

UNCERTAINTY ANALYSIS OF MICRO DIFFERENTIAL PRESSURE SENSOR USING INTERVAL ANALYSIS

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Abstract- A methodology for robust design analysis of micromechanical systems using interval methods is presented by considering piezo resistive micro differential pressure sensor with uncertainty in its parameters. The proposed method guides the design of micro sensor to achieve a robust and reliable design in a most efficient way. The uncertainty analysis is carried out numerically using Coventorware and analytically using Intlab.

Index terms: Uncertainty Analysis, Interval Analysis, Micro differential Pressure sensor, Uncertainty Analysis, Coventorware, Intlab

I. INTRODUCTION

The major design objectives in any device design, is to meet the required functional parameters and the reliability of the device. The functional parameters depend on the geometry of the structure, material properties and process parameters. The major difficulty the designer faces is the dimensions and properties used in the simulation of the MEMS devices can not be exactly followed during fabrication. In order to overcome this problem, the designer must test the device in simulation for bound of parameters involved in it. This paper demonstrates the use of interval method to assess the piezo resistive micro differential pressure sensor under the presence of manufacturing and process uncertainties. The uncertainty analysis is carried out numerically using Coventorware and analytically using Intlab.

Pressure sensors presently constitute the largest market segment of mechanical MEMS devices [1-7]. The silicon differential pressure sensor that can be fabricated using silicon surface micromachining, monolithically integrated with twin diaphragms, is described in paper [2]. The advances in MEMS technology, however, have led to the introduction of solid-state sensors giving rise to enhanced functionality and performance [8]. The choice for using piezoresistive pressure sensors has been done mainly due to their relatively straightforward technological implementation [9]. For ultra miniaturized sensors, piezoresistive detection is often preferred instead of capacitive detection due to the lower cost, the scaling characteristics, and the possibility to have the amplifier separate from the sensor [10-11]. Differential pressure sensors have two requirements such as high static pressure and high differential over-range pressure, applied to the sensors, which is caused by the mis-operation of valves, must be considered in the design. Therefore, conventional silicon sensors for these applications must be mounted in a strong high-pressure vessel, and the output terminals need to be hermetically sealed. Also, a complex mechanical structure is required to protect each silicon diaphragm from high differential pressure [2]. Different techniques to compensate for the temperature dependency have been reported including laser trimming of the resistors, the use of external resistors, material compositions and self-compensation bridge configurations. The self-compensating bridge configuration requires precisely matched piezoresistors, which, at least in theory, give a temperature independency due to cancellation of the common signal in the output signal from the bridge [12 - 16]. By measuring the temperature with, for instance, a pressure-insensitive piezoresistor, the output signal can be adjusted by a correction of the signal. The correction value can be taken from a look-up table containing calibration values. A capacitive pressure sensor for differential pressure with an overload capability without the need for an external overload protection is presented [17]. The sensing element consists of a PyredsilicodPyrex sandwich and is fabricated in silicon/glass technology using anodic bonding. To increase the overpressure range the sensor element is mounted in a pre-stressed mounting fixture, which was optimized numerically. A differential pressure sensor which has protectors on both sides for over range pressure is presented [18]. There are two narrow gaps on both sides of a square diaphragm on the sensor. When the over range pressure is applied, the diaphragm contacts the planes which face the narrow gaps and is protected from fracturing. In this work, the differential pressure sensor is designed with an over-range protection structure. The sensor has two diaphragms with

piezoresistors working complementarily to each other due to the respective pressures. The sensor design is optimized for high output with improved sensor linearity, and reduced errors caused by changes in ambient temperature and static pressure.

Uncertainty analysis is a technique by which one can determine, with good approximation, whether a system will work within raw specification limits when the parameters vary between their limits. If the influencing variables are uncertain, a direct consequence is that the response parameters are uncertain as well. Interval Analysis is a technique used to estimate the bounds on various model outputs based on the bounds of the model inputs and parameters. Every uncertain parameter can be described by an interval [4, 5, and 6]. In this work, Interval Analysis has been applied to micro differential pressure sensor

II. MODELING AND DESIGN OF A MICRO DIFFERENTIAL PRESSURE SENSOR

a. Analytical Modeling

The piezoresistive pressure sensor (Figure 1) measures the applied pressure on one side of the diaphragm. The stress change in the diaphragm causes the resistance change of the piezoresistor.

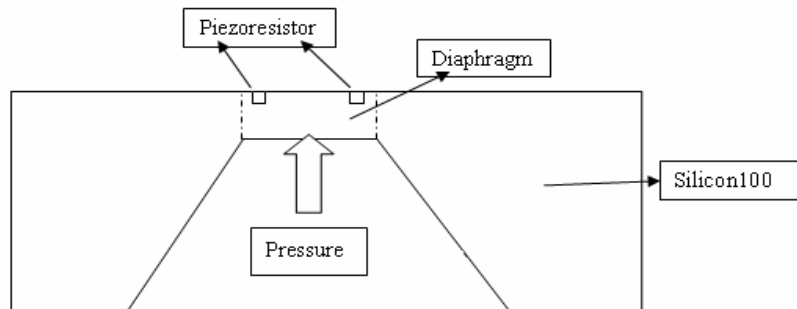


Figure 1. A Typical structure of piezoresistive pressure sensor

The deflection of a uniformly loaded square diaphragm with clamped edge is given by

$$Y_0 = 0.0141 \times \left(\frac{p \times a^4}{E \times h^3} \right) \times (1 - \nu^2) \quad (1)$$

where Y_0 is the deflection at center of the diaphragm,

p is the applied pressure,

a is the dimension of side of the square diaphragm,
 E is the modulus of elasticity of diaphragm material,
 h is the thickness of the diaphragm, and
 ν is the Poisson's ratio.

The maximum longitudinal stress σ_l is at the edge and it is given by

$$\sigma_l = \left(\frac{3}{16}\right) \times \frac{p \times a^2}{h^2} \quad (2)$$

The maximum tangential stress σ_t is at the center and it is given by

$$\sigma_t = \left(\frac{3}{32}\right) \times (1 + \nu) \times \left(\frac{p \times a^2}{h^2}\right) \quad (3)$$

Figure 2 shows the Wheatstone bridge arrangement of piezoresistors. The longitudinal stress σ_l and transverse stress σ_t is experienced by the piezoresistors R1 and R3 respectively. Then, the piezoresistors R2 and R4 experience longitudinal stress σ_t and transverse stress σ_l which are rotated 90° compared with the stresses experienced by R1 and R3.

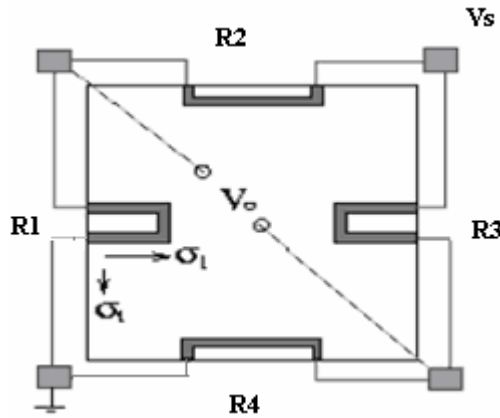


Figure 2. Wheatstone bridge of piezoresistive pressure sensor

The piezoresistive coefficients π_l and π_t are given by

$$\pi_l = \frac{\pi_{11} + \pi_{12} + \pi_{44}}{2} \quad \& \quad \pi_t = \frac{\pi_{11} + \pi_{12} - \pi_{44}}{2} \quad (4)$$

In a cubic semiconductor, the matrix of piezoresistive coefficients contains only three independent values, conventionally labeled as π_{11} , π_{12} , and π_{44} . For a diffused resistor subjected to

longitudinal and transverse stress components σ_l and σ_t , respectively, the resistance change is given by

$$\frac{\Delta R}{R} = \pi_l \sigma_l + \pi_t \sigma_t \quad (5)$$

σ_t can be related with σ_l via $\sigma_t = \nu \sigma_l$ where ν is the Poisson ratio. The differential output voltage V_0 of an ideally balanced bridge with assumed identical (but opposite in sign) resistance change, ΔR , in response to an applied pressure P, on the sensor is given by

$$V_0 = \left(\frac{\Delta R}{R} \right) \times V_s \quad (6)$$

The differential pressure can be the difference between pressures between the two diaphragms and the differential voltage can be the difference between the voltage experienced by the two signal conditioning circuits i.e. the Wheatstone bridge circuits.

The analytical simulation is carried out in MATLAB. Table 1 shows the analytical simulation results of differential pressure sensor.

b. Numerical Modeling

The numerical design is done using the FEM design tool Coventorware. Silicon<100> of thickness 400 μm is deposited and anisotropically etched backside to get the diaphragm1 of thickness 10 μm . The piezoresistors are ion implanted on the diaphragm for transduction. The silicon of 2 mm thickness is deposited and etched to get high range over protection layer. Again Silicon <100> of thickness 400 μm is deposited and anisotropically etched front side to get the diaphragm2 of thickness 10 μm .

Table 1: Analytical Results of Micro Differential Pressure Sensor

Pressure applied to diaphragm1, P1 (kPa)	Pressure applied to diaphragm2, P2 (kPa)	Diaphragm1 displacement, Y1 (μm)	Diaphragm2 displacement, Y2 (μm)	Differential Voltage, Vo (mV)
10	20	62.463	124.926	6.8987
20	40	124.926	249.852	13.7973
30	60	187.389	374.778	20.6960
40	80	249.852	499.704	27.5947
50	100	312.315	624.630	34.4933

The square diaphragm is of size $(500 \times 500 \times 10) \mu\text{m}^3$. The wafer thickness of $400 \mu\text{m}$ and sidewall angle of -35.3° is considered. The piezoresistors are of size $(70 \times 8 \times 1) \mu\text{m}^3$. The diaphragm structure for differential pressure is created by specifying process step in the process editor with the mask layout. The model is meshed with mapped mesh of linear element size 50. Architect is a module in MEMS design and simulation software CoventorWare. It is used to find the optimized location of Piezoresistors on the diaphragms.

The sensor has two diaphragms which work complementarily to each other by the applied differential pressure [3]. The output of Piezoresistors in Wheatstone bridge configuration is conditioned using operational amplifiers and the same is being simulated in Architect, which is a module in Coventorware. The stress results got from MemMech analysis from designer is carried to architect module for signal conditioning. Figure 3 shows the 3D model of the sensor. Figure 4 shows the Schematic in Architect. Figure 5 shows the diaphragm deflection after numerical simulation. Table 2 shows the numerical results of micro differential pressure sensor.

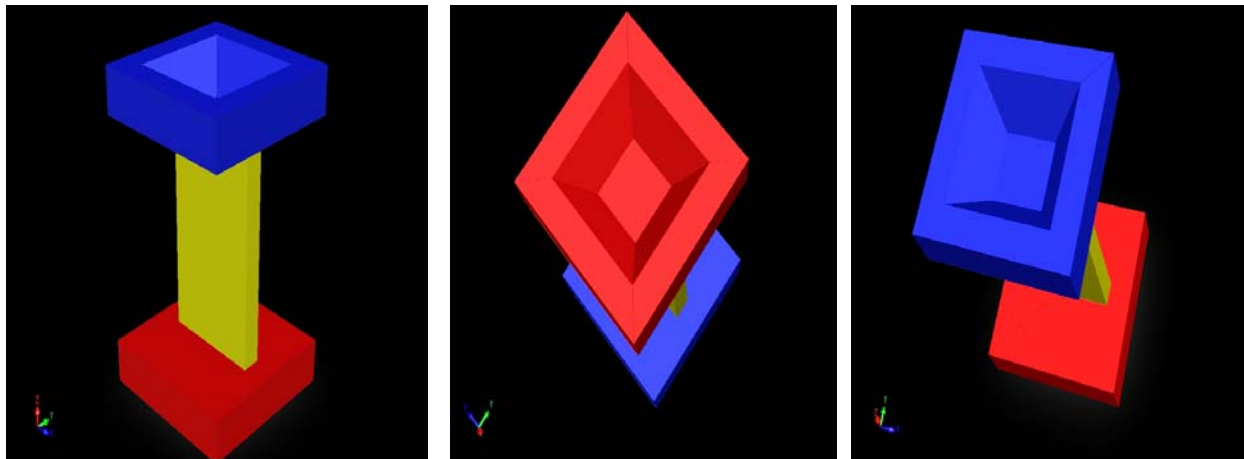


Figure 3. Topview and Bottom view of Differential Pressure Sensor

Table 2: Numerical Results of Micro Differential Pressure Sensor

Pressure applied to diaphragm1, P1 (kPa)	Pressure applied to diaphragm2, P2 (kPa)	Diaphragm1 displacement, Y1 (μm)	Diaphragm2 displacement, Y2 (μm)
10	20	65.202	130.303
20	40	130.303	230.179
30	60	195.298	389.575
40	80	260.179	518.442
50	100	324.940	646.733

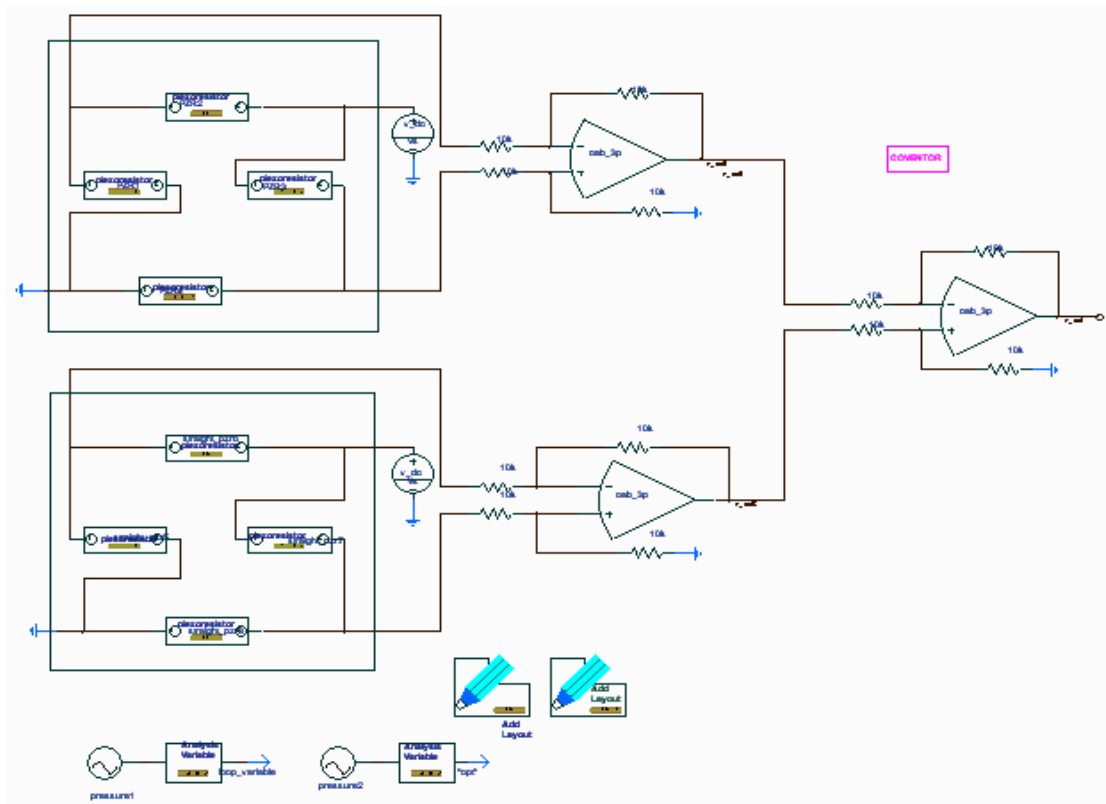


Figure 4. Schematic in Architect

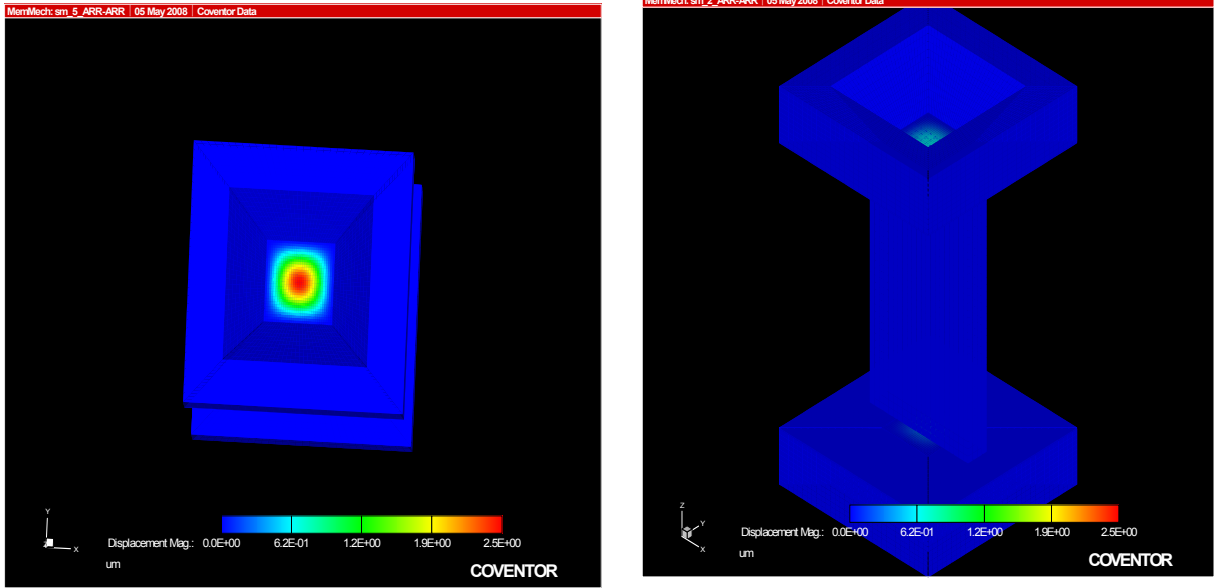


Figure 5. Diaphragm Deflection after simulation in Coventorware

III. DESIGN UNDER UNCERTAINTY

The deflection of a uniformly loaded square diaphragm with clamped edge under uncertainty is given by

$$\Delta Y_0 = 0.0141 \left(\frac{p \times \Delta a^4}{\Delta E \times \Delta h^3} \right) (1 - \nu^2) \quad (7)$$

The maximum longitudinal stress $\Delta \sigma_l$ is at the edge and under uncertainty it is given by

$$\Delta \sigma_l = \left(\frac{3}{16} \right) \times \frac{p \times \Delta a^2}{\Delta h^2} \quad (8)$$

The maximum tangential stress $\Delta \sigma_t$ is at the center and under uncertainty it is given by

$$\Delta \sigma_t = \left(\frac{3}{32} \right) (1 + \nu) \left(\frac{p \times \Delta a^2}{\Delta h^2} \right) \quad (9)$$

where ΔY_0 is the deflection at center of the diaphragm under uncertainty, P is the applied pressure under uncertainty, Δa is the dimension of side of the square diaphragm under uncertainty, ΔE is the modulus of elasticity of diaphragm material under uncertainty, Δh is the thickness of the diaphragm under uncertainty, and ν is the Poisson's ratio.

Interval Analysis is a technique used to estimate the bounds on various model outputs based on the bounds of the model inputs and parameters. In the interval method approach, uncertain parameters are assumed to be unknown but bounded and each of them has upper and lower limits without a probabilistic structure. Every uncertain parameter is described by an interval [6]. An interval is a close set in R , which included the possible range of a number. In this paper, an interval will be represented by the ordered pair $[a, b] = \{x: a \leq x \leq b\}$ where 'a' is the lower limit of the interval and 'b' is the upper limit of the interval and 'a' and 'b' are real numbers. The number is known to lie between values but the exact value is unknown. Interval arithmetic is an elegant tool for practical work with inequalities, approximate numbers, error bounds, and more generally with certain convex and bounded sets.

Let $x = [a, b]$ and $y = [c, d]$ be two interval numbers, a and c are lower limits, b and d are upper limits and a, b, c, d are real.

1. Addition: $x + y = [a, b] + [c, d] = [a + c, b + d]$
2. Subtraction: $x - y = [a, b] - [c, d] = [a - d, b - c]$
3. Multiplication: $xy = [\min(ac, ad, bc, bd), \max(ac, ad, bc, bd)]$
4. Division: $1/x = [1/b, 1/a]$

When the pressure of 10 kPa is applied to diaphragm1 (which is at top) and pressure of 20 kPa is applied to diaphragm2 (which is at bottom), the analytical differential output voltage with nominal values for all the parameters is 6.8986v. When the same analysis is carried out for uncertainties in the range of 5% to the parameters like diaphragm thickness, diaphragm side and Modulus of elasticity, the differential output voltage is [1.6455, 12.4292] v. On the similar line, considering a single dimensional parameter for example thickness, the sensor is analyzed with $\pm 5\%$ of the nominal value of the thickness and the same procedure is repeated for the diaphragm side and Modulus of elasticity, considering each parameter individually at a time.

For carrying out uncertainty analysis numerically, the vary analysis is performed in Coventorware, with all the dimensional parameters and material properties 5% less than the nominal value and 5% more than the nominal values. The differential output voltage is calculated analytically by inputting all inputs in interval using INTLAB. INTLAB is a tool for calculating interval arithmetic's using MATLAB. Table 3 shows the dimensions of diaphragm. Table 4 shows the diaphragms displacement, y_1 , y_2 , differential voltage v. The pressure range of (0-50) kPa is applied to diaphragm1 and (0-100) kPa is applied to diaphragm2. Figure 6 shows the Input

Output graph (a) for nominal values of all the parameters, (b) when all the parameters varied by 5% (c) when only diaphragm side varied by 5% (d) when only Youngs Modulus varied by 5% (e) when only diaphragm thickness varied by 5%, when (0-50) & (0-100) kPa applied to diaphragm1&2 respectively. The results obtained numerically are tabulated in Table 5.

Table 3: Dimensions of the Diaphragm

Parameters	Nominal Value
Square Diaphragm side (um)	500
Thickness of Diaphragm (um)	10
Modulus of Elasticity of Diaphragm (MPa)	1.3018e5
Poisson Ratio	0.278

Table 4: Results of Pressure Sensor under Uncertainty

Parameter	Parameter Interval	Analytical Simulation			
		Y1 (um)	Y2 (um)	V (v)	Sensitivity (mv/v/10kPa)
Nominal value	-	62.463	124.926	6.8986	0.6868
Interval to all the parameters	5%	[41.85, 93.21]	[83.712, 86.430]	[1.645, 12.429]	[0.164, 1.242]
Diaphragm Thickness, h (µm)	[9.5, 10.5]	[53.95, 72.85]	[107.91, 145.70]	[4.261, 9.639]	[0.426, 0.963]
Diaphragm side, a (µm)	[475, 525]	[48.45, 79.92]	[96.907, 159.84]	[4.240, 9.591]	[0.424, 0.959]
Modulus of Elasticity, E (MPa)	[123671, 136689]	[59.48, 65.75]	[118.97, 131.50]	[6.898, 6.898]	[0.686, 0.686]

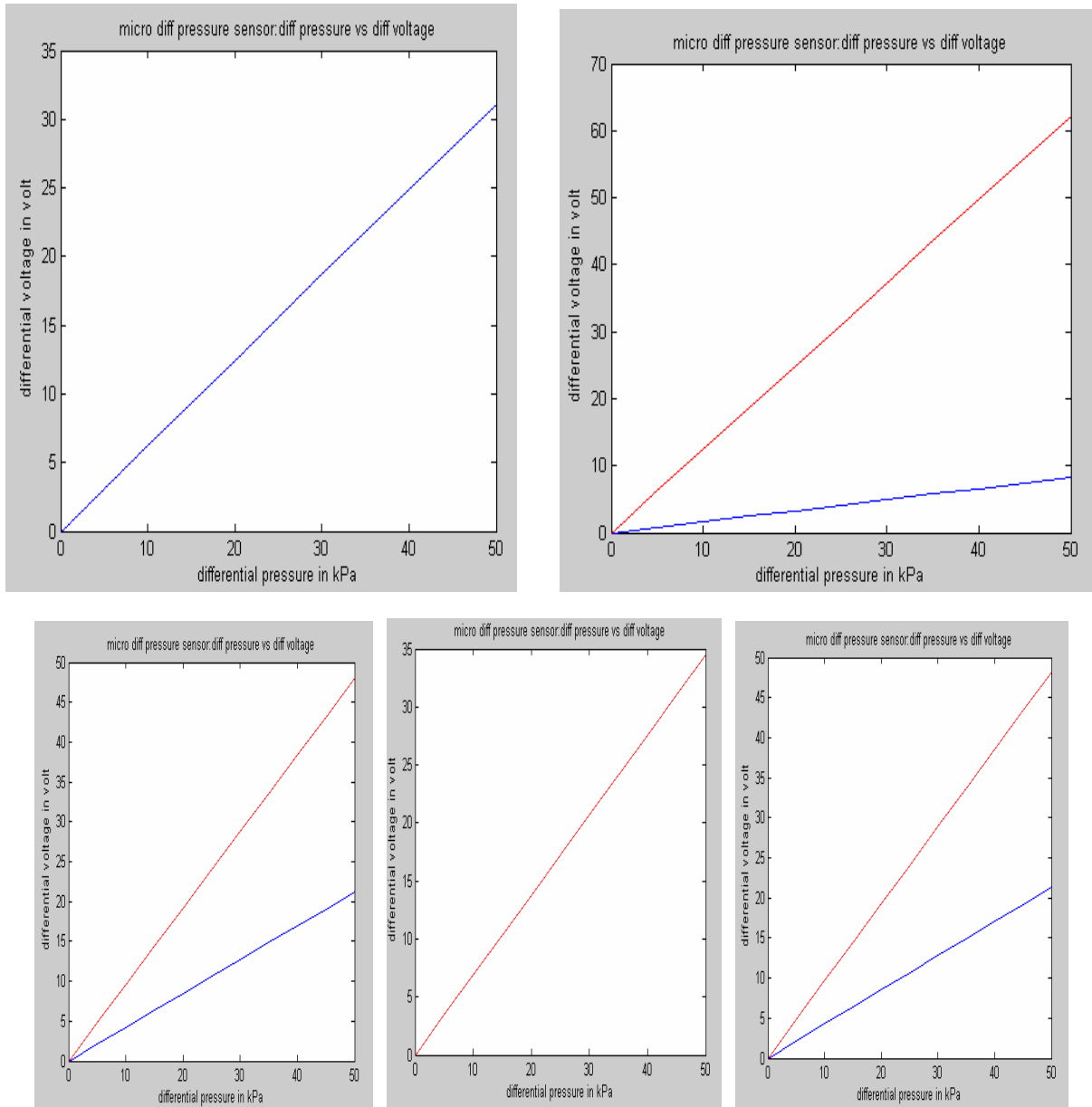


Figure 6. Input Output graph (a) for nominal values of the parameters, (b) when all the parameters varied by 5% (c) when only diaphragm side varied by 5% (d) when only young's modulus varied by 5% (e) when only diaphragm thickness varied by 5%

Table 5: Results of Pressure Sensor under Uncertainty

Parameters	Parameter Interval	Numerical Simulation	
		Y1 (μm)	Y2 (μm)
Nominal value	-	65.202	130.303
Interval to all the parameters	5%	[63.773, 66.059]	[127.446, 132.018]
Diaphragm Thickness, $h(\mu\text{m})$	[9.5, 10.5]	[75.2196, 56.5011]	[150.304, 112.934]
Diaphragm side, $a(\mu\text{m})$	[475, 525]	[53.325, 78.926]	[106.580, 157.780]
Modulus of Elasticity, $E(\text{MPa})$	[123671, 136689]	[67.6304, 62.9466]	[135.153, 125.799]

IV. CONCLUSION

This paper presented a methodology for analysis of the effects of parameter uncertainty on the variability of functional properties of micro pressure sensor. The uncertainty analysis for piezoresistive micro differential pressure sensor is carried out numerically using Coventorware and analytically using Intlab. The results indicate that the sensor output is most sensitive to diaphragm side and thickness and least sensitive to Modulus of elasticity. From the results, the tolerance given to diaphragm side and thickness need to be tightly controlled. A powerful method called Interval analysis could be implemented for any micro devices to analyze their uncertainty and reliability.

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