PARAMETERS OPTIMIZATION OF PASSIVE VEHICLE SUSPENSION BASED ON INVARIANT POINTS THEORY

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Abstract- Two methods for parameters optimization of traditional passive suspension based on the invariant point theory are presented in this paper. Firstly, the two degree-of-freedom dynamical model for a quarter vehicle suspension is established. Then all the invariant points for the acceleration of

vehicle body, suspension deformation, and tire displacement are obtained through the invariant equation associated with Laplace transform. Secondly, two optimization methods for the system parameters of passive suspension are presented, and the design objectives are to designate the invariant points as the local maximum points of the amplitude-frequency curves so as to improve the control performance of vibration isolation. At last, the control performances of the two improved passive suspension systems are compared with those of the original passive suspension, the active suspension, and the semi-active suspension by some performance indexes. The results show that the two optimization methods could greatly improve the control performance of passive suspension systems.

Index terms*:* **Vehicle suspension, invariant points, parameters optimization, passive control**

I.   INTRODUCTION

With the development of the modern vehicle engineering, the riding comfort has attracted more and more attention from the scientists and engineers [1-8]. As a key device, the suspension system is important to affect not only the driving security but also the riding comfort to passengers. Because the vibration isolation parameters of passive suspension are fixed, it is difficult to obtain the satisfactory isolation results in whole-frequency range. Due to the limitation of passive suspension, the semi-active or active suspension is presented and studied to improve the riding comfort. A lot of works have concentrated on this subject, where different control strategies and controllable dampers are adopted in vehicle suspension.   In recent years, many scholars have conducted a lot of studies on active and semi-active vibration isolation algorithm and experiments, and have made some important conclusions [9-18]. There are some shortcomings for active suspension and semi-active suspension, such as large energy consumption, bad reliability and so on, so that these two kinds of controllable suspensions have not been widely applied in vehicle engineering.

Among the evaluation methods for riding comfort, the most important indicator is the acceleration of vehicle body. The vehicle suspension is an important functional device to reduce the acceleration of vehicle body. The better control performance could be achieved by reasonably selecting suspension parameters, so that the riding comfort and driving security would be improved. The passive suspension is more reliable than the active suspension and semi-active suspension, and its cost is much lower. Accordingly, the study on improving the control performance of passive suspension is still meaningful.

In this paper two optimization methods for system parameters of passive suspension are presented, where the invariant points theory [4-6] of the amplitude-frequency curves is adopted and the appropriate constrain conditions for the amplitude-frequency curves are considered. The paper is organized as follow. In Section 1 the dynamical model is established and all the invariant points are studied. Two optimization methods for the system parameters of passive suspension are introduced in Section 2. Based on an example of real vehicle suspension, the comparisons between the control performances of the improved passive suspensions with those of the original passive suspension, the active suspension and the semi-active suspension are presented in Section 3 through three different performance indexes. Finally the conclusions are drawn in Section 4.

II.   MODEL OF VEHICLE SUSPENSION AND THE INVARIANT POINTS

The two degree-of-freedom model of a quarter vehicle suspension is taken as the study object, whose prototype and mechanic model are is shown in Fig.1. According to Newtonian second law, the differential equations could be established as follows

$$m_s \ddot{x}_s + c_s(\dot{x}_s - \dot{x}_t) + k_s(x_s - x_t) = u, \tag{1}$$

$$m_t \ddot{x}_t + c_s(\dot{x}_t - \dot{x}_s) + k_s(x_t - x_s) + k_t(x_t - x_r) = -u, \tag{2}$$

where m_s, m_t, k_s, k_t, c_s and u are the mass of vehicle body, the mass of tire, the stiffness of passive suspension, the stiffness of tire, the damping coefficient of suspension and the acting

force respectively. If u equals to 0, the differential equations will be for passive suspension system. The state variables x_s, x_t and x_r stand for suspension displacement, tire displacement and road input respectively.

Adding Eq. (1) and Eq. (2) together, the invariant equation of suspension is given as

$$m_s \ddot{x}_s + m_t \ddot{x}_t - k_t (x_r - x_t) =).$$

(3)
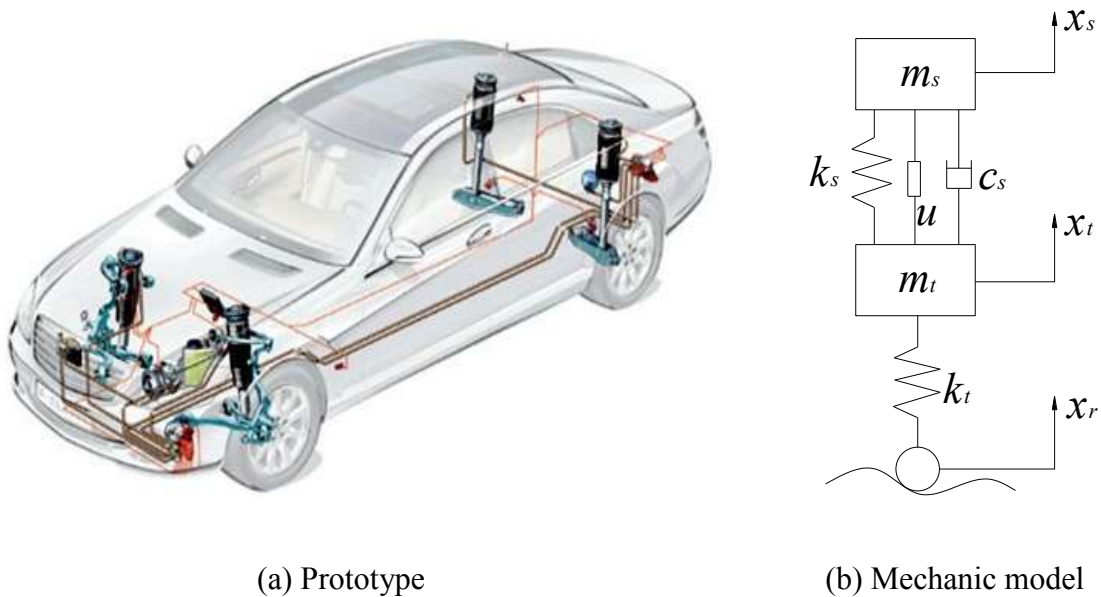

(a) Prototype   (b) Mechanic model

Figure 1. Prototype and mechanic model for a quarter vehicle

It could be found that there are no suspension parameters in this equation, which means that the active, semi-active and passive suspensions are all under the control of the invariant equation. Supposing the initial condition is zero, the Laplace transform of Eq. (3) could be obtained

$$m_s \ddot{X}_s(s) + m_t \ddot{X}_t(s) - k_t [X_r(s) - X_t(s)] =).$$

(4)

If the road roughness is used as input, three transfer functions that describe control performance could be defined. The transfer function for riding comfort is

$$H_{\ddot{x}s-r}(s) = \frac{\ddot{X}_s(s)}{X_r(s)}.$$

(5)

The transfer function for suspension deformation is

$$H_D(s) = \frac{X_s(s) - X_t(s)}{X_r(s)}. \tag{6}$$

The transfer function for tires deformation is

$$H_T(s) = \frac{X_t(s) - X_r(s)}{X_r(s)}. \tag{7}$$

Based on Eq. (4) to Eq. (7) and considering $s = j\omega$ ($j^2 = -$), the relationships between the three transfer functions can be established after simplification and classification

$$m_s H_{\ddot{x}s-r}(j\omega + k_t - n_t\omega)H_T(j\omega = n_t\omega , \tag{8}$$

$$- n_s\omega\ H_D(j\omega + k_t - m_s + n_t)\omega\ |H_T(j\omega = m_s + n_t)\omega , \tag{9}$$

$$- k_t - m_t + n_s)\omega\ |H_{\ddot{x}s-r}(j\omega = n_t\omega +) \ (k_t - n_t\omega)H_D(j\omega . \tag{10}$$

From Eq. (8) to Eq. (10), it could be observed when one of the three transfer functions is designated; the other two transfer functions can be completely determined by these constraint conditions. Therefore, a particular performance indicator should not be emphasized solely so that other performance indicators will be ignored in the process of designing suspension.

From Eq. (8), the invariant point of the modulus of transfer function or amplitude-frequency equation for riding comfort can be obtained

$$\left|H_{\ddot{x}s-r}(\omega)\right| = \frac{k_t}{m_s}, \tag{11}$$

where

$$\omega = \sqrt{k_t / m_t}. \tag{12}$$

From Eq. (9) or Eq. (10), the invariant point of the amplitude-frequency equation for suspension deformation can be obtained

$$\left|H_D(\omega)\right| = \frac{n_s + n_t}{m_s}, \tag{13}$$

where

$$\omega = \sqrt{\frac{k_t}{m_s + n_t}} . \tag{14}$$

In Eq. (9), there is not any other invariant point for tire deformation except for the invariant

point $H_T(0) =)$. That is to say, the wheels will vary with road tightly around $\omega \div)$.

Furthermore, it is found that if other types of transfer functions are adopted, there is still an

invariant point at ω for the transfer function of the riding comfort, with the only different form

in the invariant point. Meanwhile, the situations of the invariant points for other two transfer

functions, i.e. the transfer functions for the suspension deformation and tire deformation, are

similar to the transfer function for the riding comfort.

For example, three other types of transfer functions could be defined to describe the control

performance if the road speed is used as input. The transfer function for riding comfort is

$$H_{\ddot{x}s-r}(s) = \frac{\ddot{X}_s(s)}{\dot{X}_r(s)} . \tag{15}$$

The transfer function for suspension deformation is

$$H_D(s) = \frac{X_s(s) - X_t(s)}{\dot{X}_r(s)} . \tag{16}$$

The transfer function for tires deformation is

$$H_T(s) = \frac{X_t(s) - X_r(s)}{\dot{X}_r(s)} . \tag{17}$$

If Eq.(15) to Eq.(17) are plugged into Eq.(4), one could obtain the relationships between the three

transfer functions after simplification and classification

$$m_s H_{\ddot{x}s-r}(j\omega) + (k_t - n_t\omega^2)H_T(j\omega) = - m_t\omega , \tag{18}$$

$$- n_s\omega^2 H_D(j\omega) + k_t - m_s + n_t)\omega^2]H_T(j\omega) = - (m_s + n_t)\omega , \tag{19}$$

$$[k_t - m_t + n_s)\omega]H_{\ddot{x}s-r}(j\omega = ik_t\omega -) (k_t - n_t\omega)H_D(j\omega . \tag{20}$$

There are still invariant points at ω and ω_- obtained from Eq. (18) and Eq. (19) for the

transfer functions of body acceleration and suspension deformation, but the invariant points will become $\sqrt{m_t k_t}/m_s$ and $(m_s + n_t)^{3/2}/(m_s \sqrt{k_t})$. Therefore, whatever the forms of transfer functions are taken, the values of invariant points only depend on the suspension parameters m_s, m_t and k_t, and they have nothing to do with k_s and c_s. That means all the amplitude-frequency curves of the active, semi-active and passive suspension systems will pass through the same invariant points.

III.   OPTIMIZATION OF PARAMETERS OF PASSIVE SUSPENSION

For two degree-of-freedom passive suspension system that $u = $) in Eq. (1) and Eq. (2), the results can be obtained through Laplace transform

$$m_s X_s(s)s^2 + c_s s X_s(s) - c_s s X_t(s) + k_s X_s(s) - k_s X_t(s) =) , \tag{21}$$

$$m_t s^2 X_t(s) + c_s s X_t(s) - c_s s X_s(s) + k_s X_t(s) - k_s X_s(s) + k_t X_t(s) - k_t X_r(s) = 1 . \tag{22}$$

It could be obtained

$$X_s = \frac{k_t(k_s + {}_s s)X_r}{[- - {}_s - {}_s s)^2 + k_s + {}_s s + n_s s^2)(k_s + {}_t + {}_s s + n_t s^2)]} , \tag{23}$$

$$X_t = \frac{k_t(k_s + {}_s s + n_s s^2)X_r}{[- - {}_s - {}_s s)^2 + k_s + {}_s s + n_s s^2)(k_s + {}_t + {}_s s + n_t s^2)]} . \tag{24}$$

Based on Eq. (5) and Eq. (6) one could get the amplitude-frequency equations of the acceleration of vehicle body and suspension deformation as

$$\left| H_{\ddot{x}s- r}(\omega \right. = \frac{k_t \omega \sqrt{k_s^2 + {}_s^2 \omega}}{\sqrt{c_s^2 \omega \; k_t - m_s + n_t)\omega \;|^2 + k_s^2 - k_s - n_s \omega \;|(k_s + {}_t - n_t \omega \;|]^2}} , \tag{25}$$

$$\left| H_D(\omega \right. = \frac{k_t m_s \omega}{\sqrt{m_s^2 \omega \;(k_t - n_t \omega \;)^2 - !k_s m_s \omega \;(k_t - n_t \omega \;)(k_t - m_s + n_t)\omega \;) + {}_s^2 [k_t - m_s + n_t)\omega \;|^2 + {}_s^2 [k_t \omega - m_s + n_t)\omega \;|^2}} . \tag{26}$$

Because there are invariant points at ω and ω for Eq. (25) and Eq. (26), the invariant points

could be considered as the local maximum points of the amplitude-frequency equations to restrain the shape of the amplitude-frequency curves. This will make the control performance of vibration isolation for passive suspension improved. Two design methods for optimizing the system parameters of passive suspension, based on the abovementioned thought, are given as follows.

a. Method 1

The first method is based on the principle directly, that is to say, the two invariant points are respectively designed as local maximum points of the amplitude-frequency equations for vehicle body acceleration and suspension deformation. Letting the derivatives of $\left|H_{\ddot{x}s-r}(\omega\right.$ at $\omega = \sqrt{k_t/m_t}$ and $\left|H_D(\omega\right.$ at $\omega = \sqrt{k_t/(m_s + m_t)}$ equal to zero respectively, one can obtain

$$c_s^2 k_t + k_s(-k_t m_s + k_s m_t) = 0,\tag{27}$$

$$-k_t m_s m_t + k_s(m_s + m_t)^2 = 0.\tag{28}$$

Then the optimal suspension parameters by method 1 are

$$k_s = \frac{k_t m_s m_t}{(m_s + m_t)^2},\tag{29}$$

$$c_s = \sqrt{\frac{k_t m_s^3 m_t (m_s + m_t)}{(m_s + m_t)^4}}.\tag{30}$$

The obtained k_s and c_s could make the two invariant points as the local maximum points of the amplitude-frequency curves, which will improve the control performance.

b. Method 2

In method 2, the amplitude-frequency curve for vehicle body acceleration is designed as local maximum at ω and ω simultaneously, and the amplitude-frequency curve of suspension

deformation is done similar to that of the vehicle body acceleration.

Differentiating $\left|H_{\ddot{x}s-r}(\omega\right.$ and $\left|H_D(\omega\right.$ at $\omega = \sqrt{k_t/m_t}$, one can obtain

$$Y_1 = c_s^2 k_t + c_s(-c_t m_s + c_s m_t) , \tag{31}$$

$$Y_2 = c_s^2 k_t(m_s + m_t) + k_s m_t(-c_t m_s + c_s m_t) . \tag{32}$$

Similarly, differentiating $\left|H_{\ddot{x}s-r}(\omega\right.$ and $\left|H_D(\omega\right.$ at $\omega_- = \sqrt{k_t/(m_s + n_t)}$, one could get

$$Y_3 = - k_s^2(m_s + n_t)[-c_t m_s m_t + c_s(m_s + n_t)_2] + c_s^2 k_t[- k_s(m_s + n_t)^2 + c_t m_s(m_s + m_t)], \tag{33}$$

$$Y_4 = - c_t m_s m_t + c_s(m_s + n_t)^2 . \tag{34}$$

If the above-mentioned four equations are ordered to equal to zero and solving the system of algebraic equations, one could obtain the optimal suspension parameters. Unfortunately, there are only two unknowns in the four equations, so that the solutions should only be solved under the least squares criterion. The objective function is selected as

$$F = a_1 Y_1^2 + a_2 Y_2^2 + a_3 Y_3^2 + a_4 Y_4^2 , \tag{35}$$

where a_1 a_2 a_3 a_4 are positive weighting coefficients for the local maximum values. After the initial values and value ranges of k_s and c_s are given, the solutions under the least squares criterion can be solved by using some optimization software, such as the optimal toolbox in MatLab. The obtained stiffness and damping of passive suspension could make objective function as small as possible.

IV. PERFORMANCE SIMULATION

A group system parameters for a real vehicle suspension are given as m_s=400 kg, m_t=30 kg, k_t=130000 N/m. The initial parameters for suspension stiffness and damping coefficient are k_s=13000 N/m and c_s=2000 N s/m.

Based on method 1, one could obtain the optimal parameters as

$$k' = 8437 \text{ N/m}, \quad c' = 1833 \text{ N s/m}. \tag{36}$$

Based on method 2 and selecting $a_i = \quad (i = ,2,3,4)$, the optimal parameters are shown as

$$k' = 6999.4 \text{ N/m and} \quad c' = 1265 \text{ N s/m}. \tag{37}$$

Three ways are adopted to compare the vibration isolation performances of the improved passive suspensions with those of original passive suspension, active suspension and semi-active suspension. The first way is to compare the theoretical transfer function curves for two improved passive suspensions with those of the active and original passive suspension, and the results are shown in Fig. 2 to Fig. 4.   Due to the strong nonlinearity, the theoretical transfer function curves for the semi-active suspension could not be presented here.
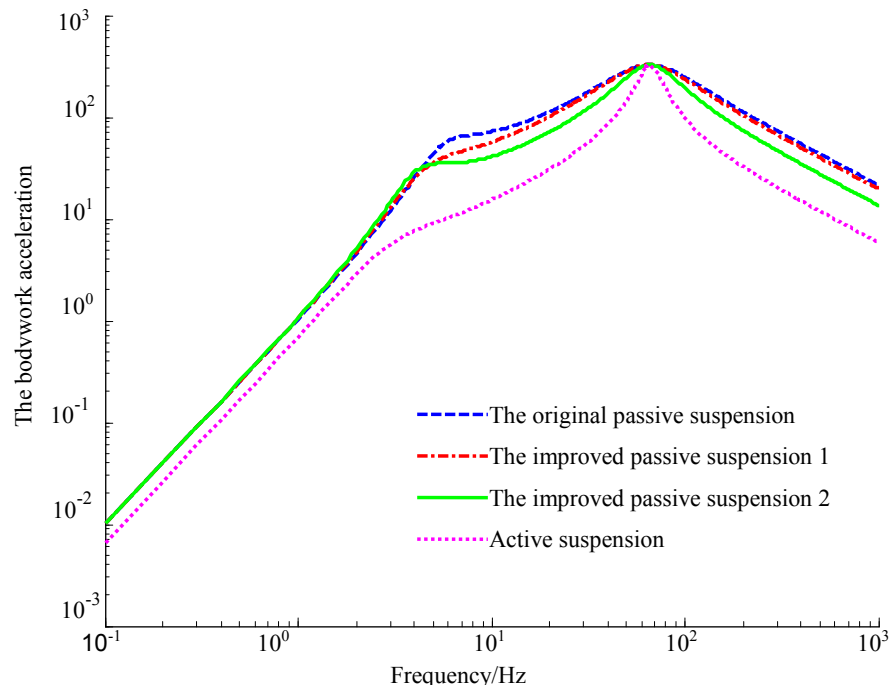

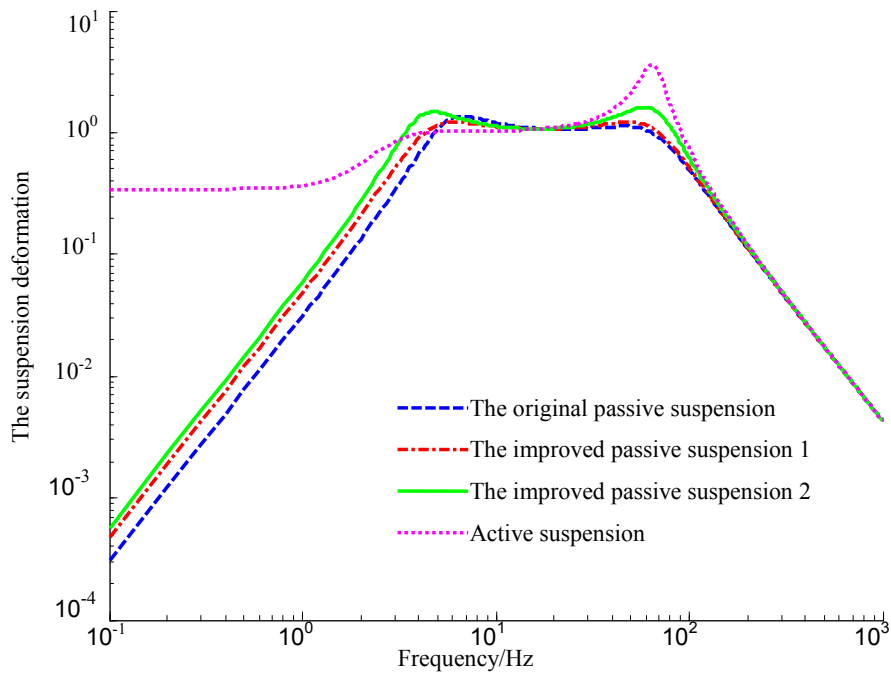Figure 2.   Comparison of the acceleration of vehicle body

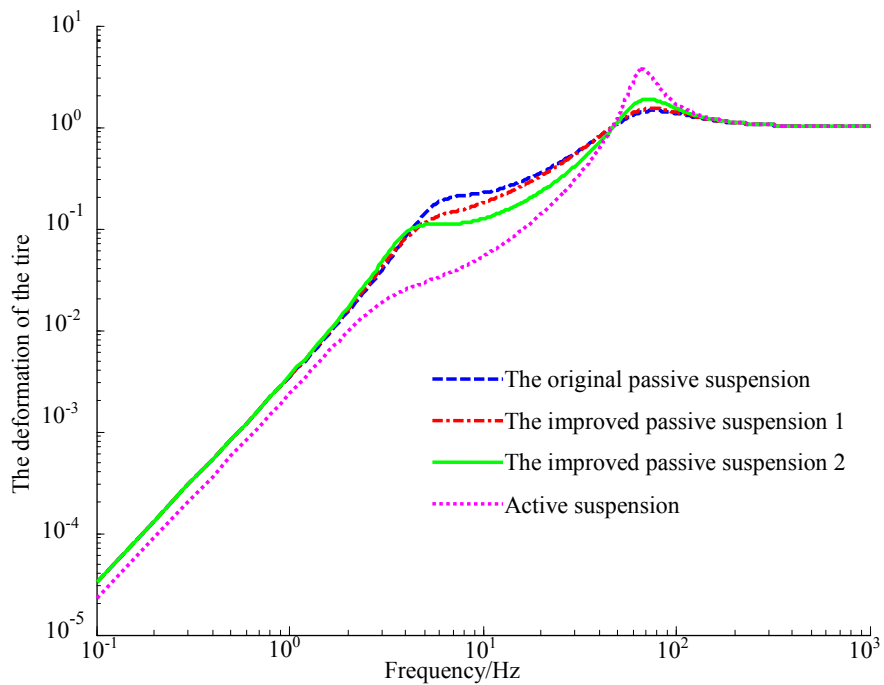Figure 3.　Comparison of the suspension deformation

Figure 4.　Comparison of the tire deformation

The second way is to numerically study the frequency response of five suspensions, including

two improved passive suspensions, the original passive suspension, the active and the semi-active suspension, where the deterministic sine road excitation with amplitude as 0.01m and different frequencies is adopted. The square value for vehicle body acceleration, suspension deformation and tire deformation are used as evaluation indicators after they become stable. The results are illustrated in Fig.5 to Fig 7.

The third way is to study the statistical responses of the abovementioned five kinds of suspension to the stochastic excitation. Based on the model for road surface suggested in references [7-8], stochastic excitation of road roughness with different levels (Grade A, B, C, D, E) are constructed for numerical integration and then the statistical properties of system response to stochastic excitation of five kinds of road roughness are summarized in Table 1.

The conclusions can be drawn based on Fig. 2 to Fig. 7 and Table 1.

(1) The vehicle body accelerations of the two improved passive suspensions have been significantly reduced compared with those of original suspension, so that the riding comfort for passive suspension is enhanced remarkably. This means the main design objectives of the two presented methods has been achieved and the two presented methods could improve the control performance of the passive suspension.

(2) From the observation of the suspension deformation, it could be found that the suspension deformations of two improved suspensions are slightly bigger than those of original suspension, but it is still in the design range for suspension. In addition, it could be found that the suspension deformations of two improved suspensions are still smaller than those of active and semi-active suspension. This could guarantee the improved passive suspensions have much feasibility in vehicle engineering.

(3) From the observation of tire deformation, the two improved passive suspensions are similar to the original passive suspension. They all are superior to the semi-active and active suspension, especially at high-frequency band. This means it is much easier to keep the tire life of the passive suspensions than the semi-active and active suspension.

Although the experimental verification could not be fulfilled to certify the effectiveness of the

presented optimization methods, the results by numerical simulation present much more reliable information. Accordingly, these optimization methods may be applied in vehicle engineering or other similar engineering fields.
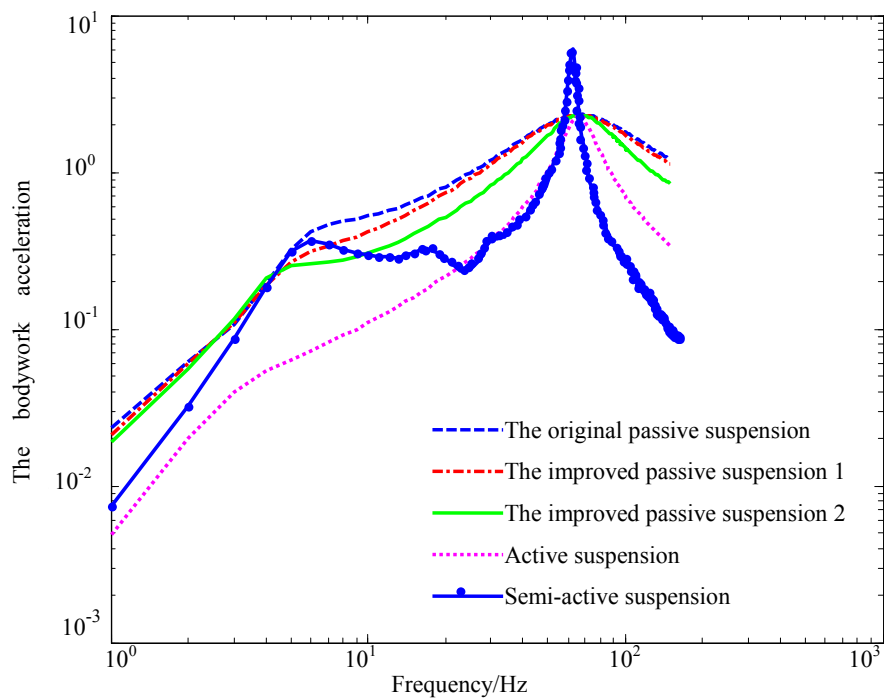

Figure 5.   Comparison of the acceleration of vehicle body
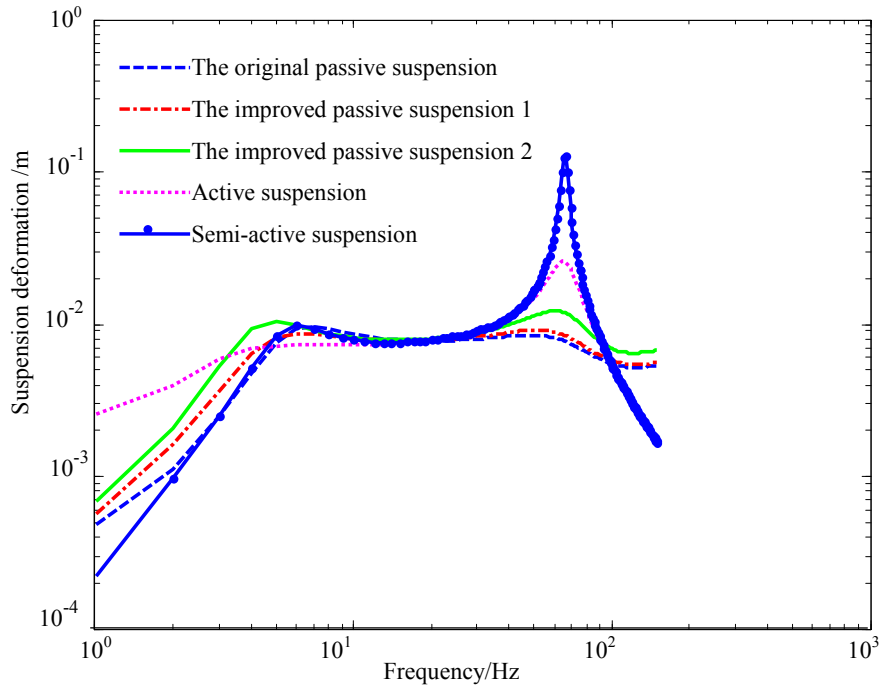
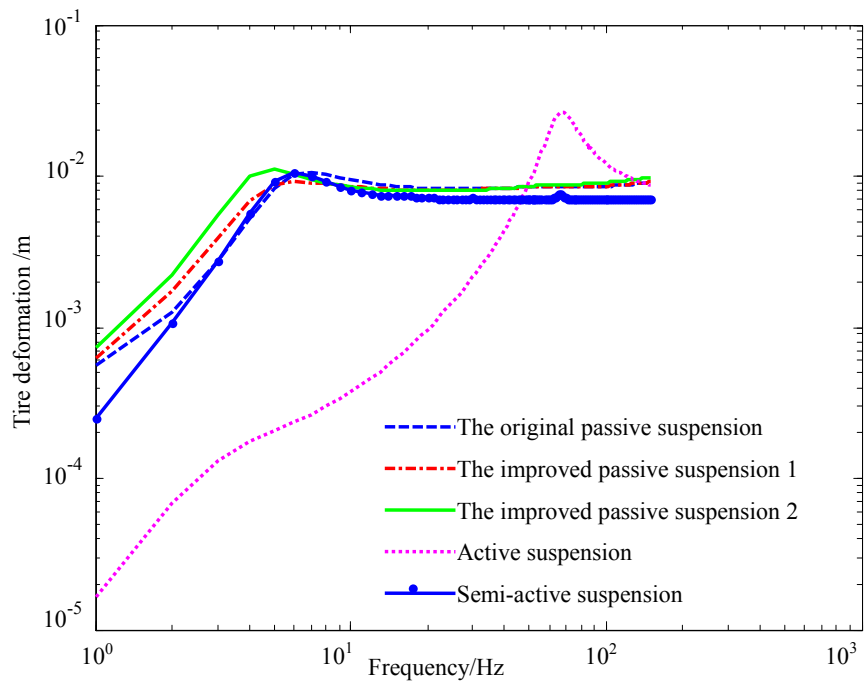Figure 6.   Comparison of the suspension deformation

Figure 7.   Comparison of tire deformation

Table 1 . The mean square value of response to the road levels

Road levels	Types of suspension	Vehicle body acceleration $/(m^2 \cdot s^-)$	Suspension deformation $/ 10^- \cdot m^2$	Tire deformation $/ 10^- \cdot m^2$
A-class road	Original passive suspension	0.2684	0.0027	0.0030
	Improved passive suspension 1	0.2501	0.0028	0.0029
	Improved passive suspension 2	0.2103	0.0033	0.0034
	Semi-active suspension	0.1909	0.0042	0.0036
	Active suspension	0.1754	0.0035	0.0015
B-class road	Original passive suspension	0.5408	0.0054	0.0060
	Improved passive suspension 1	0.5044	0.0056	0.0059
	Improved passive suspension 2	0.4235	0.0066	0.0068
	Semi-active suspension	0.4013	0.0086	0.0070
	Active suspension	0.3534	0.0070	0.0031
C-class road	Original passive suspension	1.0743	0.0108	0.0119
	Improved passive suspension 1	1.0018	0.0111	0.0118
	Improved passive suspension 2	0.8434	0.0131	0.0135
	Semi-active suspension	0.7948	0.0173	0.0148
	Active suspension	0.7015	0.0139	0.0061
D-class road	Original passive suspension	2.1474	0.0216	0.0239
	Improved passive suspension 1	1.9984	0.0221	0.0236
	Improved passive suspension 2	1.6806	0.0260	0.0269
	Semi-active suspension	1.5769	0.0349	0.0299
	Active suspension	1.4079	0.0278	0.0124
E-class road	Original passive suspension	4.3302	0.0434	0.0480
	Improved passive suspension 1	4.0361	0.0444	0.0472

Improved passive suspension 2	3.3945	0.0523	0.0538
Semi-active suspension	3.2505	0.0692	0.0564
Active suspension	2.8165	0.0555	0.0248

V.   CONCLUSIONS

Based on the invariant equation, the invariant points of amplitude-frequency equations for vehicle body acceleration, suspension deformation and tire deformation of passive suspension are obtained. According to the invariant point theory, two design methods for optimizing the system parameters of passive suspension are presented. Then the performance indicators of improved passive suspensions are compared with those of original passive suspension, active suspension and semi-active suspension analytically and numerically. The results show that the control performance of the two improved suspensions can be greatly improved. These methods can also be applied to solve other problems about vibration isolation in engineering, and they provide a simple way to design the system parameters for the appropriate engineering structure of vibration isolation.

ACKNOWLEDGEMENT

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