Residential aggregator risk constrained profit maximization under demand response program

by

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Abstract

This thesis proposes a Mixed Integer Non-Linear Programming (MINLP) stochastic energy model for an energy aggregator operating in the US distribution systems energy markets. Day-ahead, real-time and spot markets are considered as trading market options for the aggregator. When trading in real-time and spot markets; the aggregator faces multiple risks coming from load variability and uncertain market price. Deciding the selling price to be offered to the aggregator's customers is a challenge for the aggregator. Uncertainties are modeled via stochastic programming and quantified via Conditional Value at Risk (CVaR). The aggregator's optimal day-ahead selling prices to be offered to customers under real-time and spot prices uncertainty are determined by solving the proposed stochastic model. Changing the hourly prices offered to customers will change their hourly consumption resulting in a load redistribution during the day. Savings for the aggregator and customers will be gained by shifting the customers load to a lower price periods during the day. A case study is implemented to show the validity of the proposed model and influence of the aggregator in the distribution systems energy market.

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Dedication

To my Mother and Father for their endless support.

To my sister Shireen for her endless encouragement and support during my MS study.

To my brothers and sisters.

To my niece Razan and nephews Adam and Ibraheem

Chapter 1

Introduction

Grid modernization is the key change to the current electric grid, it will improve, reform the old energy business models, and it will create new ones [1]. According to Patricia Hoffman, the US Assistant Secretary for Electricity Delivery and Energy Reliability "in order to manage American's energy consumption more efficiently and cost-effectively, recover from disruption more quickly and remain globally competitive, we must modernize our nation's electric grid" [2]. As the current electric grid is aging and can't support the nation's growth it is a necessity to address the challenges of grid modernization [2]. So increasingly the nation's need for a modernized grid is at the center of scientific research, political conversation and people's concerns.

1.1 Grid Modernization

The US market grid modernization varies from state to state since each state has its own policies, regulations and energy market rules [1]. According to the grid modernization index published in December 2018 by Gridwise alliance, California and Illinois are the leaders in grid modernization and Texas is moving toward grid modernization [1] as shown in Figure 1.1.

This thesis seeks to address the challenges envisioned by the aggregator trading with customers in the modernized electric grid. The goal of this research is to create an optimal pricing mechanism for the aggregator trading for customers to participate in the US energy market. This thesis investigates optimal trading strategies to account for real-time market price and customers' energy fluctuations with underlying uncertainties.

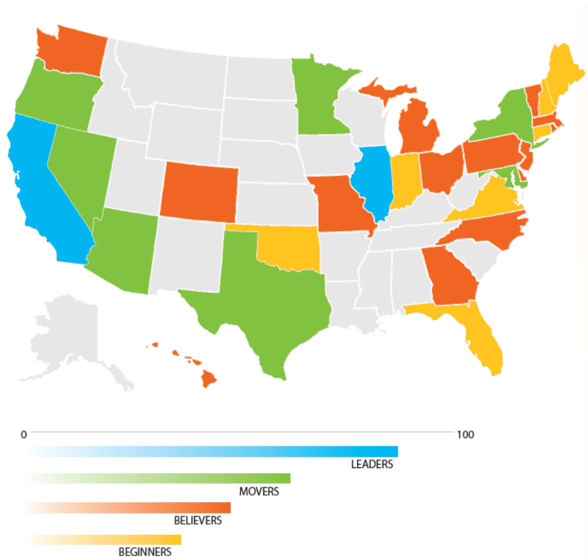

Figure 1.1 Grid modernization index in the US states [1]

Resiliency, reliability, security, affordability, flexibility and sustainability are the desired characteristics of the modernized electric grid [2]. Electric grid restructuring means that the current electric grid will be changed, from the vertically integrated structure to a more economical and efficient grid ,where all users such as in the distribution level can participate in the electric market [3].This thesis will focus on addressing concepts related to affordability of the restructured electric grid, at the distribution level. Affordability means that the grid should be able to sustain and improve the nation's economy [2]. This thesis will discuss improving the nation's economy by addressing the challenges of cost, profit and risk that may face the customers of electricity and the electricity firms, interested in participating in the new restructured distribution systems electric grid.

Trading in the electric market with residential customers is still not a widely adopted concept and varies based on location. Aggregator companies have been proposed for the market to manage customers participation in the electric market. "Aggregator is an entity or a firm that combines customers into buying groups" [3]. The aggregator can also be defined as the middle man between the customers and the Independent System Operator (ISO) to manage customers participation into the competitive electric market [4].

1.2 Research Motivation

The research has mainly been motivated from the idea of enabling end-users to participate in the current modernized, restructured electric grid aided by aggregators. The research will mainly focus on the aggregator's operation and challenges in the current competitive market. How would the aggregator reduce customers' payments while making profit from managing customers energy needs? Additionally, the aggregator is a new entity, it becomes one of the key market entities in the new electric grid market. In this thesis a model for the aggregator participating in the energy market by aggregating customers into buying groups is proposed. The aggregator's aim is to determine the hourly selling prices for customers that would insure the aggregator's maximum profit and minimize energy cost for customers.

1.3 Research Questions

Question 1: How the aggregator's operation in a competitive market can be modeled?

Question 2: What day-ahead prices should be offered by the aggregator to their customers?

Question 3: How to maximize the aggregator's profit and minimize its risk under real-time and pool price uncertainty along with customers load variability?

Question 4: How to minimize customers' payments for their energy consumption within the restructured electric markets?

1.4 Literature Review

This section addresses related work on the subjects of Demand Response (DR) and Load shifting, Electric Aggregator, Risk Management and Stochastic Programming.

1.4.1 Related Work on Demand Response

Grid modernization utilizes the demand resources in the restructured electric grid economy and efficiency by applying demand response program. According to the ISO-New England, 2015 regional electricity outlook, "Efforts to modernize the grid opens up new approaches to demand resources (including energy efficiency, demand response, and distributed generation) and for coordination planning, operations, and pricing between the wholesale and retail sectors." [2] This section will address previous work on demand response program applied to residential demand and managed by the aggregator.

Demand Response (DR) is proposed to reform the consumption of energy during a specified period, when the supply is rare or expensive by reducing or shifting load to periods when the supply of energy is at low-cost [5]. A risk constrained optimization model with demand response has been proposed for profit maximization for microgrid aggregator in [6]. Authors assumed that the microgrid aggregator's objective is to determine optimal hourly energy bid to be submitted to the day-ahead market while offering predefined retail prices for their customers.

1.4.2 Related Work on the Aggregator Risk Management and Stochastic Programming

In literature, there is a considerable amount of published work on electricity retailers that is similar in operation to the aggregator and their operation in the electricity market. Retailers and the aggregator is fairly new entities in the electricity market [3]. Typically, the aggregator is interested in aggregating customers into buying groups [3]. A considerable amount of research on the aggregator operating in the context of electric vehicle (EV) is found as in reference [7].

The aggregator considered for this thesis operates in residential distribution markets serving home energy needs. A brief description of available research considering the aggregator operating in residential markets interacting with customers under demand response programs. For example, in reference [8] the authors propose a stochastic linear programming model for price taking retailers constructing bidding curves in the Nord pool market to minimize the cost in day-ahead and regulating markets. In reference [9] the author presents a stochastic programming model for retailers trading with customers, encouraging them to shift their load to lowest price periods at the same time minimizing the aggregator risk trading in the pool electricity market. The author considered six aggregate time periods and considered planning on a monthly basis. The author also considered time of use tariff (TOU) which could be used to trade with customers in two periods during a month. The author used the CVaR risk model to quantify the risk originating from profit variation when trading in the pool market. In reference [10] the authors proposed optimal involvement for power producers in future markets by using the conditional value at risk (CVaR) as a coherence risk measure to hedge against profit variation. The authors proposed their model for retailers interested in trading in future markets. The authors in reference [4] proposed a stochastic energy model for electric vehicle aggregators participating in day-ahead markets and used the CVaR index to hedge against the risk of uncertainties imposed from wind volatility and EV load fluctuations. In reference [11] authors proposed optimal selling price and energy procurement strategies for retailers in electricity markets. The authors in reference [12] surveyed the decision makers of electricity retailers and reported that the elasticity of the demand is best utilized under real-time pricing (RTP).

This thesis will discuss the aggregator decision on optimal day-ahead prices offered to customers who agreed to participate in a demand response program. Also, this work will use customer demand elasticity while ensuring maximized profit for the aggregator and minimized energy cost for customers.

1.5 Contributions of This Thesis

The contribution for this thesis can be stated as the following:

1. Finding an optimal solution for the hourly selling prices, offered by the aggregator to customers, applying demand response program, while hedging the risk using CVaR methodology, under real-time prices uncertainty and customers load fluctuations.
2. A heuristic procedure to optimally generate enough scenarios for real-time market prices, based on probability distribution for each hour of the price, using long time historical data.
3. Technical case study based on Pecan Street, a US residential area in Austin, Texas, under ERCOT day-ahead and real-time market prices.

Chapter 2

The Aggregator Mathematical Model

2.1 The Aggregator Market Model

The aim of this thesis is to decide the expected real-time market price that will be offered to customers by the aggregator. In addition, it aims to address the problem of the aggregator participates in day-ahead, real-time and spot markets.

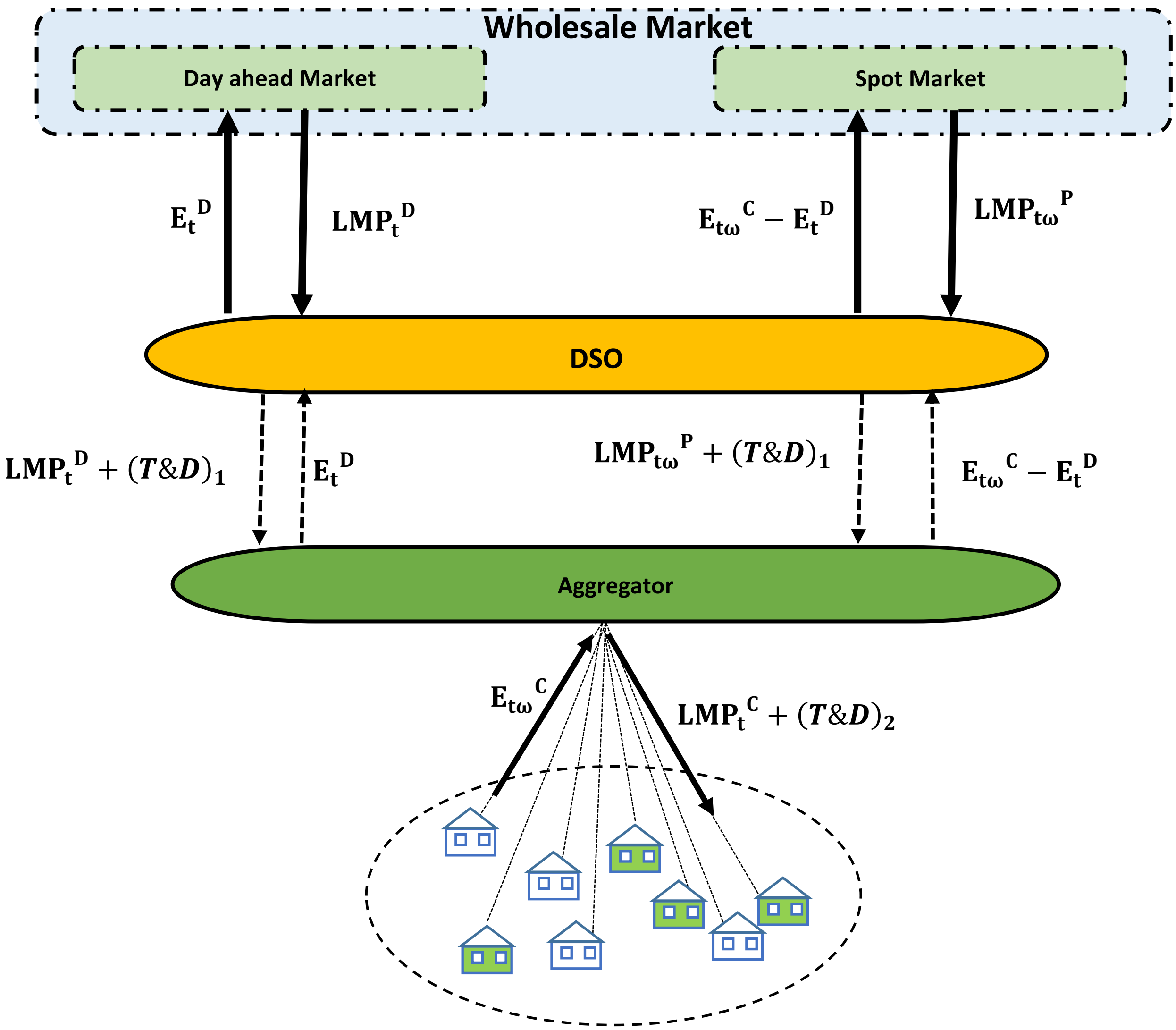

Figure 2.1 Aggregator model before change selling prices

The aggregator's energy trading is considered to be under uncertain real-time price and uncertain customers load. The aggregator market model is illustrated in Figure 2.1.

Figure 2.1 is a schematic for the aggregator market model shows the flow of information in the proposed market model for the aggregator. It shows the market prices and energy bids between market entities before the actual real-time market occurs. The model is the day-ahead market model for the aggregator, since all the quantities shown in the Figure 2.1 are known to the aggregator either as fixed known quantity such as the day-ahead prices ${\mathrm{LMP}_t}^{\mathrm{D}}$ or as a random variable such as the spot market prices ${\mathrm{LMP}_t}^{\mathrm{P}}$. The aggregator determines forecasted load ${\mathrm{E}_{t\omega}}^{\mathrm{C}}$ for the next day and based on that determines the amount $({\mathrm{E}_t}^{\mathrm{D}})$ for the day-ahead contract with the Distribution Systems Operator (DSO). The aggregator usually would like the contracted amount ${\mathrm{E}_t}^{\mathrm{D}}$ with DSO to be equal the expected customer load ${\mathrm{E}_{t\omega}}^{\mathrm{C}}$, but under certain circumstances such as the customer's load variability, the contracted amount with DSO, ${\mathrm{E}_t}^{\mathrm{D}}$ may not match the customers load, ${\mathrm{E}_{t\omega}}^{\mathrm{C}}$. In this case the aggregator will buy the additional energy at unknown and ,usually, very high price from the spot market.

Since this work considers that the aggregator is a price taker company, then it will not affect the wholesale market price; all the prices are based on the wholesale market prices with extra mark-up charges added to accommodate for transmission and distribution charges $(\mathrm{T\&D})$. To illustrate more, the aggregator will charge the customers for their energy consumption based on the day-ahead price. The selling price passed to customers by the aggregator is ${\mathrm{LMP}_t}^{\mathrm{C}}$ plus the transmission and distribution charges $(\mathrm{T\&D})_2$. The price passed to the aggregator by DSO is the wholesale market price ${\mathrm{LMP}_t}^{\mathrm{D}} + (\mathrm{T\&D})_1$. It should be noted that the transmission and distribution charges $(\mathrm{T\&D})$ assigned by DSO to the aggregator and by the aggregator to customers are different since each part in the market will try to maximize its profit by adding some extra charges to the selling price.

The excess energy that might result from the mismatch between actual customers' load and the aggregator contract with DSO $\left(\mathrm{E_{t\omega}}^{\mathrm{C}} - \mathrm{E_t}^{\mathrm{D}}\right)$ will be paid by the aggregator to the DSO at the spot market price $\mathrm{LMP_{t\omega}}^{\mathrm{P}}$ plus extra transmission and distribution charges $(\mathrm{T\&D})_1$. This work assumes that the aggregator will take the risk of purchasing the excess energy in the spot market and the customers will not be affected by the spot market price uncertainty since the customers are being charged by the aggregator a fixed price which is the day-ahead market price plus $(\mathrm{T\&D})_2$ charges. The aggregator will try to change the selling price offered to customers to encourage them to change their load to lower price periods during the day to achieve higher savings and reduce the risk of participating in the spot market. This work assumes that the customer's energy cost will always be less at the end of the day by participating in the model proposed to give the customer an incentive to shift their load during the day. Mathematical detail in section 2.2 shows how the aggregator would be able to control the customer's behavior (load shifting) by controlling the selling price either by increasing or decreasing it during specific hours in the day. To illustrate these concepts further, Figure 2.2 illustrates the market model and its entities when the aggregator changes the selling prices offered to customers.

In Figure 2.2, in order to achieve higher profit for the aggregator while insuring less energy cost for customers, the aggregator will change the selling price to the customers by $\pm\,\Delta\mathrm{LMP_t}^{\mathrm{C}}$; thus the prices offered to customer will be $\mathrm{LMP_t}^{\mathrm{C}} \pm\,\Delta\mathrm{LMP_t}^{\mathrm{C}}$. The aggregator still considers adding the same T&D charges as in the day-ahead model shown in Figure 2.1. The customers will respond to the new price by changing their load according to their price elasticity of demand and either increase or decrease the load by $\pm\Delta\mathrm{E_{t\omega}}^{\mathrm{C}}$. The model assumes that the total energy consumed by the customers would be fixed during the day and the customers are only allowed to shift their load from one hour to another hour within the same day. The new load of the customers will become $E_{t\omega}{}^{C} \pm \Delta E_{t\omega}{}^{C}$. By changing the load the extra energy that the aggregator will buy from the spot market price will be changed and it will be equal to $\left(E_{t\omega}{}^{C} \pm \Delta E_{t\omega}{}^{C} - E_{t}{}^{D}\right)$. It becomes a very challenging problem for the aggregator to decide how the new selling price might be offered to customers in a way to avoid very high prices in the spot market

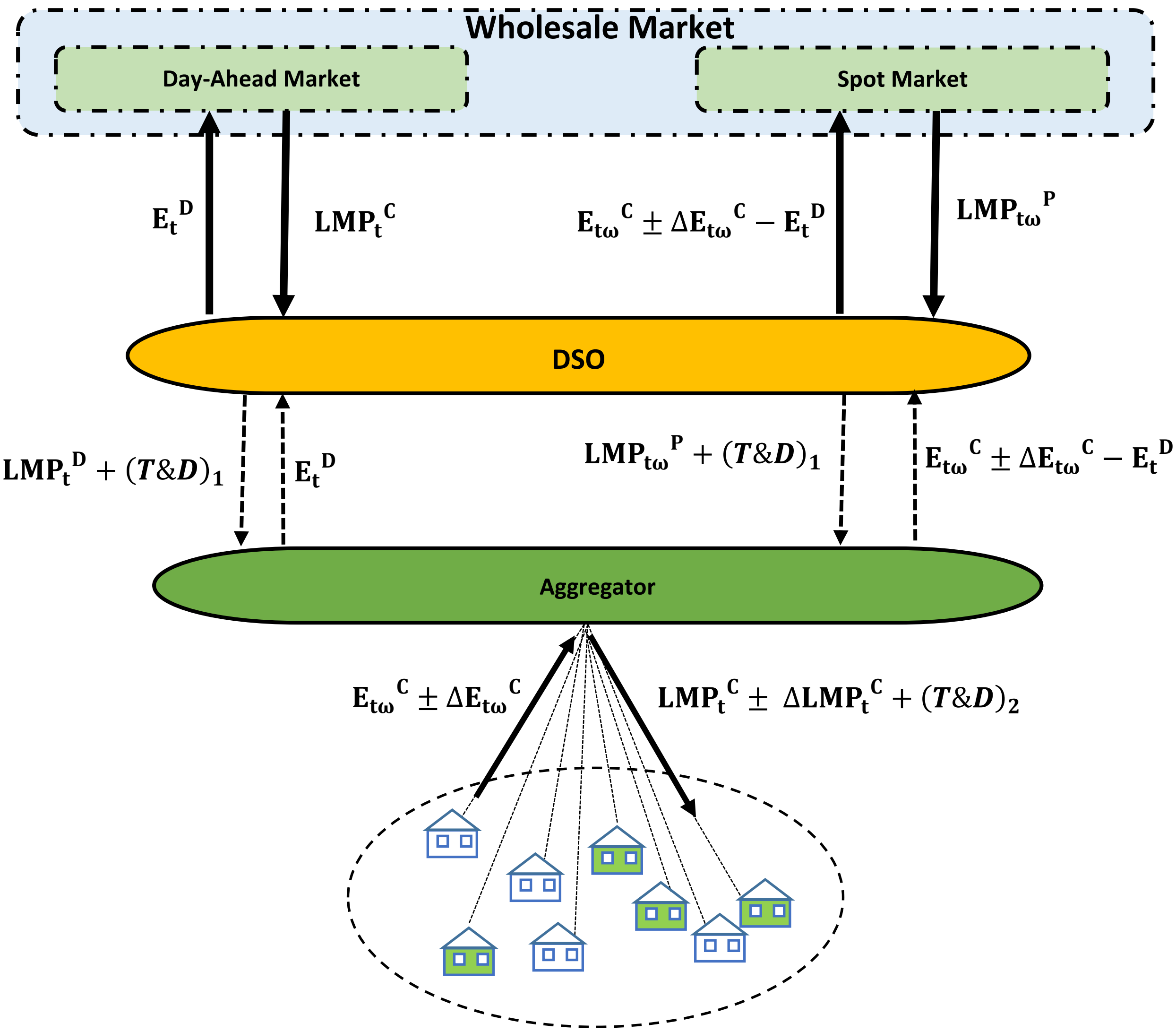

Figure 2.2 Aggregator Model change selling price

and ensure lower costs for customers while maintaining its own profit. In summary, the aggregator has to determine new selling prices to be offered to customers under the following assumptions as shown in the Figure 2.2:

1. Energy contracts between aggregator and customers for day-ahead market will be on an hourly basis.
2. Customers are considered to have elastic demand and they participate in the Demand Response (DR) Program on an hourly basis in the one-day framework.
3. Customers' energy should be fixed during a day and customers may shift their loads within the same day.
4. Customers' energy cost after changing the selling prices should always be less than the cost based on day-ahead prices.
5. The aggregator will see their profit increases.
6. The aggregator objective is to maximize the profit difference between the base profit (before changing selling prices) and the new profit (after changing selling prices).
7. The aggregator's second objective is to maximize the Conditional Value at Risk (CVaR) to accommodate for market price uncertainty and customers' load variability.

2.2 The Aggregator Mathematical Model

Since real-time price and loads have uncertainties, the aggregator must cover the risk by offering hourly price to customers to minimize risk while maximizing profit. To cover all possibilities, the aggregator selects N_{ω} scenarios of different conditions of prices and loads.

The aggregator's objective and constraints are listed in equations (1)-(16). (1) shows the objective function to maximize the conditional value at risk (CVaR). This function represents the difference between the value at risk and the payoff for each price and load scenario. The CVaR represents the mean value of the lowest profit scenarios. This index, determined by the aggregator, is based on the aggregator type and whether the aggregator is a risk taker or risk averse. Alpha confidence levels take values between zero and one; higher values of α means lower risk. Constraint in equation (2) requires that the total change of energy during a day should be zero and customers can shift their load from one hour to another hour in response to the price change.

Typically, the aggregator is interested in shifting customer energy consumption to lower price periods by controlling the hourly prices of electricity. Constraint in equation (3) shows that the cost of energy for customers after changing the selling prices should be less than those based on original selling prices. Equation (4) shows that the new price after change should be always positive. Equation (5) guarantees that the total hourly energy consumed cannot be negative. Constraint (6) implies load ramp up and down limits. Equations (7) and (8) includes the CVaR index into the optimization problem. Auxiliary parameter u_ω is a positive quantity representing the distance between the value at risk and the profit of a specific scenario. If the value is greater than the profit of a specific scenario, u_ω has a positive value indicating the scenario's risk. The higher values of u_ω implies a higher risk of a specific scenario. Otherwise u_ω is equal to zero if a specific scenario's profit has a value greater than the value at risk, which indicates a risk-free scenario.

Equations (9)-(16) model the penalty value that the aggregator might pay when deviating from the contracted value with DSO.

Solving the optimization problem using equations (1) – (16) will give the optimal hourly price variation $\Delta \mathrm{LMP}_t^{\mathrm{C}}$ and the price that will be offered to customers by the aggregator under load and price uncertainties. This model is a decision tool for electricity aggregator to find optimal hourly prices offered to customers within the day-ahead market.

$$\underset{\Delta LMP_t{}^C, \forall t, \zeta, u_\omega \forall \omega}{\text{Maximize}} \quad \zeta - \frac{1}{(1-\alpha)} \sum_{\omega=1}^{N_\omega} \pi_\omega u_\omega \quad \cdots (1)$$

$$\sum_{t=1}^{N_t} \frac{\varepsilon_{t\omega}{}^C \, \Delta LMP_t{}^C \, E_{t\omega}{}^C}{LMP_t{}^C} = 0; \quad \forall \omega \ \cdots (2)$$

$$\sum_{t=1}^{N_t} \left(E_{t\omega}{}^C + \frac{\varepsilon_{t\omega}{}^C \, \Delta LMP_t{}^C \, E_{t\omega}{}^C}{LMP_t{}^C} \right) \left(LMP_t{}^C + \Delta LMP_t{}^C \right) - \sum_{t=1}^{N_t} E_{t\omega}{}^C \, LMP_t{}^C \leq 0; \quad \forall \omega \ \cdots (3)$$

$$LMP_t{}^C + \Delta LMP_t{}^C \geq 0; \ \forall t \ \cdots (4)$$

$$E_{t\omega}{}^C + \frac{\varepsilon_{t\omega}{}^C \, \Delta LMP_t{}^C \, E_{t\omega}{}^C}{LMP_t{}^C} \geq 0; \forall t, \forall \omega \ \cdots (5)$$

$$-aE_{t\omega}{}^C \leq \frac{\varepsilon_{t\omega}{}^C \, \Delta LMP_t{}^C \, E_{t\omega}{}^C}{LMP_t{}^C} \leq aE_{t\omega}{}^C; \ \forall t, \forall \omega \ \cdots (6)$$

$$u_\omega \geq \zeta - \sum_{t=1}^{N_t} \pi_\omega \, \Delta LMP_t{}^C \, E_{t\omega}{}^C (1 + \varepsilon_{t\omega}{}^C) - \sum_{t=1}^{N_t} \pi_\omega \frac{\varepsilon_{t\omega}{}^C \left(\Delta LMP_t{}^C \right)^2 E_{t\omega}{}^C}{LMP_t{}^C}$$

$$- \sum_{t=1}^{N_t} \pi_\omega P_{t\omega}{}^{RH} \, LMP_{t\omega}{}^R \left(-E_{t\omega}{}^C - \frac{\varepsilon_{t\omega}{}^C \, \Delta LMP_t{}^C \, E_{t\omega}{}^C}{LMP_t{}^C} + E_t{}^D \right)$$

$$- \sum_{t=1}^{N_t} \pi_\omega P_{t\omega}{}^{RL} \, LMP_{t\omega}{}^R \left(-E_{t\omega}{}^C - \frac{\varepsilon_{t\omega}{}^C \, \Delta LMP_t{}^C \, E_{t\omega}{}^C}{LMP_t{}^C} + E_t{}^D \right)$$

$$- \sum_{t=1}^{N_t} \pi_\omega P_{t\omega}{}^{DH} \, LMP_{t\omega}{}^R \left(E_{t\omega}{}^C - E_t{}^D \right)$$

$$- \sum_{t=1}^{N_t} \pi_\omega P_{t\omega}{}^{DL} \, LMP_{t\omega}{}^R \left(E_{t\omega}{}^C - E_t{}^D \right); \;\; \forall \omega \; \cdots (7)$$

$$u_\omega \geq 0; \;\; \forall \omega \; \cdots (8)$$

$$\left(E_{t\omega}{}^C - E_t{}^D \right) - M_{t\omega} \, x_{t\omega}{}^B \leq 0 \, ; \; \forall t, \forall \omega \; \cdots (9)$$

$$\left(E_{t\omega}{}^C - E_t{}^D \right) + m_{t\omega} \, x_{t\omega}{}^B \geq m_{t\omega} \, ; \; \forall t, \forall \omega \; \cdots (10)$$

$$\left(E_{t\omega}{}^C + \Delta E_{t\omega}{}^C \right) - M_{t\omega} \, y_{t\omega}{}^B \leq E_t{}^D; \; \forall t, \forall \omega \; \cdots (11)$$

$$\left(E_{t\omega}{}^C + \Delta E_{t\omega}{}^C \right) + m_{t\omega} \, y_{t\omega}{}^B \geq m_{t\omega} + E_t{}^D \, ; \; \forall t, \forall \omega \; \cdots (12)$$

$$P_{t\omega}{}^{DH} = A_t{}^{DH} \, x_{t\omega}{}^B \; \cdots (13)$$

$$P_{t\omega}{}^{DL} = A_t{}^{DL}(1 - x_{t\omega}{}^{B}) \cdots (14)$$

$$P_{t\omega}{}^{RH} = A_t{}^{RH}\, y_{t\omega}{}^{B} \cdots (15)$$

$$P_{t\omega}{}^{RL} = A_t{}^{RL}(1 - y_{t\omega}{}^{B}) \cdots (16)$$

1. $P_{t\omega}{}^{DH}$: Penalty variable. Applied for Load deviation before changing selling price. $A_t{}^{DH}$ is the penalty coefficients multiplier associated to $P_{t\omega}{}^{DH}$ Applied if the load is higher than the energy contract with DSO.
2. $P_{t\omega}{}^{DL}$: Penalty variable. Applied to Load deviation before changing selling prices. $A_t{}^{DL}$ is the penalty coefficients multiplier associated to $P_{t\omega}{}^{DL}$. Applied if the load is lower than the energy contract with DSO.
3. $P_{t\omega}{}^{RH}$: Penalty variable. Applied for Load deviation after changing selling prices. $A_t{}^{RH}$ is the penalty coefficient multiplier associated to $P_{t\omega}{}^{RH}$ Applied if the load is higher than the energy contract with DSO.
4. $P_{t\omega}{}^{RL}$: Penalty variable. Applied for Load deviation After changing selling prices. $A_t{}^{RL}$ is the penalty coefficient multiplier associated to $P_{t\omega}{}^{RL}$ Applied if the load is Lower than the energy contract with DSO.
5. $x_{t\omega}{}^{B}$, $y_{t\omega}{}^{B}$: Binary variables take values either 0 or 1. Depends on the result of a set of if condition statements. $X_{t\omega}{}^{B}$ will be equal one if load deviation is positive before changing selling price. $X_{t\omega}{}^{B}$ will be equal zero if load deviation is negative before changing selling price. While $y_{t\omega}{}^{B}$ will be equal one if load deviation is positive after changing selling price. And $y_{t\omega}{}^{B}$ will be equal zero if load deviation is negative after changing selling price. The aggregator considers load deviation to be equal to the

difference between customers' load (before/after changing selling prices) and contracted energy with DSO.

2.3 Nomenclature

This section will address the nomenclature used in the aggregator mathematical models and through the rest of the thesis. Any extra symbols used and not mentioned here will be explained in the same section where it has been used.

2.3.1 Indices

ω: Scenario Index

t: Time index

2.3.2 Notations

C: Customer

D: Day − ahead market

R: Real − time market

DH: Notation used for Penalty applied in **D**ay-ahead when customers load is **H**igher than contracted energy with DSO.

DL: Notation used for Penalty applied in **D**ay-ahead when customers load is **L**ower than contracted energy with DSO.

RH: Notation used for Penalty applied in **R**eal-Time when customers load is **H**igher than contracted energy with DSO.

RL: Notation used for Penalty applied in **R**eal-Time when customers load is **L**ower than contracted energy with DSO.

2.3.3 Constants

a: Ramping limits $[0:1]$

α: Confidence level $[0:1]$

$\pi_{\omega} = \dfrac{1}{N_{\omega}}$: Probability of scenario occurrence

2.3.4 Sets

N_{ω}: Number of Scenarios

N_t: Number of time periods

2.3.5 Parameters

LMP_t^{D}: Day − ahead locational marginal price [$/MWh]

LMP_t^{C}: Day − ahead customers selling price [$/MWh]

E_t^{D}: Day − ahead energy hourly contracts [MWh] with DSO

LMP_t^{P}: Penalty prices in [$/MWh]for excess energy in spot market

$\varepsilon_{t\omega}^{C}$: Price elasticity of the demand

2.3.6 Variables

ΔLMP_t^{C}: Change in hourly selling price [$/MWh]

$\Delta E_{t\omega}^{C}$: Change in hourly energy consumption [MWh]

ζ: Value at risk [$]

u_{ω}: Positive variable. indicating active and inactive profit scenarios

$P_{t\omega}^{DH}$: Penalty variable for day − ahead profit when load is higher than contract

$P_{t\omega}^{DL}$: Penalty variable for day − ahead profit when load is lower than contract

$P_{t\omega}^{RH}$: Penalty variable for Real − time profit when load is higher than contract

$P_{t\omega}^{RL}$: Penalty variable for Real − time profit when load is lower than contract

2.3.7 Random Variables

$LMP_{t\omega}^{R}$: LMPs in Real − Time Market [$/MWh]

$E_{t\omega}^{C}$: Energy consumption of customers [MWh] in Day − ahead market

2.3.8 Binary Variables

$x_{t\omega}^{B}$: Binary variable applied when $P_{t\omega}^{DH}$ is active at initial profit

$y_{t\omega}{}^{B}$: Binary variable applied when $P_{t\omega}{}^{RH}$ is active at new profit

Chapter 3

Modeling Market Price Behavior

All the data used for electricity prices in this thesis are all based on The Electric Reliability Council of Texas (ERCOT) market. "ERCOT is the Independent Organization certified by the Public Utility Commission of Texas (PUCT) for the ERCOT Region" [13]. This work will study day-ahead and real-time market prices for decision making by the aggregator. The following sections will analyze both markets and will propose a probabilistic model for real-time price that can be used by the aggregator to forecast the real-time price and accommodate the uncertainties coming from price fluctuations.

3.1 Day-Ahead Prices

Day-ahead market is defined as the forward market to schedule resources for the upcoming day on an hourly basis [3]. Market clearing prices in the day-ahead market are cleared based on the bids of the market participants. The market prices and energy quantities are usually cleared by an Independent System Operators (ISO's). The process of bidding in the wholesale market and the process of clearing the market quantities is out of our scope and this work will only focus on the relationship between day-ahead and real-time market price and how to utilize this relationship to predict the prices offered to customers based on the proposed model in Chapter 2 for the uncertainties associated with the market prices.

The day-ahead prices have been collected from ERCOT market for four years starting from 2014 to 2017. Price data has been processed during the summer period (May 15th to September 15th) of each year and for working days only. Weekends and federal holidays data

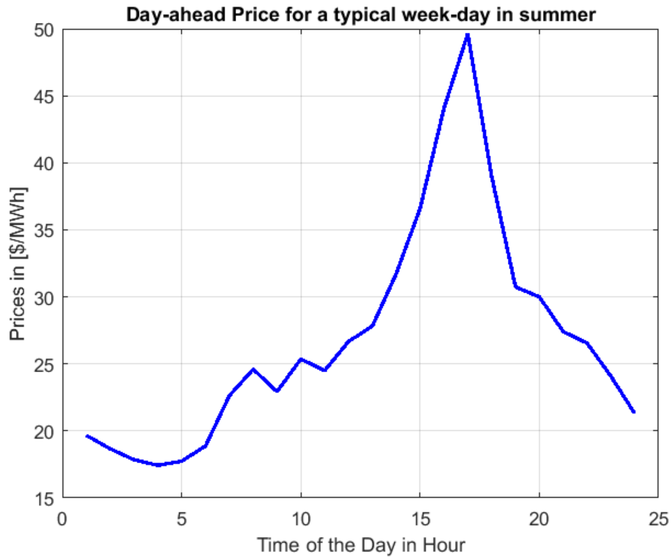

Figure 3.1 Day-ahead price for a typical week day in summer

have been excluded as the prices are highly correlated with customers' behavior, since weekday prices may differ from holidays prices in terms of peak and off-peak periods.

Figure 3.1 illustrates the price behavior on a typical day in the day-ahead market. The prices can be categorized into three periods: first is the morning off-peak period from the hour ending at 1:00 AM to the hour ending at 1:00 PM. The second period is the on-peak period from the hour ending at 2:00 PM to the hour ending at 8:00 PM. Third is the second off-peak period from the hour ending at 9:00 PM to the hour ending at 12:00 AM. Prices during the on-peak period are usually high compared to the off-peak periods. Thus, the aggregator will change the selling price offered to customers to avoid buying more energy during the on-peak period. It is worthwhile to mention in this section that it is possible for the price of electricity to be negative for short periods of time during a day. This might happen when the demand is less than generation due to renewable energy power plants such as wind farms, and the power generators might decide to sell energy to the grid for free because of generators' ramping constraints, but the T&D charges may still apply.

3.2 Real-Time Prices

The main purpose of real-time market is to make the generation equal the demand in real-time [3]. Therefore, real-time market prices cannot be known in advance until the real-time market occurs since the actual demand should be known to clear the market prices in real-time.

Expected real-time market prices will affect the aggregator decision on selling price for customers, because the aggregator will need to purchase any excess energy (contract deviation) from the real-time market. Real-time prices are uncertain in nature as they are correlated with uncertain variable demand but, in general, prices follow three periods based on the level of loads.

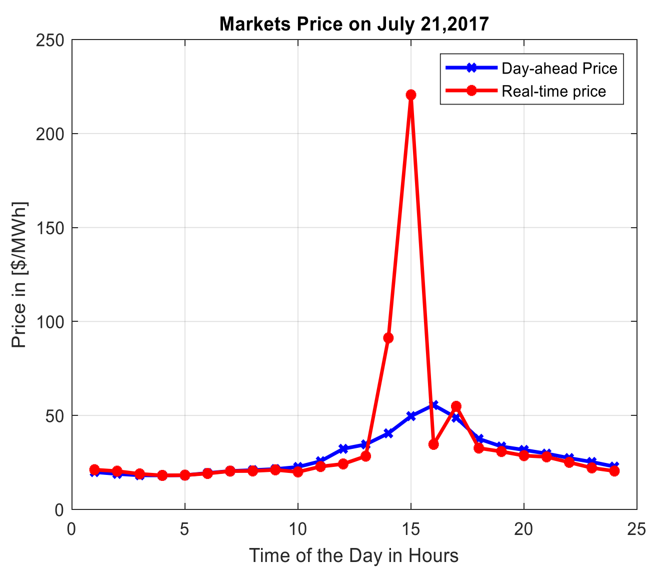

Figure 3.2 Day-ahead and real-time markets price for a typical day in summer

3.3 Probability Density Estimate for Market Prices

The procedure followed to obtain the probability distribution for each hour for the price difference between real-time and day-ahead prices will be described in detail. The difference in price between real-time and day-ahead at each hour for each of the days in the study has been calculated. After that, histogram plotted for the price difference data for each hour. There was a total of 613 days in the study period. It is found that the price difference distribution can be estimated to be a Gaussian distribution for each hour with zero mean and different standard deviation for each hour. Plots in Figures 3.4 to 3.10 indicate hourly price differences for each hour based on the collected data. The price data can fluctuate between very high and very low, as much as1000 times the normal prices. To take into account the very high prices it has been decided to truncate the very high values that rarely happen. The standard deviation estimates before and after applying truncation for the data is shown in Figure 3.3.

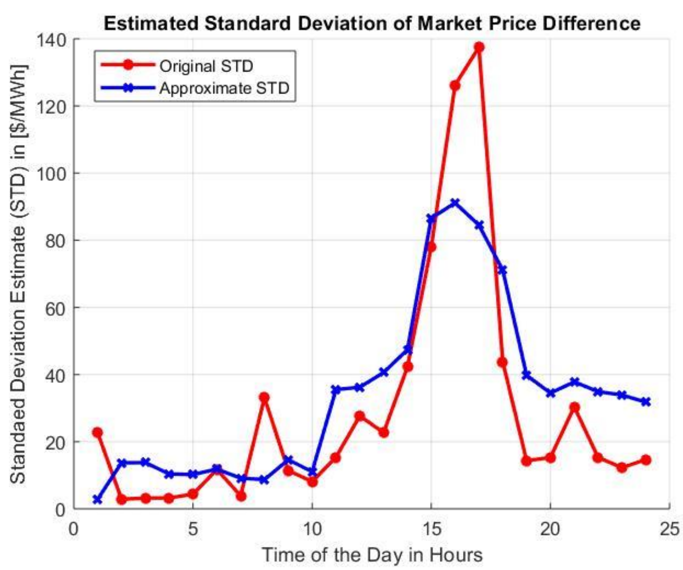

Figure 3.3 STD estimate for the difference between real-time and day-ahead markets price

The procedure followed to truncate the data as follows. Divided the day into slots based on the characteristics of the electricity prices. First is the off-peak period from 12:00AM to 10:00 AM, shoulder one period from 11:00 AM to 1:00 PM. peak from 2:00 PM to 8:00 PM, and shoulder two period from 9:00 PM to 11:00 PM. The allowed fluctuation for the off-peak period is within ± 50 \$/MWh, for the shoulder periods ± 100 \$/MWh while for the peak period it is ± 200 \$/MWh. After finishing all the steps mentioned above the distribution for the price difference at each hour has been obtained as shown in the following plots.

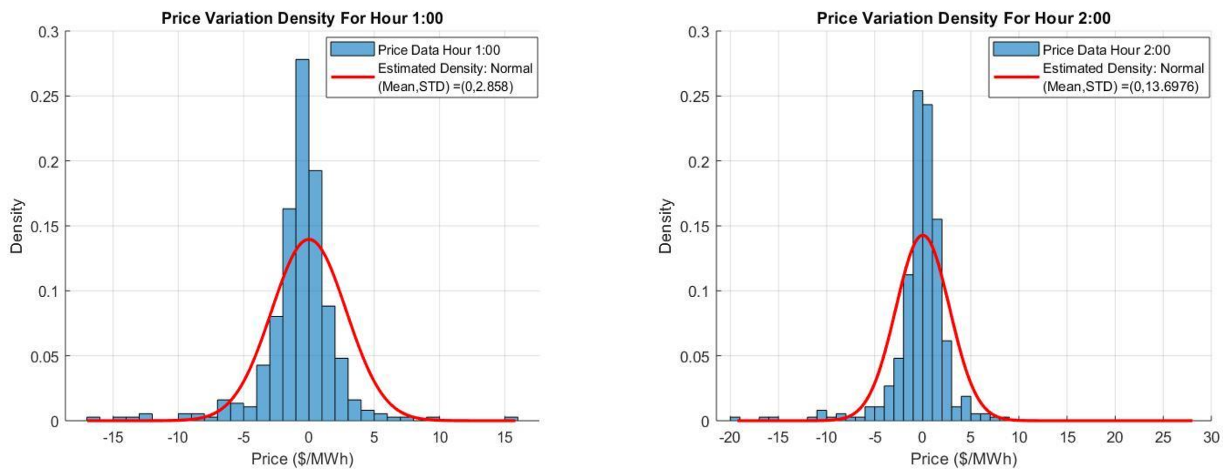

Figure 3.4 Off-peak period price difference density estimate

It can be seen that the standard deviation of the price is very high during the peak period from 2:00 PM to 8:00 PM due to the nature of the demand and price relationship. Peak period is characterized by very high demand, accordingly the electricity price will have very high variability during this period.

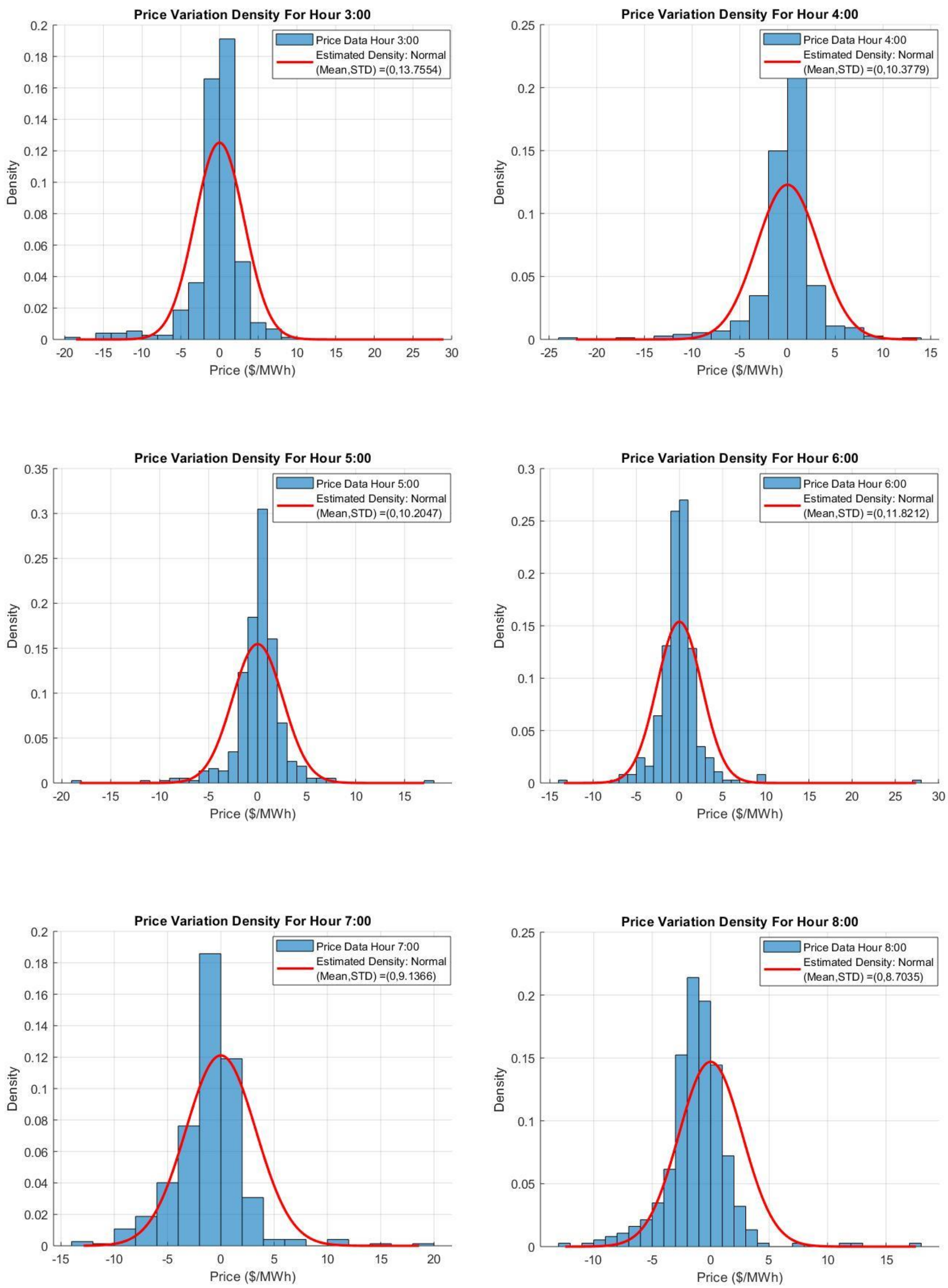

Figure 3.5 Off-peak period price difference density estimate

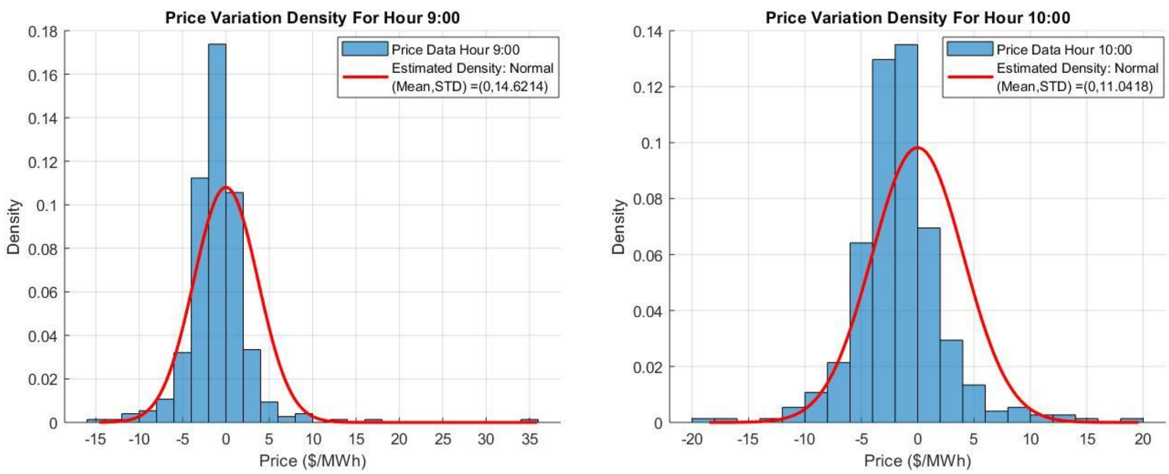

Figure 3.6 Off-peak period price difference density estimate

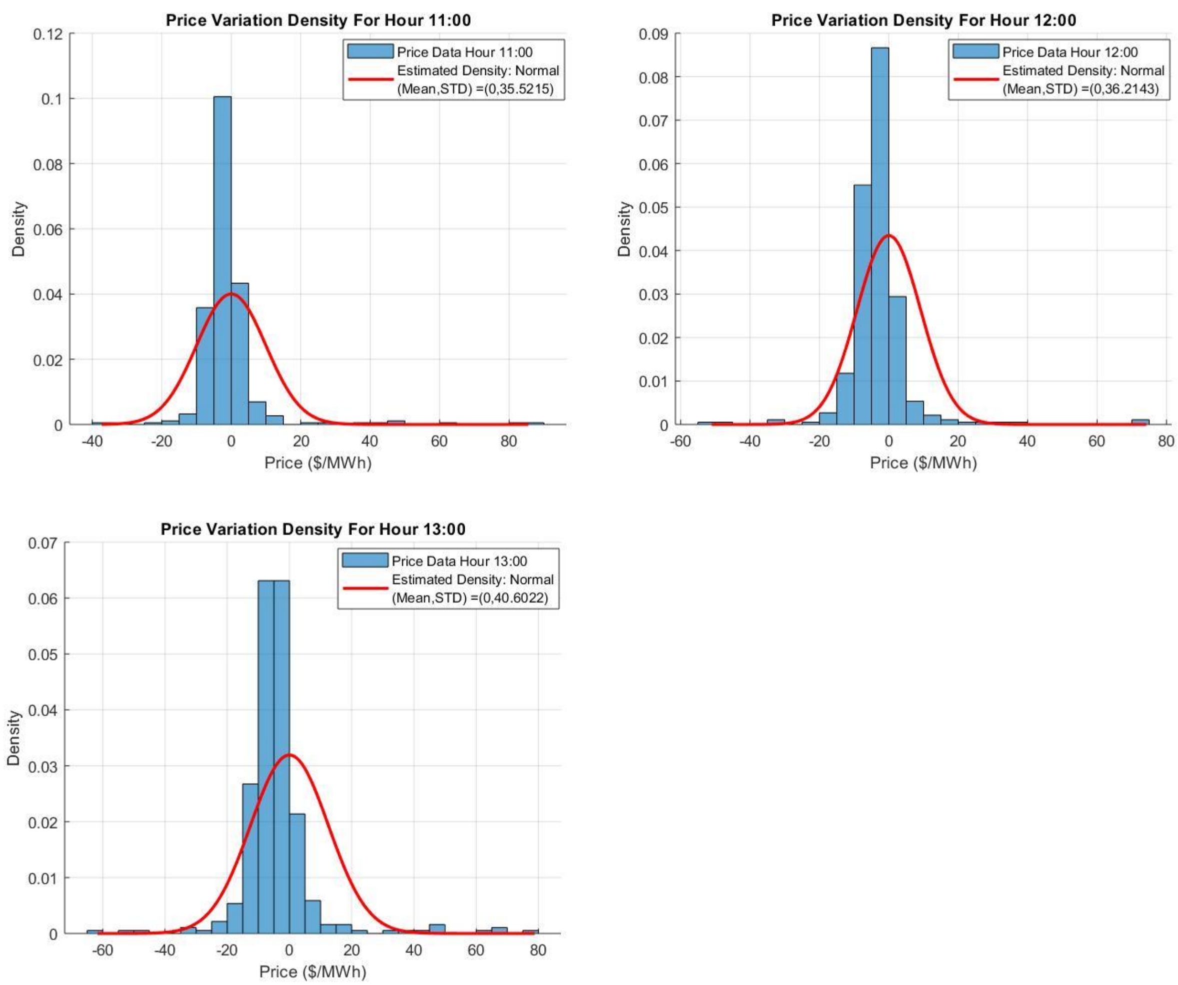

Figure 3.7 Shoulder one period price difference density estimate

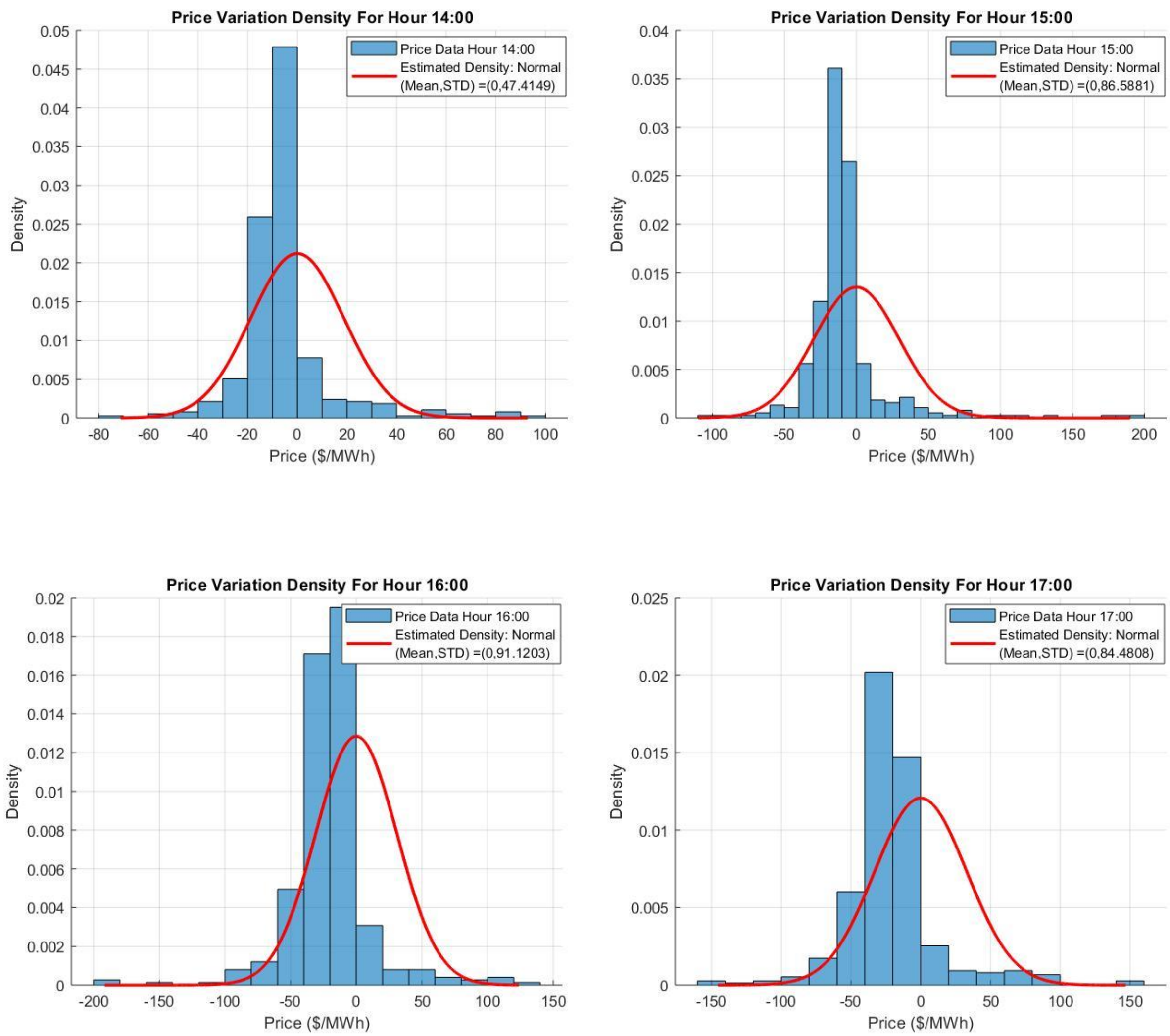

Figure 3.8 Peak period price difference density estimate

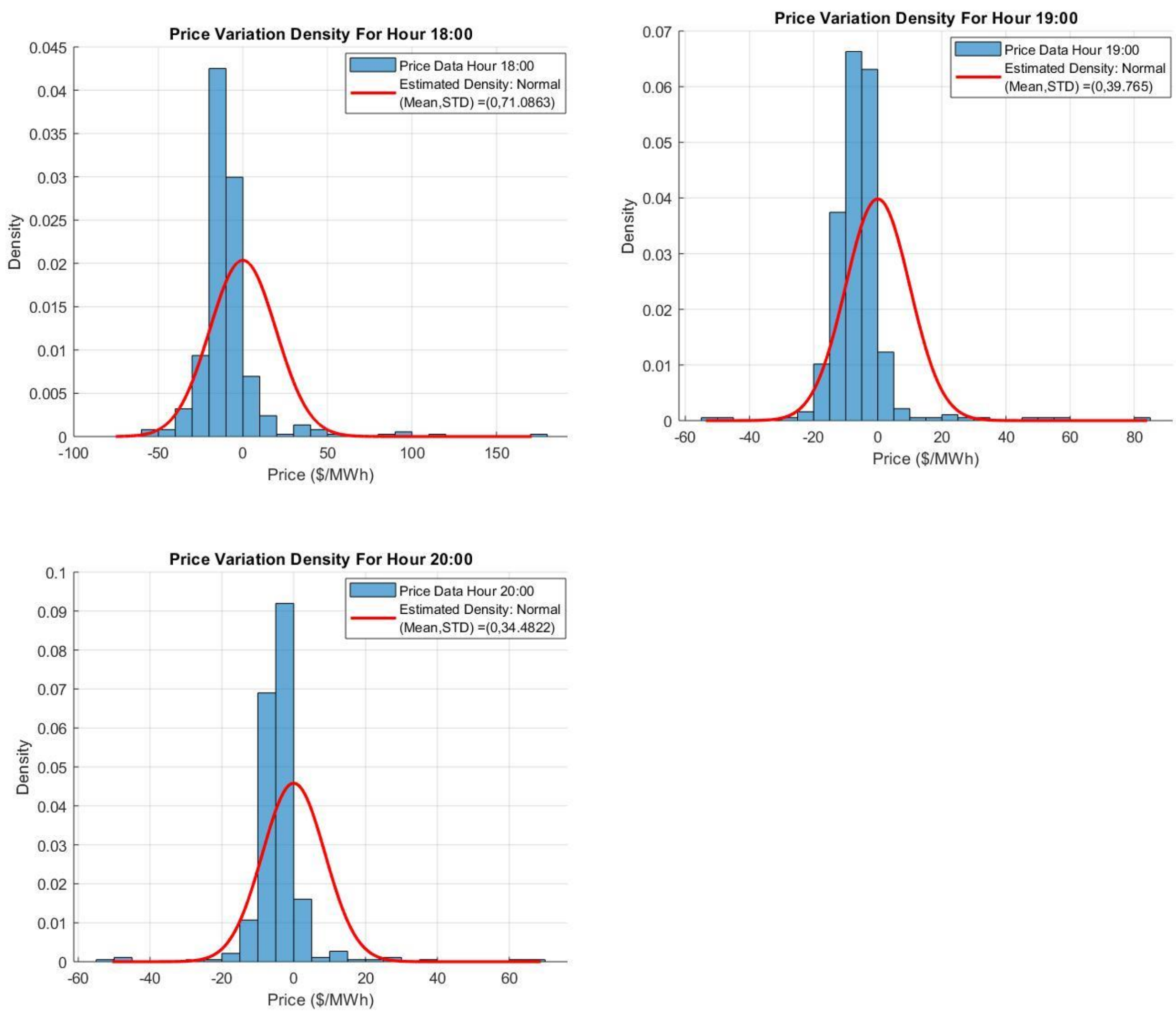

Figure 3.9 Peak period price difference density estimate

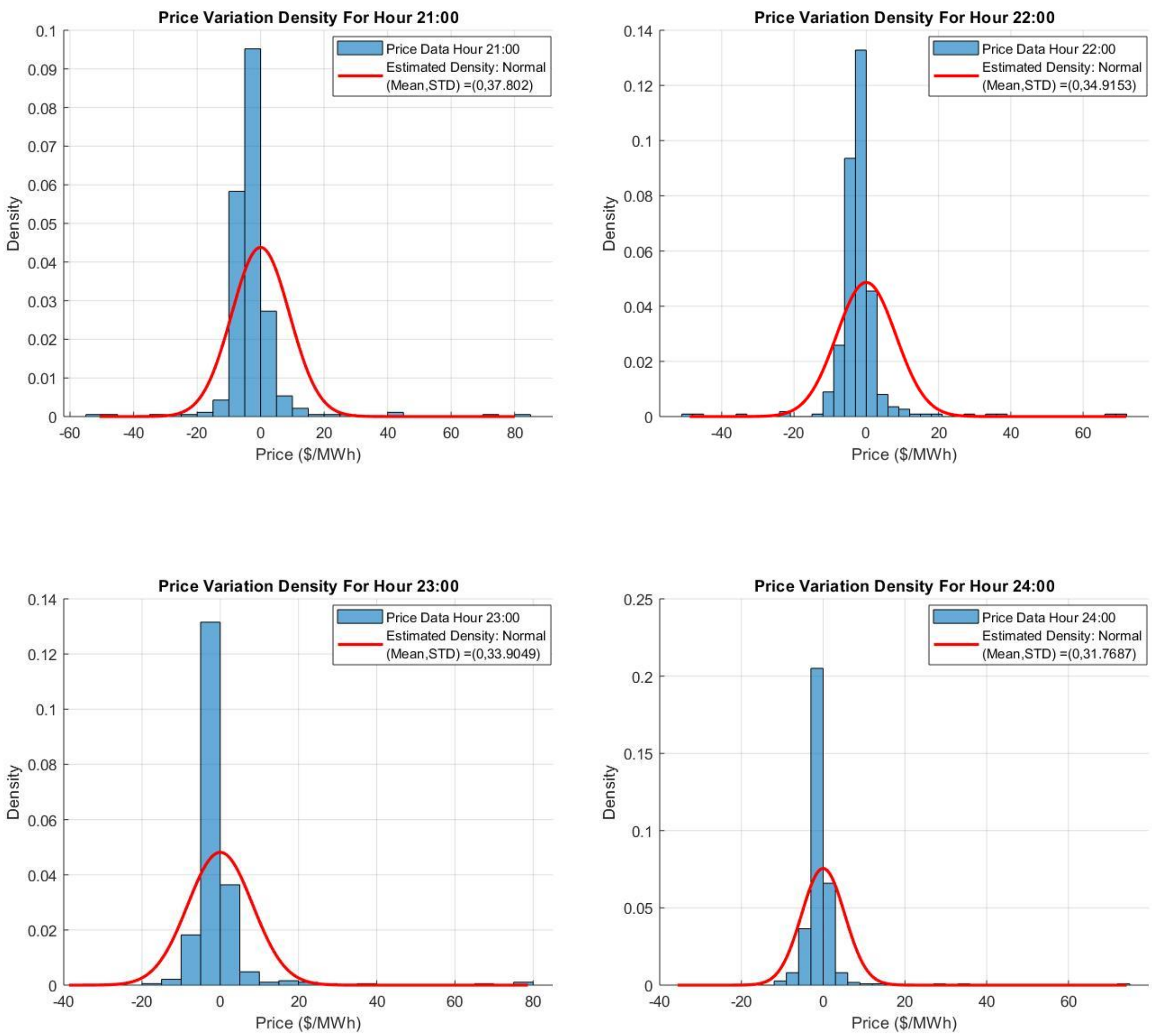

Figure 3.10 Shoulder two period price difference density estimate

Chapter 4

Customers' Load

This thesis will consider only residential loads. The aggregator would like to aggregate homes into buying groups based on their consumption pattern. The real-time load will be often unequal to the day-ahead load and the risk of trading in the spot market for excess energy will be addressed through creating multiple scenarios for the load that would represent the most possible load scenarios that might be faced by the aggregator. The following sections and the next Chapter show the proposed procedure.

4.1 Load Data Collection

The load data in this thesis was collected from Pecan street, which is an actual residential area in Austin, Texas, Pecan Street. The data are available for interested readers at Pecan Street website [14]. The access to the data requires an educational or business license.

Next, the data collected are for 100 homes. The challenging part is to find complete data without missing entries. Many homes were viewed and checked for the summer 2017 until a complete data set for 100 homes was obtained. The time frame considered is for the summer of 2017. From the study and analysis for the data, it has been found that the load profiles can be categorized into five groups by comparing to the average value of the aggregate load of the 109 load profiles, one for each day in the study duration. The first group is above the average value of the aggregate load, 35.78% of the days had this feature for all hours as shown in Figure 4.1. The second group is below the average value with 29.36% of days. The third group is approximately equal to the average value with a percentage of 5.5%. Next is a group of profiles having load values above the average value during the morning off-peak period from hour 12:00

AM to 10:00 AM, and then after the on-peak following 8:00 PM, load values fall below the average values. This group of profiles was found to be in 11.93% of the total profiles. The last group has values below the average value during the morning off-peak period and then after the on-peak period (after hour 8:00 PM) the load was found to be higher than the average values. This group of profiles represents 17.43% of the total load profiles.

4.2 Load Profiles Above the Average Value

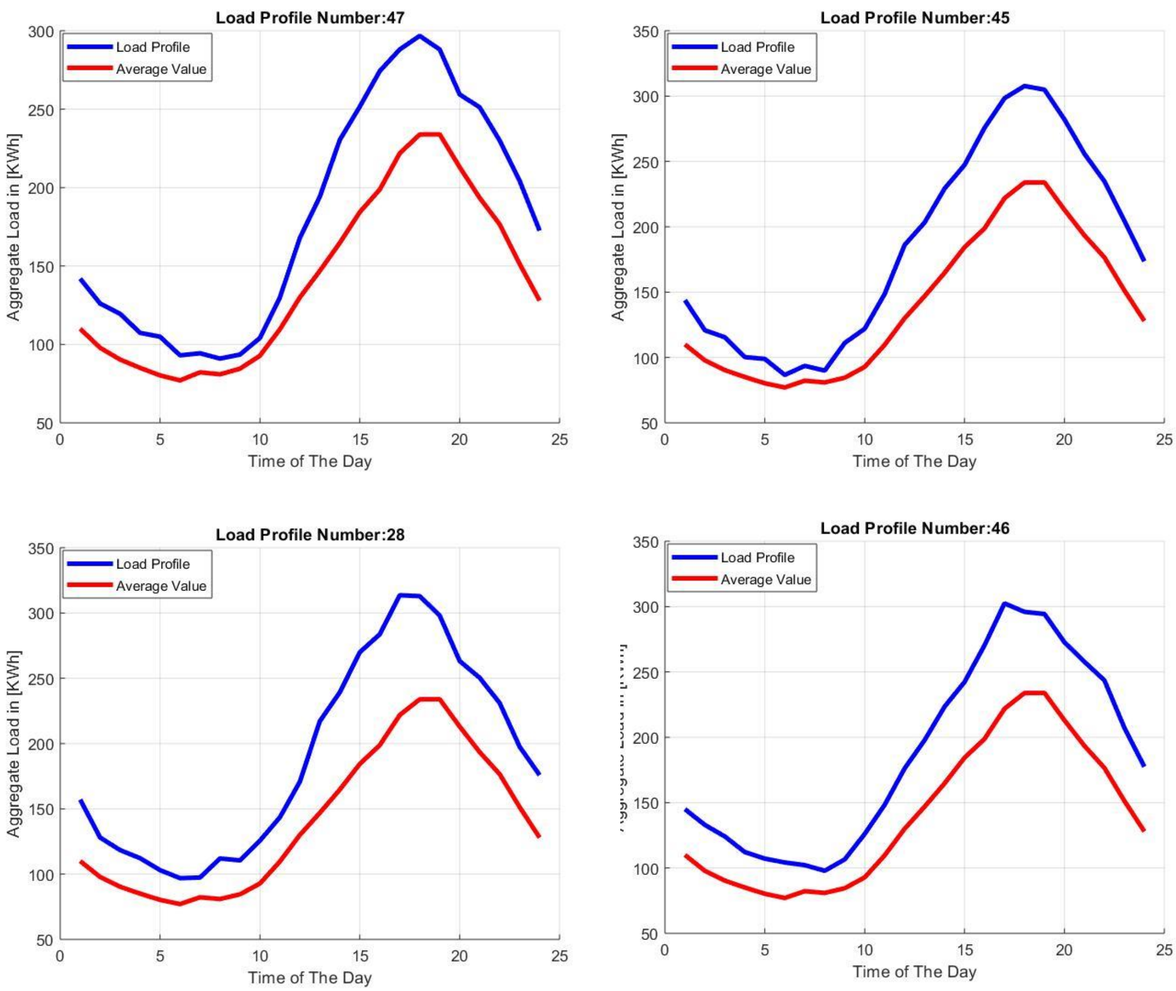

Figure 4.1 Samples of load profiles above the average value

Figure 4.1 shows four different load profiles selected randomly from the load profiles group above the average value.

4.3 Load Profiles Below the Average Value

Figure 4.2 shows four different load profiles selected randomly from the load profiles group below the average value. First plot on the top left corner is load profile number 13 next right is a plot for load profile number 11; the second row from left is load profile 4 and last plot is load

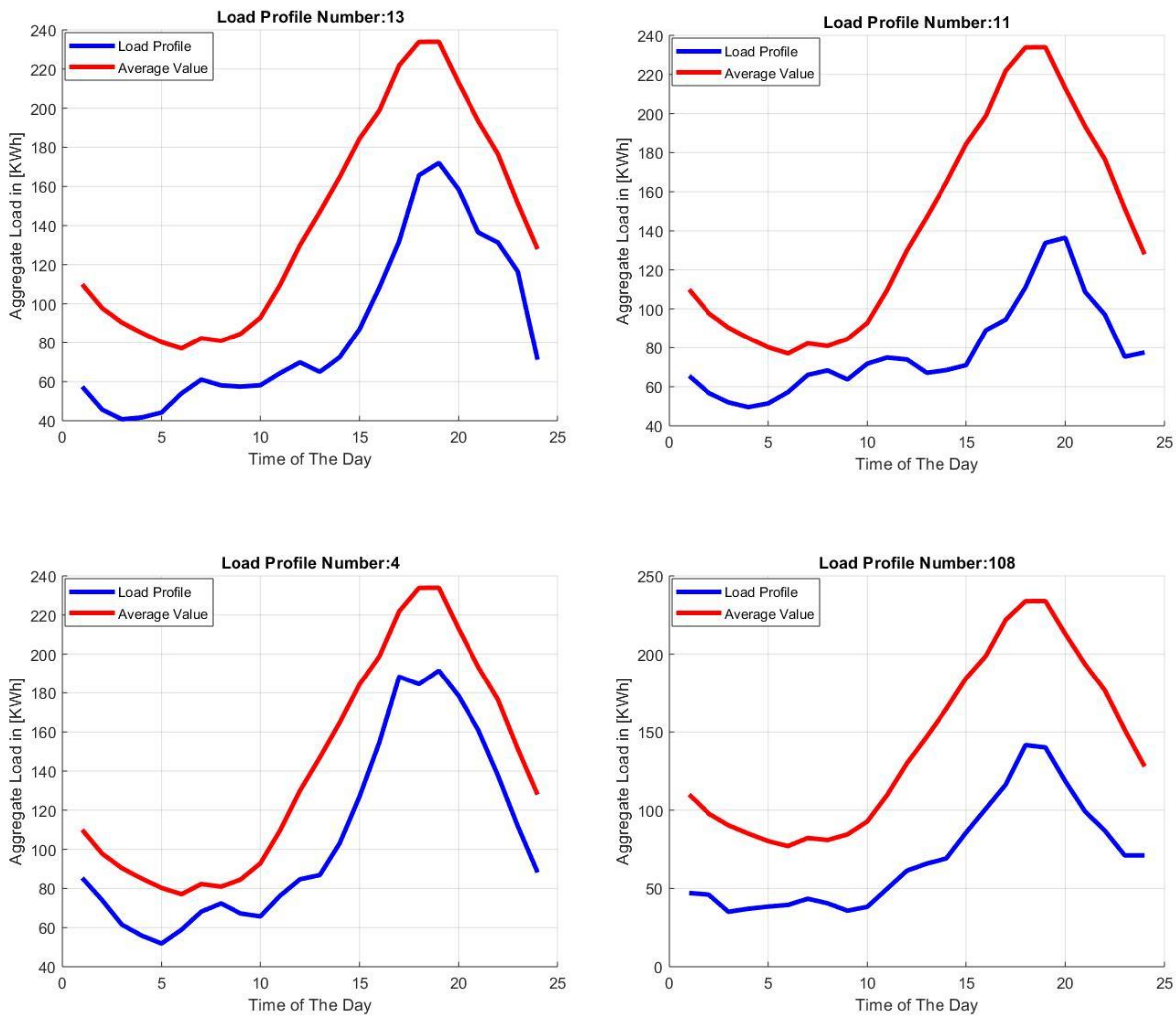

Figure 4.2 Samples of load profiles below the average value

profile number 108. All the plot shows that the load is approximately lower bounded by 47KWh and upper bounded by approximately 247 KWh. The rest of load profiles in this group are approximately within this range.

4.4 Load Profiles Approximately Equal the Average Value

Figure 4.3 shows four different load profiles are selected randomly from the load profiles group equal to the average value.

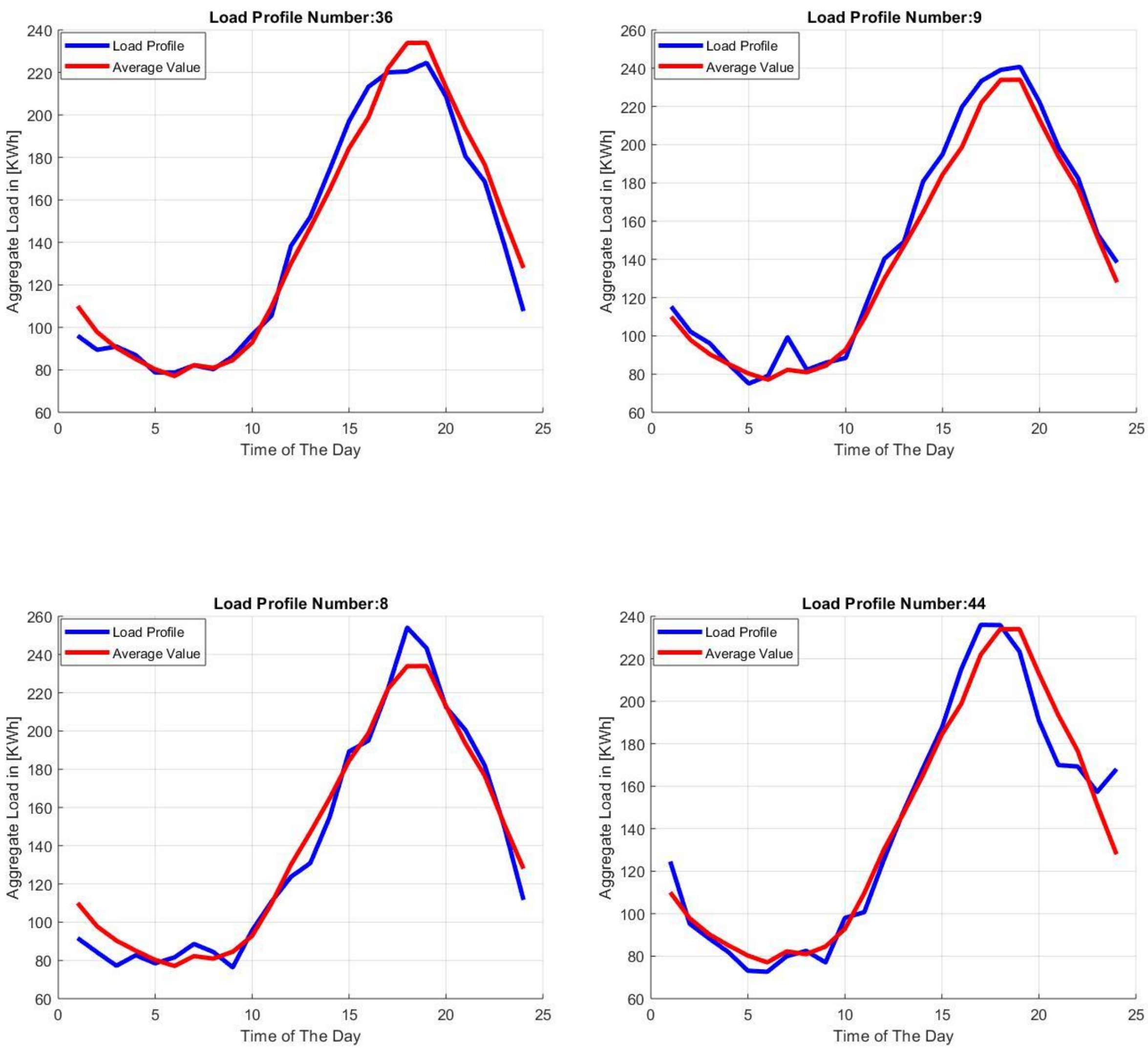

Figure 4.3 Samples of load profiles approximately equal the average value

4.5 Load Profiles Above and Below the Average Value

Figure 4.4 shows four different load profiles that are selected randomly from the load profiles of the group above and below the average value.

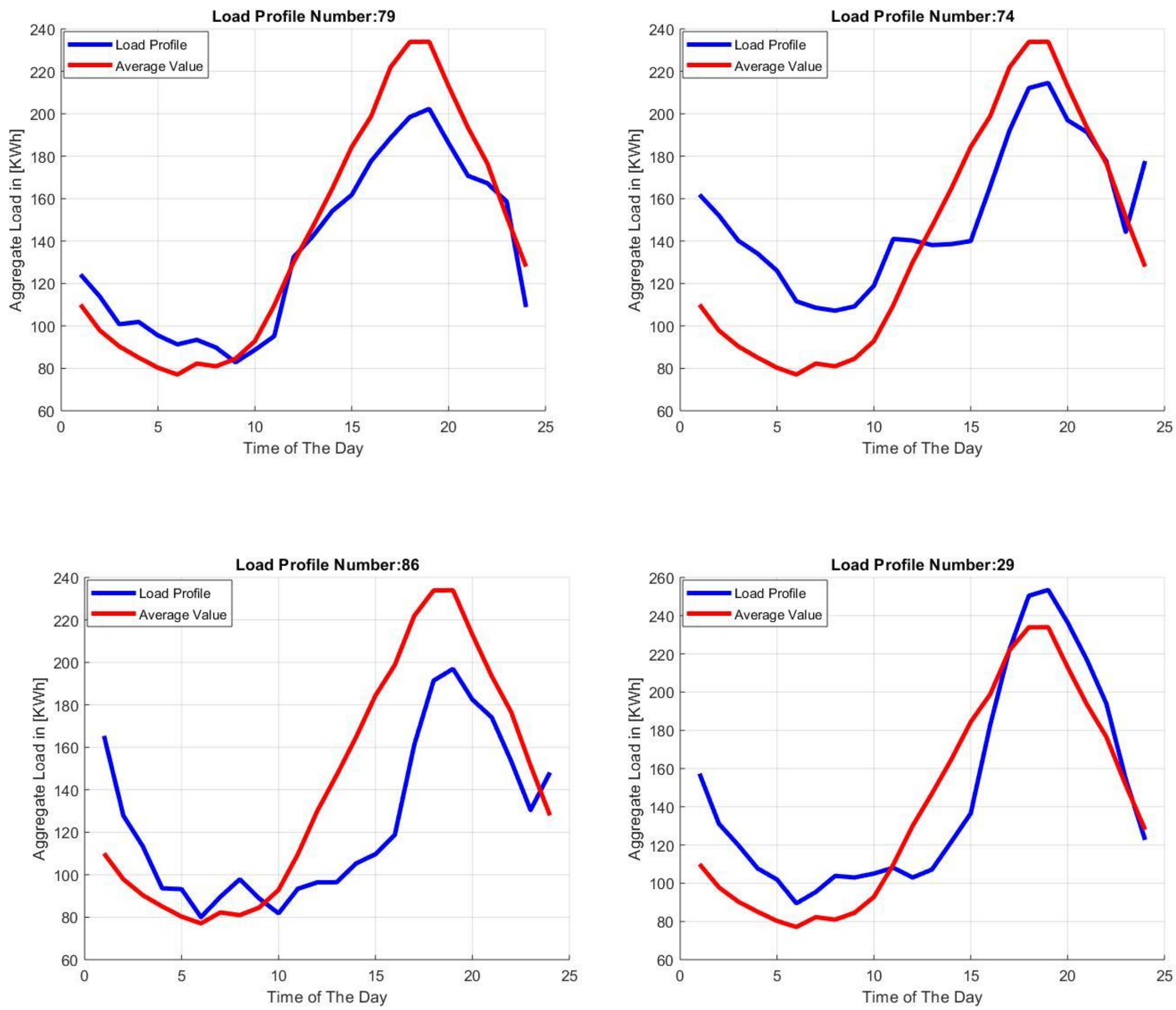

Figure 4.4 Samples of load profiles above and below the average value

4.6 Load Profiles Below and Above the Average Value

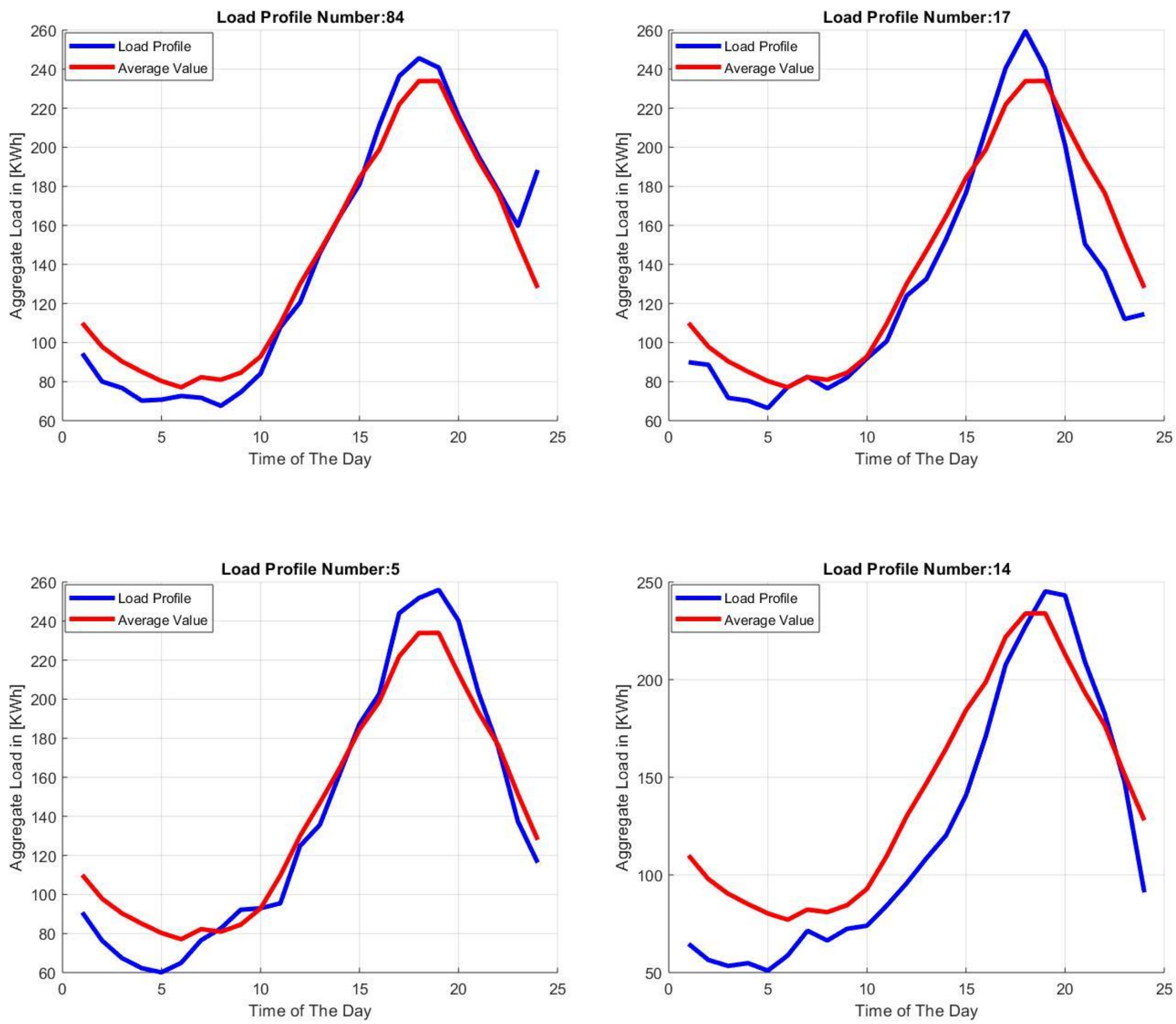

Figure 4.5 Samples of load profiles below and above the average value

Figure 4.5 shows four different load profiles are selected randomly from the load profiles group below and above the average value.

Chapter 5

Scenario Generation for Real-Time Price and Residential Load

5.1 Heuristic Algorithm to Generate Real-Time Prices

This section will propose a heuristic procedure for real-time price generation based on ERCOT market behavior obtained in the previous section. This procedure generates samples from a Gaussian probability distribution for each hour in a day using the computed mean and standard deviation has been found in Chapter 3. The following is our algorithm:

Algorithm Real-Time Price (RTP)

Step (0): Collect real-time and day-ahead prices then classify them based on seasonality and working or non-working days.

Step (1): Obtain the difference between real-time and day-ahead prices.

Step (2): For each hour fit a gaussian distribution $N \sim (m_t, \sigma_t)$ for the difference.

Step (3): Receive Mean and STD for the error at each hour.

Step (4): Initialize $t \leftarrow 1$ and $\omega \leftarrow 1$.

Step (5): if $t \leq 24$

Randomly generate ${LMR_{t\omega}}^R: N \sim (m_t, \sigma_t)$, such that

$$ {LMP_t}^D - 3\sigma_t \leq {LMR_{t\omega}}^R \leq {LMP_t}^D + 3\sigma_t $$

Step (6): if $\omega < N_\omega$

$\omega \leftarrow \omega + 1$

elseIf $\omega = N_\omega$

$t \leftarrow t + 1$

Go to Step (5)

End

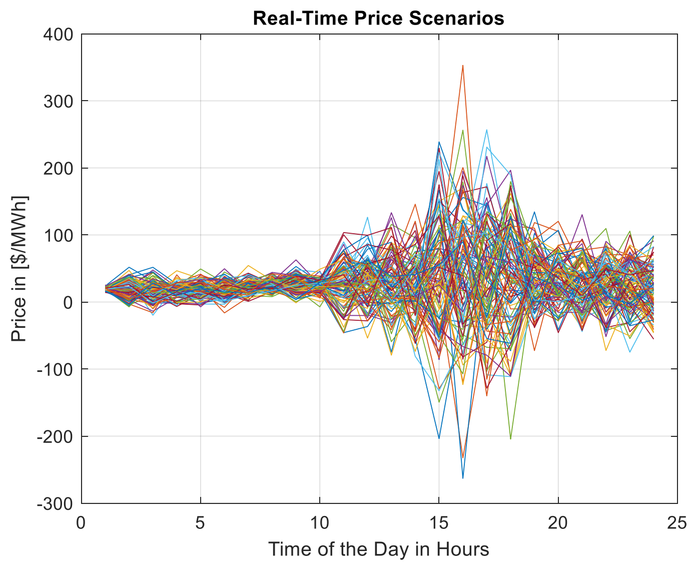

Figure 5.1 Real-time price scenarios using the proposed algorithm

Since normal distribution have been used to describe the data, the randomly generated values must have both positive and negative values. Also, 99% of the generated values must be within +/- 3 sigma from the mean values of the prices. Figure 5.2 shows the validation of the method used.

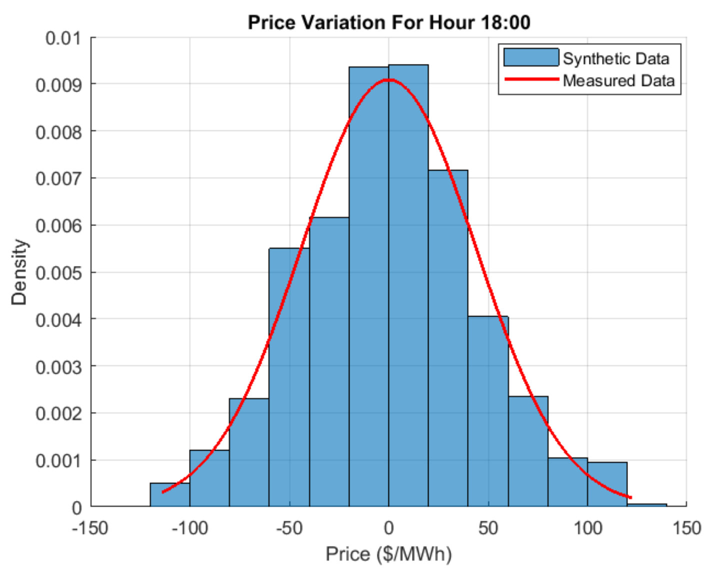

Figure 5.2 Price Variation density for a selected hour by method proposed

Figure 5.2 shows that the random generated data for hour 18:00 which is following the normal distribution given by the following equation:

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{1}{2}\left(\frac{x-m}{\sigma}\right)^2\right]$$

Where m is the mean value and it's equal to 0 and σ is the standard deviation which is equal to 71.0863 $/MWh. The same procedure was followed to validate the rest of the hours of the day.

5.2 Scenario Generation Procedure for the load

The load variability is taken based on an hourly basis from data collected from Pecan Street server for a residential area in Texas Austin to represent behavior of the load during summer period on working days. Customers' response to electricity is modeled via the price elasticity of the demand based on the US market. Markets considered are for short run trading and on an hourly basis.

This section will be dedicated to generating scenarios for the load profiles from the 109 days of the summer period. The load profiles are compared to the aggregator energy contract with DSO, which is considered to be the mean value of the 109 load profiles. Load profiles can be above the contract with DSO or below, or equal, or a mix as showed in the previous section. The aggregator load profile selection will be random to take into account the load variability. All possible load profiles scenarios are shown in Figure 5.3:

The total number of profiles above the average value of the aggregate load is 39 out of 109 profiles with 32 profiles below the average value, 13 above and below and 19 below and above the average value, while 6 profiles are approximately equal the average value.

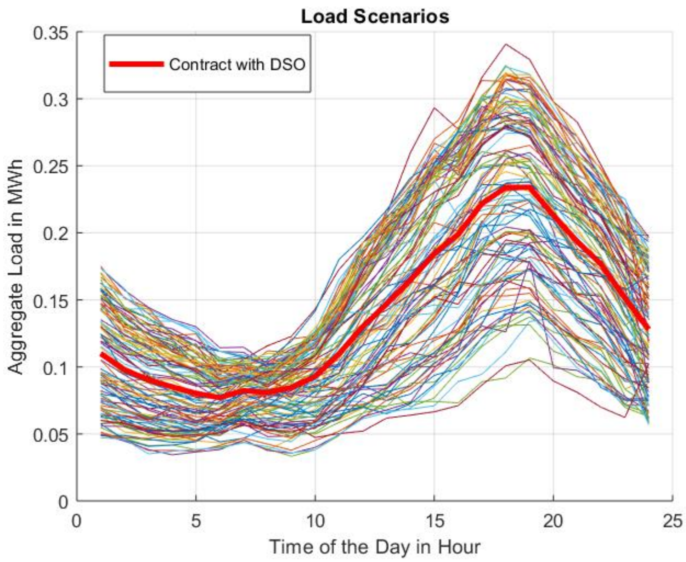

Figure 5.3 Load Scenarios in MWh

To generate scenarios for the load this work considers the percentage representation value of each profile group mentioned. For example, if the need to generate 10 scenarios for the load it should be decided how many profiles should be selected from each group out of 109 total profiles as the following:

1. Number of Profiles Above the Average Value

$$\mathrm{N_A} = \frac{39}{109} \times 10 \cong 4 \text{ profiles}$$

Where $\mathrm{N_A}$ is number of load profiles should be selected above the average value.

2. Number of Profiles Below the Average Value

$$\mathrm{N_B} = \frac{32}{109} \times 10 \cong 3 \text{ profiles}$$

Where $\mathrm{N_B}$ is number of load profiles should be selected below the average value.

3. Number of Profiles Equal the Average Value

$$N_E = \frac{6}{109} \times 10 \cong 1 \text{ profile}$$

Where N_E is number of load profiles should be selected equal the average value.

4. Number of Profiles Above and Below the Average Value

$$N_{AB} = \frac{13}{109} \times 10 \cong 1 \text{ profile}$$

Where N_{AB} is number of load profiles should be selected above and below the average value.

5. Number of Profiles Below and Above the Average Value

$$N_{BA} = \frac{19}{109} \times 10 \cong 1 \text{ profile}$$

Where N_{BA} is number of load profiles should be selected below and above the average value.

Another way to generate scenarios for the load is to consider that all load profiles are uniformly distributed between 1-109 and start randomly generating numbers taking values between 1-109, generate numbers up to the total number of scenarios required, such as the 10 scenarios. If the generated number from the uniform distribution turns out to be 98, then profile number 98 should be picked and fill scenario number 1 with the selected profile until 10 load scenarios have been generated.

Also, this work considered equal selection of the load profiles from each group. For example, if 10 scenarios should be selected, then pick randomly two profiles from each of the five groups mentioned earlier until getting the required 10 scenarios.

The next chapter will explore different case studies based on the data and procedures explained for the scenario generation for both price and load.

Chapter 6

Results

This chapter discusses different study cases based on the proposed model for the aggregator and the collected data shown previously. The aggregator stochastic model implemented and solved using SCIP solver [15] within the General Algebraic Modeling (GAMS) platform [16]. If the load is higher, the aggregator will have to purchase electricity in the spot market to make up for this difference, and spot market prices can be very high. The spot price considered is five times the real-time price during the peak period unless otherwise mentioned. The following three scenarios can be considered, but for this work is dedicated to study the first scenario:

1- There is a penalty if the load exceeds, but no penalty if load is lower than the contract value.
2- There is a penalty if the load deviates (higher or lower) from the contracted values.
3- There is a penalty if the load is higher than the contracted value, but there is a reward if it is lower than the contracted value.

The specifications of the penalty values are purely dependent on the aggregator's judgment and former experience in energy trading in the market. The aggregator has to guess penalty values when purchasing excess energy from the spot market, based on prior experience in the electric market. The expected volume deviation cost in $[\$/\mathrm{hr}]$ can be described by the following equation:

$$\text{Expected Volume Deviation Cost} = \sum_{\omega=1}^{N_\omega} \pi_\omega \, \mathrm{LMP}_t^{\,P}\left[\left(E_t^{\,C} + \Delta E_{t\omega}^{\,C}\right) - E_t^{\,D}\right]$$

Where $\mathrm{LMP}_t^{\,P}$ is the spot market price, and $\left[\left(E_t^{\,C} + \Delta E_{t\omega}^{\,C}\right) - E_t^{\,D}\right]$ is the load deviation from the contracted value in real-time at each hour. The next section discusses the simulation conditions used to simulate case study number one, where 100 real-time price scenarios were generated, and load profile is above the average value.

6.1 Simulation Conditions

Study cases in this chapter consider the aggregator working in Austin, Texas, with a group of 100 homes from Pecan Street. After solving the proposed model, the aggregator is interested in determining the price variation for each hour and the new selling price offered to customers after solving the proposed model. Our standard case study conditions, unless other information is stated, are 20% confidence level, 20% ramping limit and 40% demand elasticity. It was compiled and solved on a Windows-based server with Intel Xeon processor rated at 2.40 GHz with 256 GB installed memory RAM. A default parameter setup was used to initialize the SCIP solver. Initial points for the solver allowed the GAMS initialization for all variables to start with zeros. Solutions of all cases returned optimal results with zero non-optimality, zero infeasibilities, zero unbounded, and zero errors. The simulation parameters are summarized in Table 6.1 and 6.2 below. Table 6.1 indicates the number of scenarios being used for real-time price, confidence level, ramping limit and T&D charges. All these parameters are the choice of the aggregator and can be chosen based on the aggregator's priorities and trade-off between profit and risk. Table 6.2 shows the penalty multiplier. The penalty multiplier is a multiplier used to penalize the aggregator when the load deviates from the contracted value in day-ahead market. The spot market price that the aggregator will pay for the deviation is the real-time price times the multiplier factor shown in Table 6.2.

Table 6.1 Simulation Parameters

#	Simulation Parameter	Value
1	**Number of Scenarios**	100
2	**Time Periods**	24
3	**Confidence Level**	20%
4	**Ramping Limit**	20%
5	**T&D Charges Payed by Customers**	+$70
6	**T&D Charges Payed by the aggregator**	+$30
7	**Elasticity**	-40%
8	**Probability of a scenario**	1/100

Table 6.2 Penalty Multiplier Coefficient

Penalty Multiplier Coefficient				
#	A_t^{DH}	A_t^{DL}	A_t^{RH}	A_t^{RL}
1	1	0	1	0
2	1	0	1	0
3	1	0	1	0
4	1	0	1	0
5	1	0	1	0
6	1	0	1	0
7	1	0	1	0
8	1	0	1	0
9	1	0	1	0
10	1	0	1	0
11	1	0	1	0
12	1	0	1	0
13	1	0	1	0
14	5	0	5	0
15	5	0	5	0
16	5	0	5	0
17	5	0	5	0
18	5	0	5	0
19	5	0	5	0
20	5	0	5	0

21	**1**	**0**	**1**	**0**
22	**1**	**0**	**1**	**0**
23	**1**	**0**	**1**	**0**
24	**1**	**0**	**1**	**0**

Where:

$\mathrm{A_t}^{\mathrm{DH}}$: Penalty constant for day − ahead profit when load is higher than contract

$\mathrm{A_t}^{\mathrm{DL}}$: Penalty constant for day − ahead profit when load is lower than contract

$\mathrm{A_t}^{\mathrm{RH}}$: Penalty constant for Real − time profit when load is higher than contract

$\mathrm{A_t}^{\mathrm{RL}}$: Penalty constant for Real − time profit when load is lower than contract

6.2 Case Study I: Load Profiles are Above Contract with 100 Scenarios for Real-Time Price

Table 6.3 shows the aggregator contract with DSO in the day-ahead in [MW]. The aggregator used the average value of the aggregate load of the 100 homes to contract with DSO. The aggregator proposed the average value since the load is random and cannot be known to the aggregator until real-time market. To avoid contracting more or contracting less it is an appropriate assumption to contract the average value for the following day. If the aggregator contracted more and didn't use the energy contracted, it might lose money in this case for not being refunded for the excess energy contracted. Or, if it contracted less, the aggregator would buy more energy in the spot market at a very high price which would affect its profit.

Table 6.3 Contract with DSO for 24 hours period

Contract with DSO in [MW]											
t1	**t2**	**t3**	**t4**	**t5**	**t6**	**t7**	**t8**	**t9**	**t10**	**t11**	**t12**
0.11	0.10	0.09	0.09	0.08	0.08	0.08	0.08	0.08	0.09	0.11	0.13
t13	**t14**	**t15**	**t16**	**t17**	**t18**	**t19**	**t20**	**t21**	**t22**	**t23**	**t24**
0.15	0.16	0.18	0.20	0.22	0.23	0.23	0.21	0.19	0.18	0.15	0.13

Load values for the case under study are shown in Table 6.4 below. The load shown in Table 6.4 is for 100 homes for 109 days combined at each specified hour of the day. The one load profile is chosen randomly from 39 load profiles above the average value which is mentioned in Chapter 5 section 5.2.

Table 6.4 Customers aggregate load values in [MWh]

Customers Aggregate Load Values in [MWh]											
t1	**t2**	**t3**	**t4**	**t5**	**t6**	**t7**	**t8**	**t9**	**t10**	**t11**	**t12**
0.16	0.14	0.13	0.12	0.10	0.10	0.09	0.10	0.11	0.12	0.14	0.17
t13	**t14**	**t15**	**t16**	**t17**	**t18**	**t19**	**t20**	**t21**	**t22**	**t23**	**t24**
0.19	0.23	0.24	0.25	0.27	0.28	0.28	0.27	0.25	0.23	0.20	0.20

The day-ahead prices shown in Table 6.5 are selected from ERCOT market for Tuesday September 12th, 2017. The day selected is a working summer day. Assumption is that the aggregator is a price taker company, so it will not affect the wholesale price in the day-ahead market.

Table 6.5 Day-ahead price for 24 hours period

Day-ahead Price in [$/MWh] at the Time of the Day in Hours											
t1	**t2**	**t3**	**t4**	**t5**	**t6**	**t7**	**t8**	**t9**	**t10**	**t11**	**t12**
19.67	18.67	17.85	17.43	17.74	18.87	22.62	24.6	22.93	25.36	24.5	26.67
t13	**t14**	**t15**	**t16**	**t17**	**t18**	**t19**	**t20**	**t21**	**t22**	**t23**	**t24**
27.82	31.73	36.6	44.05	49.64	39	30.74	30	27.41	26.55	24.08	21.3

Real-Time price scenarios are generated using MATLAB and randomly generated from normal random generator with mean and standard deviation stated in Chapter 3. Figure 6.1 shows 100 price scenarios generated randomly from the Gaussian distribution. Price scenarios are plotted for 24 hours to show the possible variations in real-time price during a given day. Buying in real-time market contains a high level of risk to the aggregator since the price is fluctuating in uncertain manner based on the real-time demand and real-time generation.

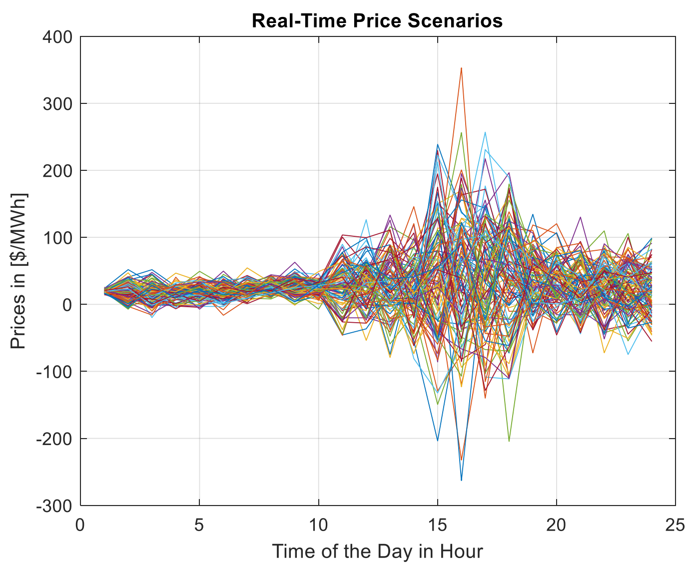

Figure 6.1 Real-Time Price Scenarios

The next sections will show simulation results for this case study. The aggregator is interested in determining the price variation at each hour of the day and the new real-time selling price to be offered to customers. The price variation at each hour will produce an hourly load variation per real-time price scenario, based on the customer elasticity. The aggregator is interested in finding the mean values of energy variation resulting from changing selling price. For example, if the price increased by +20% the load should decrease by 80% when the price elasticity of the demand is -40%. This work will show the customers savings and prove the validity of the proposed model where energy cost for customer is less with the new selling price. Also, the proposed model will achieve a higher profit for the aggregator when trading with customers under the new selling price.

6.2.1 Case Study I: Hourly Price Variation

The results show the aggregator would decrease the selling price for off-peak periods while increasing it in the peak period to motivate customers to shift their load to the low-price periods during a day.

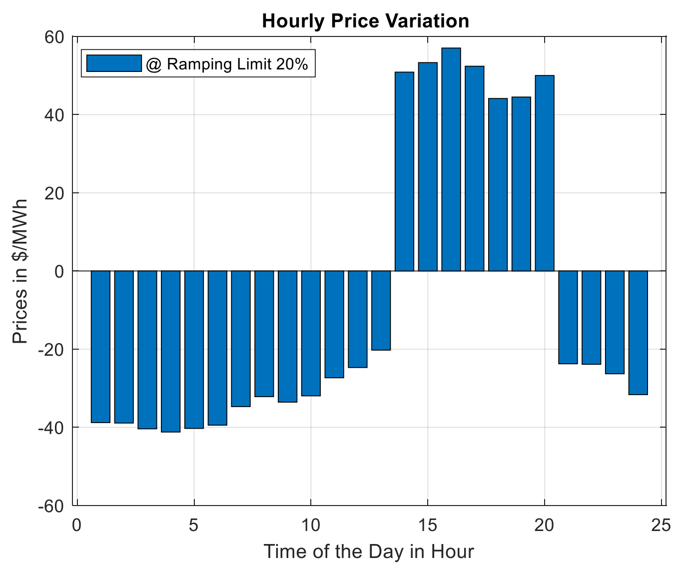

Figure 6.2 Hourly price variation for the selected case study

Tables 6.6, 6.7, and 6.8 show the price variation at each hour of the day. Price variation for the first and second off-peak periods are shown in Tables 6.6 and 6.8 while the on-peak period price variation is shown in Table 6.7 below. The price variations depend on the period of the day; on-peak period has a positive increase while off-peak periods have a negative increase. This is because the price of electricity tends to be higher during on-peak period while it is lower during off-peak periods.

Table 6.6 Hourly price variation for the first off-peak period

First Off-peak Period												
Hourly price variation in [$/MWh] at the Time of the Day in Hours												
t1	**t2**	**t3**	**t4**	**t5**	**t6**	**t7**	**t8**	**t9**	**t10**	**t11**	**t12**	**t13**
-38.78	-38.90	-40.38	-41.20	-40.27	-39.44	-34.69	-32.14	-33.55	-31.95	-27.32	-24.70	-20.24

Table 6.7 Hourly price variation for the On-peak period

On-peak Period						
Hourly price variation in [$/MWh] at the Time of the Day in Hours						
t14	**t15**	**t16**	**t17**	**t18**	**t19**	**t20**
50.86	53.30	57.02	52.37	44.12	44.49	50.00

Table 6.8 Hourly price variation for the second off-peak period

Second Off-peak Period			
Hourly price variation in [$/MWh] at the Time of the Day in Hours			
t21	**t22**	**t23**	**t24**
-23.75	-23.88	-26.30	-31.63

The negative sign in the first and second off-peak periods indicates that the aggregator will decrease the selling price at those hours, while the positive sign for the price variation in the peak period indicates that the aggregator will increase the selling price with an amount shown in Table 6.7. The new selling price shown in Figure 6.3 is the new selling price that will be offered by the aggregator to the customers. The new selling price will change the load at each hour

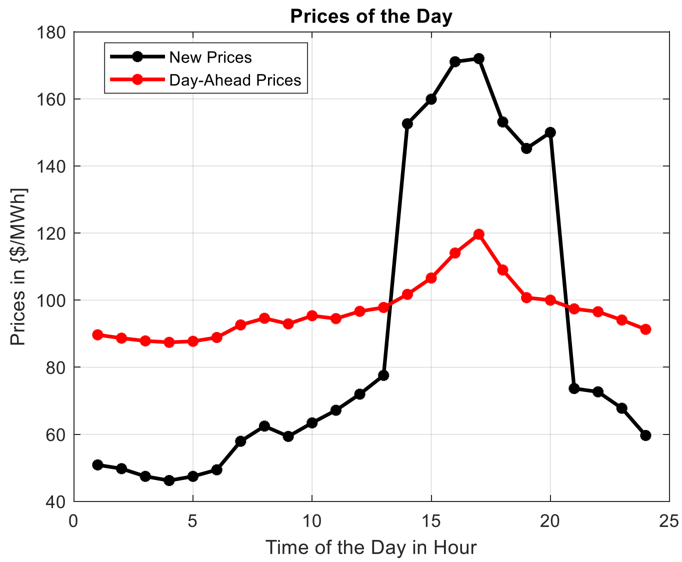

Figure 6.3 New and old selling prices

by a percentage determined by the price elasticity of the demand which is in our case is –40%. The negative sign of the elasticity is to indicate the inverse relationship between the price and demand. As the price of electricity increases the demand will decrease and vice versa. That means if a 10% increase in the price happens the demand will decrease by 4%. This will be shown in section 6.2.2.

6.2.2 Case Study I: Hourly Mean Energy Variation

Since customers are interested in achieving higher savings, they would be interested in shifting their load from the peak periods to the off-peak periods. Changing the selling price will yield a change in customers' energy values, since there are multiple scenarios for real-time price the solution of customers load will be scenario-based also.

Table 6.9 Mean energy variation for the first off-peak period

First Off-peak Period												
Mean energy variation in [MWh] at the Time of the Day in Hours												
t1	**t2**	**t3**	**t4**	**t5**	**t6**	**t7**	**t8**	**t9**	**t10**	**t11**	**t12**	**t13**
0.03	0.03	0.02	0.02	0.02	0.02	0.01	0.01	0.02	0.02	0.02	0.02	0.02

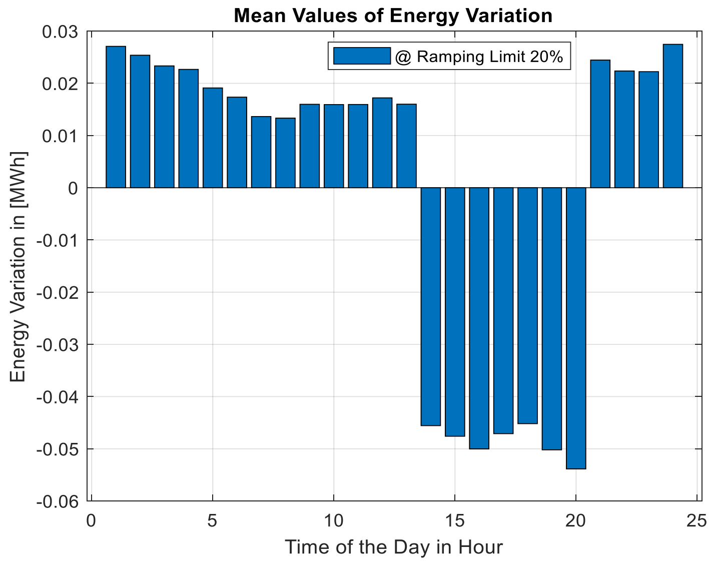

Figure 6.4 Mean energy variation in MWh

The aggregator is interested in knowing the mean energy variation of all the scenarios at each hour of the day as shown in Figure 6.4.

Table 6.10 Mean energy variation for the On-peak period

On-peak Period						
Mean energy variation in [MWh] at the Time of the Day in Hours						
t14	**t15**	**t16**	**t17**	**t18**	**t19**	**t20**
-0.05	-0.05	-0.05	-0.05	-0.05	-0.05	-0.05

Table 6.11 Mean energy variation for the second off-peak period

Second Off-peak Period			
Mean energy variation in [MWh] at the Time of the Day in Hours			
t21	**t22**	**t23**	**t24**
0.02	0.02	0.02	0.03

6.2.3 Case Study I: Customers Savings

The total amount paid by customers to the aggregator in real-time decreased compared to day-ahead payment. The expected payment for the selected day was $432.19 with day-ahead prices. Alternately, after changing the selling price by the aggregator, the amount is $411.64, as customers responded by shifting their loads. The implied customer savings is around $20.55 per day which represents a 4.75% decrease in energy cost for customers. Figure 6.5 shows the cost change for customers on an hourly basis.

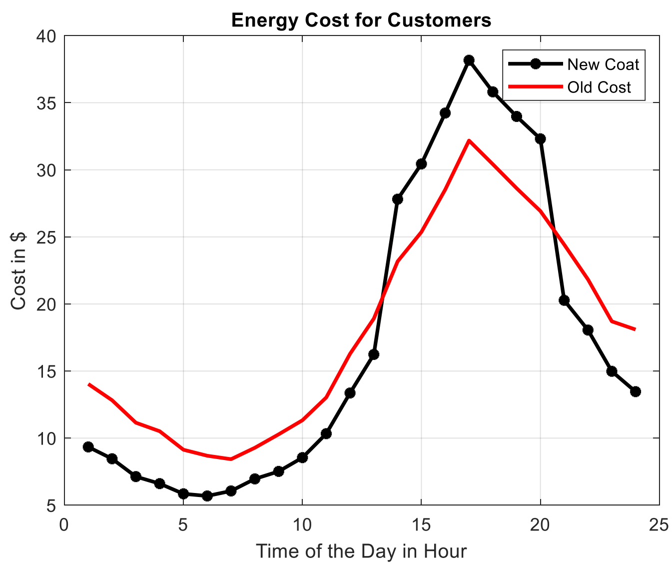

Figure 6.5 Energy cost hourly for customers

6.2.4 Case Study I: The Aggregator's Profit

The aggregator's profit also increases. The old expected profit for the aggregator is $59.79 and after changing selling price it becomes $131.64. This represents 120.2% increase with respect to the base profit. The aggregator may increase the profit by allowing higher values for ramping limits and lower values of confidence levels.

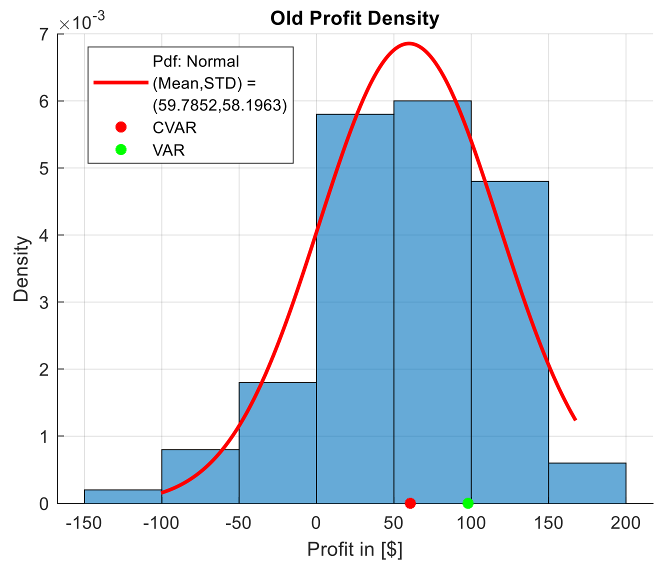

Figure 6.6 Aggregator old profit density

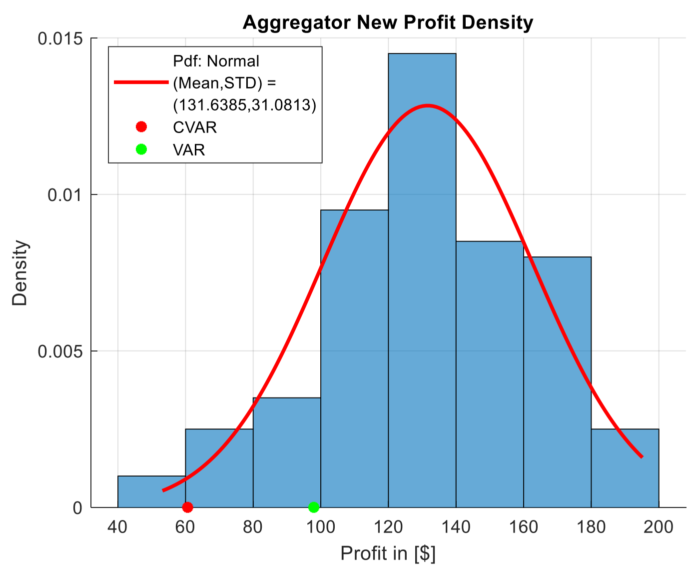

Figure 6.7 Aggregator new profit density

Profit density at scenarios considered for real-time price and load resulted to be normally distributed as shown in Figures 6.6 and 6.7. Profit distribution before and after change selling price show an increase in the expected profit.

6.2.5 Case Study I: The Aggregator Risk Measure

The risk measures, CVaR and the value at risk showing the trade-off between risk and profit of the aggregator, are explained in this section. The relationship between those values is given by an efficient frontier plot. Efficient frontier is a set of boundary optimal portfolios under a specific level of expected profit and for a specific risk level [17]. The portfolios below the efficient frontier plot are suboptimal and may not provide enough profit at the specified level of risk [17]. Efficient frontier plot represents the relationship between the aggregator expected profit and CVaR at different values of α confidence levels. As the confidence level increases, the expected profit decreases as the aggregator is not willing to take a higher risk. At higher level of confidence levels (lower risks) the aggregator tends to increase the selling price, the load will decrease and, thus, the profit will decrease, the opposite behavior is expected at lower level of confidence levels.

Efficient frontier plot can be used to find the most efficient portfolios, to be considered by the aggregator. Efficient frontier plot will give the aggregator a decision frame work, on how to decide between risk and profit, for a specific portfolio. If the aggregator is a profit seeker it may operate at lower level of α confidence level, that means a higher level of risk. If the aggregator is a risk-averse, then it would select a higher level of confidence level as high as α 95%. Sometimes and based on a pure decision by the aggregator it would be optimal to operate at the tangent point on the efficient frontier plot, the tangent point is a balance point between having a good level of profit and acceptable level of risk, a point could be at 50% confidence level.

- The expected profit at confidence level 20% is calculated using the following equation:

$$\text{Expected Profit } (\mu) = \text{E[Profit]} = \text{Prob}(\omega) \times \sum_{i=1}^{N_\omega} \text{Profit}_\omega$$

$$= \frac{1}{100} \times \left(\text{Prof}_{\omega_1} + \text{Prof}_{\omega_2} + \text{Prof}_{\omega_3} + \cdots + \text{Prof}_{\omega_{100}}\right)$$

$$\text{Expected Profit} = \ \$(131.64)$$

Expected profit at different confidence levels is found in the same manner above. The results are shown in the Table 6.12.

Table 6.12 The aggregator risk measure values

Confidence level	VAR ($)	CVAR ($)	Expected Profit ($)
0%	162.9413	71.8546	131.6398
10%	113.9146	65.69365	131.6386
20%	97.93031	60.61339	131.6385
30%	82.67599	56.66242	131.531
40%	76.68412	52.83375	131.5392
50%	70.82631	48.70042	131.3321
60%	61.39538	44.34934	131.2271
70%	55.99212	39.48365	131.1049
80%	35.1465	27.12581	128.6139
90%	35.1465	27.12581	128.6139

Relationship between expected profit and the conditional value at risk (CVaR) is shown in Figure 6.8. It can be used as a useful tool to hedge against the risk. The expected profit is less at high values of α with a minimum risk since the CVaR values are less, while the expected profit is higher at lower values of α with a higher risk as the CVaR values are higher.

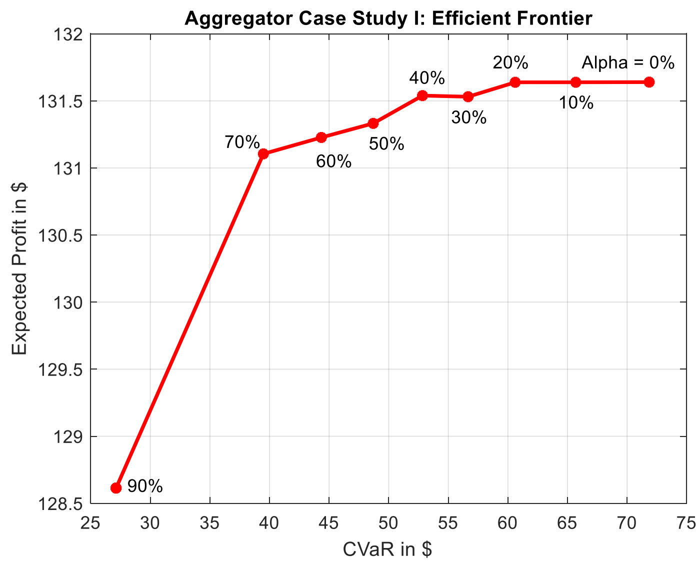

Figure 6.8 Aggregator case study I: efficient frontier

6.3 Case Study II: Random Load Profiles (From All Possible Groups with Equal Probable Selection) and Random Real-time Price

A more general and practical case study in which the load and price are random is discussed in this section. The simulation parameters are summarized in Table 6.13. Table 6.13 which show the number of scenarios being used for real-time price and customers load, confidence level, ramping limit and T&D charges. The penalty multipliers are kept the same as case study I.

Table 6.13 Simulation Parameters

#	Simulation Parameter	Value
1	**Number of Scenarios**	25
2	**Time Periods**	24
3	**Confidence Level**	20%
4	**Ramping Limit**	20%
5	**T&D Charges Payed by Customers**	+$70
6	**T&D Charges Payed by the aggregator**	+$30
7	**Elasticity**	-0.4
8	**Probability of a scenario**	1/25

Also, day-ahead price and energy contract with DSO are kept the same as the previous case. Therefore, five real-time price scenarios are generated using the reported mean and standard deviation in Chapter 3 and based on the proposed algorithm in Chapter 5 to generate scenarios. Real-time price scenarios are plotted as shown in Figure 6.9 below. Customers load scenarios are picked randomly from each load profile groups discussed in Chapter 4. Equal probable selection method of load profiles is used in this case study. The load scenarios are one load profile higher, one is lower, one is equal, one is higher and lower, and one is lower and higher than the contracted value with DSO. The average of 109 load profiles is selected to contract with DSO and used for this simulation. Figure 6.10 shows the load scenarios considered for this case study.

Figure 6.9 Real-Time price scenarios

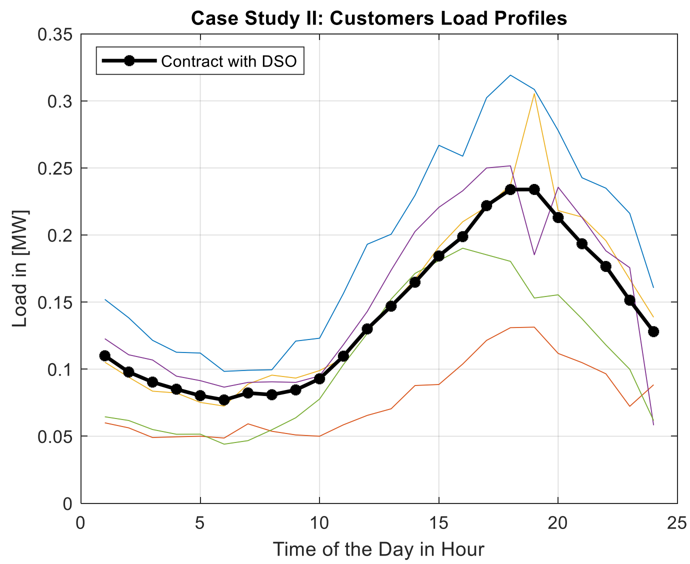

Figure 6.10 Customer load scenarios

6.3.1 Case Study II: Hourly Price Variation

The aggregator tends to decrease the selling price for off-peak periods while increasing it in during peak period to motivate customers to shift their load to the low-price periods during a day, as shown in Figure 6.11, which is similar to the previous case. However, there are some hours in the mornings where the price is increased slightly. Tables 6.14, 6.15, and 6.16 indicate the price variation at each hour of the day.

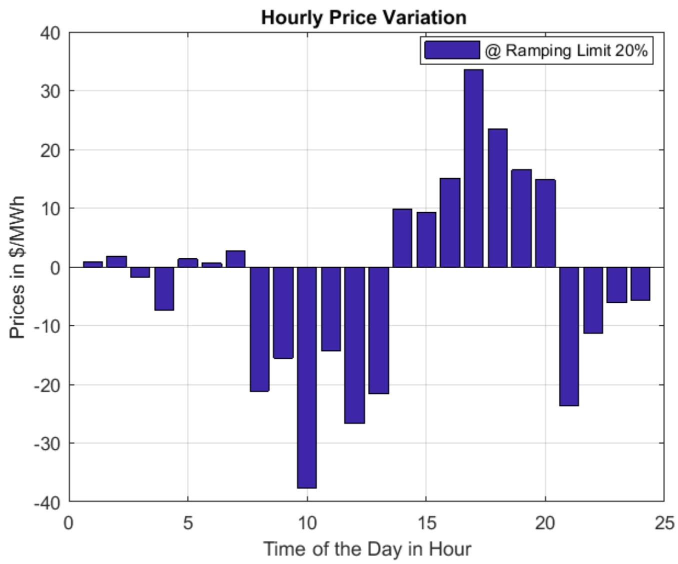

Figure 6.11 Hourly price variation for the selected case study

Table 6.14 Hourly price variation for the first off-peak period

First Off-peak Period												
Hourly price variation in [$/MWh] at the Time of the Day in Hours												
t1	**t2**	**t3**	**t4**	**t5**	**t6**	**t7**	**t8**	**t9**	**t10**	**t11**	**t12**	**t13**
0.93	1.85	-1.76	-7.42	1.32	0.63	2.80	-21.13	-15.52	-37.71	-14.24	-26.66	-21.64

Table 6.15 Hourly price variation for the On-peak period

On-peak Period						
Hourly price variation in [$/MWh] at the Time of the Day in Hours						
t14	**t15**	**t16**	**t17**	**t18**	**t19**	**t20**
9.78	9.30	15.05	33.62	23.55	16.51	14.84

Table 6.16 Hourly price variation for the second off-peak period

Second Off-peak Period			
Hourly price variation in [$/MWh] at the Time of the Day in Hours			
t21	**t22**	**t23**	**t24**
-23.65	-11.39	-6.07	-5.77

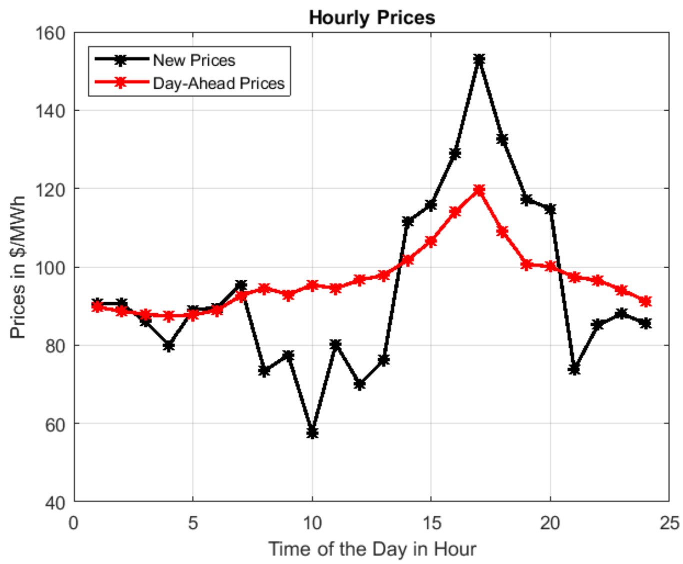

Figure 6.12 New and old selling prices offered to customers

6.3.2 Case Study II: Hourly Mean Energy Variation

Figure 6.13 shows the mean energy variation at each hour of the day, which are similar to the previous case. However, this case is showing a higher price and load fluctuation due to the added randomness in the load.

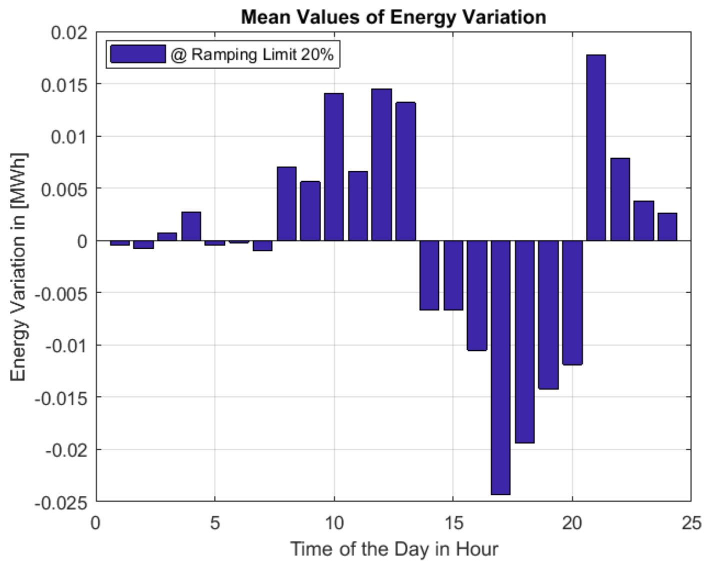

Figure 6.13 Mean energy variation in MWh

Table 6.17 Mean energy variation for the first off-peak period

First Off-peak Period												
Mean energy variation in [MWh] at the Time of the Day in Hours												
t1	**t2**	**t3**	**t4**	**t5**	**t6**	**t7**	**t8**	**t9**	**t10**	**t11**	**t12**	**t13**
-0.00042	-0.00077	0.000667	0.002654	-0.00046	-0.0002	-0.00093	0.007043	0.005599	0.014067	0.00657	0.014488	0.013178

Table 6.18 Mean energy variation for the On-peak period

On-peak Period						
Mean energy variation in [MWh] at the Time of the Day in Hours						
t14	**t15**	**t16**	**t17**	**t18**	**t19**	**t20**
-0.0066	-0.00661	-0.01051	-0.02428	-0.01935	-0.0142	-0.01186

Table 6.19 Mean energy variation for the second off-peak period

Second Off-peak Period			
Mean energy variation in [MWh] at the Time of the Day in Hours			
t21	**t22**	**t23**	**t24**
0.017706	0.007869	0.003772	0.002567

6.3.3 Case Study II: Customers Savings

Total amount paid by customers to the aggregator decreased compared to those based on day-ahead prices. The expected payment for the selected day was $321.98 with day-ahead prices. Alternately, after the aggregator changed the selling price, the amount became $319.79. The customer savings are around $2.19 per day, which represents a percentage decrease in customers energy cost by 0.68% from the original base case cost.

6.3.4 Case Study II: The Aggregator Profit

In this case study again the aggregator's profit increased. The old expected profit for the aggregator is $26.74 and after changing selling price, it becomes $38.3, which represents 43.23% increase with respect to the base profit. While the percent increase is substantial the base profit as well as new profit are much smaller compared to the previous case.

6.3.5 Case Study II: The Aggregator Risk Measures

The aggregator problem for this case is solved for multiple values of α confidence level and the results are shown in Table 6.20. The efficient frontier (Figure 6.14) obtained to study the relationship between the expected profit and the conditional value at risk at different values of α.

Table 6.20 The aggregator risk measures

Confidence level	VAR ($)	CVAR ($)	Expected Profit ($)
0%	33.51256	11.93498	38.67288
10%	26.81792	10.01632	38.43439
20%	26.11183	7.932567	38.30633
30%	19.49018	5.71519	36.5856
40%	11.31025	4.354889	33.87494
50%	11.0699	2.981852	33.76379
60%	3.521301	1.670664	29.69744

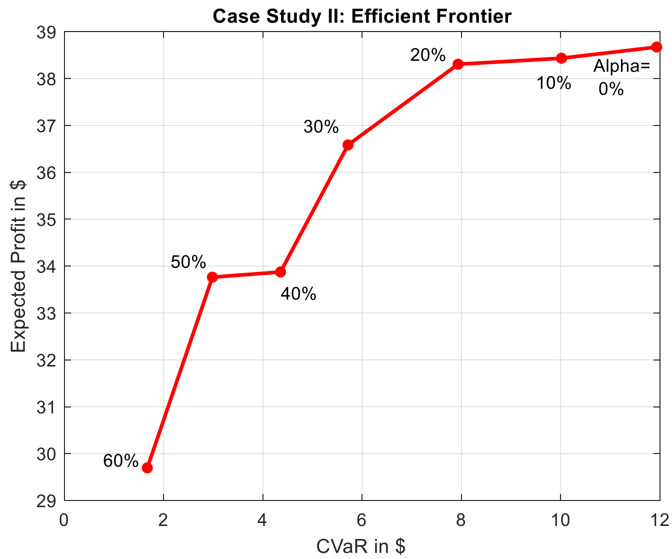

Figure 6.14 Aggregator case study II, efficient frontier

For confidence level of $\alpha = 0\%$ the aggregator is concerned about achieving maximum expected profit without concerning about risk. In this case, profit was found to be \$38.67 with a higher CVaR value of \$11.94 which implies a higher risk. Increasing the confidence level decreases the expected profit and reduces the CVaR value, which implies a lower risk. For example, with $\alpha = 60\%$ the expected profit is \$29.70 and the CVaR value is \$1.67. Comparison of the expected profit at $\alpha = 0\%$ with $\alpha = 60\%$ shows that the expected profit has decreased by 23.2% while the CVaR value decreased by 86.01%.

6.3.6 Case Study II: Increased Number of Real-time Price Scenarios

This case study focuses on adding more randomness to the real-time market price and study the effect of increasing number of real-time price scenarios on the selling price to customers, while keeping all simulation parameters the same as case study II. The simulation parameters are shown in Table 6.21 and Figure 6.15 shows the real-time market price scenarios.

Table 6.21 Simulation Parameters

#	Simulation Parameter	Value
1	**Number of Scenarios**	100
2	**Time Periods**	24
3	**Confidence Level**	20%
4	**Ramping Limit**	20%
5	**T&D Charges Payed by Customers**	+$70
6	**T&D Charges Payed by the aggregator**	+$30
7	**Elasticity**	-0.4
8	**Probability of a scenario**	1/100

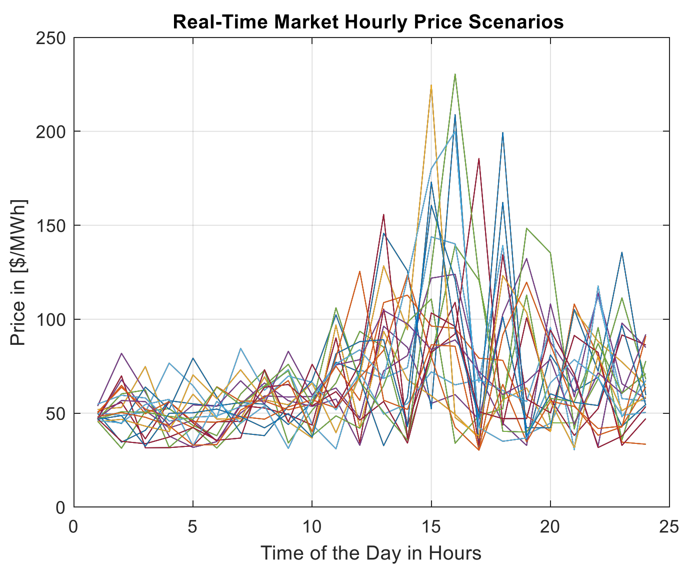

Figure 6.15 Real-time market price scenarios

Simulation results are shown in Figures 6.16,6.17,6.18,6.19. Figure 6.16 shows the hourly price variation while Figure 6.17 shows the mean energy variation. The price fluctuation increases as the real-time price randomness increases. Figure 6.18 shows the resulted new selling prices offered to customers.

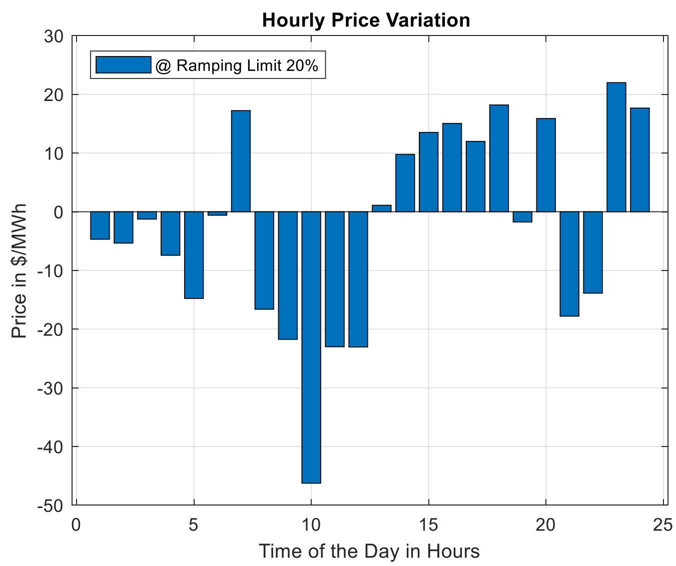

Figure 6.16 Hourly price variation

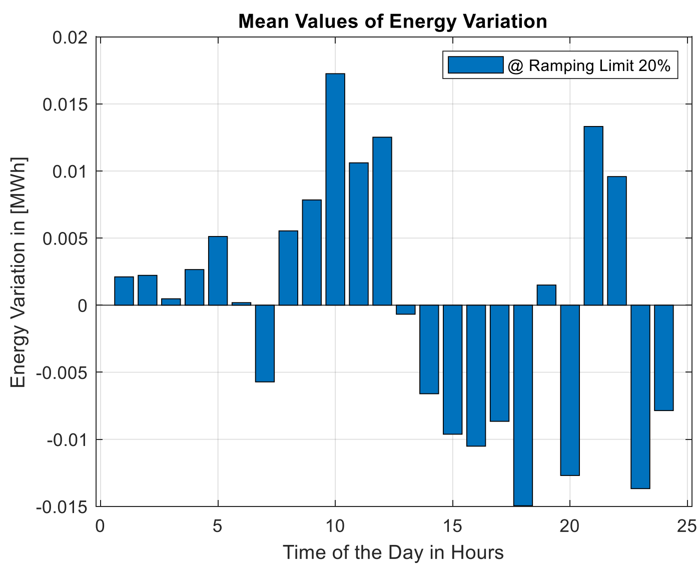

Figure 6.17 Mean energy variation with

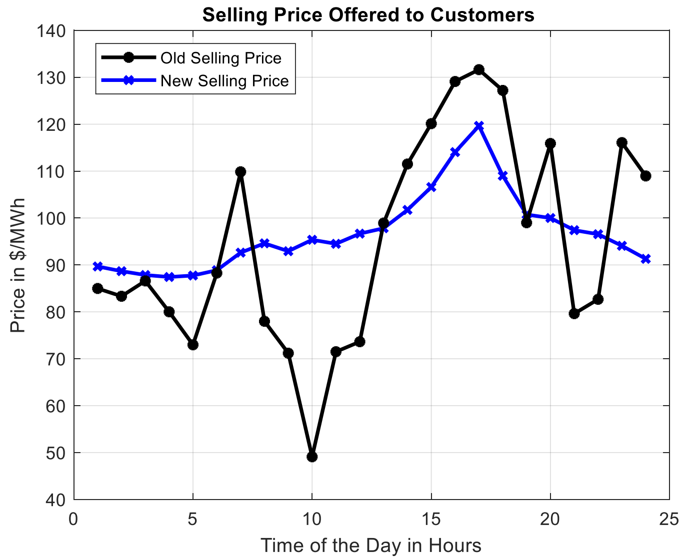

Figure 6.18 New and old selling prices offered to customers

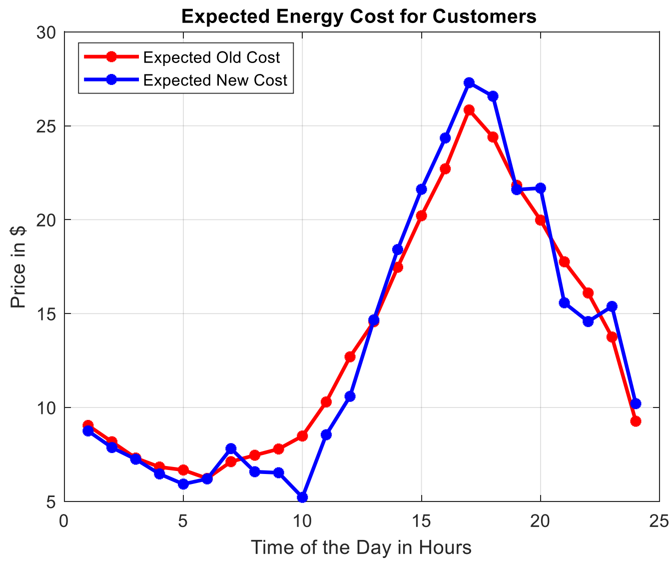

Figure 6.19 Expected energy cost for customers

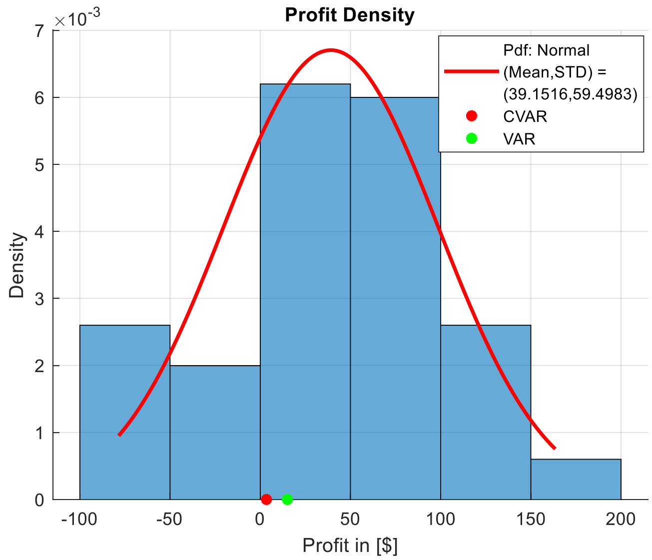

Figure 6.20 Old profit density for

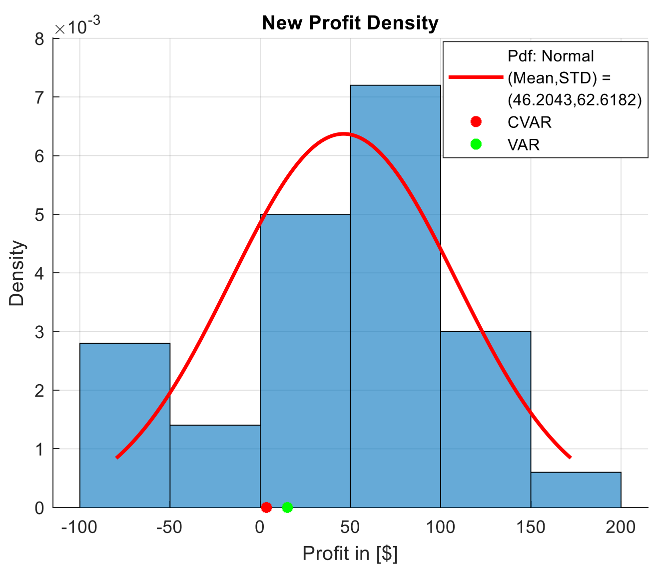

Figure 6.21 New profit density for aggregator

Customers' expected payments to the aggregator are shown in Figures 6.19. The expected new cost for customers is $319.66 while the old expected cost is $321.98 which represents a 0.721% decrease in customers payments. The expected profit before and after changing selling price are shown in Figures 6.20 and 6.21, which are $39.15 and $46.20, respectively. This represents a percentage increase in the profit by 18.01%.

Figure 6.22 shows the aggregator efficient frontier for this case with 100 scenarios for the problem. At higher confidence level the expected profit decreases and the risk also decreases. While at lower confidence levels the expected profit increases and the risk increases.

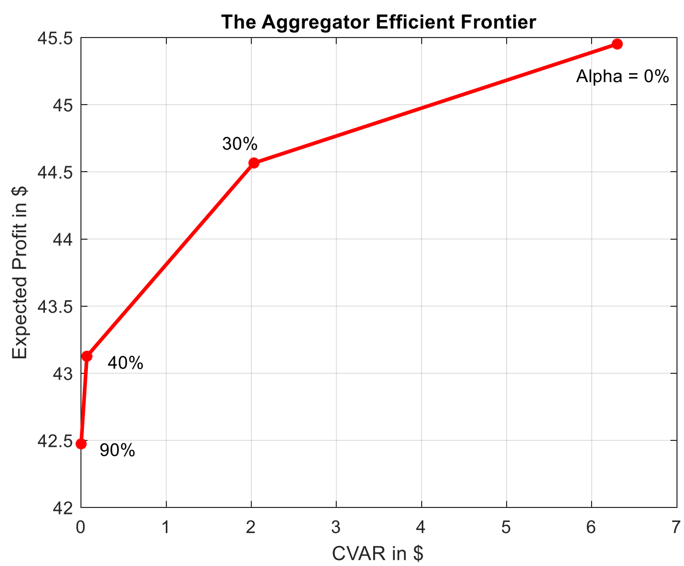

Figure 6.22 The aggregator efficient frontier

6.4 Case Study III: Load Profiles Selected in Proportion to Occurrences in the Data Set

Case study III will consider 10 load profiles and 10 samples for real-time price. The load profiles are picked randomly in proportion to occurrences in the data set explained in Chapter 4. The simulation conditions are kept the same as previous cases. Figure 6.23 shows the real-time price scenarios, while Figure 6.24 shows the load profiles selected for this case study and Table 6.22 lists the number of samples selected from each load profile groups. Simulation results are shown in Figures 6.25 to 6.28.

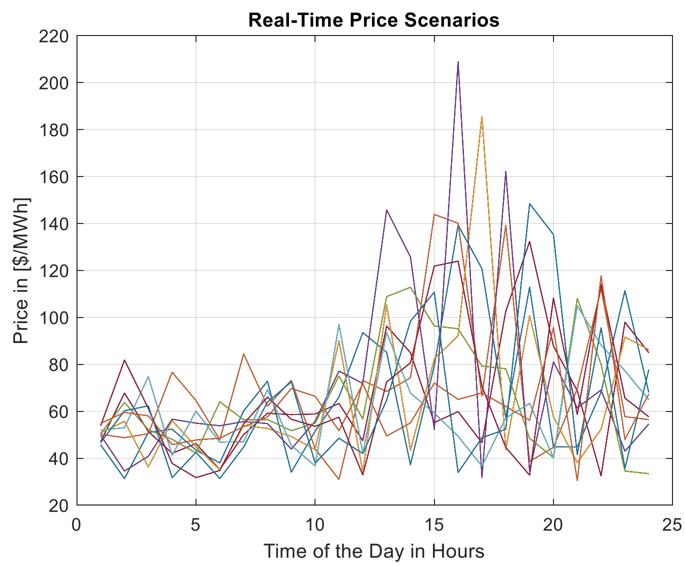

Figure 6.23 Real-time price scenarios

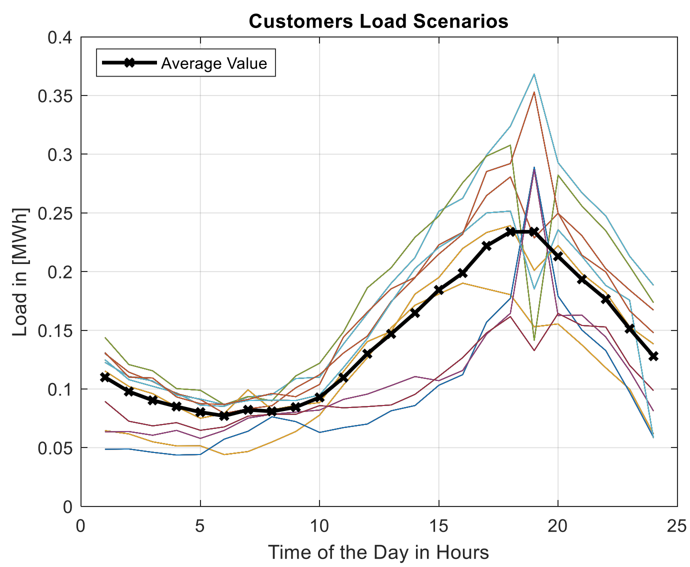

Figure 6.24 Customers load scenarios

Table 6.22 Number of samples from each load profile groups

Type of load profile	Number of load profiles
Above average	$\frac{39}{109} \times 10 \cong 4$
Below average	$\frac{32}{109} \times 10 \cong 3$
Equal average	$\frac{6}{109} \times 10 \cong 1$
Above and below	$\frac{13}{109} \times 10 \cong 1$
Below and above	$\frac{19}{109} \times 10 \cong 1$

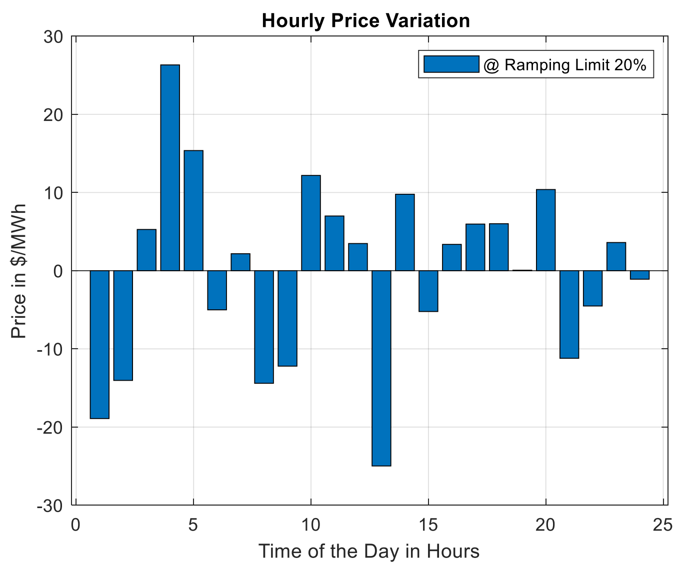

Figure 6.25 Hourly price variation

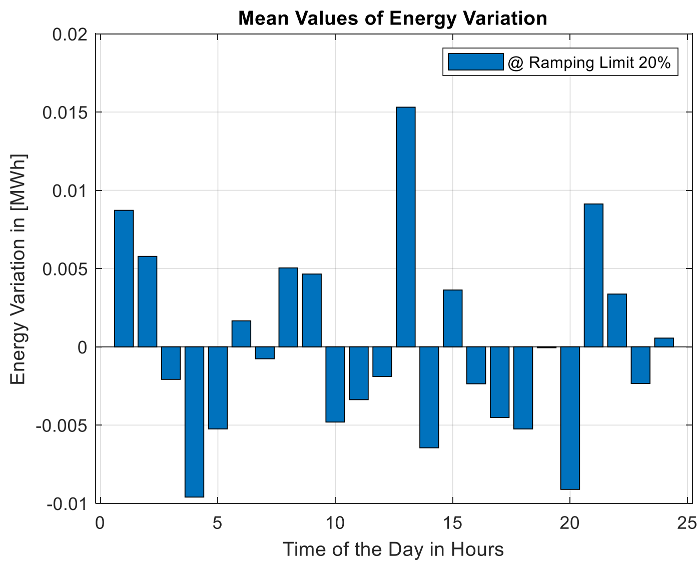

Figure 6.26 Mean energy variation

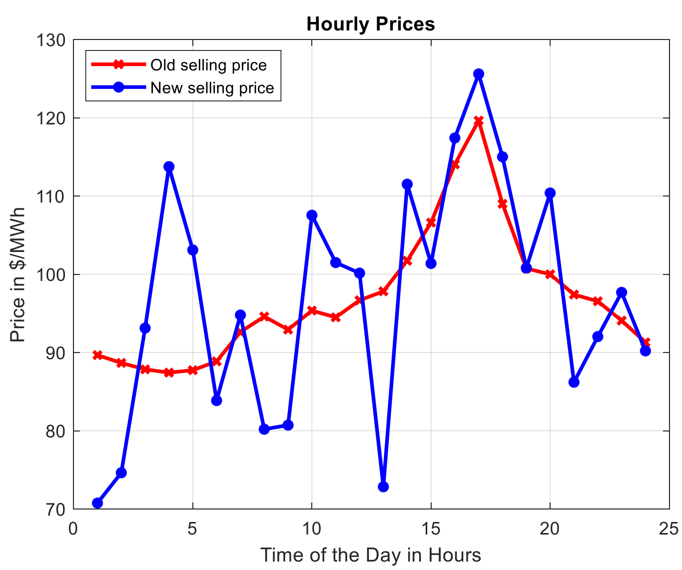

Figure 6.27 New and old selling prices offered to customers

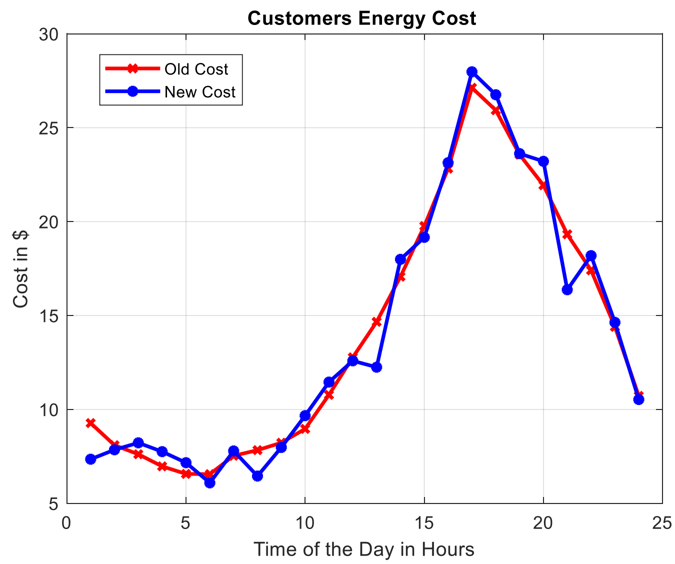

Figure 6.28 Expected energy cost for customers

Figure 6.28 shows the old and new expected cost for customers. The old expected cost is \$335.91 and after change selling price it became \$334.19 which represents a decrease in the expected cost for customers by 0.51%.

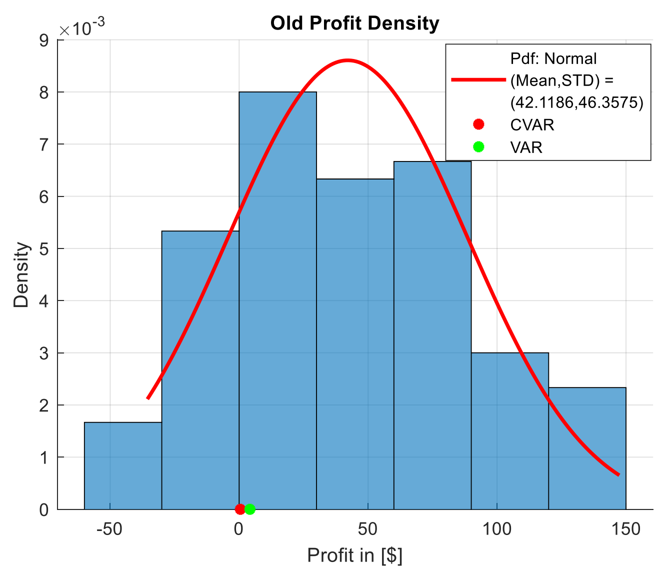

Figure 6.29 Old profit density

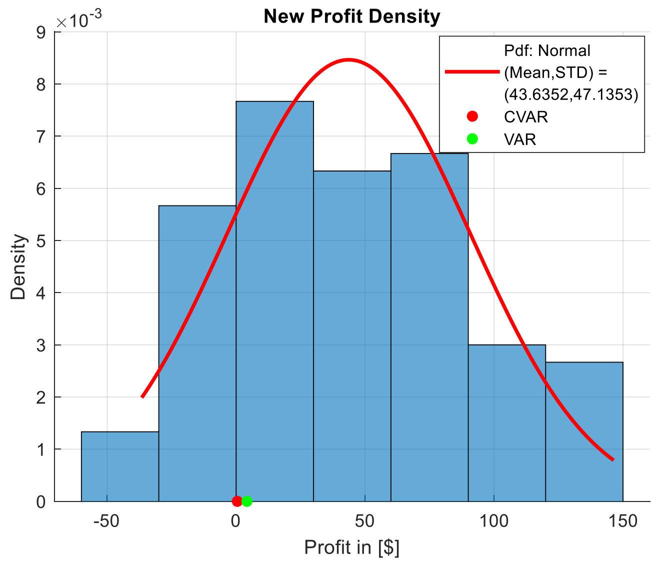

Figure 6.30 New profit density

The old and new profit density in Figures 6.29 and 6.30 show an increase in the expected profit for the aggregator by 3.61%. The old expected profit is $42.12 while the new expected profit after changing selling price is $43.64.

The aggregator risk measure is shown in Figure 6.31, the expected profit decreases as the confidence level increases, less risk for the aggregator is achieved when trading with a very high confidence level. Achieving a higher expected profit or a minimum risk is then a choice for the aggregator, as those values are controlled by the level of confidence and the confidence level is the aggregator choice of operation.

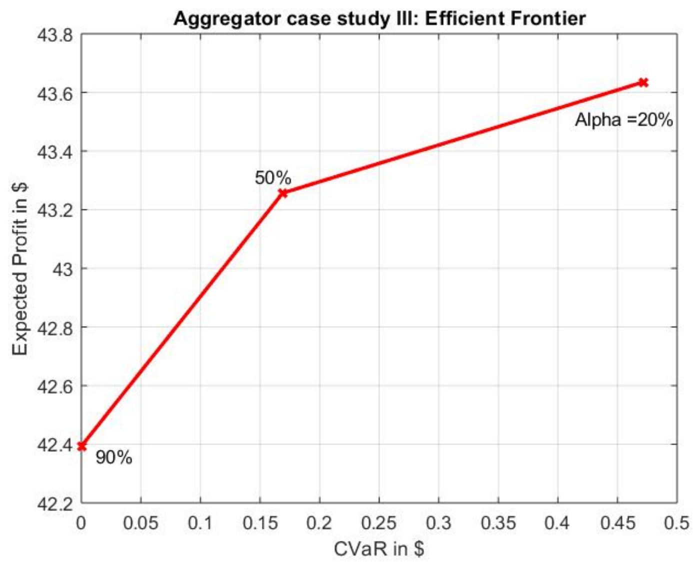

Figure 6.31 Aggregator case study III: Efficient Frontier

Chapter 7

Conclusions and Future Work

This thesis proposed a Mixed Integer Non-Linear Programming (MINLP) stochastic model for the aggregator to participate in real-time market through a demand response program. The case studies indicate that the model is able to achieve higher savings for customers and higher profit for the aggregator while getting a better tradeoff for the aggregator in terms of profit and penalties.

The aggregator in our proposed model copes with two major challenges. First is the real-time price uncertainty and the customers' load variability. This work characterized real-time price by Gaussian probability density function at each hour and generated enough scenarios for real-time price using the proposed model.

Customers' load uncertainty is modeled based on actual data from Pecan Street in Austin, Texas for summer 2017. The resultant 109 load profiles exhibit five different behaviors when comparing them to the average aggregate load value of the 109 profiles.

The model is solved using SCIP solver within the General Algebraic Modeling (GAMS) platform for different study cases and based on the CVaR risk model. The study cases show the validity of our model to customers and the aggregator by procuring lower cost for customers and achieving higher profit for the aggregator. The number of scenarios considered in this simulation were limited by the ability of GAMS solver. Although the change in energy cost to customers and profit for the aggregator are small, they yield positive outcomes. With larger number of customers in the portfolio, the aggregator can expect to make higher profit.

Future work would be integrating the physical grid and studying the effect of changing selling price on parameters like voltages and currents. Also, future work might consider customers having on-grid photovoltaic units equipped with smart inverters and customers can trade in the market for ancillary services. The work can also be extended to a different type of load such as industrial, commercial and agriculture loads. Further the work can be extended to include the optimal bidding for the aggregator in the real-time market within the proposed uncertainties of price and load.

References

[1] G. Alliance, "Grid Modernization Index 2018 Key Indicator for Changing Electric Grid," 2018 2018.

[2] U. D. o. E. . "Grid Modernization Multi-Year Program Plan," ed: Rep DOE, Washington, DC, 2015.

[3] M. Shahidehpour, H. Yamin, and Z. Li, *Market operations in electric power systems: forecasting, scheduling, and risk management*: John Wiley & Sons, 2003.

[4] H. Wu, M. Shahidehpour, A. Alabdulwahab, and A. Abusorrah, "A game theoretic approach to risk-based optimal bidding strategies for electric vehicle aggregators in electricity markets with variable wind energy resources," *IEEE Transactions on Sustainable Energy,* vol. 7, pp. 374-385, 2016.

[5] D. Muthirayan, D. Kalathil, K. Poolla, and P. Varaiya, "Mechanism design for demand response programs," *arXiv preprint arXiv:1712.07742,* 2017.

[6] D. T. Nguyen and L. B. Le, "Risk-constrained profit maximization for microgrid aggregators with demand response," *IEEE Transactions on smart grid,* vol. 6, pp. 135-146, 2014.

[7] S. I. Vagropoulos, D. K. Kyriazidis, and A. G. Bakirtzis, "Real-time charging management framework for electric vehicle aggregators in a market environment," *IEEE Transactions on Smart Grid,* vol. 7, pp. 948-957, 2016.

[8] S.-E. Fleten and E. Pettersen, "Constructing bidding curves for a price-taking retailer in the Norwegian electricity market," *IEEE Transactions on Power Systems,* vol. 20, pp. 701-708, 2005.

[9] R. Garcia-Bertrand, "Sale prices setting tool for retailers," *IEEE Transactions on Smart Grid*, vol. 4, pp. 2028-2035, 2013.

[10] A. J. Conejo, R. Garcia-Bertrand, M. Carrion, Á. Caballero, and A. de Andres, "Optimal involvement in futures markets of a power producer," *IEEE Transactions on Power Systems*, vol. 23, pp. 703-711, 2008.

[11] A. Hatami, H. Seifi, and M. Sheikh-El-Eslami, "Optimal selling price and energy procurement strategies for a retailer in an electricity market," *Electric Power Systems Research*, vol. 79, pp. 246-254, 2009.

[12] J. Yang, J. Zhao, F. Luo, F. Wen, and Z. Y. Dong, "Decision-making for electricity retailers: A brief survey," *IEEE Transactions on Smart Grid*, vol. 9, pp. 4140-4153, 2018.

[13] E. R. C. O. T. Inc. (2019, 2-18-2019). *Market Prices*. Available: http://www.ercot.com/

[14] P. Street. (2019, 2-16-2019). *Pecan Street*. Available: https://www.pecanstreet.org/

[15] R. A. Zuse Institute Berlin (ZIB) in cooperation with TU Darmstadt, and University of Erlangen-Nürnberg, "SCIP " 2019.

[16] GAMS. (2019). *General Algebraic Modeling (GAMS)* Available: https://www.gams.com/

[17] A. Ganti. (Apr 1, 2019). *Efficient Frontier*. Available: https://www.investopedia.com/terms/e/efficientfrontier.asp

Appendix A - Mathematical Derivation of the Aggregator Model

$$\begin{gathered}\text{Expected Profit}\\ \text{(Before changing the selling Price)}\end{gathered}$$

$$= \sum_{\omega=1}^{N_\omega}\sum_{t=1}^{N_t} \pi_\omega \mathrm{LMP_t}^{C} \mathrm{E_{t\omega}}^{C} - \sum_{\omega=1}^{N_\omega}\sum_{t=1}^{N_t} \pi_\omega \, \mathrm{LMP_{t\omega}}^{R} \mathrm{E_t}^{D}$$

$$- \sum_{\omega=1}^{N_\omega}\sum_{t=1}^{N_t} \pi_\omega \left(\mathrm{P_{t\omega}}^{DH} + \mathrm{P_{t\omega}}^{DL}\right) \mathrm{LMP_{t\omega}}^{R} \left(\mathrm{E_{t\omega}}^{C} - \mathrm{E_t}^{D}\right)$$

$$\text{Penalty binary indicator } = \begin{cases} \mathrm{x_{t\omega}}^{B} = 1, & \left(\mathrm{E_{t\omega}}^{C} - \mathrm{E_t}^{D}\right) > 0 \\ \mathrm{x_{t\omega}}^{B} = 0, & \left(\mathrm{E_{t\omega}}^{C} - \mathrm{E_t}^{D}\right) < 0 \end{cases}$$

Which can be translated into the following MIP representation:

$$\left(\mathrm{E_{t\omega}}^{C} - \mathrm{E_t}^{D}\right) - \mathrm{M_{t\omega}} \, \mathrm{x_{t\omega}}^{B} \leq 0 \; ; \; \forall t, \forall \omega$$

$$\left(\mathrm{E_{t\omega}}^{C} - \mathrm{E_t}^{D}\right) + \mathrm{m_{t\omega}} \, \mathrm{x_{t\omega}}^{B} \geq \mathrm{m_{t\omega}} \; ; \; \forall t, \forall \omega$$

$$\mathrm{M_{t\omega}} = \mathrm{abs}\left(\mathrm{E_{t\omega}}^{C} - \mathrm{E_t}^{D}\right)$$

$$\mathrm{m_{t\omega}} = -\mathrm{abs}\left(\mathrm{E_{t\omega}}^{C} - \mathrm{E_t}^{D}\right)$$

$$\begin{gathered}\text{Expected Profit}\\ \text{(After changing the selling Price)}\end{gathered}$$

$$= \sum_{\omega=1}^{N_\omega}\sum_{t=1}^{N_t} \pi_\omega \left(\mathrm{LMP_t}^{C} + \Delta \mathrm{LMP_t}^{C}\right)\left(\mathrm{E_{t\omega}}^{C} + \Delta \mathrm{E_{t\omega}}^{C}\right) - \sum_{\omega=1}^{N_\omega}\sum_{t=1}^{N_t} \pi_\omega \, \mathrm{LMP_{t\omega}}^{R} \mathrm{E_t}^{D}$$

$$- \sum_{\omega=1}^{N_\omega}\sum_{t=1}^{N_t} \pi_\omega \left(\mathrm{P_{t\omega}}^{RH} + \mathrm{P_{t\omega}}^{RL}\right) \mathrm{LMP_{tw}}^{R} \left(\mathrm{E_{t\omega}}^{C} + \Delta \mathrm{E_{t\omega}}^{C} - \mathrm{E_t}^{D}\right)$$

$$\text{Penalty binary indicator } = \begin{cases} \mathrm{y_{tw}}^{B} = 1, & \left(\mathrm{E_{t\omega}}^{C} + \Delta \mathrm{E_{t\omega}}^{C}\right) > 0 \\ \mathrm{y_{tw}}^{B} = 0, & \left(\mathrm{E_{t\omega}}^{C} + \Delta \mathrm{E_{t\omega}}^{C}\right) < 0 \end{cases}$$

Which can be translated into the following MIP representation:

$$\left(E_{t\omega}{}^{C} + \Delta E_{t\omega}{}^{C} - E_{t}{}^{D}\right) - M_{t\omega}\, y_{t\omega}{}^{B} \leq 0\ ;\ \forall t, \forall \omega$$

$$\left(E_{t\omega}{}^{C} + \Delta E_{t\omega}{}^{C} - E_{t}{}^{D}\right) + m_{t\omega}\, y_{t\omega}{}^{B} \geq m_{t\omega}\ ;\ \forall t, \forall \omega$$

$$M_{t\omega} = \mathrm{abs}\left(E_{t\omega}{}^{C} + a\, E_{t\omega}{}^{C} - E_{t}{}^{D}\right)$$

$$m_{t\omega} = -\mathrm{abs}\left(E_{t\omega}{}^{C} - a\, E_{t\omega}{}^{C} - E_{t}{}^{D}\right)$$

Where:

$M_{t\omega}$: is an upper bound on $\left(E_{t\omega}{}^{C} + \Delta E_{t\omega}{}^{C} - E_{t}{}^{D}\right)$

$m_{t\omega}$: is a lower bound on $\left(E_{t\omega}{}^{C} + \Delta E_{t\omega}{}^{C} - E_{t}{}^{D}\right)$

Using Price Elasticity of the Demand write the load variation as the following:

$$\varepsilon_{t\omega}{}^{C} = \frac{\Delta E_{t\omega}{}^{C} / E_{t\omega}{}^{C}}{\Delta LMP_{t}{}^{C} / LMP_{t}{}^{C}}$$

$$\Delta E_{t\omega}{}^{C} = \frac{\varepsilon_{t\omega}{}^{C}\, \Delta LMP_{t}{}^{C}\, E_{t\omega}{}^{C}}{LMP_{t}{}^{C}}$$

$$\begin{array}{c}\text{Change in}\\ \text{Profit}\end{array} = \begin{array}{c}\text{Expected Profit}\\ \text{(After changing the selling Price)}\end{array} - \begin{array}{c}\text{Expected Profit}\\ \text{(Before changing the selling Price)}\end{array}$$

$$\begin{aligned}\begin{array}{c}\text{Change in}\\ \text{Profit}\end{array} = &\sum_{\omega=1}^{N_\omega}\sum_{t=1}^{N_t}\pi_\omega\left(LMP_t{}^C + \Delta LMP_t{}^C\right)\left(E_{t\omega}{}^C + \Delta E_{t\omega}{}^C\right) - \sum_{\omega=1}^{N_\omega}\sum_{t=1}^{N_t}\pi_\omega\ LMP_{t\omega}{}^R E_t{}^D \\ &- \sum_{\omega=1}^{N_\omega}\sum_{t=1}^{N_t}\pi_\omega\left(P_{t\omega}{}^{RH} + P_{t\omega}{}^{RL}\right) LMP_{tw}{}^R\left(E_{t\omega}{}^C + \Delta E_{t\omega}{}^C - E_t{}^D\right) \\ &- \sum_{\omega=1}^{N_\omega}\sum_{t=1}^{N_t}\pi_\omega LMP_t{}^C E_{t\omega}{}^C + \sum_{\omega=1}^{N_\omega}\sum_{t=1}^{N_t}\pi_\omega\ LMP_{t\omega}{}^R E_t{}^D \\ &+ \sum_{\omega=1}^{N_\omega}\sum_{t=1}^{N_t}\pi_\omega\left(P_{t\omega}{}^{DH} + P_{t\omega}{}^{DL}\right) LMP_{tw}{}^R\left(E_{t\omega}{}^C - E_t{}^D\right)\end{aligned}$$

$$\begin{aligned}\begin{array}{c}\text{Change in}\\ \text{Profit}\end{array} = &\sum_{\omega=1}^{N_\omega}\sum_{t=1}^{N_t}\pi_\omega LMP_t{}^C E_{t\omega}{}^C + \sum_{\omega=1}^{N_\omega}\sum_{t=1}^{N_t}\pi_\omega LMP_t{}^C \Delta E_{t\omega}{}^C + \sum_{\omega=1}^{N_\omega}\sum_{t=1}^{N_t}\pi_\omega \Delta LMP_t{}^C E_{t\omega}{}^C \\ &+ \sum_{\omega=1}^{N_\omega}\sum_{t=1}^{N_t}\pi_\omega \Delta LMP_t{}^C \Delta E_{t\omega}{}^C - \sum_{\omega=1}^{N_\omega}\sum_{t=1}^{N_t}\pi_\omega\ LMP_{t\omega}{}^R E_t{}^D \\ &- \sum_{\omega=1}^{N_\omega}\sum_{t=1}^{N_t}\pi_\omega\left(P_{t\omega}{}^{RH} + P_{t\omega}{}^{RL}\right) LMP_{t\omega}{}^R\left(E_{t\omega}{}^C + \Delta E_{t\omega}{}^C - E_t{}^D\right) \\ &- \sum_{\omega=1}^{N_\omega}\sum_{t=1}^{N_t}\pi_\omega LMP_t{}^C E_{t\omega}{}^C + \sum_{\omega=1}^{N_\omega}\sum_{t=1}^{N_t}\pi_\omega\ LMP_{t\omega}{}^R E_t{}^D \\ &+ \sum_{\omega=1}^{N_\omega}\sum_{t=1}^{N_t}\pi_\omega\left(P_{t\omega}{}^{DH} + P_{t\omega}{}^{DL}\right) LMP_{t\omega}{}^R\left(E_{t\omega}{}^C - E_t{}^D\right)\end{aligned}$$

$$\begin{aligned}\text{Change in Profit} =& \cancel{\sum_{\omega=1}^{N_\omega}\sum_{t=1}^{N_t}\pi_\omega LMP_t{}^C E_{t\omega}{}^C} + \sum_{\omega=1}^{N_\omega}\sum_{t=1}^{N_t}\pi_\omega LMP_t{}^C \Delta E_{t\omega}{}^C + \sum_{\omega=1}^{N_\omega}\sum_{t=1}^{N_t}\pi_\omega \Delta LMP_t{}^C E_{t\omega}{}^C \\ &+ \sum_{\omega=1}^{N_\omega}\sum_{t=1}^{N_t}\pi_\omega \Delta LMP_t{}^C \Delta E_{t\omega}{}^C - \cancel{\sum_{\omega=1}^{N_\omega}\sum_{t=1}^{N_t}\pi_\omega LMP_{t\omega}{}^R E_t{}^D} \\ &- \sum_{\omega=1}^{N_\omega}\sum_{t=1}^{N_t}\pi_\omega \left(P_{t\omega}{}^{RH} + P_{t\omega}{}^{RL}\right) LMP_{t\omega}{}^R \left(E_{t\omega}{}^C + \Delta E_{t\omega}{}^C - E_t{}^D\right) \\ &- \cancel{\sum_{\omega=1}^{N_\omega}\sum_{t=1}^{N_t}\pi_\omega LMP_t{}^C E_{t\omega}{}^C} + \cancel{\sum_{\omega=1}^{N_\omega}\sum_{t=1}^{N_t}\pi_\omega LMP_{t\omega}{}^R E_t{}^D} \\ &+ \sum_{\omega=1}^{N_\omega}\sum_{t=1}^{N_t}\pi_\omega \left(P_{t\omega}{}^{DH} + P_{t\omega}{}^{DL}\right) LMP_{t\omega}{}^R \left(E_{t\omega}{}^C - E_t{}^D\right)\end{aligned}$$

$$\begin{aligned}\text{Change in Profit} =& \sum_{\omega=1}^{N_\omega}\sum_{t=1}^{N_t}\pi_\omega LMP_t{}^C \Delta E_{t\omega}{}^C + \sum_{\omega=1}^{N_\omega}\sum_{t=1}^{N_t}\pi_\omega \Delta LMP_t{}^C E_{t\omega}{}^C + \sum_{\omega=1}^{N_\omega}\sum_{t=1}^{N_t}\pi_\omega \Delta LMP_t{}^C \Delta E_{t\omega}{}^C \\ &- \sum_{\omega=1}^{N_\omega}\sum_{t=1}^{N_t}\pi_\omega \left(P_{t\omega}{}^{RH} + P_{t\omega}{}^{RL}\right) LMP_{t\omega}{}^R \left(E_{t\omega}{}^C + \Delta E_{t\omega}{}^C - E_t{}^D\right) \\ &+ \sum_{\omega=1}^{N_\omega}\sum_{t=1}^{N_t}\pi_\omega \left(P_{t\omega}{}^{DH} + P_{t\omega}{}^{DL}\right) LMP_{t\omega}{}^R \left(E_{t\omega}{}^C - E_t{}^D\right)\end{aligned}$$

$$\begin{aligned}\text{Change in Profit} =& \sum_{\omega=1}^{N_\omega}\sum_{t=1}^{N_t}\pi_\omega LMP_t{}^C \Delta E_{t\omega}{}^C + \sum_{\omega=1}^{N_\omega}\sum_{t=1}^{N_t}\pi_\omega \Delta LMP_t{}^C E_{t\omega}{}^C + \sum_{\omega=1}^{N_\omega}\sum_{t=1}^{N_t}\pi_\omega \Delta LMP_t{}^C \Delta E_{t\omega}{}^C \\ &- \sum_{\omega=1}^{N_\omega}\sum_{t=1}^{N_t}\pi_\omega \left(P_{t\omega}{}^{RH} + P_{t\omega}{}^{RL}\right) LMP_{t\omega}{}^R \left(E_{t\omega}{}^C + \Delta E_{t\omega}{}^C - E_t{}^D\right) \\ &+ \sum_{\omega=1}^{N_\omega}\sum_{t=1}^{N_t}\pi_\omega \left(P_{t\omega}{}^{DH} + P_{t\omega}{}^{DL}\right) LMP_{t\omega}{}^R \left(E_{t\omega}{}^C - E_t{}^D\right)\end{aligned}$$

$$\begin{aligned}\text{Change in Profit} &= \sum_{\omega=1}^{N_\omega}\sum_{t=1}^{N_t}\pi_\omega \Delta E_{t\omega}{}^{C}\left(LMP_t{}^{C} + \Delta LMP_t{}^{C}\right) + \sum_{\omega=1}^{N_\omega}\sum_{t=1}^{N_t}\pi_\omega \Delta LMP_t{}^{C} E_{t\omega}{}^{C} \\ &- \sum_{\omega=1}^{N_\omega}\sum_{t=1}^{N_t}\pi_\omega\left(P_{t\omega}{}^{RH} + P_{t\omega}{}^{RL}\right) LMP_{t\omega}{}^{R}\left(E_{t\omega}{}^{C} + \Delta E_{t\omega}{}^{C} - E_t{}^{D}\right) \\ &+ \sum_{\omega=1}^{N_\omega}\sum_{t=1}^{N_t}\pi_\omega\left(P_{t\omega}{}^{DH} + P_{t\omega}{}^{DL}\right) LMP_{t\omega}{}^{R}\left(E_{t\omega}{}^{C} - E_t{}^{D}\right)\end{aligned}$$

$$\begin{aligned}\text{Change in Profit} &= \sum_{\omega=1}^{N_\omega}\sum_{t=1}^{N_t}\pi_\omega \Delta E_{t\omega}{}^{C}\left(LMP_t{}^{C} + \Delta LMP_t{}^{C}\right) + \sum_{\omega=1}^{N_\omega}\sum_{t=1}^{N_t}\pi_\omega \Delta LMP_t{}^{C} E_{t\omega}{}^{C} \\ &- \sum_{\omega=1}^{N_\omega}\sum_{t=1}^{N_t}\left(\pi_\omega P_{t\omega}{}^{RH}\, LMP_{t\omega}{}^{R} + \pi_\omega P_{t\omega}{}^{RL}\, LMP_{t\omega}{}^{R}\right)\left(E_{t\omega}{}^{C} + \Delta E_{t\omega}{}^{C} - E_t{}^{D}\right) \\ &+ \sum_{\omega=1}^{N_\omega}\sum_{t=1}^{N_t}\left(\pi_\omega P_{t\omega}{}^{DH}\, LMP_{t\omega}{}^{R} + \pi_\omega P_{t\omega}{}^{DL}\, LMP_{t\omega}{}^{R}\right)\left(E_{t\omega}{}^{C} - E_t{}^{D}\right)\end{aligned}$$

$$\begin{aligned}\text{Change in Profit} &= \sum_{\omega=1}^{N_\omega}\sum_{t=1}^{N_t}\pi_\omega \frac{\varepsilon_{t\omega}{}^{C}\,\Delta LMP_t{}^{C}\, E_{t\omega}{}^{C}}{LMP_t{}^{C}}\left(LMP_t{}^{C} + \Delta LMP_t{}^{C}\right) + \sum_{\omega=1}^{N_\omega}\sum_{t=1}^{N_t}\pi_\omega \Delta LMP_t{}^{C} E_{t\omega}{}^{C} \\ &- \sum_{\omega=1}^{N_\omega}\sum_{t=1}^{N_t}\left(\pi_\omega P_{t\omega}{}^{RH}\, LMP_{t\omega}{}^{R} + \pi_\omega P_{t\omega}{}^{RL}\, LMP_{t\omega}{}^{R}\right)\left(E_{t\omega}{}^{C} + \frac{\varepsilon_{t\omega}{}^{C}\,\Delta LMP_t{}^{C}\, E_{t\omega}{}^{C}}{LMP_t{}^{C}}\right. \\ &\left. - E_t{}^{D}\right) + \sum_{\omega=1}^{N_\omega}\sum_{t=1}^{N_t}\left(\pi_\omega P_{t\omega}{}^{DH}\, LMP_{t\omega}{}^{R} + \pi_\omega P_{t\omega}{}^{DL}\, LMP_{t\omega}{}^{R}\right)\left(E_{t\omega}{}^{C} - E_t{}^{D}\right)\end{aligned}$$

$$
\begin{aligned}
\text{Change in Profit} = {} & \sum_{\omega=1}^{N_\omega}\sum_{t=1}^{N_t} \pi_\omega \frac{\varepsilon_{t\omega}{}^{C}\,\Delta LMP_t{}^{C}\,E_{t\omega}{}^{C}}{\cancel{LMP_t{}^{C}}}\cancel{LMP_t{}^{C}} + \sum_{\omega=1}^{N_\omega}\sum_{t=1}^{N_t} \pi_\omega \frac{\varepsilon_{t\omega}{}^{C}\,\Delta LMP_t{}^{C}\,E_{t\omega}{}^{C}}{LMP_t{}^{C}}\Delta LMP_t{}^{C} \\
& + \sum_{\omega=1}^{N_\omega}\sum_{t=1}^{N_t} \pi_\omega \Delta LMP_t{}^{C} E_{t\omega}{}^{C} \\
& - \sum_{\omega=1}^{N_\omega}\sum_{t=1}^{N_t}\left(\pi_\omega P_{t\omega}{}^{RH}\,LMP_{t\omega}{}^{R} + \pi_\omega P_{t\omega}{}^{RL}\,LMP_{t\omega}{}^{R}\right)\Bigg(E_{t\omega}{}^{C} + \frac{\varepsilon_{t\omega}{}^{C}\,\Delta LMP_t{}^{C}\,E_{t\omega}{}^{C}}{LMP_t{}^{C}} \\
& - E_t{}^{D}\Bigg) + \sum_{\omega=1}^{N_\omega}\sum_{t=1}^{N_t}\left(\pi_\omega P_{t\omega}{}^{DH}\,LMP_{t\omega}{}^{R} + \pi_\omega P_{t\omega}{}^{DL}\,LMP_{t\omega}{}^{R}\right)\left(E_{t\omega}{}^{C} - E_t{}^{D}\right)
\end{aligned}
$$

$$
\begin{aligned}
\text{Change in Profit} = {} & \sum_{\omega=1}^{N_\omega}\sum_{t=1}^{N_t} \pi_\omega\,\Delta LMP_t{}^{C}\,E_{t\omega}{}^{C}\left(1 + \varepsilon_{t\omega}{}^{C}\right) + \sum_{\omega=1}^{N_\omega}\sum_{t=1}^{N_t} \pi_\omega \frac{\varepsilon_{t\omega}{}^{C}\left(\Delta LMP_t{}^{C}\right)^2 E_{t\omega}{}^{C}}{LMP_t{}^{C}} \\
& - \sum_{\omega=1}^{N_\omega}\sum_{t=1}^{N_t}\left(\pi_\omega P_{t\omega}{}^{RH}\,LMP_{t\omega}{}^{R} + \pi_\omega P_{t\omega}{}^{RL}\,LMP_{t\omega}{}^{R}\right)\Bigg(E_{t\omega}{}^{C} + \frac{\varepsilon_{t\omega}{}^{C}\,\Delta LMP_t{}^{C}\,E_{t\omega}{}^{C}}{LMP_t{}^{C}} \\
& - E_t{}^{D}\Bigg) + \sum_{\omega=1}^{N_\omega}\sum_{t=1}^{N_t}\left(\pi_\omega P_{t\omega}{}^{DH}\,LMP_{t\omega}{}^{R} + \pi_\omega P_{t\omega}{}^{DL}\,LMP_{t\omega}{}^{R}\right)\left(E_{t\omega}{}^{C} - E_t{}^{D}\right)
\end{aligned}
$$

$$
\begin{aligned}
\text{Change in Profit} = & \sum_{\omega=1}^{N_\omega}\sum_{t=1}^{N_t} \pi_\omega\, \Delta LMP_t{}^C\, E_{t\omega}{}^C (1+\varepsilon_{t\omega}{}^C) + \sum_{\omega=1}^{N_\omega}\sum_{t=1}^{N_t} \pi_\omega \frac{\varepsilon_{t\omega}{}^C \left(\Delta LMP_t{}^C\right)^2 E_{t\omega}{}^C}{LMP_t{}^C} \\
& - \sum_{\omega=1}^{N_\omega}\sum_{t=1}^{N_t} \pi_\omega P_{t\omega}{}^{RH}\, LMP_{t\omega}{}^R E_{t\omega}{}^C - \sum_{\omega=1}^{N_\omega}\sum_{t=1}^{N_t} \pi_\omega P_{t\omega}{}^{RH}\, LMP_{t\omega}{}^R \frac{\varepsilon_{t\omega}{}^C\, \Delta LMP_t{}^C\, E_{t\omega}{}^C}{LMP_t{}^C} \\
& + \sum_{\omega=1}^{N_\omega}\sum_{t=1}^{N_t} \pi_\omega P_{t\omega}{}^{RH}\, LMP_{t\omega}{}^R E_t{}^D - \sum_{\omega=1}^{N_\omega}\sum_{t=1}^{N_t} \pi_\omega P_{t\omega}{}^{RL}\, LMP_{t\omega}{}^R\, E_{t\omega}{}^C \\
& - \sum_{\omega=1}^{N_\omega}\sum_{t=1}^{N_t} \pi_\omega P_{t\omega}{}^{RL}\, LMP_{t\omega}{}^R \frac{\varepsilon_{t\omega}{}^C\, \Delta LMP_t{}^C\, E_{t\omega}{}^C}{LMP_t{}^C} + \sum_{\omega=1}^{N_\omega}\sum_{t=1}^{N_t} \pi_\omega P_{t\omega}{}^{RL}\, LMP_{t\omega}{}^R E_t{}^D \\
& + \sum_{\omega=1}^{N_\omega}\sum_{t=1}^{N_t} \pi_\omega P_{t\omega}{}^{DH}\, LMP_{t\omega}{}^R E_{t\omega}{}^C - \sum_{\omega=1}^{N_\omega}\sum_{t=1}^{N_t} \pi_\omega P_{t\omega}{}^{DH}\, LMP_{t\omega}{}^R E_t{}^D \\
& + \sum_{\omega=1}^{N_\omega}\sum_{t=1}^{N_t} \pi_\omega P_{t\omega}{}^{DL}\, LMP_{t\omega}{}^R E_{t\omega}{}^C - \sum_{\omega=1}^{N_\omega}\sum_{t=1}^{N_t} \pi_\omega P_{t\omega}{}^{DL}\, LMP_{t\omega}{}^R E_t{}^D
\end{aligned}
$$

$$
\begin{aligned}
\text{Change in Profit} = & \sum_{\omega=1}^{N_\omega}\sum_{t=1}^{N_t} \pi_\omega\, \Delta LMP_t{}^C\, E_{t\omega}{}^C (1+\varepsilon_{t\omega}{}^C) + \sum_{\omega=1}^{N_\omega}\sum_{t=1}^{N_t} \pi_\omega \frac{\varepsilon_{t\omega}{}^C \left(\Delta LMP_t{}^C\right)^2 E_{t\omega}{}^C}{LMP_t{}^C} \\
& + \sum_{\omega=1}^{N_\omega}\sum_{t=1}^{N_t} \pi_\omega P_{t\omega}{}^{RH}\, LMP_{t\omega}{}^R \left(-E_{t\omega}{}^C - \frac{\varepsilon_{t\omega}{}^C\, \Delta LMP_t{}^C\, E_{t\omega}{}^C}{LMP_t{}^C} + E_t{}^D\right) \\
& + \sum_{\omega=1}^{N_\omega}\sum_{t=1}^{N_t} \pi_\omega P_{t\omega}{}^{RL}\, LMP_{t\omega}{}^R \left(-E_{t\omega}{}^C - \frac{\varepsilon_{t\omega}{}^C\, \Delta LMP_t{}^C\, E_{t\omega}{}^C}{LMP_t{}^C} + E_t{}^D\right) \\
& + \sum_{\omega=1}^{N_\omega}\sum_{t=1}^{N_t} \pi_\omega P_{t\omega}{}^{DH}\, LMP_{t\omega}{}^R \left(E_{t\omega}{}^C - E_t{}^D\right) \\
& + \sum_{\omega=1}^{N_\omega}\sum_{t=1}^{N_t} \pi_\omega P_{t\omega}{}^{DL}\, LMP_{t\omega}{}^R \left(E_{t\omega}{}^C - E_t{}^D\right)
\end{aligned}
$$

Appendix B - DATA

Tables B.1 to B.4 summarize the estimated standard deviation of the price difference distribution at each hour.

Table B.1 Price Difference Standard Deviation Estimate for Off-Peak Period

Off-peak period 12:00 AM – 10:00 AM									
Price Difference Standard Deviation Estimate in [$/MWh]									
t1	t2	t3	t4	t5	t6	t7	t8	t9	t10
2.88	2.88	3.18	3.24	4.34	11.68	3.92	33.09	11.40	8.11

Table B.2 Price Difference Standard Deviation Estimate for Shoulder One Period

Shoulder one period 11:00 AM – 1:00 PM		
Price Difference Standard Deviation Estimate in [$/MWh]		
t11	t12	t13
15.35	27.66	22.68

Table B.3 Price Difference Standard Deviation Estimate for On-Peak Period

Peak period 2:00 PM – 8:00 PM						
Price Difference Standard Deviation Estimate in [$/MWh]						
t14	t15	t16	t17	t18	t19	t20
42.22	78.12	126.25	137.48	43.80	14.31	15.28

Table B.4 Price Difference Standard Deviation Estimate for Shoulder Two Period

Shoulder two period 9:00 PM – 12:00 AM			
Price Difference Standard Deviation Estimate in [$/MWh]			
t21	t22	t23	t24
30.3B	15.30	12.26	14.66

Table B.5 Load Profiles examples for Off-peak period

Off-peak period 12:00 AM – 10:00 AM										
Load Profiles Above the Average Value [KWh]										
Profile/Hr	t1	t2	t3	t4	t5	t6	t7	t8	t9	t10
28	157.11	127.99	118.40	112.19	103.07	96.88	97.39	111.99	110.53	125.63
45	143.91	120.82	115.48	100.32	98.92	86.66	93.61	89.98	111.12	121.96
46	145.01	132.96	124.10	112.17	107.16	104.23	102.19	97.87	106.59	126.04
47	141.91	126.04	119.52	107.37	104.94	93.04	94.34	90.99	93.52	104.11

Table B.6 Load Profiles examples for Shoulder one

Shoulder one period 11:00 AM – 1:00 PM			
Load Profiles Above the Average Value [KWh]			
Profile/Hr	t11	t12	t13
28	143.55	170.82	217.15
45	148.61	186.09	203.29
46	148.33	176.20	197.82
47	129.95	167.95	194.27

Table B.7 Load Profiles examples for Peak period

Peak period 2:00 PM – 8:00 PM							
Load Profiles Above the Average Value [KWh]							
Profile/Hr	t14	t15	t16	t17	t18	t19	t20
28	239.31	269.94	283.71	313.62	312.88	521.26	263.19
45	229.25	247.30	275.72	298.35	307.69	141.48	282.14
46	223.42	242.36	270.44	302.46	295.96	260.78	272.71
47	230.45	251.63	274.29	288.11	296.72	248.73	259.36

Table B.8 Load Profiles examples for Shoulder two period

Shoulder two period 9:00 PM – 12:00 AM				
Load Profiles Above the Average Value [KWh]				
Profile/Hr	t21	t22	t23	t24
28	250.36	231.06	197.60	175.83
45	255.87	234.80	204.61	173.60
46	257.79	243.58	207.29	177.46
47	251.19	230.03	204.30	172.56

Off-peak period 12:00 AM – 10:00 AM										
Load Profiles Above the Below Value [KWh]										
Profile/Hr	t1	t2	t3	t4	t5	t6	t7	t8	t9	t10
4	85.37	74.11	61.49	55.90	51.85	58.89	68.14	72.37	67.23	65.71
11	65.51	56.79	51.97	49.54	51.42	57.17	66.05	68.35	63.68	71.81
13	57.39	45.75	40.74	41.66	44.19	53.95	61.06	58.07	57.43	58.13
108	47.10	46.07	35.11	37.02	38.41	39.47	43.44	40.52	35.79	38.27

Table B.9 Load Profiles examples for Off-peak period

Table B.10 Load Profiles examples for Shoulder one

Shoulder one period 11:00 AM – 1:00 PM			
Load Profiles Above the Below Value [KWh]			
Profile/Hr	t11	t12	t13
4	76.31	84.68	86.86
11	74.95	73.96	67.12
13	64.36	69.86	65.00
108	49.83	61.37	65.89

Table B.11 Load Profiles examples for Peak period

Peak period 2:00 PM – 8:00 PM							
Load Profiles Above the Below Value [KWh]							
Profile/Hr	t14	t15	t16	t17	t18	t19	t20
4	103.10	126.95	154.65	188.32	184.52	230.92	178.45
11	68.49	71.01	89.05	94.50	111.16	276.56	136.47
13	72.49	86.94	108.30	132.02	165.73	246.71	158.38
108	69.16	85.65	100.98	116.35	141.61	165.87	118.61

Table B.12 Load Profiles examples for Shoulder two period

Shoulder two period 9:00 PM – 12:00 AM				
Load Profiles Above the Below Value [KWh]				
Profile/Hr	t21	t22	t23	t24
4	161.11	137.75	111.89	88.22
11	108.80	97.05	75.39	77.55
13	136.54	131.37	116.53	71.21
108	99.17	86.89	71.10	71.10

Table B.13 Load Profiles examples for Off-peak period

Off-peak period 12:00 AM – 10:00 AM										
Load Profiles Below and Above the Average Value [KWh]										
Profile/Hr	t1	t2	t3	t4	t5	t6	t7	t8	t9	t10
5	90.78	76.38	67.42	62.25	60.09	65.05	76.57	82.64	92.16	92.92
14	64.68	56.46	53.38	54.83	50.98	58.74	71.39	66.44	72.41	74.00
17	89.90	88.61	71.66	70.21	66.45	77.14	82.50	76.47	82.09	91.89
84	94.42	79.99	76.75	70.25	70.76	72.63	71.70	67.59	74.52	84.08

Table B.14 Load Profiles examples for Shoulder one

Shoulder one period 11:00 AM – 1:00 PM			
Load Profiles Below and Above the Average Value [KWh]			
Profile/Hr	t11	t12	t13
5	95.45	124.88	135.69
14	84.32	95.70	108.51
17	100.68	123.93	132.52
84	107.82	120.61	145.90

Table B.15 Load Profiles examples for Peak period

Peak period 2:00 PM – 8:00 PM							
Load Profiles Below and Above the Average Value [KWh]							
Profile/Hr	t14	t15	t16	t17	t18	t19	t20
5	161.73	187.33	202.74	244.00	251.80	228.04	240.18
14	120.36	140.80	171.20	207.69	227.31	294.41	243.12
17	153.07	176.73	209.08	240.53	259.48	211.70	201.10
84	164.62	181.02	211.21	236.38	245.62	122.84	215.99

Table B.16 Load Profiles examples for Shoulder two period

Shoulder two period 9:00 PM – 12:00 AM				
Load Profiles Below and Above the Average Value [KWh]				
Profile/Hr	t21	t22	t23	t24
5	203.60	175.54	137.40	116.29
14	209.21	182.55	147.65	91.05
17	150.57	136.69	112.03	114.55
84	195.53	177.99	159.70	188.39

Table B.17 Load Profiles examples for Off-peak period

Off-peak period 12:00 AM – 10:00 AM										
Load Profiles Equal the Average Value [KWh]										
Profile/Hr	t1	t2	t3	t4	t5	t6	t7	t8	t9	t10
8	91.65	84.29	77.21	82.70	78.48	81.60	88.61	84.42	76.37	95.67
9	115.31	102.17	96.03	84.86	75.06	79.14	99.29	82.26	86.05	88.42
36	96.04	89.42	90.97	86.90	78.76	78.74	82.11	80.29	86.32	96.49
44	124.56	95.21	88.33	81.87	73.12	72.65	79.93	82.50	77.04	98.00

Table B.18 Load Profiles examples for Shoulder one

Shoulder one period 11:00 AM – 1:00 PM			
Load Profiles Equal the Average Value [KWh]			
Profile/Hr	t11	t12	t13
8	110.72	123.68	130.88
9	114.90	140.32	149.12
36	105.46	138.25	152.02
44	100.72	125.16	147.71

Table B.19 Load Profiles examples for Peak period

Peak period 2:00 PM – 8:00 PM							
Load Profiles Equal the Average Value [KWh]							
Profile/Hr	t14	t15	t16	t17	t18	t19	t20
8	154.97	189.24	194.97	221.78	254.08	317.56	212.48
9	180.86	194.97	219.72	233.29	239.15	201.07	222.23
36	174.23	197.10	213.23	220.03	220.45	202.57	208.70
44	167.95	187.57	215.13	235.98	235.82	111.11	190.91

Table B.20 Load Profiles examples for Shoulder two period

Shoulder two period 9:00 PM – 12:00 AM				
Load Profiles Equal the Average Value [KWh]				
Profile/Hr	t21	t22	t23	t24
8	200.55	181.96	150.11	111.64
9	198.22	182.34	153.10	138.38
36	180.59	168.62	139.23	107.65
44	169.94	169.30	157.30	168.09

Table B.21 Load Profiles examples for Off-peak period

Off-peak period 12:00 AM – 10:00 AM										
Load Profiles Above and Below the Average Value [KWh]										
Profile/Hr	t1	t2	t3	t4	t5	t6	t7	t8	t9	t10
29	157.34	131.12	119.96	107.74	101.91	89.47	95.40	103.83	103.00	105.04
74	161.86	152.09	140.20	134.11	126.02	111.63	108.57	107.19	109.18	118.92
79	124.25	113.79	100.79	101.89	95.50	91.28	93.44	89.81	82.83	88.69
86	165.41	127.79	113.40	93.59	93.20	80.01	89.57	97.84	88.86	81.80

Table B.22 Load Profiles examples for Shoulder one

Shoulder one period 11:00 AM – 1:00 PM			
Load Profiles Above and Below the Average Value [KWh]			
Profile/Hr	t11	t12	t13
29	108.11	102.96	107.18
74	141.00	140.28	138.12
79	95.18	132.37	142.40
86	93.41	96.41	96.47

Table B.23 Load Profiles examples for Peak period

Peak period 2:00 PM – 8:00 PM							
Load Profiles Above and Below the Average Value [KWh]							
Profile/Hr	t14	t15	t16	t17	t18	t19	t20
29	121.74	136.56	182.56	222.25	250.43	335.84	236.45
74	138.55	140.01	165.57	192.06	212.12	-27.27	196.94
79	154.18	161.80	177.73	188.56	198.47	65.15	186.15
86	105.33	109.67	118.85	161.09	191.42	329.41	182.48

Table B.24 Load Profiles examples for Shoulder two period

Shoulder two period 9:00 PM – 12:00 AM				
Load Profiles Above and Below the Average Value [KWh]				
Profile/Hr	t21	t22	t23	t24
29	216.80	193.96	154.49	122.68
74	191.34	177.59	144.32	177.72
79	170.77	167.27	158.55	108.86
86	174.05	153.51	130.43	148.14

Appendix C - GAMS Code

```
$Title: The Aggregator Model
SETS
w Scenarios          /w1*w25/
t Time Periods     / t1 * t24/
;
SCALARS
Alpha     Confidence Level/0.95/
Nw        Number of scenarios /25/
a         Ramping limits   /0.2/
b         T&D Charges for Customers  /70/
c         T&D Charges for The aggregator /30/
;
PARAMETERS
Piw(w)            Probability of occurrence of scenario w
Ect(t,w)          Energy Consumption of Customers [MWh]
ED(t)             Day-ahead Contract between DSO and The aggregator
[MWh]
LMPC(t)           LMP Price offered to customers in Day-ahead market [$
per MWh]
LMPD(t)           LMP Price in Day ahead market [$ per MWh]
LMPRP(t,w)        LMP for excess energy in spot market [$ per MWh]
LMPR(t,w)         Locational Marginal Price in Real Time Market [$ per
MWh]
ec(t,w)           Elasticity of Customers at time t in scenario w
NewProfit(w)      Profit in [$] Per Scenario After Change selling Price
OldProfit(w)      Profit Before Change Selling Price
NewPrices(t)      Selling Price After Change
CostForCustAft(t,w) Cost of Energy for Customers After Change Selling
Prices
CostForCustBef(t,w) Cost of Energy for Customers Before Change Selling
Prices
NewLoad(t,w)        New Load After Change Prices
DiffLoadCont(t,w)   Difference between new load and contract
;
ec(t,w) = -0.4;
Piw(w) = 1/(Nw);
$call GDXXRW ED.xlsx trace=3 par=ED rng=ED!a1 rdim=1 cdim=0
$call GDXXRW LMPC.xlsx trace=3 par=LMPC rng=LMPC!a1 rdim=1 cdim=0
$call GDXXRW LMPD.xlsx trace=3 par=LMPD rng=LMPD!a1 rdim=1 cdim=0
$call GDXXRW LMPR.xlsx trace=3 par=LMPR rng=LMPR!a1 rdim=1 cdim=1
$call GDXXRW Ect.xlsx trace=3 par=Ect rng=Ect!a1 rdim=1 cdim=1
$GDXIN ED.gdx
```

```
$LOAD ED
$GDXIN
$GDXIN LMPC.gdx
$LOAD LMPC
$GDXIN
$GDXIN LMPD.gdx
$LOAD LMPD
$GDXIN
$GDXIN LMPR.gdx
$LOAD LMPR
$GDXIN
$GDXIN Ect.gdx
$LOAD Ect
$GDXIN
LMPC(t) = b+LMPC(t);
LMPR(t,w) = abs(LMPR(t,w));
LMPRP(t,w) = LMPR(t,w);
LMPR(t,w) = c+LMPR(t,w);
Display Ect,ec,ED,LMPC,LMPR,Piw;

*********************************************************************
***********            DECLARATION OF VARIABLES            **********
*********************************************************************
VARIABLES
CVAR     CVaR: objective function
DLMPC(t) Change in Hourly Price
Zeta     Value at Risk: Auxiliary variable used to calculate CVaR
Uw(w)    Auxiliary variable used to calculate CVaR
u(t,w)   Selection of load higher than contract before change
x(t,w)   Selection of load higher than contract after change
P1(t,w)  Penalty if load higher than contract before change
P2(t,w)  Penalty if load lower than contract before change
P3(t,w)  Penalty if load higher than contract after change
P4(t,w)  Penalty if load lower than contract after change
;

*********************************************************************
***********   MATHEMATICAL CHARACTERIZATION OF VARIABLES   **********
*********************************************************************
POSITIVE VARIABLES
Uw(w)    Positive Variable = VAR - Profit_w
;
binary VARIABLE u(t,w);
binary VARIABLE x(t,w);
parameter ub1(t,w);
```

```
parameter lb1(t,w);
parameter ub2(t,w);
parameter lb2(t,w);
ub1(t,w) =       abs(Ec(t,w)-ED(t));
lb1(t,w) =      -abs(Ec(t,w)-ED(t));
ub2(t,w) =       abs(Ec(t,w)+a*Ec(t,w)-ED(t));
lb2(t,w) =      -abs(Ec(t,w)-a*Ec(t,w)-ED(t));
parameter A1(t)
/
t1        1
t2        1
t3        1
t4        1
t5        1
t6        1
t7        1
t8        1
t9        1
t10       1
t11       1
t12       1
t13       1
t14       5
t15       5
t16       5
t17       5
t18       5
t19       5
t20       5
t21       1
t22       1
t23       1
t24       1
/;
parameter A2(t)
/
t1        0
t2        0
t3        0
t4        0
t5        0
t6        0
t7        0
t8        0
t9        0
```

```
t10       0
t11       0
t12       0
t13       0
t14       0
t15       0
t16       0
t17       0
t18       0
t19       0
t20       0
t21       0
t22       0
t23       0
t24       0
/;

parameter A3(t)
/
t1        1
t2        1
t3        1
t4        1
t5        1
t6        1
t7        1
t8        1
t9        1
t10       1
t11       1
t12       1
t13       1
t14       5
t15       5
t16       5
t17       5
t18       5
t19       5
t20       5
t21       1
t22       1
t23       1
t24       1
```

```
/;

parameter A4(t)
/
t1      0
t2      0
t3      0
t4      0
t5      0
t6      0
t7      0
t8      0
t9      0
t10     0
t11     0
t12     0
t13     0
t14     0
t15     0
t16     0
t17     0
t18     0
t19     0
t20     0
t21     0
t22     0
t23     0
t24     0
/;
EQUATIONS
Obj                     Obj to Max CVAR
TotalVariationInLoad    Change In load DeltaP
ChangeInCost            Change In Cost
TotalPrice              Total Retail Price
EnergyConsumed          Total Energy Consumed After Change
Rampinglimit1           Ramp Down limit
Rampinglimit2           Ramp up limit
RiskModelConstraint1    Risk Equation 1
RiskModelConstraint2    Risk Equation 2
PenaltyConstraint1
PenaltyConstraint2
PenaltyConstraint3
PenaltyConstraint4
PenaltyMultiplier1
```

```
PenaltyMultiplier2
PenaltyMultiplier3
PenaltyMultiplier4
;
Obj.. CVAR =e= (Zeta - ((1/(1-alpha))*sum(w,Piw(w)*Uw(w))));
TotalVariationInLoad(w)..
sum(t,(ec(t,w)*DLMPC(t)*Ect(t,w)/LMPC(t)))=e=0;
ChangeInCost(w)..
sum(t,((Ect(t,w)+(ec(t,w)*DLMPC(t)*Ect(t,w)/LMPC(t)))*(LMPC(t)+DLMPC(t
)))-(Ect(t,w)*LMPC(t))) =l=0;
TotalPrice(t).. (LMPC(t)+DLMPC(t))=g=0;
EnergyConsumed(t,w)..
(Ect(t,w)+(ec(t,w)*DLMPC(t)*Ect(t,w)/LMPC(t)))=g=0;
Rampinglimit1(t,w)..  (ec(t,w)*DLMPC(t)*Ect(t,w)/LMPC(t)) =g= -
a*Ect(t,w);
Rampinglimit2(t,w)..  (ec(t,w)*DLMPC(t)*Ect(t,w)/LMPC(t)) =l=
a*Ect(t,w);
RiskModelConstraint1(w)..  Uw(w) =g= (Zeta -
(sum(t,(DLMPC(t)*Ect(t,w)*(1+ec(t,w)))
+((ec(t,w)*power(DLMPC(t),2)*Ect(t,w)/LMPC(t)))
+(P3(t,w)*LMPRP(t,w)*(-Ect(t,w)-
(ec(t,w)*DLMPC(t)*Ect(t,w)/LMPC(t))+ED(t)))
+(P4(t,w)*LMPRP(t,w)*(-Ect(t,w)-
(ec(t,w)*DLMPC(t)*Ect(t,w)/LMPC(t))+ED(t)))
+(P1(t,w)*LMPRP(t,w)*(Ect(t,w)-ED(t)))
+(P2(t,w)*LMPRP(t,w)*(Ect(t,w)-ED(t))))) );
RiskModelConstraint2(w).. Uw(w)=g=0;
PenaltyConstraint1(t,w)..   ((Ect(t,w)-ED(t))-ub1(t,w)*u(t,w))=l=0;
PenaltyConstraint2(t,w)..   ((Ect(t,w)-
ED(t))+lb1(t,w)*u(t,w))=g=lb1(t,w);
PenaltyConstraint3(t,w)..
((Ect(t,w)+(ec(t,w)*DLMPC(t)*Ect(t,w)/LMPC(t))-ED(t))-
ub2(t,w)*x(t,w))=l=0;
PenaltyConstraint4(t,w)..
((Ect(t,w)+(ec(t,w)*DLMPC(t)*Ect(t,w)/LMPC(t))-
ED(t))+lb2(t,w)*x(t,w))=g=lb2(t,w);
PenaltyMultiplier1(t,w).. P1(t,w)=e=(A1(t)*u(t,w));
PenaltyMultiplier2(t,w).. P2(t,w)=e=(A2(t)*(1-u(t,w)));
PenaltyMultiplier3(t,w).. P3(t,w)=e=(A3(t)*x(t,w));
PenaltyMultiplier4(t,w).. P4(t,w)=e=(A4(t)*(1-x(t,w)));

MODEL AggregatorDA /ALL/;
option iterlim = 1e8;
option reslim = 1e10;
```

```
Option MINLP = SCIP;
SOLVE AggregatorDA USING MINLP MAXIMIZING CVAR;
NewProfit(w) =
sum(t,(LMPC(t)+DLMPC.l(t))*(Ect(t,w)+(ec(t,w)*DLMPC.l(t)*Ect(t,w)/LMPC
(t))))
            -sum(t,LMPR(t,w)*ED(t))
            -sum(t,(P3.l(t,w)+P4.l(t,w))*LMPRP(t,w)*(Ect(t,w)-
ED(t)+(ec(t,w)*DLMPC.l(t)*Ect(t,w)/LMPC(t))));

OldProfit(w) = sum(t,LMPC(t)*Ect(t,w))
            -sum(t,LMPR(t,w)*ED(t))
            -sum(t,(P1.l(t,w)+P2.l(t,w))*LMPRP(t,w)*(Ect(t,w)-ED(t)));

NewPrices(t) = LMPC(t)+DLMPC.l(t);
CostForCustAft(t,w)=
(Ect(t,w)+(ec(t,w)*DLMPC.l(t)*Ect(t,w)/LMPC(t)))*(LMPC(t)+DLMPC.l(t));
CostForCustBef(t,w)= (Ect(t,w)*LMPC(t));
NewLoad(t,w) = (Ect(t,w)+(ec(t,w)*DLMPC.l(t)*Ect(t,w)/LMPC(t)));
DiffLoadCont(t,w) =  NewLoad(t,w)-ED(t);
Display
P1.l,P2.l,P3.l,P4.l,u.l,x.l,DLMPC.l,CVAR.l,Zeta.l,Uw.l,EnergyConsumed.
l,TotalPrice.l,NewProfit,OldProfit,NewPrices,CostForCustAft,CostForCus
tBef,NewLoad,DiffLoadCont;
Execute_Unload "Results(6-23-
2019).gdx",P1,P2,P3,P4,u,x,DLMPC,NewPrices,LMPC,Ect,EnergyConsumed.l,U
w,CVAR,Zeta,a,Alpha,ec,NewProfit,OldProfit,LMPR,ED,CostForCustAft,Cost
ForCustBef,NewLoad;
Execute 'GDXXRW.EXE Results(6-23-2019).gdx var=u rng=BinaryVarD1!a1';
Execute 'GDXXRW.EXE Results(6-23-2019).gdx var=x rng=BinaryVarR1!a1';
Execute 'GDXXRW.EXE Results(6-23-2019).gdx var=P1 rng=P1!a1';
Execute 'GDXXRW.EXE Results(6-23-2019).gdx var=P2 rng=P2!a1';
Execute 'GDXXRW.EXE Results(6-23-2019).gdx var=P3 rng=P3!a1';
Execute 'GDXXRW.EXE Results(6-23-2019).gdx var=P4 rng=P4!a1';
Execute 'GDXXRW.EXE Results(6-23-2019).gdx var=DLMPC rng=DLMPC!a1';
Execute 'GDXXRW.EXE Results(6-23-2019).gdx par=LMPC  rng=LMPC!a1';
Execute 'GDXXRW.EXE Results(6-23-2019).gdx par=NewPrices
rng=NewPrices!a1';
Execute 'GDXXRW.EXE Results(6-23-2019).gdx par=LMPR rng=LMPR!a1';
Execute 'GDXXRW.EXE Results(6-23-2019).gdx par=ED rng=ED!a1';
Execute 'GDXXRW.EXE Results(6-23-2019).gdx par=Ect  rng=Ect!a1';
Execute 'GDXXRW.EXE Results(6-23-2019).gdx par=NewLoad
rng=NewLoad!a1';
Execute 'GDXXRW.EXE Results(6-23-2019).gdx equ=EnergyConsumed
rng=EnergyVariation!a1';
Execute 'GDXXRW.EXE Results(6-23-2019).gdx var=CVAR rng=CVAR!a1';
```

```
Execute 'GDXXRW.EXE Results(6-23-2019).gdx var=Zeta rng=Zeta!a1';
Execute 'GDXXRW.EXE Results(6-23-2019).gdx var=Uw rng=Uw!a1';
Execute 'GDXXRW.EXE Results(6-23-2019).gdx par=a rng=a!a1';
Execute 'GDXXRW.EXE Results(6-23-2019).gdx par=Alpha rng=Alpha!a1';
Execute 'GDXXRW.EXE Results(6-23-2019).gdx par=ec rng=Elasticity!a1';
Execute 'GDXXRW.EXE Results(6-23-2019).gdx par=NewProfit
rng=NewProfit!a1';
Execute 'GDXXRW.EXE Results(6-23-2019).gdx par=OldProfit
rng=OldProfit!a1';
Execute 'GDXXRW.EXE Results(6-23-2019).gdx par=CostForCustAft
rng=NewCostCus!a1';
Execute 'GDXXRW.EXE Results(6-23-2019).gdx par=CostForCustBef
rng=OldCostCus!a1';
```