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## **ANALYSIS OF POSSIBILITIES FOR THE USE OF VOLUME-DELAY FUNCTIONS IN THE PLANNING MODULE OF THE TRISTAR SYSTEM**

**Summary.** Travel time is a measure commonly used for traffic flow modelling and traffic control. It also helps to evaluate the quality of traffic control systems in urban areas. Traffic control systems that use traffic models to predict changes and disruptions in vehicle flows have to use vehicle speed-prediction models. Travel time estimation studies the effects of traffic volumes on a street section at an average speed. The TRISTAR Integrated Transport Management System, currently being deployed across the Tri-City (Gdansk, Sopot, Gdynia), is almost completed and data obtained from the System can be useful for the development of prediction models. A procedure for travel speed model selection for the Tri-City street network is presented in this paper. Matching of chosen volume-delay functions to the data obtained from the TRISTAR has been tested. Analyses have shown insufficient matching of functions that does not justify the possibility of their use in traffic control due to variability in different conditions of traffic, weather and, in the case of an incident, which justifies the need for further research aimed at satisfying matching of functions depending on the above-mentioned factors.

### **1. INTRODUCTION**

Control and traffic management systems that use modern technologies provide better management of the existing transport infrastructure. American and European studies prove that ITS solutions offer significant benefits for the transport network including better safety and traffic flow, which translates into reduced fuel consumption (and fewer emissions of harmful substances into the environment) [1]. Because transport in urban areas is extremely susceptible to all sorts of disruptions (accidents, stationary traffic, closing of road sections), efficient transport management requires tools for collecting up-to-date traffic data and information (some of this information should then be passed on to drivers and passengers).

For this purpose, road infrastructure is equipped with various kinds of sensors for measurement of basic traffic parameters. The information generated by these devices provides a basis for decision-making on a regular basis to minimize the effects of a particular incident.

Decisions taken by road authorities affect the entire transport network. A wrong decision has its consequences and may reduce the overall efficiency of transport networks. This affects transport costs and may deteriorate safety under certain circumstances. For this reason, transport management departments must have adequate tools to support their decisions. Multi-level urban transport models are the answer, providing support in areas of strategic, tactical and operational control [2].

The models should be supplied with data from ITS. This ensures that the results are reliable and offer a valuable support tool for transport authorities in cities. One of the answers that transport

models are expected to provide is how an incident in the network will alter traffic assignment on the road network. In this way, appropriate control scenarios can be prepared in advance to minimize the impact of the event on the transport network. The main factor influencing traffic assignment on the network in transport models is travel time, mapped by the so-called volume-delay functions. The choice of these functions is extremely important because it significantly affects subsequent decisions taken in the control and management of transport in the area covered by the control system.

The Planning Module supported by Multi-level Transport Model plays a significant role in the TRISTAR system. It is possible to use data from databases of the TRISTAR system more effectively by using traffic modeling tools at multiple levels of detail. The authors have developed mechanisms to power the traffic models directly from the database of TRISTAR. It required verification of VDF functions used in the models to better reflect the traffic parameters under different conditions. This will be taken into account in the models (e.g. occurrence of heavy rainfall), and thus the traffic control algorithms can be adjusted for signaling with the use of simulation (modeling) tools.

## 2. APPLICATION AND REVIEW OF THE VOLUME-DELAY FUNCTIONS

In the macroscopic approach, traffic flow is described by parameters such as traffic volume, traffic density and speed. The relationship between these parameters, density, speed and volume, is described by the so-called fundamental diagram of traffic flow [3,4]. The objective of transport modelling system authors is to provide the best possible description of traffic flows. One of the most important elements of such a model is the so-called volume-delay function (VDF). This function refers to a fundamental diagram and aims to describe the relationship between the speed of traffic flow and its volume. The correct determination of the volume-delay function is very important due to its strong effect on the results and, as a consequence, on the reliability of the model.

In the classic approach, transport models are constructed using a four-phase methodology with volume-delay function elements used in two phases of the four-phase classical model:

- when modelling a non-pedestrian modal split journey, where the modal split function is a function of the quotient of travel costs of particular modes of transport [5]; travel cost is usually accepted as the product of travel time of individual transport and generalized public transport travel time;
- in traffic assignment, especially macroscopic and sometimes mesoscopic models. This is because the network seeks equilibrium as formulated by Wardrop [6]; it describes the relationship between the choice of route to complete the journey between A and B, and the travel time on different routes.

Research on volume-delay functions was carried out in the 1950s. Most of the functions showed a highly simplified Speed–Volume relationship. The linear function (later modified) by Irwin Dodd and Von Cube is an example [7]. In subsequent years, many new, non-linear volume-delay functions were developed (Smock, Soltman or Overgaard) [7,8,9]. Today, when reviewing the literature, we can find many different volume-delay functions. Some of these functions, thanks to their versatility, have become popular and are used worldwide. This does not mean that the work on the development of new VDF models was abandoned. In the literature, there are various functions that were created while developing a travel model for a particular area [10, 11]. In such cases, usually, there is no verification of the correctness of such a function in another area for which there is no certainty that the relationship determined for the specific case will show the characteristics of another area in an appropriate manner.

In the 1990s, Spiess characterized the elements that a desired volume-delay function should have: [12]:

- the value of the function should increase with volume;
- the value of the function for the volume equal to 0 should be equal to the travel time at the speed of the free flow and for the volume equaling capacity, time should be twice as long;
- the function should have a non-decreasing derivative;

- the derivative of the function at a flow equaling capacity should equal  $\alpha$ . The parameter  $\alpha$  is equivalent to the exponent in the BPR function and defines the intensity of the changes in the travel time when the capacity is exceeded;
- the derivative of the function should be less than  $M\alpha$ , where  $M$  is a positive variable. It is associated with a reduction of the function slope when the capacity is exceeded to reduce the value adopted by the function; and
- the value of the derivative for volume equal to 0 should be greater than 0.

Evaluation of the function should not take more computing time than that for the BPR function. Currently, the most commonly used functions are compound functions that are based on the following:

- free speed of vehicles established depending on the geometry and traffic organization;
- optimal speed of traffic flow (at maximum volume) depending on the class and geometrical parameters of the road and traffic organization;
- changes in speed when capacity is exceeded.

The shape of the change curve of vehicle speed between the points defining free and optimal speed is determined using travel model parameters. Statistical methods (nonlinear regression) are applied. Below, the most popular VDF functions used in traffic models are presented.

Abbreviations used in the description of the function are as follows:

- T – travel time,
  - $T_0$  – travel time in free flow,
  - V – traffic volume,
  - $C_p$  – practical capacity with PRS section equal to C,
  - C – full capacity
- $\alpha, \beta, \gamma, \varepsilon, \sigma, J, c$  – estimated parameters of the model.

The BPR function developed by the US Bureau of Public Roads (BPR), from which its name is derived, is the best known and most widely used function in transport modelling. It has many variants that differ in shape when capacity is exceeded [13].

BPR:

$$T_{cur} = T_0 \left( 1 + \alpha \left( \frac{V}{C} \right)^\beta \right) \quad (1)$$

BPR2:

$$T_{cur} = T_0 \left( 1 + \alpha \left( \frac{V}{C} \right)^\beta \right) \quad \text{for } V < C \quad (2)$$

$$T_{cur} = T_0 \left( 1 + \alpha \left( \frac{V}{C} \right)^{\beta'} \right) \quad \text{for } V > C \quad (3)$$

BPR3:

$$T_{cur} = T_0 \left( 1 + \alpha \left( \frac{V}{C} \right)^\beta \right) \quad \text{for } V < C \quad (4)$$

$$T_{cur} = T_0 \left( 1 + \alpha \left( \frac{V}{C} \right)^\beta \right) + (V - C) * \gamma \quad \text{for } V > C \quad (5)$$

The Conical Function was published in the 1990s by H. Spiess as an extension of the BPR function. It offers a better match when capacity is exceeded [12]:

$$T_{cur} = T_0 \left( 2 + \sqrt{\alpha^2 \left( 1 - \frac{V}{C} \right) + \beta^2} \right) - \alpha \left( 1 - \frac{V}{C} \right) - \beta + V * \gamma \quad (6)$$

The INRETS Function, developed by the French National Transport Research Institute, is as follows:

$$T_{cur} = T_0 \left( \frac{1,1 - \alpha * \left( \frac{V}{C * c} \right)}{1,1 - \left( \frac{C}{C * c} \right)} \right) \quad \text{for } V < C * c \quad (7)$$

$$T_{cur} = T_0 \left( \frac{1,1 - \alpha * \left( \frac{V}{C * c} \right)^2}{0,1} \right) \quad \text{for } V > C * c \quad (8)$$

The DAVIDSON Function, developed by the Davidson from the queuing theory [14], is as follows:

$$T_{cur} = T_0 \left( 1 + J \left( \frac{V}{C - V} \right) \right) \quad (9)$$

The VATZEK Function developed and tested by Jastrzębski [11] is as follows:

$$T_{cur} = T_0 * \left( 1 + \alpha * \left( \left( \frac{V}{C} - \sigma \right)^\beta + \sigma^\beta \right) + \varepsilon * \frac{V}{C} \right) \quad \text{for } V \leq C \quad (10)$$

$$T_{cur} = T_0 * \left( 1 + \alpha * \left( \left( \frac{V}{C} - \sigma \right)^\beta + \sigma^\beta \right) + \varepsilon * \frac{V}{C} + (V - C) * \gamma \right) \quad \text{for } V \leq C \quad (11)$$

### 3. DATA-COLLECTION METHOD

Thanks to modern technology, road traffic can be measured quite precisely. Measuring devices are an indispensable element of ITS systems. Some of the most common sensors are inductance, infrared and radio waves operated. They can not only identify the vehicle but also estimate the basic parameters (instantaneous speed and length). Vision techniques (ANPR cameras), on the other hand, can identify and track a vehicle by license plates between network sections fitted with a measuring device that also establishes its section (travel) speed.

Some of the technology discussed above was used in the TRISTAR – Integrated Transport Management System in the Tri-City (Gdańsk Gdynia and Sopot). Devices have been installed for automatic registration of multiple parameters including the speed of vehicles. The system comprises a number of fundamental devices. The traffic measurement station is one of the elements of a traffic light controller. It consists of inductive sensors (2 measuring loops 1 m apart from each other) mounted on each lane of the junction (entry and exit). It can measure parameters such as the following:

- vehicle direction;
- vehicle speed;
- vehicle length;
- distance between vehicles;
- time spent at the station.

ANPR cameras are devices for recognizing vehicle registration numbers. Because the devices are deployed at the Tri-City's critical junctions, it is possible to estimate the time of travel between measurement points. Each measurement point allows for the collection of data such as the following:

- time at the measurement point
- time at the next measurement point, i.e. the time between the two points.

Bluetooth/Wi-Fi Scanners have been installed on roads covered by the road accident detection system. They detect devices equipped with Bluetooth/Wifi technology (phones, hands free sets). Their principle of operation is identical to that of ANPR cameras. Detected devices have their own individual registration number: the MAC. Speed over sections is determined based on the detection of the time difference between the station's measuring devices. Due to the low accuracy of the solution, it can only be used to detect variations in the speed of flows and in addition to the ANPR system for estimating the travel time.

Since the various measuring devices became operational, the data that they collect have been saved and stored in TRISTAR's data warehouse. It provides easy reference to specific historic data and helps to map specific traffic situations within the system's catchment. Thanks to this, changes in urban transport can be studied.

#### 4. DATA ANALYSIS

The TRISTAR system data became fully available to researchers from the beginning of this year. No data quality analysis has been carried out. The only factor that was checked before the beginning of VDF modelling was comparison with the fundamental traffic flow diagram. Analysis of Fig. 1-3 shows that the shape of the data is as should be expected.

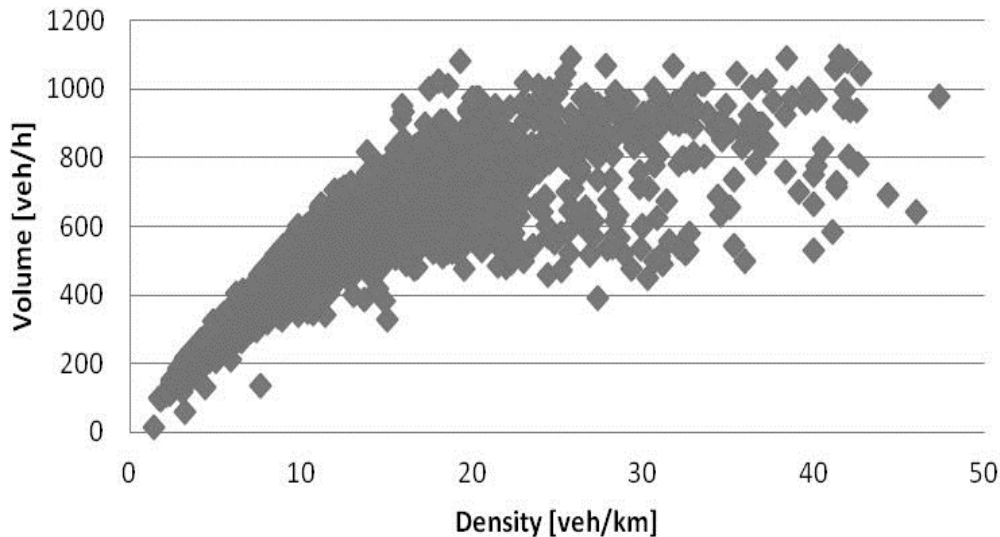


Fig. 1. Volume – Density diagram

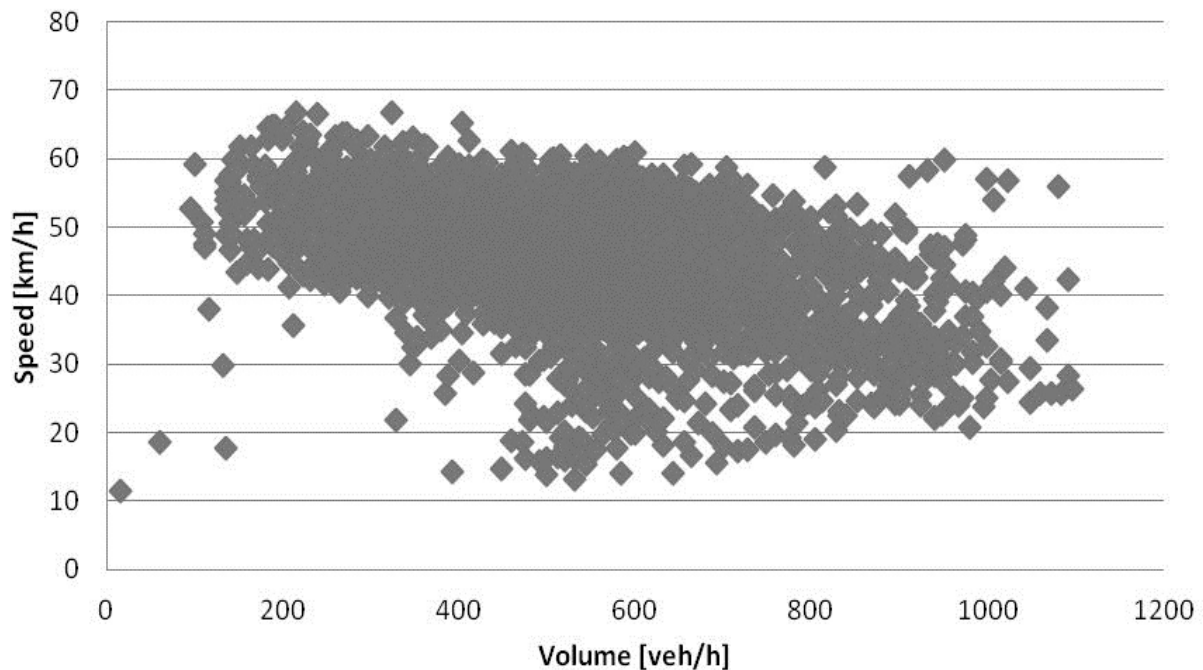


Fig. 2. Speed – Volume diagram

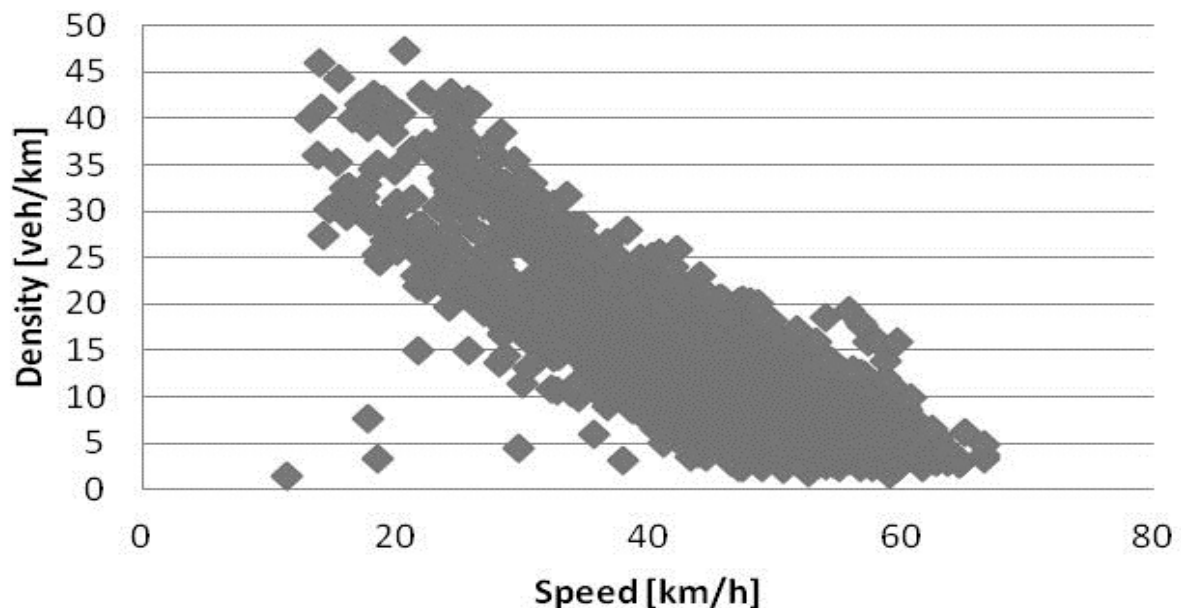


Fig. 3. Density – Speed diagram

To evaluate the volume-delay function, data from the measurement stations of the TRISTAR System were used. The test site covered Gdynia's main roads with 37 signalized junctions in total included in the traffic control system. The data were obtained from a data warehouse from October 2014 for indicative days of the week (i.e. Tuesday, Wednesday, Thursday). The data were aggregated into intervals of 15 minutes, which yielded approx. 50 thousand measurements on each lane of a junction.

Data that were used for the purpose of this article were collected from most of the GdyniaTraffic Measurement Stations (TMS) that were implemented in the TRISTAR system (70 TMS in Gdynia and more than 150 in the Tri-City area).



Fig. 4. The area covered by the TRISTAR in Gdynia system with marked analyzed sections of streets

Table 1

Analyzed sections of streets

Route	Cross-section	Class (Polish)	Status
Morska Section Cisowa	2x2	G	National
Morska Section Grabówek	2x2	G	Regional
Wielkopolska Section Mały Kack	2x2	Z	Regional
Wielkopolska Section Karwiny	2x2	Z	Regional

## 5. VDF MODELING

Free flow speed is the speed selected by a driver in the absence of the influence of other vehicles. In the literature, there are different approaches for determining free flow speed. In one of them, free flow speed is determined based on the velocity of vehicles in free flow traffic i.e. at low density in the section (10 vehicles/km) and the localization of a vehicle from the others (distance measured by time) [15]. The other approach is to define free flow speed (FFS) as speed that is chosen on the basis of the law limit [16].

The authors decided to use the first approach to establish free flow speed. Using the data from the traffic measurement stations in the first approach, the researchers estimated the FFS as  $\sim 60$  km/h for all analyzed roads and used this value for further analysis. These values were used to estimate VDF parameters. Using the established free flow speed value, however, the shape of the models differed from what the literature suggests. There is a significant decline in velocity when the value for density is low. On analyzing this, it was proved that collection of data at night influences the results. In Poland, the daytime (5.00AM-11.00PM) speed limit in built-up areas is 50 km/h and increases to 60 km/h at night. By analyzing the cumulative distribution of velocities in different sectors (see Fig. 5) during day and night, we can observe that the graphs shift by 10 km/h. This suggests that current legislation influences how drivers react at different times of the day. Based on these conclusions, it was decided not to include data gathered when the maximum speed limit applied. As we continued to establish the free flow speed for the 85% quantile of daytime measurements including measurements for density less than 5 vehicles/km, the result was  $\sim 50$  km/h for every sector of the roads analyzed. This value was established as the free flow speed in further modelling of the sector's drag.

The next element required for correctly marking the shape of the sector's drag is its capacity. Several different approaches for establishing this value may be found in the literature:

- according to the VDF rules coined by Spiess [12], the velocity at the point of capacity should equal half of the free speed. Capacity can hence be calculated based on the vehicles' velocity;
- establishing theoretical capacity e.g. using the HCM guidelines [17];
- making an assumption that practical capacity is the quantile of the maximal density per hour.

In the case of Gdynia, the capacity was calculated in a manner similar to the last approach while taking into consideration the traffic control system, which, in the macro scale, adjusts the parameters of area control every 5 minutes. It is impossible to establish the theoretical capacity for a junction with traffic lights. This is why the TRISTAR traffic control module that implements surveillance cameras was used. Based on the archived visual data, the researchers identified specific hours during which traffic density was approaching full capacity. The density measured during those specified hours was assumed to be the practical capacity. This value for all chosen road sectors is  $\sim 1000$  vehicles/h/lane.

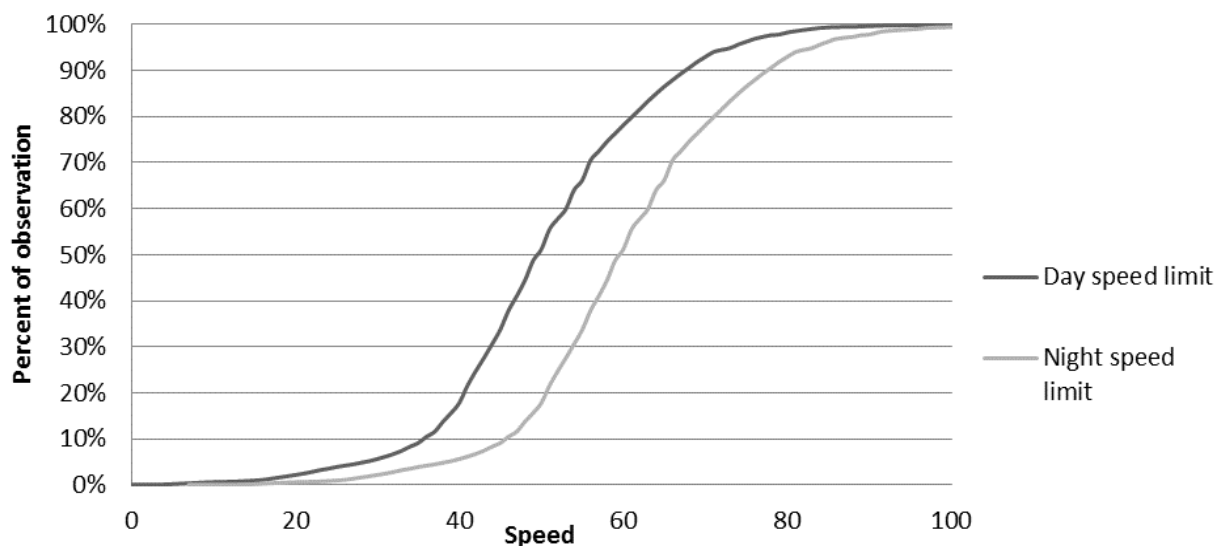


Fig. 5. Cumulative distribution of speed for two speed limits

The parameters of the sector's drag function were determined using STATISTICA software. Three most commonly used functions were modelled: BPR, CONICAL and INRETS. The results of the estimation of the functions' parameters are shown in Table 2. An exemplary graph is shown in Fig. 6. It is clear that of all available options, the CONICAL function has the best match, but insufficient for use for function in the current traffic control, requiring higher accuracy.



Table 2

Results of the VDF estimation for sections of streets

Parameter	$\alpha$	$\beta$	$\gamma$	$c$	R	MSE
<i>Morska Section Cisowa</i>						
BPR	-0,31	1,52	–	–	0,63	16,7
CONICAL	2,34	1,42	-0,002	–	0,65	15,6
INRETS	-5,4	–	–	-0,03	0,61	16,9
<i>Morska Section Grabówek</i>						
BPR	-0,48	1,39	–	–	0,78	17,9
CONICAL	2,34	1,42	-0,002	–	0,79	17,6
INRETS	-3,4	–	–	-0,1	0,75	17,8
<i>Wielkopolska Section Mały Kack</i>						
BPR	-0,31	1,73	–	–	0,70	13,6
CONICAL	2,43	1,39	-0,001	–	0,76	16,1
INRETS	-9,59	–	–	-0,02	0,68	16,1
<i>Wielkopolska Section Karwiny</i>						
BPR	-0,35	1,23	–	–	0,83	8,02
CONICAL	1,68	1,71	-0,013	–	0,83	8,01
INRETS	-8,19	–	–	-0,03	0,82	8,56

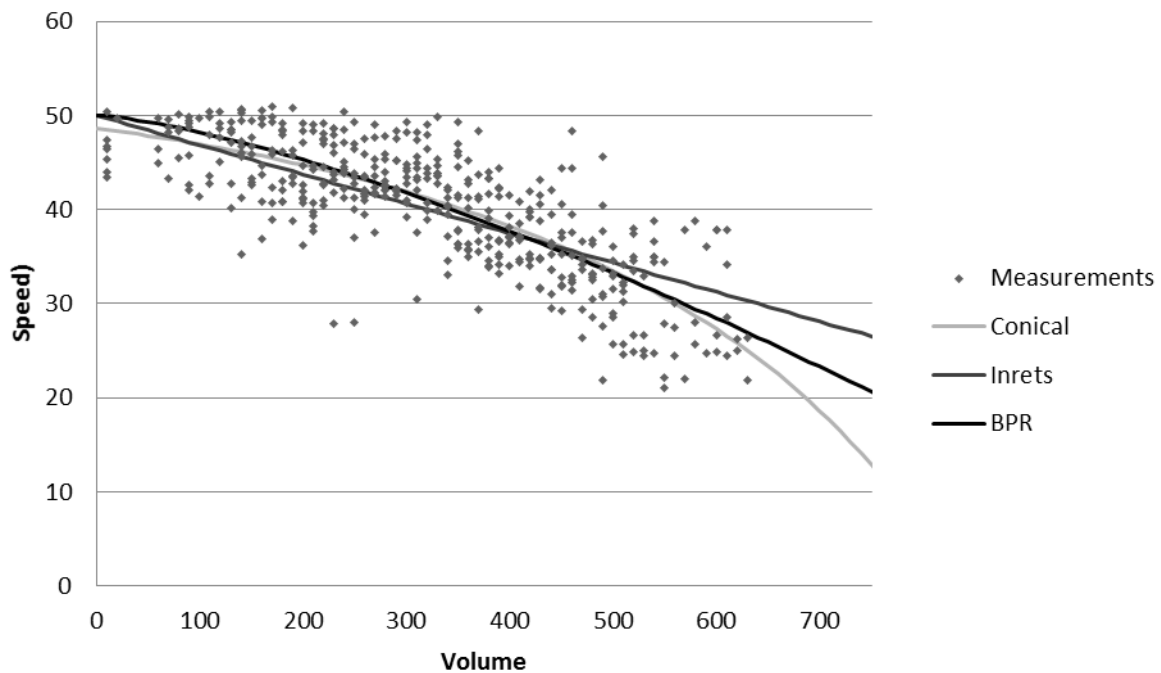


Fig. 6. Example of estimated VDF models

To validate the developed models, a test data set was chosen from the TRISTAR system's historical databases. The chosen data were from October 2015. Researchers sought to check data from a similar time period of the year. Thanks to this approach, the factors that influence driver behavior and occur only at specific times of the year could be taken into account.

Researchers developed new model parameters from a new data set and compared them with the original model. The comparison is shown below in Fig. 7 and Tab. 3.

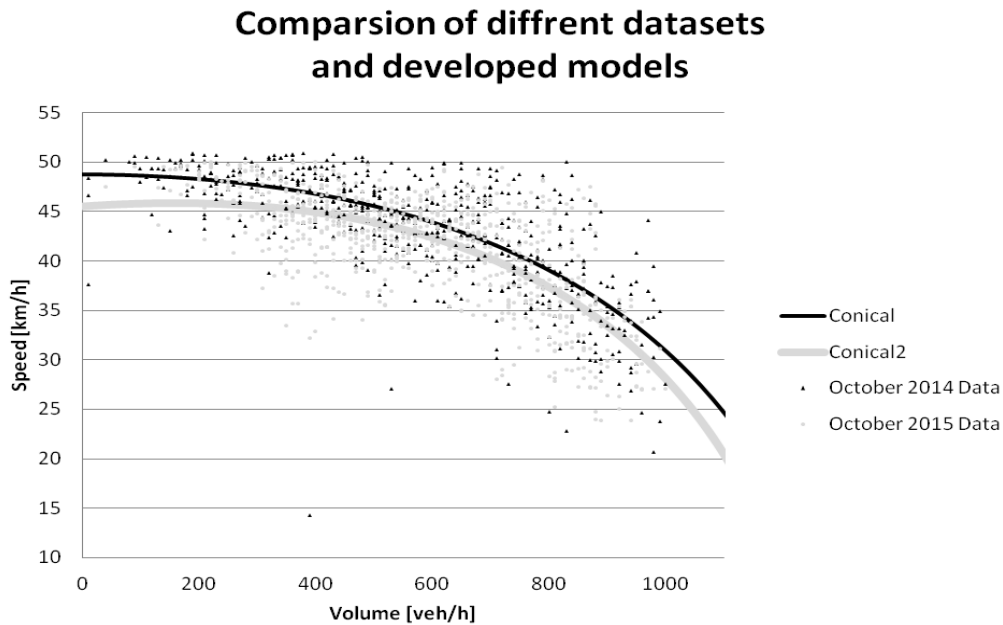


Fig. 7. Comparison of different datasets and developed models, Wielkopolska route section Mały Kack

Table 3

Comparison of estimated parameters for selected roads

<b>Wielkopolska - Mały Kack Section</b>				
	Estimated Parameters		Correlation	
	2014	2015	2014	2015
$\alpha$	2,342513	2,236203	0,78	0,77
$\beta$	1,427936	1,546694		
$\gamma$	-0,002358	-0,002202		
<b>Morska - Cisowa Section</b>				
	Estimated Parameters		Correlation	
	2014	2015	2014	2015
$\alpha$	2,431314	2,718506	0,75	0,7
$\beta$	1,399088	1,455178		
$\gamma$	-0,001382	-0,001438		

The estimated parameters from the October 2015 data set are very similar to the parameters estimated from the October 2014 data. Also, the correlation level is similar in both functions developed. To compare two data sets, the Kolmogorov–Smirnov test was used. The result of this test showed that the confidence interval of both data sets meshed together; thus, the difference in the chosen data is not significant.

## 6. CONCLUSIONS

This paper presents the first attempts to model travel time using data gathered by the ITS system. Free speed was concluded to be almost equal to the current speed limit. This fact and the current legislation (the speed limit in built-up areas varies depending on the time of the day) were found to influence the results. At lower densities, two point clouds may be observed for velocities averaging

50 km/h and 60 km/h. The sector's drag functions determined have an R compliance level of 60-80%. Of all functions analyzed, it was established that the CONICAL provides best results since its shape most closely resembled the measured points, but was insufficient for use for the function in the current traffic control, requiring higher accuracy. The initial results suggest that the models analyzed achieved a very low level of compliance with the measured values, and thus cannot be used in traffic models used for traffic control. This is related to overgeneralization of dependence on traffic parameters (density, traffic volumes, speed), which alter dynamically depending on the time of the day (traffic saturation), but also depend on the weather conditions, incidents, etc. This was confirmed by the analysis of a multiple range test that compared a set of average values. On analyzing the data of seven representative days and one special day (31.10), it was found that there are several consistent groups. Also, when the data from another year were used to validate the models developed, the Kolmogorov–Smirnov test for two samples showed that the difference in the samples checked was not significant. Analysis of the factors (traffic incidents, road works, drivers' behavior) that could affect the difference in the shape of the models on low traffic volume and volume near the capacity will be the subject of future researches.

More research, however, is needed to provide measurement errors under various traffic conditions and to find satisfactory VDF function adjustments depending on the traffic, weather conditions and other factors.

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