



## MODELING AND CONTROLLER DESIGN OF PNEUMATIC ACTUATOR SYSTEM WITH CONTROL VALVE

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*Abstract- Pneumatic actuators offer several advantages over electromechanical and hydraulic actuators for positioning applications. Nonetheless, pneumatic actuators are subject to high friction forces, dead band and dead time, which make fast and accurate position control difficult to achieve. This research paper presents the process of controller identification, design, modeling and control for pneumatic actuator system. System Identification approach is used with the purpose to estimate the mathematical model of pneumatic actuator system and for controller design. Data collection of input and output signal of the system has been performed from experiment procedure. This data is used for estimate the model by selecting Auto-Regressive Exogenous (ARX) model as a model structure. The accepted model is based validation test namely as residual correlation, Akaike Final Prediction Error and best fit percentage. Different control schemes such as PID and LQR (Linear Quadratic Regulator) have been applied for controller design. PID controllers with Ziegler Nichols tuning are enabled to provide good performance in various systems. The effects of Tustin transformation, zero order hold and discrete model are tested in PID controller designed. The methodology for this paper combines off-line model*

*based on analysis with on-line iteration. Different external loads are added in order to investigate the effectiveness to the designed controllers in real time system. The tracking performance of the closed loop system is satisfied which offers considerable robustness even on a slight increase in load. The results obtained in the experiment are successful to prove that the output signals which with the controller are almost the same for both simulation and experimental modes.*

**Index terms:** Pneumatic actuator system, ARX model, System identification, PID controller, LQR controller

## I. INTRODUCTION

Pneumatic actuator, are also known as man-made or artificial muscles for robotic field, it has been the main concern in several researched papers in the past. Pneumatic systems are extensively used in the automation of production machinery and in the field of automatic controllers. Relatively high force output-to-weight ratios, comparatively low cost and cleanliness, pneumatic actuators are well suited for a number of industrially relevant tasks ranging from point-to-point positioning to high-accuracy servo positioning and force control. Of course, pneumatic actuators are presently implemented not just in industrial machinery but also in medical instruments in assisting our daily life. By comparing with hydraulic servomotors, pneumatic actuators are safe and reliable to be implemented in term of environmental concern as well as personnel protection. Nevertheless, pneumatic actuators are difficult in designing due to complex nonlinear dynamics, air compressibility and the parasitic effects of actuator friction.

Pneumatic servos have advantages over hydraulic, which compromise environment contaminations as well as manufacturing cost. Pneumatic actuators are suited for use in difficult surroundings such as nuclear environment. Ambient temperature change has no significant effects on pneumatic actuation systems, gases are not subjected to the temperature limitations of hydraulic fluids; the actuator exhaust gases need not be collected, so fluid return lines are not necessary and long term storage is not required due to pneumatic systems are virtually dry and inorganic materials need be used. Pneumatic actuators are on demand for industrial applications due to its high power rate (torque-squared to inertia ratio) as well as significant weight advantage. (Markov *et al*, 2009) ever mentioned that the classical PID controller law is not competitive enough for a nonlinear application system which requires significantly high accuracy in

positioning and actuator rigidity under different external loads. Nonetheless, Linear Neural Model Based Predictive Controller (LNMBPC) is synthesized with the combination of Model-Based Predictive Control (MBPC) and Artificial Neural Network (ANN) to improve the system response experimentally over classical PID model within the control loop of pneumatic actuator control system.

(Jihong *et al.*, 1999) proposed a precise position control strategy for servo pneumatic actuator systems. This control strategy has been applied in combination with a modified PID controller to a pusher mechanism in the packaging of confectionery products. Both the positioning and the time accuracy required by the production task were achieved using such a control strategy.

To enhance the dynamic characteristic of pneumatic servo drives (Nagarajan *et al.*, 1985) described an approach based on an outer decision loop, which modifies the command issued to an existing closed loop drive. Experimental results are presented which show that the scheme can improve the quality of response with the respect of positioning time and overshoot.

In another research, (Sy Najib *et al.*) modified the PID controller with the addition of nonlinear gain (NPID). Nonlinear gains are adjusted automatically based on the generated errors feedback to the controller. (Sy Najib *et al.*) proved that the performance of the system is significantly enhanced with the respect of system robustness against the load changes.

Due to the limitation of conventional PID controller to nonlinear systems, (Amin *et al.*, 2011) suggested, LQR is a control scheme that provides the best possible performance with respect to some given measure of performance. The LQR design problem is to design a state feedback controller  $K$ , is able to be reduced. Feedback gain matrix is developed which minimized the objective function with the purpose to achieve some compromise between the use of control effort, the magnitude, and the speed of response that will guarantee a stable system.

(Jian-Bo, He *et al.*, 1998) stated that LQR has a very nice robustness property. If the process is of the single-input and single-output (SISO), then the control system has at least the phase margin of 60 degrees and the phase margin of infinity. In his paper, (Jian- Bo, He *et al.*, 1998) added that LQR solution is appropriate to develop an optimal tuning algorithm for processes with time delay.

## II. METHODOLOGY

### A. HARDWARE IMPLEMENTATION

The pneumatic actuator that is used in the research composed of double-acting actuator, 5/3 proportional directional control valve and mass payload as illustrated in Figure 1.

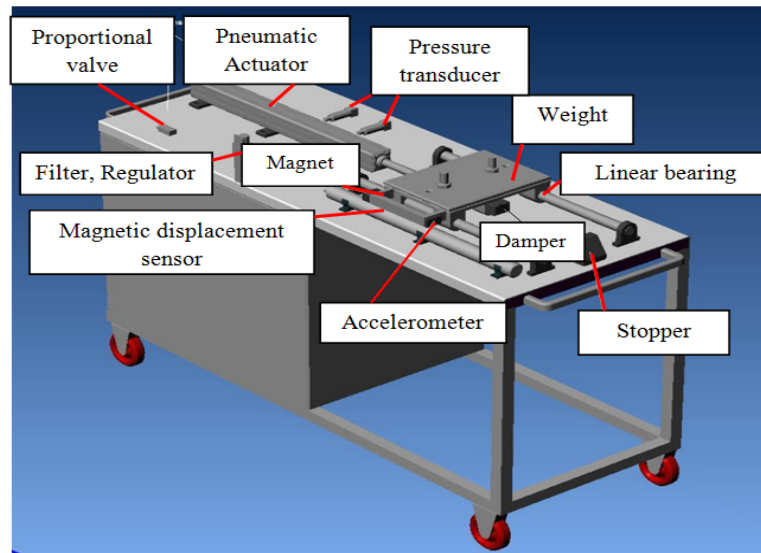


Figure 1. Basic structure of experimental pneumatic apparatuses

Pneumatic actuator is driven by Jun Air compressor which offers vibration level, minimum noise, longer life time and higher pressure. National Instrument Data Acquisition System, DAQ (Model 182556F-01REV2) is connected to the sensors and actuators to the computer with the aim to build and exact control system and set parameters for control algorithms to establish a custom user interface. Typically, DAQ is used to convert analog waveform into digital values for processing after the error detection between end-device set-point command signal and actual device monitoring feedback signal are detected by Proportional Pneumatic Control Valve LS-C10.

### B. SOFTWARE IMPLEMENTATION

System Identification Toolbox as shown in Figure 2 is a graphical user interface (GUI) for estimating and analyzing linear and nonlinear models in the System Identification. It is used to construct the mathematical models of dynamic system based on the user-defined input and measured output. The user is allowed to use frequency domain or time domain input output data

to build discrete time and continuous time transfer functions, process models and state space models. Identification toolbox provides maximum likelihood, prediction-error minimization (PEM), subspace system identification, and other identification approaches. User can implement the identified model to predict or forecast the system response as well as for the simulation in Simulink.

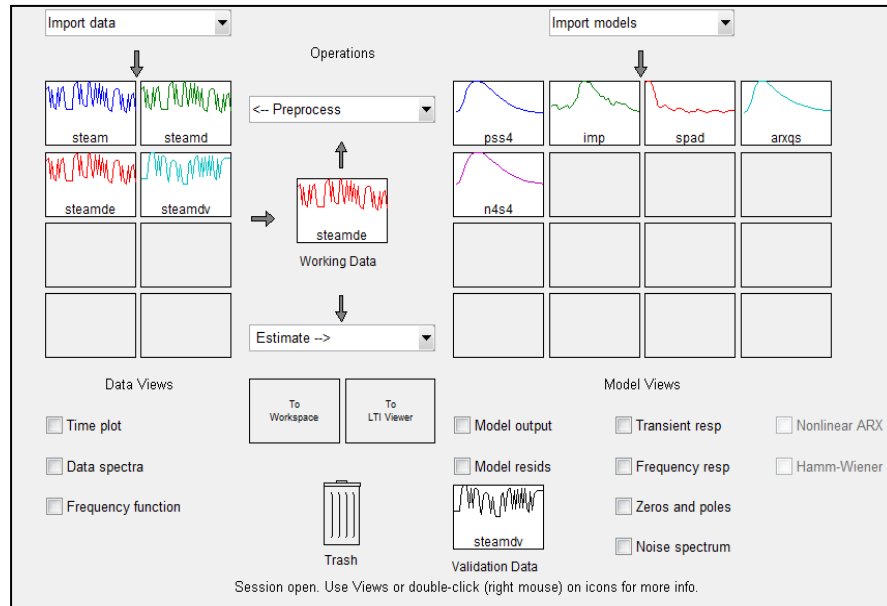


Figure 2. MATLAB System Identification Toolbox

### C. CONTROLLER DESIGNS

Proportional-integral-derivative (PID) controller is a generic control loop feedback mechanism which is employed in industrial control systems. The PID controller calculates the difference in between the desired value and measured output, which is known as system error. There is a tendency for the controller to minimize the error by having a feedback system to compensate the gap between the desired set point and measured output. A mathematical description of the PID controller as below,

$$u(t) = K_p \left[ e(t) + \frac{1}{T_i} \int_0^t e(\tau) d\tau + T_d \frac{de(t)}{dt} \right] \quad (1)$$

where,  $u(t)$  is the input signal to the plant model before the summation of disturbance  $d(t)$ , the error signal  $e(t)$  is defined as  $e(t) = r(t) - y(t)$ , and  $r(t)$  is the preset input signal. PID controller can be adjusted to tune the coefficients of proportional, integral and derivative gain with the purpose to compensate the feedback error. Ziegler Nichol tuning method is implemented by inserting the calculated parameters into PID block in Simulink. According to the philosophy suggested by Ziegler and Nichols, ultimate gain,  $K_c$  can be obtained by increasing the gain constant in proportional mode until a sustained oscillation takes place before the tuning process. Nonetheless, Linear Quadratic Regulator is an optimal controller, which is concerned with operating a dynamic system at minimum cost. Generally, the system dynamics are expressed by a set of linear differential equations whereas the cost is defined by a quadratic function, which is known as LQ problem. The term "cost" is often described as a sum of the deviations of key measurements from the desired values. Hence, the functionality of this algorithm is to adjust the controller setting to ensure that the undesired deviations are minimized. LQR is a method in modern control theory that implemented the state-space techniques to control and optimize the systems. However, the conventional control theory is relied on either the input-output association or the transfer function. In the configuration of LQR, the state-space models for the LQR process control are derived as follows:

$$\dot{x}(t) = Ax(t) + Bu(t) \quad (2)$$

$$y(t) = Cx(t) + Du(t) \quad (3)$$

$$u(t) = -Kx(t) + v(t) \quad (4)$$

$$\dot{x}(t) = (A - BK)x(t) + Bv(t) = A_c x(t) + Bv(t) \quad (5)$$

The optimal LQR is often defined more generally and compromises of determining the controller transfer-matrix  $C(s)$  that minimizes,

$$J_{LQR} = \int_0^{\infty} z'(t)Qz(t) + \rho u'(t)Ru(t)dt \quad (6)$$

where,  $Q$  is known as  $l \times l$  symmetric positive-definite matrix, whereas  $R$  is a  $m \times m$  symmetric positive-definite matrix, and  $\rho$  is a positive constant. . Parameters  $R$  and  $Q$  are chosen to balance the step input to the system and to optimize the cost function respectively. In the LQR controller

design, assume  $R = 1$  and  $Q = C' * C$ . In order to reduce the steady state error of the system output,  $Nbar$  is added after the reference input. State vector is multiplied by the feedback vector,  $K$  to compensate or reduce the system error after the addition of constant gain,  $Nbar$ .  $Nbar$  can be determined by implementing the user-defined function in m-file code. Eventually, the performances of PID and LQR controllers are compared for analysis purposes.

### III. RESULTS AND DISCUSSION

#### A. MODEL ESTIMATION

Data is imported to the System Identification Toolbox for various data processing, model estimation, and model analysis tasks. Input signal (7) is applied to the system in order to observe the differences between the estimated results and the actual outcomes to the system. Figure 3 illustrates the input and output signals with the duration of 100 seconds for model identification.

$$y(t) = 40 [0.87 (\cos (2\pi 0.5t) + 1.25 \cos(2\pi 0.2t + 2.5\cos (2\pi t)))] \quad (7)$$

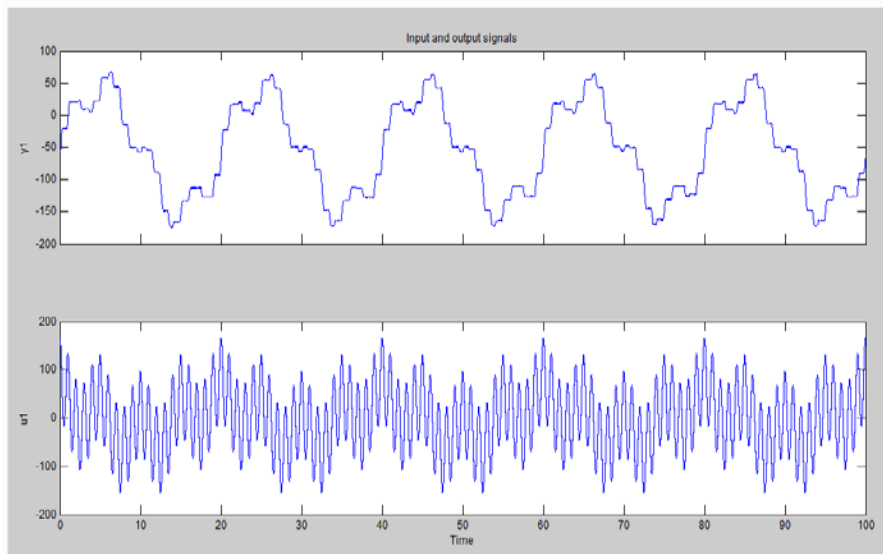


Figure 3 Input and output signals

ARX model is selected as the model structure. With the purpose to justify the best estimations and performances for the system before the controllers are designed. Offset value and time interval (Table 1) are highlighted to determine the best performance by varying the coefficients.

Table 1. Percentage of best fit by varying offset values and time interval

Offset Value	Best Fit (%)		
	$T_s = 0.01$	$T_s = 0.03$	$T_s = 0.05$
-0.090000	91.62	88.60	92.30
-0.100000	85.52	83.45	81.10
-0.080000	84.72	81.99	91.17
-0.091000	92.43	88.52	88.03
-0.091500	91.91	87.99	86.24
-0.090500	92.07	87.15	87.29
-0.090750	92.75	89.04	89.04
-0.090875	93.05	88.77	88.06

$T_s$  = Sampling time

Refer to Table 1, 93.05 percent is the best estimation with 10 ms time interval and -0.090875 offset value. These results are acceptable due the best fit percentage is exceeding 90 percent. The sampling rate must be sufficiently large so that the analog signal can be reconstructed from the samples with sufficient accuracy. Interval time must be sufficiently small so that no information will lose. ARX model order is adjusted with the purpose to investigate the best fit percentage as state in Figure 4, 5 and 6.



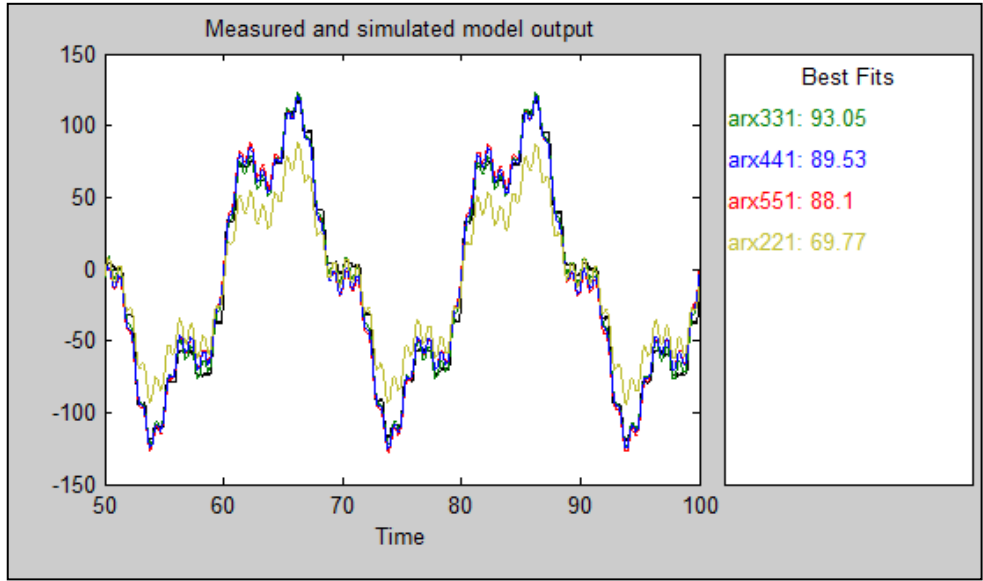


Figure 4 Best fit percentage with different ARX model orders ( $T_s = 10$  ms)

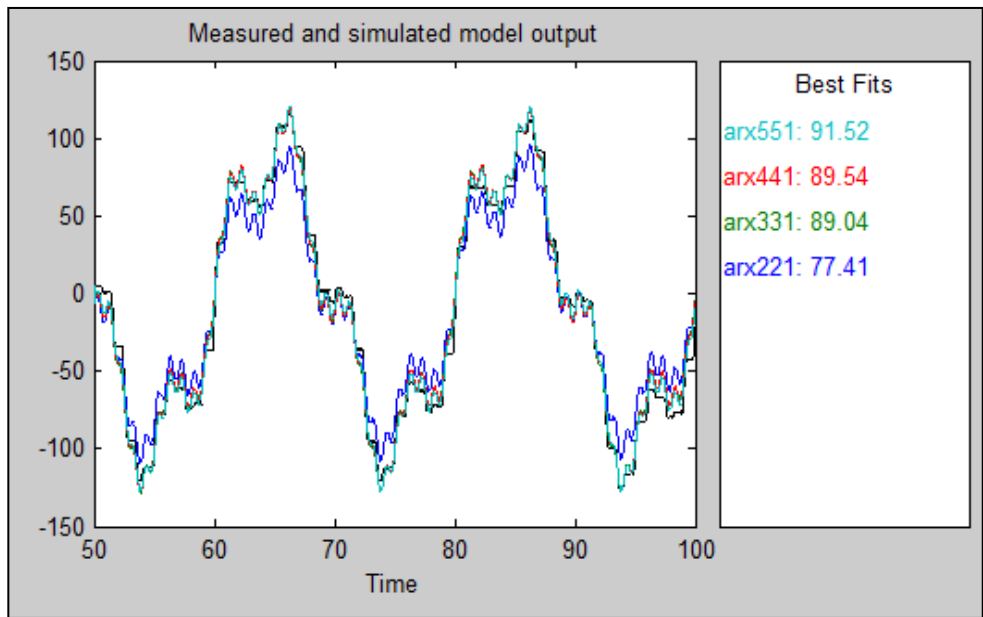


Figure 5 Best fit percentage with different ARX model orders ( $T_s = 30$  ms)

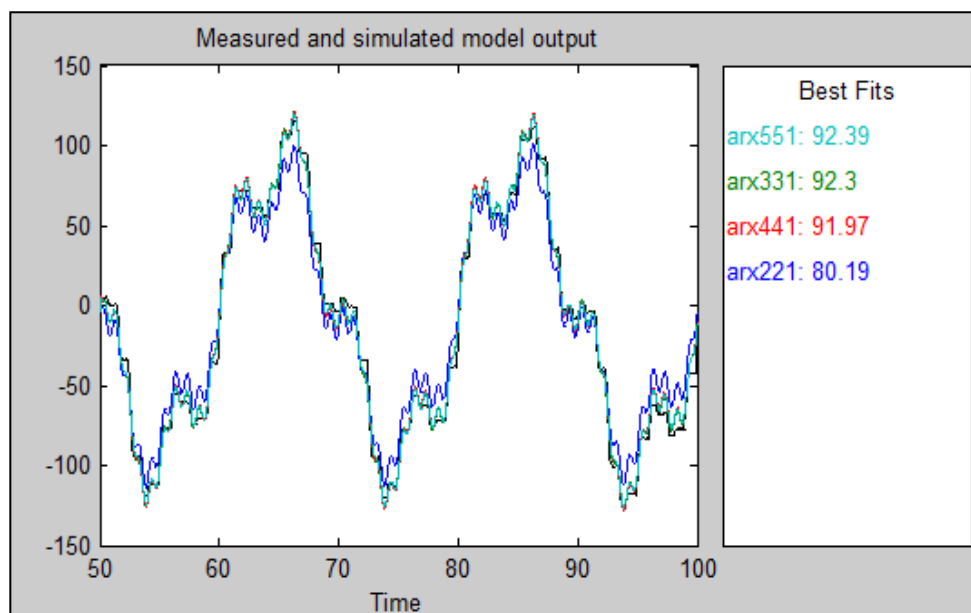


Figure 6 Best-fit percentage with different ARX model order ( $T_s = 50$  ms)

According to results obtained, obviously, ARX331 model with 10 millisecond intervals (Figure 4) has the greatest best-fit percentage (93.05%) compares to others (Figure 5 and Figure 6). Even though ARX551 model with 50 millisecond intervals (Figure 6) is sufficiently high in the best fit percentage, yet it is not chosen due to 5th order model is difficult for modeling and controller design. ARX331 model is selected and constructed based on the Akaike's Final Prediction Error (FPE), loss function and best-fit criterion, which are indicated in Table 2. ARX551 model has the smallest value of FPE and loss function; nonetheless, it is not appropriate for controller design due the percentage of best fit is below 90%. In addition, model a fifth order model structure is relatively difficult even though it has better performance.

Table 2 Model estimation and validation

<b>ARX Model</b>  <b>T<sub>s</sub> = 10 ms</b>	<b>Model Estimation and Validation</b>		
	<b>FPE</b>	<b>Loss Function</b>	<b>Best Fits (%)</b>
<b>ARX 221</b>	0.261192	0.260775	69.77
<b>ARX 331</b>	0.1539	0.15352	93.05
<b>ARX 441</b>	0.142806	0.142351	89.53
<b>ARX 551</b>	0.141974	0.141408	88.1

Polynomial model is obtained in the form of discrete time equations which are stated in (8) and (9).

$$A(z) = 1 - 1.037z^{-1} - 0.5855z^{-2} + 0.6222z^{-3} \quad (8)$$

$$B(z) = 0.2511z^{-1} - 0.5032z^{-2} + 0.255z^{-3} \quad (9)$$

Transfer function for ARX model is represented as follows,

$$G(q) = \frac{B(z)}{A(z)} = \frac{0.2511z^2 - 0.5032z + 0.255}{z^3 - 1.037z^2 - 0.5855z + 0.6222} \quad (10)$$

ARX331 model with 10 milliseconds is selected for controller design. It is applicable to represent the pneumatic actuator system due to the minimum FPE and loss function. ARX331 model is significantly high in the best-fit percentage or similarity in between the simulated outputs and the true values.

ARX331 model is tested for the system stability by investigating the residual performance as stated in Figure 7. A good estimated model should have an autocorrelation function within the confidence interval, indicating that the residuals are not correlated. However for the independent test in residual analysis, residuals are uncorrelated with previous inputs for good model validation.

The signal fluctuation is insignificant to the residual analysis. Hence, based on the results in Figure 7, the autocorrelation and cross correlation are satisfied with the tolerance range  $\pm 1\%$ . Step input is used to determine the model stability as plotted in Figure 8. Obviously, output response for the system is over damped when the step input is imported into the ARX331 model. In Figure 7, output response is non-minimum phase in the condition without the feedback loop and the controller to compensate the steady state error.

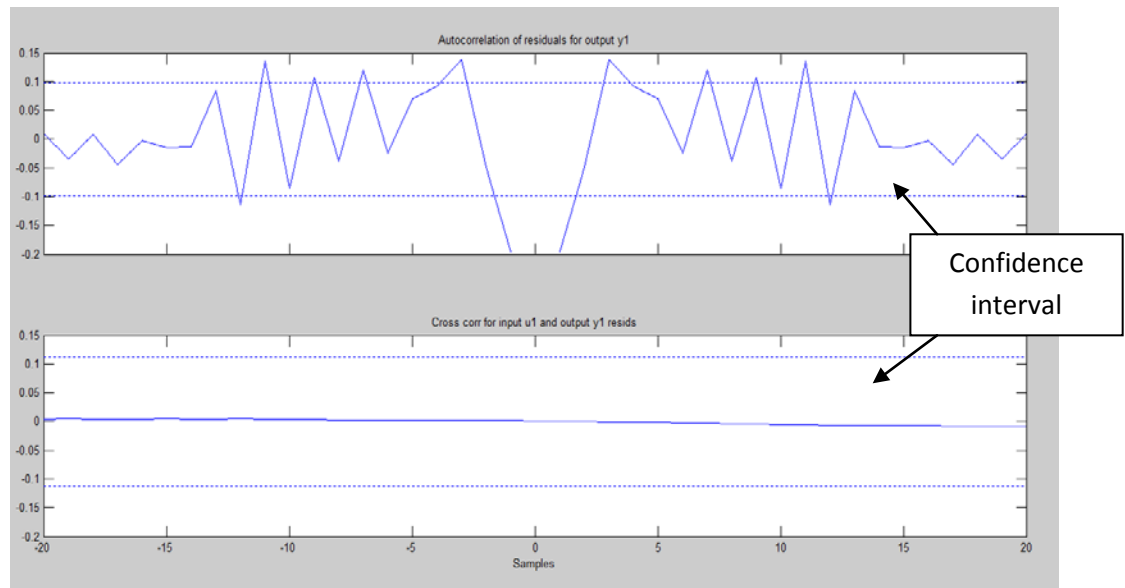


Figure 7. Residual analysis

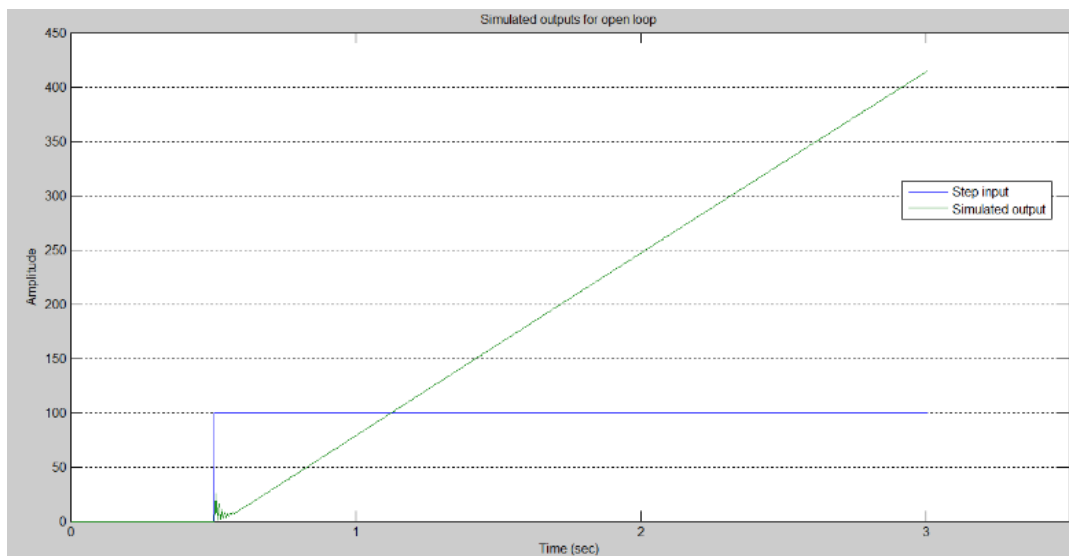


Figure 8. Output response (Open loop system)

## B. CONTROLLER DESIGNS

ARX331 model is simulated to investigate the output response by designing the PID and LQR controllers. Initially, the open loop transfer function for ARX331 is in discrete model, transformation into the continuous time model is needed to determine the best performance of ARX331 after the application of controller. Bilinear transformation and zero order hold techniques are implemented to convert the discrete time model into the continuous time model. The transfer functions are transformed into continuous time model after the conversion of Bilinear transformation and zero order hold are stated in (11) and (12) respectively.

$$G_{Bilinear}(s) = \frac{1.216s^3 + 245.1s^2 - 515.6s + 27950}{s^3 - 1564s^2 + 32820s - 10600} \quad (11)$$

$$G_{ZOH}(s) = \frac{0.4941s^3 + 15820s^2 - 24880s + 1808000}{s^4 + 73.53s^3 + 100500s^2 + 2123000s - 685700} \quad (12)$$

Based on the parameters of  $K_p$ ,  $K_i$  and  $K_d$  in Table 3, PID controller is connected in series to the system for the performance testing. Square wave signal is imported to the system for the verification of different PID controller designs. Figure 9, 10 and 11 represented the output response of the system.

Table 3. Parameters of  $K_p$ ,  $K_i$  and  $K_d$  in Ziegler Nichols tuning methods

Controller Type	Continuous time model ( $K_c = 12.312$ , $T_c = 0.6$ )			Discrete time model ( $K_c = 5.0752$ , $T_c = 2$ )		
	$K_p$	$K_i$	$K_d$	$K_p$	$K_i$	$K_d$
<b>P</b>	6.156	-	-	2.538	-	-
<b>PI</b>	5.596	11.19	-	2.306	3.045	-
<b>PID</b>	7.387	24.624	0.5404	3.043	3.043	0.761
<b>P.I.R</b>	8.618	35.91	0.770	1.940	2.425	0.582

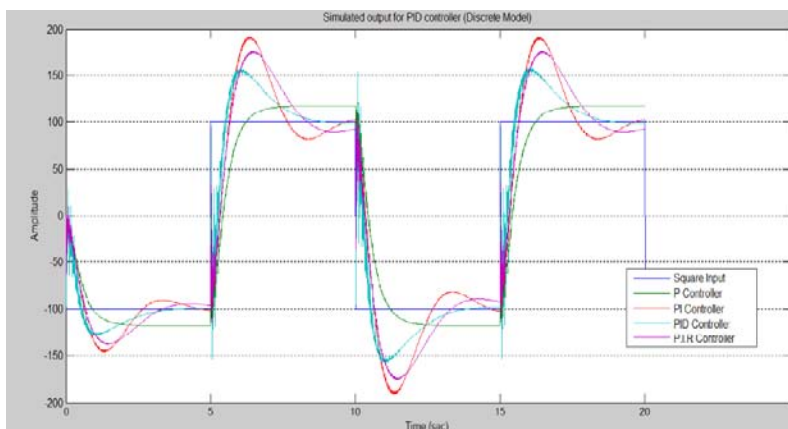


Figure 9. Simulated outputs for different controllers in discrete time model

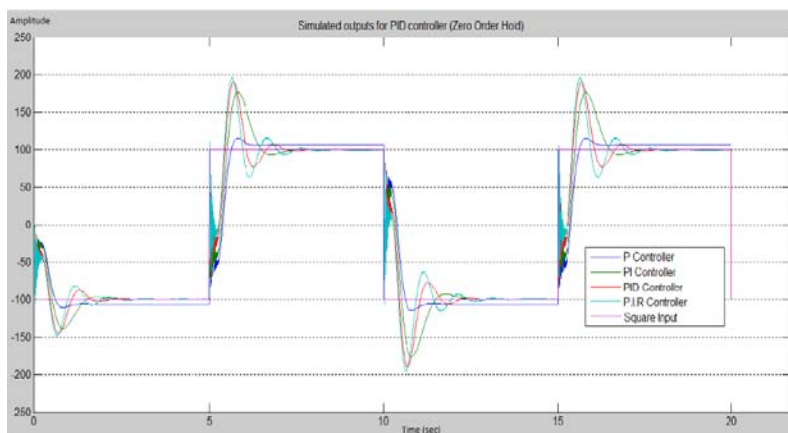


Figure 10. Simulated outputs for different controllers in continuous time model (Zero Order Hold)

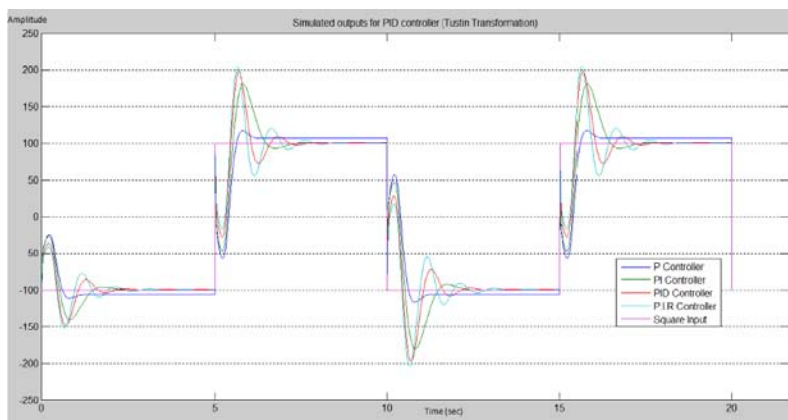


Figure 11. Simulated outputs for different controllers in continuous time model (Bilinear Transformation)

By observation, P controller is sufficient and reliable to be implemented in the system controlling as well as steady state error compensation. Figure 12 summarizes the performances of simulated outputs in Figure 9, 10 and 11 with the respect of settling time, rising time, steady state error, and the percentage of overshoot. P controller is taken as a reference for both continuous and discrete time models.

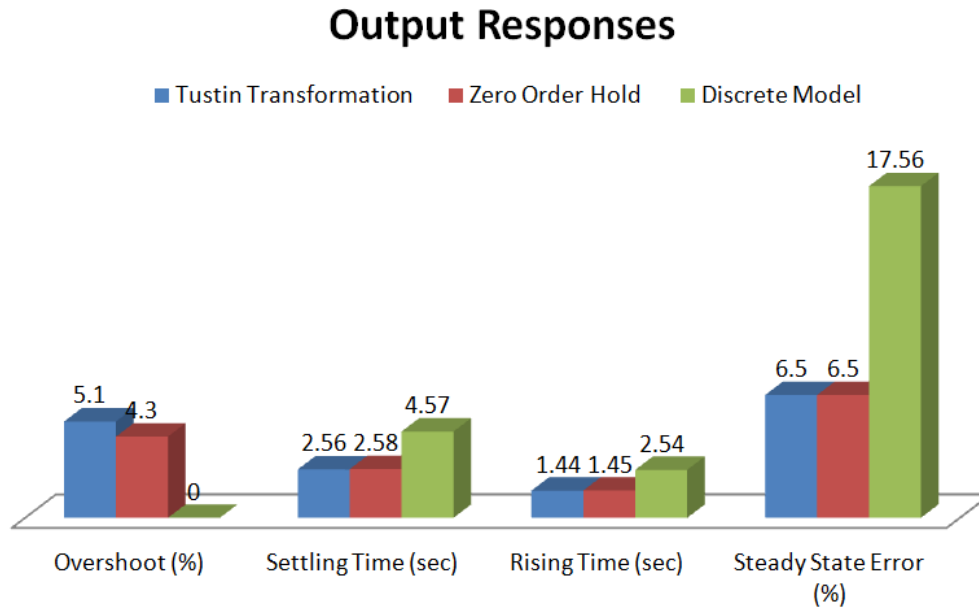


Figure 12. Output responses between continuous and discrete time models

Bilinear transfer function is chosen for the system modeling even though its percentage of overshoot is slightly higher than zero order hold. Figure 11 as a reference, it shows that the output responses of Bilinear transformation are not affected by machining vibration or distortion compare to zero order hold in Figure 10. Bilinear transformation method rounds any time delay to the nearest multiple of the sampling time. Therefore, P controller with the Bilinear transfer function is used for online testing with the connection to the pneumatic actuator as shown in Figure 13 and 14.

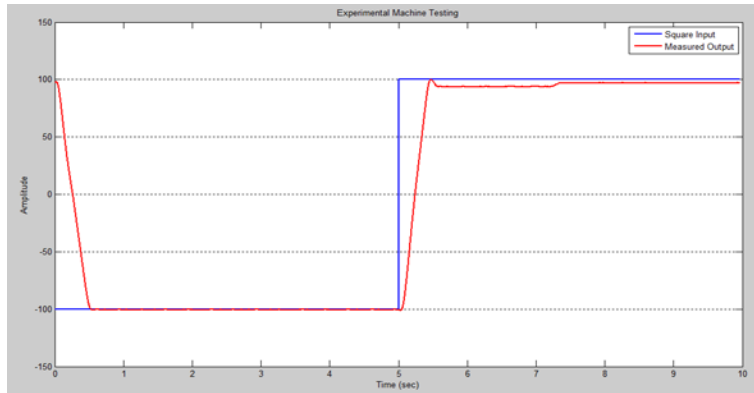


Figure 13 P controller online testing (Square wave input)

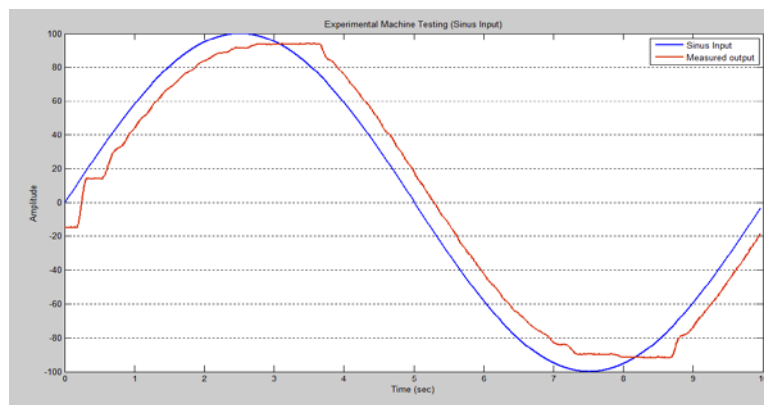


Figure 14. P controller online testing (Sine wave input)

From the Figure 4.13, it is observed that:

- i. Settling time,  $t_s = 0.81$ seconds
- ii. Rise time,  $t_r = 0.6$  seconds
- iii. Percentage overshoot = 4.25%
- iv. Steady state error = 2.3%

Meanwhile for the Figure 14, sine wave signal is imported to the system with the purpose to determine the measured output for the machine testing. Clearly, output response is 50.4 degrees out of phase compared to the reference input. In order to test the capability of the P controller in positioning control, additional loads with different weights are added to the system. Square wave signal is taken as the reference input to check the accuracy of output response as illustrated in Figure 15.



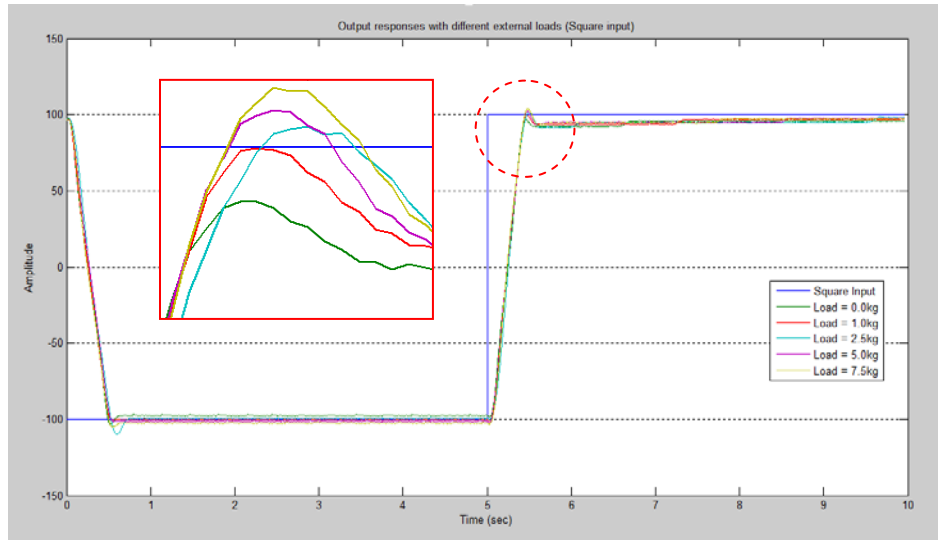


Figure 15 Output responses with different external loads

Apparently by observing the output responses in Figure 15, it shows that the external load with the highest weight has the most significant overshoot in the system. External loads with 7.5kg results the largest percentage of overshoot compare to the others. Nonetheless, P controller is applicable in the system even though the external loads are varied due to the steady state error is still within the range of 5 percent. In order to test the performance of PID controller, LQR controller is implemented in position control. Feedback vectors,  $K$  and the value of  $Nbar$  are stated as follows. In Figure 16, it compares the performance in between PID and LQR controller for positioning control.

$$\text{Feedback vector, } K = [2.6017 \quad 0.8530 \quad 0.4174 \quad 2.4979]$$

$$Nbar = 1.0695$$

LQR controller provides zero steady state error and percentage of overshoot in positioning control compare to PID controller. However, LQR controller is distorted by machining vibration or noise initially. Besides, PID controller response time is significantly fast yet it cannot fully compensate the steady state error back to the system. Regardless of that, both controllers are applicable in positioning control for the pneumatic actuator system. Table 4 summarizes the performance of PID and LQR controllers for positioning control.

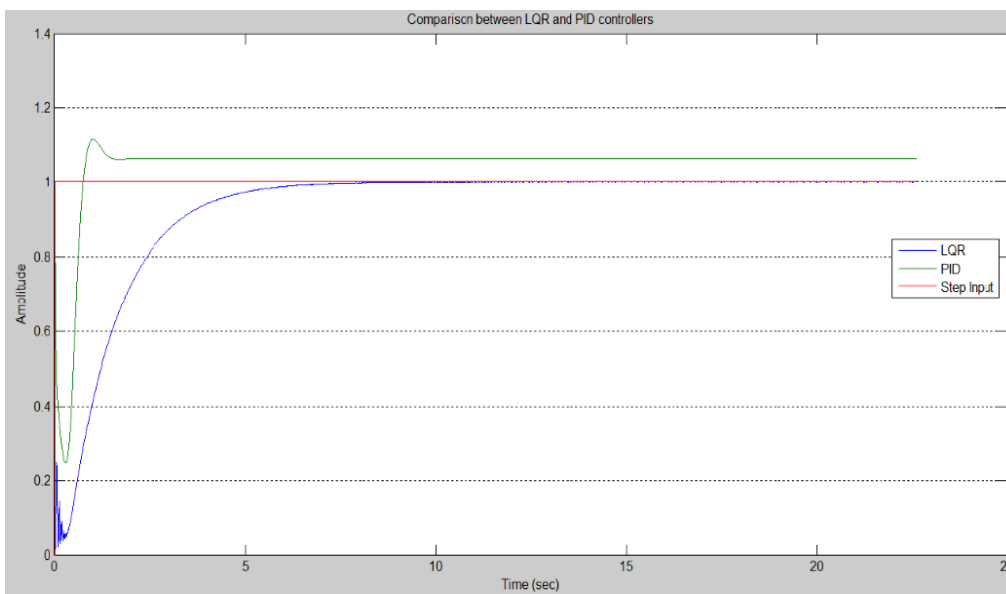


Figure 16. Comparison between LQR and PID controller

Table 4. Difference between LQR and PID controllers

	<b>LQR Controller</b>	<b>PID Controller</b>
<b>Rise time, <math>t_r</math> (s)</b>	6.25	1.34
<b>Settling time, <math>t_s</math> (s)</b>	6.97	1.56
<b>Steady state error (%)</b>	0.00	6.21
<b>Overshoot (%)</b>	0.00	4.72

## VI. CONCLUSIONS

As a conclusion, identification system provides a convenient method to control a nonlinear system by using linear controllers. It has been successfully applied to pneumatic actuator system to establish the best linear discrete model to the system. ARX model structure is selected for system modeling and controller design. PID controller is designed to the system with the reference of Ziegler Nichols tuning method. P controller with the Bilinear transfer function is applied for online testing. Step and sine inputs are injected with the purpose to determine the system response according to the tracking performance of input and output. PID controller is capable to improve the system robustness against the slight change in loads. External load with the highest weight has the most significant overshoot in the system. Both PID and LQR controllers are applicable to enhance the system performance. On real time control, output response is almost similar to the reference input for the system positioning control.

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