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CHARACTERISTIC OF A CRITICAL NETWORK ARC IN A SERVICE SYSTEM

Summary. Vulnerability of a transportation network influences, importantly, the function of public service systems constituted on the network. To be able to study the importance of an individual network arc for proper functionality of the system, we suggested a characteristic function of the arc transit time elongation together with its upper and lower estimations. We also suggested and verified an algorithm that enables the determination of parameters of the characteristic estimation, and we provide a reader with a computational study performed with real road networks.

1. INTRODUCTION

Performance of each service system depends on reliability of the associated transportation network [4, 8-10]. In a real transportation network, various random events may occur, which can considerably elongate the time necessary to traverse the affected arc [7]. The effect of individual arc collapse was studied in [4, 9, 10], and some measures such as "Network robustness index" and "Network trip robustness" were also suggested. The individual arc elongation may or may not influence performance of the service system. It depends on the position of the arc with respect to paths, which are or can be used for transport from service centers to the system users [3]. Within this paper, we focus on identification of the network arcs, which have decisive impact on a given service system performance in the case when traversing time of the arcs is elongated owing to some detrimental event. We suggest a characteristic of an individual arc that enables the estimation of the impact of arc failure on the system performance, and we also present an algorithm that enables the determination of parameters of the characteristic. Furthermore, we provide a classification of possible characteristics.

The remainder of the paper is organized as follows. The next section introduces notions connected with performance of a service system. The third section contains the description of the algorithm for arc characteristic determination, together with a suggestion of characteristic classification. The fourth section presents the numerical experiments with the algorithm performed on models of real road networks. The final section summarizes the obtained findings.

2. SERVICE SYSTEM AND ARC CHARACTERISTIC

The considered service system is designed to provide users spread over some geographical area with some specific service (e.g. medical first aid, fireman service etc.). The system users are concentrated in dwelling places, which represent nodes of the transportation network connecting the dwelling places and some other sites, where a service facility may be located. Structure of a service system is determined by a deployment of service centers, from which the given kind of service is provided. It is assumed that the shortest path from the nearest service center to a serviced user location is used [1-3, 6]. The system is represented by a set I_1 of p service center locations and by a set J of user locations, where volume b_j of demand is associated with the user location j. We assumed that the transportation network used for servicing of user demands is symmetrical, which means that each link connecting pair of nodes can be traversed in both directions. Symbol t_{ij} denotes the time length of the shortest path from i to j in the underlying network. The set of associated arcs forming the path is denoted by P_{ij}. Let us denote $ass(j) \in I_1$ the center, which provides user j with service. Then, the original total transport performance TP^* of the service system can be computed according to (1).

$$TP^* = \sum_{j \in J} b_j t_{ass(j),j} \tag{1}$$

Let us denote (u, v) an arc with traversing time t_{uv}. We will deal with the impact of elongation of t_{uv} by Δ on the total transport performance of the system. We realize that arc (u, v) may or may not be the element of a path $P_{ass(j),j}$ for some $j \in J$. If the arc does not belong to any of the paths, it cannot influence the transport performance and it will be called irrelevant. A relevant arc can belong to several paths. We denote $J^{uv} \subseteq J$ the set of $j \in J$, for which $(u, v) \in P_{ass(j),j}$. Now, we try to analyze the progress of transport performance $TP(\Delta)$ with increasing value of Δ representing an elongation of the traversing time t_{uv}. If the value of Δ is near to zero, the value $TP(\Delta)$ will grow linearly with a slope (derivative) B_1, because each original path $P_{ass(j),j}$ for any $j \in J^{uv}$ stays the shortest one. The starting slope B_1 of the piece-wise linear increasing concave function $TP(\Delta)$ equals the sum of all demands b_j of all $j \in J^{uv}$, which have no equivalent alternative path to a service center. The equivalent alternative path means a path that does not contain the arc (u, v) and has the same length as the original shortest path $P_{ass(j),j}$. When Δ reaches the value, for which some of the paths result in not being the shortest one, then the associated user j will be serviced by the shortest path in the updated network to some service center from I_1, and this path will not contain the arc (u, v). Then the slope of $TP(\Delta)$ behind the enlarged Δ decreases by b_j. This situation can repeat several times during increase Δ up to some value e_2, when the slope of $TP(\Delta)$ decreases last time (see Fig. 1).

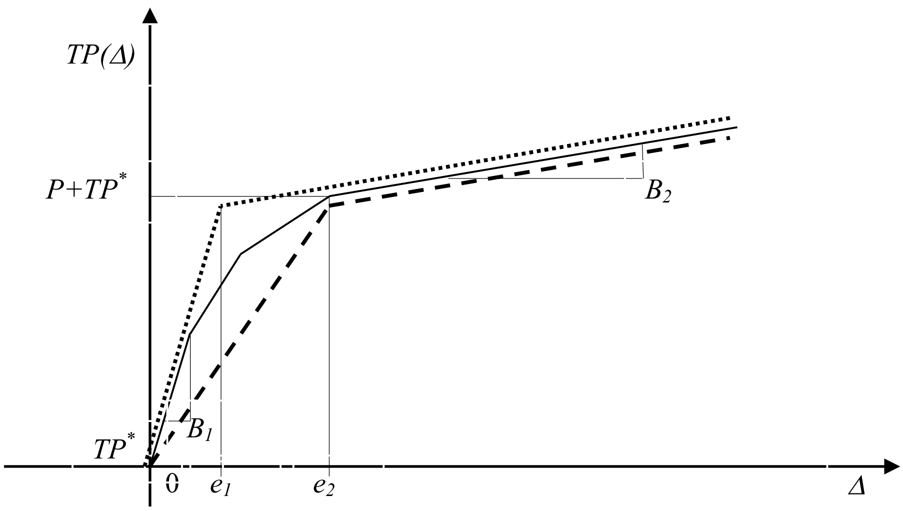

Fig. 1. Progress of the transportation performance $TP(\Delta)$ depending on elongation Δ is depicted by a full line; the upper and lower bounds of the progress are depicted by dotted and dashed lines, respectively

When Δ exceeds the value e_2, the slope of $TP(\Delta)$ stays constant. This final slope will be denoted by B_2. The value of B_2 is the sum of all user demands b_j of users $j \in J^{uv}$, which can be connected to a service center only by a path containing the arc (u, v). The symbol P denotes the value of $TP(e_2)$ for the elongation $\Delta = e_2$. Elongation of the original traversing time t_{uv} behind the value e_2 affects only the users, who cannot avoid using the arc (u, v) on arbitrary path to a service center. We have to note that

a special relation holds for a symmetrical network. If any user from J^{uv} has an alternative path not containing the arc (u, v), then all users from J^{uv} have their own alternative paths.
The values of e_2, B_1, B_2 and P are sufficient to determine graphs of the upper or lower estimation of the characteristic $TP(\Delta)$-TP^*. The value of e_1 can be computed according to (2) under assumption $B_1 > B_2$.

$$e_1 = \frac{P - B_2 e_2}{B_1 - B_2} \tag{2}$$

In the case $B_1=B_2$, the characteristic is a linear function with the slope B_1. In this case, we set e_2= e_2=0. It follows from the network symmetricity that if $B_1 > B_2$, then $B_2 = 0$.

The relevant arcs satisfying $B_1 > 0$ can be classified according to coefficients B_1 and B_2. Three basic classes of the relevant arcs will be distinguished.

a) $B_1 > B_2=0$ and $e_1 = e_2$; piece-wise linear concave function limited from above

b) $B_1 > B_2=0$ and $e_1 < e_2$; piece-wise linear concave function limited from above

c) $B_1 = B_2$; linear growing function

An algorithm, which enables to determine the values of e_2, B_1, B_2 and P, is introduced in the next section.

3. COMPUTATION OF THE ARC CHARACTERISTIC PARAMETERS

The presented algorithm uses Bellman, Ford and Moor algorithm [5] to compute the shortest paths from all users to the nearest service center in the adjusted network. The resulting nearest center to a user j will be denoted by $\underline{ass(j)}$ and the time length of the associated path will be denoted by $t'_{\underline{ass(j)},j}$. Let T be enough big constant, e.g. sum of all arc traversing times is satisfactorily big. Then, the algorithm for determination of arc (u, v) characteristic parameters can be stated as follows:

0. Set B_1=0, B_2=0 and P=0.

1. Change evaluation t_{uv} of the arc (u, v) so that the updated evaluation equals $t_{uv}+T$.

2. Compute new shortest paths from each $j \in J^{uv}$ to the nearest service center. The results of the computation are the subscript $\underline{ass(j)}$ of the nearest service center and the length $t'_{\underline{ass(j)},j}$ of the new path.

3. Process all users j from J^{uv} in the following way:
   If $t'_{\underline{ass(j)},j} \geq t_{ass(j),j}+T$ then
   update $B_2 = B_2+b_j$
   else update $B_1 = B_1+b_j$
   $e_2 = max\{ e_2, t'_{\underline{ass(j)},j} - t_{ass(j),j}\}$
   $P = P+(t'_{\underline{ass(j)},j} - t_{ass(j),j}) b_j$.

4. If B_1= 0 then
   set e_1=0 and e_2=0
   else update $e_1=P/B_1$
   $B_1 = B_1+ B_2$
   $P = P+B_2 e_2$.

5. Terminate.

After the algorithm has been performed, the upper and lower bounds of the characteristic of arc (*u, v*) are defined according to the following definitions:

If $\Delta<e_1$ then $Uc(\Delta)=B_1\Delta$ else $Uc(\Delta)=B_1\, e_1+ B_2\Delta$.

If $\Delta<e_2$ then $Lc(\Delta)=(P/e_2)\Delta$ else $Lc(\Delta)=P+ B_2\Delta$.

4. COMPUTATIONAL STUDY

Within this section, we present a computational study focused on real network arc characteristics regarding a given public service system.

The used benchmarks were derived from the real emergency healthcare system, which was originally implemented in selected regions of Slovakia. For each self-governing region, i.e. Bratislava (BA), Banská Bystrica (BB), Košice (KE), Nitra (NR), Prešov (PO), Trenčín (TN), Trnava (TT) and Žilina (ZA), all cities and villages (communities representing elements of the set J) with corresponding number b_j of inhabitants were taken into account for each j from J. The coefficients b_j were rounded to hundreds. These sub-systems cover demands of all communities – towns and villages spread over the particular regions by a given number of ambulance vehicles. In the benchmarks, the set of communities represents both the set J of users' locations and also the set I of possible service center locations. The cardinalities of these sets vary from 87 to 664 according to the considered region. The number p of located centers was derived from the original design, and it varies from 22 to 46. Individual subnetwork parameters are described in Table 1.

Table 1

Parameters of the individual networks, in which the service systems were designed

Region	$\|J\|$	$\|I\|$	p
BA	87	87	25
BB	515	515	46
KE	460	460	38
NR	350	350	36
PO	664	664	44
TN	276	276	26
TT	249	249	22
ZA	315	315	29

The final set of center locations was obtained by solving weighted p-median problem for each self-governing region. Then, the set of relevant network arcs was identified and upper and lower estimations of the characteristic for each relevant arc were determined regarding the designed service system. The set of relevant arcs was partitioned into three classes according to the types of characteristics mentioned at the end of Section 2. The classes a) and c) contain such arcs, for which the upper and lower characteristic estimations coincide and thus upper and lower estimations describe exactly the arc characteristic. The class b) contains network arcs, in which upper and lower estimations differ. The mean of upper and lower characteristic estimations were computed for each arc of the class b) on characteristic domain from 0 to e_2 to evaluate mean of maximal deviation of the estimations from the actual characteristic.

To perform the algorithm described in the previous sections, the programming environment NetBeans 8.1 was used and the associated algorithms were programmed in Java language and the experiments were run on a PC equipped with the Intel® Core™ i7 5500U processor with the following parameters: 2.4 GHz and 16 GB RAM.

The achieved results are summarized in the Table 2, where "Rel. arcs" means the number of relevant arcs that were found. The denotations "a) arcs", "b) arcs" and "b) arcs" denote columns where the number of arcs of type a), b) and c) are plotted, respectively. The column denoted by "avg. dif. [%]" contains average of difference of upper and lower characteristic estimations on characteristic domain from 0 to e_2 for arcs of the class b) expressed in percentage of P.

Table 2

Parameters of the individual subnetworks, in which the public service systems were designed

Region	Rel. arcs	a) arcs	b) arcs	c) arcs	b) avg. dif. [%]
BA	83	49	17	17	15.17
BB	593	191	201	201	17.86
KE	523	185	171	167	17.81
NR	408	194	148	66	17.26
PO	753	177	295	281	18.39
TN	302	105	91	106	18.18
TT	301	166	90	45	16.91
ZA	370	105	114	151	16.51

The results of the computational study performed on real transportation networks of individual self-governing regions of the Slovak Republic show that there is a considerable portion of c-class arcs in each transportation network. The size of the portion is approximately one-third of the total number of arcs. It means that these arcs have no detour in the given network, and thus these arcs deserve a special attention by the network administrator.

It can be noticed that the portion of a-class arcs represent also one-third of the total number of arcs, which means that our upper estimation of the arc characteristic is equivalent to the exact arc characteristic.

Regarding upper and lower estimations of the b-class arc characteristic, the values reported in the last column of the table indicate that the characteristic values may be at most by 19 percent lower than the suggested upper estimation.

5. CONCLUSIONS

This paper was focused on means of network arc importance investigation, where the importance is considered to be relative regarding a given public service system. A smart algorithm was suggested to find parameters describing the upper and lower estimations of the characteristics of relevant arcs. These estimations enable the evaluation of the importance of an individual arc for proper functioning of the service system. The obtained parameters enable the partitioning of the set of relevant arcs into several classes and separate the cut-arcs from the arcs that have an alternative if the arc collapses. This way, we presented a tool for transportation network analysis from the point of performance of a service system. The attached computational study demonstrates the use of this algorithm for real road network analysis and also gives a level of accuracy of the lower and upper estimations regarding the exact arc characteristic.

The future research in this field will be focused on exploitation of the suggested algorithm and associate arc evaluations for construction of an effective set of detrimental scenarios destined for robust service system designing.

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