# GUIDANCE-BASED ON-LINE MOTION PLANNING FOR AUTONOMOUS HIGHWAY OVERTAKING 

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#### Abstract

In the context of intelligent transportation, this paper presents a novel on-line trajectorygeneration method for autonomous lane changing. The proposed scheme is guidance based, realtime applicable, and ensures safety and passenger ride comfort. Based on the principles of Rendezvous Guidance, the passing vehicle is guided in real-time to match the position and velocity of a shadow target (i.e., rendezvous with) during the overtaking manoeuvre. The shadow target's position and velocity are generated based on real-time sensory information gathered about the slower vehicle ahead of the passing vehicle as well as other vehicles which may be travelling in the passing lane. Namely, the guidance principle is also used to prevent any potential collision with these obstacle vehicles. The proposed method can be used as a fully autonomous system or simply as a driver-assistance tool. Extensive simulations and experiments, some of which are presented herein, clearly demonstrate the tangible efficiency of the proposed method.


Keywords: Intelligent Transportation, Autonomous Vehicle Overtaking, Collision Avoidance

## I. INTRODUCTION

Intelligent transportation systems have been widely researched in the past two decades by the academic community as well as automotive manufacturers for increased safety, passenger comfort, traffic congestion, etc. [1]. Although manufacturers have concentrated their efforts on developing technologies to help drivers, academic interest on the subject matter has primarily been on the autonomy of driving. In this context, our focus in this paper is specifically on the autonomy of the lane-changing manoeuvre: An effective on-line time-optimal motion-planning method is presented herein for the safe and comfortable overtaking of a slow-moving vehicle travelling on a two-lane highway.

The overtaking manoeuvre is one of the critical actions that a driver performs while travelling on a highway. Errors in this decision-making process, typically caused by driver failure to accurately and timely interpret information about other vehicles in close proximity, have often resulted in catastrophic accidents [2]. In order to eliminate such errors, or at least minimize their impact, and increase the level of safety, the vehicles of the future would have to incorporate intelligent algorithms that will allow them to accurately consider all aspects of a lanechanging/overtaking manoeuvre. A number of real-time issues would need to be addressed; (i) calculating proximities to other vehicles, (ii) determining when the lane-change manoeuvre should start, and (iii) developing optimal and safe trajectories. The last two issues are addressed in this paper.

Majority of autonomous-driving research has been on lane following, as part of promoting driver-assistance systems, (e.g., [3-12]). Limited research, however, has been carried out on lane changing, though, primarily proposing non-real-time solutions that are commonly based on lanefollowing approaches (e.g., [13-21]). Since, these systems have not been primarily designed for lane changing, or vehicle overtaking, they usually yield non-smooth lane transitions. One may furthermore note that the few papers that have addressed the smoothness issue have paid little attention to collision avoidance [22, 30].

A vehicle's acceleration (lateral, longitudinal, and vertical) and angular motion (roll, pitch, and yaw) directly contribute to ride comfort (or discomfort), which are often compared to set standard metrics. In this context, comfort disturbance has been classified as: (i) direct disturbance caused by a sudden motion of the vehicle, and (ii) indirect disturbance, commonly,
caused by high lateral accelerations and/or lateral jerks while negotiating transition curves [31]. The limits for lateral and axial acceleration, while negotiating a transition curve, have typically been set to $1.25 \mathrm{~m} / \mathrm{s}^{2}$ and $5 \mathrm{~m} / \mathrm{s}^{2}$, respectively, with a mean comfort rating of 2.5 [32, 33]. This paper is primarily concerned with the indirect type of comfort disturbance.

Another important factor in lane changing is the maintenance of a safe distance during the manoeuvre. Although studies on the calculation of a minimum safe distance, for a collision-free overtaking manoeuvre, have differed on their recommendations, they commonly assumed worstcase assumptions scenarios (e.g., [18, 34, and 35]). In the absence of information on the passing vehicle's performance ability (including braking capability) and road conditions, most studies recommend a (worst-case scenario) minimum safe (closing) distance based on 2 seconds of driving separation as an ideal value for preventing most accidents under emergency conditions.

An effective trajectory planner for lane-changing must address both of the abovementioned issues of comfort and safety. In achieving an optimal manoeuvre, the vehicle should, thus, be guided in a way that ensures minimum passing time and avoids any potential collisions. In this context, there exist three levels of vehicle guidance: geometric rule, guidance law, and vehicle control [36]. The first is simply a rule that one needs to obey to follow a target. The guidance law is the algorithm that implements the geometric rule. Vehicle control is concerned with the dynamics of the vehicle.

The utilization of (missile) guidance-based techniques in the on-line motion planning for autonomous robotic vehicles was first proposed by our research group in the late 1990s [37, 38]. One may note that, such methods have also been proposed specifically for autonomous undersea and aerial vehicles (e.g., [39, 40]). Missile-guidance techniques are, typically, classified into five main categories [36, 41]: Line-Of-Sight (LOS) guidance; Pure Pursuit (PP); Proportional Navigation Guidance (PNG); Optimal Guidance (OG); and, other guidance methods including the use of differential game theory. Missile-guidance laws assume that the future trajectory of the target is completely defined either analytically or by a probabilistic model [42-44].

The PNG law uses the homing triangle for computing the acceleration of an interceptor pursuing an evading target. The homing triangle is defined by the interceptor, the target, and the point of interception. This control law makes the interceptor's acceleration normal to its path and proportional to the rate of change of the LOS vector to the target. Due to its low computational
requirements, simplicity of on-board implementation, and time optimality characteristics, PNG has been the most widely used guidance technique [45].

The abovementioned methods provide interception of a target, i.e., positional matching. The need for velocity matching as well has resulted in a new class of guidance methods, commonly referred to as Rendezvous-Guidance (RG) methods. A PNG-based RG method for the docking problem of two space vehicles was proposed in [46]. In [47], the use of exponential-type guidance was suggested for asteroid rendezvous. The problem of rendezvous with an object capable of performing evasive manoeuvres in order to avoid rendezvous was addressed in [48]. The utilization of RG-based techniques in the on-line motion planning for autonomous robotic vehicles was also proposed by our research group (e.g., [49, 50]).

In conclusion to the above discussion, it can be noted here that this paper presents a novel time-optimal RG-based on-line trajectory (i.e., time-phased path) planning algorithm for the guidance of a pursuer vehicle overtaking a slower vehicle on a highway setting in the presence of other (obstacle) vehicles travelling in the passing lane.

## II. OPTIMAL OVERTAKING MANOEUVRE

## A. Problem Definition

Let us first consider the simplest highway overtaking scenario, namely, where a vehicle (hereafter referred to as the pursuer, $P$ ) is driving with a velocity $v_{p}$, while in front of it, another vehicle (hereafter referred to as the obstacle in the driving lane, $O_{D}$ ) is travelling with a slower velocity, $\boldsymbol{v}_{\boldsymbol{o d}}$, (i.e., $\boldsymbol{v}_{\boldsymbol{p}}>\boldsymbol{v}_{\boldsymbol{o d}}$ ). There exists no (obstacle) vehicle in the passing lane that would influence the overtaking manoeuvre, which can be performed by the pursuer in three phases: (i) move from the driving lane to the passing lane, (ii) travel in the passing lane and, thereafter, (iii) return to the driving lane.

In the more complex scenario, one could be forced to consider another (obstacle) vehicle in the passing lane, $O_{P}$, which would not allow the $P$ to immediately overtake $O_{D}$ due to safety considerations. In this case, an additional velocity-adjustment phase would need to be included: during this phase, $P$ would adjust its velocity according to the velocity of $O_{D}$ until the passing lane becomes free of obstacles, Figure 1.


Figure 1: The Overtaking Manoeuvre.

## B. Proposed Solution Methodology

The proposed on-line motion-planning method, determines a pursuer vehicle trajectory to perform an optimal overtaking manoeuvre based on the Rendezvous Guidance (RG) technique [50]. RG has been shown analytically to yield an optimal solution for rendezvous with nonmanoeuvring targets: which can be assumed to be the case for vehicles travelling on highways. However, there still exist two major issues that restrict the use of RG law in trajectory planning for our cases. First, RG is designed for matching velocity with a target and not to overtake it. Therefore, a target needs to be defined in our case. Second, there would be numerous constraints on the motion of a pursuer vehicle, which would not exist for spaceships rendezvous manoeuvres.

In order to address the first issue, we introduce herein the concept of a shadow target, $S$, which will be used to guide the pursuer, $P$, during all the phases of the overtaking manoeuvre. The location of $S$ is defined according to the obstacle vehicle, $O_{D}$, that is being overtaken. In order to address the second issue, this paper uses the RG method proposed in [46-50] for robotic (autonomous vehicle) interception: namely, information about $P, O_{D}$, and $O_{P}$ is used to generate a single acceleration command for the pursuer, $P$, to avoid $O_{P}$ and overtake $O_{D}$ in a time-optimal manner. This acceleration command is calculated based on velocity-matching capability with $S$ keeping in mind the constraints imposed due to pursuer vehicle dynamics and passenger comfort. The concept of shadow target is also utilized for obstacle avoidance in the passing lane to ensure a collision free path.

## Trajectory Based on RG Law

Let us consider a two-dimensional engagement geometry, in which $P$ and $S$ are moving at velocities $\boldsymbol{v}_{\boldsymbol{p}}$ and $\boldsymbol{v}_{\boldsymbol{s}}$, respectively. An imaginary line joining the pursuer vehicle and the target is referred to herein as the Line-of-Sight (LOS). The angle formed by the LOS with the fixed reference, $\lambda$, is defined by

$$
\begin{equation*}
\lambda=\tan ^{-1} \frac{h}{l}, \tag{1}
\end{equation*}
$$

where $h$ is the distance between $P$ and $S$ in the lateral direction and $l$ is distance in axial direction, the length of $L O S$ is defined as a range, $\boldsymbol{r}$, connecting $P$ to $S$.

The parallel-navigation law [36] states that the direction of LOS should remain constant relative to a non-rotating frame, while, the interceptor (pursuer) approaches the object (target). Namely, the relative velocity, $\dot{\boldsymbol{r}}$, between the pursuer and the target should remain parallel to the LOS, $\boldsymbol{r}$, at all the times. If this rule holds throughout the motion of the pursuer, the distance between the pursuer and the target would decrease until they collide.

The parallel-navigation law is expressed by the following two relationships

$$
\boldsymbol{r} \times \dot{\boldsymbol{r}}=0,
$$

and

$$
\begin{equation*}
\boldsymbol{r} \cdot \dot{\boldsymbol{r}}<0 . \tag{3}
\end{equation*}
$$

Equation (2) ensures that $\boldsymbol{r}$ and $\dot{\boldsymbol{r}}$ remain collinear, while (3) ensures that $P$ is not receding from $S$. The above equations can be solved in a parametric form to yield

$$
\begin{equation*}
\dot{\boldsymbol{r}}=-a \boldsymbol{r}, \tag{4}
\end{equation*}
$$

where $a$ is a positive real number. The instantaneous relative velocity can then be written in terms of the pursuer and target velocities, $\boldsymbol{v}_{\boldsymbol{p}}$ and, $\boldsymbol{v}_{s}$ as follows:

$$
\begin{equation*}
\dot{r}=v_{s}-v_{p} . \tag{5}
\end{equation*}
$$

Substituting (4) into (5) and solving for the pursuer velocity yields

$$
\begin{equation*}
\boldsymbol{v}_{p}=\boldsymbol{v}_{s}+a \boldsymbol{r} \tag{6}
\end{equation*}
$$

The goal of the proposed trajectory planner is to obtain an optimal pursuer velocity command according to the parallel navigation law for the next command instant. The value of $\boldsymbol{r}$ is obtained
based on the data received from proximity sensors on the pursuer vehicle. Substituting this vector into (6) would result in a locus for the pursuer's velocity vectors, $\boldsymbol{v}_{\boldsymbol{p}}$, all lying on a semiline parameterized by $a$. This semi-line is referred to herein as the Rendezvous Line ( $R L$ ), Figure 2. The end-points of the velocity vectors show the positions of $S$ and $P$, after one unit of time has passed, should they adopt the corresponding velocities. If $P$ continually adopts a velocity command that falls on the instantaneous $R L$, the direction of LOS remains constant and positional matching between $P$ and $S$ is guaranteed.


Figure 2: The Rendezvous Line ( $R L$ ).

The next task is to find the value of $a$, such that velocity matching is also assured. Let us assume that the acceleration capability of the pursuer in this direction is given by $A$. The simultaneous reduction of velocity and position differences in the direction of LOS for rendezvous may, then, be written as:

$$
\left\{\begin{array}{c}
\dot{\boldsymbol{r}}_{\max }^{\text {rend }}-A t_{r}=0,  \tag{7}\\
\boldsymbol{r}-\dot{\boldsymbol{r}}_{\max }^{\text {rend }} t r+1 / 2 A t_{r}^{2}=0,
\end{array}\right.
$$

where $\dot{\boldsymbol{r}}_{\max }^{\text {rend }}$ is the magnitude of the maximum allowable closing velocity, and $t_{r}$ is the remaining time-to-intercept from the current instant. The maximum instantaneous allowable closing velocity is obtained by solving (7):

$$
\begin{equation*}
\dot{\boldsymbol{r}}_{\max }^{r e n d}=\sqrt{2 \boldsymbol{r} A} \tag{8}
\end{equation*}
$$

The maximum closing velocity, as imposed by the frequency of velocity command generation by the trajectory planner for a fast asymptotic interception, is given by

$$
\begin{equation*}
\dot{\boldsymbol{r}}_{\max }^{c r}=\boldsymbol{r} / n . \Delta t \tag{9}
\end{equation*}
$$

The value of $n$ above is determined empirically. The final allowable closing velocity component of the velocity command is, then, obtained by considering (8) and (9) simultaneously:

$$
\begin{equation*}
\boldsymbol{v}_{\max }^{\text {rel }}=\min \left\langle\dot{\boldsymbol{r}}_{\max }^{\text {rend }}, \dot{\boldsymbol{r}}_{\max }^{c r}\right\rangle . \tag{10}
\end{equation*}
$$

The end points of all velocity command vectors on $R L$ that have a closing velocity component smaller than $\boldsymbol{v}_{\max }^{\text {rel }}$ constitute a line segment extending from $\boldsymbol{v}_{\boldsymbol{p}}=\boldsymbol{v}_{\boldsymbol{s}}$ to $\boldsymbol{v}_{\boldsymbol{p}}=\boldsymbol{v}_{\boldsymbol{p}, \text { max }}\left(=\boldsymbol{v}_{\boldsymbol{s}}+\boldsymbol{v}_{\max }^{\text {rel }}(\boldsymbol{r} /\|\boldsymbol{r}\|)\right)$. This set of points is referred to herein as the Rendezvous Set (RS), Figure 3.


Figure 3: The Rendezvous Set (RS).

The velocity represented by $\boldsymbol{v}_{\text {max }}^{\text {rel }}$ in Figure 3 may not be achievable by $P$ within the time interval $\Delta t$ due to constraints of vehicle dynamics. Therefore, a Feasible Velocity Region $(F V R)$, Figure 4, representing all velocities physically reachable by $P$ within the time interval $\Delta t$ is defined herein. Assuming that the current heading angle of $P$ is $\delta$, and considering all the kinematic and dynamic constraints of $P$, the velocity selected for $P$, for the time interval $\Delta t$, is
the component of the $R S$ within $F V R$ with the maximum value represented by $\boldsymbol{v}_{\boldsymbol{p}}\left(t_{i}+\Delta t\right)$. It is, thus, concluded that if $P$ adopts the velocity commands from within the $R S$ with the largest allowable velocity components, then, a time-efficient interception can be achieved.


Figure 4: Generation of Pursuer Velocity Command.

Let us assume that the maximum value for the lateral acceleration, $a_{Y \text { max }}$, is defined by

$$
\begin{equation*}
a_{Y \max }=\frac{v_{p}^{2}}{K h} 2 \sin ^{2} \vartheta\left(1+\frac{\cos \vartheta}{\sqrt{K^{2}-\sin ^{2} \vartheta}}\right), \tag{11}
\end{equation*}
$$

where $K=\boldsymbol{v}_{p} / v_{s}, h$ is the width of the lane and $\vartheta$ is the maximum angle the pursuer vehicle can turn with the given set of variables.

## Modification of the RG Algorithm

The RG method described above is further modified below to yield better overtaking times while remaining within the constraints of passenger comfort. One may note that the limitation on lateral acceleration does not allow $P$ to travel at its optimum velocity by limiting the angular acceleration that it can achieve: namely, RG selects an angular acceleration value that ensures the velocity of $P$ remains on $R L$, even though the vehicle has the capability of selecting a higher value of velocity from the $F V R$.

In the case of a target moving on unknown trajectory, if $P$ tries to achieve a velocity greater than the rendezvous velocity a situation may arise wherein $S$ is turning away from the direction
in which the velocity is increased. This could lead to an increase in the rendezvous time instead of a reduction. However, in the case of an overtaking manoeuvre, the behaviour of the vehicle moving on a highway is predictable. Using this information, the velocity of $P$ can be increased in the forward direction to reduce the overtaking time. However, as noted earlier, the same reduction in time would not be possible if the behaviour of $S$ is unknown.

Taking advantage of this predictability, we define herein a Velocity Line (VL) which originates from the start point of the $R L$ and makes an angle $\vartheta$ with the fixed reference, Figure 5. Now, if $P$ were to select and use a velocity command from $V L$ instead of $R L$, a more time time-efficient overtaking could be achieved.


Figure 5: The Velocity Line.

## III. IMPLEMENTATION

As discussed above, a shadow target, $S$, is utilized herein for the guidance of the pursuer vehicle, $P$. The location of $S$ is dictated by the location of the (obstacle) vehicle in the driving lane, $O_{D}$, and varied according to the stage of overtaking manoeuvre. Once the manoeuvre starts, the position and velocity information is passed on to the RG algorithm which constructs a $R S$, as shown in Figure 6 and checks whether the maximum closing velocity $\boldsymbol{v}_{\max }^{\text {rel }}$ is within the $F V R$. If $\boldsymbol{v}_{\text {max }}^{\text {rel }}$ is not within the $F V R$, an optimal velocity from $R S$ is required for the next time instance. This velocity is also required to be within the Feasible Velocity Set (FVS), formed by the
intersection of $V L$ and $F V R$. For time optimal rendezvous, we select the velocity from within $F V S$ that takes the $P$ nearest to $S$, which corresponds to the velocity $v_{1}$ or $v_{2}$, Figure 6.


Figure 6: Pursuer Velocity Command.

As mentioned in the Problem Statement sub-section above, an obstacle vehicle may be present in the passing lane, $O_{P}$. Thus, the overtaking manoeuvre should be considered under two possible scenarios:

Scenario 1: As a first step, when $P$ is 2.5 s behind $O_{D}$, it checks for obstacles in the passing lane. If there is no obstacle vehicle in the passing lane, $P$ continues to travel in the driving lane until the distance between $P$ and $O_{D}$ is 2 s . At this point, a shadow target, $S$, is created in the passing lane - as shown in Figure 7 by Positions 1, 2, and 3, for Phases 1, 2, and 3 of overtaking, respectively. During the complete manoeuvre, the velocity of $S$ is chosen as the original velocity of $P, \boldsymbol{v}_{s t}$, at the start of overtaking. As discussed previously, all position and velocity matching objectives for $P$ (with $S$ ) are achieved via the proposed RG algorithm.


Figure 7: Shadow Target Positions for Scenario 1.

Scenario 2: As in Scenario 1, as a first step, when $P$ is 2.5 s behind $O_{D}$, it checks for obstacles in the passing lane. If there is an obstacle vehicle, $O_{P}$, in the passing lane and the gap available between $O_{P}$ and $\mathrm{O}_{D}$ is deemed as unsafe for overtaking, the pursuer must 'wait' until $O_{P}$ first overtakes $O_{D}$. In this case, first, a shadow target is created in the driving lane, 2 s behind $O_{D}$ having a velocity equal to the velocity of $O_{D}$, Position 1, Figure 8 . Once $O_{P}$ clears $O_{D}$, the pursuer vehicle may start the overtaking manoeuvre. However, unlike in Scenario 1, the velocity of the shadow target at Positions 2 and 3 is set to either to the original velocity of $P$, $\boldsymbol{v}_{\boldsymbol{s}}=\boldsymbol{v}_{\boldsymbol{s t}}$, or to the velocity of $O_{P}, \boldsymbol{v}_{\boldsymbol{s}}=\boldsymbol{v}_{\boldsymbol{o p}}$, whichever is less.


Figure 8: Shadow Target Positions for Scenario 2.

## IV. SIMULATIONS

A large number of simulations were carried out incorporating combinations of various pursuer and obstacle initial positions and velocities. The results clearly showed the viability of the proposed RG method in guiding the pursuer vehicle in an optimal, comfortable, and collisionfree manner during the overtaking manoeuvre. Our on-line method was also shown to be comparable to the off-line technique proposed in [35] for Scenario 2 discussed above - with our method showing even some improvement.

Two simulation cases are discussed below: In Example 1, Scenario 2 above is considered, where the obstacle vehicle in the driving lane is moving with a sinusoidal velocity - namely, the objective herein is to illustrate that our proposed method, unlike most other methods in the literature, can cope with variations in obstacle velocity by adjusting its own velocity in real-time. In Example 2, our on-line method is also shown to be comparable to the off-line technique proposed in [35] for Scenario 2 discussed above - with our method showing even some
improvement. However, in this example, the obstacle vehicle in the driving lane is moving with a constant velocity, since the technique proposed in [35] cannot cope with variations in obstacle velocity.

## Example 1

In this example, an obstacle vehicle, $O_{P}$, in the passing lane is nearby the pursuer, $P$, and as such an overtaking manoeuvre is not immediately feasible. The slower vehicle in the driving lane, $O_{D}$, is moving with a sinusoidal velocity - its velocity is oscillating between 18 and $22 \mathrm{~m} / \mathrm{s}$ (i.e., about $10 \%$ variation about its mean velocity of $20 \mathrm{~m} / \mathrm{s}$ ), Figure 9 . Similarly, the obstacle vehicle in the passing lane, $O_{P}$, is also moving with a sinusoidal velocity - its velocity is oscillating between 23 and $27 \mathrm{~m} / \mathrm{s}$, Figure 9. Due to presence of $O_{P}, P$ first undertakes a collision-avoidance manoeuvre - its velocity is reduced to match the velocity $O_{D}$ at shadow target, $S$, Position 1 in Figure 8. Once the path of $P$ becomes collision free for overtaking, the RG method initiates the overtaking manoeuvre which is completed in about 30.5 s, Figure 10 and Table 1.


Figure 9: Velocity Profiles of Obstacle Vehicles.


Figure 10: Simulation Results for Example 1.

Table 1: Overtaking Parameters for Example 1.

|  | Proposed RG Technique |
| :--- | :---: |
| Total Time | 30.5 s |
| Distance Travelled | 800 m |
| Maximum Lateral Acceleration | $1.1 \mathrm{~m} / \mathrm{s}^{2}$ |
| Maximum lateral Deceleration | $0.41 \mathrm{~m} / \mathrm{s}^{2}$ |
| Maximum Axial Acceleration | $2.5 \mathrm{~m} / \mathrm{s}^{2}$ |

## Example 2

In this example, our on-line RG method is compared to the off-line technique proposed in [35]. Example 1 is repeated. However, herein, $O_{D}$ is moving with a constant velocity of $20 \mathrm{~m} / \mathrm{s}$ and $O_{P}$ is moving with a constant velocity of $25 \mathrm{~m} / \mathrm{s}$ since the technique proposed in [35] cannot cope with variations in obstacle velocity. The results are shown in Figures 11 and 12 and Table 2.

Table 2: Basic Overtaking Parameters for Example 2 - A Comparison.

|  | Proposed RG Method | Method Proposed in [35] |
| :--- | :---: | :---: |
| Total Time (s) | 30 | 35 |
| Distance Travelled (m) | 800 | 900 |



Figure 11: Simulation Results for Example 2: Using the Proposed RG Method.


Figure 12: Simulation Results for Example 2: Using the Method Proposed in [35 ].

## V. EXPERIMENTS

The proposed RG method was tested via a number of experiments, incorporating combinations of various pursuer and (constant) obstacle velocities. The results show that the vehicle behavior observed during the experiments is very similar to simulated behavior. Two examples are included in this paper: Experiment 1 considers a case with no obstacle in the passing lane and Experiment 2 considers the case with an obstacle.

The hardware specifications for the experimental set-up are given in Table 3. The software for the experiments run on a Pentium IV 1.6 GHz processor PC and included three primary modules: image acquisition and processing, trajectory planning, and communication modules, respectively. In our set-up, an analog CCD camera captures the entire image of the workspace. The vision algorithm, then, extracts the positional information of all the objects in the workspace. This information is sent to the trajectory planner, where an acceleration command is calculated in real-time for the pursuer vehicle.

Table 3: Experimental Hardware

| Component | Characteristics |
| :--- | :--- |
| Pursuer and Obstacle <br> Vehicles | Miabot PRO BT v2 Differential-Drive mobile <br> Robots with Bluetooth Communication |
| CCD Camera | Resolution: $640 \times 480$ pixels <br> Lens Focal Length: 6 mm <br> Vertical Distance from Floor: 3000 mm |
| Floor Workspace | $2740 \times 1500 \mathrm{~mm}$ |

o Robotic Vehicles: Three Miabot PRO BT v2 differential-drive mobile robots were used in the implementation of the proposed methodology. The robot motors are driven by $6 \times 1.2 \mathrm{~V}(\mathrm{AA})$ cells through a low-resistance driver I.C. with a slow-acting current limit at about 5 A . Maximum speed of an unloaded motor is in the range of 6000 to 8000 rpm . The motor shafts drive the wheels through an $8: 1$ gearing. The motors incorporate quadrature encoders giving 512 position-pulses per rotation. The wheels are 52 mm in diameter; one encoder pulse corresponds to just under 0.04 mm of movement.
o Communication System: A Bluetooth card enabled the robotic vehicle to communicate with the host PC. The MIABOT-BT Bluetooth board is equipped with fixed communication settings 19200 baud ( 8 bits, 1 stop bit, no parity). A PC Bluetooth dongle plugs into the USB port on the PC. This can support wireless links with up to 7 robots at once.

## Experiment 1

In this experiment, $P$ is required to overtake $O_{D}$ with no obstacle vehicle, $O_{P}$, being present in the passing lane. Due to the dimensions of the robotic vehicle $(80 \times 80 \mathrm{~mm})$ and the availability of limited workspace, the width of the lane was set as 160 mm , the velocity of $P$ was set to $8 \mathrm{~mm} / \mathrm{s}$, and the velocity of $O_{D}$ was set to $6 \mathrm{~mm} / \mathrm{s}$. The shadow target, $S$, positions for this experiment are shown in Figure 13. The simulation and experimental results are shown in Figures 14 and 15, respectively. The experiments were repeated three times under identical conditions.


Figure 13: Shadow Target Positions for Experiment 1.


Figure 14: Simulation Results for Experiment 1.



Figure 15: Experimental Results for Experiment 1.

## Experiment 2

In this experiment, an obstacle vehicle, $O_{P}$, is present in the passing lane and an overtaking manoeuvre by $P$ is not immediately feasible. $O_{P}$ is moving with a constant velocity of $8 \mathrm{~mm} / \mathrm{s}$
and $O_{D}$ is moving with a constant velocity of $6 \mathrm{~mm} / \mathrm{s}$. The starting velocity of $P$ is $10 \mathrm{~mm} / \mathrm{s}$. The positions of the shadow target are shown in Figure 16.

Due to the presence of $O_{P}, P$ first undertakes a collision-avoidance manoeuvre by reducing its velocity and ensuring a safe distance between itself and $O_{D}$. Once $O_{P}$ is ahead of $O_{D}, P$ starts the overtaking manoeuvre. The simulation and experimental results are shown in Figures 17 and 18, respectively. The experiments were repeated three times under identical conditions.


Figure 16: Shadow Target Positions for Experiment 2.


Figure 17: Simulation Results for Experiment 2.


Figure 18: Experimental Results for Experiment 2.

## CONCLUSIONS

In this paper, a novel guidance-based on-line trajectory-planning algorithm is presented for autonomous ground vehicle overtaking manoeuvres in dynamic highway environments. The focus has been on three primary aspects: (i) time-optimal overtaking, (ii) obstacle avoidance, and (iii) passenger comfort. The proposed algorithm uses a modified Rendezvous Guidance method to obtain optimal vehicle acceleration commands for the overtaking manoeuvre. A shadow-target concept is utilized to adjust in real-time the driving parameters of the pursuer vehicle in response to changes in the driving parameters of the obstacle vehicles in the driving land as well as in the passing lane - this ability presents the primary novelty of our proposed method in contrast to previous off-line methods presented in the literature. Numerous simulations and experiments have verified the proposed methodology to be robust and time efficient.

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