



## EXPERIMENTAL AND THEORETICAL VALIDATION METHOD FOR ESTIMATION OF STRAIGHTNESS DEVIATION AND ASSOCIATED UNCERTAINTY IN CNC-CMM MEASUREMENT

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*Abstract- Geometrical deviation in intelligent metrology system is an important factor in precision engineering. Estimating the deviation and associated uncertainty in straightness feature remains a necessary requirement to ensure highly accurate metrology method especially in CNC-CMM measurement. Optimization in straightness measurement using soft algorithm techniques is a widely encountered solution in coordinate metrology. In this work, straightness deviation has been measured precisely in 2D and in 3D using slab surface by CNC-CMM at the Egyptian national metrology institute (NIS). The work has been investigated experimentally and theoretically analyzed. The straightness deviation and its uncertainty results from 2D measurement have been estimated experimentally. The straightness deviation result of the 3D discrete points measurements have been analyzed theoretically using the standard Particle Swarm Optimization (PSO) algorithm. The probability density distribution of the measured straightness was calculated using a Sequential Monte Carlo (SMC) technique. A*

*probability density histogram is obtained with an expanded measurement uncertainty based on coverage factor  $k$  equals 2 providing confidence level 95%. The computational results of straightness deviation and expanded uncertainty have been also estimated for 3D discrete point measurements. Comparison with relevant report showed agreement with our result since we used a computationally efficient modified SMC technique and PSO algorithm. The results of the straightness deviations and associated expanded uncertainties for both 2D and 3D measurements have been discussed and compared. They were found to be suitable for the proposed validation method. This work confirms that the developed strategic alternative methodology can be achieved successfully. Systematic acquisition of CNC-CMM data is another contributing factor for improving the required accuracy in measurement. Moreover, the confidence in the proposed hybrid validation method for estimating the straightness deviation with associated uncertainty has been achieved.*

**Index terms:** CNC-CMM, straightness, uncertainty estimation and validation method.

### **Nomenclature**

2D	two dimensions
3D	three dimensions
$\alpha; \beta$	angle
$\alpha_j^0; \beta_j^0$	constant
$ab$	measured distance
$d$	variation
$f$	diameter, straightness deviation
$l$	CNC-CMM resolution
$n$	repetition number of 2D measurement
$u_c$	combined standards uncertainty of 2D measurement
$u_l$	uncertainty due to CMM resolution in measurement
$u_{MPE}$	uncertainty due to maximum permissible error of CNC-CMM
$u_{MPPE}$	uncertainty due to maximum permissible probing error of CNC-CMM
$u_r$	uncertainty due to repeatability
$\nu_r$	$= (n-1)$
CMM	coordinate measuring machine
CNC	computer numerical control

GUM	guide to the expression of uncertainty in measurement
ISO	International Standards Organization
$K$	coverage factor
LSQ	least square fitting
$MPE$	maximum permissible error of CNC-CMM machine
$MPPE$	maximum permissible probing error of CNC-CMM
NIS	National Institute for Standards
$^{\circ}\text{C}$	temperature degree, Celsius
PDF	probability density functions
PSO	particle swarm optimization algorithm
SD	standard deviation
SMC	sequential Monte Carlo technique
$U_{exp)2D}$	expanded uncertainty due to 2D measurement
$U_{exp)3D}$	expanded uncertainty due to 3D measurement
$V$	degree of freedom
X-	X-axis
Y-	Y-axis
Z-	Z-axis

## I. INTRODUCTION

The growing role of simulation software, used in the intelligent metrology systems to increase the quality of innovative products, requires a greater focus on verification and validation methods. Dimensional metrology is a strongly supporting science in manufacturing processes and quality control of engineering products [1-3]. Geometrical form accuracy of engineering surfaces plays an important role in modern industry using CNC-CMM, especially in geometrical shape measurement for automotive, airplane, reactors and aerospace technologies [4-6]. Implanted engineered biomaterial in-vitro or in-vivo of human is also needs more accurate surface to achieve high compatible in interface interact [7-8]. Straightness feature in coordinate metrology is a very *necessary* parameter in surface characterization, especially in straight line form and flatness measurements. Any increase in straightness deviation has a serious impact on the

performance and inspection of mechanical systems [5-6]. Straightness deviation is also an important feature of alignment for rotating parts. Optimization in straightness form deviation in measurement using many algorithm techniques is a newly introduced methodology in dimensional metrology as it has been indicated in the GUM guidelines [9-13]. In GUM, propagation of uncertainty and the characterization of the output quantity by a Gaussian distribution are recommended [14]. Also the standard uncertainty would be calculated on the basis of a first-order Taylor series approximation of the mathematical model. If the model is nonlinear, the estimate of the standard uncertainty provided by GUM might be unreliable [15]. On the other hand, sometimes may need necessary correction of measurement result according to ISO standards. The correction can apply by compensate for a known systematic measurement error to be suitable for real estimate of associated uncertainty [16].

The aimed accuracy in straightness measurement should be lying in the dimensional tolerances using CMM machine. The developed measurement strategies, linearity error, straightness errors (translation and rotational errors) are sources that have to be controlled and counted. In order to quantify the uncertainties due to many repeated positions/traces linear translational, there must be different sources of variations listed with their associated probability density functions. Then, Monte Carlo techniques are performed for two purposes; the first is to simulate measurement methodology and the second is to optimize measurement uncertainty. Sequential Monte Carlo (SMC) technique is more reliable for dynamic systems such as CNC-CMM than Monte Carlo (MC) technique. SMC technique was used as a reliable and accurate tool for mobile robot localization dynamically in distributed intelligent environments by Qian et al [17]. SMC technique has been used as a *useful software tool* for a wide variety of simulation-based in coordinate metrology and standard Particle Swarm Optimization (PSO) algorithm were computationally used for optimal measurement allocation of straightness feature [9-10]. Standard PSO is introduced as a computational optimization algorithm by Kennedy and Eberhart [18] in 1995. While, the hybrid method based on PSO algorithm and SMC technique, may be intelligent and preferable for leading to accurate convergence [19-20]. Mingzhao et al [11] evaluated the spatial straightness error using coordinates investigation by Multilateration algorithm. Arencibia et al [12] reported on a simplified analytical model to estimate measurement uncertainty in CMM. However the model entails several corrections due to temperature fluctuations and differences in thermal expansions. Estimation of the CMM uncertainty can also be performed

numerically as reported by Zhu et al [13]. They evaluated the spatial straightness using linear complex chebyshev approximation. Nevertheless the numerical calculation using the Monte Carlo statistical approach seems to be the method of choice for most researchers specially when combined with the GUM guidelines [15].

Straightness geometric tolerance zone is specified to include all of the deviation sources such as waviness or other surface imperfections. Our proposed straightness measurement strategy depends on PSO code for estimating the least diameter of a cylinder that accommodates possible measured points on the surface. The repeated measurements produce a set of optimized diameters corresponding to a set of  $(X, Y)$  coordinates for the cylinder associated with each element of the diameter set. This is followed by  $10^6$  Monte-Carlo trails to produce a probability distribution for the measured straightness error using probability density functions (PDF) for the averages of  $(X, Y)$  as the main inputs for the Monte-Carlo code in MATLAB environment. So, we expect the SMC technique with PSO algorithm represent a powerful hybrid genetic model.

Briefly, 2D measurement has been performed and studied experimentally and 3D measurement was made for theoretical computations of straightness feature on a slab surface of CNC-CMM. The results of straightness deviation and associated expanded uncertainty for both 2D and 3D measurements have been discussed and compared. The proposed alternative methodology in this work was planned to find an accurate solution for straightness deviation and expended uncertainty for 3D using PSO Optimization algorithm and SMC technique. The measurement result was verified using ISO GUM. The result for SMC technique using 3D was validated by comparison with previously reported result and experimental result of 2D measurement.

## II. EXPERIMENTAL STUDY USING CNC-CMM

Straightness measurement has been measured precisely using CNC-CMM at NIS laboratory in Egypt. This measurement is investigated within two procedures. The first one was the verification of CNC-CMM at standard conditions, while the second was the experimental setup for straight line runs parallel to the  $X$ -axis of CMM.

### a.i CMM verification method

The environmental conditions of CNC-CMM testing room have been adjusted in the range of standard specifications at  $20 \pm 0.5$  °C. The performance verification of CNC-CMM was tested

according to common standards [6, 21]. The verification test procedures were already implemented by measured standards based on ceramic reference sphere and by comparing the measured value with the specified measurement uncertainty. Table 1 comprises the specifications of the CMM machine set-up and strategy of measurements.

TABLE 1. CMM set-up and measurement strategy

CMM strategic parameters	Specifications
Master probe radius	3.9999 mm
Reference sphere radius	14.9942 mm with SD =0.0002 mm
Used long probe radius	4.0003 mm with SD = 0.0002 mm
Machine travelling speed	30 mm/s
Probe scanning speed	10 mm/s
Straightness points	step width = 10 mm
Fitting technique	LSQ

#### a.ii Experimental setup

The experimental setup for straightness measurement is limited to CMM runs parallel to the X-axis for both the 2D and 3D measurements.

##### a.ii.i Straightness measurement in 2D

Straightness measurement for a certain position on the granite slab surface is investigated. CNC-CMM strategy for automatic measurement is used at Gaussian-filter. Fifteen repeated measurement on the 2D straight line *ab* of 150 mm which involved 16 points on X-axis as shown in Fig.1 is conducted.

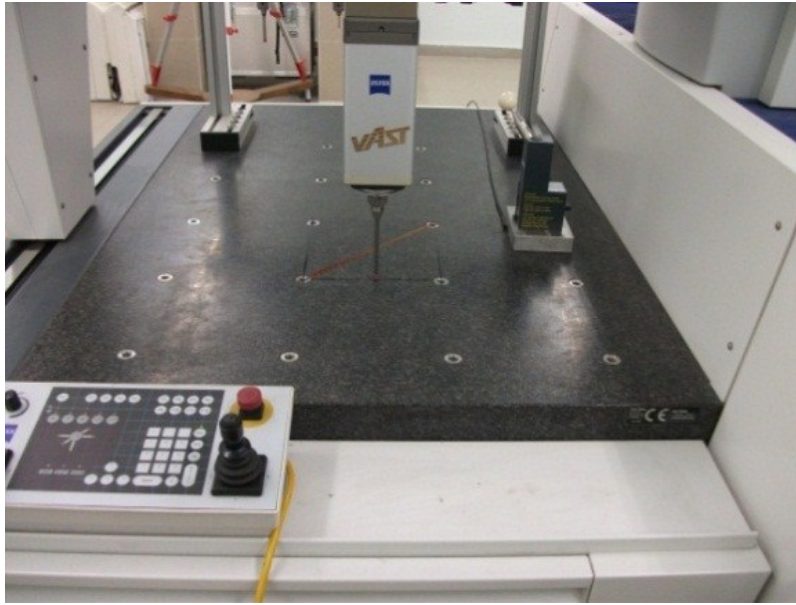


Figure 1. Straightness measurement in 2D using coordinate measuring machine

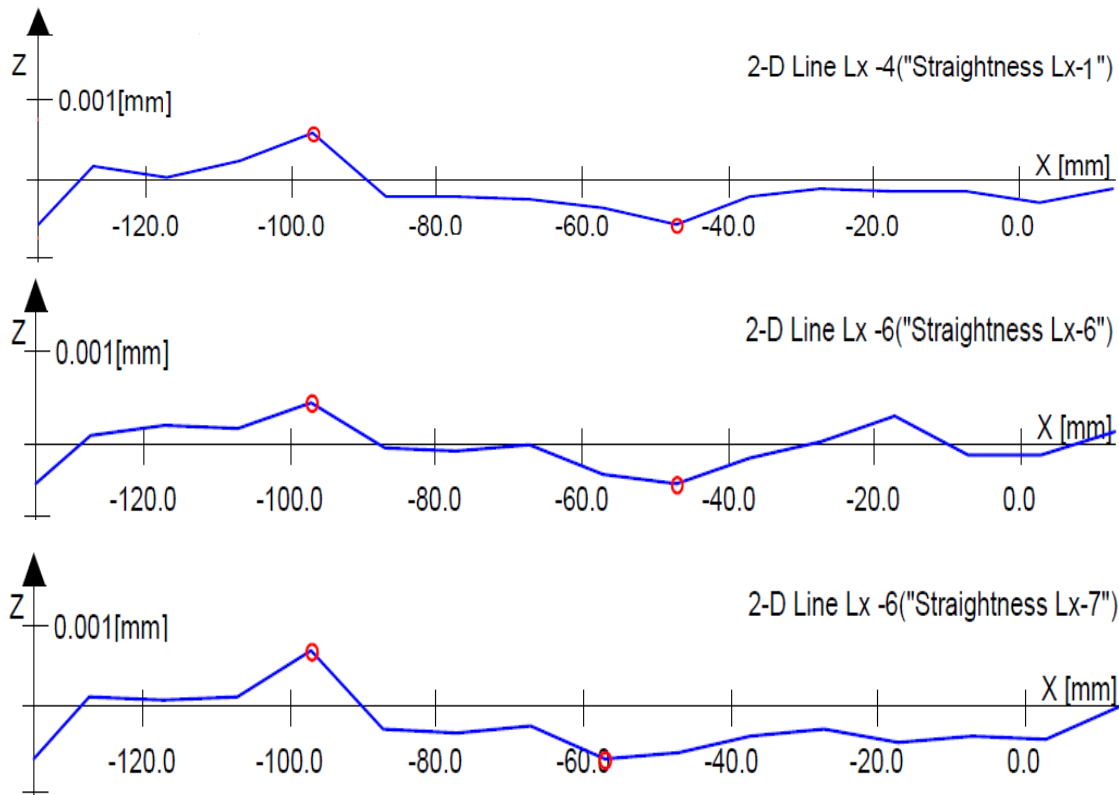


Figure 2. Typical result of 2D straightness measurement

Figure 2 shows the typical experimental result of CNC-CMM scan for the 2D straightness result. The result of the repeated straightness deviation is calculated and the average of straightness deviation is found to equal to be 1.00  $\mu\text{m}$ , while standard deviation ( $SD$ ) was within 0.14  $\mu\text{m}$ .

#### a.ii.ii Uncertainty in the 2D CMM measurement

The expanded uncertainty in the 2D CMM measurement for straightness deviation is studied based on GUM [12]. Combined and expanded uncertainty for the experimental results is estimated. The repeatability, least reading (sensitivity) of CMM and probing errors are considered as a components of both *Type A* and *Type B* uncertainty. The uncertainty component due to repeatability  $u_r$  is:

$$u_r = SD/\sqrt{n} = 0.14/\sqrt{15} = 0.0362 \mu\text{m}$$

The CMM straightness measurement has least reading of 0.1  $\mu\text{m}$  which represents the CMM resolution  $l$ . The measured data of CMM straightness measurement has sensitivity error 0.1  $\mu\text{m}$  which is represented the CMM resolution  $l$ . Digital instrumentation such as CMM by its nature, sampling rate, etc., provides a discrete number, but the true value could lie within the range  $\pm(l/2)$ , where  $l$  is the resolution of the CMM display [22]. Therefore we must take this fact into account when we estimate the uncertainty. Assuming a rectangular distribution then  $u_l$  will be:

$$u_l = (l/2) / \sqrt{3} = 0.05/\sqrt{3} = 0.0289 \mu\text{m}$$

The maximum permissible error ( $MPE$ ) of CNC-CMM is 0.9  $\mu\text{m}$ . A rectangular distribution is assumed, the uncertainty component due to machine error is:

$$u_{MPE} = 0.9 / \sqrt{3} = 0.5196 \mu\text{m}$$

The maximum permissible probing error ( $MPPE$ ) of CNC-CMM is 0.5  $\mu\text{m}$ . A rectangular distribution is assumed, the uncertainty component due to probing error is:

$$u_{MPPE} = 0.5 / \sqrt{3} = 0.2887 \mu\text{m}$$

During the implementation of experimental work, the environmental temperature in the NIS laboratory was controlled within  $20 \pm 0.5$   $^{\circ}\text{C}$ . The uncertainty component from temperature is



expected to have a negligible effect. This is due to the type of CMM granite slab surface with short distance in straightness measurement. Thus, the combined standard uncertainty  $u_c$  of measured straightness deviation using CNC-CMM is calculated as follows:

$$\begin{aligned}
 u_c &= \sqrt{((Type A)^2 + (Type B)^2)} \\
 &= \sqrt{((u_r)^2 + (u_l)^2 + (u_{MPE})^2 + (u_{MPEP})^2)} \\
 &= \sqrt{((0.0362)^2 + (0.0289)^2 + (0.5196)^2 + (0.2887)^2)} \\
 &= \sqrt{((0.0013) + (0.0009) + (0.2700) + (0.0834))} \\
 &= \sqrt{0.3556} \\
 &= 0.5963 \mu\text{m}
 \end{aligned}$$

The coverage factor  $K$  is an important value for estimating the expanded uncertainty. The *coverage factor* depends on the *degree of freedom*  $V$ . In order to determine the coverage factor  $K$ , it is necessary to calculate the effective degree of freedom  $V$ , using Welch-Satterthwaite formula as follows [22]:

$$\begin{aligned}
 V &= u_c^4 / ((u_r^4/v_r) + (u_l^4/v_l)) \\
 \text{where } v_r &= (n-1) = 14; \quad v_l = \infty \\
 V &= v_r (u_c/u_r)^4 \\
 &= 14 (0.5963/0.0362)^4 \\
 &= 14 (16.4724)^4 \\
 &> 20
 \end{aligned}$$

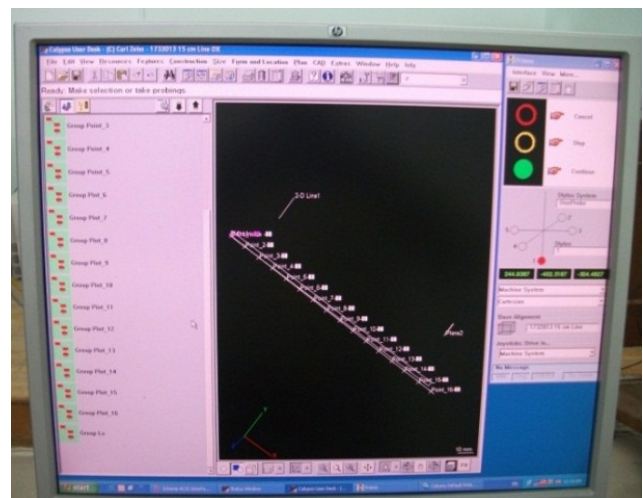
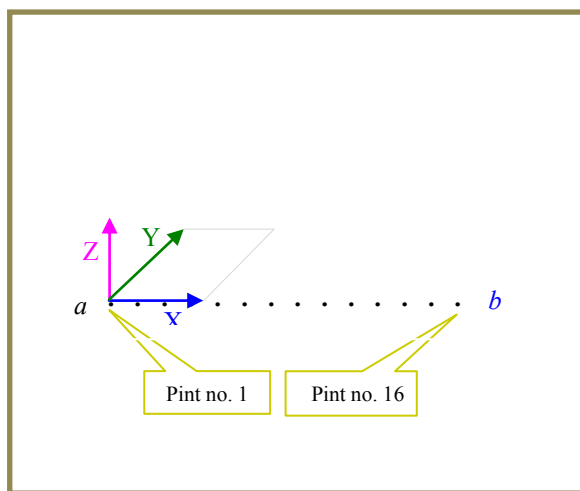
Assuming the coverage factor  $K$ , is equal to 2.0 providing *level of confidence* of approximately 95% [15; 21-22].

$$U_{Exp)2D} = K (u_c) = 2.0 (0.5963) = 1.193 \mu\text{m}$$

The expanded uncertainty in straightness measurement using CNC-CMM has been estimated (1.04  $\mu\text{m}$ ). Therefore, the 2D straightness deviation measurement using CNC-CMM scan is 1.00  $\pm$  1.19  $\mu\text{m}$ .

a.ii.iii Straightness measurement in 3D

CNC-CMM program has been made of auto repeated measurements, resulting in 16 sets of data points with 30 point repetitions per set, giving rise to the 3D discrete points on a straight line “ $ab=150$  mm” with interval 10 mm. Thus for each set  $i = 1, 2, \dots, 16$ , the point  $(X_{ij}, Y_{ij}, Z_{ij})$  has been repeatedly measured 30 time, i.e.  $j = 1, 2, \dots, 30$ . For example the set  $i = 1$  consists of the following points:  $(X_{11}, Y_{11}, Z_{11}), \dots, (X_{1j}, Y_{1j}, Z_{1j}), \dots, (X_{1,30}, Y_{1,30}, Z_{1,30})$ . In other words, strategically method makes probing of the straightness measurement using PRISMO CMM with repeated measurement 30 times for each 16 points of the same line “ $ab=150$  mm” as shown in Fig. 3a. For instance the actual value of 3D discrete point\_1-1 is  $(x=-137.7023, y=-574.0024; z=-0.0013)$  while the nominal value of point\_1-1 is  $(x=-137.7000, y=-574.0000; z=0.0000)$ . In other words the line ‘ $ab$ ’ runs parallel to the x-axis, see Fig. 3.



(a) An schematic of straightness 16 points distribution in X- direction

(b) CMM monitoring of straightness 3D measurement in X- direction

Figure 3. Distribution of the data points (16 sets of 3D discrete points with 30 repeated measurements for each set) on line ‘ $ab$ ’ = 150 mm, parallel to x-axis, measured using CMM.

In the following discussion we assume the geometrical location of 3D measured points to be represented relative to the axis of a horizontal cylinder parallel to the direction of X-axis on CMM machine as shown in Fig.4.

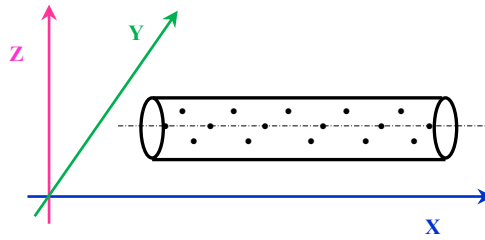


Figure 4. Schematic of virtual spatial for straightness 16 points deviation

### III. THEORETICAL ANALYSIS

In this paper we used research strategy based on two main procedures (methodologies) for the data validations: the PSO and the SMC. The first one was using standard Particle Swarm Optimization (PSO) algorithm for optimization of experimental 3D measurement points which assumed represents the geometrical location of horizontal cylindrical axis. The second procedure was to study the distribution for the PDF using Sequential Monte Carlo (SMC) technique for straightness measurement. The hybrid validation method using these computational procedures has been implemented in five consecutive steps that will be described in details as follows:

#### a.i Calculation of straightness deviation using standard PSO algorithm

Particle Swarm Optimization algorithm is an efficient optimization tool for nonlinear continuous optimization problems, combinatorial optimization problems, and mixed-integer nonlinear optimization problems [23]. PSO algorithm has been used to overcome the limitations of standard SMC technique. In the following section, can be describe the implementation of a PSO code in MATLAB environment for the Monte Carlo calculation of the special straightness in the CMM measurement of line 'ab' described in figure 1 and 3. For the PSO algorithm, we used the open source code called "SwarmOps for Matlab" which accessed in 2015 [24]. The proposed hybrid method based on PSO algorithm and SMC technique achieves the best solution for leading to faster convergence [20]. Thus, we implemented the following steps:

Step 1- An analytical model was bearing in consideration and built according to straightness standard GBT11336-2004. In this model, a virtual cylindrical form is introduced to contain all CMM measured as 3D discrete points as shown in Fig.4. The minimum diameter of the cylinder is called virtual spatial straightness deviation.

Step 2- Modification of the fitness equation within the “myproblem” file in the SwarmOps folder so that this equation takes the following form [9]:

$$fitness = R_j = \left( [Y_{ij} - Y_j^0 + X_{ij} \tan \alpha_i^0]^2 + [Z_{ij} - Z_j^0 + X_{ij} \tan \beta_j^0]^2 \right)^{0.5} \quad (1)$$

where in equation (1),  $R_j$  is the minimum radius of the cylinder that encompasses the whole data points  $(X_{ij}, Y_{ij}$  and  $Z_{ij})$  for  $i = 1, 2, \dots, 16$ ;  $(Y_j^0, Z_j^0)$  is the coordinates for the center of the cylinder base, and  $(\alpha_j^0, \beta_j^0)$  are the angles between the central axis of the cylinder and the reference line for measuring the  $(X_{ij}, Y_{ij}$  and  $Z_{ij})$  coordinates. The cylinder is depicted schematically based on the location of the central axis of the tilted cylinder as shown in Fig.3 and Fig.4, for a line parallel (actually tilted by  $(\alpha, \beta)$  angles, but these angles are relatively small so we will ignore them for simplicity) to the  $Z$ -axis, while in our case the measured “ $ab$ ” as a 16 discrete points who composed them straight line runs parallel to the  $X$ -axis for a slab surface of CMM.

Step 3- The procedure in step-2 gives rise to 30 results for the straightness defined as the minimum *diameter* as:

$$f_j(Y_j^0, Z_j^0, \alpha_j^0, \beta_j^0) = 2 \text{ Min. Max. } R_j$$

where *Min.* and *Max.*, respectively, refer to the minimum and maximum MATLAB program functions which return the *maximum*  $R_j$  of the cylinder that contains all the measured points. Then the PSO code searches for the set  $(Y_j^0, Z_j^0, \alpha_j^0, \beta_j^0)$  that *minimizes*  $R_j$ . It is worth mentioning that at this stage we have 30 values for  $f_j$  at  $j = 1, 2, \dots, 30$  with average value given by:

$$\bar{f} = \sum_{j=1}^{30} \frac{f_j(Y_j^0, Z_j^0, \alpha_j^0, \beta_j^0)}{30} \quad (2)$$

In the following we define the averages  $\pm$  the corresponding uncertainties for the quantities  $Y_j^0, Z_j^0, \alpha_i^0, \beta_j^0$  as follows:

$$\bar{Y}_0 \pm \delta Y_0 = \sum_{j=1}^{30} \frac{Y_j^0}{30} \pm \sqrt{\sum_{j=1}^{30} \frac{(Y_j^0 - \bar{Y}_0)^2}{30}} \tag{3}$$

$$\bar{Z}_0 \pm \delta Z_0 = \sum_{j=1}^{30} \frac{Z_j^0}{30} \pm \sqrt{\sum_{j=1}^{30} \frac{(Z_j^0 - \bar{Z}_0)^2}{30}} \tag{4}$$

$$\bar{\alpha}_0 \pm \delta \alpha_0 = \sum_{j=1}^{30} \frac{\alpha_j^0}{30} \pm \sqrt{\sum_{j=1}^{30} \frac{(\alpha_j^0 - \bar{\alpha}_0)^2}{30}} \tag{5}$$

$$\bar{\beta}_0 \pm \delta \beta_0 = \sum_{j=1}^{30} \frac{\beta_j^0}{30} \pm \sqrt{\sum_{j=1}^{30} \frac{(\beta_j^0 - \bar{\beta}_0)^2}{30}} \tag{6}$$

Figure 5 shows  $Y_{avg}$  and  $Z_{avg}$  versus  $X_{avg}$  from real values of 3D discrete set of 16 measured points for straightness. These quantities are defined in the discussion of step 3. The data analysis of 3D discrete points has an error  $\pm 0.0001$  mm, which is the approximate CMM sensitivity. Table 2 shows the optimized average values corresponding to quantities in equations 2–6.

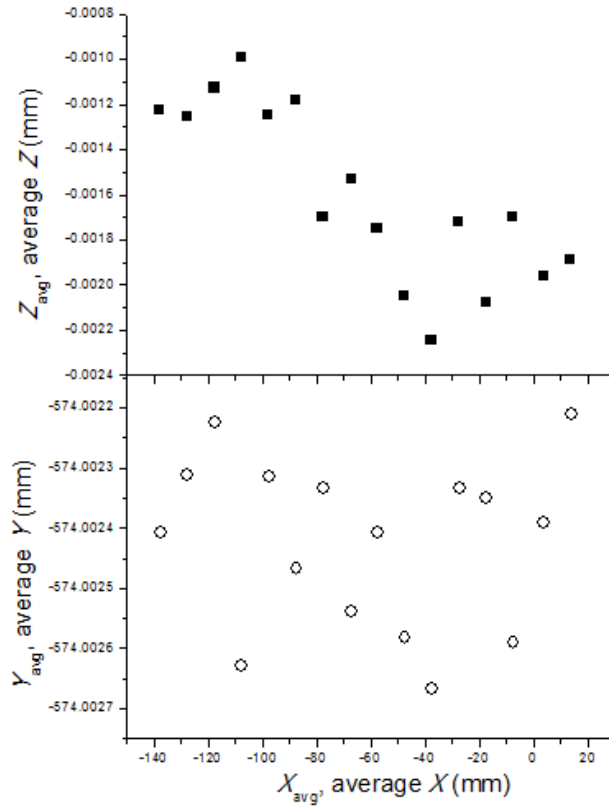


Figure 5. Average measured points in Z- and Y-axis of straight line as a function of the average X points

Table 2. PSO optimized straightness deviation ( $f$ ) and the PSO parameters used for minimizing (optimizing)  $f$ .

Optimization parameters	Value
$\bar{f}$ (mm)	$(9.08 \pm 0.7) \times 10^{-4}$
$\bar{Z}_0 \pm \delta Z_0$ (mm)	$(-2.1 \pm 0.3) \times 10^{-3}$
$\bar{Y}_0 \pm \delta Y_0$ (mm)	$-574.0024 \pm 0.0005$
$\bar{\alpha}_0 \pm \delta \alpha_0$ (rad)	$(1.0 \pm 0.1) \times 10^{-6}$
$\bar{\beta}_0 \pm \delta \beta_0$ (rad)	$(1.0 \pm 0.1) \times 10^{-5}$

From data analysis in Table 2 it is clear that all the data points in Fig.4 are contained within the cylinder with diameter  $\bar{f}$  and circular base center at  $\bar{Y}_0 \pm \delta Y_0$  and  $\bar{Z}_0 \pm \delta Z_0$ , indicating an adequately successful PSO simulation.

a.ii Calculation of uncertainty using SMC technique

Step 4- This is the Monte Carlo simulation step in which we calculated the values of the probability density function ( $PDF_f(\xi)$ ) for the straightness values evaluated using equation 1 as follows:

$$f_{PDF} = \left( \left[ \xi_Y - \xi_{Y0} + \xi_X \tan \xi_{\alpha0} \right]^2 + \left[ \xi_Z - \xi_{Z0} + \xi_X \tan \xi_{\beta0} \right]^2 \right)^{0.5} \quad (7)$$

where  $\xi_{Y0}$ ,  $\xi_{Z0}$ ,  $\xi_{\alpha0}$  and  $\xi_{\beta0}$  represent the *random normal* distribution functions corresponding to  $\bar{Y}_0$ ,  $\bar{Z}_0$ ,  $\bar{\alpha}_0$ , and  $\bar{\beta}_0$  (defined in equations 3→6).  $\xi_Y$ ,  $\xi_X$  and  $\xi_Z$  are the *random uniform* distribution functions corresponding to the values of the average measured  $X$ ,  $Y$ , and  $Z$  coordinates that maximize the straightness (as discussed in the above step 3 for given values  $\bar{Y}_0$ ,  $\bar{Z}_0$ ,  $\bar{\alpha}_0$  and  $\bar{\beta}_0$ ). This is simply done by running the MATLAB *for loop* to evaluate the straightness from equation 7 for all of the 16 values of the averages of:

$$X_{avg} = \sum_{j=1}^{30} \frac{X_{ij}}{30}, Y_{avg} = \sum_{j=1}^{30} \frac{Y_{ij}}{30}, \text{ and } Z_{avg} = \sum_{j=1}^{30} \frac{Z_{ij}}{30},$$

then determining the values ( $X$ ,  $Y$  and  $Z$ ) for maximum  $f_{PDF}$  at given values of  $\bar{Y}_0, \bar{Z}_0, \bar{\alpha}_0$  and  $\bar{\beta}_0$ . The uniform distributions are assumed to cover the range  $\pm\lambda = \pm 0.0001$  mm about all measured points, where this range approximately represents CMM measurement resolution. Thus, Ref. [25] leads us to get these distributions forms as follows:

$$\xi_{Y0} = \delta Y_0 * \text{randn}(1, M) + \bar{Y}_0 \quad ; \quad \xi_Y = [(y_+ - y_-) \text{rand}(1, M)] + y_-$$

$$\xi_{Z0} = \delta Z_0 * \text{randn}(1, M) + \bar{Z}_0 \quad ; \quad \xi_X = [(x_+ - x_-) \text{rand}(1, M)] + x_-$$

$$\xi_{\alpha 0} = \delta \alpha_0 * \text{randn}(1, M) + \bar{\alpha}_0 \quad ; \quad \xi_Z = [(z_+ - z_-) \text{rand}(1, M)] + z_-$$

$$\xi_{\beta 0} = \delta \beta_0 * \text{randn}(1, M) + \bar{\beta}_0$$

where  $y_{\pm} = Y_{\pm} \lambda$  ( $\lambda = 0.0001$  mm) and  $x_{\pm}$  and  $z_{\pm}$  have similar forms.

Step 5- Here the MATLAB random functions **randn**(1,  $M$ ) and **rand**(1,  $M$ ) return arrays of  $M$  random values (i.e., 1-by- $M$  matrix of values) values respectively drawn from the standard normal distribution (i.e., **randn**() function) and the uniform distribution (i.e., **rand** () function). In order to obtain 95% coverage interval in the  $PDF_f(\xi)$  that is suggested by Ref. [15] number of Monte Carlo iterations for random number generation should be  $M = 10^6$ . The two endpoints of the 95% statistical coverage interval for the  $PDF_f(\xi)$  are, using **prctile**() MATLAB function, given as: **prctile**( $PDF_f(\xi)$ , 2.5) and **prctile**( $PDF_f(\xi)$ , 97.5)

According to both experimental and theoretical work, in the following section we will present the analysis of our result based on the proposed validation method using the relations that mentioned in the previous five steps.

Feeding the data in Table 2 into equation no.7 through the random distribution forms (see step 4) one can get the  $PDF_f(\xi)$  versus  $f_{PDF}$  as shown in Fig.6(a) which is a histogram with the frequency

representing the probability distribution  $PDF_f(\xi)$  of the straightness. The  $PDF_f(\xi)$  is obtained with Monte Carlo simulation trials  $M = 10^6$  which is known to give a distribution with 95% coverage level. Indeed, this shows that the mathematical models have successfully recovered the zone of spatial straightness errors. From Fig.6(a) the endpoints for the coverage interval are (using MATLAB **prctile** functions) as follows:

$$\mathbf{prctile} (PDF_f(\xi), 97.5) = 0.0027 \text{ mm}; \quad \mathbf{prctile} (PDF_f(\xi), 2.5) = 0.0005 \text{ mm}$$

These give rise to the uncertainty for the straightness distribution was shown in the following equation:

$$\text{Uncertainty} = \mathbf{prctile} (PDF_f(\xi), 97.5) - \mathbf{prctile} (PDF_f(\xi), 2.5) = 0.0022 \text{ mm} \quad (8)$$

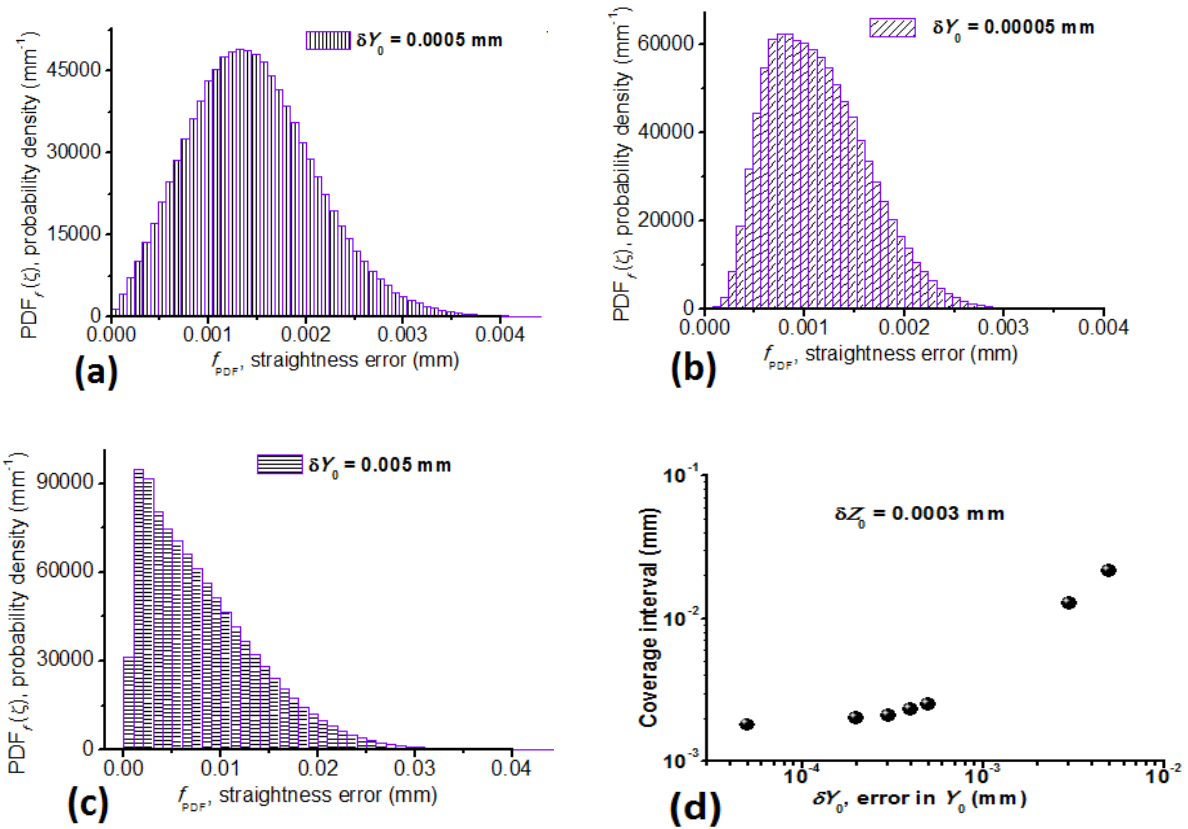


Figure 6. The probability density at  $\delta Z_0 = 0.0003 \text{ mm}$  as a function of the straightness error for (a)  $\delta Y_0 = 0.0005 \text{ mm}$  which is corresponding to error in the optimized  $\bar{Y}_0$  value in Table 2.



Manually reducing and increasing  $\delta Y_0$  by one order of magnitude changes the data in (a) to those in (b) and (c), respectively. (d) The resulting change in the coverage interval as  $\delta Y_0$  changes. The coverage interval in Fig. 6(d) represents the computed expanded uncertainty of the discrete points for 3D spatial measurement,  $U_{exp)3D}$ . The expanded uncertainty in straightness measurement using CMM has been calculated (2.2  $\mu\text{m}$ ). The 3D straightness deviation measurement using CMM discrete set of points is  $0.91 \pm 2.2 \mu\text{m}$  as in Table 2 and equation 8.

#### IV. VALIDATION OF THE PROPOSED METHOD

The proposed evaluation method of straightness measurement has been implemented into a computational program using real 3D measurement points (x, y and Z) for CMM slap surface and in 2D CNC-CMM measurement. To study the efficiency and validate the result from the proposed hybrid method, comparative studies were carried out between two cases as follows:

***In the first case of comparison:*** it can be seen that the above spatial deviation of the computed straightness deviation is around one order of magnitude (0.0005 mm) smaller (better) than the previously reported result ( $\sim 0.004$  mm) in Ref. [11]. Our improved computational methodology of the straightness deviation is expected to mainly be due to the repeatability in measurement of the data points as mentioned in step 2 at the section 3.1. The repeated measurement is expected to implicitly incorporate errors due to temperature fluctuations and other sources of errors. The validity of our approach may be justified based on the good accuracy in the straightness measurement signified by the small spatial extend of the straightness as can be seen from Fig 4.

Generally speaking our approach, which mainly relies on PSO calculations with input measured quantities having uncertainties derived from standard deviations of measured coordinates, can be regarded as a robust means of judging on the accuracy of the manufactured engineering components such as machine shafts and engine cylinders with pistons. The effect of the error  $\delta Y_0$  in the optimized midpoint ( $\bar{Y}_0$ ) on the data result in Fig. 5(a) is depicted in Fig. 5(b) and 5(c). It is clear that the histogram is skewed to the left and right as we increase and decrease  $\delta Y_0$  from its optimized value in Fig. 5(a). This behavior is generally seen from a view point of the coverage interval versus  $\delta Y_0$  shown in Fig. 5(d). Here we can see that at small  $\delta Y_0$  values the coverage interval is almost fixed near its optimized value calculated in equation (8). However, Fig. 5(d)

shows that the coverage interval is very sensitive to variation in  $\delta Y_0$  only as  $\delta Y_0$  deviates largely from its optimized value in Fig. 5(a). Similar behavior is observed for  $\delta Z_0$ .

***In the second case of comparison:*** 3D straightness deviation using proposed computational method for evaluation is calculated equal to  $0.91 \pm 2.20 \mu\text{m}$ . While in straightness deviation using the experimental method of 2D measurement is estimated equal to  $1.00 \pm 1.19 \mu\text{m}$ . In the other word, the average of 2D straightness deviation measurement using CNC-CMM scan is  $1.00 \pm 1.19 \mu\text{m}$ , while 3D straightness deviation measurement using SMC technique combined with PSO algorithm is  $0.91 \pm 2.20 \mu\text{m}$ . The difference between the two results is within the uncertainty estimated and it can be said that they are insignificant deferent. Thus, it can be said, the proposed hybrid method for estimating the straightness deviation is presented and found suitable for both 2D and 3D measurement. Comparison between 2D experimental result and 3D theoretical result of straightness deviation with expanded uncertainty is shown in the Fig.7. The validation by using these two types of comparison insures the confidence in the proposed method for estimating the deviation in the straightness measurement.

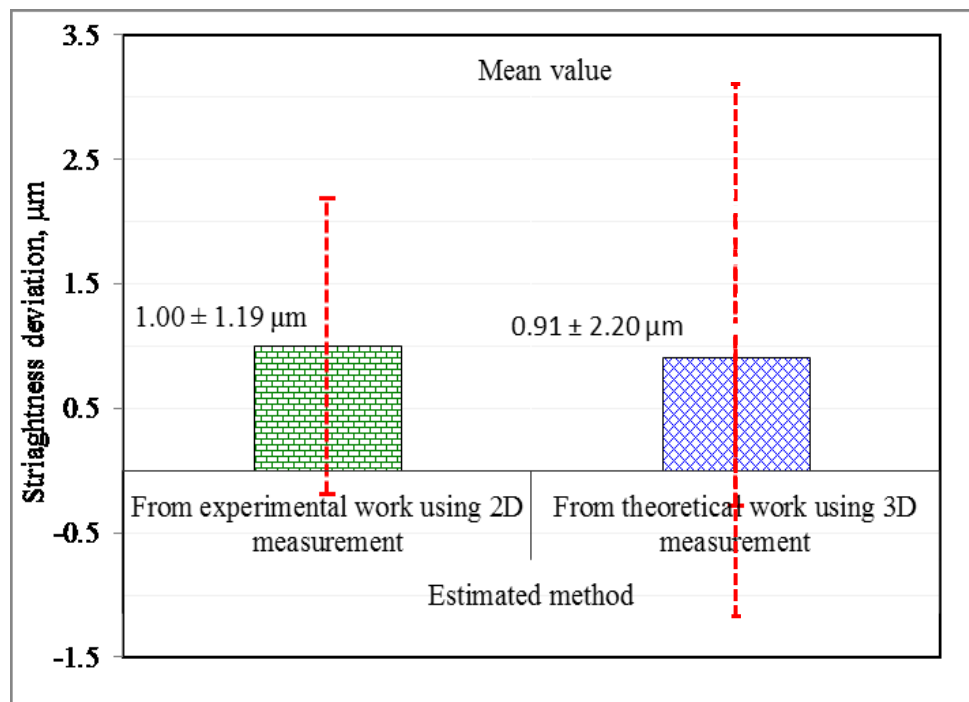


Figure 7. The straightness deviation and uncertainty for traditional 2D and proposed 3D methods

## V. CONCLUSIONS

Result analysis of the developed estimation method of straightness deviation in CMM measurement have been presented, discussed, interpreted and compared. The following conclusions are derived:

- A solution is found for estimation the straightness deviation and uncertainty of CMM straightness measurement on slab surface using a proposed hybrid methodology involving Sequential Monte Carlo (SMC) technique combined with Particle Swarm Optimization (PSO) algorithm. An uncertainty is estimated to be around one order of magnitude lower (better) than the calculated by Multilateration algorithm [11].
- The achieved accuracy is associated with the systematic analysis using PSO and SMC strategic methodology. Therefore the uncertainty in straightness measurement insures the ability of the proposed hybrid validation method to accompany CMM for reliable measurement and calibration of engineering parts. Hence, the efficiency and performance of the proposed methodology has practically been verified for CMM measurement.
- The evaluation result of the 3D estimation analysis using proposed method is valid compared to the 2D evaluation of the experimental measurement using CNC-CMM without any significant deferent.
- Estimating the straightness deviation and associated uncertainty have been successfully investigated theoretically and experimentally.
- Eventually, the proposed hybrid validation method for estimation the straightness deviation and uncertainty for CMM measurement is powerful tool and flexible to be applied.

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## REFERENCES

- [1] Salah H.R. Ali, Method of Optimal Measurement Strategy for Ultra-High-Precision Machine in Roundness Nanometrology, *Int. Journal on Smart Sensing and Intelligent Systems, Vol.8, No.2, pp.896-920, June 2015.*

- [2] Xiang Hua, Zhang Jinjin and Bin Lei, A New Three-Dimension Spatial Location Algorithm of Wireless Sensor Network, *Int. Journal on Smart Sensing and Intelligent Systems, Vol.9, No.1, pp.233-255, March 2016.*
- [3] Salah H.R. Ali and Omar M. Mohamd, Dimensional and Geometrical Form Accuracy of Circular Pockets Manufactured for Aluminum, Copper and Steel Materials on CNC Milling Machine using CMM, *Int. Journal of Engineering Research in Africa, Vol.17, pp.64-73,2015.*
- [4] Haining Zhang and Fu Bai, Automatic Measurement of Shape Parameters for Hydraulic Torque-Converter, *Int. Journal on Smart Sensing And Intelligent Systems, Vol.9, No.1, pp.89-107, March 2016.*
- [5] Salah H.R. Ali, Sarwat Z.A. Zahwi and Hassan H. Dadoura, Proposed Metrological Method for Identifying Automotive Brake Discs, *SAE Int.Journal of Fuels and Lubricants, Section: Surface Engineering and Automotive Tribology, Vol.8, Issue 1, pp.41-49,2015.*
- [6] Salah H.R. Ali, H.H. Dadoura and M.K. Bedewy, Identifying Cylinder Liner Wear using Precise Coordinate Measurements, *Springer, Int. Journal of Precision Engineering and Manufacturing, Vol.10, No.5, pp.19-25, Dec.2009.*
- [7] Salah H.R. Ali and Sarwat Z.A. Zahwi, CT Measurement and Geometrical Shape Analysis for Human Shoulder Glenoid, *Int. Review of Mechanical Engineering (I.RE.M.E.), Vol.8, No.2, pp.370-378, March 2014.*
- [8] Salah H.R., Developed Design of Humeral Head Replacement using 3D Surface Mapping, *Latvian Journal of Physics and Technical Sciences,(JPTS), Physics in Engineering, Vol.51, Issue 6, pp.41-55, 2014, Latvia. ISSN (Online) 0868-8257, DOI: 10.1515/lpts-2014-0035.*
- [9] Wen Xiulan , Xu Youxiong , Li Hongsheng, Wang Fenglin and Sheng Danghong, Monte Carlo Method for the Uncertainty Evaluation of Spatial Straightness Error based on New Generation Geometrical Product Specification, *Chinese Journal of Mechanical Eng., Vol.25, Issue 5, pp.875-881, 2012.*
- [10] Hennebelle F., Coorevits T. and Bigerelle M., Optimization of the Straightness Measurements on Rough Surfaces by Monte Carlo Simulation, *Wiley Periodicals Inc., The Journal of Scanning Microscopic, Vol.36(1), pp.161-169, Feb.2014.*
- [11] Mingzhao He, Xiaoyou Ye, Jianshuang Li and Xiaochuan Gan, Evaluation of Spatial Straightness Error using LaserTRACER, *Proc. SPIE 8759, Eighth Int. Symposium on*

*Precision Engineering Measurement and Instrumentation*, pp.1-5, January 2013.  
doi: 10.1117/12.2014642

- [12] Rosenda V. Arencibia, Claudio C. Souza, Henara L. Costa and Antonio P. Filho, Simplified Model to Estimate Uncertainty in CMM, *Journal of the Brazilian Society of Mechanical Sciences and Engineering*, Vol.37, Issue 1, pp.411-421, 2015.
- [13] LiMin Zhu, Ye Ding and Han Ding, Algorithm for Spatial Straightness Evaluation using Theories of Linear Complex Chebyshev Approximation and Semi-infinite Linear Programming, *J. Manuf. Sci. Eng.*, Vol.128(1), pp.167-174,2005.
- [14] Salah H.R. Ali and Hossam M. Sidki, CNC-CMM Measurement Accuracy and Accompanied Uncertainty at Different Alignment Positions of Long GBs, *Int. Review of Automatic Control (I.R.E.A.CO.)*, Vol.7, No.5, pp.485-491,2014.
- [15] BIPM, Joint Committee for Guides in Metrology, Evaluation of measurement data-Supplement 1 to the “Guide to the expression of uncertainty in measurement”-Propagation of distributions using a Monte Carlo method, *JCGM 101*, 2008.
- [16] JCGM 200:2012 International Vocabulary of Metrology – Basic and general concepts and associated terms (VIM), 3<sup>rd</sup> Edition Joint Committee for Guides in Metrology, Geneva, Switzerland, pp.1-108, 2012.
- [17] Kun Qian, Xudong Ma, Xian Zhong Dai and Fang Fang, Spatial-temporal Collaborative Sequential Monte Carlo for Mobile Robot Localization in Distributed Intelligent Environments, *Int. Journal on Smart Sensing And Intelligent Systems*, Vol.5, No.2, pp.295-314, June 2012.
- [18] James Kennedy and Russell Eberhart, Particle Swarm Optimization, *Proceedings of IEEE Int. Conference on Neural Networks*, Vol.4, pp.1942-1948, Nov.27- Dec. 1, 1995, Perth, Western Australia (WA).
- [19] S.; Ahmad, A.R.; Abdel-Rahman, E.M.; Naqvi, T., "A PSO Accelerated Immune Particle Filter for Dynamic State Estimation, 2011 Canadian Conference on Computer and Robot Vision (CRV), Newfoundland, Canada, IEEE Computer Society, pp.72-79, 25-27 May 2011.
- [20] Ali, S., Khalafalla, M., Naeim, I., and Zahwi, S., Proposed Validation Method for the Uncertainty Estimation of CMM Straightness Measurement Using PSO Algorithm and SMC Technique, *SAE Technical Paper 2016-01-0285*, pp.1-6, 2016, doi:10.4271/2016- 01-0285.

- [21] ISO 10360-6, Coordinate Metrology, International Standard: Geometrical Product Specifications (GPS)-Acceptance and Reverification Tests for Coordinate Measuring Machines (CMM)-Part 6: CMMs used for Measuring Size, 2001, Switzerland.
- [22] The American Association for Laboratory Accreditation (A2LA), P103d – Annex: Policy on Estimating Measurement Uncertainty for Construction Materials & Geotechnical Testing Labs, pp.1-28, Sep. 2010. The website on: [https://www.a2la.org/policies/A2LA\\_P103d.pdf](https://www.a2la.org/policies/A2LA_P103d.pdf)
- [23] Jingzhao Yang, Guoxi Li, Baozhong Wu, Jingzhong Gong, Jie Wang and Meng Zhang, Efficient Methods for Evaluating Task-Specific Uncertainty in Laser-Tracking Measurement, *MAPAN*, Vol.30, Issue 2, pp.105-117,2015.
- [24] SwarmOps, Numerical & Heuristic Optimization for Matlab, Version 1.1, 2015. The website on: <http://www.hvass-labs.org/projects/swarmops/matlab/>
- [25] Han Jiale, Li En, Tao Bingjie and Lv Ming, Reading Recognition Method of Analog Measuring Instruments based on Improved Hough Transform, 10<sup>th</sup> Int. Conference on Electronic Measurement & Instruments (ICEMI), China, Vol.3, pp.337-340, 16-19, 2011.