# Signal Processing Methods for Heart Rate Detection Using the Seismocardiogram

A Thesis Submitted
to the College of Graduate Studies and Research
in Partial Fulfillment of the Requirements
for the Degree of Master of Science
in the Division of Biomedical Engineering
University of Saskatchewan

by

**Mahsa Mafi**

Saskatoon, Saskatchewan, Canada

# Permission to Use

# Abstract

Cardiac diseases are one of the major causes of death. Heart monitoring and diagnostic techniques have been developed over decades to address this concern. Monitoring a vital sign such as heart rate is a powerful technique for detecting heart abnormalities (e.g., arrhythmia). This work presents novel heart rate detection methods, which are both robust and adaptive compared to existing heart rate detection methods. Two different experimental data sets, with varying operating conditions, were used in validating the proposed methods.

In this work, utilized methods for heart rate detection include Signal Energy Thresholding (SET), Empirical Mode Decomposition (EMD) and Empirical Wavelet Transform (EWT). To the best of the author's knowledge, this work is the first to use EMD and EWT for heart rate detection from Seismocardiogram (SCG) signal. Obtained result from applying SET to ECG signal is selected as our ground truth. Then, all three methods are used for heart rate detection from the SCG signal. The average error of SET method, EWT and EMD respectively 13.9 ms, 13.8 ms and 16 ms. Based on the obtained results, EMD and EWT are promising techniques for heart rate detection and interpretation from the SCG signal.

Another contribution of this work is arrhythmia detection using EWT. EWT provides us with the instantaneous frequency changes of the corresponding modes to ECG signal. Based on the estimated power spectral density of each mode, power spectral density of arrhythmia affected ECG is higher (more than $50dB$) compared to the power spectral density of a normal ECG which is less than $20dB$. This provides the potential for arrhythmia detection using EWT.

# Acknowledgements

I would like to express my great gratitude to my dear supervisors, Dr. Babyn and Dr. Bui for their invaluable suggestions, support and encouragement throughout my graduate program at university of Saskatchewan.

My deepest gratitude goes to my family, for their unwavering love and encouragement throughout my life.

Finally, I truly appreciate the help from everyone involved in this research study. Additionally, the completion of this thesis would not have been possible without the continued support and encouragement from my brother Ramin and my best friend Hussein. Thank you for everything.

# Table of Contents

# List of Tables

# List of Figures

# List of Abbreviations

| | |
|---|---|
| AO | Aortic valve Opening |
| BCG | Ballistocardiogram |
| bpm | beat per minute |
| CAD | Coronary Artery Disease |
| ECG | Electrocardiogram |
| EMD | Empirical Mode Decomposition |
| ETT | Exercise Tolerance Test |
| EWT | Empirical Wavelet Transform |
| FFT | Fast Fourier Transform |
| HPF | High Pass Filter |
| HR | Heart Rate |
| IF | Instant Frequency |
| IMF | Intrinsic Mode Function |
| LPF | Low Pass Filter |
| LVH | Left Ventricle Hypertrophy |
| MC | Mitral valve Closure |
| MEMS | Micro Electro Mechanical Systems |
| MIT-BIH | Massachusetts Institute of Technology-Beth Israel Hospital |
| MRI | Magnetic Resonance Imaging |
| PF | Peak Frequency |

| | |
|---|---|
| PS | Power Spectrum |
| PSD | Power Spectral Density |
| RMSSD | Root Mean Square of Successive Differences |
| SET | Signal Energy Thresholding |
| SCG | Seismocardiogram |
| SDANN | Standard Deviation of Average N-N intervals |

# 1. Introduction

Cardiac diseases are one of the major causes of death in the world [1,2]. Therefore early diagnosis of cardiac disease before progression to a severe stage is important. Long term measurement of vital signs such as heart rate is the most common way to monitor the cardiovascular function [3]. Tracking the heart rate (HR) is a powerful technique that can be used to diagnose some heart abnormalities such as arrhythmia [3, 4]. Numerous non-invasive monitoring/diagnostic techniques have been developed for HR monitoring including cardiac imaging (echocardiography, magnetic resonance imaging (MRI), Electrocardiography (ECG), Ballistocardiography (BCG) and Seismocardiography (SCG). These medical techniques can be used for diagnosis of cardiac diseases [5–10]. Before describing these non-invasive medical techniques, a brief introduction of the heart and its performance is presented in next section.

## 1.1 Heart Anatomy and Physiology

The heart is a muscular organ which sits within a fluid-filled cavity called the pericardial cavity. The wall of pericardial cavity is covered with special membrane known as the pericardium. The pericardium has several functions including keeping the heart contained in the chest cavity, preventing the heart from overexpanding when blood volume increases and holding heart in position by limiting its motions [11]. The heart is located in thorax between two lungs. As shown in Fig. 1.1 it has four chambers and four valves including:

- Two atria

- Two ventricles
- Atrioventricular valves: Tricuspid valve (right side), Mitral valve (left side)
- Semilunar valves: Pulmonic valve (right side), Aortic valve (left side)

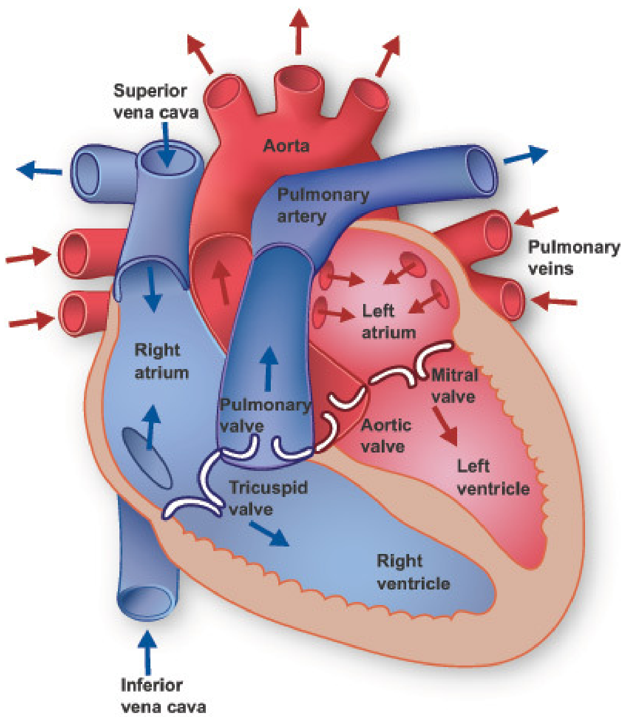


**Figure 1.1** Anatomy of the heart including chambers, valves, arteries and coronaries. Blue components indicate de-oxygenated blood pathways and red components indicate oxygenated pathways. Retrieved with permission from [12]

The right side of the heart receives de-oxygenated blood from the body and sends it to the lungs while the left side of the heart receives oxygenated blood from the lungs and sends it to the body. Before blood leaves each chamber, it passes through a set of valves. Valves are leaflets that act as one-way inlets for blood coming into a ventricle and one-way outlets for blood leaving a ventricle. Valves are essential to prevent backward flow of blood. In the following, we will discuss the conduction system of the heart which describes the function of the heart as a pumping organ. Also, cardiac cycle which refers to a complete heartbeat and its related physiological cardiac events will be discussed.

### 1.1.1 Conduction System of the Heart

The heart is capable of setting its own rhythm and has control over conducting this rhythm throughout its structures [13]. Only 1% of the cardiac muscle cells in heart are involved in initiation of the conduction system which set the pace for the rest of the cardiac muscle cells [14]. These cardiac cells, known as pacemakers (Fig. 1.2) are categorized as the following,

- Sinoatrial node (SA node)
- Atrioventricular node (AV node)
- AV bundle

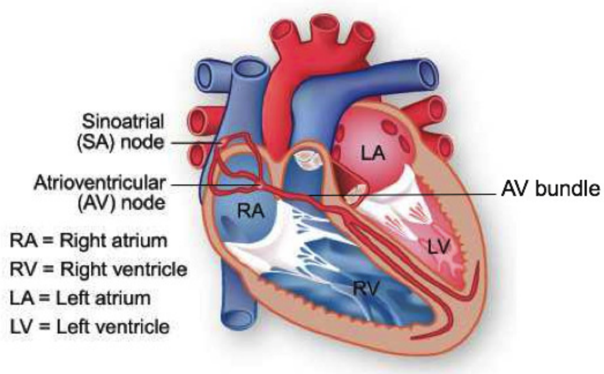


**Figure 1.2** Location of cardiac pacemakers in heart. Retrieved with permission from [15]

The cardiac cycle begins with depolarization of the SA node as the main pacemaker, in the upper right atrium [16]. This cycle follows by the action potential through the atria which originates an electric impulse. This electrical impulse is the cause of atria contraction. Then the impulse travels to the left atrium and down the interatrial septum to the AV node. This is the secondary pacemaker to the heart. The AV node slows down these impulses while they continue traveling down a common pathway branching off into the right and left bundle-branches and eventually to the ventricles and causing them to contract [16]. ECG records the summation of the spread of the electrical potentials.

### 1.1.2 Heartbeat

A heartbeat consists of complete action of the entire heart which can be categorized into two-part pumping action. It usually takes about a second [17]. The first part is atria contraction due to the originated electric impulse. This contraction pushes blood through the tricuspid and mitral valves filling the ventricles. This part of the two-part pumping phase is longer and is called *diastole* (see Fig 1.3).

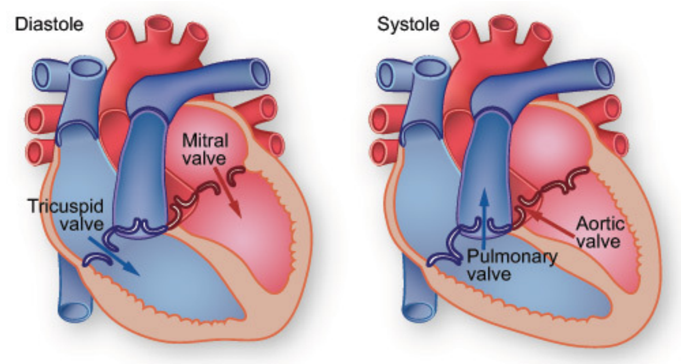


**Figure 1.3** Blood flow illustrated in both systole and diastole parts. Retrieved with permission from [18]

The second part of the pumping phase begins when the ventricles are full of blood. The electrical impulses from the SA node travel along a pathway of cells to the ventricles, causing them to contract. This is called *systole* (see Fig 1.3). As the tricuspid and mitral valves close tight to prevent backward flow of blood, the pulmonary and aortic valves are pushed open. The right ventricle sends blood into the lungs to pick up oxygen, while oxygen-rich blood flows from the left ventricle to the heart muscle and other parts of the body.

After blood moves into the pulmonary artery and the aorta, the ventricles relax, and the pulmonary and aortic valves close. The lower pressure in the ventricles causes the tricuspid and mitral valves to open, and the cycle begins again. This cardiac cycle is repeated over and over again and increases in speed with physical activity and decreases while resting. The normal heartbeat at rest condition is about 60 to 80 beats per minute (bpm) but this can vary depending on age or physiological conditions [19, 20]. Resting heart rate in children is higher. Also, it is usually lower in people who are physically active. Sometimes, heart rate can be irregular due to a problem in heart performance. This abnormality called arrhythmia in which the heart can beat too fast, too slow, or with an irregular rhythm [21].

Contraction and expansion of the heart chambers during systole and diastole produces the mechanical movements of the heart. These mechanical movements exist in all three dimensions. SCG provides the measurement of these movements using an accelerometer.

## 1.2 Non-invasive Cardiac Monitoring/Diagnostic Techniques

Non-invasive cardiac imaging provides information about the structure and function of the heart by capturing cardiac image sequences [22]. Invasive methods, which require catheters to be inserted into the heart through blood vessels in the leg, can detect coronary artery disease. On the other hand, noninvasive tests are safe and easier to perform than invasive studies but also are capable of detecting some heart conditions including coronary artery disease.

One of the reasons that our research focus area is non-imaging techniques (ECG, BCG and SCG) is the longer term potential for monitoring. In the following, non-invasive monitoring/diagnostic techniques of ECG, BCG and SCG which are non-imaging are discussed.

ECG is one of the most commonly used cardiac monitoring/diagnostic tools. It records the electrical activity of the heart which provides the information on heart performance. Unlike ECG, both BCG and SCG are based on measurements of mechanical signals. While BCG records the vibrations of the body caused by shifts in the center of mass of blood in the arterial system, SCG itself measures the vibration of the heart beat to the chest wall caused by heart contraction [1]. Seismocardiography is a method of recording precordial[1] acceleration signals induced by heart contractions, chest wall movements, and respiration. Both BCG and SCG main frequency

[1]Precordium is a region of thorax immediately in front of or over the heart

components are below 20 Hz (infrasonic[2] range/low frequency) [24]. According to the signal frequency components, BCG is classified as an infrasonic signal. SCG can extend to 30 Hz which has a small overlap with the audible frequency (sonic) range (20-2000 Hz).

ECG measurements are frequently based on electrodes attached to body [25–27]. In order to measure any electrical activity of the heart at least two electrodes[3] are required which are placed on the chest [28]. In most BCG measurements a piezoelectric[4] pressure sensor is used which is usually placed in a bed or chair's back/seat [30–32]. Fig. 1.4 presents the simultaneous recording of both ECG and BCG signals in the supine position.

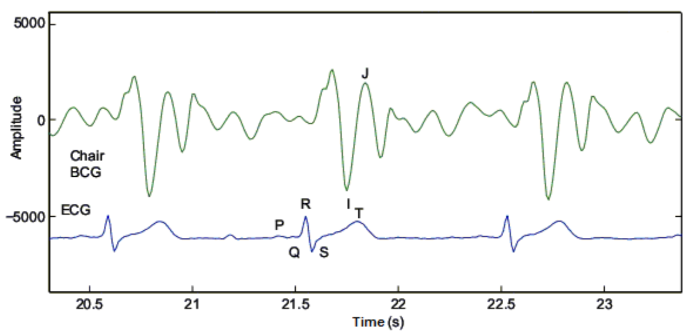


**Figure 1.4** IJ peaks in BCG and R wave in ECG is part of the cardiac cycle phase in which the ventricular contraction happens. Retrieved with permission and edited [33].

SCG measurements are done using a 1D or 3D accelerometer often placed on the sternum [9, 34]. ECG electrode placement usually increases the stress level in patients compared to BCG measurement sensor which is usually placed on the bed

[2]Infrasonic signal is caused by heart muscles ejection and fraction in each heartbeat and has low frequency components(below 20 HZ). Sonic signal is caused by heart valvular sound and includes higher frequency components (20-2000 Hz) [23]

[3]In the traditional ECG recording, 12 electrodes are required which are typically placed on the upper body

[4]A piezoelectric sensor is a device that is able to measure changes in pressure, acceleration, strain or force by converting them to an electrical charge [29].

or chair [32, 35]. For SCG measurement only one sensor is required. In conclusion, SCG and BCG measurement methods compared to ECG might be less stressful for the patient.

As the SCG measurement sensor is placed on the sternum and is closer to the heart, this gives a better representation of heartbeat signal compared to BCG. Furthermore, SCG provides us with mechanical information of the heart, such as ventricles' movements, which is a different aspect of cardiac information compared to the traditional cardiac electrical information gained by ECG. The mentioned facts above make SCG signal processing an interesting area for more investigation [36–39].

SCG was first introduced into clinical medicine by J. Zanetti in which a 1D accelerometer was utilized to collect the Z axis (perpendicular to heart) acceleration [40]. A 3D accelerometer collects the acceleration of heart in three axes of X (right-left), Y (head-foot) and Z (back-front). Z-axis is the main focus of most SCG research studies since the maximum force generated by the heart is in Z direction. Therefore, Z component provides more cardiac information [1]. SCG is usually recorded with ECG simultaneously since ECG is required as a time reference (RR interval[5]). Fig. 1.5 and Fig. 1.6 respectively indicate ECG and SCG Z-axis waveforms and the sensor placement on chest.

SCG reflects the mechanical state of heart which provides extra information of cardiac events in each heartbeat. SCG device is inexpensive, portable, safe and can be easily performed. These are advantageous characteristics of an SCG device as a medical tool. On the other hand, SCG does not have a prevalent clinical usage due to its interpretation complexity.

Many applications of SCG using information extracted from Z-axis have been investigated [1, 36, 42, 43]. The applications of SCG can be classified in two main groups: i) monitoring cardiac vital signals and ii) diagnosis of heart abnormalities. Examples of the first group include Ramos-Castro et al. [38], where the possibility of

[5]Ventricular contraction begins at R peak and RR interval is the duration of one cardiac cycle.

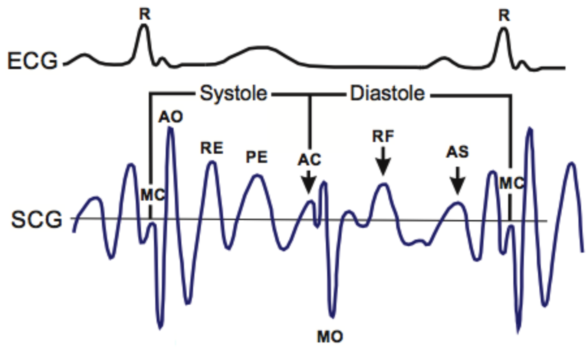


**Figure 1.5** SCG waveform includes both systolic points which are AO (Aortic valve Opening), RE (Rapid ventricular ejection), PE (Peak Ventricular Ejection), AC (Aortic Valve Closure) and diastolic points which are MO (Mitral Valve Opening), RF (Rapid Ventricular Filling), AS (Atrial Systole), MC (Mitral Valve Closure). Retrieved with permission and edited [41]

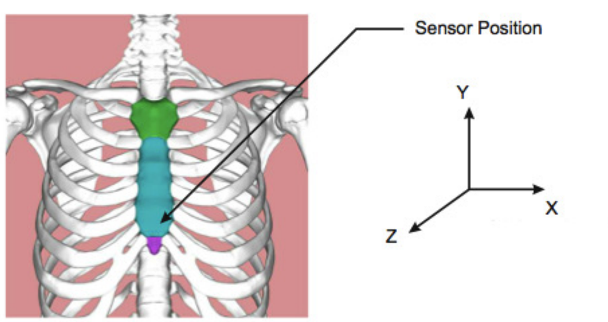


**Figure 1.6** Sensor position and 3 directions of SCG's X,Y and Z axes

obtaining a good estimation of heart rate and its variability using SCG is presented. An overview to the second application includes the work by Richard A. Wilson et al. [10]. The authors have investigated the diagnostic accuracy of SCG in Coronary Artery Diseases (CAD). There are different publications addressing these two aspects which will be reviewed in the next chapter.

According to the literature, SCG has been utilized in tracking changes of the cardiovascular system [38]. The most commonly performed methods for the heart rate detection using SCG signal are algorithms based on autocorrelation and thresholding of the signal energy. These algorithms can be easily implemented into embedded hardware systems which makes them suitable for real-time analysis [38, 44]. Autocorrelation algorithms applied to an SCG signal compute heart rate frequency using a mathematical statistic autocorrelation function and a peak detector is used to extract signal peaks [44, 45]. Signal energy thresholding algorithm [44, 45] steps are as following:

- The signal filtering using high-pass and low-pass Butterworth filters
- Obtaining the energy of the filtered signal
- Thresholding the signal energy for maximum peaks extraction
- Calculating heart rate based on the time interval between each two consecutive peaks

More details about the briefly described methods above will be covered in Chapter 4. Signal pre-processing and processing are respectively the next stages after collecting the raw data. In most research studies, the signal pre-processing stage usually happens through hardware implementation (e.g., Low pass filter for removing baseline wander and respiration signal) [4, 34, 38]. Furthermore for multiple signal recording purposes (e.g., ECG and SCG), synchronization is performed within the hardware and leads to simultaneous recorded signals for further processing. Described pre-processing parts of filtering and synchronization affects the quality of whole process after having the recorded data including signal processing stage and obtained results. Therefore proper implementation of these parts within hardware is critical. In conclusion, data collection algorithms are one of the challenging and most important part of recording cardiac data which does not fit in scope of this thesis since we have utilized recorded data originating from two research groups (more details on Chapter 3). Based on the mentioned facts above, we don't have access to raw data but the pre-processed data. In the next section, typical process of cardiac diagnosis and its advantages and disadvantages will be briefly discussed. It will also be compared to other cardiac diagnosis methods.

## 1.3 Motivation

A patient experiencing a heart abnormality may plan to visit a doctor. Typically vital signs of the patient such as heartbeat, ECG and blood pressure are measured by the doctor and maybe sent to a specialist for further analysis. Waiting for results may take weeks or even longer. In some cases, after receiving the test results additional specialized tests, such as wearing a Holter monitor[6] for a day or more, may be required. Patient convenience and the result preparation time are two potential concerns following a typical diagnosis process. In order to address these concerns, numerous methods and devices for monitoring vital signs have been developed which not only are wearable, portable and designed for long time monitoring, but also provide real-time interpretation of the recorded signals. Therefore, patient can be at home while wearing a vital signs monitoring device, which enables simultaneous analysis of results. As discussed before, some of these non-invasive methods which can detect HR include electrocardiogram, ballisocardiogram and seismocardiogram.

[6]A Holter monitor is a wearable device that continuously records the heart's rhythms and is usually worn for 24 - 48 hours during normal activity [46]

## 1.4 Problem Statement

In regards with heart rate detection, lots of studies have been completed. These studies proposed different kinds of HR estimation methods mostly using ECG signal features. There are few studies which utilized usefulness of SCG signal in heart rate detection as well as ECG signal.

These HR detection methods can be categorized as either linear/non-linear or adaptive/non-adaptive. Each category has its own advantages and disadvantages. In the first category, linear methods have less computational complexity but when they are used for non-linear signal (e.g., ECG OR SCG) processing would cause the missing some of the signal's important information. On the other hand, non-linear approaches provides reliable information about signals but they are not capable of tracking changes in high frequencies which is considered as a limitation of thesis approaches.

In the second category, adaptive methods such as Fast Fourier Transform (FFT) provides a reliable frequency response of stationary signals but it is not capable of localizing the instantaneous frequency changes. Instantaneous frequency (IF) is one of the essential signal parameters which provides important information about the time-varying spectral changes in non-stationary[7] signals.and it is applied to various applications such as seismic, radar, sonar, communications and biomedical applications [8] [47]. Considering the dynamic changes of heart behaviour and its non-stationary nature, adaptive methods, whose basis functions are directly derived from the signal itself, can perform better in tracking changes of heart rate including its rapid variation compared to non-adaptive methods.

Therefore, it is really important to choose an appropriate method for the processing of the non-stationary signals including ECG, SCG and heartbeat. According

[7]Non-stationary signal is a signal that its frequency contents change with time

[8]As an example, for the echo-location systems of bats the IF plays an important role as a time-varying parameter which defines the location of the signals spectral peak when it varies with time [47]

to their non-stationary natures, an adaptive method suits these signals processing effectively. To address this concern, highly adaptive methods of Empirical Mode Decomposition (EMD) and Empirical Wavelet Transform (EWT) were proposed for HR detection which will be discussed in further detail in Chapter 3.

Lots of studies were accomplished regarding both detection and classification of arrhythmia. Since arrhythmia affects the frequency content of an ECG signal, proposed spectrum analysis methods are suitable for arrhythmia detection in frequency domain. Non- adaptive spectrum analysis methods such as FFT are not able to track the instantaneous frequency changes of the input signal. Therefore, obtained results based on FFT may miss some spectrum information. On the other hand, EWT is a new approach of building adaptive wavelets and is capable of tracking the instantaneous frequency changes. This advantage, highlights the potential of EWT in arrhythmia detection. The summary of the missing gaps is as below:

- Few adaptive heart rate detection algorithms have been developed using SCG signal.
- The capability of Empirical Wavelet Transform (EWT) in cardiac abnormality detection such as arrhythmia has not been investigated.

## 1.5 Objectives of the Thesis

The purpose of this study is to propose solutions to the missing gaps. In the following the objectives of this thesis is described and discussed briefly.

- Developing new algorithms for HR detection using SCG signal based on adaptive methods
- Investigating the possibility of using Empirical Wavelet Transform (EWT) method for arrhythmia detection

### 1.5.1 Heart Rate Detection Using SCG Signal

For this objective, Signal Energy Thresholding (SET), Empirical Mode Decomposition (EMD) and Empirical Wavelet Transform (EWT) are the algorithms that have been used for HR detection using SCG signal. The heart rate calculated from applied SET to ECG signal is taken as the ground truth. Then the obtained estimated HR results from adaptive methods of EMD and EWT were evaluated by comparing to the ground truth.

In this research, we used the cardiac data from two data sets with different sample frequencies and recording lengths. The first and second data sets, respectively, include 11 and 5 young (20-25 year old) healthy men. These cardiac data include simultaneously recorded ECG and SCG signals in sitting position. After the signal processing phase, evaluation of the applied methods required. Based on the obtained results and their statistical analysis, applying EMD and EWT to SCG are reliable alternatives for the estimation of HR.

### 1.5.2 Arrhythmia Detection Using Empirical Wavelet Transform

EWT is one of the recent signal processing methods which suits non-stationary signals such as ECG and SCG [48, 49]. Since EWT decomposes the input signal into several components with different frequency bands, it gives the advantage of detecting cardiac changes within each frequency component. Therefore it has the potential to be used for the detection of cardiac abnormalities such as arrhythmia which is the focus of the second objective. For this purpose, we used the Massachusetts Institute of Technology (MIT) arrhythmia database. EWT was applied to both normal and arrhythmia affected ECG signals. Then the power spectral density of each frequency component was estimated using a correlogram method and the results of normal data was compared to the abnormal one. According to the obtained results, power spectral density of arrhythmia affected ECG is higher ($\geq 50dB$) compared to the power spectral density of a normal ECG ($\leq 20dB$).

## 1.6 Thesis Organization

Chapter 2 includes relevant literature survey of SCG signal hardware, its applications, signal processing methods of heart rate detection and arrhythmia detection methods. In Chapter 3 SCG signal processing methods for heart rate detection and arrhythmia detection method will be discussed. Chapter 4 presents the results of the proposed signal processing methods and their evaluation. In chapter 5 contributions and the limitations of this research study are discussed and finally chapter 6 provides conclusion and the future work.

# 2. Literature Survey

In this chapter, the development of seismocardiography in terms of both hardware and signal processing methods is discussed. Heart rate and arrhythmia detection methods are also covered within this chapter. SCG is a prime technology for monitoring of heart health as it requires simply a mechanical connection to patient's chest. SCG applications can be classified in two main groups of *diagnosis of heart abnormalities* and *monitoring of heart performance* as previously mentioned in Chapter 1. Before talking about the potential applications of SCG, its hardware development will be discussed briefly.

## 2.1 SCG Hardware Development

An SCG device includes an acceleration sensor which is typically placed on the sternum to measure the acceleration of heart due its vibration. During the last 2-3 decades, many developments have been proposed regarding SCG's hardware. Some of them are discussed in this section.

In 1990, seismocardiography was first introduced into clinical medicine and commercialized by Zanetti et al. [40, 41]. They recorded SCG signal using an analog 1D ultra-low frequency acceleration transducer. They also recorded ECG simultaneously along with SCG to have a timing reference.

Initially, a 1D accelerometer was utilized and provided cardiac information on just the Z axis. It was also was a bit heavy, big and uncomfortable for patient's chest. Addressing these concerns, very small and light accelerometers called micro electromechanical systems (MEMS)[1] have been applied which are capable of measuring acceleration in all three axes.

Another important development of SCG's hardware is the wearable SCG [37,39,50] which can be used for long term monitoring and cardiac assessment in daily life since it is portable. This development is classified as SCG's monitoring application which is discussed in section 2.2.

## 2.2 SCG Diagnostic Applications

J. Zanetti et al. [40, 41] showed that SCG can be useful detecting left ventricular (LV) changes due to heart abnormalities [9]. Additionally, it was found that comparison of SCG waveforms before and after exercise can be a promising method for detection of Coronary Artery Disease (CAD) and assessment of cardiac function [40, 51].

Richard A. Wilson et al. in 1993 [10] performed a multicenter study to compare diagnostic accuracy of SCG with ECG for diagnosis of CAD during exercise testing. They recorded SCG signal using an analog ultra-low frequency acceleration transducer placed on participant's sternum and also utilized simultaneous ECG recording as a timing mark for signal analysis [10]. Based on their results, SCG offered significantly better sensitivity for detecting CAD compared to ECG (73% vs 48%) without loss of specificity (78% vs 80%).

Korzeniowska-Kubacka et al. in 2005 [36] compared the diagnostic accuracy of SCG with an exercise tolerance test (ETT) for diagnosis of ischemia in patients with angiographically proved coronary artery disease. Ischemia is a condition in which the blood flow to a part of the body is restricted eading to a shortage of both oxygen and glucose in that part [52]. They recorded SCG signal using an accelerometer placed on patient's sternum in the supine position. Based on their results, SCG is more

[1] MEMS is a technology that combines computers with tiny mechanical devices such as sensors, valves, gears, mirrors, and actuators embedded in semiconductor chips

sensitive for detecting ischemia compared to ETT (61.1% vs 44.2%).

In simultaneous recording of ECG and SCG, SCG can provide additional mechanical information of heart. Using this advantageous feature, Patrick Neary et al. in 2011 [53] showed that the simultaneous measurement of SCG alongside ECG is a legitimate alternative for diagnosis of cardiac abnormalities such as left ventricle hypertrophy (LVH) in athletes when Echo and cardiac MRI are precluded.

## 2.3 SCG Monitoring Applications

Jerosch-Herold et al. in 1999 [54] demonstrated that SCG is an MRI compatible technology permitting for monitoring of left ventricle (LV) function during stress testing during a MRI procedure. Since ECG signal is distorted in a magnetic field, a mechanical signal like SCG can be a reliable replacement of ECG while being used with MRI.

Korzeniowska et al. in 2007 [55] showed that SCG is a practical technique for monitoring systolic and diastolic left ventricular (LV) function in CAD patients who undergo a training program. Based on their results, the training program caused a significant improvement in the physical capacity and cardiac performance in CAD patients with exercise-induced left ventricular dysfunction.

Castiglioni et al. in 2007 [37] proposed the novel idea of wearable seismocardiography for long term monitoring. The system they developed could provide statistically consistent estimations of both heart sound related vibrations and recoil movements. They recorded SCG signal using a triaxial analog MEMS accelerometer placed on the left clavicle.

Tavakolian et al. in 2010 [56] demonstrated that SCG as a monitoring tool can estimate hemodynamic parameters such as stroke volume and systolic time intervals. They used suprasternal pulsed Doppler [57] and impedance cardiogram (ICG) [58] as the reference methods. Based on their results, SCG gives accurate measurements of systolic time intervals compared to reference methods. They also showed the obtained results from SCG has consistency for such measurements. Therefore, SCG is capable of detecting changes such as a sudden drop in stroke volume during a time period.

Di Rienzo et al. [39,50] proposed a wearable SCG tool for the assessment of cardiac mechanics dynamic features, such as systolic time intervals, in daily life and tested the applicability of this method. They recorded SCG signal using a digital triaxial MEMS accelerometer located on the sternum.

## 2.4 Signal Processing Methods for Heart Rate Detection

These methods can be categorized into different classes based on whether 1) they are *linear* or *non-linear*, and 2) if they use *adaptive* or *non-adaptive* techniques. Both linear and non-linear analysis have their own advantages and limitations. Since HR is a non-stationary signal, any linear analysis has the potential risk of underestimating or even missing a great amount of information content [59]. On the other hand, complex nonlinear approaches are not capable of tracking changes in high non-stationary context of RR interval series [49]. According to the second classification, HR detection algorithms are either adaptive (e.g. EMD, EWT) or non-adaptive (e.g. short-term Fourier transform).

EMD is an adaptive technique which decomposes a complex signal into several frequency components called Intrinsic Mode Functions (IMFs) that do not overlap in frequency [60]. EWT is another adaptive signal processing method proposed by Jerome Gilles in 2013 [48]. Considering the dynamic changes of heart behaviour and its non-stationary nature, adaptive methods of EMD and EWT whose basis functions are directly derived from the signal itself, can perform better in tracking changes of heart rate including its rapid variation compared to non-adaptive methods [61, 62]. Adaptive algorithms of EMD and EWT will be discussed in more details in Chapter 3.

The heart rate from ECG, SCG and BCG signals can be detected by various signal processing methods. In the following, some of the signal processing algorithms for HR detection using ECG, BCG and SCG signals are summarized. These algorithms include autocorrelation and thresholding of the signal energy [38, 45], EMD [59, 63], wavelet transform [64, 65], artificial neural networks [66, 67], adaptive filtering [68, 69] and the template matching approach [70].

Parak et al. proposed the application of three statistical methods of autocorrelation, signal energy thresholding and peaks detection in energy signal envelope on both ECG and BCG signals for heart rate detection purpose [4, 44, 45]. They also showed that practical methods for heart rate detection from BCG signals are the algorithms based on autocorrelation and signal energy thresholding since they are less complicated compared to the other methods in terms of hardware implementation [44, 71]. This advantage allows for real-time BCG signal processing using autocorrelation and SET methods [44]. Steps of the signal energy thresholding method as used in [38, 44] which happens after preprocessing phase are as following:

- Signal filtering using 4th order Butterworth filters
- Calculation of filtered signal's energy for R peaks extraction
- Calculation of heart rate based on the time distance between each two consecutive R peaks

Ramos-Castro et al. compared HR indices estimated from SCG signal with the ones calculated using RR series obtained from ECG [38]. Heart rate from ECG signal was obtained as mentioned above. In order to estimate heart rate using SCG signal, they used the z-axis of the accelerometer data. Then, the signal was filtered with a fourth order Butterworth band pass filter with cutoff frequencies of 6 Hz and 25 Hz respectively. After filtering, the signal energy is estimated and compared with a threshold and heart rate is calculated using extracted time-intervals. Finally, HR time and frequency domain parameters [72] were estimated for both ECG and SCG signals and compared. Based on their results, applying SET to an SCG signal is a promising signal processing method for heart rate detection.

Laurin et al. [73] indicated that HR time and frequency domain parameters obtained with SCG interbeat intervals are valid and can be used without ECG corroboration. They also recommended that AO marker (see Fig. 1.5) is the best alternative for HR detection, since it is obtainable without the use of another signal like ECG to identify heartbeats. HR parameters for short-term recording (5 minutes or less) have been described in Table 2.1.

| | Parameter | Index: ECG time-interval R-R | SCG time-interval AO-AO | MC-MC |
|---|---|---|---|---|
| Time Domain | **SDNN (ms)** | Standard deviation of all interbeat intervals | | |
| Time Domain | **RMSSD (ms)** | Square root of the mean of the sum of squares of differences between adjacent interbeat intervals | | |
| Freq. Domain | **HF norm %** | High frequency power in normalized units HF/(Total Power - VLF[2]) ×100 | | |
| Freq. Domain | **LF/HF** | Ratio of power in low frequency range (0.04-0.15 Hz) to power in high frequency range (0.15-0.4 Hz) | | |

**Table 2.1** Time-domain and frequency domain parameters of HR for short-term recording ($\leq$ 5 minutes) for both ECG and SCG time intervals.

Bu et al. [3] performed EMD on ECG signal in order to extract both heart rate and respiration signals. They calculated the peak frequency of each IMF ($h_i$) in order to select the IMFs corresponding to respiration and heartbeat. The IMF whose peak frequency, ($PF_i(i = 1, 2, ..., n)$), is in the range of 0.1-0.5 Hz is determined as a component of respiration, while the range for heartbeat is 1.0-10 Hz. After selecting the corresponding IMFs, both heartbeat ($x(h)$) and respiration ($x(r)$) signals are reconstructed. The following equations mathematically describe this procedure

[2]Power in very low frequency range $\leq$ 0.04 Hz

$$x(r) = \sum_i h_i \qquad (PF_i \in [0.1, 0.5]Hz) \tag{2.1}$$

$$x(h) = \sum_i h_i \qquad (PF_i \in [1.0, 10]Hz) \tag{2.2}$$

where $h_i$ stands for IMF, $n$ is the total number of IMFs, $PF_i$ is the peak frequency function, $x(r)$ is the respiration function and $x(h)$ is the heartbeat function.

Garcia-Gonzalez et al, [74] proposed 4 heartbeat detectors using SCG which was based on continuous wavelet transform or bandpass filtering. The detectors were capable of adapting their parameters to the morphology of the signal by estimating mean heart rate and the bandwidth of the heartbeat signal. They recorded SCG signals from 17 healthy volunteers using a triaxial accelerometer. Based on their results, the standard deviation of the error for all detectors in the obtained RR time series was around 2 ms and the percentage of obtained RR time intervals with an higher error than 30 ms was around 3.5%. Therefore using proposed detectors, measured SCG in a quiet environment makes it a promising alternative for the ECG's substitute to obtain reliable HR parameters.

### 2.4.1 HR Detection Methods in this Research Study

In this research, SET was applied to ECG signal and was used as the ground truth. Even though this method gives a good estimation of heart rate, it has its own limitations. Since ECG and SCG signals are non-linear/non-stationary and their morphology may change during a time period, a non-adaptive method like SET which utilizes pre-defined filters is not always able to give an accurate estimation of heart rate. Filtering and thresholding in SET is not unique and applicable for all input signals and needs to be reset for some input signals as necessary.

Considering the fact that HR is a non-stationary signal which is the result of many nonlinearly interacting processes [59], any linear analysis may underestimate or miss a great amount of information content [59]. In this case we need a method that is capable of analyzing a nonstationary data. Empirical mode decomposition (EMD) is a signal processing technique well suited for nonlinear/nonstationary data [60].

EWT is another adaptive method suitable for nonlinear/nonstationary data. In this algorithm, different modes of a signal are extracted by an appropriate designed wavelet filter bank. Based on the obtained results this method compared to the classic EMD is more useful and practical [48]. Both EMD and EWT methods have their own advantages and disadvantages which will be discussed in more detail in Chapter 4.

To the best of author's knowledge, both EMD and EWT methods have not yet been applied to SCG signal for heartbeat detection. In this thesis, adaptive techniques have been used for SCG signal processing to extract heart rate. Chapter 4 covers the details about the proposed methods.

## 2.5 Arrhythmia Detection Methods

An extensive numbers of studies have been published regarding cardiac abnormality detection using ECG signals. In this section arrhythmia detection methods using ECG signal will be covered briefly.

Himanshu Gothwal et al. in 2011 [75] presented a method for the classification of heart beats according to different types of arrhythmia based on the extracted features from ECG signal. For feature extraction and heart beat classification respectively FFT and Artificial Neural Network were utilized. Based on their obtained results, the proposed method has a better efficiency compared to the previously proposed methods.

Shreya Das et al. in 2011 [76] proposed a method for investigating the deffering frequency content in normal vs. abnormal ECG signals. Periodgram was the method that was used to calculate the power spectral density in both normal and abnormal ECG signals. Based on their results, the amplitude of the frequency components of an abnormal ECG is higher compared to the normal ECG.

EWT is another signal processing method which has the potential to be used for further signal processing purposes such as cardiac abnormality detection. Another contribution to this research work is using EWT in ECG signal processing for arrhythmia detection purpose.

# 3. Methodology

This chapter covers the details of the proposed methods in this research work along with the utilized data sets phase. The first section describes the utilized data sets and the required procedures for obtaining the demanded input signal whereas the rest of the sections discuss the proposed signal processing methods, challenges and limitations.

## 3.1 Utilized Data Sets

In this section, utilized data sets and the relevant recording information are discussed. In this research work, three data sets have been used that all of them are digital input signals. For heart rate detection using SCG signal, two different data sets were used. The sample frequency of each data set is respectively 1000 and 5000 Hz. From data set 1 which belongs to a research group [34] and data set 2 which were obtained from PhysioNet [74] respectively, eleven and five healthy young men (20-25 year old) with a normal heart condition were selected. It should be noted that most of the existent SCG data sets are exclusive and can not be easily reached at reliable online databases such as Physionet which cause some limitations in terms of data set size and quality.

For both data sets, ECG Lead II[1] (see Fig. 3.1) and SCG Z-axis (see Fig. 1.6) signals were each recorded through a channel and synchronized. Of course for each data set, other vital signals such as respiration were collected and synchronized as

[1] Lead II is the voltage difference between the left leg (LL) and right arm (RA) electrodes (LL - RA)

well which do not fit the focus area of this research. The collected simultaneous data from both data sets have been recorded in supine position. Each recording span is one minute and these signals were recorded while the participants had the least possible movement during recording time. Movement causes the distortion of the original signal and therefore extraction of the correct information will be an issue.

For arrhythmia detection using Empirical Wavelet Transform (EWT), utilized data set, Massachusetts Institute of Technology-Beth Israel Hospital (MIT-BIH), was obtained from Physionet [77] which includes both normal and arrhythmia affected ECG signals with the sample frequency of 360 Hz.

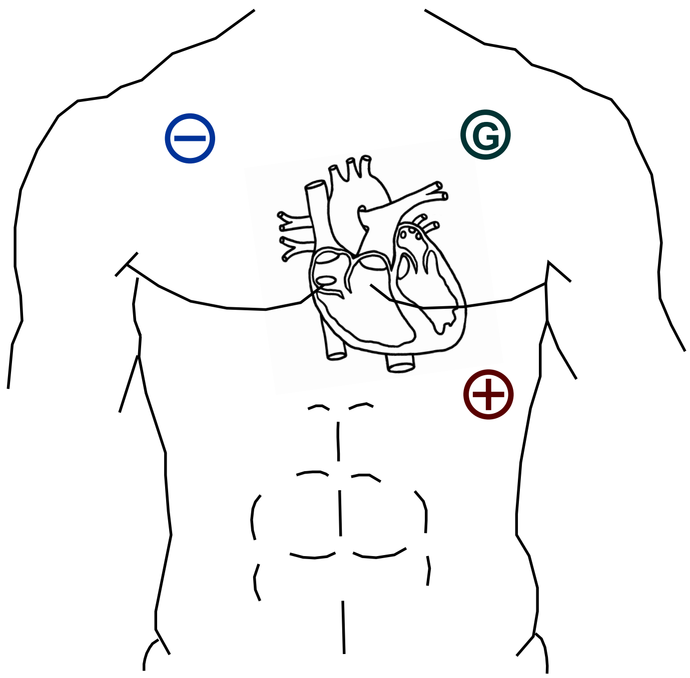


**Figure 3.1** Electrode placement locations

The utilized data sets were preprocessed with several methods. For instance, unnecessary low frequency components such as baseline wandering, respiration and high frequency components such as noise had been removed by filtering the raw data. Therefore, it limits the access to the raw data for applying other pre-processing methods. The span of the recorded signals causes another limitation in case the full-length of the signals are being used. In order to significantly reduce the computational complexity of the proposed methods, a solution was proposed in which the span of each data span was divided in to the shorter lengths. Therefore, multiple 10 second spans were selected from each data and the final result for each data is the average of these 10 second spans. Fig. 3.2 shows 10 second recording of ECG and 3-axes SCG of a participant.

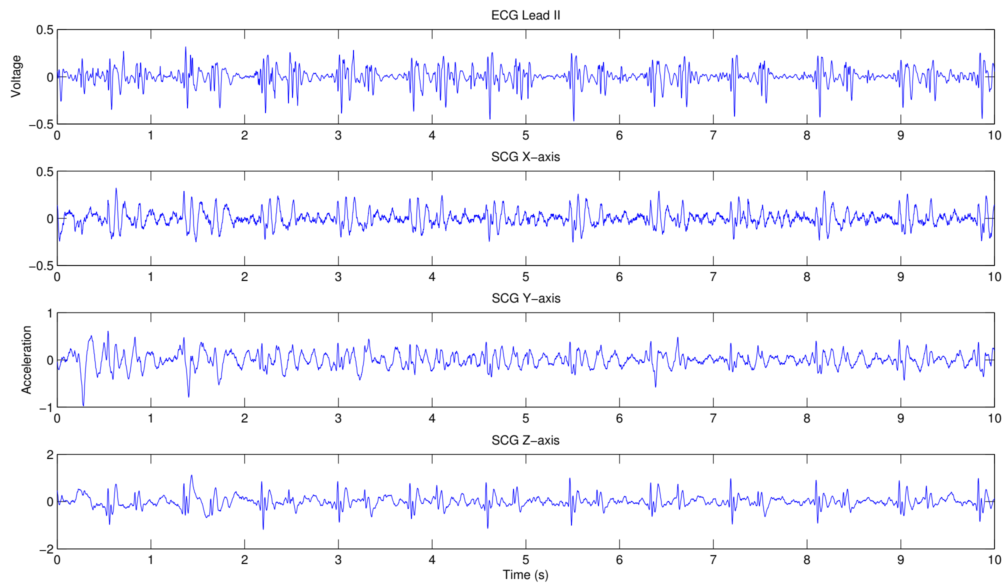


**Figure 3.2** Multi-axis SCG and the corresponding ECG signal samples versus time

## 3.2 Methods

In this thesis, the following methods are proposed:

Ground truth:

- Modified version of signal energy thresholding (SET)

Proposed adaptive methods:

- Application of Empirical mode decomposition (EMD) for heart rate detection using SCG signal
- Application of Empirical wavelet transform (EWT) for heart rate detection and arrhythmia detection using both ECG and SCG signals

All the algorithms are implemented in MATLAB. Relevant information of these methods will be covered in the following sections.

## 3.3 Modified Signal Energy Thresholding

SET has a rather simple mathematical model which has been implemented and applied to the input signal. In this method, HR is estimated by thresholding the signal energy which is calculated after the signal is filtered using 4th order Butterworth filters [45]. Butterworth filters are having an advantage of providing maximally flat frequency response with no ripples in the pass band [78]. Additionally the calculation of Butterworth filter is simpler compared to other filters with similar characteristics. This simplicity combined with a level of performance makes Butterworth filters an ideal filtering option for different kinds of signals.

One of the limitations of SET method is the filtering part which is the same for different input signals (ECG/SCG data sets). Filtering part of SET has been modified by defining Butterworth filters with variable cut-off frequencies. This modification improves the performance of SET while different data sets are being used as the input signal. In this research study, two data sets were used for the first objective which is heart rate detection. For each data set, the filtering step is different in terms of cut-off frequency selection which has been obtained experimentally. Fig 3.3 is a block diagram which illustrates steps required to perform SET.

Since input signals are digital, they are discrete in both time and amplitude. Variables in all following equations are functions of a discrete time parameter ($t = \frac{n}{F_s},\ 0 \leq n \leq N$) which itself is a function of number of samples ($n$) and the sampling frequency($F_s$). In the following, the general formulation involved within SET steps will be presented.

### 3.3.1 Steps of Modified SET

After filtering ECG and SCG, the energy of these filtered signals is calculated as the following,

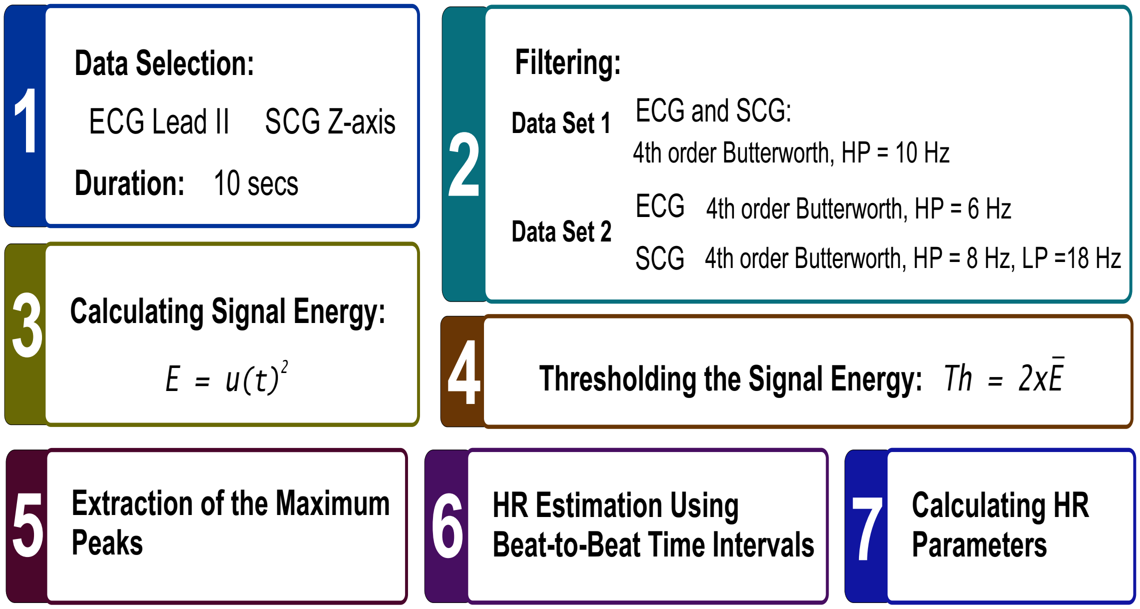


**Figure 3.3** Block diagram of SET steps. The second block (filtering) differs for each data set. For example, in data set 1 ECG and SCG signals were filtered by 4th order highpass Butterworth filter with the cut-off frequency of 10 Hz. On the other hand in data set 2, ECG signal was filtered by 4th order highpass Butterworth ( cutt-off freq = 6 Hz) and SCG signal was filtered using 4th order highpass and lowpass Butterworth filters (HP cut-off freq = 8 Hz, LP cut-off freq = 18 Hz)

$$E(t) = u(t)^2 \tag{3.1}$$

where $E(t)$ and $u(t)$ are respectively the signal energy and the filtered signal. In order to extract signal peaks from ECG and SCG signals, the signal energy is thresholded based on an empirical equation (3.2).

$$TH = 2 \times \bar{E}(t), \tag{3.2}$$

where $TH$ is the threshold parameter and $\bar{E}$ is average of energy signal and factor 2 is selected empirically. After extracting signal peaks, heart rate can be calculated based on beat per minute (bpm) using the interbeat time intervals from both signals.

$$\text{HR} = \frac{60}{\text{Interbeat-interval(sec)}} \quad (bpm) \tag{3.3}$$

The HR signal obtained using ECG is considered as our ground truth. Since the filtering and thresholding parts of SET may need to be changed for different input signals, some pre-defined filters with different cut-off frequencies (LP Butterworth cut-off frequencies are 6, 8, 10 and HP Butterworth cut-off frequencies are 10, 15, 18) have been selected based on their given output results. Then, these filters were applied to all input signals and the best result was selected as the final estimated heart rate for each signal.

### 3.3.2 Challenges and Limitations of Modified SET

SET can provide reliable results in short intervals (*e.g.* 10 sec) but it has its own limitations. These limitations, challenges and the their relevant solutions are addressed and sorted based on the steps of the SET block diagram (Fig. 3.3). The first challenge in this method lies in selection of an appropriate input signal which is the 10 second span of each data. Each input signal should match the following criteria:

- The length of each span must be 10 seconds
- Maximum peaks should not be at the beginning or the end of the selected span
- Level of noise and other unnecessary components in each span must match the rest otherwise two options are recommended for the highly noisy signals : a) further pre-processing b) selecting another span with the fair level of noise

The second challenge of SET is the filtering part which is not unique since the cut-off frequency may vary for each input signal. In other words, this method is not adaptive therefore some of the employed parameters such as cut-off frequency need to be changed for each distinct data set which justifies the different defined filtering cut-off frequencies for each data set in step 2. Developing an adaptive filtering method has the potential of being a future contribution.

The last but not the least challenge is about the maximum peak extraction (step 5). If it is assumed that earlier steps specifically filtering step has been done appropriately, maximum peaks could be extracted with a proper thresholding which is theoretically ideal but practically it does not happen since the filtering part is quite challenging and sometimes it produces several maximum peaks with a very short time distance from each other ($\leq 1ms$). In this case, the maximum of the peaks or the average of the maximum peaks, if they have the same magnitude will be selected as the maximum peak.

## 3.4 Heart Rate Detection Using Empirical Mode Decomposition

EMD algorithm decomposes input signal into different modes of IMFs with separate spectral bands. This decomposition algorithm is based on successive removal of elemental signals which estimates the IMFs. Given any signal, the IMFs are found by an iterative procedure called a sifting algorithm [60], [48]. Parameter selection of the EMD algorithm is challenging. An inappropriate selection of input parameters can lead to a large number of IMFs (10-15) which makes their interpretation complicated.

In this study, EMD is applied to the SCG-Z axis and the IMFs corresponding to heartbeat are determined according to their peak frequencies. The IMF whose peak frequency is in the range of 1-10 Hz is considered as a heartbeat component. Then, we can reconstruct the signal of heartbeat by accumulating the respective components. Finally, heart rate is calculated by finding the time distance between the maximum peaks of the extracted heartbeat signal. Fig. 3.4 shows the block diagram of the applied EMD on ECG and SCG signals. In the next subsection, general formulation of EMD algorithm and heartbeat extraction will be described.

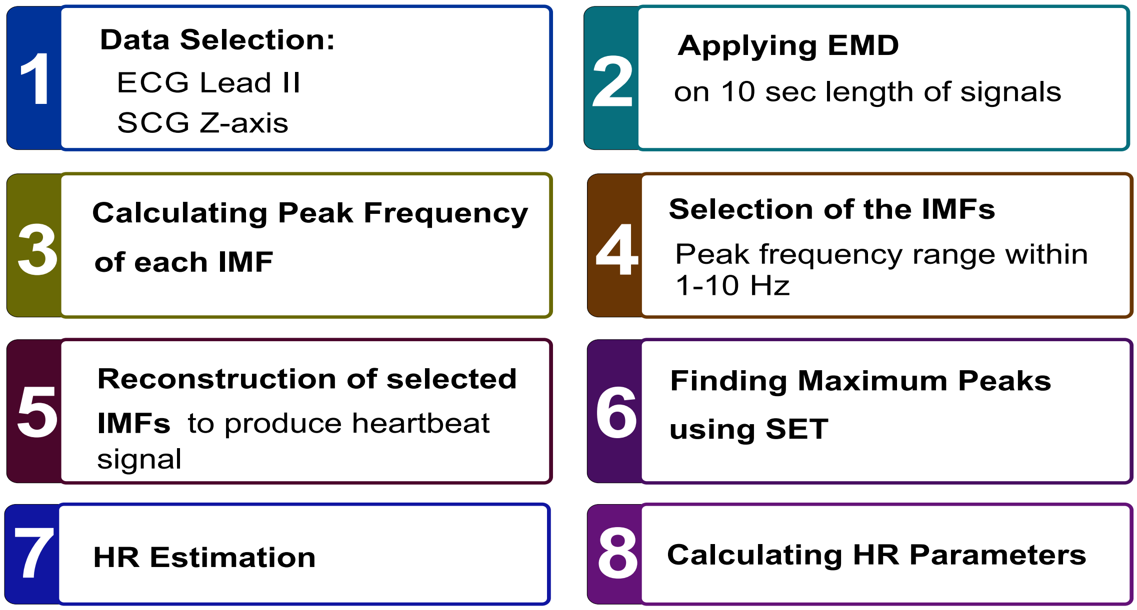


**Figure 3.4** Block diagram of applied EMD steps for HR detection

### 3.4.1 Heartbeat Extraction Procedure

After decomposing the input signal $f(t)$ into several IMFs, the peak frequency of each IMF was obtained. As mentioned previously, the IMFs whose peak frequencies are in the range of $1-10$ Hz, were determined as a component of heartbeat. After selecting the corresponding IMFs, the heartbeat $h(t)$ signal was reconstructed as (3.4)

$$h(t) = \sum_{k} f_k \qquad (PF_k \in [1.0, 10]Hz) \tag{3.4}$$

where $PF_k$ denotes the $k^{\text{th}}$ element of the peak frequency parameter.

### 3.4.2 Challenges and Limitations of EMD

The main issue with the EMD approach is its lack of a rigorous theoretical framework, even though its adaptability seems useful for many applications (*e.g.* signal spectrum analysis). Even though the EMD algorithm is highly adaptable and is able to extract the non-stationary part of the original function, it can only address some specific problems. For example when the signal is noisy, the information provided of the frequency spectrum of the decomposed IMFs are not reliable.

One of the challenges regarding the data selection (step 1) of the EMD block diagram, is the length of the input signal. Since EMD has high computational complexity, input signals with long length cause an issue by increasing the processing time. Therefore, 10 second spans were selected as the input signal of SET. Also the noise level of the input signal matters as if the signal to noise ratio (SNR) is less than 5dB the output result is not reliable. To address this concern, noisy data spans were eliminated from the input signals.

In the following, the steps of the EMD algorithm has been described.

### 3.4.3 EMD General Formulation

Input signal ($f(t)$) is decomposed using EMD through a sifting process which estimates IMFs [60]. Two conditions are considered in this sifting process: (a) the number of IMF extrema and zero-crossings must differ at most by one, (b) the mean value between the upper and lower envelops must be close to zero. EMD process [79] involves the following steps:

1. Finding all the local maxima, $M_i, i = 1, 2, ...$ and minima, $m_k, k = 1, 2, ...;$.
2. Computing the corresponding interpolating (The interpolating function is a cubic spline [80]) signals $M(t) = f_M(M_i, t$ and $m(t) = f_m(m_k, t)$. These signals are the upper and lower envelopes of the input signal $f(t)$.
3. Let $e(t) = \frac{M(t)+m(t)}{2}$ which in the local mean value.
4. Subtracting $e(t)$ from the signal: $f(t) = f(t) - e(t)$.
5. Returning to step (1)—stop when $f(t)$ remains nearly unchanged.
6. Once obtaining an IMF,$f_k(t)$, removing it from the signal $f(t) = f(t) - f_k(t)$ and return to (1) if $f(t)$ has more than one extremum (neither a constant nor a trend).

The sifting process decomposes $f(t)$ into locally orthogonal modes that are zero-mean oscillatory components. After the EMD process, $f(t)$ can be expressed as

$$f(t) = \sum_{k=1}^{n} f_k(t) \tag{3.5}$$

## 3.5 Heart Rate Detection Using Empirical Wavelet Transform

EWT is another signal decomposition method in which each mode is extracted using an appropriate wavelet filter bank [81]. In fact, EWT builds a family of wavelets adapted to the processed signal [48]. The decomposition process of EWT includes the segmentation the Fourier spectrum which is adaptive to the input signal. The spectrum of an IMF in EMD is signal dependent, *i.e.* of compact support and centered around a specific frequency. In EWT, also separate portions of the spectrum correspond to different modes *e.g* centered around a specific frequency and of compact support. In the following heart rate extraction using EWT has been described. Parameter selection of the EWT algorithm is challenging and is discussed in next section.

EWT was applied to SCG-Z axis signal and the corresponding modes to heartbeat were determined with the similar method was used in EMD and heart rate was obtained with the same process in EMD as well. Fig. 3.5 shows the block diagram of the applied EMD on ECG and SCG signals.

In general wavelet has the advantage that it does not require any predefined cutoff frequency for detection and it provides reliable results while being applied on symmetric (non-dynamical) signals. Most of the methods including wavelet based approach are non-adaptive but EWT is the only *fully adaptive wavelet transform* which is highly sensitive to noise and its results are reliable.

EMD automatically estimates the number of modes while this number can be changed for a better segmentation in EWT. EWT gives a more consistent decomposition while, generally, the EMD generates too many modes which are sometimes difficult to interpret. Another advantage of EWT compared to EMD is that EWT

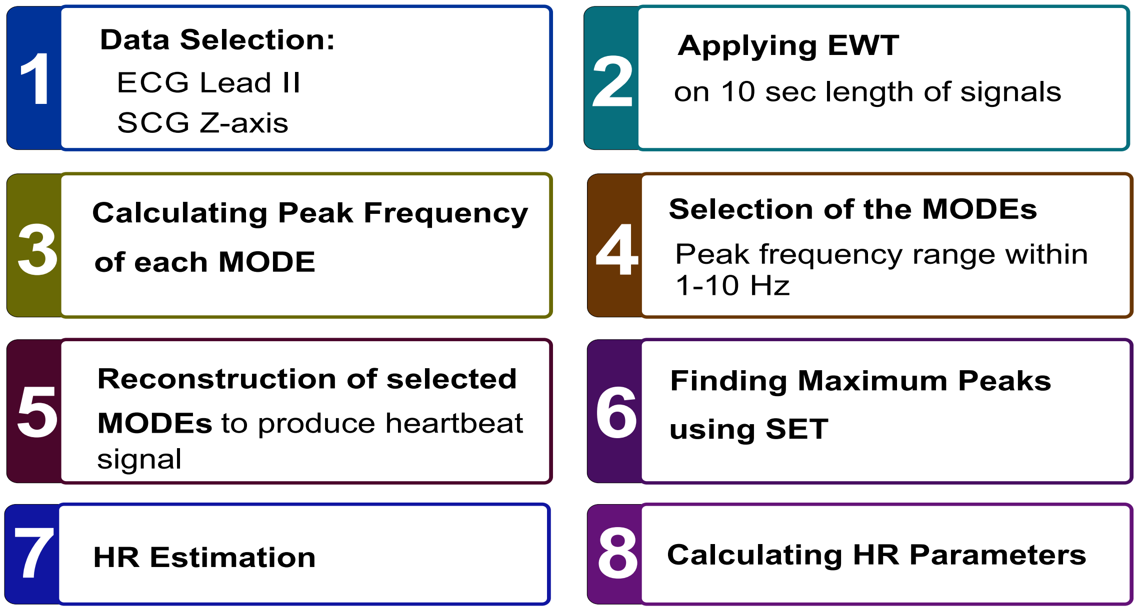


**Figure 3.5** Block diagram of EWT steps

can be adapted to classic wavelet formalism for a better understanding of its process. The mathematical formulation of EWT has been expanded in appendix.

### 3.5.1 Challenges and Limitations

Unlike EMD that its interpolating[2] function (cubic spline[3]) is fixed, EWT presents multiple functions not only for interpolation but also for detecting the local maxima/minima. There is no doubt that the mentioned feature highlights the advantage of EWT but on the other hand, it makes it more challenging as well. The parameter selection phase of EWT is manually in the software (MATLAB). In order to approach an appropriate parameter selection the following criteria are applied:

- Resolution
- Processing time

[2]Interpolation is the process of defining a function that takes on specified values at specified points.

[3]Cubic spline interpolation is a form of interpolation where the interpolant is a special type of polynomial called a spline

There is a trade-off between the resolution and the processing time of the output results.

### 3.5.2 EWT General Formulation

The process of estimating the number of bands in EWT is as the following:

- By assuming the number of segments ($N$) is given total number of $N+1$ boundaries are needed (0 and $\pi$ are the defined boundaries so basically $N+1$ boundaries required to be found)
- To find such boundaries, the local maxima in the spectrum is detected and sorted in decreasing order (0 and are $\pi$ excluded)
- After finding the local maxima ($M$) two cases can appear:
  - $M \geq N$: the algorithm found enough maxima to define the wanted number of segments, then only the first $N-1$ maxima are kept.
  - $M < N$ the signal has less modes than expected, all the detected maxima are kept and $N$ needs to be reset to the appropriate value.

Having maxima (plus 0 and $\pi$), the boundaries of each segment $\omega_n$ is defined as the center between two consecutive maxima. The next step after segmentation is building a tight frame set of empirical wavelets. The following proposition indicates the obtaining of a tight frame.

**Proposition** : If $\gamma < min_n(\frac{\omega_n+1-\omega_n}{\omega_n+1+\omega_n})$, then the set $\{\phi_1(t)\}, \{\varphi_n(t)\}_{n=1}^{N}$ is a tight frame of $L^2(\mathbf{R})$.

$\gamma$ is the ration of $\frac{\tau_n}{\omega_n}$ in which $\tau_n = \frac{T_n}{2}$. $T_n$ is the transition phase which is equal to $\frac{2\pi}{\omega_n}$.

## 3.6 Advantages and Disadvantages of all three Methods

SET provides reliable information in time-domain such as heart rate estimation. It is not adaptive but it can be used as the peak detector. As mentioned in Chapter 2, SET has been used for heart rate estimation using both ECG and SCG signals by other research groups. In this research, SET was mainly used as a ground truth (applied SET to ECG result) and also as a peak detector method.

Both EMD and EWT are non-linear adaptive signal processing methods which suit the non-stationary signals such as heartbeat. Even though the estimated heart rate results using EMD and EWT in terms of accuracy is not significantly better compared to the other relevant results but these algorithms provide an effective way of analyzing the instantaneous frequency of signals. These features highlight the significant difference between the proposed HR detection methods in this research compared to the previous presented HR detection methods. On the other hand, unlike other methods EMD and EWT provides the heartbeat signal.

Another disadvantage of adaptive methods of EMD and EWT is their computational complexity which limits the utilized length of the input signal. The computational complexities of both EMD and EWT are discussed in the following.

### 3.6.1 Computational Complexity

The system configuration that was used for the implementation of EMD and EWT is as below:

- Processor Speed: 1.4 GHz
- Number of Processors: 1
- Total Number of Cores: 2
- System Version: OS X 10.9.5
- Utilized software : MATLAB

Table 3.1 compares the approximate processing time of EMD and EWT methods. The

| **Number of input data points** | **EMD** | **EWT** |
|---|---|---|
| 10000 | $2-5$ min | 1 min |
| 50000 | $10-30$ min | $1-2$ min |
| 60000 | $15-40$ min | $1-3$ min |
| 300000 | $1-3$ hours | $2-10$ min |

**Table 3.1** Approximate processing time of both EMD and EWT

factors that affect the processing time of EMD and EWT include: number of input data points, number of operations in each stage, number of iterations. Since EMD produces higher number of modes compared to EWT, the number of its iterations is higher which makes its processing time longer.

## 3.7 Arrhythmia Detection Using Power Spectral Density

The abnormality in which heart has an irregular rhythm is called arrhythmia. Cardiac arrhythmia is categorized into different types including ventricular tachycardia, atrial/ventricular fibrillation, atrial arrhythmia and sinus tachycardia. In this study, types of cardiac arrhythmia that occurs with an irregular beat such as ventricular tachycardia and ventricular/atrial fibrillation are the major focus of the proposed arrhythmia detection method. There are a lot of publications [59,75,76] regarding the both detection and classification of different types of arrhythmia. Most of these research works estimated the Power Spectral Density (PSD) using different methods for arrhythmia detection. PSD describes the signal power distribution over the frequency. Computation of PSD is done directly by the method called FFT (periodogram) or computing autocorrelation function and then transforming it (correlogram). PSD is sensitive to the frequency oscillation of the input signal which makes it a useful tool for detecting sudden changes such as irregular beat in frequency spectrum. In other words, PSD tells us at which frequency ranges variations are strong and that might be quite useful for further analysis.

Since the EWT method provides instantaneous frequency information of input signals, calculating the PSD of each mode will be able to detect the sudden changes in frequency domain. Therefore, EWT was applied to the obtained data from Physionet and the PSD of each band then estimated for the further analysis. Fig. 4.25 illustrates the steps of the proposed arrhythmia detection method. The method which was used

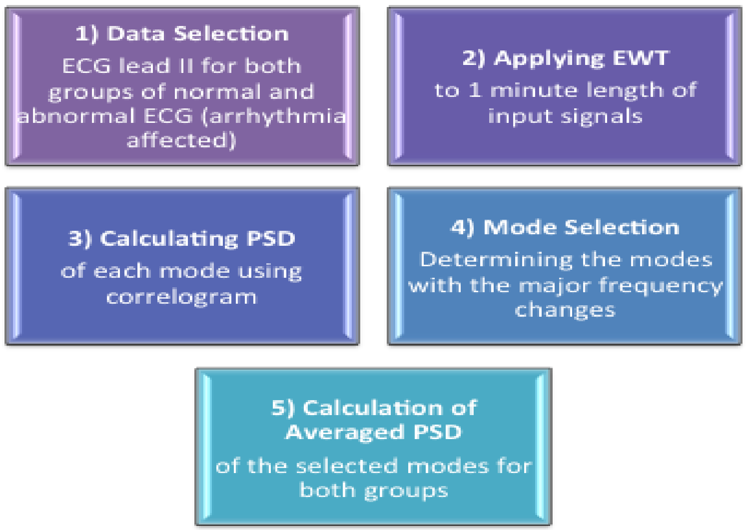


**Figure 3.6** Block diagram of arrhythmia detection method steps

for PSD calculation is called *correlogram*. A correlogram is an image of correlation statistics. In time series analysis, a correlogram is also known as an autocorrelation plot. Mathematical formulation of both PSD and correlogram has been given in the appendix.

### 3.7.1 Challenges and Limitations

Even though correlegram has quite low computational complexity as a PSD estimation method, it has its own limitations. A windowing technique is used for the estimation of PSD in correlogram. There is a trade-off between spectral resolution and variance of the spectral estimates for most windowing techniques which means low variance implies loss of resolution and high resolution implies high variance. Therefore to achieve a more accurate spectral estimate, longer length of the input signal is required.

Another limitation of correlogram method is the noise level of the input signal. If the input signal is a highly noisy, correlogram can not provide an accurate and reliable spectral estimate.

In this research study, in order to overcome these challenges, two solutions are proposed. The first solution is using long-length input signals (1 minute span) to provide more accurate results. To address the second limitation, for decomposition of the input signal, EWT is used which provides reliable results while the input signal is noisy. However the proposed method in this study is able to detect the arrhythmia affected ECG signal , it is in its preliminary stage. Classification of different types of arrhythmia using a better PSD estimation method is one of the future contributions.

# 4. Results

In the previous Chapter, we discussed the proposed methods in this research including their advantages and limitations. In this Chapter, we will present the results of each performed method on ECG and SCG signals. As mentioned in Chapter 3 we have two data sets for the first objective in which both ECG Lead II and SCG Z-axis have been recorded simultaneously. We didn't have access to raw data only the pre-processed data. Results obtained by applied SET to ECG signal is our ground truth that has been used for the evaluation of other results. In the following the obtained results of each method will be discussed individually.

## 4.1 Heart Rate Detection Results by Modified SET

ECG and SCG signals of both data sets have been shown in Fig. 4.1 and Fig. 4.2. SET or signal energy thresholding has 3 steps of filtering, signal energy calculating and thresholding the signal energy. In data set 1, ECG and SCG were first filtered with 4th order high pass Butterworth with the cutoff frequencies of 10Hz and 8Hz respectively. Then they were filtered with 4th order low pass Butterworth with the cut-off frequencies of 20Hz and 18Hz respectively. In the next step, energy of the filtered signals were calculated. Last step was threholding both signals energies for peak extraction. Having the peak-to-peak time distance ,the heart rate can be estimated. Fig. 4.3 and Fig. 4.4 show the HR graph related to each data set (1 subject from each data set was selected).

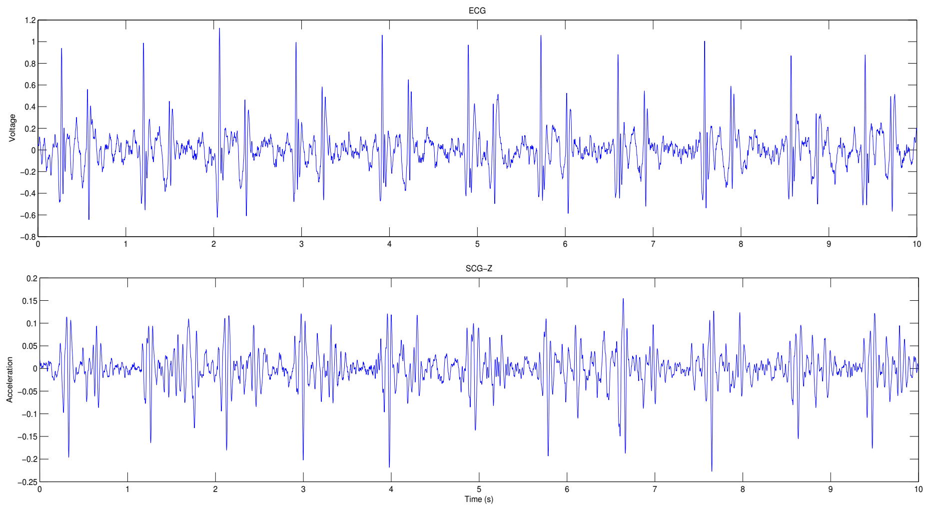


**Figure 4.1** ECG and SCG-Zaxis of subject11, data set 1 with the sample frequency of 1000 Hz

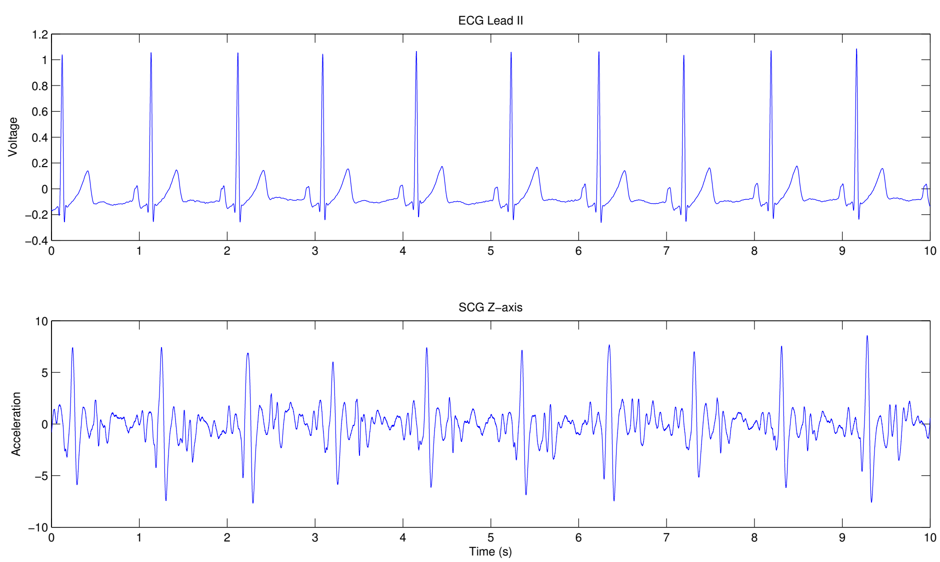


**Figure 4.2** ECG and SCG-Zaxis of subject5, data set 2 with the sample frequency of 5000 Hz

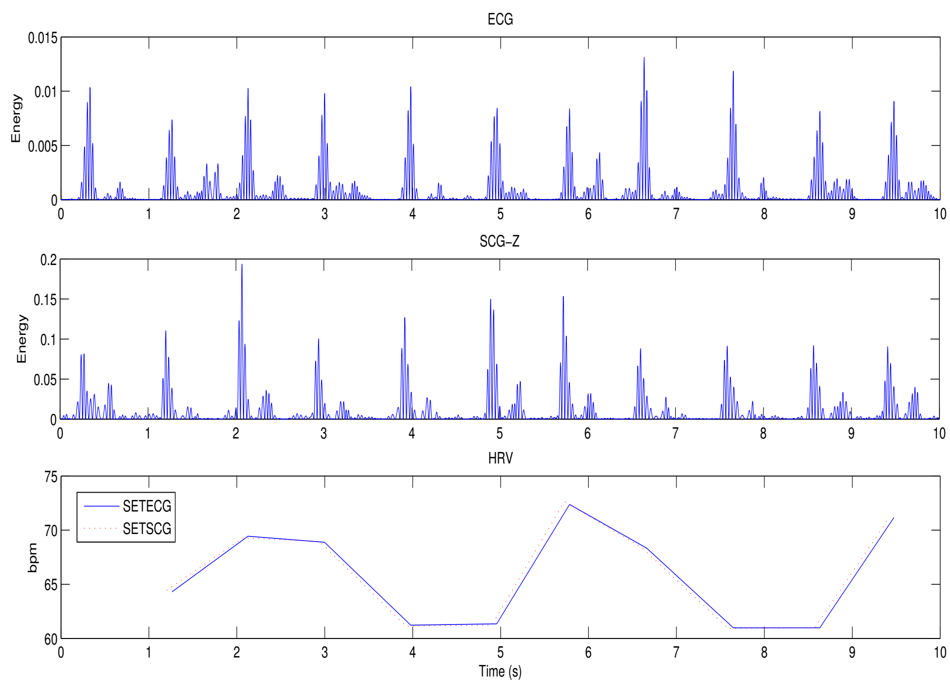


**Figure 4.3** 1st and 2nd graph from above presents energy signals of ECG and SCG-Z. Last graph shows the heart rate estimated using SETECG (solid blue) and SETSCG (dashed red) methods

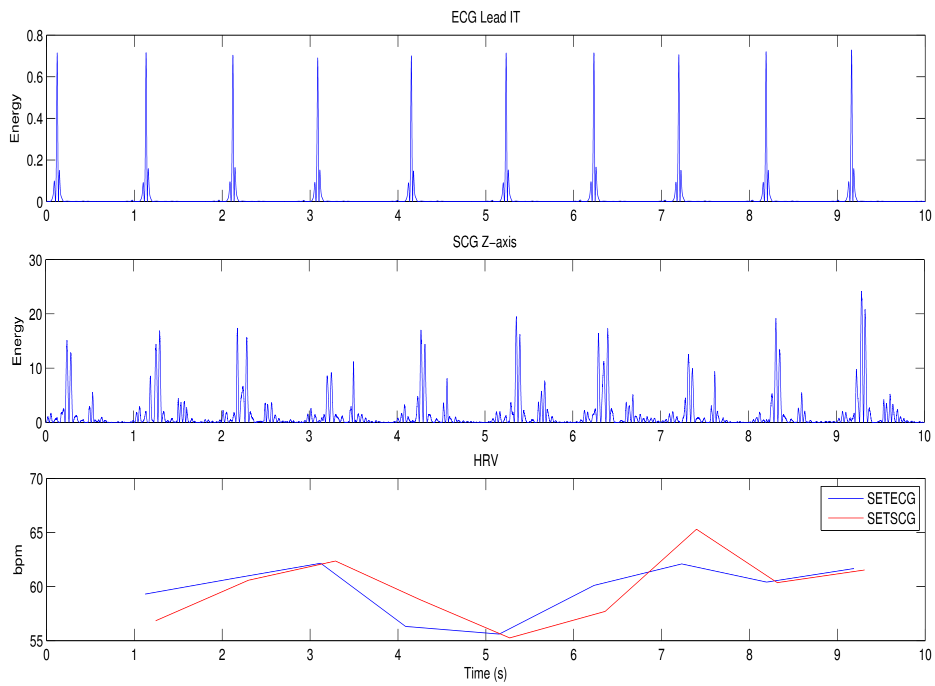


**Figure 4.4** Heart rate estimation using SETECG (solid blue) and SETSCG (solid red) methods

According to Fig. 4.3 and Fig. 4.4, estimated heart rate graph of SETSCG tracks the estimated heart rate graph of SETECG (ground truth). The existing delay between the heart rate graphs is due to the difference in ECG and SCG peaks location.

## 4.2 Heart Rate Detection Results by EMD

EMD is an adaptive method which decomposes a signal into different modes (with different frequency bands). It was applied to SCG signals of data set 1 and decomposed most of them into 12 modes. Fig. 4.5 shows different modes of a SCG-Zaxis signal whose peak frequencies were obtained. Then, IMF5, IMF6 and IMF7 with peak frequencies between the range of 1 to 10 Hz were selected as heartbeat components. Fig. 4.6 show the power spectra of the selected IMFs. Using these IMFs, the heartbeat signal was constructed as shown in Fig. 4.7.

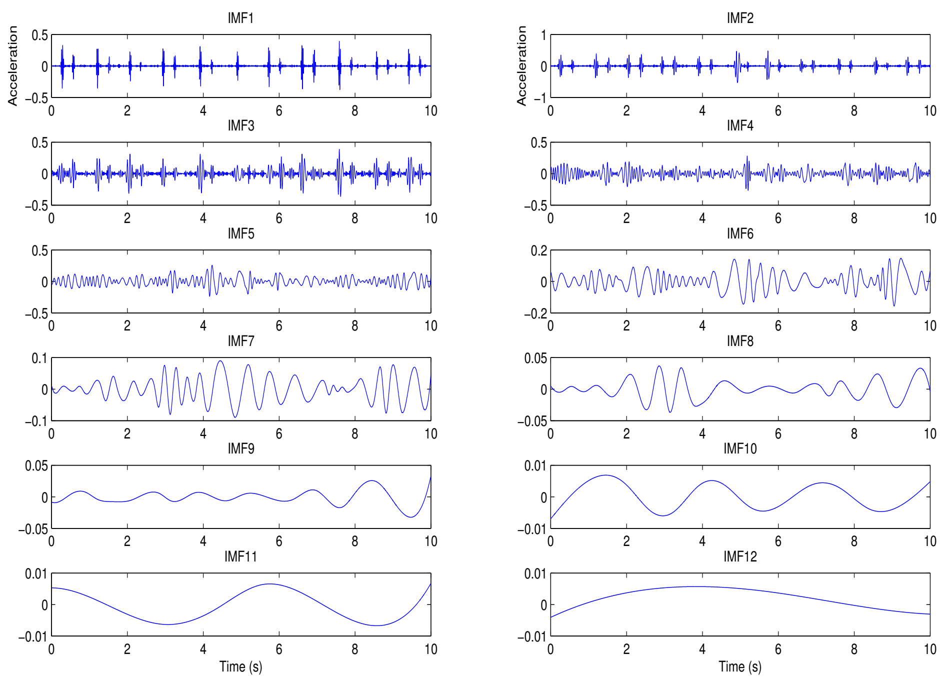


**Figure 4.5** EMD decomposed SCG-Zaxis into 12 modes. Last 4 IMFs are trends and first 4 IMFs are noise.

The next step is to estimate the heart rate from the heartbeat signal which was done using SET as a peak detection method. The heartbeat signal was filtered using

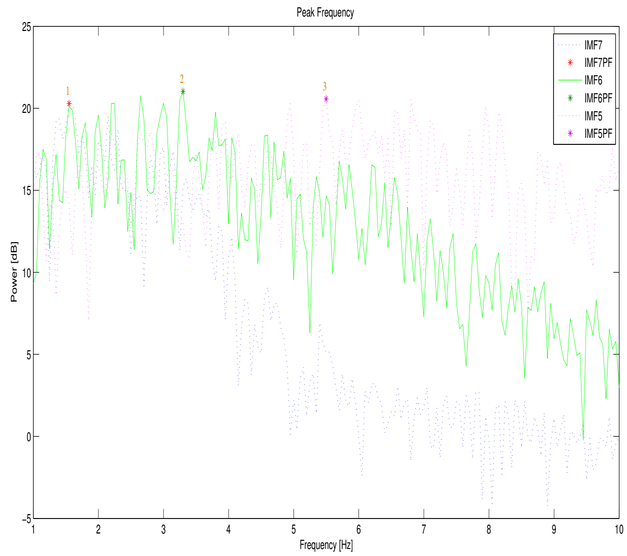


**Figure 4.6** Power Spectra of the IMFs corresponding to heartbeat

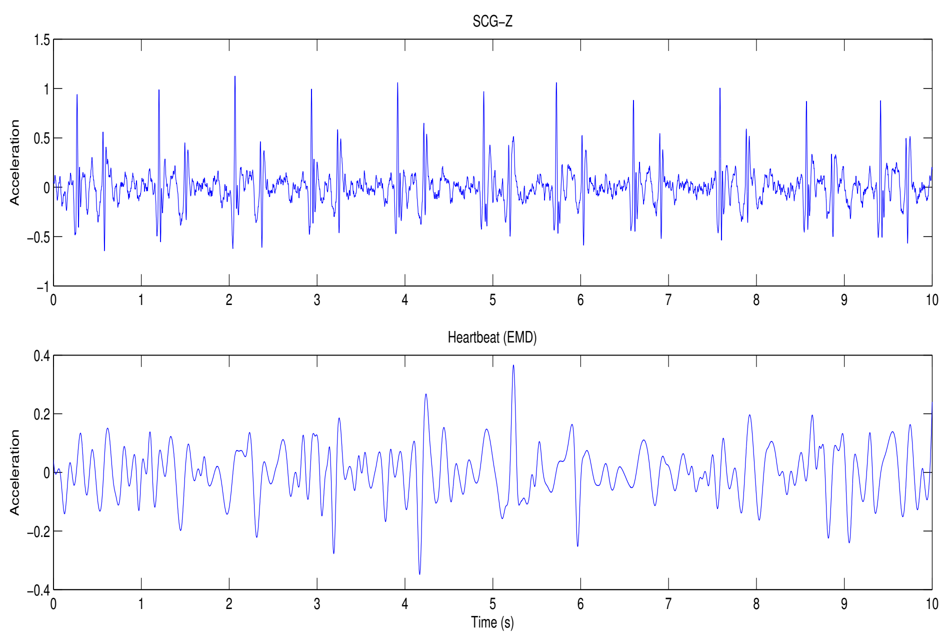


**Figure 4.7** Heartbeat signal constructed using EMD

4th order high pass and low pass Butterworth filters with the cut-off frequencies of 2 Hz and 10 Hz respectively. Then, energy of the filtered signal was calculated and thresholded. Fig. 4.8 shows the HR graph estimation using heartbeat signal.

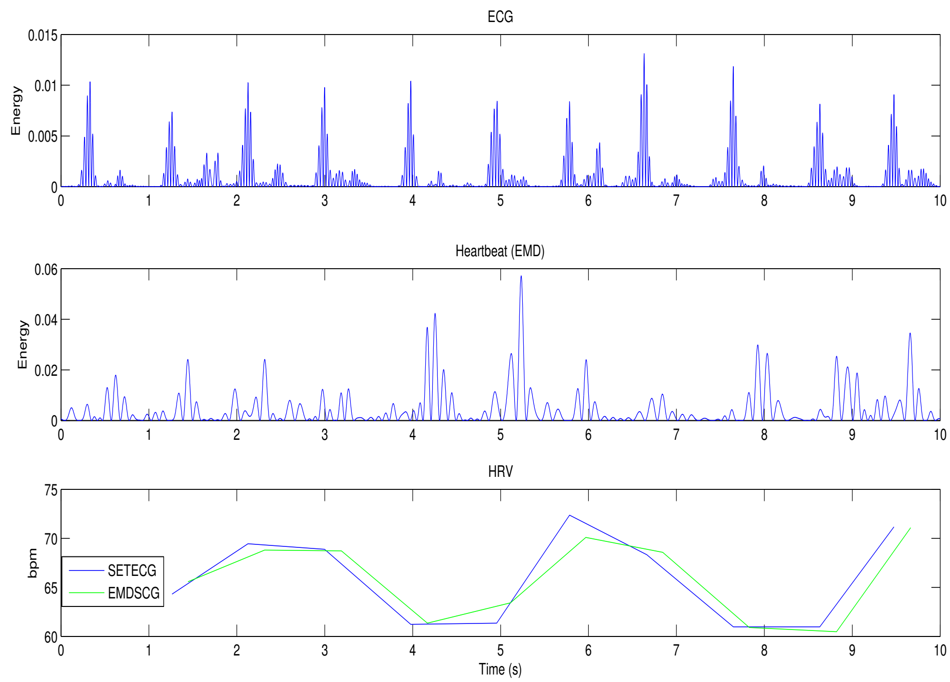


**Figure 4.8** Heart rate estimated using SETECG (solid blue) and EMDSCG (solid green)

According to Fig. 4.3 and Fig. 4.4, estimated heart rate graph of EMDSCG tracks the estimated heart rate graph of SETECG (ground truth) with a delay due to the difference in ECG and heartbeat signal peaks location.

## 4.3 Heart Rate Detection Results by EWT

Like EMD, EWT is also an adaptive method which decomposes a signal into different modes. It was applied to SCG signals of both data sets. Fig. 4.9 and Fig. 4.10 show the modes of applied EWT to the SCG-Zaxis signals of a subject for each data set. EWT decomposed these signals into 5 and 9 modes respectively (data set 1 and data set 2). Peak frequency of the signals were then obtained and Mode (2-4) from data set 1 and Mode (2-8) from data set 2 with peak frequencies between the range of 1 to 10 Hz were selected as heartbeat components. Fig. 4.11 and Fig. 4.12 respectively show the power spectra of the selected IMFs for each data set. Constructed heartbeat signals corresponding each data set are as shown in Fig. 4.13 and Fig. 4.14.

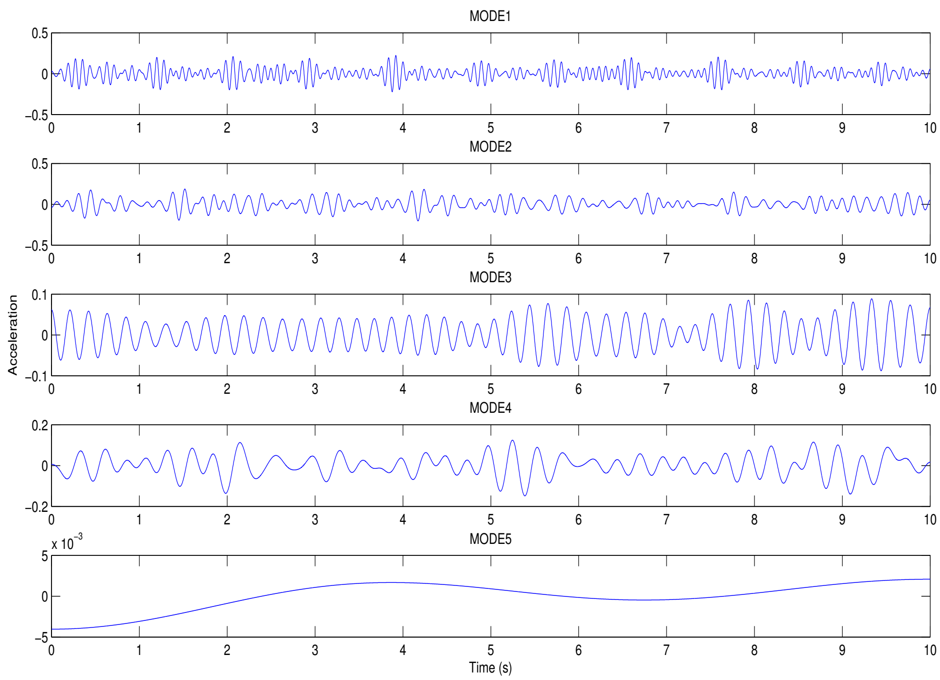


**Figure 4.9** EWT Modes of a subject from data set 1

Then SET was applied to the heartbeat signals obtained from both data sets. Heartbeat signals were filtered using 4th order high pass and low pass Butterworth filters with the cut-off frequencies of 2 Hz and 10 Hz respectively. Fig. 4.15 and Fig. 4.16 show the HR graph estimated from heartbeat signals. Fig. 4.17 and Fig. 4.18 illustrate the HR graph using all the proposed methods. Based on the current results in this study, EWTSCG method gives the more similar heart rate graph to SETECG compared to SETSCG methods. In fact, the estimated heart rate graph of EWTSCG tracks the estimated heart rate of SETECG with a delay which is due to the ECG and SCG signals peaks locations as mentioned in previous sections.

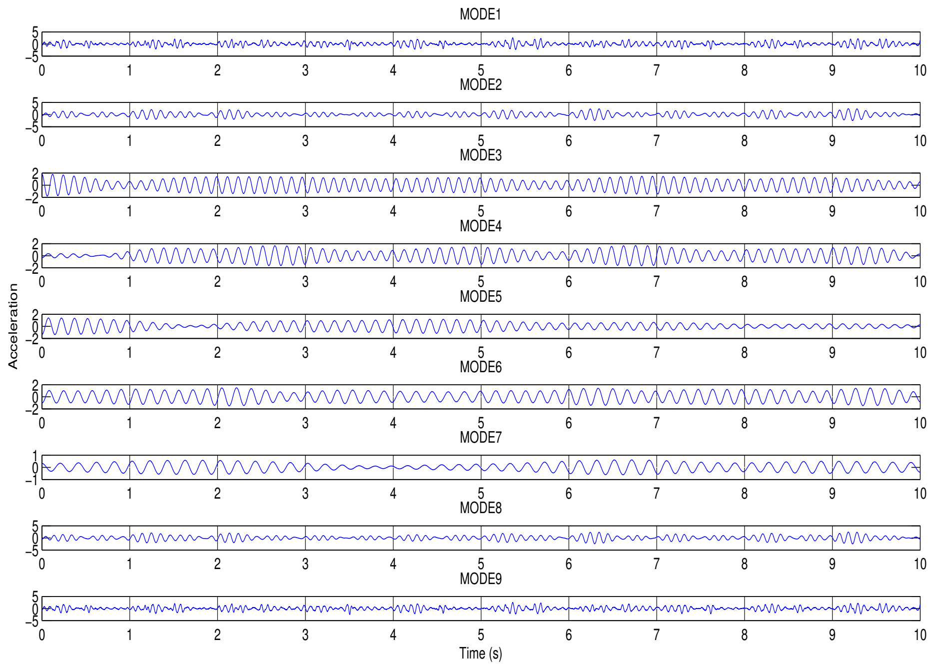


**Figure 4.10** EWT Modes of a subject from data set 2

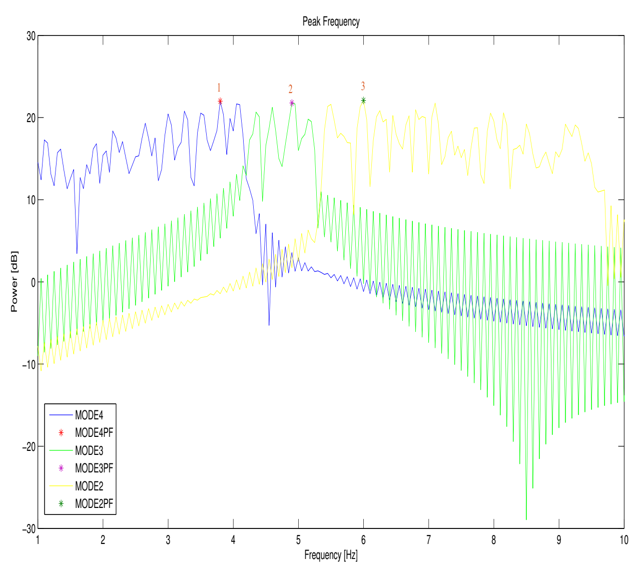


**Figure 4.11** Power Spectra of the Modes corresponding to heartbeat (datase1)

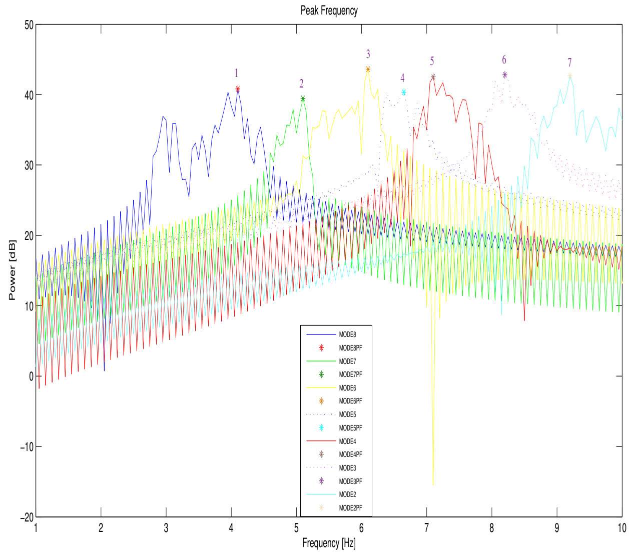


**Figure 4.12** Power Spectra of the Modes corresponding to heartbeat (datase2)

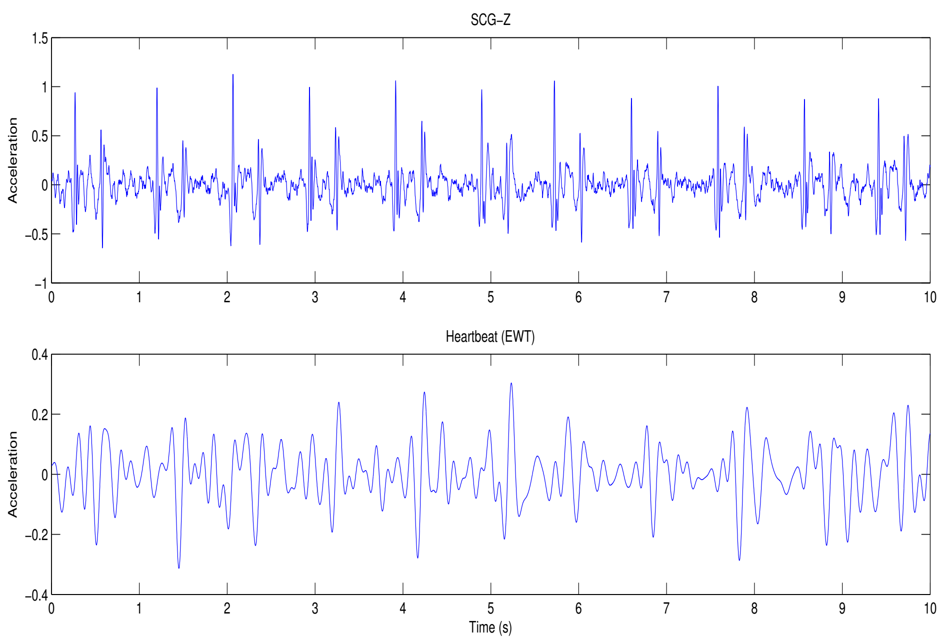


**Figure 4.13** Heartbeat signal constructed using EWT (data set 1)

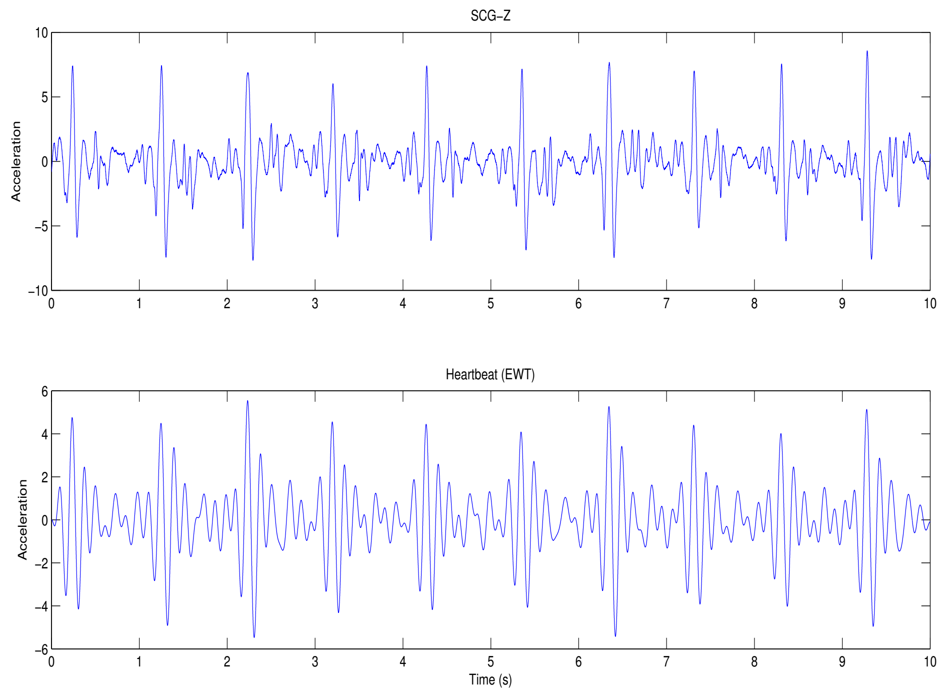


**Figure 4.14** Heartbeat signal constructed using EWT (data set 2)

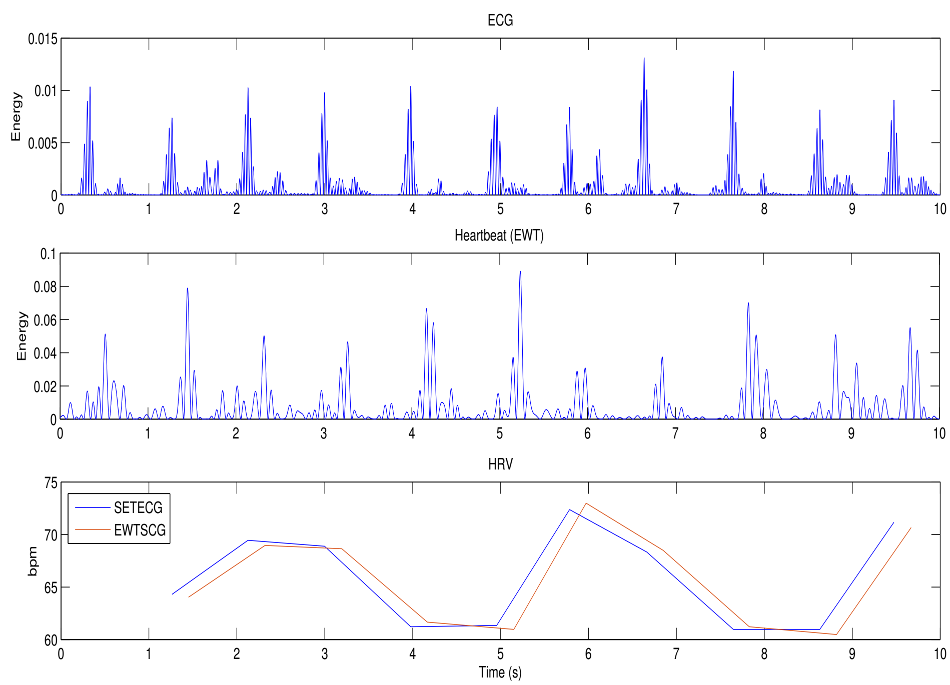


**Figure 4.15** Heart rate estimated using SETECG (solid blue) and EWTSCG (solid brown) (data set 1)

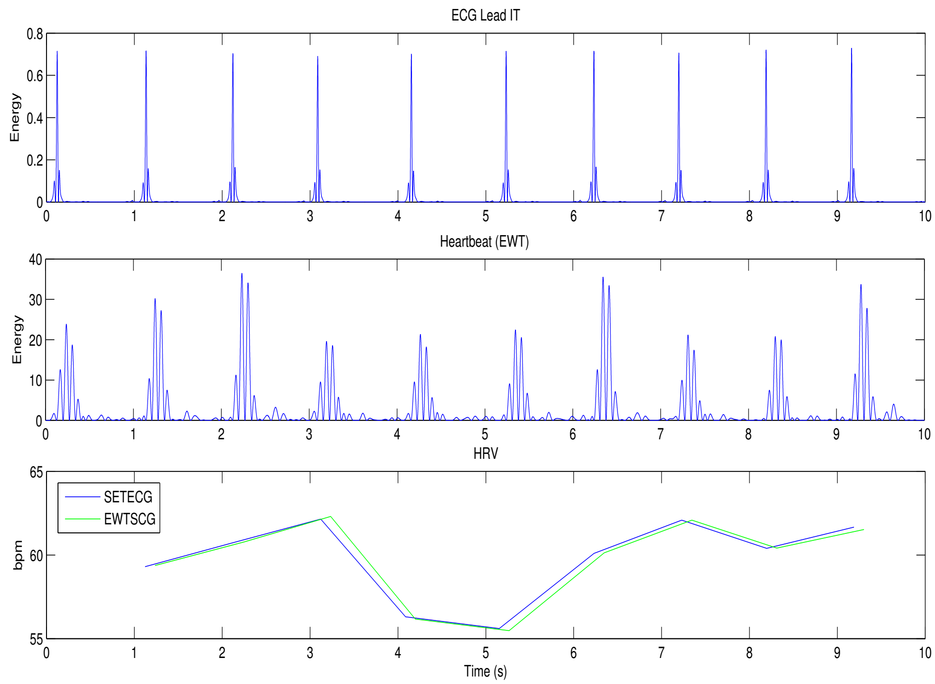


**Figure 4.16** Heart rate estimated using SETECG (solid blue) and EMDSCG (solid green) (data set 2)

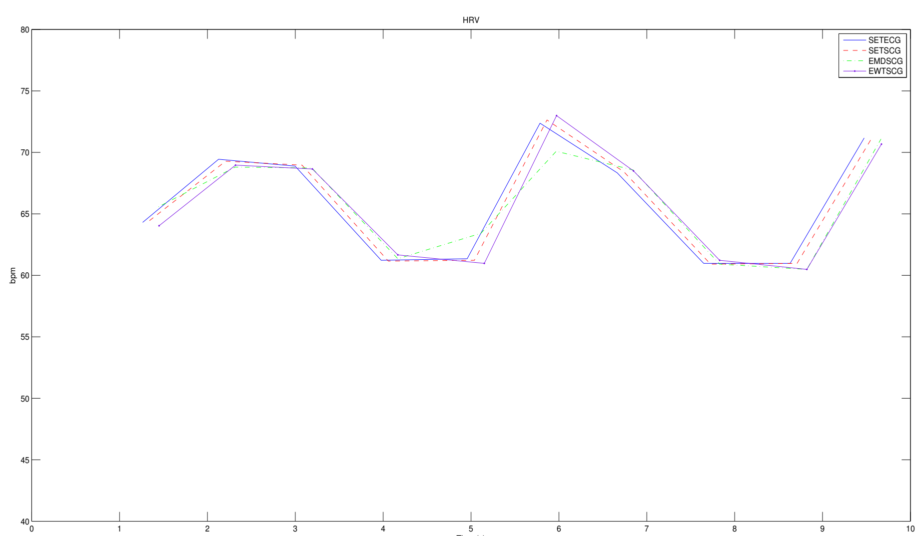


**Figure 4.17** Heart rate estimated using SETECG (solid blue), SETSCG (dashed red), EMDSCG (dashed green), EWTSCG (solid purpel) (data set 1)

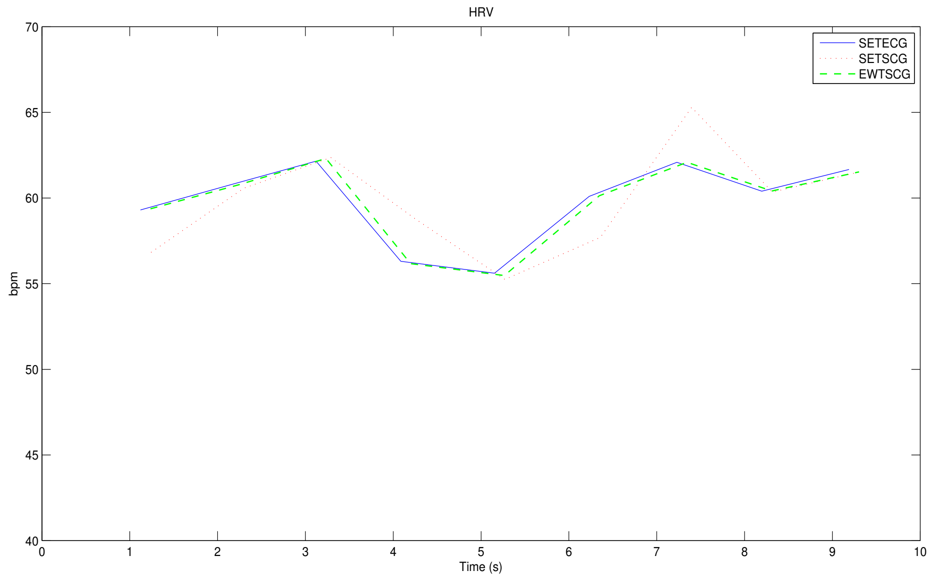


**Figure 4.18** Heart rate estimated using SETECG (solid blue), SETSCG (solid red), EWTSCG (dotted green) (datase2)

## 4.4 Heart Rate Parameters

In this section, time-domain HR parameters including SDANN, RMSSD, Mean (NN (interbeat interval)) , MEAN HR have been calculated for each method. Table. 4.1 and Table. 4.2 show HR time-domain parameters for data set 1 and data set 2 respectively.

| Time domain Parameters | SET ECG | SET SCG | EMD SCG | EWT SCG |
|---|---|---|---|---|
| | mean , std | mean , std | mean , std | mean , std |
| Mean NN (s) | 0.8170 , 0.0673 | 0.8176 , 0.0699 | 0.8129 , 0.0668 | 0.8135 , 0.0632 |
| Mean HR (bpm) | 74.1570 , 6.2605 | 74.0698 , 6.5644 | 74.5086 , 6.2889 | 74.3905 , 6.4656 |
| SDANN (ms) | 46.0328 , 23.0917 | 46.7712 , 15.5057 | 47.16550 , 14.5983 | 45.1962 , 18.6880 |
| RMSSD (ms) | 59.0150 , 28.0917 | 51.1320 , 18.9177 | 52.4303 , 18.9177 | 57.9653 , 14.7807 |

**Table 4.1** Time-domain parameters of HR for data set 1

In table 4.1 the estimated amounts of Mean NN and Mean HR parameters by all 3 methods of SET SCG, EMD SCG and EWT SCG have a high correlation with the estimated amounts of these parameters by the ground truth method of SET ECG.

Also estimated SDANN parameter by EWT SCG has a higher correlation with the estimated SDANN by SET ECG compared to EMD SCG.

| Time domain Parameters | SET ECG | SET SCG | EWT SCG |
|---|---|---|---|
| | mean , std | mean , std | mean , std |
| Mean NN (s) | 0.9408 , 0.0396 | 0.9385 , 0.0397 | 0.9390 , 0.0401 |
| Mean HR (bpm) | 63,8992 , 2.5979 | 64.2827 , 2.608 | 64.1399 , 2.602 |
| SDANN (ms) | 23.3049 , 10.3488 | 25.7166 , 10.9261 | 22.9109 , 11.1249 |
| RMSSD (ms) | 29.632 , 10.1121 | 32.1320 , 14.8382 | 27.7509 , 13.9900 |

**Table 4.2** Time-domain parameters of HR for data set 2

In table 4.2 the estimated amounts of Mean NN and Mean HR parameters by all 2 methods of SET SCG and EWT SCG have a high correlation with the estimated amounts of these parameters by the ground truth method of SET ECG. Also estimated SDANN and RMSSD parameters by EWT SCG have a higher correlation with the estimated SDANN and RMSSD by SET ECG compared to SET SCG.

In conclusion, the proposed HR detection methods in this study provide promising results even in short-term recordings. Therefore they can be used for both short and long term heart rate monitoring.

## 4.5 Evaluation of All the Proposed Heart Rate Detection Methods

The average of interbeat intervals and heart rate for each method has been shown in Fig. 4.19, Fig. 4.20, Fig. 4.21 and Fig. 4.22. In the following, discussed methods were evaluated by comparing to our ground truth (SETECG). For this purpose, the average of interbeat intervals for each method was obtained compared to the interbeat interval average of the ground truth. The difference between the averaged interbeat interval of ground truth with other methods averaged interbeat interval is defined as interbeat interval error.

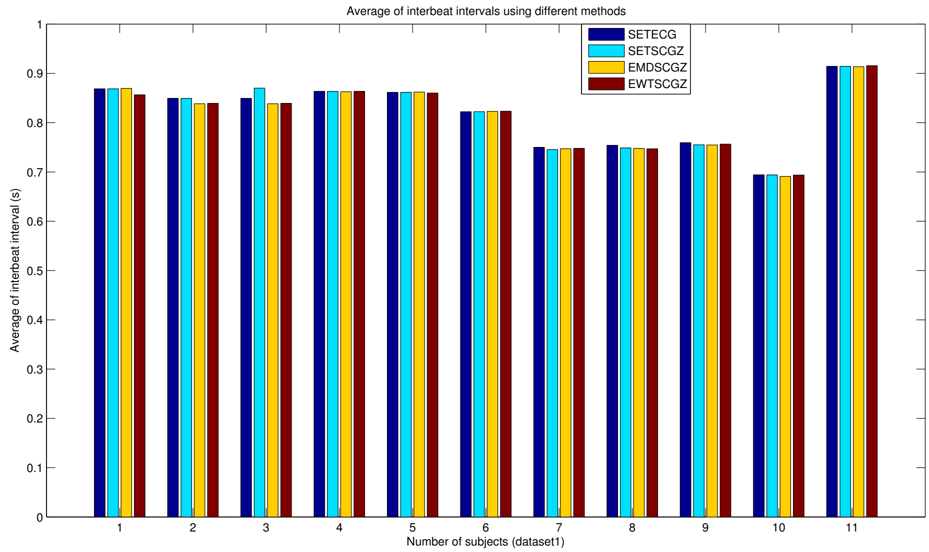


**Figure 4.19** Averaged interbeat intervals (data set 1)

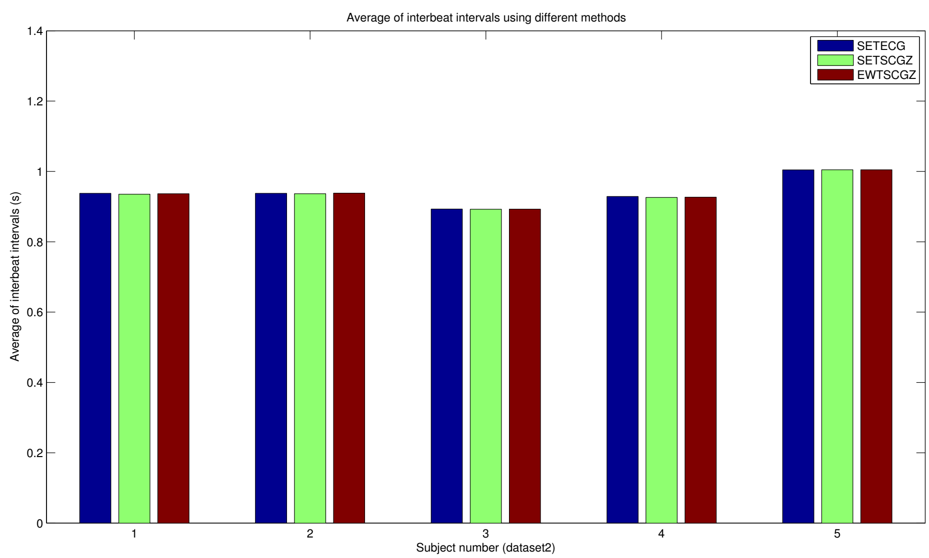


**Figure 4.20** Averaged interbeat intervals (data set 2)

Table. 4.3 provides us with the error information by presenting the mean and standard deviation of each method's error for both data sets. Fig. 4.23 and Fig. 4.24 show the bar graphs of interbeat interval error for both data sets. Based on the obtained results the average error of SET method is 12.5 ms and 17 ms, the average error of EWT is 16 ms and 9 ms for data set 1 and data set 2 respectively. The

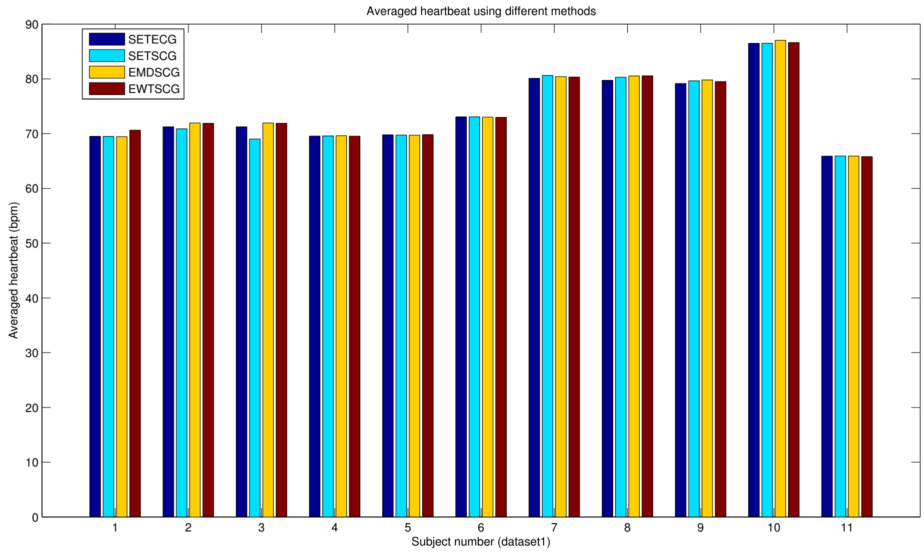


**Figure 4.21** Averaged heart rate (data set 1)

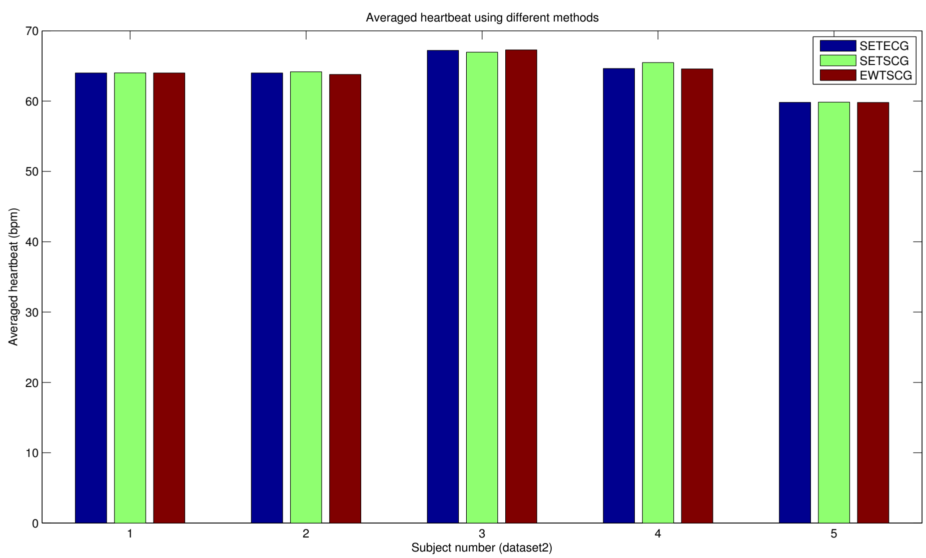


**Figure 4.22** Averaged heart rate (data set 2)

average error of EMD for data set 1 is 16 ms.

According to the relevant literature (other HR detection methods) range of inter-beat interval error is approximately 5 to18 ms [38, 44, 74].

| Heartbeat Interval Error (ms) | SET SCG | | EMD SCG | | EWT SCG | |
|---|---|---|---|---|---|---|
| **data set 1** | 9.0364 | 3.5308 | 11.3636 | 5.0393 | 11.0545 | 5.0014 |
| **data set 2** | 12.0516 | 5.1966 | - | - | 6, 7231 | 2.4238 |

**Table 4.3** Averaged error of all three methods

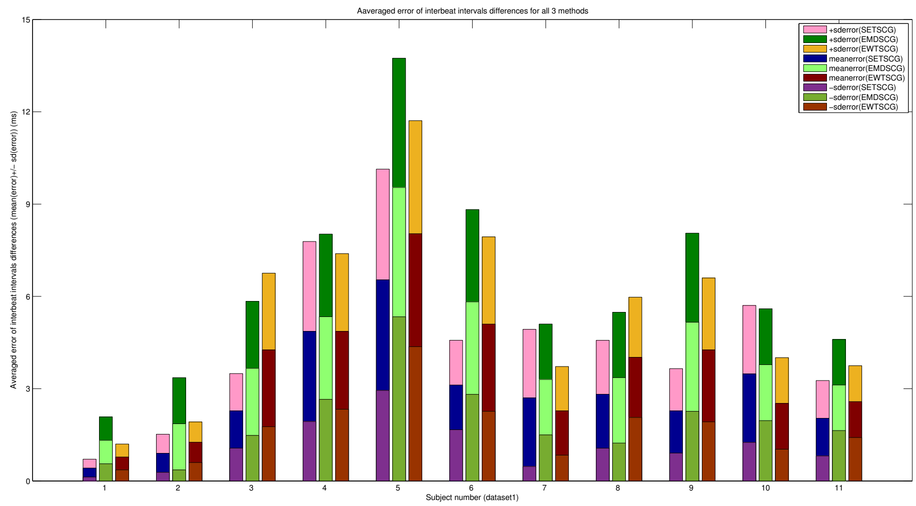


**Figure 4.23** Averaged error of interbeat interval differences (data set 1)

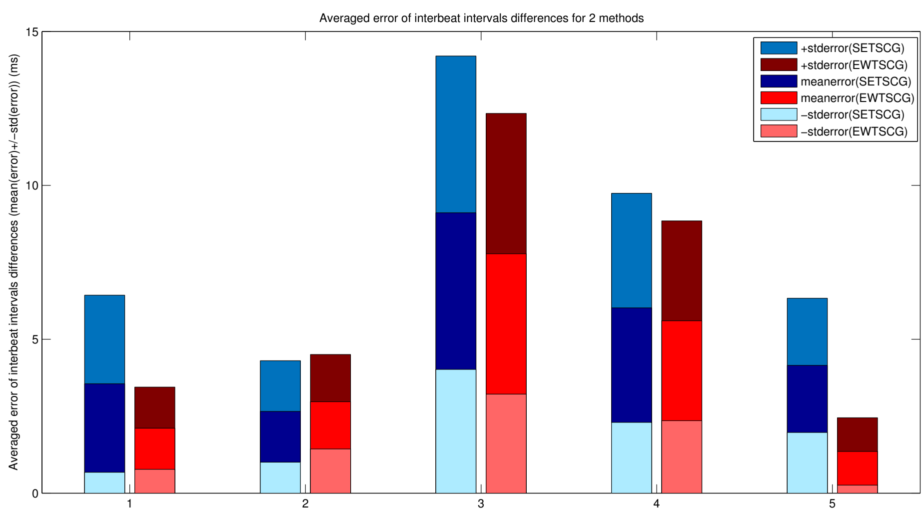


**Figure 4.24** Averaged error of interbeat interval differences (data set 2)

## 4.6 Power Spectrum of EWT Bands in Normal vs. Arrhythmia Affected ECG Signals

EWT, when applied to the ECG signal, decomposes it into different modes with different frequency components. Power Spectral Density (PSD) of each mode gives specific information about its frequency oscillation. PSD can be calculated using different methods. In this research, using *correlogram method* PSD of each input ECG signal and its modes were estimated. Input ECG signals are two categories of normal and arrhythmia affected signals. After estimation of PSD for each mode of ECG signal, the average of PSD is obtained for both categories. For this purpose, we obtained our data from PhysioNet [77]. 5 subjects for each group were selected for further analysis from the data set. Fig. 4.25 shows normal and abnormal (affected by arrhythmia) ECG signals.

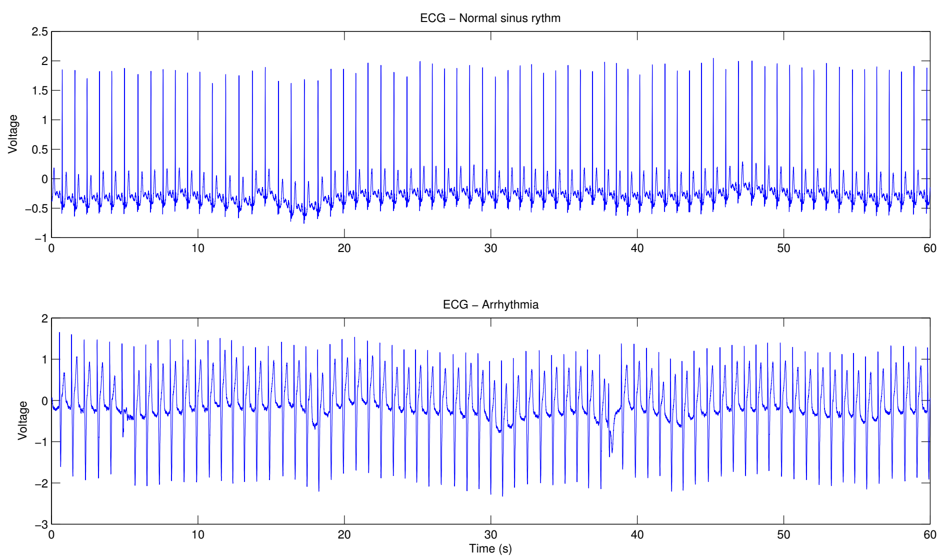


**Figure 4.25** Upper figure shows a normal ECG and the other figure shows affected ECG signal

Fig. 4.26 and Fig. 4.27 present the different modes of normal ECG signal and the arrhythmia one respectively. Fig. 4.28 and Fig. 4.29 show the estimated power spectral density of each mode in both normal and affected ECG signal.

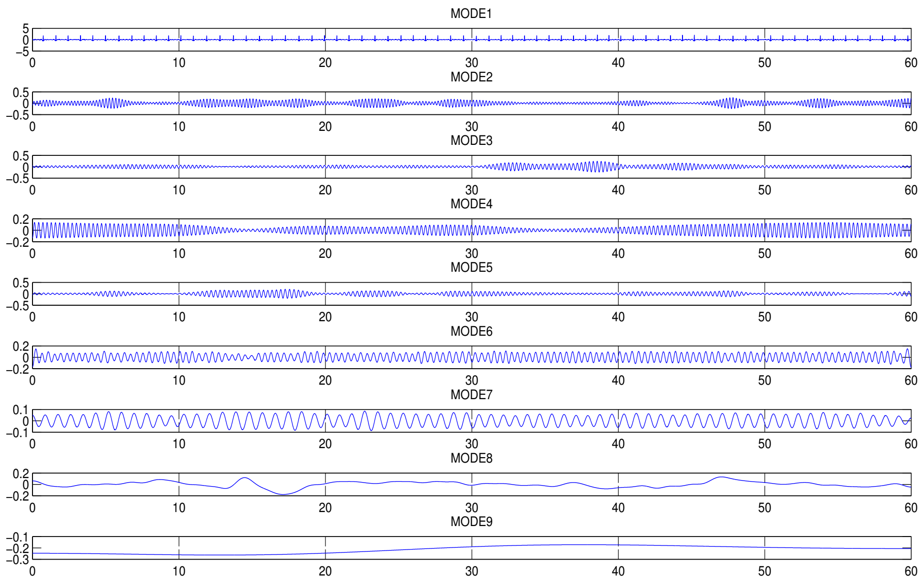


**Figure 4.26** EWT of a normal ECG signal which has 9 bands

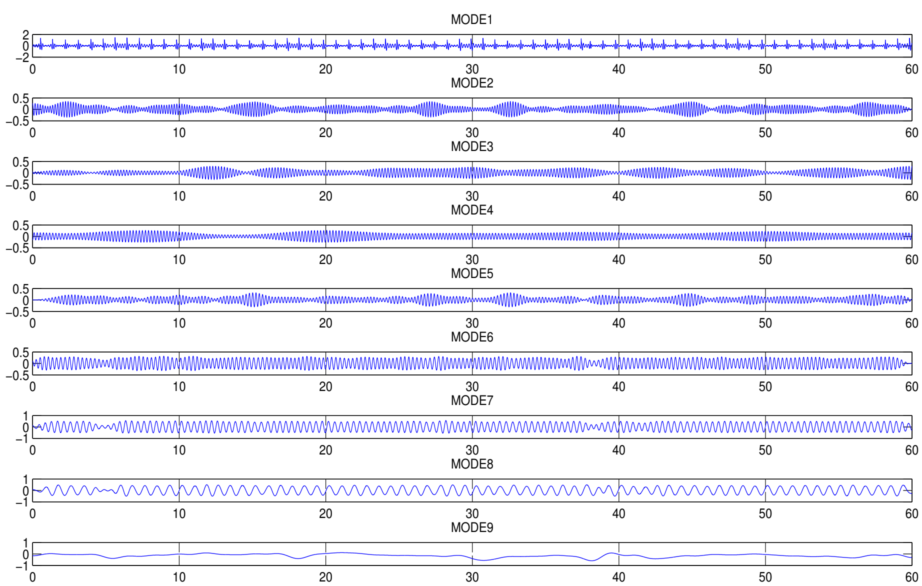


**Figure 4.27** EWT of affected ECG signal which has 9 bands

Based on the obtained results, the average PSD of a normal ECG signal is 20dB or less and the average PSD of an abnormal ECG signal affected by arrhythmia is at least 50dB. Therefore the power spectral density of abnormal ECG compared to the

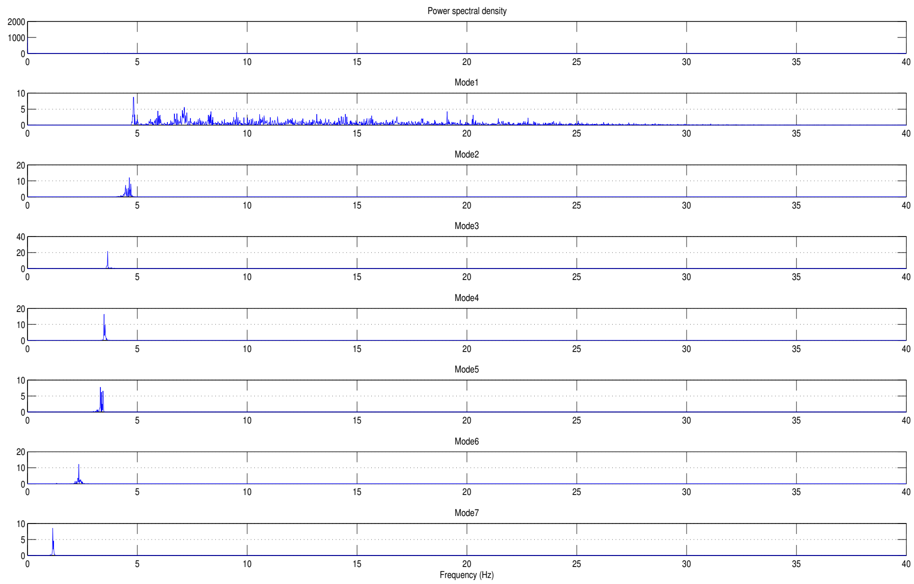


**Figure 4.28** PSD of normal ECG signal's modes using correlogram

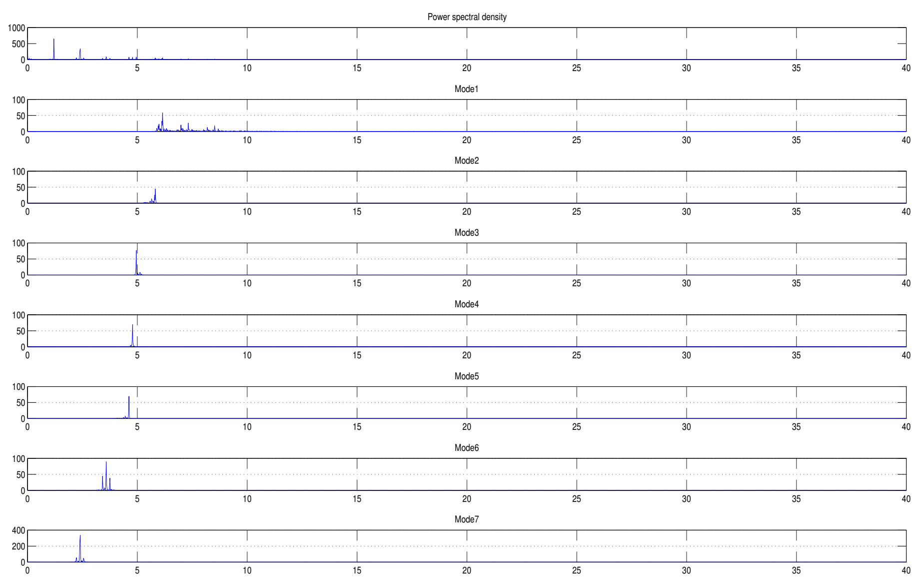


**Figure 4.29** PSD of affected ECG signal's modes using correlogram

normal ECG is higher. Also specific modes 4-7 of EWT reflect the major frequency changes that can be used for further analysis.

The proposed arrhythmia detection method in this study is a promising approach for diagnosing arrhythmia affected ECG. Using a different PSD estimation method is one of the improvement that can be applied which makes the arrhythmia classification as one of the potential future contributions.

# 5. Discussion

In this chapter the statement of the problem and presented results which are the contributions of this research study have been summarized and compared to other relevant studies. Another section of this chapter discusses the limitations of this study and the proposed solutions.

## 5.1 Contributions

In regards with heart rate detection, a large number of studies can be found since it is an active research field. These studies proposed different kinds of HR estimation methods mostly using ECG signal features. Further studies highlight the usefulness of SCG signal in heart rate detection.

These methods are classified as whether linear/non-linear or adaptive/non-adaptive. Each category has its own advantages and disadvantages. In the first category, linear methods have less computational complexity but when they are used for non-linear signal (e.g., ECG OR SCG) processing would cause the missing of the signal's important information. On the other hand, non-linear approaches provides reliable information about signals but they are not capable of tracking changes in high frequencies which is considered as a limitation of theses approaches.

In the second category, adaptive methods such as FFT provides a reliable frequency response of stationary signals but it is not capable of localizing the instantaneous frequency changes. Considering the dynamic changes of heart behaviour and its non-stationary nature, adaptive methods whose basis functions are directly derived from the signal itself, can perform better in tracking changes of heart rate including its rapid variation compared to non-adaptive methods.

### 5.1.1 Heart Rate Detection from SCG Using EMD and EWT

Based on the non-stationary nature of heartbeat, adaptive methods of EMD and EWT are appropriate approaches for the further analysis of this signal. The first contribution of this thesis is the proposed HR detection methods based on EMD and EWT algorithms which are highly adaptive. To the best of author's knowledge these methods were not being applied to SCG signals for heart rate detection yet which highlights the novelty of this contribution.

On the other hand, some of proposed HR detection methods by other research groups provide an accurate and reliable estimation of heart rate even though they are not adaptive. In most cases, these approaches are quite complicated and own a high computational complexity which question their robustness.

One of the other differences with the the other HR detection methods is the construction of the heartbeat signal using both EMD and EWT and heart rate was estimated using it.

### 5.1.2 Arrhythmia Detection from ECG Using EWT

Arrhythmia is caused an irregular beat which reflects in the frequency content by a sudden change. Different kind of studies have been done regarding arrhythmia detection and classification. According to the literature, the power spectrum of abnormal (arrhythmia affected) ECG is higher than a normal one. As mentioned before EWT is able to track the instant frequency changes which makes it an appropriate approach for arrhythmia detection. On the other hand power spectral density of a signal is a powerful tool for tracking the frequency oscillation of a signal which can be estimated using different methods such as periodogram and correlogram.

The second contribution of this thesis is using EWT in arrhythmia detection. It was applied to both normal and arrhythmia affected ECG signals and decomposed each input signal into several modes. Then, correlogram was used for PSD estimation to reflect the difference between the normal and abnormal (arrhythmia affected) ECG signals in terms of frequency content. The obtained results of this contribution recommend that EWT is a reliable method for arrhythmia detection. The results provided are in the preliminary stage since the proposed method can only be used as a general classifier of normal vs. abnormal ECG signals. The classification of different types of arrhythmia is one of the future directions.

## 5.2 Limitations and Challenges

Like any other research studies, this study also includes some limitations and challenges. Some of them have specifically been addressed in Chapter 3. These limitations and challenges are listed as below:

- Lack of access to the raw data
- Lack of access to the arrhythmia affected SCG signal
- Filtering step of SET is a limitation for this method since it is not applicable to different ECG/SCG input data sets
- Computational complexities of the adaptive HR detection methods which make a trade-off between the processing time and the resolution of the obtained results

### 5.2.1 Solutions

Proposed solutions to the mentioned limitations above have been sorted as the following:

- Since there was no access to the raw data, preprocessed data was used
- Since there was no access to the abnormal (arrhythmia affected) SCG data set, abnormal ECG data set was used.

- Multiple Butterworth filters with variable cut-off frequencies were defined in order to make SET HR detection method applicable for different ECG/SCG input data sets.
- Short spans of the utilized data sets were taken as the input signals to reduce the processing time of the proposed HR detection methods to the least possible level.

# 6. Conclusion and Future Work

In this chapter the research steps are summarized by discussing each step briefly. Then, potential future contributions of this work will be addressed.

## 6.1 Research Summary

Cardiac diseases are one of the major causes of death. Heart monitoring/diagnostic techniques have been developed over decades to address this concern. Monitoring a vital sign such as heart rate is a powerful technique for heart abnormalities detection (e.g., arrhythmia). After having the raw data, the heart rate from ECG, SCG and BCG signals can be detected by various signal processing methods which have been discussed in chapter 2.

The novelty of this work is that offers new heart rate detection methods which are both robust and adaptive and suit the non-stationary signals. Utilized data sets (data set 1, data set 2) in this study have been provided from two sources. Data set 1 has not been released in PhysioNet and belongs to a research group and data set 2 has been obtained from PhysioNet. Each data set includes several simultaneous recorded signals that ECG and SCG Z-axis are the focus of this thesis. Utilized data sets, have been pre-processed and therefore there was no access to the raw data.

In this research study, proposed methods for heart rate detection include modified SET, EMD and EWT. Using EMD and EWT for heart rate detection using SCG signal one of the contributions of this study. Results obtained from applied SET to ECG signal was selected as the ground truth. Then all three methods were used for heart rate detection from the SCG signal and were compared to the ground truth for further evaluation. Since the nature of heartbeat signal is non-stationary, recommended methods of EMD and EWT give more efficient interpretation of heart rate.

As previously mentioned in chapter 3, adaptive methods of EMD and EWT which are suitable for nonlinear/nonstationary signals, decompose a signal into its different frequency components. After obtaining the peak frequencies of each mode, the corresponding modes to heartbeat with the frequency range of 1-10 Hz are selected and used to construct the heartbeat signal. Then, heart rate is estimated using the heartbeat signal. Time-domain parameters of heart rate have been estimated for all proposed methods and were compared to ground truth. The average error of SET method is 12.5 ms and 17 ms, the average error of EWT is 16 ms and 9 ms for data set 1 and data set 2 respectively. The average error of EMD for data set 1 is 16 ms. Based on the obtained results, EMD and EWT are promising techniques for heart rate detection and interpretation from the SCG signal.

Another contribution of this work is the arrhythmia detection using EWT since it provides us with the instantaneous frequency changes of the corresponding modes to ECG signal. Based on the obtained results, power spectral density of arrhythmia affected ECG is more than $50dB$ and is higher compared to the power spectral density of a normal ECG which is less than $20dB$). In the following, future contributions will be discussed.

## 6.2 Future Directions

In this section, potential future contributions have been briefly discussed as the following.

### 6.2.1 Cardiac Information of the 2 Other Axes of SCG (X, Y)

In this thesis, the information extracted from Z axis of SCG which is perpendicular to the heart was interpreted. The maximum force generated by the heart is in

Z direction and therefore, Z component provides more cardiac information. Considering the fact that collecting data from all three axes will lead to a more complete interpretation of cardiac events, highlights the importance of the extracted data from X and Y axes. The future contribution is to use X and Y axes for heart rate detection and compare the obtained results with the current Z axis results. Furthermore, the extracted information from all three axes will be combined for achieving a reliable interpretation of heart rate.

### 6.2.2 Arrhythmia Detection Using SCG

The focus of this thesis was mainly the HR detection using adaptive methods and the utilized data sets belong to the subjects with normal heart condition. Proposed heart rate detection methods will be performed on more data sets to get a more reliable result. Applying the adaptive method of EWT to SCG for heart abnormality detection such as arrhythmia is also an interesting topic for the future research direction. Based on EWT's specific properties (e.g. being time adaptive and highly sensitive to noise), it has a high potential of applicability in heart abnormality detection.

# Appendix

# Mathematical Equations of the Proposed Methods

## A.1 EWT

The empirical wavelets are defined as bandpass filters with the center frequency of $\omega_n$. In order to obtain these wavelets, the empirical scaling function and the empirical wavelets are respectively defined as the following equations A.1 and A.2 (where $0 < \gamma < 1$) [48]:

$$A.\hat{\phi}_n(\omega) = \begin{cases} 1 & \text{if } |\omega| \leq (1-\gamma)\omega_n \\ \cos\left[\frac{\pi}{2}\beta\left(\frac{1}{2\gamma\omega_n}(|\omega| - (1-\gamma)\omega_n)\right)\right] & \text{if } (1-\gamma)\omega_n \leq |\omega| \leq (1+\gamma)\omega_n \\ 0 & \text{otherwise,} \end{cases} \tag{A.1}$$

$$\hat{\psi}_n(\omega) = \begin{cases} 1 & \text{if } (1+\gamma)\omega_n \leq |\omega| \leq (1-\gamma)\omega_{n+1} \\ \cos\left[\frac{\pi}{2}\beta\left(\frac{1}{2\gamma\omega_{n+1}}(|\omega| - (1-\gamma)\omega_{n+1})\right)\right] & \text{if } (1-\gamma)\omega_{n+1} \leq |\omega| \leq (1+\gamma)\omega_{n+1} \\ \sin\left[\frac{\pi}{2}\beta\left(\frac{1}{2\gamma\omega_n}(|\omega| - (1-\gamma)\omega_n)\right)\right] & \text{if } (1-\gamma)\omega_n \leq |\omega| \leq (1+\gamma)\omega_n \\ 0 & \text{otherwise.} \end{cases} \tag{A.2}$$

The function $\beta(x)$ is an arbitrary function of $C^k([0,1])$ and is defined as equation A.3

$$\beta(x) = \begin{cases} 0 & \text{if } x \leq 0 \\ & \text{and } \beta(x) + \beta(1-x) = 1 \qquad \forall x \in [0,1]. \\ 1 & \text{if } x \geq 1. \end{cases} \tag{A.3}$$

The definition of the EWT as a classic wavelet transform is as $\mathcal{W}_f^{\mathcal{E}}(n,t)$. Equations A.4 presents he detail coefficients by the inner products with the empirical wavelets:

$$\mathcal{W}_f^{\mathcal{E}}(n,t) = \langle f, \psi_n \rangle = \int f(\tau)\overline{\psi_n(\tau - t)}d\tau = \left(\hat{f}(\omega)\overline{\hat{\psi}_n(\omega)}\right)^{\vee} \tag{A.4}$$

Equation A.6 indicates the approximation coefficients by the inner product with the scaling function:

$$\mathcal{W}_f^{\mathcal{E}}(0,t) = \langle f, \phi_1 \rangle = \int f(\tau)\overline{\phi_1(\tau - t)}d\tau = \left(\hat{f}(\omega)\overline{\hat{\phi}_1(\omega)}\right)^{\vee}, \tag{A.5}$$

Knowing the definitions of $\hat{\psi}_n(\omega)$ and $\hat{\phi}_1$ in equation A.1 and A.2 $f(t)$ can be reconstructed as equation A.6

$$f(t) = \mathcal{W}_f^{\mathcal{E}}(0,t) \star \phi_1(t) + \sum_{n=1}^{N} \mathcal{W}_f^{\mathcal{E}}(n,t) \star \psi_n(t) = \left(\hat{\mathcal{W}}_f^{\mathcal{E}}(0,\omega)\hat{\phi}_1(\omega) + \sum_{n=1}^{N} \hat{\mathcal{W}}_f^{\mathcal{E}}(n,\omega)\hat{\psi}_n(\omega)\right)^{\vee}. \tag{A.6}$$

Following this formalism, the empirical mode $f_k$ is given by the following equations A.7 and A.8

$$f_0(t) = \mathcal{W}_f^{\mathcal{E}}(0,t) \star \phi_1(t), \tag{A.7}$$

$$f_k(t) = \mathcal{W}_f^{\mathcal{E}}(k,t) \star \psi_k(t). \tag{A.8}$$

## A.2 PSD

Let $\{x(n)\}$ be an input random signal,

$$E\{x(n)\} = 0, \qquad r(k) = E\{x(n)\, x^*(n-k)\}. \tag{A.9}$$

First definition of PSD [82]:

$$P(e^{j\omega}) = \sum_{k=-\infty}^{\infty} r(k)e^{-j\omega k}, \tag{A.10}$$

$$r(k) = \frac{1}{2\pi}\int_{-\pi}^{\pi} P(e^{j\omega})e^{j\omega k}\, d\omega. \tag{A.11}$$

Second definition of PSD [82]:

$$P(e^{j\omega}) = \lim_{N\to\infty} E\left\{ \frac{1}{N}\left|\sum_{n=0}^{N-1} x(n)e^{-j\omega n}\right|^2 \right\}. \tag{A.12}$$

## A.3 Correlogram

Correlogram is obtained from the first definition of PSD as below [83]:

$$\hat{P}_C(e^{j\omega}) = \sum_{k=-N+1}^{N-1} \hat{r}(k)e^{-j\omega k}. \tag{A.13}$$

in which

$$\hat{r}(k) = \begin{cases} \frac{1}{N}\sum_{i=k}^{N-1} x(i)x^*(i-k), & k \geq 0, \\ \hat{r}^*(-k), & k < 0. \end{cases} \tag{A.14}$$

# References

[1] M. Paukkunen, M. Linnavuo, and R. Sepponen, "A portable measurement system for the superior-inferior axis of the seismocardiogram," *J Bioengineer & Biomedical Sci 3: 123. doi*, 2013.

[2] S. F. Sheet—Populations, "International cardiovascular disease statistics," *American Heart Association*, 2004.

[3] N. Bu, N. Ueno, and O. Fukuda, "Monitoring of respiration and heartbeat during sleep using a flexible piezoelectric film sensor and empirical mode decomposition," in *Engineering in Medicine and Biology Society, 2007. EMBS 2007. 29th Annual International Conference of the IEEE*, pp. 1362–1366, IEEE, 2007.

[4] J. Parak, J. Dvorak, and J. Havlik, "Device for long term measurement of heart rate," in *Proceedings of the 4th International Symposium on Applied Sciences in Biomedical and Communication Technologies*, p. 19, ACM, 2011.

[5] S. Stern, D. Tzivoni, and Z. Stern, "Diagnostic accuracy of ambulatory ecg monitoring in ischemic heart disease.," *Circulation*, vol. 52, no. 6, pp. 1045–1049, 1975.

[6] D. Jabaudon, J. Sztajzel, K. Sievert, T. Landis, and R. Sztajzel, "Usefulness of ambulatory 7-day ecg monitoring for the detection of atrial fibrillation and flutter after acute stroke and transient ischemic attack," *Stroke*, vol. 35, no. 7, pp. 1647–1651, 2004.

[7] O. Inan, M. Etemadi, R. Wiard, L. Giovangrandi, and G. Kovacs, "Robust ballistocardiogram acquisition for home monitoring," *Physiological measurement*, vol. 30, no. 2, p. 169, 2009.

[8] J. Nickerson, G. Humphreys, R. Deterling, T. Fleming, and J. Mathers, "Di-

agnosis of coarctation of the aorta with the aid of the low frequency, critically damped ballistocardiograph," *Circulation*, vol. 1, no. 4, pp. 1032–1036, 1950.

[9] D. M. Salerno and J. Zanetti, "Seismocardiography for monitoring changes in left ventricular function during ischemia.," *CHEST Journal*, vol. 100, no. 4, pp. 991–993, 1991.

[10] R. A. Wilson, V. S. Bamrah, J. Lindsay Jr, M. Schwaiger, and J. Morganroth, "Diagnostic accuracy of seismocardiography compared with electrocardiography for the anatomic and physiologic diagnosis of coronary artery disease during exercise testing," *The American journal of cardiology*, vol. 71, no. 7, pp. 536–545, 1993.

[11] P. A. Iaizzo, *Handbook of cardiac anatomy, physiology, and devices.* Springer Science & Business Media, 2009.

[12] "Heart anatomy." `http://www.texasheart.org/HIC/Anatomy/anatomy2.cfm/`, 1996. Accessed: 2015-01-25.

[13] A. M. Katz, *Physiology of the Heart.* Lippincott Williams & Wilkins, 2010.

[14] "Heart and Cardiovascular System of the Upper Torsoheart." `http://www.innerbody.com/image/card01.html#full-description/`, 1999. Accessed: 2015-01-30.

[15] "Cardiac pacemakers." `http://www.texasheart.org/HIC/Anatomy/conduct.cfm/`, 1996. Accessed: 2015-01-30.

[16] E. N. Marieb and K. Hoehn, *Human anatomy & physiology.* Pearson Education, 2007.

[17] A. Harhad and N. Souag, "Cardiac motion estimation using factorization method from echography images,"

[18] "The heartbeat." `http://www.texasheart.org/HIC/Anatomy/systole.cfm/`, 1996. Accessed: 2015-01-30.

[19] J. P. Finley and S. T. Nugent, "Heart rate variability in infants, children and young adults," *Journal of the Autonomic Nervous System*, vol. 51, no. 2, pp. 103–108, 1995.

[20] L. Bernardi, F. Valle, M. Coco, A. Calciati, and P. Sleight, "Physical activity influences heart rate variability and very-low-frequency components in holter electrocardiograms," *Cardiovascular research*, vol. 32, no. 2, pp. 234–237, 1996.

[21] J. A. Kastor, *Arrhythmias*. WB Saunders Company, 2000.

[22] J. H. Mieres, A. N. Makaryus, R. F. Redberg, and L. J. Shaw, "Noninvasive cardiac imaging.," *American family physician*, vol. 75, no. 8, pp. 1219–1228, 2007.

[23] F. Refet, C. Van Hoof, R. Puers, *et al.*, *Biopotential readout circuits for portable acquisition systems*. Springer Science & Business Media, 2008.

[24] K. Tavakolian, B. Ngai, A. P. Blaber, and B. Kaminska, "Infrasonic cardiac signals: Complementary windows to cardiovascular dynamics," in *Engineering in Medicine and Biology Society, EMBC, 2011 Annual International Conference of the IEEE*, pp. 4275–4278, IEEE, 2011.

[25] S. Manoli, "Ecg electrode pad," Apr. 22 1986. US Patent 4,583,549.

[26] A. Baba and M. Burke, "Measurement of the electrical properties of ungelled ecg electrodes," *International journal of biology and biomedical engineering*, vol. 2, no. 3, pp. 89–97, 2008.

[27] R. Anderson, U. Sartipy, and J. Jakobsson, "Use of conventional ecg electrodes for depth of anaesthesia monitoring using the cerebral state index: a clinical study in day surgery," *British journal of anaesthesia*, vol. 98, no. 5, pp. 645–648, 2007.

[28] N. V. Thakor and J. G. Webster, "Ground-free ecg recording with two electrodes," *Biomedical Engineering, IEEE Transactions on*, no. 12, pp. 699–704, 1980.

[29] P. R. Dahl, J. J. Gerardi, and G. A. Hickman, "Piezoelectric sensor," Mar. 9 1993. US Patent 5,191,791.

[30] T. Koivistoinen, S. Junnila, A. Varri, and T. Koobi, "A new method for measuring the ballistocardiogram using emfi sensors in a normal chair," in *Engineering in Medicine and Biology Society, 2004. IEMBS'04. 26th Annual International Conference of the IEEE*, vol. 1, pp. 2026–2029, IEEE, 2004.

[31] J. Holcik and J. Moudr, "Mathematical model of seismocardiogram," in *World Congress on Medical Physics and Biomedical Engineering 2006*, pp. 3415–3418, Springer, 2007.

[32] O. Postolache, G. Postolache, and P. Girão, "New device for assessment of autonomous nervous system functioning in psychophysiology," in *Medical Measurement and Applications, 2007. MEMEA'07. IEEE International Workshop on*, pp. 1–5, IEEE, 2007.

[33] J. Alametsä, A. Värri, J. Viik, J. Hyttinen, and A. Palomäki, "Ballistocardiogaphic studies with acceleration and electromechanical film sensors," *Medical engineering & physics*, vol. 31, no. 9, pp. 1154–1165, 2009.

[34] A. Dinh, "Design of a Seismocardiography Using Tri-Axial Accelerometer Embedded with Electrocardiogram," *The World Congress on Engineering and Computer*, 2010.

[35] G. G. Berntson, J. T. Cacioppo, K. S. Quigley, and V. T. Fabro, "Autonomic space and psychophysiological response," *Psychophysiology*, vol. 31, no. 1, pp. 44–61, 1994.

[36] I. Korzeniowska-Kubacka, M. Bilińska, and R. Piotrowicz, "Usefulness of seismocardiography for the diagnosis of ischemia in patients with coronary artery disease," *Annals of noninvasive electrocardiology*, vol. 10, no. 3, pp. 281–287, 2005.

[37] P. Castiglioni, A. Faini, G. Parati, and M. Di Rienzo, "Wearable seismocardiography," in *Engineering in Medicine and Biology Society, 2007. EMBS 2007. 29th Annual International Conference of the IEEE*, pp. 3954–3957, IEEE, 2007.

[38] J. Ramos-Castro, J. Moreno, H. Miranda-Vidal, M. Garcia-Gonzalez, M. Fernández-Chimeno, G. Rodas, and L. Capdevila, "Heart rate variability analysis using a seismocardiogram signal," in *Engineering in Medicine and Biology Society (EMBC), 2012 Annual International Conference of the IEEE*, pp. 5642–5645, IEEE, 2012.

[39] M. Di Rienzo, E. Vaini, P. Castiglioni, G. Merati, P. Meriggi, G. Parati, A. Faini, and F. Rizzo, "Wearable seismocardiography: Towards a beat-by-beat assessment of cardiac mechanics in ambulant subjects," *Autonomic Neuroscience*, vol. 178, no. 1, pp. 50–59, 2013.

[40] J. Zanetti *et al.*, "Seismocardiography: A new technique for recording cardiac vibrations. concept, method, and initial observations," *Journal of cardiovascular technology*, vol. 9, no. 2, pp. 111–118, 1990.

[41] J. M. Zanetti and D. M. Salerno, "Seismocardiography: a technique for recording precordial acceleration," in *Computer-Based Medical Systems, 1991. Proceedings of the Fourth Annual IEEE Symposium*, pp. 4–9, IEEE, 1991.

[42] M. Di Rienzo, P. Meriggi, F. Rizzo, E. Vaini, A. Faini, G. Merati, G. Parati, and P. Castiglioni, "A wearable system for the seismocardiogram assessment in daily life conditions," in *Engineering in Medicine and Biology Society,EMBC, 2011 Annual International Conference of the IEEE*, pp. 4263–4266, 2011.

[43] A. Dinh, S. Ko, P. Neary, and D. M. Quarrie, "A Low Cost Sensing Device to Detect Cardiac Timing and Function," *Mixed-Signals, Sensors and Systems Test Workshop (IMS3TW), 2012 18th International*, pp. 101–104, 2012.

[44] J. PARAK, "Heart rate detection from ballistocardiogram," 2012.

[45] J. Parák and J. Havlík, "Ecg signal processing and heart rate frequency detection methods," *Proceedings of Technical Computing Prague*, vol. 8, p. 2011, 2011.

[46] J. P. DiMarco and J. T. Philbrick, "Use of ambulatory electrocardiographic (holter) monitoring," *Annals of internal medicine*, vol. 113, no. 1, pp. 53–68, 1990.

[47] B. Boashash, "Estimating and interpreting the instantaneous frequency of a signal. i. fundamentals," *Proceedings of the IEEE*, vol. 80, no. 4, pp. 520–538, 1992.

[48] J. Gilles, "Empirical wavelet transform," *Signal Processing, IEEE Transactions on*, vol. 61, no. 16, pp. 3999–4010, 2013.

[49] M. Varanini, A. Macerata, M. Emdin, and C. Marchesi, "Non linear filtering for the estimation of the respiratory component in heart rate," in *Computers in Cardiology 1994*, pp. 565–568, IEEE, 1994.

[50] M. Di Rienzo, P. Meriggi, E. Vaini, P. Castiglioni, and F. Rizzo, "24h seismocardiogram monitoring in ambulant subjects," in *Engineering in Medicine and Biology Society (EMBC), 2012 Annual International Conference of the IEEE*, pp. 5050–5053, IEEE, 2012.

[51] D. M. Salerno, J. M. Zanetti, L. A. Green, M. R. Mooney, J. D. Madison, and R. A. Van Tassel, "Seismocardiographic changes associated with obstruction of coronary blood flow during balloon angioplasty," *The American journal of cardiology*, vol. 68, no. 2, pp. 201–207, 1991.

[52] J. W. Gofman, W. Young, and R. Tandy, "Ischemic heart disease, atherosclerosis, and longevity," *Circulation*, vol. 34, no. 4, pp. 679–697, 1966.

[53] J. P. Neary, D. S. MacQuarrie, V. Jamnik, N. Gledhill, S. Gledhill, and E. F. Busse, "Assessment of mechanical cardiac function in elite athletes.," *Open Sports Medicine Journal*, 2011.

[54] M. Jerosch-Herold, J. Zanetti, H. Merkle, L. Poliac, H. Huang, A. Mansoor, F. Zhao, and N. Wilke, "The seismocardiogram as magnetic-field-compatible alternative to the electrocardiogram for cardiac stress monitoring," *The International Journal of Cardiac Imaging*, vol. 15, no. 6, pp. 523–531, 1999.

[55] I. Korzeniowska-Kubacka, M. Bilińska, and R. Piotrowicz, "Influence of physical training on cardiac performance in patients with coronary artery disease and exercise-induced left ventricular dysfunction.," *Acta cardiologica*, vol. 62, no. 6, pp. 573–578, 2007.

[56] K. Tavakolian, A. P. Blaber, B. Ngai, and B. Kaminska, "Estimation of hemodynamic parameters from seismocardiogram," in *Computing in Cardiology, 2010*, pp. 1055–1058, IEEE, 2010.

[57] J. Lewis, L. Kuo, J. Nelson, M. Limacher, and M. Quinones, "Pulsed doppler echocardiographic determination of stroke volume and cardiac output: clinical validation of two new methods using the apical window.," *Circulation*, vol. 70, no. 3, pp. 425–431, 1984.

[58] W. Kubicek, R. Patterson, and D. Witsoe, "Impedance cardiography as a non-invasive method of monitoring cardiac function and other parameters of the cardiovascular system*," *Annals of the New York Academy of Sciences*, vol. 170, no. 2, pp. 724–732, 1970.

[59] R. Balocchi, D. Menicucci, E. Santarcangelo, L. Sebastiani, A. Gemignani, B. Ghelarducci, and M. Varanini, "Deriving the respiratory sinus arrhythmia from the heartbeat time series using empirical mode decomposition," *Chaos, Solitons & Fractals*, vol. 20, no. 1, pp. 171–177, 2004.

[60] N. E. Huang, Z. Shen, S. R. Long, M. C. Wu, H. H. Shih, Q. Zheng, N.-C. Yen, C. C. Tung, and H. H. Liu, "The empirical mode decomposition and the hilbert spectrum for nonlinear and non-stationary time series analysis," *Proceedings of the Royal Society of London. Series A: Mathematical, Physical and Engineering Sciences*, vol. 454, no. 1971, pp. 903–995, 1998.

[61] K. Watanabe, T. Watanabe, H. Watanabe, H. Ando, T. Ishikawa, and K. Kobayashi, "Noninvasive measurement of heartbeat, respiration, snoring and body movements of a subject in bed via a pneumatic method," *Biomedical Engineering, IEEE Transactions on*, vol. 52, no. 12, pp. 2100–2107, 2005.

[62] D. Friedrich, X. L. Aubert, H. Fuhr, and A. Brauers, "Heart rate estimation on a beat-to-beat basis via ballistocardiography-a hybrid approach," in *Engineering in Medicine and Biology Society (EMBC), 2010 Annual International Conference of the IEEE*, pp. 4048–4051, IEEE, 2010.

[63] X. Cao, H. Guo, and J. Tang, "Heart rate extraction of ballistocardiogram based on hilbert-huang transformation," in *World Congress on Medical Physics and Biomedical Engineering May 26-31, 2012, Beijing, China*, pp. 619–622, Springer, 2013.

[64] J. Jin, X. Wang, S. Li, and Y. Wu, "A novel heart rate detection algorithm in ballistocardiogram based on wavelet transform," in *Knowledge Discovery and Data Mining, 2009. WKDD 2009. Second International Workshop on*, pp. 76–79, IEEE, 2009.

[65] V. Pichot, J.-M. Gaspoz, S. Molliex, A. Antoniadis, T. Busso, F. Roche, F. Costes, L. Quintin, J.-R. Lacour, and J.-C. Barthélémy, "Wavelet transform to quantify heart rate variability and to assess its instantaneous changes," *Journal of Applied Physiology*, vol. 86, no. 3, pp. 1081–1091, 1999.

[66] C. Bruser, K. Stadlthanner, A. Brauers, and S. Leonhardt, "Applying machine learning to detect individual heart beats in ballistocardiograms," in *Engineering in Medicine and Biology Society (EMBC), 2010 Annual International Conference of the IEEE*, pp. 1926–1929, IEEE, 2010.

[67] U. R. Acharya, P. S. Bhat, S. S. Iyengar, A. Rao, and S. Dua, "Classification of heart rate data using artificial neural network and fuzzy equivalence relation," *Pattern Recognition*, vol. 36, no. 1, pp. 61–68, 2003.

[68] C. Bruser, K. Stadlthanner, S. de Waele, and S. Leonhardt, "Adaptive beat-to-beat heart rate estimation in ballistocardiograms," *Information Technology in Biomedicine, IEEE Transactions on*, vol. 15, no. 5, pp. 778–786, 2011.

[69] M. P. Tarvainen, S. D. Georgiadis, P. O. Ranta-aho, and P. A. Karjalainen, "Time-varying analysis of heart rate variability signals with a kalman smoother algorithm," *Physiological measurement*, vol. 27, no. 3, p. 225, 2006.

[70] J. Shin, B. Choi, Y. Lim, D. Jeong, and K. Park, "Automatic ballistocardiogram (bcg) beat detection using a template matching approach," in *Engineering in Medicine and Biology Society, 2008. EMBS 2008. 30th Annual International Conference of the IEEE*, pp. 1144–1146, IEEE, 2008.

[71] M. Jacobson, "Auto-threshold peak detection in physiological signals," tech. rep., DTIC Document, 2001.

[72] M. Malik, "Heart rate variability," *Current opinion in cardiology*, vol. 13, no. 1, pp. 36–44, 1998.

[73] A. Laurin, A. Blaber, and K. Tavakolian, "Seismocardiograms return valid heart rate variability indices," in *Computing in Cardiology Conference (CinC), 2013*, pp. 413–416, IEEE, 2013.

[74] M. A. García-González, A. Argelagós-Palau, M. Fernández-Chimeno, and J. Ramos-Castro, "A comparison of heartbeat detectors for the seismocardiogram," in *Computing in Cardiology Conference (CinC), 2013*, pp. 461–464, IEEE, 2013.

[75] H. Gothwal, S. Kedawat, R. Kumar, *et al.*, "Cardiac arrhythmias detection in an ecg beat signal using fast fourier transform and artificial neural network," *Journal of Biomedical Science and Engineering*, vol. 4, no. 04, p. 289, 2011.

[76] S. Das and M. Chakraborty, "Comparison of power spectral density (psd) of normal and abnormal ecg," in *IJCA, Special Issue on 2nd National Conference-Computing, Communication and Sensor Network,(2): pp-10-14*, 2011.

[77] A. L. Goldberger, L. A. Amaral, L. Glass, J. M. Hausdorff, P. C. Ivanov, R. G. Mark, J. E. Mietus, G. B. Moody, C.-K. Peng, and H. E. Stanley, "Physiobank, physiotoolkit, and physionet components of a new research resource for complex physiologic signals," *Circulation*, vol. 101, no. 23, pp. e215–e220, 2000.

[78] N. Rastogi and R. Mehra, "Analysis of butterworth and chebyshev filters for ecg denoising using wavelets," *IOSR Journal of Electronics and Communication Engineering*, vol. 6, no. 6, pp. 37–44, 2013.

[79] G. Rilling, P. Flandrin, P. Goncalves, *et al.*, "On empirical mode decomposition and its algorithms," in *IEEE-EURASIP workshop on nonlinear signal and image processing*, vol. 3, pp. 8–11, NSIP-03, Grado (I), 2003.

[80] S. McKinley and M. Levine, "Cubic spline interpolation," *College of the Redwoods*, vol. 45, pp. 1049–1060, 1998.

[81] G. Strang and T. Nguyen, *Wavelets and filter banks.* SIAM, 1996.

[82] P. Stoica and R. L. Moses, *Introduction to spectral analysis*, vol. 1. Prentice hall Upper Saddle River, 1997.

[83] E. Paparoditis and D. N. Politis, "Nonlinear spectral density estimation: thresholding the correlogram," *Journal of Time Series Analysis*, vol. 33, no. 3, pp. 386–397, 2012.