DETERMINING THE STEP-CHANGE CONDUCTIVITY PROFILES WITHIN LAYERED METAL STRUCTURES USING INDUCTANCE SPECTROSCOPY

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Abstract – This paper presents an inverse method for determining the conductivity distribution of a flat, layered conductor using a multi-frequency electromagnetic sensor based on phase signature alone. Eddy current sensors are used in a wide range of non-destructive testing (NDT) applications. Single frequency sensors are very common, however, the potential of an eddy current sensor with spectroscopic techniques offer the ability to extract depth profiles and examine more fully the internal structure of the test piece. In this paper, we found a simplified model that can estimate the phase signature of a cylindrical coil above a conductor with an arbitrary conductivity profile. This simplified model improves the computational efficiency by many fold compared to the complete analytic solution. For inverse solution, a simplex search method was used to fit a set of multi-frequency phase values in a least-squared sense. Experimental eddy-current tests are performed by taking the difference in inductance of the coil when placed in free space and next to a layered conductor over the range 100Hz -1MHz. Good estimates for the conductivity profile from experimental and simulated data were obtained.

Index terms: Multi-frequency electromagnetic sensor, simplified model, conductivity distribution, layered conductor

I. INTRODUCTION

Determining the conductivity profile of a conductor is important in a range of technological applications such as coating, surface treatment and quality inspection. One approach is to measure the direct current (DC) impedance map and estimate the conductivity [1-2]. Another approach is eddy current inspection, which infers the conductivity profile from impedance (or inductance) data [3-9]. The principle of these inverse methods [5-9] is to fit the inductance data (measured or simulated) to a complete analytical solution using simplex search. Therefore, these methods limit their reconstructed results to certain profiles. For example, only monotonic profile of a conductor and the conductivity of single piecewise constant surface layer on metal plates have been measured in this way. Yin et al [10] developed an inverse method which can in principle reconstruct arbitrary continuous conductivity profile from inductance spectroscopic measurements. However, the regularization method used in [10] tends to smooth the abrupt changes in conductivity, making it unsuitable for imaging step-change conductivity profiles. In addition, since a complex analytical model was used, the computation process is extremely slow even for a moderate number of layers, which limits its use for imaging complex conductivity profiles. To solve this problem, this paper presents an inverse method that uses the phase information alone and develops a very fast forward model, which allows a fast inverse method to be realized by which the profile that produces the closest phase signature to the measurements can be quickly identified. In addition, the phase signature is strongly dependent on the electromagnetic property of the metal structure and virtually immune to lift-off variations. Traditional eddy current methods relying on amplitude information can suffer from undesirable effects due to variations in the distance (lift-off) between the sensor and the test piece in field operation. Lift-off effect of an eddy current sensor above a metallic plate was discussed in [11-12].

There are two major computational problems in the reconstruction process: the forward problem and the inverse problem. The forward problem is to calculate the frequency-dependent measurements for a layered conductor with an arbitrary conductive profile. The inverse problem is to determine the conductive profile from the frequency-dependent measurements.

II. METHODS

This section describes the method to obtain the conductivity profile. First, the complete forward model is presented. Then, the simplified model is deduced. Further, the inverse problem is solved. Finally, the numerical and experimental results are given to verify our method.

A. The complete forward model

The analytical solution for the inductance of a right-cylindrical air-cored coil placed above a finite number of layers with constant conductivity and permeability in each layer has been given by Cheng [13]. In this paper, only non-magnetic layered conductors will be dealt with, therefore, the relative permeability for all layers are assumed to be that of the free space, i.e. 1. Figure 1 shows the schematic diagram of the model. The base of the coil is at a height of h_1 above the surface and the top of the coil is at h_2. The coil parameters of importance are number of turns N, inner and outer radii r_1 and r_2 and coil length $L= h_2- h_1$. Note that the space below layer 1 is free space. In the forward model, Layer 0 is treated as a layer with infinite thickness. In practice, the inductance change induced by the layered conductor (1a) is compared with free space (1b). This arrangement reduces common measurement errors and facilitates subsequent reconstruction algorithm.

Cheng [13] considered a similar geometry without subtracting the coil impedance in free space. He first determined the impedance of a single turn delta-function filament by solving corresponding differential equations, and then derived the impedance for a right-cylindrical air-cored coil by superstition, assuming that the current density is uniform over the cross-section of the coil. In this paper, the results of Cheng are presented in a slightly different form to express the coil inductance difference for cases (1) the layered conductor and (2) free space.

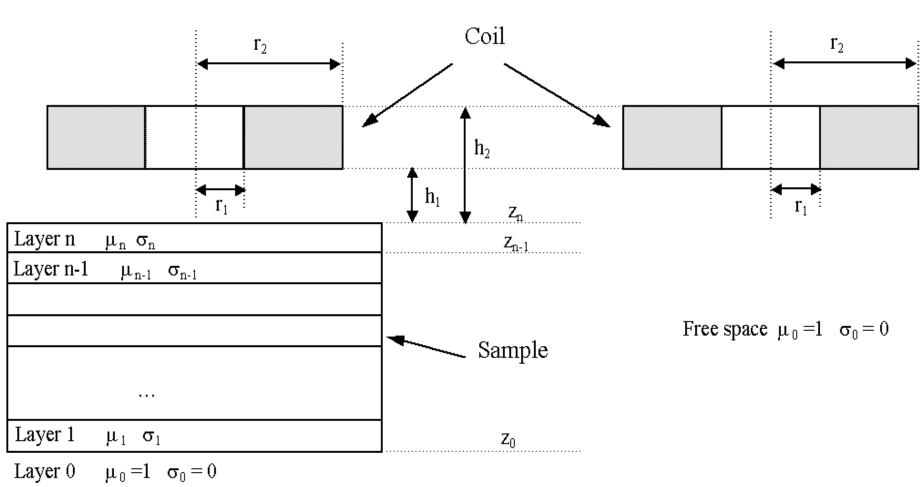


Figure 1. Schematic diagram of the model (a) the sensor placed next to the layered conductor (b) the sensor placed in free space

$$\Delta L(\omega) = K\int_0^\infty \frac{P^2(\alpha)}{\alpha^6} A(\alpha)\phi(\alpha)d\alpha \tag{1}$$

where,
$$\phi(\alpha) = \frac{\mathbf{U}_{12}}{\mathbf{U}_{22}} \tag{2}$$

$$\mathbf{U} = \mathbf{H}_n \cdot \mathbf{H}_{n-1} \cdot \mathbf{H}_{n-2} \cdot \ldots \mathbf{H}_0 \tag{3}$$

$$\mathbf{H}_k = \frac{1}{2}\begin{bmatrix} (1+\frac{\mu_{k+1}\alpha_k}{\mu_k\alpha_{k+1}})e^{(\alpha_{k+1}-\alpha_k)\cdot z_k}, & (1-\frac{\mu_{k+1}\alpha_k}{\mu_k\alpha_{k+1}})e^{(\alpha_{k+1}+\alpha_k)\cdot z_k} \\ (1-\frac{\mu_{k+1}\alpha_k}{\mu_k\alpha_{k+1}})e^{(-\alpha_{k+1}-\alpha_k)\cdot z_k}, & (1+\frac{\mu_{k+1}\alpha_k}{\mu_k\alpha_{k+1}})e^{(\alpha_k-\alpha_{k+1})\cdot z_k} \end{bmatrix} \tag{4}$$

$$A(\alpha) = (e^{-\alpha h_1} - e^{-\alpha h_2})^2 \tag{5}$$

$$P(\alpha) = \int_{\alpha r_1}^{\alpha r_2} xJ_1(x)dx \tag{6}$$

$$K = \frac{\pi\mu_0 N^2}{(h_1 - h_2)^2(r_1 - r_2)^2} \tag{7}$$

$$\alpha_k = \sqrt{\alpha^2 + j\omega\sigma_k\mu_k} \tag{8}$$

where α is a spatial frequency variable; **U** and **H** are transfer matrices; K is a pre-factor and $J_1(x)$ is a first-order Bessel function of the first kind. The interface between layers k and k+1 occurs at a depth z_k. Also μ_k and σ_k denotes the permeability and conductivity of layer k. We number the layers starting from the base of the material; that is base material is layer number 0. There are a total of n+1 layers. For a layered conductor with smoothly-varying and continuous conductivity profile, the continuous profile can be replaced by a piecewise continuous approximation consisting of n layers of constant conductivity and permeability, which is determined from the values for the continuous profile in the middle of that layer. As n increases, the inductance calculated using this method converges to the real solution [14]. In this paper, equations (1-8) can be used directly since we only deal with conductors with a finite number of layers with constant conductivity and permeability

B. The simplified model

Equations (1-8) are an accurate description of the forward problem. However, to evaluate them in practice involves integration over a large zone and the computation process is slow. Therefore, a simplified model is sought to increase the speed of the computation process. The strategy is to separate the magnitude and phase response, only the latter of which is believed to be strongly dependent on the conductivity distribution for a given coil.

To identify the simplified model, firstly, two limiting cases ω=0 and ∞ are considered. Setting ω=0 gives the inductance change for zero-frequency. The real part and imaginary part of the inductance change are zero, which means the non-magnetic conductor causes no inductance change and the magnetic flux penetrates the plate as in free space. In the limit of arbitrarily large frequency, the inductance change is given by $\Delta L = -\Delta L_0$, where $\Delta L_0 = K\int \frac{P^2(\alpha)}{\alpha^6} A(\alpha) d\alpha$. ΔL_0 is dependent on lift-off for a given coil, but is independent of the conductivity distribution, which corresponds to the situation that the incident magnetic flux is totally excluded from the plate.

The simplification of the complete model is to evaluate the phase term $\phi(\alpha)$ at α_0 and take it outside of the integral.

$$\Delta L(\omega) = \phi(\alpha_0)\Delta L_0 \qquad (9)$$

This operation originates from the fact that $\phi(\alpha)$ varies slowly with α compared to the rest of the integrand, which reaches its maximum at a characteristic spatial frequency α_0. α_0 is defined to be one over the smallest dimension of the coil.

Note that the phase term $\phi(\alpha_0)$ solely depends on the conductivity profile of the conductor, and totally accounts for the frequency-dependent phase signature. ΔL_0 contributes to the strength of the signal, but is not related to the phase components. Therefore, $\phi(\alpha_0)$ can be used to approximate the phase signature of a coil instead of equations (1-8).

C. The inverse problem

The inverse problem in this case is to determine the conductive profile from the frequency-dependent phase measurements (phase signature). A simplex search method is used to find the conductivity profile to fit phase values (measured or simulated) in a least-squared sense.

Definition of the problem:

1) $p_0 \in \mathbf{R}^m$ are the observed phases arranged in a vector form and m is the number of frequencies at which the phase measurements are taken (i.e. the number of phases observed)

2) $\boldsymbol{\sigma} \in \mathbf{R}^{2n}$ is a conductivity distribution described by two sets of parameters: the conductivities $(\sigma_1, \sigma_2, \ldots \sigma_n)$, and the boundary coordinates $(z_1, z_2, \ldots z_n)$. The base layer, i.e. layer 0 is set to free space.

3) $\mathbf{f} : \mathbf{R}^{2n} \rightarrow \mathbf{R}^m$ is a function mapping a conductivity distribution into a set of m approximate phase observations.

4) $\phi = \frac{1}{2}(\mathbf{f} - p_0)^T(\mathbf{f} - p_0)$ represents the squared error of the measured and estimated phases.

Note that $\mathbf{f}$ is a function of conductivity distribution $\boldsymbol{\sigma}$ under fixed measurement arrangements. The problem is to find a point $\boldsymbol{\sigma}^*$ that is a global minimum of ϕ. Since a simplified model, which is easy and fast to evaluate, has been obtained, a simplex search method can be used in the inverse solution.

The eddy current signal can be interpreted as a complicated extended spatial integral transform of the conductivity profile that is filtering away the information of quickly changing profile features in the signal, the step-change profile features is weakly embedded and normally difficult to reconstruct [15]. We will consider two kinds of problems. First, we consider the case when the boundary coordinates are known, but conductivities are to be determined. Second, we consider the case when the conductivities σ_i are given and the boundary coordinates z_i are to be determined. We limit the number of layers to 3, which is adequate for most practical applications such as coating thickness gauging and coating adhesion monitoring.

III. NUMERICAL AND EXPERIMENTAL RESULTS

In numerical simulations and experiments, the following coil geometry parameters were used:

Parameter	Value
Number of turns N	10
Inner diameter r_1	20mm
Outer diameter r_2	20.1mm
Coil length $L = h_2 - h_1$	2 mm = 2.2mm-0.2mm

Table 1. Coil geometry and model parameters

Figure 2 shows the differential inductance (subtracted from free space) of a layered conductor whose conductivity profile is shown in figure 3. The difference between the calculated data and the measured data is due to imperfect modelling of the coil, one factor being the undesirable capacitance of the coil.

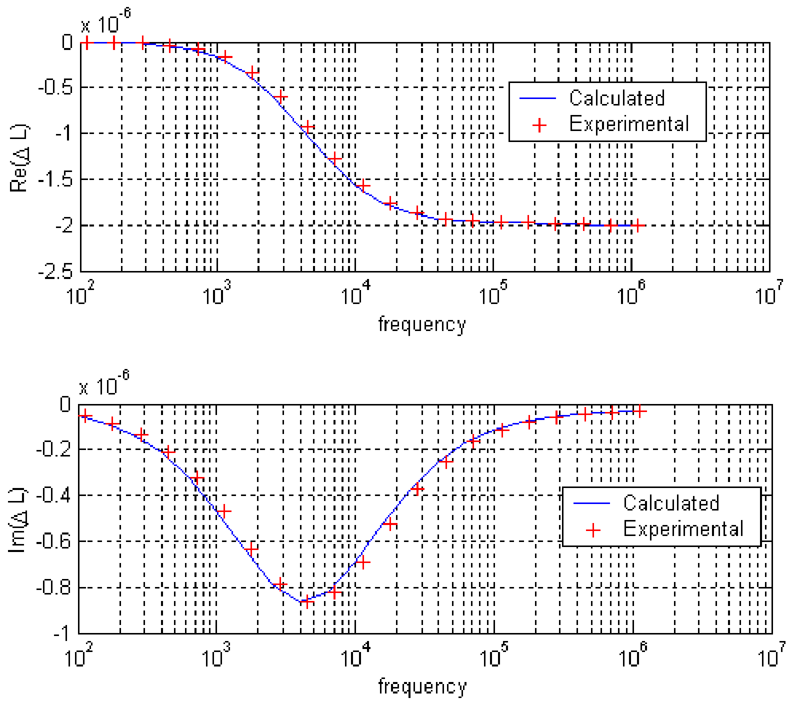


Figure 2. The real and imaginary parts of the inductance change for the sample whose conductivity profile is shown in Figure 4 (= layered – free space)

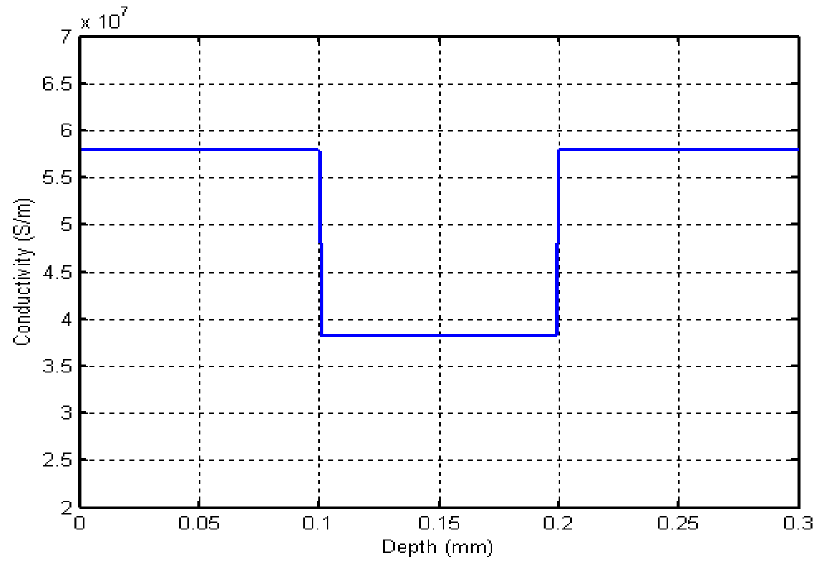
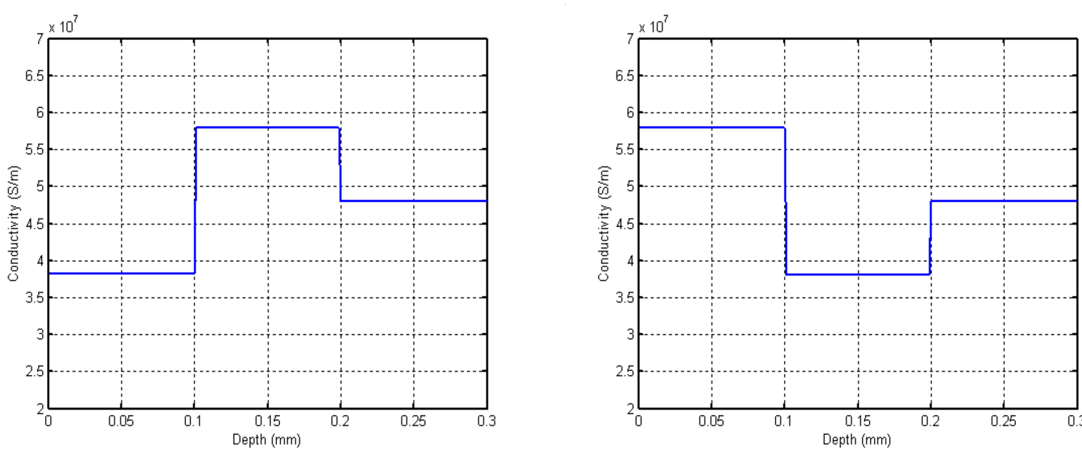
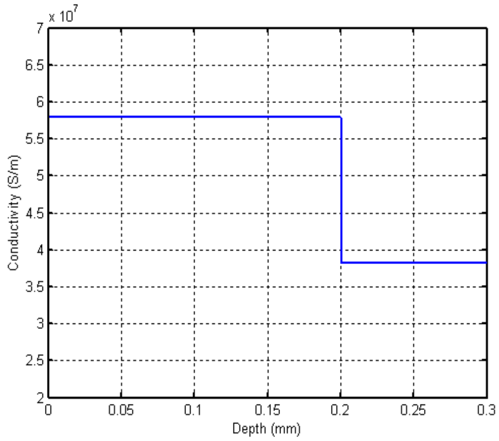


Figure 3. Nominal conductivity profile for a sample prepared by stacking a series of thin flat foils

A. Inversions of simulated data

The utility of the inverse method was tested firstly using the inductance of samples, which was obtained through simulation by solving the complete forward problem numerically. The profiles used in simulations were chosen as monotonic and non-monotonic ones. Figure 4 shows four profiles for simulation data.





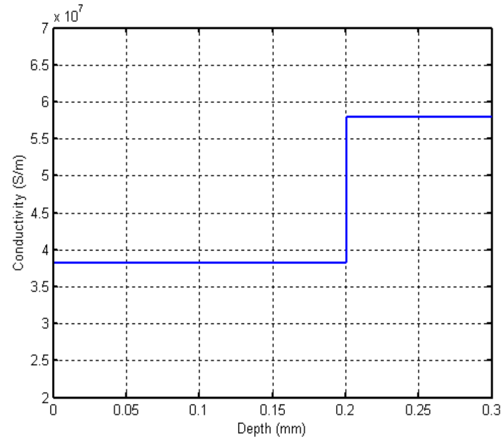


Fig. 4. Simulated conductivity profiles

First, the boundary coordinates are determined based on known conductivities. The results are shown in Table 2.

Table 2. Estimated and actual layer coordinates

	Profile 1		Profile 2		Profile 3		Profile 4	
	Actual	Estimated	Actual	Estimated	Actual	Estimated	Actual	Estimated
z1	0.1mm	0.095mm	0.1mm	0.097	-		-	
z2	0.2mm	0.208mm	0.2mm	0.209	0.2mm	0.214	0.2mm	0.206
z3	0.3mm	0.312mm	0.3mm	0.315	0.3mm	0.320	0.3mm	0.319

Second, the conductivities are determined based on known boundary coordinates. The results are shown in Table 3.

Table 3. Estimated and actual conductivities

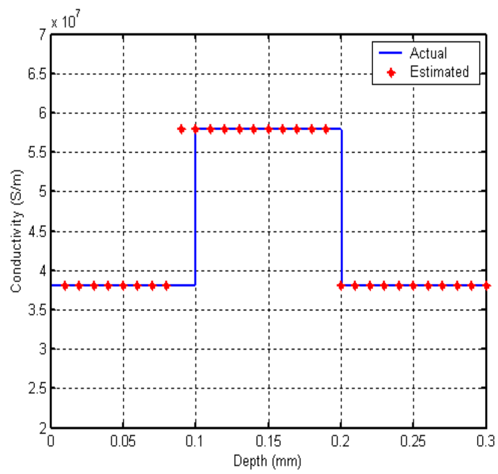
	Profile 1		Profile 2		Profile 3		Profile 4	
	Actual (10^7S/m)	Estimated (10^7S/m)	Actual (10^7S/m)	Estimated (10^7S/m)	Actual (10^7S/m)	Estimated (10^7S/m)	Actual (10^7S/m)	Estimated (10^7S/m)
σ 1	3.8	3.74	5.8	5.73	5.8	5.76	3.8	3.82
σ 2	4.8	5.62	3.8	4.01	5.8	5.69	3.8	3.90
σ 3	5.8	4.87	4.8	4.67	3.8	3.91	5.8	5.68

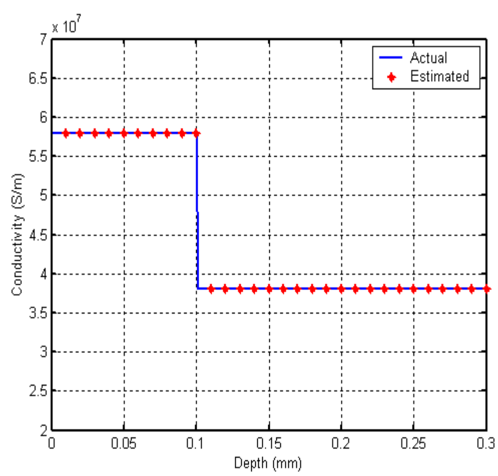
The estimated conductivities and boundary coordinates agree reasonably well with the actual ones for all cases. This confirms the potential of this method of being able to reconstruct more complex conductivity profiles than monotonic ones.

The errors between the actual and the estimated boundary coordinates are mainly due to two factors: 1, the approximate nature of the simplified forward model; 2, the eddy current phenomenon, i.e. eddy currents decay quickly from the surface to the inside of a metal piece.

B. Inversions of measured data

As an experimental verification of this inverse method, samples with step conductivity profiles were prepared by stacking a series of thin flat foils. Since eddy currents flow parallel to the surface, the effects of non-perfect electrical between foil layers were thought to be negligible. Measurements were made using an impedance analyser (SL1260) over the frequency range 100Hz -1MHz in swept-frequency mode. The inductance change was obtained by subtracting the inductance of the layered conductor from the air at each frequency. The reconstructed profiles based on measurements are shown in Figure 5. The nominal conductivity values were used for the copper and aluminium foils.





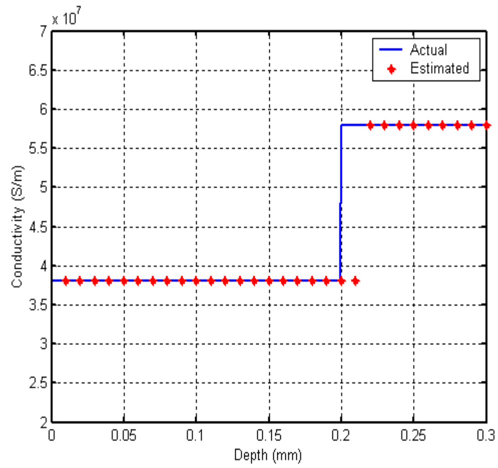


Figure 5. Actual and reconstructed profiles by measurements

IV. CONCLUSIONS

In this paper, a method is presented which has the potential to reconstruct complex step-change conductivity profile for a flat non-magnetic conductor from inductance spectroscopic measurements. A simplified model was found by approximating the complete forward analytic solution. In inverse solution, a simplex search method was used to find the conductivity profile to fit phase signatures (measured or simulated) in a least-squared sense. Conductivity profiles have been reconstructed from simulated and measured data, which verified this method.

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