix} \delta^T(k+1) \ z^T(k+1) \ e^T(k-d) \end{bmatrix}^T,$$
$$\Sigma = \begin{bmatrix} \Delta & \Sigma_1 \\ 0 & I_N \otimes (\hat{A} - LC_1) \end{bmatrix},$$
$$\Sigma_1 = \begin{bmatrix} -S[\mathcal{L}_{\mathcal{G}} \otimes (\hat{B}\bar{D}C_1\hat{A}^{d-1})] \\ \mathcal{L}_{\mathcal{G}} \otimes (\bar{B}C_1\hat{A}^{d-1}) \end{bmatrix}.$$

Based on the previous derivation, protocol (18) can solve the consensus problem of system (13) if and only if Σ is Schur stable which implies that Δ and $\hat{A} - LC_1$ are Schur stable. □

Corollary 1: *For networked singular multi-agent system (4) with $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ and the communication delay d, protocol (5) solves the admissible consensus problem if and only if the following conditions hold:*

(i)

$$\text{rank} \begin{bmatrix} E & 0 & 0 \\ A & E & B \end{bmatrix} = \text{rank}(E) + n, \tag{25}$$

$$\text{rank} \begin{bmatrix} E & A \\ 0 & E \\ 0 & C \end{bmatrix} = \text{rank}(E) + n; \tag{26}$$

(ii) *There exist matrices F, $\bar{A}$, $\bar{B}$, $\bar{C}$, $\bar{D}$, L and positive definite matrices $P_\Delta > 0$, $P_L > 0$ such that the following matrix inequalities hold:*

$$\Delta^T P_\Delta \Delta - P_\Delta < 0, \tag{27}$$

$$(\hat{A} - LC_1)^T P_L (\hat{A} - LC_1) - P_L < 0, \tag{28}$$

where Δ is defined as in Theorem 1. $\hat{A}$ and C_1 are defined by (14) and (8), respectively.

Proof. It can be conclude from Lemma 2 that (25) and (26) hold if and only if each agent of system (4) is Y-controllable and Y-observable. According to the proof of Theorem 1, it suffices to show that the condition (ii) holds if and only if Δ and $\hat{A}-LC_1$ are Schur stable as follows. It can be obtained from Lyapunov stability theory that Δ and $\hat{A}-LC_1$ are Schur stable if and only if there exist positive definite matrices $P_\Delta > 0$ and $P_L > 0$ such that matrix inequalities (27) and (28) hold, simultaneously. □

Corollary 2: *For networked singular multi-agent system (4) with $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ and the communication delay d, if each agent of system (4) is Y-controllable and Y-observable, $(\hat{A}, C_1)$ is detectable, and there exist $\bar{D}$ and $P_\Gamma > 0$ such that*

$$\Gamma^T P_\Gamma \Gamma - \Gamma < 0 \tag{29}$$

holds, then protocol (5) can solve the admissible consensus of system (4), where

$$\Gamma = I_{N-1} \otimes \hat{A} - (\mathcal{L}_{22} - \mathbf{1}_{N-1}\mathcal{L}_{12}) \otimes (\hat{B}\bar{D}C_1),$$

$\mathcal{L}_{12},\ \mathcal{L}_{22},\ \mathbf{1}_{N-1}$ *is defined as in Theorem 1.*

Proof. Since $(\hat{A}, C_1)$ is detectable, there exists a matrix L such that $(\hat{A} - LC_1)$ is Schur stable. Choose matrices $\bar{A}, \bar{B}, \bar{C}$ to guarantee that $\bar{A}$ is Schur stable, and $\hat{B}\bar{C} = 0$ or $\bar{B}C_1 = 0$. According to Lyapunov stability theory, it can be concluded from (29) that Γ is Schur stable. Thus, using Theorem 1, it can be obtained that the admissible consensus problem of system (4) is solved by protocol (5). □

Corollary 3: *For networked singular multi-agent system (4) with $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ and the communication delay d, if each agent of system (4) is Y-controllable and Y-observable, $(\hat{A}, C_1)$ is detectable, the topology $\mathcal{G}$ has a directed spanning tree, and there exists $\bar{D}$ such that $\hat{A} - \lambda_i\hat{B}\bar{D}C_1,\ i \in \mathcal{V}\setminus\{1\}$, are Schur stable, then protocol (5) can solve the admissible consensus of system (4), where $\lambda_i,\ i \in \mathcal{V}\setminus\{1\}$, are the non-zero eigenvalues of $\mathcal{L}_\mathcal{G}$.*

Proof. Similar to the proof processes of Theorem 1, one has eigenvalues of $I_{N-1} \otimes \hat{A} - (\mathcal{L}_{22} - \mathbf{1}_{N-1}\mathcal{L}_{12}) \otimes (\hat{B}\bar{D}C_1)$ are given by all eigenvalues of $\hat{A} - \lambda_i\hat{B}\bar{D}C_1,\ i \in \mathcal{V}\setminus\{1\}$. Since $\hat{A} - \lambda_i\hat{B}\bar{D}C_1,\ i \in \mathcal{V}\setminus\{1\}$, are Schur stable, it can be concluded from the proof of Corollary 2 that the conclusion of this corollary holds. □

Remark 5: When networked singular multi-agent system (4) satisfies the preconditions in above Theorem and corollaries, it can be obtained from that networked singular multi-agent system (4) achieving consensus via protocol (5) based on NPCS depend not only on the topology of networked singular multi-agent system (4) but also the structure properties of each agent dynamics. Obviously, protocol (5) can compensate for communication delays effectively.

Based on Corollary 3, the following algorithm is provided to design predictive protocol (5) associated with F and protocol (18), which implies that Problem 1 will be solved under some reasonable assumptions.

Algorithm 1: *Input: the matrices $E,\ A \in \mathbb{R}^{n\times n},\ B \in \mathbb{R}^{n\times q},\ C \in \mathbb{R}^{m\times n},\ \mathcal{A} = [a_{ij}] \in \mathbb{R}^{N\times N}$ and $d \in \mathbb{R}$;*
Output: the gain matrices $F,\ L,\ \bar{A},\ \bar{B},\ \bar{C}$ and $\bar{D}$.

(a) *Carry out the singular value decomposition of the matrix E by*

$$E = U \begin{bmatrix} \Sigma & 0 \\ 0 & 0 \end{bmatrix} V^T$$

and obtain the orthogonal matrices $U,\ V \in \mathbb{R}^{n\times n}$, the diagonal positive definite matrix $\Sigma \in \mathbb{R}^{r\times r}$ such that

$$\Sigma = \mathrm{diag}\{\sigma_1,\ \sigma_2,\ \cdots,\ \sigma_r\} > 0$$

where $\sigma_i, i = 1, 2, \cdots, r$ are the non-zero singular values of E;

(b) *Compute the matrices P and Q by*

$$P = \mathrm{diag}(\Sigma^{-1},\ I_{n-r})U^T,\ Q = V$$

(c) *Compute the matrices $\hat{A}_{11} \in \mathbb{R}^{r\times r}$, $\hat{A}_{12} \in \mathbb{R}^{r\times(n-r)}$, $\hat{A}_{21} \in \mathbb{R}^{r\times(n-r)}$, $\hat{A}_{22} \in \mathbb{R}^{(n-r)\times(n-r)}$, $B_1 \in \mathbb{R}^{r\times q}$, $B_2 \in \mathbb{R}^{(n-r)\times q}$, $C_1 \in \mathbb{R}^{m\times r}$ $C_2 \in \mathbb{R}^{m\times(n-r)}$ by*

$$\begin{bmatrix} \hat{A}_{11} & \hat{A}_{12} \\ \hat{A}_{21} & \hat{A}_{22} \end{bmatrix} = PAQ, \quad \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} = PB,\ \begin{bmatrix} C_1 & C_2 \end{bmatrix} = CQ;$$

(d) *Choose a matrix $F \in \mathbb{R}^{n\times q}$ such that $\det(\hat{A}_{22} + B_2FC_2) \neq 0$;*

(e) *Compute the matrices $A_{11},\ A_{12},\ A_{21},\ A_{22},\ \hat{A}$ and $\hat{B}$ by*

$$A_{11} = \hat{A}_{11} + B_1FC_1,\ A_{12} = \hat{A}_{12} + B_1FC_2,$$

$$A_{21} = \hat{A}_{21} + B_2FC_1,\ A_{22} = \hat{A}_{22} + B_2FC_2,$$

$$\hat{A} = A_{11} - A_{12}A_{22}^{-1}A_{21},\ \hat{B} = B_1 - A_{12}A_{22}^{-1}B_2;$$

(f) *Compute Laplacian matrix $\mathcal{L}_{\mathcal{G}}$ by (1) and the non-zero eigenvalues λ_i of $\mathcal{L}_{\mathcal{G}}$, $i \in \mathcal{V} \setminus \{1\}$;*

(g) *If $(\hat{A},\ C_1)$ is detectable, solve the unique positive-definite solution to Riccati equation*

$$\hat{A}P\hat{A}^T - P - \hat{A}PC_1(I + C_1PC_1^T)^{-1}C_1P\hat{A}^T + I = 0, \tag{30}$$

and set $L = \hat{A}PC_1(I + C_1PC_1^T)^{-1}$. Otherwise, go back Step (c) to choose F again;

(h) *Choose matrices $\bar{A} \in \mathbb{R}^{n_c\times n_c}$, $\bar{B} \in \mathbb{R}^{n_c\times m}$ and $\bar{C} \in \mathbb{R}^{q\times n_c}$ such that $\bar{A}$ is Schur stable, and $\hat{B}\bar{C} = 0$ or $\bar{B}C_1 = 0$;*

(i) *Choose a matrix $\bar{D} \in \mathbb{R}^{q\times m}$ such that $\hat{A} - \lambda_i\hat{B}\bar{D}C_1,\ i \in \mathcal{V} \setminus \{1\}$ are Schur stable. Then output the matrices $F,\ L,\ \bar{A},\ \bar{B},\ \bar{C}$ and $\bar{D}$.*

Remark 6: According to Lemma 3, the condition which each agent of system (4) is Y-controllable and Y-observable can guarantee the existence of F in step (d) of Algorithm 1. Moreover, it can be concluded from (Duan, 2010, Theorem 7.9) that the set

$$\mathcal{N}_{(\hat{A}_{22}, B_2, C_2)} = \left\{F|\det(\hat{A}_{22} + B_2FC_2) \neq 0\right\}$$

is a Zariski open set. Thus, solving a gain matrix from the set $\mathcal{N}_{(\hat{A}_{22}, B_2, C_2)}$ can often be easily sought by a "trial-and-test" procedure.

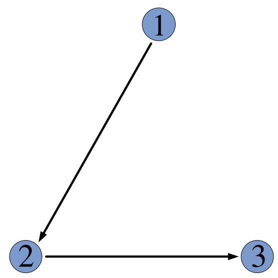

Figure 1. The communication topology

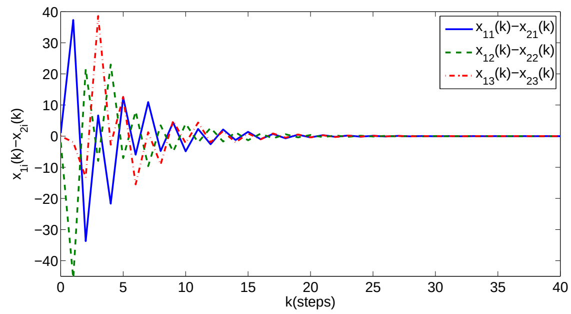

Figure 2. State difference trajectories of agents based on delayed states

5. A numerical example

Consider networked singular multi-agent system (4) composed of $N = 3$ agents, where

$$E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \; A = \begin{bmatrix} 1 & 1 & -1 \\ -3.5 & -1.5 & 1 \\ -2 & 0 & 0 \end{bmatrix}, \; B = \begin{bmatrix} 1 & 0 & 1 \\ -1 & 1 & 2 \end{bmatrix}^T, \; C = \begin{bmatrix} 4 & 2 & 1 \\ 2 & 1 & 1 \end{bmatrix}.$$

The communication delay $d = 3$ and the communication topology is described by a weighted digraph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$, where $\mathcal{V} = \{1, 2, 3\}$, $\mathcal{E} = \{(1, 2),\ (2, 3)\}$ and $\mathcal{A} = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$. Figure 1 shows its communication topology. According to the steps in Algorithm 1, the following can be obtained:

$$F = I_2, \bar{A} = -0.1, \; \bar{B} = \begin{bmatrix} 1 & -2 \end{bmatrix}, \; \bar{C} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \bar{D} = \begin{bmatrix} 0.9 & 0.8 \\ 1.9 & -0.2 \end{bmatrix}.$$

Case one: Time-delay information is used to control the system directly. Thus, adopt the protocol:

$$u_i(k) = Fy_i(k) + \bar{D} \sum_{j=1}^{N} a_{ij} [y_j(k - d) - y_i(k)].$$

The simulation results are presented in Fig. 2 and Fig. 3, which indicates networked singular multi-agent system (4) achieves admissible consensus. Meanwhile, it is easy to see that achieving

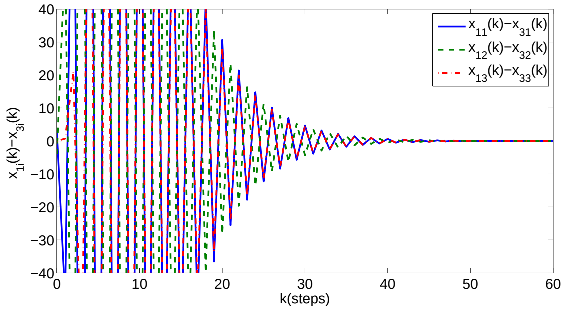

Figure 3. State difference trajectories of agents based on delayed states

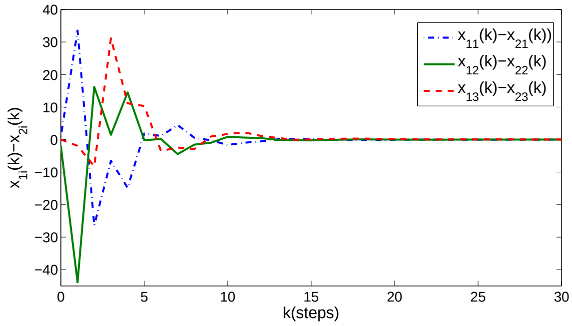

Figure 4. State difference trajectories of agents based on NPCS

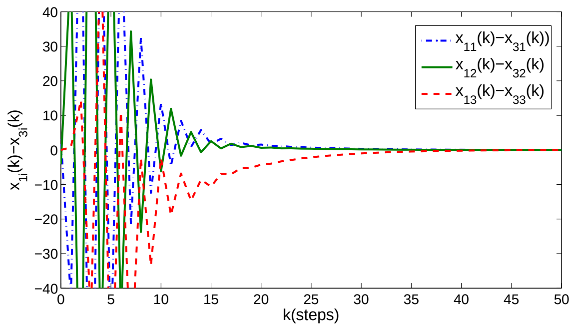

Figure 5. State difference trajectories of agents based on NPCS

consensus costs 53 steps.

Case two: Based on NPCS, adopt the protocol designed in this paper:

$$z_i(k+1) = \bar{A}z_i(k) + \bar{B}(\sum_{j=1}^{N} a_{ij}\delta_j(k|k-d)),$$

$$u_i(k) = Fy_i(k) + \bar{C}z_i(k) + \bar{D}(\sum_{j=1}^{N} a_{ij}\delta_j(k|k-d)),$$

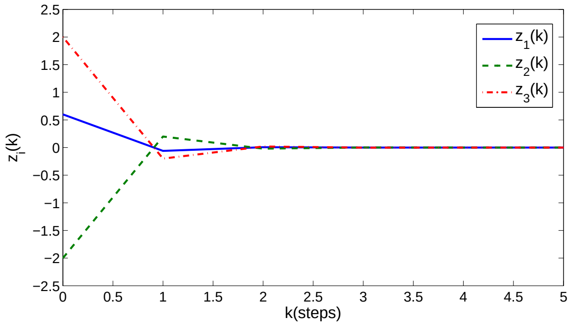

Figure 6. Trajectories of protocol states based on NPCS

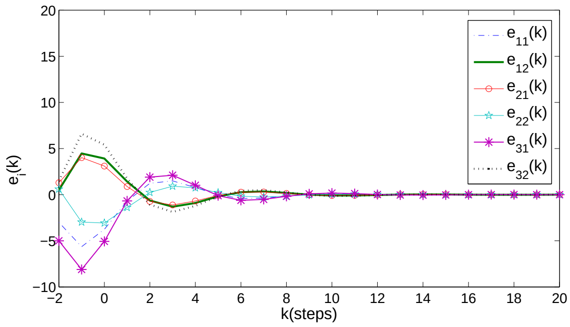

Figure 7. Trajectories of estimate error based on NPCS

where $\delta_j(k|k-d) = \hat{y}_{j1}(k|k-d) - \hat{y}_{i1}(k|k-d)$. The simulation results are presented in Fig. 4 to Fig. 7. Fig. 4 and Fig. 5 show state trajectories of networked singular multi-agent system (4), which indicates that networked singular multi-agent system (4) achieves admissible consensus via observer-based predictive protocol (18). Fig. 6 and Fig. 7 present state trajectories of protocol (18) and error trajectories $e_i(k)$ tend to zero, respectively. According to Definition 1, Problem 1 is solved by protocol (18) of system (4). Moreover, achieving consensus costs 41 steps. It is clearly that achieving consensus based on the predictive protocol designed in this paper is faster than the method used time-delay information.

6. Conclusion

For networked singular multi-agent systems with directed topologies and communication delays, the admissible consensus problem via predictive protocols has been solved. Due to only the information of outputs is available through the shared network with communication delays, an observer-based NPCS has been employed to compensate for communication delays actively. Based on the dynamic compensator and NPCS, a novel protocol has been proposed. The provided simulation results have demonstrated the effectiveness of compensation for communication delays successfully. In this paper, the obtained results only serve as a stepping stone to investigate singular multi-agent systems with more complicated topologies. The future research will study singular multi-agent systems with time-varying networked delays, stochastic or switching topologies, and agents described by switching systems, hybrid systems or Markovian jump systems (Li et al., 2015a,b), and so on.

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