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LATENT CLASS MODELS IN FINANCIAL DATA ANALYSIS

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1. INTRODUCTION

Latent variables and unobservability problems represent a preferred topic in financial literature, where the core variable, the risk, is typically unobservable. Most widespread and relevant financial models specify a factor structure for asset returns, where unobservable factors are frequently evaluated by resorting to factor analysis methodologies. Studies related to the number of factors, their interpretation and correlation with observable macroeconomic variables have caused much scientific and operational discussion during the last thirty years.

Although of great importance, risk measurement does not exhaust the topic of unobservability in financial data analysis. A further important and still unresolved question regards portfolio choice, where the investment decision is observed, but the decision process remains unobserved. More specifically, investors are subject to constraints that are generally unobserved, such as the minimum financial level to enter the stock or bond global markets, and constraints which are unobservable, such as investor preferences, information costs and invisible barriers to foreign investments. These constraints represent restrictions on the shares of wealth invested in financial assets, which are typically categorical variables. Furthermore, these constraints are not fixed for all investors, but are a function of economic and demographic characteristics of households (such as gender, level of education, percentile of wealth, etc.) which play a major role in the composition of portfolios (Guiso and Paiella, 2003).

Because of these peculiarities, factor analysis is not a feasible solution in order to deal with the measurement of portfolio choice constraints. Among different proposals of statistical methodology, a valid alternative is represented by latent class analysis, a factor analytic analogue for categorical data which identifies an unobserved set of latent classes that explains the relationship among a set of observed categorical variables (McCutcheon, 1987). The latent classes constitute a set of mutually exclusive and exhaustive categories, within which the observed variables are independent of one another. This is a requirement referred to as the axiom of "local independence" (Lazarsfeld and Henry, 1968) and can be seen as the defining characteristic of latent structure models. In every latent structure model it is assumed that observed associations between manifest variables depend on the relationship between latent and manifest variables. Thus, local independence assumes that if we hold the latent variable constant, manifest variables should be statistically independent from each other (Heinen, 1996).

Latent class models are traditionally applied to psychometric and genetic problems, but they can be extremely useful also in order to solve many questions about portfolio choices; in the following we propose to apply latent class analysis to international investment decisions.

The systematic lack of international diversification may be due to unobservable exogenous constraints, which prevent a group of investors from achieving complete diversification of their portfolios. Modelling the distinction between international investors and non international investors might give a plausible explanation of the bias.

In this paper we provide a new latent class framework based on household survey data which, by exploring the possibility to analyze the evident inconsistency of international asset pricing models, reconciles theory and empirical measures of international optimal portfolio diversification.

2. LATENT CLASS ANALYSIS OF FINANCIAL MARKETS

In order to explain the usefulness of latent class analysis for portofolio choices let i be an investor who faces a problem of portfolio choice according to his preferences represented by utility function U_i and individual risk aversion parameter θ_i. For sake of semplicity, suppose there are only two financial assets. Let w_{ij} the observed fraction of the wealth invested by investor i in financial asset j ($j=1, 2$) and x_{ij} the unobserved restriction of this fraction. This implies that investor i can not hold a wealth share in asset j higher than x_{ij}, that is $w_{ij} \leq x_{ij} \leq 1$. The remaining wealth share is invested in the other financial asset.

Now suppose that the unobserved restriction variable x_{ij} assumes, accordingly to latent class requirements, only m discrete values, ($C_1=0$, C_2, ..., C_{m-1}, $C_m=1$). When $x_{ij} = C_1 = 0$, the i-th investor is precluded from any investment in financial asset j. It follows necessarily that $w_{ij} = 0$. When $x_{ij} = C_2$, the i-th investor is allowed to invest in asset j, but cannot hold a wealth share higher than C_2, that is $w_{ij} \leq C_2$. Similarly, when $x_{ij} = C_3$, the i-th investor is allowed to invest in asset j, but cannot hold a wealth share higher than C_3, that is $w_{ij} \leq C_3$. Finally, when $x_{ij} = C_m = 1$, the i-th investor has no restriction on the investment in asset j, thus implying that w_{ij} can assume any value between 0 and 1.

The relationship between the unobserved restriction variable x and the observed wealth share invested w can be summerized by the following contingency table, where f_{kk} represents the population share with restriction C_k and investment $C_{k-1} < w \leq C_k$.

TABLE 1

Contingency table: x (unobserved restriction on wealth share) vs w (observed wealth share invested)

x / *W*	With complete restriction $C_1=0$	With partial restriction C_2	With partial restriction C_3	…	Without restriction $C_m=1$
Without asset $W= C_1=0$	f_{11}	f_{12}	f_{13}	…	f_{1m}
With asset $0< w \leq C_2$	0	f_{22}	f_{23}	…	f_{2m}
With asset $C_2< w \leq C_3$	0	0	f_{33}	…	f_{3m}
…	…	…	…	…	…
With asset $C_{m-1}< w \leq C_m=1$	0	0	0	…	f_{mm}

Note the contingency table represented in Table 1 is a square matrix with null observed frequencies in the lower off-diagonal triangle, that is $f_{rs}=0$ for all r>s, due to the restriction $w_{ij} \leq x_{ij}$.

In order to correctly compare the investor's effective portfolio with the risk-return efficient one it is important to consider the restrictions on portfolio weitghs deriving from the constraints. If the restrictions are not taken into account in the evaluation of the frontier, any comparison leads to over-estimating the gap between the empirical portfolio and the theroretical one. Moreover, if we allow investors heterogeneity (the constraints are not the same for all people) we have a "family" of frontier investment opportunities corresponding to different sets of constraints. Therefore, for the statistical analysis of empirical financial choices we ought to classify the population of investors into the *m* unobserved sub-groups, which, in latent class analysis, correspond to latent classes.

A special case of this framework is for m=2, that is when we have a dichotomous latent variable. It follows that investors can only belong to two sub-groups. The first group is represented by those investors who are not allowed to invest in asset *j*, that is $w_{ij} = x_{ij} = C_1 = 0$. The other sub-group has no constraints in investing in financial asset *j*, that is $x_{ij} = C_1 = 1$; for the second group it follows $w_{ij} = 0$ if the *i*-th investor decides not to hold financial asset *j* or $w_{ij} > 0$ if the *i*-th investor decides to hold financial asset *j*. In this special case contingency table *(x,w)* reduces to Table 2.

TABLE 2

Contingency table in the binary case: x (unobserved restriction on wealth share) vs w (observed wealth share invested)

x / *w*	With complete restriction $C_1=0$	Without restriction $C_2=1$
Without asset *w=0*	f_{11}	f_{12}
With asset *w>0*	0	f_{22}

The framework of Table 2 can be very useful if we are interested in investors participation in specific financial markets, such as the stock market, the financial derivative market, the foreign exchange market, etc., since most of investors do not hold those instruments and only a small number of investors hold a very small share of their wealth in those financial markets.

According to Table 2, the population of investors is classified into two groups (m=2): such a classification is represented by a dichotomous latent variable X defined as follows

$X_i = C_1 = 0$ complete restriction

$X_i = C_2 = 1$ without restriction

A natural indicator for this latent class variable is represented by a dichotomous variable Y_i that equals *0* when the *i*-th investor does not effectively hold the *j*-th asset and that equals *1* when the *i*-th investor holds asset *j*:

$Y_i = 0 \;\; if \;\; w_i = 0$ without asset *j*

$Y_i = 1 \;\; if \;\; w_i > 0$ with asset *j*

When the *i*-th investor is precluded to any investment in asset *j*, that is $X_i = C_1 = 0$, it follows that $Y_i=0$, that is the *i*-th investor won't hold asset *j*. On the other side, when the *i*-th investor is not precluded from investing in asset *j*, that is $X_i = C_2 = 1$, it follows $Y_i=1$ if the *i*-th investor decides to hold asset *j* or $Y_i=0$ if the *i*-th investor decides not to hold asset *j*. Such a relationship can be summarized by Table 3 which is subject to the restriction, $f_{00} + f_{01} + f_{11} = 1$ (i.e. $f_{10} = 0$).

TABLE 3

Latent class variable X *and investment in asset j* Y

Y \ X	X	With restrictions	Without restrictions
Y		0	1
Without asset j	0	f_{00}	f_{01}
With asset j	1	0	f_{11}

It is important to note that, in our approach, latent class analysis, which is traditionally used as an explorative method, is considered in the perspective of a confirmative methodology.

3. THE MODEL

As a first step of the analysis we consider a vector Z of k variables related to social and demographic characteristics of the investors, and a vector W of h variables regarding economic and financial characteristics of the investors.

The starting point is the model depicted by the path diagram in Figure 1: the k exogenous variables Z, representing social and demographic characteristics, are assumed not to be influenced by the other variables; variables regarding economic and financial characteristics are influenced by the latter variables and directly influence the latent class variable X. This implies that there is no direct relationship between social and demographic investor characteristics and the existence of restrictions on the investment in asset j: latent classes, X, are assumed to be the only direct determinant of j-th asset holding, Y.

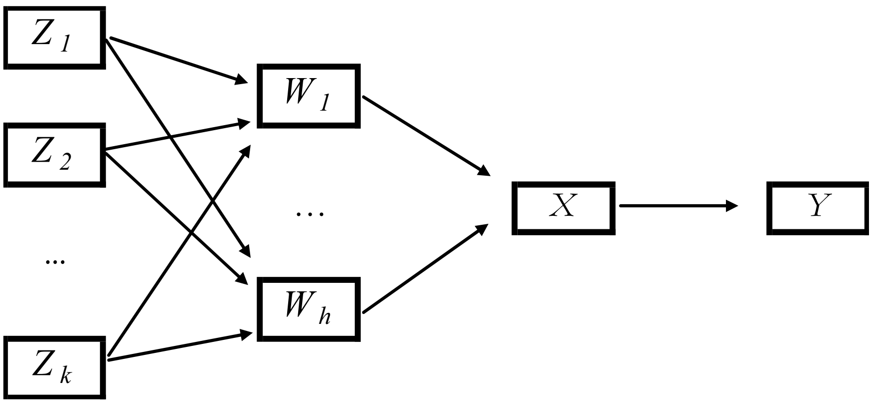

Figure 1 – Latent class model of financial choices.

The model illustrated in Figure 1 can be expressed in equations 1 and 2 in terms of conditional response probabilities and loglinear parameters, respectively. All parameters are restricted to the usual identifying restrictions; the conditional probabilities sum to one where appropriate and the loglinear parameters add up to zero whenever they are summed over any of their subscripts.

$$\pi_{zwxy}^{ZWXY} = \pi_{z}^{Z} \cdot \pi_{wz}^{W|Z} \cdot \pi_{xw}^{X|W} \cdot \pi_{yx}^{Y|X} \tag{1}$$

$$\ln(F_{zwxy}^{ZWXY}) = \lambda + \lambda_{y}^{Y} + \lambda_{x}^{X} + \lambda_{w}^{W} + \lambda_{z}^{Z} + \lambda_{xy}^{XY} + \lambda_{wx}^{WX} + \lambda_{zw}^{ZW} \tag{2}$$

Conditional probabilities $\pi_{yx}^{Y|X}$ are the probabilities that an investor of latent class x will be at level y of the observed indicator variable Y.

3.1. *Estimation and evaluation of latent class model*

In order to obtain maximum likelihood estimates for the latent class model (1), the following equations (3)-(5):

$$\pi_{zwyx}^{ZWYX} = \pi_{zx}^{Z|X} \cdot \pi_{wx}^{W|X} \cdot \pi_{yx}^{Y|X} \cdot \pi_{x}^{X} \tag{3}$$

$$\pi_{zwy}^{ZWY} = \sum_{x} \pi_{zwyx}^{ZWYX} \tag{4}$$

$$\sum_{x} \pi_{x}^{X} = \sum_{z} \pi_{zx}^{Z|X} = \sum_{w} \pi_{wx}^{W|X} = \sum_{y} \pi_{yx}^{Y|X} = 1 \tag{5}$$

have to satisfy equations (6)-(9):

$$\pi_{x}^{X} = \sum_{zwy} f_{zwy} \cdot \pi_{xzwy}^{X|ZWY} \tag{6}$$

$$\pi_{zx}^{Z|X} = \frac{\sum_{wy} f_{zwy} \cdot \pi_{xzwy}^{X|ZWY}}{\pi_{x}^{X}} \tag{7}$$

$$\pi_{wx}^{W|X} = \frac{\sum_{zy} f_{zwy} \cdot \pi_{xzwy}^{X|ZWY}}{\pi_{x}^{X}} \tag{8}$$

$$\pi_{yx}^{Y|X} = \frac{\sum_{zw} f_{zwy} \cdot \pi_{xzwy}^{X|ZWY}}{\pi_{x}^{X}} \tag{9}$$

where f_{zwy} identifies the observed probability relative to cell *zwy* in the observed variable crosstabulation *ZWY* (McCutcheon, 1987).

The specification of the latent class model requires the definition of a structural constraint regarding the relationship between the latent classes, *X*, and its indicator, *Y*, that is the conditional probability that the indicator variable *Y=1*, given that latent class *X=0*, equals zero, $\pi_{10}^{Y|X} = 0$.

In the following, parameters of the latent class models are estimated by an iterative maximum-likelihood procedure, called Expectation-maximization (E-M) algorithm, which is robust with respect to the initial values (Dempster *et al.*, 1977; Scott, 1993). The E-M estimation starts by replacing the class variable (*X*) by its expectated value. Then by using a set of reasonable starting values, the expectation for *X* and the maximization for conditional probabilities and class proportions alternate until convergence.

Several criteria have become more or less standard in the evaluation of latent class models. For a latent class model the Pearson chi-square (χ^2) or the likelihood ratio chi-squared (L^2) can be used for comparing the observed frequencies of the response patterns with the expected frequencies under the fitted model. Latent class models that lead to expected cell frequencies that are too far from the observed cell frequencies are deemed unacceptable or implausible. Models with more parameters usually provide a better fit to the data. More parsimonious models tend to have a somewhat poorer fit. Thus, the usual task is to find the most parsimonious model that has an acceptable fit to the observed data.

However, as the number of manifest variables increases, the frequency table of the response patterns become sparse and that invalidates the *p*-values obtained from the χ^2 and the log-likelihood ratio tests are less. Solutions to this problem when a model is fitted in a multi-way contingency table with binary variable are proposed by Reiser and Lin (1999) and Bartholomew and Leung (2002). No formal statistical test has been yet obtained for sparse contingency tables in case of multiple response data, therefore Pearson chi-square and the log-likelihood ratio chi-square can only be used as indicators for bad fit.

Model selection among LCA models with different number of parameters can be obtained by the use of information criteria, parametric resampling, etc.. Information criteria are probably the most convenient methods from a computing efficiency viewpoint as they require much less computing efforts than other methods such as parametric resampling.

The Akaike information criterion (AIC) was one of the earliest proposals of information criteria. The AIC has the following form:

$$AIC = L^2 - 2 \cdot df$$

where L^2 is the likelihood chi-squared ratio and *df* is the total number of free parameters in the model. Woodruffe (1982) showed that the AIC is not theoretically consistent, consequently AIC will not select the correct model when the sample size N approaches infinity.

The Bayesian information criterion (BIC) was proposed by Schwartz and has the following form:

$$BIC = L^2 - df \cdot \log N$$

The BIC has a consistent property (Haughton, 1988) that leads to a correct model choice as $N \to \infty$. Bozdogan (1987) derived a consistent version of AIC (CAIC) from the Kullback-Leibler information measure:

$$CAIC = L^2 - df \cdot (\log N + 1)$$

Because the CAIC has more severe penalty on over-parameterization than the BIC or the AIC, the CAIC will tend to favor a model with fewer parameters.

4. LATENT CLASS ANALYSIS OF EQUITY HOME BIAS

In the framework of latent class methodology, we resort to the binary case illustrated in Tables 2 and 3, and to the model presented in the previuos paragraph in order to deal with an important unresolved problem in financial literature: the measurement of equity home bias. The binary case is a suitable framework to deal with international portfolio choice since most of investors decide not to hold foreign equities and a small number of investors hold a very small share of their wealth in foreign financial markets.

For thirty years financial literature has been looking for convincing explanations of systematically low international investments: share of foreign assets held by domestic investors is much lower than expected by risk-return efficient portfolios. This puzzle is usually called the 'equity home bias' in finance (Lewis, 1999) and, in macroeconomic literature, 'consumption home bias' or 'lack of risk sharing' (Gardini *et al.*, 2001; Cavaliere *et al.*, 2005).

This empirical lack of consistency may be due to the statistical properties of consumption and asset prices or to implausible hypothesis of the theoretical models. Lewis has suggested the greater variability in stock returns relative to consumption as an explanation of the bias (Lewis 2000). Others papers have attributed the bias to the incompleteness of asset data, which should include also transaction and information costs and the international diversification obtained by means of domestic shares of multinational firms. However, including transaction costs (Rowland, 1999), or information costs (Ahearne *et al.*, 2004) is not enough to explain the bias; even other adjustments to the measures of financial returns (Glassman and Riddick, 2001) or the inclusion of the diversification obtained through multinational firms (Salehizadeh, 2003; Rowland and Tesar, 2004) do not give a full explanation of the bias.

Starting from the failure of all these empirical explanations of the bias we propose a different approach based on the distinction between two classes of investors: international investors and non-international investors. The theoretical model assumes that all investors hold fully diversified portfolios, but actually most of investors do not hold international assets. The hypothesis that all individuals hold complete portfolios is necessary for the validity of the aggregate equilibrium solution in financial theory, but it does not correspond to the actual working of financial markets. Infact, the costs of international diversification, which include international taxes, informational costs and other barriers to trade foreign equities (Cooper and Kaplanis, 1986) largerly vary across investors and may be sufficiently high then many investors may be induced to keep their savings at home.

Household data show that the participation in the international financial market is very limited. The analysis based on aggregate portfolio data and macroeconomic measure of the equity home bias might be inconsistent because of aggregation bias in aggregate portfolio data. We use micro data, a choice that we regard as essential for correctly evaluating and measuring the macro result about the presence of the equity home bias.

Traditionally, the equity home bias is measured by following two basic rules: (a) the whole population of investors is considered as a block, and (b) the observed portofolio is compared to the mean-variance efficient one, where no restrictions are allowed. However, the Markowitz efficient frontier lies far above the efficient portfolio effectively available to the investors because of the existence of unobservable constraints on portfolio weights. Especially for increasing risk values, the Markowitz efficient frontier represents only a theoretical reference and it illustrates investment opportunities which are incompatible with standard trading activities. In this case the performace of the observed portfolio is compared with that of a portfolio that could not be accessible to all investors. This result is entirely due to the inadequacy of the hypothesis of the theoretical model, but it would be traditionally interpreted as a lack of efficiency. Therefore, it is not correct to compare the observed portfolio with the mean-variance efficient portfolio for all investors considered as a single group without restrictions.

In order to take account of unobservable restrictions on portfolio composition we can analyze data through a latent class model, which allows us to test for the existence of two groups (m=2) of investors: on one hand, the sub-group of investors who are completely prevented from any kind of investment in foreign assets, and, on the other hand, the sub-group of investors who are not prevented from investing in foreign assets. The explicit consideration of two categories of investors might allow to go over the failure of standard asset pricing model.

By referring to Table 3, it is important to note that for investors with a combination of variables $(Y,X)=(0,0)$, the equity home bias is completely explained by unobserved constraints, while, for investors with combinations $(Y,X)=(0,1)$ and $(Y,X)=(1,1)$, the equity home bias is not exclusively attributable to unobserved constraints. This implies that it may be very important to classify the population of investors who do not effectively hold foreign equities into two subgroups: one sub-group made of investors who are not precluded from the investment in foreign assets (potential international investors with potentially unexplained home bias), and the other group made of investors who are actually precluded from any investment in foreign assets (explained home bias).

5. THE DATA

The model has been applied to the 2002 wave of the Survey of Household Income and Wealth (SHIW), which is run every two years by the Bank of Italy. The SHIW collects data on real and financial wealth and on several demographic variables for a representative sample of 8011 Italian households. For a detailed description of the survey and the related methodological issues see Brandolini and Cannari (1994), Brandolini (1999) and D'Alessio *et al.* (2004).

The selection of variables represents a key issue in empirical latent class analysis. The information set has been identified by referring to a previuos research (Costa *et al.*, 2005), where effects of socio-demographic and geographical characteristics on households welfare (measured as income inequality) are evaluated by resorting to a recursive partitioning method based on classification trees.

Data on social and demographic characteristics of investors available in the SHIW include age, gender, household size, geographical area of residence and education, while economic and financial characteristics refer to number of banks used, total real assets, total financial assets. Tables 6-12 in the Appendix illustrate the main features of Italian households financial wealth distribution by these variables. It is immediate to observe a strong concentration toward domestic financial assets: national assets represent southern 99.07% of total financial wealth. Furthermore, international investment is virtually absent in southern Italy and in age classes <30 and 30-40. Households with less than 3 members shows a higher participation in international financial market. The effect of education is quite strong: the group with university degree holds 53% ot total international assets. With regard to economic characteristics, Italian households using more than one bank hold 31.57% of financial wealth. Finally, a wealth effect is also well documented: participation in international financial markest is limited to higher percentiles of real and financial wealth.

A further crucial step in latent class analysis is related to the classification of the selected variables, that is the identification of the relevant levels of each informative variable. Also this choice has been performed by means of a classification tree methodology (Costa *et al.*, 2005), which represents one of the most powerful and widespread methodologies for detecting relevant splitting points. The variables of the SHIW have been reclassified as follows:

– *Age of head of household* (T): less than or equal to 40; between 40 and 60; over 60 years.
– *Sex of head of household* (S): male; female.
– *Household size* (C): less than or equal 2 members; more than 2 members.
– *Geographical area of residence* (G): north; center; south and islands.
– *Education of head of household* (E): middle school or less; high school; university degree or more.
– *Number of banks used by household* (M): one bank; more than one bank.
– *Total real assets* (R): less than the 85th percentile; more than the 85th percentile.
– *Total financial assets* (F): less than or equal to the 75th percentile; between the 75th and the 95th percentile; more than the 95th percentile.

The complete dataset is therefore composed by the foreign security holding, Y, the set of $k=5$ variables related to social and demographic characteristics of the households (T,S,C,G,E), and the set of $h=3$ variables regarding economic and financial characteristics of the households (M,R,F).

6. LATENT STRUCTURE OF INVESTMENT DECISIONS STRATEGIES

Given the information provided by the model specified in paragraph 3 and the latent class variable X introduced in paragraph 4, the basic latent class model for the above data is stylised in Figure 2.

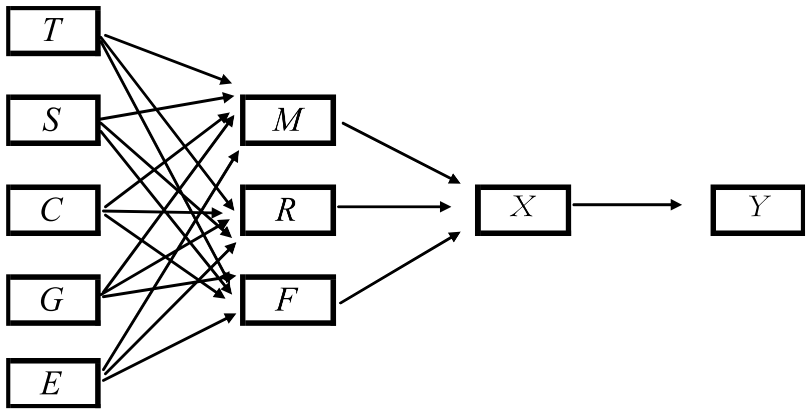

Figure 2 – Basic latent class Model 0.

This simple specification (*Model 0*) includes neither interaction effects between social and demographic variables nor interaction effects between financial and economic variables.

The corresponding expressions in terms, respectively, of conditional probabilities (10) and loglinear parameters (11) are:

$$\begin{aligned}\pi_{tscgemrfxy}^{TSCGEMRFXY} &= \pi_t^T \cdot \pi_s^S \cdot \pi_c^C \cdot \pi_g^G \cdot \pi_e^E \cdot \pi_{mt}^{M|T} \cdot \pi_{ms}^{M|S} \cdot \pi_{mc}^{M|C} \cdot \pi_{mg}^{M|G} \cdot \pi_{me}^{M|E} \cdot \\ &\pi_{rt}^{R|T} \cdot \pi_{rs}^{R|S} \cdot \pi_{rc}^{R|C} \cdot \pi_{rg}^{R|G} \cdot \pi_{re}^{R|E} \cdot \pi_{ft}^{F|T} \cdot \pi_{fs}^{F|S} \cdot \pi_{fc}^{F|C} \cdot \pi_{fg}^{F|G} \cdot \pi_{fe}^{F|E} \cdot \\ &\pi_{xm}^{X|M} \cdot \pi_{xr}^{X|R} \cdot \pi_{xf}^{X|F} \cdot \pi_{yx}^{Y|X}\end{aligned} \tag{10}$$

$$\begin{aligned}\ln(F_{tscgemrfxy}^{TSCGEMRFXY}) &= \lambda + \lambda_t^T + \lambda_s^S + \lambda_c^C + \lambda_g^G + \lambda_e^E + \lambda_m^M + \lambda_r^R + \lambda_f^F + \lambda_x^X + \lambda_y^Y \\ &+\lambda_{mt}^{MT} + \lambda_{ms}^{MS} + \lambda_{mc}^{MC} + \lambda_{mg}^{MG} + \lambda_{me}^{ME} + \lambda_{rt}^{RT} + \lambda_{rs}^{RS} + \lambda_{rc}^{RC} + \lambda_{rg}^{RG} + \lambda_{re}^{RE} \\ &+\lambda_{ft}^{FT} + \lambda_{fs}^{FS} + \lambda_{fc}^{FC} + \lambda_{fg}^{FG} + \lambda_{fe}^{FE} + \lambda_{xm}^{XM} + \lambda_{xr}^{XR} + \lambda_{xf}^{XF} + \lambda_{yx}^{YX}\end{aligned} \tag{11}$$

This model can be generalized by relaxing the hypothesis of independence between either social-demographic and economic-financial variables. Therefore we analyze a sequence of models, characterized by increasing complexity, in order to evaluate the effects of interactions between explanatory variables on the probability to have restrictions on foreign assets investment (see Table 13 in the Appendix).

Our first step is to introduce interactions between social-demographic variables: in *Model 1* we allow $\lambda_{tc}^{TC} \neq 0$, $\lambda_{sc}^{SC} \neq 0$, $\lambda_{cg}^{CG} \neq 0$, $\lambda_{ts}^{TS} \neq 0$, $\lambda_{sg}^{SG} \neq 0$, $\lambda_{tg}^{TG} \neq 0$.

We consider also interactions of the third order: in *Model 2* we allow $\lambda_{tsc}^{TSC} \neq 0$, $\lambda_{tcg}^{TCG} \neq 0$, $\lambda_{tsg}^{TSG} \neq 0$, $\lambda_{scg}^{SCG} \neq 0$ and of the fourth order (in *Model 3* $\lambda_{tsgc}^{TSGC} \neq 0$), still achieving an improvement of the model. A further relevant step is represented by the inclusion, in *Model 4,* of a relation which measures the influence of social-demographic variables on education ($\lambda_{et}^{ET} \neq 0$, $\lambda_{es}^{ES} \neq 0$, $\lambda_{ec}^{EC} \neq 0$, $\lambda_{eg}^{EG} \neq 0$).

Finally, we allow for interaction effects between age and education, and between age and sex, on real and financial assets (in *Model 5* we include $\lambda_{rte}^{RTE} \neq 0$, $\lambda_{rts}^{RTS} \neq 0$ for real assets and $\lambda_{fte}^{FTE} \neq 0$, $\lambda_{rts}^{FTS} \neq 0$ for financial assets) and also interaction between age and sex, and between age and household size on education (in *Model 6* $\lambda_{ets}^{ETS} \neq 0$, $\lambda_{etc}^{ETC} \neq 0$).

Obviously it might be possible to further generalize the latent class model by including more variables, connections and interaction effects, but we believe that our specification can be considered satisfactory, characterized by a reasonably complete informative content.

The goodness of fit of the seven models are judged by the AIC, the BIC and the consistent AIC (CAIC) statistics based on L^2. Table 13 in the Appendix reports also the likelihood-ratio statistic (L^2), the Pearson chi-square (χ^2) and the number of degrees of freedom (*df*).

7. THE RESULTS

Results show that the model can be substantially improved by relaxing the independence hypothesis between the four social and demographic variables (age, sex, geographical area of residence and household size), with the exception of the inclusion of the fourth order interaction. A further significant increase in the goodness-of-fit of the model is obtained by including the association between the social-demographic variables and educational level of the head of the household (*Model 4*). Although the AIC statistic has been always substantially increased moving from *Model 0* to more informative models, the BIC and CAIC statistics show a slight worsening when we allow interaction effects between age and education, and between age and sex, on real and financial assets (*Model 5*). Anyway, all of the evaluation criteria unequivocally support the conclusion that *Model 6* is preferable to the others.

From the parameter estimates of the final *Model 6*, in Table 4 we want first of all stress the results related to the contingency table (Table 3) illustrated in paragraph 2. The probability to be exposed to complete restrictions, that is $X_i=0$, is 0.8918: more than 89% of total households is completely precluded from investing in foreign assets. For the 10.8% of households without restrictions precluding investment in foreign assets, (i.e. $X_i=1$), we can observe f_{01}=0.0982, related to investors without foreign assets, and f_{11}=0.0100, related to investors with foreign assets. The equity home bias is completely explained by unobserved constraints for the 89% of households beeing precluded from the investment in foreign assets, while it could be unexplained for the remaining 10.8% (9.8% of households are not subject to restrictions but do not hold foreign assets; 1% of households hold foreing assets[1]).

[1] Obviously also for those holding foreign assets it could be possible to observe equity home bias due to inefficient diversification strategies.

TABLE 4

Latent class variable X *and investment in foreign assets* Y *in Model 6*

Y \ X		With restrictions 0	Without restrictions 1
Without foreign assets	0	0.8918	0.0982
With foreign assets	1	-	0.0100

The analysis of the connections between the latent variable X and the social, demographic, economic and financial characteristics of the households allows powerful insights into the nature of the constraints for Italian households. Table 14 in the Appendix reports posterior probabilities of class membership $\pi_{xz}^{X|Z}$ and $\pi_{xw}^{X|W}$ obtained from conditional probabilities and latent class probabilities π_{x}^{X} for final Model 6. Posterior probabilities $\pi_{0z}^{X|Z}$ and $\pi_{0w}^{X|W}$ are the probabilities that a household with to the z-th attribute of variable Z (or the w-th attribute of variable W) is costrained in his investment decision in foreign assets ($X=0$), while posterior probabilities $\pi_{1z}^{X|Z}$ and $\pi_{1w}^{X|W}$ are the probabilities that an household with the z-th attribute of variable Z (or to the w-th attribute of variable W) has no restrictions to investment in foreign assets ($X=1$).

The results illustrated in Table 14 of the Appendix suggest that all demographic and economic variables substantially affect the latent classes. The area of residence and the education level appear to play a major role with respect to household size, age and sex of head of household. In particular, for an household living in the south, the probability to be a potential international investor is 0.0531, and it becomes 0.1096 and 0.1473 for households living, respectively, in central and northern Italy. For middle school (or less) educated head of household, the probability to be a potential international investor is 0.0787, and it increases to 0.1611 and to 0.2245 for college and university education levels respectively. This result supports the idea that managing international portfolio is information intensive and requires a degree of intellectual ability that lead to invisible information-related barriers to entry into foreign financial markets. The importance of the area of residence might indicate strong differences in financial development betweeen areas of the country.

With regard to economic and financial characteristics, all three variables included in Model 6 appear to be strongly significant in order to detect the group of potential international investors. An informative indicator for the identification of the latent classes is also the number of banks: when the household deals with just one bank, the probability to be a potential international investor is mil, while, when dealing with more than one bank, the probability becomes 0.1393, thus underlining the fact that banks are a relevant channel of useful information on international financial markets.

When the value of total real assets of the household is less than or equal to the

85th percentile, the probability to be a potential international investor is 0.0775, while, when the value of total real assets exceeds that threshold, the probability becomes 0.2826. Finally, financial assets play the most important role in defining the group of potential international investors, in line with the view that minimum investment requirements and monetary transaction costs are important sources of costs in the international diversification. In particular, when the value of total financial assets of the household is less than or equal to the 75th percentile, the probability to be a potential international investor is 0.0589. When financial assets are between the 75th and 95th percentile, such a probability shifts to 0.1593, but, when total financial assets exceed the 95th percentile, the probability of membership to potential international investors group becomes 0.6460.

Globally, we can define the household profile with the highest probability to be foreign assets investor: male, aged between 40 and 60 years, with a large household, resident in northern Italy and with a university degree. To such a profile corresponds a probability to be a potential international investor equal to 0.333. Analogously, the household profile consistent with the highest probability to be prevented from investment in foreign assets is marked by the following characteristics: less than or equal to 40 years old, female, with a small family, resident in the south or islands and with a middle school education or less. To such a household profile corresponds a probability to be a non-international investor equal to 0.983.

In order to subdivide the whole population in the two sub-groups, the household membership has been simulated on the basis of the posterior probabilities of class membership. Table 5 reportes the observed fractions of wealth w invested by Italian households in 2002 and the risk-return efficient fractions of wealth w^* for the whole population of investors and for the two sub-groups identified by the simulation. In Table 15 in the Appendix some details about the series used in efficient portfolio computation are provided. Some simplifying hypothesis are introduced and therefore values w^* are to be considered only as reference points.

TABLE 5

Observed and efficient portfolios

	With restrictions		Without restr.		Total	
	obs	eff	obs	eff	obs	eff
	w	w^*	w	w^*	w	w^*
National stocks	0.0611	0.0454	0.0997	0.0173	0.0791	0.0135
National bond and funds	0.1570	0.2108	0.2441	0.3809	0.1975	0.2990
National other	0.7819	0.7438	0.6361	0.5261	0.7140	0.6273
Foreign stocks	0	0	0.0056	0.0758	0.0026	0.0601
Foreign bonds funds	0	0	0.0126	0	0.0058	0
Foreign other	0	0	0.0019	0	0.0009	0
Mean (annual)	3.41	3.48	4.22	4.61	3.79	4.08
Standard deviation	0.2575	0.2575	0.4281	0.4281	0.3366	0.3366
Equity home bias	-		*73.5%*		*84.5%*	

Usually, equity home bias is evaluated for the population as a whole by comparing the sitxh and the seventh columns Table 5. Following our proposal, equity home bias should be evaluated by taking account of the two subgroups of investors separately by comparing the first column with the second and the third with the fourth.

A traditional index of equity home bias (*HB*) is substantially based on the comparison of the observed share of wealth invested by Italian investors in foreign equities, w_f, with the weight that foreign equites have in the risk-return efficient portfolio, w_f*:

$$HB = 1 - \frac{w_f}{w_f^*}$$

The HB index equals 1 in case of total home bias; it equals zero when home bias is absent.

According to this measure (see last raw of Table 6), home bias for the whole population is 84.5 per cent, while, after taking the unobservable portfolio contraints into account, the index is equal to 73.5 per cent, thus indicating that 13 per cent of the observed equity home bias is attributable to these constraints.

8. CONCLUSIONS

In this paper we propose to deal with the unobservability problem in portfolio choice through a latent class analysis approach. The goal of this methodology is to identify latent classes representing groups of investors subject to different constraints on their assets. Such latent classes are determined through the relationship among a set of observed categorical variables representing socio-demographic and economic characteristics of the investors and one manifest variable measuring the effective portfolio choice.

We apply this methodology to international investment decisions in order to shed some light on equity home bias puzzle. The basic idea is to classify the population of investors who do not effectively hold foreign equities into two subgroups: one made of investors who are not precluded from the investment in foreign assets (potential international investors with potentially unexplained home bias), the other of investors who are actually prevented from any investment in foreign assets (home bias explained by inobserved constraints).

The methodology proposed is applied to the 2002 wave of the Survey of Household Income and Wealth by the Bank of Italy. The results show that 90 percent of households who do not hold foreign assets, are completely prevented from investing in foreign assets. They represent the 89 percent of the whole population. Of the remaning 11 percent of households without restrictions precluding investment in foreign assets, we observe that 10 percent do not invest in foreign equities. These results imply that equity home bias is completely explained for 89 percent of households, while it could be unexplained for 11 percent of households.

It is important to stress that, in order to correctly measure equity home bias, it is necessary to detect investors who are precluded from operating on foreign markets (89% of total households). With respect to the existing literature, where investors are considered as a single group and equity home bias is measured by referring to all households, we introduce a strategic distinction.

The results also provide estimates of the effects of social, demographic, economic and financial characteristics on the probability of being an international investor. Globally it is possible to define the household profile consistent with the higher probability to be an international investor: between 40 and 60 years old, male, with a large houshold, resident in the northern Italy, and with a university degree.

We finally propose an evaluation of the equity home bias based on the distinction between non-international and potential international investors and show that considering this distinction gives powerful insights of the equity home bias: on the basis of traditional procedures home bias reaches a level of 84.5 per cent against the 73.5 per cent of our approach, thus indicating the relevant role of unobserved constraints.

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RIASSUNTO

Modelli a classe latente per l'analisi di dati finanziari

Nel lavoro si propone un modello a classi latenti per l'analisi delle decisioni di investimento internazionale. La specificazione adottata consente di valutare l'effetto di numerose variabili sociali, demografiche, economiche e finanziarie sulla probabilità, non osservabile, di essere investitori internazionali. Le misure tradizionali impiegate per valutare l'equity home bias non tengono conto della presenza di operatori razionati sugli investimenti all'estero. Al contrario, l'analisi a classe latente consente di individuare la distinzione, non osservabile, tra investitori internazionali e investitori che non possono accedere ai mercati esteri e, pertanto, di valutare il ruolo di questi vincoli non osservabili sull'equity home bias.

SUMMARY

Latent class models in financial data analysis

This paper deals with optimal international portfolio choice by developing a latent class approach based on the distinction between international and non-international investors. On the basis of micro data, we analyze the effects of many social, demographic, economic and financial characteristics on the probability to be an international investor. Traditional measures of equity home bias do not allow for the existence of international investment rationing operators. On the contrary, by resorting to latent class analysis it is possible to detect the unobservable distinction between international investors and investors who are precluded from operating into international financial markets and, therefore, to evaluate the role of these unobservable constraints on equity home bias.

APPENDIX

TABLE 6

Italian households financial wealth by household size, 2002, percentages

2002	≤ 2	3-4	> 4	Tot
National stocks	3.08	4.57	0.26	7.91
National bonds and funds	8.87	10.11	0.77	19.75
National other	32.85	33.11	5.44	71.40
Foreign stocks	0.15	0.09	0.02	0.26
Foreign bonds and funds	0.28	0.29	0.01	0.58
Foreign other	0.03	0.06	0.00	0.09
Tot	45.27	48.23	6.51	100.00

TABLE 7

Italian households financial wealth by geographical area of residence, 2002 , percentages

2002	North	Center	South	Tot
National stocks	6.18	1.22	0.52	7.91
National bonds and funds	15.02	3.85	0.88	19.75
National other	40.83	13.13	17.44	71.40
Foreign stocks	0.14	0.12	0.00	0.26
Foreign bonds and funds	0.56	0.03	0.00	0.58
Foreign other	0.09	0.00	0.00	0.09
Tot	62.80	18.36	18.84	100.00

TABLE 8

Italian households financial wealth by age of the head of the household, 2002 , percentages

2002	≤ 30	31-40	41-50	51-60	61-70	>70	Tot
National stocks	0.16	1.48	1.68	2.47	1.00	1.12	7.91
National bonds and funds	0.59	2.59	4.18	4.12	4.75	3.53	19.75
National other	3.01	11.68	13.78	14.53	14.80	13.60	71.40
Foreign stocks	0.01	0.02	0.05	0.10	0.03	0.06	0.26
Foreign bonds and funds	0.00	0.07	0.24	0.05	0.13	0.10	0.58
Foreign other	0.00	0.00	0.00	0.02	0.07	0.00	0.09
Tot	3.77	15.83	19.93	21.27	20.79	18.41	100.00

TABLE 9

Italian households financial wealth by education of the head of the household, 2002 , percentages

2002	Less than middle school	Middle school	High school	University degree	Tot
National stocks	0.32	1.82	3.61	2.16	7.91
National bonds and funds	1.69	4.48	7.93	5.64	19.75
National other	15.46	20.26	19.35	16.32	71.40
Foreign stocks	0.02	0.05	0.05	0.14	0.26
Foreign bonds and funds	0.00	0.10	0.17	0.31	0.58
Foreign other	0.00	0.00	0.05	0.04	0.09
Tot	17.49	26.72	31.16	24.63	100.00

TABLE 10

Italian households financial wealth by number of banks used, 2002 , percentages.

2002	1	> 1	Tot
National stocks	4.34	3.57	7.91
National bonds and funds	12.20	7.55	19.75
National other	51.38	20.02	71.40
Foreign stocks	0.06	0.20	0.26
Foreign bonds and funds	0.41	0.17	0.58
Foreign other	0.04	0.05	0.09
Tot	68.43	31.57	100.00

TABLE 11

Italian households financial wealth by total real wealth percentiles, 2002 , percentages

2002	≤ 50	50 – 75	75 – 85	85 – 95	>95	Tot
National stocks	0.91	1.23	1.36	1.88	2.52	7.91
National bonds and funds	2.61	4.32	2.90	4.55	5.37	19.75
National other	21.26	15.67	7.97	11.40	15.11	71.40
Foreign stocks	0.04	0.02	0.02	0.16	0.02	0.26
Foreign bonds and funds	0.01	0.20	0.04	0.12	0.22	0.58
Foreign other	0.00	0.00	0.03	0.00	0.05	0.09
Tot	24.84	21.43	12.33	18.11	23.30	100.00

TABLE 12

Italian households financial wealth by total financial assets percentiles, 2002 , percentages

2002	≤ 50	50 – 75	75 – 85	85 – 95	>95	Tot
National stocks	0.05	0.45	0.67	2.22	4.51	7.91
National bonds and funds	0.06	1.04	1.89	5.42	11.35	19.75
National other	4.19	10.71	8.48	14.02	34.00	71.40
Foreign stocks	0.01	0.01	0.02	0.05	0.18	0.26
Foreign bonds and funds	0.01	0.02	0.01	0.19	0.36	0.58
Foreign other	0.00	0.00	0.00	0.00	0.09	0.09
Tot	4.32	12.23	11.07	21.90	50.48	100.00

TABLE 13

Latent class analysis evaluation criteria for 2002 shiw data

		L^2	χ^2	*AIC*	*BIC*	*CAIC*	*Df*
0	Model in equation (10)	5735,3	9558,1	653,3	-17104,7	-19645,7	2541
1	as Model 0 with $\lambda_{tc}^{TC} \neq 0$, $\lambda_{sc}^{SC} \neq 0$, $\lambda_{cg}^{CG} \neq 0$, $\lambda_{ts}^{TS} \neq 0$, $\lambda_{sg}^{SG} \neq 0$, $\lambda_{tg}^{TG} \neq 0$	3430.5	6853,6	-1625,5	-19292,6	-21820,6	2528
2	as Model 1 with $\lambda_{tsc}^{TSC} \neq 0$, $\lambda_{tcg}^{TCG} \neq 0$, $\lambda_{tsg}^{TSG} \neq 0$, $\lambda_{scg}^{SCG} \neq 0$	3310,7	6599,7	-1721,3	-19304,5	-21820,5	2516
3	as Model 2 with $\lambda_{tsgc}^{TSGC} \neq 0$	3295,1	6706,9	-1728,9	-19284,2	-21796,2	2512
4	as Model 3 with $\lambda_{et}^{ET} \neq 0$, $\lambda_{es}^{ES} \neq 0$, $\lambda_{ec}^{EC} \neq 0$, $\lambda_{eg}^{EG} \neq 0$	2605.4	5446,4	-2394,6	-19866,0	-22366,0	2500
5	as Model 4 with $\lambda_{rte}^{RTE} \neq 0$, $\lambda_{rts}^{RTS} \neq 0$, $\lambda_{fte}^{FTE} \neq 0$, $\lambda_{rts}^{FTS} \neq 0$	2505.9	5093,6	-2458,1	-19803,7	-22285,7	2482
6	as Model 5 with $\lambda_{ets}^{ETS} \neq 0$, $\lambda_{etc}^{ETC} \neq 0$	2362.4	4917,8	-2585,6	-19875,4	-22349,4	2474

TABLE 14

Posterior probabilities of class membership - model 6 estimated on 2002 SHIW data (: restriction improsed)*

	Variable	Probabilities X 0	1
T	*Age of head of household*	$\pi_{xt}^{X\|T}$	
	Less than or equal to 40	0.9160	0.0840
	Between 40 and 60	0.8729	0.1271
	Over 60 years	0.8967	0.1033
S	*Sex of head of household*	$\pi_{xs}^{X\|S}$	
	Male	0.8784	0.1216
	Female	0.9148	0.0852
C	*Household size*	$\pi_{xc}^{X\|C}$	
	Less than or equal to 2 members	0.9088	0.0912
	More than 2 members	0.8758	0.1242
G	*Geographical area*	$\pi_{xg}^{X\|G}$	
	North	0.8527	0.1473
	Center	0.8904	0.1096
	South and islands	0.9469	0.0531
E	*Education of head of household*	$\pi_{xe}^{X\|E}$	
	Middle school or less	0.9213	0.0787
	High school	0.8389	0.1611
	University degree or more	0.7755	0.2245
M	*Number of banks used*	$\pi_{xm}^{X\|M}$	
	One bank	1.0000	0.0000
	More than one bank	0.8607	0.1393
R	*Total real assets*	$\pi_{xr}^{X\|R}$	
	Less than or equal to the 85^{th} percentile	0.9225	0.0775
	More than the 85^{th} percentile	0.7174	0.2826
F	*Total financial assets*	$\pi_{xf}^{X\|F}$	
	Less than or equal to the 75^{th} percentile	0.9411	0.0589
	Between the 75^{th} and the 95^{th} percentile	0.8407	0.1593
	More than the 95^{th} percentile	0.3540	0.6460
Y	*Holding of foreign assets*	$\pi_{xy}^{X\|Y}$	
	No	0.9008	0.0992
	Yes	0.0000*	1.0000*
		π_{x}^{X}	
	Latent class probabilities	0.8918	0.1082

TABLE 15

Italian households financial investments monthly returns in Euros 2003-2005

	mean	std. dev.	min	max	kurt
National stocks Datastream Italian stock market index	1.03	3.64	-7.68	9.21	2.92
National bonds and funds MSCI Italian Bond index	0.53	1.04	-1.99	2.16	2.61
National other 3 months Italian Treasury Bills	0.18	0.02	0.16	0.24	7.48
Foreign stocks Datastream world stock market index	0.94	3.55	-9.67	7.18	3.50
Foreign bonds and funds MSCI world Bond index	0.12	1.28	-1.95	2.75	2.09
Foreign other 3 months US Treasury Bills	0.15	0.08	0.07	0.32	2.04