Journal of Applied Statistics

Publication details, including instructions for authors and subscription information:
http://www.tandfonline.com/loi/cjas20

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Published online: 23 Apr 2013.

To cite this article: Luca De Angelis & Leonard J. Paas (2013) A dynamic analysis of stock markets using a hidden Markov model, Journal of Applied Statistics, 40:8, 1682-1700, DOI: 10.1080/02664763.2013.793302

To link to this article: http://dx.doi.org/10.1080/02664763.2013.793302

Journal of Applied Statistics, 2013
Vol. 40, No. 8, 1682–1700, http://dx.doi.org/10.1080/02664763.2013.793302

A dynamic analysis of stock markets using a hidden Markov model

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(*Received 29 October 2010; accepted 3 April 2013*)

This paper proposes a framework to detect financial crises, pinpoint the end of a crisis in stock markets and support investment decision-making processes. This proposal is based on a hidden Markov model (HMM) and allows for a specific focus on conditional mean returns. By analysing weekly changes in the US stock market indexes over a period of 20 years, this study obtains an accurate detection of stable and turmoil periods and a probabilistic measure of switching between different stock market conditions. The results contribute to the discussion of the capabilities of Markov-switching models of analysing stock market behaviour. In particular, we find evidence that HMM outperforms threshold GARCH model with Student-*t* innovations both in-sample and out-of-sample, giving financial operators some appealing investment strategies.

Keywords: stock market pattern analysis; regime-switching; hidden Markov model; financial crises; market stability periods

1. Introduction

Investors and analysts tend to classify stock market fluctuations into two categories, namely positive and negative market phases which sometimes are referred to as bull and bear market regimes. However, the reality of stock markets could be more complex than this dichotomous specification and uncertainty about regime-switching may lead to poor investment decisions. In particular, due to the strong fluctuations that occur, the negative market phase may be characterized by rapid switching between different regimes, alternating regimes characterized by large decreases in stock prices with regimes characterized by large price increases. Investors might be particularly interested in knowing which regime they are experiencing and which is the most likely regime to occur next. This knowledge would facilitate investment decisions during both bull and bear market phases.

The main objective of this paper is to analyse the stock market dynamic pattern detecting the different regimes which characterize stock markets and thus disaggregating the positive and negative markets into many market phases. We aim to focus on each market regime which is

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characterized by a probability distribution with a mean return and a volatility level that can help us to achieve relevant insights into the dynamics and regime-switching of stock markets. In particular, we provide an analysis of the US stock market during a period of more than 20 years distinguishing different market regimes and focusing on transition probabilities of switching between different market phases. Specifically, we analyse one of the most relevant US stock market index: the S&P 500. Our approach allows us to achieve two main goals. First, we detect regime-switching in stock markets with a particular focus on the mean returns which characterize each regime and separate periods of market stability, represented by one particular regime which can be interpreted as the bull market phase, from turmoil periods and crises which are characterized by strong fluctuations between positive and negative regimes, thus obtaining a disaggregation of the so-called bear market phases. Second, we exploit the information provided by the different market regime features and regime-switching for answering two pressing questions which arise during crises: (i) Are we able to detect the end of a crisis and the switch to a stable period? (ii) How can we support investment decision-making processes during such periods? We find empirical evidence of a fairly prompt recognition of the end of the turmoil phases and the beginning of the stable market regime and we show that our approach provides interesting insights and accurate predictions for the US stock market behaviour which can be exploited for setting profitable investment strategies. Quite surprisingly, we find evidence that it performs better than a main reference in stochastic volatility (SV) modelling such as the threshold GARCH (T-GARCH) model [33] with Student-*t* innovations.

Since the seminal work by Hamilton [18], Markov-switching approaches have been applied frequently to analyse stock price index data. The initial autoregressive framework, with unobserved changes in regimes modelled by a first-order Markov chain, has been extended [19] with an autoregressive conditional heteroskedasticity (ARCH) specification with regime-switching, governed by an unobserved Markov chain (SWARCH), which can evaluate changes in stock market volatilities. This method prevents an excess of persistence, which ARCH models impute to stock index volatility, and improves forecasting performance. Further extensions of the Markov-switching model specification [19] investigate changes in stock market volatility [[6,12,16,17,20,25,29], among many others]. This article offers another type of extension that can model stock index dynamics.

Previous research mainly has investigated conditional variances, categorizing time periods according to variances in stock market indexes, such that periods with similar volatilities have relatively high probabilities of being allocated to the same category. Although risk is obviously a fundamental factor in finance and a primary concern for market operators, investment decisions are usually based on expected returns. In this work, our purpose is to classify time periods on the basis of mean returns, which provide crucial proxies of expected returns. Accordingly, we investigate conditional means that characterize different stock market regimes, for which purpose we undertake an exploratory analysis of the (nonlinear) latent stochastic process that underlies the observed time-series of the stock market return distribution, using a hidden Markov model (HMM) [5,8,13]. The latent process comprises a discrete number of states that can be interpreted as different market regimes. Therefore, as our first research contribution, we provide a probabilistic classification of each time observation into market regimes, according to the value of the observed return at that time and the correlation structure of the series. The HMM is a powerful tool for classifying time-series into homogenous discrete latent states and aims both to cluster observations with respect to their conditional means in order to maximize the distance between these mean values and to account for time-dependence structure in the data [34]. Therefore, the classification procedure tends to minimize the variability within each state and, for this reason, the measure of volatility of each latent state (i.e. market regime) will be somewhat controlled by the method.[1]

Furthermore, prior financial literature has addressed the topic of conditional means only marginally, claiming that ‘it is well known that conditional means are hard to estimate’ [2] and thus rarely investigated them in depth. Some authors who consider differences in variances find similar or non-significantly different means across regimes that could be constrained to be equal [2,12]. But an accurate evaluation of conditional means might improve time-series classification. Stable periods, crises, and financial bubbles should be characterized by significantly different mean returns, and our analysis reveals that in the US stock market, the conditional means differ statistically significantly across time periods.

We, therefore, undertake an endogenous detection of different market phases, which contributes to extant research involving Markov-switching models that usually identify the number of market regimes *a priori*, often predetermining the number of latent states that characterize the unobservable Markov chain. For example, a study might select two predetermined latent states that represent a low-volatility regime (i.e. the bull market phase) and a high-volatility regime (i.e. the bear market phase) [22,23,28]. However, model complexity and the many parameters that must be estimated in SWARCH and MS-GARCH specifications make it impractical to include more than three/four latent states [19,29]. As an alternative, we identify the number of latent states by turning to statistical procedures and the more parsimonious HMM model, which introduces an approach for detecting the different regimes which characterize stock markets, allowing for more than two (or four) latent states. Therefore, an HMM enables a researcher to focus on dynamics and regime switches across different stock market phases, which offers valuable insights for financial variables analyses and guides investors in their investment decision-making process also in periods of high-volatility and during financial crises. We show that HMM results are particularly suitable for defining profitable investment strategies also with respect to SV models such as the T-GARCH.

In Section 2, we formally introduce the applied HMM. Section 3 includes the theoretical framework for our application of HMM, followed by a discussion in Section 4 of the analysed data, analysis, and results. We conclude in Section 5.

2. Hidden Markov model

In this section, we briefly describe existing proposals of financial analyses based on the HMM and we introduce model assumptions, parameter estimation and model selection. Furthermore, we outline the classification procedure used for allocating the time observations to the estimated latent states.

2.1 *Application of the HMM in financial market analysis*

Recent work has exploited the potential of the HMM to analyse the stock market return distribution as well as a variety of financial variables [24]. Among others, Rydén *et al.* [31] show that an HMM reproduces most of the stylized facts for daily return distributions. Bulla and Bulla [7] generalize the HMM for capturing also the slow decay in the autocorrelation function of the squared returns, allowing for a more flexible distributions of the sojourn time in visited regimes. In their book, Zucchini and MacDonald [34] use univariate and multivariate HMMs for analysing the return distributions of different stocks. Moreover, Dias *et al.* [11] investigate stock market dynamics using a mixture of HMMs. In this paper, we propose to apply the HMM for obtaining an accurate modelling of the dynamics of stock market return series which allows the detection of financial crises, as well as stable periods, thus enormously facilitating investment decisions, as shown in Section 4.5. Furthermore, unlike all the papers listed above which assume two or, at most, three regimes, in this paper we do not set *a priori* the number of regimes, but we estimate it allowing for a (latent) Markov chain with more than three states (see Section 2.3).

Other authors [3,4,21,34] focus on the volatility process of returns deriving a direct link between SV models and the HMM. Since it is a known fact that SV models are difficult to fit, Bartolucci and De Luca [3,4] and Langrock *et al.* [21] circumvent this problem by proposing a method which approximates the SV likelihood using an HMM formulation. A different approach to model the volatility process using an HMM is proposed by Rossi and Gallo [29] where changes in volatility occur according to a tridiagonal transition probability matrix. With respect to this strand of research, in this paper, we focus on expected returns rather than volatilities thus enabling a simple and intuitive method to support investment decision-making process. Specifically, we exploit the ability of the HMM to classify time observations into homogenous regimes characterized by significantly different conditional means. This approach is shown to be useful for defining a profitable investment strategy.

2.2 *Model assumptions and parameter estimation*

Let $\mathbf{z} = (z_1, \ldots, z_t, \ldots, z_T)'$ where z_t denote the return observation of a stock market index at time t ($t = 1, \ldots, T$). The HMM analyses $f(\mathbf{z})$, or the probability density function of the return distribution of the market index over time, using a latent transition structure defined by a first-order Markov process. For each time point t, the model defines a single discrete latent variable denoted by y_t, which consists of S latent classes (usually referred to as latent states). In this paper, we assume that

$$z_t | y_t = k \sim N(\mu_k, \sigma^2), \tag{1}$$

i.e. that the observed return at time t, conditional on the latent state k occupied at time t, for $k = 1, \ldots, S$, is distributed as a Gaussian with mean μ_k and variance σ^2 which is assumed as constant across latent states. The restriction on the conditional variance (to be equal across states) allows the model to cluster observations into latent states on the basis of the conditional means and first-order correlation structure of the data (see also note 1). This is an important point of this paper since we suggest a simple, intuitive and easy-to-interpret method that allows investors and pratitioners to base investment decisions on mean returns. Therefore, with respect to Markov-switching and SV models discussed above, in our approach, the variability of the observed process is accounted for by the regime-switching of the latent process y_t which is defined by S regimes characterized by different means and the same variance. Note also that the normal distribution assumption could appear simplistic. However, the mixture of normal distributions implied by the HMM provides an effective solution to the well-known skewness and excess of kurtosis of the stock return distribution [11].

Thus the HMM includes T latent variables and can be specified as

$$f(\mathbf{z}) = \sum_{y_1=1}^{S} \sum_{y_2=1}^{S} \cdots \sum_{y_T=1}^{S} f(y_1, \ldots, y_T) f(\mathbf{z}; y_1, \ldots, y_T), \tag{2}$$

where

$$f(y_1, \ldots, y_T) = f(y_1) \prod_{t=2}^{T} f(y_t | y_{t-1}), \tag{3}$$

and

$$f(\mathbf{z}; y_1, \ldots, y_T) = \prod_{t=1}^{T} f(z_t | y_t). \tag{4}$$

Equation (2) reveals that the model is a mixture, with S^{T} latent classes (mixture components). In the other mixture models, $f(\mathbf{z})$ can be obtained by marginalizing with respect to the latent variables. Because the y are discrete variables, Equation (2) is a weighted average of probability densities $f(\mathbf{z}; y_1, \ldots, y_T)$, where the latent class membership probabilities (or prior probabilities)

$f(y_1, \ldots, y_T)$ are weights [26]. Furthermore, Equations (3) and (4) depict the conditional independence assumption implied by the HMM, which can simplify the density functions $f(y_1, \ldots, y_T)$ and $f(\mathbf{z}; y_1, \ldots, y_T)$. Equation (3) implies an additional model assumption, namely, that $f(y_1, \ldots, y_T)$ follows a first-order Markov process. Thus, latent state y_t is associated with y_{t-1} and y_{t+1} only. Furthermore, $f(y_1)$ denotes the (latent) initial-state probability function which is assumed to coincide with the stationary distribution of the Markov chain. According to Equation (4), the return observation at time t is independent of observations at other time points, conditional on the latent state occupied at time t. Moreover, $f(z_t|y_t)$ is assumed to be normally distributed as stated in Equation (1).

Furthermore, $f(y_t|y_{t-1})$ denotes the latent transition probability function, which provides the probability of being in a particular latent state at time t, conditional on the state occupied at the previous time point, $t-1$. Assuming a homogenous transition process with respect to time, we achieve a latent transition matrix, $\mathbf{P}$, of dimension $S \times S$ where the generic element $p_{jk} = \Pr\text{ob}(y_t = k|y_{t-1} = j)$ denotes the probability of switching from latent state j at time $t-1$ to latent state k at time t, for $j, k = 1, \ldots, S$.

This HMM can be considered a restricted version of the regime-switching autoregressive model proposed by Hamilton [18], which restricts the autoregressive components to 0. Using the specification provided by Equations (2)–(4), we intend to model the latent stochastic process y_t to gain insights into stock market dynamics and a specific focus on the different conditional means $\mu_k = \mu(z_t|y_t = k)$ for $k = 1, \ldots, S$, obtained as

$$\mu_k = \frac{\sum_{t=1}^{T} z_t f(y_t = k|z_t)}{\sum_{t=1}^{T} f(y_t = k|z_t)}, \tag{5}$$

where $f(y_t = k|z_t)$ denotes the posterior probability of being in latent state k at time t, conditional on the observation at time t.

For the parameter estimation, we maximize (the logarithm of) the likelihood function (LL) given in Equations (2)–(4) according to the expectation–maximization (EM) algorithm [10] for the vector of parameters $\boldsymbol{\theta} = (\boldsymbol{\delta}, \mathbf{P}, \boldsymbol{\mu}, \sigma^2)$, where, for $k = 1, \ldots, S$, $\boldsymbol{\delta}$ is the S-dimensional vector containing the initial-state probabilities, $\delta_k = f(y_1 = k)$, $\boldsymbol{\mu}$ is the S-dimensional vector containing the conditional means μ_k, whereas σ^2 is a scalar. However, the iterative procedure of the EM algorithm is often impractical for estimating an HMM. For the expectation step, it must compute and store S^{T} entries of the joint posterior latent distribution, $f(y_1, \ldots, y_T|\mathbf{z})$, so computational time increases exponentially with T, and even a moderate time-series length may prevent the convergence of the algorithm. We, therefore, use a variant of the EM algorithm, called the forward-backward or Baum–Welch algorithm [5], as extended by Paas *et al.* [27] for an application to data sets with multiple observed indicators and implemented in the Latent GOLD 4.5 computer program [32].

The forward–backward algorithm exploits the conditional independence assumption of the HMM to compute the joint posterior latent distribution by estimating the missing data, which in the HMM are unobserved state memberships. This estimation is realized by computing the expected value of the log-likelihood function, given the current parameter values and the observed data. The maximization step uses standard maximum likelihood estimation methods for complete data to update the model parameters. The algorithm cycles between these steps until it reaches a previously defined convergence criterion.

2.3 *Model selection, class membership and prediction*

Model selection involves the choice of the number of latent states S, which in our framework represents the number of market regimes. This extension of existing approaches addresses their inability to estimate Markov switching models with $S > 4$ because of their complexity [19,29]. The choice of the appropriate number of latent classes is based on the Akaike information criterion (AIC) [1]:[2]

$$\text{AIC} = -2\text{LL} + 2N\text{Par},$$

where $N\text{Par} = S(S + 1)$ is the number of free model parameters which includes $S(S - 1)$ transition probabilities, $S - 1$ initial probabilities, S conditional means and 1 variance.

For our analysis, stock index returns are the indicators z_t, for $t = 1, \dots, T$. Each z_t is classified into one latent state according to the estimated posterior probabilities. That is, z_t is allocated to latent state j if $\hat{f}(y_t = j|z_t) > \hat{f}(y_t = k|z_t)$ for every $k = 1, \dots, S$.[3] In this modal classification, time points with a similar development are more likely to be allocated to the same latent state than are those with highly divergent developments.

The estimation of the posterior probabilities also allows the computation of the conditional means in Equation (5). Therefore, the prediction of the stock market return at time $T + 1$ given the information available until time T can be obtained as [34]

$$\hat{z}_{T+1|T} = E[z_{T+1}|y_T = j] = \sum_{k=1}^{S} p_{jk}\hat{\mu}(z_T|y_T = k), \tag{6}$$

under the assumption of homogeneity on the transition process over time.

3. Theoretical framework

The use of the HMM for these purposes is promising, because financial markets are characterized by frequent changes in regimes. If stock market index returns are subject to discrete changes in regimes, including periods in which the dynamic pattern of the series differs markedly, a nonlinear model should exploit the time path of the observed series to draw inferences about a set of discrete latent states [18]. Different market regimes thus should be characterized by different means and standard deviation values or – using the terminology of the portfolio theory framework – by different risk–return profiles. During a financial crisis, the stock market experiences a strong negative mean return, and the standard deviation, used as a proxy of risk, is large. During more stable phases, stock returns fluctuate around a constant mean, and the standard deviation of the index value is lower. Different regimes in different time periods imply the ability to cluster time observations, according to the similarity in the dynamics of the index value and the volatility of that index [15]. Time periods with more (less) similar index dynamics have a higher (lower) probability of being allocated to the same cluster. Moreover, empirical analyses clearly show stock returns are characterized by asymmetry and larger kurtosis than the Gaussian distribution [14], which invalidates inferences. By modelling regime changes using a mixture of normal distributions, HMM provides an effective solution to these issues [11].

The HMM applied in this study classifies different observations into a limited set of regimes, on the basis of stock market price index dynamics. For example, a week characterized by a strong decline in the stock market price index may be allocated to the large value decrease market regime, whereas weeks defined by small changes likely appear in the stable market regime. Switches between regimes are modelled as a Markov process. In order to contribute to the debate on the poor out-of-sample forecast capability of Markov-switching models [9], we show that HMM can be employed to define profitable investment strategies and outperforms an SV model commonly used in stock market analysis such as the T-GARCH with Student-t innovations (see Section 4.5).

4. Empirical analysis using the HMM

In this section, the proposed model is applied to the US stock market index S&P 500. First, the data description is provided. Next, empirical results and applications of the estimated model are discussed. Finally, the estimated model is exploited for supporting investment decision-making processes.

4.1 *Data description*

Our analysis is based on weekly returns for the US stock market price index S&P 500, calculated as the percentage achieved in the relative variation in index prices,[4] $p_t : z_t = (p_t - p_{t-1})/p_{t-1} \times 100$. The data set covers the period from 5 January 1990 to 9 April 2010, which includes $T = 1058$ time points. As Figure 1 shows, our data set includes at least three periods with high volatility, which reflect stronger fluctuations and rapid changes from positive to negative peaks: prior to 1991, from 1997 to 2003, and after 2008. Our data also contain several stable periods, such as those from 1992 to 1997 and from mid-2003 to the end of 2007. According to NBER-defined business cycles,[5] the total study period contains three crisis periods: the 'savings and loan' crisis (July 1990–March 1991), the Internet bubble burst and the September 11 attacks (March 2001–November 2001), and the credit crisis (December 2007–June 2009). A pressing question during such periods is when the economic situation might improve. Therefore, we apply the HMM to discriminate endogenously the stable from the crisis periods, disaggregating the 'bull-bear' market dichotomization, and recognize the end of a crisis, according to the mean returns of the stock market price index.

Table 1 contains the different values of the mean returns and standard deviations for the entire time sample and five subperiods, which can be associated with low or high volatility market phases: According to the standard deviation values, the five subperiods are characterized by different levels of variability. In particular, the levels differ greatly for periods II and IV and periods I, III, and V, as well as across the three high-volatility phases. The latter finding implies that each financial crisis creates its own peculiarities.

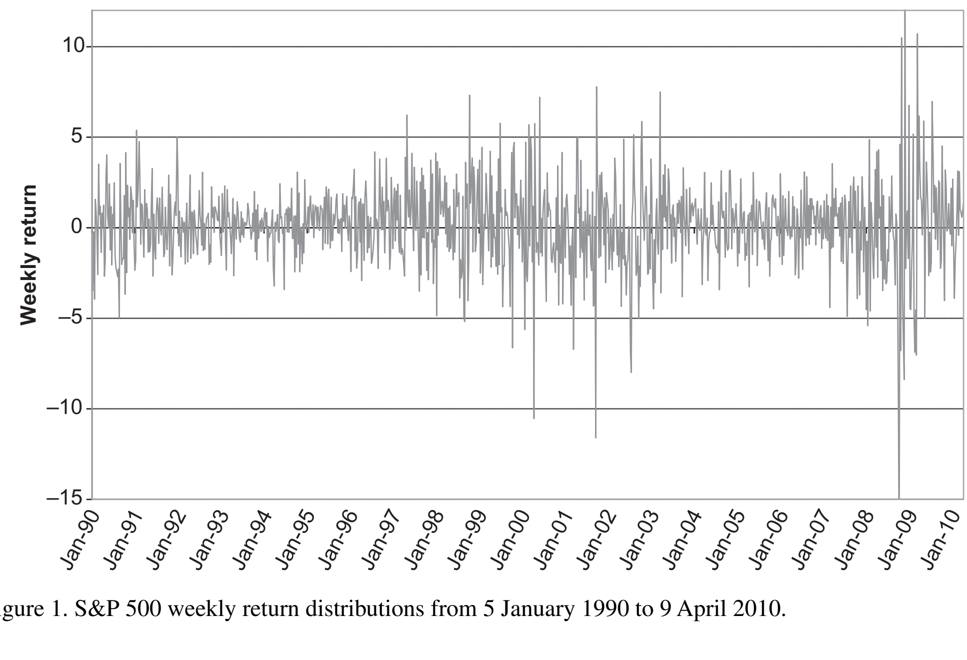

Figure 1. S&P 500 weekly return distributions from 5 January 1990 to 9 April 2010.

Table 1. Mean, standard deviation, minimum, maximum, skewness, kurtosis, and Jarque–Bera test of S&P 500 index in different periods.

Period	Mean return	Standard deviation	Min	Max	Skewness	Kurtosis	Jarque–Bera test
Full sample (1 May 1990–9 April 2010, $T = 1058$)	0.1381	2.3571	−18.20	12.03	−0.4873	6.0027	1608.7**
Period I (high volatility) (1 January 1990–15 February 1991, $T = 59$)	0.1000	2.3330	−4.98	5.39	0.1090	−0.6003	1.003
Period II (low volatility) (22 February 1991–21 March 1997, $T = 318$)	0.2465	1.3754	3.42	5.02	0.0753	0.1637	0.655
Period III (high volatility) (28 March 1997–16 May 2003, $T = 321$)	0.0970	2.7890	−11.60	7.78	−0.2653	1.2159	23.54**
Period IV (low volatility) (23 May 2003–20 July 2007, $T = 218$)	0.2328	1.4353	−4.41	3.54	−0.2761	0.2753	3.458
Period V (high volatility) (27 July 2007–9 April 2010, $T = 142$)	−0.1724	3.8928	−18.20	12.03	−0.3969	3.6494	74.39**

Note: **Significant at 1%.

Table 2. Log-likelihood function, number of parameters, and AIC criterion of the HMM from 1 to 8 latent states for S&P 500.

Number of latent states	LL	*N*Par	AIC
1	−2405.29	2	4814.58
2	−2358.32	6	4728.64
3	−2308.43	12	4652.86
4	−2282.53	20	4605.07
5	−2258.16	30	4576.31
6	−2236.47	42	4556.94
7	−2216.98	56	4545.95
8	−2208.67	72	4561.34

The Jarque–Bera normality test results are significant for the entire data set, implying a significant difference between the observed and a normal distribution. We also can reject the normality assumption for subperiods III and V, according to the Jarque–Bera test. Therefore, the HMM may be a desirable alternative to traditional financial econometric models, because it accounts for both asymmetry and more kurtosis than a normal distribution.

4.2 *Model estimation and class profiling*

We estimate the HMM for 1–8 latent states ($S = 1, \dots, 8$) and provide, in Table 2, the maximum log-likelihood function, number of estimated parameters, and AIC values. According to the AIC

Table 3. Sizes, return means, within-state variabilities, and Jarque–Bera tests of seven latent states for S&P 500 index.

Latent state	Size	Return mean (standard error)	Within-state variability
1	0.0041	−12.432 (0.730)	3.952
2	0.0142	−6.035 (0.483)	1.370
3	0.1901	−1.939 (0.201)	1.327
4	0.4959	0.247 (0.065)	1.399
5	0.2435	1.091 (0.212)	1.220
6	0.0491	4.529 (0.375)	1.177
7	0.0031	11.072 (0.842)	0.833
Full sample	1.000	0.143	2.351

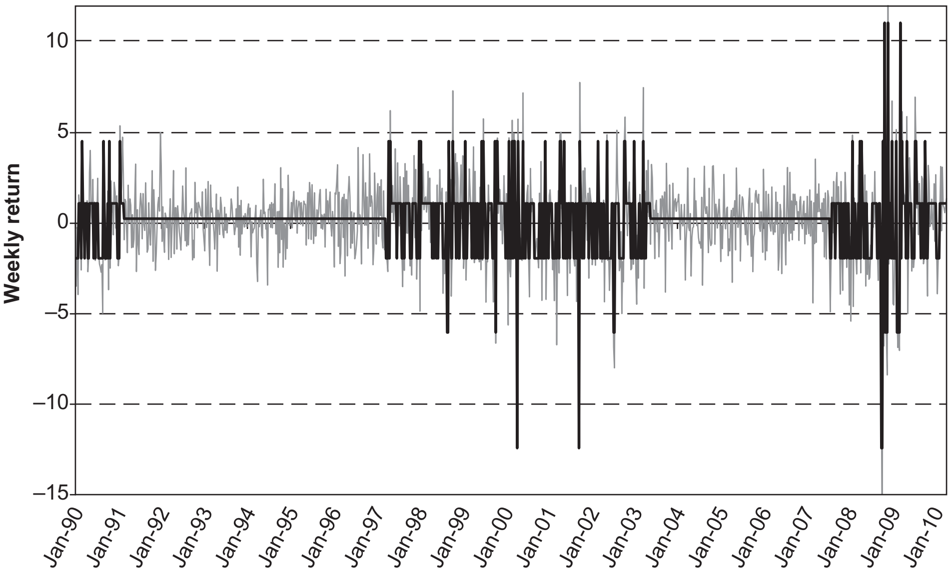

Figure 2. S&P 500 and HMM estimated time series.

criterion, the HMM with seven latent states provides the best fit to the data. In our framework, these latent states represent seven different stock market regimes. According to the return means in each state, the S&P 500 index reveals three negative and four positive regimes; in Table 3, we label the seven market regimes using the return means. For example, latent state 1 has an average return of −12.43% and constitutes 0.41% of the $T = 1058$ analysed weeks.

As Table 3 shows, the HMM can define different regimes of the stock market. The return means differ significantly across latent states, according to both the Wald test ($W = 846.27$, df $= 6$, $p < 0.001$) and ANOVA ($F = 360.83$, df $= 6; 1051$, $p < 0.001$), so we reject the null hypothesis of equality between conditional means.[6] Furthermore, the variability within each latent state is relatively low, according to the similar standard deviation values in Table 3, with the exception of latent state 1, which represents the biggest stock market drops.

Figure 2 displays the weekly return time-series of the S&P 500 index and that obtained through the seven-state HMM. The estimated series is plotted using the latent state return means. The HMM approximates the observed time series of the S&P 500 index quite accurately. Moreover, it detects two stable periods, corresponding to latent state 4, as represented by the straight lines from 1 March 1991 to 14 March 1997 and from 23 May 2003 to 20 July 2007 (Figure 2). The HMM

Table 4. Latent transition matrix for S&P 500 index.

j/k	1	2	3	4	5	6	7
1	0.2462	0.0049	0.0049	0.0049	0.0050	0.7290	0.0050
2	0.0016	0.4686	0.0053	0.0017	0.1975	0.1172	0.2081
3	0.0066	0.0004	0.4594	0.0002	0.3360	0.1973	0.0001
4	0.0000	0.0000	0.0043	0.9953	0.0002	0.0001	0.0000
5	0.0068	0.0077	0.3310	0.0108	0.6360	0.0076	0.0001
6	0.0005	0.0909	0.3620	0.0014	0.4434	0.1014	0.0004
7	0.0065	0.3266	0.6235	0.0065	0.0136	0.0168	0.0065

results also show that the three periods, characterized by high volatility, include frequent switches between regimes with positive and negative conditional means. These three periods correspond to the three crises and recessions we noted previously, though the 2001 crisis was preceded by a period of turmoil that started in 1997, which may indicate a spillover of the Asian crisis to the US stock market [30].

4.3 *Latent transition analysis*

In the transition probability matrix estimated by the HMM in Table 4, the transition probabilities define the stock market regime-switching. The values on the diagonal represent state persistence, that is, the probabilities of remaining in a particular market regime. The modal latent state 4 has high persistence ($p_{44} = 0.9953$) and represents the stable market regime. As Figure 2 reveals, this result indicates that the US stock market tended to remain in that regime ($T = (1 - p_{44})^{-1} \approx 213$ weeks). The off-diagonal p_{jk} values indicate the probabilities of market regime-switching. It is quite likely that the S&P 500 index switches from a very negative phase to a period of fast growth ($p_{16} = 0.7290$), whereas the opposite switch is unlikely to occur ($p_{61} = 0.0005$).

The probabilities in Table 4 thus underline some important features of market regime-switching. First, for latent states 1–5, the transition probabilities p_{jj} are relatively high, whereas state 6 has a persistance probability of 0.10 and for latent state 7 persistance is unlikely ($p_{77} < 0.01$). Second, when the S&P 500 declines (states 1 or 2) at time t, at time $t + 1$, the market may continue in a negative phase ($p_{11} = 0.246$ and $p_{22} = 0.427$) or switch to a positive regime ($p_{16} = 0.729$, $p_{25} = 0.198$, $p_{26} = 0.117$, and $p_{27} = 0.208$). The other states rarely occur after state 1 or 2.

Overall, 31 of the 49 transition probabilities are less than 0.05 in the transition matrix, which indicates most regime switches are very unlikely for the S&P 500 index. Accordingly, our results offer interesting insights for future market phase predictions. Possible applications of these insights by investors are further discussed in Section 4.5.

4.4 *Recognition of the stable market phase*

The model also can predict a stable period, after a previous stable period or after a period in which the market was not categorized in the stable latent phase 4. The latent state characterized by a moderate positive mean return is most common and has a persistence probability of close to 1. These features denote a stable market regime, as mentioned in Section 4.2. In these periods, which correspond to subperiods II and IV in Table 1, the stock price index value does not experience large and frequent changes. The ranges between the minimum and the maximum returns of the S&P 500 index in periods II and IV in Table 1 are 8.44 and 7.95, respectively, and the standard deviations are 1.375 and 1.435. In contrast, periods I, III, and V in Table 1 are characterized by ranges of 10.37, 19.38, and 30.23 and standard deviations of 2.333, 2.789, and 3.746, respectively. Therefore, the time points classified into latent state 4 can be interpreted as belonging to a low-volatility period

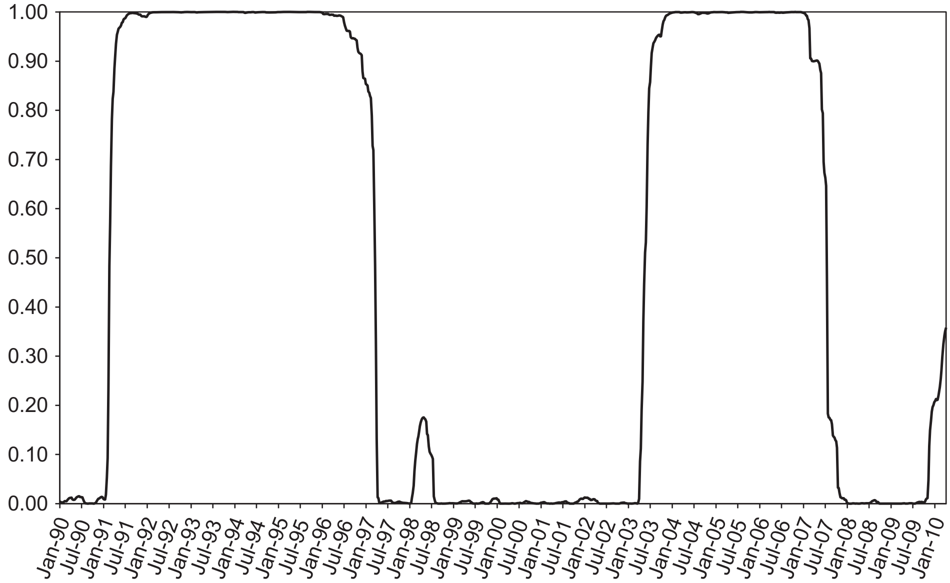

Figure 3. Estimated posterior probabilities for latent state 4.

of market stability which investors and analysts may define as the ‘bull market’ phase, whereas the other six latent states refer to the high-volatility periods I, III, and V, characterized by frequent switches between high- and low-return regimes (the so-called ‘bear market’).

In Figure 3, the estimated posterior probabilities for the latent state 4, $\hat{f}(y_t = 4|z_t)$, underline the high level of confidence with which the HMM determines the two stable periods characterized by a low level of volatility: Of the 536 observations classified into latent state 4, only 38 have a posterior probability less than 0.90. In other words, the probability of remaining in latent state 4 across time points is quite high, as correctly predicted by the model.

To evaluate the model’s capability to detect a stable period after a period of crisis, we estimate the HMM with seven latent states for shorter time series. The beginning of the second stable regime (period IV in Table 1) provided by the HMM, when applied to the entire time series, starts on May 23, 2003. We use the crisis before May 2003 to evaluate the model’s capacity to detect a stable period, because period II (Table 1) is preceded by a very short unstable period in our data set. We assess how many weeks of stability are required to detect the end of the financial crisis, which the HMM estimates as 23 May 2003. Therefore, we first estimate the model using data from 5 January 1990 to 23 May 2003, and then from 5 January 1990 to 30 May 2003, and so on. A stable period emerges when multiple weeks, latest in time, are allocated to the stable latent state 4.

Our analysis reveals that HMM can detect the stable market phase within 13 weeks of 23 May 2003. That is, a period containing only stable regimes after 23 May 2003, appears when we use the data set with stock index returns from 5 January 1990 to 22 August 2003. In the analysis in which we included fewer than 13 weeks, the last few observations are not allocated to the stable latent state 4. This feature of the HMM is potentially useful for detecting when the financial crisis that started in 2007 will end. That is, by the time we concluded our analysis (9 April 2010), there were not 13 consecutive weeks allocated to the stable latent state 4; the crisis had not ended by 9 April 2010. However, Figure 3 shows that the posterior probability for latent state 4 increased in the most recent observations in our sample, reaching $\hat{f}(y_{4/9/10} = 4|z_{4/9/10}) = 0.358$, though still not representing the modal state.

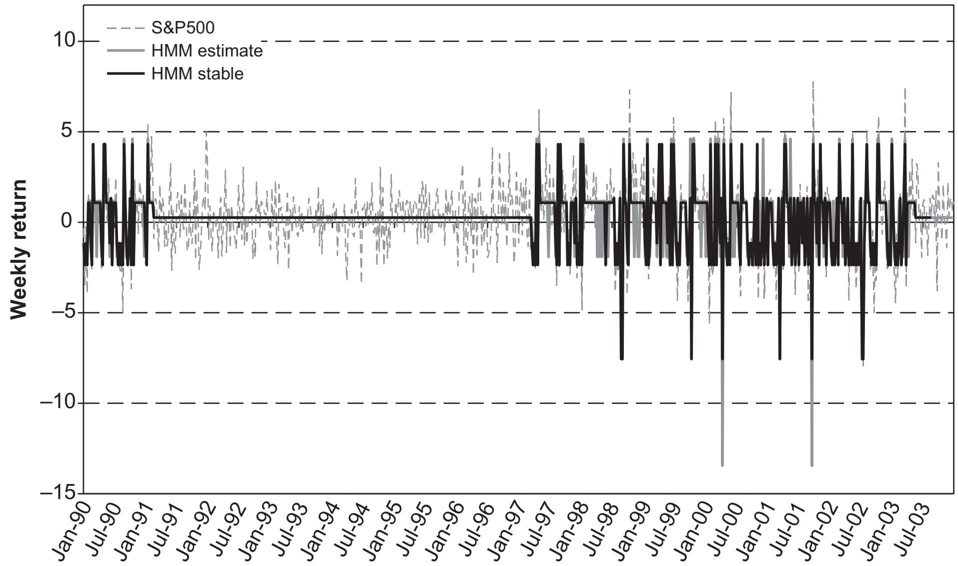

Figure 4. S&P 500 index return distribution, overall HMM estimates, and HMM estimates for the stable regime (HMM stable).

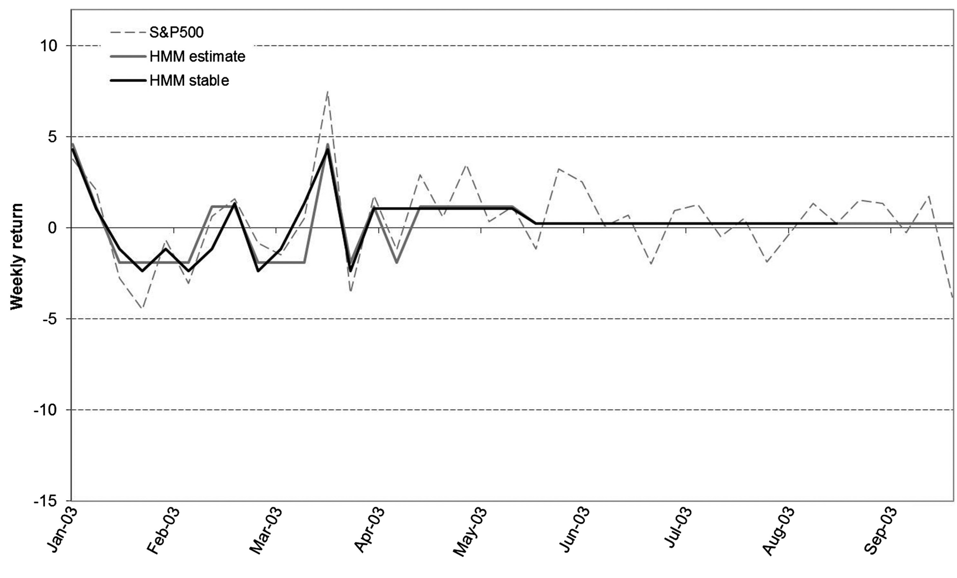

Figure 5. Close-up of S&P 500 index return distribution, overall HMM estimates, and HMM estimates for the stable regime (HMM stable).

Of course, great care should be taken in interpreting the results of this application of the HMM. Each crisis has idiosyncratic characteristics, which implies that different periods of stability may be required to detect different crises. This topic remains for further study.

Other model characteristics also emerge from the analysis for predicting the end of the crisis that occured prior to May 2003. Figures 4 and 5 compare the original time-series with respect to the HMM estimate derived from the whole data set and the estimate of an HMM with seven latent states applied to the data from 5 January 1990 to 22 August 2003. The return means of the HMM estimate, based on the shorter time series, differ slightly from the means of the overall HMM estimated time series. Nevertheless, latent state memberships derived from the shorter time series are almost the same as the HMM estimates achieved with the entire data set. In particular, the observations from 1 March 1991 to 14 March 1997, can be allocated to the stable regime (latent state 4, Table 3) in both data sets, in support of the robustness of the HMM classification procedure and its power to detect low-volatility stable periods without referring directly to the analysis of any volatility measure. However, after only five weeks, the HMM can identify the beginning of a 'potential' stable period; it classifies the previous 12 weekly return observations into latent state 5. Despite a low transition probability ($p_{54} = 0.0108$ in Table 4), this latent state is the regime that the stock market experiences just before switching to the stable latent state 4.[7]

This feature underlines an interesting behaviour of the S&P 500, which tends to stabilize and consolidate after a positive regime. Our analysis instead shows that once the stable market phase ends, instability occurs for quite a long period in the three crises in our data set. For instance, the high-volatility period III in Table 1 has approximately the same length of the stable period II. This feature can be generalized to other crises.

4.5 *Supporting investment decision-making processes using an HMM*

In Sections 4.2 and 4.3, we reported on the means of the latent states and the latent transition matrix for the S&P 500 (Tables 3 and 4). In this section, we exploit the information provided by the conditional means and the transition probabilities to evaluate how the HMM can support investment decision-making processes. In particular, we investigate the utility of the estimated model to guide the decision of investing on the S&P 500 stock market index the following week (one-step ahead). We evaluate this capability both in-sample and out-of-sample, thus giving insights about the HMM usefulness in forecasting framework. Therefore, we evaluate the HMM performance with respect to a model which is commonly used to analyse stock market indexes, namely the T-GARCH(1,1) model [33] with Student-*t* innovations which captures both asymmetry and non-normality. This model can be specified by a system comprising the conditional mean equation which, in our case, considers an autoregressive component of order 1

$$z_t = \mu + \phi_1 z_{t-1} + u_t$$

and the conditional variance equation

$$h_t = \mathrm{Var}(u_t) = \omega + \alpha_1 u_{t-1}^2 + \beta_1 h_{t-1} + \gamma_1 u_{t-1}^2 I_{t-1},$$

where $I_t = 1$ if $u_t < 0$ and 0 otherwise and the conditional distribution of errors is a Student-*t* with $\upsilon > 2$ degrees of freedom. The estimates of the T-GARCH model with Student-*t* innovations are reported in Table 5.

First, we compare the forecast accuracy provided by the two models. Table 6 collects the mean squared errors (MSEs) and mean absolute errors (MAEs) for the seven-state HMM and the T-GARCH model for both the in-sample and out-of-sample periods. From the results reported in Table 6, one can clearly observe that the forecast obtained by the HMM is much more accurate than the forecast obtained using the T-GARCH(1,1) model with Student-*t* innovations. The superiority of the HMM over the T-GARCH model is also evident from the scatter plots depicted in Figure 6, where we observe a closer correspondence between the HMM forecasts (on the *y*-axis) and the actual observations (on the *x*-axis). However, it must be noted that the forecast of the T-GARCH model is achieved using the conditional mean equation but not the conditional variance equation, i.e. the forecast of the volatility process. In the following, we develop an analysis where we

Table 5. Estimates for the T-GARCH(1,1) with Student-t innovations model.

	Parameter	Coefficient (standard error)
Mean equation	μ	0.2161 (0.0549)
	ϕ_1	−0.1130 (0.0320)
Variance equation	ω	0.1029 (0.0347)
	α_1	0.0202 (0.0213)
	β_1	0.8822 (0.0237)
	γ_1	0.1456 (0.0316)
Degrees of freedom (Student-t)	υ	12.129 (3.7596)
Average sample variance	$\bar{h}$	5.7141

Table 6. MSEs and MAEs for the predicted return values of the HMM and the forecasts of the T-GARCH(1,1) with Student-t innovations model.

	HMM		T-Garch(1,1) Student-t	
	MSE	MAE	MSE	MAE
Full sample	1.517	0.934	5.503	1.679
Out-of-sample forecast (16 April 2010–27 August 2010)	1.347	0.931	9.488	2.506
Crisis period (28 December 2007–27 March 2009)	2.041	1.010	23.66	3.494

consider both the equations of the T-GARCH model and where we assess whether the forecasts provided by the models can be exploited for setting a profitable investment strategy.

In particular, we compare the outcomes of three investment strategies, two of whom exploit the information provided by the estimated models (seven-state HMM for Investor A and T-GARCH(1,1) model for Investor B), whereas the third strategy, adopted by Investor C, does not. Specifically, at time t, Investor A decides whether investing 1000$ in the US stock market (represented by the S&P 500 index) the following week (time $t+1$) or not. In particular, her/his decision is based on the expected (monetary) value conditional on the latent state j occupied at time t, $E[z_{t+1}|y_t = j]$, which corresponds to the prediction of $z_{T+1|T}$ given in Equation (6).

If $E[z_{t+1}|y_t = j] > 0$, i.e. the expected value computed as a weighted average of the conditional means where the transition probabilities serve as weights is positive, then Investor A will invest 1000$ the next week with payoff given by $1000\$ \times (1 + z_{t+1}/100)$; otherwise, if $E[z_{t+1}|y_t = j] < 0$, she/he will decide to leave the money on her/his deposit and, thus, saving 1000$. Then, the outcome in the latter case will be 1000$. The expected values for the decision criterion followed by Investor A are reported in Table 7, as well as the corresponding investment strategy for each latent state $j = 1, \dots, 7$ occupied at time t.[8] Therefore, this strategy suggests to invest when the stock market is experiencing a negative or the stable regime and not to invest when the S&P 500 return is positive since $E[z_{t+1}|y_t = j] < 0$ for $j = 5, 6, 7$. This result suggests the effectiveness of the well-known 'trade against the trend' investment strategy. On the other hand, the investment decision-making process for Investor B is based on the static one-step ahead forecasts obtained by the estimates of the T-GARCH model with Student-t innovations reported in Table 5. In particular, at time t, Investor B bases her/his decision on both the equations provided by the model trying to maximizing the expected return as well as minimizing the risk. In particular, Investor B decides to invest 1000$ at time $t+1$ if the forecasted conditional mean value is positive and the forecasted conditional variance is lower than the average sample variance reported in Table 5 ($\hat{z}_{t+1|t} > 0 \cap \hat{h}_{t+1|t} < \bar{h}$) and does not invest otherwise.[9] Finally, Investor C follows a 'naïve' approach and she/he invests 1000$ every week without evaluating any additional information and thus earning (or losing) $1000 \times (1 + z_{t+1}/100)$ dollars a week.

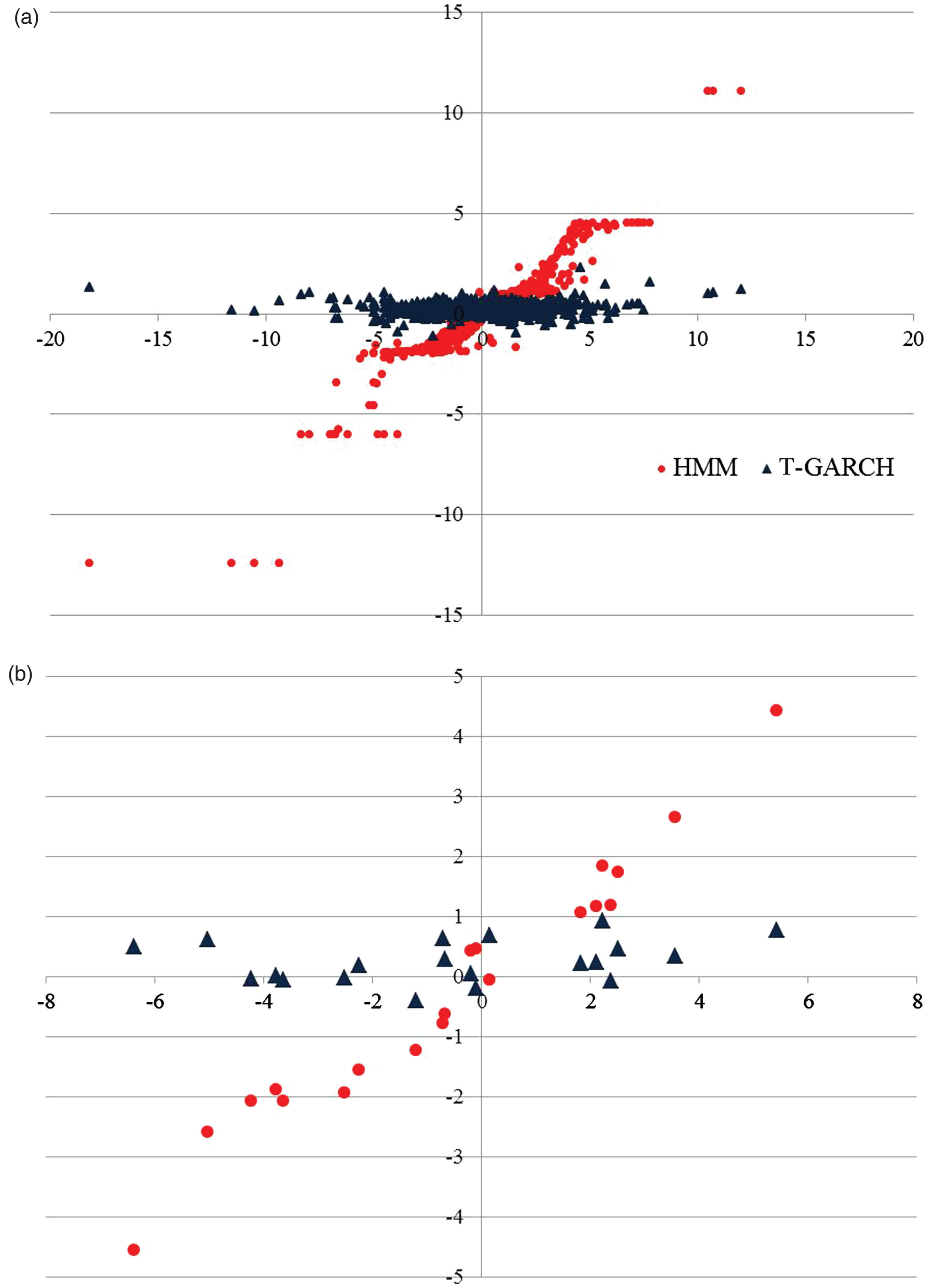

Figure 6. Evaluation of the forecast accuracy of the seven-state HMM and the T-GARCH with Student-t innovations model. (a) In-sample forecasts and (b) out-of-sample forecasts.

The results for the three decision-making processes are illustrated in Table 8. Specifically, the means and the net profits for the investment strategies based on the HMM, the T-GARCH and the naïve approach are reported with respect to different time windows: the whole analysed data set, the out-of-sample period of 20 weeks (from 16 April 2010 to 27 August 2010) subsequent to the last observation of the data set (in this case, $t + 1 = T + 1, T + 2, \ldots, T + 20$), and from

Table 7. Results for the expected value criterion and respective investment strategy decisions (Investor A) for each latent state.

Criterion\state k	1	2	3	4	5	6	7
Expected value (decision)	0.264 (INV)	0.193 (INV)	0.286 (INV)	0.238 (INV)	−0.041 (NO)	−0.309 (NO)	−3.096 (NO)

Notes: INV, Invest 1000$ next week; NO, do not invest 1000$ next week.

Table 8. Mean and net profit for the investment strategies based on the expected values of the HMM, the forecasts of the T-GARCH(1,1) with Student-t innovations model, and the 'naïve' approach (Investors A, B, and C, respectively).

	HMM		T-GARCH(1,1) Student-t		Naïve	
	Mean	Net profit	Mean	Net profit	Mean	Net profit
Full sample	1001.75	1847.70	1001.49	1574.57	1001.43	1516.61
Out-of-sample forecast (16/04/2010–27/08/2010)	1000.54	10.79	997.77	−44.66	994.73	−105.31
Crisis period (28/12/2007–27/03/2009)	998.76	−80.70	997.36	−171.49	992.07	−515.29

28 December 2007 to 27 March 2009 which corresponds to the sharpest phase of the crisis of 2007/2008 and which is characterized by the negative mean return of −0.793% (−52.34% in 66 weeks) and a high standard deviation level (4.807). The results in Table 8 show that the naïve investment decision-making process followed by Investor C leads to a mean of 1001.43$ as the outcome for the entire analysed data set which corresponds to a net profit of 1516.61$. However, Investor C achieves negative outcomes for the out-of-sample window with a mean value below the 1000$ invested amount and a net loss of 105.31$. Moreover, as expected, the naïve investment strategy leads to a disastrous outcome during the financial crisis of 2007/2008: the net loss Investor C had in the period from 28 December 2007 to 27 March 2009 is 515.29$.

As we expected, outcomes for Investors A and B are better than the ones achieved by Investor C, thus highlighting that effective investment decision-making processes must be based on the information provided by a suitable model. The results reported in Table 8 show that, adopting the expected value criterion in conjunction with the seven-state HMM estimates, Investor A is able to achieve a mean of 1001.75$ and a net profit of 1847.70$ if we consider the entire data set window length. This outcome is preferable to the one achieved by Investor B (mean = 1001.49$ and net profit = 1574.57$). Furthermore, if we investigate the other two periods analysed in Table 8, the better performance of the strategy adopted by Investor A and, hence, the usefulness of the HMM in supporting investment decisions for time $t+1$ appears to be evident. On the contrary of the case of Investor B, outcomes for out-of-sample forecast are positive for Investor A: a mean of 1000.54$ against 997.77$ and a net profit of 10.79$ against −44.66$, respectively.

The latter result clashes with the main theoretical findings by Dacco and Satchell [9] that although most regime-switching techniques give a good in-sample fit, they are usually outperformed out-of-sample. Moreover, the information provided by the HMM are particularly useful during the analysed crisis period: the mean is 998.76$ which corresponds to a net loss of 80.70$ against a net loss of 171.49$ suffered by Investor B. This result highlights the importance of evaluating market regime-switching during periods characterized by high-volatility and the benefit of exploiting its potential for decision-making strategies.

On the basis of the results of our example, we can affirm that regime forecasting achieved by means of the HMM provides interesting insights and accurate predictions for the US stock market behaviour which can be exploited for setting a profitable investment strategy.

5. Discussion and conclusions

We have investigated the dynamic patterns of stock markets by exploiting the potential of the HMM for defining different market regimes and providing transition probabilities for regime-switching. On the basis of the AIC, we find empirical evidence for an HMM with seven latent states for the US S&P 500 index. The regimes, represented by the seven latent states, are clearly defined and characterized by different return means. Therefore, we show that stock markets can be analysed by referring to a simple and flexible model specification with a specific focus on conditional means that differ significantly and substantially across latent states. Our approach represents an efficient alternative to the more sophisticated but much less flexible Markov-switching models that attempt to evaluate the conditional variance with a restricted number of latent states [29] and without consideration of conditional means [2].

The HMM endogenously detects crises, including the 1990–1991 US recession, the turmoil of 1997–1999 and 2000–2001, and the crisis that started in late 2007. It also detects two long, stable periods between these crises. A stable market regime is defined by a particular latent state, characterized by a moderate positive return mean and a high state persistence probability, comparable to the low-volatility regime achieved in volatility-based Markov-switching models and interpreted as the 'bull' market phase. Furthermore, the model distinguishes relatively moderate fluctuations in stable periods from stronger fluctuations during periods of crisis. With respect to volatility, our approach describes the fluctuations during high-volatility periods with six latent states and therefore enhances understanding of crises, in terms of switching between regimes with low and high (conditional) mean returns. That is, the HMM provides straightforward insights into high-volatility regimes, disaggregating the 'bear' market phase. This feature cannot be achieved by Markov-switching volatility approaches that are useful for defining periods characterized by a high conditional variance value but cannot investigate fluctuations within these periods.

With regime characterization and latent transition probabilities, we can achieve two additional important goals. First, with HMM, we recognize the beginning of stable periods within 13 weeks. This feature may provide a highly pertinent opportunity to detect the end of the current financial crisis that started in 2007. Despite some preceding positive weeks, the crisis had not ended by April 2010. The HMM also enables us to recognize the beginning of a crisis promptly, based on a switch from the stable market phase represented in our analysis by latent state 4 to one of the other six latent states. Unstable periods last for many weeks before 'bouncing back' to a new stable phase. Second, with HMM, we can derive crucial insights into investment decision-making processes. By exploiting the information provided by the model, we resort to the expected value criterion for setting a profitable investment strategy which, against expectations, is shown to be more effective than the strategy defined on the basis of the T-GARCH(1,1) model with Student-t innovations for both in-sample and out-of-sample forecasts. This empirical evidence clashes with the findings that regime-switching techniques does not provide an adequate out-of-sample performance [9].

Additional studies should apply this methodology to other periods and countries as well to determine if the latent states we have found, as well as our other findings, hold in different circumstances.

Acknowledgements

Both authors express their gratitude to the editor of *Journal of Applied Statistics* and to the two anonymous reviewers for their useful suggestions. De Angelis gratefully acknowledges funding from the Italian Ministry of Education, University and Research (MIUR) through PRIN project 'Multivariate statistical models for risk assessment'.

Notes

1. In our HMM approach, we do not directly estimate conditional variances which are restricted to be equal across states. Instead, we obtain measures of the variability within each latent state. We are thus able to achieve two goals: (i) avoiding the estimation of conditional variances allows the model to cluster observations into latent states on the basis of the conditional means and first-order correlation structure of the data, thus attaining a reliable and easily interpretable classification of the stock market return distribution; (ii) restricting the conditional variances enables us to decrease the number of parameters to be estimated. It must be noted that the model we adopt allows us to easily discriminate between periods characterized by low and high-volatility (see Section 4.2).
2. Markov chain order identification in HMM remains an unresolved issue (see [[18, Chapter 15] for a recent discussion]), and there are several concerns about the robustness and reliability of information criteria. We also agree with the concerns about a uncritical use of these indicators. However, we believe that they can contribute to current procedures, for which the choice of latent states is somewhat arbitrary.
3. The posterior probabilities are estimated using the properties of the forward and backwards probabilities. In particular, the posterior probability for the latent state j is given by $f(y_t = j|z) > f(z, y_t = j)/f(z)$ (see [34] for further details).
4. For weekly frequency of the data, the simple net returns z_t we use in this paper are approximately equal to the (continuously compounded) log-returns obtained as $z_t = \log(p_t) - \log(p_{t-1})$ and which are sometimes used in financial studies. Therefore, the results are not affected by this choice.
5. Source: http://www.nber.org/cycles.html.
6. The Wald statistic tests the significance of sets of parameters. The general expression for the Wald test statistic is given by $W = (C'\varphi)'(C'\Sigma C)^{-1}(C'\varphi)$, where the tested set of linear constraints is $C'\varphi = 0$ and Σ is the estimated variance–covariance matrix. This test is distributed as a chi-square where the number of degrees of freedom equals the number of contraints. In our analysis, we test the equality of conditional means between states and, thus, we constrain the means of six latent states to be equal to the one of the remaining latent state. Hence, the number of degrees of freedom of both Wald and ANOVA tests reported in the text is 6. The (one-way) ANOVA test for comparing conditional means is performed using latent state membership as factor.
7. However, p_{54} is the highest transition probability with respect to the other transition probabilities p_{j4} for $j \neq 4$ (see Table 4), and latent state 5 is the last visited regime before the switch to latent state 4 in both cases in the analysed data set.
8. The results obtained for the expected (monetary) value criterion are equivalent to the ones achieved using another traditional criterion in decision theory, namely the expected opportunity loss which suggests the strategy associated to the lowest weighted expected value for the regret.
9. The criterion adopted by Investor B provides the best outcomes using the forecasts given by the estimated T-GARCH model.

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