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# Appendix

## A method for handlebars ballast calculation in order to reduce vibrations transmissibility in walk behind tractors

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## I vibration mode - Horizontal deformation

$K_{h_{TOT}} = \dfrac{F_h}{y_{h_{TOT}}}$ : horizontal bending stiffness of the whole structure,

$y_{h_{TOT}} = B_1 + B_2 + B_3 \mathrm{d}' + \left( A_{13} \mathrm{d}'^2 - A_{14} \mathrm{d}'^3 \right) \cos\left(\gamma - \theta\right)$ : lateral displacement of the handles horizontal ends,

$$y_{h_1} = \theta x + \left[ A_5 x^2 - A_6 x^3 \right] \cos\theta \qquad x \in \left[0, \mathrm{b}\right],$$

$$y_{h_2} = B_1 + \left[ A_9 s^2 - A_{10} s^3 \right] \cos\theta + \theta s + \left[ A_3 \mathrm{b} - A_4 s^2 \right] s \qquad s \in \left[0, \mathrm{c}\right],$$

$$y_{h_3} = B_1 + B_2 + B_3 t + \left[ A_{13} t^2 - A_{14} t^3 \right] \cos\left(\gamma - \theta\right) \qquad t \in \left[0, \mathrm{d}'\right],$$

$\theta = \left[ A_1 \mathrm{a} - A_2 \mathrm{a}^2 \right]$, $B_1 = \theta \mathrm{b} + \left[ A_5 \mathrm{b}^2 - A_6 \mathrm{b}^3 \right] \cos\theta$, $B_2 = \left[ A_9 \mathrm{c}^2 - A_{10} \mathrm{c}^3 \right] \cos\theta + \left[ \theta + A_3 \mathrm{b} - A_4 \mathrm{b}^2 \right] \mathrm{c}$,

$B_3 = \left[ \theta + A_3 \mathrm{b} - A_4 \mathrm{b}^2 + A_7 \mathrm{c} - A_8 \mathrm{c}^2 \right]$, $A_1 = \dfrac{F_h}{G_1 I_1} \left[ \left( \mathrm{c} + \mathrm{d} \right) \cos\beta + \dfrac{\mathrm{b}}{\sqrt{2}} + \mathrm{a} \sin\alpha \right]$, $A_2 = \dfrac{F_h}{2 G_1 I_1} \sin\alpha$,

$A_3 = \dfrac{F_h}{E_2 I_2} \left( \mathrm{c} + \mathrm{d} + \mathrm{b} \right)$, $A_4 = \dfrac{F_h}{2 E_2 I_2}$, $A_5 = \dfrac{F_h}{2 E_2 I_2} \left( \mathrm{c} + \mathrm{d} + \mathrm{b} \right)$, $A_6 = \dfrac{F_h}{6 E_2 I_2}$, $A_7 = \dfrac{F_h}{E_3 I_3} \left( \mathrm{c} + \mathrm{d} \right)$,

$A_8 = \dfrac{F_h}{2 E_3 I_3}$, $A_9 = \dfrac{F_h}{2 E_3 I_3} \left( \mathrm{c} + \mathrm{d} \right)$, $A_{10} = \dfrac{F_h}{6 E_3 I_3}$, $A_{11} = \dfrac{F_h}{2 E_a I_a} \cos\gamma \mathrm{d}'$, $A_{12} = \dfrac{F_h}{4 E_a I_a} \cos\gamma$,

$A_{13} = \dfrac{F_h}{4 E_a I_a} \cos\gamma \mathrm{d}'$, $A_{14} = \dfrac{F_h}{12 E_a I_a} \cos\gamma$ . $l_1 = \mathrm{a} \sin\alpha + \dfrac{\mathrm{b}}{\sqrt{2}} + \mathrm{c} \cos\beta$,

$l_2 = \mathrm{a} \sin\alpha + \dfrac{\mathrm{b}}{\sqrt{2}} + \left( \mathrm{c} + \mathrm{d} \right) \cos\beta$,

$K_{h_p} = \dfrac{F_h}{\left[ B_1 + \left( A_9 \mathrm{c}^2 - A_{10} \mathrm{c}^3 \right) \cos\theta + \theta \mathrm{c} + \left( A_3 \mathrm{b} - A_4 \mathrm{b}^2 \right) \right]}$ : horizontal bending stiffness of the pillar.

$Kr = \dfrac{F_L L^2}{\Delta y_L \left( F_L \right)}$ : torsional bending stiffness of the elastic silent-block.

## II vibration mode - Vertical deformation

$K_{v_{TOT}} = \frac{F_v}{y_{v_{TOT}}}$ : vertical bending stiffness of the whole structure,

$y_{v_{TOT}} = B_6 + A_{23}\mathrm{d'} + A_{24}\mathrm{d'}^2 - A_{25}\mathrm{d'}^3$ : lateral displacement of the vertical ends of the handles,

$y_{v_1} = A_{15}x^2 - A_{16}x^3 \quad x \in [0,\mathrm{a}]$, $y_{v_2} = B_4 + A_{17}s - A_{18}s^2 - A_{19}s^3 \quad s \in [0,\mathrm{b}]$,

$y_{v_3} = B_5 + A_{20}n + A_{21}n^2 - A_{22}n^3 \quad n \in [0,\mathrm{c}]$, $y_{v_4} = B_6 + A_{23}t + A_{24}t^2 - A_{25}t^3 \quad t \in [0,\mathrm{c}]$,

$B_4 = A_{15}\mathrm{a}^2 - A_{16}\mathrm{a}^3$, $B_5 = B_4 + A_{17}\mathrm{b} + A_{18}\mathrm{b}^2 - A_{19}\mathrm{b}^3$ , $B_6 = B_5 + A_{20}\mathrm{c} + A_{21}\mathrm{c}^2 - A_{22}\mathrm{c}^3$ ,

$A_{15} = \frac{F_v}{2E_1I_1}\left[\sin^2\alpha\,\mathrm{a} + (\mathrm{c}+\mathrm{d})\cos\beta\sin\alpha + \frac{\mathrm{b}}{\sqrt{2}}\sin\alpha\right]$, $A_{16} = \frac{\sin^2\alpha F_v}{6E_1I_1}$,

$A_{17} = \frac{F_v}{E_1I_1}\left\{\left[\frac{\sin^2\alpha\,\mathrm{a}}{2} + \left((\mathrm{c}+\mathrm{d})\cos\beta + \frac{\mathrm{b}}{\sqrt{2}}\right)\right]\mathrm{a}\right\}\frac{1}{\sqrt{2}}$, $A_{18} = \frac{F_v}{2\sqrt{2}E_2I_2}\left[(\mathrm{c}+\mathrm{d})\cos\beta + \frac{\mathrm{b}}{\sqrt{2}}\right]$,

$A_{19} = \frac{F_v}{12E_2I_2}$, $A_{20} = \frac{F_v}{E_1I_1}\left\{\left[\frac{\sin^2\alpha\,\mathrm{a}}{2} + \left((\mathrm{c}+\mathrm{d})\cos\beta + \frac{\mathrm{b}}{\sqrt{2}}\right)\right]\mathrm{a}\right\}\cos\beta + \frac{F_v}{E_2I_2}\left[(\mathrm{c}+\mathrm{d})\mathrm{b}\cos\beta\sin\alpha + \frac{\mathrm{b}^2}{2\sqrt{2}}\right]\cos\beta$,

$A_{21} = \frac{F_v}{E_3I_3}\left[(\mathrm{c}+\mathrm{d})\frac{\cos^2\beta}{2}\right]$, $A_{22} = \frac{F_v}{6E_3I_3}\cos\beta$,

$$A_{23} = \left\{\begin{array}{c} \frac{F_v}{2E_1I_1}\sin^2\alpha\,\mathrm{a} + \frac{F_v}{E_1I_1}\left[(\mathrm{c}+\mathrm{d})\cos\beta + \frac{\mathrm{b}}{\sqrt{2}}\right]\mathrm{a} + \frac{F_v}{E_2I_2}\left[(\mathrm{c}+\mathrm{d})\mathrm{b}\cos\beta + \frac{\mathrm{b}^2}{2\sqrt{2}}\right] + \\ \frac{F_v}{E_3I_3}\mathrm{c}\left[(\mathrm{c}+\mathrm{d})\cos\beta + \frac{\mathrm{c}}{2}\right] + \frac{F_v\mathrm{de}}{2G_aI_a} \end{array}\right\}\cos\beta \quad ,$$

$A_{24} = \frac{F_v\mathrm{d'}\cos\beta}{2E_aI_a}$, $A_{25} = \frac{F_v\cos\beta}{12E_aI_a}$.

$K_{v_p} = \frac{F_h}{\left[B_5 + A_{20}\mathrm{c} + A_{21}\mathrm{c}^2 - A_{22}\mathrm{c}^3\right]}$ : vertical bending stiffness of the pillar.

$A_{26} = \sin\alpha$, $A_{27} = \frac{1}{\sqrt{2}}$, $A_{28} = \mathrm{a}\sin\alpha$, $A_{29} = \frac{\mathrm{b}}{\sqrt{2}} + \mathrm{a}\sin\alpha$, $A_{30} = \cos\beta$, $A_{31} = A_{29} + \mathrm{c}\cos\beta$.

**III vibration mode - Torsional deformation**

$K_{t_{TOT}} = \frac{F_v}{y_{t_{TOT}}}$ : torsional bending stiffness of the whole structure,

$y_{t_{TOT}} = B_7 + \mathrm{d}'\theta_2 + A_{34}\mathrm{d}'^2 + A_{33}\mathrm{d}'^3$ : torsional displacement of the handles end,

$y_{t_1} = t\theta_1 + A_{32}t^2 - A_{33}t^3 \quad t \in [0,\mathrm{e}]$, $y_{t_2} = B_7 + s\theta_2 - A_{34}s^2 - A_{33}s^3 \quad s \in [0,\mathrm{d}']$,

$B_7 = \mathrm{e}\theta_1 + A_{32}\mathrm{e}^2 - A_{33}\mathrm{e}^3$, $\theta_1 = \frac{F_v(\mathrm{e} + \mathrm{d}'\sin\gamma)}{G_3 I_3}\left(\frac{\mathrm{b}}{\sqrt{2}} + \mathrm{c}\right)$, $\theta_2 = \frac{F_v \mathrm{de}}{2G_a I_a}$, $A_{32} = \frac{F_v \mathrm{e}}{4E_a I_a}$,

$A_{33} = \frac{F_v}{12E_a I_a}$, $A_{34} = \frac{F_v \mathrm{d}'}{4E_a I_a}$.