Keywords: model; locomotive; train flow; optimization; transportation problem

Petr KOZLOV*
Research and Production Holding STRATEG
Nizhegorodskaya 32, building 15, 109029, Moscow, Russia
Sergey VAKULENKO
Moscow State University of Railway Transport
Obrazcova 9, building 9, 127055, Moscow, Russia
Nikolay TUSHIN, Elena TIMUKHINA
Ural State University of Railway Transport
Kolmogorova 66, 620034, Ekaterinburg, Russia
**Corresponding author*. E-mail: laureat_k@mail.ru

MODEL TO CALCULATE THE OPTIMAL MODE OF TRAIN LOCOMOTIVES TURNOVER

Summary. A model to calculate the optimal work modes of train locomotives while serving train flows is proposed. The model is a further development of the dynamic transportation problem. Parameters of train movement and locomotive utilization are published. Both locomotive turnover and train schedules are produced. Useful utilization of locomotives increases from 70% to 90–95%. This allows the reduction of several thousands of train locomotives.

1. INTRODUCTION

The problem of rational utilization of locomotives is quite relevant at the present time. It is necessary to find the best trade off. When there is an excess of locomotives, we have less train delays, but high expenditures, connected with locomotive servicing. When there is a lack of locomotives, the converse is true.

Considering only operational planning covering several hours, it is impossible to produce from separate time intervals the effective work mode as a whole. Therefore, in practice, locomotive turnover schedules are developed for 7–10 days considering the service area structure and the volume of train flow. Usually, such calculations are accompanied by hand-made locomotive turnover schedules. Considering a service area with several hundred locomotives and a period of several days, however, there will be a great number of variants of train locomotive turnover schedules. This is the reason the optimization model is required.

2. LITERATURE REVIEW

Since the 1980s an extensive number of articles have been dedicated to the problem of managing train locomotives. In [1] Booler tries to solve a scheduling model using linear programming but he tests the method only on small instances (10–50 trains). Wright in [2] underlines that the approach in [1] is not suitable for more realistic situations (100–500 trains). In 1999 Canadian scientists assigned multiple types of locomotives to a fixed timetable [3]. They developed the branch-first, cut-second method to solve the integer programming model. Cordeau et al. in [4] describe a Benders' decomposition method for the assignment of locomotives and cars. An alternative approach to solve the problem of locomotive assigning was proposed by a group of scientists from Princeton University. Their idea is based on the formulation of the original problem as a dynamic programming problem [5-10]. In 2012 Powell et al., in [11], describe an application called Princeton Locomotive And Shop Management system (PLASMA), which claimed to solve the problem of assigning locomotives to trains over a planning horizon capturing a high level of detail. Important improvements in building realistic locomotive assigning models are described by authors in [12-14]. Ahuja et al. in [12] describe a heuristic based on very large-scale neighborhoods to find near-optimal schedules for locomotives, which considers breakups and the desire for weekly patterns in the flows of locomotives. In [13, 14] authors provide models to solve a locomotive routing problem capturing a number of constraints with an adaptation of their large neighborhood search strategy.

Currently, in Russia, it is rather difficult to use foreign train locomotive optimization models and systems because of the following problems:

1. the interface of foreign models and systems is not adopted to Russian users.
2. the high cost of optimization systems (e.g. PLASMA).
3. the necessity to adopt a model for the Russian railway features of infrastructure, technological process, and the information environment, which, in real life, can create a large number of problems to be solved for the full reconstruction of a model.

To overcome these difficulties in Russia a domestic optimization system is developed , which is adapted to the features and volume of work in the Russian railways.

3. DESCRIPTION OF THE MODEL USED IN THE OPTIMIZATION SYSTEM

This article describes the system of train locomotive turnover optimization «Labyrinth». The optimization model in this system is based on the dynamic transportation problem. The solution of the dynamic transportation problem is reduced to a static one with reproduction in time [15]. The problem has been already discussed by the authors in the scientific press [16]. This article presents a further development of the proposed approach. The model described in [16] turned into a system of the locomotive turnover optimization that can become an optimizing unit in the corresponding automatic control systems.

The railway station layout represented in the model consists of three parts – the input sector (index ***i*),** the output sector (index ***j*)**, and the makeup origin point (index ***q*).**

Trains (makeups with locomotives) or reserve locomotives arrive at the input sector and depart from the output sector. At the origin point makeups appear according to the scheduled train departures. The connection of locomotives and makeups also takes place at this point. Fig. 1 shows the process of locomotive and makeup connection. In this case the reserve locomotive arriving from the previous station is denoted as (variable $y_{ab}(t-\tau_{ab})$). Next, the locomotive moves to the point ***q*.** If there is a makeup (variable $x_{qq}(t) \neq 0$), it connects with the locomotive, forming a new train. The train moves to the output sector (variables $y_{qj}(t-\tau_{qj})$ and $x_{qj}(t-\tau_{qj})$) and can subsequently depart to the following station C (variables $y_{bc}(t)$ and $x_{bc}(t)$).

If the locomotive is not required, it waits in the input sector (variable $y_{ii}(t)$).

A new concept «pool of free locomotives» (index ***z*)** is introduced in the model. Locomotives can appear in it at the beginning of the calculation period or depending on necessity. Therefore, experiments to determine the number of locomotives required in the conditions of given train flow movement parameters can be carried out. Locomotives from the pool can arrive at all parts of the station (variables $y_{zi}(t)$, $y_{zq}(t)$ and $y_{zq}(t)$).

The running time between points is denoted by $\boldsymbol{\tau}$.

For example, $y_{ab}(t-\tau_{ab})$ means the number of locomotives arriving at station **B** at moment **t**, but departing from station ***A*** at running time τ_{ab} earlier.

The transit passing of a station is displayed as a joint movement of the locomotive and the makeup (Fig. 2).

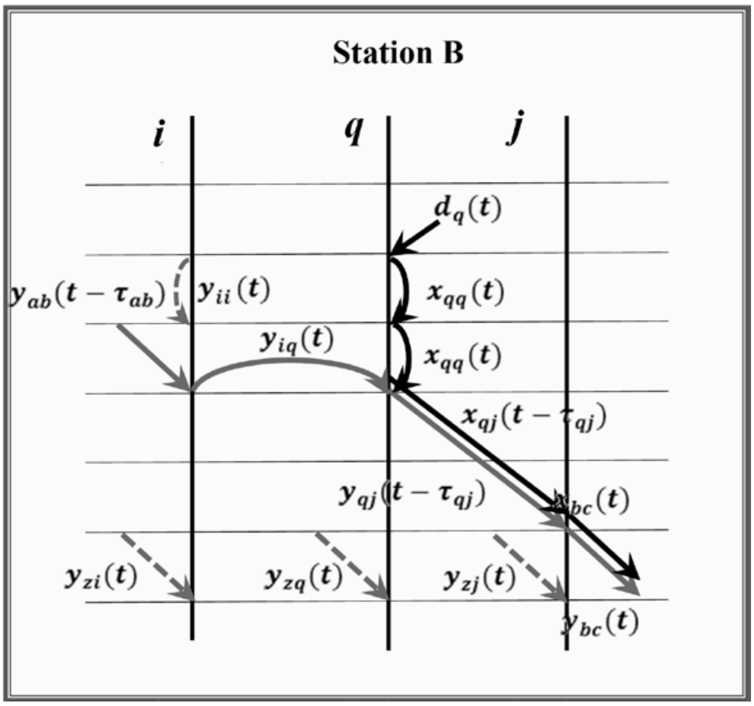

Fig. 1. Locomotive and makeup connection scheme: *i* – input sector, *j* – output sector, *q* – makeup origin point

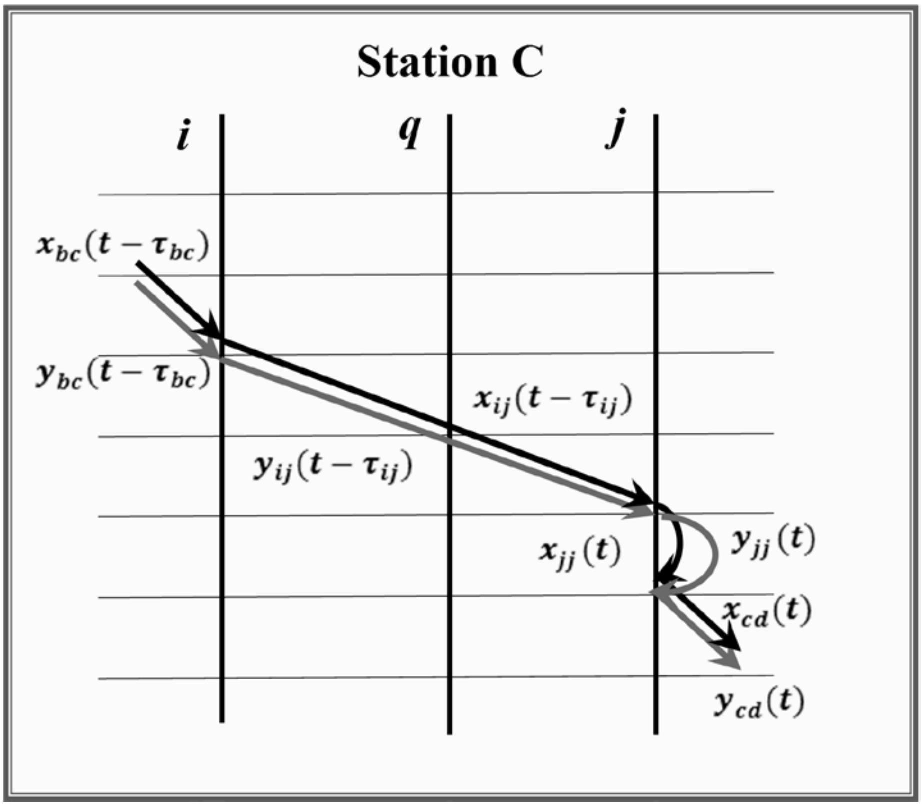

Fig. 2. Scheme showing a non-stop passing of a station

Fig.2 shows the joint arrival of a locomotive (variable $y_{bc}(t-\tau_{bc})$) and a makeup (variable $x_{bc}(t-\tau_{bc})$). They move from the input sector into the output sector (variables $x_{ij}(t-\tau_{ij})$ and $y_{ij}(t-\tau_{ij})$). Delays (variables $x_{jj}(t)$ and $y_{jj}(t)$) appear because of the impossibility to dispatch a train. Train departure is displayed by variables $x_{cd}(t)$ and $y_{cd}(t)$.

Time of arrival without delays is assigned for each train. The arrival is fixed by variable $d_i(t^*)$ (Fig. 3).

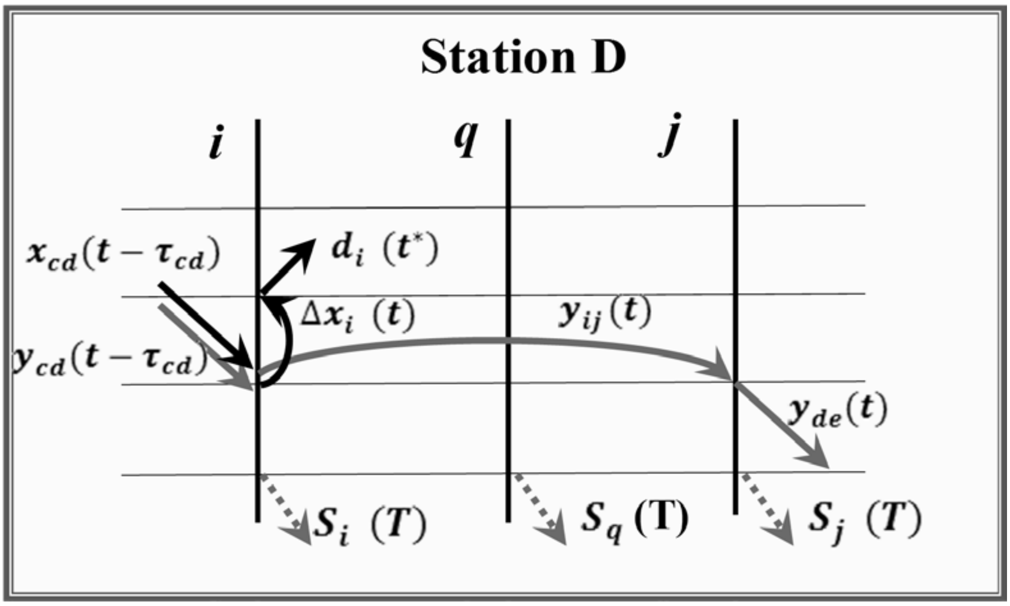

Fig. 3. Scheme of a train arrival at the terminal station: $\Delta x_i(t)$ – delay

If the train arrives later, a delay variable $\Delta x_i(t)$ is formed. In this case the locomotive either runs empty (variables $y_{ij}(t)$ and $y_{de}(t)$) or waits in the input or output sector (variables $y_{ii}(t)$ or $y_{jj}(t)$, see Fig. 1).

In the dynamic transportation problem the source and the drain should be given. For the flows that have not reached the drains up to time *T*, artificial drains are introduced (variables $S(T)$).

3.1. Basic equations

The dynamics of the makeups in the input sector are as follows:

$$x_{ab}(t-\tau_{ab}) = x_{ij}(t) + \Delta x_i(t) + d_i(t) + S_i(T),\ 0 \le t \le T \tag{1}$$

The dynamics of the makeups in the output sector are as follows:

$$x_{jj}(t) = x_{jj}(t-1) + x_{qj}(t-\tau_{qj}) + x_{ij}(t-\tau_{ij}) - x_{bc}(t) - S_j(T) \tag{2}$$

The display of the train delays is as follows:

$$\Delta x_i(t) = x_{ab}(t-\tau_{ab}) - x_{ij}(t) - d_i(t) \tag{3}$$

$$\Delta x_i(t-1) = \Delta x_i(t) - d_i(t) \tag{4}$$

where $\Delta x_i(t)$ is the number of trains being late at the moment t.

The representation of waiting for the locomotive is as follows:

$$x_{qq}(t) = x_{qq}(t-1) + d_q(t) - x_{qj}(t) - S_q(T) \tag{5}$$

The balance of the locomotives in the input sector is as follows:

$$y_{ii}(t)=y_{ii}(t-1)+y_{zi}(t)-y_{ij}(t)-y_{iq}(t)+y_{ab}(t-\tau_{ab}) \tag{6}$$

The new locomotive either arrives from the previous station or is taken from the pool.

Dynamics of the locomotives in the output sector are as follows:

$$y_{jj}(t)=y_{jj}(t-1)+y_{ij}(t-\tau_{ij})+y_{zj}(t)+y_{qj}(t-\tau_{qj})-y_{bc}(t) \tag{7}$$

Locomotive balance in the makeup origin point is as follows:

$$y_{iq}(t)+y_{zq}(t)=y_{qj}(t) \tag{8}$$

3.2. Additional restrictions

The impossibility for the makeup to move into the output sector without a locomotive is as follows:

$$x_{qj}(t)=y_{qj}(t) \tag{9}$$

The makeup in the output sector can only be with a locomotive, as follows:

$$x_{jj}(t)\le y_{jj}(t) \tag{10}$$

The impossibility to dispatch a makeup without a locomotive is as follows:

$$x_{ab}(t)\le y_{ab}(t) \tag{11}$$

3.3. Functional

$$\begin{aligned}&\sum_{t=0}^{T}(c^c\sum_k(x_{jj}^k(t)+x_{qq}^k(t))+c^o\sum_k\Delta x_i^k(t)+c_\beta^c\sum_k(y_{ii}^k(t)+\\&+y_{ij}^k(t)+y_{jj}^k(t))+\sum_\eta c_\eta(y_\eta(t)-x_\eta(t))\to min\end{aligned} \tag{12}$$

where η is service area number, k is railway station number, c^c is hour cost of the makeup idle time, c^o is hour cost of the train idle time, c_β^c is hour cost of locomotive utilization. The costs c^c, c^o, and c_β^c are set for one simulation time-step; c_η is cost of one locomotive–kilometer, multiplied by the length of the service area $\boldsymbol{\eta}$.

3.4. The meaning of the terms of the functional

$c^c\sum_k(x_{jj}^k(t)+x_{qq}^k(t))$ is cost of the makeup idle time (all costs are measured during one simulation time-step)

$c^o\sum_k\Delta x_i^k(t)$ is penalty for the train delay.

$c_\beta^c\sum_k(y_{ii}^k(t)+y_{ij}^k(t)+y_{jj}^k(t))$ is cost of locomotive utilization. Technical operations at the station are shown by $y_{ij}^k(t)$.

$\sum_\eta c_\eta(y_\eta(t)-x_\eta(t))$ is cost of the locomotive when it is empty running. In this equation, costs, connected with the makeup movement, are subtracted from the whole train costs.

4. RESULTS OBTAINED USING THE OPTIMIZATION SYSTEM

Testing of the optimization system was carried out on the Gorkovskaya Railway Druzhinino-Vekovka service area (Fig. 4).

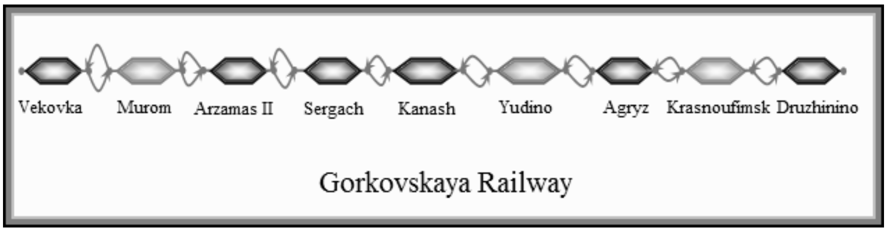

Fig. 4. Locomotive service area scheme

A set of calculations was performed and some results from one of the sets are shown below. The calculation period was 9 days. The first day and the last two days were discarded as transitional because of the necessity to consider only stable periods.

First of all, locomotive utilization parameters are provided (Fig. 5)

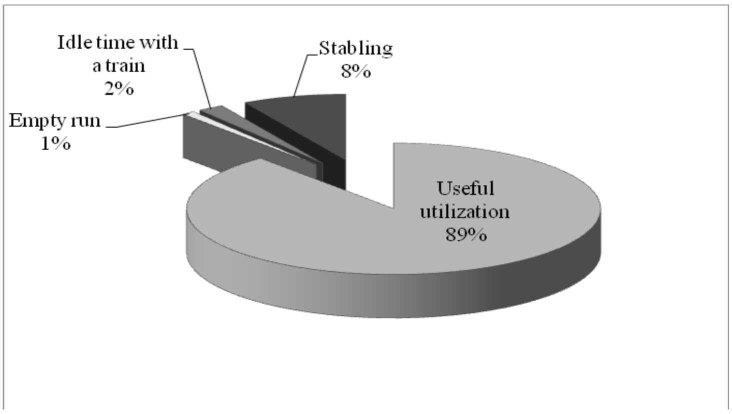

Fig. 5. Locomotive utilization

Fig. 5 shows that empty running is minimized. It is impossible to achieve such results using manual control because there are billions of variants of locomotive movements during several days. Idle time of the train includes time for technical operations and the idle time connected with overtakings.

It is possible to see the work parameters of each locomotive (Tab. 1).

The contents of the table can be sorted by any column. It is also possible to open the locomotive turnover schedule from each line (Fig. 6).

Moreover, a considerable number of locomotives do not have empty runs at all (Fig. 7).

Useful employment does not include idle time of the train because of overtakings.

Similar information about train movement is also provided (Fig. 8).

As can be seen, waiting for a locomotive is also minimized. The optimization system provides information about train movement in each direction (Fig. 9).

The system also produces the train schedule and provides the parameters for each train (Fig. 10).

5. CONCLUSION

The optimization system allows carrying out different experiments including the calculation of the optimal number of locomotives in conditions of given cost parameters, determination of the best locomotive location at the beginning of the calculation period with the given train flow structure, the evaluation of the influence on train movement depending on the number of locomotives, and so on.

If there is a need, it is possible to connect the optimization system to the corresponding automatic control systems to use in operative planning.

Table 1

Locomotives work parameters

Locomotive	Useful utilization	Stabling	Idle time with a train	Empty run	Number of trains
2461	[5] 16:08	01:36	04:00	00:00	6
2453	[5] 16:08	03:04	02:48	00:00	6
2496	[5] 16:00	04:16	02:16	00:00	8
2507	[5] 15:36	02:56	03:20	00:00	6
2559	[5] 15:36	01:52	02:40	00:32	5
2454	[5] 15:20	01:12	06:24	00:00	6

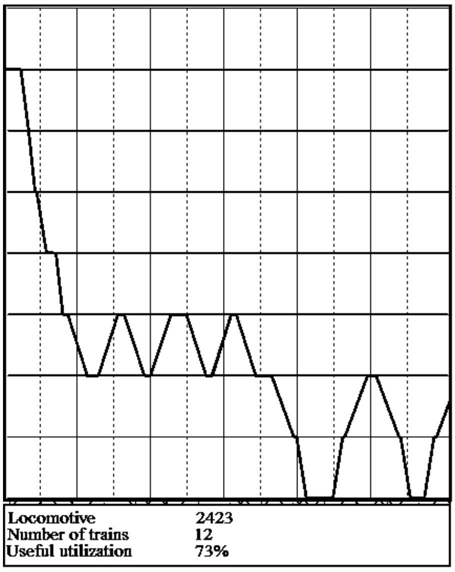

Fig. 6. Locomotive turnover schedule (the black line shows the idle time and empty running)

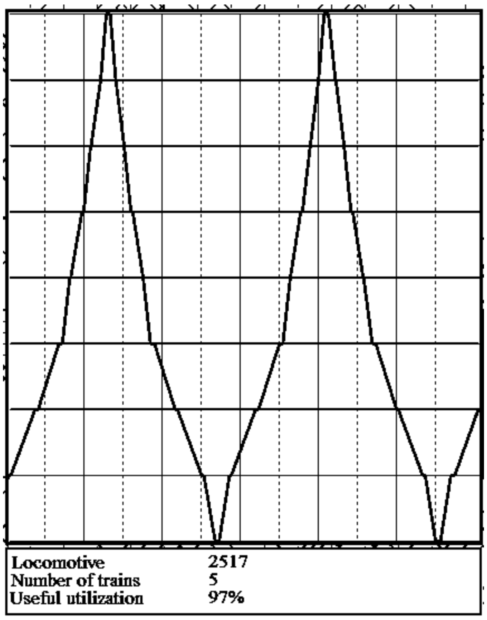

Fig. 7. Locomotive turnover schedule without empty running

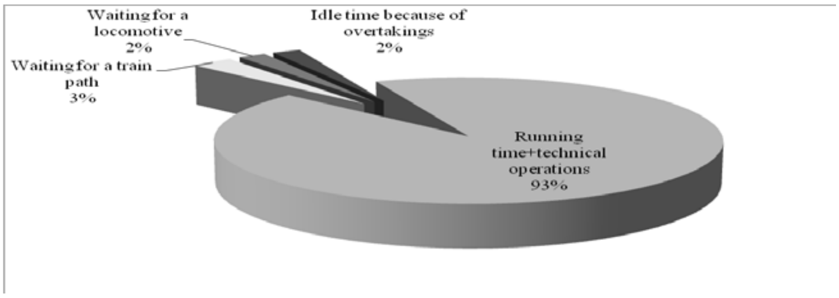

Fig. 8. Train flow movement parameters

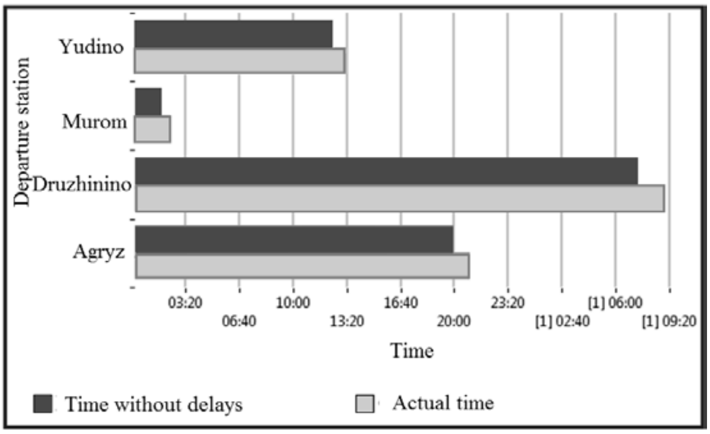

Fig. 9. Train delays applying to the separated train flow streams

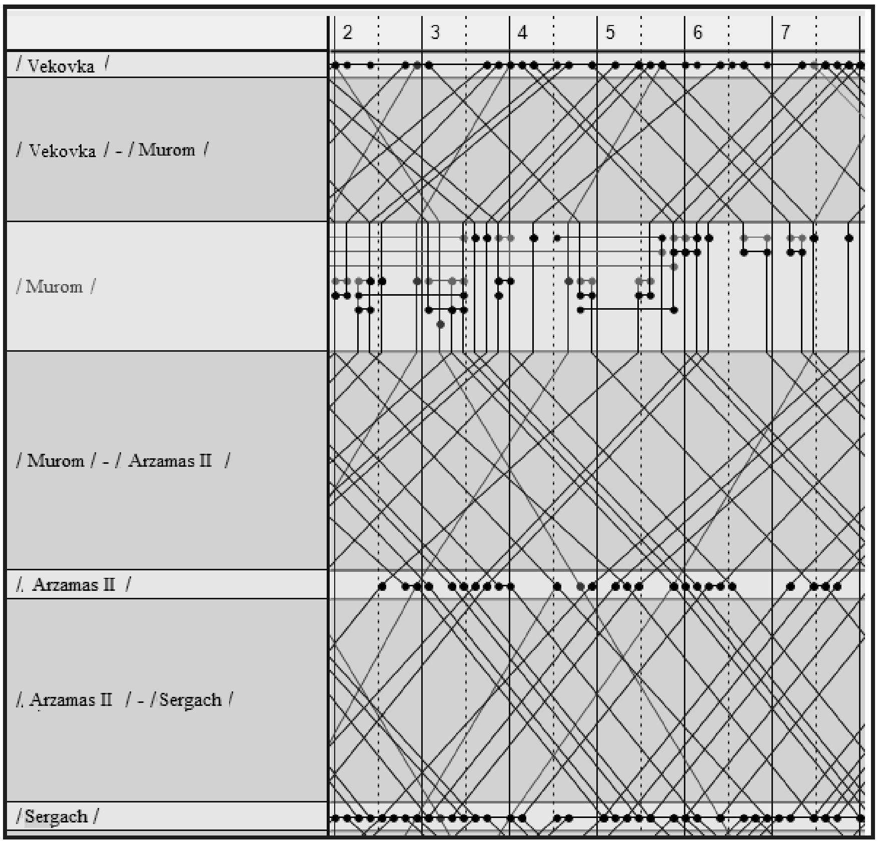

Fig. 10. Train schedule

References

1. Booler, J.M.P. The solution of a railway locomotive scheduling problem. *Journal of the Operational Research Society*. 1980. Vol. 31. No. 10. P. 943-948.
2. Wright, M.B. Applying stochastic algorithms to a locomotive scheduling problem. *Journal of the Operational Research Society*. 1989. Vol. 40. No. 2. P. 187-192.
3. Ziarati, K. & Soumis, F. & Desrosiers, J. & Solomon, M.M. A branch-first, cut-second approach for locomotive assignment. *Management Science*. 1999. Vol. 45. No. 8. P. 1156-1168.
4. Cordeau, J.F. & Soumis, F. & Desrosiers, J. A benders decomposition approach for the locomotive and car assignment problem. *Transportation Science*. 2000. Vol. 34. No. 2. P. 133-149.
5. Powell, W.B. & Shapiro, J.A. & Simao, H.P. An adaptive dynamic programming algorithm for the heterogeneous resource allocation problem. *Transportation Science*. 2002. Vol. 36. No. 2. P. 231-249.
6. Powell, W.B. & Topaloglu, H. Stochastic programming in transportation and logistics. *Handbooks in Operations Research and Management Science*. 2003. No. 10. P. 555-636.
7. Powell, W.B. Dynamic models of transportation operations. *Handbooks in Operations Research and Management Science*. 2003. No. 11. P. 677-756.
8. Powell, W.B. & Topaloglu, H. Fleet management. In: Wallace, S.W. & Ziemba, W.T. (eds.) *Applications of Stochastic Programming*. SIAM & MPS. 2005. P. 185-215.
9. Topaloglu, H. & Powell, W.B. Dynamic-programming approximations for stochastic time-staged integer multicommodity-flow problems. *INFORMS Journal on Computing*. 2006. Vol. 18. No. 1. P. 31-42.
10. Powell, W.B. & Bouzaiene-Ayari, B. & Simao, H.P. Dynamic models for freight transportation. In: C. Barnhart, & L.G. (Eds.) *Handbook in Operations Research and Management Science*. Amsterdam: North-Holland. 2007. Vol. 14. P. 285-365.
11. Powell, W. & Bouzaiene-Ayari, B. & Hall, S. & Lawrence, C. & Cheng, C. & Das, S. & Fiorillo, R. *Strategic, tactical and real-time planning of locomotives at Norfolk Southern using approximate dynamic programming*. Tech report. 2012.
12. Ahuja, R.K. & Liu, J. & Orlin, J.B. & Sharma, D. & Shughart, L.A. Solving real-life locomotive scheduling problems. *Transportation Science*. 2005. No. 39. P. 503-517.
13. Vaidyanathan, B. & Ahuja, R.K. & Liu, J. & Shughart, L.A. Real-life locomotive planning: new formulations and computational results. *Transportation Research Part B: Methodological*. 2008. No. 42. P. 147-168.
14. Vaidyanathan, B. & Ahuja, R.K. & Orlin, J.B. The locomotive routing problem. *Transportation Science*. 2008. No. 42. P. 492-507.
15. Козлов, П.А. *Теоретические основы, организационные формы, методы оптимизации гибкой технологии транспортного обслуживания заводов черной металлургии*. D. Sc. thesis. Липецк: ЛПИ. 1986. 377 p. [In Russian: Kozlov, P.A. *Theoretical basis, organizational forms, methods to optimize the flexible technology of the ferrous industry transportation service*. D. Sc. thesis. Lipetsk: LPI.]
16. Козлов, П.А. & Вакуленко, С.П. Расчет оптимальных режимов работы локомотивов при обслуживании поездопотоков. *Транспорт Урала*. 2016. No 1. P.3-8. [In Russian: Kozlov, P.A. & Vakulenko, S.P. Calculation of the optimal locomotives work modes at servicing train flows. *Ural Transport*].

Received 11.12.2015; accepted in revised form 30.08.2017