Language in argumentation and solution of problems with graphs

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The paper investigates undergraduates' argumentations to justify their answers to elementary calculus problems involving the recognition of relationships among graphs, verbal texts and formulas. The examination of the texts produced over more than ten years highlights serious language difficulties and suggests that we cannot exclude that language is a key factor for the quality of arguments. The main goal of this study is to gain a better understanding of how language difficulties (depending on both competence and attitudes) affect argumentations.

Key words: language, problem solving, register, argumentation

Introduction

The paper focuses on how undergraduates justify their answers to elementary calculus problems involving relationships among graphs, verbal texts and formulas, in the frame of an introductory mathematics course delivered in Italian to biology freshman students. The course is short (48 hours of lectures and 24 hours of optional tutoring sessions) and taught by two instructors, one of whom is the author. It is usually attended by more than 400 students, coming from various regions of Italy and Eastern Europe.The students' language competence is much varied. Due to the goals, the attendance, and the lack of time, I did not develop a standard course moving from the basic definitions of Calculus to get to theorems and applications, but focused instead on a few concepts such as graph symmetries and slope. Most of the tests administered as working material or examination papers require the linking of different representations of functions (symbolic expressions, graphs, verbal texts, tables of numbers). Through these activities participants are systematically asked to explain and justify their answers in writing, even informally. This requirement is aimed at discouraging guessing or rote learning and help the students to better understand the concepts involved. Morgan (1998) provides a description and discussion of the 'writing-to-learn' paradigm, and highlights the benefits of the use of writing as a means for learning. Very appropriately, she suggests (with the expression 'learning-to-write') that nobody needs to assume that students of any age have achieved the level of linguistic competence required in order to produce texts adequate to their goals, and challenges the assumption, more or less implicit in a number of studies, that language naturally develops and there is no need for deliberate language teaching (Morgan, 1998, pp. 37-49).

The scrutiny of the papers written by students over the years highlights serious difficulties with language and suggests that we cannot exclude at all that language competence is a key factor for the quality of arguments. If we admit that there is a link between language and thought, there is no reason at all to rule out the hypothesis that the quality of the texts a subject can produce or interpret could deeply affect the quality of her/his thinking, and thus of the arguments s/he can produce.

An investigation of this topic requires dealing with the texts involved as objects, not only as means to develop a discourse within a given context. The focus should be on the organization of the texts, not just on the corresponding communication process.

The main goal of my research is to get a better understanding of how language difficulties (depending on both competence and attitudes) affect the production of arguments by undergraduates. In this specific study I focus on problems involving graphs.

Theoretical framework

From the end of WW2 research on argumentation has produced a large number of papers from a wide range of perspectives. Some researchers, such as Crawshay-Williams (1957) and van Eemeren et al. (1996), in different ways, have underlined the role of context in argumentation and the links between argumentation and language. Crawshay-Williams (1957, p.3), for example, claims that his work on argumentation "enquires how we use language as an instrument of reason " and argues that "[i]t is only possible to determine whether an empirical statement is true or false if the context of the statement is known." On the other hand, van Eemeren et al. (1996) relate the theory of argumentation to the pragmatic theory of speech acts (Austin, 1962), which takes into account not just the *propositional content* of a statement (i.e. the part of its meaning, based on vocabulary and grammar, that allows the receiver to identify the referents and possibly to establish whether the statement is true or false), but also the *speech act* (i.e., the fact of expressing a proposition in a specific context, which conveys also speaker's (or writer's) beliefs, attitudes and commitments, possibly influencing the hearer's (or reader's) ones. Toulmin's framework (2003), on the contrary, although it is widely adopted in research on argumentation in the context of mathematics education, seemingly pays very little attention to language and context.

As far as language is concerned, I adopt Halliday's (2004) account of the relationship between scientific language and science and his Systemic Functional Linguistics (SFL) (Halliday, 1985, 2004; Leckie-Tarry, 1995; O'Halloran, 2005). Halliday, whose research is in the field of pragmatics started by Austin (1962), argues that there is no learning of science without some learning of its language (2004, p. 160). The adoption of the SFL framework is justified by the opportunity of focusing on the functions of language in mathematics education, where the needs for effective representations of concepts and their relationships and algorithms is unavoidably at odds with those of effective communication. Multisemioticy is an important feature too, as the interplay among verbal, figural and symbolic representations is stronger in mathematics than in other fields.

In order to analyze the protocols, I am using the idea of *register* as a linguistic variety related to use (Halliday & Hasan, 1990). An enlightening discussion on registers in an SFL framework has been provided by Leckie-Tarry (1995). Morgan (1998) and Ferrari (2004) have applied this idea to mathematical language. Any individual has at her/his disposal a range of registers that s/he uses according to circumstances. The most relaxed registers, used in spoken (but sometimes also written) everyday communication are classified as *colloquial*, while those adopted in written (but sometimes also spoken) communication among educated people, for example in institutional, educational, literary, research contexts are referred to as *literate*.

Colloquial registers, in short, are characterized by their strong dependence on the *context of situation* (i.e., according to Leckie-Tarry, 1995, the space and time in which the exchange takes place, the participants, …), which allows participants to negotiate meanings and makes it unnecessary to produce accurate and unambiguous statements from the beginning. Literate registers are less related to the context of situation. In colloquial registers the meaning of words is mainly taken from previous experience, and most often much precision is not required to achieve the goals of the exchange, whereas in literate ones words have precise meanings, based on definitions (the so called *lexicalization*). In colloquial registers syntax is customarily relaxed, while in literate ones it is tighter. In colloquial registers there is an extensive use of *iconicity*, i.e. the analogy between the form or organization of a representation and its meaning. Iconicity is opposed to *arbitrariness* and can involve factors such as order (the order of facts matches the sequence of the representations). In literate registers representations are less iconic and more conventional. As a consequence, the interpretation and production of texts and representations in colloquial registers are quite unstable, since they depend on factors difficult to control (how the subject is accustomed to use words, how s/he interprets images, the mental models s/he uses in place of the definitions, the personal experiences s/he tries to recall, …). In literate registers, the lesser dependence on the specific situation, the reference to defined meanings (thus more objective and verifiable) and the role of syntax (objective and verifiable too) make the interpretation and production of texts more stable.

Even a quick analysis of what is described above should make it clear that most of the registers used in mathematical settings share the features of a literate register in an extreme way: in mathematical registers the interpretation of a text depends little on the context of situation in which it is produced. Here I am not referring to the processes of learning or communicating mathematics, but on the organization of mathematical texts, as they can be found in any mathematics textbook from primary to graduate schools. The dependence of mathematical language on the *context of culture* (any kind of systems of knowledge related to the participants and the topics of the exchange), on the contrary, is very strong (think of definitions, conventions, theorems, …), as well as lexicalization and conventionality (there are not many other semantic domains where definitions play as important a role as in mathematics). The same holds for syntax: in a mathematical text, either symbolic or not, a minor variation (e.g., the displacement of a parenthesis or of a comma) can deeply change its meaning. The interpretation of texts in mathematical registers is stable: in some cases it can be performed automatically. The use of colloquial registers is essential for learning as well: nobody could ever learn anything if s/he should use literate registers only. So, in learning mathematics the trouble is not the use of colloquial registers, but the failure to adopt literate ones when necessary.

Methodology

A large number of argumentative texts produced by freshman students to justify their answers to problems involving the interpretation of graphs, both in examinations and in tutoring sessions (including online ones) have been scrutinized. In this paper I take into consideration only texts produced for one specific examination. To understand the argumentations it is necessary to regard them related to the problem-solving context they are produced within, considering the solutions produced as well. This study is not aimed at testing a particular model but rather at understanding the difficulties of a relevant number of students with different backgrounds, cultures, attitudes, and levels of competence, also in order to improve our teaching and tutoring strategies. For these reasons I have used a large number of protocols taken from a real examination, as most often the weakest students are not willing to take part in other activities, such as special tutoring sessions.

Some of the participants have been interviewed after the test. For each participant I tried to classify answers and errors, if any, such as: use of pseudo-rules or of mathematically inappropriate models, wrong reading of the data, miscalculations, and language errors. I have also classified the kind of text produced (basically, the register adopted, by means of the indicators suggested by Leckie-Tarry, 1995) to see if and how linguistic competence might have affected the answers. In some of the excerpts both the original Italian text and an English translation are given. The kind of analysis I want to carry out does not allow me to refer to an English translation only, which, even if it may convey with fair approximation the ideational component of the text, unavoidably it cannot but fail in conveying other aspects of the text, such as register or improper uses.

It is never easy to understand whether an error depends on the language (e.g., a proper idea wrongly expressed), on contents (e.g., a wrong idea truly expressed) or on both. For example, the (wrong) claim that function *g* below is decreasing in [0, 10] might depend on a poor understanding of the corresponding definition, or on the improper use of ‘decreasing’, or even on a wrong interpretation of the graph. This in turn might be affected by the everyday use of the same words. The analysis of cohesive devices (i.e., the linguistic resources used to link the parts of the text), as carried out by Alarcon and Morales (2001) is a classical way to deal with argumentation in a SFL setting. In the analysis of the protocols, I have applied two criteria: the appropriate use of cohesive devices (contrasted to improper use or no use at all) and the vocabulary (lexical vs colloquial use of words).

The problem

Here I focus on problems involving graphs, such as problems requiring to associate a formula to a graph, or a graph to a formula, or to link the graph of a function to the graph of its derivative. All the protocols (about 200) used in this study come from the following problem.

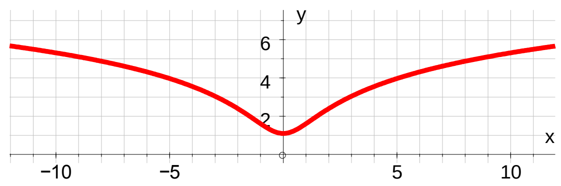
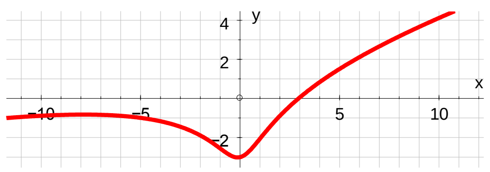
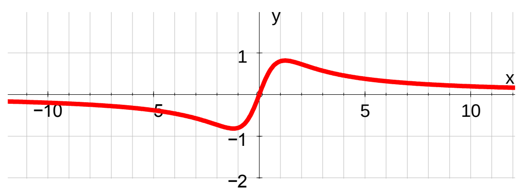
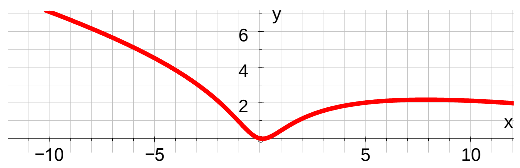
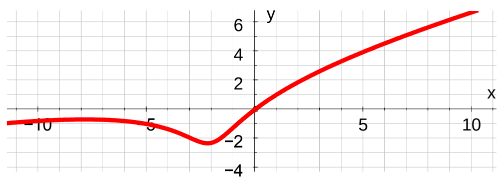
Consider the graphs A, B, C, D drawn below and choose three of them which, in the interval displayed, do not correspond to the derivative of the function *g* drawn on the right. Justify your answer.	*g* 
A) 	B) 
C) 	D) 

Table 1. The problem

The problem is in negative form, as participants are required to identify three graphs that do not correspond to the derivative of the function *g*. It is manifest that in a problem of this sort it is not possible, from the scrutiny of the graphs only, to decide that a graph does correspond to the derivative of a given function. On the other hand, it is possible, in many cases, to decide that a graph does *not* correspond to the derivative of a function. Considerations of this kind hold for a great deal of mathematical problems involving graphs. The negative wording of the problem has proved a source of trouble although it explicitly refers to the need for excluding three graphs.

I have regarded as acceptable all the answers excluding the three appropriate graphs with appropriate justifications, i.e. argumentations where the properties of *g* and of the graphs the answer is based on are explicitly mentioned. For example, a text like

(1) "*g* is increasing in (0, +∞), so its derivative must be positive in the same interval, so I exclude graph A, which is partly negative in the same interval."

has been considered a sufficient justification in order to rule out graph A, although the student has made no explicit reference to any theorem or rule. Actually, it is uncommon among freshman students to find explicit reference to some general property to justify an inferential step.

Outcomes

Although this is not a quantitative study, I often give some quantitative indication about the size of the groups adopting some behaviors. It might be interesting, from a teaching perspective, to know if a behavior is adopted by a small group of students or it is more general.

Correct answers equipped with acceptable arguments usually range from 20 % to 40 %, according to the sample and the task. In this experiment correct answers have been a bit less than 25 %.

Student A02 marks the graphs A, B, D and gives the following argument:

> "Non corrispondono, perché g è in positivo mentre A, B, C sono sia in positivo che in negativo."
>
> ["Do not match, because *g* is in positive while A, B, C are both in positive and in negative."]

First of all, the argument, which seems aimed at ruling out graphs A, B, C, is inconsistent with the marks on the diagram, which rule out graphs A, B, D. The argument adopted seems to fit graph D better than graph C, and one can imagine that the subject wrote 'C' in the argument by mistake. Errors of this kind are quite frequent. Second, the subject only deals with what s/he is looking at and makes no reference to mathematical properties connected to the problem, nor any attempt to link to each other the data s/he has mentioned. In other words, the argument is completely bounded within the context of situation, with no attempt to put it in a framework of knowledge, i.e., in a context of culture. Third, the text is inaccurate: the main verb has no subject, the expressions "in positivo", "in negativo", which are circumstantials of (spatial) location, are used in place of the more correct attributes ('positivo', 'negativo') and the expression "sia in positivo che in negativo" is vague.

Student A03 marks graphs B, C, D and produces the following argument:

> "in x>0 la fne della derivata dev'essere positiva quindi non è sicuramente la B, in x<0 la fne della derivata dev'essere negativa (decrescente) quindi non può essere la D e la C non è sempre crescente nell'intervallo (0,+∞). Penso sia il grafico a."

'fne' is an informal abbreviation for 'funzione' (function). In order to help reading, I translate it as the whole word.

> [“in x>0 the {function} of the derivative must be positive so surely it is not B, in x<0 the {function} of the derivative must be negative (decreasing) so it cannot be D and C is not always increasing in the interval (0, +∞). I think it is graph a.”]

In this text some connection is hinted at but not developed, the student states (in her/his way) that the derivative must be positive for x>0 and negative for x<0 but s/he does not explain why. S/he seemingly identifies “increasing” with “positive” and (explicitly) “decreasing” with “negative”, as s/he rules out graph B which is positive for x>0 but not increasing in most part of that interval. Moreover, s/he inconsistently does not rule out graph A, which is the only one with negative values for some x>0. The identification of “increasing” with “positive” and “decreasing” with “negative” may depend on poor understanding of the subject matter, but most likely it has linguistic roots, as this student seemingly makes no distinction between the words and most likely s/he refers to everyday-life uses, according to which “positive trend” might mean “increasing trend”.

Student A36 marks graphs A, B, C (with some erasures) and writes:

> “La funzione tra [0, +∞[*f(x)*>0 quindi la funzione è crescente quindi la B non è crescente. *[erased words]* funzione *g(x)* è tutta positiva da da]-∞,0] è decrescente mentre da [0,+∞[la funzione è crescente l’unico che cresce sempre di più è la D.”

> [“The function between [0,+ ∞[*f(x)*>0 so the function is increasing so B is not increasing. *[erased words]* function *g(x)* is all positive from from]-∞,0] is decreasing while from [0,+ ∞"[the function is increasing the only one that always increases is D.”]

The linguistic quality of this text is very poor. There is a bad coordination between the verbal and symbolic parts, the given function is referred to as ‘f ’ instead of using its proper name ‘g’, the conjunction “quindi” [“so”] is used twice inappropriately, in the second occurrence to introduce some data taken from a graph. There are a number of erasures and repetitions, and some of the last clauses are linked neither by discourse markers nor by punctuation.

Student A39 marks graphs A, C, D and writes:

> “Escludo la C perché nell’intervallo (10; 0), la funzione decresce perciò la sua derivata dovrà essere negativa. Escludo la D perché la funzione è pari mentre il grafico D è dispari. Escludo la A perché la funzione g è crescente nell’intervallo [0;10) e quindi il grafico A dovrebbe essere positivo mentre è negativo per $x\in[0;3]$.”

> [“I rule out C because in the interval (10; 0) the function decreases so its derivative will be negative. I rule out D because the function is even whereas graph D is odd. I rule out A because function g is increasing in the interval [0; 10) and so graph A should be positive, whereas it is negative for $x\in[0;3]$.”

In this case the choice of graphs is the correct one. Most likely in the expression (10;0) the subject has forgotten to write the sign ‘-’ before ‘10’ (although other participants wrote reversed intervals too). The motivation to rule out D is inappropriate, for it would have been necessary to recall that the derivative of an even function, if any, is an odd function and that graph D does not correspond to an odd function but it is neither odd nor even). On the contrary, the subject proceeds by analogy (f even $\Rightarrow f'$ even), missing the classification of graph A: actually s/he claims it is odd. Maybe s/he means that it is not even, but is misguided by the meaning of odd/even in the frame of integers.

Student A17 marks graphs A, C, D and writes:

> "Non corrispondono i grafici A-C-D. Possiamo escludere il grafico C perché per esempio nell'intervallo (-10;0), la nostra funzione *g* risulta decrescente mentre in quel tratto il grafico C risulta positivo (dovrebbe invece essere negativo). Possiamo escludere la A perché per esempio nell'intervallo (0;3), la funzione *g* risulta crescente mentre il grafico A in quell'intervallo è negativa anzi ché positiva. Escludiamo anche il grafico D perché nella funzione *g* la concavità è verso il basso tra (1;5) quindi nello stesso intervallo il grafico dovrebbe essere decrescente mentre la D è crescente."
>
> ["Graphs A-C-D do not correspond. We can rule out graph C because, for example in the interval (-10; 0), our function *g* results decreasing while in that stretch graph C results positive (it should be negative instead). We can rule out A because, for example in the interval (0;3), function *g* results increasing while graph A in that interval is negative instead of being positive. We rule out graph D too because in function *g* the concavity is downwards between (1; 5) so in the same interval the graph should be decreasing, while D is increasing."]

This excerpt has been reported in order to underline the difference between those who can use language in a mathematical setting and those who cannot. The text of A17 is not perfect, but language for her/him is a tool good enough to understand the problem, find a solution and justify it. The text is explicitly organized with conjunctions and discourse markers ("while", "so", "for example", "instead") and each statement is equipped with its own domain of validity ("…in the interval (-10; 0) …"). The general properties the argument is based on are not explicitly mentioned, but the subject adds some remarks that highlight the connections between the parts of her/his argumentation and make it unambiguous ("… it should be negative …", " … while D is increasing."). Although the subject does not write down some general rule or property, s/he underlines the critical points of her/his argumentation and uses language (including grammar) to organize and clarify her/his answer.

In the optional interviews performed in the week following the experiment, subjects A02, A03, A36, and A39 were not able to reconstruct their thinking and explain their answers. This is a general behavior: a great number of students cannot reconstruct the meaning of the text they have produced, even if they have it before them and are given time to read it with no pressure.

Discussion

The protocols examined above have been chosen as representatives of diffused patterns of argumentation. In particular it is worthwhile to remark:

The lack or improper use of connectives and discourse markers (i.e., of cohesive devices) is a serious problem: the links between the clauses are not made explicit or are expressed in a vague and improper way; even if the subjects, while writing down, may have some nice idea in mind, the lack of an explicit and effective objectification through language, prevents them from reconstructing and developing it afterwards. Behaviors of this kind are common.

Some students (such as A02) seem not to be able to recall the necessary pieces of knowledge and work on the data of the problem by creating pseudo-rules (e.g., g increasing/positive/even $\Rightarrow g'$ increasing/positive/even). Models of this kind are very robust. It is possible that these models are consequence of the practice of not interpreting the text of a word problem in order to reconstruct the problem situation, but to search for keywords that might suggest the proper.

The difficulties mentioned above all increase the instability of the processes of interpretation and production of texts, which might explain some apparently inconsistent behaviors; an example is protocol A39: the student answers correctly and correctly rules out graphs A and C reasoning on the basis of known properties of functions; to rule out D as well, s/he properly focuses on the evenness of g, but, probably in the attempt to apply the pseudo-rule "g even $\Rightarrow g'$ even" claims that D is odd; a number of students (more than 30 % of the sample in this experiment) correctly rule out A and C but use wrong or inconsistent arguments to rule out D too; the fact that in order to rule out D some 'rule' different from "g increasing $\Rightarrow g'$ positive" is required is enough to trouble the subjects and induce them to provide wrong answers.

Although much research is needed to determine the exact role of language in argumentation processes, it seems to me that the outcomes of this study suggest that it cannot be disregarded at all, in spite of the fact that a number of current studies on argumentation do not take the role of language into account. On the other hand, SFL seems a promising framework to better understand students' linguistic behaviors in a mathematical setting, disregarding neither the factors related to interpersonal communication nor those related to the specific features of mathematical language.

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