



ADAPTIVE INTELLIGENT INVERSE CONTROL OF NONLINEAR SYSTEMS WITH REGARD TO SENSOR NOISE AND PARAMETER UNCERTAINTY (MAGNETIC BALL LEVITATION SYSTEM CASE STUDY)

Yaghoub Pour Asad¹, Afshar Shamsi², Hoda Ivani³ and Jafar Tavoosi⁴

1. Faculty of Electrical Engineering, Urmia University of Technology, Urmia, Iran
 2. Faculty of Electrical Engineering, Tabriz University, Tabriz, Iran
 3. Faculty of Engineering, University of Sistan and Baluchestan, Zahedan, Iran
 4. Young Researchers and Elite Club, Ilam Branch, Islamic Azad University, Ilam, Iran
- Email: y.pourasad@uut.ac.ir, jtavoosi@aut.ac.ir

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Abstract-Type-2 Fuzzy Neural Networks have tremendous capability in identification and control of nonlinear, time-varying and uncertain systems. In this paper the procedure of designing inverse adaptive type-2 fuzzy neural controller for online control of nonlinear dynamical systems will be presented. At first the structure of a novel class of Interval Type-2 Nonlinear Takagi-Sugeno-Keng Fuzzy Neural Networks (IT2-NTSK-FNN) will be presented. There is a class of nonlinear function of inputs in the consequent part of fuzzy rules. This IT2-NTSK-FNN comprises seven layers and the fuzzification is done in two first layers including type-2 fuzzy neurons with uncertainties in the mean of Gaussian membership functions. Third layer is rule layer and model reduction occurs in fourth layer via adaptive nodes. Fifth, sixth and seventh layers are consequent layer, centroid rules' calculation layer and output layer respectively. For training the network backpropagation (steepest descend) method with adaptive training rate is used. Finally, three methods including online adaptive inverse controller based on IT2-NTSK-FNN, IT2-TSK-FNN (linear consequent part) and Adaptive Neuro-Fuzzy Inference System (ANFIS) are employed to control of a magnetic ball levitation system. External disturbances and uncertainty in parameters are considered in the model of magnetic ball levitation system. Simulation results show the efficacy of the proposed method.

Index terms: Nonlinear Type-2 Fuzzy, Adaptive Inverse control, Magnetic ball levitation System

I. INTRODUCTION

Computational intelligence is one of the effective and high performance methods in modeling and identification of an unknown system. High computation ability, adaptability and parallel processing are the important advantages of neural networks. A neural network can create a mapping between its input and output spaces by a set of connection weights and activation functions. Nowadays fuzzy neural net have attracted much attention because of they have simple topological structure, they have locally tuned neurons and they have ability to have a fast learning algorithm in comparison with other fuzzy systems. The applications of computational intelligence and intelligent algorithms are considerably growing. After introduction of fuzzy logic by Zadeh and its first application in control; intelligent systems with pioneering fuzzy logic quickly spread to varieties of engineering contexts especially in control [1-3]. Ten years later, Zadeh introduced type-2 fuzzy logic which obviated some drawbacks of type-1 fuzzy logic. In type-1 fuzzy logic, the membership value is non-fuzzy or crisp which hardens exact determination of this value, nevertheless the membership values in type-2 fuzzy logic are fuzzy sets. As an instance type-2 fuzzy logic can be useful in cases where linguistic variables are inexact and uncertain or different results may be interpreted from some conditions close to each other [4]. Type-1 FLC is unable to handle the linguistic and numerical uncertainties which are associated with dynamic unstructured environment. But type-2 fuzzy sets have the capability to determine the exact membership function for a specified fuzzy set [5]. Type-2 fuzzy logic with its capability and flexibility more than type-1 fuzzy logic has been fully considered in recent ten years [6-11]. Castillo and Melin discussed type-2 fuzzy logic and systems in details [12]. In [13,14] type-1 fuzzy neural network is used to control of robot arm and water bath systems. Tavoosi and Badamchizadeh proposed type-2 Takagi-Sugeno-Kang fuzzy neural network with linear consequent part [15]. Rule pruning was the novelty of that paper. Higher learning speed was goal by reducing the parameter in both antecedent and consequent parts.

Casrto et al. presented three different structures of type-2 fuzzy neural networks [16]. They proposed two fuzzifying methods (type-2 fuzzy neural and adaptive node) and two reduction method (Karnik-Mendel algorithm and adaptive layers) and used backpropagation method for network training. In [17], type-2 fuzzy neural network by use of fuzzy clustering is used for structural identification and updating parameters of conditions while backpropagation method

is used for updating parameters of result. They confessed that fuzzy clustering method is not appropriate for online identification and control. In recent years, variety of methods has been suggested for training type-2 fuzzy neural networks such as Genetic Algorithm (GA) [18] and Particle Swarm Optimization (PSO) [19]. By daily growing research on type-2 fuzzy systems, these systems have found excessive applications such as time series prediction [20], linear motor control [21], system identification and modeling [22], sliding mode control [23], pattern recognition [24], and robot control [25].

Adaptive inverse control using computation intelligence for nonlinear dynamical system has received much attention in recent years. The aim of this technique is to inversely identify the dynamic of the process using its outputs as inputs of the model [26]. Li et al. suggested inverse control method using type-2 fuzzy neural network for controlling Cable-Driven Parallel Mechanism (CDPM) [27]. In this work, interval type-2 fuzzy neural network is used for inverse identification and then trained network is used as offline controller. In [28], ANFIS inverse control is compared with fuzzy control method to control the level of the water in a tank. Kadhim has shown that ANFIS inverse control operates faster than fuzzy controller and has no steady state error. Juang and Chen used recurrent fuzzy neural network implemented on FPGA to inverse control temperature of bath water [29]. They compared inverse control of water temperature using inverse fuzzy neural controller with neural network controller and they showed that fuzzy neural controller outperforms neural controller. In [30], adaptive inverse optimal control is used to control a magnetic levitation system. Experimental results indicate effectiveness of this method for controlling magnetic levitation system. Panduro et al has used sliding mode to control magnetic levitation system [31]. They used a combination of Output Regulation Theory (ORT) and sliding mode controllers however this method imposes a non-zero steady state error to the position of the levitated object. Some studies have been done in fuzzy control of magnetic levitation. In [32] a linear model of magnetic levitation system is controlled by simple type-2 fuzzy controller. In this paper single input of type-2 fuzzy controller is the sign of distance .The simulation shows better performance of the proposed controller compared to an IT2FLC and IT1FLC controller. Salim and Karsli presented the difference between the performance of fuzzy logic control (FLC) and LQRC for the same linear model of magnetic levitation system [33]. Their results of simulation show that the fuzzy logic controller had better performance than the LQR control. There are some studies about type-1 fuzzy control of magnetic levitation in [34-36]. In [37] robust adaptive inverse control of a class of nonlinear systems with prandtl-ishlinskii hysteresis model has been presented.

It is clear that nonlinear model has better performance than linear model for nonlinear system identification [38]. So this paper uses nonlinear then part in each fuzzy rules. There are few studies about nonlinear consequent (then) part in fuzzy systems up to now. In the following some of the works in this area are reviewed. Moodi and Farrokhi proposed T-S model with nonlinear consequent to reduce the number of rules in a fuzzy system [39]. The consequent part of each rule is assumed to contain a linear part plus a sector-bounded nonlinear term. A priori it seems that this method increases the complexity of the fuzzy model, whereas it decreases the number of rules and at the same time increases the model accuracy. Abiyev et al. presented a Nonlinear Neuro-Fuzzy Network (NNFN) for equalization of channel distortion [40]. Their NFNN is constructed by using fuzzy rules that incorporate nonlinear functions. Sometimes linear then part fuzzy systems need more rules, during modeling complex nonlinear processes in order to obtain the desired accuracy. Increasing the number of the rules leads to the increasing the number of neurons in the hidden layer of the network. To improve the computational power of neuro-fuzzy system, they used nonlinear functions in the consequent part of each rule.

This paper presents a novel nonlinear type-2 fuzzy system with nonlinear then part in fuzzy rules for magnetic levitation system control. Both nonlinear type-2 fuzzy system and control strategy are the novelty of this paper. The proposed type-2 fuzzy system has seven layers. The present study organized as follows: in section 2 type-2 fuzzy logic and systems will be briefly introduced and then in section 3 a structure of IT2-NTSK-FNN is given. Also procedure of designing inverse type-2 fuzzy neural controller for nonlinear dynamical systems is explained. Nonlinear dynamic of magnetic ball levitation system is described in section 4. In section 5 procedure of designing adaptive inverse type-2 fuzzy neural network controller using sugeno model is characterized. Simulation results are given in section 6 to show the efficiency of proposed method. Finally the study is summarized in section 7.

II. Type-2 Fuzzy Logic and Systems

In type-1 fuzzy sets, the membership degree is a crisp number, but in type-2 fuzzy sets, the membership degree is a type-1 fuzzy number. In some systems such as time-series prediction, the exact membership degree is determined in a very difficult manner due to their complexity and their noisy information [41].

In general, a type-2 fuzzy set has the following form [42]:

$$\tilde{A} = \int_{x \in X} \mu_{\tilde{A}}(x) / x = \frac{\int_{x \in X} [\int_{\mu \in J_x} \frac{f_x(\mu)}{\mu}]}{x} \quad (1)$$

In equation (1) \tilde{A} is a type-2 fuzzy set, $\mu_{\tilde{A}}(x)$ is initial membership function, J_x is sum of initial membership value for $x \in X$ and $f_x(\mu) \in [0,1]$ is secondary membership function. Presentation of a type-2 fuzzy set requires dealing with three dimensions and calculations in these systems are very numerous and complicated. As an alternative interval type-2 fuzzy sets are defined. Interval type-2 fuzzy sets are special case of general type-2 fuzzy sets when following condition is hold:

$$f_x(\mu) = 1, \forall \mu \in J_x \subseteq [0,1]$$

With above condition initial membership functions are in the form of interval sets. For presenting a Gaussian interval type-2 fuzzy membership function there are two ways: uncertain mean and uncertain standard deviation. In case of uncertain mean, standard deviation (width) of Gaussian function has a fixed value σ while mean of the function is not fixed and can take any value in the interval $[m1, m2]$. Similarly in case of uncertain standard deviation the center of Gaussian function is fixed at m but width of the function changes in the interval $[\sigma_1, \sigma_2]$. Figure 1 illustrates the Gaussian functions in two cases.

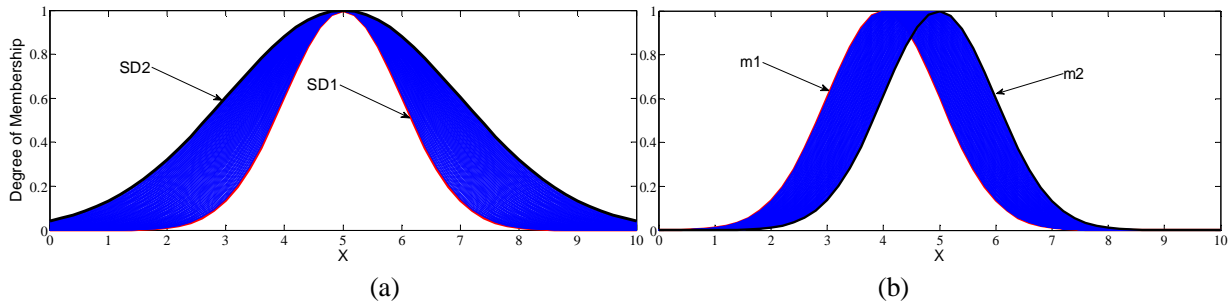


Figure 1. Gaussian membership function in case of uncertain mean (a) and uncertain standard deviation (b)

The region of uncertainty in type-2 fuzzy functions is called Footprint Of Uncertainty (FOU) [43]. Hatched parts in figure1 are FOU of type-2 membership functions. In type-2 fuzzy systems lower and higher bounds of membership functions are called Lower Membership Functions (LMF) and Upper Membership Functions (UMF) respectively.

The main difference between type-1 and type-2 fuzzy systems is the form of their membership functions. Despite the fact that the output of a type-1 fuzzy system is a type-1 fuzzy set, a crisp value can be obtained by defuzzification of the system output. In type-2 fuzzy systems on the other hand, outputs are type-2 fuzzy sets whose outcome after defuzzification are

type-1 fuzzy sets. The procedure of converting type-2 fuzzy sets to type-1 fuzzy sets is named "type reduction" which is a very important issue in analysis of type-2 fuzzy systems [43]. Configuration of a type-2 fuzzy system is shown in figure 2.

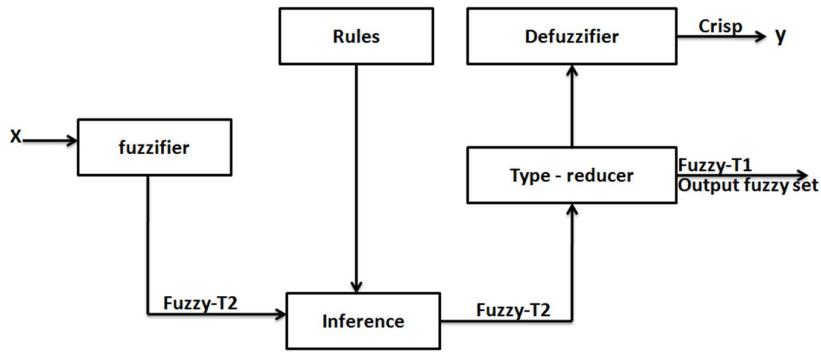


Figure 2. Configuration of type-2 fuzzy system

The structure of type-2 fuzzy systems are similar to type-1 fuzzy systems and as shown in figure 2 the only extra part in type-2 fuzzy systems is type reduction block.

III. INTERVAL TYPE-2 TAKAGI-SUGENO-KANG FUZZY NEURAL NETWORK

In IT2-NTSK-FNN like type-1 fuzzy neural networks, outputs are in the form of polynomials of inputs nevertheless outputs and their coefficients in type-1 fuzzy neural networks are crisp values but in type-2 fuzzy neural networks the outputs and their coefficient are type-1 fuzzy values [44]. Proposed IT2-NTSK-FNN is composed of seven layers that in first two layers an interval type-2 fuzzy neuron is used for fuzzifying. Type reduction occurs in fourth layer using adaptive weights. Structure of the proposed type-2 fuzzy neural network with TSK model is illustrated in figure 3.

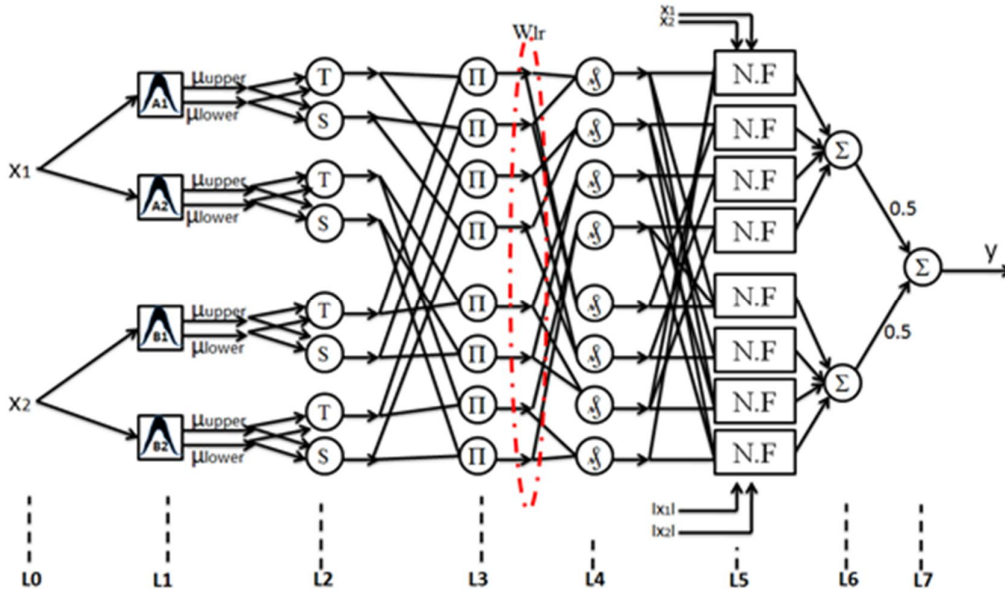


Figure 3. Structure of the proposed IT2-NTSK-FNN

A fuzzy rule for type-2 TSK fuzzy model is written as:

$$R^k : \text{if } x_1 \text{ is } \tilde{A}_1^k \text{ and } \dots \text{ and } x_n \text{ is } \tilde{A}_n^k \text{ then } \tilde{y}_k = C_{k,0} + C_{k,1}\phi_{k,1}(x_1) + \dots + C_{k,n}\phi_{k,n}(x_n)$$

In above rule $X = \{x_1, x_2, \dots, x_n\}$ is input of the system, $\{\tilde{A}_1^k, \tilde{A}_2^k, \dots, \tilde{A}_n^k\}$ are type-2 membership functions, \tilde{y}_k is the system output and has a fuzzy value and

$$C_{k,i} \in [c_{k,i} - s_{k,i}, c_{k,i} + s_{k,i}], \quad i = 1, 2, \dots, n$$

are type-1 fuzzy sets and coefficients of result part where $c_{k,i}$ and $s_{k,i}$ indicate mean and width of Gaussian membership functions. It is worth mentioning that subsystems with the above description are from the class of so-called Φ -systems [45]. In Φ -systems,

$$\phi_k \in OL := \{\varphi: \mathbb{R} \rightarrow \mathbb{R} \mid \forall s, t \in \mathbb{R}, |\varphi(s) + \varphi(t)| \leq |s + t|\}$$

It is clear from the description that the nonlinearity of this class of systems is odd and 1-Lipschitz. The standard saturation and the hyperbolic tangent (popular activation function in neural network) are examples of this type of nonlinear systems. The discrete-time recurrent artificial neural network is a special case of Φ -systems [46,47]. Furthermore, results related to this class of nonlinear systems have potential applications in the classical problems related to uncertain nonlinearities such as Lur'e systems [48].

Now all layers are considered in details:

Layer 0: This is input layer. There are as many nodes as the number of inputs.

Layer 1: This is the fuzzifying layer. The output of this layer is as follows:

$${}^1\mu_{k,i}(x_i, [\sigma_{k,i}, {}^1m_{k,i}]) = e^{-.5\left(\frac{x_i - {}^1m_{k,i}}{\sigma_{k,i}}\right)^2} \quad (2)$$

$${}^2\mu_{k,i}(x_i, [\sigma_{k,i}, {}^2m_{k,i}]) = e^{-.5\left(\frac{x_i - {}^2m_{k,i}}{\sigma_{k,i}}\right)^2} \quad (3)$$

Layer 2: in this layer by means of T-norm and S-norm the output of a type-2 fuzzy neuron which is the upper and lower bounds of membership function is calculated.

$$\underline{\mu}_{k,i}(x_i) = {}^1\mu_{k,i}(x_i) \times {}^2\mu_{k,i}(x_i) \quad (4)$$

$$\bar{\mu}_{k,i}(x_i) = {}^1\mu_{k,i}(x_i) + {}^2\mu_{k,i}(x_i) - \underline{\mu}_{k,i}(x_i) \quad (5)$$

Layer 3: This is rules layer. In this layer AND operation is done as following:

$$\underline{f}^k = \prod_{i=1}^n \underline{\mu}_{k,i} \quad ; \quad \bar{f}^k = \prod_{i=1}^n \bar{\mu}_{k,i} \quad (6)$$

Layer 4: In this layer type reduction is accomplished using weighted adaptive nodes and right and left values of their fire point are computed as

$$f_l^k = \frac{\bar{w}_l^k \bar{f}^k + \underline{w}_l^k f^k}{\bar{w}_l^k + \underline{w}_l^k} \quad ; \quad f_r^k = \frac{\bar{w}_r^k \bar{f}^k + \underline{w}_r^k f^k}{\bar{w}_r^k + \underline{w}_r^k} \quad (7)$$

Layer 5: This is called result layer.

$$y_l^k = c_{k,0} + c_{k,1}\phi_{k,1}(x_1) + \dots + c_{k,n}\phi_{k,n}(x_n) - s_{k,0} - s_{k,1}|\phi_{k,1}(x_1)| - \dots - s_{k,n}|\phi_{k,n}(x_n)| \quad (8)$$

$$y_r^k = c_{k,0} + c_{k,1}\phi_{k,1}(x_1) + \dots + c_{k,n}\phi_{k,n}(x_n) + s_{k,0} + s_{k,1}|\phi_{k,1}(x_1)| + \dots + s_{k,n}|\phi_{k,n}(x_n)| \quad (9)$$

Layer 6: There are two nodes in this layer which calculate the centroid of the whole result rules.

$$\hat{y}_l = \frac{\sum_{k=1}^M f_l^k y_l^k}{\sum_{k=1}^M f_l^k} \quad (10)$$

$$\hat{y}_r = \frac{\sum_{k=1}^M f_r^k y_r^k}{\sum_{k=1}^M f_r^k} \quad (11)$$

Layer 7: this layer has one node which calculates output of the network.

$$\hat{y} = \frac{\hat{y}_l + \hat{y}_r}{2} \quad (12)$$

For training the neural network back propagation algorithm is utilized. In this algorithm output of the network is computed for every input and then error is obtained. Suppose input-output pairs of the system for the training the network are known as

$$\{(x_p: t_p)\} \forall p = 1, \dots, q$$

Now output error of the system is defined as:

$$e_p = t_p - \hat{y}_p \quad (13)$$

$$E_p = \frac{1}{2} e_p^2 = \frac{1}{2} (t_p - \hat{y}_p)^2 \quad (14)$$

$$E = \sum_{p=1}^q E_p \quad (15)$$

In above relations q is the number of all input-output pairs for network training. Updating rules of unknown parameters in IT2-NTSK-FNN are described in equations (16) to (19) as follows:

$$new \underline{w}_l^k = old \underline{w}_l^k + \eta * 0.5 * e_p * \frac{y_l^k - \hat{y}_l}{\sum_{j=1}^M f_l^j} * \frac{f_l^k - \bar{f}_l^k}{\bar{w}_l^k + \underline{w}_l^k} \quad (16)$$

$$new \bar{w}_l^k = old \bar{w}_l^k + \eta * 0.5 * e_p * \frac{y_l^k - \hat{y}_l}{\sum_{j=1}^M f_l^j} * \frac{\bar{f}_l^k - f_l^k}{\bar{w}_l^k + \underline{w}_l^k} \quad (17)$$

$$new \underline{w}_r^k = old \underline{w}_r^k + \eta * 0.5 * e_p * \frac{y_r^k - \hat{y}_r}{\sum_{j=1}^M f_r^j} * \frac{f_r^k - \bar{f}_r^k}{\bar{w}_r^k + \underline{w}_r^k} \quad (18)$$

$$new \bar{w}_r^k = old \bar{w}_r^k + \eta * 0.5 * e_p * \frac{y_r^k - \hat{y}_r}{\sum_{j=1}^M f_r^j} * \frac{\bar{f}_r^k - f_r^k}{\bar{w}_r^k + \underline{w}_r^k} \quad (19)$$

In above equations k = 1, 2, ..., M is the number of rules. Updating rules of unknown parameters in result part is given in below:

$$new c_{k,i} = old c_{k,i} + \eta * 0.5 * e_p \left[\frac{f_r^k}{\sum_{k=1}^M f_r^k} + \frac{f_l^k}{\sum_{k=1}^M f_l^k} \right] * \phi_{k,i}(x_i) \quad (20)$$

$$new c_{k,0} = old c_{k,0} + \eta * 0.5 * e_p \left[\frac{f_r^k}{\sum_{k=1}^M f_r^k} + \frac{f_l^k}{\sum_{k=1}^M f_l^k} \right] \quad (21)$$

$$new s_{k,i} = old s_{k,i} + \eta * 0.5 * e_p \left[\frac{f_r^k}{\sum_{k=1}^M f_r^k} - \frac{f_l^k}{\sum_{k=1}^M f_l^k} \right] * |\phi_{k,i}(x_i)| \quad (22)$$

$$new s_{k,0} = old s_{k,0} + \eta * 0.5 * e_p \left[\frac{f_r^k}{\sum_{k=1}^M f_r^k} - \frac{f_l^k}{\sum_{k=1}^M f_l^k} \right] \quad (23)$$

Updating rules of unknown parameters related to condition section (center and width of Gaussian membership functions) are given as following:

$${}^1 m_{k,i}^{new} = {}^1 m_{k,i}^{old} + \eta * 0.5 * e_p \left[\frac{y_l^k - \hat{y}_l}{\sum_{j=1}^M f_l^j} * \frac{\partial f_l^k}{\partial {}^1 m_{k,i}} + \frac{y_r^k - \hat{y}_r}{\sum_{j=1}^M f_r^j} * \frac{\partial f_r^k}{\partial {}^1 m_{k,i}} \right] \quad (24)$$

$${}^2 m_{k,i}^{new} = {}^2 m_{k,i}^{old} + \eta * 0.5 * e_p \left[\frac{y_l^k - \hat{y}_l}{\sum_{j=1}^M f_l^j} * \frac{\partial f_l^k}{\partial {}^2 m_{k,i}} + \frac{y_r^k - \hat{y}_r}{\sum_{j=1}^M f_r^j} * \frac{\partial f_r^k}{\partial {}^2 m_{k,i}} \right] \quad (25)$$

$$\sigma_{k,i}^{new} = \sigma_{k,i}^{old} + \eta * 0.5 * e_p \left[\frac{y_l^k - \hat{y}_l}{\sum_{j=1}^M f_l^j} * \frac{\partial f_l^k}{\partial \sigma_{k,i}} + \frac{y_r^k - \hat{y}_r}{\sum_{j=1}^M f_r^j} * \frac{\partial f_r^k}{\partial \sigma_{k,i}} \right] \quad (26)$$

The whole algorithm is accomplished in this manner: First each input is applied to the system and using equations (2) to (12) output of the network is computed and by means of equation (13) error is obtained. Then by use of equations (16) to (26) unknown parameters are adjusted

in such a way that output error between IT2-NTSK-FNN and real system is minimized. Lyapunov function is used to guarantee of learning algorithm convergence. Let lyapunov function is as eq. (27).

$$V_p(k) = E_p(k) = \frac{1}{2} e_p^2(k) = \frac{1}{2} (t_p(k) - \hat{y}_p(k))^2 \quad (27)$$

Eq. (28) shows the change of layapunov function.

$$\Delta V_p(k) = V_p(k+1) - V_p(k) = \frac{1}{2} (e_p^2(k+1) - e_p^2(k)) \quad (28)$$

Next moment error is calculated from eq. (29) [40].

$$e_p(k+1) = e_p(k) + \Delta e_p(k) \cong e_p(k) + \left[\frac{\partial e_p(k)}{\partial W} \right]^T \Delta W \quad (29)$$

In eq. (29), ΔW is parameter changing where $W = [\sigma_{k,i}, {}^1m_{k,i}, {}^2m_{k,i}, c_{k,i}, s_{k,i}, \gamma_i]$

Back Propagation (BP) algorithm is used to update the unknown parameters in IT2-NTSK-FNN is described in eq. (30).

$$W(k+1) = W(k) + \Delta W(k) = W(k) + \eta * \left(-\frac{\partial E_p(k)}{\partial W} \right) \quad (30)$$

Where,

$$\frac{\partial E_p(k)}{\partial W} = -e_p(k) * \frac{\partial \hat{y}}{\partial W} \quad (31)$$

Eq. (28) can be rewritten as eq. (32).

$$\begin{aligned} \Delta V_p(k) &= \frac{1}{2} (e_p^2(k+1) - e_p^2(k)) \quad (32) \\ &= \frac{1}{2} [(e_p(k+1) - e_p(k))] * [(e_p(k+1) + e_p(k))] \\ &= \frac{1}{2} \Delta e_p(k) [2(e_p(k)) + \Delta e_p(k)] \\ &= \Delta e_p(k) \left[e_p(k) + \frac{1}{2} \Delta e_p(k) \right] \\ &= \left[\frac{\partial e_p(k)}{\partial W} \right]^T * \eta * e_p(k) * \frac{\partial \hat{y}(k)}{\partial W} * \left\{ e_p(k) + \frac{1}{2} \left[\frac{\partial e_p(k)}{\partial W} \right]^T * \eta * e_p(k) * \frac{\partial \hat{y}(k)}{\partial W} \right\} \\ &= - \left[\frac{\partial \hat{y}(k)}{\partial W} \right]^T * \eta * e_p(k) * \frac{\partial \hat{y}(k)}{\partial W} * \left\{ e_p(k) - \frac{1}{2} \left[\frac{\partial \hat{y}(k)}{\partial W} \right]^T * \eta * e_p(k) * \frac{\partial \hat{y}(k)}{\partial W} \right\} \\ &= -\eta * (e_p(k))^2 \left| \frac{\partial \hat{y}(k)}{\partial W} \right|^2 * \left[1 - \frac{1}{2} \eta * \left| \frac{\partial \hat{y}(k)}{\partial W} \right|^2 \right] \end{aligned}$$

In order that $\Delta V_p(k) < 0$, the eq. (33) must be satisfied

$$0 < \eta < \frac{2}{\left| \frac{\partial y(k)}{\partial W} \right|^2} \quad (33)$$

If for every parameter $W = [\sigma_{k,i}, {}^1m_{k,i}, {}^2m_{k,i}, c_{k,i}, S_{k,i}, \gamma_i]$, its η held in eq. (33) then convergence is guaranteed. For example in updating ${}^1m_{k,i}$, $\eta_{{}^1m_{k,i}}$ must be held in

$$0 < \eta_{{}^1m_{k,i}} < \frac{2}{\left| \frac{\partial y(k)}{\partial {}^1m_{k,i}} \right|^2}.$$

IV. MAGNETIC BALL LEVITATION SYSTEM

Magnetic ball levitation System (MLS) is very unstable, nonlinear and complex system that can be applied in many application area such as in high speed transport, magnetic bearing system, vibration isolation, levitation of wind power generation and fusion Energy Materials processing in magnetic ball levitation furnaces. The purpose of the controller is to keep the steel ball suspended in air, at the nominal equilibrium position by controlling the current in the magnet.

Magnetic ball levitation system in this paper is a second-order unstable nonlinear system. A sketch of this system is shown in figure 4.

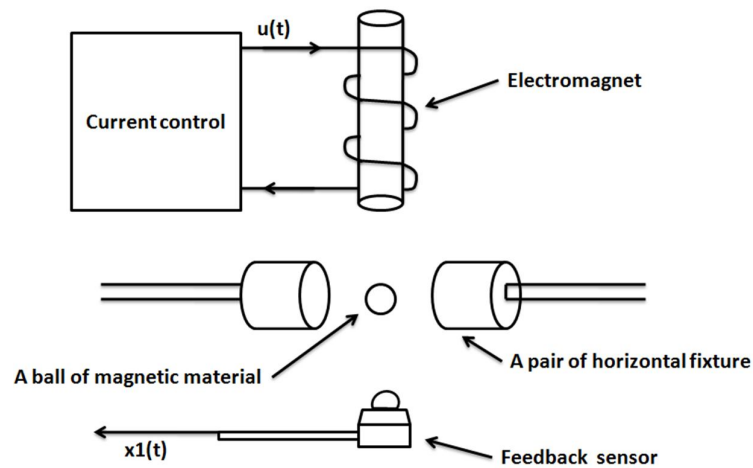


Figure 4. Schematic view of magnetic ball levitation system

As shown in figure 4 the magnetic ball levitation system is composed of four parts: a ball of magnetic material, a pair of magnets to fix the ball horizontally, an electromagnet and a sensor. The goal is levitating the ball in a fixed position with specific distance from sensor. Control signal is the applied voltage to the electromagnet whose increase (decrease) cause increase (decrease) of the magnetic field produced by electromagnet and avoids the ball to stick to the coil (fall). Sensor measures the distance at every sample time.

Nonlinear dynamic of magnetic ball levitation system is given by [49]:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -\frac{k}{m}x_2 + g - \frac{L_0 a u^2}{2m(a+x_1)^2} \end{cases} \quad (27)$$

In (27), x_1 (m) is the distance between ball and coil, x_2 (m/s) is relative velocity of ball, $m = 0.1$ kg is mass of ball, $k = 0.001$ N/m/s is friction coefficient, $g = 9.806$ is gravitational force of earth, $L_0 = 0.02$ H is inductance of the electromagnet, $a = 0.005$ m is a fixed constant and u (A) is input current (control signal).

V. DESIGNING ADAPTIVE INVERSE CONTROL USING IT2-NTSK-FNN

Abilities of type-2 fuzzy neural networks in approximation of functions capacitate them to identify and control variety of systems using these networks. In this section two different methods for designing adaptive inverse control using IT2-NTSK-FNNis represented. The structures of these two types are shown in figure 4 and 5.

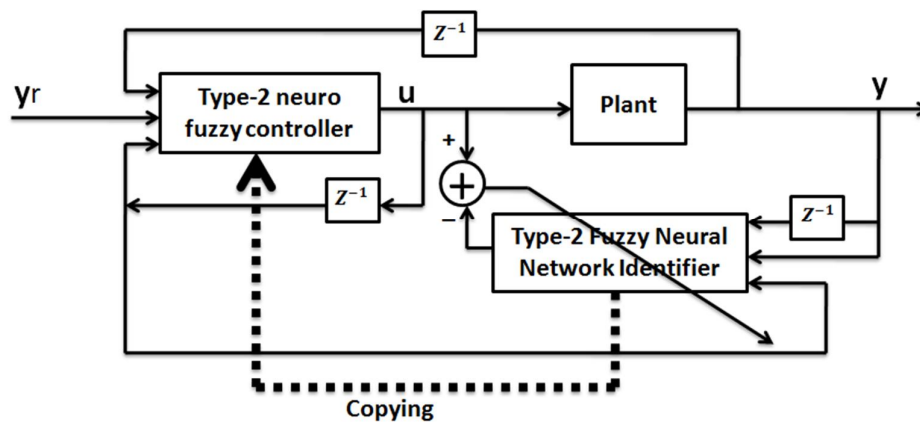


Figure 4. Adaptive inverse control (first structure)

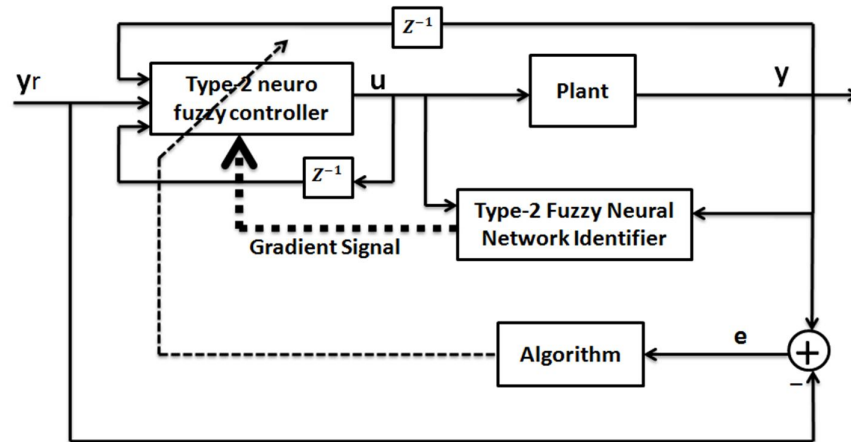


Figure 5. Adaptive inverse control (second structure)

In both structures y is the system output, u is control signal and y_r is reference signal. As it's clear two type-2 fuzzy neural networks are used, one for identification and the other one as controller. Identifier network is used for adjusting parameters of controller network. In first structure a type-2 fuzzy neural network identifies the inverse model of the system and simultaneously acts as controller. In this structure the goal is to minimize control signal and consequently reduce the difference between system output and reference signal. In second structure a type-2 fuzzy neural network as an identifier calculates the gradient of the output to the input ($\frac{\partial y}{\partial u}$) at each step. Here error between system output and reference output and also obtained gradient value are used to train the type-2 fuzzy neural network controller online [50].

VI. SIMULATION RESULTS

In this section simulation results of adaptive inverse control of magnetic ball levitation system using type-2 fuzzy neural network is given and then first structure of inverse control will be applied online. For simulations two inputs for type-2 fuzzy neural network identifier and controller are used. The structure of adaptive inverse control using type-2 fuzzy neural network is shown in figure 6.

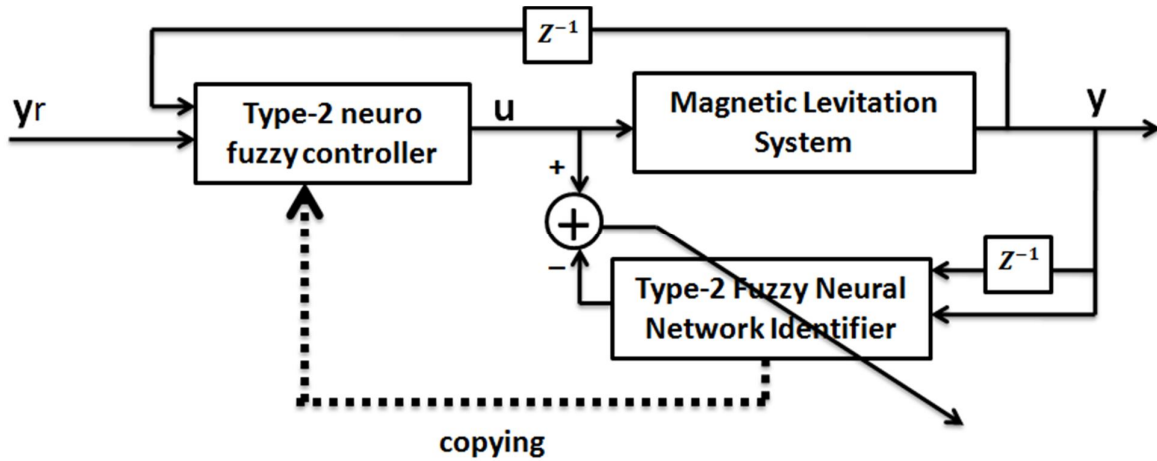


Figure 6. Structure of adaptive inverse control of magnetic ball levitation system using type-2 fuzzy neural network

In the structure of inverse controller used for magnetic ball levitation system (figure 6) inputs of type-2 fuzzy neural network identifier are $y(k)$ and $y(k - 1)$ and inputs of type-2 fuzzy neural network controller are $y_r(k)$ and $y(k - 1)$. For each one of inputs three type-2 Gaussian membership functions with uncertainty in their centers are assigned. By optimally choosing three rules from nine rules, the number of rules can be reduced without significant effect on the result. By reducing the number of rules, the number of parameters in result part is decreased and therefore speed of type-2 fuzzy neural network in online applications is significantly escalated.

In figure 7, simulation results for desired ball position (reference signal), position of the ball using adaptive inverse control based on ANFIS, IT2-TSK-FNN and IT2-NTSK-FNN are illustrated. Desired value of the ball is 5 cm in first 10 seconds and 4cm in the following 10 seconds.

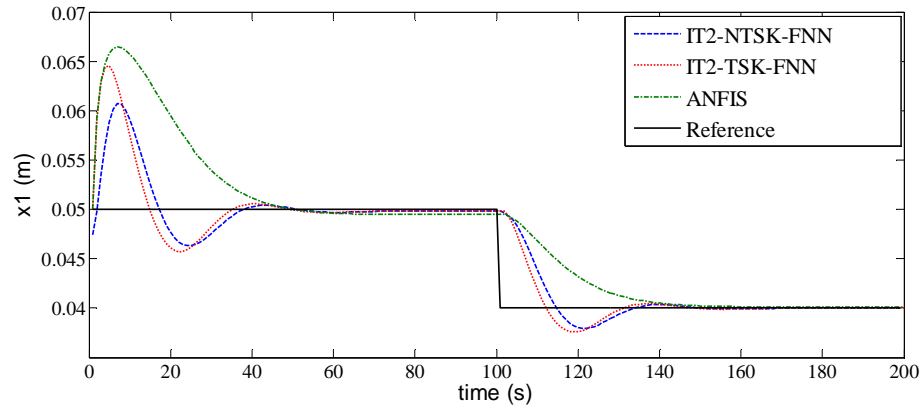


Figure 7. Desired position of the ball, ball position using adaptive inverse control based on ANFIS, IT2-TSK-FNN and IT2-NTSK-FNN

In figures 8, 9 and 10, simulation results for desired ball position (reference signal), position of the ball using adaptive inverse control based on ANFIS, IT2-TSK-FNN and IT2-NTSK-FNN regard to noise, uncertainty in $m \pm 15\%$ and $\pm 35\%$ are illustrated respectively.

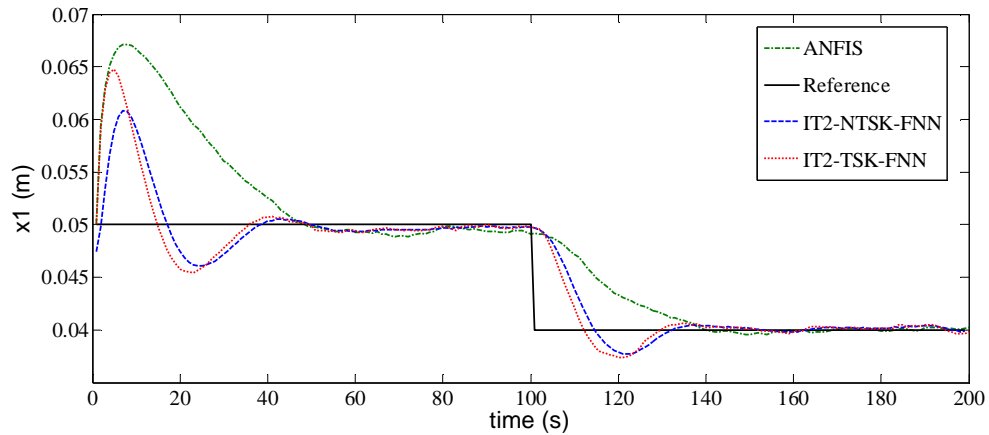


Figure 8. Desired position of the ball, ball position using adaptive inverse control based on ANFIS, IT2-TSK-FNN and IT2-NTSK-FNN regard to noise

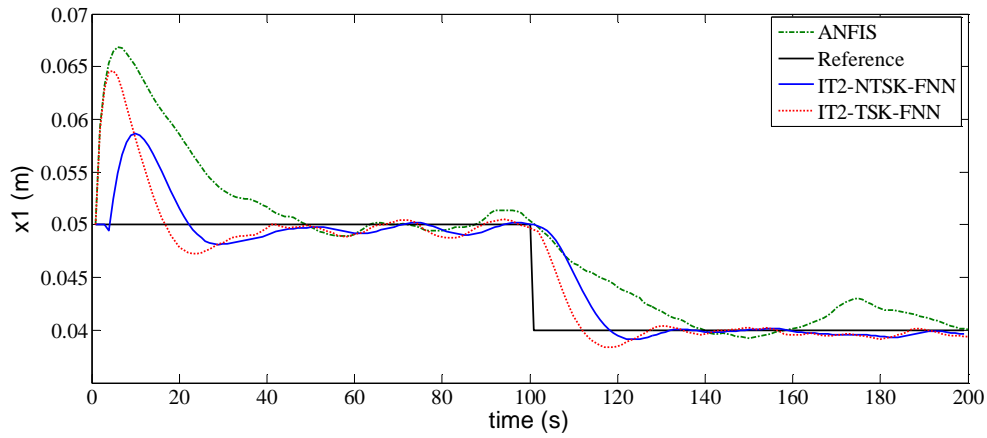


Figure 9. Desired position of the ball, ball position using adaptive inverse control based on ANFIS, IT2-TSK-FNN and IT2-NTSK-FNN regard to parameter uncertainty in $m \pm 15\%$.

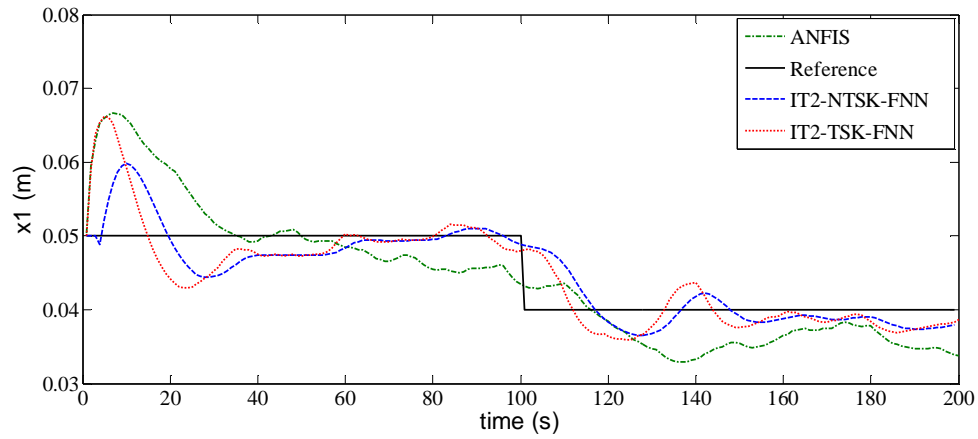


Figure 10. Desired position of the ball, ball position using adaptive inverse control based on ANFIS, IT2-TSK-FNN and IT2-NTSK-FNN regard to parameter uncertainty in $m \pm 35\%$.

Control signal (coil current) using adaptive inverse control based on ANFIS, IT2-TSK-FNN and IT2-NTSK-FNN is shown in figure 11.

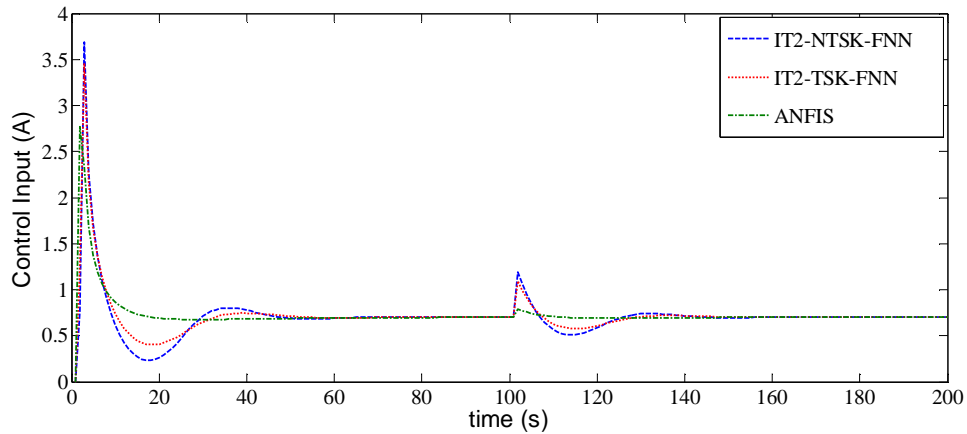


Figure 11. Control signal (coil current) using adaptive inverse control based on ANFIS, IT2-TSK-FNN and IT2-NTSK-FNN without noise and uncertainty

As it is seen in figure 7 position of the ball for adaptive inverse control based on IT2-NTSK-FNN has peaks with less magnitude than adaptive inverse control based on ANFIS and IT2-TSK-FNN and maintains the ball in the vicinity of desired point. Figure 8 shows that the results of all three controllers are almost identical given that there is noise. Figures 9 and 10 show that type-1 fuzzy system (ANFIS) is not suitable controller in uncertain systems especially if there is large uncertainty. In all figures it is clear that IT2-NTSK-FNN has better performance than IT2-TSK-FNN because of nonlinear system with nonlinear terms requires a nonlinear controller. Although IT2-TSK-FNN is also nonlinear controller but in inverse identification phase, the IT2-NTSK-FNN with nonlinear then part can better identify inverse of nonlinear system.

In first two seconds gravitational force of the earth pulls down the ball and the coil uses high current to avoid the ball from falling (figure 11). About third second system achieves its steady state and the coil current approaches a constant value to levitate the ball in 5cm. In tenth second the ball is ordered to elevate 1 cm toward the coil to reach the distance of 4cm so there is a peak in coil current right after step command. After reaching steady state coil current approaches to constant value of 0.71A.

VII. CONCLUSION

In this paper online adaptive inverse control using IT2-NTSK-FNN is utilized to control a magnetic ball levitation system. This method doesn't need any background knowledge about the system; so simultaneously an IT2-NTSK-FNN identifies the system dynamic and also in the middle of training this network is used as controller. Due to simplification of the structure and development of online applications of this network in identification and control, rules reduction algorithm (from nine to three) is employed. Capability of IT2-NTSK-FNN in modeling uncertainties has escalated the efficiency of this method in identification of inverse dynamic of systems more than type-1 fuzzy neural networks. Simulation results indicate the prominence of IT2-NTSK-FNN over ANFIS in adaptive inverse control of magnetic ball levitation system for following a reference signal. In addition, as shown in figure 7, the tracking error of the proposed controller is zero and has less settling time than ANFIS adaptive inverse controller.

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