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CONTRASTING TRENDS IN FIRM VOLATILITY: THEORY AND EVIDENCE

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ABSTRACT

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Contrasting Trends in Firm Volatility: Theory and Evidence *

David Thesmar† Mathias Thoenig‡

Abstract

Over the past decades, the real and financial volatility of listed firms has increased, while the volatility of private firms has decreased. We first provide panel data evidence that, at the firm level, sales and employment volatility are impacted by changes in the degree of ownership concentration. We then construct a model with private and listed firms where risk taking is a choice variable at the firm-level. Due to general equilibrium feedback, we find that an increase in stock market participation or integration in international capital markets generate opposite trends in volatility for private and listed firms. This pattern cannot be replicated by alternative comparative statics exercises, such as an increase in product market competition, an increase in product market size, an increase in the fraction of listed firms, or a decrease in aggregate volatility.

1 Introduction

Over the past decades, the idiosyncratic volatility of publicly listed US firms has risen considerably, be it computed from real or financial variables.[1] Existing explanations focus on product market competition.[2] Indeed the US, like many other countries over the past thirty years, have experienced profound deregulation of many industries, a dramatic increase in international competition, and an acceleration in the pace of innovation on product markets. Yet privately held firms experienced the opposite movement. In a recent paper, Davis et al (2006) show that employment growth volatility of non listed firms has decreased by about 50% between the early 1980s and the late 1990s. In our French census data, we find similar evolutions (see figure 1). The volatilities of listed and private firms have evolved in opposite directions, a fact that competition-based theories cannot explain.

This paper proposes an explanation for these contrasting trends. Our starting point is that, at the firm level, risk taking is a choice variable which is affected by risk sharing among shareholders

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[1] Campbell et alii (2000) find that the volatility of stock returns, after filtering out aggregate shocks, has been multiplied by four. Brandt et alii (2007) argue that this trend disappears when they include the more recent period of volatility decrease. Their study, however, stops in 2005. When more recent years (and months) are included, the trend reappears. The 10 year rolling standard deviation of sales or employment growth has doubled between 1955 and 2000 (Comin and Philippon, 2005, Irvine and Pontiff, 2007). Compared to the 1950s, industry leaders are now three times more likely to lose their preeminence.

[2] See Thesmar and Thoenig (2000), Comin and Philippon (2005), Gaspar and Massa (2006), Irvine and Pontiff (2007).

(Helpman and Razin, 1978, Saint-Paul, 1993, Obstfeld, 1994). Recently the degree of risk sharing among owners of listed firms has risen dramatically because of increased stockmarket participation (Guiso et alii, 2002), rise in institutional ownership (French, 2008), and international capital market integration (Chari and Henry, 2004). Against this background, publicly listed firms have taken on more operating risk by adopting ambitious, but risky, projects. This could explain the pattern of rising volatility for listed firm. But could this lead to a *decrease* in volatility of privately held firms? Our theoretical analysis addresses this question.

To this purpose, we provide a model of endogenous risk taking by listed and private firms in the presence of aggregate and idiosyncratic uncertainty.[3] Listed firms are owned by diffused shareholders (the "investors"), while private firms have only one owner (the “entrepreneur”) who therefore bears all of idiosyncratic risk. Even though they have fully distinct shareholders, listed and non listed firms interact in our model: they compete on the labor market, and on product markets segmented à la Dixit Stiglitz. In this setup, our main comparative static exercise relates to a increase in stock market participation: Following an exogenous increase the number of investors, we show that investors effective tolerance to risk increases and public firms respond by becoming more volatile and, on average, more productive. The demand for production factors increases and this props up factors real prices. This impacts negatively the profits of privately held firms and entrepreneurs become poorer. Since entrepreneurs have decreasing absolute risk aversion,[4] their risk tolerance decreases and private firms reduce their risk taking. To sum up, an increase in stock market participation generates two constrasting trends in firm volatility.

To gain tractability, we then study a log-linearised version of our model, assuming small aggregate and idiosyncratic shocks. This approach allows us to show that in spite of constrasting changes in their volatilities, listed and private firms both experience a drop in their real profits : even though listed firms become more efficient on average, competition prevents them from passing these gains to their shareholders. We also find that the utilities of investors and private entrepreneurs declines while workers' utilities go up.

Finally, we use our approximated model to generate additional comparative statics, such as an increase in the number of listed firms, or a decrease in aggregate uncertainty. In our model, none of these alternative comparative statics generate the dual trend in firm volatility that we seek to explain. In our last extension, we also study the impact of international capital market integration. We do this

[3] In our model risk taking affects the firm-level demand curve only. It is clear that risk taking also affects the firm-level supply curve. From a theoretical perspective, both channels deliver very similar results and we thus choose to consider only the first channel. As a consequence our anlaysis remains silent on productivity evolution. However it is clear that our theoretical mechanism and our empirical results are perfectly compatible with the technology view elaborated in Comin and Mulani (2005) and Comin and Philippon (2005) (who show that sales per worker growth has also experienced an increase in volatility for listed firms).

[4] We assume that agents have CRRA utility in final consumption.

because financial integration is viewed by some economists as a more significant source of risk sharing among investors than stock market participation (see discussion in Guiso et al, 2003). For instance, Chari and Henry (2004) show that stock markets liberalizations increase stock prices by about 15%, of which 6.8% can be attributed to improved risk sharing. To this purpose we extend our baseline model to the case with two countries and imperfectly correlated aggregate shocks. We find that capital market integration, by enhancing risk sharing among investors, is able to generate contrasting trends in firm volatility.

Beyond providing an unified rationale behind the opposite trends in firm volatility, our model also contributes to the literature on firm risk taking and capital market development (Saint-Paul, 1993, Obstfeld, 1994, Acemoglu and Zilibotti, 1997, Perroti and Von Thadden, 2006) in several ways. First, we augment existing models by introducing firms that do not access capital markets and explicitly model imperfect competition. Our model, in contrast to Saint-Paul (1993) or Obstfeld (1994), does not have the unsettling property that countries with more developed, or integrated, financial markets, should display more aggregate output uncertainty (but less income fluctuations) as they specialize: it is a well established fact that more developed economies have lower GDP growth volatility (see Koren and Tenreyro, 2008 for a discussion).

Before developing the model, we present in Section 2 some suggestive evidence on French data. Like Davis et al (2006), we find evidence of opposite trends in idiosyncratic volatility: the volatility of listed firms increases strongly, while the volatility of non listed firms declines. We also provide micro-level evidence that firms with more diversified shareholders tend to become more volatile. Given the influence of the theories referred to above, such evidence is surprisingly lacking in the empirical literature, apart from very few papers. Using a sample of publicly listed firms, Sraer and Thesmar (2007) find that family firms are less volatile than non family firms. In their sample of privately held Italian companies, Michellacci and Schivardi (2008) find that the dispersion of productivity is smaller among family firms, a fact that they interpret as indicative of lower volatility. Such existing evidence is cross sectional only and is therefore subject to strong omitted variable biases. Here we exploit the panel dimension which allows us to identify the within firm correlation between *changes* in shareholder concentration and *changes* in firm level volatility. This allows to test our core theoretical assumption, namely that risk taking is a decision variable at the firm level and depends on the degree of ownership concentration.

The rest of the paper is devoted to the theoretical analysis. In Section 3, we present the closed economy model and derive some comparative static properties. In Section 4, we make the assumption that aggregate and idiosyncratic shocks are small, and linearize the model: it allows us to present additional properties of the equilibrium and comparative statics. In Section 5, we present the linearized model with capital market integration. We conclude in Section 6.

2 Firm Level Volatility: Trends and Determinants

This Section presents supporting evidence based on French data. We describe our data in Section 2.1 and provide evidence on the opposite trends of volatility for listed and private firms in Section 2.2. Finally in Section 2.3 we successfully test the mechanism at the core of our theory by showing that, in the panel dimension, a *decrease* in ownership concentration is accompanied by an *increase* in firm level idiosyncratic volatility.

2.1 Data

Our sample is composed of all French firms, active at some point between 1984 and 2004, that (i) were never state owned[5] and (ii) whose total sales exceed 30 million euros or whose labor force exceeds 500 employees, *at least three years* during the period. Each of these firms is tracked throughout the period. This leads to an unbalanced panel of all large and many medium sized businesses in the French economy, be they privately held or publicly listed.

For all these firms, information is gathered from two sources: accounting and ownership data. Accounting data come from tax files used by the Ministry of Finance to collect the corporate tax (BRN data), available from 1978 to 2004. The BRN data represents the universe of all French firms with more than 1 million euros of annual turnover. In terms of variables, BRN provides us with the balance sheet, profit statement and employment of these firms. This data source is used to construct three variables: the 4 digit industry, total employment, total turnover. We end up with 148,789 observations (some 5,722 firms per year), corresponding to 9,294 different firms since there is both entry to and exit from the sample.

Ownership information are obtained from the Financial Relation Survey (LIFI in French), conducted each year from 1984 to 2004 by the French Statistical office. Only 64,275 observations (some 3,200 firms per year) can be found in LIFI. From this survey we build two variables. The first one, $LIST$, is a dummy equal to 1 when the firm belongs to a publicly listed business group, or is itself listed on the French stock market. For business group membership, we define as the group leader, the firm that owns, directly or indirectly, at least 50% of a given firm's equity. It is retrieved using LIFI survey and, for indirect ownership, an algorithm developed at the statistical office. We then checked, by hand, whether, each year between 1984 and 2004, the firm itself or its group leader, was listed on the French stock market. In our 1984-2004, sample, such directly or indirectly listed firms make up 13% of all observations which corresponds on average to 815 firms out of 5,972 firms each year. Most

[5] This screen automatically removes privatized firms.

of these firms are indirectly listed through their group leader.

Our second variable based on the LIFI survey is the *INDIV* dummy which measures ownership concentration. From LIFI, we get for each firm, the fraction of equity that is held by: (i) French individual; (ii) foreign individuals; (iii) French firms; (iv) foreign firms; (v) employees; (vi) the state; and (vii) by "unknown" companies or persons. *INDIV* is equal to 1 when the self reported fraction of equity held by French individuals lies above 50%. This variable is not always reported, so the number of observations for which we have such information is 64,275 (3,213 obs per year). Given that the survey LIFI asks for ownership by "known" individuals, which cannot be too numerous, we believe that *INDIV* captures properly the set of firms that are controlled by a restricted set of shareholders.

2.2 Trends in Firm Level Volatility

Our first measure of firm level volatility, $rolvol_{it}$, is the rolling standard deviation of sales growth as standardly used in the literature[6]. For firm i at date t, with a growth rate of g_{it}, we compute:

$$rolvol_{it} = \frac{1}{10}\sum_{t'=t-4}^{t'=t+5}(g_{it'})^2 - \left(\frac{1}{10}\sum_{t'=t-4}^{t'=t+5} g_{it'}\right)^2$$

This measure requires the firm to be present for 10 years in a row, from 4 years before t, until 5 years after t. Moreover, our census data containing outliers due to hand-typing errors, mergers, and the presence of smaller, more volatile firms, we trim extreme values of $rolvol_{it}$. Last, we compute for each year the mean of $rolvol_{it}$ separately for the groups of listed and non listed firms. The time evolution of these two figures are reported in Figure 1. The volatility of listed firms increases from 15% to 18%, while the volatility of non listed firms declines by about 1%. While there is a pattern of opposite trends in our French data, the trends are smaller than what is reported in the US-based study by Davis et alii. One possible reason is that we use firm-level data while Davis et alii (2006) exploit plant level data. Plants being sub-units of firms, they are significantly more volatile (two times more volatile in Davis et alii' sample). Another issue with $rolvol_{it}$ is potential contamination by aggregate volatility whose declining trend may mask changes in idiosyncratic volatility in our French data. Those concerns lead us to consider an alternative measure of firm volatility.

Our second measure of volatility is firms' reaction to industry shocks. Based on the methodology developed by Thesmar and Thoenig (2007) and Sraer and Thesmar (2007) we estimate the following

[6] For recent empirical studies using this kind of rolling window measure, see Comin and Philippon (2005), Comin and Mulani (2006) and Davis et alii (2006). More generally the literature has used two different measures of firm volatility: i/ with the standard deviation of the time series performance of a firm over a (rolling) window; ii) with the cross-sectional dispersion of firm performance across firms. The evolution of these two measures may in principle be different. The time series volatility measure is better in that it removes the average growth rate of the firm and hence eliminates the bias in the evolution of firm volatility caused by a change in the distribution of the firm's growth potential.

model, for firm i, in industry s at date t:

$$\log sales_{ist} = \alpha_i + \beta_{it}. \log \overline{sales}_{ist} + X_{it} + \varepsilon_{it} \tag{1}$$

under the constraint that β_{it}, the time-varying sensitivity to industry sales, is conditioned on $LIST_{it}$, the listing status of firm i over the period

$$\beta_{it} = \sum_{T=1984}^{T=2004} \delta_T^P.1_{\{t=T\}} \times (1 - LIST_{it}) + \sum_{T=1984}^{T=2004} \delta_T^L.1_{\{t=T\}} \times LIST_{it} \tag{2}$$

where $\overline{sales}_{ist}$ is the sum of sales in industry s (excluding firm i);. X_{it} are control variables; α_i is a firm fixed effect; and the coefficients (δ_T^P, δ_T^L) capture the time evolution of β_{it} for private and publicly listed firms.

We estimate (1)-(2) using OLS, including year dummies interacted with $LIST_{it}$ as the controls X_{it}. Results are reported in Figure 2 where the time evolution of $(\delta_T^P,\ \delta_T^L)$ are represented. For listed firms, the sensitivity to industry shocks goes from 0.05 to 0.15 over the period, while it decreases from 0.17 to 0.01 for non listed firms. The opposite trends of volatility between the two groups of firms is very large in our data when measured through this method.[7]

2.3 Ownership Concentration and Firm Level Volatility

We test now the impact of ownership concentration on volatility. In order to circumvent omitted variable biases, we exploit the panel dimension of our dataset. We first use our measure of firms' reaction to industry shocks. We thus estimate equation (1) but instead of imposing (2), we assume now that β_{it} is conditioned on ownership concentration:

$$\beta_{it} = \alpha + \lambda.INDIV_{it} + \mu. \log(sales_{it}) + \phi_s + \varepsilon_{it} \tag{3}$$

where INDIV_{it} is the dummy variable described above, equal to 1 when the firm reports that "known individuals" own at least 50% of the equity; α is a constant term and ϕ_s is a two digit industry dummy and ε_{it} is an error term clustered at the firm level. Results are reported in Table 1, where columns 1-3 focus on sales reactions, while columns 4-6 focus on employment reactions to industry shocks. Comparing the results in columns 1 and 4 where we impose $\mu = \phi_s = 0$ in equation (3), to the results in columns 2 and 5, it appears that the sensitivity of sales and employment to shocks is lower for firms with concentrated ownership. It decreases from 0.09 to 0.02 for the sensitivity of sales, and from -0.12 to -0.18 for employment. In columns 3 and 6 we further control for industry and size effects by first regressing INDIV_{it} on $\log(\text{sales}_{it})$ and the ϕ_s, and then interacting the residual with industry shocks. We find the same order of magnitude.

[7] This divergence is strongly statistically significant, when for instance we impose the δ_T to follow different trends and test for equality.

There are limitations to the above approach. First, it does not identify properly employment dynamics which are negatively correlated with sales shocks in the data; probably because it fails to account for the inertia in employment. Second, and most importantly, this approach does not allow to include firm fixed effects in equation (3). This requires to find an alternative measure for idiosyncratic volatility which varies, at the firm level, in the time-dimension. To this purpose we implement a recent methodology developed in Morgan, Rime and Strahan (2004) and Castro, Clementi and MacDonald (2008) which proxies volatility with the absolute deviation to conditional expected sales growth.

In a first stage we run the following regression, for firm i at date t:

$$\frac{sales_{it} - sales_{it-1}}{sales_{it-1}} = \alpha_i + \beta.INDIV_{it} + \delta_t + \varepsilon_{it} \tag{4}$$

where sales growth is the left hand side variable α_i is a firm fixed effect and δ_t is year dummy designed to capture aggregate volatility. We use our OLS estimate of (4) to retrieve $|\widehat{\varepsilon}_{it}|$, the absolute deviation of sales growth from its conditional mean.

In a second stage we estimate the following equation:

$$|\widehat{\varepsilon}_{it}| = \gamma_i + \lambda.INDIV_{it} + \delta_t + \nu_{it} \tag{5}$$

where γ_i is a firm fixed effect; δ_t is a year dummy; and ν_{it} is an error term that we allow to be correlated across observations of a same firm.

OLS estimates of various forms of equation (5) are reported in Table 2. Panel A uses the estimate of sales growth volatility, while panel B reproduces the exercise for employment growth. In both cases, growth rates are windsorized prior to running regression (4). The specification in column 1 (panels A and B) is estimated without the firm fixed effect γ_i in equation (5); the estimated coefficient λ is statistically significant at the 1% threshold for sales volatility and 5% threshold for employment volatility; in both cases the coefficient is negative and this shows that in the cross section of firms, a concentrated ownership impacts negatively volatility. The effect is economically sizeable. For sales growth volatility, the sample mean of $|\widehat{\varepsilon}_{it}|$ is 10.7 basis points and the coefficient λ equal to -1.7. Hence, the mean idiosyncratic volatility in sales growth is lower by some 15% for firms with a concentrated ownership. The effect is around 10% for employment volatility. This relation is consistent with Michelacci and Schivardi (2008) who find in their Italian data that the dispersion of productivity growth is smaller for family firms than for non family firms.

In column 2, for panels A and B, the firm fixed effects γ_i are included: in that case, the λ coefficient is identified on firms that transit between the state $INDIV_{it} = 1$ (being controlled by individuals) and the state $INDIV_{it} = 0$. On the cross-sectional sample of 2,254 firms (60,120 obs.), only 350 firms change of $INDIV_{it}$ over the period. In spite of this demanding identification strategy, these panel estimates are statistically significant. With respect to the cross sectional specification of column 1, the estimated coefficient λ is now reduced by three times for sales growth volatility but is unchanged for employment volatility. Finally, in column 3, we control for firm size by including $\log(\text{sales}_{it})$ in equation (5). Our panel results are not affected.

These panel results offer supporting evidence to our core theoretical assumption, namely that risk taking is a decision variable at the firm level and depends on the degree of ownership concentration.

3 The Baseline Model

In this section, we present the baseline model and derive some general properties.

3.1 Set-Up

We consider a static economy populated by $\widetilde{L}$ workers, each of them supplying one unit of labor. A measure N of firms compete imperfectly on the product market: N_L firms are listed on the stockmarket while each of the remaining $N_P \equiv N - N_L$ firms are held by a single *entrepreneur*, who does not supply labor. Among the $\widetilde{L}$ workers, there are I *investors* who trade the stocks of listed firms with each other.[8] There are two sources of risk: an idiosyncratic shock on firm-level demand and an aggregate supply shock on labor supply. There are three periods[9]. At date 1, each firm i implements a strategy s_i that indirectly affects the mean and variance of future profits. At date 2, investors trade stocks and the financial market clears. At date 3, uncertainty is revealed; the product and the labor markets clear; the workers receive wages; firm owners (investors or entrepreneurs) receive the profits.

3.1.1 Preferences and Technology

Each agent k in the economy has a utility $U(C_k)$ with constant[10] relative risk aversion $\gamma > 0$. The consumption index C_k is a composite of the consumptions $y_{k,i}$ of goods supplied under monopolistic competition by firms $i \in [0; N]$: $C_k = \left(\int_0^N \tilde{\delta}_i^{1/\sigma} y_{k,i}^{(\sigma-1)/\sigma} di\right)^{\sigma/(\sigma-1)}$ where $\sigma \geq 2$ by assumption.[11] The random coefficients $\tilde{\delta}_i > 0$ correspond to good i specific demand shifter. A convenient feature of this Dixit Stiglitz index is that aggregating individuals consumptions is simple in spite of heterogeneity in individual incomes between the three groups of agents populating this economy (pure workers, entrepreneurs, investors). Indeed the total demand $\tilde{y}_i$ addressed to each monopoly i is obtained

[8] For expositional simplicity we assume that entrepreneurs do not supply labor. Relaxing this assumption does not affect our results.

[9] Inverting period 1 and period 2 would not change the result.

[10] Most of our results are robust to assuming a DRRA utility.

[11] In our theory, the assumption $\sigma > 2$ is crucial because it forces our main aggregate variable M to positively depend on firm level decisions (see equation (9)). This assumption is fully consistent with the empirical estimates of the elasticity of substitution based on microdata (Head and Ries 2001; Broda and Weinstein 2006) and with recent estimates based on macro data (Imbs and Méjean 2008). Moreover it is now standard in the macro literature with heterogenous industries or firms (as it is the case in our model) to calibrate σ with values above 4 (for recent examples, see Atkeson and Burnstein (2008), Corsetti, Dedola and Leduc (2008)).

by aggregating consumptions $y_{k,i}$ over the whole population and standard computations lead to : $\tilde{y}_i = \tilde{\delta}_i E P^{\sigma-1}/p_i^{\sigma}$, where p_i is the monopoly price charged by firm i, $E \equiv \int_0^{\widetilde{L}+N_P} E_k dk$ is the aggregate nominal expenditure and P is a price index equal to: $P \equiv \left(\int_0^N \tilde{\delta}_j.p_j^{1-\sigma} dj\right)^{1/1-\sigma}$.

On the supply side, we assume that the total number of workers is random and equal to $\widetilde{L} = \widetilde{A}L$ where $\tilde{A}$ is positive with mean 1 and variance σ_A^2. We interpret $\widetilde{A}$ as an aggregate supply shock.[12] Each firm i hires l_i workers at nominal wage w, and produces according to the constant returns to scale technology $y_i = l_i$. We implicitly rule out entry on the product market by fixing exogenously[13] the number of firms at N.

3.1.2 Strategies of Firms

At date 1, each firm i implements a strategy $s_i \geq 0$ at a cost $C(s_i)$ in real terms, which affects its demand shifter $\tilde{\delta}_i$ in the following way[14]:

$$\tilde{\delta}_i = 1 + s_i \tilde{d}_i \tag{6}$$

where $\tilde{d}_i$ are i.i.d. positively distributed shocks with mean 1 and variance σ_d^2. The cost function $C(.)$ is increasing, convex with $C(0) = C'(0) = 0$ and $C'(\infty) = \infty$. For private firms, the optimal strategy s_i maximizes entrepreneur's expected utility; for listed firm, it maximizes the date 2 stock price.

Assumption (6) is motivated by the literature on growth and finance (see for instance Saint-Paul (1993), Obstfeld (1994), Acemoglu and Zilibotti (1997)). In existing theories, enhanced risk sharing reduces the costs of risk taking for the representative firm, which has the effect of increasing the volatility of aggregate output. Hence, financially developed and integrated economies should be more volatile, which is a counterfactual prediction (Koren and Tenreyro, 2008). In our model, this does not happen (see section 3.2.1). Moreover our view of risk taking is precisely microfounded with firm level demands. Here the strategies s_i can be interpreted as a choice of *customization* by the firm. Customized goods can be highly valuable, but their demand is difficult to predict because of erratic preferences. This interpretation is close to the view developed in standard models of advertising (Schmalensee 1974), in models with consumer inertia (Fishman and Rob (2003)) and in Piore and Sabel (1984)'s vision of flexible manufacturing.

[12] An alternative modelization would be to take $\widetilde{A}$ as a productivity shock. This does not change the heart of the analysis but makes exposition more complex. Moreover we assume that for all $\widetilde{A}$ we have $\widetilde{A}L > I$. Hence the number of investors is not affected by aggregate uncertainty.

[13] In an unreported extension of the model (available upon request from the authors) we endogenize N. In that case, the mechanisms are similar except that now the general equilibrium feedback is channelized by N rather than by M.

[14] All our results can easily be generalized to any functional form $\widetilde{\delta}_i = G(s_i, \widetilde{d}_i)$ such as $G(.,.)$ is positive, increasing and quasi concave in both arguments with the following log-supermodularity condition: $\partial^2 \log G / \partial s_i \partial \widetilde{d}_i > 0$.

3.2 Solving the model

We solve this model backwards, starting with labor and product market clearing conditions in period 3; stock trading in period 2, and strategy choice in period 1.

3.2.1 Product and Labor Market Equilibria

At date 3, the aggregate labor supply shock $\widetilde{A}$ and final demand shocks $\widetilde{d_i}$ are revealed[15]. Each firm i charges an optimal monopoly price p_i in order to maximize its profits in real terms $\tilde{\pi}_i = \tilde{y}_i(p_i - w)/P - C(s_i)$. As it is standard in this monopolistic competition setting, N is assumed to be sufficiently large such that the marginal effect of p_i on the price index P is negligible. Solving this standard maximization problem leads to a constant markup over marginal cost $p_i = w/(1-1/\sigma)$ and a labor demand $l_i = \tilde{\delta}_i E w^{-\sigma} P^{\sigma-1}$. Aggregating labor demands across firms and using the definition of P we get the labor market clearing condition $\tilde{A}L = (1-1/\sigma)E/w$. This yields the real wage $w/P = (1-1/\sigma)(\int_0^N \tilde{\delta}_j dj)^{1/(\sigma-1)}$ and the real spending $E/P = \tilde{A}L(1-1/\sigma)^{\sigma}(\int_0^N \tilde{\delta}_j dj)$. Together with (6) we then obtain real profits at equilibrium

$$\tilde{\pi}_i = \frac{\tilde{A}}{M}\left(1 + s_i \tilde{d}_i\right) - C(s_i) \tag{7}$$

where $M \equiv (\sigma/L)\left(\int_0^N \tilde{\delta}_j dj\right)^{(\sigma-2)/(\sigma-1)}$. We label M the degree of *market pressure* because it measures the extent to which labor market pressure feedbacks on real wage and firm profits. Indeed, using the definition of M, we can rewrite the real wage as an increasing function of M :

$$w/P = \omega M^{\frac{1}{\sigma-2}} \tag{8}$$

where ω is a combination of exogenous parameters equal to $\omega \equiv (1-1/\sigma)(L/\sigma)^{1/\sigma-2}$.

The interpretation of the negative impact of M on profits in equation (7) is the following: When all firms face very high demand shocks $\widetilde{\delta}_j$, aggregate demand for labor is high; this props up real wage and this reduces profits[16].

Noticing that the shocks $\tilde{d}_j$ are *i.i.d*, the law of large numbers leads to the following formula for M:

$$M = \frac{\sigma}{L} N^{\frac{\sigma-2}{\sigma-1}} \left[1 + \frac{N_P}{N} s_P + \frac{N_L}{N} s_L\right]^{\frac{\sigma-2}{\sigma-1}} \tag{9}$$

[15] The shocks are assumed to be independent: this simplifies drastically the analysis. In reality, however, shocks are correlated in non-trivial ways. Schumpeterian dynamics leads some firms to win at the same time as others loose. Franco and Philippon (2006) find evidence on this. This negative correlation of shocks is also at the root of the mechanisms that Comin and Mulani (2005) use to link firm volatility with aggregate volatility.

[16] In fact, this effect competes with another force: due to Dixit-Stiglitz preferences, larger $\widetilde{\delta}_j$ by competitors increases real spending E/P and profits for all firms. This second effect is however dominated under our assumption that the elasticity of substitution is large enough (*ie.* $\sigma > 2$).

where (s_P, s_L) correspond to the risky strategies for private and listed firms respectively. Note that all firms within each group are identical and therefore adopt identical strategies.

From the previous computations we get the relationship between (s_P, s_L) and volatility. Indeed, from the labor demand, we get the size adjusted volatility of sales at the firm level:

$$\frac{var\,(y_i)}{E^2\,(y_i)} = \sigma_A^2 + \sigma_d^2 \frac{(1+\sigma_A^2)}{(1+1/s_i)^2} \tag{10}$$

which is increasing in s_i.This formula allows us to interpret s_i as a measure of risk taking at the firm level. With respect to aggregate volatility we look at the size-adjusted volatility of real spending E/P; simple computations show that this does not depend on (s_P, s_L) and is always equal to σ_A. Hence, our model makes no prediction on the evolution of aggregate volatility.

3.2.2 Stock Trading and Prices

At date 2, each investor initially owns a potentially unbalanced portfolio of N_L/I shares of listed firms. Stock trading allows her to rebalance optimally her portfolio.[17] All listed firms being ex-ante symmetric, their equilibrium holding of each stock is exactly $1/I$. The Euler condition of the underlying portfolio choice problem leads to the following standard expression for the stock price of firm i:

$$\rho_i = E\left[U'(\tilde{R}_I)\widetilde{\pi}_i\right]/E\left[U'(\tilde{R}_I)\right] \tag{11}$$

where $\tilde{R}_I$ is date 3 real income of the representative investor (all investors are identical). It is made of the real wage and the dividends paid by stocks owned: $\tilde{R}_I = w/P + (1/I)\int_0^{N_L}\widetilde{\pi}_j dj$. Taking (7) and (8) and N_L being large, we get:

$$\tilde{R}_I = \omega M^{\frac{1}{\sigma-2}} + \frac{N_L}{I}\left[\tilde{A}\left(1+s_L\right)/M - C(s_L)\right] \tag{12}$$

The investor real income can be decomposed into labor income and financial income. A larger aggregate supply shock $\tilde{A}$ increases financial income while market pressure M increases the labor component but decreases the financial component. The financial income is unaffected by idiosyncratic demand shocks $\tilde{d}_i$ because of efficient risk diversification through the stockmarket.

3.2.3 Strategy Choice of Privately Held Firms

We first analyze the strategy choice of private firms. The date 3 real income of the entrepreneur owning private firm i is equal to:

$$\tilde{R}_{E,i} = \tilde{A}(1+s_i\tilde{d}_i)/M - C(s_i) \tag{13}$$

[17] The safe investment is assumed non tradable to simplify exposition.

Entrepreneurs do not supply labor and their income $\tilde{R}_{E,i}$ corresponds to the profits of their firm. It is affected by the idiosyncratic shock $\tilde{d}_i$ because the entrepreneur is underdiversified.[18] At date 1 the entrepreneur chooses her optimal strategy s_p taking the expected market pressure M as given so as to maximize her expected utility $s_p = \arg\max_{s_i} E\left[U(\tilde{R}_{E,i})\right]$. Omitting the index i, the first order condition of this problem can be rewritten as:

$$C'(s_p) = \frac{1}{M} + cov\left[\frac{U'(\tilde{R}_E)}{EU'(\tilde{R}_E)}, \frac{\tilde{A}.\tilde{d}}{M}\right] \tag{14}$$

The optimal s_P equalizes the LHS marginal cost of risk taking to the RHS marginal benefit of risk taking. Benefit is composed of the marginal increase in expected income, *i.e.* $E(\tilde{A}.\widetilde{d}/M) = 1/M$, corrected for the marginal increase in risk exposure. This risk correction term is negative because the marginal increase in revenue, $\tilde{A}.\widetilde{d}/M$, is negatively correlated with marginal utility, $U'(\tilde{R}_E)$: a marginal increase in s is less desirable for a risk averse entrepreneur than for a risk neutral one, because it generates the most income when it is the least needed. We show in appendix A.2 that the LHS (resp. RHS) is increasing (resp. decreasing) in s such that there is one and only one interior solution to equation (14). It is in general not possible to find a closed form solution for s_P except in a CARA-gaussian framework. Yet we can infer the following partial equilibrium property:

Lemma 1 *Risk Taking by private firms, s_P, is a decreasing function of Market Pressure, M*

Proof. See Appendix A.2. ■

The RHS of (14) highlights how market pressure M impacts the marginal benefit of risk taking and consequently the optimal s_P. The overall effect results from three counterbalancing forces. First, an increase in M reduces the expected marginal benefit of raising s_P; this tends to decrease s_p. The second effect goes in the opposite direction: entrepreneur real income being in $\widetilde{A}/M$, an increase in M smooths profits across states of nature and reduces the elasticity of income to risk; this reduces the magnitude of the (negative) risk exposure term and this increases s_P. Last, a larger M reduces real income and therefore, utility being CRRA, this increases absolute risk aversion; this tends to decrease s_p. Lemma 1 shows that the first and third effects dominate the second. Interestingly, it is also possible to show that when the third effect is absent (ie. assuming CARA utility), the second effect dominates the first effect, such that an increase in M tends to increase s_P: in that case there is no reasonable comparative static exercise which is able to generate a pattern of diverging trends in volatility for private and listed firms.[19] Hence, the assumption that absolute risk aversion is decreasing

[18] For instance, Moskowitz and Vissing-Jorgensen (2003) provide evidence of entrepreneurial underdiversification. They document that owners of private stock in the Survey of Consumer Finances place on average half of their wealth in private stocks, most of it corresponding to a single, actively managed firm (see their Table 2, p. 751).

[19] For the analytical results derived in such a CARA-gaussian setup, see Thesmar and Thoenig (2006). Moreover existing evidence from the experimental and empirical literature shows that the assumption of constant absolute risk aversion is not empirically relevant.

will prove crucial to the ability of our model to replicate the opposite volatility trends.

3.2.4 Strategy Choice of Listed Firms

At date 1, the shareholders of listed firms choose an optimal strategy s_L, so as to maximize the date 2 stock price: $s_L \equiv \arg\max_{s_i} \rho_i$ taking the expected market pressure M as given. Omitting the index i and considering (11) and (12), the FOC of this problem is given by:

$$C'(s_L) = \frac{1}{M} + cov\left[\frac{U'(\tilde{R}_I)}{EU'(\tilde{R}_I)}, \frac{\tilde{A}}{M}\right] \tag{15}$$

At the optimum, the marginal cost equals the marginal gain of risk taking which can be decomposed into an expected marginal gain corrected for the marginal increase in risk exposure. The interpretation is thus similar to the private firm case (14) except that now idiosyncratic risk $\tilde{d}$ is completely diversified. Combined with the definition of investors income $\tilde{R}_I$, equation (15) implicitly defines s_L as a function of market pressure M and number of investors I. In appendix A.3, we show that this problem admits a unique interior solution and we establish the following partial equilibrium property:

Lemma 2 *Risk Taking by listed firms s_L is an increasing function of Stockmarket Participation I*

There exists I_0 such that, for all $I > I_0$, Risk Taking by listed firms s_L is unambiguously a decreasing function of Market Pressure M.

Proof. See Appendix A.3. ■

In contrast to the case of private firms, it is not possible to show that s_L is always decreasing in M, unless I is large enough. As for private firms there are three forces. First, a larger M reduces expected marginal benefit of risk taking; second, it also reduces risk exposure; third, it impacts investors' risk aversions but now in an ambiguous way. Indeed from the definition of $\tilde{R}_I$ it appears that a larger M reduces financial income but increases labor income because real wages benefit from labor market competition (see equations (8) and (12)). As a consequence M has a contrasted impact on real income and thus on risk aversion. As noticed in the entrepreneur case, this third effect is crucial to establish that s is decreasing in M. The overall impact is therefore ambiguous. However, for a large number of investors I, the investor real income depends less on financial income and is thus less exposed to aggregate risk. This makes marginal utility constant across states of nature: the risk exposure term in equation (15) shrinks to zero and only the expected marginal gain effect remains significant; this tends to decrease the optimal s_L. In the loglinearized version of the model in Section 4, we are be able to drop the assumption $I > I_0$.

The impact of I on s_L follows from the previous discussion: larger I reduces risk exposure by decreasing the share of financial income in investor total income.

3.3 Equilibrium and Comparative Statics

The general equilibrium is the solution of a system of three equations:

Definition 1 *The equilibrium consists of* (M, s_P, s_L) *such that:*

1. M *depends positively on* s_L *and* s_P *through equation (9)*
2. s_P *depends negatively on* M*, through equation (14)*
3. s_L *depends negatively on* M*, through equation (15)*

This is a rational expectation equilibrium. In period 1, each firm takes the expected M as given, and sets up its optimal strategy choice. Given the above definition, the equilibrium value of market pressure can be written as the solution to the fixed point problem:

$$M = \frac{\sigma}{L} N^{\frac{\sigma-2}{\sigma-1}} \left[1 + \frac{N_L}{N} s_L(\underset{(-)}{M}, \underset{(+)}{I}) + \frac{N_P}{N} . s_P(\underset{(-)}{M}) \right]^{\frac{\sigma-2}{\sigma-1}} \tag{16}$$

where s_L and s_P both decrease in M by virtue of Lemmas 1 and 2. The LHS of this equation corresponds to the 45° line while the RHS is positive and decreasing in M as long as $I > I_0$: hence the equilibrium exists and is unique. From this simple expression, we easily get the following comparative static properties:

Proposition 3 *Assume* $I > I_0$*. Then the equilibrium* (M, s_L, s_P) *exists and is unique. And:*

1. An increase in stock market participation I *leads to an increase in market pressure* M*, an increase in risk taking for listed firms,* s_L*, and a decrease in risk taking for private firms,* s_P

2. An increase in the number of firms N *leads to an increase in* M *and a decrease both in* s_L *and* s_P

3. An increase in market size L *leads to a decrease in* M *and an increase both in* s_L *and* s_P

Proof. See Appendix A.4. ■

Let us begin with parts 2 and 3. An increase in the number of firms N increases market pressure M which in turn tends to decrease risk taking for all firms. Market size L makes profits bigger; this decreases market pressure M and increases risk taking for all firms. In the two cases, risk taking for both categories of firms evolve in the same direction. The general intuition is straightforward: insofar as competition or market size affect both types of firm in the same fashion, they should, in equilibrium, behave in the same way.

Part 1 of proposition 3 focuses on the effect of stock market participation interpreted in our model as an increase in the number of investors I. As I increases, this directly induces listed firms to increase risk taking s_L. This in turn tends to increase competitive pressure M: listed firms face now on average higher demand, they exert more pressure on wages, and reduce profits for other firms. As an indirect consequence, private firms reduce their level of risk taking s_P. This effect is also there for listed firms, but it is dominated by the direct effect.

The above proposition thus suggests that increased stockmarket market participation, more than competition or market size expansion, is a natural candidate to explain the divergence in volatility trends. As it turns out, participation in the US has increased dramatically over the past decade, in large part because of the development of mutual funds and individual retirement accounts such as 401k. For instance, Favilukis (2008) reports that the fraction of households owning stocks directly or indirectly has risen from 33% to 43% between 1982 and 2004 (see also evidence from Guiso et al, 2003). In France, the number of shareowner has gone up from less than two millions in 1978 to more than 6 million in 2006, most of the rise taking place during the mass privatization of the 1980s (NYSE Euronext - SOFRES, 2007).

The above results are, however, obtained under the parameter restriction that $I > I_0$. In addition, it is not possible to obtain closed form solutions that would allow us to fully characterize the equilibrium, and derive more comparative static properties. This is why we approximate the model in the following Section.

4 A closed form version of the model with small shocks

In this section we derive an approximated version of the model by assuming small aggregate and idiosyncratic shocks. Beyond providing closed form solutions for the main endogenous variables (M, s_L, s_P), the benefit of this approach is that it allows to derive additional results, in particular on international risk sharing.

4.1 Loglinearization

Assuming small variations of $(\widetilde{A},\widetilde{d}_i)$ around their means, we loglinearize the system around the deterministic equilibrium which corresponds to the special case where shocks take their mean value: $\widetilde{A} = \widetilde{d}_i = 1$. In the deterministic equilibrium, private and listed firms face similar incentives because the risk adjustment terms in (14)-(15) are both equal to zero; and therefore they choose the same strategy s_0 which satisfies:

$$C'(s_0) = 1/M_0 \tag{17}$$

where market pressure M_0 is given by equation (16):

$$M_0 = \frac{\sigma}{L} N^{(\sigma-2)/(\sigma-1)} \left(1 + s_0\right)^{(\sigma-2)/(\sigma-1)} \tag{18}$$

Since C is convex, it is clear that there exists one and only one deterministic equilibrium (M_0, s_0) which solves (17)-(18).

In the stochastic equilibrium, for each variable x of our model, we denote x_0 its value in the deterministic equilibrium, and $\widehat{x} \equiv (x - x_0)/x_0$ its percentage deviation from x_0. In Appendix B.2 we derive the first order Taylor expansion of the equilibrium (s_L, s_P, M) as defined by equations (14)-(15)-(16) where incomes $(\tilde{R}_E, \tilde{R}_I)$ are given by (12)-(13). We obtain:

$$\begin{aligned} \varepsilon_C \hat{s}_P &= -\hat{M} - \gamma\Omega_P \\ \varepsilon_C \hat{s}_L &= -\hat{M} - \gamma\Omega_L \\ \hat{M} &= m_0 \left(\frac{N_L}{N}\hat{s}_L + \frac{N_P}{N}\hat{s}_P \right) \end{aligned} \tag{19}$$

where γ is relative risk aversion, ε_C is the elasticity of marginal cost C' to s, $m_0 \equiv s_0(\sigma - 2)/(\sigma - 1)$ is a parameter capturing the elasticity of market pressure to risk taking, and (Ω_P,Ω_L) are the *risk exposures* of owners of private and listed firms:

$$\begin{aligned} \Omega_P &= \theta_{E,A} \cdot \sigma_A^2 + \theta_{E,d} \cdot \sigma_d^2 \\ \Omega_L &= \theta_{I,A} \cdot \sigma_A^2 \end{aligned} \tag{20}$$

where $\theta_{E,A} \equiv (1 + s_0)/R_{E0}M_0$; $\theta_{E,d} \equiv s_0/R_{E0}M_0$; $\theta_{I,A} \equiv (N_L/I)(1 + s_0)/R_{I0}M_0$.

The weights $(\theta_{E,A}; \theta_{E,d})$ correspond to the shares of entrepreneur income which are affected by the aggregate and the idiosyncratic shocks; the weight $\theta_{I,A}$ is the share of investor income which is affected by the aggregate shocks. Implicitly we have $\theta_{I,d} = 0$ because investors are fully sheltered from idiosyncratic risk. We also see that $\theta_{E,A} > \theta_{I,A}$: investors, who earn a deterministic labor income, have a smaller exposure to aggregate shocks than entrepreneurs.

Looking at equations (19) and (20), a few interesting features emerge. First, the elasticities of $\widehat{s}_L$ and $\widehat{s}_P$ to $\widehat{M}$ are similar: in the approximated equilibrium, the first order impact of market pressure on risk taking transits only through the expected marginal benefit channel (see the discussion in section (3.2.3) and (3.2.4)). Second, m_0 can be interpreted as the strength of the general equilibrium feedback. Third, both $\widehat{s}_P$ and $\widehat{s}_L$ are decreasing functions of risk exposures (Ω_P,Ω_L) : a reduction in the share of incomes that is exposed to risk generates a direct increase in risk taking.

Inverting the equilibrium system (19), we get the following closed form solutions:

$$\hat{s}_P = \frac{\gamma}{m_0 + \varepsilon_C} \cdot \left[-\Omega_P + \frac{m_0}{\varepsilon_C}\frac{N_L}{N}(\Omega_L - \Omega_P) \right] \tag{21}$$

$$\hat{s}_L = \frac{\gamma}{m_0 + \varepsilon_C} \cdot \left[-\Omega_L + \frac{m_0}{\varepsilon_C}\frac{N_P}{N}(\Omega_P - \Omega_L) \right] \tag{22}$$

$$\hat{M} = -\frac{\gamma m_0}{m_0 + \varepsilon_C} \cdot \left[\frac{N_P}{N}\Omega_P + \frac{N_L}{N}\Omega_L \right] \tag{23}$$

where it appears that risk taking by private firms is a decreasing function of entrepreneurs' risk exposure, but an increasing function of investors' risk exposures. This second effect is channelized by the feedback effect of market pressure on risk taking. This is how our model generates the pattern of opposite trends in volatility for listed and private firms.

4.2 Stock Market Participation

This first proposition summarizes the impact of an increase in stock market participation:

Proposition 4 *A positive shock on stock market participation I :*

1. increases market pressure, M, and risk taking by listed firms s_L; but decreases risk taking by private firms, s_P.

2. increases the real wage and the utility of workers

3. decreases expected real profits for all firms and the utilities of investors and entrepreneurs

Proof. see Appendix B.4. ■

The first point of proposition 4 is already established in proposition 3 except that now the result does not hinge on a condition on I. Regarding points 2 and 3, we know from equation (8) that the real wage is an increasing function of $\hat{M}$. And closed form solution for expected profits is obtained by linearizing expression (7):

$$E\tilde{\pi}_L = E\tilde{\pi}_P = \pi_0 - \frac{1+s_0}{M_0}.\hat{M}$$

where π_0 is profit in the deterministic equilibrium. Since we compute a first order approximation, the envelope theorem applies and wipes out the terms in $(\hat{s}_P, \hat{s}_L)$. Hence the only equilibrium variable that affects expected profits is market pressure $\hat{M}$. Stock market participation (or anything that increases market pressure in our model) reduces real profits of both types of firms to the same extent. Using our French data presented in Section 2, we draw in Figure 3 the evolution of the Return on Assets of the two categories of firms:[20] both groups exhibit the same downward trend, even though we saw earlier that patterns of volatility diverge sharply. This confirms that intuition based on partial equilibrium reasoning may be misleading. Even though listed firms take on more risk to increase their profitability, in general equilibrium, their ROA decreases.

A last outcome of interest is the equity premium. To this purpose we assume that there exists a risk free security with a positive but very small exogenous return r such that investors real income, $\tilde{R}_I$, is still given by (12). In appendix B.3, we loglinearize the stock price (11) and find the following expression for the equity premium:

$$EP = r\gamma.\theta_{I,A}.\theta_{E,A}.\sigma_A^2 \tag{24}$$

[20] We compute ROA by dividing operating income through net assets. Such accounting variables are available from our French dataset because the unit of observation is a firm, with its own financial statement. This constrats with the data exploited by Davis et al (2006), which is plant-level and therefore has no financial statement.

which is, through $\theta_{I,A}$, a decreasing function of stockmarket participation I : as more and more investors can share the aggregate risk, the premium that stocks demand tends toward zero. Thus, besides predicting a diverging pattern in firm volatility, our theory predicts that the equity premium should have declined over the past thirty years so as to compel listed firms to take on more aggregate and idiosyncratic risk. There exists such evidence, at least for the US. Using very different methodologies, Pastor and Stambaugh (2001) and Fama and French (2002) both conclude that the US equity premium has declined by about 100 bp over the past decades.

4.3 The Great Moderation

It is a well established fact that aggregate volatility has declined significantly in most developed countries over the past three decades (see Gali and Gambetti (2008) for a recent survey). The determinants of this "Great Moderation" are still not clear: it could be luck, stabilizing monetary policy (Clarida et al. (2000)) or improvements in inventory management (Kahn et al (2002)). As mentioned in Section 3.2.1, our theory has no predictive power about size adjusted GDP volatility. Hence, we assume hereafter that the decrease in aggregate volatility is exogenous and structural: in our model, this corresponds to an exogenous decline in σ_A^2:

Proposition 5 *A negative shock on aggregate volatility* σ_A^2 *:*

1. increases risk taking by private firms, s_P*, and market pressure,* M

2. increases (resp. decreases) risk taking by listed firms, s_L*, if* $\omega M_0^{\frac{1}{\sigma-2}}$ *is sufficiently large (resp. small).*

3. decreases real profits and increases real wage.

Proof. see Appendix B.4. ■

The predicted effect of a dampening of macroeconomic shocks differs from the impact of stock market participation. In our model, this tends to systematically *increase* risk taking by entrepreneurs, which goes against the evidence presented in this paper and by Davis et al (2006). The effect on listed firms is, however, ambiguous. This comes from two conflicting forces. First, equation (19) makes clear that the direct effect of a reduction in σ_A^2 is to increase risk taking by both listed and private firms. Yet, because entrepreneurs are more exposed to aggregate risk than investors ($\theta_{E,A} > \theta_{I,A}$), private firms increase their level of risk by more than listed firms[21]. Second, since all firms tend to increase risk taking, the resulting increase in market pressure M results in higher wages, and lower expected profits. This reduces incentives to take risk, by the same amount for listed and private held firms. On balance, the first effect always dominates for private firms, but not always for listed ones.

[21] This is here a consequence of the assumption that $N_L < I$: because there are more shareholders than listed firms, the share of risky income for entrepreneurs is larger than what it is for investors.

4.4 IPO Wave

In many developed countries, the fraction of the economy that is listed on the stockmarket has significantly increased. For instance, Fama and French (2004) report that the number of new listings in the US rose from 156 per year in the 1970s, to 549 per year in the 1980-2001 period: this rise in the annual number of IPOs can be related to the creation of NASDAQ in the mid 1970s.[22] In our model, this can be interpreted as an increase in N_L, holding $N = N_P + N_L$ constant.

Proposition 6 *A rise in the number of listed firms N_L, holding the total number of firms $N = N_L + N_P$ constant:*

1. *decreases risk taking by listed firms, s_L*
2. *has an ambiguous effect on risk taking by private firms, s_P, and market pressure, M*

Proof. see Appendix B.4. ■

In our model, an IPO wave has the effect of reducing risk taking by listed firms, which runs opposite to available evidence from the US and France. There are two effects. The first, direct, effect, is that an increase in N_L leads to a reduction in risk sharing, for investors only (Ω_L increases, Ω_P is unaffected). This reduces risk taking in listed firms, while entrepreneurial firms do not change. The second, general equilibrium, effect comes through market pressure and is ambiguous: reduced risk taking tends to dampen market pressure, but the increase in N_L/N increases average risk taking through a composition effect as listed firms are riskier. Hence, the overall effect on market pressure is ambiguous. What proposition 6 demonstrates is that the first effect always dominates for listed firms.

5 The Globalization of Capital Markets

In the preceding analysis, we show that the opposite trends in firm volatility is due to the improvement in risk sharing which follows, within our model, from an increase in stock market participation. Yet, an increase in stock market participation does not increase risk sharing and reduce the equity premium in all models. In practice, Guiso, Haliassos and Jappelli (2003) recall that, since wealth is concentrated, marginal shareholders tend to hold very small portfolios, and do therefore not contribute much to global risk sharing.[23] They even note that participation may *increase* the equity premium: lowering

[22] Similar reforms were undertaken in European countries, as with the creation of the Second Marché and the Nouveau Marché in France, the Frankfurter Neue Markt in Germany, and the London AIM. These reforms had the effect of increasing the number of publicly held firms in the 1990s, albeit to a small extent. Taking a longer view, the picture reverses itself in continental Europe. For instance, Bozio (2003) finds that the number of listed firms in France decreases from about 30 per million inhabitants in the 1950s to only 15 in 2000.

[23] For instance, Kopczuk and Saez (2004) find, using estate tax files, that the top 2% of the wealth distribution in the US owns in 2000 more than 25% of aggregate wealth, and probably a larger fraction of outstanding corporate equity. Given that, according to Guiso et al (2002), 93% of the households present in the top quartile of the wealth distribution already own some equity, additional participation has to come from relatively poor households.

participation costs for example will mechanically add investors that are less wealthy and more risk averse than existing stockholders. Since the equity premium is determined by the preferences of the marginal investors, it may very well increase.

In this Section, we consider a less controversial mechanism to the improvement in risk sharing: capital markets integration. In a recent paper, Chari and Henry (2004) look at the effect of stockmarket liberalizations around the world. They find that, on average, stock prices increase by about 15%. Part of this increase can be explained by a straightforward decline in the risk free rate, reflecting more abundant capital in liberalized stock markets. But approximately two fifth of it (6.8 percentage points) is explained by the fact that domestic firms' stocks covariate much less with the world market portfolio than with the domestic one (by a factor of 200!).

5.1 The Model With Two countries

We therefore expand the baseline model of Section 3 by considering a second, identical country ("foreign"). The world now has $2I$ investors, $2\widetilde{L}$ workers, $2N_L$ listed firms (half of them domestic, half of them foreign) and $2N_P$ privately held firms. To clarify exposition, we assume that investors do not supply labor.[24] The number of goods is now $2N = 2N_P + 2N_L$ and we allow both countries to trade goods; by assumption there are no trading costs. The only ex post difference between the two countries comes from the realization of their aggregate shock on labor supply. Labor supply is $\widetilde{L}_D = \widetilde{A}_D.L$ in the domestic country, and $\widetilde{L}_F = \widetilde{A}_F.L$ in the foreign country. The shocks $\widetilde{A}_D$ and $\widetilde{A}_F$ are identically distributed and assumed uncorrelated to simplify exposition (but this assumption is not necessary). All the computational details are given in Appendix C.

Let us start with period 3. Product and labor markets clear and the profits of a domestic firm operating on the world market is now given by:

$$\tilde{\pi}_{D,i} = \frac{\widetilde{B}_D}{M}.(1 + s_i.\tilde{d}_i) - C(s_i) \tag{25}$$

where the subscript D is an index for domestic firms, and

$$\widetilde{B}_D \equiv \widetilde{A}_D. \left[\frac{1}{2} + \frac{1}{2}(\widetilde{A}_F/\widetilde{A}_D)^{(\sigma-1)/\sigma}\right]^{1/(\sigma-1)}$$

This is very similar to the expression in a closed economy (equation (7)): the only difference is the term of trade effect $\widetilde{A}_F/\widetilde{A}_D$ which affects domestic profits positively (see Coeurdacier (2008) for a discussion). Following a positive relative supply shock in the domestic economy, domestic production

[24] In this model with trade, real wages become random because of the terms-of-trade effect. This makes computations somewhat more cumbersome, although still feasible thanks to the linearisation. In particular, portfolio composition is not symmetric anymore between domestic and foreign stocks, because domestic workers seek to hedge the terms-of-trade effect. We have verified that including labor income in investors' revenues does not affect our results (computations are available from the authors upon request).

and income expand; but foreign and domestic goods being imperfect substitutes, domestic demand for foreign goods increases relatively more than foreign production; so the domestic terms of trade decrease and so do profits.

The aggregate shocks $(\widetilde{A}_D, \widetilde{A}_F)$ being identically distributed, the ex-ante strategy choice s_L (resp. s_P) is similar for listed firms (resp. private) in both countries. As a consequence the equilibrium level of market pressure is now given by:

$$M = \frac{\sigma}{2L}(2N)^{\frac{\sigma-2}{\sigma-1}}\left[1 + \frac{N_L}{N}s_L + \frac{N_P}{N}s_P\right]^{\frac{\sigma-2}{\sigma-1}}$$

Ad date 1, the optimal strategy choices are very close to those in closed economy (see sections 3.2.3 and 3.2.4). Entrepreneurs choose s_P so as to maximize their expected utility:

$$C'(s_P) = \frac{1}{M} + cov\left[\frac{U'(\tilde{R}_{D,E})}{EU'(\tilde{R}_{D,E})}, \frac{\widetilde{B}_D.\widetilde{d}_i}{M}\right]$$

where $\tilde{R}_{D,E} = \tilde{\pi}_{D,i}$ is the income of domestic entrepreneurs at date 3.

And investors choose s_L so as to maximize the stock price:

$$C'(s_L) = \frac{1}{M} + cov\left[\frac{U'(\tilde{R}_I)}{EU'(\tilde{R}_I)}, \frac{\widetilde{B}_D}{M}\right] \tag{26}$$

where $\tilde{R}_I$ is the income of investors in period 3. $\tilde{R}_I$ depends on investors' trading opportunities in date 2, which in turn depends on whether capital markets in the two countries are integrated or not.

5.2 Trade in Goods, No Asset Trade

As a benchmark, we look at the case where there is no trade in assets. Only domestic investors can purchase domestic stocks, and domestic investors cannot purchase foreign stocks. The solution to equation (26) is the same as in Section 3.2.4, at the difference that now the aggregate shock is $\widetilde{B}_D$ instead of $\widetilde{A}_D$. After linearization around the deterministic equilibrium, the equilibrium equations in $\widehat{s}_L, \widehat{s}_P, M$ are identical to (19) except that risk exposures Ω_P and Ω_L are given by:

$$\begin{aligned}\Omega_P &= \theta_{E,A}.\sigma_B^2 + \theta_{E,d}.\sigma_d^2 \\ \Omega_L &= \theta_{I,A}.\sigma_B^2\end{aligned} \tag{27}$$

where $\sigma_B^2 = [1-(2\sigma-1)/2\sigma^2]\sigma_A^2 < \sigma_A^2$. Compared to the closed economy, international trade provides some diversification via the terms of trade effect, since the terms of trade tend to appreciate when domestic productivity is low.

5.3 Trade in Goods and Assets

We first compute the stock price when investors can freely buy foreign stocks. Because aggregate shocks are i.i.d., in equilibrium the representative investor holds half of her wealth in domestic stocks, and half in foreign stocks. Hence, the investor's income is given by:

$$\tilde{R}_I = \frac{N_L}{I}\left[\frac{\widetilde{B}_D + \widetilde{B}_F}{2}\frac{(1+s_L)}{M} - C(s_L)\right]$$

Combined with equations (26) and linearizing around the deterministic equilibrium, the equilibrium equations in $\widehat{s}_L, \widehat{s}_P, M$ are identical to (19) except that now risk exposures Ω_P and Ω_L are given by:

$$\begin{aligned}\Omega_P &= \theta_{E,A}.\sigma_B^2 + \theta_{E,d}.\sigma_d^2 \\ \Omega_L &= \theta_{I,A}.\frac{\sigma_B^2 + \sigma_{D,F}}{2}\end{aligned} \tag{28}$$

where $\sigma_{D,F}$ is the covariance between trade adjusted country shocks $\widetilde{B}_D$ and $\widetilde{B}_F$. Since these two shocks are not perfectly correlated, it is easy to deduce that $(\sigma_B^2 + \sigma_{D,F})/2 < \sigma_B^2$.

Proposition 7 *Capital market integration :*

1. *increases risk taking by listed firms, s_L, and market pressure, M*
2. *decreases risk taking by private firms, s_P*
3. *increases the real wage and the utility of workers*
4. *decreases real profits and the utilities of investors and entrepreneurs*

The effects at work are the same as for stock market participation. Capital market integration enhances risk sharing among investors, which increases listed firms risk taking. Market pressure increases and this reduces profits: this forces non listed firms to scale back on their risk taking.

5.4 Calibration

The change in size adjusted volatilities v_P and v_L that results from capital markets integration can easily be computed by linearizing equation (10) and plugging it into expression (28). We obtain:

$$\begin{aligned}\Delta v_L &= \lambda.\gamma.v_0\frac{1}{m_0+\varepsilon_C}.\left(1+\frac{N_P}{N}.\frac{m_0}{\varepsilon_C}\right).\theta_{I,A}.\frac{\sigma_B^2}{2}.(1-\rho) \\ \Delta v_P &= -\lambda.\gamma.v_0\frac{1}{m_0+\varepsilon_C}.\left(1+\frac{N_L}{N}.\frac{m_0}{\varepsilon_C}\right).\theta_{I,A}.\frac{\sigma_B^2}{2}.(1-\rho)\end{aligned} \tag{29}$$

where ρ is the correlation between $\widetilde{B}_D$ and $\widetilde{B}_F$; the parameter v_0 is the size adjusted volatility as given by equation (10) with $s \equiv s_0$, and $\lambda = [s_0/(1+s_0)] \times \left[1-\left(\sigma_A^2/v_0^2\right)\right]$.

From the above expression, it is clear that:

$$\frac{-\Delta v_P}{\Delta v_L} = \frac{\varepsilon_C/m_0 + N_L/N}{\varepsilon_C/m_0 + N_P/N} \tag{30}$$

In the US, Moskowitz and Vissing-Jorgensen (2002) estimate that the market value of listed firms is larger than the value of privately held ones. If this is the case, equation (30) predicts a decrease of privately held firm volatility that is larger in magnitude than the observed increase in listed volatility. This is consistent with evidence from Davis et al (2006): for instance, their Figure 5 reports an increase in listed firm volatility by some 5 percentage points, along with a decrease of private firm volatility by nearly 10 percentage points. The size of the divergence is smaller in France (see Section 2), which is consistent with the fact that the share of listed equity in total corporate equity is only about 30% in this country (Banque of France data allow to separate out listed and non listed equity).

To see whether improvement in risk sharing has a sizeable impact on firms risk taking, a simple calibration exercise can be implemented on $\Delta v_L - \Delta v_P$. From equations (29), we obtain that:

$$\frac{\Delta v_L - \Delta v_P}{v_0} = \frac{\lambda}{\varepsilon_C}.\gamma.\theta_{I,A}.\frac{\sigma_B^2}{2}.(1-\rho)$$

Regarding relative risk aversion we assume $\gamma = 4$. From Imbs (2004), we take for aggregate volatility $\sigma_B^2 = 0.25\%$ (the mean standard deviation of GDP growth is 5% in his data) and for the mean correlation across GDP growths we take $\rho = 0.2$. From the Federal Reserve's Flows of Funds data, we obtain that directly and indirectly held equities are approximately 30% of US households' net worth (see Table B100e), so $\theta_{I,A} = 30\%$. This leads to

$$\frac{\Delta v_L - \Delta v_P}{v_0} = \frac{\lambda/\varepsilon_C}{1000} \tag{31}$$

From these figures, it appears clearly that the cost function C needs to be almost linear (*ie.* the elasticity $\varepsilon_C \tilde{} 0$) to ensure that $(\Delta v_L - \Delta v_P)/v_0$ responds significantly to capital markets integration.

It is possible to roughly calibrate λ/ε_C using our microeconomic estimations (in section 2.3) where we find that a switch from "being closely held" to "being more widely held" has on average a positive impact on volatility of approximately 10%. In our theoretical model, such a switch corresponds to the differential of size adjusted volatility between listed and private firms: $(v_L - v_P)/v_0 = 10\%$.

Linearizing (10) and taking $\hat{M}$ as given, this leads to:

$$\begin{aligned} \frac{v_L - v_P}{v_0} &= \lambda.\left(\widehat{s}_L - \widehat{s}_P\right) \qquad (32) \\ &= \frac{\lambda}{\varepsilon_C}.\gamma.\left[\theta_{E,d}.\sigma_d^2 + \left(\theta_{E,A} - \theta_{I,A}\right).\sigma_A^2\right] \end{aligned}$$

Volatility increases because there is less exposure both to idiosyncratic risk and aggregate risk. From Moskowitz and Vissing-Jorgensen (2002), we take $\theta_{E,d} = \theta_{E,A} = 50\%$. As above we set $\theta_{I,A} = 30\%$ and $\sigma_A^2 = 0.5\%$. Last we assume that idiosyncratic risk has the same magnitude than aggregate risk $\sigma_d^2 = 0.25\%$ (though there is no easy way of calibrating this last parameter). From our empirical estimates we know that $(v_L - v_P)/v_0 = 10\%$. Together with (32) we deduce that:

$$\lambda/\varepsilon_C \simeq 14 \tag{33}$$

Combining (31) and (33) this simple calibration exercise suggests that capital market integration, in our model, should be responsible for a divergence in volatility by about 1.4% of the initial volatility. This accounts for half of the divergence found in our French data (see Figure 2).

6 Conclusion

This paper is motivated by the fact that volatility of listed and non listed firms have experienced opposite trends over the past few decades. Our starting point, that we first convincingly test on a panel data of French firms, relies on the insight of the development literature: risk sharing among investors should promote corporate risk taking. We extend existing models by (1) including a class of firms that do not benefit from risk sharing, and (2) modelling product market competition. We find that an increase in risk sharing, through capital market integration or rising stock market participation, can generate opposite trends in volatility for private and listed firms. The model is also used to investigate the impact of alternative determinants of firm volatility, such as an increase in product market size, in the number of firms, in the fraction of listed firms or a decrease in aggregate volatility. All these alternative comparative statics generate counterfactual trends in firm volatility.

We do not argue that there was no improvement in risk sharing among non listed firms owners. The private equity market has been incredibly dynamic over the past 10 years (Kaplan and Stromberg, 2008): many firms that were family controlled are now owned by funds whose investors (institutions) are well diversified. We believe that this movement is more recent than the liberalization of stock market and capital market. But this is gathering strength as time goes by. Privately held firms may indeed become riskier, possibly reversing the opposite trends that this paper seeks to explain. We plan on studying these new developments in future research.

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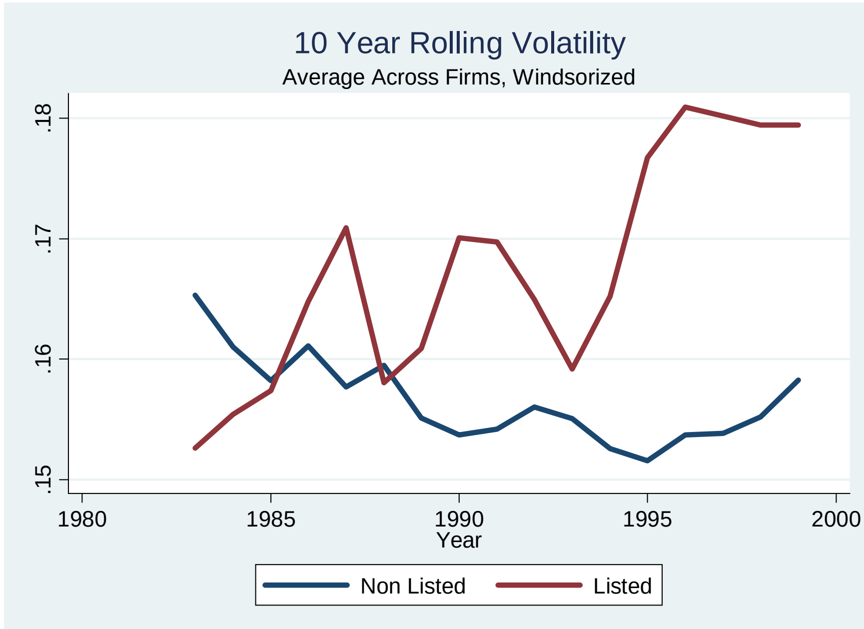

Figure 1: Uncertainty in Listed and Non Listed Firms: Rolling standard Deviation

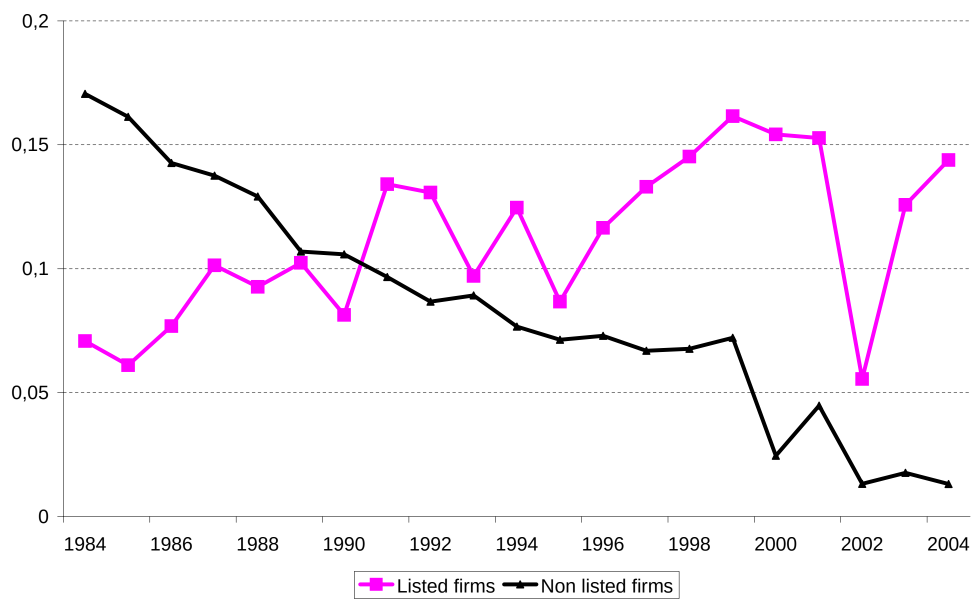

Figure 2: Sensitivity to Industry Shocks in Public and Private Firms

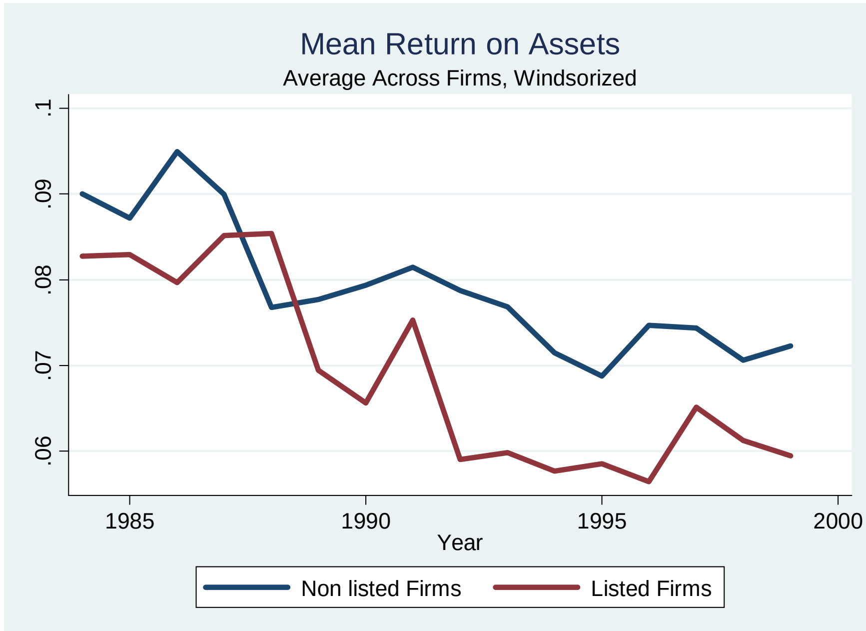

Figure 3: Mean Profitability: Comparing Listed and Non Listed Firms

Table 1: Sales Response to an Industry Shock and Ownership Concentration

	log(Sales)			log(Employment)		
	Model 1	Model 2	Model 3	Model 1	Model 2	Model 3
$\log\widehat{\text{sales}}_{st}\times$	-	-0.07***	-0.41***	-	-0.06***	-0.50***
Known indiv. ownership $> 50\%$		(0.01)	(0.11)		(0.01)	(0.10)
$\log\widehat{\text{sales}}_{st}$	0.09***	0.05***	0.10***	-0.06***	-0.12**	-0.08***
	(0.01)	(0.01)	(0.02)	(0.01)	(0.01)	(0.02)
Known individual	-	1.31***	11.02***	-	1.06***	12.93***
ownership $> 50\%$		(0.24)	(2.28)		(0.25)	(2.07)
$\log(\text{sales}_{i0})\times\log\widehat{\text{sales}}_{st}$	No	No	Yes	No	No	Yes
Industry$\times\log\widehat{\text{sales}}_{st}$	No	No	Yes	No	No	Yes
Firm FE, Year FE	Yes	Yes	Yes	Yes	Yes	Yes
Observations	106,998	54,589	47,763	104,310	53,399	47,633

Source: Tax files and Financial relation survey (INSEE) over the 1984-1999 period. The dependant variable is the logarithm of the firm sales in columns 1-3 and firm employment in columns 4-6. $\log\widehat{\text{sales}}_{st}$ stands for industry-level sales, excluding firm i 's own sales. In columns 3 and 6, the ownership concentration dummy is first regressed on firm log sales and 2 digit industry dummies. Then, the residual of this regression is interacted with industry sales. Standard errors correct for observation level heteroskedasticity using the White's method.

Table 2: Ownership Concentration and Risk Taking

	Model 1	Model 2	Model 3
Panel A: Dependent variable = Sales growth residual ($\times$100)			
Direct ownership	-1.7***	-0.5**	-0.5**
of known individuals > 50%	(0.2)	(0.2)	(0.2)
log(sales)	-	-	-1.8***
			(0.2)
Year FE	Yes	Yes	Yes
Firm FE	No	Yes	Yes
Observations	60,120	60,120	60,120
Panel B: Dependent Variable = Employment residual ($\times$100)			
Direct ownership	-0.5**	-0.6***	-0.6***
of known individuals > 50%	(0.1)	(0.2)	(0.2)
log(sales)	-	-	-0.9***
			(0.1)
Year FE	Yes	Yes	Yes
Firm FE	No	Yes	Yes
Observations	58,184	58,184	58,184

Source: Tax files and Financial relation survey (INSEE) on 1984 - 2004. The dependent variable is obtained through the following procedure. First, we run

$$Y_{it} = \alpha_i + POST_t + POST_t \times Listed_{it} + \varepsilon_{it}$$

where POST is equal 1 after 1990. Second, we take $\nu_{it} = |\varepsilon_{it}|$ as our measure of volatility. In panel A, Y_{it} is the annual sales growth. In panel B, Y_{it} is annual employment growth.

A Proofs in the Benchmark Model

A.1 A Useful Lemma

We first demonstrate a Lemma that will be used in most of the proofs of the baseline model

Lemma 8 *Assume $h(\tilde{A}, \widetilde{d})$ is positive and strictly increasing in $\tilde{A}$ and weakly increasing in $\widetilde{d}$.*

Furthermore, assume that either (1) $\partial f(\tilde{A}, \widetilde{d})/\partial d = 0$, or (2) $f(\tilde{A}, \widetilde{d})$ is such that there exists a unique $d^*(\tilde{A})$ such that $0 = f(\tilde{A}, d^*(\tilde{A}))$.

Last assume that $\partial f(\tilde{A}, d)/\partial \tilde{A} = -f(A^*, \tilde{d})h\left(A^*, d\right)/A^*$.

Then: $E\left[f(\tilde{A}, \widetilde{d})\right] = 0 \Rightarrow E\left[h(\tilde{A}, \widetilde{d}) \times f(\tilde{A}, \widetilde{d})\right] > 0$

Proof. Assume first that $\partial f(\tilde{A}, \widetilde{d})/\partial d \neq 0$. By assumption, for each $\tilde{A}$:

$$\begin{cases} \tilde{d} \leq d^*(\tilde{A}) \Rightarrow f(\tilde{A}, \tilde{d}) \leq 0 \\ \tilde{d} > d^*(\tilde{A}) \Rightarrow f(\tilde{A}, \tilde{d}) > 0 \end{cases}$$

and since $h(\tilde{A}, \tilde{d})$ is decreasing in $\tilde{d}$, we can deduce that, $\forall \tilde{A}$,

$$\begin{cases} \tilde{d} \leq d^*(\tilde{A}) \Rightarrow f(\tilde{A}, \tilde{d})h\left(\tilde{A}, \tilde{d}\right) \leq f(\tilde{A}, \tilde{d})h(\tilde{A}, d^*(\tilde{A}))) \\ \tilde{d} > d^*(\tilde{A}) \Rightarrow f(\tilde{A}, \tilde{d})h\left(\tilde{A}, \tilde{d}\right) \leq f(\tilde{A}, \tilde{d})h(\tilde{A}, d^*(\tilde{A}))) \end{cases}$$

As a consequence it is easy to see that, $\forall \tilde{A}$,:

$$E\left[f(\tilde{A}, \tilde{d})h\left(\tilde{A}, \tilde{d}\right) \mid \tilde{A}\right] \leq \hat{f}(\tilde{A})h(\tilde{A}, d^*(\tilde{A})) \tag{34}$$

where $\hat{f}(\tilde{A}) \equiv E\left[f(\tilde{A}, \tilde{d}) \mid \tilde{A}\right]$.

At this stage we need to show that there exists a unique A^* such that

$$\tilde{A} \leq A^* \Longleftrightarrow \hat{f}(\tilde{A}) \leq 0 = \hat{f}(A^*)$$

At least one A^* exists, by virtue of the intermediate value theorem and the fact that, by assumption, $E\left[\hat{f}(\widetilde{A})\right] = 0$. This A^* is unique because $\hat{f}$ is locally increasing in $\tilde{A}$ in A^*. To see why, we compute the first derivative of $\hat{f}$:evaluated in A^*

$$\frac{d\hat{f}}{dA}(A^*) = \int \frac{\partial f(A^*, d)}{\partial A} dG_d = -\frac{1}{A^*} \int f(A^*, d)h\left(A^*, d\right) dG_d$$

by assumption. Then:

$$\begin{aligned} \frac{d\hat{f}}{dA}(A^*) &\geq -\frac{1}{A^*}\left[\int_{d \leq d^*(A^*)} f(A^*, d)h\left(A^*, d\right) dG_d + \int_{\tilde{d} > d^*(A^*)} f(A^*, d)h\left(A^*, d\right) dG_d\right] \\ &\geq -\frac{h^*}{A^*} \int f(A^*, \tilde{d}) dG_d = -\frac{h^*}{A^*}\hat{f}(A^*) = 0 \end{aligned}$$

where $h^* \equiv h(A^*, d^*(A^*))$. As a consequence $\hat{f}$ is locally increasing function around A^*, thus A^* is unique.

Coming back to (34), the existence and unicity of A^* implies that:

$$\begin{cases} \tilde{A} \leq A^* \Rightarrow \hat{f}(\tilde{A}) \leq 0 \Rightarrow \hat{f}(\tilde{A}).h(\tilde{A}, d^*(\tilde{A})) \leq \hat{f}(\tilde{A})h^* \\ \tilde{A} \geq A^* \Rightarrow \hat{f}(\tilde{A}) \geq 0 \Rightarrow \hat{f}(\tilde{A}).h(\tilde{A}, d^*(\tilde{A})) \leq \hat{f}(\tilde{A})h^* \end{cases}$$

Hence, we have an upper bound for $E\left[f(\tilde{A}, \tilde{d}).h\left(\tilde{A}, \tilde{d}\right)\right]$:

$$E\left[f(\tilde{A}, \tilde{d})h\left(\tilde{A}, \tilde{d}\right)\right] \leqslant h^*.E\left[\hat{f}(\tilde{A})\right] = h^*.E\left[f(\tilde{A}, \tilde{d})\right] = 0$$

which proves the Lemma if $\partial f(\tilde{A}, \tilde{d})/\partial d \neq 0$.

If $\partial f(\tilde{A}, \tilde{d})/\partial d = 0$, the proof is the same, except that $\hat{f} = f$ so the first part is irrelevant (until equation (34)). QED ■

A.2 Proof of Lemma 1

The FOC (14) may be rewritten:

$$0 = E\left[U'\left(\tilde{R}_E\right).\left(\frac{\tilde{A}.\tilde{d}_i}{M} - C'(s_P)\right)\right] \tag{35}$$

This maximization problem is well defined because the SOC of this problem is negative:

$$SOC \equiv E\left[-C''(s_P)U'\left(\tilde{R}_E\right) + \left(\frac{\tilde{A}.\tilde{d}_i}{M} - C'(s_P)\right)^2 U''\left(\tilde{R}_E\right)\right] < 0 \tag{36}$$

Since $C'(0) = 0$, the first order derivative is strictly positive in $s_P = 0$ and negative for large s_P. This ensures the existence of an interior solution.

Given the SOC, the first derivative of s_P w.r.t. M has the same sign as:

$$\Delta \equiv \underbrace{-\frac{C'(s_P)}{M} EU'\left(\tilde{R}_E\right)}_{<0} - \frac{\gamma}{M} E\left[\underbrace{U'\left(\tilde{R}_E\right)\left(\frac{\tilde{A}.\tilde{d}_i}{M} - C'(s_P)\right)}_{\equiv f(\tilde{A},\tilde{d})} . \underbrace{\left(\frac{1}{\tilde{R}_E}\frac{d\tilde{R}_E}{dM}\right)}_{\equiv I(\tilde{A},\tilde{d})}\right]$$

The first component in Δ is negative. It is straightforward to show that f and I satisfy the conditions required by Lemma 8. This proves the result.■

A.3 Proof of Lemma 2

Given that $\tilde{R}_I$ is not affected by idiosyncratic risk, the FOC (15) may be rewritten as:

$$0 = E\left[U'\left(\tilde{R}_I\right)\left(\frac{\tilde{A}.\tilde{d}_i}{M} - C'(s_L)\right)\right] \tag{37}$$

the SOC of this problem is satisfied since $C'' > 0$. Since $C'(0) = 0$, the first order derivative is strictly positive in $s_L = 0$ and negative for large s_L. This ensures the existence of an interior solution. It also ensures that profits are never negative: if they are in some states of nature, expected utility is equal to $-\infty$. What ensures that it can be greater than $-\infty$ is that it is positive for $s_L = 0$.

Step 1.

We first show that s_L is increasing in I. Given the second order condition of this problem, it is the case if and only if:

$$\Omega \equiv \frac{\partial}{\partial I}\left\{E\left[U'\left(\tilde{R}_I\right)\left(\frac{\tilde{A}.\widetilde{d}_i}{M}-C'(s_L)\right)\right]\right\}>0$$

rewriting Ω we find:

$$\Omega=\gamma E\left[\underbrace{U'\left(\tilde{R}_I\right)\left(\frac{\tilde{A}.\widetilde{d}_i}{M}-C'(s_L)\right)}_{\equiv f(\tilde{A})}.\underbrace{\left(-\frac{1}{\tilde{R}_I}.\frac{d\tilde{R}_I}{dI}\right)}_{\equiv I(\tilde{A})}\right]$$

It is easy to see that $I(\tilde{A})$ is positive and increasing in $\tilde{A}$. It is easy to see that f satsifies the properties required by Lemma 8. From the first order condition, we know that $Ef(\tilde{A})=0$, which implies that $\Omega>0$. This proves the first point of Lemma 2.

Step 2.

We then look at the conditions under which s_L is decreasing in M. Given the SOC of the problem, it is the case if and only if:

$$\Omega \equiv -\frac{1}{M^2}E\left[U'\left(\tilde{R}_I\right).\tilde{A}\right]+\gamma.E\left[U'\left(\tilde{R}_I\right)\left(\frac{\tilde{A}.\widetilde{d}_i}{M}-C'(s_L)\right).f(\widetilde{A})\right]<0$$

is negative. Notice that:

$$\begin{aligned} f(\widetilde{A}) &= \frac{1}{M}.\frac{N_L}{I}.\left(\frac{\tilde{A}.\left(1+s\widetilde{d}_i\right)}{M}\right).\left[\omega M^{\frac{1}{\sigma-2}}+\frac{N_L}{I}.\left(\frac{\tilde{A}.\left(1+s_i\widetilde{d}_i\right)}{M}-C(s_L)\right)\right]^{-1} \\ &\quad -\frac{1}{\sigma-2}.\frac{1}{M}.\omega M^{\frac{1}{\sigma-2}}.\left[\omega M^{\frac{1}{\sigma-2}}+\frac{N_L}{I}.\left(\frac{\tilde{A}.\left(1+s_i\widetilde{d}_i\right)}{M}-C(s_L)\right)\right]^{-1}\end{aligned}$$

as I goes to infinity:

$$f(\widetilde{A})=-\frac{1}{\sigma-2}.\frac{1}{M}$$

thus:

$$\begin{aligned}\Omega &= -\frac{1}{M^2}E\left[U'\left(\tilde{R}_I\right).\tilde{A}\right]-\frac{1}{\sigma-2}.\frac{1}{M}E\left[I(\widetilde{A})\right] \\ &= -\frac{1}{M^2}E\left[U'\left(\tilde{R}_I\right).\tilde{A}\right]<0\end{aligned}$$

when I goes to infinity. By continuity, there exists an I_0 such that $I>I_0 \Rightarrow \Omega<0$ which proves the proposition.■

A.4 Proof of Proposition 3

First, notice that M is the solution of the fixed point problem (16). Given Lemmas 2 and 1 hold, the right hand side of (16) is decreasing in M. Hence, the equilibrium M is unique and can be thought of as the intersection of the 45° line and the RHS of (16). We now prove the three points of Proposition 3.

1. When ϕ increases, the RHS of equation (16) shifts up. This ensures that M increases. s_P depends on M only, so it decreases. If s_L was also decreasing, then M would also increase which leads to a contradiction: s_L therefore increases.

2 and 3. Assume that N increases of L decreases: the RHS of (16) shifts upward. The equilibrium M increases, which reduces both s_P and s_L.■

B Linearising the Closed Economy Model

We loglinearize around their deterministic values the two Euler conditions (14)-(15) and the general equilibrium equation (16).

B.1 Log Linearizing Euler Equations: The General Case

Both Euler conditions take the following form $E\left[F(\tilde{\boldsymbol{\theta}}, \mathbf{x})\right] = 0$ where $F(.)$ is differentiable, x is the vector of endogenous variables (s_P, s_L, M), $\tilde{\boldsymbol{\theta}}$ is the stochastic vector $(\tilde{A}, \tilde{d})$ which is distributed in the neighborhood of its mean $\boldsymbol{\theta}_0 = (1, 1)$. A second order-Taylor expansion of the Euler condition in $\tilde{\boldsymbol{\theta}}$ around $\boldsymbol{\theta}_0$ leads to:

$$
\begin{aligned}
0 &= E\left[F(\tilde{\boldsymbol{\theta}}, \mathbf{x})\right] \\
&\simeq E\left[F(\boldsymbol{\theta}_0, \mathbf{x}) + \sum_i (\tilde{\theta}_i - \theta_{i,0}) \frac{\partial F(\boldsymbol{\theta}_0, \mathbf{x})}{\partial \tilde{\theta}_i} + \frac{1}{2} \sum_{i,j} (\tilde{\theta}_i - \theta_{i,0})(\tilde{\theta}_j - \theta_{j,0}) \frac{\partial^2 F(\boldsymbol{\theta}_0, \mathbf{x})}{\partial \tilde{\theta}_i \partial \tilde{\theta}_j}\right] \\
&= F(\boldsymbol{\theta}_0, \mathbf{x}) + \sum_{i,j} \frac{\sigma_{i,j}}{2} \frac{\partial^2 F(\boldsymbol{\theta}_0, \mathbf{x})}{\partial \tilde{\theta}_i \partial \tilde{\theta}_j}
\end{aligned}
$$

where $\sigma_{i,j}$ corresponds at the variance-covariance terms. Then we develop at the *first-order only* this equation in $\mathbf{x}$ around $\mathbf{x_0}$ and we find

$$
0 = F(\boldsymbol{\theta}_0, \mathbf{x}_0) + \sum_k (x_k - x_{k,0}) \frac{\partial F(\boldsymbol{\theta}_0, \mathbf{x}_0)}{\partial x_k} + \sum_{i,j} \frac{\sigma_{i,j}}{2} \frac{\partial^2 F(\boldsymbol{\theta}_0, \mathbf{x}_0)}{\partial \tilde{\theta}_i \partial \tilde{\theta}_j} + \sum_k (x_k - x_{k,0}) \frac{\sigma_{i,j}}{2} \frac{\partial^3 F(\boldsymbol{\theta}_0, \mathbf{x}_0)}{\partial \tilde{\theta}_i \partial \tilde{\theta}_j \partial x_k}
$$

In the previous equation the terms $(x_k - x_{k,0})\sigma_{i,j}$ are dominated by the terms in $(x_k - x_{k,0})$ and $\sigma_{i,j}$, so we can ignore them. This also justifies why a first order expansion in $\mathbf{x}$ is sufficient while a second order expansion in $\tilde{\boldsymbol{\theta}}$ is necessary (Tille and Van Wincoop, 2007). Moreover from the deterministic equilibrium FOC we get $F(\boldsymbol{\theta}_0, \mathbf{x}_0) = 0$. This leads to the following approximated Euler equation:

$$
\sum_k x_{k,0}.\hat{x}_k.F'_{x_k}(\boldsymbol{\theta}_0, \mathbf{x}_0) = -\frac{1}{2}. \sum_{i,j} \sigma_{i,j} F''_{\tilde{\theta}_i \tilde{\theta}_j}(\boldsymbol{\theta}_0, \mathbf{x}_0) \tag{38}
$$

which also shows why the log deviation in equilibrium variables $\hat{x}_k$ are of the order of the variances $\sigma_{i,j}$ (hence second order in the log deviation of $\widetilde{\theta}$).

B.2 Linearising Equilibrium Conditions (14)-(15)-(16)

Step 1.

We start with the Euler equation of privately held firms:

$$F(\theta, \mathbf{x}) \equiv U'\left(\frac{\tilde{A}.\left(1+s_P.\tilde{d}_i\right)}{M} - C(s_P)\right).\left(\frac{\tilde{A}.\tilde{d}_i}{M} - C'(s_P)\right)$$

By definition, $1/M_0 - C'(s_0) = 0$ and $R_{E,0} = (1+s_0)/M_0 - C(s_0)$. Hence the derivatives of F simplify into:

$$\begin{aligned}
F'_s(\boldsymbol{\theta}_0, \mathbf{x}_0) &= -C''(s_0).U'\left(R_{E,0}\right) \\
F'_M(\boldsymbol{\theta}_0, \mathbf{x}_0) &= -\frac{1}{M_0^2}.U'\left(R_{E,0}\right)
\end{aligned}$$

and

$$\begin{aligned}
F''_{AA}(\boldsymbol{\theta}_0, \mathbf{x}_0) &= 2.\frac{1+s_0}{M_0^2}.U''(R_{E,0}) \\
F''_{dd}(\boldsymbol{\theta}_0, \mathbf{x}_0) &= 2.\frac{s_0}{M_0^2}.U''(R_{E,0})
\end{aligned}$$

Using formula (38) and the fact that $\sigma_{Ad} = 0$, we find that:

$$U'\left(R_{E,0}\right).\left(C''(s_0).s_0.\widehat{s} + \frac{\widehat{M}}{M_0}\right) = \frac{U''\left(R_{E,0}\right)}{M_0}.\left(\frac{1+s_0}{M_0}.\sigma_A^2 + \frac{s_0}{M_0}.\sigma_d^2\right)$$

given that $C'(s_0) = 1/M_0$ and rearranging we find:

$$\left(\varepsilon_C.\widehat{s} + \widehat{M}\right) = -\gamma.\underbrace{\left(\frac{1}{R_{E,0}}.\frac{1+s_0}{M_0}.\sigma_A^2 + \frac{1}{R_{E,0}}.\frac{s_0}{M_0}.\sigma_d^2\right)}_{\equiv \Omega_P}$$

with $\varepsilon_C = C''(s_0).s_0/C'(s_0)$.

Step 2.

The derivation of $\widehat{s}_L$ is similar and we skip it to save space.

Step 3.

Equilibrium on the labor market is given by (16):

$$\begin{aligned}
M_0.\left(1+\widehat{M}\right) &= \frac{\sigma}{L}N^{\frac{\sigma-2}{\sigma-1}}\left[1+s_0+s_0\frac{N_L}{N}\widehat{s}_L + s_0\frac{N_P}{N}\widehat{s}_P\right]^{\frac{\sigma-2}{\sigma-1}} \\
&\simeq \underbrace{\frac{\sigma}{L}N^{\frac{\sigma-2}{\sigma-1}}.\left(1+s_0\right)^{\frac{\sigma-2}{\sigma-1}}}_{\equiv M_0}.\left[1+\underbrace{\frac{\sigma-2}{\sigma-1}\frac{s_0}{1+s_0}}_{\equiv m_0}\left(\frac{N_L}{N}\widehat{s}_L + \frac{N_P}{N}\widehat{s}_P\right)\right]
\end{aligned}$$

which leads to the expression of $\widehat{M}$ in the text.

B.3 Additional Equilibrium Variables

Step 1.

Expected profits of entrepreneurs are given by:

$$\begin{aligned} E\tilde{\pi}_P &= \frac{1}{M}(1+s_P) - C(s_P) \\ &\simeq \frac{1+s_0}{M_0}.\left(1-\widehat{M}\right).\left(1+\frac{s_0}{1+s_0}\widehat{s}_P\right) - C(s_0) - C'(s_0)s_0.\widehat{s}_P \\ &\simeq \pi_0 + \widehat{s}_P.s_0\underbrace{\left(\frac{1}{M_0} - C'(s_0)\right)}_{=0 \text{ by definition}} - \frac{1+s_0}{M_0}.\widehat{M} \end{aligned}$$

which proves the result. The computation for $E\tilde{\pi}_L$ is identical.

Step 2.

The expected utility of entrepreneurs is given by:

$$EU(\tilde{\pi}_P) \simeq U(E\tilde{\pi}_P) + \frac{1}{2}.U''(E\tilde{\pi}_P).V\tilde{\pi}_P$$

with:

$$\tilde{\pi}_P \simeq \pi_0 - \frac{1+s_0}{M_0}.\widehat{M} + \frac{1+s_0}{M_0}.\widehat{A} + \frac{s_0}{M_0}.\widehat{d} + \frac{s_0}{M_0}.\widehat{A}.\widehat{d}$$

hence:

$$V\tilde{\pi}_P = \left(\frac{1+s_0}{M_0}\right)^2 \sigma_A^2 + \left(\frac{s_0}{M_0}\right)^2 .\sigma_d^2$$

Plugging $V\tilde{\pi}_P$ and $E\tilde{\pi}_P$ back into the utility formula we get:

$$\begin{aligned} EU(\tilde{\pi}_P) &\simeq U(\pi_0) - U'(\pi_0)\frac{1+s_0}{M_0}.\widehat{M} \\ &\quad -\frac{1}{2}.U''(E\tilde{\pi}_P).\left[\left(\frac{1+s_0}{M_0}\right)^2 \sigma_A^2 + \left(\frac{s_0}{M_0}\right)^2 .\sigma_d^2\right] \\ &\simeq U(\pi_0).\left[\begin{array}{c} 1-(1-\gamma).\frac{1}{R_{E,0}}.\frac{1+s_0}{M_0}.\widehat{M} \\ -\frac{1}{2}\gamma(1-\gamma)\left[\left(\frac{1}{R_{E,0}}.\frac{1+s_0}{M_0}\right)^2 .\sigma_A^2 + \left(\frac{1}{R_{E,0}}.\frac{s_0}{M_0}\right)^2 .\sigma_d^2\right] \end{array}\right] \end{aligned}$$

the differentiation of the other expected utilities follows similar lines and we do not report them to save space.

Step 3.

The equity premium is derived from the asset pricing condition (11):

$$\lambda\rho = E\left[U'\left(\widetilde{R}_I\right).\widetilde{\pi}_L\right]$$

To fix λ, we use the demand for safe asset:

$$\lambda = E\left[U'\left(\widetilde{R}_I\right).r\right]$$

hence, the equilibrium price is given by:

$$E\left[U'\left(\widetilde{R}_I\right).\left(\widetilde{\pi}_L - r\rho\right)\right] = 0$$

Given that the supply of safe asset is negligible, investor's income is still given by:

$$\begin{aligned}
\widetilde{R}_I &= \omega M^{\frac{1}{\sigma-2}} + \frac{N_L}{I}.\left(\frac{\widetilde{A}.\left(1+s_L\right)}{M} - C(s_L)\right) \\
&\simeq \omega M_0^{\frac{1}{\sigma-2}} + \frac{\omega}{\sigma-2} M_0^{\frac{1}{\sigma-2}}.\widehat{M} + \frac{N_L}{I}.\left(\pi_0 - \frac{1+s_0}{M_0}.\widehat{M} + \frac{1+s_0}{M_0}.\widehat{A}\right) \\
&\simeq \underbrace{\left(\omega M_0^{\frac{1}{\sigma-2}} + \frac{N_L}{I}.\pi_0\right)}_{\equiv R_{I,0}} \left[1 - \underbrace{\frac{1}{R_{I,0}}.\left(\frac{\omega}{\sigma-2} M_0^{\frac{1}{\sigma-2}} - \frac{N_L}{I}.\frac{1+s_0}{M_0}\right)}_{\equiv \frac{1}{\sigma-2}\theta_{I,W} - \theta_{I,A}}.\widehat{M} + \frac{1}{R_{I,0}}.\frac{N_L}{I}.\frac{1+s_0}{M_0}.\widehat{A}\right]
\end{aligned}$$

Differentiating the asset pricing condition:

$$\begin{aligned}
0 &\simeq E\left[\left(U'\left(R_{I,0}\right) + U''\left(R_{I,0}\right).\left(R_I - R_{I,0}\right)\right).\left(\pi_0.\widehat{\pi}_L - r.\rho_0.\widehat{\rho}\right)\right] \\
&\simeq E\left[\left(1 + \gamma\left(\frac{1}{\sigma-2}\theta_{I,W} - \theta_{I,A}\right).\widehat{M} - \gamma.\theta_{I,A}.\widehat{A}\right).\left(\pi_0.\widehat{\pi}_L - r.\rho_0.\widehat{\rho}\right)\right] \\
&\simeq E\left[\pi_0.\widehat{\pi}_L - r.\rho_0.\widehat{\rho} - \gamma\theta_{I,A}.\frac{1+s_0}{M_0}.\widehat{A}^2\right] \\
&\simeq -\frac{1+s_0}{M_0}.\widehat{M} - r.\rho_0.\widehat{\rho} - \gamma\theta_{I,A}.\frac{1+s_0}{M_0}\sigma_A^2
\end{aligned}$$

Hence

$$\begin{aligned}
EP &= \frac{E\widetilde{\pi}_L - r\rho}{\rho} \\
&\simeq \frac{\pi_0}{\rho_0}.\widehat{\pi}_L - r\widehat{\rho} \\
&\simeq r\gamma\theta_{I,A}.\frac{1}{r\rho_0}.\frac{1+s_0}{M_0}.\sigma_A^2
\end{aligned}$$

since $\pi_0 = r\rho_0$. This proves the result reported in the text.

B.4 Comparative Statics in the Closed Economy

Step 1: Proof of proposition 4

This comparative static is sensible because the deterministic equilibrium (s_0, s_0, M_0) is unaffected by ϕ. Ω_L is a decreasing function of I, since

$$\theta_{I,A} = \frac{1}{r + \frac{N_L}{I}.\left(\frac{1+s_0}{M_0} - C(s_0)\right)}.\left(\frac{N_L}{I}.\frac{1+s_0}{M_0}\right)$$

is decreasing in I, while Ω_P is unaffected by changes in I. The first two points of proposition 4 derive from equations (22)-(21). From (23), market pressure increases, which raises the real hourly wage given by (8). Results on profit and utilities follow.

Step 2: Proof of Proposition 5

The deterministic equilibrium is unaffected by shifts in σ_A^2. Since $\theta_{I,A} < \theta_{E,A}$, $\Omega_P - \Omega_L$ is an increasing function of σ_A. Hence, given that $\widehat{s}_P$ depends on $-\Omega_P$ and $-(\Omega_P - \Omega_L)$, $\widehat{s}_P$ is a decreasing function of σ_A^2. Written in terms of σ_A^2, equation (22) rewrites:

$$\hat{s}_L = \gamma.\frac{m_0}{m_0 + \varepsilon_C}.\left[\left(\theta_{E,A}\frac{m_0}{\varepsilon_C}.\frac{N_P}{N} - \left(\frac{m_0}{\varepsilon_C}\frac{N_P}{N} + 1\right).\theta_{I,A}\right)\sigma_A^2 + \frac{m_0}{\varepsilon_C}\frac{N_P}{N}\theta_{E,d}.\sigma_d^2\right]$$

the condition stated in the proposition ensures that the term in front of σ_A^2 is positive. Last, since $\widehat{M}$ is a decreasing function of Ω_P and Ω_L, it is also decreasing in σ_A^2.

Step 3: Proof of Proposition 6

Again, the deterministic equilibrium is not affected by shifts in N_L, taking N as a constant. $\theta_{I,A} < \theta_{E,A}$, which implie that $\Omega_P > \Omega_L$. Using expression (22), we can see deduce that $\widehat{s}_L$ is an increasing function of N_P/N, hence decreasing in N_L/N. Besides, N_L makes Ω_L increase, hence makes $\widehat{s}_L$ decrease. This proves the proposition.

C Resolution of the Open Economy Model

C.1 Computing Profits

In period 3, consumers can consume both domestic and foreign goods, without restriction. Let set:

$$\begin{aligned}
\Delta_c &\equiv \int \widetilde{\delta}_j^c dj \\
\Theta &\equiv \left[\Delta_D^{1/\sigma}.\widetilde{L}_D^{(\sigma-1)/\sigma} + \Delta_F^{1/\sigma}.\widetilde{L}_F^{(\sigma-1)/\sigma}\right]^{\sigma/(\sigma-1)}
\end{aligned}$$

for $c = D, F$. Then, optimizing profits and writing down the labor market equilibrium, we find that:

$$\begin{aligned}
\frac{p_c}{P} &= \frac{\sigma}{\sigma-1}.\frac{w_c}{P} \\
\frac{E_D + E_F}{P} &= \Theta \\
\frac{w_c}{P} &= \frac{\sigma-1}{\sigma}.\left(\frac{\Delta_c}{\widetilde{L}_c}\right)^{1/\sigma}.\Theta^{1/\sigma}
\end{aligned}$$

We solve the equilbrium by assuming that all listed firms in the domestic and foreign country choose the same strategies s_L for listed firms and s_P for non listed firms. Implicitly, we are restricting our analysis to symmetric equilibria. Under these conditions, the profit of firm i is given by equation (25).

C.2 Financial Autarky

Under financial autarky, s_P and s_L are given by the new first order conditions:

$$\begin{aligned}
0 &= E\left[U'\left(\frac{\widetilde{B}_D.\left(1 + s_P.\tilde{d}_i\right)}{M} - C(s_P)\right).\left(\frac{\widetilde{B}_D.\tilde{d}_i}{M} - C'(s_P)\right)\right] \\
0 &= E\left[U'\left(\frac{N_L}{I}.\left(\frac{\widetilde{B}_D.(1 + s_L)}{M} - C(s_L)\right)\right)\left(\frac{\widetilde{B}_D}{M} - C'(s_L)\right)\right]
\end{aligned}$$

which is formally identical to the Euler conditions in the closed economy, except that the aggregate shock is now $\widetilde{B}_D$ instead of $\widetilde{A}_D$. Hence, the linearization of these conditions is identical. The only difference is that $E\widetilde{B}_D = 1 + (1/2 - 1/4\sigma).\sigma_A^2 > 1$ even though $E\widetilde{A}_D = 1$ but the first order term vanishes in the Taylor expansion.

C.3 International Trade

The problem only changes for investors. In period 2, domestic investors now solve the following optimization problem:

$$\begin{aligned} \max EU &\left(\int x_i \pi_{D,i} di + \int x_j \pi_{F,j} dj\right) \\ \text{s.t.} & \int x_i \rho_{D,i} di + \int x_j \rho_{F,j} dj < E \end{aligned}$$

where E is their endowment: $E = \rho_D.N_L/I$. Each domestic firm's stock solves:

$$E\left[U'\left(\widetilde{R}_I\right).\pi_{D,i}\right] = \lambda \rho_i$$

the equilibrium will be symmetric, so that all investors (domestic or foreign) will hold a fraction $1/2I$ of each firm. Domestic and foreign firms will choose the same strategies. Hence,

$$\widetilde{R}_I = \frac{N_L}{I}\left(\left(\frac{\widetilde{B}_D + \widetilde{B}_F}{2}\right).\frac{(1+s_L)}{M} - C(s_L)\right)$$

Maximizing ρ_i w.r.t. s_i amounts to solving:

$$E\left[U'\left(\frac{N_L}{I}\left(\left(\frac{\widetilde{B}_D + \widetilde{B}_F}{2}\right).\frac{(1+s_L)}{M} - C(s_L)\right)\right).\left(\frac{\widetilde{B}_D}{M} - C'(s_L)\right)\right] = 0$$

using formula (38) requires to compute the various derivatives:

$$\begin{aligned} F'_s(\boldsymbol{\theta}_0, \mathbf{x}_0) &= -C''(s_0).U'(R_{E,0}) \\ F'_M(\boldsymbol{\theta}_0, \mathbf{x}_0) &= -\frac{1}{M_0^2}.U'(R_{E,0}) \end{aligned}$$

and:

$$\begin{aligned} F''_{DD}(\boldsymbol{\theta}_0, \mathbf{x}_0) &= 0 \\ F''_{DB}(\boldsymbol{\theta}_0, \mathbf{x}_0) &= 2\frac{1+s_0}{M_0^2}.U''(R_{E,0}) \\ F''_{BB}(\boldsymbol{\theta}_0, \mathbf{x}_0) &= 0 \end{aligned}$$

hence the liearised FOC is given by

$$U'(R_{E,0}).\left(C''(s_0).s_0.\widehat{s}_L + \frac{\widehat{M}}{M_0}\right) = \frac{U''(R_{E,0})}{M_0}.\left(\frac{1+s_0}{M_0}.\sigma_{DB}\right)$$

which leads to the expression in the text.