Accessing the Transverse Dynamics and Polarization of Gluons inside the Proton at the LHC

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We argue that the study of heavy quarkonia, in particular that of Υ, produced back to back with an isolated photon in pp collisions at the LHC is the best—and currently unique—way to access the distribution of both the transverse momentum and the polarization of the gluon in an unpolarized proton. These encode fundamental information on the dynamics of QCD. We derive analytical expressions for various transverse-momentum distributions that can be measured at the LHC and which allow for a direct extraction of the aforementioned quantities. To assess the feasibility of such measurements, we evaluate the expected yields and the relevant transverse-momentum distributions for different models of the gluon dynamics inside a proton.

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Introduction.—At LHC energies, the vast majority of hard reactions are initiated by the fusion of two gluons from both colliding protons. A good knowledge of gluon densities is, therefore, mandatory to perform reliable cross-section predictions, the archetypal example being the H^0 boson production. In perturbative QCD (pQCD), the production cross section of a given particle is conventionally obtained from the convolution of a hard parton-scattering amplitude squared and the *collinear* parton distribution functions (PDFs) inside the colliding hadrons, $G(x, \mu)$ or $f_1^g(x, \mu)$ for the gluon [1]. The PDF provides the distribution of a given parton in the proton as a function of its collinear momentum fraction x, at a certain (factorization) scale μ. Whereas the scale evolution of the PDFs is given by pQCD, experimental data are necessary to determine their magnitude (see e.g., Ref. [2]).

This collinear factorization, inspired by the parton model of Feynman and Bjorken, can be extended to take into account the transverse dynamics of the partons inside the hadrons. Different approaches have been proposed (unintegrated PDF, impact factors within k_T factorization, etc.). Out of these, the transverse-momentum (TM) dependent factorization is certainly the most rigorous with proofs of factorization for a couple of processes [3–6]. The further advantage of the TM dependent (TMD) formalism lies in its ability to deal with spin-dependent objects, both for the partons and the hadrons.

Much effort has been made recently to extract quark TMD distributions (TMDs in short) inside a proton from low-energy data from HERMES, COMPASS, or JLab experiments (see, e.g., Ref. [7] for recent reviews). But nothing is known experimentally about the gluon TMDs that rigorously parametrize the transverse motion of gluons

inside a proton. For an unpolarized proton, these are the distribution of unpolarized gluons denoted by f_1^g and the distribution of linearly polarized gluons $h_1^{\perp g}$ [8]. These functions contain fundamental information on the transverse dynamics of the gluon content of the proton [see the interpretation of $h_1^{\perp g}$ in Figs 1(a) and 1(b)] and are necessary to correctly describe gluon-fusion processes at all energies. Without any knowledge of these functions, it is impossible to calculate the Higgs transverse-momentum distribution accurately [9]. We, therefore, stress that a first determination of these quantities should have high priority.

In the small-x limit, the behavior of the gluon TMD f_1^g is probably connected to the unintegrated gluon distribution (UGD) [10], which has been widely studied in the
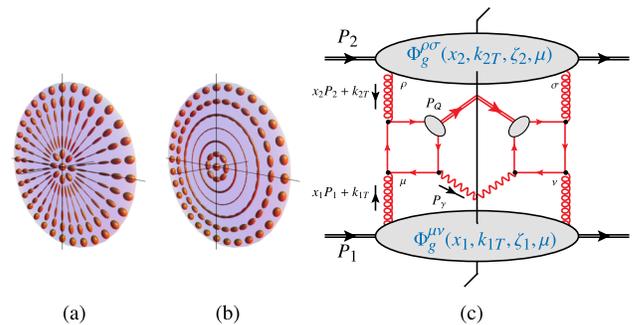
FIG. 1 (color online).   Visualization of the gluon polarization in the TM plane for a positive (a) and negative (b) Gaussian $h_1^{\perp g}$. (The ellipsoid major and minor axis lengths in the plane are proportional to the probability of finding a gluon with a linear polarization in that direction). (c) Feynman diagram for $p(P_1) + p(P_2) \rightarrow \mathcal{Q}(P_{\mathcal{Q}}) + \gamma(P_\gamma) + X$ via gluon fusion at LO in the TMD-factorization formalism.

framework of the color glass condensate (CGC) model [11–14], in k_T-factorization approaches, and as the solution of the Ciafaloni-Catani-Fiorani-Marchesini (CCFM) equation [15]. This connection is, however, less trivial than sometimes asserted, as in, e.g., Refs. [16,17]. For instance, the Weizsäcker-Williams distribution that appears in the CGC model *does* have the same operator structure as the TMD correlator [see Eq. (2)], *but* with a lightlike gauge link. The regularization of the rapidity divergence is, thus, different. Moreover, the CCFM equation does not rely on a gauge-invariant-operator definition. Nonetheless, to give some estimates of the experimental requirements, we will use various UGDs as an *Ansatz* for f_1^g and let $h_1^{\perp g}$ saturate a model-independent positivity bound derived in Ref. [8]. The latter is in accordance with k_T factorization in which full gluon polarization is implicit. In fact, this would serve as a test of the applicability of k_T-factorization methods for x close to 10^{-3}.

In this Letter, we will argue that the LHC experiments are ideally positioned to extract for the first time the gluon TMDs through the study of an isolated photon produced back to back with a heavy quarkonium. Furthermore, we show that the yields are large enough to perform such extractions with existing data at $\sqrt{s} = 7$ and 8 TeV.

Reactions sensitive to gluon TMDs.—Several processes have been proposed to measure both f_1^g and $h_1^{\perp g}$. A potentially very clean probe to extract gluon TMDs is the back-to-back production of a heavy-quark pair in electron-proton collisions $ep \rightarrow eQ\bar{Q}X$ in which the gluon TMDs appear linearly. Theoretical predictions have been provided at leading order (LO) [18] and next-to-leading order (NLO) [5] in pQCD. Such measurements could be performed at future facilities (EIC or LHeC), whose realization is, however, at best a decade away, while available HERA data on transverse-momentum imbalance of dijets (e.g., Ref. [19]) receive contributions from quark-induced subprocesses.

Back-to-back isolated photon-pair production in proton collisions $pp \rightarrow \gamma\gamma X$ is also sensitive to gluon TMDs [20] and is accessible at RHIC and the LHC but suffers from a contamination from quark-induced channels, a huge background from π^0 decays, and an inherent difficulty to trigger on such events.

Final states such as a heavy-quark pair or a dijet [18] should also be ideal candidates to probe gluon TMDs. However, once there is a color flow into the detected final state in the partonic-scattering subprocess, one cannot cleanly separate final-state interactions of this color flow from the nonperturbative TMD objects due to the non-Abelian characteristics of QCD [21]. This leads to a breakdown of TMD factorization for processes with colored final states.

This problem can be avoided in the case of the production of heavy quarkonia, provided that the heavy-quark pair is produced in a colorless state at short distances as in the color-singlet model [22] and that it is not accompanied by other—necessarily colorful—partons.

C-even quarkonium (χ_Q, η_Q) production at small TM is one of these cases where the factorization is expected to hold, as illustrated by studies both at LO [23] and NLO [24]. At low P_{QT}, η_Q and $\chi_{Q0,2}$ production proceeds without the emission of a final-state gluon and the color-octet (CO) contributions [25] are not kinematically enhanced. However, such experimental measurements are particularly difficult since they should be done at low TM, $P_{QT} \ll Q \simeq M_Q$, as required by TMD factorization. The hard scale of the process Q can only be the mass of the heavy quarkonium; hence, $Q \simeq M_Q$. The observation of low P_{QT} C-even quarkonia is likely impossible with ATLAS and CMS. LHCb may look at these down to $P_{QT} \simeq 1$ GeV, but an unambiguous gluon-TMDs determination—free of large power corrections in P_{QT}/Q—requires one to reach the sub-GeV region. Besides, this would not allow one to look at the scale evolution of the TMDs. Only two ranges can be probed: close to the charmonium and bottomonium masses.

Back-to-back quarkonium+isolated-photon production.—We propose a novel process to overcome these issues: the production of a back-to-back pair of a $^3S_1$ quarkonium Q (Υ or J/ψ) and an isolated photon $pp \rightarrow Q + \gamma + X$. Compared to the aforementioned processes, it is accessible by the LHC experiments: only the TM imbalance $\boldsymbol{q}_T = \boldsymbol{P}_{QT} + \boldsymbol{P}_{\gamma T}$ has to be small, not the individual TM, for TMD factorization to apply. In addition, the hard scale of the process Q can be tuned by selecting different invariant masses of the Q-γ pair. This allows one to look at the scale evolution of the TMDs and to greatly increase the q_T range where the TMD factorization applies with tolerable power corrections.

Previous studies [26–28] have shown that the CO contributions to inclusive $Q + \gamma$ production are likely smaller than in the inclusive case $Q + X$ (see e.g., Refs. [29–31]). (The case of $J/\psi + \gamma$ is however intriguing since state-of-the art NLO evaluations using recent non-relativistic QCD (NRQCD) fits predict negative CO cross sections [32].) The smallness of CO contributions is crucial since these would violate the TMD factorization.

As studied in Ref. [33], the CO contributions are also suppressed with respect to the color-singlet (CS) ones when the Q-γ pair is produced back to back, i.e., dominantly from $2 \rightarrow 2$ processes, although the gg fusion CS contribution [Fig. 1(c)] scales like P_{QT}^{-8}. Indeed, the P_{QT}^{-4} (fragmentation) CO contribution only appears for $q\bar{q}$ annihilation, extremely suppressed at LHC energies and, incidentally, on the order of the pure QED CS contribution (as for $J/\psi + W$ [34]). As regards gg fusion CO channels, they are subleading in P_{QT}, since they come from quark box and s-channel gluon diagrams, only via $C = +1$ CO states, such as $^1S_0^{[8]}$ or $^3P_J^{[8]}$. (For J/ψ, these CO states are known to be severely constrained if one wants to comply with e^+e^- inclusive data [35].) To substantiate this, we have computed the different CS and CO contributions in LO NRQCD, see Fig. 2. The CS yield is clearly dominant for Υ and likely above the CO one for J/ψ at the lowest Q value accessible at the LHC
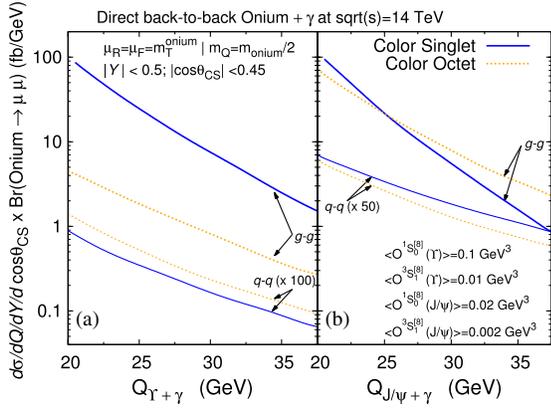
FIG. 2 (color online). Different contributions to the production of an isolated photon back to back with (a) $\Upsilon(1S)$ or (b) J/ψ from g-g and q-$\bar{q}$ fusion, from the CS and CO channels as function of the invariant mass of the pair. The curves for the q-$\bar{q}$ fusion are rescaled by a factor of 100 (respectively 50). The CO matrix elements we used are very close to those obtained in a recent LO fit of LHC data [36].

($P_{QT} \gtrsim 10$ GeV). It is also clear that this process is purely from gg fusion.

A further suppression of CO contributions can be achieved by also isolating the quarkonium (see Ref. [37]). The isolation should be efficient at large enough P_{QT} where the soft partons emitted during the hadronization of the CO heavy-quark pair are boosted and energetic enough to be detected. Experimentally, this would provide an interesting check of the CS dominance by measuring the (conventional) q_T-integrated cross section, which should coincide with the parameter-free CS prediction. This would also confirm that double-parton scattering contributions are suppressed by the isolation criteria. We emphasize that, according to our evaluations, such an isolation is not at all necessary for the Υ case.

Analytical expression for the q_T-dependent cross section.—Within TMD factorization [Fig. 1(c)], the cross section for a gluon-fusion initiated process is written, up to $\mathcal{O}(q_T^2/Q^2)$ corrections, as the convolution of a hard part with two TM-dependent correlators, i.e.,

$$d\sigma = \frac{(2\pi)^4}{8s^2} \int d^2\boldsymbol{k}_{1T} d^2\boldsymbol{k}_{2T} \delta^2(\boldsymbol{k}_{1T} + \boldsymbol{k}_{2T} - \boldsymbol{q}_T) \mathcal{M}_{\mu\rho}(\mathcal{M}_{\nu\sigma})^*$$
$$\times \Phi_g^{\mu\nu}(x_1, \boldsymbol{k}_{1T}, \zeta_1, \mu) \Phi_g^{\rho\sigma}(x_2, \boldsymbol{k}_{2T}, \zeta_2, \mu) d\mathcal{R}, \quad (1)$$

where $s = (P_1 + P_2)^2$ is the hadronic center-of-mass system (c.m.s.) energy squared and the phase space element of the outgoing particles is denoted by $d\mathcal{R}$. The hard part can be obtained as a series expansion in α_s by perturbatively calculating the partonic scattering $g(k_1) + g(k_2) \to \mathcal{Q}(P_{\mathcal{Q}}) + \gamma(P_\gamma)$, with the incoming gluon momenta given by $k_1 = x_1 P_1 + k_{1T} - k_{1T}^2/(x_1 s) P_2$ (and likewise for k_2), and subtracting the parts already contained in the gluon TMD correlators [6,38,39]. k_{1T} is a 4-vector perpendicular

to both P_1 and P_2, which has transverse components $\boldsymbol{k}_{1T}$ in the c.m.s. frame; $x_1 = q \cdot P_2 / P_1 \cdot P_2$ and $x_2 = q \cdot P_1 / P_1 \cdot P_2$, where $q = P_{\mathcal{Q}} + P_\gamma$.

Since QCD corrections to the inclusive production of a quarkonium-photon pair are known to be large [26,28], we find it useful to emphasize that this does not translate to TMD factorization. The reason is that the initial-state radiations are absorbed into the TMDs such that the hard part is free of q_T dependence and, with appropriate choices of ζ and μ, is also free of large logarithms [6,38,39]. In addition, the back-to-back (small q_T) requirement and the photon isolation in our observable further suppresses additional radiations. A LO calculation of the hard part is, therefore, sufficient for a first gluon TMD extraction.

The gluon-TMD correlator for an unpolarized proton is defined as

$$\Phi_g^{\mu\nu}(x, \boldsymbol{k}_T, \zeta, \mu) \equiv \int \frac{d(\xi \cdot P) d^2 \xi_T}{(xP \cdot n)^2 (2\pi)^3} e^{i(xP + k_T) \cdot \xi}$$
$$\times \langle P | F_a^{n\nu}(0) \left(\mathcal{U}_{[0,\xi]}^{n[-]} \right)_{ab} F_b^{n\mu}(\xi) | P \rangle |_{\xi \cdot P' = 0}$$
$$= -\frac{1}{2x} \left\{ g_T^{\mu\nu} f_1^g - \left(\frac{k_T^\mu k_T^\nu}{M_p^2} + g_T^{\mu\nu} \frac{\boldsymbol{k}_T^2}{2M_p^2} \right) h_1^{\perp g} \right\}$$
$$+ \text{supp}, \quad (2)$$

where $g_T^{\mu\nu} = g^{\mu\nu} - (P_1^\mu P_2^\nu + P_2^\mu P_1^\nu)/P_1 \cdot P_2$, M_p is the proton mass, and the gauge link $\mathcal{U}_{[0,\xi]}^{n[-]}$ renders the matrix element gauge invariant. It runs from 0 to ξ via $-\infty$ along the n direction. [n is a timelike dimensionless 4-vector with no transverse components such that $\zeta^2 = (2\,n \cdot P)^2/n^2$.] The correlator is parametrized by the two gluon TMDs discussed above, $f_1^g(x, \boldsymbol{k}_T, \zeta, \mu)$ and $h_1^{\perp g}(x, \boldsymbol{k}_T, \zeta, \mu)$ [8], and by terms that are suppressed in the high-energy limit.

The structure of the TMD cross section is then found to be

$$\frac{d\sigma}{dQ\,dY\,d^2\boldsymbol{q}_T\,d\Omega} = \frac{C_0(Q^2 - M_{\mathcal{Q}}^2)}{sQ^3 D} \{ F_1 \mathcal{C}[f_1^g f_1^g] + F_3 \cos(2\phi)$$
$$\times \mathcal{C}[w_3 f_1^g h_1^{\perp g} + x_1 \leftrightarrow x_2] + F_4 \cos(4\phi)$$
$$\times \mathcal{C}[w_4 h_1^{\perp g} h_1^{\perp g}] \} + \mathcal{O}\left(\frac{\boldsymbol{q}_T^2}{Q^2} \right), \quad (3)$$

where $d\Omega = d\cos\theta\,d\phi$ is expressed in terms of Collins-Soper angles [40] and where Q, Y, and $\boldsymbol{q}_T$ are the invariant mass, the rapidity, and the TM of the pair—the latter two to be measured in the hadron c.m.s. frame. The Collins-Soper angles describe the spatial orientation of the back-to-back photon-quarkonium pair in the (Collins-Soper) rest frame of the pair. The overall normalization is given by $C_0 = 4\alpha_s^2 \alpha_{em} e_Q^2 |R_0(0)|^2 / (3M_{\mathcal{Q}}^3)$, where $R_0(0)$ is the quarkonium radial wave function at the origin and e_Q is the heavy-quark charge. The F factors, the denominator D and the weights are found to be
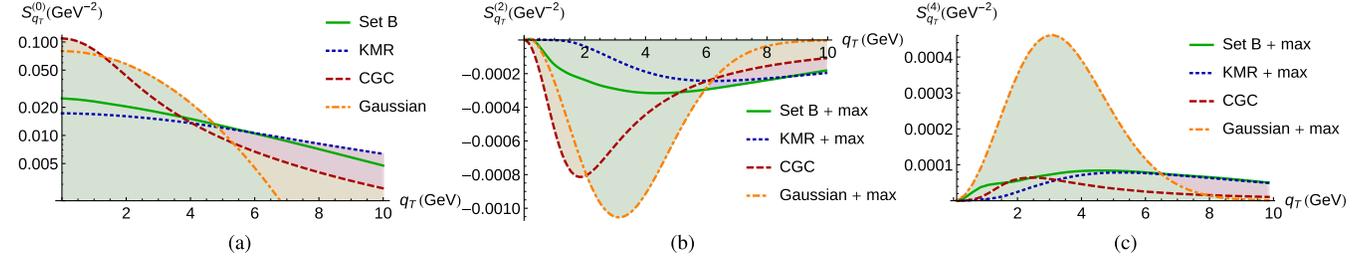
FIG. 3 (color online).   Model predictions for $\Upsilon + \gamma$ production at $Q = 20$ GeV, $Y = 0$, and $\theta = \pi/2$ at $\sqrt{s} = 14$ TeV for (a) $\mathcal{S}_{q_T}^{(0)}$, (b) $\mathcal{S}_{q_T}^{(2)}$, and (c) $\mathcal{S}_{q_T}^{(4)}$. The longitudinal momentum fractions are $x_1 = x_2 = Q/\sqrt{s} \simeq 1.4 \times 10^{-3}$.

$$F_1 = 1 + 2\alpha^2 + 9\alpha^4 + (6\alpha^4 - 2)\cos^2\theta + (\alpha^2 - 1)^2\cos^4\theta,$$

$$F_3 = 4\alpha^2\sin^2\theta, \qquad F_4 = (\alpha^2 - 1)^2\sin^4\theta,$$

$$D = ((\alpha^2 + 1)^2 - (\alpha^2 - 1)^2\cos^2\theta)^2,$$

$$w_3 = \frac{\boldsymbol{q}_T^2 \boldsymbol{k}_{2T}^2 - 2(\boldsymbol{q}_T \cdot \boldsymbol{k}_{2T})^2}{2M_P^2 \boldsymbol{q}_T^2},$$

$$w_4 = 2\left[\frac{\boldsymbol{k}_{1T} \cdot \boldsymbol{k}_{2T}}{2M_P^2} - \frac{(\boldsymbol{k}_{1T} \cdot \boldsymbol{q}_T)(\boldsymbol{k}_{2T} \cdot \boldsymbol{q}_T)}{M_P^2 \boldsymbol{q}_T^2}\right]^2 - \frac{\boldsymbol{k}_{1T}^2 \boldsymbol{k}_{2T}^2}{4M_P^4}. \quad (4)$$

where $\alpha \equiv Q/M_{\mathcal{Q}}$. The convolution is defined as

$$\mathcal{C}[wfg] \equiv \int d^2\boldsymbol{k}_{1T} \int d^2\boldsymbol{k}_{2T}\delta^2(\boldsymbol{k}_{1T} + \boldsymbol{k}_{2T} - \boldsymbol{q}_T)$$
$$\times w(\boldsymbol{k}_{1T}, \boldsymbol{k}_{2T})f(x_1, \boldsymbol{k}_{1T}^2)g(x_2, \boldsymbol{k}_{2T}^2), \quad (5)$$

where $x_{1,2} = \exp[\pm Y]Q/\sqrt{s}$.

We propose the measurement of 3 TM spectra, normalized and weighted by $\cos n\phi$ for $n = 0, 2, 4$,

$$\mathcal{S}_{q_T}^{(n)} \equiv \frac{\int d\phi \cos(n\phi) \frac{d\sigma}{dQdYd^2\boldsymbol{q}_T d\Omega}}{\int d\boldsymbol{q}_T^2 \int d\phi \frac{d\sigma}{dQdYd^2\boldsymbol{q}_T d\Omega}}, \quad (6)$$

where we will take the $\boldsymbol{q}_T^2$ integration in the denominator up to $(Q/2)^2$. These spectra separate out the three terms in Eq. (3),

$$\mathcal{S}_{q_T}^{(0)} = \frac{\mathcal{C}[f_1^g f_1^g]}{\int d\boldsymbol{q}_T^2 \mathcal{C}[f_1^g f_1^g]}, \qquad \mathcal{S}_{q_T}^{(2)} = \frac{F_3\mathcal{C}[w_3 f_1^g h_1^{\perp g} + x_1 \leftrightarrow x_2]}{2F_1 \int d\boldsymbol{q}_T^2 \mathcal{C}[f_1^g f_1^g]},$$

$$\mathcal{S}_{q_T}^{(4)} = \frac{F_4\mathcal{C}[w_4 h_1^{\perp g} h_1^{\perp g}]}{2F_1 \int d\boldsymbol{q}_T^2 \mathcal{C}[f_1^g f_1^g]}. \quad (7)$$

It is remarkable to note that the sole measurement of $\mathcal{S}_{q_T}^{(0)}$, i.e., of the cross section integrated over ϕ, allows for a clean determination of the unpolarized gluon TMD f_1^g, since $h_1^{\perp g}$ does not enter $\mathcal{S}_{q_T}^{(0)}$. If $\mathcal{S}_{q_T}^{(2)}$ or $\mathcal{S}_{q_T}^{(4)}$ can also be measured, then the linearly polarized gluon distribution $h_1^{\perp g}$ is also accessible.

Numerical results and discussion.—In our calculations, we adopt the following UGD *Ansätze* for f_1^g: the Set B0 solution to the CCFM equation with an initial distribution based on the HERA data from Refs. [41,42], the KMR

parametrization from Ref. [43], and the CGC model prediction from Refs. [11–14]. The first two depend on a factorization scale, taken to be Q, whereas the last one depends on a saturation scale taken as $Q_s = (x_0/x)^\lambda Q_0$, with $\lambda = 0.29$, $x_0 = 4 \times 10^{-4}$, and $Q_0 = 1$ GeV [44]. We have also used a simple Gaussian parametrization, as done in Ref. [45] to describe the intrinsic gluon TM, but with $\langle p_T^2 \rangle = (2.5 \text{ GeV})^2$. Our results are shown in Fig. 3(a).

For $h_1^{\perp g}$, we use the CGC model prediction of Refs. [13,14] and the maximal value from the positivity constraint $|h_1^{\perp g}| \leq 2M_P^2/\boldsymbol{k}_{1T}^2 f_1^g$ [8]. The resulting $\mathcal{S}_{q_T}^{(2,4)}$ values are plotted in Figs. 3(b) and 3(c).

From Fig. 3(a), we first conclude that measuring $\mathcal{S}_{q_T}^{(0)}$ in bins of 1 GeV should suffice to get a first determination of the shape of the unpolarized gluon distribution. As regards $\mathcal{S}_{q_T}^{(2)}$ and $\mathcal{S}_{q_T}^{(4)}$, whose magnitude is obviously smaller, one can integrate them over $\boldsymbol{q}_T^2$ (up to $(Q/2)^2$) to get the first experimental verification of a nonzero linearly polarized gluon distribution. $\mathcal{S}_{q_T}^{(2)}$ is here the most promising, as we obtain for the integrated distribution -2.9%, -2.6%, -2.5%, and -2.0% for the Gauss, CGC, SetB, and KMR *Ansatz,* respectively, whereas for the $n = 4$ distribution we obtain 1.2%, 0.7%, 0.6%, and 0.3% for the Gauss, SetB, KMR, and CGC models, respectively. We note that the q_T-integrated cross section for $\Upsilon + \gamma$ production in Fig. 2 is about 100 (50) fb/GeV at $Q = 20$ GeV for $\sqrt{s} = 14(7)$ TeV. The 20 fb^{-1} of integrated luminosity already collected at 7 and 8 TeV should be sufficient to measure the q_T shape of $\mathcal{S}_{q_T}^{(0)}$, while $\mathcal{S}_{q_T}^{(2)}$ could be measured in a single q_T bin.

Conclusion.—The production of an isolated photon back to back with a possibly isolated quarkonium in pp collisions is the ideal observable to study the transverse dynamics and polarization of the gluons in the proton along the lines of TMD factorization. The requirement for a heavy quarkonium in the final state suppresses quark-initiated reactions, making it a very clean probe of the gluon content of the proton, whereas the large scale set by the invariant mass of the pair allows a TMD-factorized description over an extensive range of q_T and, hence, an extraction of the gluon TMDs in this range. The expected yields at the LHC experiments are large enough to get the

first experimental verification of a nonzero gluon polarization in unpolarized protons. These measurements would, therefore, provide a test of the reliability of the k_T-factorization approach at $x \sim 10^{-3}$ and allow for the first extraction of the gluon TMDs in the proton.

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[1] R. Brock *et al.*, Rev. Mod. Phys. **67**, 157 (1995).
[2] P. M. Nadolsky, H.-L. Lai, Q.-H. Cao, J. Huston, J. Pumplin, D. Stump, W.-K. Tung, and C.-P. Yuan, Phys. Rev. D **78**, 013004 (2008); A. D. Martin, W. J. Stirling, R. S. Thorne, and G. Watt, Eur. Phys. J. C **63**, 189 (2009); R. D. Ball, L. Del Debbio, S. Forte, A. Guffanti, J. I. Latorre, J. Rojo, and M. Ubiali, Nucl. Phys. **B838**, 136 (2010).
[3] J. C. Collins, *Foundations of Perturbative QCD* (Cambridge University Press, Cambridge, England, 2011); S. M. Aybat and T. C. Rogers, Phys. Rev. D **83**, 114042 (2011); J. C. Collins and T. C. Rogers, Phys. Rev. D **87**, 034018 (2013); M. G. Echevarria, A. Idilbi, and I. Scimemi, J. High Energy Phys. 07 (2012) 002.
[4] X. D. Ji, J.-p. Ma, and F. Yuan, Phys. Rev. D **71**, 034005 (2005).
[5] R. Zhu, P. Sun, and F. Yuan, Phys. Lett. B **727**, 474 (2013).
[6] J. P. Ma, J. X. Wang, and S. Zhao, Phys. Rev. D **88**, 014027 (2013).
[7] V. Barone, F. Bradamante, and A. Martin, Prog. Part. Nucl. Phys. **65**, 267 (2010); U. D'Alesio and F. Murgia, Prog. Part. Nucl. Phys. **61**, 394 (2008).
[8] P. J. Mulders and J. Rodrigues, Phys. Rev. D **63**, 094021 (2001).
[9] D. Boer, W. J. den Dunnen, C. Pisano, M. Schlegel, and W. Vogelsang, Phys. Rev. Lett. **108**, 032002 (2012); D. Boer, W. J. den Dunnen, C. Pisano, and M. Schlegel, Phys. Rev. Lett. **111**, 032002 (2013); W. J. den Dunnen, D. Boer, C. Pisano, M. Schlegel, and W. Vogelsang, arXiv:1205.6931; W. J. den Dunnen, *Proc. Sci.*, DIS2013 (**2013**) 242; W. J. den Dunnen, arXiv:1311.1048.
[10] S. Catani, M. Ciafaloni, and F. Hautmann, Phys. Lett. B **242**, 97 (1990); S. Catani, M. Ciafaloni, and F. Hautmann, Nucl. Phys. **B366**, 135 (1991); S. Catani, M. Ciafaloni, and F. Hautmann, Phys. Lett. B **307**, 147 (1993).
[11] F. Dominguez, B.-W. Xiao, and F. Yuan, Phys. Rev. Lett. **106**, 022301 (2011).
[12] F. Dominguez, C. Marquet, B.-W. Xiao, and F. Yuan, Phys. Rev. D **83**, 105005 (2011).
[13] F. Dominguez, J.-W. Qiu, B.-W. Xiao, and F. Yuan, Phys. Rev. D **85**, 045003 (2012).
[14] A. Metz and J. Zhou, Phys. Rev. D **84**, 051503 (2011).
[15] B. Andersson *et al.*, Eur. Phys. J. C **25**, 77 (2002); J. R. Andersen *et al.*, Eur. Phys. J. C **35**, 67 (2004); **48**, 53 (2006).
[16] F. Hautmann and H. Jung, arXiv:1312.7875.
[17] A. V. Lipatov, G. I. Lykasov, and N. P. Zotov, Phys. Rev. D **89**, 014001 (2014).
[18] D. Boer, S. J. Brodsky, P. J. Mulders, and C. Pisano, Phys. Rev. Lett. **106**, 132001 (2011); D. Boer, P. J. Mulders, and C. Pisano, Phys. Rev. D **80**, 094017 (2009); C. Pisano, D. Boer, S. J. Brodsky, M. G. A. Buffing, and P. J. Mulders, J. High Energy Phys. 10 (2013) 024.
[19] S. Chekanov *et al.* [ZEUS Collaboration], Nucl. Phys. **B786**, 152 (2007).
[20] J.-W. Qiu, M. Schlegel, and W. Vogelsang, Phys. Rev. Lett. **107**, 062001 (2011).
[21] J. C. Collins and J. W. Qiu, Phys. Rev. D **75**, 114014 (2007); T. C. Rogers and P. J. Mulders, Phys. Rev. D **81**, 094006 (2010).
[22] C.-H. Chang, Nucl. Phys. **B172**, 425 (1980); R. Baier and R. Rückl, Phys. Lett. **102B**, 364 (1981); Z. Phys. C **19**, 251 (1983).
[23] D. Boer and C. Pisano, Phys. Rev. D **86**, 094007 (2012).
[24] J. P. Ma, J. X. Wang, and S. Zhao, Phys. Rev. D **88**, 014027 (2013).
[25] G. T. Bodwin, E. Braaten, and G. P. Lepage, Phys. Rev. D **51**, 1125 (1995); **55**5853(E) (1997); P. L. Cho and A. K. Leibovich, Phys. Rev. D **53**, 150 (1996); **53**, 6203 (1996).
[26] R. Li and J.-X. Wang, Phys. Lett. B **672**, 51 (2009).
[27] C. S. Kim, J. Lee, and H. S. Song, Phys. Rev. D **55**, 5429 (1997).
[28] J. P. Lansberg, Phys. Lett. B **679**, 340 (2009).
[29] N. Brambilla *et al.*, Eur. Phys. J. C **71**, 1534 (2011).
[30] J. P. Lansberg, Eur. Phys. J. C **61**, 693 (2009).
[31] J. P. Lansberg, Int. J. Mod. Phys. A **21**, 3857 (2006).
[32] R. Li and J.-X. Wang, arXiv:1401.6918.
[33] P. Mathews, K. Sridhar, and R. Basu, Phys. Rev. D **60**, 014009 (1999).
[34] J. P. Lansberg and C. Lorce, Phys. Lett. B **726**, 218 (2013).
[35] Y. J. Zhang, Y. Q. Ma, K. Wang, and K. T. Chao, Phys. Rev. D **81**, 034015 (2010); Z. G. He, Y. Fan, and K. T. Chao, Phys. Rev. D **81**, 054036 (2010).
[36] R. Sharma and I. Vitev, Phys. Rev. C **87**, 044905 (2013).
[37] A. C. Kraan, AIP Conf. Proc..**1038**, 45 (2008); D. Kikola, Nucl. Phys. B, Proc. Suppl. **214**, 177 (2011).
[38] X.-D. Ji, J.-P. Ma, and F. Yuan, J. High Energy Phys. 07 (2005) 020.
[39] P. Sun, B.-W. Xiao, and F. Yuan, Phys. Rev. D **84**, 094005 (2011).
[40] J. C. Collins and D. E. Soper, Phys. Rev. D **16**, 2219 (1977).
[41] H. Jung, arXiv:hep-ph/0411287.
[42] H. Jung *et al.*, Eur. Phys. J. C **70**, 1237 (2010).
[43] M. A. Kimber, A. D. Martin, and M. G. Ryskin, Phys. Rev. D **63**, 114027 (2001).
[44] F. Gelis, E. Iancu, J. Jalilian-Marian, and R. Venugopalan, Annu. Rev. Nucl. Part. Sci. **60**, 463 (2010).
[45] K. Sridhar, A. D. Martin, and W. J. Stirling, Phys. Lett. B **438**, 211 (1998).