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A VIABLE CAPACITIVE APPROACH FOR DAMAGE DETECTION OF AIRCRAFT COMPOSITE MATERIAL

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Abstract- Military and Commercial aircraft are being constructed by more and more advanced composite materials. In order to prevent catastrophic failure any damage in aircraft composite material should be detected as soon as possible. The relations of the electrode length, the electrode width, and the space between electrodes to the testing sensitivity of the uniplanar double electrodes are investigated. A three dimensions model of the uniplanar capacitive sensor with 8 electrodes is founded and the optimization for the structure parameters of the sensor with 8electrodes is carried out. According to the optimization, a uni-planar capacitive sensor, with 8-electrode on one plane substrate and a ground screen electrode and the screen between electrodes, is designed to get the corresponding capacitance information of the measured composite material slab. 2 aircraft composite material slabs, one is healthy and the other is notched, are used as a sample for the experiment of detecting the damages. The preliminary experimental results show that the measured capacitances decrease after

damage occurs in aircraft composite material and that the proposed approach can effectively detect the damage of aircraft composite material. The proposed approach is a viable technique for in-situ damage detection of aircraft composite material.

Index terms*:* Uni-planar capacitive sensor, sensitivity simulation, optimization, FEM, damage detection, composite material.

I. INTRODUCTION

Military and commercial aircraft, such as AV-8B, F16, F18, F22, the Eurofighter, A320, A340, A380, and Boeing 777, et al, are being constructed using more and more advanced lightweight composite materials. The wing skins, forward fuselage, flaperons and rudder all make use of composites [1]. With the service time of aircraft increasing, impact environmental damage as well as fatigue stress can cause fiber fracture, matrix cracking, internal delaminations, and interfacing debanding [2]. Possible damage of aircraft composite material can lead to catastrophic part failure during flight without prior visible warning and threaten flight security and pilot and people's lives. Any damage in the parts made of composite material should be detected and repaired as soon as possible. Therefore Structural health monitoring has become more and more important for military and commercial aircraft. Practically to say, the diagnosis should be made at the time when faults are still small and their effect is not yet harmful to flight security and people's lives. However, currently successful damage detection and health monitoring techniques, including radiography detection, ultrasonic testing, acoustic emission testing and magnetic resonance imaging, pulse-echo and through transmission, and piezoelectric technology [3-6], are not suitable for in-situ damage detection. Our objective is to find a capacitive approach to detect damage and the impending failures in aircraft composite material.

A uniplanar capacitive sensor with double electrodes is taken as an example. The relations of the electrode length, the electrode width, and the space between electrodes to the testing sensitivity are investigated. A structure parameter optimization for the uniplanar capacitive sensor with 8 electrodes is carried out. Moreover, a uniplanar capacitive sensor, with 8-electrode on one plane substrate and a large screen electrode and the screen between electrodes, is designed to get the corresponding electrical capacitance information of the measured composite material slab for the purpose of detecting changes that may indicate damage or degradation. A large screen electrode is utilized to reflect the electrical field for increasing sensitive depth. Due to the complexity of capacitance simulation of the uniplanar capacitive sensor with multi-electrode, a 2-D finite-element method (FEM) is employed to simulate capacitance measurements for the sensor. The health state of the measured slab can therefore be characterized by analyzing the capacitance information. Both the simulation and the preliminary experimental results show that, the proposed approach is capable of detecting damages of aircraft composite materials, and can be practically used for damage detection of aircraft composite material.

The paper is organized as follows: After a general introduction of the composite material applied in aircraft and the detecting technology, "Section II" provides sensitivity simulation of the uniplanar double electrodes capacitive sensor. The optimizing for the structure parameter of the uniplanar capacitive sensor with 8 electrodes is presented in "Section III". The details of the uniplanar capacitive sensor and capacitance simulation are described in "Section IV". "Section V" gives experiment setup. We give the results and discussion in "Section VI".

II. SENSITIVITY SIMULATION FOR THE UNIPLANAR DOUBLE ELECTRODES

If we have a configuration made up of any number of electrodes, shown in figure 1, then the capacitance between two of the electrodes (say *i* and *j*) is given by the quotient of the charge induced on one of the electrodes due to the potential difference between the two electrodes, and that difference in potential. When written as an equation this gives

$$C_{ji} = \frac{Q_{ji}}{V_j - V_i} \tag{1}$$

where, C_{ji} is the capacitance between electrodes *i* and *j*; Q_{ji} is the charge on electrode *j* induced by the potential difference (V_j - V_i); V_i is the potential on electrode *i*; and V_j is the potential *j*. This means that for all the other electrodes on electrode (except *i* and *j*) only their presence and not their potential contributes to the capacitance between the electrodes *i* and *j* [7].

Willem Chr. Heerens gives the analytical formulae of multi-electrodes capacitive sensor from the capacitive sensor with the circular cylinders and toroids geometries (shown in figure 2). In circular cylinders and toroids with rectangular cross sections and in parallel plate geometries the distance *d* between both plane parallel surfaces is used as a reference distance. Coordinates or dimensions with arbitrary index *j*, like z_j in the direction of the rotation axis or r_j going radially out from the axis are transformed into of the sector electrode A in the top surface lies inside the ring axis are transformed into ξ_j and ρ_j according to $\xi_j = \pi z_j / d$, $\rho_j = \pi r_j / d$ [7].

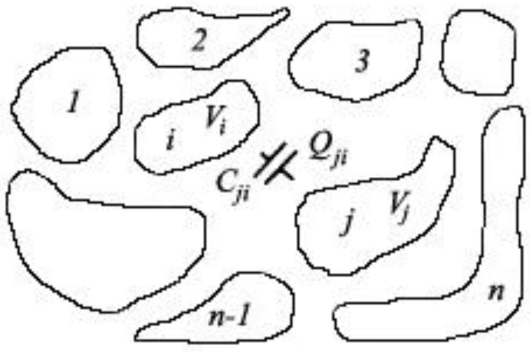

Figure 1. Fundamental representation of capacitance between conductors

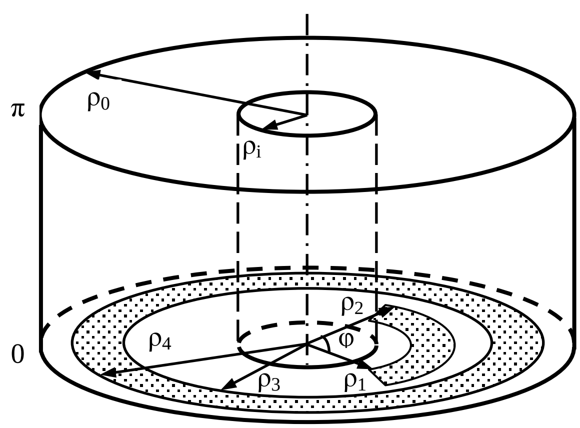

Figure 2. The capacitive sensor with the circular cylinders and toroids geometry

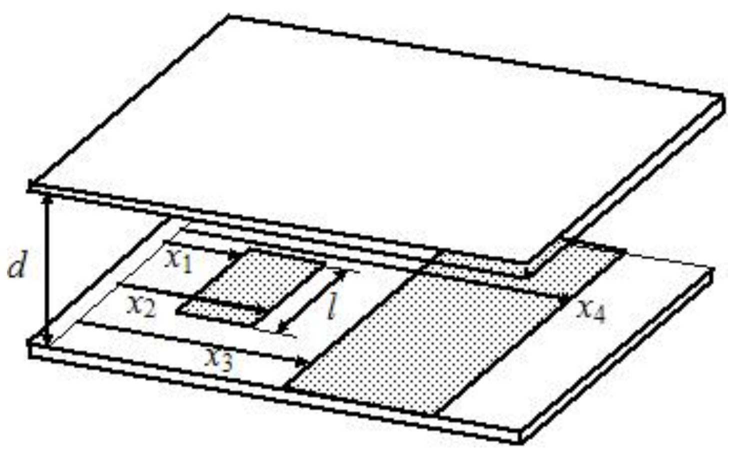

Figure 3. The uniplanar electrodes

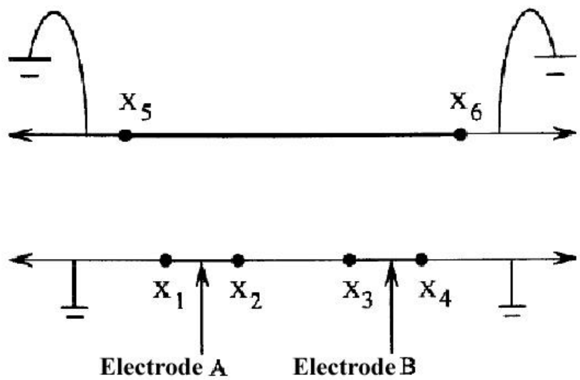

Figure 4. The simplied diagram of uniplanar electrodes

For the uniplanar electrodes are shown in figure 3 and figure 4, the capacitance between two electrodes is as follows [8]:

$$C_{AB}=\frac{\varepsilon_0\varepsilon_r l}{\pi}\ln\left(\frac{\sinh[\frac{\pi}{2d}(x_3-x_1)]\sinh[\frac{\pi}{2d}(x_4-x_2)]}{\sinh[\frac{\pi}{2d}(x_3-x_2)]\sinh[\frac{\pi}{2d}(x_4-x_1)]}\right) \quad (2)$$

where, x_1 is the distance from the left edge of electrode A to the reference axis; x_2 is the distance from the right edge of electrode A to the reference axis; x_3 is the distance from the left edge of electrode B to the reference axis; x_4 is the distance from the right edge of electrode B to the reference axis; d is the vertical distance from the object detected to electrodes A and B; l is the length of electrode A; $\varepsilon_0=8.85\times10^{-12}$ $C^2/N\cdot m^2$; ε_r is the relative permittivity.

According to figure 3, figure 4 and (2), the effect of the width and the length of the uniplanar double electrodes, and the space between two electrodes on the measured capacitance are calculated for investigating the sensitivity of uniplanar electrodes.

If the electrode space $g=x_3-x_2$, the electrode width $w=x_2-x_1=x_4-x_3$ (see figure 1), then $x_3-x_1=w+g$; $x_4-x_2=w+g$; $x_4-x_1=2w+g$. (3) can be obtained from (2)

$$C_{AB}=\frac{\varepsilon_0\varepsilon_r l}{\pi}\ln\left(\frac{\sinh^2[\frac{\pi}{2d}(\mathrm{w}+\mathrm{g})]}{\sinh(\frac{\pi}{2d}\mathrm{g})\sinh[\frac{\pi}{2d}(2\mathrm{w}+\mathrm{g})]}\right) \quad (3)$$

a. The effect of electrode width

In simulation, l=200mm, the distance from the measured object to the electrode d=50mm, is certain, the width of electrode B w=40mm. When w of electrode A changes from 0.1mm to 40mm, the relation of the calculated capacitance C between two electrodes to w is shown in figure 5 according to (3). It can be seen from figure 5 that, under the condition of the certain space the capacitances between two electrodes increase with the electrode width increment when the distance of the object measured to the electrode is great. The width variation results in great change in capacitance when the electrode width is small; after the electrode width is beyond a certain value, the capacitance increases slowly with the electrode width increment.

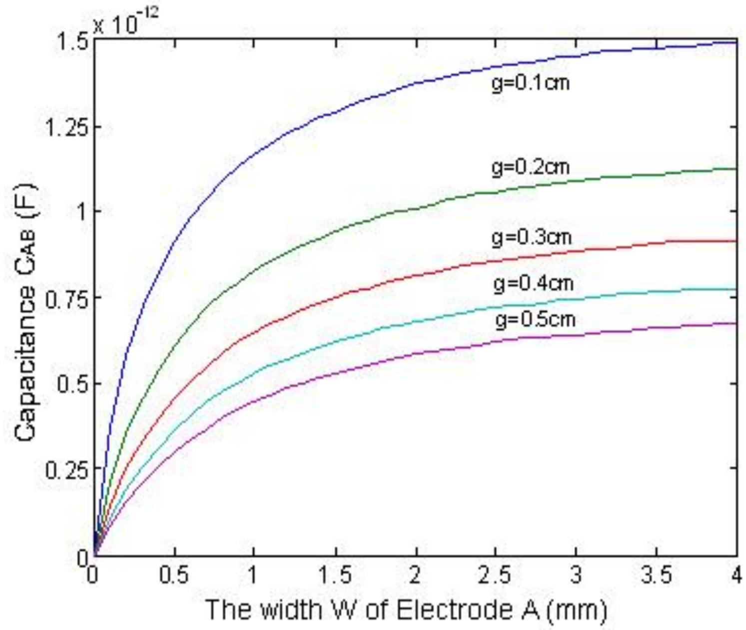

Figure 5. The relation of capacitance to the width

b. The effect of electrode space

In simulation calculation, l=200mm, w=40mm, the distance from the measured object to electrode is determined. The relation of the capacitance C between two electrodes to the electrode space g is shown in figure 6 according to (3). When d is certain, the effect of the electrode space on the capacitance is great. It can be found from figure 6 that with the electrode space increasing, although the capacitance between two electrodes decreases gradually, the capacitance changes slowly in small distance (d=1mm); and that the capacitance between two electrodes decreases rapidly with the electrode space increasing, i.e., the capacitance variation is great, which indicates that the effect of electrode space variation on the capacitance is great when *d* is certain.

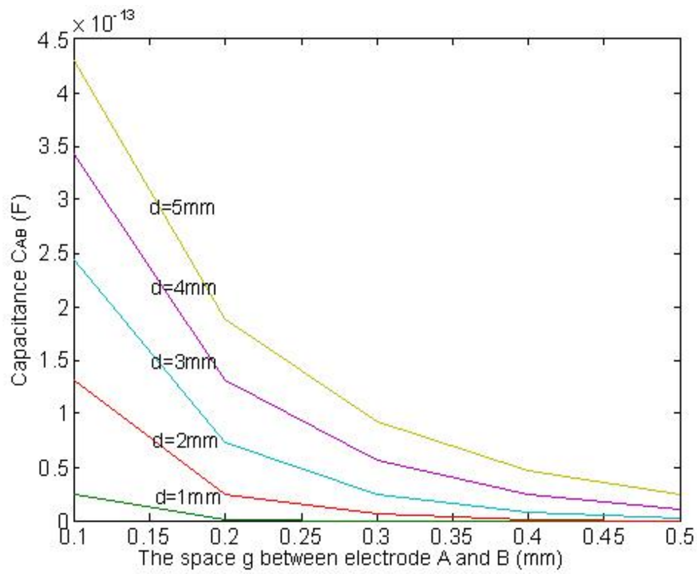

Figure 6. The relation of capacitance to the space

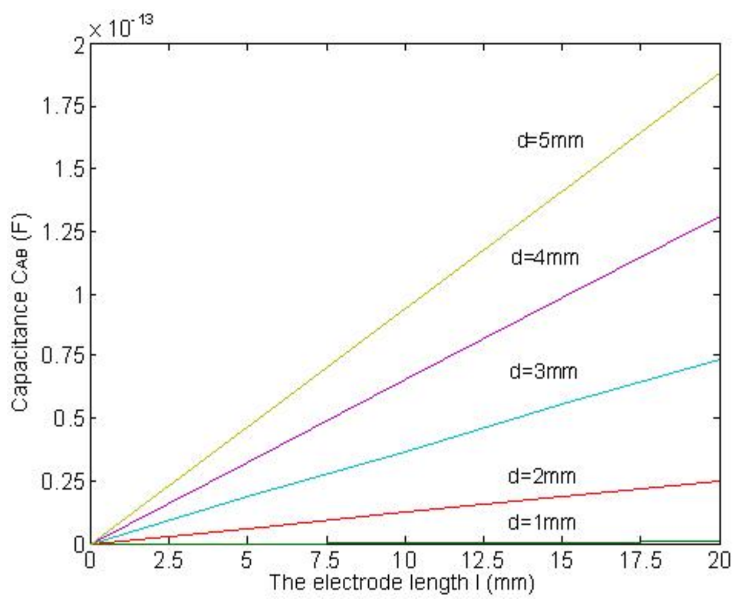

Figure 7. The relation of capacitance to the length

c. The effect of electrode length

Under the conditions of the certain electrode space, the certain width, and the certain distance from the measured object to the electrode, the relation of the computed capacitance between two electrodes to the electrode length *l* is shown in figure 7 according to (3). It is apparently known from the relation of the capacitance to *l* that the capacitance between two electrodes augments with the increasing of the electrode length. The electrode length variation results in a small change in capacitance in a small distance. The variation of the electrode length results in a great change in capacitance in a big distance from the measured object to the electrode.

III. OPTIMIZING FOR THE STRUCTURE PARAMETER OF THE UNIPLANAR CAPACITIVE SENSOR WITH 8 ELECTRODES

a. Modeling for the sensor

A three dimensions model of the uniplanar capacitive sensor with 8 electrodes is founded by finite element method (FEM), shown in figure 8. The length of electrodes is 34mm, the width of electrodes is 20mm, the space between electrodes is 3mm, the shield width outside electrodes is 10mm, the substrate thickness is 2mm, the shield width and height between electrodes are respectively 1mm and 1.5mm, and the relative permittivity is 3.45. The sensing area and the air area of the sensor are subdivided by the mesh subdivision tool in FEM. According to the different simulation demands, the subdivision meshes in the sensing area of the sensor may be set to one fifth of them in the other area. The subdivision of the uniplanar capacitive sensor with 8 electrodes is shown in figure 9.

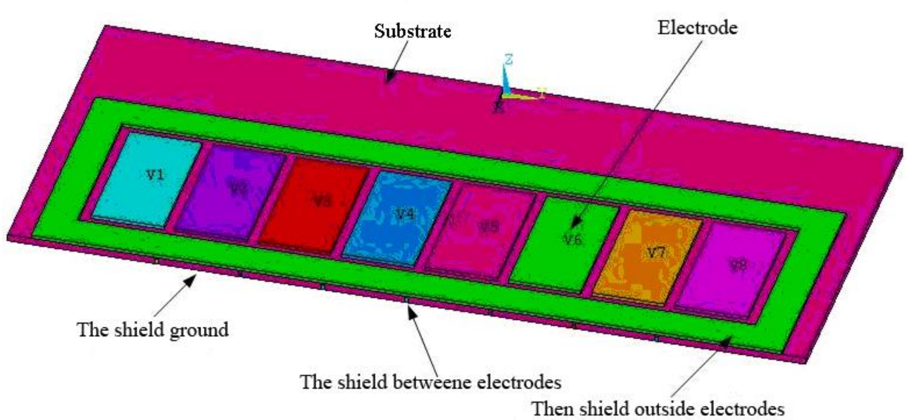

Figure 8. The model of the uniplanar capacitive sensor with 8 electrodes

The electrodes and the shield ground are respectively set to modules; the capacitances between every 2 electrodes can be calculated by FEM.

b. The determination of the optimization index

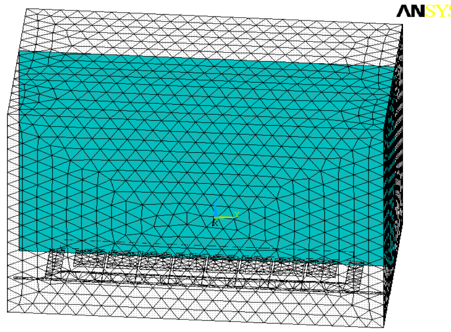

Figure 9. The subdivided meshes of the uniplanar capacitive sensor with 8 electrodes

The high sensitivity of the uniplanar capacitive sensor with 8 electrodes is demanded because the damage in the composite material component is generally very slight. The structure parameters of the electrode are needed to be optimized in order to improve the sensor sensitivity. The sensitivity and the change range of the detected capacitance are selected as the optimization index.

The sensitivity, an important index of the sensor, is defined as follows [9].

$$\Delta C / C = (C_{i,j}(obj) - C_{i,j}(air)) / C(air) \tag{4}$$

where, $C_{i,j}(obj)$ is the capacitance when the detected object is placed in the sensing area of the sensor, $C_{i,j}(air)$ is the static capacitance when the air is in the sensing area of the sensor. The bigger the $\Delta C / C$, the higher the sensitivity.

The change range of the detected capacitance is as follows [9]:

$$k_c = C_{\max} / C_{\min} \tag{5}$$

The ration of the detected maximum capacitance $C_{\max}$ to the detected minimum $C_{\min}$ is not too more, considering the design and implementation of the next C/V transformation circuit. $C_{\max}$ is the capacitance between the next electrodes when the composite material slab is placed on the sensor. $C_{\min}$ is the capacitance between No.1 and No.8 electrodes when the sensing area of the sensor is full of air.

Through a synthetical consideration, the structure parameters of the uniplanar capacitive sensor with 8 electrodes include the permittivity of the substrate, the substrate thickness, the electrode length, the electrode width, and the outer shield width. The static capacitance $C(air)$ is calculated when the detected object is air. The static capacitance $C_{i,j}(obj)$ is calculated when the detected object is the composite material slab which permittivity is 18.5, length is 197mm, width is 50mm, and thickness is 10mm.

c. Analysis and comparison of the sensitivity

The capacitances of every 2 electrodes have been calculated under different structure parameters, then the effect of the various structure parameters on the sensitivity is analyzed and compared.

1) The effect of the substrate permittivity on the sensitivity

The sensitivity change with the substrate permittivity is shown in figure 10. It is apparently known from figure 10 that the sensitivity between adjacent electrodes can increase but not much when the substrate permittivity increases, as a result, the effect of the substrate permittivity on the sensitivity is not great.

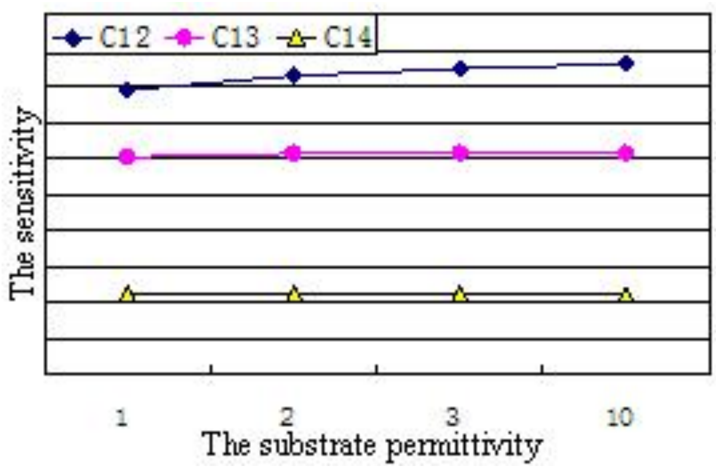

Figure 10. The effect of the substrate permittivity on the sensitivity

2) The effect of the substrate thickness on the sensitivity

The sensitivity change with the substrate thickness is shown in figure 11. It can be seen from figure 11 that the sensitivity between adjacent electrodes linearly decreases when the substrate thickness is bigger than 2mm, but the sensitivity between nonadjacent electrodes falls slightly, so the substrate thickness can be 2mm.

3) The effect of the outer shield width on the sensitivity

The effect of the outer shield width on the sensitivity is shown in figure 12. It can be found from figure 12 that the sensitivity between adjacent electrodes will first increase then decrease when the outer shield width increases. So the outer shield width can be 6mm.

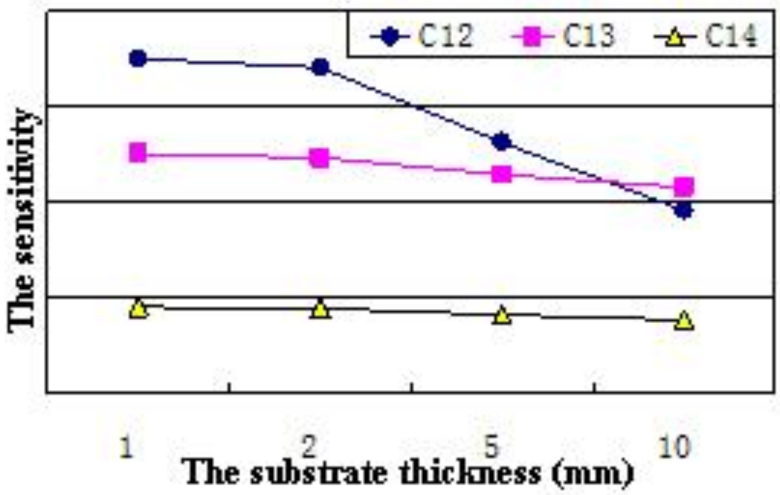

Figure 11. The effect of the substrate thickness on the sensitivity

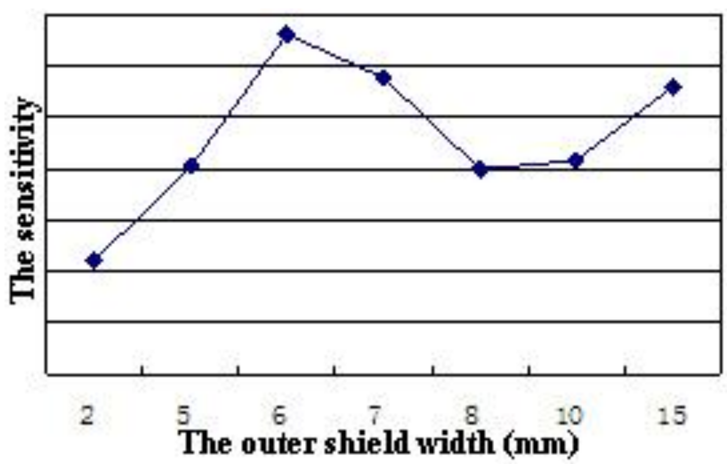

Figure 12. The effect of the outer shield width on the sensitivity

4) The effect of the electrode width on the sensitivity

The effect of the electrode width on the sensitivity is shown in figure 13. It can be found from figure 13 that the sensitivity between 2 electrodes decreases when the electrode width increases. If the electrode width is too small, the detected capacitance will be too small, the detecting difficulty increases. So the electrode width is 10mm.

5) The effect of the electrode length on the sensitivity

When the electrode width is 10mm and the outer shield width is 6mm, the effect of the electrode length on the sensitivity is shown in figure 14. It can be seen from figure 14 that with the electrode length increasing, the sensitivity between 2 electrodes falls. Because the electrode length directly decides the detecting area of the sensor, the electrode length is 25mm according to the actual demand of the damaged slab.

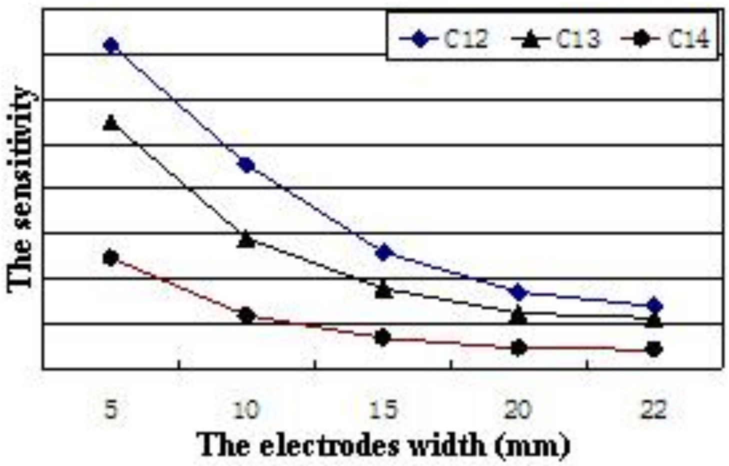

Figure 13. The effect of the electrode width on the sensitivity

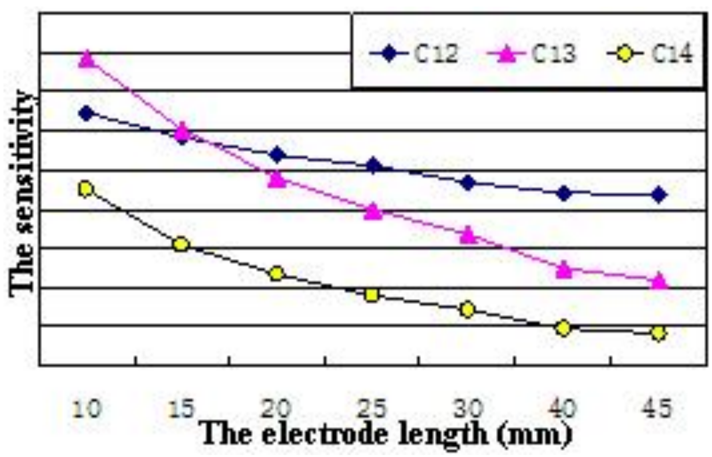

Figure 14. The effect of the electrode length on the sensitivity

d. Analysis and comparison of the change range of the detected capacitance

k_c is the change range of the detected capacitance, shown in figure 15. According to figure 15, it can be found that k_c increases when the outer shield width increases and the substrate permittivity become big. So a small outer shield width and the substrate permittivity are selected. When the substrate thickness and the electrode width increase, k_c decreases. Therefore a big substrate thickness and a long electrode should be selected. When the electrode width increases, k_c first decreases, and then increases, the electrodes width should be about 10mm.

According to the above simulation, the electrode length is 25mm, the electrode width is 10mm, the substrate thickness is 2mm, the substrate permittivity is 3.45, and the outer shield width is 6mm. the simulation indicates that the sensitivity and k_c are improved after the sensor structure parameters are optimized.

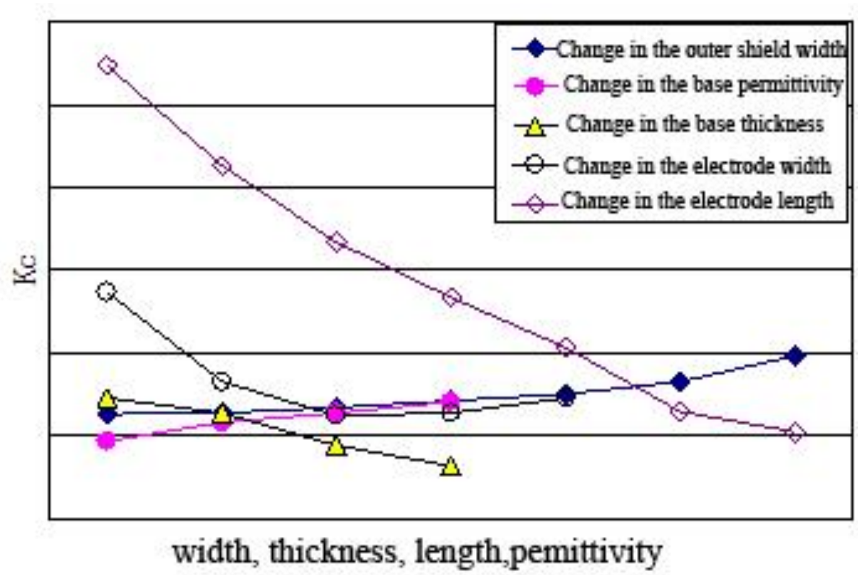

Figure 15. The change in k_c

IV. THE UNIPLANAR CAPACITIVE SENSOR AND CAPACIANCE SIMULATION

a. The uniplanar capacitive sensor

According to the above-mentioned structure parameter optimization, a uniplanar capacitive sensor with 8electrodes is manufactured. The sketch of a capacitive sensor with 8 sensing electrodes is shown in figure 16, and its actual picture is shown in figure 17. The sensor performance can be improved when the outer shield, the shield between electrodes, and the shield ground are added in the sensor. The substrate length is 250mm and the substrate width is 90mm, the substrate thickness is 1.5mm; the length of each sensing electrode is 25mm and the width is 10mm, and the space between electrodes is 13mm. the width of the outer shield is 6mm. the shield length between electrodes is 60mm, its width is 1mm, and its height is 1mm. 8 sensing electrodes are evenly placed on a uniplanar plane. The composite material sample detected is placed on the uniplanar capacitive sensor with 8 electrodes.

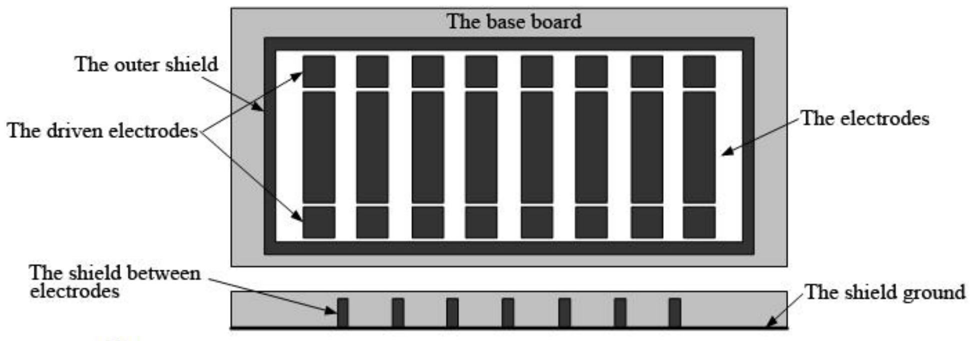

Figure 16. The uniplanar capacitive sensor structure

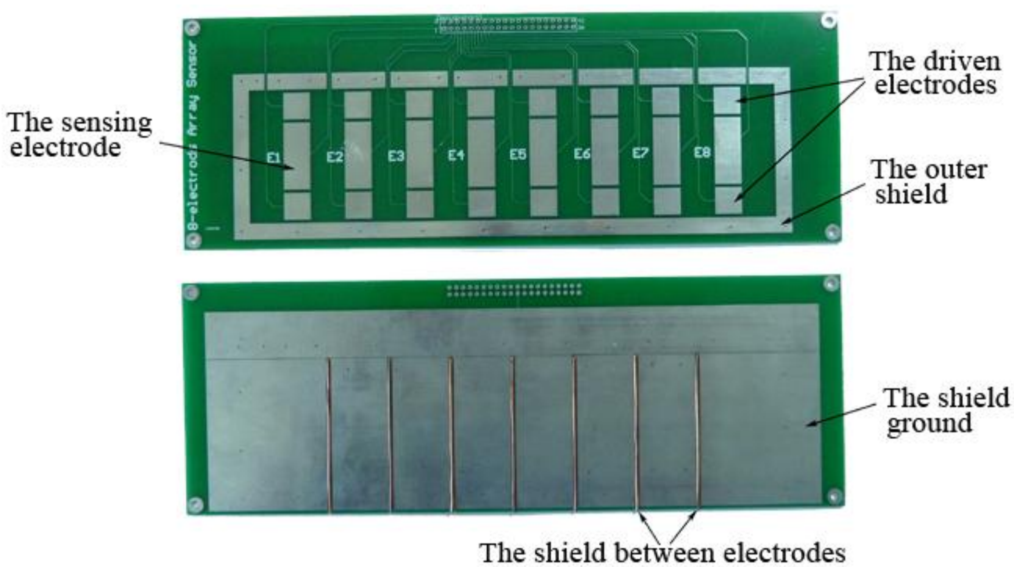

Figure 17. The actual picture of the uniplanar capacitive sensor with 8 sensing electrodes

In the sequence of capacitance measurement, one electrode (e.g. No.1) is first used as a source electrode, and applied on voltage *U*; the others, as detector electrodes, are all held at zero (virtual ground) potential. Capacitances are measured between single pairs of electrodes. A second electrode is then selected as the source electrode and the sequence is repeated until all possible electrode pair capacitances have been measured. A sensor consists of N electrodes resulting in a total of M [10] different mutual capacitances to be measured. M=N•(N-1)/2, and N is the number of electrodes located on the uniplanar plane. 28 independent inter-electrodes capacitances are measured. The electrostatic field problem of a capacitive sensor can be characterized by Laplacian equation [11].

$$\nabla \cdot (\varepsilon_0 \varepsilon_r (x, y) \nabla \Phi(x, y)) = 0 \tag{4}$$

where, ε_0 is the permittivity of free space, $\varepsilon_r(x,y)$ is the relative permittivity distribution of material in two dimensions, and $\Phi(x,y)$ is the electrical potential distribution in two dimensions. The electrical potential $\Phi(x,y)$ can be calculated by solving (4) after ε_0, $\varepsilon_r(x,y)$ and some boundary conditions (the potential values of various electrodes) are given. The analytical solution of (4) is very hard to get. In an effort to solve this problem, 2-D finite element method [12] is employed to solve (4) for obtaining the potential distribution which is used to determine the energy W_e, then capacitance according to the following formula.

$$W_e = \frac{l}{2} \int_{\Omega} \varepsilon(x, y) \left| \nabla \Phi \right|^2 d\Omega \tag{5}$$

$$C = \frac{2W_e}{U^2} \tag{6}$$

where, W_e is the energy in the electric field; U is the voltage difference between electrode pair; and l is the length of electrode, Ω is the region of two dimensions.

b. Capacitance simulation

The cross section of a certain region above the uniplanar capacitive sensor is divided into 3762 triangle elements in this paper. After the electric potential values at 3 apexes in each triangle element are computed by FEM, the energy can be obtained.

For a two dimensions electrostatic field, after the cross section is divided into n triangle elements, (7) can be further described as follows:

$$W_e = \frac{l}{2} \sum_{e=1}^{n} \int_{\Omega^e} \varepsilon \left[\left(\frac{\partial \Phi}{\partial x} \right)^2 + \left(\frac{\partial \Phi}{\partial y} \right)^2 \right] d\Omega \tag{7}$$

where, n is the number of elements, Ω_e corresponds to the region of an element. The electric potential values at 3 apexes in each triangle element are already computed by FEM above-mentioned. Therefore, the energy in electric field can be obtained, and the capacitances can be calculated.

The capacitance simulations are classified into 2 situations. One is for the un-notched composite material, which size is the same as that of the material with 3 slots, the other is for the aircraft composite material with 3 slots (shown in figure 18), which simulates the surface and subsurface defects detection. 2 sets of capacitance values are calculated in MATLAB according to discussing above.

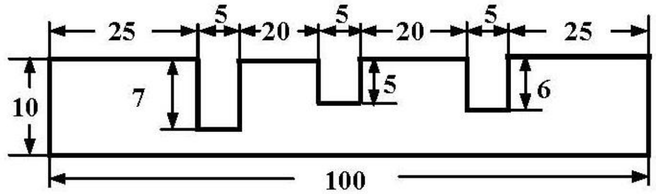

Figure 18. The notched sample (mm)

The capacitance reductions of the 3-slot-notched slab are shown in figure 19. Compared with the un-notched case, the capacitances of the material with 3 slots decrease. The capacitance lessening prognoses that the material may have subsurface damage and the structural integrity of material is not in good condition.

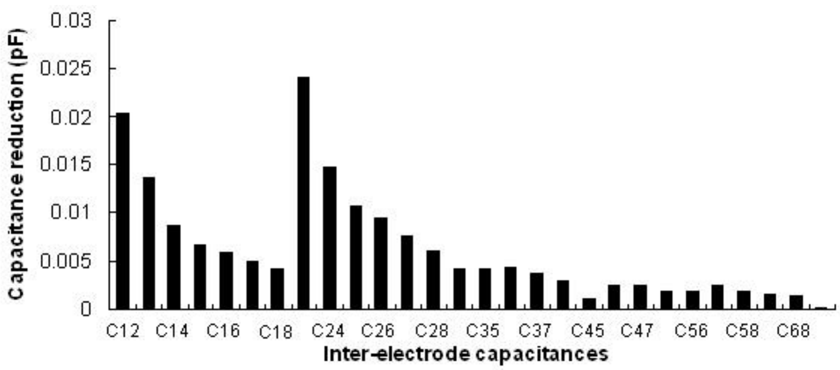

Figure 19. Capacitance reductions

According to the simulation, it is obviously seen that the capacitances will change when there are flaws in composite material. Therefore the damage in composite material can be detected through the capacitance variations.

V. EXPERIMENT SETUP

For testing purpose we have prepared two aircraft composite material slabs. One is notched, shown in figure 20, the other is an un-notched slab (not shown) whose size is the same as the notched one. In the experiment of detecting damage, these 2 samples are respectively placed on the uniplanar capacitive sensor (shown in figure 17) and evenly removed from the left to the right. The removing distance is 3mm each time.

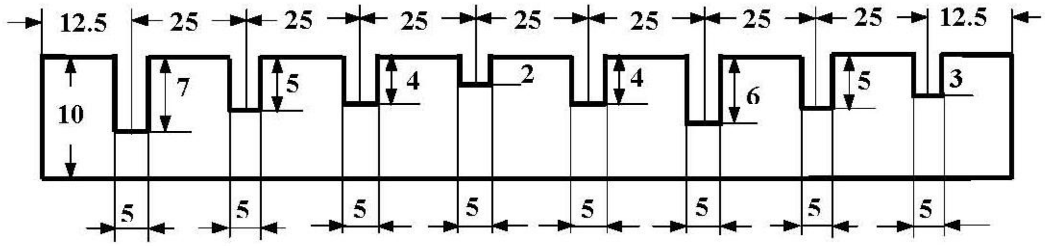

Figure 17. The notched composite material slab (mm)

4 sets of data are measured by *HP 4282A precision LCR meter*. There are 28 capacitance values in each set of data. The former 2 sets are measured at the excitation frequency of 20kHz when the healthy sample is evenly removed on the sensor, and the latter 2 sets are measured at the excitation frequency of 100kHz when the notched sample is evenly removed on the sensor. The set of capacitance reductions of the notched sample at the frequency of 20kHz is shown in figure 21, and that at the frequency of 100kHz is shown in figure 22. It is apparently found that the capacitances decrease when the sample is notched. The result is similar to the simulation.

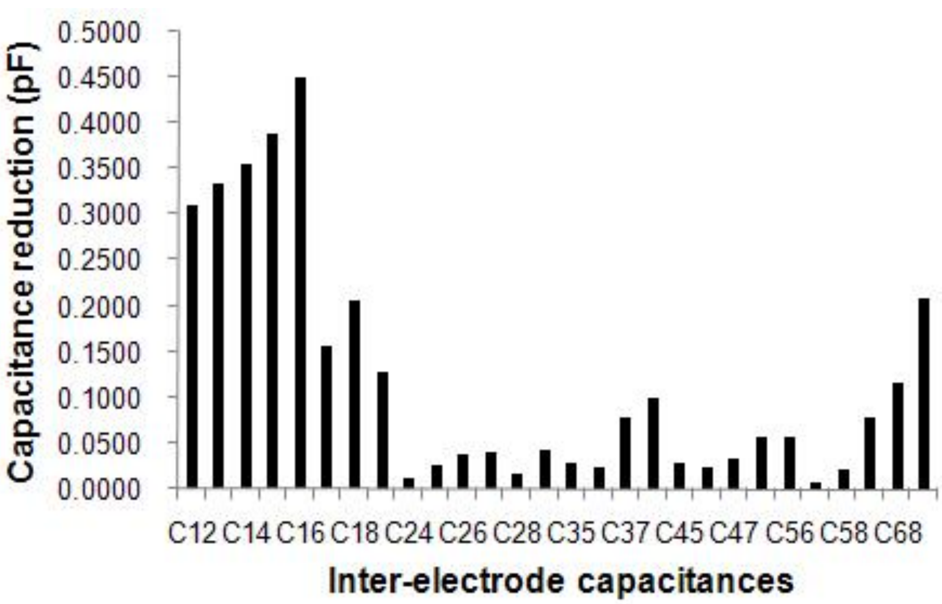

Figure 21. Capacitance reductions

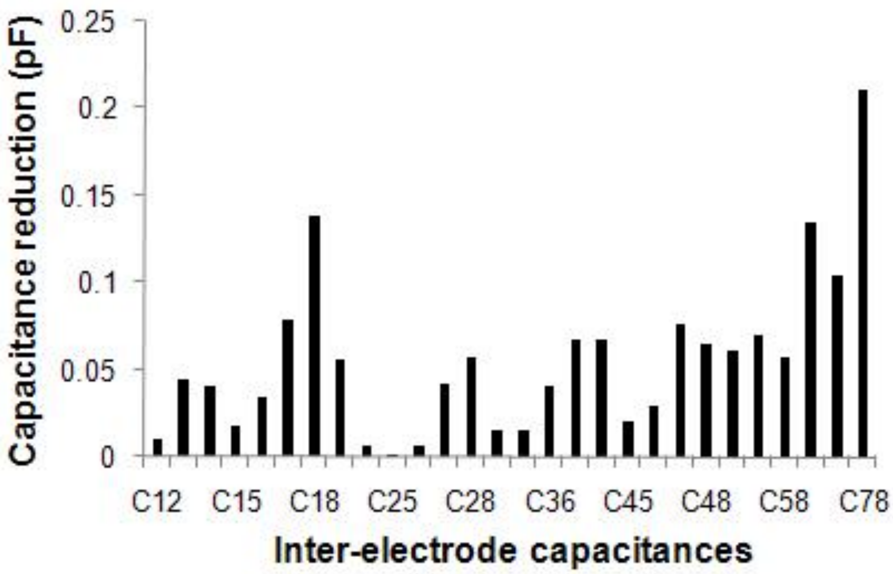

Figure 22. Capacitance reductions

VI. RESULTS AND DISCUSSION

A uniplanar capacitive approach for subsurface damage detection of aircraft composite material is reported. The sensitivity simulation for the uniplanar double electrodes is done. According to the simulation, the measured capacitances between two electrodes are affected by the electrode width, the electrode length, and the space between electrodes. The sensing distance of the capacitive sensor is also affected by those factors above-mentioned. When the space between electrodes and the distance from the measured object to the electrode are determined, the changes in electrode length and width less affect the measured capacitance; after the space becomes big and the object distance is certain, the changes in electrode length and width greatly affect the detected capacitance. When the space between electrodes is determined, the electrode length increment can enlarge the sensing distance of the uniplanar capacitive sensor.

A three dimensions model of the uniplanar capacitive sensor with 8 electrodes is founded. The optimization for the structure parameters of the sensor with 8electrodes is carried out by FEM. The electrode length and width, the substrate thickness and permittivity, and the outer shield width can differently affect the sensitivity of the sensor. According to the optimization, a uniplanar capacitive sensor with 8 sensing electrodes is manufactured. The performance of the sensor is improved.

A damage detection method for aircraft composite material is analyzed by the experiments. According to figure 21, 22, compared with the un-notched sample, the measured capacitance reduces when the sample is notched. It indicates that the subsurface damage can be detected by the capacitances reductions. Therefore, the faults of the aircraft composite material can be detected according to the capacitance variation.

The preliminary experimental results presented here demonstrate that the multi-electrode uniplanar capacitive approach is capable of detecting the damage of aircraft composite materials. The proposed approach is a viable technique for damage detection of aircraft composite material. The structure health monitoring can also be done according to the obtained capacitance variations of aircraft composite material parts.

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