



# DEFORMATION FORECAST OF FLEXIBLE MATERIAL PROCESS BY SPLINE FINITE ELEMENT METHOD AND APPLICATION

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*Submitted: Oct. 15, 2012*

*Accepted: Jan. 17, 2013*

*Published: Feb. 20, 2013*

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*Abstract- This paper provides a new computing method for the amount of deformation of flexible materials by spline finite element. Firstly, it analyzes the deformation of single yarn in the way of cubic spline interpolation, with the idea of spline finite element; it takes the flat fabric as anisotropic material to build a simple surface model with the use of bicubic interpolation, so as to deduce to the formula of flat fabric's deformational displacement under the stress. Finally, it takes the line graphic element for object to get the amount of deformation and to compensate it in quilting, test result show that the position error scope of quilting is 0.093~0.186mm, the accuracy is higher than excellence grade of quilting which refers to national standard FZ/T81005-2006.*

**Index terms:** Flexible material process, Deformation calculation, Spline finite element.

## I. INTRODUCTION

Quilting is a way of sewing the layered fabric with needles to fix the cottons inside. Due to the particularity of the processed object, the quilting digital control processing differs from general digital control processing. The object of general digital control processing is always rigid material, which is not easy to deform. While the object of quilting digital control processing is flexible material such as fabric and sponge, which is easy to deform, so the tool path is displaced, seriously affecting the accuracy and effect of processing [1][2][3]. Figure 1 is the diagram about the tool path deformation of linear graphic elements. The solid line is the ideal graphic tool path, while the dashed is the possible displacement of tool path for the material's deformation. Obviously, because of the material's deformation, the actual processing path is displaced. Therefore, in order to make the actual path approach to the ideal path, it needs to amend the coordinates of the processed graphic before or in the processing for achieving the desired results.

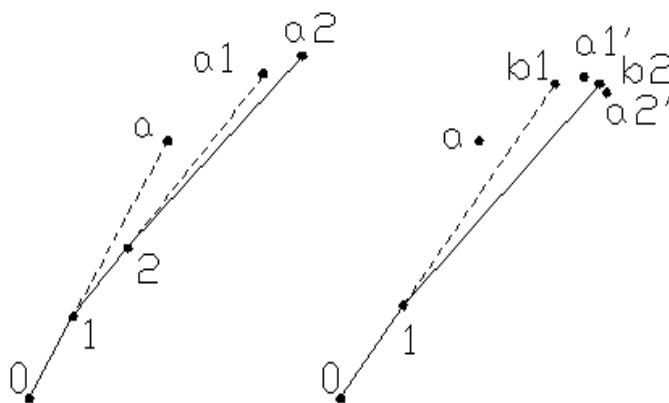


Figure1. Tool path deformation of linear graphic

## II. THE RESEARCH OF DEFORMATION CALCULATION THROUGH FINITE ELEMENT

Research of Finite Element has been applied in the rigid workpiece of the deformation compensation for some years. For example, in the paper (reference [4], [5]), authors studied the thin-walled parts' deformation, they analyzed and calculated the deformation of typical thin-walled frame spare parts by finite element analysis software. papers (reference[6], [7]) mainly

studied the deformation and impact factors of the digital control processed the structural parts of the aircraft, according to the analysis of the influence to the aircraft's structural parts of digital control processing from the cutting force to deformation, residual stress to processing deformation and fixture and layout, the condition of workpiece's processing deformation can be got by the simulation processing, so as to compensate digitally to the deformation to reach the requirement of accuracy. As for flexible materials, the current study is just to simulate the deformation of flexible materials. There are also some scholars studying the measurement of the tool path deformation and compensation. But they did not combine the model prediction together with the compensation method of deformation. Zhang Yitong[8] simulated the square fabric stretching to overhang and buckle on the round table with the use of finite element method basing on the meso-mechanics model of fabric, and carefully observed the model. Later they calculated the buckling deformation, and did the experiment of the square fabric's overhanging and bucking at the same time. J.G.Teng [9] used the finite volume method to analyze the overhanging deformation of the fabric, the method is originally used for hydrodynamics calculation, and later for the deformation of the fabric by the researchers. In the paper [10], researchers predict the deformation according to the fabric's border and the compression to adaptively choose the finite element method, experiment demonstrates that the result of finite element equation suggested by the researchers is more effective than any other methods in the past. Reference [11-16] studied the modeling of flexible material deformation by intelligent methods such as neural networks, regression analysis. These methods have obtained better simulation effect.

On the basis of the researches above, to study the dynamic deformation of soft material and the deformation in the processing by finite element theoretically works. In this essay, the spline finite element method has been used, which is to take the flat fabric as anisotropic material to analyze the deformation of the flat fabric composed of yarns, then distributed sensors are utilized to measure the pressure stressed on soft material, combining above two steps, it can calculate deformation of the soft materials while processing.

### III. THE MODEL OF DEFORMATION

As the model showed in Figure 2, to suppose there is a constant section of soft fabric in the rectangular coordinate system, the fabric's axes is coincided with the X-axis. If the yarn at  $x_i$  is

differential settlement  $y_i$ , then the fabric's deflection curve can be expressed by  $S_i(x)$  ( $i=0,1,2,\dots,m$ ).

For the special object, the fabric which is regularly composed of yarns, the spline finite element method taking advantages of the spline function and finite element method, is suitable for the analysis of the special material, fabrics' particular deformation. The spline finite element method is based on the Variational Principle and cubic B-spline function [9]. The displacement function is composed of linear product of cubic B-spline function, and both directions use the cubic B-spline function. The overall stiffness equation is established by the variational principle. This method can be applied to various structural analyses in the regular area.

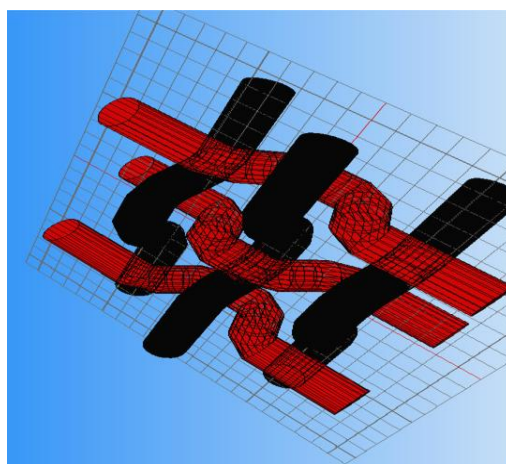


Figure 2. Cross section of the fabric

Suppose in the interval  $[a,b]$ , there is a partition  $\Delta: a = x_0 < x_1 < x_2 < \dots < x_{m-1} < x_m = b$ , and the function  $S_i(x)$  in  $[a,b]$  is named as cubic spline function( $s_3(x)$ ) if it satisfies the following conditions [18]:

①In each subinterval  $[x_{i-1}, x_i]$   $s_3(x)$  is a cubic polynomial. ②In the whole interval  $[a, b]$ ,  $s_3(x)$  is a second order continuous differentiable function, that is, at the point  $x_i$  (the node of  $s_3(x)$ ), it satisfies  $s_3^{(k)}(x_i - 0) = s_3^{(k)}(x_i + 0)$  ( $k = 0,1,2$ ). ③When an ordered sequence  $y_i$  is given, if  $s_3(x)$  satisfies the condition again:  $s_3(x_i) = y_i$ , then  $s_3(x)$  is called as an interpolation cubic spline function. If  $\omega$  is the deflection of the yarn,  $M_x$  is the flexural moment and  $E$  is bending stiffness. With the use of Fig.2, to simulate single yarn for analysis of the cubic spline finite point. The relationship between its flexural moment and deflection is as following:

$$M_x = -E \frac{d^2 \omega}{dx^2}$$

To learn more about the deformation of the yarn, the key is to find the deflection function of yarn.

The yarn is divided into  $N$  equal parts:

$$0 = x_0 < x_1 < x_2 < \dots < x_n = a$$

$$x_i = x_0 + ih, h = x_{i+1} - x_i = a/n$$

Hence, the deflection function of yarn can be approached by cubic spline function as below:

$$\omega = \sum_{i=-1}^{N+1} c_i \phi_i(x) = [\phi] \{c\}$$

$[\phi] = [\phi_{-1}, \phi_0, \phi_1, \phi_2, \dots, \phi_{N+1}]$ ,  $[c] = [c_{-1}, c_0, c_1, c_2, \dots, c_{N+1}]^T$  is an undetermined coefficient.  $\phi_i(x)$

is a set of basis function related with cubic B-spline function, that is:

$$\phi_{-1}(x) \varphi_3\left(\frac{x}{h} + 1\right)$$

$$\phi_0(x) = \varphi_3\left(\frac{x}{h}\right) - 4\varphi_3\left(\frac{x}{h} + 1\right)$$

so, the deflection function of yarn can be got if the  $c_i$  is found by the principle of minimum

potential energy:  $\frac{\partial U}{\partial \{c\}} = \{0\}$ , In the condition of small deformation, the total potential energy

function is:

$$U = \frac{1}{2} \int_0^a (\chi^T M - 2q\omega) dx$$

where,

$$\chi = -\frac{d^2 \omega}{dx^2} = -[\phi''] \{c\}$$

$$M = D\chi = D[\phi''] \{c\}$$

$$D = EI$$

The cubic spline function is substituted in the total potential energy functional formula, and the solution is:

$$U = \frac{1}{2} \{c\}^T [G] \{c\} - \{c\}^T \{k\}$$

$$[G] = \int_0^a D[\phi'']^T [\phi''] dx$$

$$\{k\} = \int_0^a [\phi]^T q dx$$

as  $EI$  is a constant, then it can be got from the formula above:

$$[G] = DA_x, A_x = \int_0^a [\phi'']^T [\phi''] dx$$

and the changed total potential energy functional formula is substituted in the minimum potential energy formula, and the result is:

$$[G]\{c\} = \{k\}$$

It is the stiffness equation of yarn,  $[G]$  stands for the stiffness matrix  $\{k\}$  is the load matrix, so:

$$\{c\} = \frac{\{k\}}{[G]} = \frac{\{k\}}{EI * A_x}$$

It can be seen that to analyze the fabric, especially the single yarn by cubic spline function is feasible. What is more, approaching the result in the use of spline is effective and with little calculation. Compared with the finite element method, it is more simple and convenient.

## 2) The Spline Finite Element Calculation of Deformation

In the fabric structure, the yarns are considered to be distributed evenly, and with the same distance between nodes, to divide the nodes the fabric area into  $m \times m$  equal parts:

$$x_i = x_0 + ih_x, h_x = a / m,$$

$$y_i = y_0 + ih_y, h_y = b / m$$

Where:  $x_i \in [0, a], y_i \in [0, b]$ . Then the fabric displacement function can be constituted by bicubic

B-spline function:

$$u = \sum_{j=-1}^{m+1} \sum_{i=-1}^{m+1} a_{ij} \Phi_i(x) \Psi_j(y) = [\Psi] \otimes [\Phi] \{A\}$$

$$v = \sum_{j=-1}^{m+1} \sum_{i=-1}^{m+1} b_{ij} \Phi_i(x) \Psi_j(y) = [\Psi] \otimes [\Phi] \{B\}$$

$$w = \sum_{j=-li=-1}^{m+1} \sum_{i=-1}^{m+1} c_{ij} \Phi_i(x) \Psi_j(y) = [\Psi] \otimes [\Phi] \{C\}$$

$\phi_i(x)$  and  $\psi_i(y)$  are basis functions related to the cubic B-spline functions. It can be seen that as  $\{A\}, \{B\}, \{C\}$  are found, then there is the fabric displacement. Total potential energy functional of fabric is as follows:

$$U = \frac{1}{4} \{\delta\}^T [G] \{\delta\} - \{\delta\}^T \{k\}$$

by Variational principle,  $[G]\{\delta\} = \{k\}$ . As the fabric is orthogonal anisotropic material [17],

$$[G] = [r]^{-1} \begin{Bmatrix} \{A\} \\ \{B\} \\ \{C\} \end{Bmatrix} = \begin{Bmatrix} k_1 \\ k_2 \\ k_3 \end{Bmatrix}$$

After simplification, the flexibility matrix is:

$$[r] = \begin{bmatrix} r_{11} & r_{12} & r_{13} & 0 & 0 & 0 \\ & r_{22} & r_{23} & 0 & 0 & 0 \\ & & r_{33} & 0 & 0 & 0 \\ & & & r_{44} & 0 & 0 \\ & & & & r_{m-1m-1} & 0 \\ & & & & & r_{mm} \end{bmatrix}$$

and the load matrix is:

$$k_1 = \int_0^a \int_0^b ([\Psi] \otimes [\phi])^T q_x dx dy$$

$$k_2 = \int_0^a \int_0^b ([\Psi] \otimes [\phi])^T q_y dx dy$$

$$k_3 = \int_0^a \int_0^b ([\Psi] \otimes [\phi])^T q_z dx dy$$

according to the material properties, the flexibility matrix is found. By the formulas above,  $\{A\}$ ,  $\{B\}$ ,  $\{C\}$  can be got, and substituted in the displacement formula, thus, the displacement of fabric can be found. The displacement of fabric is an important data for digital control processing in the quilting.

#### IV. THE SIMULATION OF DEFORMATION

To take deformation of rectangular flexible material as example, the composition of the material is polyurethane foam. The modulus of elasticity  $E = 0.3078\text{MPa}$ , Poisson coefficient  $\mu = 0.3$ , the length of a is 160mm, width b is 90mm, thickness h is 5mm. Figure 3 shows the stress-strain characteristic curve. Choose the rectangle material loading point load the points on the diagonal [19] [20], concentrated load is 10 N. Figure 4 shows deformation mechanics simulation and the theoretical results. Bending displacement distribution curve of a, b axis is respectively for the length, width of flexible material. On the basis above, to measuring six points on the surface of flexible workpiece surface of Figure 4(a). Digital pressure gauge with micrometer is used to measure point by point. Table 1 shows results of comparison between theoretical value and measured value.

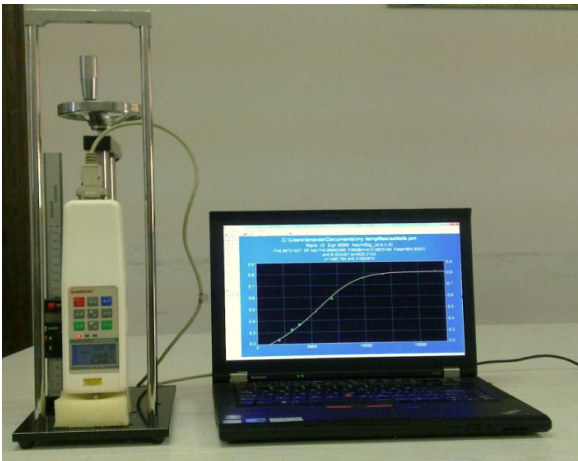
##### I. THE ANALYSIS OF PROCESSING TESTING AND RESULTS

To apply above method to quilting processing, the quilting machine and quilting processing diagram are shown in Figure 5, Figure 6 and Figure 7. Figure 5 shows the Quilting machine, it is 3-axis CNC machine tools. Figure 6 and Figure 7 shows the process of quilting of one kind of quilting pattern [21] [22].

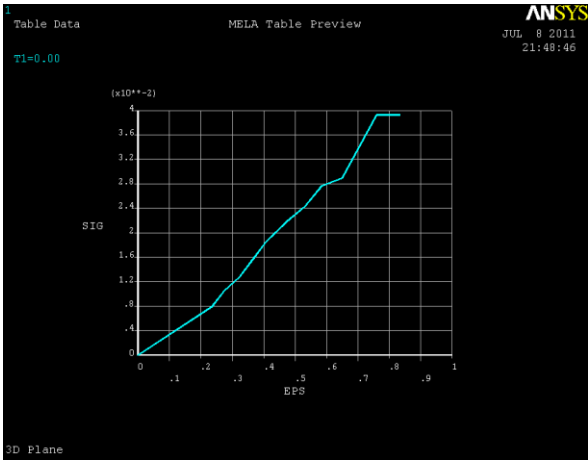
###### a. Line deformation compensation testing

The embossing mold composed of linear graphic elements is chosen to process, the processing thickness is 5mm, the largest deformation occurs at the point of the punch block moving to the lowest.



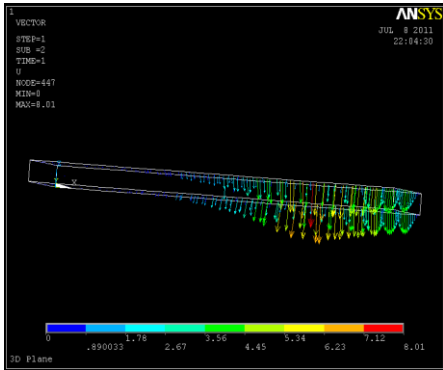


(a) Stress and strain test devices

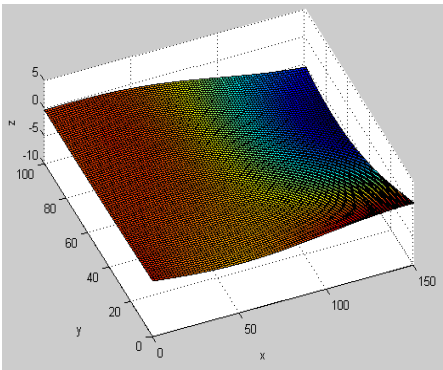


(b) The stress-strain characteristic curve

Figure 3. Material stress and strain characteristic curve

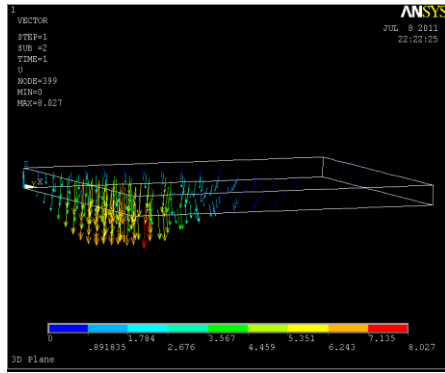


Deformation displacement vector

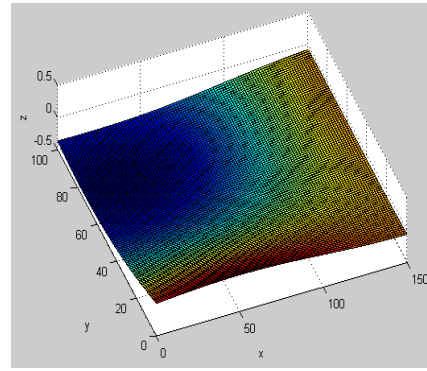


Deflection of the distribution curve

(a)  $x = 120, y = 60$



Stress distribution cloud diagram



Deflection of the distribution curve

(b)  $l = 40, k = 70$

Figure 4. Deformation displacement vector and displacement distribution diagram

Table1: Comparison between theoretical value and measured value of deformation of flexible workpiece

Serial number	X-YPlane coordinates(mm)	Theoretical value(mm)	measured(mm)	Relative deviation(%)
1	(120,60)	4.35	4.08	6.56
2	(40,70)	4.36	4.07	7.12
3	(40,20)	3.68	3.44	6.78
4	(120,20)	3.68	3.42	7.56
5	(80,45)	7.18	6.65	7.89
6	(160,45)	4.70	4.34	8.21
Average				7.35

From table 1, it can calculate results of the average of the relative deviation is only 7.35%.

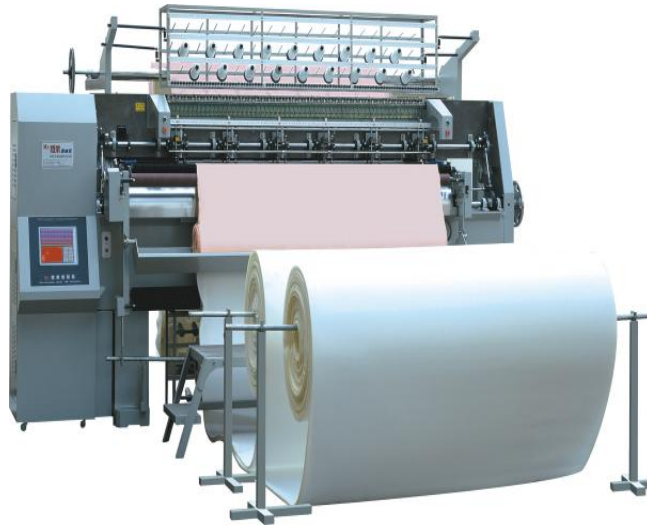


Figure 5. Quilting machine

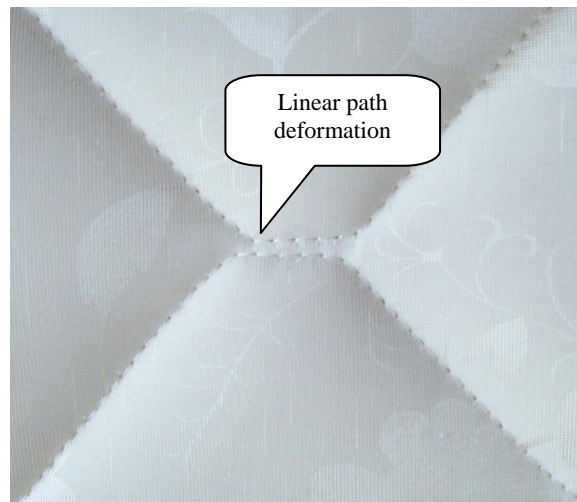


Figure 6. Processing of Quilting

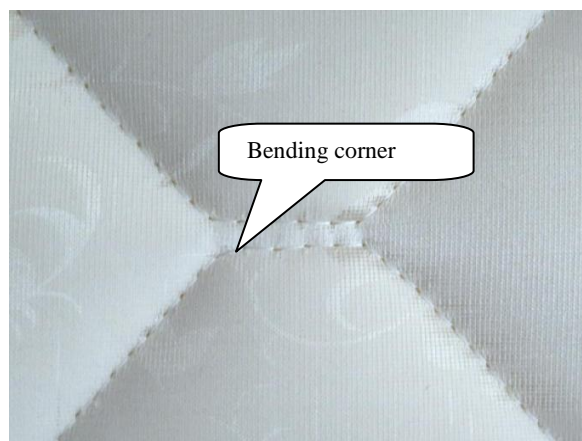


Figure 7. Quilting pattern.

The pressure can be measured by tools and substituted in the formula of deformation displacement in the Section 3 to get the amount of deformation, and the coordinates of the tool path can be amended according to the deformation displacement. Here are two groups of pictures; one is that the quilting graphic elements are processed according to the general experience without any deformation calculation (Figure 8). The other is the processed graphic elements under the combination of the actual experience and deformation compensation (Figure 9). By analyzing the two groups of pictures, it can be seen that in the first group (Figure 8), there are not any bending about the linear processing, and angle in the corner. Obviously, as the deformation, the corner of the intersection of two lines in the Fig.8 of the first group has lost its original effect to be achieved.

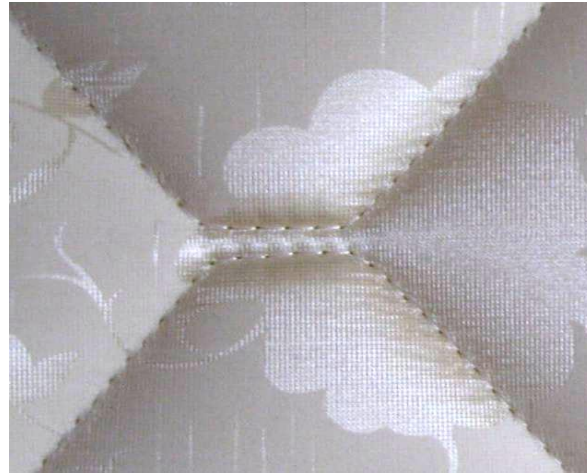


(a) Linear path deformation



(b) Bending angle of two lines

Figure 8. The diagram of tool path deformation of linear graphic elements



(a) linear path after compensation



(b) No deformation of the angle of two lines after compensation (Bending corner)

Figure 9. The pictures of linear path after compensation

b. Machine effect of different thickness of flexible workpiece

To choose one kind of flexible material which elastic modulus is 0.3078 MPa, poisson ratio is 0.3 as workpiece, the length is 20mm, width is 20mm and thickness is 1.1mm. ten groups of data are measured, after filtering to calculate by above equation, the displacement of deformation is used as the foundation of error compensation.

Six kinds of workpieces are chosen, they are constituted by high density sponge, backing material and woven cotton used as shell fabric. To set main shaft speed as 800 (r/min), feed rate is 500(mm/min), the largest deformation occurs at the point of the punch block moving to the lowest. The pressure can be measured by sensors and substituted in the formula of deformation

displacement in the Section 3 to get the amount of deformation, and the coordinates of the tool path can be amended according to the deformation displacement.

Figure 10 shows quilting result after compensating on flexible workpiece of 6.1mm thickness.

Figure 11 shows quilting result after compensating on flexible workpiece of 6.5mm thickness.

Figure 12 shows quilting result after compensating on flexible workpiece of 5.5mm thickness.

Figure 13 shows quilting result after compensating on flexible workpiece of 4.5mm thickness.

Figure 14 shows quilting result after compensating on flexible workpiece of 4.0mm thickness.

Figure 15 shows quilting result after compensating on flexible workpiece of 3.0mm thickness.

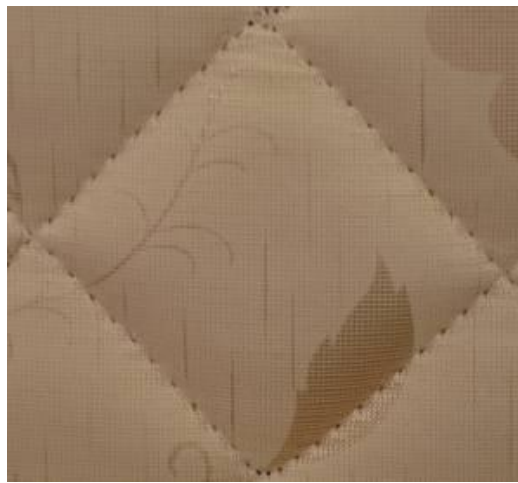


Figure 10. Quilting effect of 6.1mm thickness workpiece

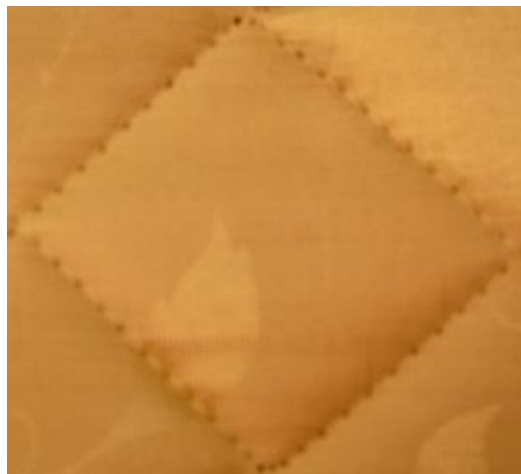


Figure 11. Quilting effect of 6.5mm thickness workpiece



Figure 12. Quilting effect of 5.5mm thickness workpiece



Figure 13. Quilting effect of 4.5mm thickness workpiece



Figure 14. Quilting effect of 3.5mm thickness workpiece

Figure 15. Quilting effect of 3.0mm thickness workpiece

Eleven sets processing data are selected to calculate the position error which processed by various workpieces (National standard FZ/T81005-2006), testing result indicates that the position error scope of quilting track is 0.093~0.186mm.

## II. CONCLUSIONS

This paper proposes the method to analyze the deformation of the flat fabric by spline finite element method, which practically solves the problem of the deviation of tool path for deformation in the flexible material's processing. Referring to national standard FZ/T81005-2006, in the graphic processing of quilting, the method is the most effective to compensate the deviation of path, thus it enhances the accuracy of the processed graphic of the flexible materials. However, this method needs to be further improved. When the thickness to be processed is increased, the accuracy of the calculation to the deformation by the method will deviate, and the materials cannot be analyzed exactly by the features of the anisotropic materials, and the deformation is tended to random directions. In view of the situations, the solutions should be random, that is the tendency equation to approach the deformation according to the regression analysis of the material's deformation data [23][24][25]. In addition, as for the thickness is increased, the response characteristics of the control system need to be improved for achieving desired effect of compensation.

## ACKNOWLEDGMENT

This work is partially supported by National Natural Science Foundation of China (No.51205069) and The Production Study Research Combination Project of Huizhou City No.2011C010002008. Thanks for the help.

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