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LT-PAM: A Ranging Method Using Dual Frequency Optical Signals

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Abstract- This paper describes a new ranging technique using optical signals. The proposed technique is called LT-PAM (Long-Term Phase Accordance Method), and it has been extended from our own ranging technique called Phase Accordance Method (PAM). LT-PAM transmits multiple sync patterns composed of two sinusoidal waves with different frequencies. Unlike chirp modulation techniques, LT-PAM transmits the two waves simultaneously and thus enables the shortening of measurement time. We have conducted experiments using two types of light sources, collimated and diffused light. The experimental results indicated that the proposed method showed a moderate level of accuracy by adjusting the measurement time. For example, LT-PAM using a light emitting diode transmitting multiple sync patterns lasting 4 ms achieved 19.5 mm standard deviation in a measurement ranging 1500 mm. We also describe the theoretical analyses related to the proposed technique and discuss possible improvements by comparing theoretical and experimental results.

Index terms: Optical distance measurement, signal processing, Long-Term Phase Accordance Method (LT-PAM), time-of-flight (TOF)

I. INTRODUCTION

Noncontact ranging techniques play an important role in various industrial situations. Therefore, many researchers have so far attempted to develop numerous techniques and systems using ultrasound, RF, cameras, etc. Among them, the optical approach is one of the more promising ways to achieve accurate measurements [1]. For optical ranging, there are two types of light sources, that is, collimated and diffused light. When we use a collimated light source such as laser light, we can conduct distance measurements to a small target spot with high signal-noise ratio (SNR) optical waves, which allow us to achieve better ranging accuracy. On the other hand, when we use a diffused light source such as LED (light-emitting diode) light, we cannot easily concentrate a transmitted wave onto a small target spot like the laser light, and this lowers the SNRs of transmitted waves and makes accurate ranging difficult. However, due to the recent market penetration of LED illumination, the use of LEDs not only for illumination or data communication [2] but also for ranging is expected to open possibilities for innovative applications such as positioning systems [3], intelligent transportation systems [4][5], etc.

The ranging technique using chirp modulation is one of the well-known methods to achieve accurate distance measurements [6][7]. A commercial product using this technique is already on

the market [8]. To measure the distance using the chirp modulation technique, a continuous frequency bandwidth is needed to conduct frequency sweeps between its minimal and maximum frequencies. Thus, instantaneous measurements using chirp modulation are difficult. For example, according to our own investigation on [8], it takes around 0.2 s to complete the transmission of one chirp signal. Therefore, it may not be suitable for conducting ranging measurements in some situations, such as identifying the distance to a rapidly moving target at a certain moment.

In this paper, we propose a new ranging technique called LT-PAM (Long-Term Phase Accordance Method). LT-PAM was extended from PAM (Phase Accordance Method), [9] which was previously proposed by the authors' group. In PAM, only one burst signal, called a sync pattern composed of two sinusoidal waves with different frequencies, is used for ranging. Like PAM, LT-PAM also uses only two sinusoidal waves, but transmits multiple sync patterns that need a longer measurement time. By adjusting the measurement time, we can achieve a required level of ranging accuracy in an easy manner.

The paper is organized as follows: In Section II, we briefly describe the PAM and LT-PAM algorithms, and their differences with the chirp modulation technique. In Section III, we report experiments using collimated and diffused light. We also evaluate the performance of LT-PAM by changing the frequency difference between the sinusoidal waves. Issues related to possible improvements for more accurate and stable ranging are discussed in Section IV. Section V presents the conclusion of the paper and future work.

II. LT-PAM: THE LONG-TERM PHASE ACCORDANCE METHOD

A . PAM: THE PHASE ACCORDANCE METHOD

Before describing LT-PAM, we present an overview of PAM, which is the core part of LT-PAM. PAM is one of the time-of-flight (TOF) ranging techniques that measure the travel time of light waves and calculate the distance between a transmitter and a receiver. Compared with many conventional TOF methods that use a light pulse, PAM uses a burst signal called a sync pattern (Figure 1(a) left) as a transmitted wave.



(a) A sync pattern (left) and a phase difference (right).



(b) A rectangular window for quadrature detection.Figure 1. Signal processing using a sync pattern in PAM

This burst signal is composed of two sinusoidal waves with different frequencies. Mathematically, a sync pattern can be expressed as:

$$s(t) = a_1 \sin(\omega_1 t + \phi_1) + a_2 \sin(\omega_2 t + \phi_2) = a_1 \sin(2\pi f_1 t + \phi_1) + a_2 \sin(2\pi f_2 t + \phi_2)$$
(1)

where a_i is the amplitude, ω_i is the angular frequency, f_i is the frequency and ϕ_i is the respective initial phase (*i*=1, 2). A sync pattern has only one time point, called ``epoch," at which the phase difference of the two composing waves becomes zero (Figure 1(a) right). The epoch is used to identify the time of arrival at the receiver. The phases ϕ_i and ϕ_2 are identified by conducting quadrature detection. The algorithm to obtain the epoch precisely is explained as follows.

Let us define an inner product of a sync pattern s(t) and $e^{j\Omega t}$ as:

$$\left\langle s(t), e^{j\Omega t} \right\rangle = \frac{1}{T} \int_{-T/2}^{T/2} s(t) e^{-j\Omega t} dt = I + jQ$$
⁽²⁾

In this equation, time *T* is the integration interval of a rectangular window as shown in Figure 1(b) and the angular frequency Ω is called the reference angular frequency. When the reference angular frequency is ω_1 , we can acquire the real and imaginary parts of $\langle s(t), e^{i\omega_1 t} \rangle$ as shown below:

$$I_{\omega_{1}} = \frac{1}{2} (\sin \omega_{1} T + 1) a_{1} \sin \phi_{1} + \frac{1}{2} \left\{ \operatorname{sinc}(\omega_{1} + \omega_{2}) \frac{T}{2} + \operatorname{sinc}(\omega_{1} - \omega_{2}) \frac{T}{2} \right\} a_{2} \sin \phi_{2}$$

$$Q_{\omega_{1}} = \frac{1}{2} (\sin \omega_{1} T - 1) a_{1} \cos \phi_{1} + \frac{1}{2} \left\{ \operatorname{sinc}(\omega_{1} + \omega_{2}) \frac{T}{2} - \operatorname{sinc}(\omega_{1} - \omega_{2}) \frac{T}{2} \right\} a_{2} \cos \phi_{2}.$$
(3)

Here, we have four unknown values, a_1 , a_2 , ϕ_1 and ϕ_2 . Therefore, by calculating the two inner product values, $\langle s(t), e^{j\omega_1 t} \rangle$ and $\langle s(t), e^{j\omega_2 t} \rangle$, we can obtain $I_{\omega_1}, Q_{\omega_1}, I_{\omega_2}$ and find these unknowns. Consequently the epoch is identified and the time delay t_e is calculated by using equation (4). Finally, the total time-of-flight, t_{roF} , of a sync pattern is obtained by equation (5):

$$t_e = \frac{\phi_1 - \phi_2}{\omega_1 - \omega_2} \tag{4}$$

$$t_{TOF} = t_w + t_e \tag{5}$$

where t_{w} is the time at the center of the window.

The duration of a single sync pattern is determined by the frequency difference between two composing waves. For example, when the sync pattern is composed of 90 and 100 MHz sinusoidal waves, its duration becomes 100 ns. The rectangular window for quadrature detection is decided accordingly.

$B \cdot LT$ -PAM

From the description in the previous section, it is clear that the ranging performance of PAM is determined by how accurately an epoch is identified. Through theoretical analyses of PAM, we derived the following mathematical expressions related to the variance of the phase difference $V_{\Delta\phi}$ between two sinusoidal waves and a standard deviation of distance measurements σ_d , respectively [10]:

$$V_{\Delta\phi} = \frac{2}{1 - \operatorname{sinc}(\omega_2 - \omega_1)T/2} \frac{W}{PT}$$
(6)

$$\sigma_d = \frac{c\sqrt{V_{\Delta\phi}}}{\omega_2 - \omega_1} \tag{7}$$

where P[W] is the signal power, W[W/Hz] is the power density of the noise, and T[s] is the ranging measurement time. Equations (6) and (7) show that the ranging accuracy of PAM depends on an SNR ($\frac{P}{W}$), which is related to features of the light sources (intensity, collimated/diffused, etc.), the distance between the light source and the target, measurement environments (noisy or noiseless), etc. Figure 2 shows the theoretical standard deviation calculated using equation (7) by changing the SNRs of received signals. Equations (6) and (7) also indicate that increasing the measurement time T improves the ranging accuracy.

This investigation on the theoretical model led us to develop LT-PAM by controlling *T*. In our current implementation of LT-PAM, concatenated multiple sync patterns are transmitted from a light source as shown in Figure 3. The measurement time *T* is set as the integration interval of a rectangular window for quadrature detection in the same manner as PAM. Unlike PAM, the multiple sync patterns are included in this window by changing the measurement time. When the frequency difference $(|f_1 - f_2|)$ is large enough, $\operatorname{sinc}(\omega_2 - \omega_1)T/2$ in equation (6) becomes so small that $V_{\Delta\phi}$ is assumed to be proportional to 1/T. Thus, if we set the measurement time to be *n* times longer, we can expect a \sqrt{n} times improvement in the standard deviation of distance measurements. The measurement time *T* can be flexibly adjusted based on the requirements or purposes of applications, for example, the required update rate of ranging.



Figure 3. Concatenated multiple sync pattern signal for LT-PAM

C. COMPARISON WITH CHIRP MODULATION TECHNIQUES

Figure 4 describes features of chirp modulation techniques and LT-PAM regarding the relationship between the frequency bandwidth and measurement time. In chirp modulation techniques conducting frequency sweeps, a frequency difference (Δf) between received and transmitted waves is identified, which corresponds to the TOF (Δt) between the transmitter and

the receiver as shown in Figure 4(a). Thus, a certain duration of frequency sweeps is needed and achieving instantaneous ranging measurements is difficult. On the other hand, LT-PAM achieves instantaneous ranging measurements because it is not required to conduct frequency sweeps and two sinusoidal waves with different frequencies are transmitted at the same time, as shown in Figure 4(b). Although LT-PAM uses multiple sync patterns that need a longer measurement time, the duration of the multiple sync patterns can be set to be sufficiently short. For example, concatenating 10,000 sync patterns, each of which lasts 100 ns, becomes a 1 ms signal. It is still a short signal compared with a chirp signal implemented in [8]. Another advantage of LT-PAM is that its ranging system configuration is less complicated, which reduces its development cost. This is because LT-PAM does not have to transmit and receive signals whose frequencies change continuously, which is required in chirp modulation techniques.



Figure .4 The time--frequency relationships in chirp and sync pattern signals

III. EXPERIMENT

A. OVERVIEW

To investigate the performance of the proposed method, three experiments were conducted, as follows.

- Experiment 1: Ranging performance of PAM and LT-PAM using collimated light (laser diode)
- Experiment 2: Ranging performance of LT-PAM using diffused light (LED)
- Experiment 3: Ranging performance of LT-PAM by changing frequency differences

By using a laser diode as the collimated light, we can transmit sync patterns with higher SNR, which improves the ranging performance. In contrast, LED as diffused light shows a relatively lower SNR, and thus its ranging performance is worse than that of the laser diode. However, as LED is safer for the eyes and is becoming a popular illumination device, its performance as a ranging device using the proposed method should be investigated to explore new and easy-to-deploy applications.

The experimental setup is shown in Figure 5. In Experiment 1, a laser diode (Opnext, Inc. HL6501MG, λ =658 nm) was used as a transmitter. Sync patterns generated by two function generators (Hewlett-Packard 8648C and Agilent N9310A) were given to the laser diode via an amplifier and an RF mounter (Asahi Systems ALTH-103BRBS). A Si avalanche photodiode (SiAPD, Hamamatsu Photonics, C5658) and Si PIN photodiode (SiPIN, Fujitsu FID08T13TX) were used as receivers for the transmitted sync patterns, respectively. An oscilloscope (Tektronix DSA70604) worked as a data recorder to perform A/D conversion of the signals from the transmitter (1.25 Gsps). The signals from the amplifier were also supplied to the oscilloscope to correctly identify the signal transmission time at the transmitter. The data recorded by the oscilloscope were transferred to a PC and the distance estimations were executed using the PAM and LT-PAM algorithms. In Experiment 2, we replaced the laser diode with an LED (OptoSupply OSRAA5111P, λ =625 nm). The rest of the equipment was as in Experiment 1. In Experiment 3, we used the laser diode from Experiment 1 and controlled the frequency difference between two sinusoidal waves by varying each frequency. The ranging performance of LT-PAM was evaluated by comparison with the theoretically predicted performance as obtained from equation (7).



B. EXPERIMENT 1: MEASUREMENTS USING COLLIMATED LIGHT

1) PAM measurement: First, we discuss the measurement performance using the PAM algorithm. A burst sync pattern lasting 100 ns was generated using two amplitude modulated signals with frequencies of 90 and 100 MHz. The SNRs of the sync pattern received with the SiAPD and SiPIN were 21.8 dB and 21.3 dB, respectively. The measurement time *T* was set to 50 ns (a half duration of one sync pattern) by adjusting the window size (Figure 1(b)).

The distance between the transmitter and the receiver units was set to 500, 1000 and 1500 mm. In each setting, the ranging measurement was conducted 500 times. To eliminate offset errors due to electronic delay introduced by cables and electronic circuits, the average value in the 500 mm distance measurement was used for calibration and set to zero.

Table 1 and 2 show the average errors and standard deviations of the measurement results. From these two tables, it can be seen that the standard deviations using the SiAPD were in the range

139--153 mm and those using SiPIN were in the range of 245--249 mm, which was consistent with values calculated using equation (7).

Distance [mm]	Average error [mm]	Standard deviation [mm]	Theoretical standard deviation [mm]
500	0.00	139	84.6
1000	-80.1	153	85.8
1500	-150.0	135	73.9

Table 1: Experiment 1: ranging results using PAM (SiAPD).

Distance [mr	n]	Average error [mm]	Standard deviation [mm]	Theoretical standard deviation [mm]
5	00	0.00	235	119
10	00	6.37	249	132
15	00	80.9	247	130

Table 2: Experiment 1: ranging results using PAM (SiPIN).

2) LT-PAM measurement: In the experiments using LT-PAM, the measurement time was set to 100 μ s and 1,000 concatenated sync patterns, each of which was the same as in the PAM measurement. The distance between the transmitter and the receiver unit was set to 500, 1000 and 1500 mm, and the measurement results for the 500 mm distance were used for calibration, which was also the same setting as in the PAM measurement.

Figure 6 shows the waveform of a signal transmitted from the laser diode and received by the SiAPD. Due to its high SNR, the waveform retained its shape (see Figure 3).

Table 3 and 4 show the average errors and standard deviations obtained through 400 measurements in each distance setting using the SiAPD and SiPIN, respectively.

The results were much improved in comparison with those obtained using PAM. In the case of the SiAPD, the standard deviations of the 500 mm distance measurement using LT-PAM and PAM were 3.12 and 139 mm, respectively, which means that by using LT-PAM a 44.6 times improvement was achieved. As the measurement time of LT-PAM was 2000 times longer than that of PAM, an improvement of 44.7 (= $\sqrt{2000}$) times was theoretically possible. Thus, the ranging results using LT-PAM achieved the theoretical upper limit almost perfectly.

Table 3 and 4 also show that the SiAPD showed better ranging performance than the SiPIN. For example, the ratios of the performance degradations using the SiAPD and SiPIN compared with the theoretical standard deviations in the 1000 mm ranging measurement were 3.01 (=3.46/1.15) and 3.62 (=6.45/1.78), respectively.

Distance [mm]	Average error [mm]	Standard deviation [mm]	Theoretical standard deviation [mm]
500	0.00	3.12	1.14
1000	-0.052	3.46	1.15
1500	-1.09	2.99	1.00

Distance [mm]	Average error [mm]	Standard deviation [mm]	Theoretical standard deviation [mm]
500	0.00	5.20	1.61
1000	12.0	6.45	1.78
1500	-0.24	5.14	1.76

Table 4: Experiment 1: ranging results using LT-PAM (SiPIN).



Figure 6. A waveform of received signals from a laser diode

C. EXPERIMENT 2: MEASUREMENTS USING DIFFUSED LIGHT

Experiment 1 using collimated light proved that LT-PAM worked properly and achieved the expected level of improvement compared with PAM. The experiment also proved that the SiAPD achieved better ranging performance than the SiPIN. Thus, we used the SiAPD as the receiver and evaluated the LT-PAM algorithm using an LED as diffused light, whose SNR was lower than collimated light. Sync patterns were generated using 21 and 11 MHz sinusoidal waves. Each sync pattern lasted 100 ns. By concatenating 40,000 sync patterns, the ranging measurement time T was set to 4 ms.

Distance	Average	Standard	SNR [dB]	Theoretical standard
[mm]	error [mm]	deviation [mm]		deviation [mm]
500	0.00	5.21	-0.715	4.63
1000	0.097	12.7	-8.81	11.8
1500	5.21	19.5	-13.8	21.0

Table 5: Experiment 2: ranging results using LT-PAM (diffused light).

Figure 7 shows the waveform of transmitted signals from the LED and of received signals by the SiAPD. Unlike the signal from the laser diode as shown in Figure 6, the received waveform in Figure 7 did not explicitly retain its original shape because of the lower SNR. Table 5 shows the ranging results using the LED. As compared with the results in Experiment 1, the average errors and standard deviations in the distance measurements worsened. However, we could confirm that the experimental results were consistent with the performance theoretically predicted through equation (7). The standard deviations obtained by the measurements were close to the theoretical values in Table 5. The standard deviations of the 500, 1000 and 1500 mm distance measurements were 5.21, 12.7 and 19.5 mm, respectively. Thus, we could confirm that the improvement observed with LT-PAM overall followed its theoretical model.

The experiments indicated that the ranging results using LED seemed acceptable for a certain class of applications, for example, indoor localization using trilateration with considerably high update rates (100--200 Hz).



Figure 7. Signal waveforms transmitted from the LED (upper) and received at the SiAPD (lower)

D. EXPERIMENT 3: MEASUREMENTS BY CHANGING FREQUENCY DIFFERENCE

As for the parameters that affect the performance of LT-PAM shown in Section II-B, the measurement time (*T*) is the most easily adjustable to improve the ranging performance. However, investigations on the validity of the theoretical model by changing the other parameters will help the design of a ranging system using LT-PAM. If the equation is confirmed to be valid and give reliable performance prediction for different parameters, it is possible, for instance, to estimate what needs to be the frequency difference between two sinusoidal waves $(|f_1 - f_2|)$ to

achieve the required ranging performance. Thus, we conducted experiments by varying the frequency differences between two waves from 10 MHz to 500 MHz as shown in Table 6. The distance between the transmitter and the receiver units was set to 500 mm.

Frequency difference [MHz]	f_1 [MHz]	f_2 [MHz]
10	90	100
20	100	120
30	100	130
40	100	140
50	110	160
100	200	300
200	300	500
300	400	700
400	500	900
500	100	600
500	300	800

Table 6: Frequency differences between two sinusoidal waves

Figure 8 shows the standard deviations obtained through 400 measurements and represented as dots. SNRs of received signals measured at individual frequency differences ranged from 14 dB to 20 dB. To clarify the relation between the frequency difference and ranging accuracy, the standard deviations obtained through the measurements were recalculated by setting SNRs of received signals at individual frequency differences to that of the 10 MHz frequency difference. Then, the calibrated standard deviation at each frequency difference was plotted as an 'x' mark in Figure 8.

The solid line means the ranging performance predicted by equation (7). This figure indicates that the experimental results and the theoretically calculated performance show the same trend: the larger the frequency difference, the smaller the standard deviation of the ranging results. However, it was also confirmed that gaps between the experimental results and the theoretically predicted performance became larger with increasing frequency difference. This seemed related to the frequency characteristics of the transmitter and receiver worsening at higher frequency

domains. The calibrated standard deviations were confirmed to more closely follow the theoretical estimation given by equation (7). The best performance achieved in this experiment was the 0.29 mm standard deviation obtained using 300 and 800 MHz sinusoidal waves (SNR: 17.5 dB).



Figure 8. Frequency differences and ranging accuracy

IV. DISCUSSION

Through the theoretical model and experiments, the following issues to be investigated have been clarified:

• Sources of errors

The experimental results proved that LT-PAM worked properly and showed the ranging performance almost expected from its theoretical model (equations (6) and (7)). However, to achieve more precise measurement, we need to detect the sources of error. There are several candidates for the errors, such as the thermal noise generated at the transmitter or receiver, or the deformation of cables connecting individual equipment devices during the measurement, etc.

As for the theoretical model, we assumed that the noise was distributed constantly in the whole frequency range, which was not true in real environments. Figure 9 shows the signal

power spectrum (90 and 100 MHz sinusoidal waves) received by the SiPIN (frequency range around 90 MHz). Theoretically, a sharp spectrum peak at 90 MHz and constant noise floors at the other frequencies were expected to be observed. However, due to the frequency response of the photodiode, its subsequent amplifier circuit and so on, the wider spectrum peak at 90 MHz and relatively large peaks around it were observed. The measurement time *T* of PAM and LT-PAM in this study were set to 100 μ s and 50 ns. Thus, the uneven noises within the range of 90 MHz ± 5 kHz and 90±10 MHz might cause the deterioration of the ranging performance of PAM and LT-PAM, respectively. One of our future investigations is to conduct more quantitative analyses of error sources including such uneven noises.



Figure 9. Power spectrum of received signals (90 and 100 MHz sinusoidal waves) using the SiPIN photodiode

Ranging ambiguity

One limitation of LT-PAM is its ranging ambiguity. Because of the periodicity of sync patterns (e.g., 100 ns when generated by 90 and 100 MHz sinusoidal waves), the ranging ambiguity occurs every 100 ns, which corresponds to every 30 m; this is inconvenient for long-distance measurements. There is a trade-off between the periodicity and ranging ambiguity: if a shorter sync pattern is generated to reduce measurement time, the ranging ambiguity increases. One idea for alleviating this problem would be to use more than two sinusoidal waves. Suppose that the frequencies of three waves s_1 , s_2 and s_3 are 90, 100, and

112.5 MHz, respectively. The periodicity of sync patterns generated by s_1 and s_2 is 100 ns and that by s_2 and s_3 is 80 ns. Therefore, by combining these two periodicities, we can reduce the ambiguity at every 400 ns, which corresponds to every 120 m. We will confirm the feasibility of this idea in our future work.

• Real-time ranging

In our current implementation, we conducted high-sampling A/D conversion (1.25 Gsps), which needs a long processing time. Achieving real-time ranging by reducing the sampling rate and retaining the accuracy level is also included in our plans for future work.

V. CONCLUSION AND FUTURE WORK

In this paper, we proposed a new ranging technique called LT-PAM, which was extended from PAM previously developed by the authors' group. We also discussed the differences between the proposed method and chirp modulation techniques. Through the experiments of LT-PAM using collimated and diffused light, we confirmed that it could achieve moderate levels of accuracy within a sufficiently short measurement time. Comparisons with the theoretical model of LT-PAM confirmed that it worked properly and showed the ranging performance to be almost as predicted. In addition to the improvement of the proposed ranging technique, we plan to explore possibilities for innovative applications and develop them using LT-PAM.

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