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# A REVIEW OF ENERGY OPTIMAL TOPOLOGY CONTROL FOR LARGE WIRELESS NETWORK USING YAO-GRAPH AND ITS VARIANTS

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***Abstract:*** ***The advent of wireless communication and networking in the last two decades has led to the need of modification in the regular graphs used as spanners. The spanner is a sub graph of the original graph which connects the essential nodes for transmission of the message. For this purpose the Omni directional antennas are usually used and the "spanner-graph" performance metric is a constant multiplied by the same of the original graph. This constant is known as the "stretch factor". One of the most common attribute for wireless communication is the energy required for faithful transmission of a message between two nodes. But the Omni directional antennas lead to interference and wastage of bandwidth and energy. The nodes of wireless communication are however neither always static nor equipped with any infrastructure. Rather the same might as well be ad-hoc and mobile. Another well used notion is that of the unit disk graph (UDG), where a node can communicate***

***only if the other nodes are within the disk of unit radius. However the graph being dynamic in nature, the nodes do not have fixed transmission radii and the same changes according to the power requirement of transmission. This work of ours mainly deals with the different graphs being used as spanners. It explains a few algorithms described in other papers which it reviewed. The associated algorithms are related to Yao graph. This graph and its modifications are discussed and it is explained how they could be used energy efficiently. The Yao-Yao graph for example can be used as a spanner for certain specific values of stretch factors.***

**Keywords: Yao-graph, Yao-Yao graph, Theta-graph, Spanner, Unit disk graph, Power, Degree bound.**

## I. INTRODUCTION

The transmission of messages in wireless communication has a lot more factors to be considered as matters of concern than wired communication. In this domain, the resources like energy are being used a lot quickly and every wireless transceiver node contains only a finite amount of the same. The graph used as a spanner should mainly have a few attributes like sparseness, bounded degree, local construction etc. The *unit disk graph* (*UDG*) is the basic concept used where a node communicates with other nodes only if they lie within the unit transmission radius of the disk centered at the source node. The concept like quasi *UDG* with the phenomenon of cages and chains for the realization of the graph is explained in the section IV of this paper. The power required for the transmission is directly proportional to some non-decreasing function of distance between the two nodes. It varies with $r^{-\alpha}$, where $\alpha$ is a constant that varies between *2* and *4*. The energy constraint is one of the major factors with which the wireless communication has to deal with. This mainly depends upon the algorithm along with the type of antenna used. The *minimum spanning tree (MST)* based broadcast is a solution that can work with directional antennas. The *MST* has an important property that the longest edge in the *MST* is the shortest among all the spanning trees. The concept of directional antennas in three different medium access protocols is also explained. In that paper, the relation between cooperation and directionality is shown for which the concept of relay and key tables are used. The broadcast tree algorithm is explained in this paper which not only helps in the cooperative transmission of messages, but also in the lifetime maximization of the battery life of the nodes. In *Yao graph*, the plane around a node is divided into equally angled non overlapping sectors. The measure of each angle is *(2*pi)/m*, where *m* is the number of sectors. Then within a sector among the nodes, the source node is connected with the node which has the smallest distance from it. The nodes might be in a dynamic state. So the extended *Yao graph* is formed along with the mutual exclusion graph for which the connection between two nodes exists only if they are within each other's transmission range. Even though, the out degree is bounded, but the in degree remains unbounded. So, a modification of this graph is made called the *Yao-Yao graph*. Here, for the source node, among the connected nodes only that incident connection remains which happens to be with the one with the shortest distance. So in a way, the final graph shows that the connection between the nodes will only remain for the ones with both sided edges. Therefore, both the in degree and out degree are bounded in this graph. The *UDG* can be formulated with the *Yao-Yao graph* for which the efficiency of energy for transfer of messages would be much better. The edge coloring is a concept briefly explained which dictates that no two adjacent edges will have the same color. A graph called the theta graph which is similar to *Yao graph* is also reviewed in this paper. In theta graphs, the division of sectors is done in the same way as *Yao-graph*. The orthogonal projections are made from each node within a sector to the counterclockwise wall of the sector. The source node is then connected to the one whose projection is closest to it. Ordered theta graph is another concept for which the ordering of the vertices in a set can led to different graphs.

The paper is organized as follows. The definitions of the key algorithms are given in Section II. In Section III, the issues related to the topic are given. Section IV offers the discussion and analysis of the individual papers. We conclude in Section V.

## II. ISSUES

A lot of important development did take place in the field of communication networks both in terms of technological advancement and theoretical development during last twenty years. First of all developments in wired network industry did lead to a saturation. There were already too many resources and too many types of equipment in the inventory of manufacturers and the so called “dot-com” revolution became a future that never happened. In this situation wireless networking technology and its various variants offered a new lease of life to communication networks community in general. Now, we are not only dealing with cellular networks and 4G and 5G technologies but we are also in sensor networks, ad hoc networks, vehicular networks, networking support for intelligent transportation systems (ITS), intelligent power grid, and computational grids. Apart from these wireless networked large scale control and cyber-physical systems, machine to machine communication in internet of things over unreliable wireless links, body area networks and related remote consultation with physicians are some of the issues that did attract the attention of research community. New technologies like MIMO, massive MIMO, steerable and adjustable multiple antenna beams, cognitive systems, network coding, directional antenna systems, cooperative communication became issues of the day. These propositions did throw in not only new research problems but a lot of fundamental questions which was still not well understood as per the framework of traditional information theory. Traditional information theory did not take care of multiuser communication, variable delay between source and destination or burstiness of traffic [1]. Neither did it addresses any possibility of cooperation or competition among nodes / agents and possibility of self-organization or phase change, micro-macro link etc which leads to some global emergent phenomena. Can there be mechanism design or inverse optimization so that desirable global emergence occurs as a consequence of local interaction patterns that we design? Can there are be a mesoscopic approach systems which takes an intermediate path between micro-analysis and macro-analysis and allows us to formulate some form of analysis of these large wireless networked multi-agent systems which reveals more subtle truth? Such questions are yet to be completely answered.

When MIMO antenna multiple beam forming and other new technologies are opening up completely new design for the physical layer itself, then the entire network design and optimization problem had to be revisited [2]. Cooperative MIMO leads us to study channel models afresh [3]. Dynamic spectrum access is a candidate technique to deal with the spectrum scarcity problem that we face in many parts of the globe. Cognitive radio, a technology that is being developed by taking inputs from fields as wide as communication theory, networking, signal processing, game theory, embedded systems, adjustable R.F. antenna design etc. may turn out to be a feasible solution for real life implementation of the same [4]. Converting wireless overhearing problem into wireless broadcast advantage (WBA) leads to the idea of cooperation in networking. This cooperation [5] may appear in the form of relay assisted forwarding [6] [7] [8] [9], energy efficient communication in ITS [10], exploiting suitable MISO channels for lifetime maximal data percolation [11], cooperation between base stations in cellular networks for interference mitigation [12], gradual appearance of cooperation in incentive based large multi hop ad hoc networks [13], in cooperative MAC layer design [14] [15].

Layering was always used as a fundamental concept in communication systems [1] [16]. However, the information transshipment process from the ingress to the egress side of the network (of possibly unreliable nodes and links) under various resource constraints, with heterogeneous flow patterns and asynchronous distributed localized controls based on delayed, partial (may be erroneous) information while optimizing multiple QOS/QOE, can be viewed as a multi objective multi constraint distributed optimization process actuated by a networked multi agent system. Therefore, the entire network design problem can be viewed as multi objective optimization problem which perhaps is mathematically reducible to sub problems each of which can be taken care of by so called *'layers'* and that's how this new layering concept got to be defined. Unlike cross layer optimization [17] which does not challenge the traditional layering concept but talks about inter-layer information exchange to arrive at global network optimization in a better manner; this optimization based stack design [18] intends to redefine the entire concept of layering based on mathematical optimization theory. Under this paradigm, certain fundamental issues like consensus optimization over multi agent systems become a matter of great interest [19] [20] [21]. The communication theorists did invent the domain of network coding [22] [23] [24] which indirectly studies similar mathematical problems [25] [26]. Therefore, network information theory [27] [28] and topological structures [29] become important tools to understand the networks of the future. One of the major issues of concern considering the $CO_2$ emissions by ICT industry [30] , mobility [31][32] and portability of the energy limited agents (nodes, wireless transceivers) and the fading and shadowing related unreliability of the channels where capacity becomes the function of the energy [33] is energy efficiency and harvesting. It is understood that for energy efficient global operation on top a large scale network of such agents (e.g., sensor network) proper topology control [29] [34] [35] is mandatory. The system of directional antennas [36] and adaptive beam forming [37] [38] also enables us in realizing power efficient interconnected networked topologies. The specific abstract graph structures which allow us to mathematize and model these lines of new technologies are the families of graphs known as Yao graphs and Theta graphs and they are variants. In a detailed survey [34], Prof. Christian Scheideler has already discussed the power spanner and other type of spanning properties as applicable to the domain of wireless networks of various graphs. However, we did feel that given the volume of new emerging works and the use of directional antennas and adaptive beam forming, the Yao graph like structures and its variants got to be revisited in a fresh survey with a lot more detailed focus while contemplating adaptation of the same in conceptualization of new network optimization problems which in corporates the effect of these new technologies.

## III. DEFINITIONS

YAO-GRAPH: Yao graph is defined to be a geometric spanning graph. From each node, a fixed number of rays suppose *t*, originates. Thus the space around a node is divided into fixed sectors with same angles. The nearest node among all the neighboring nodes in a particular sector is connected to the host node from where the rays have originated. The out degree of a particular node is reduced. But the in degree is high which could be equal to or more than the out degree. The Yao graphs act as a spanner for specific values of $t$ till $t \geq 6$. For example, for the value of $t=2, 3$ also known as the "*stretch factor*", the Yao graph does not act as a spanner.

YAO-YAO GRAPH: This is the modification of the *Yao graph*. Here, the graph formation takes place in two steps. In the first step, a specific number, suppose $k$ equally partitioned rays arise from each node in the given space forming sectors. Then the nearest neighbor in each sector is connected to the host node. In the second step, all the incoming rays in a given sector for any node except the one with the shortest distance are discarded. The disadvantage of *Yao graph* is that the in degree is high and in the worst case at the maximum can even be of $O(n)$, where, $n$ is number of nodes in the graph. But this modification, that is the Yao-Yao graph has nullified that effect and reduced the in degree of a node. Thus, in Yao-Yao graph, the maximum degree is bounded. From the paper of Nawar et al. [39], similar results for value of the *stretch factor* have been proved. We find that for the values $k=4, 6$, the Yao-Yao graph is not a spanner. Moreover, it is also shown that for value $k=4,$ Yao-Yao graph is a spanner for points in convex position. It is yet to be known whether this graph is a spanner or not even in restricted cases for $k=5$.

UNIT DISK GRAPH: The *unit disk graph* is a method where a circle or disk on every node in the Euclidean plane is considered. The radius of that circle is one. If any other neighboring node whose distance with the host is less than unity, then a connection is made between them. The message transmission can only take place from the host node with the ones which are inside the unit circle. In the paper of Fabian Kuhn et al. [40], it has been said that this is only possible if

*P=NP*. It has also been shown that if connections are made between the non-neighboring nodes, then the maximum distance between them could be √ *(3/2)-ε*, where *ε~0* as the number of nodes, n, tends to infinity.

QUASI UNIT DISK GRAPH: The real scenario in a wireless system is different from *unit disk graph*. The connection between two nodes is ambiguous. Thus, a modification of the *unit disk graph* is made. Here, the connection between two nodes are made if the distance between them is ≤*d*, where *d* is an integer between *0* and *1*. If the value of *d*is is greater than *1,* then there is no connection between the two nodes. And if the value lies between *1* and *d,* then the value is ambiguous. The proof for quasi *unit disk graph* for a symmetric graph is shown in the paper of Fabian et al. [40]. The *quasi unit disk graph* is a type of *civilized unit disk graph (UDG)*. In civilized *UDG*, a node *u* is connected to other nodes if they fall within the disk of the host node with radius *d>1*. A very interesting property called the *'cross linking'* for *quasi unit disk graph* had been explained briefly. Here, if the upper and lower bound for the radius of the disk for a node is taken to be *r* and *R*, then the property states that *R≤√2*r*.

THETA *(Θ)* GRAPH: Theta *(Θ)* graphs are similar to Yao-graphs. Here, the space around a node in a plane is divided into *k* disjoint sectors. The angle of each sector is *(2*pi)/k*. All the sectors have their apex at the host node. The resulting graph is called $\Theta_k$. Then among all the nodes in a particular sector whose projection is the angle bisection, the nearest node is joined with the host node. The spanning ratio of theta graph varies with the number of sectors made around the node in a plane. The theta graph can also be related to Delaunay triangulation. The process is described in the paper of Bose et al. [41]. The sectors are divided into two sets, positive and negative. The division was done in a pattern of one positive beside two negative sectors. The joining of other nodes if allowed is only allowed in the positive sector. This is said to be half graph. The design of theta graph is well explained in Bose et al. [41]. The host node is connected with a node closest to it in the sector. The distance is measured along the bisector of the sector. The line that joins the host node as the closet angle bisector coincides with the vertical line above the node.

ORDERED (*Θ)* GRAPHS**:** Ordered theta (*Θ*) graphs are a step advanced to theta graphs. The construction of this graph takes place in two stages. In the first stage, sectors around a node is formed such that the angle of each sector is *(2*pi)/m,* where *m* is the number of sectors. The sectors formed around a cone are non-overlapping to each other. In the second stage, the nodes with each sector points out their orthogonal projection to the counter clockwise side of the sector. Then the host node is connected to the node whose projection to the side is closest to it. The main difference of ordered theta graphs from theta graphs lies in the fact that the ordering of the vertices can produce an altogether different graph. These graphs are also used as spanners.

## IV. LITERATURE REVIEW

Various papers which we are going to discuss here did manage to address different aspects of topology control and usefulness of Yao-graph and its variants in modeling the same. Let us therefore get ahead in a systematic step by step manner discussing these issues anchored around certain significant pieces of work.

### A. EFFICIENT TOPOLOGY CONTROL FOR AD-HOC WIRELESS NETWORKS WITH NON-UNIFORM TRANSMISSION RANGES

In their work [42], the authors mainly explain the different methods used in wireless communication for transmission of messages. It states the use of *unit disk graphs* and its modification in real life scenario. The formation and the attributes of Yao-graphs are also briefly explained here. The extension of the Yao-graphs for a constant length and power *stretch factor* is used to build a spanner for another phenomenon which is included here and is called the *'mutual inclusion graphs' (MG)*. This states that the two nodes communicate only if they are in transmission range of each other. The spanner is a sub graph of the original graph which is characterized by multiplying a constant with the Euclidean distance between the source and the destination node. This constant is called the *stretch factor*. *Unit disk graphs* are modeled as the source node transferring message only if the destination node is within the disk of unit radius. The real scenario is however different. The transmission radius for different nodes is different. It is generally stated that the nodes in the system are dynamic. This helps to use the system as a multi hop one, causing it to enhance low power transmissions and spatial interference. It also provides fault tolerance and enhanced capacity. It is also generally better to use directional antenna instead of Omni-directional antenna as using the later one may cause interference and also loss of power. Directional antennas also have higher peak gain than their counterparts. Centralized algorithms are used for static wireless system which provides adaptability. But In case of dynamic system, distributed algorithms are used. The Yao-graphs are constructed by dividing the plane around a node into non-overlapping sectors. Among the nodes lying within the sector, the source node is connected with the one which has the shortest distance among all. Similar to Yao-graphs are the theta graphs, where the node is connected with the one having the smallest projection on the axis of the sector. In the formation of extended Yao-graph, the sectors or cones are half open. Initially, the source node sends a message to other nodes about its position and coordinates. It then receives the information about the location of other nodes. As a result, it connects with the one having the distance smaller than its radius within a specified sector. Then theorems are given for Yao-graphs with different values of *stretch factors*. The disadvantage Yao-graph is even though it has a bounded out degree, but the in degree is unbounded. For this, all the incoming rays for a node within a sector except the one with the smallest distance are deleted. This graph is called the *Yao-Yao graph*. Partition of Yao-graph is another method that ensures connectivity. Here, a sector is subdivided into a number of triangles and it is proved that two nodes within a region are connected in mutual inclusion graphs. The construction method for extended *Yao-Yao graph* is also shown which is similar to the process for extended Yao-graph with the method of joining the chosen links in a *Yao-Yao graph*. Then, the construction for a Yao-sink graph is said where the replacement of the one with all the links is done towards a node by a directed tree with the node acts as the sink node. Lastly, the construction of the tree of forming a Yao-sink graph is shown with the proofs for the theorems for different values of in and out degrees.

B. GEOMETRIC SPANNERS FOR WIRELESS AD HOC NETWORKS:

The main issue that was discussed in [43] was a general overview of the graphs associated in wireless networking for the transfer of messages and the constraints related with it. The general concept used to model wireless transmission by transceivers is the *unit disk graph (UDG)*. In *UDG*, a node can communicate with other nodes only if they are within its transmission range that is inside the disk of unit radius. But in real scenario, the nodes can alter their respective transmission power and therefore, it tends to change the radius of the disk graph as per the requirement. The disk graph where the radius is greater than one is known as "*civilized disk graph*". A simple method for transmitting message mentioned in the paper is *flooding*. The process involves the spreading of a message from the source node to all other nodes. But this technique involves wastage of resources and will also cause interference. A modification of the *UDG* is *the greedy perimeter stateless routing* where the source node sends the message to a sub graph of the *UDG*. But here, there is a non-zero probability of the message not reaching the destination. It reaches only if there is a path in the sub graph leading to the destination node. In order to assess the soundness of this method, a number of parameters and issues are considered like sparseness, spanner, bounded degree, planar and efficient localized construction. The sparseness of a graph depends on the number of links connected to the nodes. If the subset of a graph is used whose characterizing metric is equal to the same of the original graph up to a multiplicative constant, then the subset graph is called the spanner of the original graph. The constant that the original graph metric is multiplied with is called the *"length stretch factor"* or *"hop stretch factor*" or *"power stretch factor*" depending on the weight being the Euclidean distance or a unit value or the power required for transmission, respectively. Bounded degree refers to a node having fixed number of edges connected to its neighbors. Planar graph refers to the one where no two edges from nodes intersect each other. Localization in a graph refers to the algorithm which allows a node to determine which other nodes it should be connected to, due to the limited availability of resources. Previously, many graphs like *Relative Neighborhood graph, Gabriel graph, and Yao graph* are used for satisfying the above mentioned parameters. The working procedure and construction technique of *Connected Domination Set (CDS)* is described here as well. The *CDS* graph is a hop spanner graph. This method is a combination of the regularly used spanning graphs with the modification of *UDG*. A subset is taken from the original graph. It is called a *dominating set* if a node in the original graph is included in the subset or is adjacent to the node included in the subset. If the subset is a connected one, then it is called connected dominating set. The rest of the nodes are called *dominatees.* An *independent set* is the one where the edge does not exist between two pair of vertices. The set which cannot include any more independent vertex is called the *maximum independent set*. Then the concept of *Delaunay triangulation* is included in this algorithm. If a triangle is formed between three nodes where the circumcircle of this triangle does not contain any more nodes in its interior is called a Delaunay triangle. But if the transmission range of the edge between two vertices of the triangle formed is shorter, then it would not be possible to transmit the message in a single hop. So a modification called the *k-localized Delaunay graph* is introduced where the edges of the triangle connecting the nodes measuring one unit does not contain any other node interior to the circumcircle formed by the triangle. The *CDS* algorithm used to form the backbone consists of two phases, *clustering* and *finding connectors*. Clustering is a technique used to group a few nodes differentiating from the other nodes. In this method, a node sends a message *'I am dominator'* to its neighboring nodes. The neighboring nodes as a result sets itself as the dominatee sending the *message 'I am dominatee'* to its one-hop neighbors. This process is only possible if the nodes are aware of the ID of its neighbors. The node with the smallest ID is selected as the dominator. As a result when a node receives a message 'I am dominator' from its neighboring node, it becomes a dominatee from a white node. As a result of this process, a cluster is formed for a single dominator to several dominatees. Two lemmas are shown for a dominator having at most five dominatees and the number of dominators being constant for a node within a disk. The second step is the finding connector step. Here, a dominatee sends a message *'I am dominatee'* to its neighboring node. The neighboring node stores the message of the dominatee which is two hops away and also the information about the dominator two hops away from it for which the dominatee could be a connector between them. The dominatee sends a *message 'Try connector'* to its neighboring nodes and if it has the smallest ID, then it is the connector between the two dominators sending a message *'I am connector'* to its neighboring nodes. Finally, the initial dominatee node sends a message to its neighbor selected by it to be the connector. As a result, it acts as a connector for the dominators for which the initial node had been acting as the connector. There are at most five connectors that come up for two dominators. The relation between a connector and a dominator is just the opposite, the node with the largest weight will be the connector and the one with the smallest ID will be the dominator. The extended form of this graph is the one where all the edges from the dominatees are connected to their respective dominators. The lemmas are given for the properties of extended *CDS* such as the node degree is constant and the hop *stretch factor* and the length *stretch factor* are bounded. When the *CDS* is mixed with the property that a node can communicate with other nodes only within the unit radius of its disk, the resultant graph is called *induced connected dominating set or ICDS*. The local Delaunay triangulation on the *ICDS* is shown which is similar to the localized Delaunay graph with the fact that a node is only willing to form a triangle if the other two nodes are at a distance of a unit and they have accepted the proposals of forming triangle when they receive similar proposals. At the end, the simulation results are shown for different spanning ratios.

## C. LIFETIME OPTIMAL TOPOLOGY CONSTRUCTION FOR BROADCAST WITH COOPERATION

[11] deals with the problems the wireless communication faces while having point to point and point to multipoint communications. Among all the attributes in the message transfer, the energy consumption is one of the most important parameter. In unicast problem, the energy is considered only for the source and the destination nodes. But while in multicasting, all the nodes that lies in the path are to be considered. The *lifetime of a pathway* is known to be the time until the battery of the first node dies. The distance between two nodes in an important consideration to determine the power needed to transfer the message. The power required between two nodes varies with $r^{-\alpha}$, where r is the distance between the source and the destination node and $\alpha$ is a constant that varies between *2* and *4*. For multicast routing, the power is determined by the maximum power required to reach the farthest node. Other parameters and problems such as node lifetime and lifetime maximization are also defined. The node with the maximum cost is called the *bottleneck node* [44]. The ultimate aim of designing an algorithm is to minimize the node and link cost. In this paper, the algorithm is formulated by forming a broadcast tree which is formed by using a set of vertices and edges. A *spanning tree* is used as a *broadcast tree*. The lifetime of a broadcast tree is determined by the bottleneck node. The *'Cooperative Broadcast Lifetime Maximization Algorithm' (CBLM)* is shown to measure the lifetime of a broadcast tree. Simultaneously, another algorithm called the *'Minimum Energy Cooperative Path' (MECP)* is shown for its inability to find the maximum lifetime of a broadcast tree. The CBLM algorithm is explained with the pseudo code and an example. The algorithm has two phases, growth and the search phase. In the growth phase, the initial tree starts off with a node having a lower bound. This node includes its nearest node and in the same way as many nodes as possible are included, given that the node included is outside the broadcast tree and the cost of the node is less than the lower bound. In the second phase, every node searches for any neighboring node present in the plane and includes if any in a point to point communication. The nodes searches form other nodes based on the minimum cooperative transmission cost to form a group for the transmission. The search phase is important in the algorithm because if the growth phase fails, the search phase continues to include the nodes until all the nodes are included with the spanning area. The complexity of the search phase can be reduced in a particular manner. Instead of searching in a point to point manner for forming a group, the nodes can be sorted out on the manner of their node cost that is the ratio of power taken to transfer a message between two nodes to the initial energy provided to a node. The first few members required for forming a group for cooperative transmission is chosen. At last, the performance evaluation is done for the described algorithm and simulation results are shown.

## D. DIRECTIONAL ANTEENA AND ENERGY EFFICIENCY: AN ILLUSTRATIVE EXAMPLE

In [36], authors introduce the concept of sectored antenna and describe the function and advantages of the same. The paper overall works with the energy efficiency while transferring the message in a wireless system. If *Omni directional* antennas are used in an ad hoc network, a lot of energy is wasted. This is due to the fact that the message being transferred from the host node is received by all the nodes in the transmission range. As a result, only a small portion of the transmitted power is used. But while working with directional antennas, the message is transmitted to a few nodes that lie within a given sector. In the real scenario, the battery power of the nodes is limited and the nodes are working in a system with multi-hop connections. But in a situation with a few hopes including specified nodes, the drainage of the battery of those nodes will happen quickly and as a result will degrade the efficiency of the system. An important rule is maintained in this paper for the power requirement for the transfer of message. The power required to transfer a message between two nodes is the square of the distance between the two nodes. The paper explains an algorithm called *'Broadcast Incremental Power' (BIP) first described in* [45]. This algorithm uses Omni directional antennas and is similar to minimum spanning tree. This is further sub-divided into two categories, *RB-BIP and DBIP*. In RB-BIP, initially a broadcast tree is formed and then a beam reducing step is included to minimize the angle to cover the neighboring nodes while the transfer of message takes place. In DBIP, a broadcast tree is formed and the nodes have the capability to renew its energy while transfer and change the structure to include new nodes. The formation of broadcast tree using directional antennas is described. This phenomenon is similar to *Yao-graphs*. The plane around a node is divided into different sectors. The directional antennas are used for power efficiency. Here, the concept of *'minimum spanning tree' (MST)* is also used which consists of the minimum longest edge among all the spanning edges. The construction of the broadcast tree works in two stages. The first stage includes the use of *MST* to build a tree. And the second phase includes the use of sectored antennas for the transfer of message in the broadcast tree. The cost of an edge between two nodes is defined as the power required transferring a message between the two nodes. Initially, *Prim's or Kruskal's* algorithm is used to compute a spanning tree. Then the nodes using the formation of sectors, with the help of directional antennas determine the neighboring nodes for the final transfer. If the direction to reach the destination includes more than one path, then the node for which the edge requires minimum power to transfer is selected. The process concludes with the advantage of using *MST* and using directional antennas with more number of sectors for making the transfer energy efficient.

### E. COOPERATION AND DIRECTIONALITY: FRIEND OR FOE?

The paper we are talking about describes the interaction between cooperation and directionality for bidirectional and Omni-directional antenna in the medium access control layer [46]. Omni-directional antennas can extend the range covered by the host node but the attenuation of power also takes place. The first process is described as cooperativeness with Omni-directional antenna. The communication between the host and the destination node takes place via a server node. The host node initially finds a server which will consume lesser power to deliver the message to destination node via it than to deliver it directly from host to destination. Once it is found, the host sends a signal to the server node and the destination node for delivery. The server node in turn would inform the host and the destination node signalling that it is ready to transfer. The destination node as a result would also signal to the server and the host node indicating its readiness. The signals sent in between would include the information rate at which the transfer would take place between the three nodes. The second method is similar to the first one apart from the difference that is non-cooperative directional antenna is used. The host node sends a signal to the destination node for transfer. It in turn, receives a signal back, and then the transfer of message would take place. The two signal transfer is referred to the acknowledgement between the two nodes indicating their position. The third process is similar to the first process given that here the signal transfer does not take place in a three way manner. Initially, when the host node conveys a signal to the server node that it wants to send a message to the destination node via it, the server node sends an acknowledgement to the host node and an alert to the destination node to receive the message. The destination node in turn would reply to the host node that it is ready and the transfer to take place. The paper concludes to a point that the directionality and cooperation could work together with appropriate selection of antenna and layer to work on. However, cooperative forwarding limits the spatial reuse we may obtain in a system equipped with directional antenna when the antenna beam is narrow enough.

## F. A SPECIFIC CASE OF TOPOLOGY CONTROL: UNIT DISK GRAPH APPROXIMATION

This paper of Kuhn et al. [40] is based on *unit disk graph (UDG)* and other derived forms. *Unit disk graph* is referred as the graph where the connection between the host node and other nodes exist if any other node lies within the disk of unit radius centered at the host node. As per the paper, if only neighboring nodes are allowed to interact with each other for distance greater than *1*, then if $P \neq NP$, the distance could be as high as $\sqrt{(3/2)} - \varepsilon$, where $\varepsilon$ tends to zero for the number of nodes tending to infinity. The formulation of *quasi unit disk graph* is shown in this paper. In *quasi-UDG*, the connection between two nodes exists if the distance d between them lies between *0* and *1*. The connection is absent for distance greater than *1*, and ambiguity exists when the distance is between *d* and *1*.The *quasi-UDG* is more realistic than normal *UDG*. The paper then gives the definitions of quality and realization for *quasi unit disk graph*. The realization of a *quasi-UDG* is shown for a given graph through a grid drawing for *UDG* being NP-hard. The reduction process for a quasi-*UDG* is shown. Another important aspect that is shown in this graph is *cages and chains*. This process gives a realistic nature of the grid drawing. It includes two main clauses. The *cages* can have up to a maximum number of nodes. Two different *cages* are connected through a *hinge* which might form a *chain* and is placed inside one of the *cage*. Then the lemmas are shown for the realization of a given circle for a *quasi-UDG*. The process of *cages* and *chains* can be formed in two different ways like the clause and the variable component. For the phenomenon of cages and chains, the fig [40] is given below. Finally, the realization for the *quasi UDG* is given for different values of *d*, distance between two nodes.

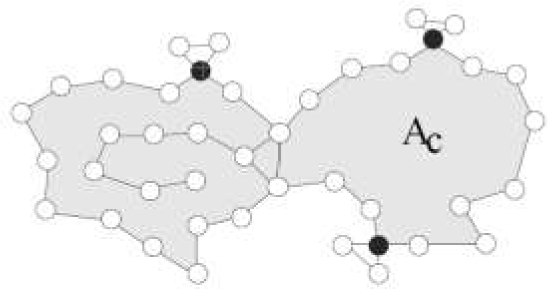


Fig.1 Formation of cages and chains [40]

## G. YAO SPANNERS FOR WIRELESS AD HOC NETWORKS

Molla offers an introduction to the spanning graphs that could be used in wireless communication [39]. Initially, it starts with a topology control problem. As described in the paper, if the prototype of the real scenario is considered, the nodes act as communicating objects in a wireless system where edges represent connections between them. In a simple case, *unit disk graph* can be considered where the connection between two nodes only exists if the Euclidean distance between them is less than unity. But in a real scenario, it is much more complicated than that. This may lead to additional interference problem. To circumvent this problem, a sub graph of the original graph is selected which connects the edges but reduces interference. The phenomenon is known as *Spanner property*. The *'relative neighborhood graph' (RNG)* is defined as the set of edges between the pair of vertices where the *lune* formed by the nodes does not contain any other node. This graph is a *planar graph* and has a *stretch factor* of *O(n)*. The *Gabriel graph* is similar to the *RNG.* But here the pair of nodes forms a circle with the edge between them as the diameter of the circle provided no other nodes lie within the circle. But the *Gabriel graph* has two disadvantages. A node in the graph has a high in *degree* and also a high *stretch factor*. The *Voronoi diagram* is a bit complicated but is much useful in communication purposes especially for the situation where a very high number of nodes are present. Here, a set of three nodes are taken in the plane which forms a triangle. Then the bisection of the three sides is done and the plane around the three nodes is divided accordingly. *Delaunay triangulation* has a process similar to the *Verona diagram*. Here, the three nodes are considered which forms a triangle. The circum circle of the triangle contains no other nodes inside it. This process is said to increase the minimum angle of the angles of the triangle considered. Then the *Yao graph* and the *Yao-Yao graph* are explained. In *Yao graph*, non over-lapping sectors are formed around a plane keeping the host node as the apex. Then, within a sector, the host node is connected with the node with which it has the least distance. The fig. shown [39] below depicts the formation of Yao-graph. But the in degree for a *Yao graph* is high. If among the nodes connected to the host node within a sector, that connection edge only remains with the one with which it has the shortest distance, so that both the in degree and the out degree are bounded a *Yao-Yao graph* is formulated. Then the proofs for spanning properties are shown for both *Yao* and *Yao-Yao graphs*. The *Yao graph* is not a *spanner* for parameter values equal to *2*, *3* and has a spanning property for the value *4* only in convex position. The *Yao-Yao graph* in not a *spanner* for parameter values equal to *4* and *6*. It is not known whether the either is a *spanner* or not for even restricted cases when parameter value is equal to *5*.

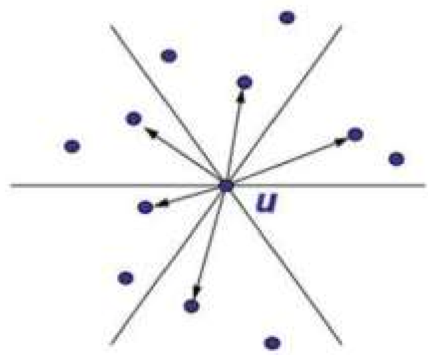


Fig.2 Selection of points in Yao-graph [39]

H. LOCAL EDGE COLOURING OF YAO-LIKE SUBGRAPHS OF UNIT DISK GRAPHS:

The main idea of their paper [47] is to introduce a method for assigning color to edges for *Yao-graph* like structures. Here, the *unit disk graphs* are explained briefly from which a subclass called *edge-wedge* graphs has been derived. A number of algorithms are also shown to stabilize this structure which is similar to other graphs like *Gabriel graphs, Yao graphs, Local Delaunay triangulations* used in wireless communications. The *edge-wedge algorithm* is based on a fact that for any two numbers (integers) *p* and *q*, if the circle around a node is divided into *2q* equally spaced wedges then each wedge can contain at most *p* points different from the source node. The nodes used here are position aware, that is they know their respective as well as other node's Cartesian coordinates. The number of colors used for edge coloring is either *2*p*q* or *2*p*q +1*. The algorithm is designed such that no two adjacent edges in the graph have same color. The minimal number of colors used in the algorithm is said to be *edge chromatic number of the graph*. The algorithm starts with the basic concept that in *a unit disk graph*, a node can communicate with other nodes only if they lie within the disk of unit radius. But in real scenario, the wireless system works better than *simple UDG*. *Spanners* as a result were introduced, in which a sub graph of the original one is considered that connects the interacting nodes but decreases interference. Commonly used spanners are *Yao graphs, Gabriel graphs, and Relative Neighborhood graphs. Edge-wedge graph* is another type of *spanner* similar to *Yao graphs*. Initially for the construction of *edge-wedge graphs*, the circle around a node is divided into *2k* equally angled wedges, where *k* is an integer. Here, at least one node incident to the source node will lie inside a wedge. The edges of the nodes are connected respectively in each wedge to the source node for the one with the shortest distance. The edges are then divided into classes. The edges lying in opposite wedges are categorized under the same class. A new concept called the *horizon distance* is introduced which is the maximum limit of a node's transmission range. So beyond that, the node is unaware of other node's coordinates. The construction of the *edge-*

*wedge graph* takes place in two stages. In the first stage, alternate colors are assigned to the edges. A single edge cannot be two colored, so the geometric nature of the plane is to be confirmed first. Accordingly, classes are formed as per the color of the edges. The plane is divided into segments based on shorter and longer paths. For an edge crossing a segment is denoted by a third color. These cutting based on the length of the edge are done by *'virtual cutting lines' (VCL).* These lines are perpendicular to the axis of the double *wedge* with a minimum distance of two. The edges that will cut these virtual cutting lines will be denoted by a third color, thus makes these lines identical to the boundary lines. Since the cutting lines are perpendicular to the side of the wedge, the same colored edges from the nodes within a wedge will be separated by a different color in between. The *Local Edge Coloring Algorithm* states that initially after sorting out the edges into different classes and ranks, the nodes should make other nodes aware of their respective positions. Then if an *edge* intersects the *VCL*, a color can be assigned, else assignment of color can be done based on the classes they are divided into. There are also a number of classes the *VCL* is divided into. The paper as a whole describes the new algorithm which is for a subclass of *Yao graphs* and the same has similarity to *unit disk graphs*. The paper ends up with calculating the lower bounds for *d* that is the Euclidean distance between two nodes and the analysis of the described algorithm for different values of *k* that is the integer into which the wedges are divided into.

I. ON CERTAIN GEOMETRIC PROPERTIES OF THE YAO–YAO GRAPHS

The *Yao-Yao graph* [48] is a modified version of the *Yao-graph*. The construction of this graph takes place in two stages. In the first stage, keeping the host node as the apex, non-overlapping sectors are formed around the plane. The angle of each sector is *(2*pi)/p*, where *p* is the number of sectors. Then among the nodes lying within a sector, the node with the shortest distance from the host node is joined with it. This forms the *Yao-graph*. But the disadvantage with *Yao-graph* is the in degree of a node is very high which can have a maximum value of *O(n)*. Thus from here, the modification is made. In the second stage, the host node only keeps the incoming ray with the smallest distance. The resultant graph is the *Yao-Yao graph*. The fig [39] shows the complete Yao-Yao graph. The advantage of *Yao-Yao graph* is that both the *in degree* and the *out degree* of each node are bounded. Similar to the spanning properties of the *Yao-graphs*, *Yao-Yao graphs* can also be used as a spanner with variable *stretch factor*. The paper deals with the spanning property of *Yao-Yao graph* on *civilized unit disk graphs* for a given *stretch factor* greater than one. It has been said that the *Yao-Yao graphs* is a spanner for *p≥8* and also for *p≥6* with a parameter *(6*p)* for a different *stretch factor*. Finally, two proofs are given for a *Yao-Yao graph* in a plane and a non-plane situation.

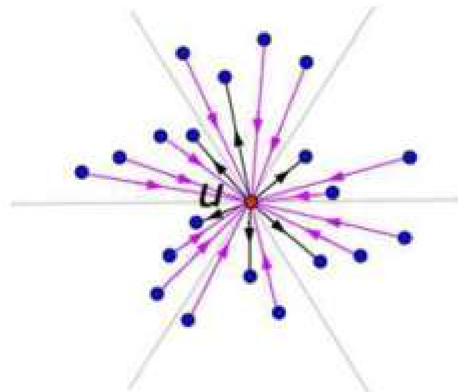


Fig.3 Formation of Yao-Yao graph [39]

J. ON THE SPANNING RATIO OF THETA-GRAPHS

The construction of *theta graphs* which is similar to *Yao-graphs* is explained in [49]. In *theta graphs*, initially, the plane around a node is divided into non-overlapping sectors. The angle of each sector is *(2*pi)/m*, where *m* is the number of sectors for a node. No two vertical lines lie parallel on the axis. Then within a sector, the source node is connected to the vertex which has the smallest distance in projection along the bisector of the sector. The bisector of a sector lies above the source node and coincides with a vertical line through the source node. In some cases, *half theta graphs* can also be constructed where positive cones are placed after two negative sectors and vice versa. Positive sectors are the ones where edges are added with one of the nodes lying within the sectors. The measurement that is taken from the source node to the ones in a particular sector depends on the angular bisector on the sector. The weight of the edge is the *Euclidean distance* between the two nodes. The resulting graph is equivalent to the *Delaunay Triangulation* whose spanning ratio is *2*. The *theta routing algorithm* is said to be the standard routing algorithm for all *theta graphs* having seven or more sectors. Then the proofs are shown for theta graphs to be spanners for different values of *stretch factors*. Lastly, the upper bounds for theta graphs with *4k + 3*, *4k + 4* and *4k + 5* are given, where *k* is an integer with value greater than *1*.

K. ORDERED THETA GRAPH

One enhancement of theta graphs is the notion of *ordered theta graphs* [41]. The construction of these graphs takes place in two stages. Initially, non-overlapping sectors are formed around the node on the given plane. Suppose *k* cones are formed, thus having an '*angle of theta= (2*pi)/k*' for each sector. Then in the second stage, within each sector, orthogonal projections are made from the nodes to the counterclockwise wall. The host node is connected to the nodes within a sector to the one having its projection on the wall closest to the host node. The fig below [41] represents the connection forming orthogonal cones. The *ordered theta graph* is different from other graphs to the fact that the change of ordering of the vertices can modify the graph formed.

The main challenge however is to find the nodes within a sector and finding the closest one. For this, *range trees* are used. One is used for each non-overlapping cone. The nodes within a sector are stored in the *range trees*. Then the minimum Y-coordinate is determined from the nodes whose X and Y coordinates are greater than the same of host node. The use of the *range trees* modifies the ordering of a *pi-ordered theta graph* to $O(k_{\Theta}*n*(logn))$, where $n$ is the number of nodes in the system. The *ordered theta graphs* can also be used as *spanners*. Then the lemmas are shown for *ordering of theta graphs*. Finally, proofs are offered for the ordering of theta graphs for *lower degrees* and *higher dimensions*.

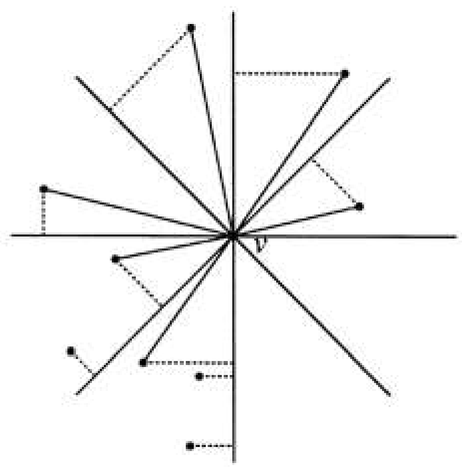

Fig.4 Formation of Cones with orthogonal angles [41]

## V. CONCLUSION

As various new technologies emerging, new understandings [50] and concepts [51] are also getting conceptualized at theoretical or mathematical plane and indeed, sometimes we are again forced to have a second thought and make a careful revisit of issues to judge the compatibility of their benefits [52]. Therefore, for adaptive beam forming and directionality enabled green wireless communication under realistic channel unreliability and interference constraints, we need to develop strong theoretical underpinnings. The civilized UDGs, Yao-graphs and theta-graphs and their variants do provide us some basic but concrete graph theoretic toolboxes for this purpose. So, we do believe that putting their properties and possibilities in a structured taxonomical manner is indeed a timely exercise, which this comprehensive survey intends to do.

## REFERENCES

[1] Anthony Ephremides, Bruce Hajek, "Information theory and communication networks: an unconsummated union", IEEE Transactions on Information Theory, Vol. 44, No. 6, pp. 2416-2434, Oct 1998.

[2] Sha Hua, Hang Liu, Mingquan Wu, S.S. Panwar, "Exploiting MIMO antennas in cooperative cognitive radio networks", IEEE INFOCOM 2011, 10-15 April, 2011, pp. 2714-2722, Shanghai, Chaina. DOI: 10.1109/INFCOM.2011.5935102.

[3] C.X. Wang, X. Hong, X. Ge, X. Cheng, G. Zhang, J. Thompson, "Cooperative MIMO channel models: A survey", IEEE Communications Magazine, Vol. 48, No. 2, pp. 80-87, Feb 2010. DOI: 10.1109/MCOM.2010.54.02668.

[4] Y.C. Liang, K.C. Chen, G.Y. Li, P. Mahonen, "Cognitive radio networking and communications: an overview", IEEE Transactions on Vehicular Technology, Vol. 60, No. 7, pp. 3386-3407, Sept 2011. DOI: 10.1109/TVT.2011.2158673.

[5] K.J. Ray Liu, Ahmed K. Sadek, Weifeng Su, Andres Kwasinski, "Cooperative Communications and Networking", Cambridge University Press, 2009. ISDN: 978-0-521-89513-2 hardback.

[6] R. Swaminathan, Rajarshi Roy, M.D. Selvaraj, "Performance analysis of triple correlated selection combining for cooperative diversity systems", IEEE ICC 2013, Budapest Hungary, pp. 5483-5488.

[7] R. Swaminathan, M.D. Selvaraj, Rajarshi Roy, "Exact Error Analysis of MPAM Signaling for a Cooperative Diversity System with Correlated Links Using Paired Error Approach", IEEE Communication Letters, Vol. 18, No. 2, pp. 273-276, Feb 2014.

[8] Soumendra Nath Datta, Saswat Chakrabarti, Rajarshi Roy, "Comprehensive Error Analysis of Multi-Antenna Decode-and-Forward Relay in Fading Channels", IEEE Communication Letters, Vol. 16, No. 1, pp. 47-49, 2012.

[9] Soumendra Nath Datta, Saswat Chakrabarti, Rajarshi Roy, "Comprehensive error performance analysis of distributed selection combining with multi-antenna amplify-and-forward relay over Nakagami-m fading channels", IET Electronics Letters, Vol. 46, No. 22, pp. 1523-1525, 2010.

[10] M.V. Kumar, R. Swaminathan, R.Roy, "Green cooperative communication techniques for intelligent transportation systems", IEEE International Conference on Signal Processing, Computing and Control, Wakhnaghat, Shimla, HP, India, 26-28 Sept., 2013, pp. 1-5. DOI: 10.1109/ISPCC.2013.6663432.

[11] Acharya, T., Chattopadhyay, S. and Roy, R. 'Maximum lifetime broadcasting in cooperative multi-hop wireless ad hoc networks', *Int. J. Ad Hoc and Ubiquitous Computing*, Vol. 6, No. 1, pp. 10–23, 2010.

[12] H. Zhang, N.B. Mehta, A.F. Molisch, Jin Zhang, Dai Huaiyu, “Asynchronous interference mitigation in cooperative base station systems”, Vol. 7, No. 1 , pp. 155-165, IEEE Transactions on Wireless Communications, Jan 2008.

[13] L.Lai, H. El Gammal, “On cooperation in energy efficient wireless networks”, IEEE Transactions on Wireless Communication, Vol. 7, No. 5, pp. 1868-1878, May 2008.

[14] Chun Nie, Pei Liu, Thanasis Korakis, Elza Erkip, Shivendra Panwar, “CoopMAX: A Cooperative MAC with Randomized Distributed Space-Time Coding for an IEEE 802.16 Network”, IEEE ICC 2009, IEEE International Conference on Communications, pp. 5212-5217.

[15] Peijian Ju, Wei Song, Dizhi Zhou, “Survey on cooperative medium access control protocols”, IET Communications 2013, Vol. 7, No. 9, pp. 893-902. DOI: 10.1049/iet-com.2012.0739.

[16] Robert G. Gallager, “Principles of Digital Communication”, Cambridge University Press, Chapter 1, Section 1.1.

[17] Xiaojun Lin, Ness B. Shroff, R. Srikant, “A Tutorial on Cross-Layer optimization in Wireless Networks ”, IEEE Journal On Selected Areas in Communication, Vol. 24, No. 8, pp. 1452-1463, Aug 2006. DOI: 11.1109/JSAC.2006.879351.

[18] Mung Chiang, Steven H. Low, A. Robert Calderbank, John C. Doyle, “Layering Optimization Decomposition: A Mathematical Theory of Network Architectures”, Proceedings of the IEEE, Vol. 95, No. 1, pp. 255-312, Jan 2007.

[19] R. Olfati-Saber, A. Fax and R.M. Murray, “Consensus and cooperation in networked multi-agent systems”, Proceedings of the IEEE, Vol. 95, No. 1, pp. 215-233, Jan 2007.

[20] A. Kshyap, T. Basar, R. Srikant, “Quantized Consensus”, Automatica, Vol. 43, No.7, pp. 1192-1203, July 2003.

[21] Shreyas Sundaram, Christoforos N. Hadjicostis, “Structural Controllability and Observability of Linear Systems Over Finite Fields with Applications to Multi-Agent Systems”, IEEE Transactions on Automatic Control, Vol. 58, No. 1, pp. 60-73, Jan 2013.

[22] Raymond W. Yung, “Information Theory and Network Coding”, Springer, Aug 2008. ISBN: 978-0-387-79233-0.

[23] Shuo-Yen Robert Li, Raymong W. Yeung, Ning Cai, “Linear Network Coding”, IEEE Transactions on Information Theory, Vol. 49, No. 2, pp. 371-381, Feb 2003.

[24] Rudolf Ahlswede, Ning Cai, Shuo-Yen Robert Li, Raymond W. Yeung, “Network Information Flow”, IEEE Transaction on Information Theory, Vol. 46, No. 4, pp. 1204-1216, July 2003.

[25] R. Koetier and M. Medard, “An algebraic approach to network coding”, IEEE/ACM Transactions on Networking, Vol. 11, No. 5, pp. 782-795. Oct 2003.

[26] R.W. Yeung, S.Y. R. Li, N. Cai and Z.Zhang, “Network Coding Theory”, in Foundations and Trends in Communications and Information theory, Now publishers Inc, Vol. 2, No. 4/5, pp. 241-381.

[27] Abbas El Gamal, Young-Han Kim, “Network Information Theory”, Cambridge University Press, Jan 2012. ISBN: 9781107008731.

[28] L-L. Xie, P.R. Kumar, “Wireless Network Information Theory”, DIMACS Workshop on Network Information Theory, March 2003.

[29] P. Santi, “Topology Control in Wireless Ad Hoc and Sensor Networks”, John Wiley and Sons, Chichester, UK, July 2005.

[30] Soumitra Dutta, INSEAD, Irene Mia, World Economic Forum (Editors), “The Global Information Technology Report 2009-2010”, ICT for sustainability.

[31] M. Grossglauser, David N.C. Tse, “Mobility increases the capacity of ad hoc wireless networks”, IEEE/ACM Transactions on Networking (TON), Vol. 10, No. 4, pp. 477-486, August 2002.

[32] Anand D. Sarwate, Alexandros G. Dimakis, “The impact of Mobility on Gossip Algorithms”, IEEE Transactions on Information Theory, Vol. 58, No. 3, pp. 1731-1742, 2012.

[33] Volkan Rodopolu, Teresa H. Meng, “Bits-per Joule Capacity of Energy-Limited Wireless Networks”, IEEE Transactions on Wireless Communications, Vol. 6, No. 3, pp. 857-865, 2007.

[34] Christian Scheideler, “Overlay Networks for Wireless Systems”. In Chansu Yu (Editor), Performance Analysis of Mobile and Ad Hoc Networks, Wireless Networks and Mobile Computing Series, Vol. 7, Nova Science Publishers, 2007.

[35] Piet Van Mieghem, “Graph Spectra for Complex Networks”, Cambridge University Press, 2011.

[36] Tamaghna Acharya, Rajarshi Roy and Samiran Chattopadhyay, “Energy-Efficient Broadcasting in Wireless Ad Hoc Networks Using Directional Antennas”, IEEE VTC, Singapore, pp. 36-40, 2008.

[37] S. Maulik, A. De, A. Bhattacharya, R. Roy, “”Dynamic Resource Allocation in SINR Constrained Cognitive Radio Network with Imperfect Channel States”, Fourth Nordic Workshop on System and Network Optimization for Wireless, SNOW, April 2-6, 2013, Yllas, Finland.

[38] U. L Wijewardhana, M. Codreanu, M. Latva-aho, "Robust Beam Former Design for underlay Cognitive Radio Network Using Worst Case Optimization", University of Oulu, Fourth Nordic Workshop on System and Network Optimization for Wireless, SNOW, April 2-6, 2013, Yllas, Finland.

[39] Nawar M. El Molla, "Yao Spanners for Wireless Ad Hoc Networks", MS thesis, CS Dept., Villanova University, 2009. (Research Supervisor: Mirela Damian)

[40] Fabian Kuhn, Thomas Moscibroda and Roger Wattenhofer, "Unit Disk Graph Approximation", DIALM-POMC'04, Philadelphia, Pennsylvania, USA, 2004.

[41] Prosenjit Bose, Joachim Gudmundsson and Pat Morin "Ordered theta graphs", Computational Geometry, No. 28, pp. 11-18, 2004. DOI:10.1016/j.comgeo.2004.01.003.

[42] Xiang-Yang Li, Wen-Zhan Song and Yu Wang, "Efficient Topology Control for Ad-Hoc Wireless Networks with Non-Uniform Transmission Ranges", Wireless Networks, Springer, Netherlands, Vol. 11, pp. 255-264, 2005.

[43] Khaled Alzoubi, Xiang-Yang Li,Yu Wang, Peng-Jun Wan, and Ophir Frieder, "Geometric Spanners for Wireless Ad Hoc Networks", IEEE Transactions on parallel and Distributed Systems, Vol. 14, No. 5, pp. 1-14, 2003.

[44] Intae Kang, Radha Pooverdran,, "Maximizing network lifetime of broadcasting over wireless stationary ad hoc networks", Vol. 10, No. 6, pp. 879-896, Dec 2005.

[45] J.E. Wieselthier, G.D. Nguyen and A. Ephremides, "On the construction of energy-efficient broadcast and multicast trees in wireless networks", IEEE INFOCOM 2000, Vol. 2, pp. 585-594, Tel Aviv, Israel.

[46] Zhifeng Tao, Thanasis Korakis, Feilu Liu, Shivendra Panwar, Jinyun Zhang, Leandros Tassiulas, "Cooperation and Directionality: Friends or Foes?", IEEE ICC, Beijing, China, pp. 4424-4430, 2008.

[47] J. Czyzowicz, S. Dobrev, E. Kranakis, J. Opatrny, J. Urrutia "Local Edge Colouring of Yao-like Sub graphs of Unit Disk Graphs", LNCS 4474, pp. 195-207, Heidelberg, Berlin, 2007.

[48] Iyad A. Kanj and Ge Xia, "On certain geometric properties of the Yao–Yao graphs", Springer, New York, USA, Nov. 2012. DOI: 10.1007/s10878-012-9570-z.

[49] Prosenjit Bose, Andre van Renssen and Sander Verdonschot, "On the Spanning Ratio of Theta-Graphs", Springer-Verlag, WADS, LNCS 8037, pp. 182-194, Heidelberg, Berlin, 2013.

[50] P. Gupta, P.R. Kumar, "The Capacity of Wireless Networks", IEEE Transactions on Information Theory, Vol. 46, No. 2, pp. 388-402.

[51] Vahid Tarokh, Nambi Seshadri and A.R. Calderbank, "Space-time codes for high data wireless communication: Performance analysis and code construction", IEEE Transactions on Information Theory,, Vol. 44, No. 2, pp. 744-765. DOI: 10.1109/18.661517.

[52] Sushant Sharma, Yi Shi, Jia Liu, Y. Thomas Hou, Sastry Kompella, Scott F. Midkiff, "Network Coding in Cooperative Communications: Friend or Foe?", IEEE Transactions on Mobile Computing, Vol. 11, No. 7, pp. 1073-1085, July 2012.