www.s2is.org
International Journal on Smart Sensing and Intelligent Systems
ISSN 1178-5608

OPTIMAL TECHNIQUES FOR SENSING ERROR MINIMIZATION WITH IMPROVED ENERGY DETECTION IN COGNITIVE RADIOS

K.Muthumeenakshi and S.Radha
SSN College of Engineering
Kalavakkam, Chennai
Tamil Nadu, India
Emails: muthumeenakshik@ssn.edu.in, radhas@ssn.edu.in

Submitted: Aug. 30, 2014 ***Accepted: Nov. 2, 2014*** ***Published: Dec. 1, 2014***

Abstract- Spectrum sensing, the problem of detecting the presence of licensed user in the channel is considered in this paper. Energy detection is best suited for the spectrum sensing when prior knowledge about the primary users is unavailable. Existing works report improved versions of energy detection which primarily focuses on maximizing the detection performance. Sensing error minimization is an important aspect of spectrum sensing that needs attention. This paper focuses on the sensing error minimization of the improved energy detection algorithm in which the decision statistic is computed using an arbitrary positive index instead of squaring operation. First, an optimum decision threshold satisfying Minimum Error Bound (MEB) is derived. Next, an optimum value of the arbitrary positive index with minimum number of samples satisfying a Target Error Bound (TEB) is derived. A thorough numerical analysis and simulations are performed and the results confirm the accuracy of the analysis.

Index terms: Spectrum sensing, Cognitive radio, Energy detection, Threshold optimization, Sensing error.

I. INTRODUCTION

Recent studies on wireless spectrum demonstrate that the wireless communication systems suffer from spectrum scarcity and inefficient spectrum usage. This observation leads to the reformation in the static spectrum assignment policies by the Federal Communications Commission (FCC) [1]. The new policy schemes would allow the vacant portions of the licensed spectrum bands to be used by the unlicensed users (called secondary users) without causing interference to the licensed users (called primary users). Cognitive radio, identified as a novel paradigm is anticipated to make this policy reformation successful.

Cognitive radios are devices that can alter its transmission / reception parameters based on the changes monitored in the environment and identify opportunities to transmit data when the incumbent is not using [2]. To achieve this intelligent functionality, cognitive radios employ a key enabling technology called spectrum sensing. Spectrum sensing techniques enable the cognitive radio to find the best available spectrum bands. The important challenge of spectrum sensing is to reliably detect the presence of primary users and not to cause harmful interference to them. A number of techniques have been proposed in the literature for spectrum sensing. Energy detection [3], Matched filter detection [4], cyclostationary feature detection [5], covariance based detection [6], Eigen value based detection [7], detection using wavelets [8], correlation based detection [9] and filter bank spectrum estimation [10] are few among them. Among these methods, energy detection is a simple and non coherent technique which compares the energy of the received signal with a pre-evaluated threshold. The performance of the existing techniques provide different trade-offs between detection accuracy, sensing time and computational complexity. But the practical applicability of these techniques depends very much on the information available about the primary signals. Energy detection is the most preferred approach for spectrum sensing when the CR is unable to gather sufficient information about the primary user signals.

The original energy detector was proposed in [11] for an unknown deterministic signal assuming a flat band-limited Gaussian channel. Recently, the energy detector proposed in [11] is being used extensively for CR spectrum sensing owing to its simplicity and lesser computational requirements. In [12], energy detection is analyzed theoretically for AWGN, Rayleigh and Nakagami channel models and expressions for the detection probability are obtained. The secondary user spectrum sensing - throughput problem is analyzed in [13]. Energy detection sensing is used and an optimal sensing time which maximizes the secondary user throughput is identified. The authors proved that for a 6 MHz channel, when the frame duration is 100ms and at 90% detection probability, the optimum sensing time is 14.2 ms. In [14], the authors proposed a blindly combined energy detection technique which does not require any information about the primary signal. The authors validated the proposed technique using wireless microphone signals and randomly generated signals and proved that their method outperforms energy detection for highly correlated signals. A detailed review of sensing algorithms and various approaches to distributed detection techniques for cognitive radio was discussed in [15].

An adaptive threshold based energy detection suitable for time varying nature of the wireless channel and primary user activities is proposed in [16]. The authors of [17] put forward the Barlett's estimate as the decision statistic for energy detection. The authors investigated the performance for unknown signals under Rayleigh and Rician fading channels. The accuracy of their method is also compared with periodogram technique and found to achieve low miss detection probability. But their technique is able to achieve low false alarm only for higher detection threshold. In [18], an energy detection based spectrum sensing is performed using Welch periodogram technique. The authors observe improved performance if the parameters of the Welch periodogram are included in the distribution of the decision statistic. They also observed that improved detection performance is achieved at the expense of increased false alarm probability under noise uncertainty. In [19], an improved version of energy detection algorithm is proposed for spectrum sensing. The improved detection scheme initially employs the traditional energy detection algorithm and confirms with additional verifications to avoid any missed detection due to instantaneous energy drops and improves the detection performance. The authors analyzed the computational complexity of the improved energy detection algorithm and found to be similar to that of the traditional energy detection algorithm. Another approach to improve the traditional energy detection algorithm is proposed in [20] and [21]. The algorithm computes an arbitrary positive power operation on the received signal to compute the decision statistic instead of squaring operation and showed better performance. The authors of [22] define a formal measure for the utilization of spectrum holes and a new adaptive sensing duration for energy detection based spectrum sensing is proposed. By dynamically changing the sensing duration, the authors ensure that more transmission time is available for the secondary users thereby improving their throughput.

Apart from these specific techniques, many hybrid detectors are also proposed which combines the advantages of two or more sensing techniques discussed above, but at the expense of increased complexity [23]. Detection performance is also well studied in the context of wireless sensor networks where target detection is a major concern. Threshold based detection techniques and error probability analysis for a non binary fault tolerant event detection for a sensor network is proposed in [24].

Most of the existing contributions on energy detection based spectrum sensing focus on maximizing the probability of detection by considering the detection problem in the context of Neyman Pearson. However, the fundamental requirement of any spectrum sensing algorithm is not to cause harmful interference to the incumbent. Hence, it is inevitable to minimize the total error probability of the cognitive radio. Thus, we analyze the improved energy detection algorithm proposed in [20] in terms of the arbitrary positive index, decision threshold and the number of samples with MEB and TEB as the design objective. Two techniques are proposed to minimize the total probability of error. In the first technique, an optimum decision threshold for the improved energy detection algorithm satisfying the MEB criterion is identified. The second technique proposes the improved energy detection with minimum number of samples satisfying a TEB criterion. The significant contributions addressed in this paper are thus summarized:

- The total probability of error for the improved energy detection algorithm is derived.
- The existence of optimum decision threshold satisfying a MEB is identified by simulations and the theoretical expression for the optimum decision threshold is derived. Further, the best choice of the arbitrary positive power used for the computation of decision statistic is found numerically by simulations.
- The minimum number of samples required to achieve a TEB is derived. The existence of an optimum value of the arbitrary positive power which minimizes the minimum number of samples is identified.
- The optimum value of the arbitrary positive power which minimizes the minimum number of samples is derived and verified numerically by simulations.

II. SPECTRUM SENSING PRELIMINARIES

The cognitive radio spectrum sensing is a binary hypothesis testing problem and can be formulated as follows.

$$y(n) = \begin{cases} w(n) & ; H_0 \\ s(n) + w(n) & ; H_1 \end{cases} \qquad n = 1,2,....N \tag{1}$$

where hypothesis H_0 denotes the absence of the primary user and hypothesis H_1 denotes the presence of the primary user. $y(n)$ is the signal received at the secondary receiver, $w(n)$ is the AWGN of variance σ_w^2 and $s(n)$ is the primary user signal assumed to be real Gaussian with variance σ_s^2. Moreover, $s(n)$ and $w(n)$ are assumed to be independent and the noise power is known a priori.

2.1 PERFORMANCE MEASURES FOR SPECTRUM SENSING

Ideally any spectrum sensing algorithm should select H_0 when the primary user is absent and H_1 when it is present. Practically, spectrum sensing algorithms are prone to errors and their performance depends on various factors such as the decision threshold, received SNR, *M*, channel conditions etc., The important performance measures used to evaluate the quality of sensing are discussed below:

- Probability of false alarm (P_f) : It is defined as the probability that the spectrum sensing algorithm declares that H_1 is true, when the primary user is actually absent. From the secondary user perspective, increase in false alarm will reduce the spectrum opportunities for them. Therefore, it is important to control the probability of false alarm for efficient secondary user spectrum utilization.
- Probability of miss detection (P_{md}): It is defined as the probability that the spectrum sensing algorithm declares that H_0 is true, when the primary user is present. From the primary user perspective, increase in miss detection will increase the interference caused

to them. Therefore, it is important to control the miss detection probability to avoid the collisions between the primary and secondary users. The complementary probability of miss detection is known to as probability of detection (P_d).

- SNR: The SNR of the received signal at the secondary user depends on the channel environment and the transmitted power of the primary user. The quality of detection improves with increase in SNR. A primary requirement of a spectrum sensing algorithm is the reliable detection of primary user signal in the low SNR regime.
- Sensing duration: Sensing duration is a very important parameter upon which the duration of secondary data transmission depends. Shorter the sensing duration, higher the secondary user throughput. However, the accuracy of the spectrum sensing algorithm also depends on the sensing duration. Hence it is desirable to achieve high performance in a short sensing duration.
- Complexity: The detection algorithms should be simple, easy to implement and should not be complex. The complexity analysis is also important for any spectrum sensing algorithm.

Based on the performance measures, a sensing algorithm is analyzed using ROC curves, SNR performance and complexity analysis which are briefly described below.

- ROC curves: ROC curve is a plot of probability of detection against probability of false alarm for varying algorithm parameters. The ROC curvature determines the detection accuracy of the algorithm. The area under an ideal ROC curve is unity which means the detection is 100% accurate. If the ROC curve moves toward the 45° diagonal of the ROC space, the detection accuracy deteriorates. Therefore, it is desirable for a good sensing algorithm to have the ROC curve closer towards the ideal curve. The ROC analysis is very important to select the optimal design parameters for the detection algorithm.
- SNR performance: The SNR performance is the plot of total probability of sensing error against increasing SNR. The total probability of sensing error is the sum of the false alarm probability and miss detection probability. It is desirable for any sensing algorithm to have a low sensing error probability for any value of SNR.
- Complexity analysis: A sensing algorithm with low complexity is always desirable. The algorithm should be designed such that it achieves high detection probability with minimum number of samples.

2.2 ENERGY DETECTION ALGORITHM

The conventional energy detector which uses squaring operation to compute the test statistic is given by

$$Y = \frac{1}{N}\sum_{n=1}^{N}\left|\frac{y(n)}{\sigma_w}\right|^2 \tag{2}$$

where Y is the test statistic and N is the number of samples used for computation. The test statistic Y is compared with a pre-evaluated threshold λ. If $Y \geq \lambda$, the decision is hypothesis H_0, otherwise hypothesis H_1. From [21] the probability density function of Y is expressed as,

$$f_Y(y) = \begin{cases} \frac{1}{2^{N/2}\Gamma(N/2)} Y^{\frac{N}{2}-1} e^{-Y/2} & ; H_0 \\ \frac{1}{2}\left(\frac{Y}{\alpha}\right)^{(N-2)/4} e^{-(Y+\alpha)/2} I_{\frac{N}{2}-1}\left(\sqrt{Y\alpha}\right) & ; H_1 \end{cases} \tag{3}$$

where $\alpha = \frac{\sigma_s^2}{\sigma_w^2}$ is the SNR, $\Gamma(.)$ is the complete Gamma function and $I_m(.)$ is the m^{th} order Bessel function of the first kind. Using central limit theorem, as N increases the test statistic approximately follow the normal distribution. The probability density function of Y is then given by,

$$f_Y(y) = \begin{cases} \frac{1}{\sqrt{2N}\sqrt{2\pi}} e^{-\frac{(Y-N)^2}{2(2N)}} & ; H_0 \\ \frac{1}{\sqrt{2N(1+\alpha)}\sqrt{2\pi}} e^{-\frac{(Y-N(1+\alpha))^2}{2(2N(1+\alpha))}} & ; H_1 \end{cases} \tag{4}$$

The error probabilities are defined as follows: The probability of missed detection, $P_{md} = P(H_0/H_1)$ and the probability of false alarm, $P_f = P(H_1/H_0)$. The complementary probability of missed detection is denoted as $P_d = P(H_1/H_1) = 1 - P_{md}$. It is required to have large P_d and low P_f for any spectrum sensing algorithm. However, there exists a trade-off between the two values. To depict the relationship between the two values, ROC curves are useful and expressed as,

$$P_d = Q\left(\frac{\sqrt{2N} Q^{-1}\left(P_f\right) - N\alpha}{\sqrt{2N(1+\alpha)}} \right) \tag{5}$$

The total error probability depends on the values of P_d, P_f and the probability of occurrence of H_0 and H_1. It is denoted by P_e and expressed as,

$$P_e = (1-P)P_f + P(1-P_d) \tag{6}$$

where P is the probability of occurrence of the primary user, i.e., $P = P(H_1)$ and $1-P = P(H_0)$.

III. IMPROVED ENERGY DETECTION

An improved version of the energy detector proposed in [21] makes use of an arbitrary positive index p to compute the test statistic instead of squaring operation. The decision statistic of the improved energy detector Y with p^{th} power summer is given by,

$$Y = \frac{1}{N}\sum_{n=1}^{N}\left|\frac{y(n)}{\sigma_w^2}\right|^p \underset{H_0}{\overset{H_1}{\gtrless}} \lambda_m \tag{7}$$

where λ_m is the modified decision threshold. For p = 2, the improved energy detection becomes the traditional energy detection. For any p, $\left|y(n)/\sigma_w\right|^p$ are independent and identically distributed random variables. Using [20], the mean and variance of $\left|y(n)/\sigma_w\right|^p$ is given by,

Under hypothesis H_0:

$$\mu_0 = \frac{2^{p/2}}{\sqrt{\pi}}\Gamma\left(\frac{p+1}{2}\right) \tag{8}$$

$$\sigma_0^2 = \frac{2^p}{\sqrt{\pi}}\left[\Gamma\left(\frac{2p+1}{2}\right) - \frac{1}{\sqrt{\pi}}\Gamma^2\left(\frac{p+1}{2}\right)\right] \tag{9}$$

Under hypothesis H_1:

$$\mu_1 = \frac{2^{p/2}(1+\alpha)^{p/2}}{\sqrt{\pi}}\Gamma\left(\frac{p+1}{2}\right) \tag{10}$$

$$\sigma_1^2 = \frac{2^p(1+\alpha)^p}{\sqrt{\pi}}\left[\Gamma\left(\frac{2p+1}{2}\right) - \frac{1}{\sqrt{\pi}}\Gamma^2\left(\frac{p+1}{2}\right)\right] \tag{11}$$

As the random variables $\left|y(n)/\sigma_w\right|^p$ follow normal distribution, the decision statistic also follow normal distribution with mean and variance values given by,

$$E(Y)=\begin{cases}\mu_0 & ;H_0\\ \mu_1 & ;H_1\end{cases} \tag{12}$$

$$Var(Y)=\begin{cases}\sigma_0^2/N & ;H_0\\ \sigma_1^2/N & ;H_1\end{cases} \tag{13}$$

The probability density of the decision statistic is hence given by,

$$f_Y(y)=\begin{cases}\dfrac{1}{\sqrt{2\pi\dfrac{\sigma_0^2}{N}}}e^{-\frac{(Y-\mu_0)^2}{2\frac{\sigma_0^2}{N}}} & ;H_0\\ \dfrac{1}{\sqrt{2\pi\dfrac{\sigma_1^2}{N}}}e^{-\frac{(Y-\mu_1)^2}{2\frac{\sigma_1^2}{N}}} & ;H_1\end{cases}$$

The expressions for the corresponding false alarm and detection probability are given by,

$$P_f=\text{Prob}\left(Y>\lambda_m/H_0\right)$$

$$P_f=\int_{\lambda_m}^{\infty} f_Y(y)/H_0\,dy$$

$$=\int_{\lambda_m}^{\infty}\frac{1}{\sqrt{2\pi\frac{\sigma_0^2}{N}}}e^{-\frac{(Y-\mu_0)^2}{2\frac{\sigma_0^2}{N}}}dy$$

$$P_f=Q\left(\frac{\lambda_m-\mu_0}{\sigma_0/\sqrt{N}}\right) \tag{14}$$

and

$$P_d=\text{Prob}\left(Y>\lambda_m/H_1\right)$$

$$=\int_{\lambda_m}^{\infty}\frac{1}{\sqrt{2\pi\frac{\sigma_1^2}{N}}}e^{-\frac{(Y-\mu_1)^2}{2\frac{\sigma_1^2}{N}}}dy$$

$$P_d = Q\left(\frac{\lambda_m - \mu_1}{\sigma_1 / \sqrt{N}}\right) \tag{15}$$

Now, the total probability of sensing error is given by,

$$P_e = (1-P)P_f + P(1-P_d)$$

$$P_e = (1-P)Q\left(\frac{\lambda_m - \mu_0}{\sigma_0 / \sqrt{N}}\right) + P\left(1 - Q\left(\frac{\lambda_m - \mu_1}{\sigma_1 / \sqrt{N}}\right)\right) \tag{16}$$

IV. OPTIMUM THRESHOLD FOR IMPROVED ENERGY DETECTION SATISFYING MEB

In the proposed algorithm, we minimize the total error probability by optimizing the decision threshold. From (16), the total probability of error P_e depends on λ_m, N, P and p and the received SNR. The variation of P_e with respect to the decision threshold can be observed in Figure 1. It is clear that there exists an optimum threshold for which the probability of error attains a minimum value for any fixed p, N, P and α. For different values of p, the optimal λ_m and the value of P_e at the optimum λ_m are different. This shows that the value of p plays a vital role in further minimizing P_e. Thus it is possible to find an optimum value of threshold for a particular p value for which P_e is minimum. In the following the expression for the optimal decision threshold is derived analytically.

The optimum value of the decision threshold satisfying a MEB is given by,

$$\lambda_m^* = \arg\min_{\lambda_m}(P_e)$$

The above equation solved by setting $\frac{\partial P_e}{\partial \lambda_m} = 0$.

$$\frac{\partial}{\partial \lambda_m}\left((1-P).Q\left(\frac{\lambda_m - \mu_0}{\sigma_0 / \sqrt{N}}\right) + P.\left(1 - Q\left(\frac{\lambda_m - \mu_1}{\sigma_1 / \sqrt{N}}\right)\right)\right) = 0$$

$$\left((1-P).\frac{\partial}{\partial \lambda_m}Q\left(\frac{\lambda_m - \mu_0}{\sigma_0 / \sqrt{N}}\right) + P.\left(-\frac{\partial}{\partial \lambda_m}Q\left(\frac{\lambda_m - \mu_1}{\sigma_1 / \sqrt{N}}\right)\right)\right) = 0$$

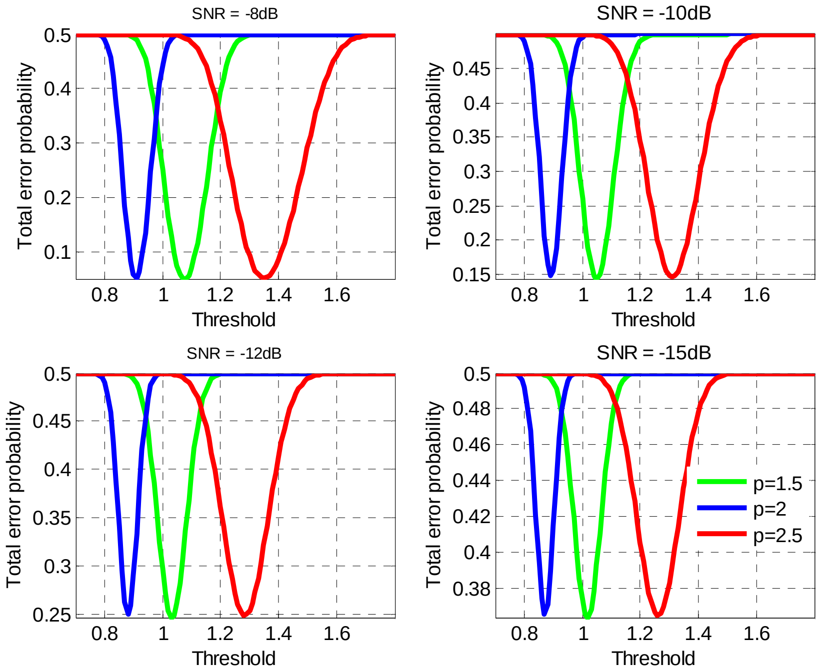

Figure 1. Total probability of sensing error against decision threshold for different fixed values of *p* and SNR, (*N*=1000, P =0.45).

We have $\frac{d}{dx}Q(x)=\frac{1}{\sqrt{2\pi}}e^{-x^2/2}$, then,

$$\left((1-P).e^{\frac{-\left(\frac{\lambda_m-\mu_0}{\sigma_0/\sqrt{N}}\right)^2}{2}}\left(\frac{\sqrt{N}}{\sigma_0}\right)+P.e^{\frac{-\left(\frac{\lambda_m-\mu_1}{\sigma_1/\sqrt{N}}\right)^2}{2}}\left(\frac{\sqrt{N}}{\sigma_1}\right)\right)=0$$

Solving, we get,

$$\lambda_m^*=\frac{-2\left(\frac{\mu_0}{\sigma_0^2}-\frac{\mu_1}{\sigma_1^2}\right)\pm\sqrt{4\left(\frac{\mu_0}{\sigma_0^2}-\frac{\mu_1}{\sigma_1^2}\right)^2-4\left(\frac{1}{\sigma_1^2}-\frac{1}{\sigma_0^2}\right)\left[\left(\frac{\mu_1^2}{\sigma_1^2}-\frac{\mu_0^2}{\sigma_0^2}\right)-2\ln\left(\frac{P}{1-P}\frac{\sigma_0}{\sigma_1}\right)\right]}}{2\left(\frac{1}{\sigma_1^2}-\frac{1}{\sigma_0^2}\right)} \tag{17}$$

Equation (17) gives the optimum value of the threshold which minimizes the probability of error over AWGN channels. The best choice of the optimum threshold for which the P_e is minimum can be chosen using (16). Equation (17) depicts that the optimum threshold depends mainly on the mean and variance of the two hypotheses. These values further depend on the arbitrary positive index *p*. Thus, the solution for λ_m and the corresponding *p*, which satisfies the MEB can be analyzed numerically and presented in section VI. It can be concluded that the proposed algorithm with optimum threshold is well suited for potential applications of cognitive radio requiring maximum accuracy like health care and military surveillance.

V. OPTIMUM '*p*' SATISFYING TEB

It is sometimes required to conduct spectrum sensing with minimum sensing time. For time bound applications where sensing duration is a critical parameter, it is necessary to have minimum number of samples for signal detection. A TEB is specified and the minimum number of samples required for sensing is computed. Then the optimum value of *p* minimizing the minimum number of samples to achieve the TEB is derived. Let the target probability of error be denoted as $\hat{P}_e$. The target probability of false alarm be denoted as $\hat{P}_f$. Then, from (16) we have,

$$\hat{P}_e = (1-P)\hat{P}_f + P\left(1 - Q\left(\frac{\lambda_m - \mu_1}{\sigma_1/\sqrt{N}}\right)\right) \tag{18}$$

$$\frac{\mu_0 + \frac{\sigma_0}{\sqrt{N}} Q^{-1}\left(\hat{P}_f\right) - \mu_1}{\sigma_1/\sqrt{N}} = Q^{-1}\left(1 - \left(\frac{\hat{P}_e - (1-P)\hat{P}_f}{P}\right)\right)$$

$$\frac{\sqrt{N}}{\sigma_1}\left[(\mu_0 - \mu_1) + \frac{\sigma_0}{\sqrt{N}} Q^{-1}\left(\hat{P}_f\right)\right] = Q^{-1}\left(1 - \frac{\hat{P}_e - (1-P)\hat{P}_f}{P}\right)$$

$$\frac{\sqrt{N}}{\sigma_1}(\mu_0 - \mu_1) + \frac{\sigma_0}{\sigma_1} Q^{-1}\left(\hat{P}_f\right) = Q^{-1}\left(1 - \frac{\hat{P}_e - (1-P)\hat{P}_f}{P}\right)$$

Solving,

$$N_{\min} = \left\lceil \left(\frac{\sigma_1}{(\mu_0 - \mu_1)} \left(Q^{-1}\left(1 - \frac{\hat{P}_e - (1-P)\hat{P}_f}{P}\right) - \frac{\sigma_0}{\sigma_1} Q^{-1}\left(\hat{P}_f\right) \right) \right)^2 \right\rceil \tag{19}$$

The minimum number of samples required to achieve a target probability or error depends on the mean and variance of the two hypotheses, which in turn depends on the arbitrary positive power *p*. Figure 2 shows the plot of the minimum number of samples determined using (19) against *p*. It is observed that there exists an optimum value of N_{min} as *p* is varied. There exists only one value of *p* to minimize N_{min} for any given *P*, $\hat{P}_f$ and $\hat{P}_e$ and SNR.

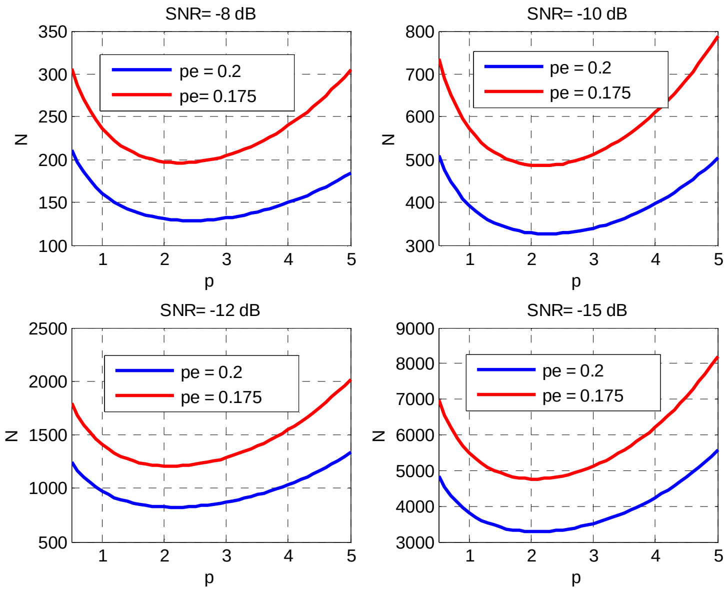

Figure.2 Minimum number of sample of improved energy detection against *p* for different fixed values of target *Pe* and SNR.

The optimal value of *p* is given by,

$$p^* = \arg\min_p \left(N_{\min}\right) \tag{20}$$

This can be obtained when $\frac{\partial N_{\min}}{\partial p} = 0$. We derive the expression for $\frac{\partial N_{\min}}{\partial p}$ in the following.

$$\frac{\partial N_{\min}}{\partial p} = 2\left(\frac{\sigma_1}{(\mu_0 - \mu_1)}C\right) \cdot \left(\left(\frac{\partial}{\partial p}\left(\frac{\sigma_1}{\mu_0 - \mu_0}\right)C\right) - \frac{\partial}{\partial p}\left(\frac{\sigma_0}{\sigma_1}\right)Q^{-1}\left(\hat{P}_f\right)\right) \tag{21}$$

where

$$C = Q^{-1}\left(1-\frac{\hat{P}_e-(1-P)\hat{P}_f}{P}\right)$$

$$\frac{\partial\sigma_0}{\partial p}=\frac{2^{\frac{p}{2}-1}}{D\sqrt[4]{\pi}}\left[D^2\ln 2+\psi\left(p+\frac{1}{2}\right)\Gamma\left(p+\frac{1}{2}\right)-\frac{1}{\sqrt{\pi}}\psi\left(p+\frac{1}{2}\right)\Gamma^2\left(p+\frac{1}{2}\right)\right] \quad (22)$$

$$\frac{\partial\sigma_1}{\partial p}=\frac{2^{\frac{p}{2}-1}(1+\alpha)^{\frac{p}{2}}}{D\sqrt[4]{\pi}}\left[D^2\ln(2+2\alpha)+\psi\left(p+\frac{1}{2}\right)\Gamma\left(p+\frac{1}{2}\right)-\frac{1}{\sqrt{\pi}}\psi\left(p+\frac{1}{2}\right)\Gamma^2\left(p+\frac{1}{2}\right)\right] \quad (23)$$

$$\frac{\partial\mu_0}{\partial p}=\frac{2^{\frac{p}{2}-1}}{\sqrt{\pi}}\left[\Gamma\left(p+\frac{1}{2}\right)\left(\ln 2+\psi\left(p+\frac{1}{2}\right)\right)\right] \quad (24)$$

$$\frac{\partial\mu_1}{\partial p}=\frac{2^{\frac{p}{2}-1}(1+\alpha)^{\frac{p}{2}}}{\sqrt{\pi}}\left[\Gamma\left(p+\frac{1}{2}\right)\left(\ln(2+2\alpha)+\psi\left(p+\frac{1}{2}\right)\right)\right] \quad (25)$$

where $D=\sqrt{\Gamma\left(p+\frac{1}{2}\right)-\frac{1}{\sqrt{\pi}}\left(\Gamma\left(\frac{p+1}{2}\right)\right)}$

and $\psi(x)$ is the Euler-psi function given by

$$\psi(x)=\frac{d}{dx}\ln\Gamma(x)$$

Substituting the equations (20)-(25), in (21) the solution to $\frac{\partial N_{\min}}{\partial p}=0$ can be obtained, which gives the optimum value of *p* that minimizes N_{min}.

VI. NUMERICAL SIMULATIONS AND ANALYSIS

To verify the accuracy of the theoretical deductions, we provide the simulated results of the improved energy detection algorithm with optimal threshold values and optimal *p*. Initially, the ROC performance of the conventional energy detector and the improved energy detector is presented. Next, a thorough analysis is performed for the optimal choices of decision threshold and *p* for varying SNR values satisfying MEB. Then the performance of the improved energy detector for MEB criterion is analyzed based on SNR values and compared with the existing algorithm. Finally, the sample complexity analysis is performed for the proposed algorithm with TEB against the existing algorithm.

The ROC performance of the conventional energy detector and the improved energy detector is shown in Figure 2 which clearly depicts the performance enhancement of the improved algorithm over the conventional algorithm.

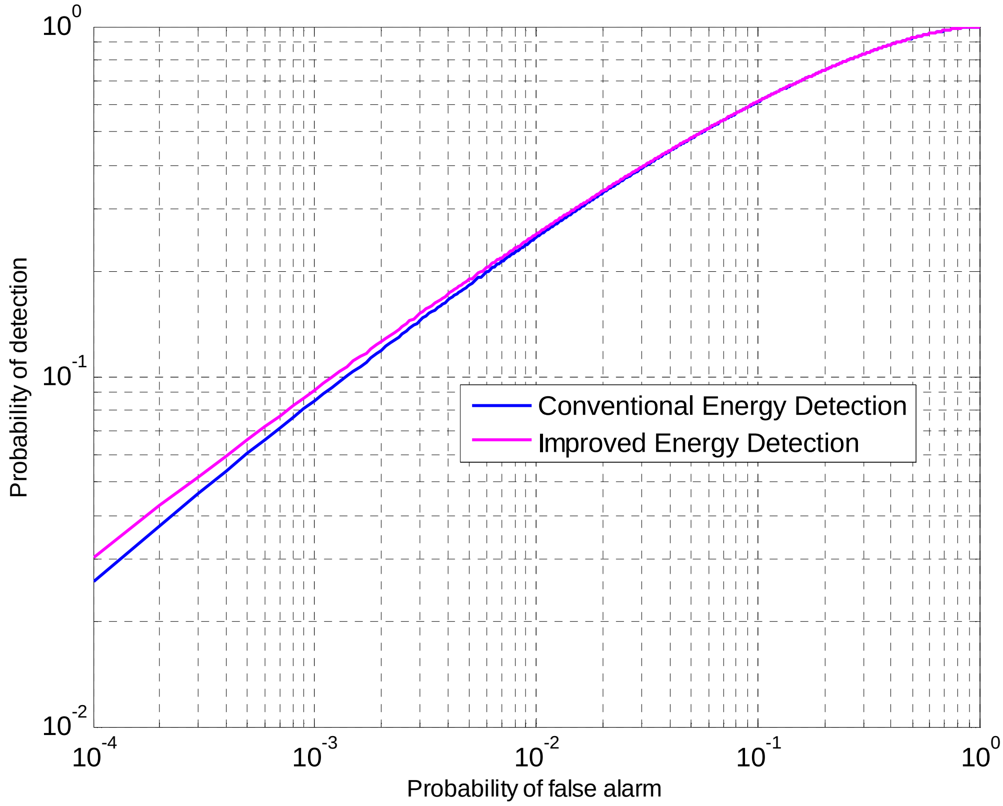

Figure.3. ROC Performance of the proposed and existing algorithms
(SNR = -15 dB, N = 1000)

Table 1 and Table 2 shows the total error probability based on the above theoretical deductions for varying N and SNR values for primary user occupancy values of 45% and 25%. These values are observed from [25], in which the spectrum occupancy measurements are observed as 45% in the cellular band typically and over the licensed bands it is 25% on an average.

Table.1 Total error probability and optimal *p* determined using optimal threshold satisfying MEB when P =45%

N	Algorithm	SNR = -5 dB		SNR = -8 dB		SNR = -10 dB		SNR = -15 dB	
		p	P_e	p	P_e	p	P_e	p	P_e
100	Proposed	2.11	0.1601	2.23	0.2917	2.33	0.3561	2.89	0.4348
	Existing	2.13	0.1602	2.56	0.3027	2.79	0.3674	3.16	0.4354
500	Proposed	1.91	0.0149	2.03	0.1202	2.05	0.2214	2.15	0.3926
	Existing	1.73	0.0560	2.02	0.1204	2.16	0.2314	2.36	0.4037
1000	Proposed	1.96	0.0011	2.01	0.0494	2.02	0.1412	2.07	0.3568
	Existing	0.65	0.0550	1.91	0.0664	2.04	0.1417	2.22	0.3748

Table.2 Total error probability and optimal *p* determined using optimal threshold satisfying MEB when P =25%

N	Algorithm	SNR = -5 dB		SNR = -8 dB		SNR = -10 dB		SNR = -15 dB	
		p	P_e	p	P_e	p	P_e	p	P_e
100	Proposed	2.45	0.1176	2.99	0.2034	3.00	0.2345	3.00	0.2499
	Existing	2.13	0.1334	2.56	0.2126	2.79	0.2485	3.16	0.2863
500	Proposed	2.03	0.0118	2.15	0.0940	2.25	0.1672	2.82	0.2477
	Existing	1.73	0.0756	2.02	0.1113	2.16	0.1730	2.36	0.2687
1000	Proposed	1.99	0.0009	2.07	0.0398	2.12	0.1112	2.38	0.2404
	Existing	1.65	0.0750	1.91	0.0813	2.04	0.1232	2.22	0.2527

Existing algorithm means the improved energy detection algorithm with the decision threshold set based on a target P_f. The proposed algorithm is the improved energy detection algorithm with the optimized threshold. For both the algorithms the optimum values of *p* and the corresponding error probability are tabulated. When the primary user occupancy is 45% and for increasing number of samples, the total error probability is low for the proposed algorithm compared to the existing algorithm. When *P* is 25%, it is observed that the probability of error is less for the proposed algorithm for any number of samples and SNR. The optimum value of *p* is also tabulated for both the algorithms. It can be observed that the optimal *p* is not the same for all the cases and different from the traditional energy detector for which p is always 2. It also depends on the collected number of samples. As the optimal p depends on SNR, N, and P, this algorithm cannot be used instantaneously for practical sensing. However, SNR can be estimated, and the information regarding P can also be predicted using techniques proposed in [26]. With this prior knowledge, optimum values of threshold and p can be computed offline and the best choice of N can be selected based on the requirement of the cognitive radio.

The performance of the proposed algorithm with optimum threshold for varying SNR is compared with the existing algorithm. It can be observed from Figure. 4, when the SNR is negative, the total error probability is high and depends on the number of samples. However, it is lower than the values obtained with the existing improved energy detection algorithm. When the value of SNR is greater than 0, the total error probability declines to values close to 0 for the proposed algorithm when compared to the existing algorithm.

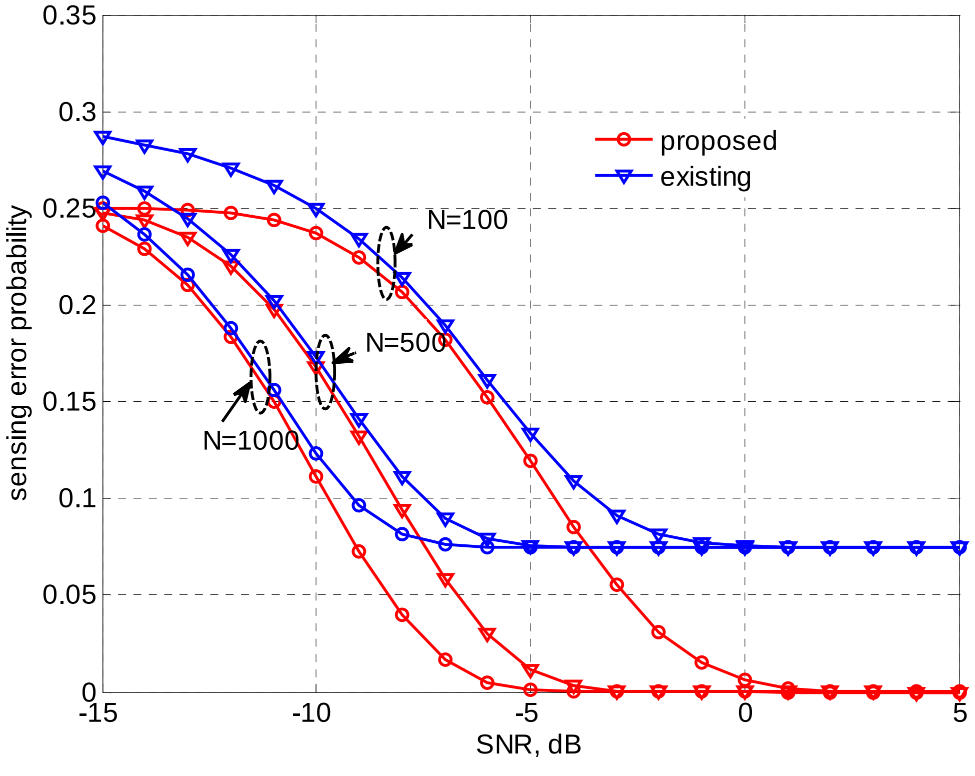

Figure. 4. Performance of the proposed and existing algorithms against SNR

Finally, the number of samples required for the proposed algorithm with a TEB is compared with the existing traditional energy detection algorithm.

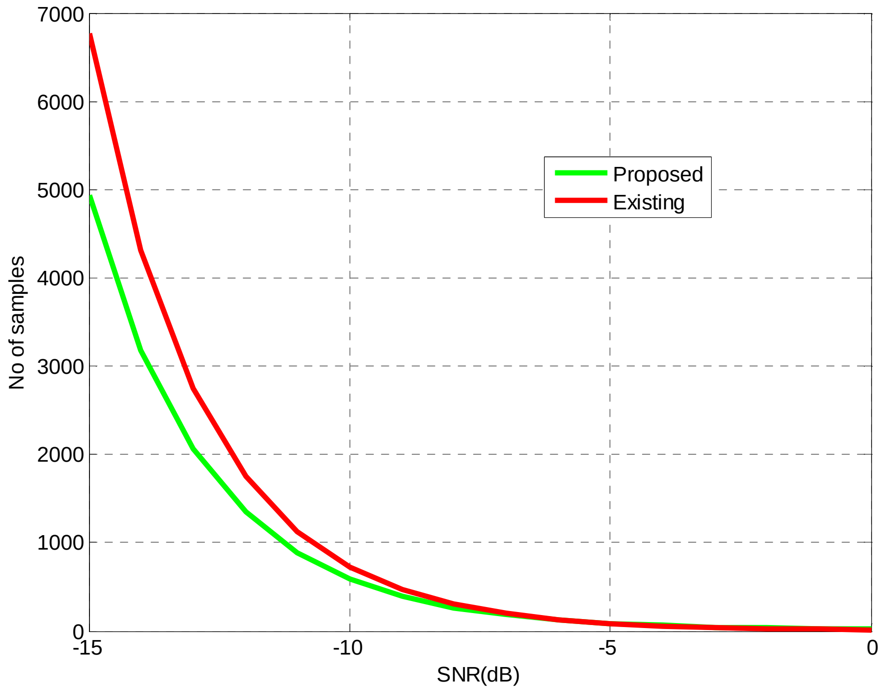

Figure 5. Sample Complexity of the proposed and existing algorithms.

The minimum number of samples required for the existing algorithm is given by [19],

$$N_{\min} = \left[\frac{1}{\alpha^2} \left(Q^{-1}\left(\hat{P}_f \right) - Q^{-1}(P_d)\sqrt{2\alpha+1} \right)^2 \right] \quad (26)$$

From (26), the sample complexity is found to be in the order of $1/\alpha^2$. Figure. 5 depicts the obtained N for the considered SNR values for both the algorithms numerically. The target probability of false alarm is set to 0.1 for both the cases. The TEB is set to 0.15 for the proposed algorithm. The curve corresponding to the proposed algorithm appears similar to the existing algorithm except for a narrow shift downwards. This means the sample complexity of the proposed algorithm also scales to the order of $1/\alpha^2$. However, it is observed that in the low SNR regime (< -5dB), the number of samples required for the proposed algorithm is lesser compared to the traditional energy detection algorithm. For SNR values greater than -5 dB, both the algorithms require very less number of samples. Nevertheless in this SNR region, it is easy to decide the presence or absence of the primary signal for any signal detection algorithm with less number of samples.

VII. CONCLUSIONS

Energy detection has gained much popularity owing to its simplicity and low computational complexity. In this paper, we proposed two implementations with improved energy detection to improve the performance of spectrum sensing. The total sensing error probability is the parameter considered for the design of the proposed techniques. First, the optimization of decision threshold for the improved energy detection is performed with MEB as the design goal. Next the optimization of the arbitrary positive index with respect to the minimum number of samples for a TEB is carried out. Numerical and simulation results validate the efficacy of the proposed algorithms thereby confirming to be superior over the existing algorithm and found to be well suited for practical spectrum sensing.

REFERENCES

[1] Federal Communications Commission, “Spectrum policy task force report”, 2002.

[2] F.Akyildiz, W.Y.Lee and S.Mohanty, “Next generation / dynamic spectrum access / cognitive radio wireless networks: A survey,” Computer Network Journal (Elsevier), vol. 50, pp 2127 – 2159, Sep 2006.

[3] D.Cabric, S.M.Mishra, and R.W.Brodersen, “Implementation issues in spectrum sensing for cognitive radios,” Conference Record of the Thirty-Eighth Asilomar Conference on Signals, Systems and Computers, vol.1, pp.772-776, Nov 2004.

[4] T.Arslan,“A survey of spectrum sensing algorithms for cognitive radio applications," IEEE Communications Surveys & Tutorials , vol.11, no.1, pp.116-130, First Quarter 2009.

[5] Kae Won Choi, Wha Sook Jeon, Dong Geun Jeong, “Sequential detection of cyclostationary signal for cognitive radio systems,” IEEE Transactions on Wireless Communications, vol.8, no.9, pp.4480,4485, Sep 2009.

[6] Zeng.Y, Liang.Y.C, “Spectrum sensing algorithms for cognitive radio based on statistical covariances”, IEEE transactions on Vehicular Technology, vol.58, no.4, pp 1804-1815, Sep 2008.

[7] Yonghong Zeng, Ying-Chang Liang, “Eigen value based spectrum sensing algorithms for cognitive radio”, IEEE transactions on communications, vol.57, no.6, pp. 1784 – 1793, June 2009.

[8] El-Khamy, S.E, El-Mahallawy, M.S, Youssef, E.S., “Improved wideband spectrum sensing techniques using wavelet-based edge detection for cognitive radio,” in Proc of International Conference on Computing, Networking and Communications, pp.418-423, Jan 2013.

[9] Yuliang Cong, Lijuan Xing, Jiaxing Bi, “Spectrum sensing algorithm based on ATSC DTV Signal Structure,” International Journal on Smart Sensing and Intelligent Systems, vol.6, no.5, pp. 2136 – 2154,Dec 2013.

[10] Smitha.K.G, Vinod.A.P, “Low Power DFT Filter bank based two stage spectrum sensing”, in Proc. of International Conference on Innovations in Information Technology, pp 173-177, March 2012.

[11] Urkowitz,H “Energy detection of unknown deterministic signals,” in Proc. Of IEEE, vol. 55, pp. 523-531,April 1967.

[12] Digham F. F., M.-S. Alouini, and M. K. Simon, “On the energy detection of unknown signals over fading channels," in Proc of IEEE Int. Conference on Communications, Anchorage, AK, USA, pp. 3575-3579, May 2003.

[13] Ying-Chang Liang, Yonghong Zeng, E.C.Y. Peh, Anh Tuan Hoang, "Sensing-Throughput Tradeoff for Cognitive Radio Networks," IEEE Transactions on Wireless Communications, vol.7, no.4, pp.1326-1337, April 2008.

[14] Zeng. Y, Liang Y. C and Zhang. R, “Blindly Combined Energy Detection for Spectrum Sensing in Cognitive Radio,” IEEE Signal Processing Letters, vol.15, p.649-652, 2008.

[15] Ramanarayanan Viswanathan, Babak Ahsant, “A review of sensing and distributed detection algorithms for cognitive radio systems,” International Journal on Smart Sensing and Intelligent Systems, vol.5, no.1, pp. 177 – 190, March 2012.

[16] Ling. X, Wu. B, Wen. H, Ho. P.H, Bao. Z and Pan. L, “Adaptive Threshold Control for Energy Detection Based Spectrum Sensing in Cognitive Radios,” IEEE Wireless Communications Letters, vol.1, no.5, pp.448-451, 2012.

[17] Gismalla. E.H and Alsusa. E “On the Performance of Energy Detection Using Bartlett's Estimate for Spectrum Sensing in Cognitive Radio Systems,” IEEE Transactions on Signal Processing, vol.60, no.7, pp.3394-3404, 2012.

[18] Martínez D. M and Andrade. A. G, “Performance evaluation of Welch's periodogram-based energy detection for spectrum sensing,” IET Communications, vol.7, no.11, pp.1117-1125.

[19] M.López-Benítez, F.Casadevall, “Improved energy detection spectrum sensing for cognitive radio,” IET Communications, vol.6, no.8, pp.785-796, May 2012.

[20] Chen.Y, “Improved energy detector for random signals in Gaussian noise”, IEEE Transactions on Wireless Communications, vol.9, no.2, pp 558-563.

[21] J.Song, Z.Feng, P.Zhang, Z.Liu, "Spectrum sensing in cognitive radios based on enhanced energy detector," IET Communications, vol.6, no.8, pp.805-809, May 2012.
[22] Treeumnuk. D & Popescu. D. C "Enhanced spectrum utilisation in dynamic cognitive radios with adaptive sensing," IET Signal Processing, vol.8, no.4, pp.339-346, 2014.
[23] Lu. L, Zhou. X, Onunkwo. U and Li. G. Y, "Ten years of research in spectrum sensing and sharing in cognitive radio," Eurasip Journal on Wireless Communication and Networking, vol.2012,:28, pp 1-16, 2012.
[24] Sean Dieter Tebje Kelly, Nagender Kumar Suryadevara, and S. C. Mukhopadhyay, "Towards the Implementation of IoT for Environmental Condition Monitoring in Homes" IEEE SENSORS JOURNAL, VOL. 13, NO. 10, OCTOBER 2013, pp. 3846-3853.
[25] Victoria Jancee, Radha.S, Nandita Das "Analysis of non binary fault tolerant event detection in wireless sensor networks," International Journal on Smart Sensing and Intelligent Systems, vol.7, no.3, pp. 1287 – 1309, September 2014.
[26] N. K. Suryadevara and S. C. Mukhopadhyay, "Determining Wellness Through An Ambient Assisted Living Environment", IEEE Intelligent Systems, May/June 2014, pp. 30-37.
[27] Habib F. Rashvand, Ali Abedi, Jose M. Alcaraz-Calero, Paul D. Mitchell, and Subhas Chandra Mukhopadhyay, Wireless Sensor Systems for Space and Extreme Environments: A Review, IEEE SENSORS JOURNAL, Vol. 14, No. 11, November 2014, pp. 3955-3970.
[28] Xiaoyuan Li; Xiang Mao; Dexiang Wang; McNair, J.; Jianmin Chen, "Primary user behavior estimation with adaptive length of the sample sequence," in Proc of IEEE Global Communications Conference 2012, pp.1308-1313, 3-7, Dec 2012.