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COMPARATIVE STUDY OF PRESSURE WAVE MATHEMATICAL MODELS FOR HP FUEL PIPELINE OF CEUP AT VARIOUS OPERATING CONDITIONS

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Abstract- High pressure (HP) Fuel Pipeline is one of the major components of Combination Electronic Unit Pump (CEUP) fuel injection system which has important role in building up of fuel pressure necessary for fuel injection. Three different 1D mathematical models of damped wave equation (WE) namely linear damped, viscous damped and damped model have been developed in MATLAB to investigate fuel pressure inside HP fuel pipeline of CEUP fuel injection system at various operating conditions of diesel engine. Lab experiments have been conducted to measure the pump side and injector side pressures by using KISTLER 4067 piezoresistive pressure sensors under controlled environment. Each model has been verified by comparing its simulated results with those of

experimentally verified AMESim numerical model of CEUP system. Model evaluation statistical techniques like "Root Mean Square Error" (RMSE) and "Index of Agreement" (IA) have been used to quantify the predicted results of each mathematical model at various operating conditions. From analytical and quantitative analysis it has been concluded that viscous damped mathematical model predicts more accurately as compared to rest of models specially at all combinations of cam rotational speeds and cam angles of 700rpm, 1100rpm and 6°CaA, 10°CaA and 14°CaA respectively. Damped mathematical model predictions have been found relatively more precise at cam angles of 6°CaA and cam rotational speeds of 900rpm and 1300rpm. Moreover linear model was accurate at cam rotational speed of 900rpm and cam angle of 14°CaA.

Index terms*: *Wave equation, Mathematical model, High pressure fuel pipeline, Finite difference, RMSE, IA.

I. INTRODUCTION

CEUP is a new type of efficient high pressure, electronic unit pump fuel injection system which is used in heavy duty vehicles and marine diesel engines. It also meets Chinese strict emission requirements [1] to restrict air pollution which is a health risk especially in urban areas [2]. During a fuel injection cycle of CEUP pressure varies between 0.5 MPa and 150 MPa [1, 3 and 4]. HP fuel pipeline between pump and mechanical injector of CEUP is one of the major components and has impact on building up and propagation of high pressure fuel [3]. Therefore, detailed experimental and theoretical study has been conducted in this paper with the support of numerical and mathematical modeling to investigate the behavior of pressure inside HP fuel pipeline at various operating conditions of diesel engine.

A number of researchers have mathematically modeled fuel pipeline using principles of mass continuity and momentum conservation [5, 6, 7 and 8]. C. D. Rakopoulos and D. T. Hountalas [5] and H-K. Lee, M. F. Russell and C. S. Bae [6] have used these principles in their pipe models of diesel fuel injection system; A. E. Catania, A. Ferrari and E. Spessa [7] have used these principles in their mathematical pipe model of inline pump system. Similarly C. Arcoumanis and R. J. Fairbrother [8] have utilized these principles in their pipe model of fuel injection equipment. Kristina Ahlin [9] has analyzed pressure inside common rail (CR) using WE with viscous damping only.

Wave propagation, damping effects, stability, decay estimates and effects of boundary conditions on WE have been studied through mathematical models by a number of researchers in last decade [10-19]. Y. Nishidate and G. P. Nikishkov [10] and M. A. Rammaha [11] investigated damped WE in their mathematical model, Ryo Ikehata [12] and Fariba Fahroo [13] worked on decay estimates whereas Brian J. McCartin [14], Jaime E. and Reinhard Racke [15] and Patrick Martinez [16] researched the stability of damped WE in their mathematical models. Oudina Assia and Djelouah Hakim [17] investigated propagation of ultrasonic waves in viscous fluids whereas P. M. Jordan, Martin R. Meyer and Ashok Puri [18] and Reinhold Ludwig and Peter L. Levin [19] have investigated the influence of viscous damping on WE in their mathematical models. Alexander Thomann [20] has used both fluid and viscous frictions in WE while discussing classical absorbing layers in a seminar.

Physical characteristics of diesel fuel i.e. density, dynamic viscosity, speed of sound and bulk modulus as a function of varying pressure and temperature have been investigated by a number of researchers [21-26]. Boban D. Nikolic, Breda Kegl, Sasa D. Markovic and Melanija S. Mitrovic [21] presented a non-destructive method for determining the speed of sound and bulk modulus of diesel depending of varying pressures. They have also presented polynomial expressions for calculating speed of sound, density and bulk modulus of diesel fuel with varying pressures upto 160MPa and more. C. C. Enweremadu, H. L. Rutto and J. T. Oladeji [22] investigated basic flow properties of biodiesel fuel and its blends with diesel. Marzena Dzida and Piotr Prusakiewicz [23] working on commercial diesel fuel measured the speed of sound at pressures and temperatures between 0.1-101MPa and 293-218K respectively. Moreover they measured density of diesel at atmospheric pressure and between 273-363K temperatures. Andre´ L. Boehman, David Morris and James Szybist [24] have measured the bulk modulus of compressibility of diesel fuel and its impact on injection timing. Mustafa E. Tat and Jon H. Van Gerpen [25] have presented correlation equations for density, speed of sound and isentropic bulk modulus while working on blend of biodiesel and diesel fuels. Whereas Wang Jun-Xiao, Lu Jia-Xiang, Zhang Jin-Yang and Zhang Xi-Chao [26] have developed empirical formulas based on variations of density, dynamic viscosity, speed of sound and bulk modulus with changing pressure during diesel fuel injection process.

In this paper experimental measured pump side pressures and injector side pressures of CEUP at various operating conditions have been used to deeply investigate pressure wave propagation inside HP fuel pipeline of CEUP using three different 1D damped mathematical models of WE developed in MATLAB namely linear damped [10-16], viscous damped [17 and18] and damped [20] models. In addition, quantitative comparisons of these mathematical models have been carried out to compare the accuracy of each model. Moreover the dynamic variation of four key fuel characteristics i.e. density, dynamic viscosity, speed of sound and bulk modulus as a function of varying pressure [21-26] during fuel injection cycle have been incorporated [26] in all mathematical models.

Predicted pressures by all mathematical models have been analytically validated by comparing with simulated and experimentally validated AMESim numerical model of CEUP at various operating conditions of diesel engine. Moreover, all mathematical results are quantitatively analyzed and compared by using model evaluation statistical techniques like “Root Mean Square Error” (RMSE) and “Index of Agreement” (IA) [30].

Analytical comparisons with AMESim numerical results show that all mathematical simulated pressure predictions are quite coherent. Whereas quantitative comparisons show that the viscous damped model [17 and 18] and damped model [20] predicted accurately at various operating conditions of diesel engine.

The rest of this paper is organized as follows. CEUP fuel injection system and its operating principle have been described briefly in Section II. AMESim numerical modeling of CEUP fuel injection system has been discussed in section III. Experimental setup and results are described in section IV. Mathematical models, their convergence and solutions are presented and explained in Section V whereas simulation results are discussed in Section VI. RMSE and IA quantitative comparisons of all results have been done in section VII. Conclusions are made in Section VIII.

II. CEUP FUEL INJECTION SYSTEM

CEUP is a HP diesel fuel injection system which mainly consists of four or more units of HP Pump unit, solenoid control unit, HP fuel pipeline and mechanical injector as shown in figure 1. In a four unit CEUP four pump units along with their solenoid control units are jointly mounted on a low pressure combination box as shown in figure 3.

As shown in figure 1 plunger of HP pump unit is cam driven and responsible for pushing-in and pushing-out of diesel fuel from plunger chamber through control valve by its downward and upward motion respectively. Plunger is reset to its rest position with the help of plunger spring. Fuel inside the plunger chamber of HP pump unit is returned back to the fuel tank by upward motion of plunger when control valve of the solenoid control unit is open. Whereas, the fuel through HP fuel pipeline is pushed towards the delivery chamber and sac chamber of the mechanical injector if the control valve is closed. Control valve is opened and closed by turning the power off and on of solenoid control unit respectively. When the control valve is closed fuel pressure inside plunger chamber, HP fuel pipeline, delivery chamber and sac chamber starts to increase with upward motion of plunger. When the fuel pressure inside delivery chamber and sac chamber surpasses closing pressure of the injector needle, it is lifted up and fuel is injected into the chamber. Injector needle is reset to its rest position with the help of needle spring.

Quantity and timing of the injected fuel and therefore pressure inside HP fuel pipeline of CEUP is controlled through solenoid control unit [1].

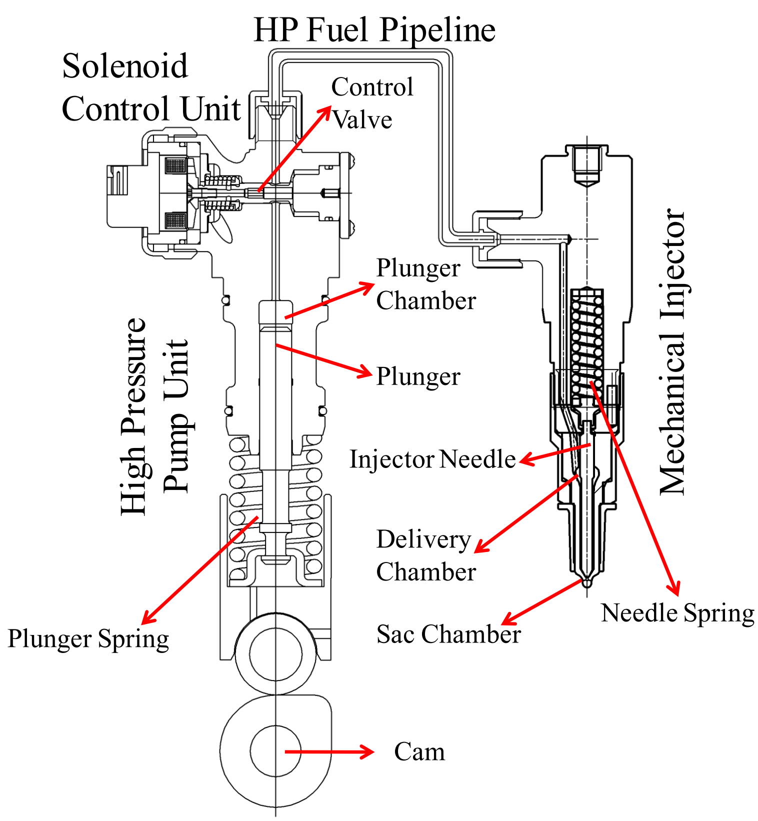


Figure 1. Schematic of CEUP fuel injection system

III. CEUP NUMERICAL MODELLING

A numerical model of CEUP fuel injection system consisting of a single unit of HP pump unit, solenoid control unit, HP fuel pipeline and a mechanical injector has been developed in AMESim numerical environment as shown in figure 2. Simulated pump side pressure and injector side pressure can be measured at locations shown in the figure 2.

AMESim numerical model has been verified by comparing simulated pump side and injector side pressures at various operating conditions of diesel engine with experimentally measured pump side pressure and injector side pressures on experimental setup shown in figure 3.

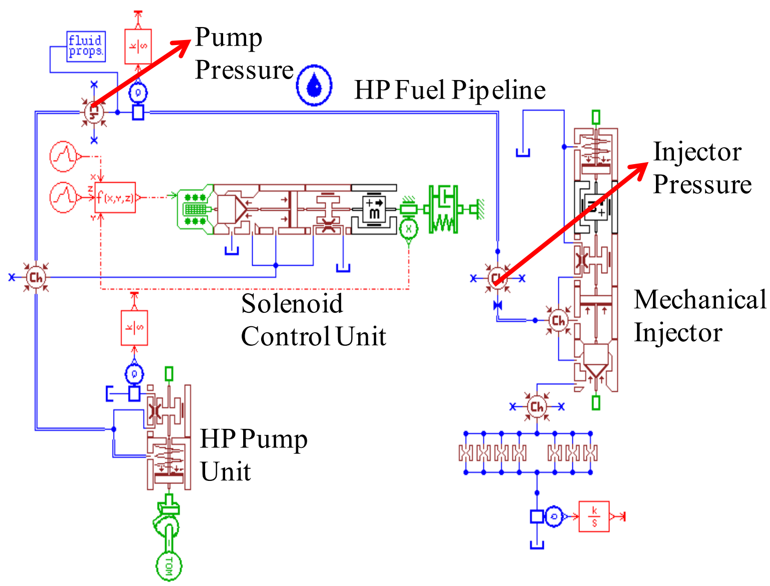


Figure 2. AMESim numerical model of CEUP fuel injection system

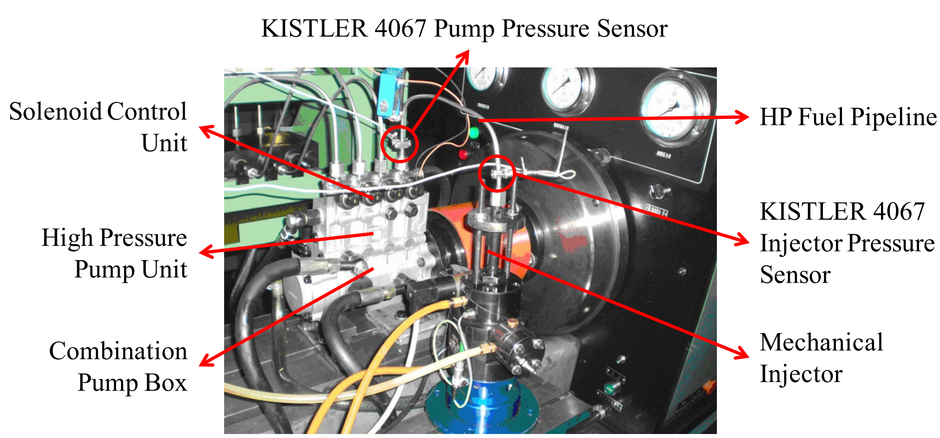


Figure 3. Experimental setup for CEUP fuel injection system

IV. EXPERIMENTAL SETUP AND RESULTS

An experimental setup as shown in figure 3 has been used to measure the pump side and injector side pressures by using KISTLER 4067 piezoresistive pressure sensors [27]. KISTLER 4067 pressure sensor is used specially for measuring pressure of hydraulic systems like fuel injection systems. During experiments only single HP pump unit, single solenoid control unit and single mechanical injector have been used as shown in figure 3.

Temperature around experimental setup has been kept at approximately room temperature through out the experiments. Sufficient time intervals of operation break have been considered between consecutive pressure measurements so that diesel fuel temperature variations may have minimal effect on measured pump side and injector side pressures.

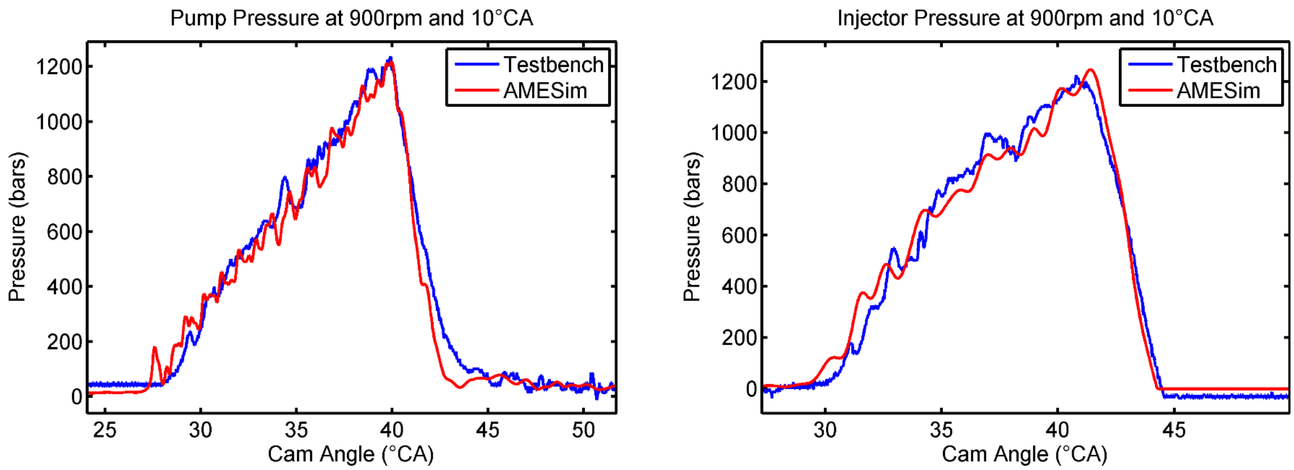
During the experiments pump and injector pressures raise up to 1500 bars depending upon the cam rotational speeds (rpm) and cam angles (°CaA).

Experimentally measured pump side pressures and injector side pressures have been used to validate the AMESim numerical model of CEUP. These pressures also been used as boundary conditions for all three mathematical models of HP fuel pipeline developed in MATLAB.

Figures 4(a and b) show comparisons of experimentally measured pump and injector pressures with AMESim numerical model at cam rotational speed and cam angle of 900rpm and 10°CaA respectively. Whereas figures 5(a and b) show comparisons at cam rotational speed and cam angle of 1300rpm and 12°CaA respectively. The results are quite coherent and validate the AMESim numerical model of CEUP with single HP pump unit and mechanical injector.

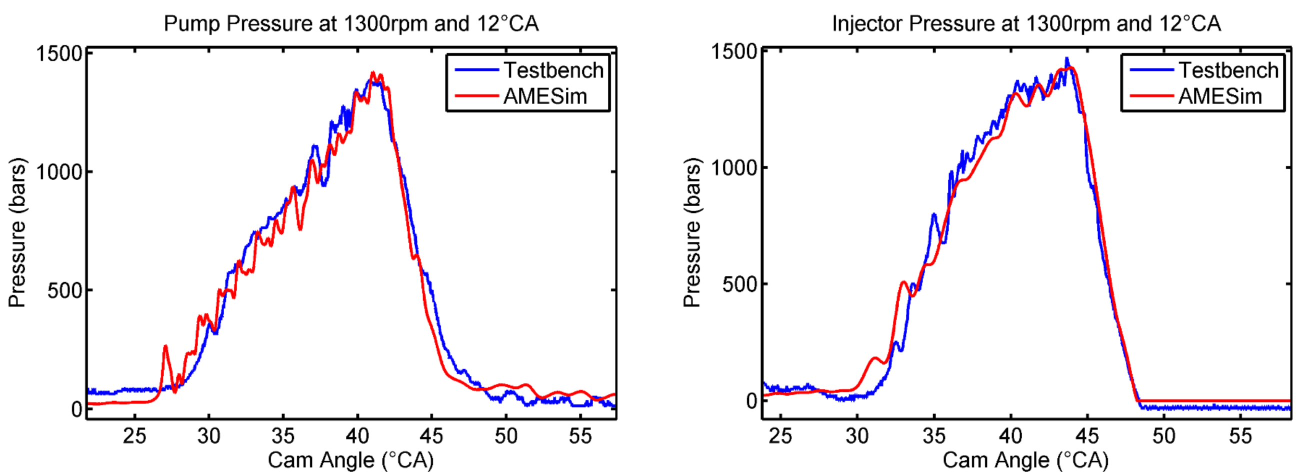
Table 1: Operating Conditions of Test Bench

Cam Rotational Speeds (rpm)	**Cam Angles (°CA)**	**Pipe Length (m)**
700, 900, 1100 and 1300	6, 10 and 14	0.47



(a) (b)

Figure 4. Experimentally measured pressures and AMESim results at cam rotational speed of 900rpm and cam angle of 10°CaA (a) pump side pressures and (b) injector side pressures



(a) (b)

Figure 5. Experimentally measured pressures and AMESim results at cam rotational speed of 1300rpm and cam angle of 12°CaA (a) pump side pressures and (b) injector side pressures

V. MATHEMATICAL MODELLING

Following assumptions have been considered for all mathematical models of pressure using WE

1. Flow of fuel is laminar.
2. Flow is in one direction i.e. from pump side to injector side.
3. Fuel is homogeneous
4. There are no air bubbles in fuel and modeling is without cavitations
5. Temperature has been considered constant.

a. Classical Undamped WE Model

Classical undamped WE without any losses is represented by equation (1) [9, 28 and 31].

$$\frac{d^2 p(x,t)}{dt^2} = c^2 \frac{d^2 p(x,t)}{dx^2} \tag{1}$$

Where p, $c=\sqrt{(B/\rho)}$, B and ρ are pressure, speed of sound, bulk modulus and density of the fuel respectively.

b. Linear Damped WE Model

Resistance experienced by fuel during its flow can be represented as a damping term [10, 12, 13, 15, 16, 32 and 33] as shown in equation (2).

$$\frac{d^2 p(x,t)}{dt^2} + b\frac{dp(x,t)}{dt} = c^2 \frac{d^2 p(x,t)}{dx^2} \tag{2}$$

Where b is damping coefficient. This model is referred to as linear damped model in the following.

c. Viscous Damped WE Model

Viscous damped WE [9, 17, 18, 19, 31, 32 and 33] is represented by a viscous damping term in a nonlinear equation (3).

$$\frac{d^2 p(x,t)}{dt^2} = c^2 \frac{d^2 p(x,t)}{dx^2} + \frac{4\eta}{3\rho}\frac{d^3 p(x,t)}{dx^2 dt} \tag{3}$$

Where η is dynamic viscosity of the fuel. Viscous losses can adequately represent all of the losses [9, 29]. This model is referred to as viscous damped model in the following.

d. Damped WE Model

A WE containing both fluid friction and viscous friction terms [20 and 33] is represented by nonlinear equation (4).

$$\frac{d^2 p(x,t)}{dt^2} + b\frac{dp(x,t)}{dt} = c^2 \frac{d^2 p(x,t)}{dx^2} + \frac{4\eta}{3\rho}\frac{d^3 p(x,t)}{dx^2 dt} \tag{4}$$

This model is referred to as damped model in the following.

Equations (1-4) are true for following ranges, initial conditions and boundary conditions mentioned in equations (5), (6) and (7) respectively.

$$x \in (0,L), t \in (0,T) \tag{5}$$

$$p(x,0) = P_{ini}, \qquad \frac{dp(x,0)}{dt} = 0 \qquad\qquad x \in (0,L) \tag{6}$$

$$p(0,t) = P_{pump}, \qquad p(L,t) = P_{inj} \qquad\qquad t \in (0,T) \tag{7}$$

Where L, T, P_{ini}, P_{pump} and P_{inj} are total length of fuel pipeline, total time of simulation/ injection process, initial pressure in pipe, pump pressure and injector pressure respectively.

e. Mathematical Modeling

Equations (1-4) with initial conditions (6) and (7) are solved by using Finite Difference Method (FD). Equations (1-4) are discretized in space and time using FD method by introducing a uniform mesh grid. The temporal domain (0,T) and spatial domain (0,L) are divided into finite number of mesh points such that

$$\begin{aligned} x_n &= (n-1)\Delta x \qquad && n = 1,2,\ldots,L \\ t_i &= i\Delta t && i = 1,2,\ldots,N \end{aligned} \tag{8}$$

Where $\Delta x = x_n - x_{n-1}$ and $\Delta t = t_i - t_{i-1}$ are the mesh sizes. Pressure predicted $p(x,t)$ by all mathematical models are calculated at these grid points at all operating conditions mentioned in Table 1.

Mesh sizes (Δx and Δt) are chosen appropriately such that all mathematical solutions converge but remain within the stability criteria of $(c\Delta t/\Delta x) \leq 1$ [9]. Most optimized mesh sizes Δx and Δt for all mathematical models have been found to be $L/10$ and $1\mu s$ respectively at all combinations of operating conditions of diesel engine mentioned in Table 1.

At first constant nominal values of ρ, η, c, B and P_{ini} are taken for all mathematical solutions at atmospheric pressure and 293K temperature as 840 kg/m^3 [21], 2.2 kg/m.s, 1360 m/s [21], 1.55 GPa [21], and 0.5 MPa respectively. But in actual these values vary with varying pressures [21-26]. Pressure inside HP fuel pipeline of CEUP varies from 5 bars to 1500 bars during fuel injection cycle as shown in figures 4(a and b) and 5(a and b), therefore dynamic variations of ρ, η, c, B are considered for all mathematical models.

f. Inclusion of Dynamic Variations of ρ, η, c and B in Mathematical Models

The cumulative effect of dynamic variations of ρ, η, c and B as a function of varying pressure during CEUP fuel injection cycle has been accommodated in all the mathematical models by using the empirical formulas derived by Wang Jun-Xiao et al. [26]. These empirical formulas of density, viscosity, speed of sound and bulk modulus are reiterated in equations (9), (10), (11) and (12) respectively.

Predicted pressure $p(x,t)$ by each mathematical model at each mesh point is recalculated and corrected using these empirical formulas.

$$\rho(x,t) = \rho_0(x,t) \times (1 + \frac{0.6 \times 10^{-9} p(x,t)}{1 + (1.7 \times 10^{-9} p(x,t))}) \tag{9}$$

$$\eta(x,t) = \eta_0(x,t) \times \exp[(\ln \eta_0(x,t) + 9.67)\{(1 + 5.1 \times 10^{-9} p(x,t))^z \times (\frac{T - 138}{T_0 - 138})^{-s}) - 1\}] \tag{10}$$

$$a(x,t) = \frac{1 + 3.23 \times 10^{-9} p(x,t)}{\sqrt{0.69 \times 10^{-9} \rho_0}} \tag{11}$$

$$K(x,t) = \frac{[1 + 3.23 \times 10^{-9} p(x,t)][1 + 3.92 \times 10^{-9} p(x,t)]}{0.69 \times 10^{-9}} \tag{12}$$

In equations (9-12) ρ_0, η_0 are density and viscosity respectively at reference room temperature T_0 (293K). a and K are speed of sound and bulk modulus respectively whereas z and s are indexes between viscosity and pressure and viscosity and temperature respectively.

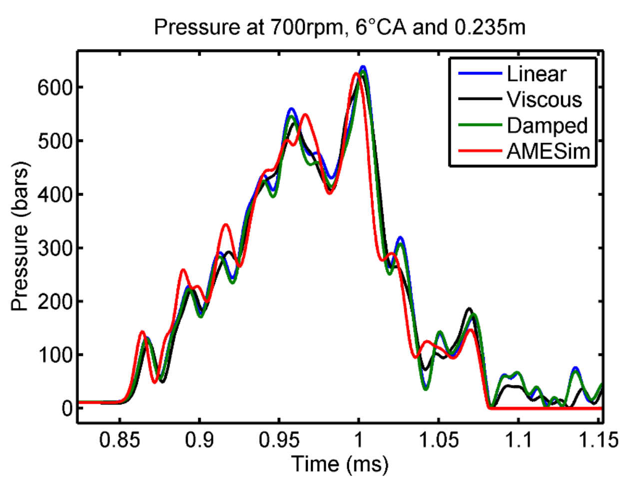
VI. SIMULATIONS AND DISCUSSIONS

All mathematical models are simulated at all combinations of operating conditions mentioned in Table 1. All simulated results of linear damped, viscous damped and damped models have been compared to AMESim numerical model for validation. The comparison show that predicted pressures by all mathematical models are quite coherent with those of AMESim numerical model as shown in figures 6-9. Simulated results in the middle of HP fuel pipeline length only have been presented and discussed.

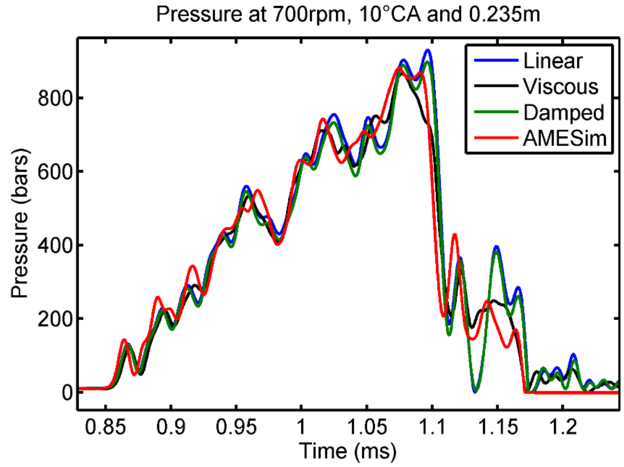
a. 700 RPM

Figures 6(a-c) show comparison of predicted pressures of linear damped, viscous damped and damped mathematical models with AMESim numerical results at 700 rpm and 6°CaA, 10°CaA and 14°CaA respectively.

The results show that the fluctuation in pressure amplitude increases with the increase of cam angle (°CaA) for all mathematical models. These fluctuations are more visible in linear damped and damped model as compared to viscous damped model at high pressures and towards the end of fuel injection cycle. For example, pressure amplitudes in linear and damped models vary sharply between 900-1200 bars at 14°CaA as shown in figure 6(c) as compared to viscous damped model. Similarly, same behavior has been observed towards the end of the fuel injection cycle between 1.12-1.17 ms at 10°CaA as shown in figure 6(b).



(a)



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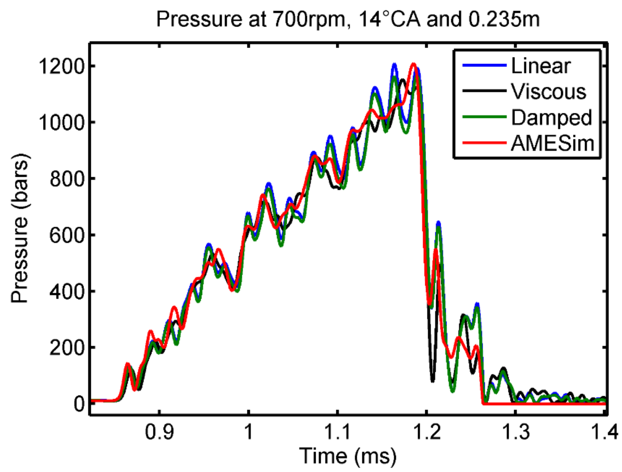


(c)

Figure 6. Comparison of Linear, Viscous, Damped Mathematical Models and AMESim Numerical Model at 700rpm and (a) 6°CaA (b) 10°CaA (c) 14°CaA

b. 900 RPM

Figures 7(a-c) show comparison of predicted pressures of linear damped, viscous damped and damped mathematical models with AMESim numerical results at 900 rpm and 6°CaA, 10°CaA and 14°CaA respectively.

An increased fluctuation in pressure amplitudes at high pressures and towards the end of fuel injection cycle has been observed in all mathematical models with the increase of cam angle from 6°CaA to 14°CaA as shown in figures 7(b) and 7(c).

Rates of pressure amplitude fluctuations with the increase of cam angle are nearly similar in all the mathematical models.

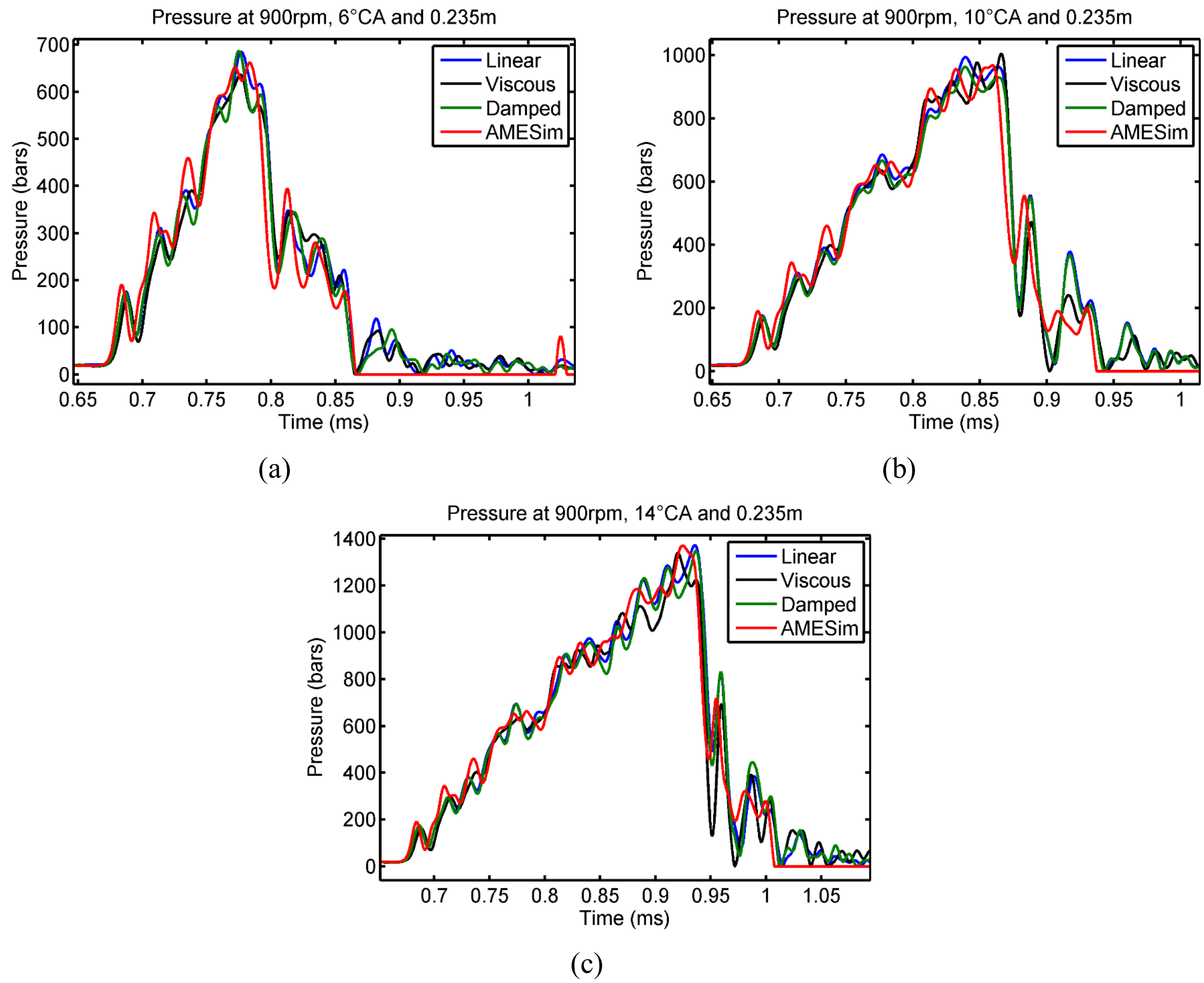


Figure 7. Comparison of Linear, Viscous, Damped Mathematical Models and AMESim Numerical Model at 900rpm and (a) 6°CaA (b) 10°CaA (c) 14°CaA

c. 1100 RPM

Figures 8(a-c) show comparison of predicted pressures of linear damped, viscous damped and damped mathematical models with AMESim numerical results at 1100 rpm and 6°CaA, 10°CaA and 14°CaA respectively.

At cam rotational speed of 1100rpm large fluctuations in pressure amplitudes have been observed at 6°CaA, 10°CaA and 14°CaA in all mathematical models as compared to similar cam angles at 700rpm and 900 rpm.

Mixed response of each model has been noted at different cam angles. For example pressure amplitude drop is recorded in viscous damped model at pressures and cam angles of ~700 bars and 6°CaA and 1200-1400 bars and 14°CaA as shown in figures 8(b) and 8(c) respectively. In addition, more fluctuation in pressure amplitudes towards the end of fuel injection cycle in viscous damped, linear damped and damped mathematical models are observed at 6°CaA, 10°CaA and 14°CaA as shown in figures 8(a), 8(b) and 8(c) respectively.

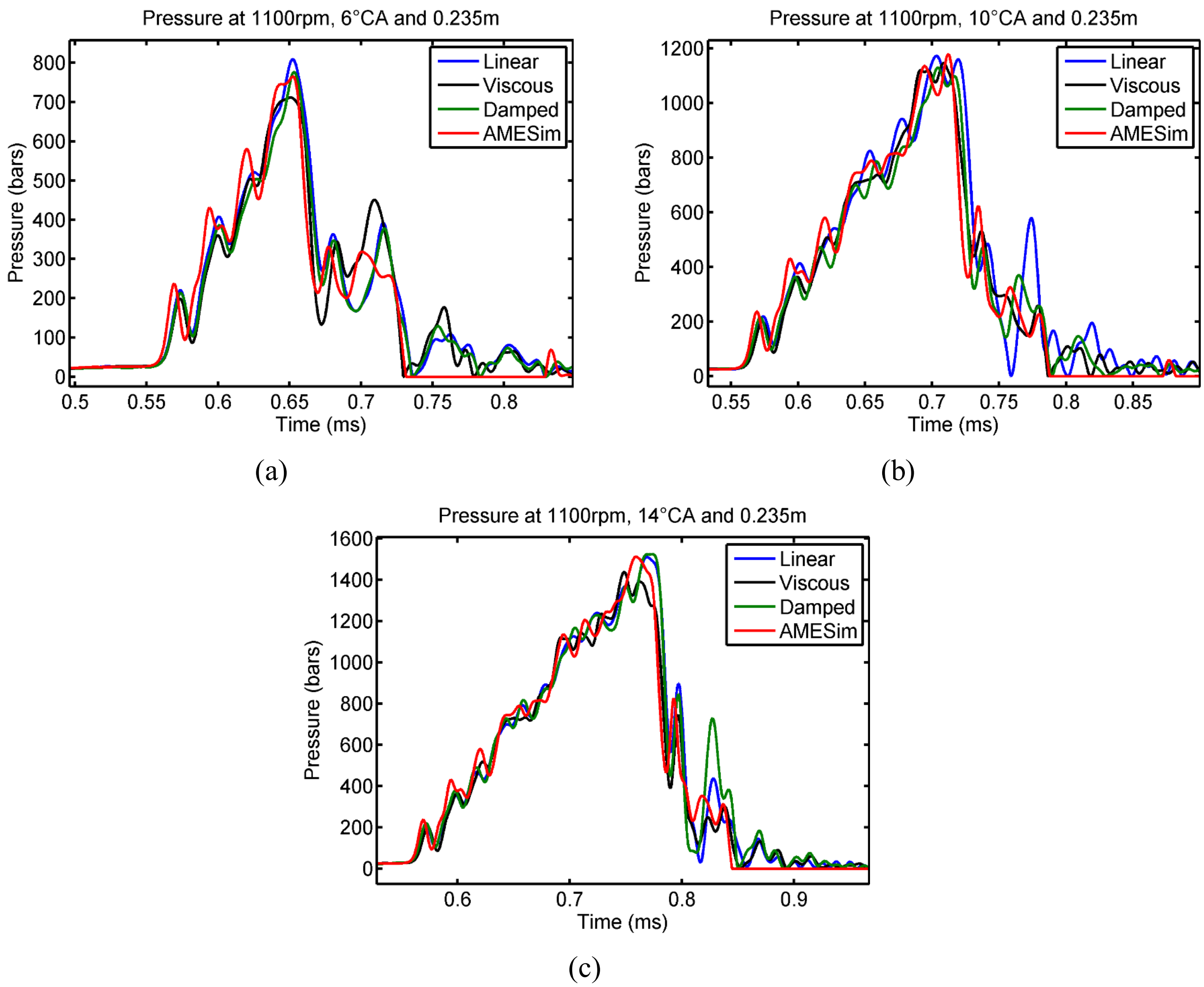
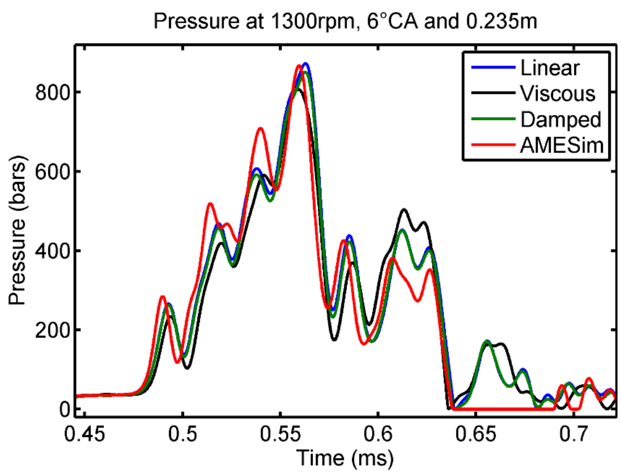


Figure 8. Comparison of Linear, Viscous, Damped Mathematical Models and AMESim Numerical Model at 1100rpm and (a) 6°CaA (b) 10°CaA (c) 14°CaA

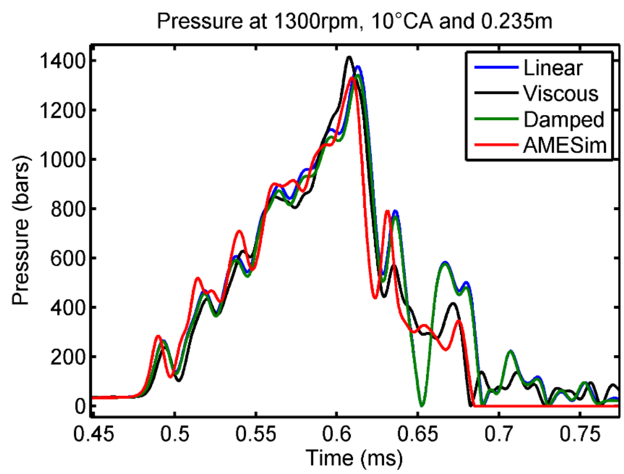
d. 1300 RPM

Figures 9(a-c) show comparison of predicted pressures of linear damped, viscous damped and damped mathematical models with AMESim numerical results at 1300 rpm and 6°CaA, 10°CaA and 14°CaA respectively.

Largest of the fluctuations in pressure amplitudes towards the end of fuel injection cycle are observed at cam rotational speed of 1300rpm and especially at high cam angle of 14°CaA.



(a)



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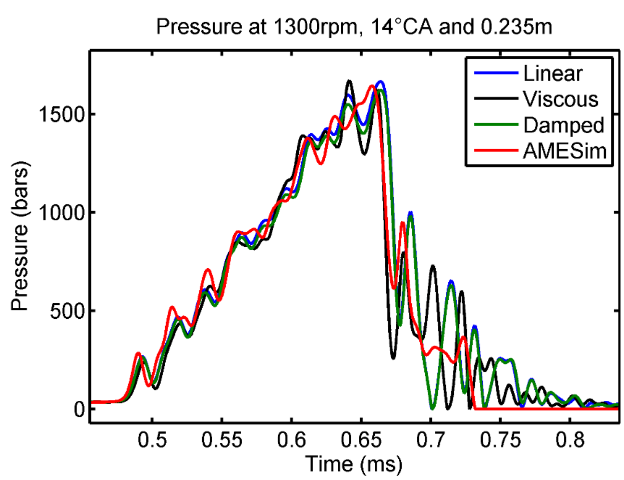


(c)

Figure 9. Comparison of Linear, Viscous, Damped Mathematical Models and AMESim Numerical Model at 1300rpm and (a) 6°CaA (b) 10°CaA (c) 14°CaA

Decrease and increase in pressure amplitudes at cam angles of 6°CaA and time between 0.53-0.55 ms and 0.61-0.53 ms respectively is recorded for all mathematical models as shown in figure 9(a). More pressure fluctuations are recorded towards the end of fuel injection cycle at cam angle of 10°CaA and time between 0.64 and 0.68 ms for linear and damped mathematical models as compared to viscous damped model. Increased fluctuations in pressure amplitudes at high pressures around 1300-1500 bars and phase change and pressure amplitude variations towards the end of fuel injection cycle around 0.69-0.73 ms are observed for all mathematical models as shown in figure 9(c).

VII. QUANTITATIVE COMPARISON

Simulated results of all mathematical models have also been compared quantitatively at all combinations of operating conditions mentioned in Table 1. using model evaluation statistical techniques like RMSE and IA [30].

a. ROOT MEAN SQUARE ERROR (RMSE)

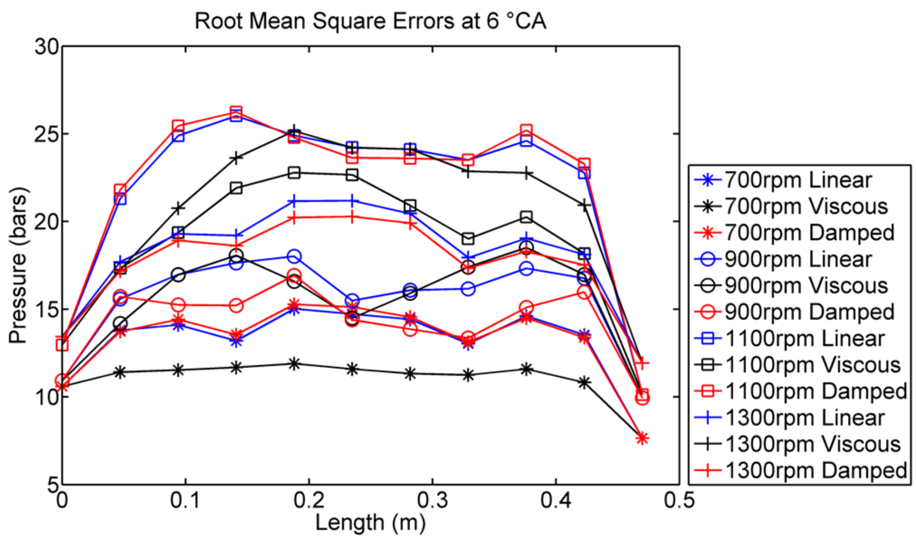
RMSE is a model evaluation technique which indicates error in model and helps in analysis of the model results. Model with lower RMSE model is accepted as a better one [30]. RMSE values have been calculated by using equation (13).

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n}(P_{AMESim,i} - P_{Model,i})^2}{n}} \tag{13}$$

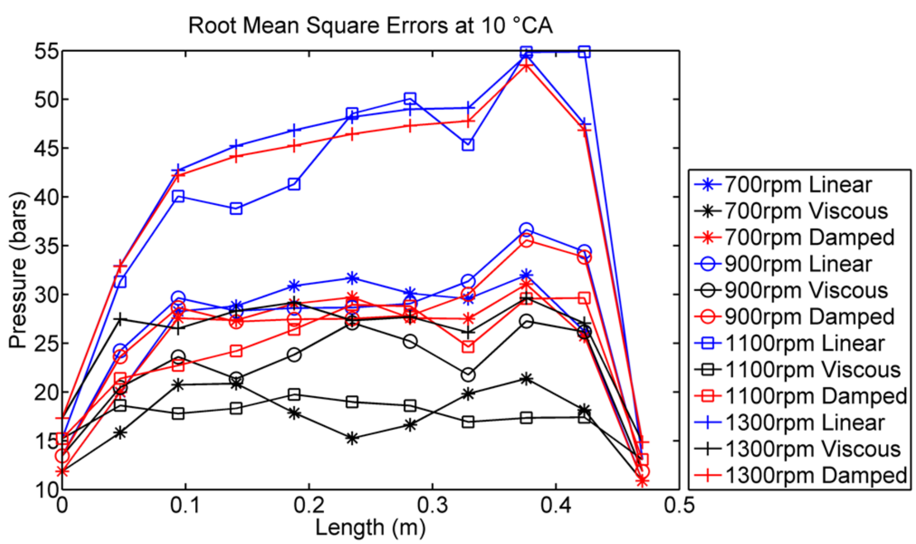
Where $P_{AMESim,i}$, $P_{Model,i}$ and n are AMESim simulated pressure, predicted pressure by mathematical model and number of elements to be compared respectively.

Figures 10(a-c) show RMSE of linear, viscous and damped mathematical models when compared to AMESim numerical model at 6°CaA, 10°CaA and 14°CaA respectively at 10 equidistant locations along the HP fuel pipeline. Maximum errors in simulated results of mathematical models in terms of RMSEs have also been summarized in Table 2. It has been observed that mathematical models are more accurate i.e. with lower RMSEs at low rpm and low °CaA.

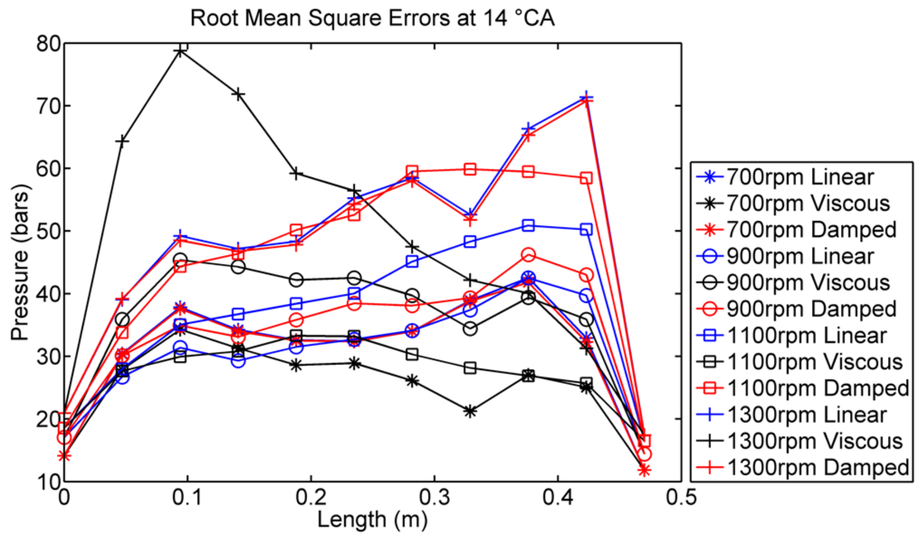
At cam angle of 6°CaA viscous damped model has lower RMSEs as compared to other models at 700rpm and 1100rpm whereas damped model has lower RMSEs at 900rpm and 1300rpm as shown in figure 10(a). Moreover at cam angle of 10°CaA viscous damped model has lower RMSEs at all cam rotational speeds as shown in figure 10(b). Whereas at cam angle of 14°CaA viscous damped model has lower RMSEs at 700rpm and 1100rpm, linear damped model is more accurate at 900rpm and damped model is more accurate as compared to other models at 1300rpm as shown in figure 10(c).



(a)



(b)



(c)

Figure 10. RMSEs of Linear, Viscous and Damped Mathematical Models as compared to AMESim Numerical Model at (a) 6°CaA (b) 10°CaA (c) 14°CaA

Table 2: Maximum RMSEs at Various Operating Conditions

Mathematical Models	Operating Conditions											
	700rpm			900rpm			1100rpm			1300rpm		
	6°CA	10°CA	14°CA	6°CA	10°CA	14°CA	6°CA	10°CA	14°CA	6°CA	10°CA	14°CA
Linear Damped	15.02	31.97	42.52	18.01	36.64	42.49	26.02	54.87	50.88	21.18	54.52	71.36
Viscous Damped	11.89	21.36	34.27	18.50	27.26	45.38	22.78	19.74	33.29	25.15	29.62	78.77
Damped Model	15.27	31.04	41.80	16.92	35.57	46.23	26.23	29.64	59.87	20.28	53.50	70.76

b. INDEX OF AGREEMENT (IA)

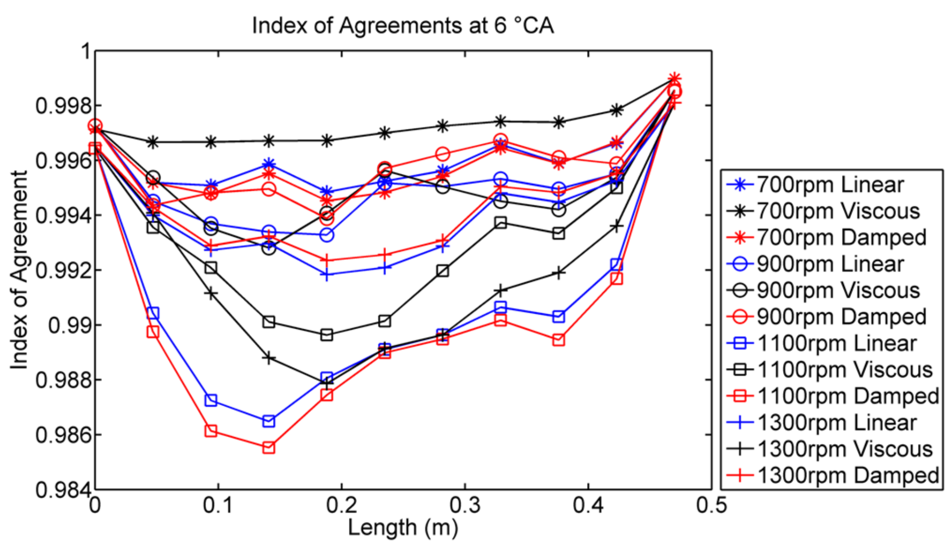
IA is also a model evaluation technique and indicates the degree of model prediction error. IA ranges from 0 to 1 where 1 indicating a perfect match and 0 indicating no agreement at all [30]. IA values have been calculated by using equation (14).

$$IA = 1 - \left[\frac{\sum_{i=1}^{n} \left(P_{AMESim,i} - P_{Model,i} \right)^2}{\sum_{i=1}^{n} \left(abs\left(P_{Model,i} - mean(P_{AMESim,i}) \right) + abs\left(P_{AMESim,i} - mean(P_{AMESim,i}) \right) \right)^2} \right] \tag{14}$$

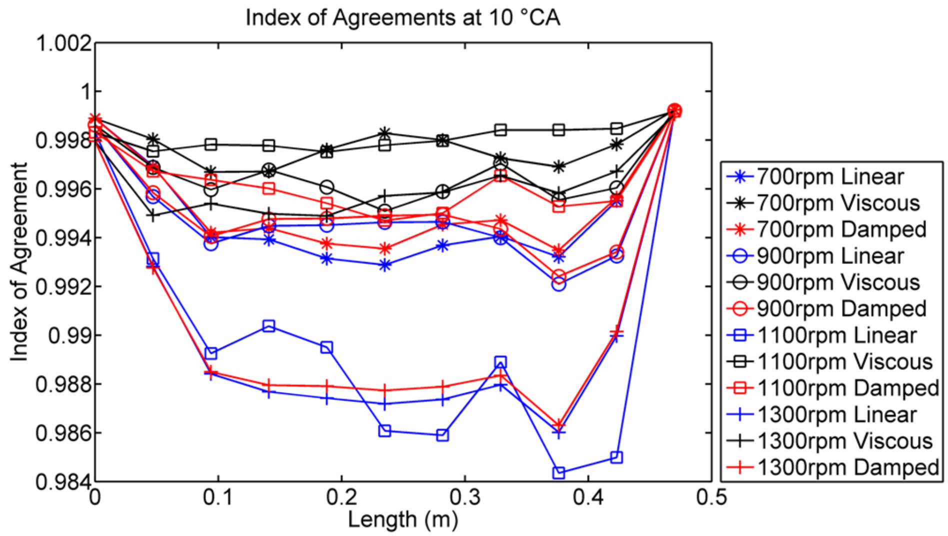
Where $P_{AMESim,i}$, $P_{Model,i}$ and n are AMESim simulated pressure, predicted pressure by mathematical model and number of elements to be compared respectively.

Figures 11(a-c) show IA of linear, viscous and damped mathematical models when compared to AMESim numerical model at 6°CaA, 10°CaA and 14°CaA respectively at 10 equidistant locations along the HP fuel pipeline. A minimum of 0.981 IA among all combinations of operating conditions and models indicates that degrees of all models predictions are quite high. Minimum IAs have also been summarized in Table 3.

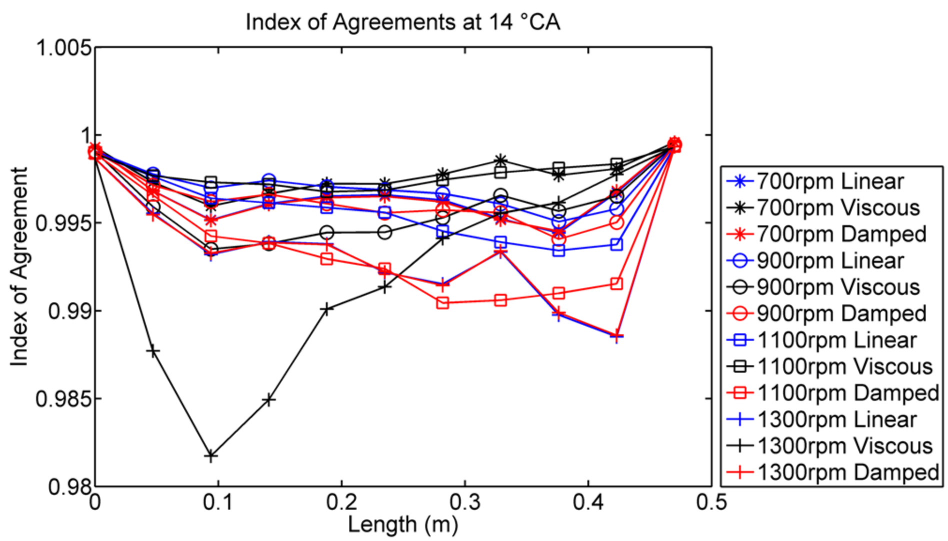
At cam angle of 6°CaA viscous damped model has higher IAs as compared to other models at 700rpm and 1100rpm whereas damped model has higher IAs at 900rpm and 1300rpm as shown in figure 11(a). Moreover at 10°CaA viscous damped model has higher IAs at all cam rotational speeds as shown in figure 11(b). Whereas at 14°CaA viscous damped model has higher IAs at 700rpm and 1100rpm, linear damped model has less error at 900rpm and damped model has less error as compared to other models at 1300rpm as shown in figure 11(c).



(a)



(b)



(c)

Figure 11. IAs of Linear, Viscous and Damped Mathematical Models as compared to AMESim Numerical Model at (a) 6°CaA (b) 10°CaA (c) 14°CaA

Table 3: Minimum IAs at Various Operating Conditions

Mathematical Models	**Operating Conditions**											
	700rpm			**900rpm**			**1100rpm**			**1300rpm**		
	6°CA	**10°CA**	**14°CA**	**6°CA**	**10°CA**	**14°CA**	**6°CA**	**10°CA**	**14°CA**	**6°CA**	**10°CA**	**14°CA**
Linear Damped	0.994	0.992	0.994	0.993	0.992	0.995	0.986	0.984	0.993	0.991	0.986	0.988
Viscous Damped	0.996	0.996	0.995	0.992	0.995	0.993	0.989	0.997	0.996	0.987	0.994	0.981
Damped Model	0.994	0.993	0.994	0.993	0.992	0.994	0.985	0.994	0.990	0.992	0.986	0.988

VIII. CONCLUSIONS

Three different 1D mathematical models of damped wave equation namely linear damped, viscous damped and damped have been developed in MATLAB to investigate the fuel pressure inside high pressure (HP) fuel pipeline of Combination Electronic Unit Pump (CEUP) fuel injection system of diesel engine at various operating conditions of diesel engine.

Experiments have been carried out in lab to measure pump and injector side pressures using KISTLER 4067 piezoresistive pressure sensors. Pressure predictions of all mathematical models inside HP fuel pipeline have been validated at various operating conditions by comparing them with those of an experimentally validated AMESim numerical model of CEUP system. Predictions of all mathematical models are quite coherent.

All mathematical results have also been quantitatively analyzed and compared by using model evaluation statistical techniques like Root Mean Square Error (RMSE) and Index of Agreement (IA).

At cam angle of 6°CaA and cam rotational speeds of 700rpm and 1100rpm viscous damped model predicts more accurately as compared to other models whereas damped model predicts more accurately at cam angle of 6°CaA and cam rotational speeds of 900rpm and 1300rpm. Moreover at cam angle of 10°CaA viscous damped model is relatively more accurately at all cam rotational speeds than rest of the models. Whereas at cam angle of 14°CaA viscous damped model has predicted more accurately at 700rpm and 1100rpm, linear damped model has predicted more accurately at 900rpm and damped model has predicted more accurately as compared to other models at 1300rpm

IX. ACKNOWLEDGEMENT

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