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The role of the massive photon decay channel for the neutrino cooling of neutron stars

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Abstract

The rôle of the massive photon decay via intermediate states of electron–electron-holes and proton–proton-holes into neutrino–anti-neutrino pairs in the course of neutron star cooling is investigated. These reactions may be operative in hot neutron stars in the region of proton pairing. The corresponding contribution to the neutrino emissivity is calculated. It varies with the temperature as $T^{3/2}e^{-m_\gamma/T}$ for $T < m_\gamma$, where m_γ is an effective photon mass in superconducting matter. Estimates show that this process appears as strong cooling channel of neutron stars at temperatures $T \simeq (10^9 - 10^{10})$ K.

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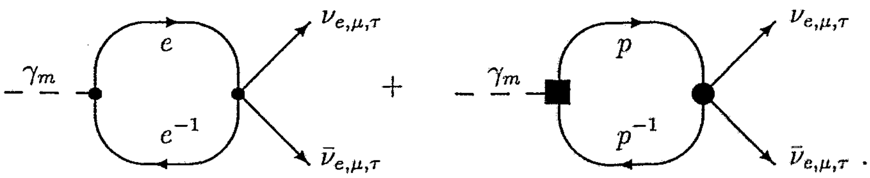
1. The EINSTEIN, EXOSAT and ROSAT observatories have measured surface temperatures of certain neutron stars and put upper limits on the surface temperatures of some other neutron stars (cf. [1, 2, 3] and further references therein). The data for the supernova remnants in 3C58, Crab, RCW103 are related to rather slow cooling, while the data for Vela, PSR2334+61, PSR0656+14 and Geminga point to an essentially more rapid cooling.

In the standard scenario of the neutron star cooling the most important channel belongs to the modified Urca process $n\,n \rightarrow n\,p\,e^-\,\bar{\nu}$ with an emissivity ε_ν^{FM} calculate, e.g., by Friman and Maxwell [4]. In ref. [4] the nucleon - nucleon interaction is treated within a model with free one-pion exchange (plus slight modifications). In a system with nucleon pairing this emissivity is suppressed by the factor $\exp(-(\Delta_n+\Delta_p)/T)$ [2], where Δ_n and Δ_p are the respective neutron and proton gaps determined by $\Delta_i(T) = \Delta_i(0)\,(T_{c,i}-T)\,T_{c,i}^{-1}\,\Theta(T_{c,i}-T)$, (here $\Theta(x)$ is the step-function; $i = \{p, n\}$, and $T_{c,i}$ is the corresponding critical temperature for nucleon pairing).

At temperatures $T \ll T_{c,p}, T_{c,n}$ the cooling is determined by the photon radiation from the neutron star surface.

We suggest here that the decay processes of massive photons (γ_m) via the electron–electron-hole (ee^{-1}) and the proton–proton-hole ($p\,p^{-1}$) intermediate states to neutrino - antineutrino pairs, $\gamma_m \rightarrow e\,e^{-1} + p\,p^{-1} \rightarrow \nu_i\bar{\nu}_i$, $i = \{e, \nu, \tau\}$, might be operative in hot neutron stars in the region of proton pairing $T < T_{c,p}$. These processes are determined by the diagrams



(1)

Fat vertices in the nucleon diagrams (1) include the nucleon-nucleon correlations.

In a Fermi system with pairing, besides the graphs (1), there are also diagrams with anomalous Green's functions of protons [5]. However, their contribution to the corresponding matrix elements is as small as $\Delta_p/\epsilon_{Fp} \ll 1$ for $T < T_{c,p} \ll \epsilon_{Fp}$ (ϵ_{Fp} is the proton Fermi energy). Within this accuracy one can drop the anomalous diagrams and use for protons in (1) the Green's functions for the normal Fermi liquid.

The contribution of the massive photon decay via the electron–electron-hole intermediate states (i.e., the first diagram in (1)) has been calculated by several authors (cf. [6] for further references) for the case of the electron gas in white dwarfs and neutron star crusts. In an electron plasma the photon acquires an effective in-medium mass which is equal to the electron plasma frequency $\omega_{Pl} \simeq 2\,e\,(3\sqrt{\pi}\rho)^{1/3}$, where e is the electron charge, and ρ_e denotes the electron density (we employ units with $\hbar = c = 1$). Therefore, the contribution to the emissivity of the mentioned process is suppressed by the factor $\exp(-\omega_{Pl}/T)$. In white dwarfs and neutron star crusts the electron density is not too large and the process is effective. In neutron star interiors the electron density is equal to the proton density ρ_p, due to the electroneutrality, which is

$$\frac{\rho_p}{\rho_0} \simeq 0.016 \left(\frac{\rho}{\rho_0}\right)^2, \tag{2}$$

where $\rho_0 \simeq 0.17$ fm^{-3} stands for the nuclear saturation density, and we used the values [4] of the neutron and proton Fermi momenta, $p_{Fn} \simeq 340(\rho/\rho_0)$ MeV and $p_{Fn} \simeq 85(\rho/\rho_0)^{2/3}$ MeV. Thus, at densities typical for the neutron star interiors, the value of the electron plasma frequency is large, i.e., $\omega_{Pl}(\rho \sim \rho_0) \sim 8$ MeV, and at temperatures $T \ll \omega_{Pl}$ the process $\gamma_m \to e\,e^{-1} \to \nu_i\bar{\nu}_i$ is strongly suppressed.

Oppositely, in a superconducting medium, formed in neutron stars at $T < T_{c,p}$, photons acquire the effective in-medium mass due to the Higgs - Meissner effect [5]

$$m_\gamma(T) = \sqrt{\frac{4\pi e^2 \rho_p^*(T)}{m_p^*}}, \quad T < T_{c,p}, \tag{3}$$

and obey the dispersion relation $\omega = \sqrt{\vec{k}^{\,2} + m_\gamma^2}$, where ω and $\vec{k}$ are respectively the frequency and the momentum of the photon, and m_p^* stands for the effective in-medium proton mass. The quantity $\rho_p^*(T) = \rho_p(T_{c,p} - T)/T_{c,p}$ denotes the paired proton density. Supposing $m_p^*(\rho_0) \simeq 0.8 m_N$ (with m_N as the free nucleon mass) with eqs. (2,3) we estimate $m_\gamma(\rho = \rho_0, T) \simeq 1.6\sqrt{(T_{c,p} - T)/T_{c,p}}$ MeV $\ll \omega_{Pl}(\rho \sim \rho_0) \sim 8$ MeV.

Due to the rather small effective photon mass in superconducting neutron star matter at $T < T_{c,p} \ll \omega_{Pl}$ one may expect a corresponding increase of the contribution of the processes (1) to the neutrino emissivity. We shall calculate the contribution of these processes to the neutrino emissivity ε_ν^γ and compare the results with the emissivity of the modified Urca process ε_γ^{FM} [4] and with the photon emissivity ε_γ^s from the star surface. We show that the processes (1) may play an important rôle in the course of neutron star cooling at temperatures $T \simeq (10^9 - 10^{10})$ K.

2. The matrix element of the diagrams (1) for the ith neutrino species ($i = \{\nu_e, \nu_\mu, \nu_\tau\}$) reads

$$\mathcal{M}^{(i)a} = -i\sqrt{4\pi}e\frac{G}{2\sqrt{2}}\Gamma_\gamma \varepsilon_\mu^a \left(T_p^{(i)\,\mu\rho} - T_e^{(i)\,\mu\rho}\right) l_\rho, \tag{4}$$

where

$$T_j^{(i)\,\mu\rho} = (-1)\,\mathrm{Tr}\int \frac{d^4p}{(2\pi)^4}\left\{\gamma^\mu i\hat{G}_j(p) W_j^{(i)\rho} i\hat{G}_j(p+k)\right\}, \quad j = \{e, p\}, \tag{5}$$

and

$$\hat{G}_j(p) = (\hat{p} + m_j)\, G_j(p) = (\hat{p} + m_j) \left\{ \frac{1}{p^2 - m_j^2} + 2\pi\, i\, n_j(p)\, \delta(p^2 - m_j^2)\Theta(p_0) \right\}$$

is the in-medium electron (proton) Green's function, and $n_j(p) = \Theta(p_{Fj} - p)$. ε_μ^a is the corresponding polarization four-vector of the massive photon with three polarization states in superconducting matter. The factor Γ_γ takes into account the nucleon-nucleon correlations in the photon vertex. The quantity $G = 1.17 \cdot 10^{-5}$ GeV^{-2} is the Fermi constant of the weak interaction. Above, l_ρ denotes the neutrino weak current. The electron and proton weak currents are determined by

$$W_e^{(i)\rho} = \gamma^\rho (c_V^{(i)} - c_A^{(i)} \gamma_5), \qquad W_p^\rho = \gamma^\rho (\kappa_{pp} - g_A \gamma_{pp} \gamma_5), \tag{6}$$

where $c_V^{(\nu_e)} = c_V^{(+)} = 1 + 4\sin^2\theta_W \simeq 1.92$ and $c_V^{(\nu_\mu)} = c_V^{(\nu_\tau)} = c_V^{(-)} = 1 - 4\sin^2\theta_W \simeq 0.08$. θ_W stands for the Weinberg angle, and $c_A^{(\nu_e)} = -c_A^{(\nu_\mu,\nu_\tau)} = 1$. The proton coupling is corrected by the nucleon-nucleon correlations, i.e., by factors κ_{pp} and γ_{pp} [7].

By integrating in eq. (5) over the energy variable, we obtain for the i-th neutrino species

$$-\, i \left(T_p^{(i)\mu\rho} - T_e^{(i)\mu\rho} \right) = \tau_t^{(i)}\, P^{\mu\rho} + \tau_l^{(i)}\, F^{\mu\rho} + \tau_5^{(i)}\, P_5^{\mu\rho}, \tag{7}$$

$$P^{\mu\rho} = (g^{\mu\rho} - \frac{k^\mu k^\rho}{k^2} + F^{\mu\rho}), \quad F^{\mu\rho} = \frac{j^\mu j^\rho}{k^2[(k\cdot u)^2 - k^2]}, \quad P_5^{\mu\rho} = \frac{i}{\sqrt{k^2}} \varepsilon^{\mu\rho\delta\lambda} k_\delta u_\lambda, \tag{8}$$

where $j^\mu = (k\cdot u)k^\mu - u^\mu k^2$, $k^\mu = (\omega, \vec{k})$, $k^2 = k_\mu k^\mu = \omega^2 - \vec{k}^2$. The four-velocity of the medium u^μ is introduced for the sake of a covariant notation. The transverse (τ_t), longitudinal (τ_l) and axial (τ_5) components of the tensors in eq. (7) render

$$\tau_t^{(i)} = \tau_{te}^{(i)} - \tau_{tp}^{(i)} = 2c_V^{(i)}(A_e + k^2 B_e) - 2c_V^{(-)} R_\kappa (A_p + k^2 B_p), \tag{9}$$

$$\tau_l^{(i)} = \tau_{le}^{(i)} - \tau_{lp}^{(i)} = 4\, k^2 [c_V^{(i)} B_e - c_V^{(-)} R_\kappa B_p], \quad \tau_5^{(i)} = \tau_{5e}^{(i)} - \tau_{5p}^{(i)} = (k^2)^{3/2} [c_A^{(i)} C_e - g_A \gamma_{pp} C_p],$$

where $R_\kappa = \kappa_{pp}/c_V^{(-)}$, and

$$A_j = \int \frac{d^3p}{(2\pi)^3} \frac{n_j(p)}{E_p^{(j)}} + \frac{k^2}{2}\left(1 + \frac{k^2}{2m_j^2}\right) m_j C_j, \; B_j = \int \frac{d^3p}{(2\pi)^3} \frac{n_j(p)}{2E_p^{(j)}} \frac{1 - \frac{(\vec{p}\vec{k})^2}{E_p^{(j)2}\vec{k}^2}}{\left(\omega - \frac{\vec{p}\vec{k}}{E_p^{(j)}}\right)^2 - \frac{k^4}{4E_p^{(j)4}}}, \tag{10}$$

$$C_j = \int \frac{d^3p}{(2\pi)^3} n_j(p) \frac{m_j}{E_p^{(j)3}} \left[\left(\omega - \frac{\vec{p}\vec{k}}{E_p^j} \right)^2 - \frac{k^4}{4E_p^{(j)4}} \right]^{-1}, \qquad E_p^{(j)} = \sqrt{m_j^2 + \vec{p}^2}.$$

Here we note that the contribution of the axial component τ_5 to the resulting neutrino emissivity is small ($\tau_5/\tau_t \sim m_\gamma^2\tau_5/\omega^2\tau_l \sim m_\gamma/m_N^*$ for protons and $\sim (m_\gamma m_e/p_{Fe}^2)\ln(p_{Fe}/m_e)$ for electrons), so that it will be dropped below.

The squared matrix element (4) for a certain neutrino species, summed over the lepton spins, and averaged over the three photon polarizations, may be cast into the following form

$$\langle\sum|\mathcal{M}^{(i)}|^2\rangle = \frac{4}{3}\pi e^2G^2 \left[\tau_t^{(i)2}\left(2\omega_1\omega_2 + 2\frac{(\vec{k}\vec{q}_1)(\vec{k}\vec{q}_2)}{\vec{k}^2}\right) - \tau_l^{(i)2}\left(\omega_1\omega_2 + \vec{q}_1\vec{q}_2 - 2\frac{(k\cdot q_1)(k\cdot q_2)}{k^2} - 2\frac{(\vec{k}\vec{q}_1)(\vec{k}\vec{q}_2)}{\vec{k}^2}\right)\right], \tag{11}$$

where $(k\cdot q_{1,2}) = \omega\omega_{1,2} - (\vec{k}\vec{q}_{1,2})$, and $\omega_{1,2}$ and $\vec{q}_{1,2}$ denote the frequencies and the momenta of the neutrino and anti-neutrino. We have also used that $\mathrm{Tr}\{l^\mu l^\nu\} = 8[q_1^\mu q_2^\nu + q_2^\mu q_1^\nu - g^{\mu\nu}(q_1\cdot q_2) - 4i\epsilon^{\mu\nu\lambda\rho}q_{1\lambda}q_{2\rho}]$.

3. The emissivity of the processes (1) is given by

$$\varepsilon_\nu^\gamma = \int \frac{d^3k}{(2\pi)^3 2\omega}\frac{d^3q_1}{(2\pi)^3 2\omega_1}\frac{d^3q_2}{(2\pi)^3 2\omega_2}\frac{\omega_1+\omega_2}{e^{\frac{\omega_1+\omega_2}{T}}-1}\sum_{i=\nu_e,\nu_\mu,\nu_\tau}\langle\sum|\mathcal{M}^{(i)}|^2\rangle(2\pi)^4\delta^4(k-q_1-q_2). \tag{12}$$

Substituting eq. (11) into eq. (12), we obtain eventually after some integrations

$$\varepsilon_\nu^\gamma = \frac{T^5}{9(2\pi)^3}\pi e^2G^2\alpha^2 I,\quad I = \int_\alpha^\infty \frac{d\xi\xi}{e^\xi - 1}\sqrt{\xi^2-\alpha^2}\left[\tau_t^2(\frac{\alpha^2}{\xi^2}) + \tau_l^2(\frac{\alpha^2}{\xi^2})\right], \tag{13}$$

where $\alpha = \frac{m_\gamma}{T}$ and

$$\tau_t^2(x) \approx 4\sum_{i=\nu_e,\nu_\mu,\nu_\tau}\left[c_V^{(-)}R_\kappa\frac{\rho_p}{2m_p^*}(1+x) - c_V^{(i)}(\frac{3}{8\pi}\rho_p)^{2/3}(1+\frac{x}{2})\right]^2, \tag{14}$$

$$\tau_l^2(x) \approx 4x^2\sum_{i=\nu_e,\nu_\mu,\nu_\tau}\left[c_V^{(-)}R_\kappa\frac{\rho_p}{2m_p^*} - c_V^{(i)}(\frac{3}{8\pi}\rho_p)^{2/3}\right]^2.$$

Some numerically small terms are dropped in eq. (14).

The integral I can be calculated analytically in two limiting cases $\alpha \ll 1$ and $\alpha \gg 1$,

$$I(\alpha\gg 1) \approx \frac{\sqrt{2\pi}}{2}\alpha^{3/2}(1+\frac{3}{2\alpha})\,e^{-\alpha}\,[\tau_l^2(1)+\tau_t^2(1)], \tag{15}$$

$$I(\alpha\ll 1) \approx 2\zeta(3)\,[\tau_l(0)+\tau_t^2(0)],\quad \zeta(3)\simeq 1.202.$$

Thus, combining eqs. (2,13-15), we obtain an estimate for the emissivity of the reactions (1) (we present here the result for $m_\gamma > T$ and for three neutrino species):

$$\varepsilon_\nu^\gamma \approx 2.6\cdot 10^{25}\,T_9^{3/2}\,e^{-\frac{m_\gamma}{T}}\left(\frac{m_\gamma}{\mathrm{MeV}}\right)^{7/2}\left(\frac{\rho}{\rho_0}\right)^{8/3}\left(1+\frac{3}{2}\frac{T}{m_\gamma}\right)[1+\eta]\,\frac{\mathrm{erg}}{\mathrm{cm^3sec^3}}, \tag{16}$$

$$\eta = 0.0003\, R_\kappa^2 \left(\frac{m_p}{m_p^*}\right)^2 \left(\frac{\rho}{\rho_0}\right)^{4/3} - 0.035\, R_\kappa \left(\frac{m_p}{m_p^*}\right) \left(\frac{\rho}{\rho_0}\right)^{2/3} . \tag{17}$$

Here the quantity T_9 stands for the temperature measured in 10^9 K. The unity in squared brackets of eq. (16) corresponds to the electron–electron-hole diagram, whereas the factor η is related to the proton–proton-hole (first term in eq. (17)) and the interference diagrams (second term in eq. (17)).

The emissivity eq. (16) varies with the temperature as $T^{3/2}\exp(-m_\gamma/T)$, whereas the emissivity of the modified Urca process varies as $T^8 \exp(-(\Delta_p + \Delta_n)/T)$ in the region of proton ($\Delta_p \neq 0$) and neutron ($\Delta_n \neq 0$) pairing. Hence, one can expect that the process $\gamma_m \to \nu\bar{\nu}$ will dominate at comparatively low temperatures, when $\Delta_p(T) + \Delta_n(T) - m_\gamma(T) > 0$ and $T < T_{c,p}$.

4. In order to perform quantitative estimates we need the values of the nucleon-nucleon correlation factors κ_{pp} and Γ_γ. According to ref. [7] one has

$$\kappa_{pp} = c_V^{(-)} - 2 f_{np} C_0 A_{nn} \Gamma(f_{nn}), \tag{18}$$

where $f_{np} \simeq -0.75$ and $f_{nn} \simeq 1.25$ stand for the constants in the theory of finite Fermi systems [8, 7]; $C_0^{-1} = m_n^* p_{Fn}/\pi^2$ is the density of states at the Fermi surface; A_{nn} is the neutron–neutron-hole loop,

$$C_0\, A_{nn} = i C_0 \int \frac{d^3p}{(2\pi)^4}\, G_n(p+k)\, G_n(p) \,\approx\, \frac{p_{Fn}^2 k^2}{6 m_n^* \omega^2}, \tag{19}$$

for values of $\omega \gg |\vec{k}| p_{Fn}/m_n^*$ of interest, and $\Gamma^{-1}(f_{nn}) = 1 - 2 f_{nn} C_0 A_{nn}$.

We note that the second term in eq. (18) is not proportional to a small factor $c_V^{(-)}$, because the nucleon - nucleon correlations also allow for the radiation of $\nu\bar{\nu}$-pairs from the nn^{-1} loop. Numerical estimates of the ratio R_κ are as follows: for $\alpha \gg 1$ $R_\kappa \simeq 1.6$ for $\rho = \rho_0$, $m_n^*(\rho_0) \simeq 0.8 m_n$ and $R_\kappa \simeq 2.1$ for $\rho = 2\rho_0$, $m_n^*(2\rho_0) \simeq 0.7 m_n$; for $\alpha \ll 1$ $R_\kappa \simeq 1$ and correlation effects are negligible. The factor Γ_γ is approximately unity, since the correction is proportional to a small proton–proton-hole loop factor (A_{pp}) being suppressed at comparatively small proton densities.

With these estimates we observe that the main contribution to the neutrino emissivity is given by the electron–electron-hole processes.

5. The ratio of the emissivity ε_ν^γ (16) to the emissivity ε_ν^{FM} of the modified Urca process yields

$$\begin{aligned} R_{FM} &= \frac{\varepsilon_\nu^\gamma}{\varepsilon_\nu^{FM}} \qquad (20) \\ &\approx 1.5 \cdot 10^4\, T_9^{-13/2}\, e^{\frac{\Delta_n + \Delta_p - m_\gamma}{T}} \left(\frac{m_\gamma}{\mathrm{MeV}}\right)^{7/2} \left(1 + \frac{3}{2}\frac{T}{m_\gamma}\right) \left(\frac{\rho}{\rho_0}\right)^2 \left(\frac{m_n^3\, m_p}{m_n^{*3}\, m_p^*}\right) [1+\eta]. \end{aligned}$$

For further estimates we need the values of the neutron and proton gaps, which unfortunately are essentially model dependent. E.g., the evaluation in ref. [9] yields $\Delta_n(0) \simeq 8.4\,T_{c,n} \simeq 0.6$ MeV, $T_{c,n} \simeq 0.07$ MeV for $3P_2$ neutron pairing at $\rho = \rho_0$, and $\Delta_p(0) \simeq 1.76\,T_{c,p} \simeq 3$ MeV, $T_{c,p} \simeq 1.7$ MeV for $1S$ proton pairing, while ref. [10] uses $\Delta_n(0) \simeq 2.1$ MeV, $T_{c,n} \simeq 0.25$ MeV and $\Delta_p(0) \simeq 0.7$ MeV, $T_{c,p} \simeq 0.4$ MeV for $\rho = \rho_0$. Employing these estimates of the zero-temperature gaps, their temperature dependence and the photon effective mass, we obtain from eq. (20) the temperature dependence of the ratio R_{FM}.

In order to find the lower temperature limit, at which the processes $\gamma_m \to \nu\bar{\nu}$ are still operative, we need to compare the value ε_ν^γ with the value of the photon emissivity from the neutron star surface, $\varepsilon_\gamma^s = 3\sigma T_s^4/R$, where σ is the Stefan-Boltzmann constant, T_s denotes the surface temperature of the star and R stands for the star radius. By employing a relation [11] between the surface and interior temperatures, we obtain

$$R_\gamma = \frac{\varepsilon_\nu^\gamma}{\varepsilon_\gamma^s} \approx 1.2 \cdot 10^9 T_9^{-0.7} e^{-\frac{m_\gamma}{T}} \left(\frac{m_\gamma}{\mathrm{MeV}}\right)^{7/2} \left(1 + \frac{3}{2}\frac{T}{m_\gamma}\right) \left(\frac{\bar{\rho}}{\rho_0}\right)^{8/3} [1+\eta], \tag{21}$$

where the star radius and the mass are supposed to be respectively 10 km and $1.4 M_\odot$, with $M_\odot$ as solar mass, and $\bar{\rho}$ is some averaged value of the density in the neutron star interior.

The ratios R_{FM} and R_γ are plotted as a function of the temperature in Fig. 1 for the both mentioned above parameter choices. We see that the processes (1) are operative in the temperature range $1\cdot 10^9$ K$\lesssim T \lesssim 8\cdot 10^9$ K for the parameter choice [9] and $1\cdot 10^9$ K$\lesssim T \lesssim 4\cdot 10^9$ K for the parameters of ref. [10].

6. A few remarks are in order:

(i) Photons with electron plasma frequency ω_{Pl} may also decay into neutrino pairs. However, the corresponding contribution to the emissivity of the neutron star interior is negligible compared to that for the modified Urca process.

(ii) The processes (1) may also occur in a charged-pion (or kaon) condensate state, however, they are suppressed due to the large value of the effective photon mass[4] $m_\gamma \simeq \sqrt{8\pi e^2 \varphi_c^2} \simeq 6$ MeV for the condensate field $\varphi_c \simeq 0.1 m_\pi \simeq 14$ MeV.

(iii) Deriving the above used value ε_ν^{FM}, one describes the nucleon-nucleon interaction essentially by the free one-pion exchange. However, in reality at $\rho > (0.5 - 1)\rho_0$ the total nucleon-nucleon interaction does not reduce to the free one-pion exchange because of the strong polarization of the medium, by which an essential part belongs to the in-medium pionic excitations [7, 12, 13, 14]. Occurring in the intermediate states of the reaction, the in-medium pions

[4]In this estimate the peculiarities of the condensate with the non-vanishing momentum [12] are for simplicity ignored.

can also decay into $e\bar{\nu}$, or first into a nucleon–nucleon-hole, which then radiates $e\bar{\nu}$, thereby substantially increasing the resulting emissivity. The other reaction channels [7, 14] in the superfluid phase with paired nucleons $n \rightarrow n_{pair}\nu\bar{\nu}$ and $p \rightarrow p_{pair}\nu\bar{\nu}$ give rise to even a larger contribution to the emissivity than that of the modified Urca process. Above we compare the value $\varepsilon_{\nu}^{\gamma}$ with the value ε_{ν}^{FM} just because the latter is used in the standard scenarios of cooling, while the mentioned in-medium processes are not yet included in computer simulation code.

7. In summary, the processes $\gamma_m \rightarrow e\,e^{-1} + p\,p^{-1} \rightarrow \nu\bar{\nu}$ might be operative in some temperature interval $T \simeq (10^9 - 10^{10})$ K, $T < T_{c,p}$, and, together with other in-medium modified processes, it should be incorporated into computer simulations of neutron star cooling.

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Figure 1: The temperature dependence of the ratios R_{FM} and R_γ at nucleon density $\rho = \rho_0$. The solid curves correspond to the parameter choice of ref. [9], whereas the dashed curves depict results with parameters of ref. [10].