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An Integration of Different Computing Approaches in Traffic Safety Analysis

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Abstract

Traffic safety is one of the crucial problems of many countries in the world. To handle this problem, a great deal of research has been conducted considering various methods. This study includes analyses of black spots using different computing approaches. Integration of cluster analysis, entropy approach and fuzzy logic approaches are used in the analyses. The conventional black spot identification method includes marking the location of each accident with a pin and investigation of black spots considering density of the pins on a map. In this study, a systematic approach is employed. Firstly, the traffic accidents data of Denizli city have been analyzed using the fuzzy clustering methods. The spots that are densely located around the cluster centers are determined as "black spots". Secondly, the safety levels of black spots' are determined by Shannon Entropy Approach considering accident types and effective factors on accident occurrence. Geometrical and physical conditions, traffic volumes, average speeds and average accident rates at around black spots are considered as effective factors on occurrence of accidents. Entropy values are calculated using these parameters. Thirdly, the safety levels are classified by both fuzzy logic and crisp approaches based on calculated entropy values. Validation of entropy approach is tested by Chi-Square and truth value methods. The results are evaluated regarding all features of the black spots, and a series of recommendations to improve traffic safety are reported.

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Keywords: traffic safety, black spot analysis, cluster, entropy, fuzzy logic

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1. Introduction

Traffic accident data can be analyzed in different ways, based on the amount and types of data. The analysis is not complicated if the data are smooth and not dispersed. But it is not an easy task if the data are scattered. Although there is not a general definition for black spots, locations where at least more than one accident occurred are treated as black spots (Meuleners et al, 2008). Based on this definition, the more the number of black spots, the more difficult their analyses become.

Several methods can be used for determination of black spots and centers (Abdel-Aty and Pande, 2007; Flahaut et al., 2003). De Luca et al. (2012) applied Bayesian model based cluster analysis on road safety management problem. Gregoriades and Mouskop (2013) studied on accident risk quantification issue. In their study, accident risk quantification was achieved through a Bayesian Networks (BNs) Model. De Pauw et al. (2014) evaluated the safety effects of an extensive black spot program that had been implemented in Flanders – Belgium. Pesic et al (2013) proposed a new method so called Benchmarked Traffic Safety Level (BTSL) to evaluate the traffic safety levels. Nghiem et al. (2016) examined the determinants of road traffic crash fatalities in Queensland for the period 1958 – 2007 using a state-space time-series model. Ghaffarı et al. (2013) proposed a new method based on the reliability analysis to identify black spots. In their study, they compared proposed method with Frequency and Emprical Bayesian methods using simulated data.

Black spots can be determined by eye using simple observations. But this simple approach can include subjective perceptions and also results obtained cannot be sensitive and scientific. Besides, other specifications of black spots should be taken into consideration for a scientific analysis. Developing countermeasures and classifying by characteristics for black spots that are intensified and covered whole area on the map is not an easy task (ITE, 1993; Murat et al., 2008). Although some black spots can have common characteristics, they can be located far away from each other. On the other hand, characteristics of black spots that are closely located to each other can be different. Therefore definition and analysis of black spots include uncertainties and conventional approaches cannot be used for this purpose (Murat et al., 2008). In this study, firstly cluster analysis approach is used for determination of black spots' centers and definition of the centers. Then Shannon entropy approach is used to determine entropy value of black spots' centers and safety levels are classified by fuzzy logic approach.

2. Study Area and Data

In this paper, the city of Denizli, a medium sized city (current population is about 700000), Turkey is studied. Traffic accident records are used in analyzing accidents. The distributions of accident data are given in Table 1.

	Type of Accident					
Intersection or Road Section (Black Spot)	Dead+Injured Accidents	Economically Damaged (ED) Accidents	ED+Injured Accidents			
Ucgen	1	229	17			
Karayolları		46	4			
Cinar	1	86	1			
Kiremitci		25	0			
Yeni Adliye		27	6			
İstasyon	1	107	12			
Sevindik		93	8			
Emniyet	1	76	9			
Ulus		94	4			
Hastane-M.Efendi		50	1			
25. cadde	1	200	26			

Table 1. Traffic Accident Data and Distribution (the figures in the table are the number of accidents).

The accident reports are provided by the Local Police Department. The following information (FHWA, 1991) are collected from the reports (Table 2).

Table 2.	The	variables	and	information	obtained.

The Variables	Descriptive statistics
Location of accident (coordinates)	1126
Type of accident	9
Date	365
Time	24 h
Accident type for participating vehicle number	8
Accident type for occurrence	11
Weather condition	7
Road and environmental conditions (signal, pavement, policeman, obstacle	11
etc.)	
Road direction type (one or two way)	5
Type of pavement	7
Pavements surface condition (dry, wet, icy etc.)	8
Problems based on roadway	8
Presence of warning sign	2
Vertical route conditions	6
Horizontal route conditions	6
Intersection conditions	7
Crossings (school crossing, pedestrian crossing, railroad crossing)	10
Other factors (narrow road, bridge, and tunnel)	7
Information about vehicles (type, model, damage condition, speed etc)	20
Information about drivers (age, sex, alcohol, usage of safety belt etc)	82
Information about passengers and pedestrians (age, sex, alcohol, usage of safety belt etc)	82
Vehicle insurance conditions	6

All of these data given above are recorded using MS Excel. Then, coordinates of each accident point are determined.

3. Methodology

In this study three methods (cluster analysis, Shannon entropy approach and fuzzy logic approach) are used for safety analysis of black spots (Fig 1). A brief information about the methods is defined in the following.



Fig. 1. Computing methods used in the study

3.1. Cluster analysis approach

In recent years, cluster analysis has been widely used in engineering application such as civil engineering, target recognition, medical diagnosis etc. In traffic safety, determination and classification of black spots is one of the crucial problems and include uncertainties. The conventional black spot identification method incorporates marking the location of each accident with a pin and investigation of black spots considering density of the pins on a map

(Moreno et al, 2007). Although some basic information can be inferred by this approach, detailed analysis cannot be made. Therefore cluster analysis approach is preferred in this study.

Cluster analysis is an unsupervised method for classifying data, i.e. to divide a given data into a set of classes or clusters. In this research the Fuzzy C-Means clustering approach is used and summarized in the following section.

3.1.1. Fuzzy C-Means Clustering

Fuzzy C-Means (FCM) clustering algorithm has been widely used and applied in different areas. The description of the original fuzzy clustering algorithm based on objective function dates back to 1973 (Bezdek, 1973; Dunn, 1974). This algorithm was conceived in 1973 by Dunn (1974) and further generalized by Bezdek (1973). Among the existing fuzzy clustering methods, the Fuzzy c-means (FCM) algorithm proposed by Bezdek (1981) is the simplest and is the most popular technique of clustering. It is an extension of the hard K-means algorithm to fuzzy framework. Grubesic (2006) explored the use of a generalized partitioning method known as fuzzy clustering for crime hot-spot detection.

FCM algorithm is extension of Hard K-means with an advantage of fuzzy set theory and contrary to the K-means method the FCM is more flexible because it shows those objects that have some interface with more than one cluster in the partition. In traditional clustering algorithms such as Hard K-Means, an element belongs fully to a cluster or not (i.e. 0 or 1). On the other hand, in Fuzzy clustering, each element can belongs to several clusters with different membership degrees. The main goal of any clustering algorithm is to determine the appropriate partition matrix U(X) of a given data set X consisting of patterns ($X = \{x_1, x_2, \dots, x_N\}$) and to find the appropriate number of clusters. The objective function and constraints can be defined as;

Objective Function

$$J(X;U,V) = \sum_{i=1}^{c} \sum_{k=1}^{N} (\mu_{ik})^{m} d^{2}(x_{k},v_{i})$$
(1)

$$V = [v_1, v_2, \dots, v_c], \quad v_i \in \mathbb{R}^n$$
⁽²⁾

Constraints

$$\sum_{i=l}^{c} u_{ik} = l \quad \forall k \in \{1, \dots, N\}$$
(3)

$$0 < \sum_{i=1}^{c} u_{ik} < N \quad \forall i \in \{1, \dots, c\}$$

$$\tag{4}$$

where, c is the number of cluster, v_i is the centroid, d is the Euclidian distance between rescaled feature vector and centroid of cluster, u_{ik} [0,1] denotes the degree of membership function of feature vector, $m[1 \infty]$ is weight exponent for each fuzzy membership and it determines the fuzziness of the clusters and controls the extent of membership shared among the fuzzy clusters. U, which is given in equation (6), is the fuzzy partition matrix which contains the membership of each feature vector in each fuzzy cluster. It should be noted that, the sum of the membership values for a cluster must be equal to 1.

$$d^{2}(x_{k}, v_{i}) = (x_{k}, v_{i})^{T} A_{i}(x_{k}, v_{i})$$

$$U = \begin{bmatrix} u_{11} & u_{1k} & \dots & u_{1N} \\ \vdots & & & \\ u_{i1} & u_{ik} & \dots & u_{iN} \\ \vdots & & & \\ u_{c1} & u_{ck} & \dots & u_{cN} \end{bmatrix}_{ctN}$$
(5)

The procedure of FCM based on iterative optimization (Bezdek, 1981) can be given as;

i) Initialize fuzzy partition matrix U or Fuzzy cluster centroid matrix V using a random number generator.ii) If the FCM algorithm is initialized with fuzzy partition matrix, the initial memberships belonging to a cluster is adjusted using equation (7).

$$u_{ik} = \frac{u_{ik}^{initial}}{\sum_{i=1}^{c} u_{ik}^{initial}} \qquad for \quad l \le i \le c, \ l \le k \le N$$

$$\tag{7}$$

ii) If the FCM algorithm is initialized with fuzzy cluster centroid matrix containing the fuzzy cluster centroid, memberships belonging to cluster is determined using equation (8).

iii) v_i fuzzy centroid is computed by equation (8),

$$v_{i} = \frac{\sum_{k=l}^{N} (u_{ik})^{m} x_{k}}{\sum_{k=l}^{N} (u_{ik})^{m}}$$
(8)

iv) The fuzzy membership (u_{ik}) is updated by equation (9),

$$u_{ik} = \frac{\left(\frac{l}{d^{2}(x_{k}, v_{i})}\right)^{l/(m-1)}}{\sum_{i=l}^{c} \left(\frac{l}{d^{2}(x_{k}, v_{i})}\right)^{l/(m-1)}} \qquad for \qquad l \le i \le c, \ l \le k \le N$$
(9)

The steps (iii) and (iv) are repeated until the change in the value of memberships between two iterations is sufficiently small level.

3.2. Entropy approach

Determination of black spots and their safety levels would be useful for prevention of future traffic accidents. But it is not an easy task. Many parameters have considerable effects on the phenomenon. On the other hand, safety level determination has uncertainty and deterministic approaches are incapable in classification (Cheng and Washington, 2005). Therefore Shannon Entropy Approach is used to determine black spots' safety levels considering accident types and effective factors on accident occurrence.

Definition of entropy is based on information theory. In information theory, entropy is a measure of the uncertainty associated with a random variable. In this concept, the term usually refers to the Shannon entropy, which quantifies the expected value of the information contained in a message, usually in units such as bits. Equivalently, the Shannon entropy is a measure of the average information content one is missing when one does not know the value of the random variable. Shannon defined the entropy as expected value of alternative conditions for a variable using a mathematical expression. Using this definition and expression, entropy of a stochastic process can easily be determined if the probability of process known. Because of many attributes, the entropy concept is accepted as an objective criterion that can be used in measuring information content of any statistical process (Bayazit, 1985). Four main entropy values (marginal, common, conditional and trans information) are used in the method for information content. Shannon defined marginal entropy; H(X) as in the following equation.

$$H(x) = -K \sum_{i=1}^{N} p(x_i) \log p(x_i)$$
⁽¹⁰⁾

3.3. Fuzzy Logic Approach

The entropy values calculated for black spot centers and other locations around the center are not certain. Although some locations have similar properties the entropy values can be different. To remove this deficiency fuzzy logic approach (as an uncertainty modeling approach) is preferred (Hawas, 2007). Safety levels of black spots are evaluated by fuzzy boundaries.

4. Analysis

4.1. ClusterAnalysis

Centers of black spots are determined by Fuzzy C-means clustering analysis defined above and shown in Table 3. The coordinates of centers and number of accidents occurred around these centers are also indicated in the table. As seen in the table, eleven centers are determined by fuzzy C-means clustering approach. The number of accidents occurred around these centers are also given in the same table.

Cluster		Coordi	Number of	
No	Black Spot Center	ack Spot Center X		Accidents
1	Ucgen	29,082	37,782	247
2	Karayolları	29,07	37,774	50
3	Cinar	29,089	37,787	88
4	Kiremitci	29,086	37,772	25
5	Yeni Adliye	29,097	29,097 37,779	
6	İstasyon	29,107	37,802	120
7	Sevindik	29,08	37,795	101
8	Emniyet	29,037	37,773	86
9	Ulus	29,099	37,792	98
10	Hastane-M.Efendi	29,089	37,756	51
11	25. cadde	29,101	37,763	227
		TOT	1126	

Table 3. Coordinates of Cluster Centers given by Fuzzy C-means Clustering analysis.

4.2. Entropy Calculations of Black Spots

Accident records of the city of Denizli are located on digital map using the corresponding coordinates. The centers of black spots that have been determined by clustering approach are considered in entropy calculations (Sekerler, 2008; Murat and Sekerler, 2009). Figure 2. (a) shows the centers of black spots determined by clustering approach and Figure 2 (b) shows a detailed sample.



Fig. 2. (a) The Black Spots' Centers determined by Fuzzy Clustering Approach; (b) Sample Detail.

In this figure, C shows the center of black spots and the numbers from 1 to 7 show sample black spots around the center. In entropy calculations, geometrical and physical conditions, traffic volumes, average speeds and average accident rates at around black spots are considered. Entropy value of the center is determined considering entropy values of corresponding black spots around the center. As seen on Fig.2 (b), the entropy value of C is calculated using entropy values of 7 black spots. These 7 black spots represent the locations where accidents (more than one) have been occurred same addresses (i.e. different locations of same streets or boulevard).

In entropy calculations of black spots four parameters are taken into account namely; geometrical and physical condition of black spot, traffic volume, average speed and accident rate of black spot. Relation of accident occurrence and degrees of influences are considered in selection of these parameters. Some information about these parameters is given in the following.

Geometrical and Physical Condition of a Black Spot:

One of the main factors on traffic accidents are related to geometrical and physical conditions of accident location (ITE, 1993; FHWA, 1982). Geometrical and physical condition includes many random variables that can be changed in different location. Therefore, this parameter is taken into account in entropy calculations.

Traffic Volume:

It is known that traffic accidents increase with increasing traffic volume. This situation is seen especially at the intersection that has geometrical problems. On the other hand, traffic volume is varied as a function of the road and the time period. It can increase as a result of special events (e.g. concerts, sport games etc). Based on these features, volume can be assumed as random variable and therefore is considered in entropy calculations of black spots.

Average Speed:

Speed and careless driving are the main contributors of traffic accidents. Most of the accidents are occurred because of these factors. Drivers can increase or decrease their speeds based on roadway and traffic conditions of locations. Speed can be affected by traffic density, number of lane, lane width, sight distance and climatic conditions. Speed has an importance on accident severity. Hence average speed values around the black spot centers are given special attention. The data are obtained from the Local Police Department of Denizli city.

Accident Rate:

Accident rates and the number of accidents contain randomness. Accident rate is another parameter used in safety level determination of black spots' centers. It is determined using the addresses where accidents occurs (Geurts et al, 2005).

The entropy calculations are made for the sections around the black spots and average values are determined for the centers of black spots (Murat, 2011). These average values are also used as the expected entropy values of each center and the spots in the environment. Steps of calculations are defined in the following.

Step 1: Aggregation of the values of parameters

$$T = GP + TV + AS + AR \tag{11}$$

where;

GP: Geometrical and Physical Condition value

TV: Traffic volume value

AS: Average speed value

AR: Accident rate value

Step 2: Determination of probability of parameters

- PGP = GP / T (12)
- PTV = TV / T(13)
- PAS = AS/T(14)
- PAR = AR / T(15)

Step 3: Calculation of information contents and entropy

$$E = -\left[\log(PGP) \times PGP + \log(PTV) \times PTV + \log(PAS) \times PAS + \log(PAR) \times PAR\right]$$
(16)

The entropy values are calculated using the steps defined above. Sample calculations and used values for certain sections are given in Table 4.

Black Spot Center	Accident Location	GP	TV (veh/h)	AS (km/h)	AR	Entropy Value
Llagon Intersection	U1	30.7	1500	80	0.10	0.29
Oegen intersection	U5	31.52	1677	80	0.25	0.27
	K3	29.89	1111	78	0.50	0.35
Karayollari Intersection	K7	30.17	1100	80	0.40	0.36
Cinor Squara	Ç2	25	700	50	0.35	0.38
Ciliai Square	Ç6	32.33	748	50	0.09	0.39
Kinemitei Meh Internetion	Ki2	30.5	652	50	0.44	0.42
Kiremitci Man. Intersection	Ki4	32	500	65	0.73	0.55
Vani Adliva Intersection	Ya1	23.5	500	75	0.87	0.55
i chi Adhye intersection	Ya3	36	525	72	0.50	0.57

Table 4. Sample Entropy Calculations for Black Spot Centers and Sections Around.

Entropy values of all black spots are evaluated and safety levels are determined. Five classes are ascertained for safety levels namely; definitely unsafe, unsafe, approximately safe, safe and definitely safe. Safety levels and related definitions are shown in Table 5.

Table 5. Safety Levels (crisp) determined by Entropy Values.

Safety Class	Definition	Interval (Entropy value)
1	Definitely Unsafe	0.22 <e<0.30< td=""></e<0.30<>
2	Unsafe	0.31 <e<0.39< td=""></e<0.39<>
3	Approximately Safe	0.40 <e<0.48< td=""></e<0.48<>
4	Safe	0.49 <e<0.58< td=""></e<0.58<>
5	Definitely Safe	0.59 <e<0.67< td=""></e<0.67<>

4.3 Fuzzy Logic based Safety Level Determination

Safety levels of black spots are evaluated by fuzzy boundaries (Fig.3).



Fig. 3. Fuzzy Logic based Safety Levels of Black Spots.

The black spots are classified by both fuzzy logic and crisp approaches and the results obtained are compared. First the result is obtained from the calculation steps defined above and then evaluated regarding the fuzzy boundaries indicated in Fig 3. Based on the comparisons, similarities and differences of outcomes are evaluated. Table 5 depicts the results of sample calculations for black spots regarding both conventional and fuzzy approaches.

4.4. Validation Search

To validate the crisp and fuzzy logic based safety level determination approaches, chi-square test is used. In this test, the calculated entropy value and fuzzy entropy value of each black spot is compared to the expected entropy value. The expected entropy value is specified by calculating average entropy value of black spots around the center and it is assigned as the value for the corresponding center. Chi-square values for the sample data set given in Table 6 are calculated as 0.05 for crisp approach and 0.065 for fuzzy approach where corresponding critical value is determined as 35.17 from chi-square table. It is found that both methods are compatible with the expected entropy values.

The safety levels assigned by the methods are also evaluated considering the truth values. The truth values that show the coherence of methods are also calculated. The truth value is assigned as 1 for the same result otherwise it is assigned as 0. Based on these calculations, 5 of 24 data provided different results for crisp approach and 4 of 24 data provided different results for fuzzy approach. Therefore, the truth value rates are calculated as 0.79 for crisp approach and 0.83 for fuzzy approach. These findings confirm that Shannon entropy approach can be used for safety level determination of black spots. Besides, in addition to crisp approach, fuzzy approach can be useful in determination of safety levels.

Black Spot	Accident	GP	TV	AS	AR	Calculated	Expected	Fuzzy
Ucgen	ut1	19.22	1100	87	0,09	0.34	0.32	0.32
	ut2	8	925	85	0.06	0.33	0.32	0.32
	ut3	18	1111	80	0.05	0.32	0.32	0.33
Vanavallam	kt1	21.14	1045	75	0.21	0.34	0.34	0.32
Karayonari	kt2	27.5	1100	77	0.33	0.35	0.34	0.35
	ct1	16	800	55	0.14	0.33	0.33	0.32
Cinar	ct2	28.5	775	47	0.07	0.36	0.33	0.35
	ct5	20.15	900	55	0.54	0.32	0.33	0.33
V:	kit1	34.42	560	55	0.78	0.50	0.50	0.53
Kiremitci	kit2	32.5	550	55	0.22	0.49	0.50	0.52
Istanuan	it1	26.29	1625	75	0.28	0.26	0.31	0.22
Istasyon	it5	15.95	1011	70	0.14	0.31	0.31	0.33
Sarrindily	st1	17	700	70	0.15	0.40	0.41	0.40
Sevinuik	st2	24.07	850	77	0.66	0.40	0.41	0.40
Empiret	et l	14.5	575	70	0.11	0.44	0.39	0.45
Emmyet	et2	18	1090	72	0.17	0.31	0.39	0.33
Ulma	utl	32.25	450	60	0.1	0.57	0.53	0.56
Ulus	ut2	22.78	511	70	0.25	0.52	0.53	0.54
Uestano	htl	7.71	600	60	0.17	0.37	0.41	0.38
Hastane	ht2	13.57	650	65	0.17	0.39	0.41	0.37
25 aadda	25ct1	36	890	72	0.43	0.42	0.43	0.45
25. caude	25ct4	14.75	600	65	0.22	0.42	0.43	0.45

Table 6. Sample Entropy Calculations regarding Conventional and Fuzzy Approaches

5. Conclusions

In this study, firstly, the black spots are determined using cluster analysis. Besides, the safety levels of black spots are determined and classified using Shannon Entropy and fuzzy logic approaches. In the scope of the study, traffic accident data obtained from the local police department in Denizli-Turkey are used. When the data are analyzed, it was found that geometrical and physical conditions, traffic volume, average speed and accident rate of black spots were effective parameters on safety level determination. Using these parameters, entropy values of black

spots are calculated separately and average entropy value is remarked for the corresponding centers. 5 levels (classes) are assigned by fuzzy logic and crisp approaches based on calculated entropy value intervals. 11 black spots' centers are taken into consideration and it is found that 5 of 11 are in the 2^{nd} safety level, 3 of 11 are in the 3^{rd} safety level and 3 of 11 are in the 4^{th} safety level. To measure the validity of the Shannon Entropy approach, the truth value method and the chi-square test were used. Both of the tests proved that Shannon Entropy approach can be used for safety level determination purposes. In addition to this, as can be seen on Table 5, fuzzy logic approach supports the results which obtained using Shannon Entropy approach. As a result, it can be concluded that fuzzy logic is an effective approach for the determination of traffic safety level.

Based on the results, it can be said that some counterplots should be planned urgently for the black spots that classified as in the 1st and 2nd levels. For instance, different types of intersections can be constructed considering geometrical conditions and traffic volumes for these black spot centers. Consequently, it can be said that safety levels for different locations can be determined using Shannon entropy and/or fuzzy logic approach defined in this paper.

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