www.s2is.org

International Journal on Smart Sensing and Intelligent Systems

ISSN 1178-5608

SPECTRUM SENSING IN COGNITIVE RADIOS UNDER NOISE UNCERTAINTY: DECISION MAKING USING GAME THEORY

K. Muthumeenakshi and S. Radha

SSN College of Engineering

Kalavakkam, Chennai

Tamil Nadu, India.

Emails: muthumeenakshik@ssn.edu.in, radhas@ssn.edu.in

Submitted: Jan. 3, 2017 ***Accepted: Feb. 9, 2017*** ***Published: Mar. 1, 2017***

Abstract- Energy detection is best suited for the detection of licensed users when prior knowledge about them is unavailable. However, the presence of noise uncertainty refrains the use of energy detection for spectrum sensing. In this paper, we propose a refined energy detection (RED) which used dual threshold in the presence of noise uncertainty, and combine the concepts from game theory to achieve further performance improvements. The secondary user payoff is defined based on the primary user activity and the strategy adopted by the secondary user. The pure strategy Nash equilibrium and the best response for the mixed strategy Nash equilibrium are analyzed for all the possible strategies adopted by the secondary user. Simulations results show the effectiveness of the proposed algorithm in terms of greater secondary user payoff and robustness against noise uncertainty.

Index terms*:* Cognitive Radio, Spectrum sensing, Energy Detection, Noise Uncertainty, Game Theory.

I. INTRODUCTION

Recent studies on wireless spectrum demonstrate that the wireless communication systems suffer from inefficient spectrum usage in the licensed portions of the spectrum [1]. Reformation in the existing spectrum assignment policies is essential such that the new policy schemes would allow opportunistic use of the licensed spectrum by the unlicensed users. Cognitive Radio (CR) offers a promising solution to overcome the inefficient spectrum usage problem and anticipated to make this policy reformation successful. The licensed users are referred as primary users and the unlicensed users as secondary users in the context of CR.

CRs are devices that can alter its parameters based on the dynamically changing environment and opportunistically occupy the licensed spectrum without causing interference to the incumbent [2]. The fundamental task called spectrum sensing enables the CR to identify the idle portions of the primary spectrum. Spectrum sensing algorithms should be efficient to identify the idle portions accurately. Moreover, reliable spectrum sensing is highly challenging in the presence of noise uncertainty. A review of literature presents a number of spectrum sensing algorithms. Energy detection [3], matched filtering [4], cyclostationary feature detection [5], covariance based detection [6], Eigen value based detection [7], detection using wavelets [8] and filter bank spectrum estimation [9] are few among them. The performance of the existing spectrum sensing algorithms provide trade-offs between detection accuracy, computational complexity and sensing time. But their practical applicability depends very much on the information available about the primary signals. Energy detection is the most preferred approach for spectrum sensing when the CR is unable to gather sufficient information about the primary user signals.

Originally, the energy detector is proposed for the detection of an unknown deterministic signal considering a flat band-limited Gaussian channel [10]. In the last decade, the energy detector proposed in [10] is being used for spectrum sensing to a large extend because of its simplicity and low computational complexity. Analytical and simulated performance analysis of energy detection for AWGN and fading channel models and various improved versions of energy detectors can be found in [11-14]. There are several contributions in which the spectrum sensing algorithms are analyzed under noise uncertainty [15 -18]. In [15], the authors describe the existence of SNR wall below which detection performance cannot be obtained effectively. The performance of energy detection under Log Normal approximation of noise uncertainty is analyzed in [16]. The authors of [17] assume a uniform distribution for noise uncertainty and analyzed the performance of energy detection. A generalized energy detector is analyzed under noise uncertainty in [18] and observed that the traditional energy detector (TED) is best suited for spectrum sensing under noise uncertainty. An optimized and improved energy detection algorithm can be found in [19]. Issues related to the future of spectrum sensing are covered in [20].

On the other hand, cooperative spectrum sensing is proposed to enhance the sensing performance of the secondary users [21]. There are numerous contributions which employ evolutionary game theory for cooperative spectrum sensing [22-23]. The authors of [22] present the spectrum sensing as an evolutionary game and develop a model for whether and when to share the sensing results for cooperative spectrum sensing. A distributed spectrum sensing game using evolutionary theory is designed for CR with heterogeneous traffic [23]. Another spectrum sensing model is formulated as an evolutionary game to study the selfish behaviour of the secondary users [24]. Most of the contributions based on game theory take advantage of cooperative spectrum sensing to achieve performance enhancements. In addition to spectrum sensing, other CR related issues such as spectrum sharing, spectrum access, security concerns are modelled using game theory [25-27].

In this paper, we first propose a refined energy detection (RED) algorithm for spectrum sensing which is superior to the TED. The RED algorithm uses an arbitrary positive p instead of squaring to compute the decision metric and also takes advantage of the past history of decision metric to improve the detection performance. Then we extend the RED algorithm suitable for spectrum sensing under noise uncertainty by incorporating a dual threshold and using game theory principles. The Bayesian belief update algorithm is employed to study the spectrum occupancy records from the past statistics. We observe significant performance improvements in terms of greater secondary user payoffs and low probability of error. The major contributions addressed in this paper are thus summarized:

1. To the best of the author’s knowledge, this is the first attempt to use the game theory based decision making for spectrum sensing in the presence of noise uncertainty.
2. First, the detection probability and false alarm probability of the proposed RED is derived. The optimum value of p maximizing the detection probability is identified.

3. The performance of the proposed scheme is analyzed in terms of the Receiver Operating Characteristic (ROC) and compared with the existing schemes. The total probability of sensing error and the sample complexity is analyzed against varying SNR.
4. A dual threshold is employed for the RED algorithm under noise uncertainty. Concepts from game theory are used to arrive at a decision. The best response is analysed for all the strategies played by the secondary user based on the primary user availability. The secondary user payoff is evaluated and compared for both with and without noise uncertainty.

II. SPECTRUM SENSING PRELIMINARIES

The CR spectrum sensing can be formulated as a binary hypothesis testing problem as follows.

$$y(n) = \begin{cases} w(n) & ; H_0 \\ s(n) + w(n) & ; H_1 \end{cases} \qquad n = 1,2,....N \tag{1}$$

where H_0 is the hypothesis denoting the absence of the primary user and H_1 is the hypothesis denoting the presence of the primary user. $y(n)$ is the signal received by the secondary user, $w(n)$ is the Additive White Gaussian Noise (AWGN) of variance σ_w^2 and $s(n)$ is the primary user signal with variance σ_w^2 and assumed to be real Gaussian. Moreover, $s(n)$ and $w(n)$ are assumed to be independent of each other. The TED uses squaring operation to compute the decision statistic, given by

$$Y = \frac{1}{N} \sum_{n=1}^{N} |y(n)|^2 \tag{2}$$

where Y is the decision statistic and N is the number of samples used for computation. Y is compared with a pre-evaluated threshold λ. If $Y \geq \lambda$, the decision is H_1, otherwise H_0. Ideally any spectrum sensing algorithm should select H_0 when the primary user is absent and H_1 when it is present. Practically, spectrum sensing algorithms are prone to errors which are classified as missed detections and false alarms. A missed detection occurs when the spectrum sensing outcome is H_0 and the primary signal is present. On the other hand, a false alarm occurs when the spectrum sensing outcome is H_1 and the primary signal is absent. A missed detection causes harmful interference to the primary user whereas false alarm results in loss of transmission opportunities to the secondary user. The sensing errors are usually represented by the following conditional probabilities, the probability of missed detection, $P_{md} = \text{Prob}(H_0 / H_1)$ and the probability of false alarm, $P_f = \text{Prob}(H_1 / H_0)$. The complementary probability of missed detection is the probability of detection given by $P_d = \text{Prob}(H_1 / H_1) = 1 - P_{md}$. It is desirable to have large P_d and low P_f for any spectrum sensing algorithm. However, there exists a trade-off between the two values. To depict the relationship between the two values, ROC curves are useful. The ROC of the TED algorithm is given by [14],

$$P_d = Q\left(\frac{\lambda - \left(\sigma_s^2 + \sigma_w^2\right)}{\sqrt{2/N}\left(\sigma_s^2 + \sigma_w^2\right)^2} \right) \tag{3}$$

where, $\lambda = \left(\sqrt{2/N}Q^{-1}\left(P_f\right) + 1\right)\sigma_w^2$.

The TED has well-known detection performance drawbacks and its performance depends on factors like SNR, N and λ. To enhance the performance of the TED algorithm, we propose the RED algorithm which is explained in the following section.

III. REFINED ENERGY DETECTION ALGORITHM FOR SPECTRUM SENSING

The enhanced energy detector (EED) proposed in [14] makes use of an arbitrary positive power p to compute the test statistic instead of squaring operation. The modified decision statistic of the improved energy detector Y_m with p^{th} power summer is given by,

$$Y_m = \frac{1}{N}\sum_{n=1}^{N} \left|y(n)\right|^p \underset{H_0}{\overset{H_1}{\gtrless}} \lambda_m \tag{4}$$

where λ_m is the modified decision threshold. For any p, $\left|y(n)\right|^p$ are independent and identically distributed random variables. Using [14], the mean and variance of $\left|y(n)\right|^p$ is given by,

Under H_0:

$$\mu_0 = \frac{2^{p/2}}{\sqrt{\pi}}\Gamma\left(\frac{p+1}{2}\right)\sigma_w^p \tag{5}$$

$$\sigma_0^2 = \frac{2^p}{\sqrt{\pi}}\left[\Gamma\left(\frac{2p+1}{2}\right) - \frac{1}{\sqrt{\pi}}\Gamma^2\left(\frac{p+1}{2}\right)\right]\sigma_w^{2p} \tag{6}$$

Under H_1:

$$\mu_1 = \frac{2^{p/2}(1+\text{SNR})^{p/2}}{\sqrt{\pi}}\Gamma\left(\frac{p+1}{2}\right)\sigma_w^p \tag{7}$$

$$\sigma_1^2 = \frac{2^p(1+\text{SNR})^p}{\sqrt{\pi}}\left[\Gamma\left(\frac{2p+1}{2}\right) - \frac{1}{\sqrt{\pi}}\Gamma^2\left(\frac{p+1}{2}\right)\right]\sigma_w^{2p} \tag{8}$$

As the random variables $|y(n)|^p$ follow normal distribution [14], the modified test statistic also follows normal distribution with mean and variance values given by,

$$E(Y_m) = \begin{cases} \mu_0 & ; H_0 \\ \mu_1 & ; H_1 \end{cases} \tag{9}$$

$$Var(Y_m) = \begin{cases} \sigma_0^2 / N & ; H_0 \\ \sigma_1^2 / N & ; H_1 \end{cases} \tag{10}$$

The P_d and P_f of the improved energy detector is given by [14],

$$\begin{aligned} P_f &= \Pr(Y_m > \lambda_m / H_0) \\ &= Q\left(\frac{\lambda_m - \mu_0}{\sigma_0 / \sqrt{N}}\right) \end{aligned} \tag{11}$$

$$\begin{aligned} P_d &= \Pr(Y_m > \lambda_m / H_1) \\ &= Q\left(\frac{\lambda_m - \mu_1}{\sigma_1 / \sqrt{N}}\right) \end{aligned} \tag{12}$$

IV. THE RED ALGORITHM AND THE OPERATING PRINCIPLE

Based on the modified decision statistic EED given by (4), the proposed RED algorithm is explained as follows. At every i^{th} sensing instant, Y_m^i is computed and compared with λ_m which is evaluated based on P_f. If $Y_m > \lambda_m$, then the decision is hypothesis H_1. If Y_m falls below λ_m, additionally a comparison with the average decision statistic of the past L sensing instants avg Y_m^j and the decision statistic of the previous sensing instant Y_m^{i-1} is performed with λ_m. If both the values exceeds λ_m, the decision is hypothesis H_1. Otherwise, the sensing outcome is hypothesis

H_0. This additional check helps to avoid any false alarms due to instantaneous signal energy drops [12]. The average signal energy of the past L sensing instants is computed as,

$$\begin{aligned} \text{avg } Y_m^j &= \text{mean}\left(T_i(y), T_{i-1}(y), \ldots T_{i-L}(y)\right) \\ &= \frac{1}{L}\sum_{j=i-L}^{i} T_j(y) \end{aligned} \tag{13}$$

As the test statistic values of the past instants $j = i$, i-1, …i-L are assumed to follow normally distribution, their average decision statistic also follow normal distribution as given by,

$$\text{avg } Y_m^j \sim Normal(\mu_{avg}, \sigma_{avg})$$

The average mean and variance of $\text{avg } Y_m^j$ can be evaluated as [12],

$$\mu_{avg} = \frac{M}{L}\mu_1 + \frac{L-M}{L}\mu_0 \tag{14}$$

$$\sigma_{avg}^2 = \frac{M}{L}\sigma_1^2 + \frac{L-M}{L}\sigma_0^2 \tag{15}$$

where M is the number of times the primary user is actually present out of the past L sensing instant. From the secondary user point of view, the value of M depend upon the sensing outcomes and may not correspond to the actual presence/absence of the primary user. Without the exact knowledge of M , the performance of the RED algorithm is difficult to predict, but the upper and lower bounds corresponding to M = 0 and $M = L$ can be analyzed. The probability of detection of the RED algorithm P_d^r is given by,

$$\begin{aligned} P_d^r = {} & P\left(T_i(y) > \lambda\right)/_{H_1} + P\left(T_i(y) \le \lambda\right)/_{H_1} \,.\, P\left(\text{avg}T_j(y) > \lambda\right)/_{H_1} \,. \\ & \left(P\left(T_{i-1}(y) > \lambda\right)/_{H_1}\right) \end{aligned}$$

$$P_d^r = P_d + (1-P_d)P_d\left\{Q\left(\frac{\lambda - \mu_{avg}}{\sigma_{avg}}\right)\right\} \tag{16}$$

The corresponding false alarm probability P_f^r is given by,

$$P_f^r = P_f + (1-P_f)P_f\left\{Q\left(\frac{\lambda - \mu_{avg}}{\sigma_{avg}}\right)\right\} \tag{17}$$

The probabilities of detection and false alarm of the RED algorithm are bounded by $P_d \le P_d^r \le 2P_d - P_d^2$ and $P_f \le P_f^r \le 2P_f - P_f^2$ as the value of Q function lies between 0 and 1. This shows that the detection probability of the RED is lower bounded by the detection probability of the EED. The degradation in false alarm probability is also observed from (17). As the probabilities depend on the value of *p*, it is possible to find an optimum *p* which maximizes $P_f \le P_f^r \le 2P_f - P_f^2$, (i.e)

$$p^* = \arg\max_p P_d^r \tag{18}$$

The solution for (18) is obtained numerically as it is difficult to obtain its solution in closed form. It is found that the p^* depends on N, SNR and P_f. Thus p^* can be computed offline for various values of SNR, N and P_f. For practical sensing, for a given N, P_f and the estimated SNR, p^* can be chosen from the offline computed values. The RED algorithm is summarized in Table 1.

Table 1. RED Algorithm

```
for every sensing instant i do
    Compute Y_m
    Compute avg Y_m^j
    Choose p* using N, SNR and P_f
      if Y_m^i > λ_m, then
       decide H_1
      else
         if avg Y_m^j > λ_m and Y_m^{i-1} > λ_m, then
            decide H_1
         else
            decide H_0
         end if
      end if
end for
```

V. RED ALGORITHM UNDER NOISE UNCERTAINTY

In this section, we describe the game theory based decision making technique for the secondary user in the presence of noise uncertainty. The threshold obtained using (11) directly depends on the noise power σ_w^2 which is difficult to estimate accurately. Practically, the average noise power σ_w^2 is known and it is different from the actual noise power σ_w^2. The noise power changes with time and location and is therefore uncertain. We assume the noise power uncertainty to be uniformly distributed in the interval [17] as given by,

$$\sigma_w^2 \in \left[(1/\rho)\hat{\sigma}_w^2,\ \rho\hat{\sigma}_w^2 \right] \tag{19}$$

where ρ is the factor describing the noise uncertainty. Under noise uncertainty, ρ >1. If noise uncertainty is absent, ρ =1. Noise uncertainty is can also be expressed in dB, denoted as *del* = $10\log_{10}\rho$. To enhance the performance of the energy detector under this environment, a dual threshold is used as described by,

$$\lambda_m^u = \left(\mu_0^{\mathrm{NU}_u} + \frac{\sigma_0^{\mathrm{NU}_u}}{\sqrt{N}} Q^{-1}\left(P_f^r\right) \right) \tag{20}$$

$$\lambda_m^l = \left(\mu_0^{\mathrm{NU}_l} + \frac{\sigma_0^{\mathrm{NU}_l}}{\sqrt{N}} Q^{-1}\left(P_f^r\right) \right) \tag{21}$$

where λ_m^u, λ_m^l are the upper and lower thresholds, $\mu_0^{\mathrm{NU}_u}$, $\mu_0^{\mathrm{NU}_u}$ are the mean values of the RED decision statistic for the upper and lower extremes of the noise uncertainty given by,

$$\mu_0^{\mathrm{NU}_u} = \frac{2^{p/2}}{\sqrt{\pi}} \Gamma\left(\frac{p+1}{2}\right) \rho^{p/2} \sigma_w^p \tag{22}$$

$$\mu_0^{\mathrm{NU}_l} = \frac{2^{p/2}}{\sqrt{\pi}} \Gamma\left(\frac{p+1}{2}\right) \frac{\sigma_w^p}{\rho^{p/2}} \tag{23}$$

Using (22) and (23), the decision regarding the presence or absence of the primary user is obtained as explained below. The RED decision statistic is compared with the upper and lower thresholds. If it exceeds λ_m^u, the decision is H_1. If it falls below λ_m^u, the decision is H_0. If it falls in between the two threshold, the secondary user choose any of the three options (i.e.,) sense again, decide H_1, decide H_0. he block diagram of the proposed RED algorithm is shown in Figure.1.

Based on the strategies, a dynamic game is formulated and explained in detail in the following section.

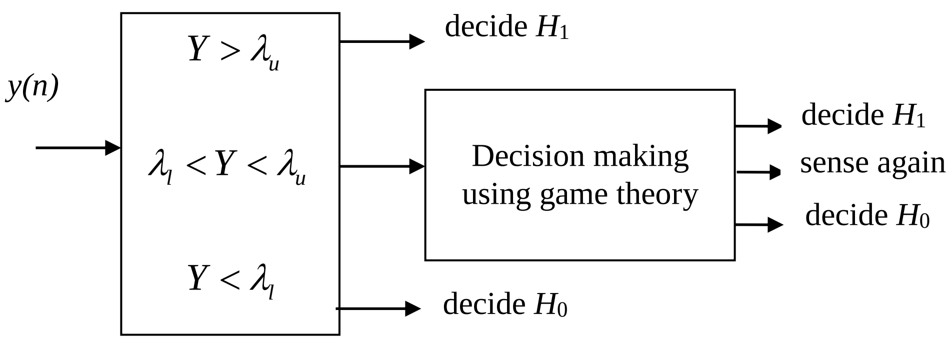

Figure. 1 Block diagram of the proposed RED algorithm

VI. DECISION MAKING USING GAME THEORY

The secondary user spectrum sensing is formulated as a decision making problem when the algorithm decision statistic falls in between the upper and lower thresholds. The primary user is the incumbent and has the right to use its spectrum when it desires. Thus the primary user is either ON or OFF based on its state of requirement. On the other hand, the secondary user's intention is use the primary spectrum to the maximum without violating the spectrum etiquette. The secondary user is uncertain about the primary user ON-OFF activity pattern. Let the probability that the primary is in the ON state be p_1. The actions taken by the primary user is from its strategy space A_p = {access, don't access}. The secondary user chooses its actions from the strategy space A_s = {sense, access, silent}. The payoff for the primary user is always the spectrum gain irrespective of the secondary user's move. The payoff matrix of the secondary user corresponding to any particular game stage is shown in Table 2. We define the following notations to formulate the secondary user payoff matrix [10].

G – denotes the spectrum gain. For example, the spectrum gain is the bandwidth attained by the secondary user.

C_s – denotes the sensing cost. It applies to the secondary user if it decides to sense the spectrum when the spectrum is actually occupied. For example, the cost refers to the energy consumed by the secondary user for sensing.

P – denotes the penalty incurred by the secondary user when it causes interference to the primary user. For example, the secondary user may be forbidden to use the spectrum for a particular amount of time.

R – denotes the reward for the secondary user and it applies if it does not cause interference to the primary user.

We assume that the relationship between the parameters follow $P > G > C_s$ and $R > C_s$. The reasons behind the constraints are as follows:

$P > G$: The interference caused to the primary user is highly unacceptable. This constraint ensures that the secondary user would not access the spectrum to avoid interference to the primary user.

$G > C_s$: This is the constraint which ensures that the secondary user gains incentives for spectrum access. Otherwise, the secondary user will be reluctant to use the spectrum if the additional sensing cost exceeds the spectrum gain.

$R > C_s$: The reason behind this constraint is to assure that the secondary user gain incentives for a correct decision.

Table 2: Secondary user payoff matrix

Secondary User	**Primary User ON (p_1)**	**Primary User OFF(1- p_1)**
Sense	$R - C_s$	$-G + C_s$
Access	$-P + G$	G
Silent	R	$-G$

The payoff matrix is analyzed for the two cases namely, the primary user if ON and OFF. When the primary user is ON, the secondary user gets a reward R if it chooses silent. Alternatively, if it chooses to sense it incurs a sensing cost C_s in addition to the reward R. Hence, the overall payoff is $R – C_s$ for this case. If the secondary user chooses to access the spectrum, it gets a penalty P for interfering with the primary user besides gaining the spectrum. In this case, the overall payoff is $– P + G$. When the primary user is OFF, the secondary user gains the spectrum if it chooses to access with the payoff *G*. If it chooses to be silent, it loses the free spectrum with a payoff of $-G$. On the other hand, if it chooses to sense, in addition to spectrum loss it incurs positive sensing cost and the total payoff is $-G+C_s$. The sensing cost is positive because when the primary user is OFF, additional sensing may result in a positive decision for the secondary user.

a. Pure Strategy Nash equilibrium analysis

A specific action that the player will follow in every possible attainable situation is defined as pure strategy. In game theory context, a particular strategy $s_i^{'}$ of player *i* is strictly dominated than his other strategy s_i if,

$$u_i\left(s_i^{'}, s_{-i}\right) < u_i\left(s_i, s_{-i}\right)$$

And weakly dominated if,

$$u_i\left(s_i^{'}, s_{-i}\right) \leq u_i\left(s_i, s_{-i}\right)$$

where $u_i(\cdot,\cdot)$ refer to the payoff of the player *i*. A two player game has unique pure-strategy Nash equilibrium if each player has a dominant strategy [27].

The primary user has the highest priority as has the pure-strategy Nash equilibrium as spectrum gain. Thus, we analyze the Nash equilibrium for the secondary user.

Lemma 1: As the primary user does not maintain one particular state (OFF or ON), pure-strategy Nash equilibrium does not exist for the secondary user.

Proof: Table 1 shows that the pure-strategy Nash equilibrium for the secondary user is choosing silent when the primary user is ON. For the second case when the primary user is OFF, the pure-strategy Nash equilibrium for the secondary user is choosing access. As the primary user does not maintain a single state, the game will not converge to unique pure-strategy Nash equilibrium for the player. Thus, we analyze the mixed-strategy Nash equilibrium for the secondary user.

b. Mixed Strategy Nash equilibrium analysis

The mixed-strategy Nash equilibrium is investigated by assigning probabilities to the pure strategy space which refers to how frequency each pure strategy is played. Here the game is a single player decision making process and hence we define the mixed strategy Nash equilibrium based on the probability of the primary user being ON. The mixed strategy space of the secondary user is defined as {sense, access, silent}. The choice of the mixed strategy such that the secondary user attains a best response is analyzed.

Lemma 2: The existence of mixed-strategy Nash equilibrium is identified in this game if, the secondary user choose access if $p_1 < \frac{2G - C_s}{R + P - 2(G - C_s)}$, choose sense if $\frac{2G - C_s}{R + P - 2(G - C_s)} \le p_1 \le \frac{1}{2}$ and choose silent if $p_1 > \frac{1}{2}$.

Proof: The game tree for a single game stage is shown in Figure.3 for the secondary user. The expected payoff for the secondary user can be calculated as follows.

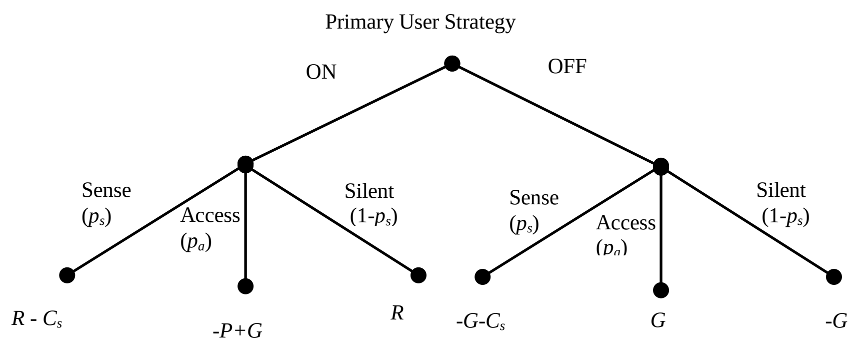

Figure. 3 Game tree showing all possible actions for the secondary user

The expected secondary user payoff for all the actions in the space A_s is given by,

$$E[sense, p_1] = p_1(R - C_s) + (1 - p_1)(-G + C_s) \quad (24)$$

$$E[access, p_1] = p_1(-P) + (1 - p_1)(R) \quad (25)$$

$$E[silent, p_1] = p_1(R) + (1 - p_1)(-G) \quad (26)$$

It should be noted that the mixed-strategy Nash equilibrium can be found by finding the best response for the secondary user as a function of probability of primary user being ON.

It can be readily obtained from the plot of the payoff function given by (24), (25) and (26) as shown in Figure.4. To find the best response, the primary user ON probability is obtained at the two relevant intersections.

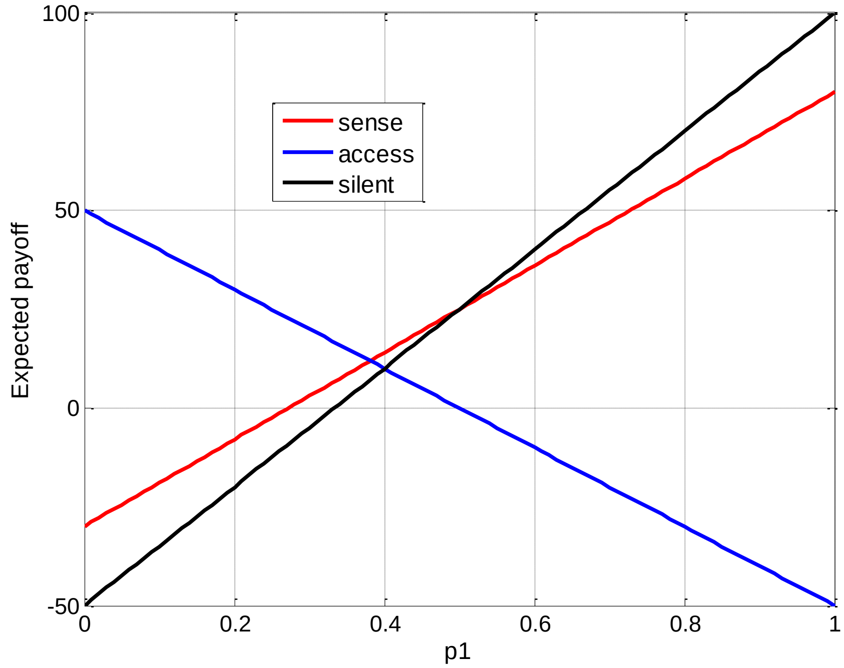

Figure. 4 Expected secondary user payoff against p_1

Equating (24) and (25),

$$p_1(R-C_s)+(1-p_1)(-G+C_s) = p_1(-P)+(1-p_1)(R)$$

$$p_1 = \frac{2G-C_s}{R+P-2(G-C_s)}$$

Equating (24) and (26),

$$p_1(R)+(1-p_1)(-G) = p_1(R-C_s)+(1-p_1)(-G+C_s)$$

$$p_1 = \frac{1}{2}.$$

The best response (BR) for the secondary user is thus given by,

$$BR = \begin{cases} \text{access if } p_1 < \dfrac{2G-C_s}{R+P-2(G-C_s)} \\ \text{sense if } \dfrac{2G-C_s}{R+P-2(G-C_s)} \le p_1 \le \dfrac{1}{2} \\ \text{silent if } p_1 > \dfrac{1}{2} \end{cases} \quad (27)$$

In practice, the value of p_1 is unknown to the secondary user. In order to find its strategy, the secondary should develop a model to estimate the value of p_1 from the past statistics. We use the basic Bayesian belief update algorithm with weighted coefficients to estimate the probability of primary user being ON [27]. The Bayesian belief update with weighted coefficients to update the belief of secondary user during any j^{th} stage is given by,

$$p_j(\mathrm{ON}) = \frac{w_1\left(\mu_d^r\right)_j\left(P_d^r\right)_{j-1}}{w_1\left(\mu_d^r\right)_j\left(P_d^r\right)_{j-1} + w_0\left(1-\mu_f^r\right)_j\left(1-P_f^r\right)_{j-1}} \tag{28}$$

where $\left(P_d^r\right)_{j-1}$ and $\left(P_f^r\right)_{j-1}$ is the detection and false alarm probability obtained using RED algorithm at the (j-1)th sensing instant, $\left(\mu_d^r\right)_j$ and $\left(\mu_f^r\right)_j$ is the detection and false alarm probability obtained for RED algorithm with noise uncertainty obtained using the upper threshold. w_0 and w_1 are the weighing factors to highlight the priority of primary user activity. If the secondary user decides that the primary user is ON due to noise uncertainty, then it loses the spectrum or additionally it may incur sensing cost if it chooses to sense. Therefore, it loses a total of $G + C_s$. Conversely, if it decides that the primary user is OFF, the secondary user may incur a total loss of $G + P$. Based on these factors, the weighing factors can be defined as,

$$w_1 = \frac{P+G}{P+2G+C_s} \tag{28}$$

$$w_0 = \frac{C_s+G}{P+2G+C_s} \tag{29}$$

Thus the strategy for the secondary user can be set based on its own belief about the primary user. To summarize, the actions taken by the secondary user are given as follows:

For the primary user,

- If there is a need to transmit, then access the spectrum.
- The probability of being ON for the primary user is p_1 and being OFF is 1-p_1.

For the secondary user,

- If there is a need to transmit,
  - Sense the spectrum and compute the decision statistic Y.
  - If $Y > \lambda_u$, then access the spectrum. The payoff is E(s) = G
  - If $Y < \lambda_l$, wait for the next sensing event. The payoff is E(s) = R
  - If $\lambda_l < Y_m < \lambda_u$, The payoff is given by Equations (10-12) with $p_1 = p_1^j$.

IV. SIMULATION RESULTS AND DISCUSSIONS

In this section, first the theoretical and simulated results of RED algorithm are presented. Then the simulated results of the RED algorithm with noise uncertainty are presented. Simulated results are obtained using Matlab with 100000 Monte Carlo iterations.

First the optimum p is identified for the RED algorithm. Then the ROC performance of the RED algorithm for the optimum p is compared with the existing algorithms. Then the total error probability ($P_{md} + P_f$) and the sample complexity of the RED algorithm against SNR is obtained numerically and compared with existing algorithms. Finally, the expected payoff for the secondary user is analyzed for varying probability of primary user being ON and the efficacy of the proposed decision making under noise uncertainty is justified.

a. Identifying optimum value of p

In Fig. 3 and Fig. 4, we obtain the curve of P_d against p for the RED and TED algorithms. It is observed for the TED algorithm that there exists a maximum P_d for a particular value of p. This is the optimum p which improves the detection performance for the given P_f, N and SNR. Similarly, a global maximum value is observed for P_d for the RED algorithm and it is greater than the TED algorithm. The optimal p also depends on the past observations (i.e), the number of times the primary signal is actually present M. This is due to the fact that the average mean and variance is a direct function of M.

Thus it can be concluded that the incorporating the past observations with TED greatly improve the detection performance. Figure. 5 and Figure. 6 also show that the optimum value of p depends on the value of P_f.

To illustrate the benefit of the operator 'p' in the RED algorithm, we evaluate the optimum p for varying P_f. As seen from Figure. 7, the optimum p is different for varying P_f. It declines as the false alarm rate increases. Thus the optimum value of p can be stored as an offline table for various spectrum sensing parameters such as SNR, P_f, and N. Based on the requirement, a perfect choice for p can be obtained quickly from the table and sensing can be performed.

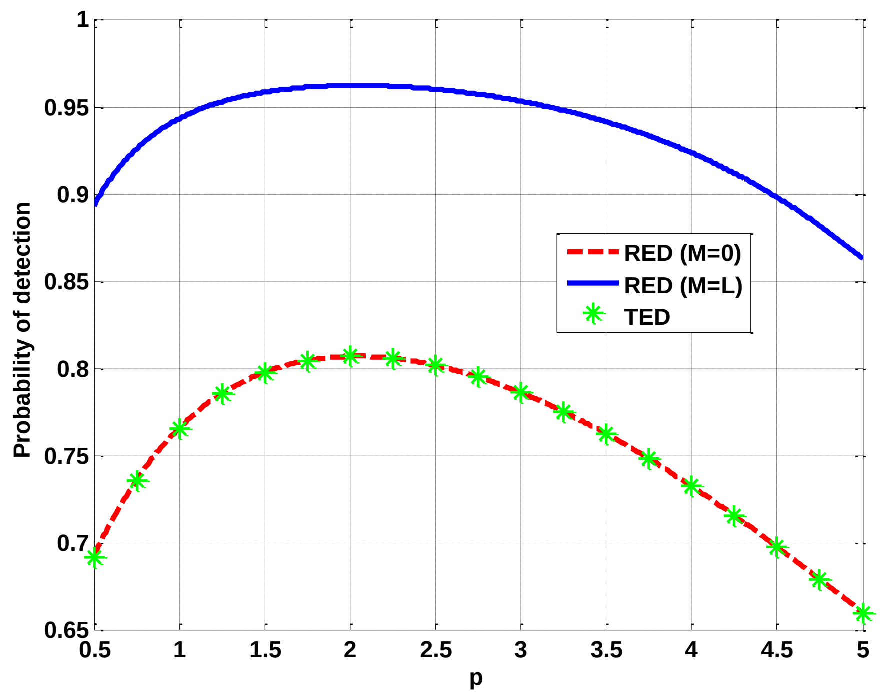

Figure.5 P_d against p for $P_f = 0.1$, $N = 1000$ & SNR = -10dB

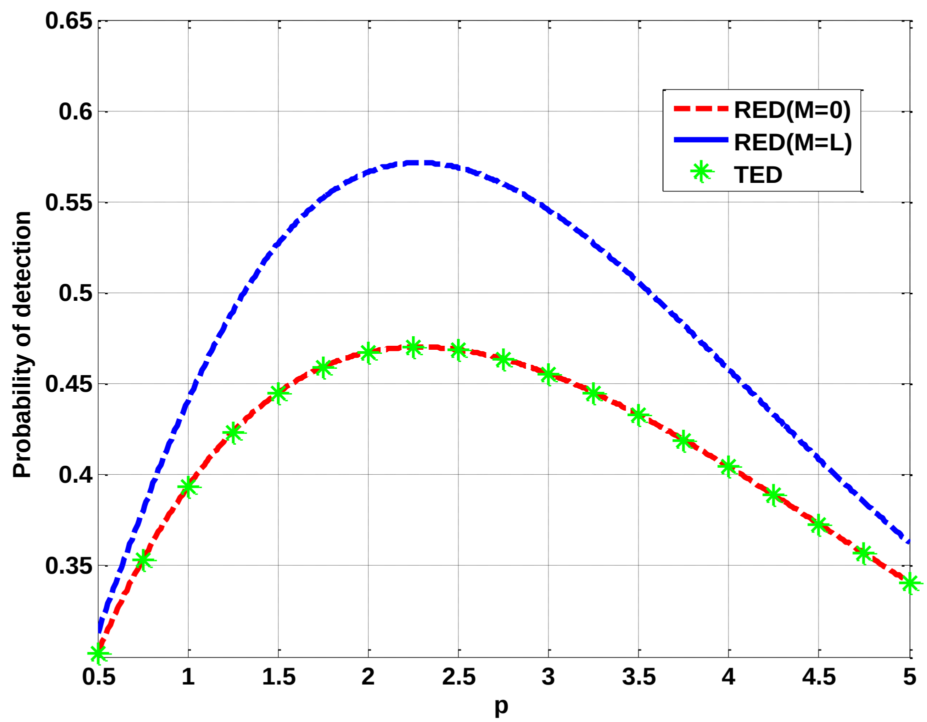

Figure.6 P_d against p for $P_f = 0.01$, $N = 1000$ & SNR = -10dB

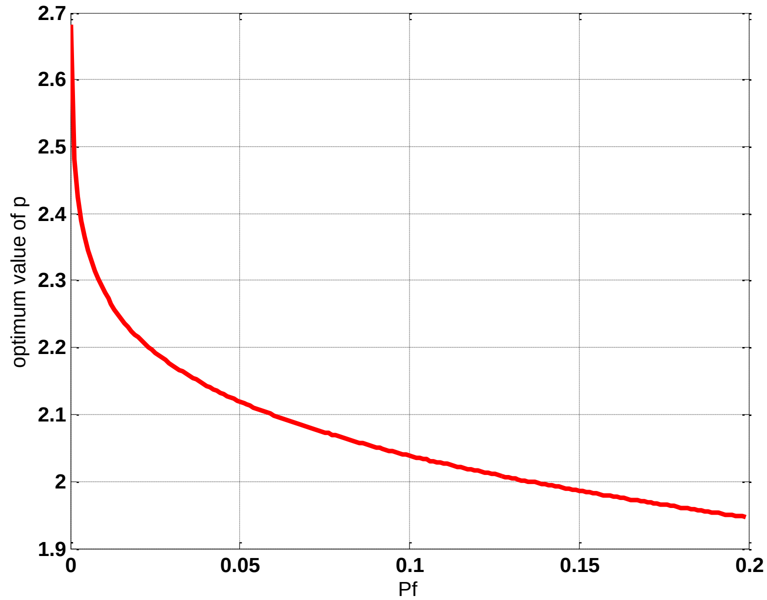

Figure.7 Optimum p against P_f, N = 1000 & SNR = -10dB

b. ROC performance of the RED Algorithm

Figure. 8 and Figure. 9 show the ROC characteristics for the proposed and the existing algorithms.

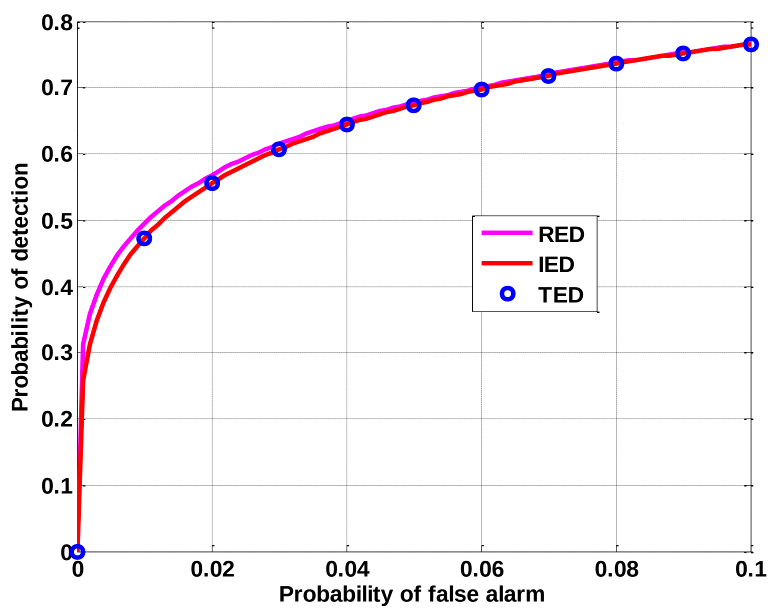

Figure. 8 ROC curves for P_f = 0.1, N = 1000, SNR = -10dB & M =0, L=10

. The ROC curves are obtained using (3), (11), (12), (16) and (17) with P_f = 0.1, SNR = -10dB, N=1000 and M = [0, L]. As it is appreciated, the performance is upper bounded by the RED algorithm. We also consider the IED is the improved energy detection algorithm proposed in [12] for comparison. When M=0, the IED performance coincides with TED because the sensing outcome is mostly based on the current sensing event. But an improved performance is obtained for the same with optimal p using RED. As M increases, errors caused due to misdetections are avoided resulting in an improved performance. For any M, the RED performance is found to be superior.

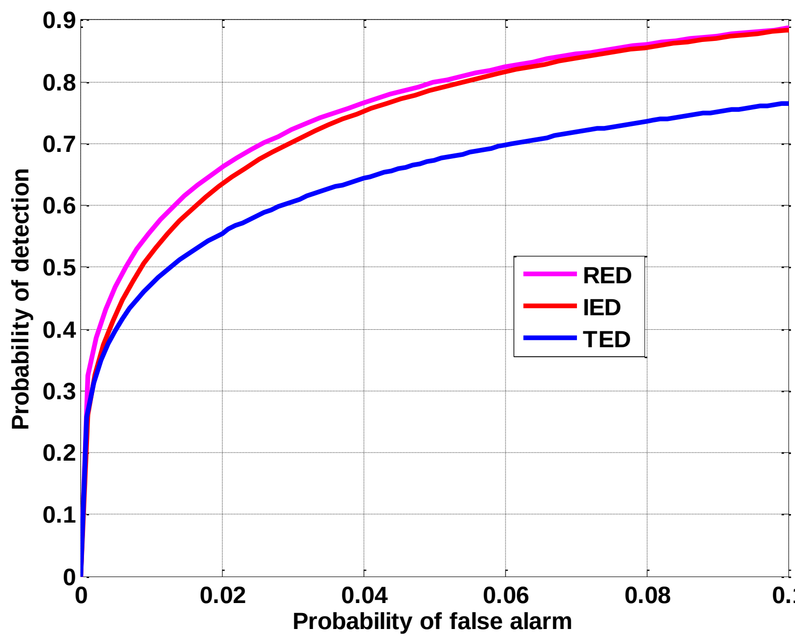

Figure. 9 ROC curves for P_f = 0.1, N = 1000, SNR = -10dB & M =L=10.

c. SNR Performance of the RED Algorithm

The total error probability is observed against SNR in Figure. 10 and Figure. 11 for optimal p, M = [0, L] and fixed P_f, N. For M = 0, the error probability obtained using RED is less for low SNR values and matches with IED and TED as SNR increases. As M > 0, the error probability of RED is the least over the entire range of observed SNR. This is because IED algorithm suffers from significant false alarm degradation.

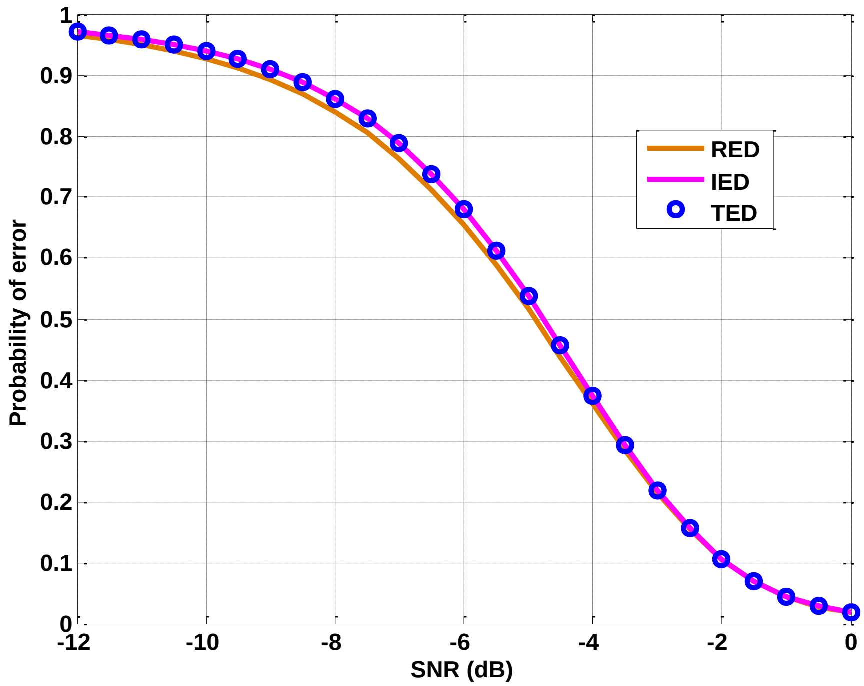

Figure. 10 Probability of error against SNR for P_f = 0.1, N = 1000, SNR = -10dB M =*0* & L=10

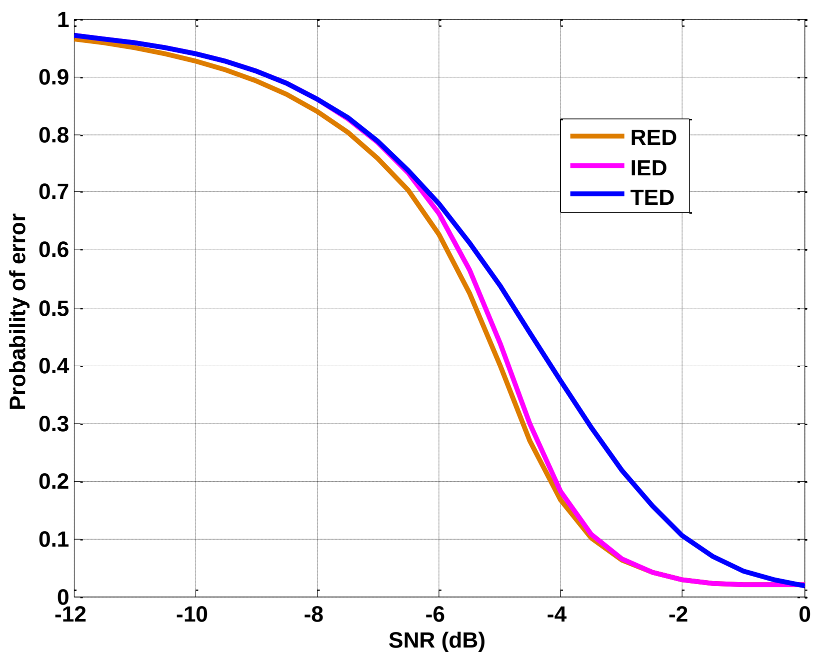

Figure. 11 Probability of error against SNR for P_f = 0.1, N = 1000, SNR = -10dB & M =L =10

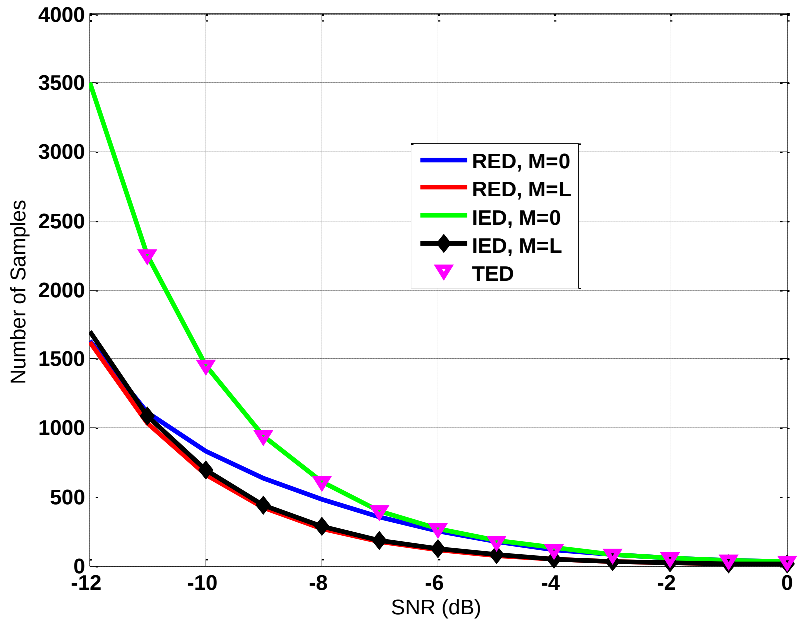

Figure. 12 Number of Samples against SNR for $P_f = 0.1$, $P_d = 0.1$, $N = 1000$

In Figure. 12, the number of samples required for all the three algorithms is observed against SNR for a target P_d and P_f. For TED, the sample complexity is determined easily using eq (3), whereas for IED and RED it is obtained numerically. It is great to note that the proposed RED method requires less number of samples over IED and TED algorithms.

d. RED Algorithm under noise uncertainty

In this section, we conduct numerical analysis and express the secondary user payoff over game stages. The parameters used the game theory analysis are assumed as follows: $P = 100$, $R = 100$, $G = 50$ and $C_s = 20$. The following probability values are assumed for the primary user activity, p_1= 0.05, 0.2, 0.35 and 0.5. The secondary user is not aware of these actual values and hence start with an initial value of $p_1 = 0.5$. As the game progresses, the secondary user updates its belief. Initially, we study the secondary user payoff under various noise uncertainty factors and compare with the TED algorithm. Figure. 13 and Figure. 14 show the secondary user average payoff per stage for the RED algorithm averaged over 100 game stages for *del* = 0.1 and 0.2 respectively. For comparison, the constant payoff obtained using TED algorithm is considered. As illustrated, the proposed game theory based RED algorithm achiever larger payoff even in the presence of noise uncertainty.

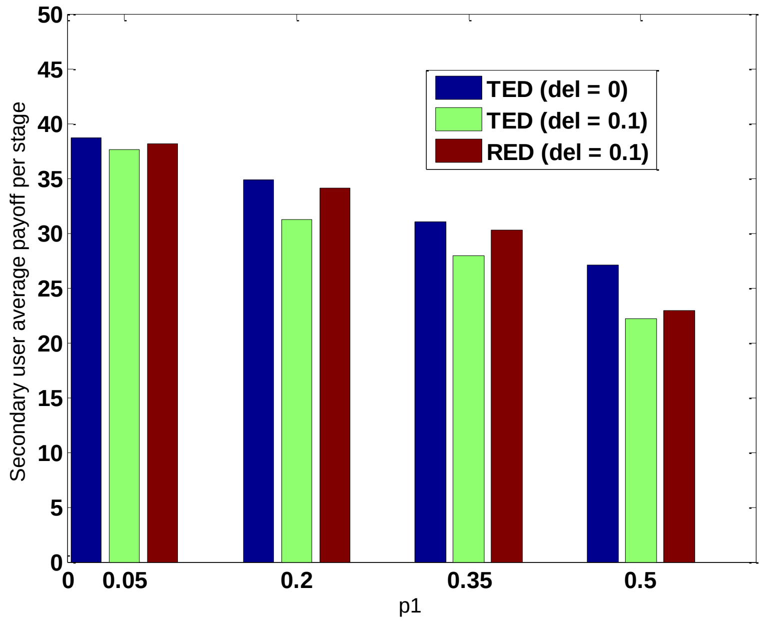

Figure. 13 Secondary user average payoff per stage for *del* = 0.1 dB (P_f = 0.1, N = 1000, SNR = -10dB)

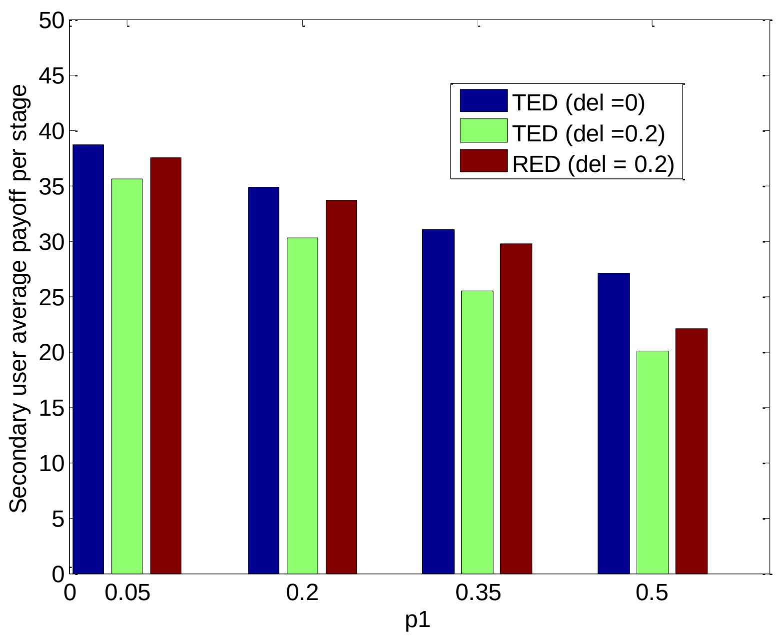

Figure. 14 Secondary user average payoff per stage for *del* = 0.2 dB (P_f = 0.1, N = 1000, SNR = -10dB)

Next, the average secondary user payoff obtained over 100 stages for increasing probability of primary user being ON (p_1) is shown in Figure, 15.

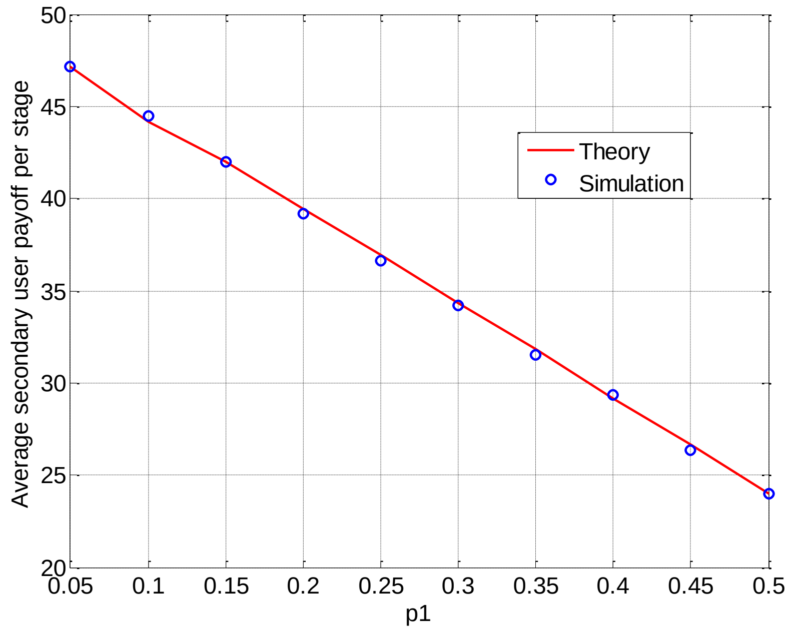

Figure. 15 Secondary user average payoff per stage against probability of primary user being ON (P_f = 0.01, N = 1000, SNR = -10dB, *del* = 0.1 dB)

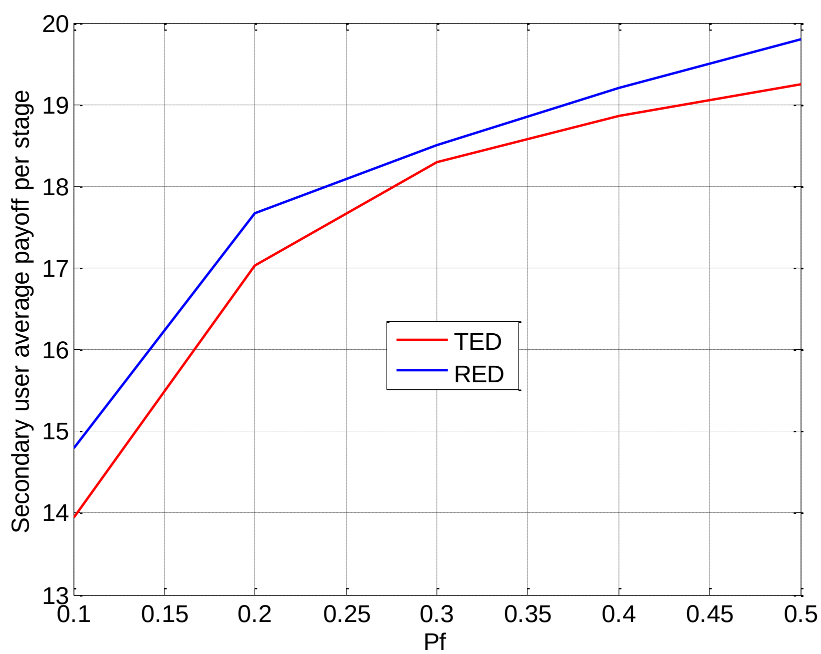

Figure. 16 Secondary user average payoff per stage against probability of false alarm (p_1= 0.5, N = 1000, SNR = -10dB, *del* = 0.5 dB)

The decrease in the average payoff against p_1 is evident as the increase in the primary user being ON will offer less spectral opportunities for the secondary user. Finally, we show that for increasing false alarm probability, the average secondary user payoff initially increases and becomes approximately constant in Figure. 16. This is because for low values of false alarm, the detection probability is also low and increases with increasing false alarm probability as evident from the ROC performance.

V. CONCLUSIONS

In this paper, a RED algorithm is proposed for spectrum sensing to enhance the performance of sensing accuracy. First, the probability of detection and false alarm of the proposed RED algorithm is derived. The optimal value of p which maximizes the detection performance is identified. The ROC performance, sensing error and sample complexity against SNR of the RED algorithm are observed and found to outperform the existing algorithms. Then the RED algorithm is extended for improving the sensing performance against noise uncertainty. A dynamic game is formulated between the primary user and the secondary user in which the primary user has the highest priority. Secondary user sensing strategies are devised when it is unable to make a hard decision regarding the state of the primary user. The expected secondary user payoff is calculated taking into account the actions played by the secondary user. The probability of choosing a particular strategy such that the secondary user payoff is maximized is analyzed. The Bayesian based belief update algorithm with weighted coefficients is used to build a decision about the state of the primary user. Simulation results validate the efficacy of the proposed RED algorithm and found to be superior over the existing schemes in terms of ROC, greater secondary user payoff and better robustness against noise uncertainty.

REFERENCES

[1] Federal Communications Commission, “Spectrum policy task force report”, 2002.

[2] F.Akyildiz, W.Y.Lee and S.Mohanty, “Next generation / dynamic spectrum access / cognitive radio wireless networks: A survey,” Computer Network Journal (Elsevier), vol. 50, pp 2127 – 2159, Sep 2006.

[3] Ying-Chang Liang, Yonghong Zeng, E.C.Y. Peh, Anh Tuan Hoang, "Sensing-Throughput Tradeoff for Cognitive Radio Networks," IEEE Transactions on Wireless Communications, vol.7, no.4, pp.1326-1337, April 2008.

[4] T.Arslan, “A survey of spectrum sensing algorithms for cognitive radio applications," Communications Surveys & Tutorials IEEE , vol.11, no.1, pp.116-130, First Quarter 2009.

[5] Kae Won Choi; Wha Sook Jeon; Dong Geun Jeong, "Sequential detection of cyclostationary signal for cognitive radio systems," IEEE Transactions on Wireless Communications, vol.8, no.9, pp.4480-4485, September 2009.

[6] Zeng.Y, Liang.Y.C, “Spectrum sensing algorithms for cognitive radio based on statistical covariances”, IEEE transactions on Vehicular Technology, 2009, pp 1804-1815

[7] Yonghong Zeng, Ying-Chang Liang, “Eigen value based spectrum sensing algorithms for cognitive radio”, IEEE transactions on communications, vol.57, issue 6, June 2009, pp. 1784 – 1793.

[8] El-Khamy, S.E, El-Mahallawy, M.S.; Youssef, E.S., "Improved wideband spectrum sensing techniques using wavelet-based edge detection for cognitive radio," International Conference

on Computing, Networking and Communications (ICNC), 2013, pp.418,423, 28-31 Jan. 2013.

[9] Smitha.K.G, Vinod.A.P, “Low Power DFT Filter bank based two stage spectrum sensing”, in Proc. of International Conference on Innovations in Information Technology, March 2012, pp 173-177.

[10] Urkowitz,H “Energy detection of unknown deterministic signals,” in Proc. of IEEE, vol. 55, April 1967, pp. 523—531.

[11] Digham F. F., M.-S. Alouini, and M. K. Simon, “On the energy detection of unknown signals over fading channels," in Conf. Rec. IEEE Int. Conf Commun. (ICC’03), Anchorage, AK, USA, May 2003, pp. 3575-3579.

[12] M.López-Benítez, F.Casadevall, "Improved energy detection spectrum sensing for cognitive radio," IET Communications, vol.6, no.8, pp.785-796, May 2012.

[13] Chen.Y, “Improved energy detector for random signals in Gaussian noise”, IEEE Transactions on Wireless Communications, vol.9, no.2, pp 558-563.

[14] J.Song, Z.Feng, P.Zhang, Z.Liu, "Spectrum sensing in cognitive radios based on enhanced energy detector," IET Communications, vol.6, no.8, pp.805-809, May 2012.

[15] Tandra, R.; Sahai, A., "SNR Walls for Signal Detection," IEEE Journal of Selected Topics in Signal Processing, vol.2, no.1, pp.4,17, Feb. 2008.

[16] Jouini, W., "Energy Detection Limits Under Log-Normal Approximated Noise Uncertainty," IEEE Signal Processing Letters, vol.18, no.7, pp.423,426, July 2011

[17] W.Yin, P.Ren and J.Cai, “Performance of energy detector in the presence of noise uncertainty in cognitive radio networks,” Wireless Networks, pp. 1-10, Aug 2012.

[18] Kalamkar, Sanket S.; Banerjee, Adrish, "On the performance of generalized energy detector under noise uncertainty in cognitive radio," National Conference on Communications (NCC), 2013, vol., no., pp.1,5, 15-17 Feb. 2013.

[19] K.Muthumeenakshi and S.Radha, "Optimal Techniques for Sensing Error Minimization with Improved Energy Detection in Cognitive Radios", International Journal of Smart Sensing and Intelligent Systems, Vol.7, No.4, Dec 2014.

[20] J. Vartiainen, M. Höyhtyä, R. Vuohtoniemi and V. L. Ramani, "The future of spectrum sensing," 2016 Eighth International Conference on Ubiquitous and Future Networks (ICUFN), Vienna, 2016, pp. 247-252.

[21] Wei Zhang; Mallik, R.K.; Letaief, K., "Optimization of cooperative spectrum sensing with energy detection in cognitive radio networks," *Wireless Communications, IEEE Transactions on* , vol.8, no.12, pp.5761,5766, December 2009.

[22] Maharjan, S.; Chau Yuen; Yong Huat Chew; Yan Zhang; Gjessing, S., "Distributed spectrum sensing for cognitive radio networks with heterogeneous traffic," 3rd International Symposium on Applied Sciences in Biomedical and Communication Technologies (ISABEL), 2010, vol., no., pp.1,5, 7-10 Nov. 2010.

[23] Ying Dai; Jie Wu, "Whether and when to share: Spectrum sensing as an evolutionary game," 8th International Conference on Cognitive Radio Oriented Wireless Networks (CROWNCOM), 2013 , vol., no., pp.68,73, 8-10 July 2013.

[24] Beibei Wang; Liu, K.J.R.; Clancy, T.C., "Evolutionary cooperative spectrum sensing game: how to collaborate?," *Communications, IEEE Transactions on* , vol.58, no.3, pp.890,900, March 2010.

[25] Zhu Ji; Liu, K.J.R., "COGNITIVE RADIOS FOR DYNAMIC SPECTRUM ACCESS - Dynamic Spectrum Sharing: A Game Theoretical Overview," *Communications Magazine, IEEE* , vol.45, no.5, pp.88,94, May 2007.

[26] Aram, A.; Sarkar, S.; Singh, C.; Kumar, A., "A coalitional game framework for cooperative secondary spectrum access," *Communication, Control, and Computing, 2008 46th Annual Allerton Conference on* , vol., no., pp.1154,1160, 23-26 Sept. 2008.

[27] Tan, Y.; Sengupta, S.; Subbalakshmi, K.P., "Primary user emulation attack in dynamic spectrum access networks: a game-theoretic approach," *Communications, IET* , vol.6, no.8, pp.964,973, May 22 2012.