# Instrumentation of CdZnTe detectors for measuring prompt gamma-rays emitted during particle therapy

Dissertationsschrift

zur Erlangung des akademischen Grades Doktor der Medizintechnologie Doctor rerum medicinalium (Dr. rer. medic.) vorgelegt der Medizinischen Fakultät Carl Gustav Carus der Technischen Universität Dresden

von

Dipl.-Ing. Philipp Födisch

aus Wolfen

Dresden 2016

- 1. Gutachter: Prof. Dr. rer. nat. habil. Wolfgang Enghardt
- 2. Gutachter: Prof. Dr.-Ing. habil. Uwe Hampel

Tag der mündlichen Prüfung: 12. Mai 2017

gez.: PD Dr. rer. nat. habil. Steffen Löck Vorsitzender der Promotionskommission

## Contents

1.	Intro	Introduction 1						
	1.1.	Aim of this work	3					
2.	Ana	log front-end electronics	5					
	2.1.	State-of-the-art	5					
	2.2.	Basic design considerations	6					
		2.2.1. CZT detector assembly	6					
		2.2.2. Electrical characteristics of a CZT pixel detector	7					
		2.2.3. High voltage biasing and grounding	9					
		2.2.4. Signal formation in CZT detectors	0					
		2.2.5. Readout concepts	2					
		2.2.6. Operational amplifier	6					
	2.3.	Circuit design of a charge-sensitive amplifier	8					
		2.3.1. Circuit analysis	9					
		2.3.2. Charge-to-voltage transfer function	2					
		2.3.3. Input coupling of the CSA	5					
		2.3.4. Noise	6					
	2.4.	Implementation and Test	8					
		Results	0					
		2.5.1. Test pulse input	1					
		2.5.2. Pixel detector	2					
	2.6.	Conclusion						
3.	Diai	tal signal processing 3	7					
-	-	Unfolding-synthesis technique						
		Digital deconvolution	9					
		3.2.1. Prior work	9					
		3.2.2. Discrete-time inverse amplifier transfer function						
		3.2.3. Application to measured signals						
		3.2.4. Implementation of a higher order IIR filter						
		3.2.5. Conclusion	-					
	33	Digital pulse synthesis						
	0.0.	3.3.1. Prior work						
		3.3.1. Prior work       5         3.3.2. FIR filter structures for FPGAs       5         3.3.3. Optimized fixed-point arithmetic       5	1					

		3.3.4.	Conclusion	60				
4.	Data interface 61							
	4.1.	State-o	of-the-art	61				
4.2. Embedded Gigabit Ethernet protocol stack				62				
	4.3. Implementation							
		4.3.1.	System overview	65				
		4.3.2.	Media Access Control	65				
		4.3.3.	Embedded protocol stack	66				
		4.3.4.	Clock synchronization	68				
	4.4.	Measu	rements and results	69				
		4.4.1.	Throughput performance	70				
		4.4.2.	Synchronization	72				
		4.4.3.	Resource utilization	74				
	4.5.	Conclu	sion	75				
5.	5. Experimental results							
	5.1.	Digital	pulse shapers	77				
		5.1.1.	Spectroscopy application	78				
		5.1.2.	Timing applications	82				
	5.2.	$\gamma$ -ray s	pectroscopy	85				
		5.2.1.	Energy resolution of scintillation detectors	85				
		5.2.2.	Energy resolution of a CZT pixel detector	90				
	5.3.	$\gamma$ -ray ti	ming	93				
		5.3.1.	Timing performance of scintillation detectors	93				
		5.3.2.	Timing performance of CZT pixel detectors	95				
	5.4.	Measu	rements with a particle beam	99				
		5.4.1.	Bremsstrahlung Facility at ELBE	99				
6.	Disc	ussion		103				
7.	Sum	mary		105				
8	7.15	ammen	fassung	107				
Ap	Appendices 10							
Α.	A. Waveform diagrams 109							
В.	B. Rear Transition Module 115							

# List of acronyms and abbreviations

3D	Three-dimensional
ADC	Analog-to-digital converter
ALM	Adaptive Logic Module
ALU	Arithmetic logic unit
AMC	Advanced mezzanine card
ARP	Address Resolution Protocol
ASIC	Application-specific integrated circuit
BGO	$Bi_4Ge_3O_{12}$ , bismuth germanate
CAR	Cathode-over-anode ratio
CFD	Constant fraction discriminator
COTS	Commercial off-the-shelf
CSA	Charge-sensitive amplifier
CZT	CdZnTe, Cadmium zinc telluride
DAQ	Data acquisition
DESY	Deutsches Elektronen-Synchrotron
DOI	Depth of interaction
DSP	Digital signal processor, Digital signal processing
ELBE	Elektronen Linearbeschleuniger für Strahlen hoher Brillianz und niedriger Emittanz
ENC	Equivalent noise charge
FCS	Frame check sequence
FIFO	First-In-First-Out
FIR	Finite impulse response
FMC	FPGA Mezzanine Card
FPGA	Field-programmable gate array
FR-4	Flame Retardant 4
FSM	Finite state machine
FWHM	Full width at half maximum
GAGG	$Gd_3Al_2Ga_3O_{12}$ :Ce, cerium-doped gadolinium aluminum gallium garnet

GBP	Gain-bandwidth product
GMII	Gigabit Media Independent Interface
GSPS	Gigasamples per second
HZDR	Helmholtz-Zentrum Dresden-Rossendorf
ICMP	Internet Control Message Protocol
IC	Integrated circuit
IFG	Interframe gap
IIR	Infinite impulse response
IP	Intellectual property
IP	Internet Protocol
LTI	Linear time-invariant
LUT	Look-up table
MAC	Media Access Control
MCH	MicroTCA Carrier Hub
MDIO	Management Data Input/Output
MicroTCA	Micro Telecommunications Computing Architecture
MSPS	Megasamples per second
MWD	Moving Window Deconvolution
Nal	Sodium iodide
OSI	Open Systems Interconnections
PCB	Printed circuit board
PC	Personal computer
PGI	Prompt gamma imaging
PGT	Prompt gamma timing
PHY	Physical layer transceiver
PLL	Phase-locked loop
PPS	Pulse per second
PTP	Precision Time Protocol
RGMII	Reduced Gigabit Media Independent Interface
RMS	Root-mean-square
RTL	Register-transfer level
RTM	Rear Transition Module
RTT	Round-trip time
SFD	Start frame delimiter
SGMII	Serial Gigabit Media Independent Interface

- SNR ..... signal-to-noise ratio
- SoC ..... System on a chip
- TCA ..... Telecommunications Computing Architecture
- TIA ..... Transimpedance amplifier
- UDP ..... User Datagram Protocol
- VHDL ..... Very High Speed Integrated Circuit Hardware Description Language
- VMEbus ...... Versa Module Europa bus

### 1. Introduction

Cancer is a deadly disease for humans. In Germany, the total number of cancer diseases increased from 338,300 in the year 1997 to 477,950 in 2012 (Batzler et al., 1999; Kaatsch et al., 2015). The quoted study claims that at the same time the associated cancer deaths remain constant slightly above 210,000 people. Regardless of how the statistics are interpreted, the decreasing cancer mortality rate is based on advances in medical science.

The successes in treating cancer are driven by a continuous improvement of medical technology. Nowadays, surgery is assisted by computers, chemotherapy benefits from computed tomography imaging, and radiation therapy involves cutting-edge technologies for particle accelerators and detector systems. An effective cure is achieved by combining the available techniques. With respect to the mentioned methodologies, radiotherapy is one of the main cancer treatments (Krause and Supiot, 2015). Therefore, a beam of high energy is generated by a particle accelerator, and is used to destroy the cancer cells in the patient's body. This can generally be a photon beam (X-rays), which is currently the standard type of radiation used for treatment, or even a particle beam (proton or ion beam). Particle beams have the same biological effect on tumor cells as photon beams, but they are aligned to stop at a determined position in the tumor, and healthy tissue behind the target is less irradiated. Thus the largest dose is deposited in the tumor, while healthy tissue is spared.

For now, there are 60 proton therapy facilities in operation around the world, where 6 are located in Germany (PTCOG, 2016). With regard to facilities under construction or in a planning stage, the total number will increase rapidly in the next years. In proton therapy, a major task is the development of a treatment plan delivering most of the dose to the tumor cells by adjusting beam parameters. An a posteriori verification of the applied dose is still challenging, as the protons are stopped inside the tissue, and transport no direct information about that location out of the patient. Consequently, information caused by interactions of protons with the target must be considered for a measurement to make progress in range verification. Monitoring the proton beam finally enables the exploitation of whole precision for proton therapy.

A promising approach for an indirect measurement of the finite range of a proton beam is based on the detection of emitted  $\gamma$ -rays. These  $\gamma$ -rays are promptly emitted as a result of nuclear reactions of protons with tissue. Thus the emission of prompt  $\gamma$ -rays correlates with the range of the proton beam and therefore with the deposited dose in the tumor (Fiedler et al., 2011). The emission spectrum is dominated by interactions of <sup>12</sup>C and <sup>16</sup>O, which is observable at 4.4 MeV and 6.1 MeV (Rohling, 2015). In order to locate the range or at

least a range shift, several methods have been proposed and are currently evaluated for clinical use. Throughout that task, a pinhole camera (Kim et al., 2009), or a knife-edge slit camera (Smeets, 2012; Richter et al., 2016) for prompt gamma imaging (PGI) have been investigated. For systems with passive collimation, "the difficulty is obviously to find a tradeoff between good spatial resolution and high detection efficiency" (Smeets, 2012). Moreover, it has been demonstrated that a time-of-flight measurement is feasible for detecting range shifts (Golnik, 2016). The prompt gamma timing (PGT) method requires "a stabilized photon detection (in terms of energy and timing) at high throughput rates" (Golnik, 2016). A further technique was proposed, which correlates the measured prompt  $\gamma$ -ray spectrum to nuclear reaction cross sections (Verburg and Seco, 2014). The data acquisition (DAQ) required a digitizer running at about 1 GSPS synchronous to the particle accelerator. At last, the most complex system for PGI is a Compton camera (Richard et al., 2009; Peterson et al., 2010; Roellinghoff et al., 2011; Kormoll et al., 2011b; Hueso-González et al., 2014; McCleskey et al., 2015; Polf et al., 2015; Taya et al., 2016). "A slight scepticism has settled down in the scientific community concerning Compton cameras for PGI. Due to the technical complexity and high radiation background, only few experimental results hint at their applicability in a clinical environment." (Hueso-González, 2016).

All quoted studies about methodology lead to the conclusion that range verification in proton therapy requires new developments in electronics. In general, a successful translation into practical clinical use-cases is possible, if nuclear instruments rise to the challenges of:

- Energy range: Prompt γ-rays are emitted at energies up to ≈ 7 MeV (Schumann et al., 2015). In case of PGI with a Compton camera, scattered events with energies below 100 keV must be also detectable by the electronics for improved event statistics (Rohling, 2015). Thus the measurement dynamic range must be extraordinarily high.
- γ-ray flux: Depending on the material and the volume of the detector, the estimated detector load is in the few-Mcps range (Pausch et al., 2016). For CdZnTe (CZT) detectors, which have been proposed for use in a Compton camera (Kormoll, 2013), the load is rather reduced. As clinical benchmarks are still ongoing work, instruments must be designed for highest possible performance in terms of dead time and achievable event throughput.
- Energy resolution: For all methods, discrimination of prompt γ-rays according to empirically defined ranges is convenient. Moreover, in case of a Compton camera, the precision of reconstructed scattering angles depends on energy resolution (Rohling, 2015). The energy resolution of the readout electronics must exceed the intrinsic resolution of the detector material. Hence, the electronic noise and processing system should not limit the overall spectroscopy performance.
- Time resolution: Regardless of the method, a timing of incident γ-rays relative to the accelerator frequency or another detector in coincidence enables filtering of usable events. In case of a time-of-flight measurement, requirements for timing are intensified to the range of picoseconds (Golnik, 2016). A precise timing demands a low-jitter clock

distribution and high bandwidth for fast signal rise times.

 Spatial resolution: For PGI, i.e. pinhole-camera, knife-edge slit camera, or Compton camera, a high segmentation of the detector enhances the precision of obtained images. But an increased spatial resolution demands an increased number of readout channels for the DAQ system, which concurrently multiplies complexity and amount of data (Kormoll, 2013).

A detector system serving all the emphasized features is necessary for an implementation of a Compton camera, but is not available. In general, before designing electronic systems, a decision between proprietary or standardized components has to be made. Proprietary systems tend to be compact and optimally adapted to a single task. On top of that design approach are application-specific integrated circuits (ASICs). For use in experimental systems, they are expensive, less versatile, and hard to access. On the contrary, standardized components benefit from their versatility due to their modular architecture at the cost of additional overhead. For nuclear physics, the Versa Module Europa bus (VMEbus) standard is widely used for instrumentation of control and experimental systems. Recent experimental results for range verification were based on electronics built with VMEbus systems (Kormoll, 2013; Hueso-González, 2016; Golnik, 2016). Moreover, an emerging trend for MicroTCA systems in the physics community is noticeable. In conjunction with field-programmable gate arrays (FPGAs), that platform seems to fit the generalized requirements of a prototype detector system for range verification, i.e. high data throughput, high signal bandwidth, and high input channel count with precise clock synchronization and distribution.

#### 1.1. Aim of this work

Starting from the development of a prototype system of a Compton camera for range verification in proton therapy (Kormoll, 2013), the integration of an ASIC-based system (Födisch et al., 2013) to that camera failed due to the limited energy range, time resolution, and throughput of the customized readout electronics. The only available ASIC for the readout of a CZT pixel detector could not withstand the requirements of a clinical scenario. With this experience and knowledge about present limitations, the development and investigation of a new prototype with commercial off-the-shelf (COTS) components was reinitialized. Electronics were replaced by highly integrated components, and analog signal processing was discarded for the purposes of digital pulse shape analysis and processing. Despite digitization, new developments in analog front-end electronics were necessary to acquire signals from a CZT pixel detector (Födisch et al., 2016a). Moreover, new algorithms for improving digital pulse processing with an FPGA have been developed (Födisch et al., 2016c) and efficiently implemented (Födisch et al., 2016d). To cope with the challenges of high data throughput and synchronization in distributed systems, an Ethernet-based interface optimized for timing applications was implemented (Födisch et al., 2016b). Finally, CZT pixel detectors were characterized for their potential use in a Compton camera application.

### 2. Analog front-end electronics

Cadmium zinc telluride (CdZnTe, CZT) radiation detectors are suitable for a variety of applications, due to their high spatial and energy resolution at room temperature. However, state-of-the-art detector systems require high-performance readout electronics. Though an ASIC is an adequate solution for the readout, requirements of high dynamic range and high throughput are not available in any commercial circuit. Consequently, this chapter describes the development of analog front-end electronics with operational amplifiers for an 8×8 pixe-lated CZT detector. For this purpose, we modeled an electrical equivalent circuit of the CZT detector with the associated charge-sensitive amplifier (CSA). Based on a detailed network analysis, the circuit design is completed by numerical values for various features such as ballistic deficit, charge-to-voltage gain, rise time, and noise level. A verification of the performance is carried out by synthetic detector signals and a pixel detector. The experimental results with the pixel detector assembly and a <sup>22</sup>Na radioactive source emphasize the depth dependence of the measured energy. After fitting the weighting potential, a correction of the energy based on the derived depth of interaction (DOI) was feasible, thus improving the energy resolution.

#### 2.1. State-of-the-art

CdZnTe is a room-temperature semiconductor material for radiation detectors (Del Sordo et al., 2009). It is available in compact detector units with highly segmented pixel layouts and is ideally suited for high energy resolution  $\gamma$ -ray spectroscopy (Verger et al., 2005) and 3D imaging (Wahl et al., 2015). As has been previously reported, CZT detectors have been used in Compton camera systems (Kormoll et al., 2011a; Hueso-González et al., 2014; Lee et al., 2016). With regard to recent investigations, they have the potential to build an imaging system for proton therapy (McCleskey et al., 2015; Polf et al., 2015; Taya et al., 2016; Golnik et al., 2016). State-of-the-art readout systems for highly segmented CZT detectors are conventionally built with an ASIC (He et al., 1999; Gan et al., 2016). Available ASICs are optimized for  $\gamma$ -ray spectroscopy. Low energy range, usually up to 2 MeV (McCleskey et al., 2015; Gan et al., 2016; Födisch et al., 2013), limited count rate capability, and poor availability and product life cycle are unsolved challenges of an ASIC-based readout system for an imaging system in proton therapy. In this environment, high energies up to 7 MeV and count rates up to 1 Mcps have to be handled (Schumann et al., 2015; Hueso-González et al., 2015; Pausch et al., 2016). Instead of using an ASIC for the readout electronics, COTS operational amplifiers have been used for the front-end electronics (Ramachers and Stewart, 2007). Along with space-saving multi-channel analog-to-digital converters (ADCs) and an FPGA, all tasks related to the signal acquisition and processing can be done with a COTS system. A programmable digital system benefits from its versatility, which is needed for the evaluation of a detector system for new applications like proton therapy. Even for application in a fixed installation, a system made of COTS components provides the advantages of proven reliability and life-cycle support.

In general, front-end electronics are the key element of overall performance of a detector system. Our goal is to maximize the dynamic range of the front-end electronics since the CZT is exposed to high-energy  $\gamma$ -rays, but also has to detect low-energy scatter events used for Compton imaging. As this is the main objective, which cannot be solved with a state-of-the-art ASIC, the timing information of an interaction must be preserved by the readout system. Most ASICs merely include simple analog signal processing (e.g. leading-edge trigger for timing and peak-hold circuit for energy information) making pulse shape analysis or advanced timing algorithms difficult or even impossible. As the design of front-end electronics is a tradeoff between signal bandwidth, noise, complexity, size, and costs, the best solution must be driven by the application. For the prototype of a Compton camera, we investigate a space-saving and simple circuit design with minimal components. The system must include at least 65 analog readout channels, which are set up with COTS voltage feedback operational amplifiers.

#### 2.2. Basic design considerations

#### 2.2.1. CZT detector assembly

For a medical imaging application, we use a CZT detector as the scattering layer in a Compton camera. A reasonable choice for this task is a pixelated CZT radiation detector from Redlen Technologies (Redlen Technologies, 2011). The detector size is  $19.42 \times 19.42 \text{ mm}^2$ with a thickness of 5 mm. Towards the continuous planar electrode on the back side, there are 64 pixel electrodes aligned in an 8×8 array on the front side. The size of a pixel pad is 2.2×2.2 mm<sup>2</sup> for all pixels except the corner pixels with 1.98×1.98 mm<sup>2</sup>. In addition, a steering grid surrounds all pixels. The inter-pixel space is 0.26 mm. The bulk device and an assembled detector are shown in fig. 2.1. The detector is mounted on a printed circuit board (PCB) with the continuous planar electrode on top. A bond wire is attached to a copper pad on the carrier PCB. Furthermore, a conductive adhesive on each pixel pad ensures the electrical connection to the pixel array on the PCB. An underfill between the pixel array and the PCB supports the adhesive connection and improves mechanical stability. We also decided to use a 3.2 mm-thick FR-4 material for the PCB to enhance the robustness of the assembly. The electrodes of the detector are accessible via rugged high-speed connectors on the bottom side. To shield the detector against visible light, a 3D-printed cap is attached above the detector on the top of the carrier board. Our front-end electronics are designed to work with this type of detector assembly, and the readout boards are plugged into the side

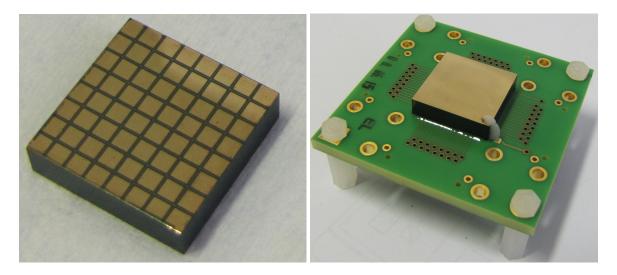


Figure 2.1.: A 5 mm thick pixelated CZT detector from Redlen Technologies (Redlen Technologies, 2011) with 8×8 electrodes (each 2.2 mm×2.2 mm) on the front side (left) and a continuous planar electrode (19.42 mm×19.42 mm) on the back side (right). The detector is mounted on a 3.2 mm-thick carrier board with rugged connectors on the bottom side.

faces of the detector carrier board (see fig. 2.22). Therefore a stacked system with arbitrary depth, as required for the evaluation of a suitable Compton camera setup, can be easily constructed. Further investigations on ruggedization of CZT detectors and detector assemblies have been presented in (Lu et al., 2015).

#### 2.2.2. Electrical characteristics of a CZT pixel detector

From the electrical point of view, a CZT detector can be modeled with the equivalent circuit shown in fig. 2.2. With an external operating voltage at the electrodes of the detector, the terminals are referred to as cathode and anode in accordance with the applied polarity. Usually, the continuous electrode is biased with a negative potential and the pixel electrodes are at ground potential. For an ideal detector material, this would force the negative charge carriers (electrons) to move towards the anode and the positive charge carriers (holes) to move towards the cathode. As a consequence of charge trapping due to structural defects, impurities, and irregularities of the material (Awadalla, 2015), the mobility and lifetime of the holes in CZT are very poor compared to the electrons (Spieler, 2005). Only the moving electrons induce a signal on the electrodes, while the portion of the signal due to the holes can be neglected. Thus, if the generated electrons move to the position-sensitive side of the detector, the overall detection performance is improved. As the readout electronics are directly connected to the electrodes, the electrical characteristics of the detector influence the dynamic behavior of the entire circuit. Finally, the network model for the readout electronics must include the electrical equivalent circuit of the detector. In general, a very simple equivalent circuit is adequate to model the properties of the detector. As summarized in fig. 2.2, it is a passive two-terminal component with a permittivity and a specific conductance. A capacitor represents the permittivity of the material and the electrical conductivity is modeled by a resistor. For the evaluated pixelated CZT detector, the capacitance can be roughly approximated by the model of the parallel-plate capacitor with an electrode area A separated by the distance d. Therefore the capacitance C is calculated by

$$C = \varepsilon_0 \varepsilon_r \frac{A}{d} , \qquad (2.1)$$

where  $\varepsilon_0$  is the vacuum permittivity and  $\varepsilon_r$  is the relative permittivity of CZT. For the detector in this study, the values are  $A = 377 \text{ mm}^2$ , d = 5 mm. In practical terms, the bias voltage does not influence the relative permittivity in the applicable voltage range up to -600 V. The capacitance of the detector is therefore largely independent of the bias voltage (Garson et al., 2007). The value of  $\varepsilon_r$  depends on the manufacturing process, but ranges from 10 to 11 (De Antonis et al., 1996; Spieler, 2005). Thus, the entire bulk capacitance is in the range from 6.6 pF to 7.4 pF. The capacitance of a single pixel can also be calculated by eq. 2.1, where the size of the pixel determines the value of *A* (Rossi et al., 2006). With the same assumption for  $\varepsilon_r$ , the pixel capacitance is in the range from 69 fF to 95 fF, including the smaller corner pixels. Besides the estimation of capacitance, the bulk resistivity of the detector is needed to model the electrical characteristics. We measured the leakage current of the assembled CZT detector from fig. 2.1 with a precision high-voltage source with current monitor (Iseg SHQ series) (iseg, 2014). This device reports a current of 10 nA $\pm$ 1 nA with a detector bias voltage of -500 V. For a homogenous material, the resistance is defined as:

$$R = \rho \frac{d}{A} \tag{2.2}$$

where  $\rho$  is the resistivity of the material. Our measurement corresponds to a resistor of 50 G $\Omega$  for the two-terminal equivalent circuit or a resistivity of 4.10<sup>11</sup>  $\Omega$ cm. This is in accordance with the values from the datasheet ( $\rho > 10^{11} \Omega$ cm) from Redlen (Redlen Technologies, 2011).

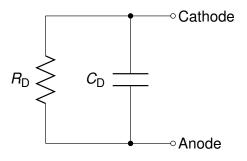


Figure 2.2.: An electrical equivalent circuit of a CZT detector. The bulk resistivity is modeled with the resistor  $R_D$ , which is typically of several tens of G $\Omega$ . The capacitor  $C_D$  represents the parallel-plate geometry of the electrodes. Its value can be estimated with eq. 2.1.

Finally, the electrical characteristics of a CZT crystal mainly depend on the manufacturing process. If they cannot be experimentally verified, the values for the components of the elec-

trical equivalent circuit can be estimated with the geometry of the detector and the constants from the literature by eqs. 2.1 and 2.2. The equivalent circuit shown in fig. 2.2 is the simplest electrical representation of the detector unit. It does not model a frequency dependency with a complex permittivity. Additionally, stray capacitances introduced by traces and the carrier board itself, cross-coupling between pixels, and any inductivities of the connectors are ignored. However, a well-designed PCB layout can minimize these effects.

#### 2.2.3. High voltage biasing and grounding

A fundamental operating condition for a CZT detector is the presence of an electric field between the electrodes. Thus, the charge carriers generated by incident radiation move towards the electrodes, and an electric current is measurable. Typical electric field strengths for CZT detectors are in the range of 1 kV/cm (Redlen Technologies, 2011). As the continuous electrode of the detector is biased with a negative voltage, the ground potential is connected to the pixelated electrodes on the opposite side. In general, the high voltage with low ripple is generated by an external power supply connected to the detector via a cable or PCB traces. To reduce any pickup noise related to electromagnetic interference, we put a high-voltage filter close to the detector electrode. This is in the simplest case a passive first-order low-pass filter (*RC* network shown in fig. 2.3). The value of the resistor *R*<sub>B</sub> should

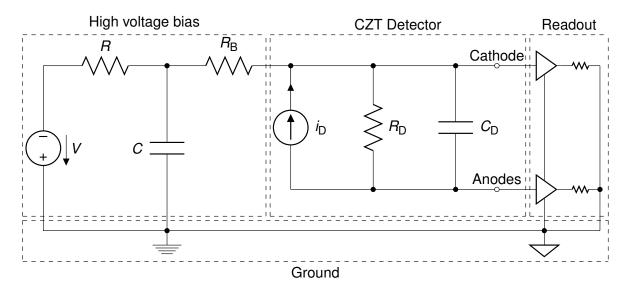


Figure 2.3.: Basic connection scheme of the CZT detector. A high voltage supply with an RC low-pass filter and a resistor  $R_B$  are used to generate the bias voltage for the cathode. The readout electronics are directly connected to the cathode and anodes of the detector. The anodes are biased by ground signal of the readout electronics, and consequently both ground potentials must be tied together.

be chosen to minimize the voltage drop across the high-voltage filter and maximize the voltage across the detector. The value of the filter capacitor C should be as high as possible to achieve the best noise filtering. According to eqs. 2.1 and 2.2, the capacitance is inversely proportional to the resistance. Therefore, the capacitor C must be chosen, such that the insulation resistance of the dielectric material is much higher than the resistance of the detector. Commercial capacitors of class 1 have an insulation resistance of more than 100 G $\Omega$ with a maximum capacitance of 10 nF. Thus, a passive first-order low-pass filter with a cutoff frequency below 1 Hz is possible (e.g.  $R = 47 \text{ M}\Omega$ , C = 10 nF). As a noise-filtered voltage is the output of the RC circuit, it cannot be directly connected to the electrode in order to bias the detector. One reason for this is that the filter capacitor C would be in parallel with the capacitance  $C_D$  of the detector. A bias resistor  $R_B$  of 47 M $\Omega$  separates the filter network from the detector. Another point that has to be taken into account is the path of current flow generated by the detector. The current should not flow into the high-voltage source. This can be ensured by choosing a high resistance for biasing the detector, so that the time constant  $R_{\rm B}C$  is much larger than the time constant of the readout electronics (Grupen and Shwartz, 2011). The active components of the readout electronics have their own power supply, which is separated from the high-voltage supply. Further, the anodes are biased with the ground potential of the readout electronics (signal ground in fig. 2.3). Both grounds have to be at the same potential and must be tied together. The electric field between the cathode and the anodes is therefore referenced to a known potential.

#### 2.2.4. Signal formation in CZT detectors

As implied, incident radiation hitting the detector generates free charge carriers. These electrons and holes move towards the electrodes because of the applied electric field. However, the generated charge is proportional to the incident  $\gamma$ -ray energy and the signal of the detector is an electric current. The induced current through an electrode is defined as

$$i = q \overrightarrow{v} \overrightarrow{E}_0 (\overrightarrow{x}), \qquad (2.3)$$

where *q* is the moving charge,  $\vec{v}$  is the instantaneous velocity of the charge *q* and  $\vec{E_0}(\vec{x})$  is the weighting field associated with the electrode at the position  $\vec{x}$  of the charge (He, 2001). The weighting potential  $\varphi_0(\vec{x})$  is defined as

$$\vec{E}_{0}(\vec{x}) = -\nabla \vec{\varphi}_{0}(\vec{x}), \qquad (2.4)$$

by setting one electrode to unit potential and all others to zero. For a parallel-plate geometry of a detector, where the widths in *x* and *y* dimensions of the electrodes are much larger than the thickness *z*, the electric field inside the detector is distributed homogeneously with a constant field strength. Therefore, the weighting field  $\vec{E_0}$  ( $\vec{x}$ ) is equal to the electric field, and, solving eq. 2.4 in the *z*-dimension, the normalized weighting potential  $\varphi_{0_c}(z)$  of the cathode is a linear function (He, 2001):

$$\varphi_{0_{\mathsf{C}}}(z) = z, 0 \le z \le 1.$$
 (2.5)

The solution for the Poisson equation in two dimensions was given by (Rossi et al., 2006; Wermes, 2006). These authors presented an equation for the calculation of the weighting

potential for a detector with a segmented electrode layout. By setting one dimension of the pixel area to zero and normalizing the thickness *z* of the detector to 1, the weighting potential  $\varphi_{0_A}$  of a single pixel with the normalized width *a* can be calculated as

$$\varphi_{0_{\mathsf{A}}}(x,z) = \frac{1}{\pi} \arctan\left(\frac{\sin(\pi z)\sinh(\pi \frac{a}{2})}{\cosh(\pi x) - \cos(\pi z)\cosh(\pi \frac{a}{2})}\right), \tag{2.6}$$

where x and z are the coordinates and  $0 \le z \le 1$ . The weighting potentials are shown in fig. 2.4. The weighting field  $E_0(z)$  under the collecting electrode (x = 0, y = 0) can be

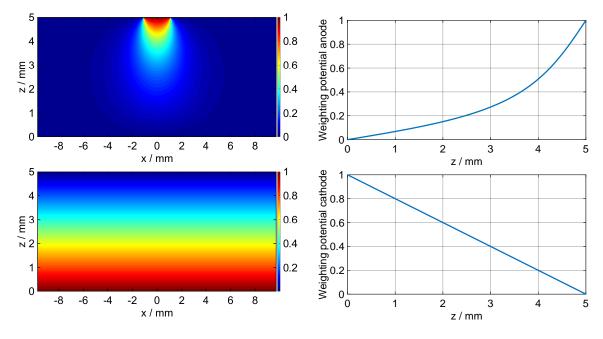


Figure 2.4.: The weighting potentials of the cathode (bottom) and pixel anode (top) of the 5 mm-thick detector. The cathode is located at position z = 0 mm. The right plots show the weighting potential under the collecting electrode (x = y = 0 mm).

calculated by solving eq. 2.4 with eq. 2.6. This results in the following equation:

$$E_0(z) = \frac{\sinh(\pi \frac{a}{2})}{\cos(\pi z) - \cosh(\pi \frac{a}{2})}.$$
 (2.7)

As we can estimate the weighting field of the detector with eq. 2.7, the next step is to calculate the expected electric current to simulate the transient behavior of the detector signals. With the assumption that the electric field E is constant and homogenous across the detector because all anodes are at the same potential, the electric field is calculated by

$$E = \frac{V}{d}, \qquad (2.8)$$

where V is the applied bias voltage and d is the thickness of the detector. Then, the velocity v of a moving charge q in the detector is calculated by

$$u = \mu_{e}E$$
 , (2.9)

where  $\mu_e$  is the mobility of the charge carriers (electrons). A typical value of  $\mu_e$  for a CZT is about 1000 cm<sup>2</sup>/(Vs) (He et al., 1998; Cho et al., 2011). With the ionization energy  $\tilde{E}_i = 4.64 \text{ eV}$  for CZT (Spieler, 2005) and the energy  $\tilde{E}_{\gamma}$  of incident radiation, the generated moving charge *q* is defined as

$$q = e \frac{\tilde{E}_{\gamma}}{\tilde{E}_{i}}$$
(2.10)

where *e* is the elementary charge. By inserting eqs. 2.7, 2.8, 2.9, 2.10 into eq. 2.3, the detector current can be numerically estimated. An example of incident radiation of  $\tilde{E}_{\gamma}$  = 511 keV is shown in fig. 2.5.

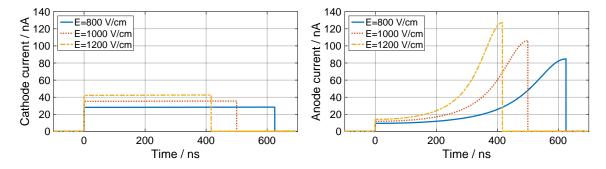


Figure 2.5.: Induced current *i* on the detector electrodes for incident  $\gamma$ -rays with an energy of 511 keV. The left plot shows the current induced on the cathode and the right plot shows the current induced on an anode pixel. The signals are calculated with eq. 2.3. An increased electric field strength *E* causes a higher current *i* with shorter drift times  $t_D$ . The generated charge remains constant.

Fig. 2.5 shows the currents flowing through the electrodes for an interaction at the cathode side of the detector. If the DOI is closer to the anode side, the total charge collection is incomplete, as the fraction of charge from the holes cannot be measured. This circumstance introduces a depth dependence and requires a correction for spectroscopic applications.

#### 2.2.5. Readout concepts

The results from fig. 2.5 give an estimate of the required sensitivity for the readout electronics. The signals are in the range of several nA (e.g. 8 nA for 100 keV at the cathode with an electric field of 1200 V/cm). In addition, the drift time  $t_D$  can be as short as 40 ns if the interaction takes place in the last 10% of the detector volume at the anode side. As the detector signals are electric currents, the simplest method for acquiring the signals is to use a shunt resistor and a measurement of voltage drop across this resistor. Finally, the measured transient voltage signal could be fed to an arbitrary signal processing system. In fact, a single resistor would be the simplest, smallest, and cheapest solution for the front-end electronics. With a resistor in the range of some M $\Omega$ , i.e. is small enough to force the detector current to flow into it, a voltage drop in the range of some mV is generated. This is sufficient for most signal processing systems. However, this concept suffers from a lack of signal bandwidth. As the left circuit in fig. 2.6 shows, the induced current will flow through  $R_S$ , but the  $R_S C_S$  time constant of the two-terminal circuit determines the bandwidth and therefore the rise time of the circuit. Moreover, the detector adds its own capacitance to the shunting elements. The -3 dB bandwidth of the output signal is given by

$$f_{-3\,dB} = \frac{1}{2\pi R_{\rm S}(C_{\rm D} + C_{\rm S})}\,,\tag{2.11}$$

where  $C_D$  is the capacitance of the detector and  $C_S$  is the shunting capacitance of the readout electronics. Even with an undersized value of 1 pF for  $C_D + C_S$  and a shunt resistor of 1 M $\Omega$ , the resulting bandwidth is only 159 kHz. However, the rise time  $t_r$  of the output pulse is proportional to the cutoff frequency  $f_c$  of a low-pass filter. For a first-order low-pass filter, the rise time from 10% to 90% of the step response is calculated by

$$t_{\rm r} = \frac{\ln(0.9) - \ln(0.1)}{2\pi f_{\rm c}} \,, \tag{2.12}$$

where  $f_c$  is the -3 dB cutoff frequency. In other words, the rise time for the given example is about 2.2  $\mu$ s. Most of the current pulses would no longer be detectable.

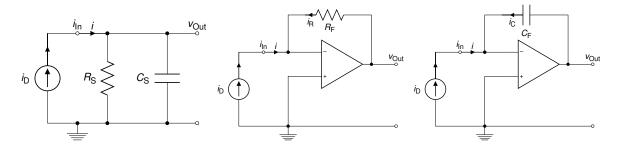
To increase the bandwidth for such a current-to-voltage converter, a transimpedance amplifier (TIA) is an appropriate solution (fig. 2.6, middle circuit). This configuration forces the generated current to flow into the negative terminal (virtual ground) of the amplifier. That means  $C_{\rm D}$  and  $C_{\rm S}$  are still present, but do not have the same impact on the time constant as the shunting readout circuit. Instead, the rise time and current-to-voltage gain of an ideal TIA are determined by the feedback network. Similarly to the example with the shunting resistor, the TIA is assumed to have a current-to-voltage gain of  $1 M\Omega$ , but the bandwidth is now limited by the parasitic capacitance of the feedback resistor  $R_{\rm F}$ . The resulting bandwidth of the TIA is about 1.6 MHz with a rise time of 220 ns, if the parasitic capacitance of the resistor is around 100 fF. This is sufficient to detect events near the cathode with drift times longer than the rise time of the TIA, but most of the events will suffer from a significant pulse amplitude loss. To further increase the bandwidth of a TIA, several methods can be applied (Brisebois, 2015), but, nevertheless, the achievable gain-bandwidth product is too low with regard to the rise times of anode signals and an adequate signal-to-noise ratio (SNR). Preserving the pulse shape by means of a current-to-voltage converter is guite challenging; however, the detector generates a charge, which can be measured with a modification of the feedback network. A single capacitor  $C_{\rm F}$  in the feedback loop of the amplifier results in a current-integrating circuit (fig. 2.6, right). Finally, the current ic through the capacitor is

$$i_{\rm D} = -i_{\rm C} = C_F \frac{dv_{\rm Out}}{dt} \,. \tag{2.13}$$

Therefore, the voltage  $v_{Out}$  at the output becomes

$$v_{\text{Out}} = \frac{-1}{C_{\text{F}}} \int i_d dt = \frac{-q}{C_{\text{F}}} + v_0.$$
 (2.14)

This results in an output voltage whose amplitude is proportional to the moving charge q generated by the detector. This circuit is referred to as the charge-sensitive amplifier (CSA).



Both the TIA and the CSA are basic negative-voltage-feedback operational amplifier circuits.

Figure 2.6.: Three different readout circuits for the conversion and amplification of the detector current  $i_D$  to a voltage  $v_{Out}$ . The left circuit converts the current to a voltage by means of a shunt resistor. The transimpedance amplifier (middle) is also used as a current-to-voltage converter, whereas the CSA (right) is used for a charge-to-voltage conversion.

As the gain of the TIA is designed with a resistor in its feedback path, the gain of the CSA is determined by its feedback capacitance. Nevertheless, a resistor has a parasitic capacitance and a capacitance also has a parasitic resistance. Thus, both circuits are adequately modeled by an operational amplifier with an *RC* feedback network. Equally importantly, an operational amplifier also introduces a shunting resistor and capacitor. These are in parallel with the impedance of the detector and abstracted by the resistor *R*<sub>D</sub> and capacitor *C*<sub>D</sub> in fig. 2.7. On the whole, the model of a CSA is a mixture of the three circuits shown in fig. 2.6.

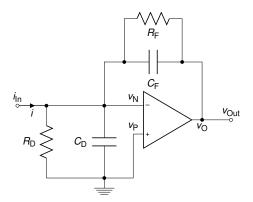


Figure 2.7.: A real configuration of a readout circuit as CSA. The shunting impedance cannot be eliminated, but the effects are reduced by the operational amplifier. The resistor  $R_F$  from the feedback path also has the parasitic capacitance  $C_F$ . This circuit will be used for further analysis.

For illustration, the simulated signal waveforms of the three readout circuits with typical values are shown in fig. 2.8. With this example, it is clearly visible that the CSA achieves the highest amplitude. As the output voltage of the CSA begins to increase at time  $t = t_0$  and reaches its peak amplitude  $v_{\text{peak}}$  at time  $t_D$  when the current flow stops, the resulting output voltage for an input current pulse with rectangle shape and constant amplitude  $i_D$  is

$$v_{\text{peak}}\left(R_{\text{F}}\right) = i_{\text{D}}R_{\text{F}}\left(1 - e^{\frac{-t_{\text{D}}}{R_{\text{F}}C_{\text{F}}}}\right).$$
(2.15)

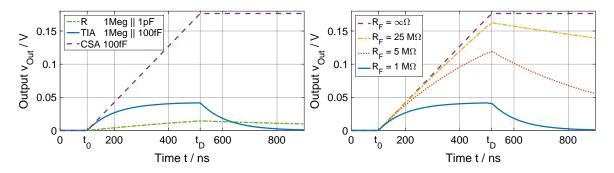


Figure 2.8.: Output signals from the three different readout circuits (left). The resistor-based current-to-voltage conversion has poor gain and bandwidth. These are improved by the transimpedance amplifier (TIA). The largest gain is achieved with the CSA. With a constant parasitic capacitance  $C_F = 100$  fF and an increased feedback resistor  $R_F$ , the TIA becomes a CSA (right).

Therefore, the maximum output voltage  $v_{max}$  for the same input signal is calculated by replacing  $R_F$  with a resistor  $R = \infty$ . Thus, we can define  $v_{peak}$  over  $v_{max}$  as the peak ratio P of the CSA with the time constant  $\tau = R_F C_F$  as

$$\frac{v_{\text{peak}}(R_{\text{F}})}{v_{\text{max}}(R)} = \frac{i_{\text{D}}R_{\text{F}}\left(1 - e^{\frac{-t_{\text{D}}}{R_{\text{F}}C_{\text{F}}}}\right)}{i_{\text{D}}R\left(1 - e^{\frac{-t_{\text{D}}}{RC_{\text{F}}}}\right)}$$
(2.16)

$$P = \lim_{R \to \infty} \frac{v_{\text{peak}}(R_{\text{F}})}{v_{\text{max}}(R)} = \frac{\tau}{t_{\text{D}}} \left(1 - e^{\frac{-t_{\text{D}}}{\tau}}\right).$$
(2.17)

Eq. 2.17 describes the attenuation of the peak amplitude. Referring to (Knoll, 2010), the degree of which the infinite time constant amplitude has been decreased is called the ballistic deficit B. With eq. 2.17, a numerical expression of B can be defined as follows:

$$B = 1 - P$$
. (2.18)

The calculations are based on the assumption that the input current pulse has a rectangular shape and a constant value during the drift time  $t_D$ , as it is seen by the cathode. If we consider a pixelated detector, where the size of the pixel is small compared to the continuous electrode and detector thickness, the current pulse as shown in fig. 2.5 only rises when the moving charge reaches the pixel electrode. However, the drift time  $t_D$  is the same as for the cathode current, but most of the charge is deposited at the end of the pulse. Thus, the anode current can be simply imagined as a rectangular pulse with a shorter drift time and higher current than the cathode signal. Consequently, the ratio of  $\tau/t_D$  is larger than the ratio of the continuous electrode, if the same CSA is used. Thus the ballistic deficit has a greater impact on the cathode signal than on anode signals. The influence of  $t_D$  and  $\tau$  is illustrated in fig. 2.9 Eq. 2.18 is essential to select an optimized feedback time constant dependent on the characteristics of the detector. Usually, the charge-to-voltage gain is a requirement imposed by the lowest measurement range of the application and is set by a carefully selected feedback capacitance (see eq. 2.14). One parameter which can be

#### 2. Analog front-end electronics

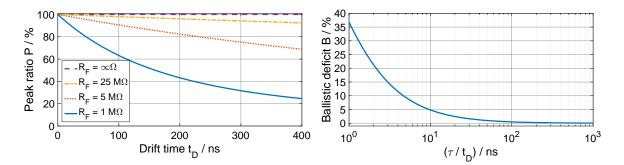


Figure 2.9.: The peak ratio *P* of the output signal from the CSA is decreased by a smaller time constant  $\tau = R_F C_F$  (left). Moreover, the ballistic deficit *B* according to eq. 2.18 depends on the time constant  $\tau$  and the drift time  $t_D$  of the moving charge (right).

optimized is the value of the feedback resistor. Of course, the best choice is a value close to infinity, since it matches the ideal CSA. However, this is practically not useful, as every current pulse from the detector forces the amplifier to integrate the charge onto its steady state voltage level. After a while, the amplifier overflows, when the output voltage reaches the level of the supply voltage (or even far below this level). Then, it cannot process another event until the feedback capacitor has discharged. A conventional method for the discharge of the capacitor uses a switch, which is added to the feedback network. This requires an additional reset logic and active control, as featured by an ASIC. A more prevalent practice is to reset the amplifier by making an appropriate choice of the feedback resistor  $R_F$ . The feedback resistor discharges the capacitor  $C_F$  with the characteristic time constant  $\tau$ . An optimally adjusted value of the resistor is a tradeoff between the ballistic deficit, count rate, and frequency response of the amplifier.

#### 2.2.6. Operational amplifier

For the design and investigation of a CSA with an operational amplifier, we will analyze the circuits in time and frequency domains. Our notation for a complex number in the frequency domain is

$$s = \sigma + j\omega$$
, (2.19)

where j is the imaginary unit,  $\sigma$  is the real part, and  $\omega$  is the imaginary part in the range of real positive values. For many circuit designs, it is useful and sufficient to analyze a network containing an operational amplifier with ideal constraints, which means the device has an infinitely high input impedance, infinite open-loop voltage gain, etc. For a realistic and detailed analysis, an operational amplifier can be modeled as shown in fig. 2.10. In contrast to the ideal operational amplifier, the input impedance  $Z_1$  is not infinite. The basic function of an operational amplifier is the amplification of the voltage drop across its positive and negative input terminals. The output voltage is therefore given by

$$v_{\rm O} = (v_{\rm P} - v_{\rm N})A$$
, (2.20)

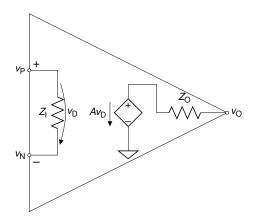


Figure 2.10.: A model of a realistic operational amplifier (Horowitz and Hill, 2015). The voltage difference  $v_D$  between the two terminals is amplified by the bandwidthlimited factor *A*. The input impedance  $Z_I$  should be as high as possible, whereas the output impedance  $Z_O$  should be close to zero.

where  $v_{P}$  is the potential at the positive terminal and  $v_{N}$  at the negative terminal. For an ideal operational amplifier, the transfer function A(s) has no frequency dependencies, so that the corresponding frequency response

$$H_{\rm A}(f) = A(0) = A_{\rm OL}$$
 (2.21)

is constant.  $A_{OL}$  is the zero-frequency open-loop gain. For a realistic operational amplifier, the frequency response of the open-loop gain has the shape of a low-pass filter. Consequently, we model the transfer function A(s) of the operational amplifier as a first-order low-pass filter in the frequency domain using

$$A(s) = \frac{A_{\rm OL}}{1+s\tau} \tag{2.22}$$

with the finite open-loop voltage gain AOL and the -3 dB cutoff frequency

$$f_c = \frac{1}{2\pi\tau} \,. \tag{2.23}$$

A characteristic parameter which simplifies the frequency response of an operational amplifier is the gain-bandwidth product (GBP). At frequencies larger than  $f_c$ , the GBP is constant for the first-order low-pass filter frequency response. At the frequency  $f_{GBP}$ , the open-loop gain equals the unity gain. With the maximum open-loop voltage gain  $A_{OL}$ , the functional relationship is given by

$$f_c A_{\rm OL} = f_{\rm GBP} \,. \tag{2.24}$$

With the given parameters  $A_{OL}$  and  $f_{GBP}$  of an operational amplifier, we can set

$$\tau = \frac{A_{\rm OL}}{2\pi f_{\rm GBP}} \tag{2.25}$$

and by setting  $s = j\omega$  with  $\omega = 2\pi f$ , the transfer function of the open-loop gain for periodic sinusoidal signals is

$$A(j2\pi f) = \frac{A_{OL}}{1 + j\frac{f}{f_{GRP}}A_{OL}}.$$
(2.26)

The frequency-dependent gain  $G_A(f)$  and phase shift  $\Phi_A(f)$  are derived from eq. 2.26.

$$G_{A}(f) = |A(j2\pi f)| = \frac{A_{OL}}{\sqrt{1 + \left(\frac{f}{f_{GBP}}A_{OL}\right)^{2}}}$$
 (2.27)

$$\Phi_{A}(f) = \angle A(j2\pi f) = -\tan^{-1}\left(\frac{f}{f_{GBP}}A_{OL}\right)$$
(2.28)

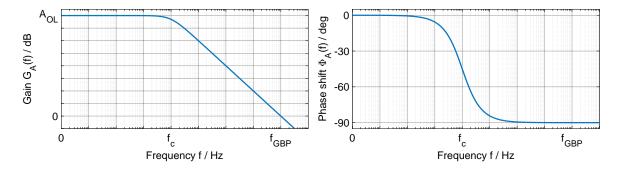


Figure 2.11.: The gain  $G_A(f)$  and the phase shift  $\Phi_A(f)$  of the frequency response of an operational amplifier with zero-frequency gain  $A_{OL}$ . The frequency response is modeled as a first-order low-pass filter with the cutoff frequency  $f_c = f_{GBP}/A_{OL}$ . The attenuation of  $A_{OL}$  is -20 dB/decade above  $f_c$ .

#### 2.3. Circuit design of a charge-sensitive amplifier

As briefly described in sec. 2.2.5, an operational amplifier integrates a charge, as there is a capacitor  $C_F$  in the feedback and the time constant of the  $R_F C_F$  network is large compared to the drift time of the moving charge at its input. In general, a two-terminal equivalent circuit adequately represents the feedback circuit. Thus, the  $R_F C_F$  network is summarized with the impedance  $Z_F$ . In accordance with fig. 2.2, the impedance of the detector is modeled with the two-terminal impedance  $Z_D$ . Both impedances are connected to the inverting terminal of the operational amplifier. In addition, there are parasitic shunt impedances between the negative and the positive input terminal of the operational amplifier. As they are in parallel with the impedance of the detector, all additional shunt impedances are absorbed by the model of  $Z_D$ . For a simplified circuit analysis, the positive terminal of the operational amplifier is grounded. If the operational amplifier requires a single supply operation, the positive terminal is biased towards the desired potential for the virtual ground. For a circuit analysis, this is negligible. Therefore, the initial circuit for the analysis is shown in fig. 2.12.

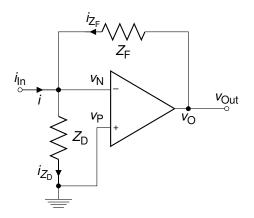


Figure 2.12.: An operational amplifier with a current input and a voltage feedback connected to the negative input terminal. With  $Z_F = C_F ||R_F$ , this configuration is used for the charge-to-voltage amplification.  $Z_D$  represents all impedances connected between the input  $i_{In}$  and ground (e.g. capacitance of the detector and parasitic capacitance of the negative input terminal).

#### 2.3.1. Circuit analysis

Kirchhoff's current law is used for the fundamental circuit analysis. For the node at the negative terminal of the operational amplifier, the sum of all currents must be zero.

$$0 = i - i_{Z_{\rm D}} + i_{Z_{\rm F}} \tag{2.29}$$

Further, the currents can be expressed in terms of the feedback impedance  $Z_F$  and shunt impedance  $Z_D$ .

$$i_{Z_{\rm F}} = \frac{(v_{\rm O} - v_{\rm N})}{Z_{\rm F}}$$
 (2.30)

$$i_{Z_{\rm D}} = \frac{v_{\rm N}}{Z_{\rm D}} \tag{2.31}$$

According to eq. 2.20, and setting  $v_{\rm P}=0$ , the voltage  $v_{\rm N}$  in eqs. 2.30 and 2.31 can be substituted with

$$v_{\rm N} = \frac{-v_{\rm O}}{A} \,. \tag{2.32}$$

The solution of eq. 2.29 with eqs. 2.30-2.32 results in the current-to-voltage transfer function

$$\frac{-v_{\rm O}}{i} = \frac{AZ_{\rm D}Z_{\rm F}}{Z_{\rm D}(A+1) + Z_{\rm F}},$$
(2.33)

where *A* is the transfer function of the operational amplifier according to eq. 2.22. Another useful method for the circuit analysis is the principle of superposition. Because the system can be described with linear equations, a superposition of all current and voltage sources is possible. That means that first, if the current source for *i* is turned off (replaced with an open circuit), the voltage source for  $v_0$  is acting alone and the resulting voltage at the negative

input terminal of the operational amplifier is

$$v_{\rm N}|_{i=0} = v_{\rm O} \frac{Z_{\rm D}}{Z_{\rm D} + Z_{\rm F}}$$
 (2.34)

Second, if the voltage source  $v_0$  is turned off (replaced with a short), the current source is acting alone and the voltage at the negative input terminal of the operational amplifier is

$$v_{\rm N}|_{v_{\rm O}=0} = i \frac{Z_{\rm D} Z_{\rm F}}{Z_{\rm D} + Z_{\rm F}}$$
 (2.35)

Finally, the superposition for the voltage node  $v_N$  is the sum of eqs. 2.34 and 2.35

$$v_{\rm N} = v_{\rm N}|_{i=0} + v_{\rm N}|_{v_{\rm O}=0}$$
 (2.36)

By inserting eqs. 2.32, 2.34 and 2.35 into eq. 2.36, the output voltage  $v_0$  can be rewritten as

$$v_{\rm O} = -A\left(v_{\rm O}\frac{Z_{\rm D}}{Z_{\rm D}+Z_{\rm F}} + i\frac{Z_{\rm D}Z_{\rm F}}{Z_{\rm D}+Z_{\rm F}}\right).$$
 (2.37)

The derived eq. 2.37 is the same as eq. 2.33, but shows the terms for the basic block structure shown in fig. 2.13 in an intuitive and easily readable form.

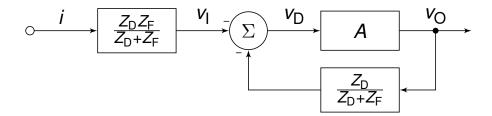


Figure 2.13.: Block diagram of the closed-loop transfer function with input current *i* and output voltage  $v_0$ . This structure shows the voltage feedback network and loop gain, which are essential for a stability analysis. The voltage difference  $v_D$  at the negative input terminal of the amplifier is the sum of the feedback voltage and an input current-dependent voltage  $v_1$ .

The basic block structure is used to identify the voltage feedback and the stability of the loop. It is obvious that the feedback network  $\beta$  is

$$\beta = \frac{Z_D}{Z_D + Z_F} \tag{2.38}$$

and the closed-loop voltage gain G is only (Horowitz and Hill, 2015)

$$G = \frac{A}{1 + A\beta} = -\frac{v_0}{v_1} \,. \tag{2.39}$$

The closed-loop gain *G* becomes  $1/\beta$  if the loop gain  $A\beta$  is much greater than one. On the contrary, the closed-loop gain becomes infinite if the loop gain  $A\beta = -1$ . At this frequency, the system tends to be unstable, and oscillates. As *A* and  $\beta$  are defined to have positive real values, the case where the loop gain becomes -1 only occurs if the loop gain is 1 and a phase

shift of 180° is introduced by the frequency response of  $A\beta$ . A phase shift of a sinusoidal signal with 180° is equal to a multiplication with -1. To make the feedback circuit stable, the phase shift therefore has to be less than 180° at the frequency  $f_i$ , where the loop gain is 1. A sufficient phase margin at the frequency  $f_i$  is required to ensure a stable operation over the entire temperature range, and also to cover tolerances of the used integrated circuits. The

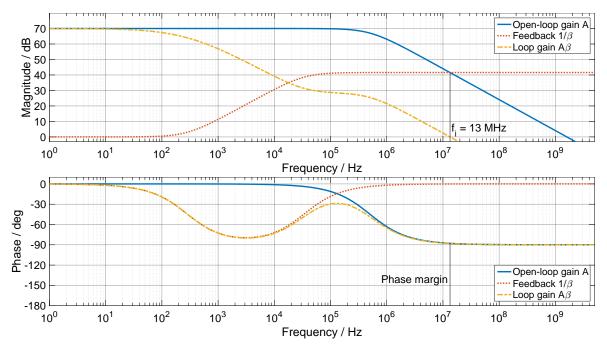


Figure 2.14.: The Bode plot of the closed-loop transfer function. The loop gain is 1 at the frequency  $f_i$  with a phase margin of roughly 90°. This is sufficient for a stable operation. See text for numerical values of the example.

example in fig. 2.14 is calculated with the parameters from table 2.1 for the OPA657. The feedback impedance was chosen to be 100 fF in parallel with  $25 M\Omega$ . Further, the detector capacitance of 6.7 pF in parallel with a  $50 G\Omega$  resistor and the parasitic input capacitance of 5.2 pF are the shunting impedance  $Z_D$ . The stability is investigated at the frequency, where the loop gain is 1. Mathematically, this intersection can be calculated by solving

$$|A(j2\pi f)\beta(j2\pi f)| = 1.$$
(2.40)

From the Bode plot shown in fig. 2.14, this point can be found at

$$\log_{10}(|A|) - \log_{10}(|\frac{1}{\beta}|) = 0.$$
(2.41)

In this plot, the reciprocal of the feedback network  $\beta$  intersects the open-loop gain at approximately 13 MHz. At this frequency, the phase shift of the loop gain is 88°, resulting in a phase margin  $\phi$  of 92°, which is sufficient for a stable operation. The phase margin  $\phi$  is calculated by

$$\phi = 180^{\circ} - (\angle A(2\pi f_i) + \angle \beta(2\pi f_i)) .$$
(2.42)

Besides the stability of the circuit, the effective input impedance also depends on the open-

loop voltage gain of the amplifier, and is changed by the feedback network (Horowitz and Hill, 2015). The effective input impedance of the CSA should be very low, to make sure that all current flows into the amplification circuit. Regarding fig. 2.12, the effective input impedance from the CSA is defined by the ratio of the voltage at the input terminal  $i_{ln}$  and the input current *i*. The voltage at the input terminal is  $v_N$ , so the effective input impedance  $Z_i^*$  is

$$Z_{\mathsf{I}}^* = \frac{v_{\mathsf{N}}}{i} \,. \tag{2.43}$$

Solving eq. 2.29 for  $v_N$  with eqs. 2.30-2.32,  $Z_I^*$  can be expressed as

$$Z_{\rm I}^* = \frac{Z_{\rm D} Z_{\rm F}}{Z_{\rm D} (A+1) + Z_{\rm F}} \,. \tag{2.44}$$

As *A* becomes infinitely large, as we assume for an ideal operational amplifier, the effective input impedance is zero. This satisfies the principle of the virtual ground. If we assume an ideal detector without a parasitic impedance from the CSA,  $Z_D$  is infinite. For this case, the effective input impedance

$$\lim_{Z_{\mathsf{D}}\to\infty} Z_{\mathsf{I}}^* = \frac{Z_{\mathsf{F}}}{A+1}$$
(2.45)

is determined by the open-loop gain and the feedback network  $Z_F$ . To make sure that all current flows into the CSA and is integrated on the feedback capacitor, the impedance from eq. 2.45 must be small compared to the shunting impedance  $Z_D$ . Consequently, an operational amplifier with a large open-loop gain is required.

#### 2.3.2. Charge-to-voltage transfer function

The CSA is designed to measure a charge with a voltage output signal, where the peak amplitude is proportional to the charge seen at the input. The relation of the output voltage  $v_0$  to the input charge Q is the charge-to-voltage transfer function

$$\frac{v_{\rm O}}{Q} = H_{\rm Q} \,. \tag{2.46}$$

As the charge is defined to be

$$\frac{dQ}{dt} = i, \qquad (2.47)$$

where *i* is the electrical current, the corresponding Laplace transformation of eq. 2.47 is

$$\mathcal{L}\left\{Q'(t)\right\} = sQ(s) = i(s).$$
(2.48)

If we replace *i* in the current-to-voltage transfer function from eq. 2.33 by eq. 2.48 and set the feedback network  $Z_{\rm F} = \frac{1}{sC_{\rm F}}$  and parasitic impedance  $Z_{\rm D} = \frac{1}{sC_{\rm D}}$  to single capacitors, then the charge-to-voltage transfer function is given by

$$H_Q = \frac{-A}{C_D + C_F(A+1)}.$$
 (2.49)

The simplification of the impedances to single capacitors is valid, as we want to investigate the frequency response of the charge-to-voltage conversion. If the feedback time constant is chosen appropriately according to eq. 2.18, there is no significant peak amplitude loss in the voltage signal. The peak amplitude of the voltage signal is therefore independent of the feedback resistor  $R_F$ . The impedance  $Z_D$  can also be simplified for this analysis, as the equivalent input resistance of the amplifier and detector is much larger than the effective input impedance according to eq. 2.44. For an ideal operational amplifier with infinite openloop gain A,  $H_Q$  from eq. 2.49 becomes  $-1/C_F$  over the entire frequency domain. Since the open-loop voltage gain of an operational amplifier is not independent of frequency, a more realistic charge-to-voltage transfer function is obtained by replacing *A* from eq. 2.49 by eq. 2.22

$$H_Q(s) = \frac{-A_{\rm OL}}{C_{\rm D} + C_{\rm F}(A_{\rm OL} + 1) + s\tau(C_{\rm D} + C_{\rm F})} \,.$$
(2.50)

The charge-to-voltage gain is then given by

$$|H_Q(j2\pi f)| = G_Q(f) = \frac{A_{OL}}{\sqrt{(C_D + C_F(A_{OL} + 1))^2 + \left(\frac{f}{f_{GBP}}A_{OL}(C_D + C_F)\right)^2}}.$$
 (2.51)

The steady state of the system is derived by calculating the zero-frequency gain:

$$G_{\rm QS} = \lim_{f \to 0} G_{\rm Q}(f) = \frac{A_{\rm OL}}{C_{\rm D} + C_{\rm F}(A_{\rm OL} + 1)}.$$
 (2.52)

Eq. 2.52 shows that a finite open-loop gain  $A_{OL}$  attenuates the measured peak voltage. The measured fraction of charge as a ratio of  $G_{QS}$  over the ideal charge-to-voltage gain  $1/C_{F}$  is given by

$$G_{\rm QS} C_{\rm F} = \frac{A_{\rm OL}}{\frac{C_{\rm D}}{C_{\rm F}} + A_{\rm OL} + 1}$$
 (2.53)

It is obvious that the measured peak amplitude is decreased by an increased detector capacitance  $C_D$  or a reduced open-loop voltage gain  $A_{OL}$ . Thus, the operational amplifier should provide a high and stable open-loop voltage gain for an improved system performance as illustrated in fig 2.15.

An equally important parameter of the CSA is the rise time of the output voltage as a reaction of a charge step at its input. As shown by eq. 2.12, the rise time is proportional to the cutoff frequency of the system. Therefore, to determine the rise time, we have to calculate its cutoff frequency, which is derived by using eq. 2.51

$$\frac{|H_Q(j2\pi f_c)|}{|H_Q(0)|} = \frac{1}{\sqrt{2}}$$
(2.54)

$$f_{\rm c} = f_{\rm GBP} \frac{\frac{C_{\rm D}}{C_{\rm F}} + (A_{\rm OL} + 1)}{A_{\rm OL}(\frac{C_{\rm D}}{C_{\rm F}} + 1)}.$$
 (2.55)

Eq. 2.55 shows, that the upper bandwidth limit is lowered with an increased fraction of the shunting capacitance  $C_D$  over  $C_F$ . The bandwidth is extended with a lower value of  $A_{OL}$ , but

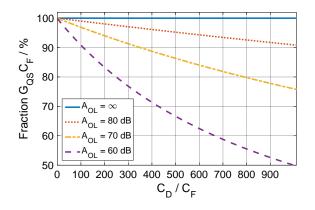


Figure 2.15.: The measured fraction of charge dependent on the ratio of input capacitance  $C_{\rm D}$  over feedback capacitance  $C_{\rm F}$ . The fraction is increased with an increased zero-frequency open-loop voltage gain  $A_{\rm OL}$  of the operational amplifier.

this will reduce the measured fraction of charge, as shown in fig. 2.15. This also means that the SNR is decreased and therefore the resolution of the charge measurement is also decreased.  $A_{OL}$  should therefore be as large as possible. For this assumption, the bandwidth of the CSA is

$$\lim_{A_{\rm OL}\to\infty} f_{\rm c} = \frac{f_{\rm GBP}}{\frac{C_{\rm D}}{C_{\rm c}} + 1} \,. \tag{2.56}$$

Eq. 2.56 shows that the cutoff frequency and therefore the rise time of the CSA are directly proportional to the gain-bandwidth product of the amplifier. The response of the transfer function from eq. 2.49 to a unit step is shown in fig. 2.16.

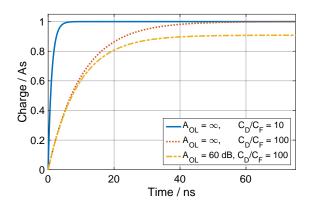


Figure 2.16.: The output signal of the CSA with a unity step at its input. The rise time is increased with an increasing ratio of  $C_D/C_F$ . The gain-bandwidth product of the operational amplifier was assumed to be  $f_{GBP} = 1$  GHz. In this example, the rise times are 17 ns (solid line), 32 ns (dashed line) and 35 ns (dotted line). The decreased rise time related to a smaller open-loop gain  $A_{OL}$  is caused by the attenuation of the peak amplitude.

#### 2.3.3. Input coupling of the CSA

As shown in fig. 2.3, at least one of the electrodes is biased at a negative high voltage. Thus, the amplifier at the cathode must be protected against the bias voltage, since it is not common for integrated circuits to operate at high input voltages in the range of several hundreds of volts. A capacitor in series to the amplifier input blocks the bias voltage of the detector, but allows the detector current to flow. This electrical circuit is shown in fig. 2.17, where the coupling capacitor  $C_B$  is represented by the impedance  $Z_B$ . In this configuration, the parasitic impedance  $Z_I$  of the operational amplifier and the detector impedance  $Z_D$  are separated by the impedance  $Z_B$ . As the influence of the coupling capacitor is not apparent at

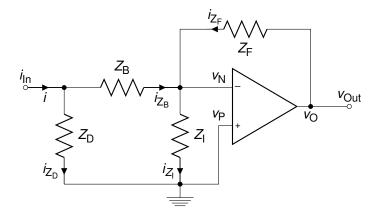


Figure 2.17.: A functionally equivalent configuration to fig. 2.12 for the charge-to-voltage amplification. The input impedance at the terminal  $i_{ln}$  is a Pi network instead of the single impedance  $Z_D$ .  $Z_B$  is a high-voltage coupling capacitor to protect the low-voltage input terminals of the operational amplifier.

first glance throughout the equation for the current-to-voltage transfer function, a calculation with an infinite loop gain turns out the simplified equation

$$\frac{-v_{\rm O}}{i} = \frac{Z_{\rm D} Z_{\rm F}}{Z_{\rm B} + Z_{\rm D}} \,. \tag{2.57}$$

To eliminate the influence of the impedance  $Z_B$ , it must be much smaller than  $Z_D$ . In the case that all impedances are represented by a single capacitor, the steady-state charge-to-voltage gain is ultimately given by

$$\frac{v_{\rm O}}{Q} = \frac{1}{C_{\rm F} \left(1 + \frac{C_{\rm D}}{C_{\rm B}}\right)}.$$
(2.58)

Eq. 2.58 shows that the coupling capacitor must be much larger than the detector capacitance to avoid a peak amplitude loss, thus improving the SNR.

#### 2.3.4. Noise

As has been pointed out, the noise caused by the electronics and the corresponding SNR characterize the quality of the CSA with respect to the achievable energy resolution. The precision of the charge measurement and the achievable timing are directly related to the SNR. Regarding both values, the front-end electronics should not limit the intrinsic resolution of the detector. Thus, to reduce the electronics noise, the noise sources of the detector system must be identified in the first step. The main source for the electronics noise is the operational amplifier. But the passive components of an electronic circuit also generate noise. The resulting noise at the output is the sum of all noise sources at the input, amplified by the noise gain. The essential noise sources of the electronics circuit are shown in fig. 2.18. The noise contribution of the operational amplifier is simplified to a model, where its noise is

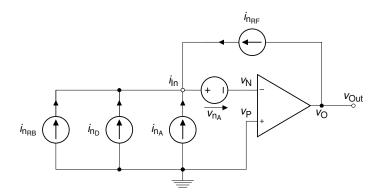


Figure 2.18.: The CSA configuration with its dominant noise sources at the input terminal  $i_{\text{In}}$ . For noise analysis, the current noise sources from the bias resistor  $(i_{n_{\text{RB}}})$ , equivalent resistor of the detector  $(i_{n_{\text{D}}})$ , feedback resistor  $(i_{n_{\text{RF}}})$ , and amplifier  $(i_{n_{\text{A}}})$  can be substituted by a single source, as they act in parallel. Finally, the noise analysis is made with a current noise source  $i_{n}$  in parallel (parallel noise) and a voltage noise source  $v_{n}$  in series (series noise) to the input terminal.

characterized by an equivalent voltage noise source  $v_{n_A}$  and current noise source  $i_{n_A}$  at the inverting input terminal. Additionally, all resistors in the system contribute to the total noise by adding thermal noise. The equivalent noise current of a resistor  $i_{n_R}$  at temperature T is given by

$$i_{\rm nR} = \frac{4kT}{R} \tag{2.59}$$

where k is the Boltzmann constant (Spieler, 2005) and the unit of noise current is  $A^2/Hz$ . The equivalent resistor  $R_D$  of the detector, the biasing resistor  $R_B$ , and the equivalent resistor  $R_F$  of the feedback impedance contribute to the total noise following eq. 2.59. Each is represented by a current source in fig. 2.18. As there are multiple noise sources in the system, they can all be absorbed into a single source for the voltage noise  $v_n$  and a single source for the current noise  $i_n$  with

$$i_{n} = i_{n_{RD}} + i_{n_{RB}} + i_{n_{RF}} + i_{n_{A}}$$
, (2.60)

as they are connected in parallel. Voltage noise sources can be summed, as they are connected in series. As we assume that all noise sources have a flat frequency spectrum (white noise), the resulting noise spectral density at the output is shaped by the noise transfer function (noise gain).

The noise spectral density is usually expressed in units of  $nV/\sqrt{Hz}$ . With regard to the block diagram of the closed-loop transfer function from fig. 2.13, the current noise source is amplified by the current-to-voltage transfer function, whereas the voltage noise source is amplified by the closed-loop voltage gain. As illustrated in fig. 2.19, the input noise current

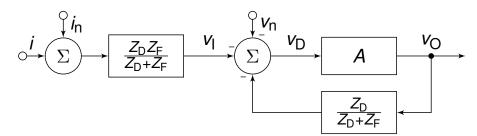


Figure 2.19.: Block diagram of the closed-loop transfer function with additional current noise source  $i_n$  and voltage noise source  $v_n$ . Both sources act at the negative input terminal of the operational amplifier but have different transfer functions.

 $i_n$  to output noise voltage  $v_{On}$  transfer function  $G_{in}$  is given by eq. 2.33 and the input noise voltage  $v_n$  to  $v_{On}$  transfer function  $G_{vn}$  is given by eq. 2.39.

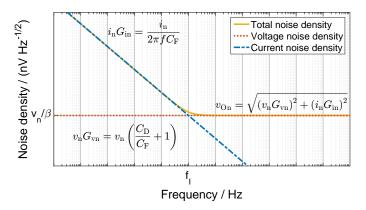


Figure 2.20.: The noise spectral density log-log plot for the ideal CSA with the capacitor  $C_F$  in the feedback network and the capacitor  $C_D$  at its input terminal. In the lower frequency range, the noise density is dominated by the current noise. Above the frequency  $f_l$ , the portion of the voltage noise dominates the total noise density  $v_{On}$ .

For a clear view on the components of the resulting spectral noise density in fig. 2.20, the calculations are based on the ideal model of the CSA. Fig. 2.20 shows the contribution of the input current noise, which has a typical 1/f shape and dominates the low frequency range (referred to as flicker noise). The noise density in the upper frequency range is dominated by the input voltage noise but remains flat. If the open-loop gain is sufficiently large, the voltage noise is amplified by the factor  $1/\beta$ . A more realistic noise spectral density is shown in fig. 2.21. This illustration emphasizes the impact of the feedback resistor  $R_F$  and the gain-

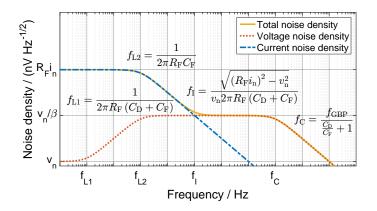


Figure 2.21.: The noise spectral density log-log plot for the CSA with the resistor  $R_F$  and capacitor  $C_F$  in the feedback network and the capacitor  $C_D$  at its input terminal. The frequency response of the current noise density has the shape of a low-pass filter, whereas the voltage noise density has the response of a band-pass filter. The time constant  $\tau = R_F C_F$  determines the low cutoff frequency  $f_{L2}$  of both responses. The voltage noise density is limited by the corner frequency  $f_c$ . Thus, the total noise density is bounded over the whole frequency range.

bandwidth product  $f_{GBP}$ . The density of the flicker noise is limited in its upper value. It has the shape of a typical low-pass filter, which is determined by the time constant  $R_F C_F$  of the feedback impedance. The component related to the voltage noise has the spectral density of a band-pass filter, where the upper-corner frequency  $f_c$  is limited by the gain-bandwidth product and the detector capacitance  $C_D$ . At zero frequency, the voltage noise is limited to the value of  $v_n$ .

The noise spectral density determines the root-mean-square amplitude of the output noise (rms noise) over a given bandwidth. As the noise spectral density is bounded over the entire frequency range, the total rms noise  $V_{\rm rms}$  is calculated by

$$V_{\rm rms} = \sqrt{\int_0^\infty \left[ \left( v_{\rm n} \, G_{\rm vn} \right)^2 + \left( i_{\rm n} \, G_{\rm in} \right)^2 \right] df} \tag{2.61}$$

An additional signal processing with filters (pulse shapers) must be adapted in accordance to the noise spectral density due to optimal results regarding the SNR.

## 2.4. Implementation and Test

A major design guideline was the reduction of the total amount of components per readout channel. This could be achieved using a single operational amplifier to build the CSA without additional gain stages. Nevertheless, this circuit covers the desired measurement range and does not need any pulse shapers for its basic operation. In order to fulfill the requirements, the detector system must process energies up to 7 MeV and should be compatible with a digitizer system with an input voltage range of 2 V. According to eq. 2.10 and the steady-state charge-to-voltage gain of the CSA from eq. 2.52, the feedback capacitance  $C_F$  should

Table 2.1.: A list of suitable commercial off-the-shelf operational amplifiers from different vendors. They are compared in terms of our selection criteria: gain-bandwidth product ( $f_{GBP}$ ), zero-frequency open-loop gain ( $A_{OL}$ ) and input impedance ( $Z_{I}$ ).  $v_{n}$  is the equivalent voltage noise source and  $i_{n}$  the current noise source. All values are extracted from the datasheets (Analog Devices, 2013; Linear Technology, 2014, 2015; Texas Instruments, 2015).

Vendor	Product	$f_{\rm GBP}/\rm MHz$	$A_{\rm OL}/{\rm dB}$	$Z_{I}  /  \Omega      pF$	$v_{\rm n}/{\rm nV\over \sqrt{\rm Hz}}$	$i_{\rm n}/\frac{{\rm fA}}{\sqrt{{\rm Hz}}}$
Analog Devices	ADA4817	1050	65	0.5 T    1.4	4	2.5
Linear Techn.	LTC6268	500	108	0.5 T    0.55	4.3	5.5
Linear Techn.	LTC6268-10	4000	108	0.5 T    0.55	4.0	7.0
Texas Instrum.	OPA657	1600	70	0.5 T    5.2	4.8	1.3

be at least 120 fF. A higher gain is acceptable, since an attenuation of the pulse amplitude can be made with less effort. Since the feedback capacitance and the electrical characteristics of the detector are fixed design parameters, the operational amplifier and the feedback resistance  $R_{\rm F}$  are the remaining components for an optimization of the readout circuit. As a first step, the drift time  $t_D$  of the moving charge must be taken into account to choose an appropriate value of the time constant  $\tau$  of the CSA. With regard to the induced currents through the anode and cathode shown in fig. 2.5, we expect rise times up to 450 ns for the cathode and up to 200 ns for the anode. Because the anode signals carry the information used for spectroscopy, the ballistic deficit from eq. 2.18 should be minimized. A value of 1% is appropriate. Therefore, the ratio  $\tau/t_{\rm D}$  must be greater than 50, which corresponds to a value of 10  $\mu$ s for the time constant  $\tau$  or 83 M $\Omega$  for the feedback resistor  $R_{\rm F}$  (the nearest matching part has 82 MΩ). With a loose constraint of 5 % ( $\tau/t_{\rm D} = 9.66$ ) for the ballistic deficit on the cathode signal, we selected a feedback resistor of  $47 M\Omega$  for that readout channel. For the implementation of the CSA with a COTS operational amplifier, we establish four important features for the parametric selection. First, the operational amplifier must have a large input impedance, which is achieved by operational amplifiers with a field-effect transistor at their input terminals. Therefore, the voltage noise gain of the feedback network  $(1/\beta)$  is reduced. Second, it must have a high open-loop voltage gain, so that the effective input impedance regarding eq. 2.44 is minimized. The third feature is that the equivalent voltage and current noise must be very low, and the fourth is that the operational amplifier must have a sufficient large gain-bandwidth product to satisfy the requirements of rise time. In a pulsed beam application, the achievable timing of the detector signals is important, and must be further investigated. For an advanced analysis, the rise time should not be limited and therefore the gain-bandwidth product should be as large as possible. In table 2.1, we list some COTS operational amplifiers from different vendors, which match the selection criteria. All operational amplifiers fulfill the requirements of a high input impedance and a relatively large gain-bandwidth product. For the first tests, we choose the OPA657 because of its larger open-loop gain and bandwidth in comparison to the ADA4817. For the second part, we choose the LTC6268-10 because of its outstanding parameters and very low parasitic capacitance. Both operational amplifiers are available in an almost pin compatible package. Consequently, the evaluation procedure can be done with the same hardware. After a first attempt with the LTC6268, which runs on our OPA657 PCB layout, we decided to populate the hardware with the LTC6268-10. Unfortunately, this circuit tends to oscillate, as also observed in (Brisebois, 2015). Thus the implementation of the CSA with the OPA657 shows the best performance for the first prototype (see fig. 2.22).

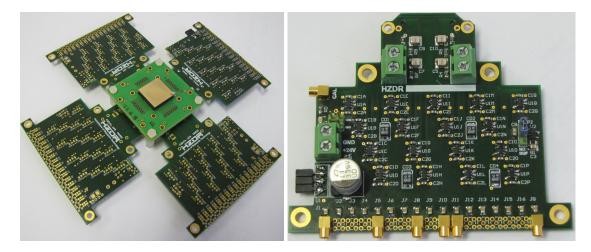


Figure 2.22.: The detector assembly with unpopulated readout boards (left) and a close-up of a populated printed circuit board with 17 CSAs (right).

Despite the higher parasitic input capacitance, a great advantage of the OPA657 over the LTC6268 is its wide supply voltage range of 12 V. This provides a larger headroom for pulse pile-ups until the output voltage of the amplifier saturates. Thus, this amplifier is best-suited for high count rates with high energies. Nevertheless, the count rate capability mainly depends on the signal processing system, which is limited by the input voltage range of the digitizer and the digital pulse processing system (Abbene and Gerardi, 2015). Moreover, any rate limitations must be determined by field experiments. The readout board contains 17 CSAs, where one channel is equipped for the readout of the cathode. All necessary functionality, including low voltage power supply, high voltage filters, biasing, and decoupling capacitors, is included on the PCB. Each CSA has a passive low-pass filter at its output. The bandwidth is limited to 21.654 MHz (16.15 ns rise time), which is sufficient for the CZT detector signals. One channel of the readout board is used for the evaluation of the CSA and contains a test input circuit. This circuit consists of a termination resistor and a series capacitor of 0.3 pF±0.05 pF for the charge injection in accordance with (Knoll, 2010). The theoretical performance parameters of the CSA are listed in table 2.2. These values are estimates and will be verified with experimental results.

## 2.5. Results

The readout board is evaluated with the test pulse input and with the detector shown in fig. 2.22. The test pulse input is sourced by a signal generator with a step voltage input. According to the current-voltage relation of a capacitor, the step voltage applied to test input

Parameter	Cathode	Anode (Pixel)	Test input			
$R_{\rm F} \parallel C_{\rm F}$	47 MΩ    100 fF	82 MΩ    100 fF	82 MΩ    100 fF			
$C_{\rm D}/C_{\rm F}$	119	53.05	55			
Ballistic deficit	95.36 %	98.79%	99.9%			
Charge fraction	96.34 %	98.32%	98.26%			
Steady-state gain	9.19 V/pC	9.71 V/pC	9.82 V/pC			
Cutoff frequency	13.84 MHz	30.11 MHz	29.08 MHz			
Rise time	25.27 ns	11.61 ns	12.03 ns			
Noise level (rms)	2.62 mV	1.82 mV	1.80 mV			
Noise level (rms), BW limited	2.01 mV	1.26 mV	1.21 mV			
ENC (rms), BW limited	1365 e	810 e	769 e			
Peak amplitude at 511 keV in CZT	162.11 mV	171.38 mV	173.33 mV			

Table 2.2.: Numerical estimation of electrical characteristics of different readout channels. The calculations refer to a CSA based on the OPA657. The value of equivalent noise charge (ENC) is given in units of the elementary charge e.

capacitor generates a current flowing into the CSA. With a varying shape of the voltage signal, arbitrary detector signals can be synthesized.

### 2.5.1. Test pulse input

The most important feature of front-end electronics is the noise performance. There are various methods to analyze the noise of a linear time-invariant (LTI) system like the CSA. A common method is described in the IEEE Std 1241-201, referred to as "Sine-wave testing and fitting" (IEEE, 2011). It is known that the response of an LTI system to a pure sinewave is a sine-wave with the same frequency, but potentially different amplitude and phase. Tests with a sine-wave have the advantage that the waveform can be generated very accurately and the interpretation is done by standardized instruments and tools. If a sine-wave is applied to the input, the output depends on the transfer function of the system and is superimposed with noise. The input sine-wave is derived a posteriori by a four-parameter sine-wave fit, and the residual is the noise level, as described in (IEEE, 2011). For this purpose, the waveform is captured by our digitizer with 100 MSPS and 14 bit resolution. The digitizer board provides an SNR of 71.3 dB at around -1 dB of full scale input (2.3 V<sub>pp</sub>). This corresponds to an rms noise level of 201  $\mu$ V. These values are measured at an input frequency of 2 MHz. With the same setup, the sine-wave test with the test input of the CSA results in an rms noise level of 1.24 mV. The result is shown in fig. 2.23. This is in accordance with the predicted values in table 2.2 for the test input.

Moreover, other tests were made to validate the pulse shape at the output of the CSA. The signal generator was set up to generate a step input with a rise time of approximately 8 ns. Fig. 2.24 shows the changes in output signals for a varying amplitude  $A_T$  of the test pulse input. The fit of the peak heights to the input amplitude shows an excellent linearity, with a maximum deviation of 2 mV. The linear fit results in a gain of 3.05, which correlates with the ratio of test input capacitance over feedback capacitance.

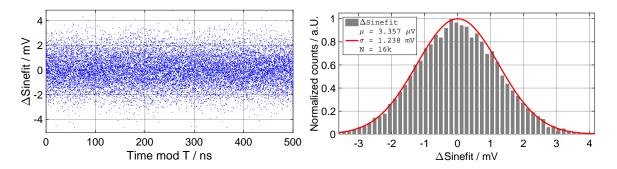


Figure 2.23.: Noise measurement of the CSA with a sine-wave test. A 2MHz sine-wave was applied to the test input and the difference between the output signal and a four-parameter sine-wave fit ( $\Delta$ Sinefit) is plotted against the period *T* of the sine-wave (left). All harmonic distortions were eliminated by the test procedure, revealing the noise level (standard deviation  $\sigma$  of  $\Delta$ Sinefit, right). Over *N*=16k samples were used.

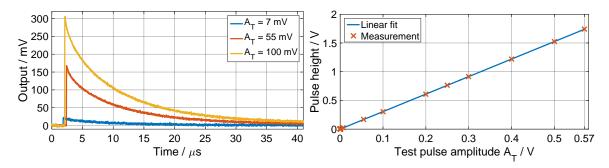


Figure 2.24.: A measurement with different rectangular pulses with amplitudes A<sub>T</sub> injected to the test input capacitance. The output signal was recorded with the 100 MSPS digitizer (left). Each pulse height is the rms value of 1k events (right). The linearity error is in the range from -2 mV to 1.5 mV.

The test pulse input was also used to evaluate the timing capabilities of the CSA. As before, the signals were captured with the digitizer and processed offline. The test pulses were synchronized to a known timing reference signal (sine-wave signal). The timing performance was obtained from the difference between the zero crossing point of the sine-wave reference and a digital constant fraction trigger (fraction = 0.2) on the output signal of the CSA. Both timestamps were calculated by software. The results of the timing measurement are shown in fig. 2.25. At high signal amplitudes, the timing performance is in the range of several picoseconds (32 ps standard deviation). This value increases with lower signal amplitudes, because the SNR decreases as well. This rough estimate of timing performance shows that the CSA does not limit the timing performance of the CZT detector, which is in the range of some nanoseconds (Meng and He, 2005).

### 2.5.2. Pixel detector

The performance of the CSA is finally evaluated with the Redlen pixel detector and the hardware shown in fig. 2.22. The detector is biased at -600 V and the signals are captured

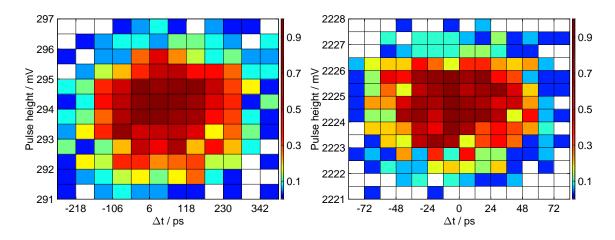


Figure 2.25.: Results of about 1k timing measurements with the 100 MSPS digitizer. The pulse height is the maximum value of the output from the CSA. The time  $\Delta t$  is the difference between the pulse trigger and the timing reference from the signal generator. The standard deviation of the measurements are  $\sigma_x = 131$  ps and  $\sigma_y = 1.15$  mV for the left plot and  $\sigma_x = 32$  ps and  $\sigma_y = 1.21$  mV for the right plot. The colormap is in logarithmic scale.

with the digitizer. The waveforms of a single pixel and the cathode are stored for an offline analysis. The first test should measure the energy on both electrodes dependent on the DOI. As shown in fig. 2.4, the relationship between the DOI and the weighting potentials should be experimentally verified with data from the detector. The DOI is measured either by the ratio of cathode-over-anode energy (Li et al., 2001) or by a direct measurement of the drift time of the moving charge. The drift time of charge correlates with the rise time of the output signal from the CSA (Verger et al., 2007). For our analysis, we calculate the rise time as the time difference between two thresholds of the rising edge of the signal. On both the cathode and the anode signal, we set the thresholds to 10% and 90% of the peak amplitude. The data was collected with a <sup>22</sup>Na radioactive source and an arbitrary selected pixel of the  $2 \times 2$  center array. The anode was used to trigger an event while the cathode signal was captured at the same time. This measurement shows the expected relationship of the DOI and the measured cathode and anode energy (fig. 2.26). The peak amplitude of the 511 keV photopeak of the cathode decreases linearly in correlation with the rise time. Events near the cathode cause longest rise times. The linear shape matches the expected weighting potential of the cathode. The pulse height of the anode signal also decreases dependent on the calculated rise time. The DOI is clearly visible along the predicted shape of the weighting potential, as shown in fig. 2.4. The expected peak amplitudes for the photopeak are in accordance with the calculated values in table 2.2. A summary of the measured key metrics for the CSA is given in table 2.3.

The correlation between the rise time of the cathode signal and the cathode-over-anode ratio (CAR) is shown in fig. 2.27. Both values carry information about the DOI. In contrast, the correlation between the cathode and anode rise time is smeared out. This is caused by uncertainties in the crossing of the low-level threshold, because of the slow rising component of the anode signal at the beginning of charge movement. The low-level trigger is difficult to

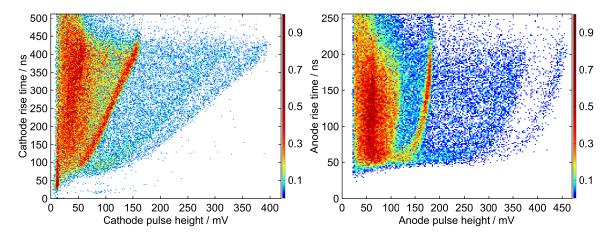


Figure 2.26.: Measurement of a <sup>22</sup>Na radioactive source with the Redlen CZT detector and the CSA readout board. The peak amplitudes of the output pulses from the CSA are plotted against the 10% - 90% rise times. The weighting potentials of the cathode (left) and an anode (right) are clearly visible along the 511 keV photopeak (158 mV for the cathode and 182 mV for the anode at rise times of 410 ns and 200 ns).

hit exactly without additional effort. However, the measurements show that the DOI can be

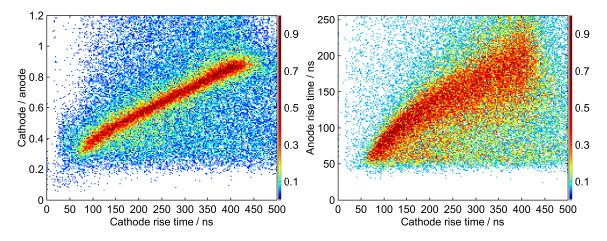


Figure 2.27.: Correlation of the cathode-over-anode ratio (CAR) with the measured rise times of the cathode pulses from the <sup>22</sup>Na radioactive source measurement (left). The rise time has a strong correlation to the CAR and therefore to the depth of interaction (DOI) in the detector volume. The correlation of the anode rise time to the DOI is smeared out (right).

calculated by the rise time of the cathode signal or the ratio of cathode-over-anode energy. In conclusion, the measurement of the cathode rise time is more precise than the anode rise time, because the cathode signal rises with a nearly linear slope. An advantage of the rise time estimation is that the information is derived from one detector signal instead of two, as required by the CAR calculation. Additionally, this calculation is error prone due to charge-sharing events on the pixelated anode side.

It is evident that the anode energy has to be corrected dependent on the DOI. There are several approaches for the depth correction, which are all carried out empirically. The results

Table 2.3.: Measured parameters of the front-end electronics. All channels are bandwidth-
limited to about 21.654 MHz by a passive RC low-pass filter. If appropriate, the
standard deviation $\sigma$ is noted for the measured value.

Parameter	Measured Channel value		Comment	
Rise time 10 % - 90 %	14.86 ns, $\sigma$ = 371 ps	Test input	Input pulse: 3.0 ns fall time, V <sub>out</sub> : 2 V <sub>pp</sub>	
Noise level (rms)	1.24 mV	Test input	Sine-wave test	
$ au_1$ (82 M $\Omega$    100 fF)	8.091 $\mu$ s $\sigma$ = 86 ns	Cathode	Waveform fit: $f(t) = e^{-t/\tau_1}$	
$ au_2$ (47 M $\Omega$    100 fF)	4.964 $\mu$ s $\sigma$ = 181 ns	Anode	Waveform fit: $f(t) = e^{-t/\tau_2}$	
Peak amplitude at 511 keV	158 mV	Cathode	Pulse height at longest drift time	
Peak amplitude at 511 keV	182 mV	Anode	Pulse height at longest drift time	
Steady-state gain	8.95 V/pC	Cathode	at 511 keV with $\tilde{E}_i = 4.64  \text{eV}$	
Steady-state gain	10.31 V/pC	Anode	at 511 keV with $\tilde{E}_i = 4.64 \mathrm{eV}$	

are achieved with best-fit functions, based on e.g. polynomial (Dönmez et al., 2007) or exponential (Hong et al., 2004; Cho and Lee, 2016) equations. Our approach is based on the weighting potential of a pixel, as this is the cause for the incorrect measurement. For depth correction, we select the events of the 511 keV photopeak along its cluster with reduced peak height and rise time, as shown in fig. 2.28. A fit of the weighting potential according to eq. 2.6 matches the data points. Thus, the derived mathematical relationship is used to correct the anode energy, which is shown in fig. 2.28. We also evaluated polynomial equations for the fitting function, resulting in comparable results with a fourth-degree polynomial (Scharnagl, 2015). The presented measurements were processes without any additional pulse shapers. This results in an energy resolution of 4.3% (FWHM) for the 511 keV photopeak. With the use of additional digital pulse shapers, this results can be improved.

## 2.6. Conclusion

This chapter has presented a circuit design and implementation of analog front-end electronics for a CZT pixel detector. Starting from the electrical equivalent circuit of the CZT, its electrical characteristics were discussed and a connection scheme presented. A short summary of the signal formation in a CZT pixel detector was provided, and the weighting potentials of the electrodes shown. Finally, these equations were applied for an analysis of the detector signals. After a comparison of different readout circuits, we focused our investigation on the CSA for the readout of the detector signals. As an ASIC-based solution is not available for the application with high  $\gamma$ -ray energies and high count rates, we designed the front-end electronics for the 8×8 pixel detector with COTS operational amplifiers.

We have shown a detailed analysis of the CSA in conjunction with the electrical model of

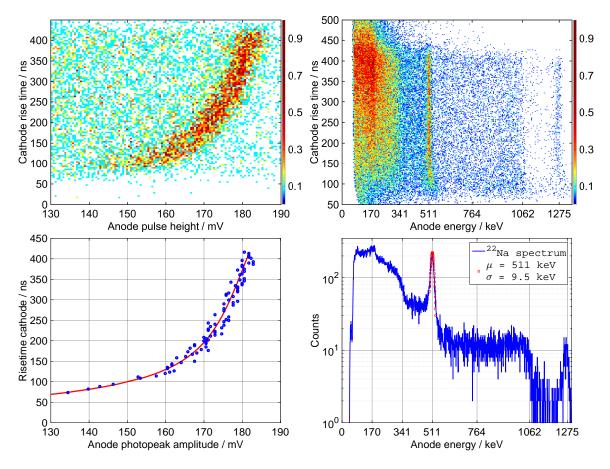


Figure 2.28.: The 511 keV photopeak along the signal rise time at the cathode with the corresponding data points used for the fit of the weighting potential (left). The corrected spectrum of the anode energy is independent of the depth of interaction (right). The selected pixel has an energy resolution of 9.5 keV (standard deviation) for the photopeak, which corresponds to 4.3 % FWHM. The spectrum is recorded without any additional pulse shapers.

the CZT detector. The limits of the design in terms of gain, bandwidth, and noise were given with exact equations and numerical values. The performance of the readout electronics was measured with synthesized detector signals from a test pulser and with a pixel detector of size  $20 \times 20 \times 5 \text{ mm}^3$  from Redlen. The measurements with the test pulse showed that the rms noise level of 1.24 mV is below the nominal intrinsic resolution of about 8 keV (FWHM at 122 keV) of the detector. Furthermore, we have shown that the SNR is also sufficient for a timing far below 1 ns, which outstrips the expected CZT performance. All results have been achieved without an additional pulse shaper. We also investigated the performance of the CSAs with the detector from Redlen, and were able to verify that the claimed depth dependence is in accordance with the calculated weighting potential of a pixel. We presented a measurement of a <sup>22</sup>Na radioactive source and showed a correction of the measured energy based on the DOI. After depth correction, we obtained an energy resolution of about 22 keV (4.3 % FWHM at 511 keV). Finally, the front-end electronics fulfill our requirements and operate from several keV up to 7 MeV with sub-nanosecond timing capabilities.

# 3. Digital signal processing

Many relationships in technical systems can be described with a sufficient accuracy by linear relations. For a time-limited observation, the parameters of the system can be regarded as constant. Such systems are referred to as linear time-invariant (LTI) systems. An LTI system has at least one input signal x(t) and one output signal y(t). The output signal is derived by a convolution of the input signal and the impulse response h(t) of the LTI system:

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau, \qquad (3.1)$$

where the impulse response h(t) is the output of the system with the Dirac delta distribution  $\delta(t)$  at its input. In general, a signal is the assignment of a function value to a time. Discretetime signals are derived by sampling a continuous-time signal, where the sampling period T is constant. To distinguish between continuous-time signals and discrete-time signals, discrete-time signals are denoted as x[n], where nT = t and x[n] = x(nT) = x(t). Thus, the convolution formula from eq. 3.1 in discrete-time domain is given by (Proakis, 1995):

$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k].$$
(3.2)

A large and powerful collection of mathematical techniques exists for calculating and analyze the characteristics of (discrete-time) LTI systems (Oppenheim and Schafer, 2007; Hoffmann, 2013). The well-known methods will be utilized and applied in the following sections of this chapter.

## 3.1. Unfolding-synthesis technique

During the period of this work, V.T. Jordanov described "a technique that allows the synthesis of virtually any pulse shape, either exactly or as a close approximation" (Jordanov, 2016). This permits an easy adaption of signal processing to the requirements of the application, e.g. optimized energy resolution, throughput, or timing performance. The approach developed in this thesis was very similar to (Jordanov, 2016), which is illustrated in fig. 3.1. If the signal of the detector is considered to have the shape of a short pulse, the output x(t)is the result of a convolution of the transfer function of the signal conditioning electronics and the Dirac delta distribution  $\delta(t)$ . After digitization of x(t), the discrete-time signal x[n] is convoluted with the inverse transfer function  $h^{-1}(t)$  of the signal conditioning electronics to

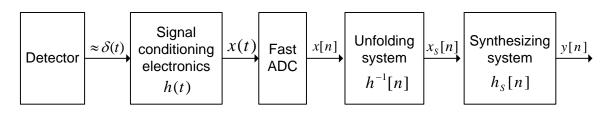


Figure 3.1.: Functional block diagram of a system using the unfolding-synthesis technique, proposed by V.T. Jordanov. The diagram is adopted from (Jordanov, 2016).

remove the frequency response of the front-end electronics. As this step reverses the initial convolution, it is referred to as deconvolution or unfolding. In general, this method can be applied to any measurement system, where the dynamic properties are known (Hessling, 2008). With our approach (Födisch et al., 2016c), the unfolding system returns a step like function  $x_S[n]$ , which can be simply considered as the integral of the detector pulse. Finally, for synthesizing an arbitrary pulse shape at the output y[n], merely the step response of the synthesizing system  $h_S(t)$  must be designed in time or frequency domain. An illustration of the signal processing chain is sketched in fig. 3.2

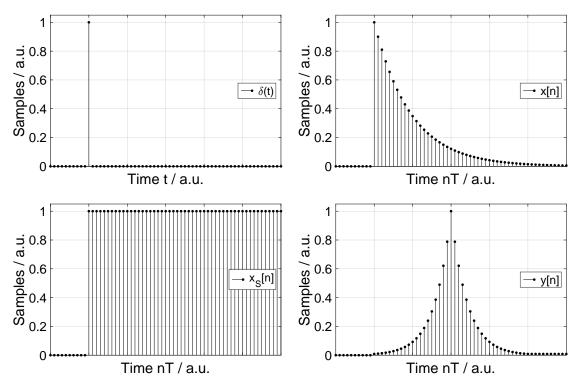


Figure 3.2.: Steps of signal processing with unfolding-synthesis technique. The Dirac delta distribution  $\delta(t)$  (top left) is convoluted with the signal conditioning electronics and digitized by the ADC (signal x[n], top right). The unfolding system generates a step-like output  $x_S[n]$  (bottom left), which is ultimately transformed to an arbitrary pulse shape y[n] by the synthesizing system (bottom right). For illustration, a cusp shape (Kowalski, 1970) is shown.

The implementation of a deconvolution system is ideally abstracted by an infinite impulse response (IIR) filter as shown in sec. 3.2 and (Födisch et al., 2016c), whereas the synthesiz-

ing system is predestined for an implementation with an finite impulse response (FIR) filter, as shown in sec. 3.3 and (Födisch et al., 2016d). The coefficients  $h_{\text{FIR}}$  of the FIR filter for an arbitrary discrete-time pulse shape  $h_p[n]$  are derived by

$$h_{\mathsf{FIR}} = h_d * h_p \,, \tag{3.3}$$

where  $h_d$  is an approximation of the first derivative in discrete-time domain, e.g.

$$y'[n] = y[n] - y[n-1],$$
 (3.4)

which corresponds to the filter coefficients  $h_d = (1, -1)$ .

## 3.2. Digital deconvolution

In the application of semiconductor detectors, the CSA is widely used in front-end electronics. The output signal is shaped by a typical exponential decay. Depending on the feedback network, this type of front-end electronics suffers from the ballistic deficit problem, or an increased rate of pulse pile-ups. Moreover, spectroscopy applications require a correction of the pulse-height, while a shortened pulse-width is desirable for high-throughput applications. For both objectives, digital deconvolution of the exponential decay is convenient. With a general method and the signals of our custom CSA for CZT detectors, we show how the transfer function of an amplifier is adapted to an IIR filter. This section investigates different design methods for an IIR filter in the discrete-time domain and verifies the obtained filter coefficients with respect to the equivalent continuous-time frequency response. Finally, the exponential decay is shaped to a step-like output signal that will be processed by the synthesizing system for an arbitrary pulse shaping.

#### 3.2.1. Prior work

A  $\gamma$ -ray detector system based on a semiconductor detector such as CZT usually consists of the detector crystal, the analog readout electronics for the amplification of the detector signal, and a pulse processing unit. Nowadays, the pulse processing is mainly integrated by an ASIC or by a digital circuit in an FPGA. As we recently showed (Födisch et al., 2016a), the front-end electronics can be appropriately implemented by a CSA with a continuous reset through an *RC* feedback circuit. This type of amplifier discharges the integrated detector current from the feedback capacitor *C* with the resistor *R*. Thus the typical signal shape with an exponential decay is seen at the output of the CSA. A well-known problem of that circuit is the ballistic deficit. This is caused by continuous discharge of the feedback capacitor, even though the current of the detector is integrated. If the ratio of the *RC* time constant over the integration time decreases, the ballistic deficit dominates the measured peak amplitude (Födisch et al., 2016a). To eliminate this effect and to reconstruct the initial charge by a deconvolution of the exponential decay, Stein et al. presented the Moving Window Deconvolution (MWD) (Stein et al., 1994, 1997; Georgiev and Gast, 1993; Georgiev et al., 1994; Stein et al., 1996). Later, Jordanov et al. (Jordanov and Knoll, 1994; Jordanov, 1994; Jordanov et al., 1994) described the same approach (Stein et al., 1996). However, both algorithms are derived by an analysis of the exponential decay in the time domain. As a result, a deconvolution of the exponential decay in the discrete-time domain is described by (Stein et al., 1996)

$$y[n] = x[n] + k \sum_{i=-\infty}^{n-1} x[i],$$
 (3.5)

where y[n] is the deconvolution of the signal x[n], which is the value of the continuous signal x(t) at the discrete time t = nT with the sampling interval T. The value of k is set to (1 - k'), where  $k' = e^{\frac{-T}{\tau}}$  is "the decay constant of the preamplifier transfer function for one sampling interval" (Stein et al., 1994). The authors proposed an alternative value  $k = \frac{T}{\tau}$  in (Stein et al., 1996) assuming  $\tau \gg T$ . Both parameters for eq. (3.5) transform the exponential decay of the signal into a step-like signal, as shown in fig. 3.3. The discrete-time signal x[n]

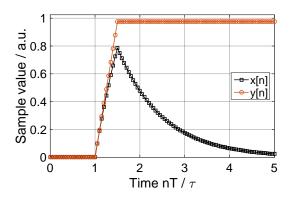


Figure 3.3.: A sampled output signal x[n] of an amplifier with a typical exponential decay at sampling interval T (simulated). The decay has the time constant  $\tau = 20T$ , and the parameter k of the deconvolution is  $k = 1 - e^{\frac{-T}{\tau}}$ . This results in a step-like signal y[n] with a corrected amplitude due to the ballistic deficit.

shown in fig. 3.3 corresponds to an output signal of an ideal CSA with a feedback resistor R and feedback capacitance C. For a rectangular shaped input current pulse with amplitude I, where the current flows in the time interval from  $t_a$  to  $t_b$ , the continuous-time output signal x(t) of the amplifier is calculated by

$$x(t) = IR\left[\left(1 - e^{\frac{t_a - t}{\tau}}\right)\theta(t - t_a) - \left(1 - e^{\frac{t_b - t}{\tau}}\right)\theta(t - t_b)\right].$$
(3.6)

Here  $\theta(t)$  is the Heaviside step function, and  $t_b > t_a > 0$ . Regarding Stein's approach for the MWD, the presented deconvolution is calculated by the recursive representation of eq. (3.5),

which is derived by

$$y[n] = y[n-1] + d$$
 (3.7)

$$x[n] + k \sum_{i=-\infty}^{n-1} x[i] = x[n-1] + k \sum_{i=-\infty}^{n-2} x[i] + d$$
(3.8)

$$d = x[n] + (k-1)x[n-1]$$
(3.9)

$$y[n] = y[n-1] + x[n] + (k-1)x[n-1]$$
(3.10)

According to the time-shifting property of the *z*-transformation (Oppenheim and Schafer, 2007)

$$x[n-k] \stackrel{\mathcal{Z}}{\longleftrightarrow} z^{-k}X(z),$$
 (3.11)

eq. (3.10) can be rewritten as

$$\frac{Y(z)}{X(z)} = \frac{1 + (k-1)z^{-1}}{1 - z^{-1}}.$$
(3.12)

It is obvious that eq. (3.12) is an equivalent of the generalized transfer function of an IIR filter, which is defined as (Oppenheim and Schafer, 2007)

$$H(z) = \frac{\sum_{k=0}^{M} b_k z^{-k}}{1 - \sum_{k=1}^{N} a_k z^{-k}}.$$
(3.13)

For further clarification, the fundamental operation of the pulse shapers described by Stein et al. (Stein et al., 1994) or by Jordanov et al. (Jordanov and Knoll, 1994) is a deconvolution of the transfer function of the amplifier and can be substituted by an IIR filter. Recently, Jordanov presented an unfolding-synthesis technique (Jordanov, 2016) that also demands an accurate deconvolution of the amplifier transfer function. Both constructed their digital algorithms intuitively by an extensive analysis of the signals in the time domain. By doing so, they neglected the established design methods for digital filters. Consequently, we will show a further analysis of the amplifier transfer function in the *s*-domain (frequency domain of continuous-time signals) and design the corresponding digital filter for the deconvolution in the *z*-domain (equivalent frequency domain of discrete-time signals). Finally, we will verify our algorithms with the signals of a CZT detector in conjunction with a CSA.

#### 3.2.2. Discrete-time inverse amplifier transfer function

The charge-to-voltage transfer function of an ideal CSA with an RC feedback network and the voltage  $v_0$  at its output is given by (Födisch et al., 2016a)

$$H(s) = \frac{v_{\rm O}}{Q} = \frac{sR}{1+sRC} \,. \tag{3.14}$$

By normalizing the charge *Q* to the feedback capacitance *C* with  $Q/C = v_Q$ , the transfer function  $H_Q$  becomes

$$H_{\rm Q}(s) = rac{v_{\rm O}}{v_{\rm Q}} = rac{s\tau}{1+s\tau}$$
, (3.15)

where  $\tau = RC$  is the characteristic time constant of the CSA. The transfer function is identical to that of a first-order high-pass filter. Therefore, the signal seen at the output of the amplifier is a convolution of the charge input signal and a high-pass filter. Moreover, as we want to reconstruct the input signal, the deconvolution of the high-pass filter is realized with the inverse transfer function of the amplifier. A deconvolution in the discrete-time domain requires an adequate approximation of  $H_Q^{-1}$  in the *z*-domain. Because the design methods for discrete-time filters that transform continuous-time filters are numerous, we will focus our investigations on a set of established methods and will test the accuracy of the transformation from the *s*-plane to the *z*-plane. At first, with the corresponding Laplace-transformation of the difference quotient of the continuous-time signal x(t)

$$sX \bullet - \circ \frac{dx(t)}{dt} = \lim_{h \to 0} \frac{x(t+h) - x(t)}{h}, \qquad (3.16)$$

the equivalent difference quotient for the discrete-time signal x(nT) is

$$\lim_{h \to T} \frac{x(nT+h) - x(nT)}{h} = \frac{x(nT+T) - x(nT)}{T}$$
$$= \frac{x[n+1] - x[n]}{T}$$
(3.17)

By using eqs. (3.11) and (3.17), the transformation of the *s*-domain to the *z*-domain is therefore derived by

$$s \longrightarrow \frac{z-1}{T}$$
, (3.18)

which is referred to as the forward difference method. In the same way, but by setting the difference quotient to  $\frac{x(t)-x(t-h)}{h}$ , the corresponding backward difference is defined by

$$s \longrightarrow \frac{z-1}{zT}$$
 (3.19)

It is clear that these design methods replace the continuous-time differentials with a discretetime difference. The exact relation of s and z in the context of the z-transformation is given by

$$z = e^{sT} \iff s = \frac{1}{T} \ln(z)$$
, (3.20)

which cannot be used for the expression of a discrete series of samples with respect to eq. (3.11). Therefore, another substitution of *s* is derived by solving the differential equation corresponding to H(s) by the approximation of an integral with the trapezoidal rule (Oppenheim and Schafer, 2007; Proakis, 1995). A replacement of *s* with

$$s \longrightarrow \frac{2}{T} \frac{z-1}{z+1}$$
, (3.21)

is referred to as a bilinear transformation. Besides the approximation of *s* with a suitable expression in terms of *z*, there exist further design techniques based on the mapping of the zeros and poles of the transfer function in the *s*-plane directly into zeros and poles in the *z*-plane (Proakis, 1995). The matched-*z* transformation, as described in (Proakis, 1995), maps the zeros  $z_k$  and poles  $p_k$  of the continuous-time transfer function H(s) to the discrete-time transfer function H(z) with the relation

$$H(s) = \frac{\prod_{k=1}^{M} s - z_k}{\prod_{k=1}^{N} s - p_k} \longrightarrow \frac{\prod_{k=1}^{M} z - e^{z_k T}}{\prod_{k=1}^{N} z - e^{p_k T}} = H(z), \qquad (3.22)$$

where T is the sampling interval. After all of the above, the transfer functions in the *z*-domain used to deconvolute the continuous-time amplifier transfer function  $H_Q$  of eq. (3.15) are summarized in tab. 3.1 (detailed results are illustrated in (Födisch et al., 2016c)).

Table 3.1.: Summary of the investigated design methods for an infinite impulse response filter. All methods rely on an approximation of the continuous-time transfer function in the *z*-domain.

Design method	Transfer function		
Forward difference, eq. (3.18)	$H_{ ext{FD}}(z) = rac{1 + \left(rac{T}{ au} - 1 ight) z^{-1}}{1 - z^{-1}}$		
Backward difference, eq. (3.19)	$H_{ m BD}(z) = rac{ig(rac{T}{ au}+1ig)-z^{-1}}{1-z^{-1}}$		
Bilinear transformation, eq. (3.21)	$\mathcal{H}_{BL}(z) = rac{\left(rac{T}{2 au}+1 ight)+\left(rac{T}{2 au}-1 ight)z^{-1}}{1-z^{-1}}$		
Matched- $z$ transformation, eq. (3.22)	$H_{MZ}(z) = rac{1-{ m e}^{-rac{T}{ au}}z^{-1}}{1-z^{-1}}$		

The transfer functions shown in tab. 3.1 are all identical in the limit  $T \ll \tau$ . But with real constraints, where  $\tau$  is chosen to be as small as possible to meet the requirements of the application, and T is chosen to be as large as possible in the range of the Nyquist frequency, the results of the presented IIR filters are slightly different. After an analysis of the filter responses in the time-domain with the example shown in fig. 3.3, it is apparent that the bilinear transformation performs best for the specified values, as the voltage step is expected to have a unity value without the presence of the ballistic deficit effect (fig. 3.4). As the methods based on the difference quotient obviously result in distorted output signals, these approaches will be neglected for our application. Moreover, the results of the matched-*z* transformation are improved if this method is extended by an additional matched gain. The

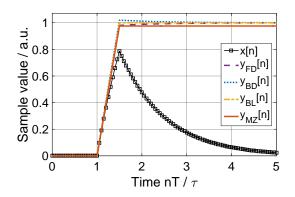


Figure 3.4.: A comparison of the results obtained by different transfer functions for the deconvolution of the signal x[n]. The filter based on the forward difference method ( $y_{FD}$ ) produces a little undershoot, whereas the backward difference ( $y_{BD}$ ) method shows an overshoot in the analogy. Further, the filters based on the bilinear ( $y_{BL}$ ) or matched-z ( $y_{MZ}$ ) transformations seem to have a flat response with respect to the exponential decay of the input signal.

discrete-time transfer function  $H_{MZ}$  is adjusted by the gain correction constant G, so that

$$G = \frac{|H(j\omega)|}{|H_{MZ}(e^{j\omega T})|}$$
(3.23)

matches the ratio of the magnitudes of the frequency responses at a specific frequency  $\omega$ . For the best gain matching of the inverse high-pass filter regarding the transfer function in the *s*-domain,  $\omega$  is set to the characteristic 3 dB corner frequency  $1/\tau$ , as carried out by a comparison of different values for  $\omega$  (Födisch et al., 2016c). Consequently, the optimum IIR filter coefficients for the ideal inverse amplifier transfer function are

$$b_n = \left\{ G\left(\frac{1}{\tau}\right), -G\left(\frac{1}{\tau}\right) e^{\frac{-\tau}{\tau}} \right\}$$
(3.24)

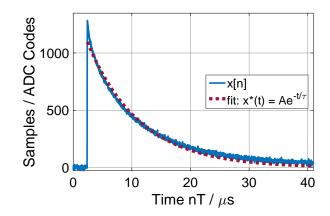
$$a_n = \{1, -1\}$$
 (3.25)

where the specific gain matching constant  $G(\omega)$  for the described system is calculated as

$$G(\omega) = \frac{2\left|\sin\left(\frac{\omega T}{2}\right)\right| \sqrt{\left(\omega T\right)^2 + 1}}{\omega \tau \sqrt{e^{\frac{-2T}{\tau}} - 2e^{\frac{-T}{\tau}}\cos(\omega T) + 1}}.$$
(3.26)

### 3.2.3. Application to measured signals

For a proof of concept, we will examine the deconvolution of the amplifier signals sampled from the developed CSAs. The amplifiers are equipped with a test pulse input so that pulses with an exponential decay and arbitrary pulse heights can be synthesized. The time constant of the amplifier is in the range of several microseconds, as the feedback resistor is chosen to be 82 M $\Omega$  and the feedback capacitance is in the range of 100 fF. The exact value of  $\tau = RC$  must be experimentally verified, because the electronic components are afflicted



with tolerances. An example of a measured pulse from the CSA is shown in fig. 3.5. The

Figure 3.5.: A sampled output signal x[n] from the CSA with a test pulse at its input. The exponential decay of the signal is clearly visible, but the curve fitting of the function  $x^*(t) = Ae^{\frac{-t}{\tau}}$  exposes a variation from the ideal shape.

values are recorded with a sampling frequency of 100 MSPS with 14 bit resolution at an input voltage range of approximately 2.3 V. A curve fit is applied to estimate the time constant  $\tau$ . For this example, the numerical value of  $\tau$  is 8.91  $\mu$ s, which corresponds to a 82 M $\Omega$ resistor in parallel with a 108.7 fF capacitor. The derived curve fit is in accordance with the expected feedback network, but obviously it does not cover all the features of the pulse shape. Further, a full analysis of the time constant by the best fit function indicates that the parameter  $\tau$  is nearly constant over the whole output voltage range. Yet at the same time, a deconvolution of the exponential decay with the estimated  $\tau$  by the matched-*z* IIR filter with gain correction reveals distortion with regard to the predicted flat step-like function. The results of the deconvolution of a measured signal with the inverse high-pass filter are shown in fig. 3.6. In terms of the proposed methods for the deconvolution, the transfer function of

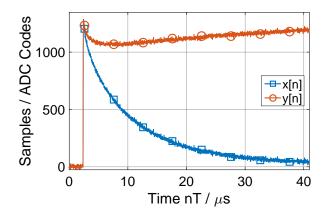
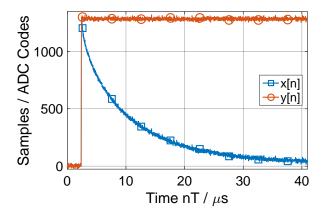


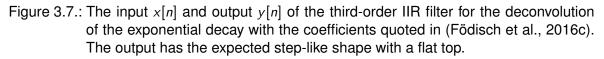
Figure 3.6.: The sampled signal x[n] and a deconvolution with the matched-*z* IIR filter with gain correction. The output of the filter y[n] traces the variations with respect to the ideal exponential decay. The anticipated flat step-like response is distorted.

the CSA cannot be approximated with the inferred model of a first-order high-pass filter from eq. (3.15). However, the costs of a detailed analysis of all parasitic components of the entire network model are much higher than the benefits. Nevertheless, the transfer function of the

### 3. Digital signal processing

CSA has been empirically found by the use of the System Identification Toolbox provided by Mathworks (MathWorks, 2016b). The tool reports the goodness of fit between the test and reference data to be 93.09% (normalized root mean square error) for a continuous-time model with one pole and one zero for the transfer function. This is improved to 96.48% by a model with four zeros and four poles. Anticipating the digital implementation of the IIR filter, we chose a model with an accuracy of 96.47% and three zeros and three poles (Födisch et al., 2016c). Furthermore, the parameters of the inverse continuous-time transfer function are transformed by the matched-*z* method and are gain-corrected at the 3 dB corner frequency. This results in an IIR filter with the coefficients quoted in (Födisch et al., 2016c). The output y[n] from the IIR filter with these coefficients, calculated by a double-precision floating point arithmetic (64 bit), is shown in fig. 3.7. In addition to the analysis in the discrete-time





domain, the characteristics of the derived IIR filter are shown in (Födisch et al., 2016c).

### 3.2.4. Implementation of a higher order IIR filter

Nowadays, digital filters are realized with general purpose digital signal processors (DSPs) or FPGAs. We will concentrate our efforts on an implementation of a higher order IIR filter based on an FPGA from Xilinx. Along with the transfer function described by eq. (3.13), the difference equation for the third-order IIR is

$$y[n] = b_0 x[n] + b_1 x[n-1] + b_2 x[n-2] + b_3 x[n-3] + \dots$$
  
$$a_1 y[n-1] + a_2 y[n-2] + a_3 y[n-3]$$
(3.27)

The difference equation can be comfortably mapped to the known Direct Form implementation structures of an IIR filter (Meyer-Baese, 2007). Still, at the same time, the coefficients need quantization. With a look at the resources provided by a state-of-the-art FPGA from Xilinx, a word length for the coefficients of 25 bits is easily realizable, since these devices have embedded 18 bit×25 bit multipliers (Xilinx, 2014). These come along with a pre-adder and a post-adder in a so-called DSP slice. The built-in logic blocks, with their multiply-accumulate

structures, are highly optimized for the requirements of digital filters. But a straightforward implementation of a digital filter with restricted word lengths for the coefficients must be evaluated with regard to quantization errors. We examined the output of the Direct Form IIR filter in the time domain with a variable word length for coefficient quantization. Even a length of 25 bits results in clear distortions. Satisfactory results were achieved only with a word length of more than 34 bits. With this implementation on a Xilinx FPGA, one multiplication is mapped to two DSP slices. Thus a Direct Form implementation based on the difference equation (3.27) requires 14 DSP slices. As we target the processing of highly segmented detectors with more than 65 readout channels, the impact on the quantization error is noticeable in an excessive consumption of slice logic resources of the FPGA. As already stated by Oppenheim, the Direct Form realization of an IIR filter is very sensitive to quantization errors, because each polynomial root of the transfer function is affected by all of the quantization errors of the coefficients (Oppenheim and Schafer, 2007). On the whole, the guantization modifies the location of the zeros and poles in the complex z-plane, and therefore the response of the filter is changed. This problem is solved by a factorization of the numerator and denominator polynomials of the transfer function in the form

$$H(z) = b_0 \prod_{k=1}^{M} \frac{1 - c_k z^{-1}}{1 - d_k z^{-1}}$$
(3.28)

where *M* is the order of the IIR filter and  $c_k$  and  $d_k$  are the zeros and poles in the complex *z*-plane. For our derived filter, all zeros and poles are real values where the imaginary part is zero. Thus, the transfer function can be rewritten to a cascade of three Direct Form filters of first-order (fig. 3.8). The coefficients  $-b_{k1}$  and  $a_{k1}$  are the roots of the numerator and

$$x \longrightarrow \underbrace{\frac{1+b_{11}z^{-1}}{1-a_{11}z^{-1}}}_{1-a_{21}z^{-1}} \longrightarrow \underbrace{\frac{1+b_{21}z^{-1}}{1-a_{21}z^{-1}}}_{1-a_{31}z^{-1}} \longrightarrow \underbrace{b_0} \longrightarrow y$$

Figure 3.8.: A Cascade Form infinite impulse response (IIR) filter of the third-order. Each block is built by a first-order Direct Form IIR filter.

denominator polynomials and must be quantized. As proven by (Crochiere and Oppenheim, 1975) and (Oppenheim and Schafer, 2007), the Cascade Form of an IIR filter is generally much less sensitive to coefficient quantization than the equivalent Direct Form implementation. Each first-order filter section of the cascade structure has the form shown in fig. 3.9. Both the feedforward path (all-zero system) and the feedback path (all-pole system) purely

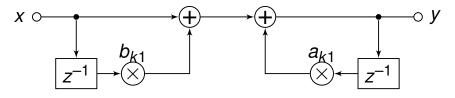


Figure 3.9.: A Direct Form implementation of a first-order infinite impulse response filter. The multiply-accumulate architecture from the built-in slices of the fieldprogrammable gate array is adapted to this basic filter structure.

match the multiply-accumulator architecture built into the FPGA. An additional register between the two paths increases the performance in terms of the maximum clock frequency, but also introduces a delay of one clock cycle.

### Results

We have implemented the presented Cascade Form structure of the third-order IIR filter in VHDL. The design was synthesized and tested with Xilinx FPGAs. Moreover, the tools reported a theoretical maximum frequency of 217 MHz for the filter running on a Kintex 7 XC7K325T. With an optional register between the feedforward path and feedback path, this value increases to 295 MHz (167 MHz on Spartan 6 XC6SLX45T). For our purposes and for most detector applications, this is sufficient. The fixed-point arithmetic uses the entire bit widths of the built-in 25 bit×18 bit multipliers. Even though the fixed-point representation of the poles and zeros is sufficient with a word length of 18 bits, the remaining 25 bits word length for the sample is absolutely required. As the output of a cascade and the word length of the feedback signal are limited to 25 bit, this truncation propagates a rounding error from each stage to the next. However, the generic VHDL design of the filter is in good agreement with our requirements while running on the Kintex 7 architecture, but on the outdated 18 bit × 18 bit architecture (e.g. Spartan 6), we observed unacceptable distortions due to rounding errors. The design of our third-order IIR filter is mapped to seven DSP slices of the Xilinx FPGA. Each Direct Form module consumes two DSP slices, while the multiplication with the gain correction constant b<sub>0</sub> consumes an additional multiplier of a DSP slice.

For illustration and in reference to the prior work of Stein and Jordanov, the step-like output of the IIR filter is turned into a trapezoidal pulse shape by applying a differentiator of the form

$$y[n] = x[n] - x[n - M], M \ge 1,$$
 (3.29)

where M is the window length and an average filter with

$$y[n] = \frac{1}{N} \sum_{k=0}^{N-1} x[n-k], \qquad (3.30)$$

where *N* is the number of samples. The implementation of the eq. (3.29) is a straightforward FIR filter. Further, the corresponding difference equation of eq. (3.30) is another direct form implementation of an IIR filter. Together, these filters implement the MWD algorithm. The results for a recorded sequence with our CSA and a CZT detector are shown in fig. 3.10. This example shows that the obtained transfer function of the CSA is sufficiently robust to be applied to an identical amplifier because the recorded sequence was captured from an arbitrarily selected pixel with another dedicated amplifier. A single pixel calibration further improves the results, since the absolute values of the electronic components vary.

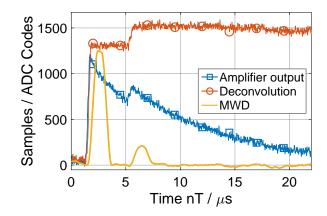


Figure 3.10.: An example of the deconvolution with the obtained inverse amplifier transfer function. Advanced pulse processing (MWD) with a differentiator (M = 128) and an average filter (N = 64) accomplishes the pulse shaping with respect to the well-known trapezoidal shaper from (Stein et al., 1994) or (Jordanov and Knoll, 1994).

### 3.2.5. Conclusion

Based on the fundamental description from (Stein et al., 1994) of the Moving Window Deconvolution, we investigated the deconvolution of an exponential decay related to an output signal of a CSA. We showed that the approach of Stein et al. for signal processing relies on an inversion of a first-order high-pass filter representing the amplifier transfer function. Thus the deconvolution can be realized with an infinite impulse response (IIR) filter. In the context of established design methods for digital filters, we derived the optimum coefficients for the demanded IIR filter. In conclusion, the matched-z transformation with a gain correction at the 3 dB corner frequency performed best while regarding the output in the time and frequency domain. Moreover, we examined the signal processing with measured signals from a CSA. We revised the apparent model of a first-order high-pass filter for the transfer function of the amplifier because it fails in terms of accuracy. After an estimation of the transfer function by experimental data, we enhanced this model to the third-order. Finally, we implemented the inverse amplifier transfer function by means of an IIR filter with an adapted Cascade Form structure. The results showed, that the proposed method for the deconvolution is realizable with little computational effort in a Xilinx Kintex 7 FPGA. The presented method and implementation of the signal processing are essential for our advanced pulse processing of the CZT detector signals with high resolution and high throughput.

## 3.3. Digital pulse synthesis

Contemporary FPGAs are predestined for the application of IIR and FIR filters. Their embedded DSP blocks for multiply-accumulate operations enable efficient fixed-point computations, in cases where the filter structure is accurately mapped to the dedicated hardware architecture. This chapter presents a generic systolic structure for high-order FIR filters, efficiently exploiting the hardware resources of an FPGA in terms of routability and timing. Although this seems to be an easily implementable task, the synthesizing tools require an adaptation of the straightforward digital filter implementation for an optimal mapping. Using the example of a symmetric FIR filter with 90 taps, we demonstrate the performance of the proposed structure with FPGAs from Xilinx and Altera. The implementation utilizes less than 1 % of slice logic and runs at clock frequencies up to 526 MHz. Moreover, an enhancement of the structure ultimately provides an extended dynamic range for the quantized coefficients without the costs of additional slice logic.

## 3.3.1. Prior work

Nowadays, FIR filters are a major application of FPGAs in the context of digital signal processing. An FPGA design is traditionally implemented in a hardware description language on the register-transfer level (RTL). Therefore, the design flow from the RTL top level down to the logic and circuit level postulates clear definitions and constraints for an optimal implementation result. As a consequence, the architecture of the device impacts the design, and a straight separation of the abstraction layers is practically impossible for an efficient realization in terms of logic utilization or speed. Although some approaches to hardware efficient filter structures exist (Meher et al., 2008; Park and Meher, 2014), the key to a valuable exploitation of the hardware resources is an adaptation of the FIR filter structure to the architecture of the FPGA. Moreover, at the early stage of filter design, a reduction of hardware complexity is possible (Mehrnia and Willson, 2016). At last, an adaptation of the mathematical modelling to the dedicated silicon (DSP blocks) maximizes performance (Xilinx, 2005) while versatility decreases.

In fact, an implementation of numerical algorithms in an FPGA, including FIR filters, is a trade-off between targeted precision, allocated logic, achievable clock frequency, and allowed latency. An operation with a fixed-point arithmetic is usually preferred if resources or speed weigh more than the highest precision. Along with the ongoing enhancement of DSP capabilities, the requirements of digital filters grow evolutionarily in accordance with the hardware. Thus, state-of-the-art FPGAs have hundreds of built-in DSP blocks, capable of fixed-point operations with a precision of at least 18 bits (Altera Corporation, 2011; Xilinx, 2014). Consequently, we will adapt a symmetric FIR filter structure to a generic FPGA architecture and meet the challenges regarding routability, timing, and precision.

Several convenient structures of FIR filters can be found in the literature, e.g. direct-form realization (Proakis and Manolakis, 2013), transposed direct-form (Meyer-Baese, 2007),

and symmetry exploiting direct-form structures for linear-phase systems (Oppenheim and Schafer, 2007). The literary work covers the basic mathematical operations and structures but bypasses the technical aspects concerning the prevalent DSP block architectures of contemporary FPGAs. Nevertheless, the major manufacturers of FPGAs support their platforms with practical explanations (Altera Corporation, 2010; Taylor, 2012; Zatrepalek, 2012).

Besides the hardware-mapped structures, an improved precision of a symmetric FIR filter without increasing the coefficient bit-widths was proposed by (Shen, 2010). The presented parallel method implementation utilizes an accumulator in combination with variable shift operations. Although FPGAs have "flexible multidata bus routing capabilities" (Shen, 2010), the suggested shift of the accumulator input contradicts the architecture of state-of-the-art embedded DSP blocks, e.g. (Altera Corporation, 2011; Xilinx, 2014). As Shen omits a benchmark with an FPGA implementation, the synthesis results of the optimized structure on a Xilinx FPGA were presented by (Yuan et al., 2012). These results show a further consumption of slice logic in addition to the DSP blocks, but neglect the specific architecture of the FPGA and do not properly include the DSP blocks in their optimization.

#### Aim of this contribution

High-order digital filters require a cascade of sub-filters for an efficient realization (Mehrnia and Willson, 2016) or, alternatively, an efficiently mapped hardware design. With regard to recent hardware architectures, a FIR filter whose length exceeds the number of cascaded DSP blocks (DSP chains) in an FPGA, can be referred to as high-order filter. A typical DSP chain includes at least several tens of DSP blocks. This section presents the methodology and implementation of a systolic FIR filter in an FPGA, which ideally matches the prevalent embedded DSP block architecture. Consequently, the direct implementation of large structures avoids the cascading into small filters. The challenges of such a design are discussed, exemplary solved, and compared to the key performance indicators of an FPGA implementation: logic utilization and clock frequency. Moreover, in reference to the "bit compression" introduced by (Shen, 2010), we adapt their parallel method and implement an improved precision of the frequency response without additional logic utilization.

### 3.3.2. FIR filter structures for FPGAs

Within the scope of high-order parallel FIR filters, it is suitable to design the filter with a linear phase. Thus, the coefficients h[k] of the FIR system of order M would satisfy the symmetry condition h[M - k] = h[k], where k = 0, 1, ..., M and the number N of coefficients and taps for the FIR system is N = M + 1 (Oppenheim and Schafer, 2007). Consequently, the generalized equation for the output y of a symmetric FIR filter with an even number N of

taps can be expressed as (Oppenheim and Schafer, 2007):

$$y[n] = \sum_{k=0}^{\frac{M-1}{2}} h[k] \left( x[n-k] + x[n-M+k] \right) \,. \tag{3.31}$$

In the case of symmetric FIR filters, the number of coefficient multipliers is essentially halved (Oppenheim and Schafer, 2007). Other types of symmetry, e.g. point symmetry or an odd number *N* of taps, are developed by an analogy with the ongoing methodology. However, the taps of the input samples x[n] are folded around half of the filter length by N/2 pre-adders, before the sums are multiplied by h[k]. Finally, the products are convoluted by N/2 - 1 post-adders. Hence, a basic arithmetic logic unit for a DSP operation incorporates a pre-adder, a multiplier and a post-adder as shown in fig. 3.11.

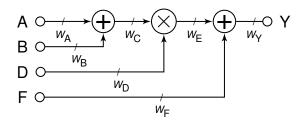


Figure 3.11.: The prevalent architecture for a DSP operation in an FPGA with pre-adder (C = A + B), multiplier  $(E = C \times D)$  and post-adder (Y = E + F). The bitwidths *W* of the inputs and outputs depend on the FPGA architecture.

Contemporary FPGA architectures embed multiply-accumulate blocks for digital filters, either by integrated hardware primitives or by synthesized logic blocks. The mapping of eq. (3.31) to parallel DSP blocks according to the direct-form implementation is convenient, but results in a large adder tree for the running sum, which slows down the maximum clock frequency of the system. If the design targets a high throughput at the highest clock frequency, a systolic structure with a pipeline will maximize the performance of the FIR filter, as long as latency is negligible. The basic structure is described in *z*-domain by using the time-shifting property of the *z*-transformation  $x[n-k] \stackrel{Z}{\leftrightarrow} z^{-k}X(z)$  (Oppenheim and Schafer, 2007) and eq. (3.31) resulting in:

$$Y = \sum_{k=0}^{\frac{M-1}{2}} h_k \left( z^{-k} + z^{-M+k} \right) X, \qquad (3.32)$$

where  $h_k = h[k]$ . Furthermore, a pipeline is embedded into the systolic structure by adding registers to the output of each running sum element. In terms of the *z*-transformation, the pipeline register is a unit delay  $z^{-1}$ . Therefore, the pipelining of the FIR filter can be ex-

pressed as:

$$Yz^{-1} = z^{-1} \sum_{k=0}^{\frac{M-3}{2}} h_k \left( z^{-k} + z^{-M+k} \right) X + z^{-1} h_{\frac{M-1}{2}} \left( z^{\frac{-M+1}{2}} + z^{\frac{-M-1}{2}} \right) X$$
(3.33)  

$$Yz^{-2} = z^{-1} \left[ z^{-1} \sum_{k=0}^{\frac{M-5}{2}} h_k \left( z^{-k} + z^{-M+k} \right) X + z^{-1} h_{\frac{M-3}{2}} \left( z^{\frac{-M+3}{2}} + z^{\frac{-M-3}{2}} \right) X \right] + z^{-2} h_{\frac{M-1}{2}} \left( z^{\frac{-M+1}{2}} + z^{\frac{-M-1}{2}} \right) X .$$
(3.34)

Eq. (3.33) illustrates the decomposition of the last term of the running sum with a pipeline register. In addition, the second pipeline stage is formed by eq. (3.34). Finally, the complete decomposition of the adder tree with (M - 1)/2 unit delays reveals the FIR filter function in terms of iteration of the basic systolic element

$$Y_{k} = z^{-1}Y_{k-1} + z^{-k}h_{k}\left(z^{-k} + z^{-M+k}\right)X$$
  
=  $z^{-1}Y_{k-1} + h_{k}\left(z^{-2k} + z^{-M}\right)X$  (3.35)

$$Y_0 = h_0 \left( 1 + z^{-M} \right) X$$
, (3.36)

where the output *Y* from eq. (3.32) is equal to the output of the last pipeline stage at position k = (M - 1)/2 from eq. (3.35). The block diagram of the symmetric systolic FIR filter is shown in fig. 3.12. A detailed mapping of this structure to the Xilinx specific architecture is shown in (Xilinx, 2005).

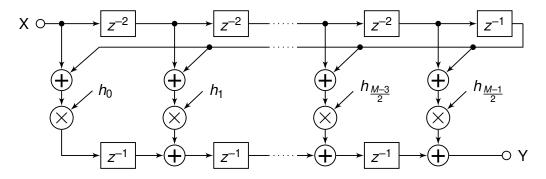


Figure 3.12.: A linear phase FIR filter of order *M* (odd number). The filter function is folded around half of the filter length *N* (even number) according to eq. (3.32), and the adder tree for the running sum is replaced by the systolic structure derived from eq. (3.35).

It is clear, that the pipeline registers at the outputs of the multiply-accumulate operations cause a delay of k clock cycles, but the derived iteration from eq. (3.35) also inserts an initial pipeline delay, as the first folded sum from eq. (3.36) is only valid after passing the tapped

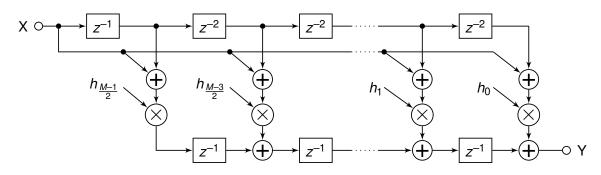


Figure 3.13.: A block diagram of the FIR filter structure from eq. (3.38) with reduced initial pipeline delay.

delay line with M stages. A modification of the fundamental filter function from eq. (3.32) brings it to

$$Y = \sum_{k=0}^{\frac{M-1}{2}} h_{\frac{M-1}{2}-k} \left( z^k + z^{-1-k} \right) X, \qquad (3.37)$$

and yields an alternative iterated function

$$Y_{k} = z^{-1}Y_{k-1} + h_{\frac{M-1}{2}-k}\left(1 + z^{-1-2k}\right)X$$
(3.38)

$$Y_0 = h_{\frac{M-1}{2}-k} \left( 1 + z^{-1} \right) X, \qquad (3.39)$$

where the initial delay is reduced to the unit delay. The corresponding block diagram is shown in fig. 3.13. The overall latency is determined by the number of DSP blocks, which in this case is N/2.

The featured symmetric systolic structure is generic so as to match the DSP block architecture of state-of-the-art FPGAs. Although the vendors put forward the systolic FIR filter, the synthesis of the structure faces two major challenges, routability and timing, which are discussed in the following paragraphs. The achieved results are highlighted afterwards.

#### Routability

Firstly, as the DSP blocks are embedded in an FPGA at dedicated locations in multiple chain-like structures, the interconnection capabilities are limited. In correlation with the filter order, the routability experiences less flexibility. Thus, successful mapping and routing of relatively large filters, compared to the number of DSP blocks, mainly depends on the ability of the development tools to exploit the DSP chain architecture. Even though a generic VHDL design permits an unrestricted synthesis, several failures were observed during the implementation process. Actually, physically separated DSP chains limit a realization of high-order FIR filters beyond the length of a DSP chain. An efficient concatenation of DSP chains is impossible for the evaluated tools without adaptation. Moreover, the mapping of the systolic structure to dedicated slices becomes more error prone, as the number of multiply-accumulate operations exceeds the total number of available DSP blocks. In this case, the

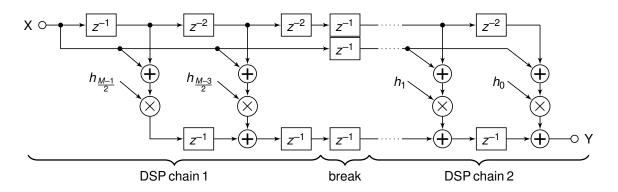


Figure 3.14.: The symmetric systolic FIR structure with additional registers for improved routability and timing. Registers between the systolic elements break the dedicated routes of DSP chains.

tools cannot map the systolic FIR filter to the DSP blocks and distributed arithmetic built on the slice logic at the same time. In conclusion, the implementation of systolic FIR filters, which overlap multiple DSP chains, requires further efforts in the design. As a manual placement and routing is inadequate, the generic structure of the filter is adapted in such a manner so as to support the routability.

The routability is rapidly improved by further pipeline registers between the cascaded running sum. Consequently, the iterated function from eq. (3.38) ultimately changes to

$$Y_{k} = \begin{cases} z^{-1}Y_{k-1} + h_{\frac{M-1}{2}-k} \left( z^{-b_{k}} + z^{-1-2k-b_{k}} \right) X \\ z^{-1}Y_{k-1}, \end{cases}$$
(3.40)  
(3.41)

where eq. (3.41) covers the case that a register is inserted at an arbitrary position k satisfying the condition (M - 1)/2 > k > 1. The elements  $b_k$  of eq. (3.40) represent the number of injected registers before position k. At least one register between the DSP blocks breaks the systolic structure to match a two-column chain architecture. Dedicated routes of DSP chains are thus replaced through slice logic utilizing a simple register (fig. 3.14). Moreover, this method also facilitates the routing of the systolic FIR structure within DSP blocks in combination with distributed arithmetic. After all of the above, the number of DSP blocks or the length of DSP chains no longer restricts the order of realizable digital filters.

### Timing

Secondly, within the scale of an FPGA, DSP chains are distantly located. Each type of device comes along with its specific physical dimensions and arrangement of configurable logic. However, the minimum clock period for the circuit primarily depends on the longest path between the logic elements. The interconnections of DSP chains can therefore be a bottleneck in terms of timing and thus limit the maximum achievable clock frequency. Even if the tools are capable of mapping systolic FIR filters spanning multiple DSP chains, the path lengths cannot be automatically reduced. In this case, a partial break with one register

or more at distinguished positions in accordance with the DSP chain lengths reduces the overall path lengths (fig. 3.14). The maximum clock frequency of the FPGA design is consequently the maximum sample rate of the filter. That limit is verified by timing constraints with the design tools.

### Results

For the proof of concept, we chose an FPGA from Xilinx (Artix 7, XC7A35T-3CSG324) and from Altera (Cyclone 5, 5CEFA5F23C6). Their DSP blocks (Altera Corporation, 2011; Xilinx, 2014) are able to perform the basic operations from fig. 3.11. The bit-widths of the generic VHDL design are adjusted to be ( $W_A$ ,  $W_B$ ,  $W_C$ ,  $W_D$ ,  $W_E$ ,  $W_F$ ) = (15, 15, 16, 18, 34, 36), where the 15-bit wide input samples ( $W_A$ ,  $W_B$ ) are adapted to our application, and, furthermore, the bit-widths of the multiplier ( $W_C$ ,  $W_D$ ) and post-adder ( $W_E$ ,  $W_F$ ) are chosen to avoid overflows and to fit into both FPGA architectures. To illustrate an example, a low-pass filter with a cutoff frequency  $f_{pass}$  to be one-tenth of the sampling frequency  $f_s$  and a stopband frequency  $f_{stop}$ to be one-eighth of  $f_s$  was selected. Furthermore, the stopband attenuation  $A_{stop}$  is chosen to be 102 dB, which corresponds to the quantization noise floor of 18 bit signed coefficients. Estimating the number  $N_{FIR}$  of taps with (Lyons, 2010)

$$N_{\rm FIR} pprox rac{A_{
m stop}}{22 \left(f_{
m stop} - f_{
m pass}
ight)}$$
, (3.42)

186 taps are required to achieve that frequency response. However, we truncated  $N_{\rm FIR}$  to 180, because the symmetry exploiting structure ultimately fits to the total 90 DSP blocks of the Xilinx device. The Altera device includes 150 DSP blocks. Finally, the straightforward structure and its adaptation with dedicated register stages at distinct positions in the data path were evaluated and compared to the state-of-the-art FIR compiler tools (Xilinx, 2015c; Altera Corporation, 2015) in tab. 3.2.

With identical configurations for the bit-widths, both tools were capable of implementing the systolic structure of the 90 taps FIR filter. Apparently, a Xilinx implementation is capable of including the tapped delay line and the output registers into DSP blocks in various ways. Therefore, the tool maps the investigated structures efficiently to the hardware architecture, consuming very less (< 0.5%) additional logic. On the contrary, the DSP architecture of Altera is not that versatile, as only one variant of the FIR filter structure results in competitive results with less than 1 % additional logic (full break with  $z^{-1}$ ). As a result, the structure with one additional register stage at the output of each systolic element performs best in terms of resource utilization.

As discussed, the timing performance is improved by further registers breaking the dedicated routes. The obtained values from the tools are shown in tab. 3.3. The best timing performance is achieved by the fully pipelined structure with two additional registers at each output. Further registers have no noticeable impact on timing. The values for the clock frequencies were obtained from the slow process corner of the timing reports. For the Xilinx

Table 3.2.: Exemplary logic utilization of 90-tap FIR filters					
90 taps	Xilinx Vivado 2016.2	Altera Quartus Prime 16.0			
systolic structure	XC7A35T-3CSG324	5CEFA5F23C6			
straightforward	LUT <sup>1</sup> : 5 / 20,800	ALM <sup>2</sup> : 1,345 / 29,080			
	Reg <sup>3</sup> .: 4 / 41,600	Reg.: 2,537 / 58,160			
	DSP <sup>4</sup> : 90 / 90	DSP: 76 / 150			
partial break $(z^{-1})$	LUT: 3 / 20,800	ALM: 1,292 / 29,080			
	Reg.: 18 / 41,600	Reg.: 2,434 / 58,160			
	DSP: 90 / 90	DSP: 76 / 150			
full break $(z^{-1})$	LUT: 60 / 20,800	ALM: 188 / 29,080			
	Reg.: 152 / 41,600	Reg.: 441 / 58,160			
	DSP: 90 / 90	DSP: 88 / 150			
full break $(z^{-2})$	LUT: 45 / 20,800	ALM: 1,009 / 29,080			
	Reg.: 3429 / 41,600	Reg.: 3736 / 58,160			
	DSP: 90 / 90	DSP: 88 / 150			
Xilinx	LUT: 3201 / 20,800				
FIR Compiler 7.2	Reg.: 4293 / 41,600	not applicable			
(Xilinx, 2015c)	DSP: 90 / 90				
Altera		ALM: 801 / 29,080			
FIR Compiler 16.0	not applicable	Reg.: 2883 / 58,160			
(Altera Corporation, 2015)		DSP: 45 / 150			

Table 3.2.: Exemplary logic utilization of 90-tap FIR filters

(1) look-up table, (2) adaptive logic module (an ALM implements a LUT), (3) register, (4) DSP block with pre-adder, multiplier, and post-adder

implementation, the timing constraints were successively increased to reach the limit. Besides the improved timing in terms of maximum clock frequency, the overall latency of the filter is increased depending on the injected register stages.

In conclusion, at least one variant of our generic design utilizes less logic and operates at a higher clock frequency than the compiler-generated implementation. The efficiency is caused by the simplicity of the approach, whereas the automated design tools include excessive configuration options.

### 3.3.3. Optimized fixed-point arithmetic

For the implementation of high-order FIR filters, the coefficient quantization becomes significant, as the granularity of coefficients increases. Thus, an increased dynamic range for the quantization of coefficients preserves the accuracy of the digital filter. The precision of a signed fixed-point operation depends on the bit-width *b* supported by the hardware. Hence, the transformation from a real coefficient *h* to the corresponding integer value  $I \in \mathbb{Z}$ , used for a signed fixed-point calculation is, in general, described as:

$$I = \operatorname{round}\left(h2^{b-1}\right). \tag{3.43}$$

#### 3. Digital signal processing

90 taps	Xilinx Vivado 2016.2	Altera Quartus Prime 16.0			
systolic structure	XC7A35T-3CSG324	5CEFA5F23C6			
straightforward	238.10 MHz	145.65 MHz			
partial break $(z^{-1})$	303.03 MHz	148.82 MHz			
full break $(z^{-1})$	476.19 MHz	218.77 MHz			
full break ( $z^{-2}$ )	526.32 MHz	231.75 MHz			
Xilinx FIR Compiler	434.78 MHz	not applicable			
Altera FIR Compiler	not applicable	213.72 MHz			

Table 3.3.: Maximum achievable clock frequencies of 90-tap FIR filters

The reversal of the fixed-point representation to a real number  $h_l$  is calculated by:

$$h_I = \frac{I}{2^{b-1}} \,. \tag{3.44}$$

The resulting rounding error is  $h - h_l$ , which is decreased by an increased bit-width *b*. An efficient method for an improved fixed-point precision with a limited bit-width *b* was shown by Shen (Shen, 2010). This method, referred to as "bit compression", performs a left shift operation on the integer value, until all redundant sign extension bits are removed. However, this equals a multiplication with  $2^Q$ , and we calculate *Q* as follows:

$$Q = \left\lfloor \log_2 \left( \frac{2^{b-1}}{|h|} \right) - (b-1) \right\rfloor .$$
(3.45)

The removal of sign extension bits increases the dynamic range of the fixed-point representation of the coefficients, but requires that all products are normalized to a common base before they are added by the accumulator. Therefore, Shen's parallel method implementation performs a normalization at the input of the running sum accumulator. Indeed, the operation is incompatible to the DSP block architecture and is therefore synthesized on distributed logic. Thus, the proposed FIR filter structure does not exploit the entire performance of a hardware mapped systolic FIR filter.

In general, the normalization of a partial term S from eq. (3.37) to a common base is realized in fixed-point representation by:

$$2^{-d_k}S = \operatorname{round}\left(h_k 2^{b-1+Q_k}\right)(z^{-k} + z^{-1-k})X$$
(3.46)

where  $Q_k$  is calculated by eq. (3.45) and  $d_k$  is a normalization factor, which must be individually calculated for each coefficient. Moreover, eq. 3.46 reveals that the multiplication with  $2^{d_k}$ (left shift operation) performs normalization and can be applied to the delayed input samples (fig. 3.15).

For our approach, the  $d_k$  left shift operation for normalization is restricted by the bit-widths of the DSP block architecture, and therefore  $Q_k$  must be limited to an upper value. The bounded value  $Q'_k$  is determined by the largest bit shift applicable to the input samples but

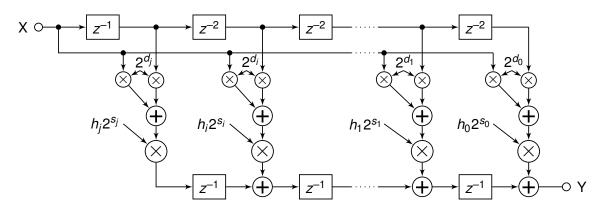


Figure 3.15.: A block diagram of the systolic FIR filter with enhanced dynamic range for fixed-point coefficients by including left shift operations.

does not exceed the bit-width  $W_C$  of the pre-adder. That method benefits from a generously designed pre-adder with regard to the bit-width. With a look at the contemporary DSP architecture from Xilinx (Xilinx, 2014), the pre-adder operates at  $W_C = 25$  bit and the multiplier supports 25 bit×18 bit operations. Thus, for 16-bit input samples and a 25-bit wide pre-adder, the limit of  $Q'_k$  is 9-bit. An example of the calculation is shown in tab. 3.4.

Table 3.4.: Exemplary numerical values for a maximized dynamic range of the coefficients quantization.

h <sub>k</sub>	Ь	1	two's-complement	$Q_k$	$Q'_k$	d <sub>k</sub>	$h_k 2^{b-1+Q'_k}$	shifted two's-complement
2.99 10 <sup>-8</sup>	18	0	00,0000,0000,0000,0000	24	9	0	2	00'0000'0000'0000'0010
–3.99 10 <sup>-5</sup>	18	-5	11'1111'1111'1111'1011	14	9	0	-2678	11'1111'0101'1000'1010
4.99 10 <sup>-3</sup>	18	654	00'0000'0010'1000'1110	7	7	2	83718	01'0100'0111'0000'0110
–3.99 10 <sup>-1</sup>	18	-52298	11'0011'0011'1011'0110	1	1	8	-104595	10'0110'0111'0110'1101

### Results

For the evaluation of the proposed structure from fig. 3.15, we designed a low-pass filter of order 179 with the window method (Lyons, 2010) based on Nuttall's window (Nuttall, 1981). Furthermore, the result of the FIR filter generated by the Xilinx FIR Compiler is compared to that of the proposed structure with additional bit shift operations and a floating point calculation. The frequency responses, which are derived by a Fourier transform of the simulated impulse responses of the filters, are shown in fig. 3.16.

A comparison of frequency responses of the compiled FIR filter structures from Xilinx and Altera with 18 bit signed coefficients reveals no significant differences. However, our proposed structure with shift operations for coefficient normalization results in an improved stopband attenuation, even though it also exploits 18 bit signed coefficient multiplier. On a Xilinx FPGA, that structure utilizes exactly the same logic resources in comparison to the equivalent variant without left shift operations. The critical path delays remain constant without reducing the maximum clock frequency.

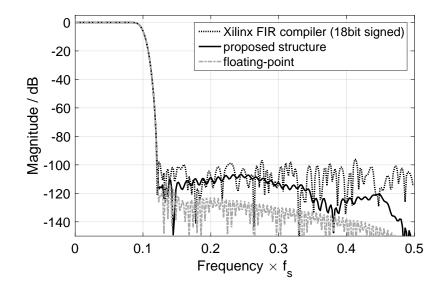


Figure 3.16.: The frequency responses of a symmetric systolic FIR filter with 90 taps depended on the numeric representation of the coefficients.

## 3.3.4. Conclusion

In this section, a systolic structure for a symmetric FIR filter was proposed, where the systolic elements ideally match the prevalent DSP block architecture of FPGAs. Thus, the derived iterative mathematical functions for the systolic structure support the mapping of a digital filter to various architectures with different constraints. Moreover, a generic FIR filter design was synthesized by the tools from Altera and Xilinx to evaluate the efficiency of the structure in terms of logic utilization and maximum clock frequency. The results confirmed that the proposed structure with additional register stages improves routability and timing of high-order FIR filters and is superior to state-of-the-art FIR compiler tools. Furthermore, to yield an increased dynamic range for coefficients quantization, we enhanced the structure by shift operations, thus improving the precision of fixed-point arithmetic. Finally, the exploitation of the entire DSP blocks enables an efficient realization of high-order FIR filters with fixed-point arithmetic in FPGAs, while utilizing less than 1 % additional slice logic and running at clock frequencies above 200 MHz.

# 4. Data interface

State-of-the-art detector readout electronics require high-throughput data acquisition (DAQ) systems. In many applications, e.g. for medical imaging, the front-end electronics are set up as separate modules in a distributed DAQ. A standardized interface between the modules and a central data unit is essential. The requirements on such an interface are varied, but demand almost always a high throughput of data. Beyond this challenge, a Gigabit Ethernet interface is predestined for the broad requirements of systems on a chip (SoC) up to large-scale DAQ systems. We have implemented an embedded protocol stack for an FPGA capable of high-throughput data transmission and clock synchronization. A versatile stack architecture for the User Datagram Protocol (UDP) and Internet Control Message Protocol (ICMP) over Internet Protocol (PTP) is presented. With a point-to-point connection to a host in a Micro Telecommunications Computing Architecture (MicroTCA) system, we achieved the theoretical maximum data throughput limited by UDP both for 1000BASE-T and 1000BASE-KX links. Furthermore, we show that the random jitter of a synchronous clock over a 1000BASE-T link for a PTP application is below 60 ps.

## 4.1. State-of-the-art

Distributed data acquisition systems are commonly spread over different fields of application in nuclear physics or medical imaging. Depending on the application, there are various requirements for the interconnections of submodules. The main challenge for an interface is the user acceptance with respect to handling and interoperability of different device types. In addition, the data throughput of the interface is an important criterion for usability and should not limit the performance of the whole DAQ system. Even though proprietary interfaces can fulfill these requirements, standardized technologies benefit from industry-proven components and are essential for reliable applications. A popular and well accepted specification is the IEEE 802.3 Standard for Ethernet (IEEE, 2015). This standard specifies the physical layer used by the Ethernet. Until now, connections up to 100 Gbit/s are specified and are going to be established by the industry. Nevertheless, for embedded systems, a Gigabit Ethernet connection is the state-of-the-art. A widespread technology is known as 1000BASE-T, which defines the 1 Gbit/s Ethernet over twisted pair copper cables. The application of Gigabit Ethernet is not restricted to the use in Local Area Networks. It also finds its way into board-to-board applications. E.g. the backplane of a MicroTCA system should implement at least one port for an Ethernet connection (PICMG, 2011), which is usually implemented as 1000BASE-KX on the physical layer. A link on the electrical backplane uses two differential pairs to establish a Gigabit Ethernet connection. With Gigabit Ethernet, the possibilities of applications range from a high speed data transfer to clock synchronization in a distributed DAQ system (Moreira et al., 2009). This work is related to the implementation and test of an embedded Gigabit Ethernet protocol stack for FPGAs. With a versatile stack architecture, we will demonstrate the performance of high-throughput data transfers with UDP and clock synchronization over the PTP. Our aim is to investigate the maximum achievable data throughput with an FPGA-based SoC as data source and a PC as receiver. For our application we need a high-throughput DAQ to cope with the count rate expected for prompt gamma imaging in particle therapy (Hueso-González et al., 2015). This will be evaluated with a 1000BASE-T and 1000BASE-KX link on a MicroTCA system. In addition, we will demonstrate the performance of a synchronized point-to-point connection as shown in (Girerd et al., 2009) with a Xilinx FPGA and different hardware for the physical layer.

## 4.2. Embedded Gigabit Ethernet protocol stack

The high-throughput data interface should connect an FPGA to a host, and should be synchronizable to a master over an Ethernet link. For this purpose, an FPGA mezzanine card was designed (Födisch et al., 2014), which equips commonly used FPGA boards with an Ethernet interface. Moreover, the data interface must be capable to operate on standardized hardware platforms, e.g. a MicroTCA advanced mezzanine card (AMC) with an FPGA and backplane Ethernet. The hardware used for the investigations is shown in fig. 4.1, and is the base for the embedded protocol stack running in an FPGA.



Figure 4.1.: The hardware used for our investigations. An FPGA mezzanine card was designed (left) for evaluation of throughput and synchronization. Additionally, the interface must be compatible to standardized hardware platforms (right), e.g. a MicroTCA advanced mezzanine card (designed by Deutsches Elektronen-Synchrotron, DESY).

# Physical layer

The embedded Gigabit Ethernet protocol stack connects to the physical layer through the data link layer regarding the Open Systems Interconnections (OSI) model as shown in fig. 4.2. Higher level functions shall be implemented above the embedded protocol stack

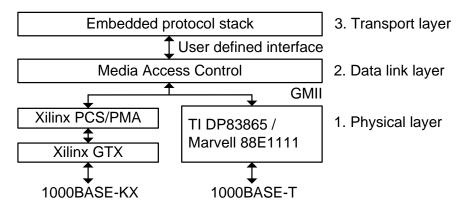


Figure 4.2.: Layer stack according to the OSI model for our embedded Gigabit Ethernet protocol stack. The higher level protocols are implemented in the transport layer. For evaluation of a 1000BASE-T link we use the external ICs DP83865 from Texas Instruments and 88E1111 from Marvell. All other components are implemented in a single FPGA.

in an application layer. The IEEE 802.3 Standard for Ethernet defines different types of copper based connections between two transceivers over the physical layer. For 1000BASE-T, it is proposed to use four pairs of wires for the signal transmission. The 1000BASE-KX technology uses two pairs for the transmission of data. The specific signaling and coding in the physical layer will be done with industry-proven integrated circuits (ICs). For the 1000BASE-T signal coding we will use the IC 88E1111 from Marvell (Marvell, 2004) and DP83865 from Texas Instruments (Texas Instruments, 2004). To access the physical layer according to the 1000BASE-KX technology, we will use a GTX Transceiver of the Xilinx Kintex 7 FPGA (Xilinx, 2015a) in combination with the "Xilinx 1G/2.5G BASE-X PCS/PMA Core" (Xilinx, 2015b). All physical layer transceivers (PHYs) have a common interface to the overlying data link layer and its Media Access Control (MAC). The MAC connects to the PHY via the Gigabit Media Independent Interface (GMII). The MAC should be implemented in the FPGA. Due to clear specifications on high data throughput and hardware, we do not intend to provide a compatibility to other PHYs with Reduced or Serial Gigabit Media Independent Interface (RGMII or SGMII) or even lower speeds as specified for 10BASE-T or 100BASE-T.

# Data link layer

The data link layer with respect to fig. 4.2 contains the MAC and a management interface. The MAC controls the access to the PHY and transmits the data in an Ethernet packet. It processes the input and output signals of the GMII with a frequency of 125 MHz. An Ethernet packet encapsulates the Ethernet frame by adding the preamble and the start frame

#### 4. Data interface

delimiter (SFD). The MAC composes (and also decomposes) the Ethernet packet with 8 bits per clock cycle (8 ns) from the Ethernet frame. This is essential for a maximum line rate of 1 GBit/s. The standard for Ethernet demands that two consecutive Ethernet packets are separated by the interframe gap (IFG) for at least 96 bit times (96 ns).

Usually all PHYs provide a management interface for the configuration of their internal register sets. The Management Data Input/Output (MDIO) interface is used for a basic link configuration (e.g. autonegotiation advertisement).

# **Transport layer**

The transport layer shall provide a stack for higher-level protocols encapsulated in the Ethernet frame. Its architecture must be easily extensible for any desired protocol in the layer stack. We target a maximized data throughput from the application layer for UDP. The theoretical data throughput of the UDP with a payload of 1472 Byte, which corresponds to a Maximum Transfer Unit (MTU) of 1500 Byte for the Ethernet frame, is 114.09 MiB/s. If the host supports jumbo frames with an MTU of 9000 Byte, the maximum data throughput is increased to 118.34 MiB/s. The embedded protocol stack should not limit the frame size of an Ethernet packet. Although various implementations of Gigabit Ethernet protocol stacks (Löfgren et al., 2005; Dollas et al., 2005; Kühn et al., 2008; Uchida, 2008; Herrmann et al., 2009; Alachiotis et al., 2010; Lieber and Hutchings, 2011; Nagy et al., 2011; Alachiotis et al., 2012; Sasi et al., 2013; Mahmoodi et al., 2014; Zhou and Yao, 2014; Batmaz and Dogan, 2015) are published, there exists no solution which achieves the theoretical maximum data throughput with UDP. Only (Uchida, 2008) reached maximum performance with a TCP/IP processor. A comparison of slice logic resources, as it is done in (Löfgren et al., 2005; Herrmann et al., 2009; Alachiotis et al., 2010; Lieber and Hutchings, 2011; Alachiotis et al., 2012; Mahmoodi et al., 2014), is not our intention, because each implementation is based on a different FPGA architecture. Whereas slice logic utilization is an important design criterion, it varies in accordance of generic configurations (e.g. FIFO depths) as well as the supported features (e.g. checksum calculations). Thus, a comparison to other implementations without the context of the application is difficult. In order to provide all the necessary functionality, we need a protocol stack that serves ARP, ICMP, PTP and UDP with the focus on maximum data throughput. A header of a protocol should be partially configurable by an user interface but also calculated automatically (e.g. length fields). All stack layers support a checksum calculation if it is required by the protocol. In terms of Löfgren's classification proposed in (Löfgren et al., 2005), our requirements belong to a "Medium UDP/IP" core.

# 4.3. Implementation

# 4.3.1. System overview

Our implementation is designed as intellectual property (IP) core with the hardware description language VHDL. It includes the MAC as well as an embedded protocol stack. For the 1000BASE-KX implementation the PHY is already included in the FPGA. As shown in fig. 4.3, the Gigabit Ethernet IP core gets its data from the application layer through a common First-In-First-Out (FIFO) interface. The asynchronous FIFO is designed to operate at

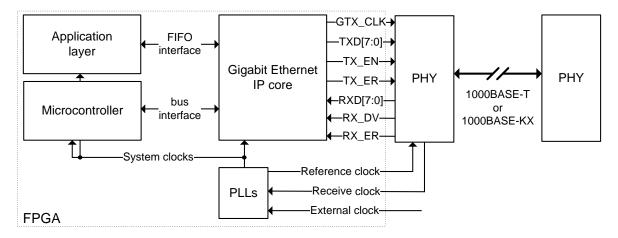


Figure 4.3.: An overview of the SoC with the embedded Gigabit Ethernet IP core and its interfaces. In case of the 1000BASE-KX implementation, the PHY will be included in the FPGA.

frequencies of 125 MHz with a bit width of 32 bit and stores at least the payload of one packet. It is used to stream the application data with high throughput into the transport layer (UDP) of the core. Another interface to the core is built with an 32 bit microcontroller (Harboe, 2016). The microcontroller with its bus-interface limits the data throughput for this interface far below the limit of a protocol. So it is used for slow-control applications over UDP, ICMP and ARP. The MDIO interface is also handled by the microcontroller and is not shown. Fig. 4.3 shows the signals of the GMII and their directions between the MAC (embedded in the Gigabit Ethernet IP core) and the PHY. The same signals will be used for the 1000BASE-KX implementation with the embedded Xilinx PHY. The system clocks as well as the necessary clocks for the PHY will be generated by built-in phase-locked loops (PLLs) of the FPGA.

# 4.3.2. Media Access Control

The functions of the MAC are restricted to the basic needs for interfacing the GMII. Fig. 4.4 shows the basic structure of the module for the transmission datapath of the MAC. For the transmission datapath, it will compose the Ethernet packet with its preamble and SFD which are initially stored in a shift register. In the following states, data is passed through this register and the arithmetic logic unit (ALU) for the checksum calculation. Finally, the 32 bit

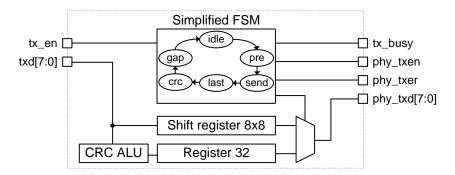


Figure 4.4.: Basic structure of the MAC for the transmission datapath. The output signals are connected to the GMII and the input is sourced by the transport layer.

frame check sequence (FCS) is added at the end of the frame. The finite state machine (FSM) of the module controls this dataflow and keeps the IFG at a programmable number of clock cycles. The module for the receiving datapath is built in the same way. It decomposes the Ethernet frame out of a received Ethernet packet and passes it to the transport layer. The MAC logic is capable of running at the speed of the transceiver clocks (125 MHz). So there is no need for additional FIFOs for clock domain crossing. This results in a deterministic latency for the complete datapath from the transport layer to physical layer and vice versa. An example of a transmitted Ethernet packet is shown in fig. A.1. The waveforms are captured with an integrated logic analyzer (Xilinx Chipscope).

# 4.3.3. Embedded protocol stack

With a look at the OSI reference model and its layers for a network communication, the stack architecture implies a dataflow from the top layer to the bottom layer. That means that the application passes its data from the transport layer to the data link layer until it is transmitted by the physical layer. So data will be "pushed" from the source to the sink and we call this dataflow as "Data-Push" model shown in fig. 4.5. The scheme in fig. 4.5 implies, that the

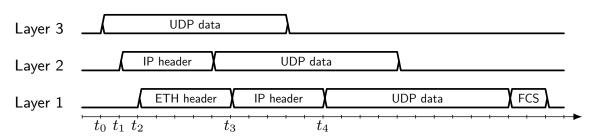


Figure 4.5.: An example of a dataflow through the stack layers driven by the "Data-Push" model

application layer has valid data which is transported through the UDP layer (layer 3). The underlying IP layer (layer 2) will start its transmission with one clock cycle delay, beginning with its own data for the IP header. The data coming from the upper layer has to be buffered in the underlying layer, while this layer sends its own data. The same situation occurs when

the IP layer passes its data to the Ethernet layer (layer 1). Finally, the dataflows initiated at time  $t_0$  and  $t_1$  are encapsulated at time  $t_4$  and  $t_3$  respectively. To keep this data valid for the latency during transmission time, data buffers are needed. As a consequence, a layer has to buffer at least the data of the overlying layer. One can also easily imagine the situation where two layers have valid data and pass it to a shared underlying layer. In this case, the number of data buffers doubles. A consistent data flow through all layers with the "Data-Push" model is handled with the appropriate number of data buffers. This model consumes additional memory for redundant data.

An alternative approach for a dataflow is shown in fig. 4.6. We call this model as "Data-Pull" model. In contrast to the "Data-Push" model from fig. 4.5, the dataflow is initiated by the

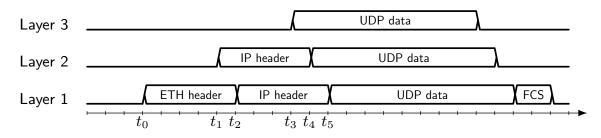


Figure 4.6.: An example of a dataflow through the stack layers driven by the "Data-Pull" model

low-level layer. The data of the overlying layers is just passed through a single register stage at the time when it is encapsulated into the frame of the underlying layer. This reduces the amount of data buffers tremendously to a single register at each interconnection. In the example shown in fig. 4.6, the latency from the UDP data in layer 3 to the time when it is encapsulated in layer 1 is reduced to two clock cycles (from  $t_3$  to  $t_5$ ). Each register stage in the underlying layer introduces one clock cycle delay. Of course the dataflow can be optimized to zero latency without additional register stages, but this will cause timing problems. Data buffers are needed in the application layers as well, but buffer redundancy in comparison to the "Data-Push" model is eliminated. The costs for this implementation are a simple arbiter and control logic and range far below those of the "Data-Push" approach. The basic scheme of the interconnections of layers is shown in fig. 4.7. All modules in the same layer N+1 pass their state to the arbiter logic. In the simplest case this is a FIFO state which indicates whether there is valid data to send or not. In case of valid data the arbiter decides which module of a layer is served first and passes this information to the underlying layer N. The module from layer N controls the dataflow of the overlying layer with its controlbus. After all, the data from layer N+1 is multiplexed to the receiving module in layer N. A real data transfer of the implemented "Data-Pull" model is shown in fig. A.2. The example in fig. A.2 shows at its initial clock cycle at position 1 that the UDP layer has valid data to send (signal "udp fifo empty" is low). In conjunction with the arbiter bus, the IP layer also reports that there is valid data to send (signal "ip fifo empty" is low). With this condition the Ethernet layer starts the transmission of data (signal "tx\_en" is high) at position 2. At position 14 the Ethernet layer pulls the data from the overlying layer by setting the signal "tx\_next\_eth" to high. The Ctrl Demux from the interconnection logic of the layers shown in

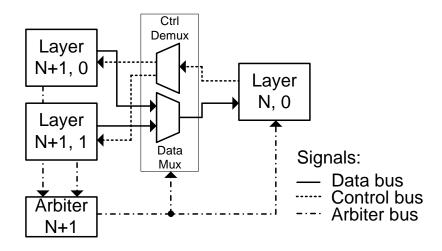


Figure 4.7.: Interconnections of layers with an arbiter and control logic. This architecture eliminates the need for redundant data buffers in a layer stack.

fig. 4.7 switches this signal to the IP layer (signal "tx\_start\_ip" is high). At the next clock cycle (position 15), the IP layer transmits its data occurring with an additional delay of one clock cycle in the frame of the Ethernet layer (signal "txd", position 16). The IP layer encapsulates the application data from the UDP layer in the same way into its frame. This can be seen by the control signals "tx\_next\_ip" and "tx\_start\_udp" at position 33 and the UDP data (signal "udp\_txd") and the IP data (signal "ip\_txd") at position 34 and 35 respectively. Finally, the MAC composes the entire packet as shown in fig. A.1.

## 4.3.4. Clock synchronization

An important issue in a distributed DAQ is a uniform clock distribution. Although a dedicated clock line is a simple and precise solution, it cannot be used for an absolute synchronization of all timestamps in the system. For this purpose an additional data signal for the transmission of a known timestamp reference is needed. The PTP offers the possibility to synchronize the timestamps over a data link. Additionally, a Gigabit Ethernet link has the property, that a transmission clock is embedded in the datastream, because the transferred data is synchronous to this reference clock. As a consequence, a receiver can recover this clock frequency. In a 1000BASE-T application, the slave recovers the clock from the master out of the data stream. This task is done by the PHY (see fig. 4.8). So it is possible to synchronize the clock signals as well as the timestamps over a single Gigabit Ethernet link. It is also known that the clock offset from a master and a slave cannot be corrected with PTP below a resolution of 8 ns (this corresponds to the transceiver clock of 125 MHz) without a phase alignment of the clocks. An accurate implementation is already done with the White Rabbit project (Moreira et al., 2009), but does not support a 1000BASE-T link by default. A synchronization over a 1000BASE-T link was done by (Girerd et al., 2009). They achieved a precision of 180 ps with the DP83865 from Texas Instruments and an FPGA from Altera. The limiting factor was the jitter of the built-in PLL of the FPGA. Our implementation is based on Xilinx FPGAs with an improved jitter. So we want to determine the absolute precision which is achievable with these devices and different ICs for the physical layer. The implemented clocking scheme is shown in fig. 4.8. Each PHY is configured with the MDIO interface to act as a master or as a slave. During the autonegotiation procedure, these configurations are advertised.

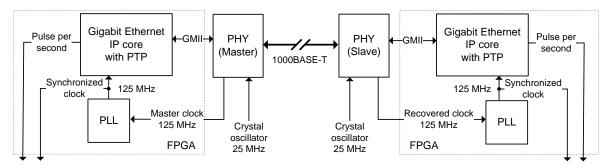


Figure 4.8.: Scheme of the clock synchronization through a point-to-point connection over a 1000BASE-T link. One PHY acts as master and embeds the clock reference into the datastream. The slave recovers a synchronized clock signal with a frequency of 125 MHz. A PLL inside the FPGA is used to build up the clock tree. Our Ethernet IP core provides synchronized timestamps and a pulse per second for the test setup.

# 4.4. Measurements and results

For our performance test on a 1000BASE-T link we use the Xilinx evaluation board SP605 equipped with a Spartan 6 (LX45T) FPGA and the PHY 88E1111 from Marvell. We also use an FPGA Mezzanine Card (FMC) equipped with two PHYs from Texas Instruments (DP83865) attached to the SP605. The host is a MicroTCA crate equipped with an AMC CPU module from Concurrent Technologies (AM 900/412-42) and a MicroTCA Carrier Hub (MCH) from N.A.T. (NAT-MCH-PHYS). The CPU module provides two 1000BASE-T ports at the front and two 1000BASE-KX ports at the backplane. A 1000BASE-KX link to the CPU is established with a Kintex 7 (325T) on the HGF-AMC from DESY/KIT through the MicroTCA backplane and the switch from the MCH. The operating system on the host is Ubuntu.

To evaluate the MAC Layer and the latency of the entire stack, we measured its output signals on the GMII. A maximum throughput is achieved if the transmit enable signal (see "phy\_txen" in fig. A.1) is high all the time except the time for the IFG. A constant latency is achieved, if the beginning of a transmission cycle and the arrival time of a packet at the receiver have a time deviation much smaller than a clock cycle. Both conditions could be experimentally verified, which indicates that the MAC layer is capable of transferring the maximum throughput with a constant latency (see fig. A.3). The measurement shown in fig. A.3 was captured with an oscilloscope during a transmission of UDP packets with a fixed payload of 20 Byte. This test was chosen to verify a maximum throughput, a constant latency of the core (see measurement "packet length" and "IFG" in fig. A.3) and the overall system latency between two PHYs (see measurement "Phy1-Phy2 latency" in fig. A.3). For

this setup we used the Marvell 88E1111 on both sides connected by a cable of 50 cm length.

# 4.4.1. Throughput performance

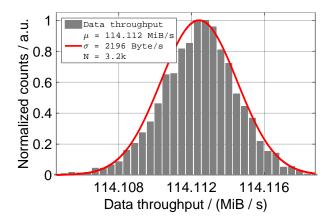
To check the performance of our FPGA implementation with a 1000BASE-T PHY, we established a point-to-point connection between the FPGA and the CPU module. The host serves a UDP socket where the incoming data throughput is measured. The data from the FPGA contain an increasing 32 bit counter value which is used to identify a missing packet or a corrupted datastream. For this measurement the throughput of the UDP on a Gigabit Ethernet link is our reference. As mentioned in sec. 4.2, this value is 114.09 MiB/s for a payload of 1472 Byte. If the payload is decreased, the data throughput decreases as well because of the increasing rate of protocol overhead. Tab. 4.1 shows the achievable data throughput dependent on the UDP payload. Additional overhead in the Ethernet packet limits the line rate. Thus the Ethernet Standard limits the MTU per frame to 1500 Byte (this

UDP payload / Byte	Data throughput / (MiB/s)	Line rate / (1 GBit/s)
8972	118.339	99.3 %
1472	114.094	95.7 %
1024	111.991	93.9 %
512	105.597	88.6 %
256	94.775	79.5 %

Table 4.1.: Theoretical data throughput dependent on the payload of a UDP packet.

results in a UDP payload of 1472 Byte), it is also common to use jumbo frames with an MTU of 9000 Byte (8972 Byte UDP payload). Our implementation supports jumbo frames and this performance will also be evaluated with the MicroTCA host. The measurement of data throughput at the host requires also a measurement of time for the corresponding amount of bytes. Because Linux is not a real-time operating system, this time measurements are above the nanosecond scale. The setup with a Linux host is sufficient for an average estimation of data throughput. Our application measures the incoming bytes on the socket, checks if data is valid and prints out the error rate and the average data throughput every ten seconds. The results of the data throughput tests with three different devices on the physical layer are shown in tab. 4.2. An example of a measurement of data throughput over 9 hours is shown in fig. 4.9. During this measurement all data were transferred without errors. The standard deviation of data throughput was 2196 Byte/s. This is caused by uncertainties in the time measurement and the latency of packet buffering in the operating system.

The results show an excellent performance up to the theoretical limit of the UDP data throughput. The values above this limits are caused by frequency uncertainties for the transmission clock. The reference values from tab. 4.1 are calculated at a clock frequency of 125 MHz. A fixed deviation of that frequency and the mentioned lack of a precise time measurement on a Linux system can cause a data throughput value above the reference value. This tests also show the importance of an efficient host as receiver. If the host is not configured appropriately, packet losses will happen. In our configuration packet losses



- Figure 4.9.: Distribution of data throughput for 1472 Byte UDP payload measured over 9 hours.
- Table 4.2.: Measured data throughput dependent on the UDP payload. The tests were performed with different hardware platforms. Data throughput is the mean value for more than 100 s.

Physical layer	Hardware	UDP payload / Byte	Data throughput / (MiB/s)
1000BASE-T	TI DP83865	8972	118.344
1000BASE-T	Marvell 88E1111	8972	118.345
1000BASE-KX	Xilinx GTX	8972	118.316
1000BASE-T	TI DP83865	1472	114.112
1000BASE-T	Marvell 88E1111	1472	114.097
1000BASE-KX	Xilinx GTX	1472	114.079
1000BASE-T	TI DP83865	1024	112.015
1000BASE-T	Marvell 88E1111	1024	111.995
1000BASE-KX	Xilinx GTX	1024	111.977
1000BASE-T	TI DP83865	512	105.643
1000BASE-T	Marvell 88E1111	512	105.641
1000BASE-KX	Xilinx GTX	512	105.619
1000BASE-T	TI DP83865	256	94.827
1000BASE-T	Marvell 88E1111	256	94.836
1000BASE-KX	Xilinx GTX	256	94.823

at the host receiver just occur at payloads smaller than approximately 256 Byte. This is caused by the excessive load of more than 388 kPackets/s. As a second test we measured the performance of the ICMP protocol layer with ordinary Ping requests from the host to the FPGA device. A Ping request is processed by an interrupt routine with the microcontroller inside the FPGA. The host generated at least 1000 Ping requests at an interval of 200 ms. The results are shown in tab. 4.3. The Ping requests were sent while the FPGA transmits UDP packets with maximum data throughput. The arbiter of the IP layer prioritizes the ICMP protocol, so that a parallel UDP data stream did not block the ICMP layer. Because of the switch of the MCH for the 1000BASE-KX backplane links, the round-trip time (RTT) for a Ping request is higher than for a point-to-point connection. If the UDP layer application was turned off, the RTT of the backplane link was decreased to 0.252 ms with 0.051 ms standard deviation. The same test was done for the ARP layer, which has a higher priority than the IP

				(,	•••••••••••••••••••••••
Physical layer	Hardware	Min / ms	Mean / ms	Max / ms	Stddev / ms
1000BASE-T	TI DP83865	0.203	0.315	0.444	0.067
1000BASE-T	Marvell 88E1111	0.189	0.307	0.547	0.066
1000BASE-KX	Xilinx GTX	0.740	1.091	1.441	0.162

Table 4.3.: The results of the ICMP layer test with Ping requests. The host generated 1000 Ping requests and the round-trip-time is measured (min, mean, and max value).

layer in the protocol stack. The Linux host generated additional ARP request with the arping command, while the FPGA handles the ICMP and UDP layer. As a result, all requests were served without losses with a RTT below 1 ms for all hardware platforms.

# 4.4.2. Synchronization

The performance of the clock synchronization is limited by the accuracy of the clock recovery system in the signal chain of the 1000BASE-T slave. Whereas the clock of the master can achieve the desired precision by choosing an appropriate clock source, the precision of the clock of the slave depends on its components in the signal chain for the clock distribution and recovery (see fig. 4.8). For our evaluation hardware, the quality of the signal chain is mainly determined by the PHY which is responsible for the clock recovery out of the datastream. The second component which influences the achievable precision is the FPGA, where the recovered clock is used for timestamp generation. Usually a PLL inside the FPGA is used to build the clock tree for all clock domains. So we want to evaluate, whether the builtin PLL limits the overall system. At first we measured the phase noise of a clock source with very low jitter which will be used as the input signal for the PLL. The phase noise is correlated with the random jitter and therefore it determines the precision of the timing system. The measurements of a low jitter clock and the performance of the built-in PLL of the FPGA (Spartan 6 LX45T) with that input is shown in fig. 4.10. These measurements were taken with a HA7062B phase noise analyzer from Holtzworth Instrumentation and the signal generator SMA100A from Rohde & Schwarz as low jitter clock source. Although there are various configurations possible for the PLL, for this measurement we set up the multiplier and the divider value to 8. The integrated phase noise of the Xilinx PLL was found out to be 6.47 ps in the range of 10 Hz to 1 MHz. Without any additional hardware, this constitutes a design limit for the precision of synchronous timestamp generation with the FPGA. To find out the random jitter of the clock synchronization over a 1000BASE-T link, we set up a point-to-point link between a master and a slave PHY and measured the clock to clock jitter in the time domain. The clock of the master triggers the measurement of time difference between the two rising edges of both clocks. The results of the measurement for the PHY DP83865 (master and slave) are shown in fig. 4.11. The clock signal is measured at an output pin of the FPGA with a frequency of 125 MHz (see fig. 4.8). It is buffered with an output register (ODDR2 primitive from Xilinx). During the measurement illustrated in fig. 4.11, the FPGA sent UDP packets with maximum throughput and PTP packets at an interval of 1 s. For another measurement, we bypassed the PLL and distributed the

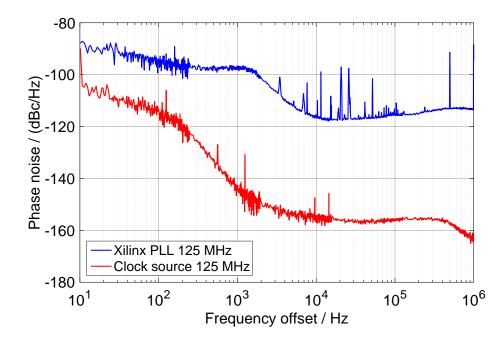


Figure 4.10.: A measurement of the phase noise of a 125 MHz low jitter clock source (red) which sources a PLL in a Spartan 6 LX45T. The corresponding output of the PLL (blue) has an integrated phase noise of 6.47 ps in the range of 10 Hz to 1 MHz.

recovered slave clock with an ordinary built-in clock buffer to the timing logic. With regard to that, the jitter was increased from 55 ps to 64 ps. Finally, we repeated the measurement of fig. 4.11 with the PHY 88E1111 for the master and the slave. With this setup we achieved a clock to clock jitter of 70 ps. In both setups the clock source of the the master was an crystal oscillator with approximately 6 ps random jitter (measured with the phase noise analyzer in the range from 10 Hz to 1 MHz). As a result, we can state that the precision is influenced by all components in the signal chain. It depends mainly on the clock recovery system of the PHY and the ability of the PLL of the FPGA to reduce random jitter. In addition to the measurement of the clock to clock jitter, we have taken measurements to estimate the precision of synchronization with timestamps generated by the master and the slave. Both devices run on the synchronized clock signal with the same frequency. An absolute synchronization of the timestamps is performed with PTP every second. Each synchronized device generates one pulse per second (PPS) at an output of the FPGA. A measurement of the time difference between the PPS signal of the master and the slave is shown in fig. A.4. The measurement was running over 13 hours in the lab and shows that the timestamps are synchronized with a random jitter of 59 ps. The digital logic for timestamp generation in the FPGA was sourced by a clock signal with a frequency of 125 MHz from the built-in PLL. For the measurements shown in fig. A.4 we used the DP83865. The same measurements were repeated with the 88E1111 PHY and resulted in a random jitter of 72 ps for the timestamp synchronization with a short-term measurement (approx. 2 hours). All measurements show a constant offset up to 8 ns which cannot be reduced with the PTP.

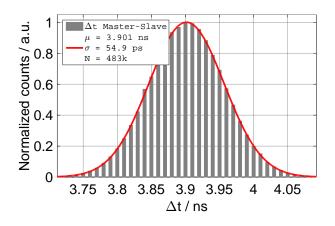


Figure 4.11.: Clock to clock jitter between the master and the slave PHY. Both are the DP83865. The standard deviation  $\sigma$  of the distribution is about 55 ps.

## 4.4.3. Resource utilization

Our Ethernet IP core can be configured with an arbitrary FIFO size for the application above the UDP layer. Our basic configuration consists of two channels for the application interface to the UDP layer. One channel is interfaced by the microcontroller and one is interfaced by a high-throughput application. On each interface there is one FIFO for the payload with a depth of at least two UDP packets. The payload size of one packet is set by generics and is by default 1472 Byte. For the support of jumbo frames this size can be easily adjusted to 8972 Byte. Larger FIFO depths and payload sizes are possible as well. The FIFOs are implemented on Dual-Port Block Memory integrated in the Xilinx FPGA. They can also be placed on distributed slice registers. The ICMP, PTP and ARP layer can store one packet to send. All receiving datapaths are configured to store one packet as well. Because there are various configurations possible, we renounce a comparison to other implementations. The resource utilization reported by the Xilinx tools for the Ethernet stack with support for ARP, PTP, ICMP and UDP is presented in tab. 4.4 and tab. 4.5, using the Xilinx ISE 14.7 tools for the implementation. The 1000BASE-T implementation with support for PTP and an MTU

Module	Slices	Slice Reg	LUTs	LUTRAM	BRAM
MAC	140	459	345	16	0
Ethernet	246	479	648	0	0
ARP	163	498	492	0	0
IP	274	546	669	0	0
ICMP	60	169	108	24	1
UDP	237	551	573	1	9
PTP	672	1890	2071	0	0
Total	1792	4592	4906	41	10

Table 4.4.: Slice Logic utilization for the Gigabit Ethernet stack with a Spartan 6 (LX45T)

of 1500 Byte on a Spartan 6 FPGA with 6822 slices occupies 1792 slices corresponding to 26.27 % (16,42 % without PTP) total occupied slices.

The implementation on Kintex 7 is designed for a 1000BASE-KX link on a MicroTCA back-

plane. This implementation uses a Xilinx IP core with a GTX transceiver as PHY. This consumes additional logic but doesn't need an external PHY. This implementation aims only at maximum data throughput and is not designed to perform a synchronization over the MicroTCA backplane. Thus, the PTP layer is not included. The 1000BASE-KX implementation

Module	Slices	Slice Reg	LUTs	LUTRAM	BRAM
GMII_to_GTX	446	997	826	71	0
MAC	94	299	276	32	0
Ethernet	137	378	419	0	0
ARP	173	498	485	0	0
IP	250	546	684	0	0
ICMP	61	169	114	24	1
UDP	198	580	580	1	5
Total	1359	3467	3384	128	6

Table 4.5.: Slice logic utilization for the Gigabit Ethernet stack with a Kintex 7 (325T)

without support for PTP and an MTU of 1500 Byte on a Kintex 7 FPGA with 50959 Slices occupies 1359 Slices corresponding to 2.67%. All reports for slice logic utilization also include several logic for internal tests and debug options.

# 4.5. Conclusion

With the need of a high-throughput UDP application, we have presented an entire stack architecture for a Gigabit Ethernet interface on an FPGA. The stack was built for the protocols UDP, ICMP, IP, ARP and PTP and can be easily extended or cut down in functionality. For a straightforward implementation we showed two basic models for the dataflow in a stacked architecture. Our embedded Gigabit Ethernet protocol stack is designed with the "Data-Pull" model to eliminate redundant buffers. A clear modular architecture for each layer with a control and arbiter logic at the interconnections keeps this implementation versatile. The underlying MAC and physical layer are also replaceable. All modules are written in VHDL and tested on Xilinx Spartan 6 and Kintex 7. We demonstrated the achievable data throughput with an UDP application on a 1000BASE-T and 1000BASE-KX link. In both cases we reached the maximum data throughput of 114.1 MiB/s with an MTU of 1500 Byte and 118.3 MiB/s with jumbo frames of 9000 Byte. The overall performance for other use cases is also excellent. Finally, we investigated the performance of a clock synchronization over a 1000BASE-T link. In dependence of the PHY, we achieved a precision of 55 ps for the clock to clock jitter between the master and the slave. An absolute synchronization of timestamps was done with PTP. The long-term test showed a standard deviation of 59 ps for the synchronized timestamps. Due to the generic data interface, the UDP/IP stack can be easily adapted to detector applications where a high data throughput is required. With regard to the requirements of precise timing applications, the relative timing is in the sub nanosecond range whereas the absolute accuracy remains in the limits of PTP.

# 5.1. Digital pulse shapers

The fundamental methods for processing signals from nuclear electronics have been described in (Kowalski, 1970; Nicholson, 1974; Spieler, 2005; Knoll, 2010). All literary work covers the concepts based on analog electronics. Nowadays, nuclear electronics are gradually adapted to digital signal processing. However, the digital counterparts of the traditional analog processing chain have to be validated experimentally.

At first, the established pulse shapers for spectroscopy applications, often referred to as slow shapers, are constructed and compared in the digital domain. Furthermore, the necessary fast shapers for extracting timing information are derived with respect to special features of the signals from CZT detectors. For this purpose, detector like stimuli were injected to the CSA (fig. 5.1), which responses with a typical exponentially decaying signal.

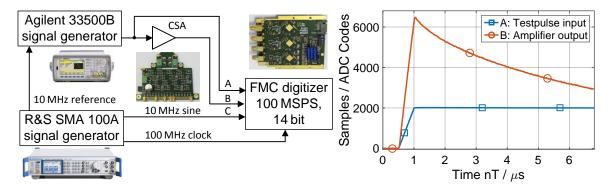


Figure 5.1.: The setup used for the evaluation of energy and timing properties of the hardware. A step input with variable rise time (signal A) is used the generate a detector like output from the CSA (signal B) for the digitizer. The entire system is sourced by a master clock generator.

The digital pulse shapers were evaluated with known test signals from the CSA with regard to spectroscopy and timing performance. Afterwards, the performance of digital pulse processing is evaluated with real detector signals from scintillation and CZT detectors. To prove the versatility of the algorithms, signals of different types of detectors were digitized and processed.

# 5.1.1. Spectroscopy application

It has been demonstrated (Hartog and Muller, 1947), that an optimum pulse shape in time domain reduces the equivalent noise charge of a pulse height analysis. In general, the well-known cusp pulse shape is regarded as the optimum pulse shape (Kowalski, 1970). "This pulse shape has become the standard by which the performance of other methods of pulse shaping are compared" (Knoll, 2010). The ideal cusp pulse shape has infinite length and only a theoretical relevance. Whereas the cusp pulse is not exactly realizable by analog electronics, digital electronics enable unlimited pulse shape synthesis. Since the shape of a cusp is described by an exponential decay with time constant  $\tau$  symmetric around the peak, the amplitude falls off the -40 dB level at a duration above  $5\tau$ . Consequently infinite cusp shape is sufficiently well approximated by an FIR filter of that length. An example is illustrated in fig. 5.2. The shown example with a test pulse at the input of the presented CSA

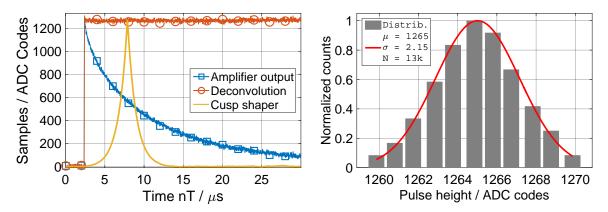


Figure 5.2.: The output of the amplifier with a test pulse of 55 mV at its input. The deconvolution of the exponential decay is implemented by an IIR filter. Finally, the output of the IIR filter is shaped by an FIR filter with arbitrary step response, which is in this case a cusp shape (left). The analysis of about 13k peak heights reveals a standard deviation of 2.15 bins.

reveals an energy resolution of 2.15 bins (standard deviation  $\sigma$ ) for a step input of 55 mV, which corresponds to an SNR of 76.64 dB at -1 dB full scale input (14 bit). The presented measurement is used as a reference for additionally evaluated digital pulse shapers. In reference to the comparison of analog pulse shaping circuits from (Kowalski, 1970), the results are normalized to the relative noise charge  $Q_{N,rel}$ , where the approximated cusp shape represents the unit value. On the basis of the established CR-(RC)<sup>n</sup> shapers mentioned in (Spieler, 2005), the *n* filter stages of the equivalent analog circuit can be implemented by an IIR filter. Increasing digital low-pass filter stages experiences no notable efforts in contrast to an analog implementation. Moreover, we investigated suitable pulse shapes based on well-known window function for digital signal processing (Heinzel et al., 2002). Window functions are implemented by an FIR filter as described in sec. 3.3.

In fig. 5.3, the kernels of the pulse shapes are adapted from the predefined window functions from the DSP system toolbox of Matlab (MathWorks, 2016a). A measurement with the same procedure as illustrated in fig. 5.2 was used to evaluate the performance of the developed

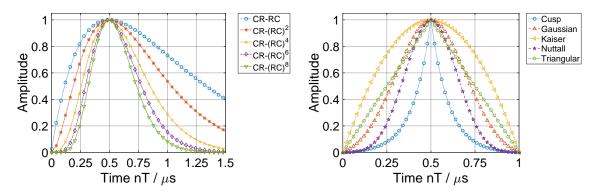


Figure 5.3.: Illustration of typical step responses of the well-known CR-(RC)<sup>n</sup> shapers implemented by an IIR filter (left) and synthesizable pulse shapes derived from established window functions (right). The window shapers are suitable for an implementation by an FIR filter.

digital pulse shapers. As mentioned, the standard deviation of the peak heights are normalized to that of the cusp shaper to obtain the relative noise charge  $Q_{N,rel}$ . The results related to noise performance are shown fig. 5.4.

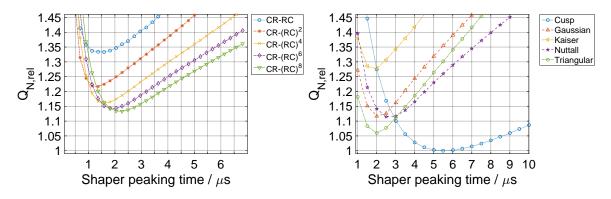


Figure 5.4.: Resulting relative noise chare  $Q_{N,rel}$  for the IIR filter based pulse shapers (left) and the FIR filter based pulse shapers (right). The relative noise charge is normalized to the optimum case of the cusp shaper.

The application of digital pulse shapers reveals the expected performance with regard to their analog equivalents. With an increasing number of cascaded IIR filter stages of the CR-(RC)<sup>n</sup> shapers, the relative noise charge decreases. On the top of that, the cusp shaper performs best, whereas only the triangular pulse shape results in an notable improvement of energy resolution. Other window based shapers, e.g. with a Gaussian pulse shape, are lined up with their performance in the range of the CR-(RC)<sup>n</sup> shapers. The numerical values are summarized in tab. 5.1.

The measured results of the digital pulse shapers are in accordance with the traditional analog implementations presented in (Kowalski, 1970). A major benefit from the digital implementation is actually that arbitrary pulse shapes can be synthesized. Even pulse shapes which are "not exactly realizable" (Kowalski, 1970) by an analog implementation can be synthesized and implemented by a digital circuit. By using the presented shapers for a peak height analysis, the energy resolution of the signal processing chain is improved, as

the analog counterparts are practically not implementable. However, the presented performance values have been derived by a signal with a fixed rise time. For realistic signals, e.g. the cathode signal of the CZT pixel detector, the rise time varies and the performance of the pulse shapers tremendously changes due to an potentially peak amplitude loss (fig. 5.6). The results of a peak height analysis with a fixed amplitude at the test input of the CSA but a varying rise time is shown in fig. 5.5.

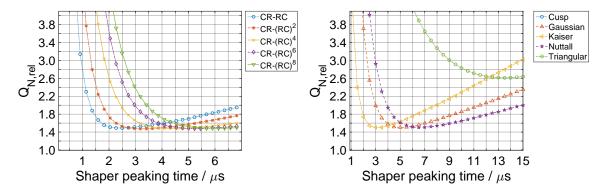


Figure 5.5.: Relative noise charge  $Q_{N,rel}$  for CZT like signals with a varying rise time up to 500 ns. The corresponding energy resolution is worsened in comparison to the measurement with a fixed rise time. The shapers with a wide spread top, e.g. Kaiser window, achieve best results.

As shown in fig. 5.5, the relative noise charge is worsened for signals with varying rise times. Obviously, the shapers with a widely spread top achieve best performance. Presuming that a flat top improves the performance, the impact of a shaper with a flat top is exemplary shown in fig. 5.6. It is obvious, that the proposed window shapers have to be adapted to the features

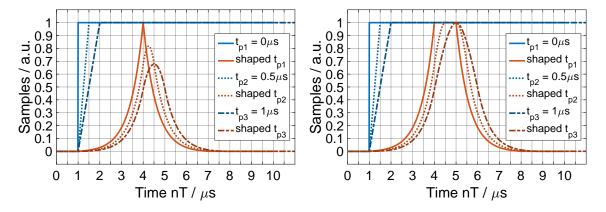


Figure 5.6.: Illustration of the amplitude loss and therefore degradation of energy resolution as a consequence of rise time variations (left). An adaption of the pulse shaper to the expected rise times eliminates the attenuation and improves energy resolution. A truncation of the pulse shape to a flat top with a duration equal to the maximum rise time is convenient (right).

of signals from a CZT. Pulse shapes with a truncated flat top equal to the duration of the maximum expected rise time performs best, as the peak amplitude is preserved. Truncated shapers and the results with the same setup from fig. 5.5 are illustrated in fig. 5.7. With the adaptation of shapers to the expected rise time variation, the energy resolution of a peak

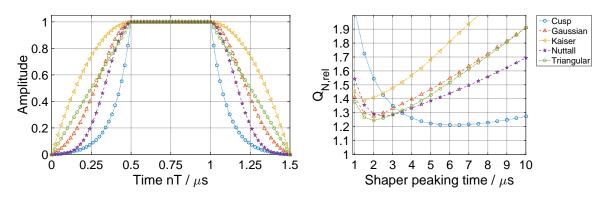


Figure 5.7.: Adaptation of the pulse shapes by truncation with a flat top (left). The resulting relative noise charge  $Q_{N,rel}$  is improved and the optimum peaking time of the shaper depends on the detector amplifier configuration.

height analysis is improved in contrast to the measurement shown in fig. 5.5. Again, the cusp shaper achieves best result, whereas the triangular (trapezoidal) shape outperforms the typical window shapers. Consequently, the trapezoidal shaper even outperforms the cusp shaper, if the relative noise charge is weighted by resource utilization in an FPGA. With a look at the achieved energy resolution (tab. 5.1), the numerical validation and the ranking of performance is comparable to the analog circuits presented in (Kowalski, 1970). But, the flexibility of an arbitrary pulse shape synthesis is inaccessible by analog electronics. Thus, for high energy resolution spectroscopy applications with a CZT pixel detector, digital pulse processing is necessary, or even indispensable with regard to varying rise times of the detector signal.

Table 5.1.: Resulting relative noise charge  $Q_{N,rel}$  and corresponding peaking time  $t_p$  of different shapers for fixed rise time signals and variable rise times up to 500 ns as expected from the CZT detector. The last column shows the results for the truncated shapers with a flat top of 500 ns. The CR-(RC)<sup>n</sup> shapers cannot be truncated by the cascaded IIR filter structure.

	Fixed ris	e time	ne Variable rise time		+ truncated shape	
Shape	tp	$Q_{N,rel}$	tp	$Q_{N,rel}$	tp	$Q_{N,rel}$
Cusp	$5.50\mu s$	1.00	15.00 $\mu$ s	7.97	$6.00\mu s$	1.21
Triangular	$2.00\mu s$	1.06	13.50 $\mu$ s	2.61	$2.00\mu s$	1.24
Nuttall	$2.50\mu s$	1.15	$6.50\mu  extsf{s}$	1.50	$2.50\mu s$	1.28
Gaussian	$2.00\mu s$	1.17	$5.00\mu  extsf{s}$	1.50	$2.00\mu s$	1.28
Kaiser	$2.00\mu s$	1.28	$3.90\mu  extsf{s}$	1.50	1.50 $\mu$ s	1.39
CR-(RC) <sup>n</sup>						
n = 1	1.59 $\mu$ s	1.33	$2.50\mu s$	1.49	-	-
n = 2	1.36 $\mu$ s	1.22	$3.18\mu  extsf{s}$	1.47	-	-
n = 4	1.59 $\mu$ s	1.16	4.32 $\mu$ s	1.48	-	-
n = 6	1.82 $\mu$ s	1.14	$5.23\mu s$	1.48	-	-
n = 8	$2.27\mu s$	1.13	6.14 $\mu$ s	1.48	-	-

# 5.1.2. Timing applications

The established methods for timestamp generation out of a voltage signal are mainly based on triggering a threshold crossing of a shaped signal. Detecting that point in time on a sharp rising edge with low noise distortion is apparently much easier and less error-prone than processing noisy signals with long rise (or fall) times. Hence, the pulse shapes derived for peak amplitude measurement, which often slow down the rising edge, are practically not suitable for extracting timing information, as the crossing point is smeared out.

In order to find the optimum timing method for CZT detector signals, a performance analysis of a digital constant fraction discriminator (CFD) applied to the unprocessed output signal of the charge sensitive amplifier is investigated first. With the same setup used for the energy measurements with the test pulse (fig. 5.1), the input and output from the CSA as well as the timing reference from the master clock source have been sampled by the digitizer. For a proof of concept, the signals were initially processed offline by a software. By doing so, the timing reference is extracted by a four parameter sine fit, such that zero crossing points are identified by the calculated phase angle of the sine wave. The digital CFD is implemented to operate on a stored sequence of samples containing the whole pulse of a triggered event. The peak value of the pulse is extracted and the position of the corresponding fraction before the peak position is used for calculating the timestamp information. A linear interpolation between the two sample points results in a timestamp accuracy better than the sampling period of 10 ns of the FMC digitizer. The intention of that experiment is to obtain the maximum achievable timing resolution with the CFD method applied to signals equal to that of a CZT pixel detector.

Depending on the amplitude of the output signal, the precision of the digital CFD method is measured by the standard deviation of the timestamp difference  $\sigma(\Delta t)$  between the zero crossing point of the sine wave and the crossing point of the fraction dependent threshold (fig. 5.8). The best fraction for the CFD is carried out experimentally and ranges about a

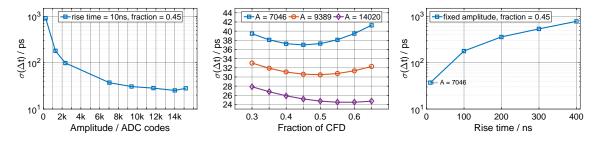


Figure 5.8.: The achievable timing precision (standard deviation  $\sigma$  of the timestamp difference  $\Delta t$ ) is improved with an increased amplitude *A* of the signal, i.e. the SNR is increased (left). Further, the precision depends on the selected fraction of the digital CFD (middle) and the rise time of the signal (right).

value of 0.45. For the evaluated signal shape, that value corresponds to that component of the signal, where the slope matches a linear shape most. An increasing SNR also improves the timing precision. Our setup achieves an resolution in the range of 30 ps, which almost

hits the timing limits of the signal generator from Agilent (Keysight Technologies, 2016). The measurement with a fixed input amplitude but varying rise time (fig. 5.8, right) confirms the assumption, that a slow rising edge degrades the timing performance. As a result, the synchronized digitizer is capable of a timing precision much better than the sampling period. Depending on the features of the detector signal, a timing far below 1 ns is possible, which is at first glance sufficient for a timing application with a CZT detector.

Besides the degradation of timing precision  $\sigma(\Delta t)$ , the varying rise times also have an impact on accuracy of timing. With a slow rise time, the absolute timestamp difference between the zero crossing of the reference signal and the threshold crossing of the detector signal is increased. Although the timewalk can be reduced by small fractions for the CFD (fig. 5.9), that effect dominates the overall timing performance excessively. Actually, the precision would be also degraded by a smaller fraction as well. An accuracy in the range of several

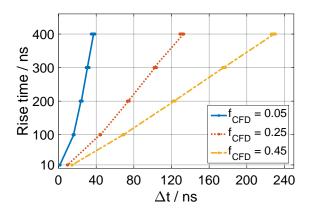


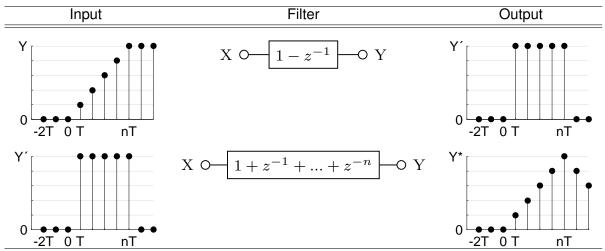
Figure 5.9.: Illustration of the rise time dependent time walk of the constant fraction discriminator (CFD). A lower fraction  $f_{CFD}$  of the CFD reduces the walk, but also lowers the precision  $\sigma$ .

tens of nanoseconds is unacceptable for most pulsed beam applications, as the time walk overlaps the repetition rate of the accelerator. To overcome the rise time dependent time walk of the detector signals, the variation in rise time has to be compensated. Several approaches have been proposed for large volume detectors (Fouan and Passerieux, 1968; Kozyczkowski and Bialkowski, 1976), but they all based on sophisticated analog circuits. In the digital domain, the rise time compensation can be constructed intuitively as sketched in tab. 5.2.

To fulfill the first claim of a fast rising edge for precise timing, a convenient method is a calculation of the first derivative of signal data (tab. 5.2, first row). By doing so, the nearly linear slope of the CSA output is transformed into a rectangular shaped signal which correlates to the current of the detector. A timestamp generated out of that signal would be independent of charge collection time, as it triggers the time when the prompt event hits the detector and therefore the current starts to flow. But on the contrary, the calculation of the derivative in time domain is also a gain for high frequency noise. With regard to practical constraints, the derived SNR is too low for a precise timing. However, the introduced noise level can be reduced by an additional low-pass filter. An averaging FIR filter transforms the assumed

rectangular current pulse into a triangular current pulse (tab. 5.2, second row). If the length of the averaging shaper equals the minimum drift time, the amplitude of the shaped signal remains constant at its maximum level.

Table 5.2.: Steps for compensating a variable rise time to a fixed one. Firstly, the output signal of the CSA is differentiated to obtain a nearly rectangular shape. Secondly, the rectangle pulse shape is averaged to reduce noise and to limit the rise time to a fixed value.



Finally, the necessary differentiation by means of coefficients of form  $b_d = (1,-1)$  and the unscaled averaging filter with *n* coefficients of value 1,  $b_a = (1,...,1)$ , can be implemented by a single FIR filter. After a convolution of both kernels, the resulting FIR filter is of form

$$b_c = (1, 0, ..., 0, -1)$$
, (5.1)

where the number of coefficients equal zero is n-1. The application of the filter from eq. 5.1 to a simulated signal is illustrated in fig. 5.10. Here, a signal with different rise times  $t_1$ ,  $t_2$ 

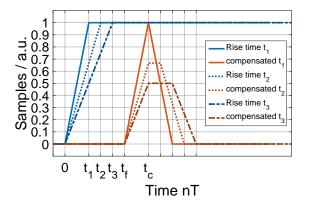
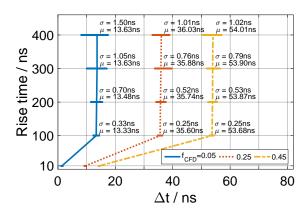


Figure 5.10.: Application of the compensation filter from eq. 5.1 to a simulated signal with different rise times  $t_1, t_2, t_3$ . The variation in rise time is compensated at the costs of an amplitude loss.

or  $t_3$  is transformed by the filter to a signal with constant rise time  $(t_c - t_f)$ . The rise time is compensated at the costs of a reduced amplitude of the output signal, which also degrades



the SNR and therefore timing precision, as illustrated in fig. 5.11.

Figure 5.11.: Resulting accuracy and precision of the digital constant fraction discriminator applied to a rise time compensated signal with a fixed amplitude. The compensation filter was chosen to compensate rise times larger than 100 ns. The accuracy is below 400 ps, while the precision is at least ten times better than the sampling period T of 10 ns.

Finally, the walk of the measurement shown in fig. 5.9 is compensated for a minimum rise time of 100 ns. With the sampling period of 10 ns, the compensation FIR filter is of length 11 with 9 zero coefficients regarding eq. 5.1. The measurement with the rise time compensation shows, that the accuracy is improved to a value of about 400 ps, whereas the precision remains almost constant in comparison to the uncompensated dataset shown in fig. 5.8.

# 5.2. $\gamma$ -ray spectroscopy

The developed hardware and algorithms should be evaluated with different detectors for  $\gamma$ -ray spectroscopy. For evaluation and calibration of the detectors, a <sup>22</sup>Na radioactive source is used. Its typical  $\beta^+$  decay causes an annihilation with an electron and produces a coincident pair of  $\gamma$ -rays with an energy of 511 keV, which can be used for energy as well as timing measurements. For an advanced analysis of the performance of the CZT pixel detector, additional radioactive sources have been used. At first, a <sup>241</sup>Am source is used to derive the energy resolution in the low energy range of 60 keV. Secondly, for linearity and a more practical relevant use case, a radium paint source was investigated. The experiments with radioactive sources should validate the performance of the algorithms with various detectors and especially reveal the characteristics of the CZT pixel detector.

## 5.2.1. Energy resolution of scintillation detectors

Several measurements with scintillation detectors were performed to evaluate the developed digital algorithms with signals of different features. Each scintillation crystal used has different characteristics regarding energy and timing resolution. The performance of various scintillation materials with a conventional setup was reported by (Roemer et al., 2015). For

our investigation, we concentrate on the application and test of the deconvolution algorithm and pulse shaping for energy measurement and timestamp recovery. For that purpose, three different scintillation detectors were chosen (tab. 5.3).

Table 5.3.: Overview of used scintillation detectors with associated readout electronics for evaluation of the digital pulse processing algorithms.

Scintillation material	Readout electronics	High-voltage
Nal	HZDR proprietary	+525 V
CeBr <sub>3</sub>	Hamamatsu R2059	-1200 V
Gd <sub>3</sub> Al <sub>2</sub> Ga <sub>3</sub> O <sub>12</sub> (GAGG)	Philips XP 2972	-1500 V

Firstly, a Nal detector with built-in amplifier from HZDR inventory was examined. The detector operates at +525 V and the output signal has the typical exponential decay with a time constant of approximately 43.46  $\mu$ s. The noise filtered signal is suitable for a peak-height analysis. It is obvious, that a spectroscopy application potentially suffers from pulse-pileups, caused by high count rates in comparison to the decay time constant. To overcome this problem, the traditional analog approach uses additional pulse shaping amplifiers as a combination of high-pass filters and low-pass filters (Knoll, 2010) to shorten the pulse-width. However, this requires highly adapted readout electronics and increases total number of components, which is unfavorable for high channel counts. Fig. 5.12 compares the pulse processing in which the left plot represents the traditional analog pulse-height analysis without pulse shortening and the right plot illustrates the application of the proposed digital pulse processing. Despite the fact of a simplified hardware, the achievable energy resolution for

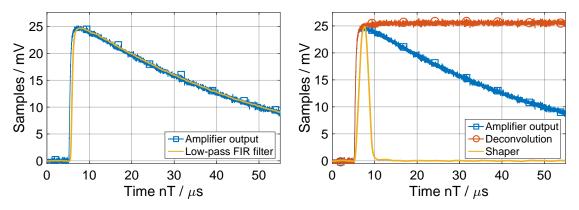


Figure 5.12.: Signals from a Nal detector and applied digital pulse processing. The left plot shows the application of a simple noise filter implemented by a 100-tap low-pass FIR filter ( $f_c = 500 \text{ kHz}$ ). The pulse-width (solid line) is ultimately short-ened by the proposed pulse shaping algorithms (right).

that type of detector signals is indeed not significantly improved by digital pulse processing (see also (Di Fulvio et al., 2016)), as long as the ballistic deficit problem is negligible and the analog processing chain matches the electrical characteristics of the detector, i.e. bandwidth and gain. The obtained pulse-height spectra from a <sup>22</sup>Na radioactive source are shown in fig. 5.13.

Secondly, the characteristics of a CeBr3 should be verified by digital signal processing meth-

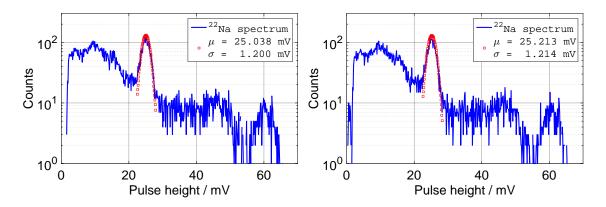


Figure 5.13.: Exemplary comparison of the classical pulse-height analysis without additional pulse shaping (left) and with the proposed deconvolution algorithm and trapezoidal shaper. The energy resolution of the Nal detector is not significantly improved and remains at approximately 11 % FWHM for the 511 keV photopeak.

ods. Therefore, the current-to-voltage gain for that detector is realized by a single  $50 \Omega$  resistor, as the photomultiplier tube provides sufficiently large gain. For this reason, the bandwidth is not significantly limited by an operational amplifier. The signals of the detector were digitized by an Agilent MSO1234 Oscilloscope with 20 GSPS and 4 GHz bandwidth. An example of a digitized waveform is shown in fig. 5.14. Fitting the exponential decay as shown in fig. 5.14 to the entire dataset of 60123 waveforms, the distribution of the characteristic light decay time results in a standard deviation of 1.2 ns with a mean value of 23.4 ns.

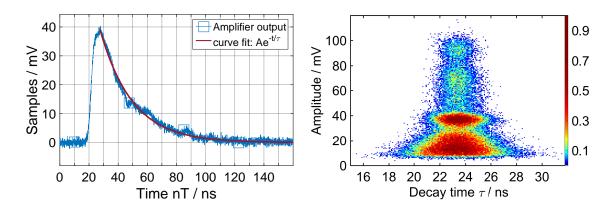


Figure 5.14.: An example of the output signals of the CeBr<sub>3</sub> detector (left). The sampled voltage signals correlates with the current generated by the photomultiplier tube. A curve fit to the exponential decay reveals the typical decay time constant  $\tau$  of the scintillation material of 23.4 ns with a standard deviation of 1.2 ns. The 511 keV photopeak of the <sup>22</sup>Na source is visible at a signal amplitude of 36.87 mV (right). The FWHM of the photopeak is 6.75 mV (18.3%).

The characteristic value of light decay time from fig. 5.14 are in accordance with the reported values of  $19.3 \text{ ns} \pm 3 \text{ ns}$  (Roemer et al., 2015),  $20.0 \text{ ns} \pm 1 \text{ ns}$  (Ra et al., 2008). A smaller value of 17.0 ns was reported by (Shah et al., 2005), but has been measured with a different method (delayed coincidence method (Bollinger and Thomas, 1961)). However,

the goodness of fit is improved by higher signal amplitudes corresponding to an increased SNR, but the peak height of the fit results in an poor energy resolution of 18.3% (FWHM, 511 keV). As the energy of the incident radiation hitting the scintillator is proportional to the total amount of light produced (Knoll, 2010), the integral of the voltage pulse is proportional to the energy. The numerical integration of that pulse can be realized by an IIR filter, and finally, pulse shaping can be applied for the peak height analysis similar to the pulse shaper for the NaI detector from fig. 5.12. A simple digital integrator is derived by applying the bilinear transformation to the Laplace operator 1/s. This results in an IIR filter with the coefficients

$$b_n = \frac{T}{2}(1,1)$$
 ,  $a_n = (1,-1)$  , (5.2)

where T is the sampling period. Along with a trapezoidal shaper, the peak height of the shaped pulse is proportional to the numerical integration of the voltage pulse (fig. 5.15). To

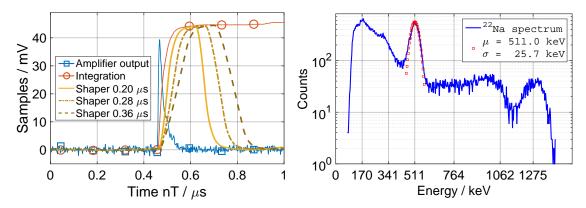


Figure 5.15.: A signal from a CeBr<sub>3</sub> detector with proper digital pulse processing (left). Different shaper length were applied to achieve an optimized energy resolution. The illustrated spectrum of a <sup>22</sup>Na radioactive source reveals an energy resolution of 11.8 % FWHM at the 511 keV photopeak (right) with a digital shaper of length 0.20  $\mu$ s.

optimize the energy resolution, different lengths of trapezoidal shapers have been applied for pulse processing. The flat top of the trapezoidal shaper was set to a constant value of about  $5\tau \approx 120 \text{ ns}$ . The averaging component of the shaper was modified in order to find the best length.

As it is not the intention to optimized the signal chain for high resolution spectroscopy with CeBr<sub>3</sub> detectors, this analysis is necessary to get a rough estimate of the energy resolution of the detector and to use the calibrated energy data for selecting events of a further timing measurement. The timing measurement with a <sup>22</sup>Na source can be improved, if an energy cut is applied to the photons with 511 keV. Thus the timing spectrum is cleaned of random coincidences or scattered events.

The methods for processing signals from a  $CeBr_3$  detector were applied to a GAGG scintillation detector. The results are illustrated in fig. 5.16

The obtained decay time  $\tau$  from the GAGG scintillator of approximately 121 ns is larger than the value of 88 ns±10 ns from the datasheet (Roemer et al., 2015; Furukawa Denshi,

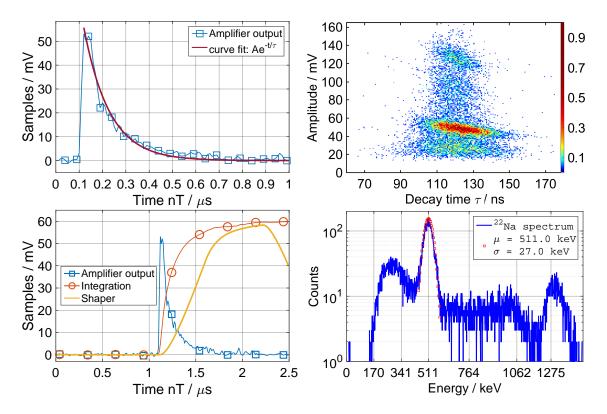
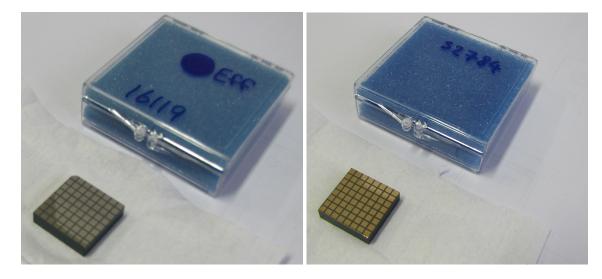


Figure 5.16.: An output signal of the GAGG detector and corresponding curve fit of the exponential decay (top left). The mean value of the decay time *τ* is 121 ns with a standard deviation of 10.1 ns (top right). The current pulse is integrated by an IIR filter and processed by a trapezoidal shaper (bottom left) for energy measurement (bottom right). The energy resolution of the 511 keV photopeak is 63.45 keV (12.42 % FWHM).

2016), but ranges in the specified tolerance. The decay time  $\tau$  was again used to set up the trapezoidal shaper for energy measurement. The flat top of the shaper is set to match the entire current integration time of about  $5\tau \approx 600$  ns. The averaging component of the shaper is set to 700 ns. Thus, the total pulse width of the shaper is 2  $\mu$ s.

The obtained energy resolution is below the reported values of 6.3% (Iwanowska et al., 2013) or 6.47% (Roemer et al., 2015). "The mismatch between the measured and the reported value is mainly due to the choice of the PMT. The XP2972 PMT is optimized for timing applications." (Golnik, 2016). This statement is also valid for the measurements with the CeBr<sub>3</sub> crystal and the Hamamatsu R2059 PMT. Actually, the timing performance of both detectors is of interest. It could be shown, that an IIR filter is suitable to perform an integration of detector signals, required for charge measurement if a current-to-voltage amplification is utilized in the readout electronics. This approach simplifies the conventional setup with a QDC with complex triggering and gating as implemented in previous prototypes of the Compton camera (Kormoll, 2013; Golnik, 2016). Thus the complexity can be reduced and throughput is increased. Further, a significant improvement of the peak height spectra with respect to analog pulse processing could not be shown, as the energy resolution was mainly limited by the readout electronics of the scintillation detectors. But, a major benefit is the easiness of adjusting the signal integrator and pulse shaper with respect to the detector.

# 5.2.2. Energy resolution of a CZT pixel detector



Most of the investigations have been done with the CZT detectors shown in fig. 5.17.

Figure 5.17.: Two CZT detectors from Redlen. These crystals were used to benchmark the key parameters of the electronics, algorithms and detector characteristics.

Both detectors were manufactured by Redlen and have a dimension of  $20 \times 20 \times 5 \text{ mm}^3$ . It is clearly visible, that both crystals came from different productions runs, as the electrode layout differs. The detector with the number 16119 (fig. 5.17, left), referred to as "CZT1" has a chamfer on the top left corner and no steering grid electrode between the pixels. The detector with the number 32784, referred to as "CZT2" has a pixel layout with a steering grid. Actually, a steering grid is not needed for the operation of a CZT, but can improve energy resolution and reduces charge-sharing effects (Jung et al., 2005). Nevertheless, recent electrode layouts of the pixel detectors from Redlen did not include a steering grid, as energy resolution is primarily improved by optimizing properties of the material (e.g. purity of the crystal) and therefore a steering grid is not needed anymore (Grosser, 2015). Consequently, the steering grid was connected to the potential of the anodes for all measurements. Besides the different electrode layouts, the detectors differ in the measured dark current by a factor of 10. For evaluation of spectroscopy performance, CZT1 with the lower dark current of about 10 nA at a bias voltage of 500 V was used.

In the first step, optimal pulse processing for best energy resolution was carried out experimentally with the methods applied to the test pulses. As the energy resolution of the CZT practically depends on the depth-of-interaction, events near the cathode side of the detector promise best signal quality. By selecting a low energy  $\gamma$ -ray source irradiating the detector from the cathode side, the interaction will mostly take place near the cathode electrode, as low energy photons are well stopped by the CZT, while attenuation decreases for higher energy photons, which are then more likely to be absorbed in the deeper regions of the material at the anode side or even cross the detector without an interaction. The cross sections of a CdZnTe compound is shown in (Kormoll, 2013; NIST, 2016). Consequently, an <sup>241</sup>Am source with a  $\gamma$ -ray emission at 60 keV was used for initial measurements of energy resolution and to find the optimal pulse shaping with respect to spectroscopy applications.

From the theoretical point of view, the optimum shaping frequency can be estimated by the intersection frequency  $f_1$  of the current and voltage noise spectral density shown in fig. 2.21. Rough estimates with assumptions on total voltage noise and current noise range about an intersection at 200 kHz, which corresponds to a rise time of  $1.75 \,\mu$ s. However, all noise sources cannot be estimated exactly, thus an experimental verification of optimum rise times for the digital pulse shapers is necessary. As demonstrated, different pulse shapes have an impact on energy resolution and require an appropriate adaptation to the detector signal. The window based pulse shapes with a flat top have been used for evaluation of energy resolution of the pixel detector (fig. 5.18). The best results are achieved with the cusp shaper

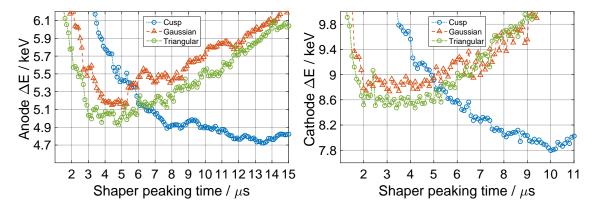


Figure 5.18.: Obtained energy resolution (FWHM) of the 60 keV photopeak from a measurement with a <sup>241</sup>Am radioactive source dependent on the applied digital pulse shaper. The results are in accordance with the test pulse setup and reveals and optimum peaking time for the triangular shaper of about 4.0  $\mu$ s for the anode and about 2.5  $\mu$ s for the cathode.

with a peaking time of  $13.5 \,\mu s$  ( $10.0 \,\mu s$ ) for the anode (cathode). As this rise time is practically not relevant for a high throughput application, the triangular (trapezoidal) shaper is considered to perform best, as it is easily implementable in an FPGA without utilizing additional multipliers. The resulting energy spectrum is illustrated in fig. 5.19.

Further investigations have been done with a <sup>22</sup>Na source with characteristic  $\gamma$ -ray emissions at 511 keV and 1275 keV. With the selected trapezoidal shaper, the uncorrected energy plot from the cathode and anode peak amplitudes reveals the discussed plot from fig. 2.26, but with an improved resolution (fig. 5.20).

After depth correction, the energy resolution of the 511 keV photopeak is about 10.81 keV. The obtained energy resolutions verify the assumption made by (Kormoll, 2013; Rohling, 2015) also for the investigated pixel detector. Both assumed the energy resolution  $R_{CZT}$  (FWHM) of a CZT detector in general as a function of the deposited energy *L* to be

$$R_{\rm CZT} = 6\,\rm keV + 0.15\,\rm keV\sqrt{L/\rm keV}\,.$$
(5.3)

A calculation for the 60 keV (511 keV) photopeak with eq. 5.3 results in an energy resolution

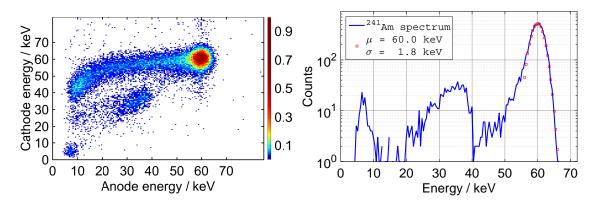


Figure 5.19.: Obtained spectrum from a measurement with the <sup>241</sup>Am radioactive source with the derived shaper constants. The bi-parametric spectrum (left) clearly highlights the 60 keV photopeak and multi-pixel events (cathode energy is larger than anode energy). The one-dimensional projection of single pixel events reveals an energy resolution of about 4.23 keV (FWHM) at 60 keV. The events in the range of 35 keV belongs to characteristic X-ray escapes of the CdZnTe compound.

of 7.2 keV (9.4 keV), which are roughly in accordance with the measured value of 4.23 keV (10.81 keV).

Moreover, to check spectroscopic linearity and detector performance at a more realistic  $\gamma$ -ray irradiation with multiple energies, the spectrum of a radium paint source was recorded, calibrated and depth corrected. The result is shown in fig. 5.22.

The peaks below 100 keV correspond to characteristic X-rays emitted by the source. The identified  $\gamma$ -ray lines were emitted by a <sup>226</sup>Ra decay. A list of detected peaks is noted in tab. 5.4. The prominent X-ray lines at about 76 keV and 85 keV belong to the characteristic

labal	tupo	Enormy	maga Energy	<u> </u>
label	type	Energy	meas. Energy	$\Delta E$
а	Cd,Te X-ray escapes	pprox 45 keV	47.1 keV	n.a.
b	Pb X-rays (K $lpha$ )	pprox 75 keV	76.4 keV	n.a.
С	Pb X-rays (K $\beta$ )	pprox 85 keV	86.8 keV	n.a.
1	$^{226}$ Ra $\gamma$ -ray	186 keV	185.2 keV	-0.8 keV
2	$^{214}$ Pb $\gamma$ -ray	242 keV	242.7 keV	+0.7 keV
3	$^{214}$ Pb $\gamma$ -ray	295 keV	294.5 keV	-0.5 keV
4	<sup>214</sup> Pb $\gamma$ -ray	352 keV	352.0 keV	-0.0 keV
5	<sup>214</sup> Bi $\gamma$ -ray	609 keV	609.6 keV	+0.6 keV
6	<sup>214</sup> Bi $\gamma$ -ray	1120 keV	1120.2 keV	+0.2 keV
7	<sup>214</sup> Bi $\gamma$ -ray	1764 keV	1761.0 keV	-3.0 keV

Table 5.4.: Detected peaks with a measurement of a radium paint source. The results were used to estimate linearity of the CZT detector upon a wider energy spectrum.

X-ray energies of Pb. Moreover, these X-rays potentially cause an interaction with the detector material, where the element Cd emits characteristic X-rays of about 23 keV and 27 keV (K $\alpha$  and K $\beta$ ) and the Te element emits X-rays of about 27 keV and 30 keV. These X-rays cannot be used for a measurement of linearity, as the energy resolution of the detector is not

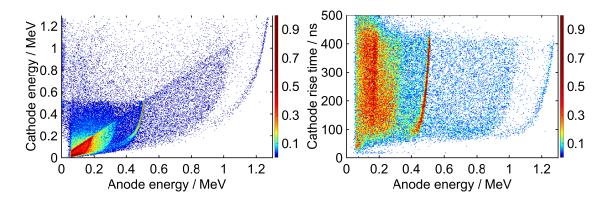


Figure 5.20.: Calibrated measurement of a <sup>22</sup>Na source illustrated as bi-parametric spectrum with cathode and anode energy (left) and visualization of depth dependency of the anode energy (right). To improve energy resolution, the anode energy must be corrected depending of the depth-of-interaction, which correlates with the cathode rise time.

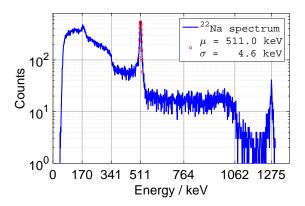


Figure 5.21.: The results of a measurement with a <sup>22</sup>Na source and optimized trapezoidal shaper after depth correction. The energy resolution of the 511 keV photopeak is 4.6 keV (standard deviation). This corresponds to 11 keV FWHM (2.2%). All events of a single pixel are shown.

as precise as needed for this exercise. But linearity can be well calculated with the known  $\gamma$ -ray emissions of the <sup>226</sup>Ra decay chain. The response of the detector shows an linearity in the range of  $\pm$ 1 keV, while ignoring the value at 1761 keV, because the statistic is not meaningful with only a few counts.

# 5.3. $\gamma$ -ray timing

#### 5.3.1. Timing performance of scintillation detectors

Even though the research targets an application of CZT detectors for prompt  $\gamma$ -ray imaging, the timing performance of scintillation detectors is of interest because of two reasons. At the one hand, the digital algorithms should be verified for various types of detector signals and on the other hand, the scintillation detectors provide an excellent reference for a coin-

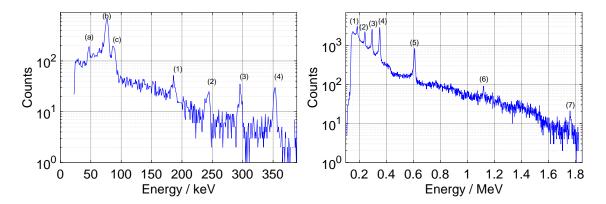


Figure 5.22.: Spectrum of a radium paint source. The energy region below 100 keV (left) is dominated by characteristic X-rays, while  $\gamma$ -ray emission were detected up to 2 MeV (right).

cidence measurement with a <sup>22</sup>Na source. Thus, a straightforward timing characterization was performed by measuring the annihilation photons emitted by the <sup>22</sup>Na source. For that, two detectors face the radioactive source with an angle of 180°. An event triggering both detectors is used to calculate the time difference between the two occurrences. For real coincidences, the variation of timestamp differences reveals the combined timing precision of both detectors. Thus, to characterize an unknown material like CZT, the characteristics of the reference detector must be well-known. As reported by (Pausch et al., 2016), CeBr<sub>3</sub> combines an excellent time resolution of 164 ps FWHM at 511 keV (Fraile et al., 2013) and very good energy resolution <4 % FWHM at 662 keV (Shah et al., 2005). While this studies utilizes highly optimized setups for this kind of detector, (Roemer et al., 2015) reports a value of 365 ps $\pm$ 50 ps for the timing resolution at 511 keV. Consequently, the CeBr<sub>3</sub> detector is assumed to be an excellent reference for coincidence measurements with the <sup>22</sup>Na source.

For timing measurements, the waveforms were recorded with 20 GSPS and 4 GHz bandwidth again, and the trigger was externally generated by an FPGA utilizing the FMC digitizer card. The measurement was performed with the oscilloscope instead of the FMC digitizer to ensure that the bandwidth did not limit timing performance. At the end, the timestamps of each event were calculated by a software implementing the digital CFD already described. That trigger method is used to evaluate the timing performance of all investigated detectors.

Several setups for the digital CFD have been tested. In general, best results have been achieved with a threshold of one fourth of the peak amplitude and an additional FIR low-pass filter adapted to the minimum rise times of the detector signals. As the bandwidth limit was set too tight, the timing resolutions were worsened, because the sharpness of the rising edge was smeared out. But if bandwidth was not proper limited, the rms noise level is relatively high and distorts the triggering timestamp. The bandwidth limit and fraction of the CFD threshold have to be individually identified depending of the shape and noise of the detector signal. The further investigations of digital pulse processing with scintillation detectors have been done to prove the versatility of the approaches. Moreover, digital timing outclasses an

analog timing circuit with regard to handling. Major efforts for analog CFD timing are related to adjusting delay times and fractions. The digital CFD technique requires no delay times, as the samples are stored in a memory and can be traced back to an individual fraction. Finally, the investigations were necessary to elaborate a well-known reference detector for timing measurements with a CZT detector. A measurement with two scintillation detectors is shown in fig. 5.23.

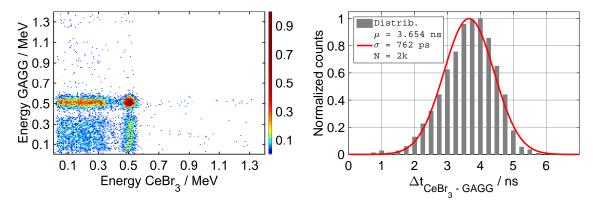


Figure 5.23.: Measurement of coincident events in the GAGG and CeBr<sub>3</sub> detector. A <sup>22</sup>Na source was placed between the two detectors centered with a distance of about 70 mm. Events with a full absorption were selected from the coincident energy spectra (left) for calculating the difference of timestamps. The fit of a Gaussian distribution to the timing measurement (right) reveals a standard deviation of 762 ps, which correspond to a timing resolution of 1.8 ns (FWHM).

The investigations of the digital timing with a GAGG and a CeBr<sub>3</sub> detector have been done to select the best timing reference for evaluating CZT timing performance. The summed timing performance of both scintillation detectors is assumed to be much better than the expected timing performance of the CZT (tab. 5.5). With regard to (Roemer et al., 2015), where the timing resolution of the GAGG (CeBr<sub>3</sub>) was estimated to be 3.9 ns (365 ps), the measured timing resolution for our setup primarily depends on the timing resolution of the GAGG detector. Moreover, our digital approach with a resolution of 1.8 ns outstrips the traditional analog setup utilized in (Roemer et al., 2015). Furthermore, the digital timing benefits from easily adaptable parameters due to varying rise times in contrast to the analog CFD timing. On the contrary, fast timing detectors are characterized by fast signal rise times, which require an increased bandwidth for the continuous-time signal and therefore an increased sampling rate for the quantized discrete-time signal. Assuming a rise time of 1 ns for a fast scintillation detector, the sampling interval of the ADC has to be 700 MSPS to digitize the signal with a bandwidth of about 350 MHz.

## 5.3.2. Timing performance of CZT pixel detectors

The characteristics (e.g. rise time and decay time) and performance (e.g. dependence of timing on energy) of suitable scintillation materials for prompt  $\gamma$ -ray timing have been studied extensively (Shah et al., 2005; Ra et al., 2008; Fraile et al., 2013; Roemer et al., 2015).

Such experimental data also exists for CZT detectors, but the published results are very diverse, as instruments (planar, coplanar, orthogonal strip, or pixel detector), methods (CFD timing, waveform fit, etc.) and setup (high voltage bias, event selection, etc.) strongly vary. E.g. (Parnham et al., 1995) reports a timing resolution of 5.3 ns (FWHM at 511 keV) for a  $3 \times 3 \times 2 \text{ mm}^3$  planar CZT detector. (Vaska et al., 2005) reports a value of 10 ns (FWHM at 511 keV) even for a thinner planar detector from the same manufacturer (eV Products) with dimensions  $5 \times 5 \times 1.4 \text{ mm}^3$ . The difference in both studies was an applied bias voltage of 600 V or rather 100 V, which corresponds to electric field strength of 3000 V/cm and 714 V/cm. All studies quoted in tab. 5.5 allow the conclusion that an increased bias voltage results in an improved timing performance for CZT detectors. At the same time, the dark current related to the bias voltage is increased. Consequently, the SNR decreases and the energy resolution is worsened. A summary of published timing measurements related to various kinds of CZT applications and setups is summarized in tab. 5.5.

Table 5.5.: Overview of timing performance of different types of CZT detectors. Besides the

bias voltage for the detector.					
Reference	Detector	Bias voltage	Timing resolution		
(Parnham et al., 1995)	planar (3×3×2 mm <sup>3</sup> )	600 V	5.3 ns (511 keV)		
	planar (5×5×10 mm <sup>3</sup> )	2000 V	18.6 ns (511 keV)		
(Amrami et al., 2000)	$4 \times 5$ pixel layout (10×12.5×5 mm <sup>3</sup> )	unknown	4-84 ns (511 keV)		
(Amrami et al., 2001)	$4 \times 4$ pixel layout (10×10×4 mm <sup>3</sup> )	unknown	16.8 ns (511 keV) with cathode signal		
	$4 \times 4$ pixel layout ( $10 \times 10 \times 4$ mm <sup>3</sup> )	unknown	5.1 ns (511 keV) with pixel signal		
(Okada et al., 2002)	planar (4×4×2 mm <sup>3</sup> )	200 V	17.0 ns (60 keV)		
	planar (4×4×2 mm <sup>3</sup> )	500 V	12.0 ns (60 keV)		
(Meng and He, 2005)	$11 \times 11$ pixel layout ( $10 \times 10 \times 10$ mm <sup>3</sup> )	1400 V	11.6 ns (511 keV)		
(Vaska et al., 2005)	planar (5×5×1.4 mm <sup>3</sup> )	100 V	10 ns (511 keV)		
	coplanar grid $(15 \times 15 \times 7.5 \text{ mm}^3)$	1000 V	21 ns (511 keV)		
(Drezet et al., 2007)	orthogonal strip $(20 \times 16 \times 0.9 \text{ mm}^3)$	500 V	1.9 ns (511 keV)		
	planar $(20 \times 16 \times 0.9 \text{ mm}^3)$	500 V	2.6 ns (511 keV)		
(Komarov et al., 2012)	9 pixel layout $(20 \times 20 \times 5 \text{ mm}^3)$	1000 V	>25 ns (511 keV)		
(Hueso-González et al., 2014)	orthogonal strip $(20 \times 20 \times 5 \text{ mm}^3)$	500 V	2.8 ns (< 12.5 MeV)		

geometry and electrode layout, an important parameter for timing is the applied bias voltage for the detector.

Our measurements with the pixel detectors were strictly made with a bias voltage of 600 V, as this results in a good energy resolution and avoids any destruction in the material due to an overvoltage. Comparable results to our setup were published by (Meng and He, 2005) and (Komarov et al., 2012). Both studies investigated the timing of a pixel detector at the cathode signal. Although their electric field strengths were larger than that we used in our setup, a timing resolution of several tens of nanoseconds seems to be reasonable for a CZT pixel detector. However, this value has to be experimentally verified with our front-end electronics, algorithms and detectors. For this purpose, the two detectors shown in fig. 5.17 were examined.

The setup for timing measurements with the CZT detector is straightforward and equal to the measurement with two scintillation detectors. A simultaneous detected event from both detectors in a relatively large coincidence window (>4  $\mu$ s) is recorded and stored by the digitizer. As a waveform-fit is not implementable with reasonable efforts in a high-throughput application (Meng and He, 2005), we also studied a linear fit based on a FIR filter (Wohsmann, 2014). But at the moment, the approach with digital pulse shapers for timing in conjunction with a digital CFD algorithm performs best with regard to SNR and robustness. The results of a measurement with the <sup>22</sup>Na source and the CZT detector in coincidence with the CeBr<sub>3</sub> detector is shown in fig. 5.24.

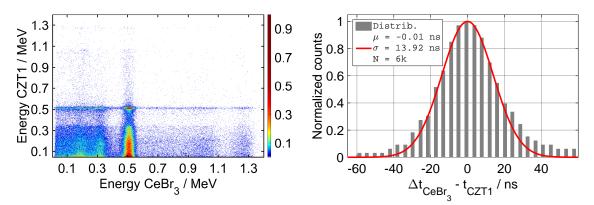


Figure 5.24.: Results of a timing measurement with a <sup>22</sup>Na source and the CZT1 pixel detector in coincidence with the CeBr<sub>3</sub> detector. The bi-parametric energy plot (left) is used to select coincident events with a full absorption of the 511 keV photopeak. The distribution of the timestamp differences of selected events with an energy of 511 keV $\pm$ 50 keV is shown in the right plot. The standard deviation  $\sigma$  of 13.92 ns corresponds to a timing resolution of 32.71 ns (FWHM).

A closer look at the dependencies of timing of the CZT detector, shown in fig. 5.25, reveals that timing performance is not significantly degraded with a lowered energy related to scattered events in the CZT detector. But with regard to measured cathode-over-anode ratio, which corresponds to the DOI in the detector, a slight S-shaped structure is visible in fig. 5.25 (right). The varying timing performance at different depths in the detector can be explained by a non-uniformity of the electric field caused by an inhomogeneous material (Butcher et al., 2013). Further investigations were made with the two crystals at a pulsed particle beam in sec. 5.4.1. Moreover, the CZT-CZT coincidence timing was measured sim-

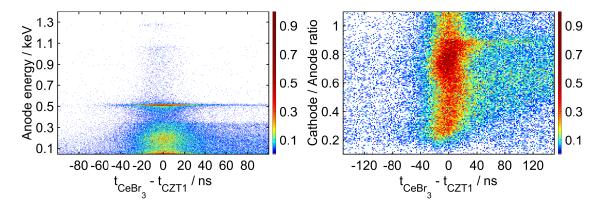


Figure 5.25.: Measured dependencies of CZT timing performance. Timing performance of scattered events with an energy below 511 keV is not significantly worsened (left). But depending on the depth-of-interaction (Cathode over anode ratio, right), a S-shaped structure is visible.

ilarly to the CZT-CeBr<sub>3</sub> setup. The measurement shown in fig. 5.26 confirms the statement, that the timing performance is not significantly worsened below 511 keV. Additionally, this claim is supported by the measurements presented in (Okada et al., 2002) with a <sup>241</sup>Am radioactive source at 60 keV. Actually, timing performance on scattered low energy photons is more important than fully absorbed photons with high energies, because only scattered photons with relatively low scattering angles, and therefore low energies, are considered to be valid events for an image reconstruction with a Compton camera.

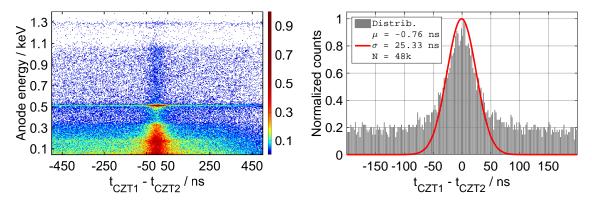


Figure 5.26.: Measured coincidence timing with the two different CZT crystals (CZT1, CZT2). The energy over timing plot reveals no significant worsening for scattered photons below 511 keV (left). The standard deviation  $\sigma$  of timing distribution for all events is 25.33 ns, which corresponds to a timing resolution of 59.53 ns (FWHM).

## 5.4. Measurements with a particle beam

## 5.4.1. Bremsstrahlung Facility at ELBE

The challenges of a prompt  $\gamma$ -ray imaging system are a high flux of high energy photons. For an image reconstruction based on prompt  $\gamma$ -ray timing, a detector with a timing resolution of several hundreds of picoseconds is desirable (Pausch et al., 2016; Golnik, 2016). A CZT pixel detector can definitely not fulfill that requirement. But the performance estimation of CZT for energies above 2 MeV or any rate limitations are still unsolved challenges of CZT detectors in general, and are also of high relevance for Compton cameras or any other potentially upcoming applications. Such experimental data was acquired at the linear electron accelerator ELBE at HZDR. At the bremsstrahlung facility of the ELBE accelerator, photons with energies up to 12.5 MeV are generated. The energy range of that bremsstrahlung photons covers the energy range of prompt  $\gamma$ -rays in a clinical proton beam. The bunches are pulses with a frequency of 13 MHz (76.92 ns). A compton camera setup equal to that described in (Hueso-González et al., 2014) was evaluated, but without a scintillation detector as absorber (fig. 5.27).

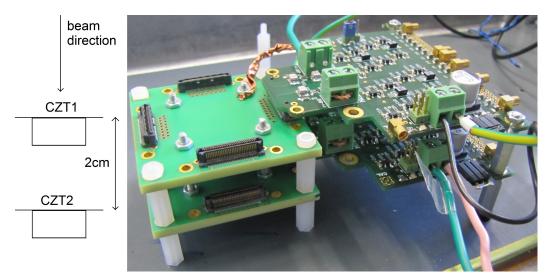


Figure 5.27.: Setup of two CZT detectors with attached readout electronics. Both detectors face the beam on their anode sides. The distance between both layers is 2 cm.

The setup was faced with the anode side towards the photon beam. Thus, high energy  $\gamma$ -rays are more likely to be absorbed on the cathode side. Firstly, the signals of the cathode and a center pixel were sampled and stored with the digitizer. In a first run, both detectors were investigated independently to suppress additional scattering events. By using the calibration obtained with a <sup>22</sup>Na source and the assumption, that the response of the detector can be extrapolated from the low energy range up to several MeV, the measured results show that CZT pixel detectors are capable of detecting photon energies up to 8 MeV. At the same time, timestamps were calculated relative to the bunch frequency of the accelerator. The results for both detectors are illustrated in fig. 5.28.

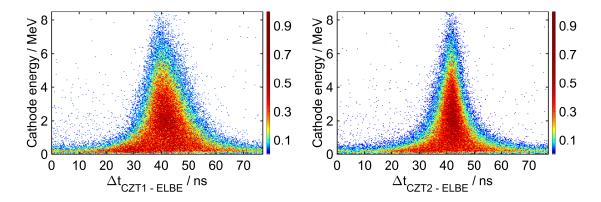


Figure 5.28.: Amplitude over timing spectrum of both CZT detectors (left: CZT1, right: CZT2). The detectors are capable of detecting photon energies up to 8 MeV. CZT2 has a better timing resolution, which needs further investigation.

It is clearly visible, that detector CZT2 has a better timing resolution over the entire energy range. By plotting the DOI, i.e. rise time of cathode signal, over timestamp difference  $\Delta t$ , a distortion of the timing spectrum due to a potentially non-uniformity of the detector crystal is apparent (fig. 5.29). As a results, it can be stated that the timing resolution depends on the

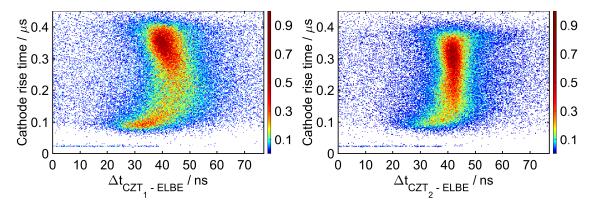


Figure 5.29.: Dependencies of timing performance on DOI, i.e. rise time of the cathode signal. CZT2 reveals an uniform timing distribution as expected (right), but timing performance of CZT1 is potentially degraded by a non-uniformity in the detector material. The measurements were performed with exactly the same readout electronics.

CZT material, as the measurements were recorded with the same readout electronics. To explain these effects, further investigations with a larger set of CZT crystal are necessary. At this time, only two detectors were available for experiments. The overall timing performance of both detectors in the relevant energy range up to 8 MeV is summarized in fig.5.30.

Besides energy and timing related characteristics of the CZT, the maximum detector load and throughput of the electronics is important for a prompt  $\gamma$ -ray imaging system. For this purpose, the photon flux of the beam was increased to a level, where the monitoring detector (3"×3" BGO) counts 638,660 events per second. At the same time, the cathode signal of the CZT1 was recorded for a duration of 5  $\mu$ s per triggered event. After digital pulse processing, a separation of events without pile-up effects is easily implementable by a leading edge

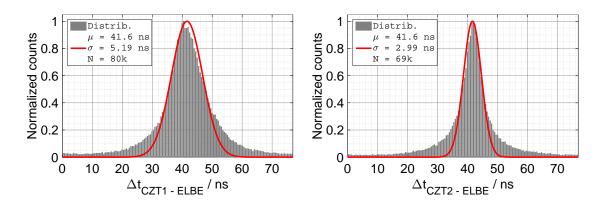


Figure 5.30.: Timing performance over the entire energy range up to 8 MeV. CZT1 (left) reveals a timing resolution of 12.2 ns (FWHM), whereas CZT2 (right) performs better with 7.03 ns (FWHM). All events of total number N are plotted. The standard deviation of the distribution is  $\sigma$ .

trigger with a constant level threshold. The results are shown in fig.5.31. By counting all

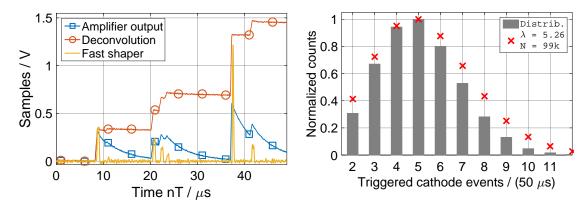


Figure 5.31.: Recorded sequence of the cathode signal at a high photon flux. A mean value of 5.083 counts per 50  $\mu$ s was extracted. The number of occurrences has the shape of a Poisson distribution with the average number of events  $\lambda$ .

detected pulses in a total number of 100k recorded sequences, a mean value of 5.083 counts per 50  $\mu$ s was obtained. This corresponds to 101,660 counts per second in the CZT pixel detector. The estimate of count rate capability is slightly increased by using the average number  $\lambda$  of the fitted Poisson distribution ( $\approx$ 105 kcps).

Finally, the total amount of coincident events in the stacked layout was estimated. A calculation of timestamp difference of both cathode trigger events instead of the difference to accelerator frequency, a coincidence timing spectrum was obtained (fig. 5.32). An event was triggered on the cathode of the detector on top of the stack, while the cathode signal of the detector in the second layer was observed. A cluster of coincident events is clearly visible in fig. 5.32. For this experiment, the monitoring detector counted about 78,000 events per second. However, the bunch structure due to random coincidences is not visible, but is potentially more prominent at higher photon fluxes. To estimate the total number of coincident events in a CZT-CZT configuration, events within the cluster of 45 ns were selected (fig. 5.33). The coincident rate for all interactions within this region is 5.34 %. With the same

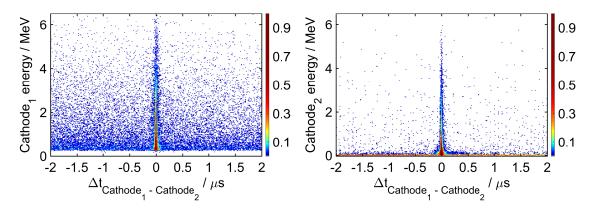


Figure 5.32.: Coincident timing spectrum of cathode triggers on both CZT detectors. The measurement contains 129,788 events.

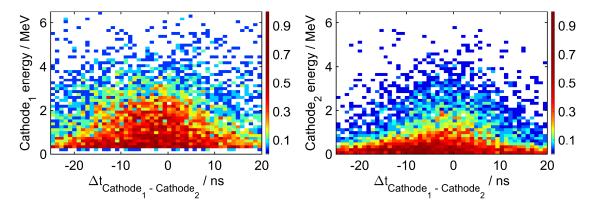


Figure 5.33.: Selected events in a coincident window of 45 ns. The total amount of events within this region is 6,926. This corresponds to a coincident rate of 5.34 %

setup, a coincident rate of about 3 % was measured with  $^{22}\mathrm{Na}$  source.

## 6. Discussion

For completeness, it has to be mentioned that the presented experiments at the ELBE beam have been repeated with a proton beam at OncoRay. Due to the higher frequency of the particle accelerator (106.3 MHz), a measurement of CZT timing performance was not satisfactory. The bunch structure was only visible for selected high energy events. But a measurement of coincident events with the stacked CZT setup verified the results obtained at ELBE bremsstrahlung beam. Moreover, the spectroscopy performance of the CZT for prompt  $\gamma$ -ray emissions at realistic scenarios has yet to be carried out.

It has been demonstrated that the developed instruments can be easily adapted to various types of detectors. However, investigations and experimental results rely on the application of CZT pixel detectors in a Compton camera. A Compton camera approach with one or two CZT detectors in the scatter layers was initially pursued, but finally rejected by the group due to several doubts. Concerns were justified by experimental results with a CZT orthogonal strip detector and several design studies based on simulation.

An outstanding result of investigations with the orthogonal strip (or cross-strip) CZT detector was a good energy resolution of about 3.3% (FWHM at 511 keV), but "the depth-ofinteraction determination was not working satisfactorily" (Kormoll, 2013). This was caused by the fact that the readout electronics were only able to trigger events near the cathode side of the detector. Consequently, the entire detector volume was not exploited, resulting in a poor efficiency of about 10% (Hueso-González et al., 2014). The subsequent experimental results with the pixel detector presented in this work, overcome the triggering issue on multiple cathodes. Moreover, redesigned electronics improved energy resolution, enabled DOI determination, and maximized the total number of useable events throughout the entire volume of the detector. It could be shown that CZT pixel detectors for application to range assessment in particle therapy require high dynamic range analog front-end electronics and digital pulse processing at high sampling rates. There is still a question mark over the applicability of an ASIC, as a suitable integrated circuit has not been developed until now.

A fundamental assumption for all design studies of a Compton camera by means of simulations was a threshold of 100 keV for CZT detectors. "In reality, a lower energy threshold of 50 keV is rather not practical for the CZT detector" (Rohling, 2015). That statement is disproved for the investigated CZT pixel detector, and even the low energy range of CZT detectors is not limited to 50 keV. The measurements with a <sup>241</sup>Am indicated that a threshold of 10 keV is realizable. Consequently, the number of valid events due to Compton scattering in the CZT detector can be increased by utilizing high dynamic range front-end electronics

### 6. Discussion

### and a pixel detector.

Finally, low efficiency and a little number of valid events attained by experiments were mainly caused by the orthogonal strip layout of the detector in combination with the readout electronics used for the first prototype. Meaningful conclusions regarding the Compton camera must be derived from experimental results with the pixel detector and digital pulse processing. Also, larger pixel detectors with a thickness of at least 10 mm or 15 mm should be taken into account for simulations and experimental setups. The lack of efficiency of a Compton camera (CC) was also confirmed by other groups. "The detection efficiency for the CC [...] were found to be too low to make the current system clinically viable" (Polf et al., 2015). But their detection system was purely designed with CZT detectors in the scatter and absorber layer. Moreover, their ASIC-based readout electronics have been developed for low energies (<2 MeV) and low count rate environments, which substantiates the demand for suitable integrated electronics.

In general, the high channel count of pixel detectors is still challenging, as long as the system simultaneously requires a large signal bandwidth (above 10 MHz). We have developed an interface card utilizing 64 readout channels at 32.5 MSPS as a Rear Transition Module (RTM) for a MicroTCA system. Signal processing is implemented in the corresponding AMC, i.e. the readout of a detector with  $8 \times 8$  pixels fits into a single slot of a MicroTCA system (fig. B.1). Final steps at construction and functional tests are currently underway. That implementation exposes the requirements on hardware for digital signal processing. The incoming data rate from that 64 channels is 24.69 GByte/s. By multiplexing two ADC channels to a serial lane with differential signaling, the interface must be clocked at 780 MHz. Such data rates can be easily handled by FPGAs (Glöckner, 2015), but increase requirements on interconnections with regard to signal integrity and the total number of pin counts.

In conclusion, all experimental data with CZT based Compton cameras suffer from limitations due to electronics. Unfortunately, sufficient experimental data with the digitalized CZT detector setup could not be obtained during this study in time. Thus the Compton camera approach cannot be abandoned until clinical experiments have been performed with appropriate electronics and high-performance grade detectors. Even if the Compton camera is not feasible for range verification in particle therapy, the proposed instrumentation and digital algorithms can be ultimately applied to any potentially satisfying method to improve overall detection performance. New developments in the application of nuclear detector systems should utilize digital sampling techniques. Furthermore, pulse processing can be implemented at a higher level of abstraction (Wohsmann, 2014; Scharnagl, 2015). Processing detector signals at an algorithmic level, either in real-time by FPGAs or offline by software, can be regarded as state-of-the-art.

## 7. Summary

**Background:** The irradiation of cancer patients with charged particles, mainly protons and carbon ions, has become an established method for the treatment of specific types of tumors. In comparison with the use of X-rays or  $\gamma$ -rays, particle therapy has the advantage that the dose distribution in the patient can be precisely controlled. Tissue or organs lying near the tumor will be spared. A verification of the treatment plan with the actual dose deposition by means of a measurement can be done through range assessment of the particle beam. For this purpose, prompt  $\gamma$ -rays are detected, which are emitted by the affected target volume during irradiation.

**Motivation:** The detection of prompt  $\gamma$ -rays is a task related to radiation detection and measurement. Nuclear applications in medicine can be found in particular for in vivo diagnosis. In that respect the spatially resolved measurement of  $\gamma$ -rays is an essential technique for nuclear imaging, however, technical requirements of radiation measurement during particle therapy are much more challenging than those of classical applications. For this purpose, appropriate instruments beyond the state-of-the-art need to be developed and tested for detecting prompt  $\gamma$ -rays. Hence the success of a method for range assessment of particle beams is largely determined by the implementation of electronics. In practice, this means that a suitable detector material with adapted readout electronics, signal and information processing, and data interface must be utilized to solve the challenges. Thus, the parameters of the system (e.g. segmentation, time or energy resolution) can be optimized depending on the method (e.g. slit camera, time-of-flight measurement or Compton camera). Regardless of the method, the detector system must have a high count rate capability and a large measuring range ( $\geq$ 7 MeV). For a subsequent evaluation of a suitable method for imaging, the mentioned parameters may not be restricted by the electronics. Digital signal processing is predestined for multipurpose tasks, and, in terms of the demands made, the performance of such an implementation has to be determined.

**Materials and methods:** In this study, the instrumentation of a detector system for prompt  $\gamma$ -rays emitted during particle therapy is limited to the use of a cadmium zinc telluride (CdZnTe, CZT) semiconductor detector. The detector crystal is divided into an 8×8 pixel array by segmented electrodes. Analog and digital signal processing are exemplarily tested with this type of detector and aims for application of a Compton camera to range assessment. The electronics are implemented with commercial off-the-shelf (COTS) components. If applicable, functional units of the detector system were digitalized and implemented in a field-programmable gate array (FPGA). An efficient implementation of the algorithms in terms of timing and logic utilization is fundamental to the design of digital circuits. The measurement

system is characterized with radioactive sources to determine the measurement dynamic range and resolution. Finally, the performance is examined in terms of the requirements of particle therapy with experiments at particle accelerators.

**Results:** A detector system based on a CZT pixel detector has been developed and tested. Although the use of an application-specific integrated circuit is convenient, this approach was rejected because there was no circuit available which met the requirements. Instead, a multichannel, compact, and low-noise analog amplifier circuit with COTS components has been implemented. Finally, the 65 information channels of a detector are digitized, processed and visualized.

An advanced digital signal processing transforms the traditional approaches of nuclear electronics in algorithms and digital filter structures for an FPGA. With regard to the characteristic signals (e.g. varying rise times, depth-dependent energy measurement) of a CZT pixel detector, it could be shown that digital pulse processing results in a very good energy resolution ( $\approx 2\%$  FWHM at 511 keV), as well as permits a time measurement in the range of some tens of nanoseconds. Furthermore, the experimental results have shown that the dynamic range of the detector system could be significantly improved compared to the existing prototype of the Compton camera ( $\approx 10 \text{ keV..7 MeV}$ ). Even count rates of  $\approx 100 \text{ kcps}$  in a high-energy beam could be ultimately processed with the CZT pixel detector. But this is merely a limit of the detector due to its volume, and not related to electronics. In addition, the versatility of digital signal processing has been demonstrated with other detector materials (e.g. CeBr<sub>3</sub>). With foresight on high data throughput in a distributed data acquisition from multiple detectors, a Gigabit Ethernet link has been implemented as data interface.

**Conclusions:** To fully exploit the capabilities of a CZT pixel detector, a digital signal processing is absolutely necessary. A decisive advantage of the digital approach is the ease of use in a multichannel system. Thus with digitalization, a necessary step has been done to master the complexity of a Compton camera. Furthermore, the benchmark of technology shows that a CZT pixel detector withstands the requirements of measuring prompt  $\gamma$ -rays during particle therapy. The previously used orthogonal strip detector must be replaced by the pixel detector in favor of increased efficiency and improved energy resolution. With the integration of the developed digital detector system into a Compton camera, it must be ultimately proven whether this method is applicable for range assessment in particle therapy. Even if another method is more convenient in a clinical environment due to practical considerations, the detector system of that method may benefit from the shown instrumentation of a digital signal processing system for nuclear applications.

## 8. Zusammenfassung

**Hintergrund:** Die Bestrahlung von Krebspatienten mit geladenen Teilchen, vor allem Protonen oder Kohlenstoffionen, ist mittlerweile eine etablierte Methode zur Behandlung von speziellen Tumorarten. Im Vergleich mit der Anwendung von Röntgen- oder  $\gamma$ -Strahlen hat die Teilchentherapie den Vorteil, dass die Dosisverteilung im Patienten präziser gesteuert werden kann. Dadurch werden um den Tumor liegendes Gewebe oder Organe geschont. Die messtechnische Verifikation des Bestrahlungsplans mit der tatsächlichen Dosisdeposition kann über eine Reichweitenkontrolle des Teilchenstrahls erfolgen. Für diesen Zweck werden prompte  $\gamma$ -Strahlen detektiert, die während der Bestrahlung vom getroffenen Zielvolumen emittiert werden.

**Fragestellung:** Die Detektion von prompten  $\gamma$ -Strahlen ist eine Aufgabenstellung der Strahlenmesstechnik. Strahlenanwendungen in der Medizintechnik finden sich insbesondere in der in-vivo Diagnostik. Dabei ist die räumlich aufgelöste Messung von  $\gamma$ -Strahlen bereits zentraler Bestandteil der nuklearmedizinischen Bildgebung, jedoch sind die technischen Anforderungen der Strahlendetektion während der Teilchentherapie im Vergleich mit klassischen Anwendungen weitaus anspruchsvoller. Über den Stand der Technik hinaus müssen für diesen Zweck geeignete Instrumente zur Erfassung der prompten  $\gamma$ -Strahlen entwickelt und erprobt werden. Die elektrotechnische Realisierung bestimmt maßgeblich den Erfolg eines Verfahrens zur Reichweitenkontrolle von Teilchenstrahlen. Konkret bedeutet dies, dass ein geeignetes Detektormaterial mit angepasster Ausleseelektronik, Signal- und Informationsverarbeitung sowie Datenschnittstelle zur Problemlösung eingesetzt werden muss. Damit können die Parameter des Systems (z. B. Segmentierung, Zeit- oder Energieauflösung) in Abhängigkeit der Methode (z. B. Schlitzkamera, Flugzeitmessung oder Compton-Kamera) optimiert werden. Unabhängig vom Verfahren muss das Detektorsystem eine hohe Ratenfestigkeit und einen großen Messbereich (≥7 MeV) besitzen. Für die anschließende Evaluierung eines geeigneten Verfahrens zur Bildgebung dürfen die genannten Parameter durch die Elektronik nicht eingeschränkt werden. Eine digitale Signalverarbeitung ist für universelle Aufgaben prädestiniert und die Leistungsfähigkeit einer solchen Implementierung soll hinsichtlich der gestellten Anforderungen bestimmt werden.

**Material und Methode:** Die Instrumentierung eines Detektorsystems für prompte  $\gamma$ -Strahlen beschränkt sich in dieser Arbeit auf die Anwendung eines Cadmiumzinktellurid (CdZnTe, CZT) Halbleiterdetektors. Der Detektorkristall ist durch segmentierte Elektroden in ein 8×8 Pixelarray geteilt. Die analoge und digitale Signalverarbeitung wird beispielhaft mit diesem Detektortyp erprobt und zielt auf die Anwendung zur Reichweitenkontrolle mit einer Compton-Kamera. Die Elektronik wird mit seriengefertigten integrierten Schaltkreisen umgesetzt. Soweit möglich, werden die Funktionseinheiten des Detektorsystems digitalisiert und in einem field-programmable gate array (FPGA) implementiert. Eine effiziente Umsetzung der Algorithmen in Bezug auf Zeitverhalten und Logikverbrauch ist grundlegend für den Entwurf der digitalen Schaltungen. Das Messsystem wird mit radioaktiven Prüfstrahlern hinsichtlich Messbereichsdynamik und Auflösung charakterisiert. Schließlich wird die Leistungsfähigkeit hinsichtlich der Anforderungen der Teilchentherapie mit Experimenten am Teilchenbeschleuniger untersucht.

Ergebnisse: Es wurde ein Detektorsystem auf Basis von CZT Pixeldetektoren entwickelt und erprobt. Obwohl der Einsatz einer anwendungsspezifischen integrierten Schaltung zweckmäßig wäre, wurde dieser Ansatz zurückgewiesen, da kein verfügbarer Schaltkreis die Anforderungen erfüllte. Stattdessen wurde eine vielkanalige, kompakte und rauscharme analoge Verstärkerschaltung mit seriengefertigten integrierten Schaltkreisen aufgebaut. Letztendlich werden die 65 Informationskanäle eines Detektors digitalisiert, verarbeitet und visualisiert. Eine fortschrittliche digitale Signalverarbeitung überführt die traditionellen Ansätze der Nuklearelektronik in Algorithmen und digitale Filterstrukturen für einen FPGA. Es konnte gezeigt werden, dass die digitale Pulsverarbeitung in Bezug auf die charakteristischen Signale (u.a. variierende Anstiegszeiten, tiefenabhängige Energiemessung) eines CZT Pixeldetektors eine sehr gute Energieauflösung ( $\approx$  2 % FWHM at 511 keV) sowie eine Zeitmessung im Bereich von einigen 10 ns ermöglicht. Weiterhin haben die experimentellen Ergebnisse gezeigt, dass der Dynamikbereich des Detektorsystems im Vergleich zum bestehenden Prototyp der Compton-Kamera deutlich verbessert werden konnte ( $\approx$  10 keV.7 MeV). Nach allem konnten auch Zählraten von  $\approx$  100 kcps in einem hochenergetischen Strahl mit dem CZT Pixeldetektor verarbeitet werden. Dies stellt aber lediglich eine Begrenzung des Detektors aufgrund seines Volumens, nicht jedoch der Elektronik, dar. Zudem wurde die Vielseitigkeit der digitalen Signalverarbeitung auch mit anderen Detektormaterialen (u.a. CeBr<sub>3</sub>) demonstriert. Mit Voraussicht auf einen hohen Datendurchsatz in einer verteilten Datenerfassung von mehreren Detektoren, wurde als Datenschnittstelle eine Gigabit Ethernet Verbindung implementiert.

**Schlussfolgerung:** Um die Leistungsfähigkeit eines CZT Pixeldetektors vollständig auszunutzen, ist eine digitale Signalverarbeitung zwingend notwendig. Ein entscheidender Vorteil des digitalen Ansatzes ist die einfache Handhabbarkeit in einem vielkanaligen System. Mit der Digitalisierung wurde ein notwendiger Schritt getan, um die Komplexität einer Compton-Kamera beherrschbar zu machen. Weiterhin zeigt die Technologiebewertung, dass ein CZT Pixeldetektor den Anforderungen der Teilchentherapie für die Messung prompter  $\gamma$ -Strahlen stand hält. Der bisher eingesetzte Streifendetektor muss zugunsten einer gesteigerten Effizienz und verbesserter Energieauflösung durch den Pixeldetektor ersetzt werden. Mit der Integration des entwickelten digitalen Detektorsystems in eine Compton-Kamera muss abschließend geprüft werden, ob dieses Verfahren für die Reichweitenkontrolle in der Teilchentherapie anwendbar ist. Auch wenn sich herausstellt, dass ein anderes Verfahren unter klinischen Bedingungen praktikabler ist, so kann auch dieses Detektorsystem von der gezeigten Instrumentierung eines digitalen Signalverarbeitungssystems profitieren.

# A. Waveform diagrams

Figure A.1.: An example of a composed Ethernet packet through the MAC layer for GMII. The transmission starts at position 3 with the preamble (signal "phy\_txd"). The MAC also adds the SFD (pos. 10), padding data (pos. 65-71) for a minimum payload length of 46 Byte and the FCS (pos. 71-75). The IFG is controlled with the tx\_busy signal.

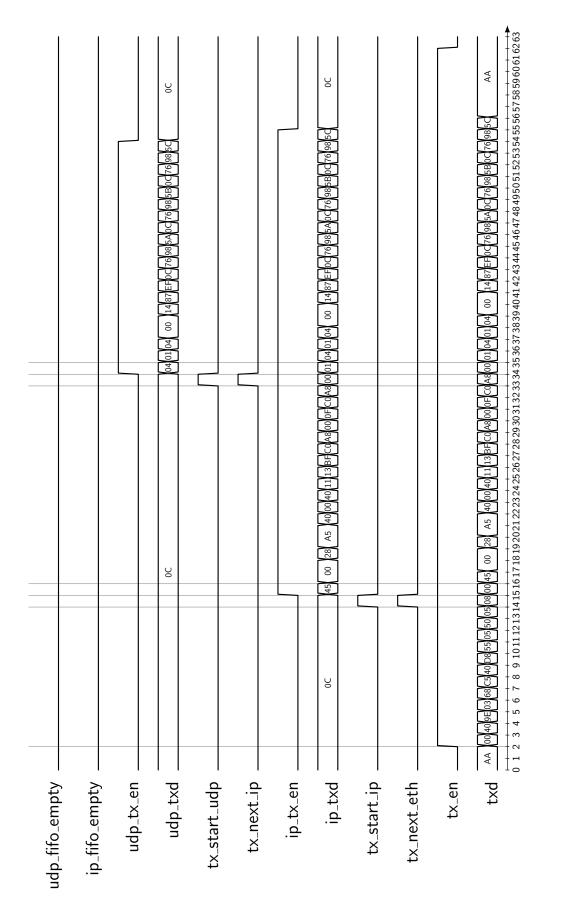


Figure A.2.: Example of the dataflow for a UDP packet through the transport layers with the "Data-Pull" model.

A. Waveform diagrams

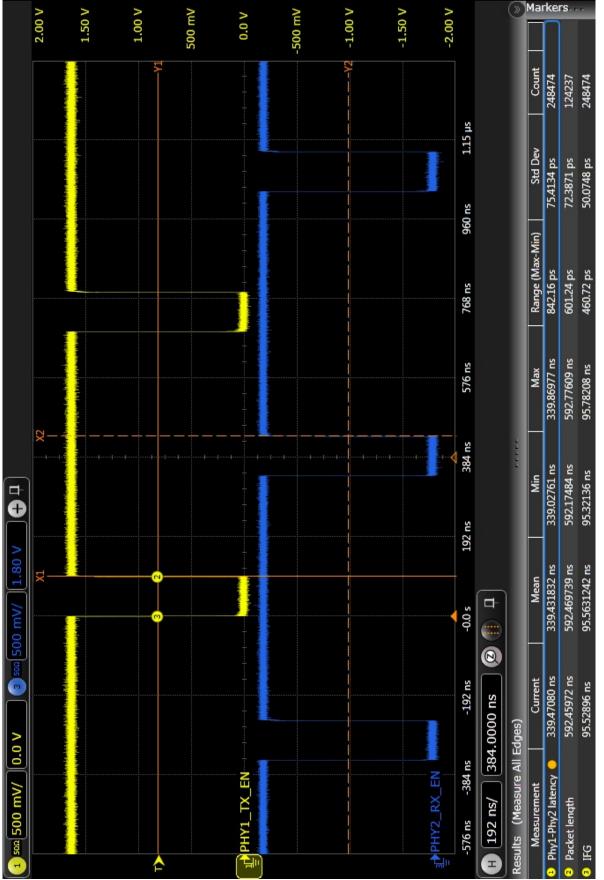


Figure A.3.: The transmission enable signal (signal "PHY1\_TX\_EN") of the transmitting MAC and the receive enable signal (signal "PHY2\_RX\_EN") of the receiving PHY. The oscilloscope measurement verifies that the MAC keeps the IFG at 96 ns while sending data with maximum throughput and constant latency. The packet length of 592 ns corresponds to a UDP payload of 20 Byte.

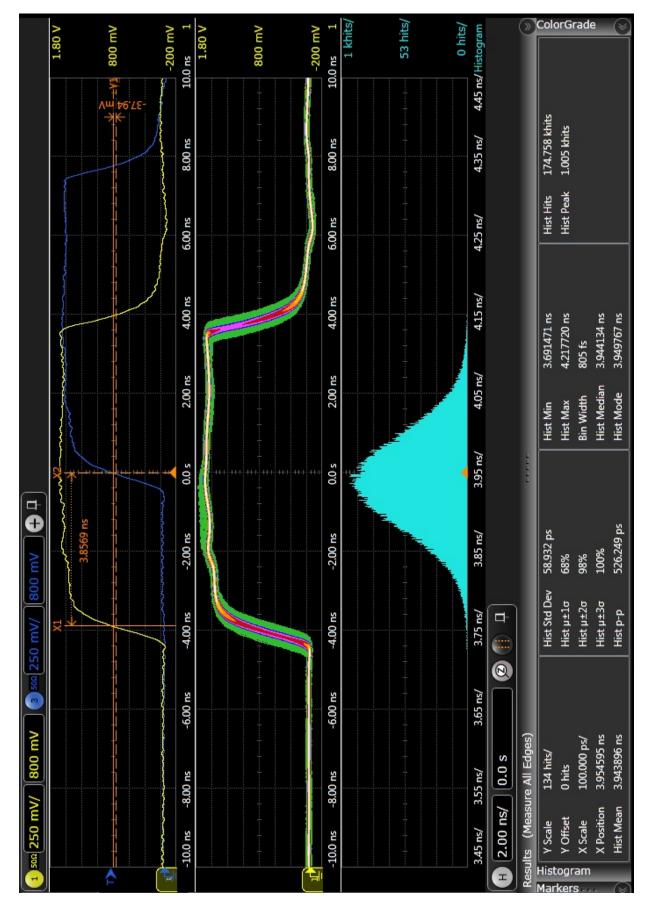


Figure A.4.: Oscilloscope measurement of the time difference between the master and the slave. Each FPGA outputs a PPS signal (for this measurement every 268.4 ms) which shows the precision of the absolute timestamp synchronization with PTP. The standard deviation of the time difference is 58.932 ps with a constant offset of about 3.94 ns

## **B. Rear Transition Module**

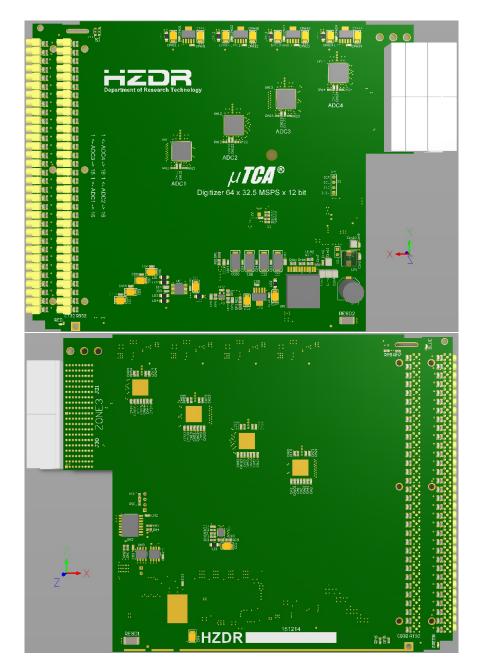


Figure B.1.: Top and bottom side of the Rear Transition Module (RTM) with 64 ADC channels for the MicroTCA implementation of the readout electronics. Each channel is capable of sampling at 32.5 MSPS (12 bit,1 V<sub>pp</sub>). Total width of the PCB is 182.5 mm and total height is 148.5 mm.

# **Bibliography**

- Abbene L and Gerardi G. 2015. High-rate dead-time corrections in a general purpose digital pulse processing system. J Synchrotron Radiat, 22(5):1190–1201. doi: 10.1107/S1600577515013776.
- Alachiotis N, Berger SA, and Stamatakis A. 2010. Efficient PC-FPGA Communication over Gigabit Ethernet. In: 10th Int Conf Computer and Information Technology, pp. 1727–1734. IEEE. doi: 10.1109/CIT.2010.302.
- Alachiotis N, Berger SA, and Stamatakis A. 2012. A versatile UDP/IP based PC↔FPGA communication platform. In: Int Conf Reconfigurable Computing and FPGAs, pp. 1–6. IEEE. doi: 10.1109/ReConFig.2012.6416725.
- Altera Corporation. 2010. Implementing FIR Filters and FFTs with 28-nm Variable-Precision DSP Architecture. White Paper WP-01140-1.0.
- Altera Corporation. 2011. Enabling High-Performance DSP Applications with Arria V or Cyclone V Variable-Precision DSP Blocks. White Paper WP-01159-1.0.
- Altera Corporation. 2015. FIR II IP Core, UG-01072, 2015.10.01.
- Amrami R, Shani G, Hefetz Y, Levy M, Pansky A, and Wainer N. 2000. PET properties of pixellated CdZnTe detector. In: Proc 22nd Ann Int Conf Engineering in Medicine and Biology Society, pp. 94–97. IEEE. doi: 10.1109/IEMBS.2000.900677.
- Amrami R, Shani G, Hefetz Y, Pansky A, and Wainer N. 2001. Timing performance of pixelated CdZnTe detectors. Nucl Instrum Methods Phys Res A, 458(3):772–781. doi: 10.1016/S0168-9002(00)00810-X.
- Analog Devices. 2013. ADA4817-1/ADA4817-2, 1 GHz FastFET Op Amps, Datasheet.
- Awadalla S. 2015. Solid-State Radiation Detectors: Technology and Applications. CRC Press.
- Batmaz B and Dogan A. 2015. UDP/IP Protocol Stack with PCIe Interface on FPGA. In: Proc Int Conf Embedded Systems and Applications, pp. 49–53. WorldComp.
- Batzler WU, Schön D, Baumgardt-Elms C, Schüz J, Eisinger B, Stegmaier C, and Lehnert M. 1999. Krebs in Deutschland: Häufigkeiten und Trends. Gesamtprogramm zur Krebsbekämpfung. Arbeitsgemeinschaft Bevölkerungsbezogener Krebsregister in Deutschland, 2 Edition.

- Bollinger LM and Thomas GE. 1961. Measurement of the Time Dependence of Scintillation Intensity by a Delayed-Coincidence Method. Rev Sci Instrum, 32(9):1044–1050. doi: 10.1063/1.1717610.
- Brisebois G. 2015. Op Amp Combines Femtoamp Bias Current with 4GHz Gain Bandwidth Product, Shines New Light on Photonics Applications. LT J Analog Innovation, 25(2):4–9.
- Butcher J, Hamade M, Petryk M, Bolotnikov AE, Camarda GS, Cui Y, De Geronimo G, Fried J, Hossain A, Kim KH, Vernon E, Yang G, and James RB. 2013. Drift Time Variations in CdZnTe Detectors Measured With Alpha Particles and Gamma Rays: Their Correlation With Detector Response. IEEE Trans Nucl Sci, 60(2):1189–1196. doi: 10.1109/TNS.2012.2234762.
- Cho HY and Lee CS. 2016. Improvement of the energy resolution in a pixellated CdZnTe detector using depth sensing based on pulse rise-time correlation. J Instrum, 11(02): C02081. doi: 10.1088/1748-0221/11/02/C02081.
- Cho HY, Lee JH, Kwon YK, Moon JY, and Lee CS. 2011. Measurement of the drift mobilities and the mobility-lifetime products of charge carriers in a CdZnTe crystal by using a transient pulse technique. J Instrum, 6(01):C01025. doi: 10.1088/1748-0221/6/01/C01025.
- Crochiere RE and Oppenheim AV. 1975. Analysis of linear digital networks. Proc IEEE, 63 (4):581–595. doi: 10.1109/PROC.1975.9793.
- De Antonis P, Morton EJ, and Menezes T. 1996. Measuring the bulk resistivity of CdZnTe single crystal detectors using a contactless alternating electric field method. Nucl Instrum Methods Phys Res A, 380(1-2):157–159. doi: 10.1016/S0168-9002(96)00335-X.
- Del Sordo S, Abbene L, Caroli E, Mancini AM, Zappettini A, and Ubertini P. 2009. Progress in the Development of CdTe and CdZnTe Semiconductor Radiation Detectors for Astro-physical and Medical Applications. Sensors, 9(5):3491–3526. doi: 10.3390/s90503491.
- Di Fulvio A, Shin T, Hamel M, and Pozzi S. 2016. Digital pulse processing for NaI(TI) detectors. Nucl Instrum Methods Phys Res A, 806:169–174. doi: 10.1016/j.nima.2015.09.080.
- Dollas A, Ermis I, Koidis I, Zisis I, and Kachris C. 2005. An Open TCP/IP Core for Reconfigurable Logic. In: Proc 13th Ann Symp Field-Programmable Custom Computing Machines, pp. 297–298. IEEE. doi: 10.1109/FCCM.2005.20.
- Dönmez B, Kim J, and He Z. 2007. 3D position sensing on UltraPeRL CdZnTe detectors. In: Nucl Sci Symp Conf Rec, pp. 420–423. IEEE. doi: 10.1109/NSSMIC.2007.4436361.
- Drezet A, Monnet O, Mathy F, Montemont G, and Verger L. 2007. CdZnTe detectors for small field of view positron emission tomographic imaging. Nucl Instrum Methods Phys Res A, 571(1-2):465–470. doi: 10.1016/j.nima.2006.10.292.
- Fiedler F, Dersch U, Golnik C, Kormoll T, Müller A, Rohling H, Schöne S, and Enghardt W. 2011. The use of prompt  $\gamma$ -rays for in-vivo dosimetry at therapeutic proton and ion beams. In: Nucl Sci Symp Conf Rec, pp. 4453–4456. IEEE. doi: 10.1109/NSSMIC.2011.6152493.

- Födisch P, Lange B, and Kaever P. 2013. Eine Ausleseelektronik für CZT-Detektoren mit dem RENA-3 IC von Nova R&D. In: 104. Tag Studiengr Elektr Instrum, pp. 135–143. DESY.
- Födisch P, Sandmann J, Lange B, and Kaever P. 2014. Taktsynchronisierung und Zeitmessung in einem verteilten Datenerfassungssystem. In: 105. Tag Studiengr Elektr Instrum, pp. 238–242. DESY.
- Födisch P, Lange B, and Kaever P. 2015. Ein VHDL basierter Gigabit Ethernet Protokollstapel für FPGAs. In: 106. Tag Studiengr Elektr Instrum, pp. 52–76. DESY.
- Födisch P, Berthel M, Lange B, Kirschke T, Enghardt W, and Kaever P. 2016a. Chargesensitive front-end electronics with operational amplifiers for CdZnTe detectors. J Instrum, 11(09):T09001. doi: 10.1088/1748-0221/11/09/T09001.
- Födisch P, Lange B, Sandmann J, Büchner A, Enghardt W, and Kaever P. 2016b. A synchronous Gigabit Ethernet protocol stack for high-throughput UDP/IP applications. J Instrum, 11(01):P01010. doi: 10.1088/1748-0221/11/01/P01010.
- Födisch P, Wohsmann J, Lange B, Schönherr J, Enghardt W, and Kaever P. 2016c. Digital high-pass filter deconvolution by means of an infinite impulse response filter. Nucl Instrum Methods Phys Res A, 830:484–496. doi: 10.1016/j.nima.2016.06.019.
- Födisch P, Bryksa A, Lange B, Enghardt W, and Kaever P. 2016d. Implementing High-Order FIR Filters in FPGAs. arXiv:1610.03360 [cs.DC].
- Fouan JP and Passerieux JP. 1968. A time compensation method for coincidences using large coaxial Ge(Li) detectors. Nucl Instrum Methods, 62(3):327–329. doi: 10.1016/0029-554X(68)90384-4.
- Fraile L, Mach H, Vedia V, Olaizola B, Paziy V, Picado E, and Udías J. 2013. Fast timing study of a CeBr<sub>3</sub> crystal: Time resolution below 120 ps at <sup>60</sup>Co energies. Nucl Instrum Methods Phys Res A, 701:235–242. doi: 10.1016/j.nima.2012.11.009.
- Furukawa Denshi. 2016. Ce:GAGG Scintillator Crystal, product information. [last update: 2014, retrieved: 03.11.2016]. URL: http://www.furukawa-denshi.co.jp.
- Gan B, Wei T, Gao W, Liu H, and Hu Y. 2016. A low-noise 64-channel front-end readout ASIC for CdZnTe detectors aimed to hard X-ray imaging systems. Nucl Instrum Methods Phys Res A, 816:53–61. doi: 10.1016/j.nima.2016.01.070.
- Garson A, Li Q, Jung IV, Dowkontt P, Bose R, Simburger G, and Krawczynski H. 2007. Leakage currents and capacitances of thick CZT detectors. In: Nucl Sci Symp Conf Rec, pp. 2258–2261. IEEE. doi: 10.1109/NSSMIC.2007.4436597.
- Georgiev A, Gast W, and Lieder RM. 1994. An analog-to-digital conversion based on a moving window deconvolution. IEEE Trans Nucl Sci, 41(4):1116–1124. doi: 10.1109/23.322868.

- Georgiev A and Gast W. 1993. Digital pulse processing in high resolution, high throughput, gamma-ray spectroscopy. IEEE Trans Nucl Sci, 40(4):770–779. doi: 10.1109/23.256659.
- Girerd C, Autiero D, Carlus B, Gardien S, Marteau J, and Tromeur W. 2009. MicroTCA implementation of synchronous Ethernet-Based DAQ systems for large scale experiments. In: 16th IEEE-NPSS Real Time Conf, pp. 22–27. IEEE. doi: 10.1109/RTC.2009.5321791.
- Glöckner T. 2015. Entwurf, Implementierung und Test eines JESD204B-IP-Cores für ein Highspeed-ADC-Board mit einem Xilinx-FPGA. Dresden University of Applied Sciences, Bachelor thesis.
- Golnik C, Bemmerer D, Enghardt W, Fiedler F, Hueso-González F, Pausch G, Roemer K, Rohling H, Schöne S, Wagner L, and Kormoll T. 2016. Tests of a Compton imaging prototype in a monoenergetic 4.44 MeV photon field – a benchmark setup for prompt gammaray imaging devices. J Instrum, 11(06):P06009. doi: 10.1088/1748-0221/11/06/P06009.
- Golnik C. 2016. Treatment verification in proton therapy based on the detection of prompt gamma-rays. Technische Universität Dresden, Dissertation.
- Grosser A. 2015. Director, Product Management, Redlen Technologies, Inc. Private communication.
- Grupen C and Shwartz B. 2011. Particle Detectors. Cambridge University Press, 2 Edition.
- Harboe Ø. 2016. The Zylin ZPU, The worlds smallest 32 bit CPU with GCC toolchain. [last update: 21.04.2015, retrieved: 03.11.2016]. URL: https://github.com/zylin/zpu.
- Hartog HD and Muller FA. 1947. Optimum Instrument Response for Discrimination against Spontaneous Fluctuations. Physica, 13(9):571–580. doi: 10.1016/0031-8914(47)90027-X.
- He Z, Knoll GF, and Wehe DK. 1998. Direct measurement of product of the electron mobility and mean free drift time of CdZnTe semiconductors using position sensitive single polarity charge sensing detectors. J Appl Phys, 84(10):5566–5569. doi: 10.1063/1.368601.
- He Z, Li W, Knoll GF, Wehe DK, Berry J, and Stahle CM. 1999. 3-D position sensitive CdZnTe gamma-ray spectrometers. Nucl Instrum Methods Phys Res A, 422(1-3):173–178. doi: 10.1016/S0168-9002(98)00950-4.
- He Z. 2001. Review of the Shockley-Ramo theorem and its application in semiconductor gamma-ray detectors. Nucl Instrum Methods Phys Res A, 463(1-2):250–267. doi: 10.1016/S0168-9002(01)00223-6.
- Heinzel G, Rüdiger A, and Schilling R. 2002. Spectrum and spectral density estimation by the Discrete Fourier transform (DFT), including a comprehensive list of window functions and some new at-top windows. [last update: 2002, retrieved: 03.11.2016]. URL: http://hdl.handle.net/11858/00-001M-0000-0013-557A-5.

- Herrmann FL, Perin G, de Freitas JPJ, Bertagnolli R, and dos Santos Martins JB. 2009. A Gigabit UDP/IP Network Stack in FPGA. In: 16th Int Conf Electronics, Circuits, and Systems, pp. 836–839. IEEE. doi: 10.1109/ICECS.2009.5410757.
- Hessling JP. 2008. A novel method of dynamic correction in the time domain. Meas Sci Technol, 19(7):075101. doi: 10.1088/0957-0233/19/7/075101.
- Hoffmann R. 2013. Signalanalyse und -erkennung: Eine Einführung für Informationstechniker. Springer.
- Hong J, Bellm EC, Grindlay JE, and Narita T. 2004. Cathode depth sensing in CZT detectors. In: Proc SPIE, Vol 5165, pp. 54–62. doi: 10.1117/12.506216.
- Horowitz P and Hill W. 2015. The Art of Electronics. Cambridge University Press, 3 Edition.
- Hueso-González F, Golnik C, Berthel M, Dreyer A, Enghardt W, Fiedler F, Heidel K, Kormoll T, Rohling H, Schöne S, Schwengner R, Wagner A, and Pausch G. 2014. Test of Compton camera components for prompt gamma imaging at the ELBE bremsstrahlung beam. J Instrum, 9(05):P05002. doi: 10.1088/1748-0221/9/05/P05002.
- Hueso-González F. 2016. Nuclear methods for real-time range verification in proton therapy based on prompt gamma-ray imaging. Technische Universität Dresden, Dissertation.
- Hueso-González F, Enghardt W, Fiedler F, Golnik C, Janssens G, Petzoldt J, Prieels D, Priegnitz M, Roemer KE, Smeets J, Vander Stappen F, Wagner A, and Pausch G. 2015. First test of the prompt gamma ray timing method with heterogeneous targets at a clinical proton therapy facility. Phys Med Biol, 60(16):6247–6272. doi: 10.1088/0031-9155/60/16/6247.
- IEEE. 2011. IEEE Standard for Terminology and Test Methods for Analogto-Digital Converters. IEEE Std 1241-2010 (Revision of IEEE Std 1241-2000). doi: 10.1109/IEEESTD.2011.5692956.
- IEEE. 2015. IEEE Standard for Ethernet. IEEE Std 802.3-2015 (Revision of IEEE Std 802.3-2012).
- iseg. 2014. High Voltage Power Supply, SHQ High Precision Series. Operator's Manual, Rev. 2014-02-13-17-34, iseg Spezialelektronik GmbH.
- Iwanowska J, Swiderski L, Szczesniak T, Sibczynski P, Moszynski M, Grodzicka M, Kamada K, Tsutsumi K, Usuki Y, Yanagida T, and Yoshikawa A. 2013. Performance of ceriumdoped Gd<sub>3</sub>Al<sub>2</sub>Ga<sub>3</sub>O<sub>1</sub>2 (GAGG:Ce) scintillator in gamma-ray spectrometry. Nucl Instrum Methods Phys Res A, 712:34–40. doi: 10.1016/j.nima.2013.01.064.
- Jordanov VT. 1994. Deconvolution of pulses from a detector-amplifier configuration. Nucl Instrum Methods Phys Res A, 351(2-3):592–594. doi: 10.1016/0168-9002(94)91394-3.
- Jordanov VT. 2016. Unfolding-synthesis technique for digital pulse processing. Part 1: Unfolding. Nucl Instrum Methods Phys Res A, 805:63–71. doi: 10.1016/j.nima.2015.07.040.

- Jordanov VT and Knoll GF. 1994. Digital synthesis of pulse shapes in real time for high resolution radiation spectroscopy. Nucl Instrum Methods Phys Res A, 345(2):337–345. doi: 10.1016/0168-9002(94)91011-1.
- Jordanov VT, Knoll GF, Huber AC, and Pantazis JA. 1994. Digital techniques for real-time pulse shaping in radiation measurements. Nucl Instrum Methods Phys Res A, 353(1-3): 261–264. doi: 10.1016/0168-9002(94)91652-7.
- Jung I, Garson AB, Perkins JS, Krawczynski H, Matteson J, Skelton RT, Burger A, and Groza
   M. 2005. Thick pixelated CZT detectors with isolated steering grids. In: Nucl Sci Symp Conf Rec, pp. 206–210. IEEE. doi: 10.1109/NSSMIC.2005.1596237.
- Kaatsch P, Spix C, Katalinic A, Hentschel S, Luttmann S, Stegmaier C, Caspritz S, Christ M, Ernst A, Folkerts J, Hansmann J, and Klein S. 2015. Krebs in Deutschland 2011/2012.
  Gesundheitsberichterstattung-Hefte, 10. doi: 10.17886/rkipubl-2015-004.

Keysight Technologies. 2016. 33500b Series Waveform Generators.

- Kim JW, Kim D, and Yim H. 2009. Pinhole Camera Measurements of Prompt Gamma-rays for Detection of Beam Range Variation in Proton Therapy. J Korean Phys Soc, 55(4): 1673–1676. doi: 10.3938/jkps.55.1673.
- Knoll GF. 2010. Radiation Detection and Measurement. John Wiley & Sons, 4 Edition.
- Komarov S, Yin Y, Wu H, Wen J, Krawczynski H, Meng LJ, and Tai YC. 2012. Investigation of the limitations of the highly pixilated CdZnTe detector for PET applications. Phys Med Biol, 57(22):7355–7380. doi: 10.1088/0031-9155/57/22/7355.
- Kormoll T, Fiedler F, Golnik C, Heidel K, Kempe M, Schoene S, Sobiella M, Zuber K, and Enghardt W. 2011a. A Prototype Compton Camera for In-Vivo Dosimetry of Ion Beam Cancer Irradiation. In: Nucl Sci Symp Conf Rec, pp. 3484–3487. IEEE. doi: 10.1109/NSSMIC.2011.6152639.
- Kormoll T, Fiedler F, Schöne S, Wüstemann J, Zuber K, and Enghardt W. 2011b. A Compton imager for in-vivo dosimetry of proton beams a design study. Nucl Instrum Methods Phys Res A, 626-627:114–119. doi: 10.1016/j.nima.2010.10.031.
- Kormoll T. 2013. A Compton Camera for In-vivo Dosimetry in Ion-beam Radiotherapy. Technische Universität Dresden, Dissertation.

Kowalski E. 1970. Nuclear Electronics. Springer Verlag, 1 Edition.

- Kozyczkowski JJ and Bialkowski J. 1976. Amplitude and rise time compensated timing optimized for large semiconductor detectors. Nucl Instrum Methods, 137(1):75–83. doi: 10.1016/0029-554X(76)90251-2.
- Krause M and Supiot S. 2015. Advances in radiotherapy special feature. Br J Radiol, 88 (1051):20150412. doi: 10.1259/bjr.20150412.

- Kühn W, Gilardi C, Kirschner D, Lang J, Lange S, Liu M, Perez T, Yang S, Schmitt L, Jin D, Li L, Liu Z, Lu Y, Wang Q, Wei S, Xu H, Zhao D, Korcyl K, Otwinowski JT, Salabura P, Konorov I, and Mann A. 2008. FPGA based compute nodes for high level triggering in PANDA. J Phys Conf Ser, 119(2):022027. doi: 10.1088/1742-6596/119/2/022027.
- Lee W, Bolotnikov A, Lee T, Camarda G, Cui Y, Gul R, Hossain A, Utpal R, Yang G, and James R. 2016. Mini Compton Camera Based on an Array of Virtual Frisch-Grid CdZnTe Detectors. IEEE Trans Nucl Sci, 63(1):259–265. doi: 10.1109/TNS.2015.2514120.
- Li W, He Z, Knoll G, Wehe D, and Berry J. 2001. Experimental results from an Imarad 8×8 pixellated CZT detector. Nucl Instrum Methods Phys Res A, 458(1-2):518–526. doi: 10.1016/S0168-9002(00)00913-X.
- Lieber P and Hutchings B. 2011. FPGA Communication Framework. In: Int Symp Field-Programmable Custom Computing Machines, pp. 69–72. IEEE. doi: 10.1109/FCCM.2011.39.
- Linear Technology. 2014. LTC6268/LTC6269 500MHz Ultra-Low Bias Current FET Input Op Amp, Datasheet 62689f.
- Linear Technology. 2015. LTC6268-10/LTC6269-10 4GHz Ultra-Low Bias Current FET Input Op Amp, Datasheet 626810f.
- Löfgren A, Lodesten L, Sjoholm S, and Hansson H. 2005. An analysis of FPGA-based UDP/IP stack parallelism for embedded Ethernet connectivity. In: 23rd NORCHIP Conf, pp. 94–97. doi: 10.1109/NORCHP.2005.1596997.
- Lu P, Gomolchuk P, Chen H, Beitz D, and Grosser A. 2015. Ruggedization of CdZnTe detectors and detector assemblies for radiation detection applications. Nucl Instrum Methods Phys Res A, 784:44–50. doi: 10.1016/j.nima.2015.01.022.
- Lyons RG. 2010. Understanding Digital Signal Processing. Addison Wesley, 3 Edition.
- Mahmoodi MR, Sayedi SM, and Mahmoodi B. 2014. Reconfigurable Hardware Implementation of Gigabit UDP/IP Stack Based on Spartan-6 FPGA. In: 6th Int Conf Information Technology and Electrical Engineering, pp. 1–6. IEEE. doi: 10.1109/ICITEED.2014.7007955.
- Marvell. 2004. 88E1111 Datasheet, Integrated 10/100/1000 Ultra Gigabit Ethernet Transceiver.
- MathWorks. 2016a. MATLAB and DSP System Toolbox.

MathWorks. 2016b. MATLAB and System Identification Toolbox.

McCleskey M, Kaye W, Mackin D, Beddar S, He Z, and Polf J. 2015. Evaluation of a multistage CdZnTe Compton camera for prompt  $\gamma$  imaging for proton therapy. Nucl Instrum Methods Phys Res A, 785:163–169. doi: 10.1016/j.nima.2015.02.030.

- Meher PK, Chandrasekaran S, and Amira A. 2008. FPGA Realization of FIR Filters by Efficient and Flexible Systolization Using Distributed Arithmetic. IEEE Trans Signal Process, 56(7):3009–3017. doi: 10.1109/TSP.2007.914926.
- Mehrnia A and Willson AN. 2016. FIR Filter Design Using Optimal Factoring: A Walkthrough and Summary of Benefits. IEEE Circuits Syst Mag, 16(1):8–21. doi: 10.1109/MCAS.2015.2510178.
- Meng L and He Z. 2005. Exploring the limiting timing resolution for large volume CZT detectors with waveform analysis. Nucl Instrum Methods Phys Res A, 550(1-2):435–445. doi: 10.1016/j.nima.2005.04.076.
- Meyer-Baese U. 2007. Digital Signal Processing with Field Programmable Gate Arrays. Springer, 3 Edition.
- Moreira P, Serrano J, Wlostowski T, Loschmidt P, and Gaderer G. 2009. White Rabbit: Sub-Nanosecond Timing Distribution over Ethernet. In: Int Symp Precision Clock Synchronization for Measurement, Control and Communication, pp. 1–5. IEEE. doi: 10.1109/ISPCS.2009.5340196.
- Nagy F, Hegyesi G, Valastyán I, Imrek J, Király B, and Molnár J. 2011. Hardware Accelerated UDP/IP Module for High Speed Data Acquisition in Nuclear Detector Systems. In: Nucl Sci Symp Conf Rec, pp. 810–813. IEEE. doi: 10.1109/NSSMIC.2011.6154544.

Nicholson PW. 1974. Nuclear Electronics. John Wiley & Sons Ltd.

- NIST. 2016. XCOM: Photon Cross Sections Database. National Institute of Standards and Technology. [last update: 04.10.2016, retrieved: 03.11.2016]. URL: https://www.nist.gov/pml/xcom-photon-cross-sections-database.
- Nuttall A. 1981. Some windows with very good sidelobe behavior. IEEE Trans Acoust, 29 (1):84–91. doi: 10.1109/TASSP.1981.1163506.
- Okada Y, Takahashi T, Sato G, Watanabe S, Nakazawa K, Mori K, and Makishima K. 2002. CdTe and CdZnTe detectors for timing measurements. IEEE Trans Nucl Sci, 49(4):1986– 1992. doi: 10.1109/TNS.2002.801709.
- Oppenheim AV and Schafer RW. 2007. Discrete-Time Signal Processing. Prentice Hall, 3rd International Edition.
- Park SY and Meher PK. 2014. Efficient FPGA and ASIC Realizations of a DA-Based Reconfigurable FIR Digital Filter. IEEE Trans Circuits Syst II, Exp Briefs, 61(7):511–515. doi: 10.1109/TCSII.2014.2324418.
- Parnham KB, Eissler EE, Jovanovic S, and Lynn KG. 1995. A Study of the Timing Properties of Cd<sub>0.9</sub>Zn<sub>0.1</sub>Te. In: Nucl Sci Symp Conf Rec, pp. 136–138. IEEE. doi: 10.1109/NSSMIC.1995.504194.

- Pausch G, Petzoldt J, Berthel M, Enghardt W, Fiedler F, Golnik C, Hueso-González F, Lentering R, Roemer K, Ruhnau K, Stein J, Wolf A, and Kormoll T. 2016. Scintillator-Based High-Throughput Fast Timing Spectroscopy for Real-Time Range Verification in Particle Therapy. IEEE Trans Nucl Sci, 63(2):664–672. doi: 10.1109/TNS.2016.2527822.
- Peterson SW, Robertson D, and Polf J. 2010. Optimizing a three-stage Compton camera for measuring prompt gamma rays emitted during proton radiotherapy. Phys Med Biol, 55 (22):6841–6856. doi: 10.1088/0031-9155/55/22/015.
- PICMG. 2011. MicroTCA enhancements for rear I/O and precision timing (PICMG-MTCA-4-R1.0). PCI Industrial Computer Manufacturers Group (PICMG).
- Polf JC, Avery S, Mackin DS, and Beddar S. 2015. Imaging of prompt gamma rays emitted during delivery of clinical proton beams with a Compton camera: feasibility studies for range verification. Phys Med Biol, 60(18):7085–7099. doi: 10.1088/0031-9155/60/18/7085.
- Proakis JG. 1995. Digital Signal Processing: Principles, Algorithms and Applications. Prentice Hall International, 3 Edition.
- Proakis JG and Manolakis DK. 2013. Digital Signal Processing. Pearson Education Limited, pearson New International Edition.
- PTCOG. 2016. Particle therapy facilities in operation and in a planning stage. Particle Therapy Co-Operative Group (PTCOG). [last update: October 2016, retrieved: 03.11.2016]. URL: http://www.ptcog.ch.
- Ra S, Kim S, Kim HJ, Park H, Lee S, Kang H, and Doh SH. 2008. Luminescence and Scintillation Properties of a CeBr<sub>3</sub> Single Crystal. IEEE Trans Nucl Sci, 55(3):1221–1224. doi: 10.1109/TNS.2008.920253.
- Ramachers Y and Stewart DY. 2007. Energy resolution improvement in room-temperature CZT detectors. J Instrum, 2(12):P12003. doi: 10.1088/1748-0221/2/12/P12003.
- Redlen Technologies. 2011. M1757 CZT Radiation Detector datasheet.
- Richard MH, Chevallier M, Dauvergne D, Freud N, Henriquet P, Le Foulher F, Letang JM, Montarou G, Ray C, Roellinghoff F, Testa E, Testa M, and Walenta AH. 2009. Design Study of a Compton Camera for Prompt *γ* Imaging During Ion Beam Therapy. In: Nucl Sci Symp Conf Rec, pp. 4172–4175. IEEE. doi: 10.1109/NSSMIC.2009.5402293.
- Richter C, Pausch G, Barczyk S, Priegnitz M, Keitz I, Thiele J, Smeets J, Stappen FV, Bombelli L, Fiorini C, Hotoiu L, Perali I, Prieels D, Enghardt W, and Baumann M. 2016.
  First clinical application of a prompt gamma based in vivo proton range verification system.
  Radiother Oncol, 118(2):232–237. doi: 10.1016/j.radonc.2016.01.004.
- Roellinghoff F, Richard MH, Chevallier M, Constanzo J, Dauvergne D, Freud N, Henriquet P, Le Foulher F, Létang J, Montarou G, Ray C, Testa E, Testa M, and Walenta A. 2011.

Design of a Compton camera for 3D prompt- $\gamma$  imaging during ion beam therapy. Nucl Instrum Methods Phys Res A, 648:S20–S23. doi: 10.1016/j.nima.2011.01.069.

- Roemer K, Pausch G, Bemmerer D, Berthel M, Dreyer A, Golnik C, Hueso-González F, Kormoll T, Petzoldt J, Rohling H, Thirolf P, Wagner A, Wagner L, Weinberger D, and Fiedler F. 2015. Characterization of scintillator crystals for usage as prompt gamma monitors in particle therapy. J Instrum, 10(10):P10033. doi: 10.1088/1748-0221/10/10/P10033.
- Rohling H. 2015. Simulation studies for the in-vivo dose verification of particle therapy. Technische Universität Dresden, Dissertation.
- Rossi L, Fischer P, Rohe T, and Wermes N. 2006. Pixel Detectors: From Fundamentals to Applications. Springer.
- Sasi A, Saravanan S, Pandian SR, and Sundaram RS. 2013. UDP/IP stack in FPGA for hard real-time communication of Sonar sensor data. In: Ocean Electronics (SYMPOL), pp. 1–6. IEEE. doi: 10.1109/SYMPOL.2013.6701940.
- Scharnagl C. 2015. Entwurf eines numerischen Algorithmus als digitale Schaltung und Vergleich der Implementierungsmöglichkeiten in einem FPGA. Dresden University of Applied Sciences, Diploma thesis.
- Schumann A, Petzoldt J, Dendooven P, Enghardt W, Golnik C, Hueso-González F, Kormoll T, Pausch G, Roemer K, and Fiedler F. 2015. Simulation and experimental verification of prompt gamma-ray emissions during proton irradiation. Phys Med Biol, 60(10):4197–4207. doi: 10.1088/0031-9155/60/10/4197.
- Shah K, Glodo J, Higgins W, van Loef E, Moses W, Derenzo S, and Weber M. 2005. CeBr<sub>3</sub> scintillators for gamma-ray spectroscopy. IEEE Trans Nucl Sci, 52(6):3157–5159. doi: 10.1109/TNS.2005.860155.
- Shen Z. 2010. Improving FIR Filter Coefficient Precision [DSP Tips & Tricks]. IEEE Signal Process Mag, 27(4):120–124. doi: 10.1109/MSP.2010.936777.
- Smeets J. 2012. Prompt gamma imaging with a slit camera for real time range control in particle therapy. Université Libre de Bruxelles, Dissertation.
- Spieler H. 2005. Semiconductor Detector Systems. Oxford University Press.
- Stein J, Scheuer F, Gast W, and Georgiev A. 1996. X-ray detectors with digitized preamplifiers. Nucl Instrum Methods Phys Res B, 113(1-4):141–145. doi: 10.1016/0168-583X(95)01417-9.
- Stein J, Georgiev A, Büchner A, and Gast W. 1994. Circuit arrangement for the digital processing of semiconductor detector signals. United States Patent 5,307,299.
- Stein J, Georgiev A, Büchner A, and Gast W. 1997. Schaltungsanordnung für die digitale Verarbeitung von Halbleiterdetektorsignalen. Europäische Patentschrift EP0550830B1.

- Taya T, Kataoka J, Kishimoto A, Iwamoto Y, Koide A, Nishio T, Kabuki S, and Inaniwa T. 2016. First demonstration of real-time gamma imaging by using a handheld Compton camera for particle therapy. Nucl Instrum Methods Phys Res A, 831:355–361. doi: 10.1016/j.nima.2016.04.028.
- Taylor AP. 2012. Ins and Outs of Digital Filter Design and Implementation. Xcell, 78:36-41.
- Texas Instruments. 2004. DP83865 Gig PHYTER V 10/100/1000 Ethernet Physical Layer, Datasheet SNLS165B.
- Texas Instruments. 2015. OPA657 1.6-GHz, Low-Noise, FET-Input Operational Amplifier, Datasheet SBOS197F.
- Uchida T. 2008. Hardware-Based TCP Processor for Gigabit Ethernet. IEEE Trans Nucl Sci, 55(3):1631–1637. doi: 10.1109/TNS.2008.920264.
- Vaska P, Bolotnikov A, Carini G, Camarda G, Pratte JF, Dilmanian FA, Park SJ, and James RB. 2005. Studies of CZT for PET applications. In: Nucl Sci Symp Conf Rec, pp. 2799– 2802. IEEE. doi: 10.1109/NSSMIC.2005.1596916.
- Verburg JM and Seco J. 2014. Proton range verification through prompt gamma-ray spectroscopy. Phys Med Biol, 59(23):7089–7106. doi: 10.1088/0031-9155/59/23/7089.
- Verger L, Ouvrier-Buffet P, Mathy F, Montemont G, Picone M, Rustique J, and Riffard C. 2005. Performance of a new CdZnTe portable spectrometric system for high energy applications. IEEE Trans Nucl Sci, 52(5):1733–1738. doi: 10.1109/TNS.2005.856716.
- Verger L, Gros d'Aillon E, Monnet O, Montémont G, and Pelliciari B. 2007. New trends in  $\gamma$ -ray imaging with CdZnTe/CdTe at CEA-Leti. Nucl Instrum Methods Phys Res A, 571 (1-2):33–43. doi: 10.1016/j.nima.2006.10.023.
- Wahl CG, Kaye WR, Wang W, Zhang F, Jaworski JM, King A, Boucher YA, and He Z. 2015. The Polaris-H imaging spectrometer. Nucl Instrum Methods Phys Res A, 784:377–381. doi: 10.1016/j.nima.2014.12.110.
- Wermes N. 2006. Pixel Vertex Detectors. arXiv:physics/0611075 [physics.ins-det].
- Wohsmann J. 2014. Entwurf und Implementierung eines digitalen Signalverarbeitungsalgorithmus für die Ausleseelektronik eines CdZnTe-Pixeldetektors. Dresden University of Applied Sciences, Diploma thesis.
- Xilinx. 2005. DSP: Designing for Optimal Results. Xcell Publications.
- Xilinx. 2014. 7 Series DSP48E1 Slice, UG479 (v1.8).
- Xilinx. 2015a. 7 Series FPGAs GTX/GTH Transceivers, UG476.
- Xilinx. 2015b. 1G/2.5G Ethernet PCS/PMA or SGMII v15.0, PG047.
- Xilinx. 2015c. FIR Compiler v7.2, PG149.

- Yuan J, Feng QY, and Wang D. 2012. Design of High-Precision FIR Filter Based on Verilog HDL. Adv Mat Res, 433-440:5198–5202. doi: 10.4028/www.scientific.net/AMR.433-440.5198.
- Zatrepalek R. 2012. Using FPGAs to Solve Tough DSP Design Challenges. Xcell, 78: 42–47.
- Zhou S and Yao L. 2014. Gigabit Ethernet Data Transfer Based on FPGA. In: Trustworthy Computing and Services: Int Conf ISCTCS, pp. 290–296. Springer Berlin Heidelberg. doi: 10.1007/978-3-662-43908-1\_37.

# Danksagung

Ich danke Herrn Prof. Dr. Wolfgang Enghardt für die Überlassung des Themas und die Betreuung der Arbeit.

Bei Herrn Prof. Dr. Uwe Hampel bedanke ich mich für die Übernahme des Zweitgutachtens.

Ein besonderer Dank gilt Herrn Prof. Dr. Peter Kaever für die fachliche Betreuung der Arbeit sowie die Freiheiten in Bezug auf die Umsetzung des Themas. Dir mir gebotenen Möglichkeiten zur Erarbeitung der Forschungsergebnisse sind von unschätzbarem Wert.

Danken möchte ich außerdem Herrn Prof. Dr. Jens Schönherr für die konstruktive Zusammenarbeit und die Betreuung und Begutachtung der Diplomarbeiten.

Ich danke Bert Lange herzlichst für die tägliche Unterstützung sowie den stetigen und ehrlichen Gedankenaustausch.

Mein Dank gilt auch Marc Berthel für die nächtliche Unterstützung während der Experimente mit den Teilchenbeschleunigern und die Hilfe beim Aufbau und Test der Detektoren. Außerdem danke ich Dr. Thomas Kormoll dafür, dass er mich über CZT Detektoren unterrichtet hat.

Ebenfalls möchte ich mich bei Dr. Christian Golnik, Dr. Fernando Hueso-González und Katja Römer für die technische Unterstützung während der Strahlzeiten bedanken.

Ich danke auch Dr. Andree Büchner und Timo Kirschke für Ihre wertvollen Erfahrungen in der analogen Schaltungstechnik.

Meinen Eltern Dr. Holger Födisch und Astrid Födisch danke ich für sämtliche Entlastungen, die ich in den letzten Jahren annehmen durfte.

Mein größter Dank gilt meiner (zukünftigen) Frau Doreen. Dafür, dass du mir Rückhalt und Verständnis entgegengebracht hast, und immer auf mich gewartet hast. Ich danke Dir auch dafür, dass Du Dich so liebevoll um unsere Kinder Amelie und Ferdinand sorgst.

### Technische Universität Dresden Medizinische Fakultät Carl Gustav Carus Promotionsordnung vom 24.10.2014

### Erklärungen zur Eröffnung des Promotionsverfahrens

- 1. Hiermit versichere ich, dass ich die vorliegende Arbeit ohne unzulässige Hilfe Dritter und ohne Benutzung anderer als der angegebenen Hilfsmittel angefertigt habe; die aus fremden Quellen direkt oder indirekt übernommenen Gedanken sind als solche kenntlich gemacht.
- Bei der Auswahl und Auswertung des Materials sowie bei der Herstellung des Manuskripts habe ich Unterstützungsleistungen von folgenden Personen erhalten: Prof. Dr. W. Enghardt, Prof. Dr. P. Kaever
- 3. Weitere Personen waren an der geistigen Herstellung der vorliegenden Arbeit nicht beteiligt. Insbesondere habe ich nicht die Hilfe eines kommerziellen Promotionsberaters in Anspruch genommen. Dritte haben von mir weder unmittelbar noch mittelbar geldwerte Leistungen für Arbeiten erhalten, die im Zusammenhang mit dem Inhalt der vorgelegten Dissertation stehen.
- 4. Die Arbeit wurde bisher weder im Inland noch im Ausland in gleicher oder ähnlicher Form einer anderen Prüfungsbehörde vorgelegt.
- 5. Die Inhalte dieser Dissertation wurden in folgender Form veröffentlicht:

[1] Födisch P, Lange B, and Kaever P. 2013. Eine Ausleseelektronik für CZT-Detektoren mit dem RENA-3 IC von Nova R&D. In: 104. Tag Studiengr Elektr Instrum, pp. 135–143. DESY

[2] Födisch P, Sandmann J, Lange B, and Kaever P. 2014. Taktsynchronisierung und Zeitmessung in einem verteilten Datenerfassungssystem. In: 105. Tag Studiengr Elektr Instrum, pp. 238–242. DESY

[3] Födisch P, Lange B, and Kaever P. 2015. Ein VHDL basierter Gigabit Ethernet Protokollstapel für FPGAs. In: 106. Tag Studiengr Elektr Instrum, pp. 52–76. DESY

[4] Födisch P, Berthel M, Lange B, Kirschke T, Enghardt W, and Kaever P. 2016a. Charge-sensitive front-end electronics with operational amplifiers for CdZnTe detectors. J Instrum, 11(09):T09001. doi: 10.1088/1748-0221/11/09/T09001

[5] Födisch P, Lange B, Sandmann J, Büchner A, Enghardt W, and Kaever P. 2016b. A synchronous Gigabit Ethernet protocol stack for high-throughput UDP/IP applications. J Instrum, 11(01):P01010. doi: 10.1088/1748-0221/11/01/P01010

[6] Födisch P, Wohsmann J, Lange B, Schönherr J, Enghardt W, and Kaever P. 2016c.
Digital high-pass filter deconvolution by means of an infinite impulse response filter.
Nucl Instrum Methods Phys Res A, 830:484–496. doi: 10.1016/j.nima.2016.06.019

[7] Födisch P, Bryksa A, Lange B, Enghardt W, and Kaever P. 2016d. Implementing High-Order FIR Filters in FPGAs. arXiv:1610.03360 [cs.DC]

- 6. Ich bestätige, dass es keine zurückliegenden erfolglosen Promotionsverfahren gab.
- 7. Ich bestätige, dass ich die Promotionsordnung der Medizinischen Fakultät der Technischen Universität Dresden anerkenne.
- 8. Ich habe die Zitierrichtlinien für Dissertationen an der Medizinischen Fakultät der Technischen Universität Dresden zur Kenntnis genommen und befolgt.
- Ich bin mit den "Richtlinien zur Sicherung guter wissenschaftlicher Praxis, zur Vermeidung wissenschaftlichen Fehlverhaltens und f
  ür den Umgang mit Verst
  ö
  ßen" der Technischen Universit
  ät Dresden einverstanden.

Hiermit bestätige ich die Einhaltung der folgenden aktuellen gesetzlichen Vorgaben im Rahmen meiner Dissertation (Nicht angekreuzte Punkte sind für meine Dissertation nicht relevant.)

- das zustimmende Votum der Ethikkommission bei Klinischen Studien, epidemiologischen Untersuchungen mit Personenbezug oder Sachverhalten, die das Medizinproduktegesetz betreffen
- □ die Einhaltung der Bestimmungen des Tierschutzgesetzes
- □ die Einhaltung des Gentechnikgesetzes
- ☑ die Einhaltung von Datenschutzbestimmungen der Medizinischen Fakultät und des Universitätsklinikums Carl Gustav Carus.