# Combining absolute and relative information in studies on food quality 

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#### Abstract

A common problem in food science concerns the assessment of the quality of food samples. Typically, a group of panellists is trained exhaustively on how to identify different quality indicators in order to provide absolute information, in the form of scores, for each given food sample. Unfortunately, this training is expensive and time-consuming. For this very reason, it is quite common to search for additional information provided by untrained panellists. However, untrained panellists usually provide relative information, in the form of rankings, for the food samples. In this paper, we discuss how both scores and rankings can be combined in order to improve the quality of the assessment.


Keywords: Consensus evaluation; Absolute information; Relative information.

## 1 Introduction

We consider the problem in which several panellists are asked to score a food sample on a given ordinal scale, the goal being to reach a consensus evaluation of the sample. This problem commonly appears in food science, for instance, when identifying the degree of spoilage $[1,2]$ or when evaluating the appearence $[3$, 4] of a given sample. Unfortunately, training and (subsequently) collecting information from panellists usually carries big expenses. For this reason, there usually is a limited amount of data available to reach a consensus evaluation. It is thus quite common to invoke untrained panellists and to gather some additional information [5]. However, untrained panellists are obviously not as skilled as trained panellists, and might be unable to accurately evaluate a given sample. Since it is a conceptually easier task, untrained panellists are then just asked to rank different samples according to their personal appreciation. In this paper, we propose to combine both types of information in order to improve the quality of the assessment. Moreover, we illustrate our proposal by discussing an experiment concerning the freshness of raw Atlantic salmon (Salmo salar) [6].

The remainder of the paper is organized as follows. In Section 2, we recall the well-known notions of median and Kemeny median. Section 3 is devoted to the introduction of a method for reaching a consensus evaluation of the given samples while combining both scoring and ranking information. We end with some conclusions in Section 4.

## 2 Preliminaries

### 2.1 Obtaining a consensus vector of scores

We consider the setting where $n_{T}$ trained panellists are asked to assign a score on a $k$-point scale to each (food) sample in a set $X=\left\{x_{1}, \ldots, x_{n}\right\}$ of $n$ (food) samples. The goal is to agree on the consensus score that should be assigned to each of the samples based on the scores provided by the trained panellists. For any $i \in\left\{1, \ldots, n_{T}\right\}$, we denote by $\mathbf{s}_{i}$ the vector of scores assigned by the $i$-th panellist. The scale we use throughout this paper is shown in Figure 1.


Fig. 1. Example of a 5-point scale, where the extreme scores of " 1 " and " 5 " represent spoiled and fresh, respectively, and the intermediate score of " 3 " represents a neutral response of neither spoiled nor fresh.

A common method for determining the consensus vector of scores is based on the minimization of a distance, i.e., the consensus vector of scores $\mathbf{s}^{*}$ should satisfy

$$
\mathbf{s}^{*}=\underset{\mathbf{s} \in\{1, \ldots, k\}^{n}}{\arg \min } \sum_{i=1}^{n_{T}} d\left(\mathbf{s}, \mathbf{s}_{i}\right),
$$

where $d$ is a fixed distance function on the set of vectors of scores. Note that there can be multiple minimizers $\mathbf{s}^{*}$.

One could note that several examples of this procedure are commonly used in practice. For instance, when we consider the sum of zero-one distances ${ }^{1}$ over

[^0]all components, the preceding method amounts to identifying the mode(s). Similarly, when we consider the sum of $\ell_{1}$-distances ${ }^{2}$, the preceding method amounts to identifying the median(s), and, when we consider the sum of $\ell_{2}$-distances ${ }^{3}$, it amounts to identifying the mean(s). One could note that the latter method presumes the existence of a certain notion of distance between labels, something that is not advisable in case the considered scale is defined by abstract words [7].

Example 1. Consider the set of $n=4$ samples $X=\left\{x_{1}, x_{2}, x_{3}, x_{4}\right\}$ and the vectors of scores on the fixed 5 -point scale provided by $n_{T}=10$ trained panellists shown in Table 1. Note that these data come from a real-life dataset concerning an experiment on raw Atlantic salmon (Salmo salar) [6].

|  | $\mathbf{s}_{1}$ | $\mathbf{s}_{2}$ | $\mathbf{s}_{3}$ | $\mathbf{s}_{4}$ | $\mathbf{s}_{5}$ | $\mathbf{s}_{6}$ | $\mathbf{s}_{7}$ | $\mathbf{s}_{8}$ | $\mathbf{s}_{9}$ | $\mathbf{s}_{10}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $x_{1}$ | 5 | 4 | 5 | 2 | 3 | 3 | 5 | 5 | 2 | 4 |
| $x_{2}$ | 5 | 2 | 1 | 2 | 4 | 2 | 5 | 4 | 3 | 3 |
| $x_{3}$ | 2 | 1 | 5 | 2 | 2 | 4 | 3 | 2 | 2 | 2 |
| $x_{4}$ | 3 | 1 | 2 | 1 | 2 | 2 | 2 | 3 | 3 | 1 |

Table 1. The scores assigned to samples $x_{1}, x_{2}, x_{3}$ and $x_{4}$ by the trained panellists.
For each of the 625 possible vectors of scores, we compute the sum of $\ell_{1}$ distances to the vectors of scores provided by the trained panellists. We conclude that the vector of scores that minimizes this value is $\mathbf{s}^{*}=(4,3,2,2)$. As expected, this vector coincides with the one obtained by identifying the median for each of the samples.

### 2.2 Obtaining a consensus ranking

We consider the setting where $n_{U}$ untrained panellists are asked to rank all the samples on the considered set $X=\left\{x_{1}, \ldots, x_{n}\right\}$ of $n$ samples. Untrained panellists are asked to provide a complete ranking of the samples, however, they are allowed to express ties in case they consider two or more samples to be equally suitable. The goal is to agree on the consensus ranking that should be assigned to each of the samples based on the scores provided by the trained panellists. For any $i \in\left\{1, \ldots, n_{U}\right\}$, we denote by $\precsim i$ the ranking assigned by the $i$-th panellist, which can be split into the usual symmetric $\sim_{i}$ and antisymmetric $\prec_{i}$ parts. We denote by $\mathcal{W}$ the set of all rankings (with ties) on $X$.

[^1]A common method for determining the consensus ranking is due to Ke meny [8] in which a consensus ranking $\precsim^{*}$ is one that satisfies

$$
\precsim^{*}=\underset{\precsim \in \mathcal{W}}{\arg \min } \sum_{i=1}^{n_{U}} K(\precsim, \precsim i),
$$

where $K\left(\precsim^{1}, \precsim^{2}\right)$ denotes the Kemeny distance between two rankings $\precsim^{1}$ and $\precsim^{2}$. We recall that the Kemeny distance ${ }^{4}$ between two rankings is computed as follows. For each pair of samples $\left\{x_{u}, x_{v}\right\}$, if both rankings agree on the order of the samples, we write down 0; if, in one ranking, $x_{u}$ is ranked above $x_{v}$ (or $x_{v}$ is ranked above $x_{u}$ ) and, in the other ranking, $x_{u}$ and $x_{v}$ are tied, we write down 1 ; and, if, in one ranking, $x_{u}$ is ranked above $x_{v}$ and, in the other ranking, $x_{v}$ is ranked above $x_{u}$, we write down 2. After writing down the numbers for all $n(n-1) / 2$ possible pairs, the Kemeny distance between the two rankings equals the sum of these numbers.

Example 2. Consider the same set of $n=4$ samples $X=\left\{x_{1}, x_{2}, x_{3}, x_{4}\right\}$ of Example 1, and the rankings provided by $n_{U}=28$ untrained panellists shown in Table 2. Note that these data also come from the experiment on raw Atlantic salmon (Salmo salar) in [6].

| $i$ | $i \mid \precsim_{i}$ | $i$ |  |  | $\precsim_{i}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | $1 \mid x_{2} \sim x_{4} \prec x_{3} \sim x_{2}$ | 15 |  | $\prec x_{1}$ | $\prec x_{4} \prec x_{3}$ |
| 2 | $2 x_{4} \prec x_{3} \prec x_{2} \prec x_{1}$ | 16 |  | $x_{4} \prec x_{3}$ | $x_{2} \prec x_{1}$ |
| 3 | $3 x_{4} \prec x_{2} \prec x_{3} \sim x_{1}$ | 17 |  | $\prec x_{3}$ | $x_{2} \prec x_{1}$ |
| 4 | $x_{1} \prec x_{4} \prec x_{3} \prec x_{2}$ | 18 |  | $x_{4} \prec x_{3}$ | $\sim x_{2} \prec x_{1}$ |
| 5 | $5 x_{1} \sim x_{4} \prec x_{3} \prec x_{2}$ | 19 |  | $x_{2} \prec x_{4}$ | $\prec x_{3} \prec x_{1}$ |
| 6 | $6 x_{4} \prec x_{3} \sim x_{1} \sim x_{2}$ | 20 |  | $x_{4} \prec x_{3}$ | $\prec x_{2} \prec x_{1}$ |
| 7 | $7 x_{4} \sim x_{3} \prec x_{1} \sim x_{2}$ | 21 |  | $x_{1} \sim x_{4}$ | $\prec x_{3} \sim x_{2}$ |
| 8 | $8 x_{4} \prec x_{1} \sim x_{2} \prec x_{2}$ | 22 |  | $x_{1} \sim x_{2}$ | $\prec x_{3} \prec x_{4}$ |
| 9 | $x_{1} \prec x_{3} \sim x_{2} \prec x_{4}$ | 23 |  | $x_{4} \prec x_{3}$ | $\prec x_{2} \prec x_{1}$ |
| 10 | $0 x_{2} \prec x_{4} \prec x_{1} \prec x_{3}$ | 24 |  | $x_{4} \prec x_{1}$ | $\prec x_{2} \prec x_{3}$ |
| 11 | $1 x_{4} \prec x_{2} \prec x_{3} \prec x_{1}$ | 25 |  | $x_{4} \prec x_{2}$ | $\prec x_{3} \prec x_{1}$ |
| 12 | $2 x_{4} \prec x_{1} \prec x_{3} \sim x_{2}$ | 26 |  | $x_{1} \prec x_{4}$ | $\prec x_{3} \prec x_{2}$ |
| 13 | $3 x_{2} \sim x_{4} \sim x_{3} \prec x_{1}$ | 27 |  | $x_{4} \prec x_{2}$ | $\prec x_{3} \prec x_{1}$ |
|  | $4 x_{2} \prec x_{4} \prec x_{1} \prec x_{3}$ |  |  | $x_{4} \prec x_{2}$ | $\prec x_{3} \prec x_{1}$ |

Table 2. The rankings expressed by the untrained panellists.
For each of the 75 possible rankings, we compute the sum of Kemeny distances to the rankings provided by the untrained panellists. We conclude that the ranking that minimizes this value is $\precsim^{*}=x_{4} \prec x_{3} \sim x_{2} \prec x_{1}$.

[^2]
## 3 Combining scores and rankings

We now consider the setting where $n_{T}$ trained panellists each have assigned a score to each of the $n$ samples in $X$ and $n_{U}$ untrained panellists each have ranked the $n$ samples in $X$. The goal is to combine both types of information in order to improve the quality of the assessment of the consensus vector of scores and/or ranking. We propose to consider a combination of the median and the Kemeny median.

### 3.1 Improving the quality of the assessment of a consensus vector of scores

To compute the 'distance'5 between each possible vector of scores $\mathbf{s}$ and the rankings provided by the untrained panellists, we define the set $\theta_{\mathbf{s}}$ of all possible rankings that do not contradict $\mathbf{s}$, as follows:

$$
\begin{equation*}
\theta_{\mathbf{s}}=\left\{\precsim \in \mathcal{W} \mid(\forall i, j \in\{1, \ldots, n\})\left(\mathbf{s}(i)<\mathbf{s}(j) \Rightarrow x_{i} \prec x_{j}\right)\right\} . \tag{1}
\end{equation*}
$$

Note that the set $\theta_{\mathbf{s}}$ is always non-empty.
Incorporating the rankings provided by the untrained panellists into the vectors of scores provided by the trained panellists requires defining a cost function. Thus, we define a convex combination of the 'distances' associated with the vectors of scores provided by the trained panellists and the rankings provided by the untrained panellists, as follows:

$$
\begin{equation*}
C_{\alpha}(\mathbf{s})=\frac{\alpha}{B_{T}} \sum_{i=1}^{n_{T}} d_{1}\left(\mathbf{s}, \mathbf{s}_{i}\right)+\frac{(1-\alpha)}{B_{U}} \min _{\precsim \in \theta_{\mathbf{s}}} \sum_{i=1}^{n_{U}} K\left(\precsim, \precsim_{i}\right) . \tag{2}
\end{equation*}
$$

where $B_{T}=n_{T} \cdot n \cdot(k-1)$ and $B_{U}=n_{U} \cdot n \cdot(n-1)$ are normalizing constants, and $\alpha \in[0,1]$ is a parameter that controls the influence of the scoring and ranking information. In particular, larger values of $\alpha$ give more importance to the trained panellists, whereas smaller values of $\alpha$ give more importance to the untrained panellists.

Finally, we consider the consensus vector(s) of scores to be the minimizer(s) of Eqs. (2) for a fixed $\alpha$, as follows:

$$
\begin{equation*}
\mathbf{s}_{\alpha}^{*}=\underset{\mathbf{s} \in\{1, \ldots, k\}^{n}}{\arg \min } C_{\alpha}(\mathbf{s}) \tag{3}
\end{equation*}
$$

Note that there can be multiple minimizers $\mathbf{s}_{\alpha}^{*}$ for the same $\alpha$.

[^3]Example 3. We continue with the data from Examples 1 and 2. To determine the consensus score that should be assigned to each of these samples, we consider the problem defined by Eq. (3) by computing $C_{\alpha}(\mathbf{s})$ for each of the 625 vectors of scores. For simplicity, we show one computation for the vector of scores $\mathbf{s}=$ $(4,3,2,2)$, which was determined as the consensus vector of scores in Example 1, with $\sum_{i=1}^{10} d_{1}\left(\mathbf{s}, \mathbf{s}_{i}\right)=34$. The distances associated with the vectors of scores are bounded by the upper bound $B_{T}=10 \cdot 4 \cdot 4=160$, whereas the distances associated with the rankings are bounded by the upper bound $B_{U}=28 \cdot 4 \cdot 3=$ 336. Now, we consider the set $\theta_{\mathbf{s}}$ of all possible rankings that do not contradict s. Since the score of $x_{1}$ is the largest, $x_{1}$ is ranked above the other samples. Similarly, $x_{2}$ is ranked above $x_{3}$ and $x_{4}$. Since the scores of $x_{3}$ and $x_{4}$ are equal, any of the following cases applies: $x_{3}$ is ranked above $x_{4}, x_{3}$ and $x_{4}$ are tied, and $x_{4}$ is ranked above $x_{3}$, as follows:

$$
\theta_{(4,3,2,2)}=\left\{\begin{array}{l}
x_{4} \prec x_{3} \prec x_{2} \prec x_{1}, \\
x_{3} \sim x_{4} \prec x_{2} \prec x_{1}, \\
x_{3} \prec x_{4} \prec x_{2} \prec x_{1}
\end{array}\right\} .
$$

We compute the sum of the Kemeny distances between each $\precsim \in \theta_{\text {s }}$ and the rankings provided by the untrained panellists. The results are summarized in Table 3.

| $\precsim$ | $\sum_{i=1}^{28} K\left(\precsim, \precsim_{i}\right)$ |
| :---: | :---: |
| $x_{4} \prec x_{3} \prec x_{2} \prec x_{1}$ | 109 |
| $x_{3} \sim x_{4} \prec x_{2} \prec x_{1}$ | 129 |
| $x_{3} \prec x_{4} \prec x_{2} \prec x_{1}$ | 153 |

Table 3. Sum of Kemeny distances between each ranking $\precsim$ that does not contradict $\mathbf{s}=(4,3,2,2)$ and the rankings provided by the untrained panellists.

Finally, we select the ranking that minimizes the sum of Kemeny distances among those in $\theta_{\mathbf{s}}$ and compute $C_{\alpha}(\mathbf{s})$ as follows:

$$
\begin{aligned}
C_{\alpha}(\mathbf{s}) & =\frac{\alpha}{160} \sum_{i=1}^{10} d_{1}\left(\mathbf{s}, \mathbf{s}_{i}\right)+\frac{(1-\alpha)}{336} \min \left(\begin{array}{l}
\sum_{i=1}^{28} K\left(x_{4} \prec x_{3} \prec x_{2} \prec x_{1}, \precsim_{i}\right), \\
\sum_{i=1}^{28} K\left(x_{3} \sim x_{4} \prec x_{2} \prec x_{1}, \precsim_{i}\right), \\
\sum_{i=1}^{28} K\left(x_{3} \prec x_{4} \prec x_{2} \prec x_{1}, \precsim_{i}\right)
\end{array}\right) \\
& =\frac{109}{336}-\frac{6016}{53760} \alpha .
\end{aligned}
$$

After computing $C_{\alpha}(\mathbf{s})$ for each of the 625 possible vectors of scores $\mathbf{s}$, we illustrate in Figure 2 all the $\mathbf{s}_{\alpha}^{*}$ that minimize $C_{\alpha}(\mathbf{s})$ for at least one value of $\alpha \in[0,1]$. One should note that, for $\alpha=0$, there will always be multiple minimizers $\mathbf{s}_{0}^{*}$ associated with all vectors of scores that are not contradicted by the Kemeny median. Since we know from Example 2 that $\precsim^{*}=x_{4} \prec x_{3} \sim x_{2} \prec x_{1}$ is the Kemeny median, we illustrate (in black) all the vectors of scores $\mathbf{s}_{0}^{*}$ that are
not contradicted by this $\precsim^{*}$. These vectors of scores form a fan-shaped pattern starting at $\alpha=0$ since, at the left end, they all result in the same value $C_{0}(\mathbf{s})$, and, at the right end, they result in (mostly) different values $C_{1}(\mathbf{s})$.


Fig. 2. Illustration of the vectors of scores $\mathbf{s}_{\alpha}^{*}$ that minimize $C_{\alpha}(\mathbf{s})$ for $\alpha \in[0,1]$.

Since we do not intend to rely only on the rankings provided by the untrained panellists, we ignore the minimizers for $\alpha=0$. The obtained minimizers $\mathbf{s}_{\alpha}^{*}$ are summarized as follows:

$$
\mathbf{s}_{\alpha}^{*}=\left\{\begin{array}{clr}
\{(4,2,2,2)\} & , \text { if } 0<\alpha<\frac{50}{71}  \tag{4}\\
\{(4,2,2,2),(4,3,2,2)\} & , \text { if } & \alpha=\frac{50}{71} \\
\{(4,3,2,2)\} & , \text { if } & \alpha>\frac{50}{71}
\end{array}\right.
$$

We deduce based on the vectors of scores provided by the trained panellists that $x_{2}$ is ranked at a better position than $x_{3}$ and $x_{4}$, since the former is assigned a higher score than the former in the consensus vector of scores for large values of $\alpha$. However, incorporating the rankings provided by the untrained panellists hints that these samples are similar.

### 3.2 Improving the quality of the assessment of a consensus ranking

To compute the 'distance' between each possible ranking $\precsim$ and the vectors of scores provided by the trained panellists, we define the set $\varphi_{\precsim}$ of all possible vectors of scores that do not contradict $\precsim$, as follows:

$$
\begin{equation*}
\varphi_{\precsim}=\left\{\mathbf{s} \in\{1, \ldots, k\}^{n} \mid(\forall i, j \in\{1, \ldots, n\})\left(x_{i} \precsim x_{j} \Rightarrow \mathbf{s}(i) \leq \mathbf{s}(j)\right)\right\} . \tag{5}
\end{equation*}
$$

Note that the set $\varphi_{\prec}$ is always non-empty.
The convex combination of the 'distances' associated with the vectors of scores provided by the trained panellists and the rankings provided by the untrained panellists is now defined as follows:

$$
\begin{equation*}
D_{\alpha}(\precsim)=\frac{\alpha}{B_{T}} \min _{\mathbf{s} \in \varphi_{\precsim}} \sum_{i=1}^{n_{T}} d_{1}\left(\mathbf{s}, \mathbf{s}_{i}\right)+\frac{(1-\alpha)}{B_{U}} \sum_{i=1}^{n_{U}} K(\precsim, \precsim i) . \tag{6}
\end{equation*}
$$

Finally, we consider the consensus ranking(s) to be the minimizer(s) of Eqs. (6) for a fixed $\alpha$, as follows:

$$
\begin{equation*}
\precsim_{\alpha}^{*}=\underset{\precsim \in \mathcal{W}}{\arg \min } D_{\alpha}(\precsim) . \tag{7}
\end{equation*}
$$

Note that there can be multiple minimizers $\precsim_{\alpha}^{*}$ for the same $\alpha$.
Since $\alpha \in[0,1]$ can take infinite values, it will be impossible to compute $\mathbf{s}_{\alpha}^{*}$ (resp. $\precsim_{\alpha}^{*}$ ) for each $\alpha$. Therefore, bearing in mind that, for any fixed vector of scores $\mathbf{s}$ (resp. ranking $\precsim$ ), the corresponding $f(\alpha):=C_{\alpha}(\mathbf{s})\left(\right.$ resp. $\left.g(\alpha):=D_{\alpha}(\precsim)\right)$ can be visualized as a line, we can compare the lines of each possible pair of vectors of scores (resp. rankings). When comparing two lines, we distinguish three cases: there are no points of intersection, there is exactly one point of intersection, or both lines coincide. These facts can then be used to analytically compute $\mathbf{s}_{\alpha}^{*}$ and $\precsim_{\alpha}^{*}$ as a function of $\alpha$.

Example 4. We continue with the data from Example 3. To determine the consensus ranking of the samples, we consider the problem defined by Eq. (7) by computing $D_{\alpha}(\precsim)$ for each of the 75 rankings. For simplicity, we show one computation for the ranking $\precsim=x_{4} \prec x_{3} \sim x_{2} \prec x_{1}$, which was determined as the consensus ranking in Example 2, with $\sum_{i=1}^{28} K\left(\precsim, \precsim_{i}\right)=99$. Now, we consider the set $\varphi_{\precsim}$ of all possible vectors of scores that do not contradict $\precsim$.

$$
\varphi_{x_{4} \prec x_{3} \sim x_{2} \prec x_{1}}=\left\{\mathbf{s} \in\{1, \ldots, 5\}^{4} \mid \mathbf{s}(4) \leq \mathbf{s}(3)=\mathbf{s}(2) \leq \mathbf{s}(1)\right\} .
$$

We compute the sum of the $\ell_{1}$-distances between each $\mathbf{s} \in \varphi_{\precsim}$ and the vectors of scores provided by the untrained panellists. We note that the vector of scores among those in $\varphi_{\precsim}$ that minimizes the sum of the $\ell_{1}$-distances is $(4,2,2,2)$ with $\sum_{i=1}^{10} d_{1}\left(\mathbf{s}, \mathbf{s}_{i}\right)=\tilde{36}$. Finally, we obtain:

$$
D_{\alpha}(\precsim)=\frac{113}{336}-\frac{5984}{53760} \alpha .
$$

After computing $D_{\alpha}(\precsim)$ for each of the 75 possible rankings $\precsim$, we illustrate in Figure 3 all the $\precsim_{\alpha}^{*}$ that minimize $D_{\alpha}(\precsim)$ for at least one value of $\alpha \in[0,1]$. One should note that, for $\alpha=1$, there will always be multiple minimizers $\mathbf{s}_{1}^{*}$ associated with all rankings that are not contradicted by the median. Since we know from Example 1 that $\mathbf{s}^{*}=(4,3,2,2)$ is the median, we illustrate all
the rankings $\precsim_{1}^{*}$ that are not contradicted by this $\mathbf{s}^{*}$. These rankings form a fan-shaped pattern starting at $\alpha=1$ since, at the right end, they all result in the same value $D_{1}(\precsim)$, and, at the left end, they result in (mostly) different values $D_{0}(\precsim)$.


Fig. 3. Illustration of the rankings $\precsim_{\alpha}^{*}$ that minimize $D_{\alpha}(\precsim)$ for $\alpha \in[0,1]$.

Since we do not intend to rely only on the scores provided by the trained panellists, we ignore the minimizers for $\alpha=1$. The obtained minimizers $\precsim_{\alpha}^{*}$ are summarized as follows:

$$
\mathbf{s}_{\alpha}^{*}=\left\{\begin{array}{llr}
\left\{x_{4} \prec x_{3} \sim x_{2} \prec x_{1}\right\}, & \text { if } & \alpha<\frac{50}{71},  \tag{8}\\
\left\{\begin{array}{l}
x_{4} \prec x_{3} \sim x_{2} \prec x_{1} \\
\left.x_{4} \prec x_{3} \prec x_{2} \prec x_{1}\right\},
\end{array},\right. \text { if } & \alpha=\frac{50}{71}, \\
\left\{x_{4} \prec x_{3} \prec x_{2} \prec x_{1}\right\}, & \text { if } & \frac{50}{71}<\alpha<1 .
\end{array}\right.
$$

We deduce based on the rankings provided by the untrained panellists that $x_{2}$ and $x_{3}$ are similar. However, incorporating the vectors of scores provided by the trained panellists hints that sample $x_{2}$ might be ranked above sample $x_{3}$.

### 3.3 Discussion

A deeper analysis of the results of the preceding subsections shows that both trained and untrained panellists agree that samples $x_{1}$ and $x_{4}$ are, respectively, the best and worst samples in $X=\left\{x_{1}, x_{2}, x_{3}, x_{4}\right\}$. However, there is a disagreement with regard to samples $x_{2}$ and $x_{3}$. While trained panellists considered sample $x_{2}$ to be better than sample $x_{3}$, untrained panellists did not see significant
differences between both samples. Thus, as can be concluded from both Eqs. (4) and (8), samples $x_{2}$ and $x_{3}$ result to be similar ( $\mathbf{s}^{*}(3)=\mathbf{s}^{*}(2)$ and $x_{3} \sim^{*} x_{2}$ ) for smaller values of $\alpha$ (i.e., in case more importance is given to the untrained panellists), whereas sample $x_{2}$ results to be better than sample $x_{3}$ ( $\mathbf{s}^{*}(3)<\mathbf{s}^{*}(2)$ and $x_{3} \prec^{*} x_{2}$ ) for larger values of $\alpha$ (i.e., in case more importance is given to the trained panellists).

## 4 Conclusions

In this paper, we have discussed how to combine absolute and relative information in order to improve the quality of the assessment of food samples. In particular, we have proposed a method based on a convex combination of the distances associated with the median and the Kemeny median. We have illustrated the use of this method using real-life examples, where the freshness of Atlantic salmon was studied, and showed the influence of combining scores and rankings on obtaining the consensus vector of scores and consensus ranking.

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[^0]:    ${ }^{1}$ The zero-one distance function is defined as $d_{0}\left(s, s^{\prime}\right)=0$ if $s=s^{\prime}$ and $d_{0}\left(s, s^{\prime}\right)=1$ otherwise.

[^1]:    ${ }^{2}$ The $\ell_{1}$-distance function is defined as $d_{1}\left(s, s^{\prime}\right)=\left|s-s^{\prime}\right|$.
    ${ }^{3}$ The $\ell_{2}$-distance function is defined as $d_{2}\left(s, s^{\prime}\right)=\left(s-s^{\prime}\right)^{2}$.

[^2]:    ${ }^{4}$ When the rankings contain no ties, the Kemeny distance is equal to the double of the Kendall distance [9].

[^3]:    ${ }^{5}$ Note that we write the word 'distance' between quotation marks since we are comparing objects of a different nature, and, thus, we are lacking the semantics associated with the mathematical formalization of a distance (metric).

