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SPATIAL GENERALIZATION OF THE EXPLOSION PROCESS MATHEMATICAL MODEL USING QUASICONFORMAL MAPPINGS METHODS

The mathematical model of the explosive process impact on the environment developed on the basis of the classical fluid theory which defines the borders of crater, the compressed and unperturbed sections of the soil created as an explosion result is generalized for the case of a three-dimensional medium. The algorithm of the corresponding boundary value problem numerical solving by quasiconformal mappings numerical methods and a stepwise parameterization of the characteristics of the environment and process and its program realization are presented. Numerical experiments were performed on the basis of the developed algorithm and the obtained results were analyzed.

Key words: explosion processes, quasiconformal mappings, three-dimensional medium, mathematical modeling, inverse problems.

К. М. МАЛАШ, А. Я. БОМБА

ПРОСТОРОВЕ УЗАГАЛЬНЕННЯ МАТЕМАТИЧНОЇ МОДЕЛІ ПРОЦЕСУ ВИБУХУ МЕТОДАМИ КВАЗІКОНФОРМНИХ ВІДОБРАЖЕНЬ

Математична модель впливу вибухового процесу на середовище, розроблена на основі класичної рідинної теорії, що визначає межі утворюваних у досліджуваному середовищі внаслідок дії вибуху вирвів, впресованої та незбуреної ділянок грунту на основі апріорно відомих вибухової сили та розміру і форми заряду і початкового розподілу коефіцієнта проникності досліджуваного середовища та його критичних значень, узагальнена на випадок трьохвимірного простору. На базі розробленої математичної моделі створено алгоритм числового розв’язування відповідної крайової задачі з використанням числових методів квазіконформних відображень та поетапної параметризації характеристик досліджуваного середовища і вибухового процесу та описана його програмна реалізація. На основі розробленого алгоритму проведено ряд числових експериментів та проаналізовано отримані результати.

Ключові слова вибухові процеси, квазіконформні відображення, трьохвимірне середовище, математичне моделювання, обернені задачі.

Е. Н. МАЛАШ, А. Я. БОМБА

ПРОСТРАНСТВЕННОЕ ОБОБЩЕНИЕ МАТЕМАТИЧЕСКОЙ МОДЕЛИ ВЗРЫВА МЕТОДАМИ КВАЗИКОНФОРМНЫХ ОТОБРАЖЕНИЙ

Математическая модель влияния взрывного процесса на среду, разработанная на основе классической жидкостной теории, определяющая границы образованных в результате взрыва воронок, впрессованного и невозмущенного участков почвы, обобщена на случай трехмерной среды. Приведены алгоритм численного решения соответствующей краевой задачи с использованием численных методов квазиконформных отображений и поэтапной параметризации характеристик среды и процесса и его программная реализация. На основе разработанного алгоритма проведены численные эксперименты и проанализированы полученные результаты.

Ключевые слова: взрывные процессы, квазиконформные отображения, трехмерная среда, математическое моделирование, обратные задачи.

Introduction. Society is increasingly using the earth's crust upper layers at the present stage of its development. Underground transport systems are being created, semi-underground or underground structures are being built, minerals are extracted from hard-to-reach places.

Common thing to all the types of work mentioned above is the need for creating the large cavities, which is fairly expensive and costly in time. Therefore, the issues of developing new methods of creating large cavities in the environment, particularly the soil, and improving the existing ones are very relevant today. One of the most perspective ways of developing such methods is by using the explosive processes which leads to a significant acceleration and cheapening of the execution work. In addition to the areas described above explosive processes are widely used in providing the materials with necessary engineering properties and testing their strength under the adverse external factors influence [1].

There are several mathematical models used to study explosive processes nowadays, among which should be mentioned elastic, solid-liquid, liquid, and others [1]. A series of works is devoted to the development of the explosion process mathematical model based on a liquid theory, particularly, the model problem of studying the explosive process impact on an anisotropic medium which aims to determine the boundaries of the formed crater, the pressed and unperturbed zones of the soil is solved in [2]; the external boundary of the studied area is further determined in [3]; the optimal explosive power and the bookmark location, which is necessary to obtain the maximum possible crater size, are determined in [4].

This article deals with the generalization of the explosive process mathematical model, which is considered in the papers described above, for the three-dimensional medium.

Problem statement. The investigated part of the medium (soil) in which the charge of a given form with a constant quasipotential on it is placed is modeled as a two-bounded spatial domain, bounded by the two closed smooth surfaces: $S^* = \left\{ z : f^*(x, y, z) = 0 \right\}$ (the investigated domain external boundary) and $S_* = \left\{ z : f_*(x, y, z) = 0 \right\}$ (the charge boundary) (Fig. 1). The particles' motion process caused by an explosion is described using the motion equation $\vec{\upsilon} = k\, grad \varphi$ and the continuity equation $div \vec{\upsilon} = 0$, where $\vec{\upsilon} = \left(\upsilon_x(x, y, z),\ \upsilon_y(x, y, z),\ \upsilon_z(x, y, z) \right)$ is the particle velocity, $\varphi = \varphi\left(x, y, z\right) = -P / \rho$ is the quasipotential of formed field, ρ is the medium (soil) density, $P = \int\limits_0^{t_0} p dt$ is the pressure pulse, $k = k(|\, grad \varphi)\,|) = k(I)$ is the medium permeability coefficient which characterizes the ability of its particles to break off [5]. The potential on the studied domain contours is considered to be known a priori $\varphi\big|_{S_*} = \varphi_*$, $\varphi\big|_{S^*} = \varphi^*$, $-\infty < \varphi_* < \varphi^* < +\infty$. The explosion process is modeled taking into account the mutual influence of the potential gradient $I = \sqrt{\varphi_x^2 + \varphi_y^2 + \varphi_z^2}$ and the value of the medium permeability coefficient $k = k(I)$. Clarification of the last one is carried out according to the following formula: $k = k_0 + \frac{1}{2}\beta\left(I - I^*\right)\left(\left(I - I^0\right) + \left|I - I^0\right|\right)$, where the parameter β is determined based on the experimental data,

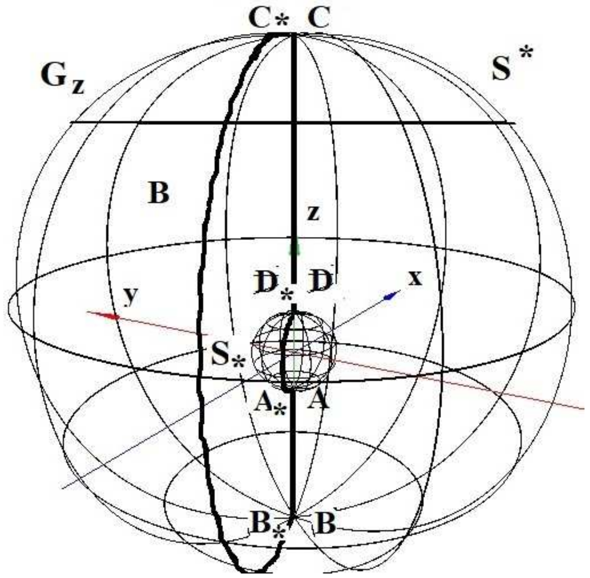

Fig. 1 – Schematic representation of the studied area.

I^0, I^* – are the critical gradient values, which characterize the delay and separation of particles (we determine the boundaries of the crater, the pressed and unperturbed sections of the soil in which the explosion occurred on their basis).

Fig. 2 shows the section of the investigated area, where area 1 is the crater, 2 is the pressed zone and the 3 is the unperturbed one. A line in which the value of the potential gradient $I = I^*$ separates the crater and the pressed soil zones, $I = I^0$ separates the pressed and unperturbed ones.

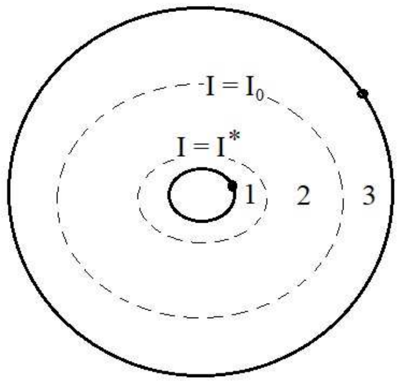

Fig. 2 – Schematic representation of investigated domain cross-section.

Mathematical model. We introduce a couple of functions $\psi = \psi(x, y, z)$ and $\eta = \eta(x, y, z)$ spatially complex conjugate with function $\varphi = \varphi(x, y, z)$ and such that the condition $k\, grad\, \varphi = grad\, \psi\, grad\, \eta$ is fulfilled. We make a conditional incision Γ with the shores $ABCD$ and $A_*B_*C_*D_*$ along one of the flow surfaces in the studied region G_z thereby reducing it to a single-connected domain [6]. We obtain the problem on a quasi-conformal mapping $\omega = \omega(z) = \varphi(x, y, z) + i\psi(x, y, z) + j\eta(x, y, z)$ domain $G_z^0 = G_z / \Gamma$ to the corresponding complex quasipotential region $G_\omega = \{\omega = \varphi + i\psi + j\eta : \varphi_* < \varphi < \varphi^*,\ 0 < \psi < Q_*,\ 0 < \eta < Q^*\}$ with unknown parameters Q_* and Q^*:

$$\begin{cases} k\dfrac{\partial \varphi}{\partial x} = \dfrac{\partial \psi}{\partial y}\dfrac{\partial \eta}{\partial z} - \dfrac{\partial \psi}{\partial z}\dfrac{\partial \eta}{\partial y}; \\ k\dfrac{\partial \varphi}{\partial y} = \dfrac{\partial \psi}{\partial z}\dfrac{\partial \eta}{\partial x} - \dfrac{\partial \psi}{\partial x}\dfrac{\partial \eta}{\partial z}; \\ k\dfrac{\partial \varphi}{\partial z} = \dfrac{\partial \psi}{\partial x}\dfrac{\partial \eta}{\partial y} - \dfrac{\partial \psi}{\partial y}\dfrac{\partial \eta}{\partial x}, \end{cases} \quad (x, y, z) \in G_z^0; \tag{1}$$

with the boundary conditions

$$\varphi|_{S_*} = \varphi_*,\ \ \varphi|_{S^*} = \varphi^*,\ \ \psi|_{ABCD} = 0,\ \ \psi|_{A_*B_*C_*D_*} = Q_*,\ \ \eta|_{ABCD} = 0,\ \ \eta_{A_*B_*C_*D_*} = Q^*,\ Q_*Q^* = Q. \tag{2}$$

We turn to the inverse boundary value problem on the quasiconformal mapping $z = z(\omega) = x(\varphi, \psi, \eta) + iy(\varphi, \psi, \eta) + jz(\varphi, \psi, \eta)$ of the quasicomplex potential domain G_ω to the physical region G_z^0 to find the real $x = x(\varphi, \psi, \eta)$ and imaginary $y = y(\varphi, \psi, \eta)$, $z = z(\varphi, \psi, \eta)$ parts of the equation of the flow line characteristic function [7]. In this case, the conditions of *the Cauchy-Riemann* type acquire the form

$$\begin{cases} \dfrac{\partial x}{\partial \varphi} = k(\dfrac{\partial y}{\partial \psi}\dfrac{\partial z}{\partial \eta} - \dfrac{\partial z}{\partial \psi}\dfrac{\partial y}{\partial \eta}); \\ \dfrac{\partial y}{\partial \varphi} = k(\dfrac{\partial z}{\partial \psi}\dfrac{\partial x}{\partial \eta} - \dfrac{\partial x}{\partial \psi}\dfrac{\partial z}{\partial \eta}); \\ \dfrac{\partial z}{\partial \varphi} = k(\dfrac{\partial x}{\partial \psi}\dfrac{\partial y}{\partial \eta} - \dfrac{\partial y}{\partial \psi}\dfrac{\partial x}{\partial \eta}); \end{cases} \quad (\varphi, \psi, \eta) \in G_\omega. \tag{3}$$

The equation of the boundary surfaces of the domain:

$$f_*(x(\varphi_*, \psi, \eta), y(\varphi_*, \psi, \eta), z(\varphi_*, \psi, \eta)) = 0;\ f^*(x(\varphi^*, \psi, \eta), y(\varphi^*, \psi, \eta), z(\varphi^*, \psi, \eta)) = 0;$$
$$0 < \psi < Q_*,\ \ 0 < \eta < Q^*. \tag{4}$$

The conditions of "gluing" on a conditional cut:

$$x(\varphi, 0, \eta) = x(\varphi, Q_*, \eta),\ \ y(\varphi, 0, \eta) = y(\varphi, Q_*, \eta),\ \ z(\varphi, 0, \eta) = z(\varphi, Q_*, \eta);$$
$$\lim_{\psi \to 0+0} \frac{\partial x}{\partial \psi} = \lim_{\psi \to Q_* - 0} \frac{\partial x}{\partial \psi},\ \lim_{\psi \to 0+0} \frac{\partial y}{\partial \psi} = \lim_{\psi \to Q_* - 0} \frac{\partial y}{\partial \psi},\ \lim_{\psi \to 0+0} \frac{\partial z}{\partial \psi} = \lim_{\psi \to Q_* - 0} \frac{\partial z}{\partial \psi};$$
$$\varphi_* < \varphi < \varphi^*,\ \ 0 < \eta < Q^*. \tag{5}$$

$$x(\varphi, \psi, 0) = x(\varphi, \psi, Q^*),\ \ y(\varphi, \psi, 0) = y(\varphi, \psi, Q^*),\ \ z(\varphi, \psi, 0) = z(\varphi, \psi, Q^*);$$
$$\lim_{\eta \to 0+0} \frac{\partial x}{\partial \eta} = \lim_{\psi \to Q^* - 0} \frac{\partial x}{\partial \eta},\ \lim_{\eta \to 0+0} \frac{\partial y}{\partial \eta} = \lim_{\eta \to Q^* - 0} \frac{\partial y}{\partial \eta},\ \lim_{\eta \to 0+0} \frac{\partial z}{\partial \eta} = \lim_{\eta \to Q^* - 0} \frac{\partial z}{\partial \eta};$$
$$\varphi_* < \varphi < \varphi^*,\ \ 0 < \psi < Q_* \tag{6}$$

must also be fulfilled along with the conditions of quasiorthogonality at the boundaries of the domain

$$\frac{\partial f_*}{\partial x}\frac{\partial x}{\partial \psi}+\frac{\partial f_*}{\partial y}\frac{\partial y}{\partial \psi}+\frac{\partial f_*}{\partial z}\frac{\partial z}{\partial \psi}=0,\quad \frac{\partial f_*}{\partial x}\frac{\partial x}{\partial \eta}+\frac{\partial f_*}{\partial y}\frac{\partial y}{\partial \eta}+\frac{\partial f_*}{\partial z}\frac{\partial z}{\partial \eta}=0,\quad \frac{\partial f^*}{\partial x}\frac{\partial x}{\partial \psi}+\frac{\partial f^*}{\partial y}\frac{\partial y}{\partial \psi}+\frac{\partial f^*}{\partial z}\frac{\partial z}{\partial \psi}=0\,,$$

$$\frac{\partial f^*}{\partial x}\frac{\partial x}{\partial \eta}+\frac{\partial f^*}{\partial y}\frac{\partial y}{\partial \eta}+\frac{\partial f^*}{\partial z}\frac{\partial z}{\partial \eta}=0. \qquad (7)$$

Numerical solving algorithm. The algorithm for the problem numerical solving is constructed (similar to [7]) in the following way. We construct an equable orthogonal grid $G_\omega=\{(\varphi_i,\psi_j,\eta_k):\quad \varphi_i=\varphi_*+i\Delta\varphi,\ i=\overline{0,n+1};$ $\psi_j=j\Delta\psi,\ j=\overline{0,m+1};\quad \eta_k=k\Delta\eta,\ k=\overline{0,l+1};\ \ \Delta\varphi=\dfrac{\varphi^*-\varphi_*}{n+1};\ \ \Delta\psi=\dfrac{Q_*}{m+1};\ \ \Delta\eta=\dfrac{Q^*}{l+1};\ \ \gamma=\dfrac{\Delta\varphi}{\Delta\psi\Delta\eta};\ \ n,m,l\in N\}$ in domain. The system of equations (3) is approximated by the following difference equations using a "cross" type calculation scheme:

$$\begin{cases} x_{i,j,k}=x_{i-1,j,k}+\frac{1}{4}k_{i,j,k}\,\gamma((y_{i,j+1,k}-y_{i,j-1,k})(z_{i,j,k+1}-z_{i,j,k-1})-(y_{i,j,k+1}-y_{i,j,k-1})(z_{i,j+1,k}-z_{i,j-1,k}); \\ y_{i,j,k}=y_{i-1,j,k}+\frac{1}{4}k_{i,j,k}\,\gamma((x_{i,j,k+1}-x_{i,j,k-1})(z_{i,j+1,k}-z_{i,j-1,k})-(x_{i,j+1,k}-x_{i,j-1,k})(z_{i,j,k+1}-z_{i,j,k-1}); \\ z_{i,j,k}=z_{i-1,j,k}+\frac{1}{4}k_{i,j,k}\,\gamma((x_{i,j+1,k}-x_{i,j-1,k})(y_{i,j,k+1}-y_{i,j,k-1})-(x_{i,j,k+1}-x_{i,j,k-1})(y_{i,j+1,k}-y_{i,j+1,k}); \\ i=\overline{1,n},\quad j=\overline{1,m},\quad k=\overline{1,l}, \end{cases} \qquad (8)$$

where $x_{i,j,k}=x(\varphi_i,\psi_i,\eta_i),\ \ y_{i,j,k}=y(\varphi_i,\psi_i,\eta_i),\ \ z_{i,j,k}=z(\varphi_i,\psi_i,\eta_i)$ are the coordinates of the node grid G_z, $k_{i,j,k}$ is the value of the coefficient k for the corresponding node (this value is specified at each iteration step). The nodes that are placed at the conditional section are determined as follows (automatically ensuring the implementation of the "gluing" conditions (5) – (6)):

$$\begin{cases} x_{i,0,k}=x_{i,m+1,k}=x_{i-1,0,k}+\frac{1}{4}k_{i,0,k}\,\gamma((y_{i,1,k}-y_{i,m,k})(z_{i,0,k+1}-z_{i,0,k-1})-(y_{i,0,k+1}-y_{i,0,k-1})(z_{i,1,k}-z_{i,m,k}); \\ y_{i,0,k}=y_{i,m+1,k}=y_{i-1,0,k}+\frac{1}{4}k_{i,0,k}\,\gamma((x_{i,0,k+1}-x_{i,0,k-1})(z_{i,1,k}-z_{i,m,k})-(x_{i,1,k}-x_{i,m,k})(z_{i,0,k+1}-z_{i,0,k-1}); \\ z_{i,0,k}=z_{i,m+1,k}=z_{i-1,0,k}+\frac{1}{4}k_{i,0,k}\,\gamma((x_{i,1,k}-x_{i,m,k})(y_{i,0,k+1}-y_{i,0,k-1})-(x_{i,0,k+1}-x_{i,0,k-1})(y_{i,1,k}-y_{i,1,k}); \\ i=\overline{1,n},\quad k=\overline{1,l}, \end{cases} \qquad (9)$$

$$\begin{cases} x_{i,j,0}=x_{i,j,l+1}=x_{i-1,j,0}+\frac{1}{4}k_{i,j,0}\,\gamma((y_{i,j+1,0}-y_{i,j-1,0})(z_{i,j,1}-z_{i,j,l})-(y_{i,j,1}-y_{i,j,l})(z_{i,j+1,0}-z_{i,j-1,0}); \\ y_{i,j,0}=y_{i,j,l+1}=y_{i-1,j,0}+\frac{1}{4}k_{i,j,0}\,\gamma((x_{i,j,1}-x_{i,j,l})(z_{i,j+1,0}-z_{i,j-1,0})-(x_{i,j+1,0}-x_{i,j-1,0})(z_{i,j,1}-z_{i,j,l}); \\ z_{i,j,0}=z_{i,j,l+1}=z_{i-1,j,0}+\frac{1}{4}k_{i,j,0}\,\gamma((x_{i,j+1,0}-x_{i,j-1,0})(y_{i,j,1}-y_{i,j,l})-(x_{i,j,1}-x_{i,j,l})(y_{i,j+1,0}-y_{i,j+1,0}). \\ i=\overline{1,n},\quad j=\overline{1,m}, \end{cases} \qquad (10)$$

Equations (4) defining the boundaries of the investigated area are approximated as follows:

$$f_*(x_{0,j,k},\,y_{0,j,k},\,z_{0,j,k})=0;\quad f^*(x_{n+1,j,k},\,y_{n+1,j,k},\,z_{n+1,j,k})=0;$$

$$j=\overline{0,m+1},\quad k=\overline{0,l+1}. \qquad (11)$$

The conditions of the surface orthogonality of the flow and of the equipotential surfaces on the boundaries of the investigated physical region (7) are replaced by the following difference equations [6]:

$$\frac{\partial f_*(x_{0,j,k},y_{0,j,k},z_{0,j,k})}{\partial x}(x_{1,j,k}-x_{0,j,k})^{-1}=\frac{\partial f_*(x_{0,j,k},y_{0,j,k},z_{0,j,k})}{\partial y}(y_{1,j,k}-y_{0,j,k})^{-1}=$$

$$=\frac{\partial f_*(x_{0,j,k},y_{0,j,k},z_{0,j,k})}{\partial z}(z_{1,j,k}-z_{0,j,k})^{-1};$$

$$\frac{\partial f^*(x_{n+1,j,k},y_{n+1,j,k},z_{n+1,j,k})}{\partial x}(x_{n,j,k}-x_{n+1,j,k})^{-1}=\frac{\partial f^*(x_{n+1,j,k},y_{n+1,j,k},z_{n+1,j,k})}{\partial y}(y_{n,j,k}-y_{n+1,j,k})^{-1}=$$

$$=\frac{\partial f^*(x_{n+1,j,k},y_{n+1,j,k},z_{n+1,j,k})}{\partial z}(z_{n,j,k}-z_{n+1,j,k})^{-1},\quad j=\overline{0,m+1},\quad k=\overline{0,l+1}. \tag{12}$$

The quasiconformal invariant γ value is determined iteratively in the calculation process on the basis of the condition of "similarity in the small" elementary parallelepipeds of the domains G_ω and G_z^0:

$$\gamma=\frac{k_{i,j,k}}{(n+1)(m+1)(l+1)}\sum_{i,j,k=0}^{n,m,l}\gamma_{i,j,k}, \tag{13}$$

where

$$\begin{aligned}
\gamma_{i,j,k} = {} & 4(\sqrt{(x_{i+1,j,k}-x_{i,j,k})^2+(y_{i+1,j,k}-y_{i,j,k})^2+(z_{i+1,j,k}-z_{i,j,k})^2}+\\
&+\sqrt{(x_{i+1,j+1,k}-x_{i,j+1,k})^2+(y_{i+1,j+1,k}-y_{i,j+1,k})^2+(z_{i+1,j+1,k}-z_{i,j+1,k})^2}+\\
&+\sqrt{(x_{i+1,j,k+1}-x_{i,j,k+1})^2+(y_{i+1,j,k+1}-y_{i,j,k+1})^2+(z_{i+1,j,k+1}-z_{i,j,k+1})^2}+\\
&+\sqrt{(x_{i+1,j+1,k+1}-x_{i,j+1,k+1})^2+(y_{i+1,j+1,k+1}-y_{i,j+1,k+1})^2+(z_{i+1,j+1,k+1}-z_{i,j+1,k+1})^2})/\\
&/((\sqrt{(x_{i,j+1,k}-x_{i,j,k})^2+(y_{i,j+1,k}-y_{i,j,k})^2+(z_{i,j+1,k}-z_{i,j,k})^2}+\\
&+\sqrt{(x_{i+1,j+1,k}-x_{i+1,j,k})^2+(y_{i+1,j+1,k}-y_{i+1,j,k})^2+(z_{i+1,j+1,k}-z_{i+1,j,k})^2}+\\
&+\sqrt{(x_{i,j+1,k+1}-x_{i,j,k+1})^2+(y_{i,j+1,k+1}-y_{i,j,k+1})^2+(z_{i,j+1,k+1}-z_{i,j,k+1})^2}+\\
&+\sqrt{(x_{i+1,j+1,k+1}-x_{i+1,j,k+1})^2+(y_{i+1,j+1,k+1}-y_{i+1,j,k+1})^2+(z_{i+1,j+1,k+1}-z_{i+1,j,k+1})^2})\times\\
&\times(\sqrt{(x_{i,j,k+1}-x_{i,j,k})^2+(y_{i,j,k+1}-y_{i,j,k})^2+(z_{i,j,k+1}-z_{i,j,k})^2}+\\
&+\sqrt{(x_{i+1,j,k+1}-x_{i+1,j,k})^2+(y_{i+1,j,k+1}-y_{i+1,j,k})^2+(z_{i+1,j,k+1}-z_{i+1,j,k})^2}+\\
&+\sqrt{(x_{i+1,j+1,k+1}-x_{i,j+1,k+1})^2+(y_{i+1,j+1,k+1}-y_{i,j+1,k+1})^2+(z_{i+1,j+1,k+1}-z_{i,j+1,k+1})^2}+\\
&+\sqrt{(x_{i,j+1,k+1}-x_{i,j+1,k})^2+(y_{i,j+1,k+1}-y_{i,j+1,k})^2+(z_{i,j+1,k+1}-z_{i,j+1,k})^2}+\\
&+\sqrt{(x_{i+1,j+1,k+1}-x_{i+1,j+1,k})^2+(y_{i+1,j+1,k+1}-y_{i+1,j+1,k})^2+(z_{i+1,j+1,k+1}-z_{i+1,j+1,k})^2})).
\end{aligned} \tag{14}$$

$$k_{i,j,k}=k_{0i,j,k}+\frac{1}{2}\beta\left(I_{i,j,k}-I^*\right)\left(\left(I_{i,j,k}-I^0\right)+\left|I_{i,j,k}-I^0\right|\right), \tag{15}$$

Q, Q_* and Q^* are determined by formulas

$$Q=\frac{\Delta\varphi}{\gamma}(m+1)(l+1),\ Q_*=\sqrt{\frac{Q}{\tilde\gamma}},\ Q^*=\sqrt{Q\tilde\gamma},$$

where

$$\tilde\gamma=\frac{k_{i,j,k}}{(n+1)(m+1)(l+1)}\sum_{i,j,k=0}^{n,m,l}\tilde\gamma_{i,j,k};$$

$$\begin{aligned}
\tilde\gamma_{i,j,k} = {} & 4(\sqrt{(x_{i,j,k+1}-x_{i,j,k})^2+(y_{i,j,k+1}-y_{i,j,k})^2+(z_{i,j,k+1}-z_{i,j,k})^2}+\\
&+\sqrt{(x_{i+1,j,k+1}-x_{i+1,j,k})^2+(y_{i+1,j,k+1}-y_{i+1,j,k})^2+(z_{i+1,j,k+1}-z_{i+1,j,k})^2}+\\
&+\sqrt{(x_{i,j+1,k+1}-x_{i,j+1,k})^2+(y_{i,j+1,k+1}-y_{i,j+1,k})^2+(z_{i,j+1,k+1}-z_{i,j+1,k})^2}+\\
&+\sqrt{(x_{i+1,j+1,k+1}-x_{i+1,j+1,k})^2+(y_{i+1,j+1,k+1}-y_{i+1,j+1,k})^2+(z_{i+1,j+1,k+1}-z_{i+1,j+1,k})^2})/\\
&/(\sqrt{(x_{i,j+1,k}-x_{i,j,k})^2+(y_{i,j+1,k}-y_{i,j,k})^2+(z_{i,j+1,k}-z_{i,j,k})^2}+\\
&+\sqrt{(x_{i+1,j+1,k}-x_{i+1,j,k})^2+(y_{i+1,j+1,k}-y_{i+1,j,k})^2+(z_{i+1,j+1,k}-z_{i+1,j,k})^2}+\\
&+\sqrt{(x_{i,j+1,k+1}-x_{i,j,k+1})^2+(y_{i,j+1,k+1}-y_{i,j,k+1})^2+(z_{i,j+1,k+1}-z_{i,j,k+1})^2}+\\
&+\sqrt{(x_{i+1,j+1,k+1}-x_{i+1,j,k+1})^2+(y_{i+1,j+1,k+1}-y_{i+1,j,k+1})^2+(z_{i+1,j+1,k+1}-z_{i+1,j,k+1})^2}).
\end{aligned}$$

The numerical implementation algorithm is carried out by means of a stepwise parameterization of the quasiconformal invariant magnitude γ, the value of the permeability coefficient and the boundary and internal node coordinates of the region (the corresponding algorithm for the two-dimensional medium is described in detail in [6]).

Conclusions. The mathematical model of the explosion process, developed on the basis of the liquid one, is generalized to the spatial case. A number of numerical experiments were carried out on the basis of the developed algorithm that confirmed the feasibility of using the developed spatial model for solving a class of problems studying the explosion processes caused by the convex form charge (limited by a closed smooth surface). The results of distribution of the crater, pressed and unperturbed soil sections formed as a result of a charge burst, which has the form of a substantially elongated ellipsoid, in the plane $z=0$ of the studied area coincided with the results of calculations for the corresponding plane problem solved for the region which contours coincide with the intersections of the plane $z=0$ and the surfaces S_* and S^*.

The perspective of he research is modeling the explosion processes, caused by two or more charges, a cord (cylindrical) charge, taking into account the presence of essentially heterogeneous parts of the environment (for example, the foundations of structures), identifying the external boundary of the investigated area.

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