NUMERICAL ANALYSIS OF TURBULENT GAS-SOLID FLOWS IN A VERTICAL PIPE USING THE EULERIAN TWO-FLUID MODEL

A Thesis

submitted to the College of the Graduate Studies and Research

in Partial Fulfillment of the Requirements for the Degree of

Master of Science

in the Department of Mechanical Engineering at the

University of Saskatchewan

Saskatoon, Saskatchewan, Canada

By

Ashraf Uz Zaman

January, 2013

PERMISSION TO USE

In presenting this thesis in partial fulfillment of the requirements for a Master of Science degree from the University of Saskatchewan, I agree that the Libraries of this University may make it freely available for inspection. I further agree that permission for copying this thesis in any manner, in whole or in part, for scholarly purposes may be granted by the professor or professors who supervised my thesis work or, in their absence, by the Head of the Department or the Dean of the College in which my thesis work was done.

It is understood that any copying, publication, or use of this thesis or parts thereof for financial gain shall not be allowed without my written permission. It is also understood that due recognition shall be given to me and to the University of Saskatchewan in any scholarly use which may be made of any material in my thesis.

Requests for permission to copy or to make other use of material in this thesis in whole or part should be addressed to:

Head of the Department of Mechanical Engineering
University of Saskatchewan
Engineering Building, 57 Campus Drive
Saskatoon, Saskatchewan S7N 5A9
Canada

ABSTRACT

Turbulent gas-solid flows are readily encountered in many industrial and environmental processes. The development of a generic modeling technique for gas-solid turbulent flows remains a significant challenge in the field of mechanical engineering. Eulerian models are typically used to model large systems of particles. In this dissertation, a numerical analysis was carried out to assess a current state-of-the-art Eulerian two-fluid model for fully-developed turbulent gas-solid upward flow in a vertical pipe. The two-fluid formulation of Bolio *et al.* (1995) was adopted for the current study and the drag force was considered as the dominant interfacial force between the solids and fluid phase. In the first part of the thesis, a two-equation low Reynolds number $k - \varepsilon$ model was used to predict the fluctuating velocities of the gas-phase which uses an eddy viscosity model. The stresses developed in the solids-phase were modeled using kinetic theory and the concept of granular temperature was used for the prediction of the solids velocity fluctuation.

The fluctuating drag, *i.e.*, turbulence modulation term in the transport equation of the turbulence kinetic energy and granular temperature was used to capture the effect of the presence of the dispersed solid particles on the gas-phase turbulence. The current study documents the performance of two popular turbulence modulation models of Crowe (2000) and Rao *et al.* (2011). Both models were capable of predicting the mean velocities of both the phases which were generally in good agreement with the experimental data. However, the phenomena that small particles cause turbulence suppression and large particles cause turbulence enhancement was better captured by the model of Rao *et al.* (2011); conversely, the model of Crowe (2000)

produced turbulence enhancement in all cases. Rao *et al.* (2011) used a modified wake model originally proposed by Lun (2000) which is activated when the particle Reynolds number reaches 150. This enables the overall model to produce turbulence suppression and augmentation that follows the experimental trend.

The granular temperature predictions of both models show good agreement with the limited experimental data of Jones (2001). The model of Rao *et al.* (2011) was also able to capture the effect of gas-phase turbulence on the solids velocity fluctuation for three-way coupled systems. However, the prediction of the solids volume fraction which depends on the value of the granular temperature shows noticeable deviations with the experimental data of Sheen *et al.* (1993) in the near-wall region. Both turbulence modulation models predict a flat profile for the solids volume fraction whereas the measurements of Sheen *et al.* (1993) show a significant decrease near the wall and even a particle-free region for flows with large particles.

The two-fluid model typically uses a low Reynolds number $k-\varepsilon$ model to capture the near-wall behavior of a turbulent gas-solid flow. An alternative near-wall turbulence model, i.e., the two-layer model of Durbin *et al.* (2001) was also implemented and its performance was assessed. The two-layer model is especially attractive because of its ability to include the effect of surface roughness. The current study compares the predictions of the two-layer model for both clear gas and gas-solid flows to the results of a conventional low Reynolds number model. The effects of surface roughness on the turbulence kinetic energy and granular temperature were also documented for gas-particle flows in both smooth and rough pipes.

ACKNOWLEDGEMENTS

I would like to express my earnest gratitude and appreciation to my respected supervisor, Prof. Donald J. Bergstrom, for his invaluable guidance, support and encouragement throughout my work during this graduate program. His critical appraisal and suggestions have been invaluable at every stage of this research work.

I would like to extend my acknowledgement to my advisory committee members Prof. James D. Bugg and Prof. Scott D. Noble for their valuable suggestions and feedback.

I take this opportunity to thank my research colleagues of the CFD group for their assistance and my friends, other colleagues and staff at the Department of Mechanical Engineering for making my stay at the University of Saskatchewan a memorable one.

Furthermore, I would like to thank Prof. D. J. Bergstrom and NSERC for providing the financial support for this research work.

DEDICATION

To my parents, Nur Uz Zaman and Parvin Zaman, and wife, Kaniz Fatema Sharmin, for their generous support, guidance and motivation

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

LIST OF SYMBOLS

d	Particle diameter
e	Particle-particle coefficient of restitution
E_k	Wake effect
e_w	Particle-wall coefficient of restitution
g_0	Equilibrium radial distribution function
I_T	Fluctuating drag in the granular energy equation
I_k	Fluctuating drag in the kinetic energy equation
k	Turbulence kinetic energy
k_{sg}	Cross-correlation of the fluctuating velocities
m	Mass loading
P	Pressure
P_k	Production term in the kinetic energy equation
P_T	Production term in the granular temperature equation
Re_s	Particle Reynolds number
r	Radial co-ordinate
Re	Reynolds number of the flow
St	Stokes number of the flow
T	Granular temperature
u_0	Axial velocity of gas phase at the centerline
u_τ	Friction velocity of gas phase
u_z	Axial velocity of gas phase

u_z'	Axial fluctuating velocity in gas phase
v_z'	Axial fluctuating velocity in solid phase
v_z	Streamwise velocity component of solid phase
y	Wall normal co-ordinate
y_{ef}	Effective wall normal distance
y_0	Average roughness height
z	Axial co-ordinate

Greek Symbols

α_0	Solids volume fraction at maximum packing
α_g	Volume fraction of gas phase
α_s	Volume fraction of solid phase
λ_m	Mean free path of solid particles
β	Drag coefficient
ε	Dissipation of kinetic energy
ϕ	Specularity constant
ρ_g	Density of gas phase
ρ_s	Density of solid phase
σ_{rr}	Radial stress in solid phase
σ_{rz}	Shear stress in solid phase
$\sigma_{\theta\theta}$	Radial stress in solid phase
τ_D	Drag time-scale
τ_c	Collision time-scale

τ_{rz}	Shear Stress in gas phase
τ_{sg}	Time-scale for the fluctuating energy transfer
μ_{eg}	Effective viscosity of gas phase
μ_g	Viscosity of gas phase
μ_s	Viscosity of solid phase
μ_t	Turbulent viscosity of gas phase
ν_g	Dynamic viscosity of the fluid phase
ν_s	Dynamic viscosity of the solid phase
ω	Damping function for the solids stress
γ	Granular energy dissipation

Subscripts

g	Pertaining to the gas phase
mx	Pertaining to maximum packing condition
0	Pertaining to the centerline value of the element
r	Pertaining to the radial component of the element
s	Pertaining to the solid phase
z	Pertaining to the axial component of the element

Superscripts

$'$	Pertaining to fluctuating quantity of the element
$^+$	Pertaining to normalized quantity of the element

CHAPTER ONE

INTRODUCTION

1.1 Background

Turbulent gas-solid flows occur in many industrial and environmental applications, such as pneumatic transport, slurry transport, fluidized beds, and dust and particle-exhaust pollutant control systems. Appropriate modeling of gas-solid flows remains a major challenge in the field of mechanical engineering. The two-fluid formulation is a popular approach for modelling gas-particle flows that describes the motion of both phases in an Eulerian framework and is applicable to large systems of particles. However, a comprehensive model that can accurately predict a wide range of two-phase flows is not yet available. In particular, the modification of the gas-phase turbulence due to the presence of the dispersed phase, which is historically known as turbulence modulation, remains difficult to capture in a general model formulation. Most of the studies which assess the turbulence modulation models focus on the prediction of the mean velocities of both phases and the velocity fluctuation of the gas-phase although the solids volume fraction and velocity fluctuation also play a significant role. The performance of these numerical models also depends on the realistic prediction of the flow properties in the near-wall region.

1.1.1 Multiphase Flows

Multiphase flow refers to the simultaneous flow of more than one phase. Gas-solid, liquid-solid and gas-liquid flows are common examples which are broadly encountered in various industrial, environmental and energy conversion processes. In these flows, the fluid phase is typically considered as the continuous or carrier phase and the other phases are dispersed within the fluid phase. The carrier phase is responsible for conveying the constituents of the flow and the dispersed phase is often associated with the generation and modification of turbulence in the flow.

1.1.2 Modeling Approaches

The determination of the flow properties of a typical gas-solid flow continues to be a major challenge in the field of mechanical engineering. Although both phases are constituents of a single system, the response to a pressure gradient and interfacial force is different for each phase. Many practical flows in different industries including power plants are found to be multiphase where the inevitable presence of turbulence represents an additional challenge for any comprehensive model. A successful modeling technique should be capable of accurately predicting the behavior of turbulent gas-solid flows in different flow regimes.

When the constituents of a two-phase flow are in thermodynamic equilibrium, then mixture models are typically used for numerical analysis. Common mixture models include:

- The homogeneous model
- The drift flux model

When the properties of the components of a two-phase flow are distinctly different from each other, then a separate modeling approach is required for each phase. Two models available in the literature for modeling a separated flow include:

- The particle trajectory model (Eulerian-Lagrangian)
- The two-fluid model (Eulerian-Eulerian)

The particle trajectory model uses a Lagrangian framework to describe the motion of the particle-phase. This model tracks the dynamics of every single particle of the system. Therefore, massive time and effort is required for the particle trajectory model to predict a large system of particles. Conversely, the two-fluid model treats both the gas and solids phase as interpenetrating continua and describes both phases in an Eulerian framework. As it predicts the averaged motion of both phases, the two-fluid model is very useful for large systems of particles. In this study the two-fluid model was used to model the properties of both the gas and solid phases.

1.1.3 Two-Fluid Model

When the constituents of a gas-particle flow are not in thermodynamic equilibrium and the particle suspension is dense with a large number of particles, then a two-fluid approach is appropriate. The two-fluid model treats the dispersed phase as a continuum and describes its motion in an Eulerian framework. The mean transport equation of the particle phase has a form that is analogous to that of the fluid phase, where the constitutive relations are described using the kinetic theory of granular flows. Particle-particle collisions, particle-wall collisions and gas-phase turbulence cause velocity fluctuations in the particle phase which are often modeled using the concept of granular temperature.

Although both the gas and particle phase have similar forms of conservation equations (mass and momentum), the coupling effects in the mean and fluctuating velocity fields complicate the model formulation. The drag force is often considered as the dominant interfacial force and responsible for the transfer of the mean kinetic energy from the gas-phase to the particle-phase. The particle phase does not respond directly to the pressure gradient applied to the system rather it extracts energy from the fluid-phase. The particle response to the motion of the carrier phase is a function of the size of the particle and is often quantified using the Stokes number. The Stokes number is a ratio of the particle response time to the characteristic time scale of the flow. The particle response time is the time required by a single particle to reach 65% of the fluid velocity from rest while the characteristic time scale of the flow is often calculated from the hydraulic diameter divided by the bulk fluid velocity. The coupling effect of the fluctuating velocity fields of both phases is often described using the concept of turbulence modulation.

Numerical modelling of the fluid phase turbulence becomes more complicated in a fluid-particle flow where particles cause significant modification of the fluid phase turbulence. When the effect of the presence of particles on the gas-phase turbulence is considered as negligible, then the model equations become one-way coupled. On the other hand, if the presence of particles has a significant effect on the gas-phase velocity fluctuation, then a two-way coupled model is necessary. The particles are typically treated as point forces in the gas-phase kinetic energy equation when the size of the particles is significantly smaller than the Kolmogorov length-scale (Crowe *et al.*, 1996). These small particles tend to break up the turbulent eddies and thus increase the dissipation rate of the flow. However, larger particles (when the size is larger than the smallest turbulence scales) can produce wakes in the gas phase, which then affect the motion of other particles in the system and the system becomes three-way coupled. The wakes act as an additional source of turbulence in the gas-phase, and they depend on the particle size, concentration, slip velocity and Reynolds number. For flows which include both particle-fluid interactions and particle-particle interactions, a four-way coupled model is required (Elghobashi, 1994). Gore and Crowe (1989) showed that the presence of particles attenuates turbulence when the ratio of the particle diameter to the turbulence length scale is less than 0.1. Otherwise the particles enhance the turbulence.

1.1.4 Near-Wall Turbulence Models

The three-dimensional nature of turbulence is restricted in the near-wall region due to the presence of a solid surface. Therefore, the performance of a numerical study of a turbulent near-wall flow depends heavily on the choice of an appropriate turbulence model. The low Reynolds number (LRN) turbulence model has been extensively used by researchers together with the two-fluid model for the numerical prediction of gas-solid flows. However, the inability to include the effect of surface roughness limits the use of this near-wall model. Chen and Patel (1989) proposed a two-layer model for high Reynolds number flows that uses two algebraic equations in the inner-layer for the eddy viscosity and dissipation rate. Later, the model was modified by Durbin *et al.* (2001) to include the effects of surface roughness.

1.2 Literature Review

In this section, some previous studies using the two-fluid approach (including those in this research group), the model formulations they adopted, and the available experimental data are discussed.

1.2.1 The Two-Fluid Formulation

Sinclair and Jackson (1989) first made an attempt to develop a mathematical model for gas-solid laminar flow to capture the associated flow phenomena. Their model successfully implemented the concept of the interaction terms for the mean velocity of both phases and fluctuating velocity of the solids phase. Subsequent studies [Ding and Gidaspow (1990), Pita and Sundaresan (1991, 1993) and Ocone *et al.* (1993)] also investigated the interaction effects of the mean and fluctuating velocities of both phases. Significant progress was made by Louge *et al.* (1991) by including the kinetic theory of dry granular flow and incorporated the effect of gas-phase turbulence into the Eulerian two-fluid model. Their model was further developed by Bolio *et al.* (1995) who employed a two-equation $k - \varepsilon$ model to describe the gas-phase turbulence and introduced an interaction or coupling term in the turbulence kinetic energy equation to capture the effect of the particles on the gas phase turbulence. Although their model was able to capture correctly many properties of a gas-solid flow, it was unsuccessful in predicting the turbulence enhancement caused by the presence of large particles.

1.2.2 Previous In-House Research

A former M.Sc. student of Professor Bergstrom, Mr. Ajay Kumar Yerrumshetty (Yerrumshetty, 2007), made significant progress in this area as part of his M.Sc. thesis research. His work considered the numerical analysis of both gas-solid and liquid-solid flows in a vertical pipe, and liquid-solid flows in a horizontal channel using the two-fluid model proposed by Bolio *et al.* (1995) which was developed for dilute turbulent gas-solid flows. The turbulence properties of the gas-phase were modeled using the LRN $k - \varepsilon$ model of Myong and Kasagi (1990). The finite volume method of Patankar (1980) was used to discretize and solve the momentum equations for both the fluid and the solids phases. The simulations were carried out for fully-developed turbulent two-phase flows and the results were compared with the available experimental data. The mean velocity predictions for both phases were in close agreement with the experimental data of Tsuji *et al.* (1984) whereas the axial gas velocity fluctuations showed some deviations with the data.

Mr. Yerrumshetty's research also considered the numerical analysis of liquid-solid flow in both vertical pipes and horizontal channels, also using the two-fluid model of Bolio *et al.* (1995). Numerical predictions for the solids phase over a range of bulk concentrations were compared with the experimental data of Alejbegovic *et al.* (1994) and Sumner *et al.* (1990). Contradictory agreement with both sets of experimental data was observed for the prediction of the volume fraction and velocity fluctuation of the solids phase. For liquid-solid turbulent flow in a horizontal channel, the model predictions were compared with the experimental data of Salomon (1965). The mean mixture velocity was in good agreement with the experimental data, but the prediction for the solids volume fraction showed significant deviations near the bottom surface.

1.2.3 Turbulence Modulation

Louge *et al.* (1991) attempted to formulate the fluctuating drag/coupling terms for the turbulence kinetic energy and granular temperature equations based on temporal and volume averaging. Yuan and Michaelides (1992) first documented that the gas-phase turbulence is enhanced due to formation of particle wakes and the turbulence is suppressed due to the work done by the gas-phase. Bolio *et al.* (1995) used a modified version of the expression proposed by Koch (1990) in which the cross-correlation term is rigorously derived based on the particle-phase inertia and viscous forces of the fluid-phase. However, Koch's expression for the cross-correlation of the fluctuating velocities does not consider the velocity fluctuation in the fluid phase, which has a significant effect on the particle-phase velocity fluctuation.

Crowe (2000) indicated that derivations which assumed the averaged properties of the flow to be local flow properties are inconsistent. He adopted an alternative approach that described the attenuation and augmentation of the gas-phase turbulence intensity. Elghobashi (1994) would characterise the overall model as being four-way coupled. Zhang and Reese (2001, 2003) obtained only limited agreement with the experimental data using a closure which included the turbulence modulation model of Crowe (2000).

Rao *et al.* (2011) modified the fluctuating interaction terms based on a convection heat transfer analogy. Although the particle velocity fluctuation has multiple sources such as particle-particle collision, particle-wall collision, and carrier-phase velocity fluctuations, Rao *et al.* (2011) modelled the carrier-phase velocity fluctuations as the sole source-term of the particle-phase velocity fluctuations. They also considered the particles to be fully-elastic and the fluid as inviscid, thus there was no loss of energy due to dissipation. Their model uses a drag time-scale for particles with low inertia and a collision time-scale when the particles cause augmentation in the gas-phase turbulence. The hypothetical model assumed by Rao *et al.* (2011) for developing the fluctuating terms advocates the use of the cross-correlation term of Sinclair and Mallo (1998), which is a simple geometric mean of the fluctuating velocities of both phases.

1.2.4 Drag Force

The drag term is the dominant interfacial force that provides a coupling effect in the transport equation for the mean velocities of both phases. Many previous studies [Wen and Yu (1966), Ishii (1976), Ishii and Mishima (1984), Clift (1978)] have developed different models for the drag coefficient. Bolio *et al.* (1995) used an expression for the drag coefficient proposed by Ding and Gidaspo (1990). Hadinoto and Curtis (2009) documented that the flow predictions are affected by the choice of drag model. Rao *et al.* (2011) tested some recent drag models [Hill *et al.* (2001^1, 2001^2) and Benyahia *et al.* (2007)] along with the model of Wen and Yu (1966) in their comparative study.

1.2.5 Particle Stresses

Bolio *et al.* (1995) used the kinetic theory of Lun *et al.* (1984) for dry granular flow to describe the stresses developed in the solids-phase. In the model of Lun *et al.* (1984), the macroscopic behaviour of the solids-phase is captured by solving a velocity distribution function using the Boltzmann equation and the particle-particle collisions are modelled using the binary collision methodology of inelastic hard spheres. The velocity fluctuation of the solids-phase is described using the concept of granular temperature. The second-order moment equation was used to develop the transport equation of the granular temperature analogous to that of the turbulence kinetic energy equation.

The model of Lun *et al.* (1984) adopts the assumption that the velocities of two approaching particles prior to a collision are unrelated. Later, Peirano and Leckner (1998) showed that this assumption can be applied to very dilute suspension flows with very large particles where the particle motion is not affected by the gas-phase velocity fluctuation because of its high inertia. They included the effect of the interstitial fluid on the dry granular flow model for the solids-phase and employed an approximation of third order for the particle velocity probability density function.

1.2.6 Effect of Surface Roughness

Numerous studies have been performed to analyse the effect of roughness on the motion of particles in turbulent flows. Fan and Ahmadi (1993) developed a sublayer model for deposition of spherical particles from turbulent air steams in vertical ducts with smooth and rough surfaces. They modified the boundary condition for the particle capture trajectories to include the effect of surface roughness and obtained reasonable agreement with the experimental data. Li and Ahmadi (1993) carried out a numerical study to model the deposition of aerosol particles in a horizontal channel with a rough surface. Later, Zhang and Ahmadi (2000) validated the sublayer model of Fan and Ahmadi (1993) for aerosol transport and deposition in vertical and horizontal turbulent duct flows using direct numerical simulations.

Huber and Sommerfeld (1998) presented numerical predictions for dispersed gas-solid flows in different pipe elements (horizontal channel, pipe bends and vertical pipes) for a wide range of Reynolds number and mass loading. They adopted the Eulerian-Lagrangian approach for particle motion and compared the predictions of the mean and fluctuating velocity fields for both smooth and rough surfaces with measurements by phase Doppler anemometry (PDA). Lain *et al.* (2002) used their PDA measurements for a wide range of particle size and mass loading to validate a particle trajectory model in four-way coupled systems where particle-particle and particle-fluid interactions become dominant, and also documented the effect of surface roughness in horizontal channels. Later, Sommerfeld (2003) adopted a stochastic approach to model the particle-particle collisions for turbulent gas-solid flows in a horizontal channel with a rough surface and presented a detailed analysis of the particle behaviour for different boundary conditions. His model assumed that each particle collides with its fictitious collision partner and used a prescribed turbulence for the gas-phase which is decoupled from the particle motion. The second part of the study (Sommerfeld and Kussin, 2003) presents the mean and fluctuating velocity fields of the particle-phase. Further studies, such as those of Lain and Sommerfeld (2008) and Lain and Sommerfeld (2012), analyse the effects of surface roughness on different flow properties for pneumatic transport of particles in horizontal channels.

1.2.7 Experimental Data

The literature includes only a limited set of experimental measurements for turbulent gas-solid flows in vertical pipes. Maeda *et al.* (1980) and Lee and Durst (1982) provided some measurements of the mean velocities of both phases for different particle sizes. Tsuji *et al.* (1984) presented experimental results of the mean velocities of both phases and fluctuating velocities of the gas-phase for a wide range of mass loading and particle size. Later, Sheen *et al.* (1993) provided measurements of the mean velocities of both phases, fluctuating velocities of the gas-phase, Reynolds shear stress and volume concentration of the solids phase for gas-solid flows with different particle size. Jones (2001) also presented some valuable measurements for the fluctuating velocity fields of both phases.

1.3 Objectives

The overall objective of the current study was to perform a numerical analysis of fully-developed turbulent gas-solid flow in a vertical pipe using a state-of-the-art two-fluid model and document the effect of roughness on the velocity fluctuation of both the solids and gas phase.

The overall objective may be broken down as follows:

[1] To develop a computational code that implements a leading two-fluid model for turbulent gas-solid upward flow in a vertical pipe;

[2] To assess the performance of two popular turbulence modulation models by comparing the model predictions with the experimental data and identify the scope for improvement;

[3] To assess the performance of an alternative near-wall turbulence model for fully-developed gas-solid flows, and document the effect of surface roughness on the turbulence kinetic energy and granular temperature;

No experimental data is available in the literature for the upward gas-solid turbulent flow in rough pipes. Therefore, a qualitative assessment will be carried out for the results predicted from the numerical model, which will document the effect of roughness in the context of a two-fluid model.

1.4 Thesis Organization

In this dissertation, a numerical study of turbulent gas-solid flow in a vertical pipe is carried out using the Eulerian two-fluid model. The layout of the thesis consists of four chapters that include two journal manuscripts. The introduction, a summary literature review and the objectives are presented in the first chapter. Two journal manuscripts that address the second and third objectives of the thesis are presented in chapter 2 and 3. A numerical analysis to assess the performance of the turbulence modulation models of Rao et al. (2011) and Crowe (2000) is presented in chapter 2. The effects of surface roughness on the turbulence kinetic energy and granular temperature for gas-particle flows in both smooth and rough pipes are discussed in chapter

3. Chapter 4 presents a summary, some concluding thoughts that are drawn from the present results and an outline of future work.

1.6 References

Alejbegovic, A., Assad, A., Bonetto, F., and Lahey, Jr. R. T., Phase distribution and turbulence structure for solid/fluid upflow in a pipe, *International Journal of Multiphase Flow*, Vol. 20 (3), pp. 453–459 (1994)

Batchelor, G. and Green, J., The determination of bulk stress in a suspension of spherical particle to order of c^2, *J. Fluid Mech.*, Vol. 56, pp. 401–427 (1972)

Benyahia, S., Syamlal, M. and O'Brien, T., Summary of MFIX equations, *Available at www.mfix.org: http://www.mfix.org/documentation/MfixEquations2005–4-3* (2007)

Bolio, E., Yasuna, J., and Sinclair, J., Dilute turbulent gas-solid flow in risers with particle-particle interactions, *AIChE J.*, Vol. 41, pp. 1375-1388, (1995)

Chen, C. J. and Patel, V. C., Near-wall turbulence models for complex flows including separation, *AIAA Journal*, Vol. 26, pp. 641-648 (1988)

Clift, R., Grace, J. R., and Weber, M. E., Bubbles, Drops, and Particles, *Academic Press, New York* (1978)

Crowe, C. T., *Multiphase Flow Handbook*, CRC Press (2005)

Crowe, C. T., On models for turbulence modulation in fluid-particle flows, *Int. J. Multiphase Flow*, Vol. 26, pp. 710-727 (2000)

Crowe, C. T., Troutt, T. R. and Chung, J. N., Numerical models for two-phase turbulent flows, *Annu. Rev. Fluid. Mech.*, Vol. 28, pp. 11-43 (1996)

Ding, J. and Gidaspow, D., A bubbling fluidization model using kinetic theory analogy of granular flow, *AIChE J.,* Vol. 36, pp. 523-538 (1990)

Elghobashi, S., On predicting particle laden turbulent flows, *Appl. Sci. Res.,* Vol. 52 (4), pp. 309-329 (1994)

Fan, F. -G. and Ahmadi, G., A model for turbulent deposition of particles in vertical ducts with smooth and rough surfaces, *J. Aerosol Sci.,* Vol. 24, pp. 45-64 (1993)

Gidaspow, D., *Multiphase Flow and Fluidization*, 1st ed., Academic Press, San Diego, (1994)

Gore, R. A. and Crowe, C. T., Effect of particle size on modulating turbulence intensity, *Int. J. Multiphase Flow,* Vol. 15, pp. 279-285 (1989)

Hadinoto K, Curtis J. S., Numerical simulation of turbulent particle laden flow a with significant fluid to particle inertia ratio, *Ind. Eng. Chem. Res.,* Vol. 48, pp. 5874–5884 (2009)

Hill, R., Koch, D. and Ladd, J., The first effects of fluid inertia on flows in ordered and random arrays of spheres, *J. Fluid Mech.,* Vol. 448, pp. 213–241 (2001)^1

Hill, R., Koch, D. and Ladd, J., Moderate-Reynolds-number flows in ordered and random arrays of spheres, *J. Fluid Mech.,* Vol. 448, pp. 243–278 (2001)^2

Huber, N. and Sommerfeld, M., Modelling and numerical calculation of dilute-phase pneumatic conveying in pipe systems, *Powder Technol.,* Vol. 99, pp. 90–101 (1998)

Ishii, M. and Mishima, K., Two-fluid model and hydrodynamic constitutive relations, *J. Nuclear Eng. and Design,* Vol. 82 (2‑3), pp. 107‑126 (1984)

Ishii, M., Drift Flux Model and Derivation of Kinematic Constitutive Laws: Two-Phase Flows and Heat Transfer, *Vol. 1, S. Kakac and F. Mayinger (eds.),* Hemisphere Publ. Corp., Washington, DC (1976)

Johnson, P. and Jackson, R., Frictional-collisional constitutive relations for granular materials with applications to plane shearing, *J. Fluid Mech.*, Vol. 176, pp. 67–93 (1987)

Jones, E., An experimental investigation of particle size distribution effect in dilute phase gas-solid flow, *PhD thesis*, Perdue University (2001)

Koch, D., Kinetic theory for a monodispersed gas-solid suspension, *Phys. fluids,* Vol. 2, pp. 1711–1722 (1991)

Lain, S. and Sommerfeld, M., Euler/Lagrange computations of pneumatic conveying in a horizontal channel with different wall roughness, *Powder Technol.,* Vol. 184, pp. 76–88 (2008)

Lain, S. and Sommerfeld, M., Numerical calculation of pneumatic conveying in horizontal channels and pipes: Detailed analysis of conveying behaviour, *Int. J. Multiphase Flow,* Vol. 39, pp. 105–120 (2012)

Lain, S., Sommerfeld, M. and Kussin, J., Experimental studies and modelling of four-way coupling in particle-laden horizontal channel flow, *Int. J. Heat Fluid Flow,* Vol. 23, pp. 647–656 (2002)

Lee, S. and Durst, F., On the motion of particles in turbulent duct flows, *Int. J. Multiphase Flow*, Vol. 8, pp. 125–146 (1982)

Li, A. and Ahmadi, G., 1993, Deposition of aerosols on surfaces in a turbulent channel flow, *Intl J. Engng Sci.,* Vol. 31, pp. 435-445 (1993)

Louge, M., Mastorakos, E. and Jenkins, J., The role of particle collisions in pneumatic transport, *J. Fluid Mech.,* Vol. 231, pp. 345–359 (1991)

Lun, C., Numerical simulation of dilute turbulent gas-solid flows, *Int. J. Multiphase Flow,* Vol. 26, pp. 1707–1736 (2000)

Lun, C., Savage, S., Jeffery, D. and Chepurniy, N., Kinetic theories for granular flow: inelastic particles in couette flow and slightly inelastic particles in general flow field, *J. Fluid Mech.*, Vol. 140, pp. 223–256 (1984)

Maeda, M., K. Hishida, and Furutani, T., Optical measurements of local gas and particle velocity in an upward flowing dilute gas-solid suspension, *Proc. Polyphase Flow and Transport Technology Conf.*, Century 2-ETC, San Francisco (1980)

Myong H. K. and Kasagi, N., A new approach to the improvement of turbulence model for wall-bounded shear flows, *JSME Int. J. (II),* Vol. 33 (1), pp. 63–72 (1990)

Ocone, R., Sundaresen, S. and Jackson, R., Gas-particle flow in a duct of arbitrary inclination with particle-particle interactions, *AIChE J.,* Vol. 39, pp. 1261-1271 (1993)

Patankar, S.V., *Numerical Heat transfer and Fluid Flow,* Hemisphere, New York (1980)

Peirano, E. and Leckner, B., Fundamentals of turbulent gas-solid applied to circulating fluidized bed combustion, *Progr. Energy Combut. Sci.,* Vol. 24, pp. 259–296 (1998)

Pita, J. A. and Sundaresen, S., Developing flow of a gas-particle mixture in a vertical riser, *AIChE J.*, Vol. 39, Issue 4, pp. 541-552 (1993)

Pita, J. A. and Sundaresen, S., Gas-solid flow in vertical tube, *AIChE J.*, Vol. 37, pp. 1009-1018 (1991)

Rao, A., Curtis, J. S., Hancock, B. B. and Wassgren, C. (2011), Numerical simulation of dilute turbulent as particle flow with turbulence modulation, *AIChE J.*, Vol. 58 (5), pp. 1381-1396 (2011)

Salomon, D.M. Flow of suspensions in a rectangular channel. *PhD Thesis,* University of Saskatchewan (1965)

Sheen, H., Chaing, Y.-Z. and Chiang, Y. -S., Two dimensional measurements of flow structure in a two phase vertical pipe flow. *Proc. Natl. Sci. Council, Republic of China (A)*, Vol. 17, pp. 200–213 (1993)

Sinclair J. L. and Mallo, T., Describing particle-turbulence interaction in a two-fluid modelling framework, *Proc. of ASME Fluids Engineering Division Summer Meeting (FEDSM'98)*, Washington, DC, June 21–25 (1998)

Sinclair, J. and Jackson, R., Gas-particle flow in a vertical pipe with particle-particle interactions, *AIChE J.*, Vol. 35, pp. 1473-1486 (1989)

Sommerfeld, M. and Huber, N., A laser strobe technique combined with digital image analysis to study particle-wall collisions, In: Xu, Y., Wang, S. (Eds.), Modern Measuring Techniques for Multiphase Flows, *Proceedings of the International Symposium on Measurement Techniques for Multiphase Flows*, Nanjing, China, April, pp. 428-440 (1995)

Sommerfeld, M. and Kussin, J., Analysis of collision effects for turbulent gas-particle flow in a horizontal channel: part II. Integral properties and validation, Int. J. Multiphase Flow, Vol. 29, pp. 701–715 (2003)

Sommerfeld, M., Analysis of collision effects for turbulent gas-particle flow in a horizontal channel: part I. Particle transport, *Int. J. Multiphase Flow,* Vol. 29, pp. 675–699 (2003)

Sumner, R. J., McKibben, M. J. and Shook, C. A., Concentration and velocity distributions in turbulent vertical slurry flows, *Ecoulements Solide Liquide*, Vol. 2 (2): 33–42 (1990)

Tsuji, Y., Morikawa, Y. and Shiomi, H., LDV measurements of air-solid two-phase flow in a vertical pipe, *J. Fluid Mech.*, Vol. 139, pp. 417–434 (1984)

Wen, C. and Yu, Y., Mechanics of fluidization, *Chem. Eng. Progr. Symp. Ser.* Vol. 62, pp. 100–111 (1966)

Yerrumshetty, A. K., Numerical prediction of turbulent gas-solid and liquid-solid flows using two-fluid models, *M.Sc. Thesis*, University of Saskatchewan (2007)

Yuan, Z., and Michaelides, E., Turbulence modulation in particulate flows - a theoretical approach, *Int. J. Multiphase Flow*, Vol. 18, pp. 779-785 (1992)

Zhang, H. and Ahmadi, G., Aerosol particle transport and deposition in vertical and horizontal turbulent duct flows, *J. Fluid Mech.*, Vol. 406, pp. 55-80 (2000)

Zhang, Y. and Reese, J. M., A gas turbulence modulation in a two-fluid model for gas–solid flows, *AIChE J.*, Vol. 49 (12), pp. 3048–3065 (2003)

Zhang, Y. and Reese, J. M., Particle-gas turbulence interactions in a kinetic theory approach to granular flows, *Int. J. Multiphase Flow*, Vol. 27, pp. 1945-1964 (2001)

CHAPTER TWO

ASSESSMENT OF TURBULENCE MODULATION IN TWO-FLUID MODELS FOR FULLY-DEVELOPED GAS-SOLID UPWARD PIPE FLOW

Submitted as

Zaman, A. and Bergstrom, D. J., Assessment of Turbulence Modulation in Two-Fluid Models for Fully-Developed Gas-Solid Upward Pipe Flow, International J. Multiphase flows, 2012

Contribution of this chapter to the thesis

The research work presented in this chapter focuses on the second objective of the thesis. More specifically, the chapter analyzes the performance of the popular turbulence modulation models of Rao *et al.* (2011) and Crowe (2000) and provides a comprehensive assessment of the predictions for the mean and fluctuating velocity fields of both phases. Some observations include the following: the solids volume fraction profiles predicted by both models do not follow the experimental results, the model of Rao *et al.* (2011) uses an ad hoc wake model which partially capture the turbulence enhancement, and the model of Rao *et al.* (2011) produces an unrealistic fluctuating energy transfer near the wall in the granular temperature equation.

ABSTRACT

A numerical study was carried out to investigate modelling the effect of particles on the gas-phase turbulence for upward fully-developed gas-solid flow in a vertical pipe, more specifically, the performance of the turbulence modulation models of Crowe (2000) and Rao *et al.* (2011) in an Eulerian two-fluid formulation. A low Reynolds number $k - \varepsilon$ model was used for the gas phase turbulence while the kinetic theory of granular flow was used to describe the solids stresses. Although both models predict the mean velocities of the gas and solids phases reasonably well, the prediction for the gas-phase velocity fluctuation is still deficient, especially for high Stokes number flows. Furthermore, the prediction for the solids concentration, calculated from the granular temperature, is inconsistent with the experimental data in the near-wall region. The current study also reveals an inappropriate energy transfer near the wall by the modulation term in the granular energy equation. Overall, the present analysis demonstrates some significant deficiencies in present state-of-the-art turbulence modulation models, which limit their ability to fully capture the effects of particles on turbulence.

Keywords *- Gas-solid flow, two-fluid model, turbulence modulation, granular temperature, solids volume fraction*

RESEARCH HIGHLIGHTS

- Two popular turbulence modulation models are investigated
- Solids volume fraction profiles do not follow the experimental results

- Use of an ad hoc wake model to produce turbulence enhancement
- Unrealistic fluctuating energy transfer is observed near the wall in T-equation

2.1 Introduction

Turbulent gas-solid flows occur in many industrial and environmental applications, such as pneumatic transport, slurry transport, fluidized beds, dust and particle-exhaust pollutant control systems. Appropriate modelling of the gas-solid flows remains a major challenge in the field of mechanical engineering. The Eulerian two-fluid model is a popular approach for modelling of turbulent two-phase flows where both phases are treated as interpenetrating continua. The presence of dispersed particles in turbulent flow complicates the flow phenomena, since the influence of the particles on the fluid phase turbulence can be significant. The modification of the gas-phase turbulence due to the particles is historically known as turbulence modulation. Most of the studies which assess the turbulence modulation models focus on the prediction of the mean velocities of both phases and the velocity fluctuation of the gas-phase. However, less attention has been given to the behaviour of the turbulent viscosity and Reynolds shear stress and solids volume fraction.

A seminal work in the development of the two-fluid model for gas-solid flow was performed by Bolio *et al.* (1995). They extended the model proposed by Louge *et al.* (1991) to include a two-equation $k-\varepsilon$ model for the gas-phase turbulence where a coupled fluctuating drag term determines the effect of the particles on the gas-phase turbulence. The Stokes number of the flow, a ratio of the particle response time[1] to the characteristic time scale of the flow[2] is often used to define the regime of the phase-coupling. When the effect of the particles on the gas-phase turbulence is considered as negligible, then the model equations become one-way coupled. This modelling approach is applicable for flows with very low Stokes numbers and very low solids concentration. On the other hand, if the presence of particles has a significant effect on the gas-phase velocity fluctuation, then a two-way coupled model is necessary. The particles are typically treated as point forces in the gas-phase kinetic energy equation, when the size of the particles is significantly smaller than the Kolmogorov length-scale (Crowe *et al.,* 1996). These small particles tend to break up the turbulent eddies and thus increase the dissipation in the flow. However, larger particles (when the size is larger than the smallest turbulence scales) produce wakes in the gas phase which affect the motion of other particles so that the system becomes three-way coupled. The wakes act as an additional source of turbulence in the gas-phase and depend on the particle size, concentration, slip velocity and Reynolds number. For flows which include both particle-fluid interactions and particle-particle interactions, a four-way coupled model is required (Elghobashi, 1994). Gore and Crowe (1989) showed that the presence of particles attenuates turbulence when the ratio of the particle diameter to the turbulence length scale[3] is less than 0.1. Otherwise the particles enhance the turbulence. Various attempts (Crowe 2000 and Rao *et al.* 2011), have been made to develop a model that can better describe the modification of the gas-phase turbulence by particles. Most of these models are capable of predicting the mean velocities of both phases, and the predictions for the gas-phase velocity fluctuation in the axial direction show partial agreement with the experimental data. The predictions for the gas-phase velocity fluctuation become increasingly challenging for high

---

[1] Particle response time is the average time required for each solid particles to reach 65% of the fluid velocity from rest

[2] Typically the ratio of the diameter of the pipe to the bulk fluid velocity

[3] The characteristic length of the most energetic eddy in single-phase flows

Stokes number flows where particles enhance the gas-phase turbulence by generating wakes. Crowe (2000) introduced a modulation model that is capable of capturing the turbulence enhancement; his model includes an additional term to capture the conversion of mechanical work by the drag force into the turbulence kinetic energy as a function of the slip velocity. However, the work done by the drag force peaks at the wall which results in an erroneous augmentation of turbulence kinetic energy near the wall for gas-solid flows. Recently, Rao *et al.* (2011) developed a modulation model using a convection heat transfer analogy that is capable of following the experimental trend of the effect of small and large particles on the gas-phase turbulence. A modified version of the wake model of Lun (2000) was used by Rao *et al.* (2011) to capture the turbulence enhancement.

The stresses developed in the solids-phase are typically derived using the dry granular flow kinetic theory of Lun *et al.* (1984). In their model, the macroscopic behaviour of the solids-phase is captured by solving a velocity distribution function using the Boltzmann equation and the particle-particle collisions are modelled using the binary collision methodology of inelastic hard spheres. The model of Lun *et al.* (1984) assumes that the velocities of two approaching particles prior to a collision are unrelated. Peirano and Leckner (1998) showed that this assumption can be applied to dilute suspension flows with large particles where the particle motion is not affected by the gas-phase velocity fluctuation because of its high inertia. They included the effect of the interstitial fluid on the dry granular flow model for the solids-phase and employed an approximation of third order for the particle velocity probability density function. The velocity fluctuation of the solids-phase is described using the concept of granular temperature. A second-order moment equation was used to develop the transport equation of granular temperature analogous to that of the turbulence kinetic energy equation. For fully-developed flows, mass conservation is typically not used to solve for the volume fraction. Instead, the radial normal stress of the solids-phase is assumed to be approximately constant for fully-developed gas-solid flows in vertical pipes. This enables the solids volume fraction to be calculated from the radial momentum balance for the particle phase, given the value of the granular temperature.

The present study implements two different turbulence modulation models to investigate the effect of the presence of particles on the gas-phase turbulence. Primary attention was given to the prediction of the gas-phase velocity fluctuations for flows with different particle sizes and mass loadings. The model predictions were compared with four different data sets: Lee and Durst (1982), Tsuji *et al.* (1984), Sheen *et al.* (1993) and Jones (2001). The modulation term in the granular energy equation quantifies the fluctuating energy transfer between the two phases and captures the effect of the gas-phase turbulence on the solids velocity fluctuation. The simulation results show that both models are capable of predicting the gas-phase mean and fluctuating velocity profiles for low Stokes number flows where the turbulence is damped by the particles. However, the prediction of the turbulence enhancement for high Stokes number flows is still problematic due to inadequate modelling of the vortex shedding. The current study performs a comprehensive assessment of these two-fluid model formulations and identifies the scope for further improvement.

2.2 Mathematical Model

As discussed earlier, both the gas and particle phases are assumed to be interpenetrating continua and described in an Eulerian framework using a two-fluid formulation. The general continuum model of Bolio *et al.* (1995) for both the gas and particle phases, where they implemented a two-fluid model for steady fully-developed dilute gas-particle flow in a vertical pipe, was adopted for the present work. The governing and constitutive relations used for the current study are discussed in the following sections.

2.2.1 Gas-Phase Transport Equations

The motion of the Newtonian fluid is governed by the Navier-Stokes equation. The effect of the presence of a second phase on the molecular viscosity of the gas in the calculation of the viscous stress was implemented using the model of Batchelor and Green (1972). The momentum transport equation of the gas-phase for a fully-developed gas-solid flow becomes:

$$0 = -\alpha_g \frac{\partial p}{\partial z} + \frac{1}{r}\frac{\partial}{\partial r}(r\tau_{rz}) - \beta(u_z - v_z) \tag{2.1}$$

The total shear stress is given by,

$$\tau_{rz} = \mu_{eg}\frac{\partial u_z}{\partial r} - \rho_g \overline{u'_z u'_r} \tag{2.2}$$

where, the effective viscosity is,

$$\mu_{eg} = \mu_g(1 + 2.5\alpha_s + 7.6\alpha_s^2)\left(1 - \frac{\alpha_s}{\alpha_0}\right) \tag{2.3}$$

The Reynolds shear stress is calculated using an eddy viscosity model relation,

$$-\rho_g \overline{u'_z u'_z} = \mu_t \frac{\partial u_z}{\partial r} \tag{2.4}$$

where the turbulent viscosity is calculated from the turbulence kinetic energy (k) and its dissipation rate (ε),

$$\mu_t = \frac{c_\mu f_\mu \rho_g k^2}{\varepsilon} \tag{2.5}$$

Here, u_z is the gas velocity in the axial direction, v_z is the solids velocity in the axial direction, p is the pressure, α_g is the gas volume fraction, ρ_g is the density of the gas-phase, u_z' is the velocity fluctuation of the gas-phase in the axial direction, u_r' is the velocity fluctuation of the gas-phase in the radial direction, μ_g is the molecular viscosity of the gas-phase, α_s is the solids volume fraction and α_0 is the solids volume fraction at maximum packing[4].

The final term in equation (2.1) represents the drag term or the dominant interfacial force that provides a coupling effect in the transport equations of the mean velocities for both phases. Due to the large density difference, the magnitude of the other interfacial forces (i.e., lift force, Basset force etc.) are assumed to be negligible compared to the drag force. The model proposed by Wen and Yu (1966) was used to calculate the drag coefficient,

$$\beta = \frac{3}{4}\frac{\rho_g}{d} C_D \frac{\alpha_s}{\alpha_g^{2.65}} |u_z - v_z| \tag{2.6}$$

where the drag coefficient is given by,

$$C_D = \frac{24}{Re_s}(1 + 0.15 Re_s^{0.687}) \tag{2.7}$$

and the particle Reynolds number is defined as,

$$Re_s = \frac{\rho_g d |v_z - u_z|}{\mu_g} \tag{2.8}$$

where d is the particle diameter.

---

[4] For all simulations, $\alpha_0 = 0.65$ and $c_\mu = 0.09$

A low Reynolds number form of the transport equation for the turbulence kinetic energy and dissipation rate was adopted:

$$0 = \frac{1}{r}\frac{\partial}{\partial r}\left[r\alpha_g\left(\mu_{eg} + \frac{\mu_t}{\sigma_k}\right)\frac{\partial k}{\partial r}\right] + \alpha_g\mu_t\left(\frac{\partial u_z}{\partial r}\right)^2 - \alpha_g\varepsilon + I_k \tag{2.9}$$

$$0 = \frac{1}{r}\frac{\partial}{\partial z}\left[r\alpha_g\left(\mu_{eg} + \frac{\mu_t}{\sigma_\varepsilon}\right)\frac{\partial \varepsilon}{\partial r}\right] + \alpha_g c_1 f_1 \frac{\varepsilon}{k}\mu_t\left(\frac{\partial u_z}{\partial r}\right)^2 - \alpha_g c_2 f_2 \frac{\varepsilon^2}{k} + c_3 f_2 \frac{\varepsilon}{k} I_k \tag{2.10}$$

where, I_k represents the turbulence modulation term and the model constants are taken as $c_1 = 1.4, c_2 = 1.8, c_3 = 1.2, \sigma_k = 1.4$ and $\sigma_\varepsilon = 1.3$ for all simulations. The low Reynolds number model of Myong and Kasagi (1990) was adopted for which the three damping functions take the following form:

$$f_1 = 1 \tag{2.11}$$

$$f_2 = \left[1 - \frac{2}{9}exp\left\{-\left(\frac{R_T}{6}\right)^2\right\}\right]\left[1 - exp\left(-\frac{y^+}{5}\right)\right]^2 \tag{2.12}$$

$$f_\mu = \left[1 - exp\left(-\frac{y^+}{70}\right)\right]\left[1 + \frac{3.45}{\sqrt{R_T}}\right] \tag{2.13}$$

where, $y^+ = \frac{\rho_g u_\tau (R-r)}{\mu_g}$, $R_T = \frac{\rho_g k^2}{\mu_{eg}\varepsilon}$ and u_τ is the friction velocity of the gas-phase.

2.2.2 Solids-Phase Transport Equations

As discussed earlier, the two-fluid model describes the motion of the solids-phase in an Eulerian framework and the transport equations take the following form for fully-developed flows:

$$0 = -\alpha_s\frac{\partial p}{\partial z} - \frac{1}{r}\frac{\partial}{\partial r}(r\sigma_{rz}) - \alpha_s\rho_s g + \beta(u_z - v_z) \tag{2.14}$$

$$0 = \frac{1}{r}\frac{\partial}{\partial r}(r\sigma_{rr}) - \frac{\sigma_{\theta\theta}}{r} \tag{2.15}$$

where σ_{rz} is the shear stress in the radial and axial plane, ρ_s is the density of the particles, g is the gravitational acceleration, and σ_{rr} and $\sigma_{\theta\theta}$ are the normal stresses in the radial and azimuthal planes, respectively.

Various models have been developed for the stresses in the solids-phase. Lun *et al.* (1984) used the kinetic theory of dense gases to describe the constitutive relations for these stresses. In their model, all the particles are considered as hard spheres and particle-particle collisions are weakly inelastic with a constant coefficient of restitution (e). The model of Lun *et al.* (1984) for the solids-phase stresses was used for the current study and the constitutive relations are summarised in Table 2.1.

The particle motion is primarily governed by the drag force generated in the fluid-particle interactions. However, particle-particle collisions, particle-wall collisions and gas-phase turbulence all cause velocity fluctuations (v') in the solids-phase. The concept of granular temperature is often used to quantify the fluctuating kinetic energy of the solids-phase i.e.,

$$T = \frac{1}{3} v'^2 \tag{2.16}$$

The transport equation for the granular temperature for a fully-developed flow is:

$$0 = -\frac{1}{r}\frac{\partial}{\partial r}(r q_{PT}) - \sigma_{rz}\frac{\partial v_z}{\partial r} - \gamma + I_T \tag{2.17}$$

where the granular energy flux is given by,

$$q_{PT} = -\lambda \frac{\partial T}{\partial r} \tag{2.18}$$

The granular energy dissipation is evaluated using the following relation,

$$\gamma = \frac{48}{\sqrt{\pi}} \eta(1-\eta) g_o \alpha_s^2 \frac{\rho_s}{d} T^{2/3} \tag{2.19}$$

where the radial distribution function, $g_o = \frac{\alpha_0^{1/3}}{\alpha_0^{1/3} - \alpha_s^{1/3}}$, $\eta = \frac{1+e}{2}$ and λ is the thermal conductivity. The constitutive relations for the thermal conductivity based on kinetic theory are summarized in Table 2.1.

2.2.3 Turbulence Modulation

In an attempt to formulate the fluctuating drag/coupling terms, Louge *et al.* (1991) obtained the following two relations which are based on temporal and volume averaging:

$$I_k = -\alpha_g \beta \left(2k - k_{sg}\right) \tag{2.20}$$

$$I_T = \alpha_g \beta \left(k_{sg} - 3T\right) \tag{2.21}$$

where k_{sg} is the cross-correlation of the gas and solid fluctuating velocities, i.e., $k_{sg} \equiv \overline{u_z' v_z'}$. The modelling of the closure for the k_{sg} term continues to be a challenge. Bolio *et al.* (1995) used a modified version of the expression proposed by Koch (1990), which has the following form:

$$k_{sg} = \frac{4}{\sqrt{\pi}} \frac{d}{\rho_s} \frac{\beta}{\alpha_s} \frac{(v_z - u_z)^2}{\sqrt{T}} \tag{2.22}$$

Koch (1990) rigorously derived the cross-correlation term based on the particle-phase inertia and viscous forces of the fluid-phase. However, his expression for calculating k_{sg} does not consider the velocity fluctuation in the fluid phase which has a significant effect on the particle-phase velocity fluctuation.

Crowe (2000) indicated that derivations which assumed the averaged properties of the flow to be local flow properties are inconsistent. He adopted an alternative approach that led to distinct models for the coupling terms i.e.,

$$I_k = \alpha_g \beta (v_z - u_z)^2 + \alpha_g \beta \left(3T - k_{sg}\right) \tag{2.23}$$

$$I_T = \alpha_g \beta \left(k_{sg} - 3T\right) \tag{2.24}$$

In their prediction of gas-solid flow in a vertical pipe, Zhang and Reese (2003) obtained only limited agreement with the experimental data using a closure which included the turbulence modulation model of Crowe (2000).

Rao *et al.* (2011) modified the fluctuating interaction terms based on a convective heat transfer analogy where the particle-phase velocity fluctuation is caused by the gas-phase turbulence. Their analogy also considered the particles to be fully-elastic and the fluid as inviscid, thus there is no loss of energy due to dissipation. The resultant model equations are as follows:

$$I_k = -\frac{\rho_g}{\tau_{sg}}\left(2k - k_{sg}\right) + E_w \qquad (2.25)$$

$$I_T = \frac{\rho_g}{\tau_{sg}}\left(k_{sg} - 3T\right) \qquad (2.26)$$

where E_w is the wake effect (model details are given in Table 2.2) and τ_{sg} is the time scale over which the transfer of energy occurs. Their model uses $\tau_{sg} =$ drag time-scale, $\left(\tau_D = \frac{\alpha_s}{\beta\alpha_f}\right)$ for particles with low inertia and $\tau_{sg} =$ collision time-scale $\left(\tau_c = \frac{d}{24\alpha_s g_0}\sqrt{\frac{\pi}{T}}\right)$ when the particles cause augmentation in the gas-phase turbulence. The hypothetical system assumed by Rao *et al.* (2011) for modelling the fluctuating terms advocates the use of the cross-correlation term of Sinclair and Mallo (1998), which is a simple geometric mean of the fluctuating velocities of both phases:

$$k_{sg} = \sqrt{(2k)(3T)} \qquad (2.27)$$

The above expression was used with both the turbulence modulation models of Crowe (2000) and Rao *et al.* (2011) for the present study.

2.2.4 Boundary Conditions

The mean gas-velocity and turbulence kinetic energy are set to zero at the wall and the boundary condition for the dissipation rate is obtained from Myong and Kasagi (1990), but including a turbulence modulation term following Rao *et al.* (2011), i.e.,

$$\alpha_g \varepsilon = \nu_{eg} \frac{\partial^2 k}{\partial r^2} + I_k \tag{2.28}$$

The current study adopts the boundary condition formulation proposed by Johnson and Jackson (1987) for the solids mean velocity and granular temperature. The momentum flux transferred to the wall is equal to the stress in the particle assembly adjacent to the wall. These momentum transfer rates are governed by a specularity coefficient that quantifies the nature of the inelastic particle-wall collisions. The energy conducted to the wall is equal to the dissipation due to the inelastic particle-wall collisions and production at the wall. The expressions used for the boundary conditions of the particle mean velocity and granular temperature are given below:

$$\sigma_{rz} = \frac{\rho_s \pi v \phi \sqrt{T}}{2\sqrt{3}\left[\left(\frac{\alpha_0}{\alpha_s}\right) - \left(\frac{\alpha_0}{\alpha_s}\right)^{2/3}\right]} \tag{2.29}$$

$$q_{PT} = \frac{\sqrt{3}\rho_s \pi \left(1 - e_w^2\right) T^{3/2}}{4\left[\left(\frac{\alpha_0}{\alpha_s}\right) - \left(\frac{\alpha_0}{\alpha_s}\right)^{2/3}\right]} - \frac{\rho_s \pi v^2 \phi \sqrt{T}}{2\sqrt{3}\left[\left(\frac{\alpha_0}{\alpha_s}\right) - \left(\frac{\alpha_0}{\alpha_s}\right)^{2/3}\right]} \tag{2.30}$$

where, ϕ is the specularity coefficient and e_w is the particle-wall restitution coefficient.

At the centerline of the pipe, axisymmetric boundary conditions are applied i.e., the gradient of each variable is set to zero,

$$\frac{\partial \varphi}{\partial r} = 0 \tag{2.31}$$

where, φ can be any parameter, such as the gas mean velocity, solids mean velocity, solids volume fraction, turbulence kinetic energy, dissipation rate and granular temperature.

2.3 Results and Discussion

All the transport equations were discretized using the finite volume method of Patankar (1980). A thorough analysis of different grids indicated that a total of sixty control volumes provided converged results for the range of Reynolds numbers (see Table 3 for details) used in the current study. For the current study, the grid was non-uniform with refinement near the wall. The gas density (ρ_g) and molecular viscosity (μ_g) was taken as $1.2\ \frac{kg}{m^3}$ and $1.8 \times 10^{-5}\ \frac{Nm}{s}$, respectively. Since no independent studies are available for the specularity coefficient, particle-particle (e) and particle-wall restitution coefficient, the following values were used for all simulations:

$$e = 0.9, e_w = 0.9, \phi = 0.002 \tag{2.32}$$

All of the experimental studies used for comparison, i.e., Lee and Durst (1982), Tsuji *et al.* (1984), Sheen *et al.* (1993) and Jones (2001) used the Laser Doppler Anemometry (LDA) technique for velocity measurements. All of the data sets represented low Reynolds number turbulent flows, and a wide range of mass loading[5] and particle size were considered. Details of the flow conditions of the selected experimental studies are summarized in Table 3.

2.3.1 Mean Velocity

Figure 2.1 shows the effect of particle diameter, and to a much lesser degree, the mass loading on the mean velocity profiles. The predictions using both turbulence modulation models for the mean velocity profiles are generally in good agreement with the experimental data in the core region of the pipe; however, the model of Rao *et al.* (2011) yields better predictions at the

---

[5] Ratio of the bulk mass flow rate of the particle-phase to that of the fluid-phase

shoulder. It is also evident from Figure 1 that the gas velocity profile grows flatter when the mass loading increases. Both numerical models were able to capture the feature of the particle velocity being higher than the gas velocity near the wall. In Figure 2.1 (b) and (c), the predictions of both models for the solids velocity profile were in good agreement with the experimental data. However, both models over-predict the slip velocity for flows with smaller particles (Figure 2.1a) and under-predict the slip velocity for flows with larger particles (Figure 2.1d). The choice of the drag model plays a significant role in predicting the slip velocity. Figure 2.1 also indicates that the turbulence modulation model is capable of affecting the mean velocity profile of both phases.

2.3.2 Gas-Phase Streamwise Velocity Fluctuation

In the current study, the value of the turbulence kinetic energy of the gas phase was used to calculate the velocity fluctuation in the axial direction. Sheen *et al.* (1993) documented that the velocity fluctuations in the radial and azimuthal direction are approximately the same and equal to one-half of the fluctuation in the axial direction i.e., $u_r' = u_\theta' = \frac{u_z'}{2}$. This assumption then gives:

$$u_z' = \sqrt{\frac{4k}{3}} \tag{2.33}$$

Figure 2.2 presents the effect of the particles on the gas phase turbulence in terms of profiles for the streamwise velocity fluctuation (u_z'). Although some similar predictions were included in the numerical studies of Rao *et al.* (2011) and Zhang and Reese (2003), the current study presents a more complete comparison of the two turbulence modulation models. The measurements of Jones (2001) show that 70 micron particles caused a small turbulence enhancement in the core-region of the pipe, whereas for flows with 200 micron particles turbulence enhancement is experienced throughout. The model of Rao *et al.* (2011) under-predicts the axial gas-velocity fluctuation in flows with 70 micron particles, while conversely, the model of Crowe (2000) over-predicts it. Both turbulence modulation models show good agreement with the experimental data of Jones (2001) for flows with 200 micron particles which cause enhancement across the entire pipe diameter.

Figures 2.2 (c) and (d) show a comparison of the predictions of both models with experimental data for flows with small particles where turbulence attenuation is observed. For this range of Stokes number as the particle size increases, the particles respond less to the gas phase fluctuation and also cause an enhanced rate of dissipation which leads to turbulence suppression (Zhang and Reese, 2003). The model of Rao *et al.* (2010) only produces a noticeable reduction in the streamwise velocity fluctuation for the particles of diameter $d = 450\ \mu m$, Figure 2.2 (d). On the other hand, the model of Crowe (2000) predicts enhanced levels of the streamwise velocity fluctuation for both cases, even though the measurements indicate a small level of turbulence suppression.

For the cases considered in Figure 2.2 (e) and (f), the wakes generated in the flow by the particles act as an additional source of generation in the gas phase turbulence in the region near the centerline of the pipe. Although augmentation of turbulence is observed in the core section of the pipe in both flows with 800 and 1000 micron particles, attenuation of turbulence is present in the near-wall region. The model of Rao *et al.* (2011) incorporated the phenomena of turbulence augmentation via the wake model of Lun (2000). The turbulence augmentation was partly captured by the model of Rao *et al.* (2011), however the level of the numerical prediction was too high. The model of Crowe (2000) predicted excessively large values of the streamwise velocity fluctuation throughout the pipe, with peak values at the centerline; in contrast, for the clear gas case, the turbulence always peaks near the wall. Overall, based on Figure 2.2, using an appropriate choice for the cross-correlation, the model of Rao *et al.* (2011) showed the capability of producing turbulence suppression and augmentation better than the model of Crowe (2000). However, agreement with the data was, at best, mixed.

2.3.3 Gas-Phase Radial Velocity Fluctuation

The radial gas velocity fluctuation was calculated from the turbulence kinetic energy using the assumption of Sheen *et al.* (1993) i.e., $u_r' = u_\theta' = \frac{u_z'}{2}$ in a similar manner to the calculation of the axial component. This assumption gives:

$$u_r' = \sqrt{\frac{k}{3}} \tag{2.34}$$

Note that an isotropic eddy viscosity is inherently incapable of reproducing the characteristic near-wall anisotropy in the normal Reynolds stress components. The purpose for comparing the prediction for the radial gas velocity fluctuation to the experimental data is partly to assess whether the effect of particles on the turbulence kinetic energy corresponds to what is documented in the measurements for u_r'.

Figure 2.3 shows the prediction for the radial velocity fluctuation of the gas-phase. Figures 2.3 (a) and (b) compare the prediction of both Crowe (2000) and Rao *et al.* (2011) with the experimental data of Jones (2001). In both cases, the turbulence modulation model over predicted the radial gas velocity fluctuation. The data suggests that the radial component remains approximately the same compared to the velocity fluctuation for the clear gas flow. Figure 2.3 (a) and (b) also show that the model of Crowe (2000) predicts almost the same level of fluctuation in the radial direction for flows with two different particle sizes.

The data of Sheen *et al.* (1993) considered in Figures 2.3 (c), (d) and (e), provides some valuable insights regarding the radial velocity fluctuation for turbulent gas-solid flows. They showed that unlike the axial component, the component of the velocity fluctuation in the radial direction is always attenuated by the presence of particles. Sheen *et al.* (1993) also stated that the relative reduction of the velocity fluctuation for small particles is approximately the same for both the axial and the radial component at the center of the pipe. As the particle size increases the magnitude of turbulence attenuation decreases across the pipe. The model of Rao *et al.* (2011) was able to reproduce the suppression of turbulence for the flow with particle sizes of 200 and 450 μm and showed good agreement with the experimental data. However, for larger particles (800 μm) the model predicted a small turbulence enhancement in the core region of the pipe, similar to the prediction for the axial component. Similar to the case of the axial component, the model of Crowe (2000) consistently over predicts the radial component. Note that the assumption used to estimate the gas velocity fluctuation in the radial direction is overly simplified.

2.3.4 Turbulent Viscosity

Dispersed particles tend to break up the eddies developed in a turbulent flow and thereby enhance the dissipation. In an eddy viscosity model closure, the eddy viscosity characterizes the transport due to the fluctuating (small-scale) field. The success of an eddy viscosity closure model for these flows is largely dependent on its ability to correctly predict the effect of particles on the turbulent viscosity profile.

Figure 2.4 shows the effect of the particles on the turbulent or eddy viscosity. Unfortunately, no experimental data is available for comparison. Figure 2.4 (a) illustrates turbulence suppression due to small particles as predicted by the model of Rao *et al.* (2011): the suppression increases but not uniformly as the mass loading increases (32% attenuation at the centerline for flow with mass loading of 3.2). As the number of particles in the flow increases, the increased work done by the gas phase turbulence on the particles leads to a larger reduction of the turbulent viscosity. For large particles, the wake generated by each particle acts as an additional source of turbulence and greater enhancement is observed as the mass loading of the flow increases. As shown in Figure 2.4 (b), the model of Rao *et al.* (2010) correctly predicts enhanced levels of the turbulent viscosity especially in the core region of the pipe (190% augmentation at the centerline for flow with a mass loading of 3.0). Figure 2.4 (c) and (d) consider the predictions for the same flow cases using the model of Crowe (2000). In both cases, the turbulent viscosity is significantly enhanced, and the effect increases with mass loading. Figure 2.4 (c) is further evidence of the failure of the model of Crowe (2000) to produce turbulence suppression for small particles. The augmentation of turbulence produced by the model of Crowe (2000) in flows with 1000 micron particles is approximately two times the prediction of Rao *et al.* (2011) at the centerline of the pipe for all three mass loadings. In some ways, the prediction of the eddy viscosity provides a better understanding of the turbulent transport than the gas-phase velocity fluctuation in gas-solid flows.

2.3.5 Reynolds Shear Stress

Figure 2.5 compares the prediction for the Reynolds shear stress of the model of Rao *et al.* (2011) and Crowe (2000) with the experimental data of Sheen *et al.* (1993). The measurement shows that the Reynolds shear stress is reduced due to the presence of particles for all three cases. Sheen *et al.* (1993) noted that the surface of the particles creates additional boundary surface in the flow, thus causing the Reynolds shear stress to deviate for gas-solid flows, and the effect increases with mass loading. The magnitude of the modeled Reynolds stress profile is higher in flows with large particles (800 μm) where vortex shedding occurs due to the high inertial particles, but still lower than that for the clear gas flow.

Although both the turbulence modulation models (Rao *et al.* 2011 and Crowe 2000) were able to reproduce the phenomena that produce a suppression of the Reynolds shear stress due to the presence of particles, they over-predict the Reynolds shear stress in all three cases. The experimental data also reveals that, unlike clear gas flows, the Reynolds stress is reduced to zero near the centerline of the pipe for flows with small particles. However, both models were unable to capture this phenomenon. The Reynolds shear stress is a critical feature of the gas-phase turbulence which has often been over-looked in other numerical studies. The present study indicates both turbulence modulation models give similar predictions for flows with different particles sizes.

2.3.6 Source Terms in k - Equation

Figure 2.6 examines the source terms in the transport equation of the turbulence kinetic energy and in particular the relative size of the turbulence modulation term. Two different flows measured by Tsuji *et al.* (1984) with turbulence attenuation [Figure 2.6 (a) and (b)] and augmentation [Figure 2.6 (c) and (d)] were considered. Recall that for the clear gas case, the mean shear production and dissipation are dominant, and both peak near the wall. Figure 2.6 (a) and (b) considers the case of turbulence suppression; as shown in Figure 2.6 (b) the prediction of the model of Rao *et al.* (2011) gives a small (almost negligible) destruction term near the wall. The negligible magnitude of the turbulence modulation across the pipe also indicates that for flows with Stokes number less than 100, the model predicts the effect of particles on the gas-phase turbulence to be minimal.

Figure 2.6 (c) and (d) shows the prediction for the source terms when the presence of particles causes enhancement in the gas phase turbulence. The turbulence modulation term acts as a destruction term near the wall, but becomes a generation term in the core region of the pipe. The wake term itself is not smooth since different models are implemented as the particle Reynolds number varies across the pipe. The current study adopts the model of Rao *et al.* (2011) for the wake term which is a modification of the formulation proposed by Lun (2000). This allows the model to capture turbulence enhancement for flows with large particles without using the slip velocity as does the model of Crowe (2000). Although the turbulence modulation term of Rao *et al.* (2011) correctly produces damping near the wall and enhancement in the core section of the pipe, the discontinuity of the ad hoc wake model can be identified in Figure 2.6 (d) as it uses three different models A-B, C-D and E-F for different particle Reynolds number.

2.3.7 Granular Temperature

The stresses developed in the solids-phase largely depend on the magnitude of the solids velocity fluctuation; the granular temperature is commonly used as a measure of the solids-phase velocity fluctuation. Figure 2.7 presents the predictions for the granular temperature using the turbulence modulation model of Rao *et al.* (2011) for different flows measured by Jones (2001) and Tsuji *et al.* (1984).

The axial velocity fluctuation for the solids-phase is calculated assuming an isotropic distribution based on the granular temperature. Figure 2.7 indicates that overall the prediction of Rao *et al.* (2011) shows satisfactory agreement with the experimental data of Jones (2001). The magnitude of the velocity fluctuation (between 1 and 1.5 m/s) is relatively small for flows with small particles (Figure 2.7a and 2.7b). When the particle size becomes large, the magnitude increases significantly as shown in Figure 2.7 (c), and the profile increases near the wall. Large particles by causing a disturbance to the fluid-phase locally affect the motion of other particles. This three-way coupled process is captured well by the model of Rao *et al.* (2011) as the modulation term for the granular temperature equation becomes a source term through the cross-correlation expression when the wake model is activated, i.e., the gas-phase turbulence is enhanced.

Figures 2.8 (a) and (b) show the predictions for the granular temperature using the turbulence modulation model of Rao *et al.* (2011), which is partially capable of capturing the turbulence suppression and augmentation in the gas-phase due to the presence of dispersed particles. In Figure 2.8 (a), the profile for the granular temperature is nearly flat, with a small decrease near the wall. The model of Rao *et al.* (2011) indicates that the particle fluctuation decreases as the mass loading increases in flows with small particles (Figure 2.7a). As the number of particles in the system increases, the attenuation of gas-phase turbulence increases which subsequently acts as a destruction term in the granular temperature equation. In addition, an increased number of particles causes enhancement in the particle energy dissipation due to particle-particle collisions.

Figure 2.8 (b) shows the effect of mass loading on the granular temperature for flows with large particles. The gas-phase turbulence is enhanced due to the presence of large particles that generate wakes in the system and the effect increases as the number of particles increases. This flow phenomenon is captured by the modulation term in the granular temperature equation as it acts as a generation term to enhance the velocity fluctuation in the solids-phase. The two-fluid model of Rao *et al.* (2011) shows the capability of reproducing the coupling effect of the gas-phase turbulence on the granular temperature relatively well, although it uses an isotropic $k - \varepsilon$ model that is typically incapable of accurately predicting the normal stresses in the gas-phase. The profile of the granular temperature is generally flat for all cases due to strong diffusion across the pipe, but decreases near the wall in flows with large particles as is evident in Figure 2.8 (b).

2.3.8 Solids Volume Fraction

Recall that for fully-developed flow, the local solids volume fraction was calculated from an algebraic relation representing a constant normal particle stress in the radial direction based on the granular temperature field predicted by its transport equation. The normal stresses developed in the solid-phase were modeled using the expressions proposed by Lun *et al.* (1984) as described in Table 2.3.

Figures 2.9 (a), (b) and (c) compare the predictions for the solids volume fraction of the model of Rao *et al.* (2011) with the measurements of Sheen *et al.* (1993) for flows with different particle sizes. The erroneous prediction is striking in the near-wall region where the experimental data indicate that the particle concentration decreases to create a particle-free region near the wall in flows with high inertial particles. Lee and Durst (1982) documented similar behavior in their study of gas-solid flows. They noted that the extent of this particle-free region increases as the particle size increases and becomes larger than the viscous sublayer for flows with large particles. However, the state-of-the-art Eulerian two-fluid models fail to capture this behavior and instead erroneously predict that the concentration increases slightly near the wall.

Figures 2.9 (d) and (e) show the effect of mass loading on the profile of the solids volume fraction for small and large particles. The magnitude typically increases with mass loading in both cases. However, the profiles are relatively flat in all cases although a small increase is observed near the wall for flows with higher mass loadings. These comparisons emphasize that the current models do not follow the limited experimental data.

2.3.9 Source Terms in *T* - Equation

In order to understand the coupling effect of the gas-phase turbulence on the solids fluctuating velocity noted above, the influence of the turbulence modulation term in the granular energy equation was investigated. Figures 2.10 (a) and (b) show the predictions for the source terms in the granular temperature equation, i.e., the production, dissipation and turbulence modulation terms. The production term is due to the particle stress working against the mean particle shear. The inelastic particle-particle collision is responsible for the energy dissipation which remains relatively small and almost constant, partly because of the minimal variation of the solids volume fraction across the pipe.

Recall that the current study adopts the boundary condition of Johnson and Jackson (1987) where the energy conducted to the wall is equal to the energy dissipation due to particle-wall collision and production at the wall. Note that the dissipation at the wall is a function of the specularity coefficient (ϕ) and the production term is developed using a particle-wall restitution coefficient (e_w). The energy flux calculated from the boundary condition is consistent with the flux calculated using the constitutive relations.

The turbulence modulation or fluctuating drag term (I_T) acts as an additional source in the granular energy equation, and plays a significant role in capturing the effect of the gas-phase turbulence. Figure 2.10 (a) indicates that the prediction for the modulation term has a discontinuity at $y^+ \approx 50$ for flows with small particles. The model of Rao *et al.* (2011) uses a time scale based on the drag which is a function of the slip velocity for flows with Stokes number less than 100. The discontinuity represents the point where the particle mean velocity becomes higher than the gas mean velocity near the wall. The modulation term acts as a destruction term in the core region of the pipe whereas it becomes a strong source of generation near the wall. The shape of the modulation term suggests that it is dominated by the cross-correlation, which is taken as a geometric mean of the fluctuating velocities as proposed by Sinclair and Mallo (1998).

The modulation term erroneously results in a strong sink term at the wall due to the presence of a non-zero magnitude of the granular temperature (see equation 2.26). Figures 2.6 (a) and (c) show that the modulation term in the turbulence kinetic energy equation reduces to zero at the wall as both the cross-correlation and fluctuating kinetic energy reduces to zero at the wall. The modulation term in the granular energy equation should diminish to zero at the wall to accurately represent the fluctuating energy transfer. However, the model of Rao *et al.* (2011) fails to do so.

Figure 2.10 (b) shows that the modulation term acts as generation in the granular temperature equation for flows with large particles and is dominant relative to the other source terms across the pipe. As the Stokes number of the flow is much greater than 100, the model uses a collisional time scale which leads to a continuous prediction of the modulation term. The dominant nature of the modulation term explains why the models accurately predict the three-way coupling effects for flows with very high Stokes numbers. The net imbalance of the source terms in the granular temperature equation caused by the modulation term is smoothed by a strong diffusion term.

2.4 Summary and Conclusion

The current study examines the performance of two popular turbulence modulation models, specifically that of Rao *et al.* (2011) and Crowe (2000). Both models gave similar predictions for the mean velocity field for both phases, whereas the predictions for the fluctuating velocity fields were markedly different. The prediction of the model of Rao *et al.* (2011) was in better agreement than the model of Crowe (2000) with the experimental data for the gas-phase velocity fluctuation. The model of Crowe (2000) relies extensively on the mean slip and appears to erroneously predict high levels of turbulence enhancement, even when the measurements indicate turbulence suppression. The incorporation of the wake model of Lun (2000) in the turbulence modulation expression allows the model of Rao *et al.* (2011) to predict turbulence enhancement in the core region of the pipe for large particles. However, the wake model of Lun (2000) implements different expressions as the particle Reynolds number changes across the pipe. A more accurate and fundamentally correct wake model should be investigated to replace the ad hoc approach adopted by Rao *et al.* (2011).

Lee and Durst (1984) identified a particle-free region near the solid wall in flows with large particles and stated that the extent of the region varies with particle size. Sheen *et al.* (1993) observed a similar phenomenon in their measurements of particle concentration for flows with $450\,\mu m$ and $800\,\mu m$ particles. The state-of-the-art two-fluid model did not follow the experimental trend of reduced concentration close to the wall, and instead often predicted a small increase in the near-wall region. The present study suggests that the constitutive relation for the radial normal stress of the particle phase is responsible for the erroneous prediction of the solids volume fraction, and further measurements and modeling work are required to resolve this significant discrepancy.

The modulation term in the granular temperature equation represents the fluctuating energy transfer from the gas-phase. Typically the modulation term acts as a generation term in the near-wall region and produces destruction in the core region of the pipe. However, the term becomes a dominant generation term in flows with large particles, mostly due to the coupling effect of the enhanced gas-phase turbulence. The modulation term acts as a strong sink term very close to the wall because of a non-zero value of the granular temperature at the wall. This is clearly inconsistent with the fact that the gas-phase turbulence diminishes at the wall, and therefore cannot transfer energy to the solids-phase.

Most of the previous studies analyzed the ability of the model to generate turbulence suppression and augmentation by comparing the model predictions with the experimental data for the gas velocity fluctuation, in particular the streamwise component. However, the attenuation and augmentation of the gas-phase turbulence is better assessed in terms of the prediction for the turbulent transport. In this context, the current study explores the behavior of the eddy viscosity and Reynolds shear stress for flows with both small and large particles and thereby provides a more complete and insightful assessment of the two-fluid models. Further analysis of these models requires a more extensive and comprehensive database, especially for the Reynolds shear stress, solids velocity fluctuation and volume fraction. Although the present turbulence modulation models reproduce some important features of the limited experimental data, there is room for further improvement, even for as simple a flow as dilute upward fully-developed gas-particle flow.

2.6 References

Batchelor, G. and Green, J., The determination of bulk stress in a suspension of spherical particle to order of c^2, *J. Fluid Mech.*, Vol. 56, pp. 401–427 (1972)

Bolio, E., Yasuna, J., and Sinclair, J., Dilute turbulent gas-solid flow in risers with particle-particle interactions, *AIChE J.*, Vol. 41, pp. 1375-1388 (1995)

Crowe, C. T., On models for turbulence modulation in fluid-particle flows, *Int. J. Multiphase Flow*, Vol. 26, pp. 710-727 (2000)

Crowe, C. T., Troutt, T. R. and Chung, J. N., Numerical models for two-phase turbulent flows, *Annu. Rev. Fluid. Mech.*, Vol. 28, pp. 11-43 (1996)

Elghobashi, S., On predicting particle laden turbulent flows, *Appl. Sci. Res.*, Vol. 52 (4), pp. 309-329 (1994)

Gore, R. A. and Crowe, C. T., Effect of particle size on modulating turbulence intensity, *Int. J. Multiphase Flow*, Vol. 15, pp. 279-285 (1989)

Johnson, P. and Jackson, R., Frictional-collisional constitutive relations for granular materials with applications to plane shearing, *J. Fluid Mech.*, Vol. 176, pp. 67–93 (1987)

Jones, E., An experimental investigation of particle size distribution effect in dilute phase gas-solid flow, *PhD thesis*, Purdue University (2001)

Koch, D., Kinetic theory for a monodispersed gas-solid suspension, *Phys. fluids*, Vol. 2, pp. 1711-1722 (1991)

Lee, S. and Durst, F., On the motion of particles in turbulent duct flows, *Int. J. Multiphase Flow*, Vol. 8, pp. 125–146 (1982)

Louge, M., Mastorakos, E. and Jenkins, J., The role of particle collisions in pneumatic transport, *J. Fluid Mech.*, Vol. 231, pp. 345–359 (1991)

Lun, C., Numerical simulation of dilute turbulent gas-solid flows, *Int. J. Multiphase Flow*, Vol. 26, pp. 1707–1736 (2000)

Lun, C., Savage, S., Jeffery, D. and Chepurniy, N., Kinetic theories for granular flow: inelastic particles in couette flow and slightly inelastic particles in general flow field, *J. Fluid Mech.*, Vol. 140, pp. 223–256 (1984)

Myong H. K. and Kasagi, N., A new approach to the improvement of turbulence model for wall-bounded shear flows, *JSME Int. J. (II),* Vol. 33 (1), pp. 63–72 (1990)

Patankar, S.V., *Numerical Heat transfer and Fluid Flow,* Hemisphere, New York (1980)

Peirano, E. and Leckner, B., Fundamentals of turbulent gas-solid applied to circulating fluidized bed combustion, *Progr. Energy Combut. Sci.*, Vol. 24, pp. 259–296 (1998)

Rao, A., Curtis, J. S., Hancock, B. B. and Wassgren, C., Numerical simulation of dilute turbulent gas-particle flow with turbulence modulation, *AIChE J.,* Vol. 58 (5), pp. 1381-1396 (2011)

Sheen, H., Chaing, Y. -Z. and Chiang, Y. -S., Two dimensional measurements of flow structure in a two phase vertical pipe flow, *Proc. Natl. Sci. Council, Republic of China (A),* 17, 200–213 (1993)

Sinclair J. L. and Mallo, T., Describing particle-turbulence interaction in a two-fluid modelling framework, *Proc. of ASME Fluids Engineering Division Summer Meeting (FEDSM' 98),* Washington, DC, June 21–25 (1998)

Sinclair, J. and Jackson, R., Gas-particle flow in a vertical pipe with particle-particle interactions, *AIChE J.,* Vol. 35, pp. 1473-1486 (1989)

Tsuji, Y., Morikawa, Y. and Shiomi, H., LDV measurements of air-solid two-phase flow in a vertical pipe, *J. Fluid Mech.,* Vol. 139, pp. 417–434 (1984)

Wen, C. and Yu, Y., Mechanics of fluidization, *Chem. Eng. Progr. Symp. Ser.,* Vol. 62, pp. 100–111 (1966)

Zhang, Y. and Reese, J. M., A gas turbulence modulation in a two-fluid model for gas–solid flows, *AIChE J.*, Vol. 49 (12), pp. 3048–3065 (2003)

Table 2.1 Constitutive relations for the solids-phase

Lun *et al.* (1984) as modified by Bolio *et al.* (1995):

Normal stresses, $\sigma_{rr} = \sigma_{\theta\theta} = \rho_s(\omega G_{1k} + G_{1c})$ and

Shear stress, $\sigma_{zr} = -\mu_s(\omega G_{2k} + G_{2c})\frac{\partial v_z}{\partial r}$

Solid viscosity, $\mu_s = \frac{5\sqrt{\pi}}{96}\rho_s d\sqrt{T}$

where, $\omega = \frac{1}{1+\lambda_{mfp}/R}$ and $\lambda_{mfp} = \frac{d}{6\sqrt{2}\alpha_s}$

$G_{1k} = \alpha_s$, $G_{1c} = 4\eta\alpha_s^2 g_0$, $G_{2k} = \frac{1}{\eta(2-\eta)g_o}\left[1 + \frac{8}{5}\eta\alpha_s g_o(3\eta - 2)\right]$ and

$G_{2c} = \frac{8\alpha_s}{5(2-\eta)}\left[1 + \frac{8}{5}\eta\alpha_s g_o(3\eta - 2)\right] + \frac{768\alpha_s^2 g_o\eta}{25\pi}$

Conductivity: $\lambda = \lambda'(\omega G_{3k} + G_{3c})$ where, $\lambda' = \frac{25\sqrt{\pi}}{128}\rho_s d\sqrt{T}$

$G_{3k} = \frac{8}{\eta(41-33\eta)g_o}\left[1 + \frac{12}{5}\eta^2\alpha_s g_0(4\eta - 3)\right]$

$G_{3c} = \frac{96\alpha_s}{5(41-33\eta)}\left[1 + \frac{12}{5}\eta^2\alpha_s g_0(4\eta - 3) + \frac{16}{15\pi}\eta\alpha_s g_0(41 - 33\eta)\right]$

Table 2.2 Constitutive relations of the wake term

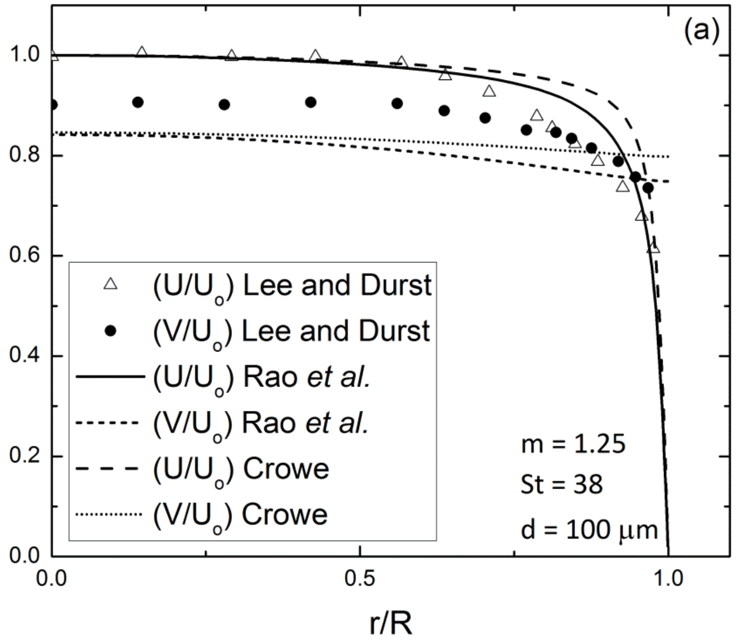
Lun (2000) modified by Rao *et al.* (2011):	
$E_w = 12\frac{C_w \nu_s \mu_t k}{d^3}$	
$Re_s = \frac{\rho_g d \lvert v_z - u_z \rvert}{\mu_g}$	
$\mu_t = 0.017 Re_s \mu_g$	for $150 \leq Re_s < 310$
$\mu_t = 1.2 + 0.000057 Re_s^2 \mu_g$	for $310 \leq Re_s < 610$
$\mu_t = 0.029 Re_s \mu_g$	for $Re_s \geq 610$
$C_w = \frac{10}{3}$	for $150 \leq Re_s < 310$
$C_w = \frac{24}{3}$	for $Re_s \geq 310$

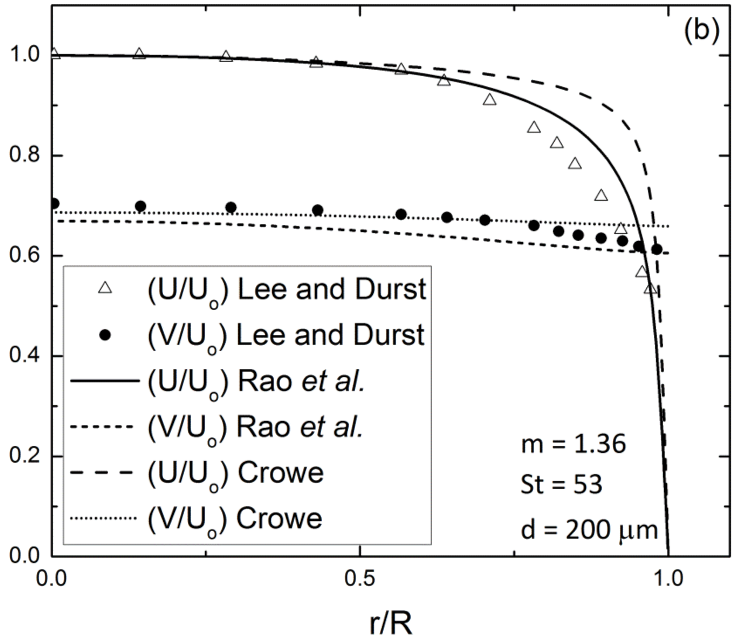
Table 2.3 Summary of the experimental data

Source	Uncertainty	Particle type	Pipe diameter (mm)	Particle diameter (μm)	Particle density (kg/m^3)		Mass loading		Reynolds number	Stokes number[6]		Centerline gas-velocity (m/s)	
Lee and	-	Glass	41.8	100	2590		1.25		14500	38		5.70	
Durst (1982)			41.8	200	2590		1.36		14000	53		5.84	
Tsuji *et al.*	-	Polystyrene	30.5	200	1020	0.5	1.30	3.2	22500	101	13.1	12.80	10.8
(1984)			30.5	1000	1020	0.6	2.00	3.0	22500	3373	13.4	12.80	13.2
Sheen *et al.*	2%	Polystyrene	52	275	1020		0.65		27000	51		9.16	
(1993)			52	450	1020		0.89		27000	138		9.22	
			52	800	1020		1.04		29000	487		9.97	
Jones (2001)	4.5%	Glass	15	70	2500		1.40		20800	70		24.00	
			15	200	2500		1.70		20800	589		25.00	
			15	500	2500		1.60		20800	3685		25.00	

---

[6] Obtained from numerical calculation


(a)
1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.5
1.0
r/R
(U/U$_o$) Lee and Durst
(V/U$_o$) Lee and Durst
(U/U$_o$) Rao *et al.*
(V/U$_o$) Rao *et al.*
(U/U$_o$) Crowe
(V/U$_o$) Crowe
m = 1.25
St = 38
d = 100 μm


(b)
1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.5
1.0
r/R
(U/U$_o$) Lee and Durst
(V/U$_o$) Lee and Durst
(U/U$_o$) Rao *et al.*
(V/U$_o$) Rao *et al.*
(U/U$_o$) Crowe
(V/U$_o$) Crowe
m = 1.36
St = 53
d = 200 μm

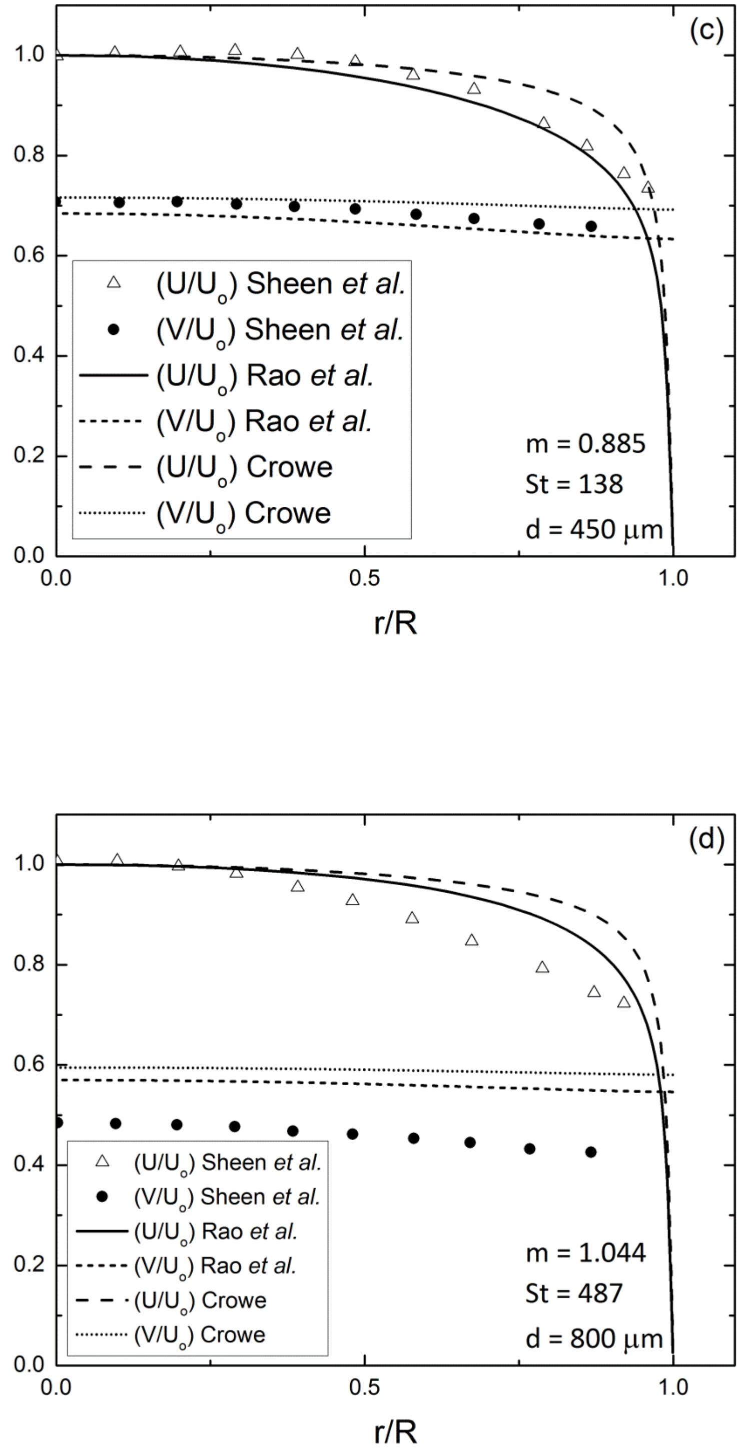
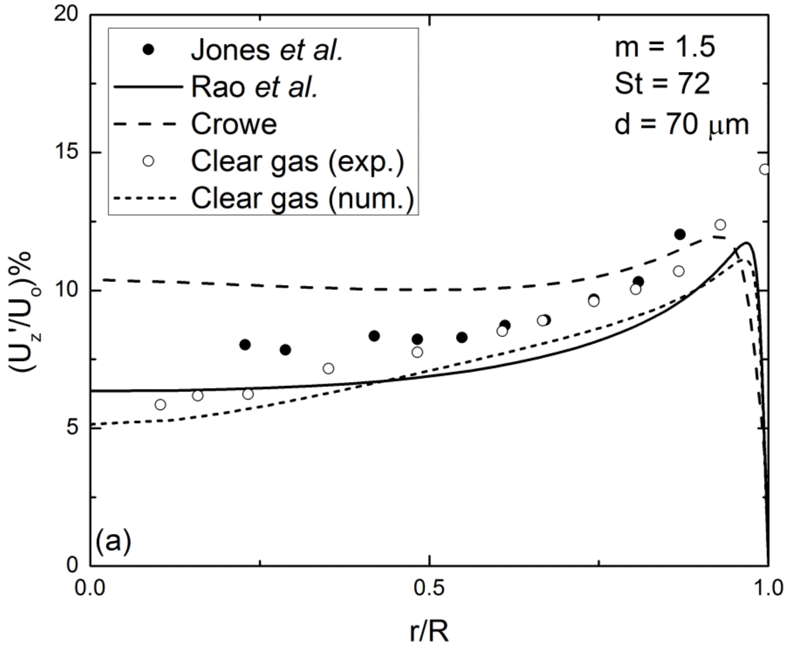
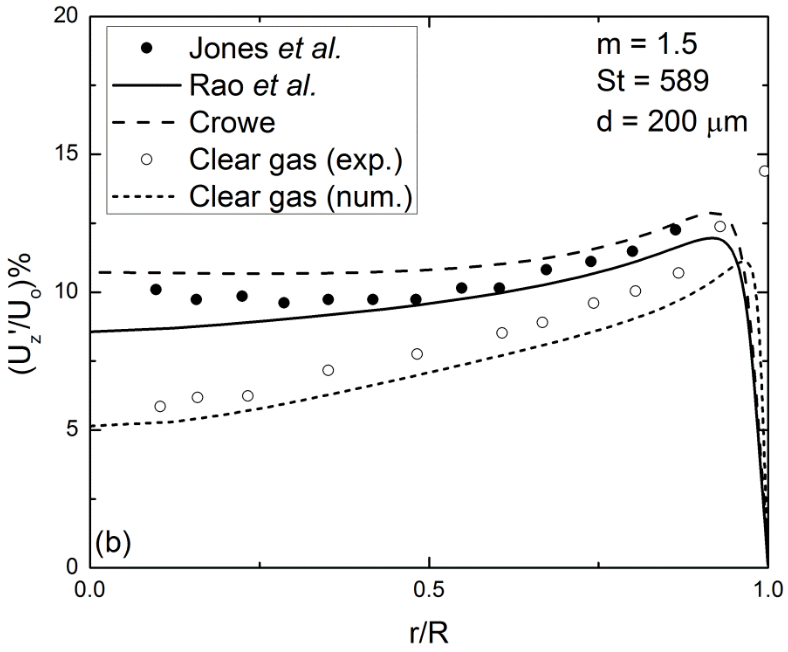
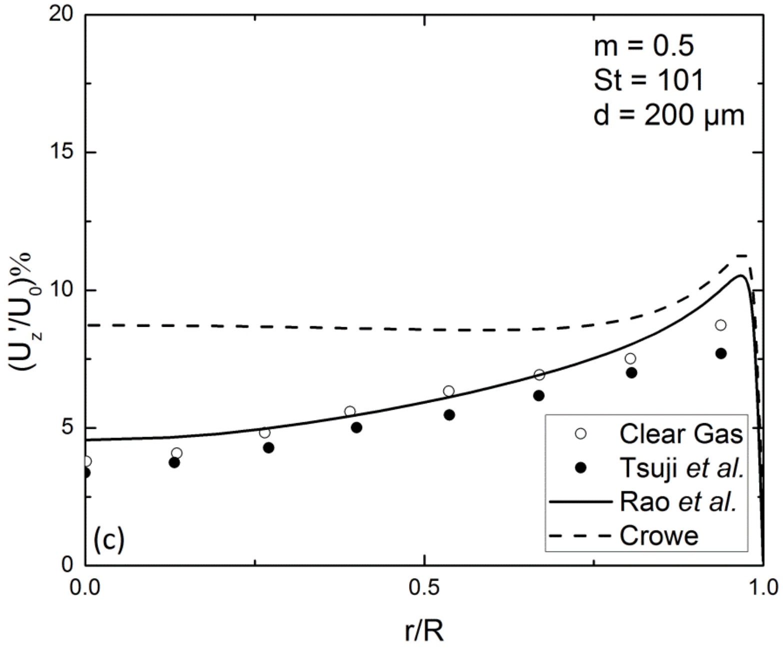


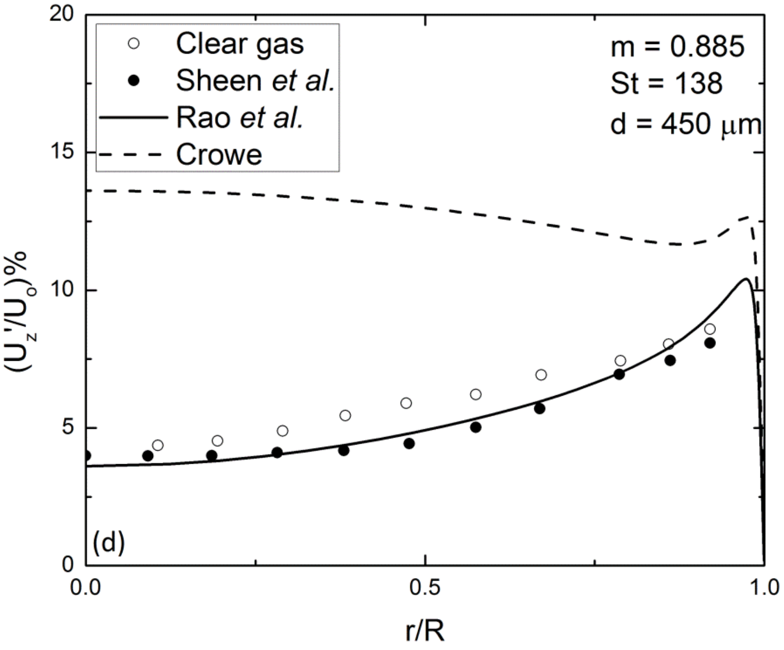
Figure 2.1 Prediction of mean velocity


20
15
10
5
0
0.0
0.5
1.0
$(U_z'/U_o)\%$
r/R
Jones *et al.*
Rao *et al.*
Crowe
Clear gas (exp.)
Clear gas (num.)
m = 1.5
St = 72
d = 70 μm
(a)


20
15
10
5
0
0.0
0.5
1.0
$(U_z'/U_o)\%$
r/R
Jones *et al.*
Rao *et al.*
Crowe
Clear gas (exp.)
Clear gas (num.)
m = 1.5
St = 589
d = 200 μm
(b)



m = 0.5
St = 101
d = 200 µm
Clear Gas
Tsuji et al.
Rao et al.
Crowe
(c)
(U_z'/U_0)%
r/R
0
5
10
15
20
0.0
0.5
1.0



Clear gas
Sheen et al.
Rao et al.
Crowe
m = 0.885
St = 138
d = 450 µm
(d)
(U_z'/U_o)%
r/R
0
5
10
15
20
0.0
0.5
1.0

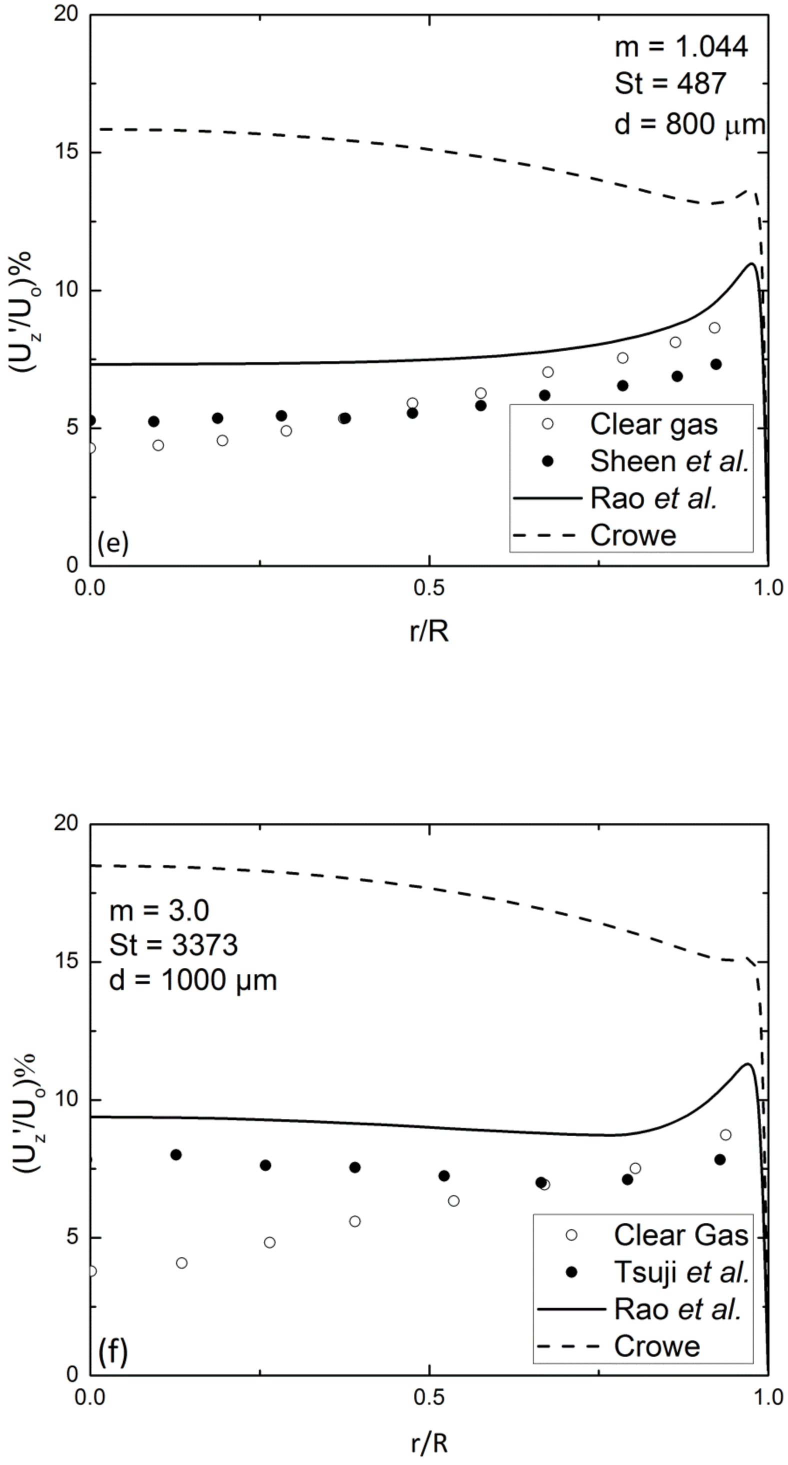
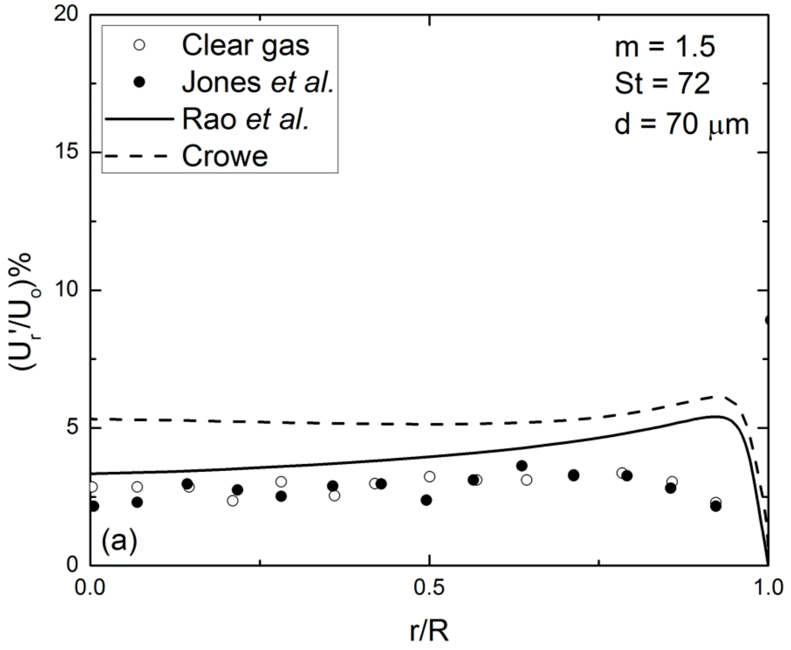
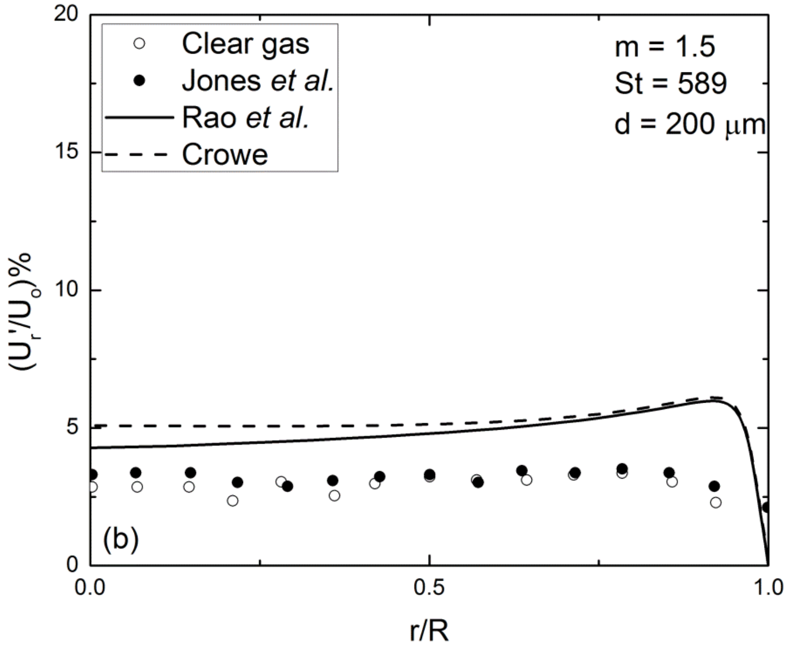


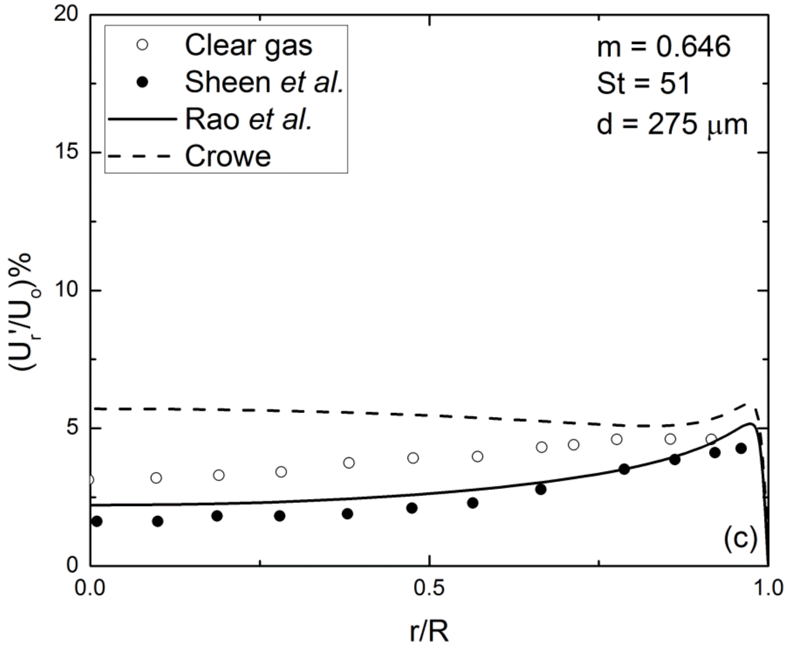
Figure 2.2 Prediction of the axial gas velocity fluctuation



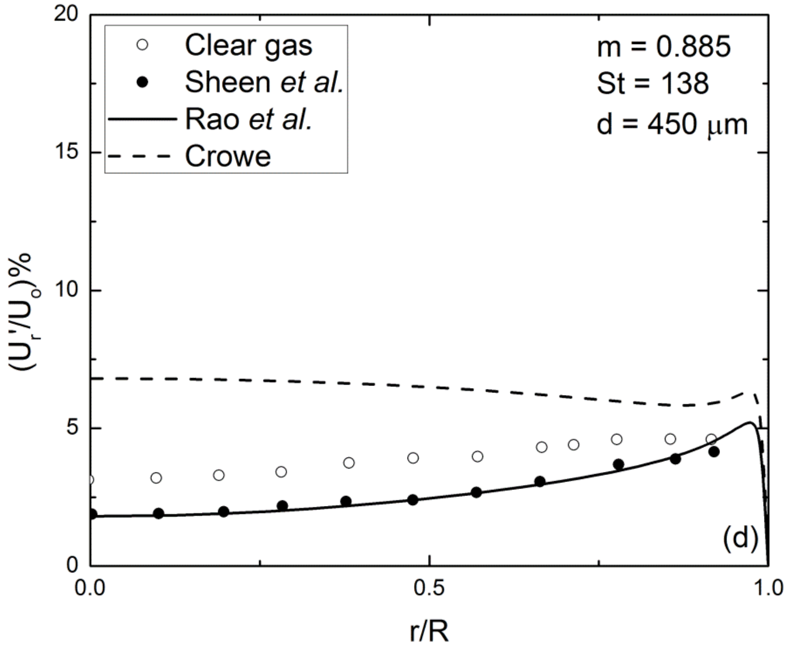
Clear gas
Jones et al.
Rao et al.
Crowe
m = 1.5
St = 72
d = 70 μm
(a)
(U_r'/U_o)%
r/R
0
5
10
15
20
0.0
0.5
1.0



Clear gas
Jones et al.
Rao et al.
Crowe
m = 1.5
St = 589
d = 200 μm
(b)
(U_r'/U_o)%
r/R
0
5
10
15
20
0.0
0.5
1.0



Clear gas
Sheen *et al.*
Rao *et al.*
Crowe
m = 0.646
St = 51
d = 275 μm
(c)
$(U_r'/U_o)\%$
r/R
0
5
10
15
20
0.0
0.5
1.0



Clear gas
Sheen *et al.*
Rao *et al.*
Crowe
m = 0.885
St = 138
d = 450 μm
(d)
$(U_r'/U_o)\%$
r/R
0
5
10
15
20
0.0
0.5
1.0

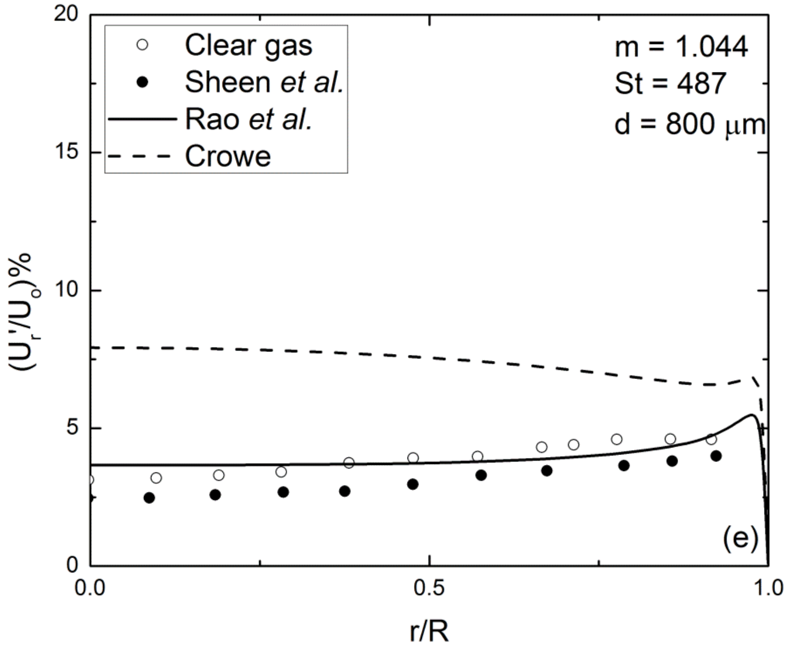
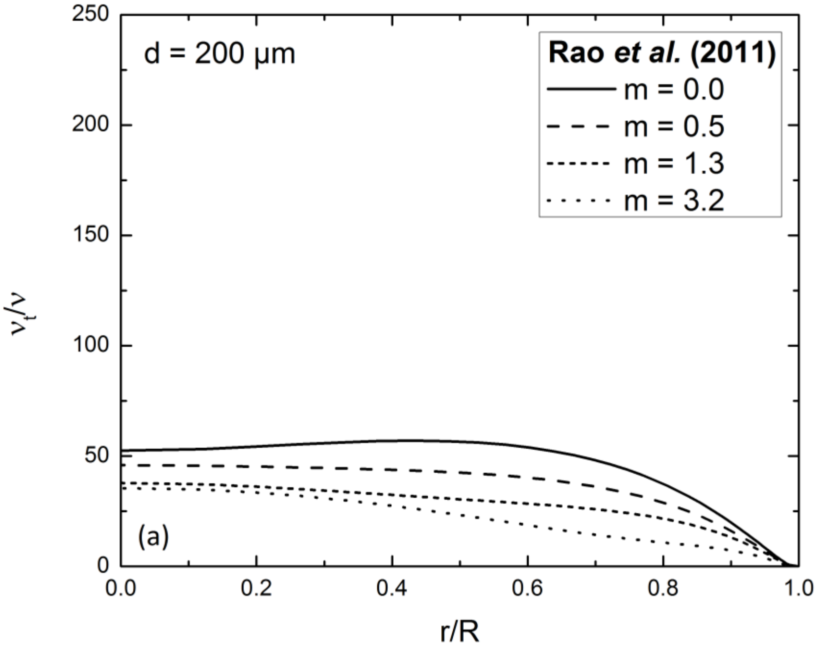
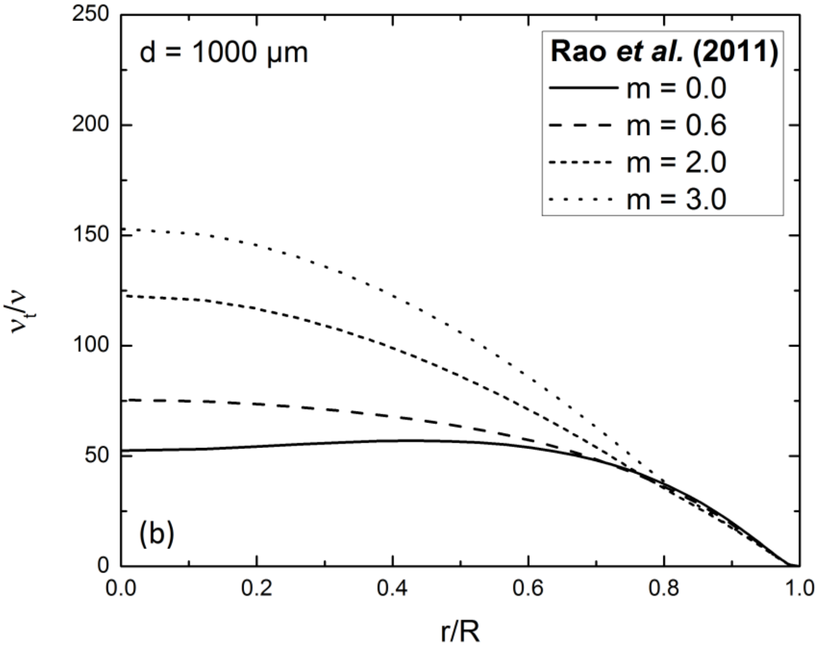
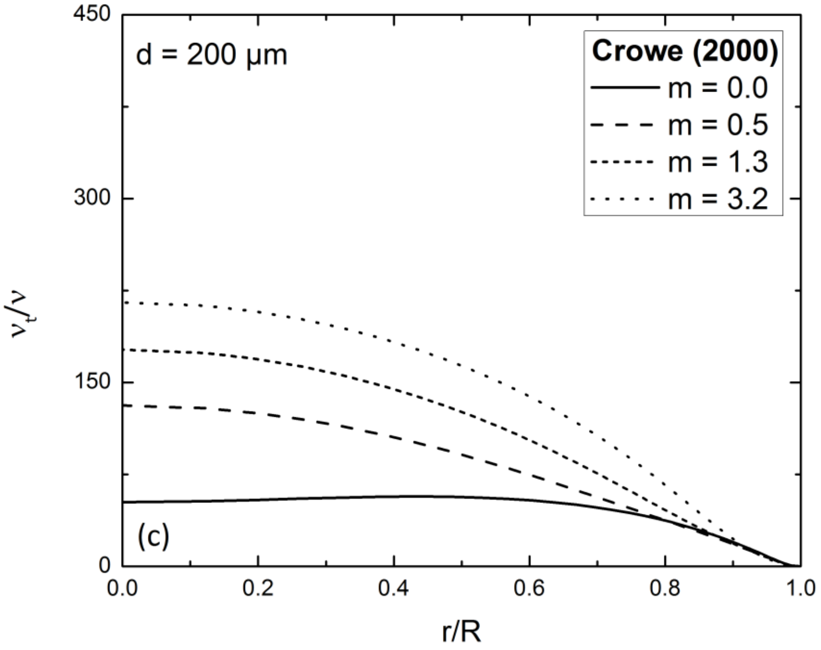


Figure 2.3 Prediction of the radial gas velocity fluctuation




d = 1000 μm
Rao et al. (2011)
m = 0.0
m = 0.6
m = 2.0
m = 3.0
(b)
ν_t/ν
r/R
0
50
100
150
200
250
0.0
0.2
0.4
0.6
0.8
1.0


d = 200 μm
Crowe (2000)
m = 0.0
m = 0.5
m = 1.3
m = 3.2
(c)
ν_t/ν
r/R
0
150
300
450
0.0
0.2
0.4
0.6
0.8
1.0

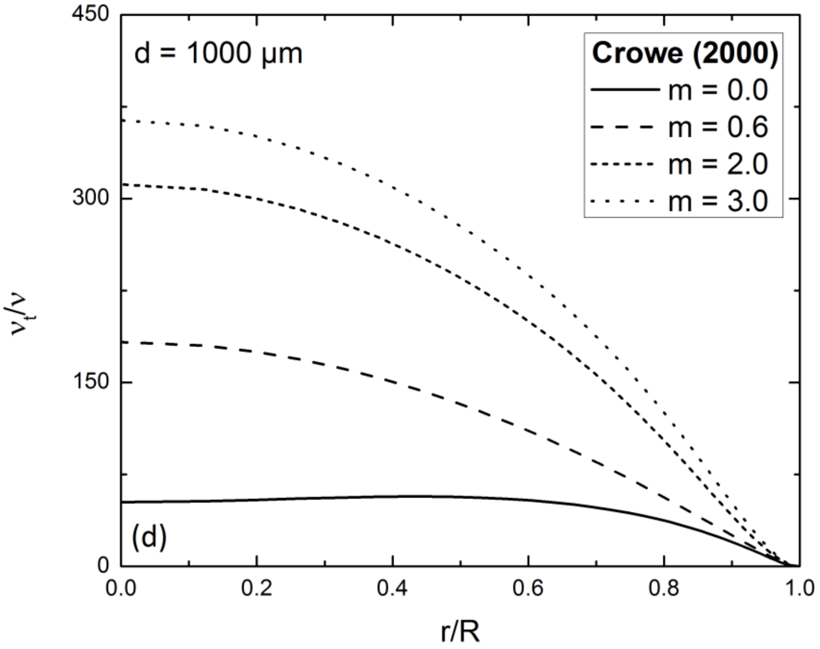
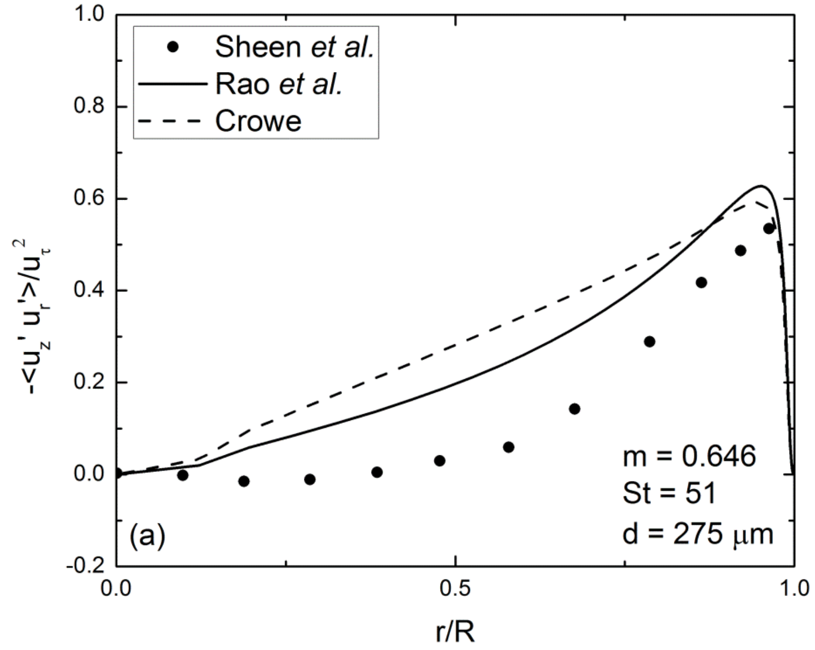


Figure 2.4 Prediction of the turbulent viscosity



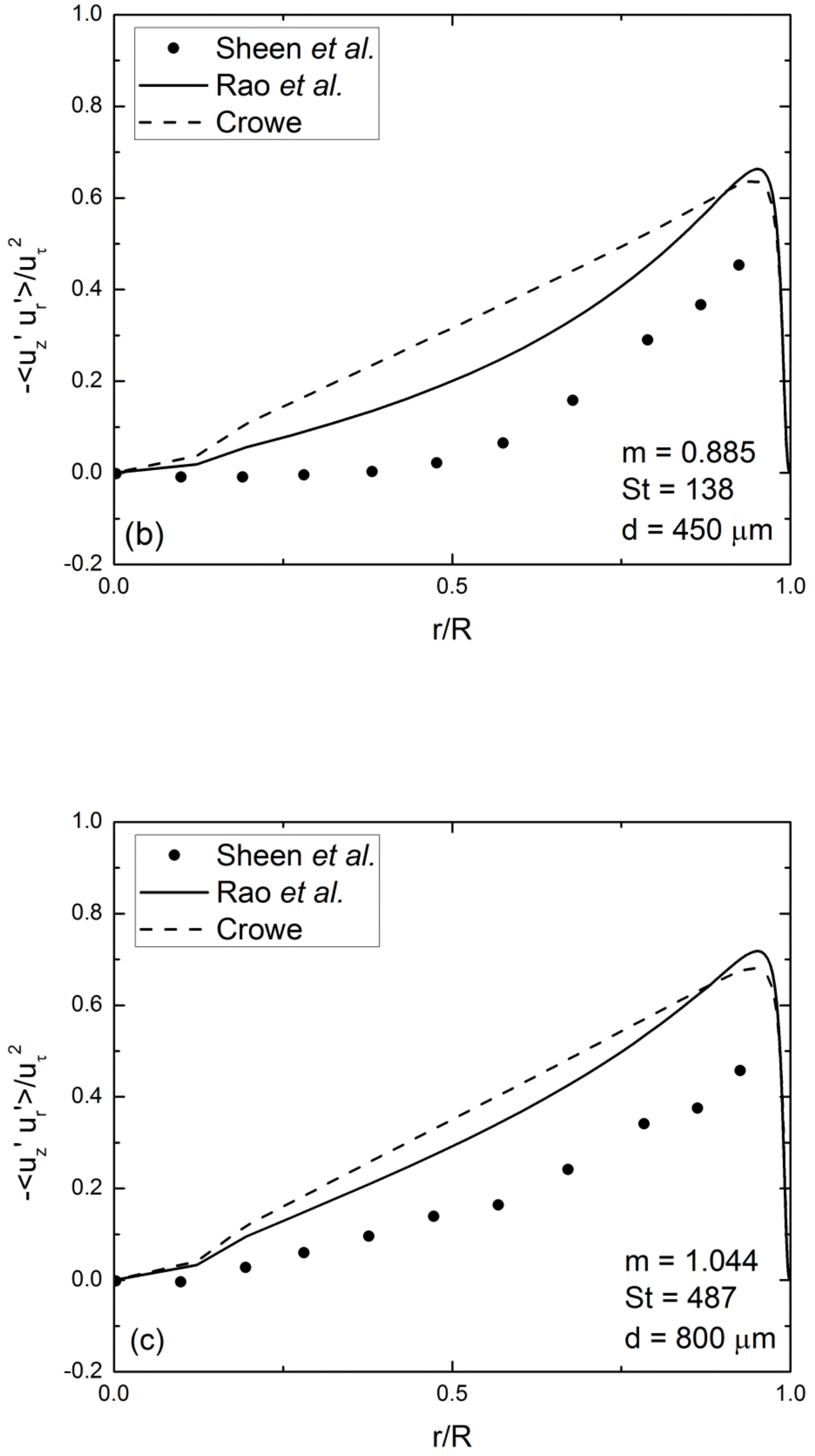
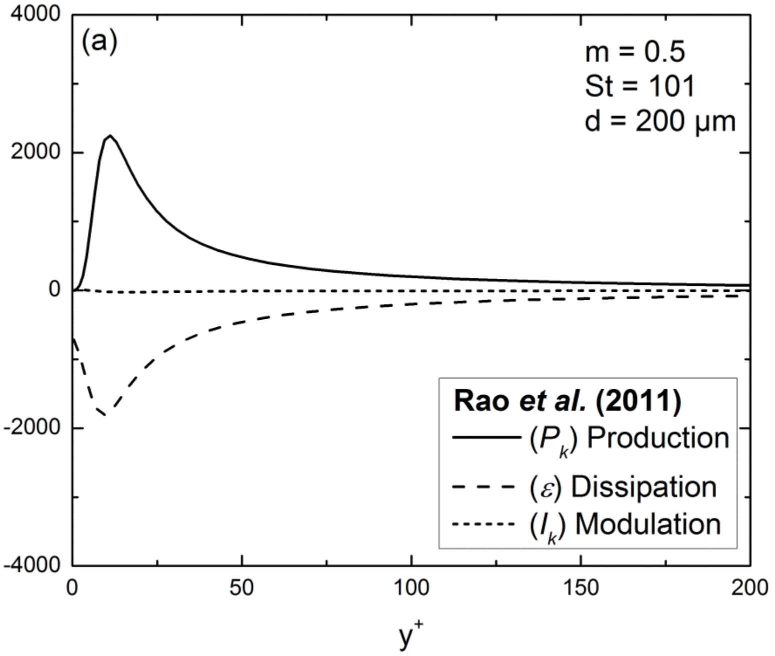
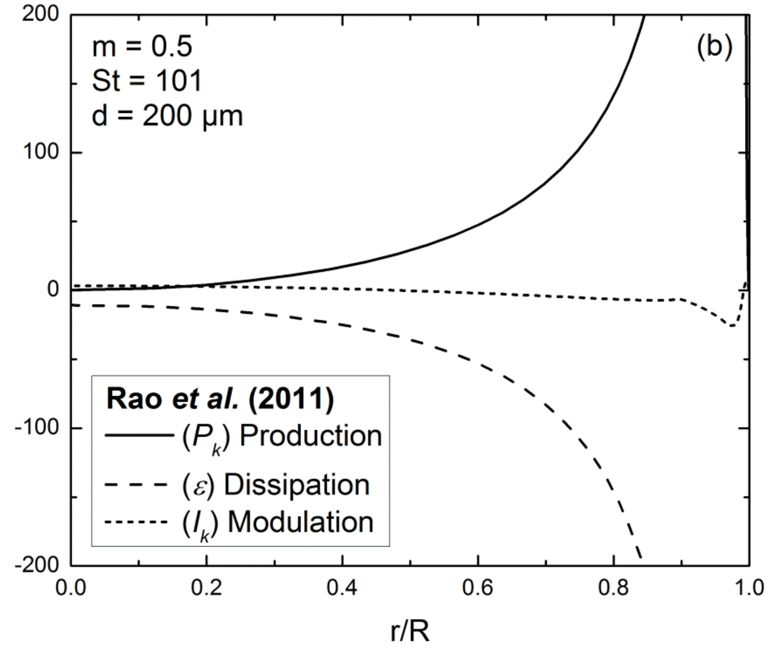


Figure 2.5 Prediction of the Reynolds shear stress


(a)
m = 0.5
St = 101
d = 200 μm
Rao et al. (2011)
(P_k) Production
(ε) Dissipation
(I_k) Modulation
4000
2000
0
-2000
-4000
0
50
100
150
200
y^+


(b)
m = 0.5
St = 101
d = 200 μm
Rao et al. (2011)
(P_k) Production
(ε) Dissipation
(I_k) Modulation
200
100
0
-100
-200
0.0
0.2
0.4
0.6
0.8
1.0
r/R

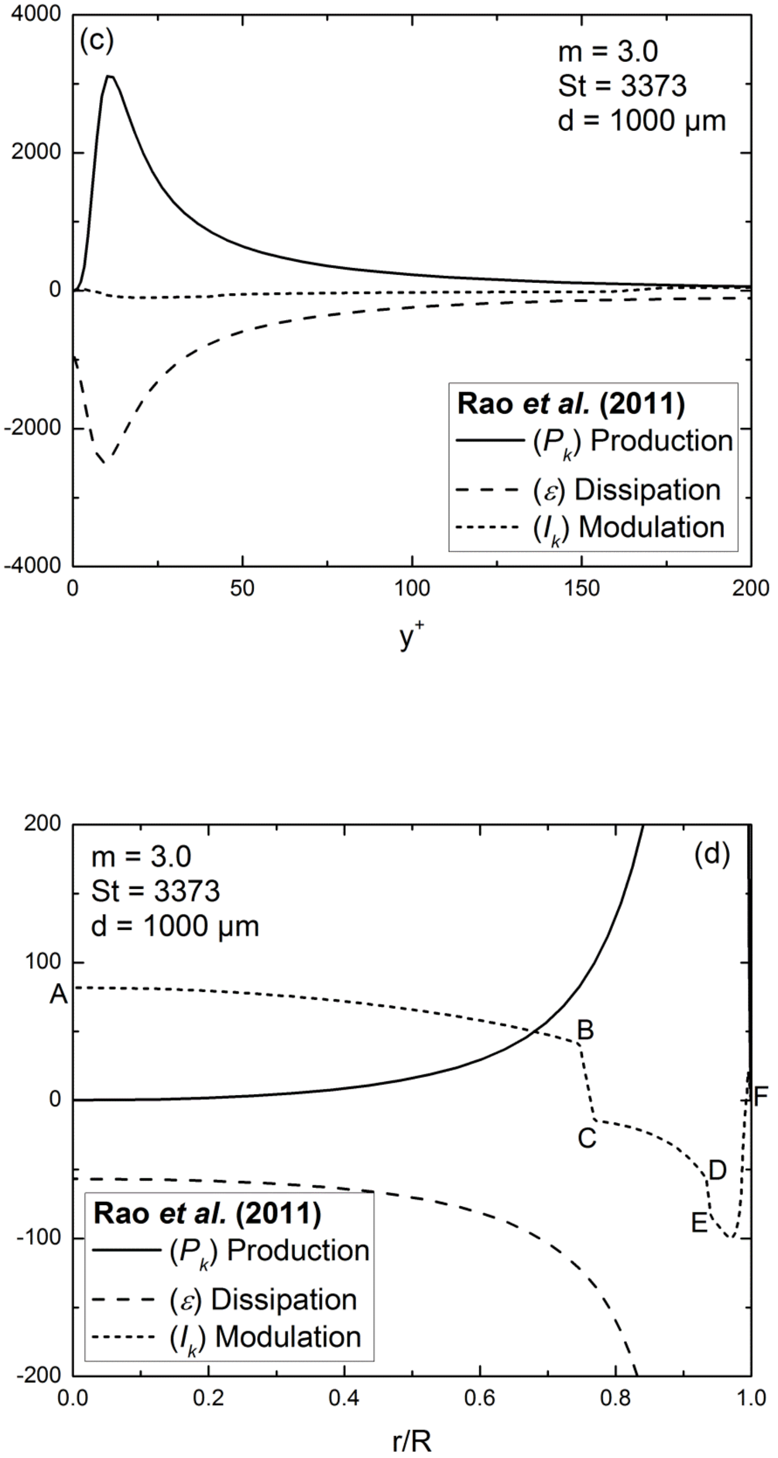
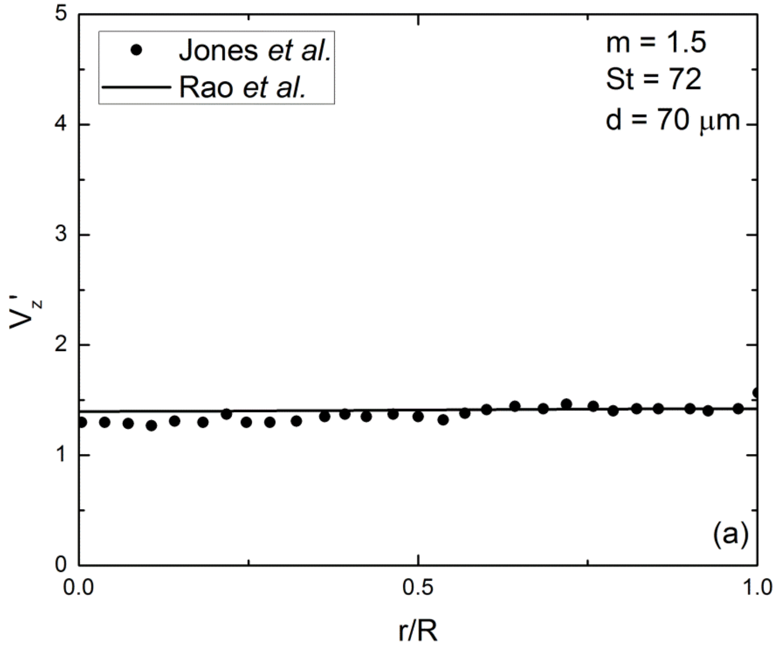
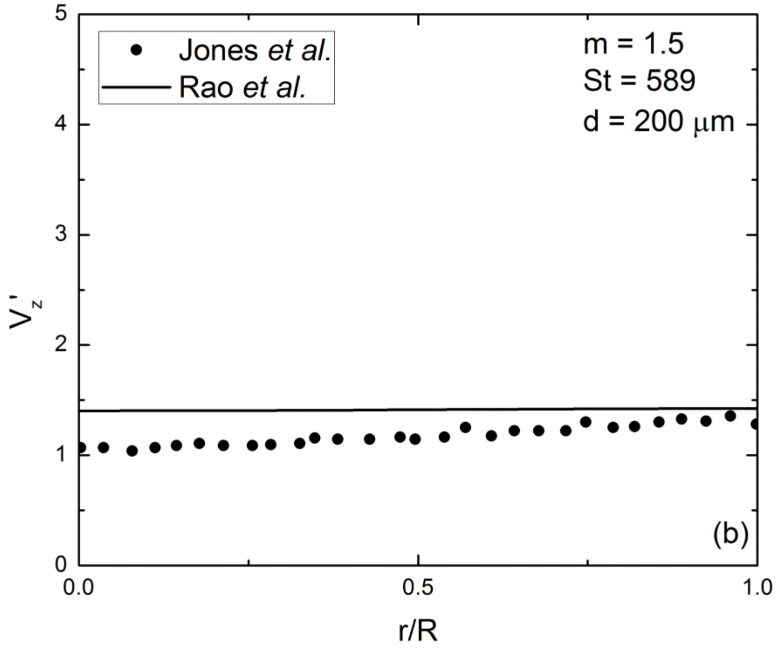


Figure 2.6 Prediction of the source terms of the k-equation


Jones *et al.*
Rao *et al.*
m = 1.5
St = 72
d = 70 μm
V_z'
r/R
0
1
2
3
4
5
0.0
0.5
1.0
(a)


Jones *et al.*
Rao *et al.*
m = 1.5
St = 589
d = 200 μm
V_z'
r/R
0
1
2
3
4
5
0.0
0.5
1.0
(b)

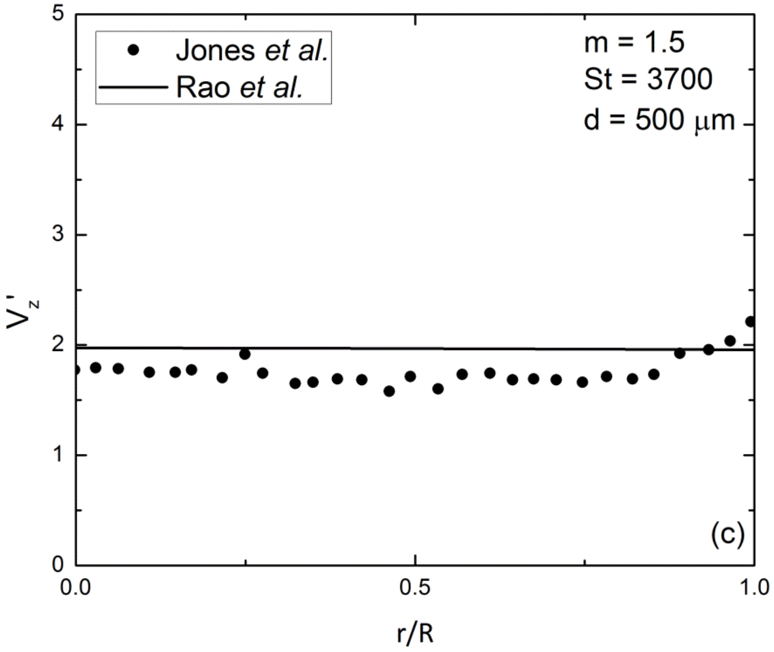
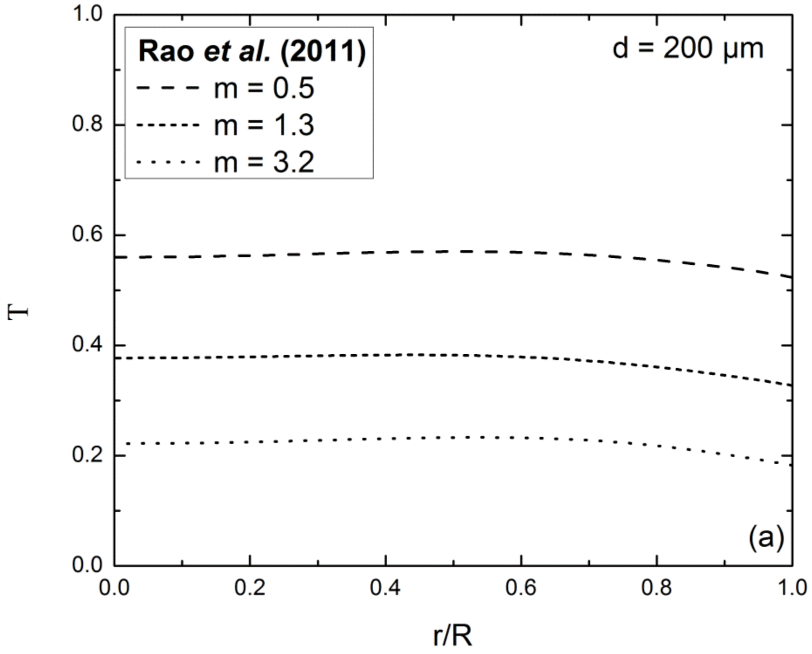


Figure 2.7 Prediction of the solids velocity fluctuation



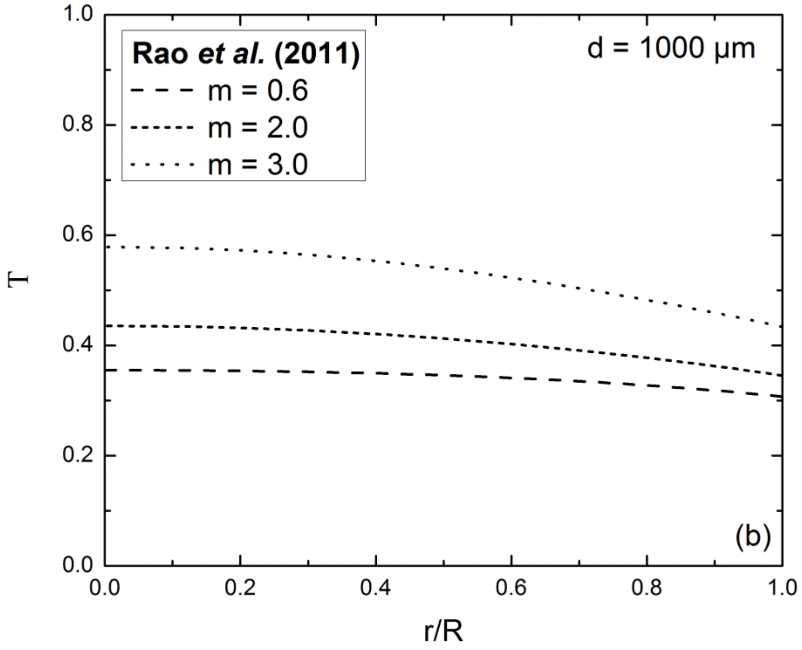
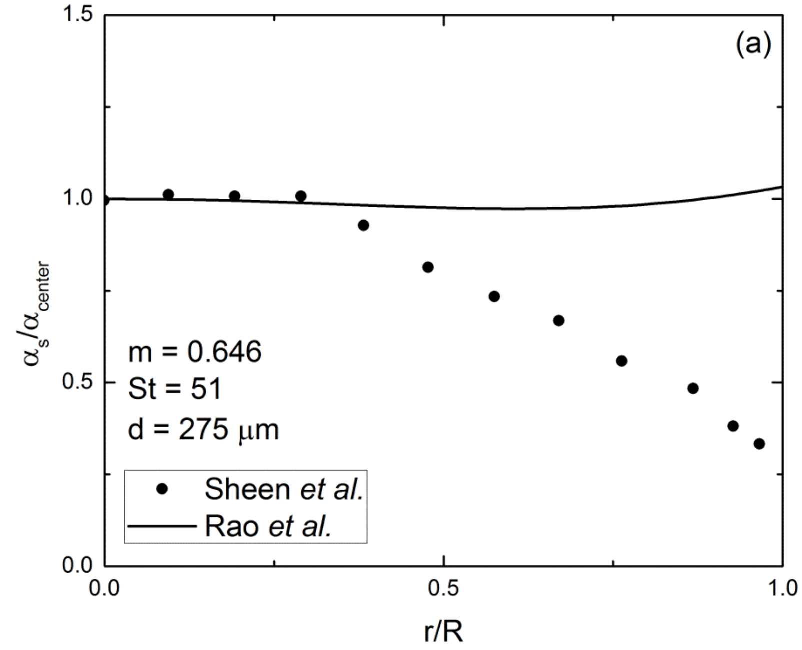
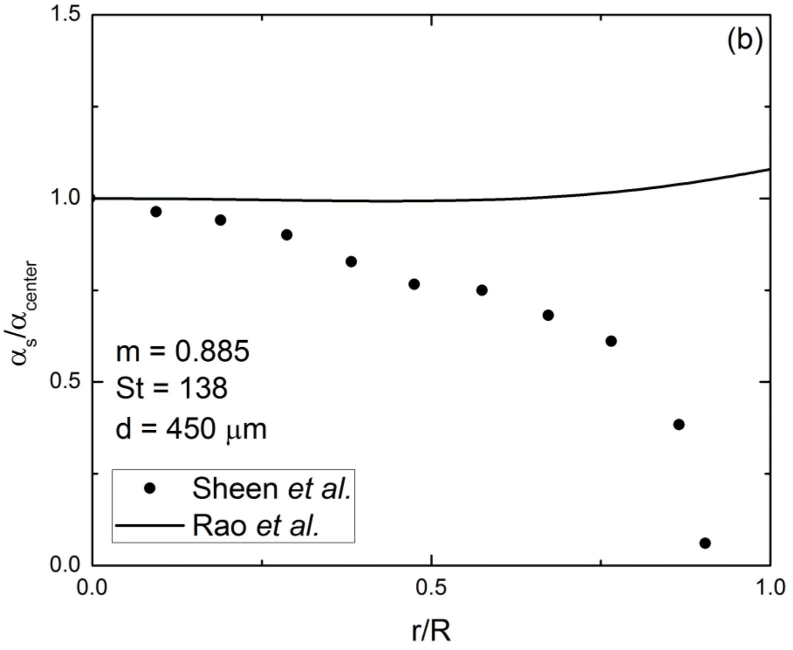
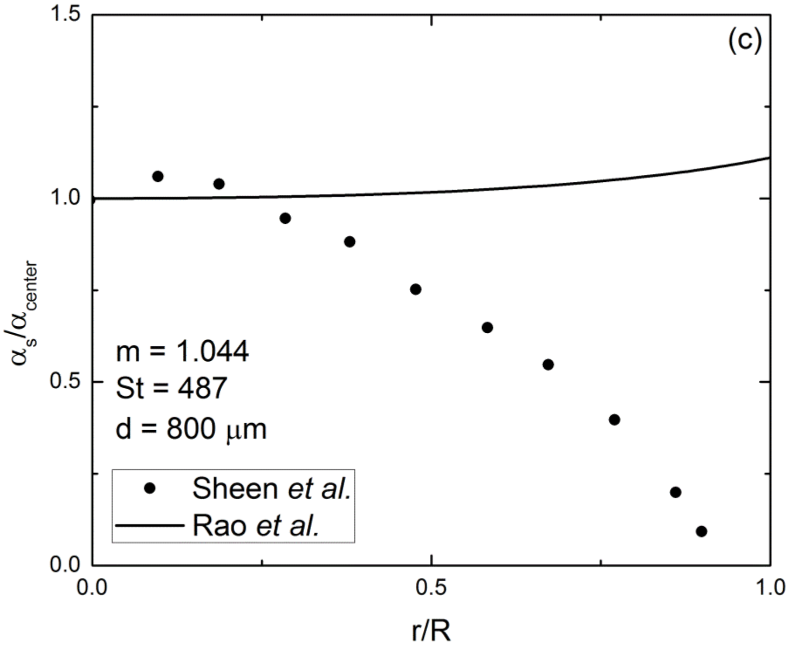


Figure 2.8 Prediction of the granular temperature




(b)
1.5
1.0
0.5
0.0
α_s/α_{center}
m = 0.885
St = 138
d = 450 μm
● Sheen *et al.*
Rao *et al.*
0.0
0.5
1.0
r/R


(c)
1.5
1.0
0.5
0.0
α_s/α_{center}
m = 1.044
St = 487
d = 800 μm
● Sheen *et al.*
Rao *et al.*
0.0
0.5
1.0
r/R

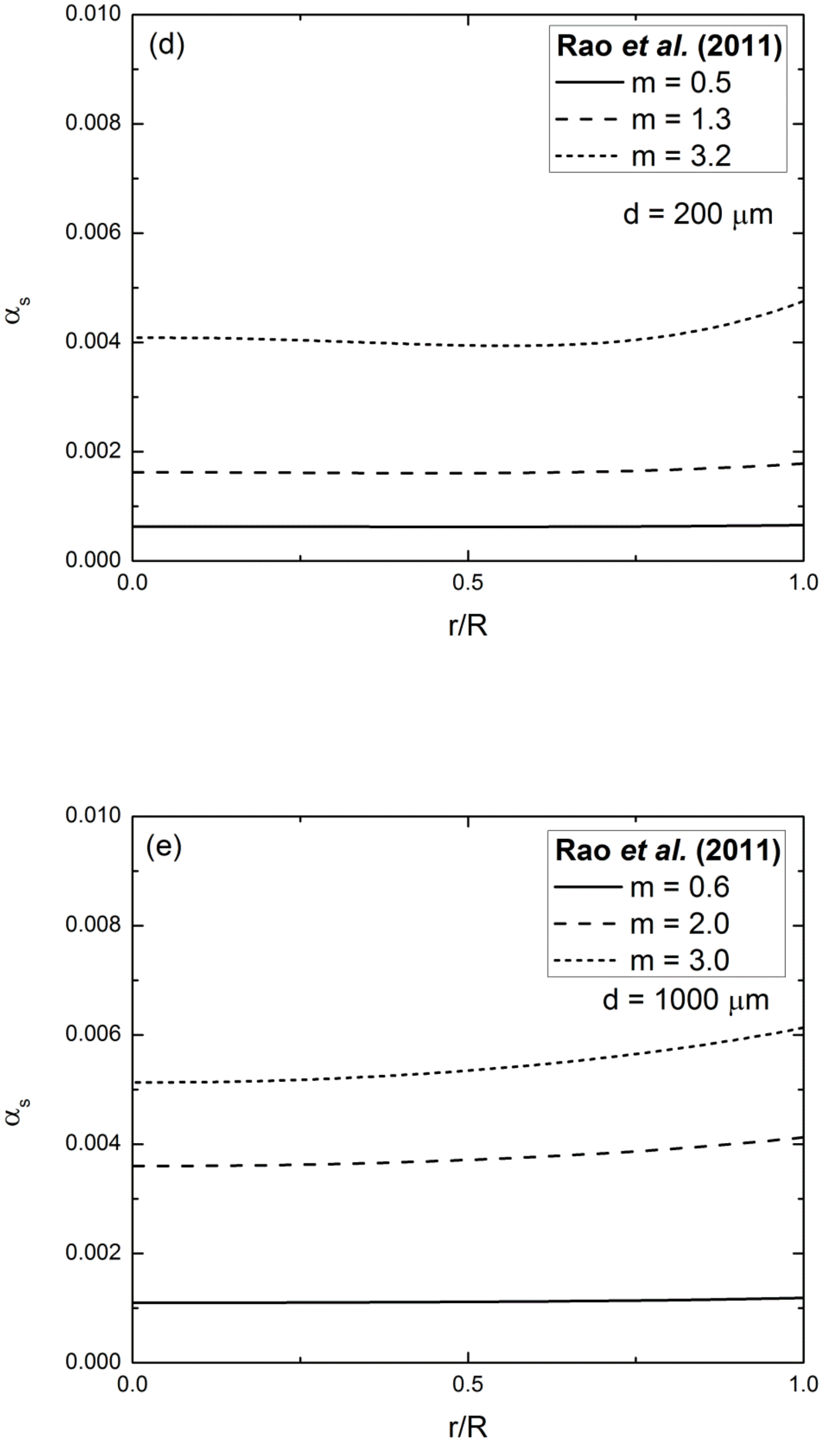


Figure 2.9 Prediction of the solids volume fraction

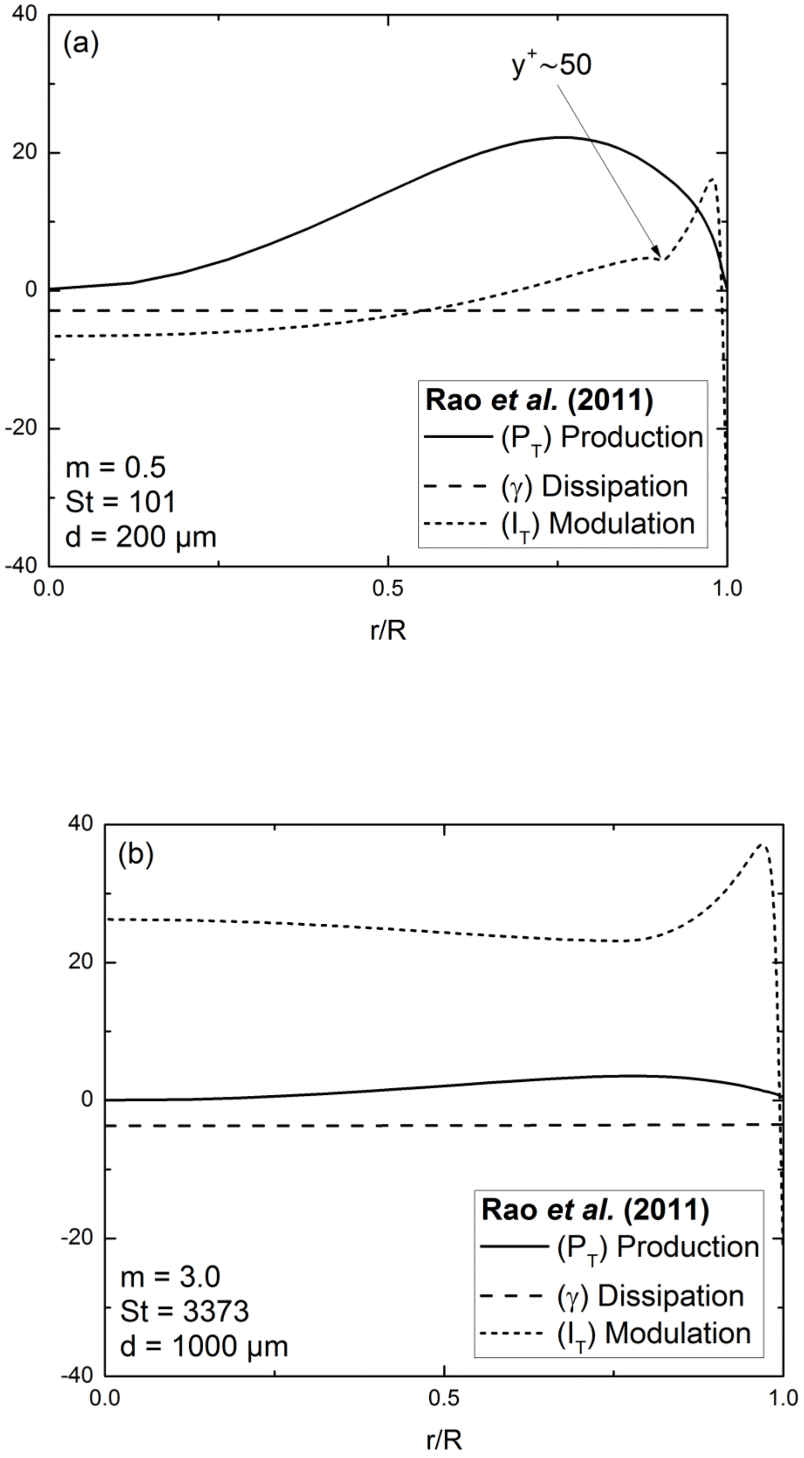


Figure 2.10 Prediction of the source terms of the T-equation

CHAPTER THREE

IMPLEMENTATION OF TWO-FLUID MODEL FOR DILUTE GAS-SOLID FLOW IN PIPES WITH ROUGH WALLS

Submitted as

Zaman, A. and Bergstrom, D. J., Implementation of two-fluid model for dilute gas-solid flow in pipes with rough walls, J. Fluids Engineering, 2012

Contribution of this chapter to the thesis

The research work presented in this chapter focuses on the third objective of the thesis. More specifically, the chapter analyzes the effects of surface roughness on the fluctuating velocity fields of both the solids and the fluid phase with special attention to the effect of roughness on the turbulence modulation terms.

ABSTRACT

A numerical study was carried out to investigate the performance of a two-layer model for predicting turbulent gas-particle flows in rough pipes. An Eulerian-Eulerian two-fluid formulation was used to model both the gas and solid phases for turbulent gas-particle flow in a vertical tube. The stresses developed in the particle phase were calculated using the kinetic theory of granular flows while the gas-phase stresses were described using an eddy viscosity model. The two-fluid model typically uses a two-equation $k - \varepsilon$ model to describe the gas phase turbulence, which includes the suppression and enhancement effects due to the presence of particles. For comparison, a two-layer model was also implemented, since it has the capability to include surface roughness. The current study examines the predictions of the two-layer model for both clear gas and gas-solid flows in comparison to the results of a conventional low Reynolds number model. The paper specifically documents the effects of surface roughness on the turbulence kinetic energy and granular temperature for gas-particle flow in both smooth and rough pipes.

Keywords - turbulence model, gas-solid flow, two-fluid model, turbulence modulation, surface roughness

3.1 Introduction

The transport of solid particles in turbulent flows occurs in many industrial and environmental processes, *e.g.*, bubbly flows, pneumatic transport, slurry transport, fluidized beds, dust and particle-exhaust pollutant control systems. Predicting the overall performance of these systems depends on the development of effective models for gas-particle flows. The two-fluid formulation is a popular approach for modelling gas-particle flows that describes the motion of both phases in an Eulerian framework and is applicable to large systems of particles. However, a comprehensive model that can accurately predict a wide range of two-phase flows is not yet available. In particular, the modification of the gas-phase turbulence due to the presence of the dispersed phase, which is historically known as turbulence modulation, remains difficult to capture in a general model formulation. The performance of these numerical models depends on the realistic prediction of the flow properties in the near-wall region.

A gas-particle flow responds to turbulence in a complex manner. The effect of particles on the gas-phase turbulence is often quantified by the Stokes number which is the ratio of the particle response time and a fluid time scale. The particle response time scale is the time required for a single particle to achieve 65% of the fluid velocity from rest and the fluid time scale is often simply the ratio of the diameter of the pipe and bulk fluid velocity. Particles tend to cause suppression in the gas-phase turbulence when the Stokes number or the particle response time is very small. Each particle contributes to the overall attenuation of turbulence, hence, the effect increases with mass loading. The inertia of the particles plays a significant role for flows with high Stokes numbers where vortex shedding exists. The wake formed by large particles acts as an additional source of turbulence in the gas-phase and the flow becomes a three-way coupled system. When the particle-particle and particle-fluid collisions also become significant, then a four-way coupled model is necessary (Elghobashi, 1994). The two-fluid model typically uses a two-equation $k - \varepsilon$ model that includes a turbulence modulation term to capture the effects of the presence of particles. Crowe (2000) first attempted to develop a modulation term that is capable of predicting the turbulence augmentation for high Stokes number flows. Recently, Rao *et al.* (2011) developed a comprehensive turbulence modulation model and the current study adopts their model.

The low Reynolds number (LRN) turbulence model has been extensively used by researchers together with the two-fluid model for the numerical prediction of gas-solid flows. However, the inability to include the effect of surface roughness limits the application of this near-wall model. An alternative model formulation that could include roughness effects would be a useful modelling technique for practical turbulent gas-solid flows, many of which include rough surfaces. Chen and Patel (1989) proposed a two-layer model for high Reynolds number flows that uses two algebraic equations in the inner-layer for the eddy viscosity and dissipation rate. The model was subsequently modified to include the effects of surface roughness by Durbin *et al.* (2001). Although a low Reynolds number turbulence model was used in the study of Rao *et al.* (2011), the two-layer model remains an equally capable near-wall modelling approach that possesses the additional feature of including the effect of surface roughness. Note that the two-layer model was initially developed for simulations of high Reynolds number turbulent flows. It is not as effective as the low Reynolds number $k - \varepsilon$ model for predicting turbulent transport in low Reynolds number flows.

Numerous studies have been performed to analyse the effect of roughness on the motion of particles in turbulent flows. Fan and Ahmadi (1993) developed a sublayer model for deposition of spherical particles from turbulent air steams in vertical ducts with smooth and rough surfaces. They modified the boundary condition for the particle capture trajectories to include the effect of surface roughness and obtained reasonable agreement with the experimental data. Li and Ahmadi (1993) carried out a numerical study to model the deposition of aerosol particles in a horizontal channel with a rough surface. Later, Zhang and Ahmadi (2000) validated the sublayer model of Fan and Ahmadi (1993) for aerosol transport and deposition in vertical and horizontal turbulent duct flows using direct numerical simulations.

Huber and Sommerfeld (1998) presented numerical predictions for dispersed gas-solid flows in different pipe elements (horizontal channel, pipe bends and vertical pipes) for a wide range of Reynolds number and mass loading. They adopted the Eulerian-Lagrangian approach for particle motion and compared the predictions of the mean and fluctuating velocity fields for both smooth and rough surfaces with measurements by phase Doppler anemometry (PDA). Lain *et al.* (2002) used their PDA measurements for a wide range of particle size and mass loading to validate a particle trajectory model in four-way coupled systems where particle-particle and particle-fluid interactions become significant, and also documented the effect of surface roughness in horizontal channels. Later, Sommerfeld (2003) adopted a stochastic approach to model the particle-particle collisions for turbulent gas-solid flows in a horizontal channel with a rough surface and presented a detailed analysis of the particle behaviour for different boundary conditions. His model assumed that each particle collides with its fictitious collision partner and used a prescribed turbulence for the gas-phase which is decoupled from the particle motion. The second part of the study (Sommerfeld and Kussin, 2003) presents the mean and fluctuating velocity fields of the particle-phase. Subsequent studies, such as those of Lain and Sommerfeld (2008) and Lain and Sommerfeld (2012), analyse the effects of surface roughness on different flow properties for pneumatic transport of particles in horizontal channels.

In this paper, the model predictions of the two near-wall turbulence models are compared for both clear gas and gas-particle flows. To begin, the current study will examine the performance of the two-layer model in the simulation of gas-particle turbulent flows in a smooth pipe and compare the results to those using a low Reynolds number model. The two-fluid model of Rao *et al.* (2011) was adopted for implementation, since it has the ability to at least partially capture the effect of particles on the gas-phase turbulence. In the second part of the paper, the two-layer model will be used to study gas-solid flow in a rough pipe. Note that the granular temperature, *i.e.*, the velocity fluctuation of the particle-phase, is also amplified by the surface roughness and acts as another source of turbulence in the gas-phase. The roughness of the pipe also modifies the particle-wall collisions and enhances the particle velocity fluctuation. Of special interest in the current study is the effect of roughness on the turbulence kinetic energy and granular temperature fields, as well as the turbulence modulation terms that couple them.

3.2 Methodology

3.2.1 Two-Fluid Model

Bolio *et al.* (1995) modified the two-fluid model of Louge *et al.* (1991) for steady fully-developed dilute gas-particle flow in a vertical pipe by introducing a low Reynolds number two-equation $k - \varepsilon$ model for the gas-phase turbulence. The kinetic theory proposed by Lun *et al.* (1984) was used for the particle-phase stresses where the Boltzmann equation was used to solve the velocity distribution function of the particle motion and binary collision methodology was employed for the inelastic particle-particle collisions. The concept of granular temperature was used to describe the velocity fluctuation of the particle-phase and the volume fraction of the particle-phase was determined using the prediction for the granular kinetic energy.

An eddy viscosity model was used for the Reynolds shear stress that uses the turbulent viscosity predicted by the turbulence model. The expression of Batchelor and Green (1972) was used to incorporate the effect of particles on the effective viscosity of the gas-phase. The transport equations for the mean velocities of both phases are coupled by the drag term. The model of Wen and Yu (1966) was used to determine the interfacial force of the gas-particle flow.

Various models have been proposed for the turbulence modulation term which represents the coupled drag of the fluctuating velocities of both phases. These fluctuating interaction terms are a function of the correlation of the velocity fluctuations of both phases. Zhang and Reese (2003) used the model of Crowe (2000) and obtained limited agreement with the experimental data. Recently, Rao *et al.* (2011) developed a set of modulation terms based on a convection heat transfer analogy for the turbulence kinetic energy and granular temperature equations. Their model uses the expression proposed by Lun (2000) for the wake generation in flows with large particles. The wake term, which becomes activated when the particle Reynolds number of the flow exceeds 150, behaves as an additional source of turbulence generation. Their model equations are as follows:

$$I_k = -\frac{\rho_g}{\tau_{sg}}\left(2k - k_{sg}\right) + E_w \tag{3.1}$$

$$I_T = \frac{\rho_g}{\tau_{sg}}\left(k_{sg} - 3T\right) \tag{3.2}$$

where τ_{sg} is the time scale over which the transfer of energy occurs. When the Stokes number of the flow is less than 100, the model adopts a drag time-scale, (τ_D); otherwise, it uses a collision time-scale (τ_c). E_w is the wake effect and the relevant equations are given in Table 1. The expression of Sinclair and Mallo (1998), which is a simple geometric mean of the fluctuating velocities of both phases,

$$k_{sg} = \sqrt{6kT} \tag{3.3}$$

was used in the current study. The complete set of governing and constitutive relations for fully-developed gas-solid flow in a cylindrical geometry are summarized in Table 3.1.

3.2.2 Near-Wall Turbulence Models

The low Reynolds number model is designed to carefully modify the turbulence parameters in the near-wall region by employing three damping functions. The ability to predict the near-wall behavior accurately for a wide range of turbulent flows makes the low Reynolds number model a popular choice of turbulence model, and it has been widely used in the two-fluid model formulation. The current study adopts the formulation proposed by Myong and Kasagi (1990), which is summarized in Table 3.2.

The two-layer model of Chen and Patel (1989), on the other hand, uses algebraic equations for both the dissipation and turbulent viscosity in the so-called inner-layer and relaxes to the standard form of the $k - \varepsilon$ model and turbulent viscosity in the outer-layer. For a smooth wall the inner layer includes the laminar sub-layer, the buffer layer and a part of the fully turbulent layer. Later, Durbin *et al.* (2001) modified the model to include the effects of surface roughness. The model equations are summarized in Table 3.2.

3.3 Results and Discussion

All the transport equations of the one-dimensional gas-solid flow were discretized using the finite volume method of Patankar (1980). A thorough analysis using different numbers of control volumes indicated that sixty control volumes is appropriate for the range of Reynolds numbers considered in the current study. Bolio *et al.* (1995) used a similar size grid in their study. For the current simulations, a non-uniform grid of sixty control volumes was used with refinement near the wall. The set of coupled transport equations was solved iteratively using an implicit tri-diagonal matrix algorithm. The local solids volume fraction was solved from an algebraic relation representing a constant normal particle stress based on the granular temperature field, which in turn was predicted from its own transport equation. The overall solution process was iterated until the results matched the centerline fluid velocity and mass loading of the experimental study. The current study adopted the boundary condition proposed by Johnson and Jackson (1987) for the particle velocity and granular temperature at the wall. Symmetry boundary conditions were used at the pipe centerline.

The near-wall turbulence models of Durbin *et al.* (2011) and Myong and Kasagi (1990) were compared for gas-solid flow predictions based on the experimental data of Tsuji *et al.* (1984) and Sheen *et al.* (1993). Next, the two-layer model (Durbin *et al.*, 2001) was used to predict the flow properties for a fully-rough wall condition. The fully-rough wall was defined by selecting the value of the so called hydrodynamic roughness length, y_o , to give an effective sand-grain roughness of

$$h_r^+ = \frac{h_r u_\tau}{\nu_g} = 90 \tag{3.4}$$

where, h_r is the average roughness height and u_τ is the friction velocity. Durbin *et al.* (2001) noted that the turbulence kinetic energy is not zero at the wall for a fully-rough surface as the origin of the wall normal coordinate (y) shifts due to roughness, and the boundary condition of the kinetic energy is interpolated between the smooth and fully-rough conditions. The effect of surface roughness on the mean and fluctuating velocity fields of both phases was analyzed for the same Reynolds number as the experimental studies which used a smooth surface. Note that the rough surface requires a higher pressure drop due to higher friction at the wall. The pressure gradient was adjusted for flows in fully-rough pipes to achieve the same Reynolds number as the smooth pipe flow. The normalized wall normal distance (y^+) for fully-rough pipes is calculated in the following manner:

$$y^+ = \frac{u_\tau y_{eff}}{\nu_g} = \frac{u_\tau (y + y_0)}{\nu_g} \tag{3.5}$$

3.3.1 Prediction for Clear Gas Flows

Figure 3.1 shows the normalized gas mean velocity (u^+)[‡‡] profiles for a Reynolds number of $Re = 22500$. As expected, the predictions generated by the low-Reynolds-number and two-layer models for flows in a smooth pipe are almost identical. For the rough pipe, lower values of u^+ were predicted by the two-layer model due to the higher friction velocity of the rough surfaces. Unlike the prediction for the smooth pipe, the velocity profile for the rough pipe is approximately logarithmic, even close to the wall.

[‡‡] $u^+ = u/u_\tau$

Figure 3.2 gives the normalized turbulence kinetic energy (k^+)[§§] prediction of the low Reynolds number and two-layer turbulence models. Although both models accurately predicted a peak near the wall, the magnitude is higher for the low Reynolds number model. The two-layer model was also used to predict the turbulence kinetic energy in a rough pipe. The two-layer model indicates that a rough surface causes enhanced turbulence in the core section of the pipe; however the peak value of the turbulence kinetic energy is lower and flatter than that for a smooth pipe.

The two-layer model uses an algebraic equation for the calculation of the eddy viscosity in the inner-layer and switches to the conventional high Reynolds number expression in the outer-layer. The eddy viscosity profile shown in Figure 3.3 reveals a discontinuity in slope in the region of the patching point, *i.e.*, when the damping function, $\left[1 - e^{-(R_y/A_\nu)}\right] = 0.95$ (Durbin *et al.*, 2001). The discontinuity becomes more pronounced as the Reynolds number decreases.

Notwithstanding small differences in the vicinity of the patching point, overall the prediction of the two-layer model is in good agreement with that of the low-Reynolds-number model. Although the peak value of the turbulence kinetic energy predicted by the two-layer model is lower than that of the low-Reynolds-number model, the magnitude of the eddy viscosity is slightly higher in the inner-layer region for the two-layer model. The magnitude of the eddy viscosity is much higher for the fully-rough pipe in the core region, mostly due to the enhanced level of turbulence kinetic energy in that region.

[§§] $k^+ = k/u_\tau^2$

Figure 3.4 presents the profile of the normalized Reynolds shear stress ($-< u'v' >^{+}$)[***] for smooth and rough pipes. The profiles predicted by both turbulence models for the smooth pipe are nearly indistinguishable. However, the Reynolds shear stress (normalized by the friction velocity) predicted by the two-layer model for the rough pipe is higher than that in the smooth pipe especially in the near-wall region. Although the friction velocity is significantly higher for the fully rough pipe, the elevated magnitude of eddy viscosity results in higher values of the normalized Reynolds shear stress.

3.3.2 Prediction for gas-particle flows

Figure 3.5 shows the mean velocity profiles for both the gas and solid phases for flows in smooth pipes. The predictions of the low Reynolds number and two-layer models are almost identical. As such the two-layer model is capable of predicting the mean gas velocities for flows with both small (Figure 3.5a) and large (Figure 3.5b) particles that are in good agreement with the experimental data. For the smaller particle (Figure 3.5a) the slip velocity is over-predicted compared to the experimental measurement of Sheen *et al.* (1993).

The gas-phase fluctuating velocity in the axial direction was calculated from the turbulence kinetic energy. It was assumed that the velocity fluctuations in the radial and azimuthal direction are approximately the same and equal to one-half of the fluctuation in the axial direction (Sheen *et al.*, 1993) *i.e.*, $u'_r = u'_\theta = \frac{u'_z}{2}$. This assumption then gives:

$$u'_z = \sqrt{\frac{4k}{3}} \tag{3.6}$$

[***] $-< u'v' >^{+} = -< u'v' >/u_\tau^2$

For the smooth wall case, Figures 3.6 (a) and (b) show that the two-layer model over-predicts the gas-phase velocity fluctuation for both particle flows. The measurements indicate turbulence suppression in both cases which is only partially captured by the low Reynolds number model. Surprisingly, the deviation in the two profiles occurs mostly in the outer-layer region where both models use the same form of the $k - \varepsilon$ model. In contrast to the single-phase case, the peak of the turbulence kinetic energy and hence value of u' predicted by both models is the same near the wall.

Typically turbulence kinetic energy tends to increase with surface roughness. For the rough pipe, the two-layer model predicted a significantly higher level for the streamwise velocity fluctuation. For the flow with 500 μm particles [Figure 3.6 (b)], the mean velocity of the particle phase (not shown) is decreased when a fully-rough surface is introduced. As the slip velocity increases, the wake term is activated in the turbulence modulation model of Rao *et al.* (2011), which then creates an additional enhancement in the core region of the pipe.

Figure 3.7 (a) shows the eddy viscosity profiles for flows with smaller particles when the turbulence in the gas-phase is attenuated as measured by Sheen *et al.* (1993). Figure 3.7 (b) shows the eddy viscosity profiles for flows with larger particles when turbulence is enhanced. In Figure 3.7 (a), apart from the discontinuity in the near-wall region, the predictions obtained by the low Reynolds number and two-layer model are similar for smooth-pipe flow. In Figure 3.7 (b), also for a smooth pipe, the two-layer model predicts a much higher value of the eddy viscosity compared to the low Reynolds number model, especially in the core region of the pipe.

Figures 3.7 (c) and (d) illustrate the effects of particles on the prediction of the eddy viscosity by the two-layer model for flows in fully-rough pipes. The phenomenon that small particles suppress the gas-phase turbulence still persists for flows in rough pipes as shown in Figure 3.7 (c). Figure 3.7 (d) represents the turbulence enhancement in the gas-phase due to the presence of large particles in fully-rough pipes. Both the wake term in the turbulence modulation formulation of Rao *et al.* (2011) and the surface roughness are responsible for the augmentation of turbulence for this flow.

Figure 3.8 shows the granular temperature predicted by the turbulence models for flows with both small and large particles. Figure 3.8 (a) shows that the two-layer model gives similar predictions to that of the low-Reynolds-number model for flows with small particles. This figure also indicates that the granular temperature increases approximately 60% for a fully-rough surface. The enhanced gas-phase turbulence kinetic energy in rough pipes acts as a generation term in the granular temperature equation. As the wake term in the turbulence modulation model of Rao *et al.* (2011) does not become activated, the cross-correlation term is responsible for enhancing the granular temperature.

A similar behavior is observed for the flows with larger particles as shown in Figure 3.8 (b). The phenomenon of large particles enhancing the gas-phase turbulence is well captured by the model of Rao *et al.* (2011) which results in an additional source term in the granular temperature equation. The particle granular temperature is further increased up to approximately 70% for flows in fully-rough pipes where both the effect of the wake and enhanced turbulence due to roughness is present.

Figure 3.9 presents the profiles of the source terms in the granular temperature transport equation for flows in both smooth and rough pipes. Figures 3.9 (a) and 3.9 (b) consider a flow with small particles. Both the production term and the magnitude of the modulation term are increased for a rough surface as shown by comparing Figure 3.9 (b) to Figure 3.9 (a). The modulation term behaves as a strong source term at the wall and as a sink term in the core section of the pipe. This behavior is consistent for both smooth and rough pipe flows, but the effect intensifies for a rough surface.

The profiles of the source term are presented for flows with large particles in Figures 3.9 (c) and (d): as before the production and modulation terms in these flows are enhanced for a rough surface. The modulation term acts as a strong source term across the entire pipe for flows in both smooth and rough pipes. The activation of the wake term supplies additional energy to the particle phase through the cross-correlation term. The production term is almost negligible compared to the modulation term in flows with large particles which is an indication of the dominant role of the wake term in implementing the effect of the gas-phase turbulence on the particle-phase velocity fluctuation.

The predicted source terms show that the magnitude of the granular energy dissipation due to particle-particle collision is increased in rough pipes for flows with both small [Figure 3.9 (a) and (b)] and large particles [Figure 3.9 (c) and (d)]. At the center of the pipe, the increase is about 90% and 120% for flows with small and large particles, respectively. This suggests that the surface roughness has a global effect on the granular temperature.

All of the results above document effects which are due to the change in the hydrodynamic field for a pipe with rough walls. Sommerfeld and Huber (1995) noted that the frequency of the particle-wall collisions is also enhanced due to roughness at the wall which subsequently augments the velocity fluctuation of the particle phase. As the particle velocity fluctuation acts as a source term in the turbulence kinetic energy equation through the cross-correlation expression, the elevated granular temperature causes a higher gas-velocity fluctuation in the near-wall region.

Recall that the boundary condition for the mean and fluctuating velocity of the particle-phase is a function of the specularity coefficient which quantifies the nature of the particle-wall collision. The boundary condition of Johnson and Jackson (1987) for the granular temperature is:

$$-\lambda \frac{\partial T}{\partial r}\bigg|_{wall} = \frac{\sqrt{3}\rho_s \pi \left(1-e_w^2\right) T^{3/2}}{4\left[\left(\frac{\alpha_0}{\alpha_s}\right)-\left(\frac{\alpha_0}{\alpha_s}\right)^{2/3}\right]} - \frac{\rho_s \pi v^2 \phi \sqrt{T}}{2\sqrt{3}\left[\left(\frac{\alpha_0}{\alpha_s}\right)-\left(\frac{\alpha_0}{\alpha_s}\right)^{2/3}\right]} \qquad (3.7)$$

The first term on the right-hand side of equation (3.7) represents the energy dissipation due to inelastic particle-wall collisions which is dependent on the particle-wall restitution coefficient (e_w). The second term determines the production at the wall which is a function of the specularity coefficient, ϕ. As there are no independent measurements available, different values of ϕ are used in different studies. Following the analysis of Bolio *et al.* (1995), the value of ϕ was set to 0.002 in the current study for all simulations. Zhang and Reese (2003) chose a higher value (0.008) for their study. For the purpose of evaluating the influence of specularity at the wall on the particle model, the value of the specularity coefficient was changed and the effect on the granular temperature was documented for flows in both smooth and rough pipes. The level of production at the wall increases as the magnitude of the specularity coefficient increases. The introduction of wall roughness causes an enhancement of the magnitude of the production term of equation (3.7) as the frequency of the particle-wall collisions increases (Sommerfeld and Huber, 1995).

The non-zero value of the granular temperature at the wall is largely determined by the value of the specularity coefficient as it determines the level of production at the wall. Figures 3.10 (a) and (b) represent the prediction of granular temperature for different values of the specularity coefficient for flows in a smooth pipe, while Figures (c) and (d) consider the prediction for a rough pipe. The velocity fluctuation in the particle phase increases with specularity for all four cases as the magnitude of the production at the wall increases. However, as the value of the specularity coefficient increases, the granular temperature tends to rise in the near-wall region for flows with small particles [Figure 3.10 (a) and (c)]; conversely, it decreases for flows with large particles [Figure 3.10 (b) and (d)]. This suggests that the production at the wall becomes more significant for flows with small particles in both smooth and rough pipes. Figure 3.10 (c) and (d) indicate that the overall magnitude of the granular temperature in rough pipes is increased in both cases for all values of the specularity coefficient.

3.4 Summary and Conclusion

The current study examines the performance of the near-wall turbulence model of Durbin *et al.* (2001) for the prediction of clear gas and gas-solid flows by comparing the results with the prediction of a popular low Reynolds number model. The model predictions were also documented for turbulent flows with small and large particles over a fully-rough surface. The two-layer model is generally able to predict the mean velocities of both phases in fully-developed gas-particle flows. The model predictions for the gas-phase velocity fluctuations follow the experimental trend and show the capability of capturing the effect of particles on gas-phase turbulence. However, the phenomenon that small particle suppress the gas-phase turbulence is better captured by the conventional low Reynolds number model.

The two-layer model was used to predict a turbulent gas-solid flow in a rough pipe. A higher pressure gradient is required to overcome additional friction imposed by a rough surface which subsequently enhances the turbulence kinetic energy of the gas-phase. The velocity fluctuation of the solids-phase is affected by the enhanced level of gas velocity fluctuation through the cross-correlation term of the turbulence modulation expression. Therefore, the turbulence kinetic energy and granular temperature increase for flows over a rough surface. These properties are also affected by the size of the particles. Small particles cause turbulence suppression even in flows over rough surfaces; conversely the large particles enhance the gas-phase turbulence by generating vortex shedding. However, the rough surface also caused a direct enhancement in the granular temperature which is attributed to the fact that the frequency of the particle-wall collision was increased. The increased wall collision frequency enhances the particle-particle collisions in the flow which is evident from the increased level of the granular energy dissipation. The increased interparticle collisions enhance the overall magnitude of the granular temperature in flows over rough surfaces. This augmentation in granular temperature acts as an additional source through the cross-correlation term for the turbulence kinetic energy. Therefore, the gas-phase velocity fluctuation increases when the fully-rough surface is introduced. Recall that small particles attenuate the gas-phase turbulence and large particles enhance it for flows in smooth pipes. Although the gas-phase fluctuation is generally enhanced in rough pipes, the overall effect of the different sized particles remains the same.

The specularity coefficient is a measure of the particle-wall collision frequency which is a function of the volume fraction of the solids-phase. Most studies depend on empirical correlations and use a constant value for the specularity coefficient. The current study showed that the specularity coefficient has a direct effect on the granular temperature for flows in both smooth and rough walls as it determines the nature of the particle-wall collisions. The size of particle also affects the modification in granular temperature due to the change in specularity coefficient. The turbulence kinetic energy of the gas-phase is also subsequently affected by the modification of the granular temperature. However, no information is found in the literature on how the specularity coefficient should change with surface roughness. The use of a constant value for the specularity coefficient independent of flow conditions, although expedient, fails to recognize the subtle influence of such factors such as particle size and surface roughness, on the velocity fluctuations of the particle phase.

Further assessment of numerical models requires a more extensive and complete database for the turbulence kinetic energy and granular temperature in upward flows in vertical pipes with rough walls. The distribution of the solids volume concentration for the particle-phase plays a significant role in determining the velocity fluctuation for both phases. The scarcity of experimental data for gas-solid flows in rough pipes is a major constraint for the validation of the capability of the two-fluid model using the two-layer turbulence model to capture the effects of roughness.

3.7 List of References

Batchelor, G. and Green, J., The determination of bulk stress in a suspension of spherical particle to order of c^2, *J. Fluid Mech.,* Vol. 56, pp. 401–427 (1972)

Bolio, E., Yasuna, J. and Sinclair, J., Dilute turbulent gas-solid flow in risers with particle-particle interactions, *AIChE J.,* Vol. 41, pp. 1375-1388 (1995)

Chen, C.J. and Patel, V.C., Near-wall turbulence models for complex flows including separation, *AIAA J.,* Vol. 26, pp. 641-648 (1988)

Crowe, C. T., On models for turbulence modulation in fluid-particle flows, *Int. J. Multiphase Flow,* Vol. 26, pp. 710-727 (2000)

Durbin, P. A., Medic, G., Seo, J.M., Eaton, J. K. and Song, S., Rough-wall modification of two-layer $k - \varepsilon$ model, *J. Fluids Eng.,* Vol. 123 (1), pp. 16-21 (2001)

Elghobashi, S., On predicting particle laden turbulent flows, *Appl. Sci. Res.,* Vol. 52 (4), pp. 309-329 (1994)

Fan, F. -G. and Ahmadi, G., A model for turbulent deposition of particles in vertical ducts with smooth and rough surfaces, *J. Aerosol Sci.,* Vol. 24, pp. 45-64 (1993)

Huber, N. and Sommerfeld, M., Modelling and numerical calculation of dilute-phase pneumatic conveying in pipe systems, *Powder Technol.,* Vol. 99, pp. 90–101 (1998)

Lain, S. and Sommerfeld, M., Euler/Lagrange computations of pneumatic conveying in a horizontal channel with different wall roughness, *Powder Technol.,* Vol. 184, pp. 76–88 (2008)

Lain, S. and Sommerfeld, M., Numerical calculation of pneumatic conveying in horizontal channels and pipes: Detailed analysis of conveying behaviour, *Int. J. Multiphase Flow,* Vol. 39, pp. 105–120 (2012)

Lain, S., Sommerfeld, M. and Kussin, J., Experimental studies and modelling of four-way coupling in particle-laden horizontal channel flow, *Int. J. Heat Fluid Flow,* Vol. 23, pp. 647–656 (2002)

Li, A. and Ahmadi, G., Deposition of aerosols on surfaces in a turbulent channel flow, *Intl J. Engng Sci.,* Vol. 31, pp. 435-445 (1993)

Louge, M., Mastorakos, E. and Jenkins, J., The role of particle collisions in pneumatic transport, *J. Fluid Mech.,* Vol. 231, pp. 345–359 (1991)

Lun, C., Numerical simulation of dilute turbulent gas-solid flows, *Int. J. Multiphase Flow,* Vol. 26, 1707–1736 (2000)

Lun, C., Savage, S., Jeffery, D. and Chepurniy, N., Kinetic theories for granular flow: inelastic particles in couette flow and slightly inelastic particles in general flow field, *J. Fluid Mech.,* Vol. 140, pp. 223–256 (1984)

Myong, H.K. and Kasagi, N., A new approach to the improvement of turbulence model for wall-bounded shear flows, *JSME Int. J. (II),* Vol. 33 (1), pp. 63–72 (1990)

Patankar, S. V., *Numerical Heat transfer and Fluid Flow,* Hemisphere, New York (1980)

Rao, A., Curtis, J. S., Hancock, B. B. and Wassgren, C., Numerical simulation of dilute turbulent as particle flow with turbulence modulation, *AIChE J.,* Vol. 58 (5), pp. 1381-1396 (2011)

Sheen, H., Chang, Y.-Z. and Chiang, Y.-S., Two dimensional measurements of flow structure in a two phase vertical pipe flow, *Proc. Natl. Sci. Council, Republic of China (A),* Vol. 17, pp. 200–213 (1993)

Sinclair, J. and Jackson, R., Gas-particle flow in a vertical pipe with particle-particle interactions, *AIChE J.*, Vol. 35, pp. 1473-1486 (1989)

Sinclair, J. L., and Mallo, T., Describing particle-turbulence interaction in a two-fluid modelling framework, *Proc. of ASME Fluids Engineering Division Summer Meeting (FEDSM'98)*, Washington, DC, June 21–25 (1998)

Sommerfeld, M. and Huber, N., A laser strobe technique combined with digital image analysis to study particle-wall collisions, In: Xu, Y., Wang, S. (Eds.), Modern Measuring Techniques for Multiphase Flows, *Proceedings of the International Symposium on Measurement Techniques for Multiphase Flows,* Nanjing, China, April, pp. 428-440 (1995)

Sommerfeld, M. and Kussin, J., Analysis of collision effects for turbulent gas-particle flow in a horizontal channel: part II. Integral properties and validation, *Int. J. Multiphase Flow*, Vol. 29, pp. 701–715 (2003)

Sommerfeld, M., Analysis of collision effects for turbulent gas-particle flow in a horizontal channel: part I. Particle transport, *Int. J. Multiphase Flow*, Vol. 29, pp. 675–699 (2003)

Tsuji, Y., Morikawa, Y. and Shiomi, H., LDV measurements of air-solid two-phase flow in a vertical pipe, *J. Fluid Mech.*, Vol. 139, pp. 417–434 (1984)

Wen, C. and Yu, Y., Mechanics of fluidization, *Chem. Eng. Progr. Symp. Ser.*, Vol. 62, pp. 100–111 (1966)

Zhang, H. and Ahmadi, G., Aerosol particle transport and deposition in vertical and horizontal turbulent duct flows, *J. Fluid Mech.*, Vol. 406, pp. 55-80 (2000)

Zhang, Y. and Reese, M., A gas turbulence modulation in a two-fluid model for gas–solid flows, *AIChE J.*, Vol. 49 (12), pp. 3048–3065 (2003)

Table 3.1

Fully-Developed Gas-Solid Flow Equations

Fluid-Phase

Momentum balance:

$$0 = -\alpha_g \frac{\partial p}{\partial z} + \frac{1}{r}\frac{d}{dr}(r\tau_{rz}) - \beta(v_z - u_z)$$

Drag Coefficient by Wen and Yu (1966):

$$\beta = \frac{3}{4}\frac{\rho_g}{d} C_D \frac{\alpha_s}{\alpha_g^{2.65}} |u_z - v_z|$$

where, $Re_s = \frac{\alpha_s \rho_g d |v_z - u_z|}{\mu_g}$ and

$C_D = \frac{24}{Re_s}(1 + 0.15 Re_s^{0.687})$ for, $Re_s < 1000$

$C_D = 0.44$ for, $Re_s < 1000$

Fluid Stresses:

$$\tau_{rz} = \mu_{eg} \frac{\partial u_z}{\partial r} - \rho_g \overline{u_z' u_r'}$$

where, Reynolds stress, $-\rho_g \overline{u_z' u_r'} = \mu_t \frac{\partial u_z}{\partial r}$ and turbulent viscosity, $\mu_t = \frac{c_\mu f_\mu \rho_g k^2}{\varepsilon}$

Effective viscosity by Batchelor and Green (1972):

$$\mu_{eg} = \mu_g(1 + 2.5\alpha_s + 7.6\alpha_s^2)\left(1 - \frac{\alpha_s}{\alpha_0}\right)$$

Turbulence kinetic energy:

$$0 = \frac{1}{r}\frac{\partial}{\partial r}\left[r\alpha_g \left(\mu_{eg} + \frac{\mu_t}{\sigma_k}\right)\frac{\partial k}{\partial r}\right] + \alpha_g \mu_t \left(\frac{\partial u_z}{\partial r}\right)^2 - \alpha_g \varepsilon + I_k$$

Dissipation:

$$0 = \frac{1}{r}\frac{\partial}{\partial z}\left[r\alpha_g\left(\mu_{eg} + \frac{\mu_t}{\sigma_\varepsilon}\right)\frac{\partial \varepsilon}{\partial r}\right] + \alpha_g c_1 f_1 \frac{\varepsilon}{k}\mu_t\left(\frac{\partial u_z}{\partial r}\right)^2 - \alpha_g c_2 f_2 \frac{\varepsilon^2}{k} + \alpha_g c_3 f_2 \frac{\varepsilon}{k} I_k$$

Wake model of Lun (2000) as modified by Rao *et al.* (2011):

$$E_w = 12\frac{C_w v_s \mu_t k}{d^3}$$

$\mu_t = 0.017 Re_s \mu_g$ for, $150 \leq Re_s < 310$

$\mu_t = 1.2 + 0.000057 Re_s^2 \mu_g$ for, $310 \leq Re_s < 610$

$\mu_t = 0.029 Re_s \mu_g$ for, $Re_s \geq 610$

$C_w = \frac{10}{3}$ for, $150 \leq Re_s < 310$

$C_w = \frac{24}{3}$ for, $Re_s \geq 310$

Solids-Phase

Momentum balance:

Axial component,

$$0 = -\alpha_s \frac{\partial p}{\partial z} - \frac{1}{r}\frac{\partial}{\partial r}(r\sigma_{rz}) - \alpha_s \rho_s g + \beta(v_z - u_z)$$

Radial component,

$$0 = \frac{1}{r}\frac{\partial}{\partial r}(r\sigma_{rr}) - \frac{\sigma_{\theta\theta}}{r}$$

Lun *et al.* (1984) as modified by Bolio *et al.* (1995):

Normal stresses,

$$\sigma_{rr} = \sigma_{\theta\theta} = \rho_s(\omega G_{1k} + G_{1c})$$

Shear stress,

$$\sigma_{zr} = -\mu_s(\omega G_{2k} + G_{2c})\frac{\partial v_z}{\partial r}$$

where, damping function, $\omega = \frac{1}{1 + \lambda_{mfp}/R}$ and mean free path, $\lambda_{mfp} = \frac{d}{6\sqrt{2}\alpha_s}$

$G_{1k} = \alpha_s$, $G_{1c} = 4\eta\alpha_s^2 g_0$, $G_{2k} = \frac{1}{\eta(2-\eta)g_o}\left[1 + \frac{8}{5}\eta\alpha_s g_o(3\eta - 2)\right]$ and

$$G_{2c} = \frac{8\alpha_s}{5(2-\eta)}\left[1 + \frac{8}{5}\eta\alpha_s g_o(3\eta - 2)\right] + \frac{768\alpha_s^2 g_o\eta}{25\pi}$$

Solid viscosity, $\mu_s = \frac{5\sqrt{\pi}}{96}\rho_s d\sqrt{T}$

Granular kinetic energy:

$$0 = -\frac{1}{r}\frac{\partial}{\partial r}(rq_{PT}) - \sigma_{rz}\frac{\partial v_z}{\partial r} - \gamma + I_T$$

Energy dissipation, $\gamma = \frac{48}{\sqrt{\pi}}\eta(1-\eta)g_o\alpha_s^2\frac{\rho_s}{d}T^{2/3}$

Energy flux, $q_{PT} = -\lambda\frac{\partial T}{\partial r}$

Conductivity: $\lambda = \lambda'(\omega G_{3k} + G_{3c})$ where, $\lambda' = \frac{25\sqrt{\pi}}{128}\rho_s d\sqrt{T}$

$$G_{3k} = \frac{8}{\eta(41-33\eta)g_o}\left[1 + \frac{12}{5}\eta^2\alpha_s g_0(4\eta - 3)\right]$$

$$G_{3c} = \frac{96\alpha_s}{5(41-33\eta)}\left[1 + \frac{12}{5}\eta^2\alpha_s g_0(4\eta - 3) + \frac{16}{15\pi}\eta\alpha_s g_0(41 - 33\eta)\right]$$

Table 3.2

Near-Wall Turbulence Models

Low-Reynolds-number model [Myong and Kasagi (1990)]:	**Two-layer model** [Durbin *et al.* (2001)]:
$f_1 = 1$	**Inner-layer**: $\varepsilon = \frac{k^{3/2}}{l_\varepsilon}$ and $\nu_t = C_\mu \sqrt{k} l_\nu$
$f_2 = \left[1 - \frac{2}{9} exp\left\{-\left(\frac{R_T}{6}\right)^2\right\}\right]\left[1 - exp\left(-\frac{y^+}{5}\right)\right]^2$	$l_\varepsilon = C_l y_{ef}\left(1 - e^{-\frac{R_y}{A_\varepsilon}}\right)$ and
$f_\mu = \left[1 - exp\left(-\frac{y^+}{70}\right)\right]\left[1 + \frac{3.45}{\sqrt{R_T}}\right]$	$l_\nu = C_l y_{ef}\left(1 - e^{-\frac{R_y}{A_\nu}}\right)$
where, $y^+ = \frac{\rho_g u_\tau (R-r)}{\mu_g}$ and $R_T = \frac{\rho_g k^2}{\mu_{eg}\varepsilon}$	where, $C_l = \frac{\kappa}{C_\mu^{3/4}} = 2.5$, $A_\varepsilon = 2C_l = 5.0$
	$y_{ef} = y + y_o$ and $A_\nu = A_\nu^0 = 62.5$
	Wall-distance Reynolds number: $R_y = \frac{y_{ef}\sqrt{k}}{\nu}$

Model constants:

$$c_\mu = 0.09, c_1 = 1.4, c_2 = 1.8, c_3 = 1.2, \sigma_k = 1.4, \sigma_\varepsilon = 1.3$$

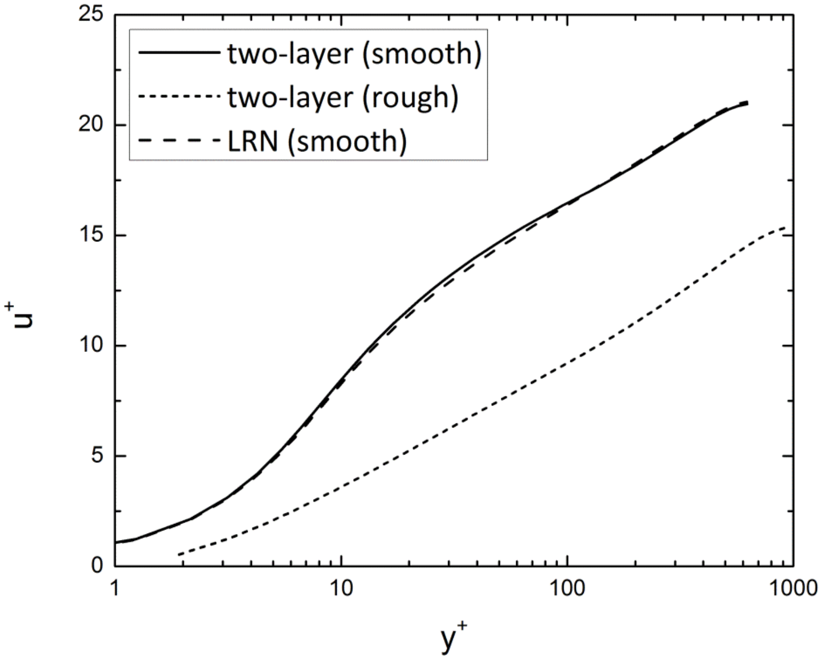


Figure 3.1 Prediction of the mean velocity

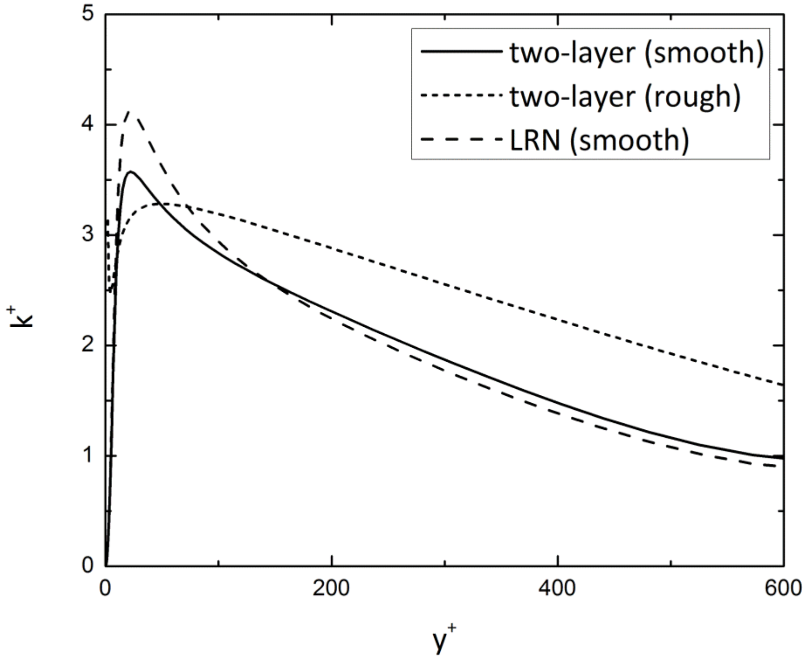


Figure 3.2 Prediction of the turbulence kinetic energy

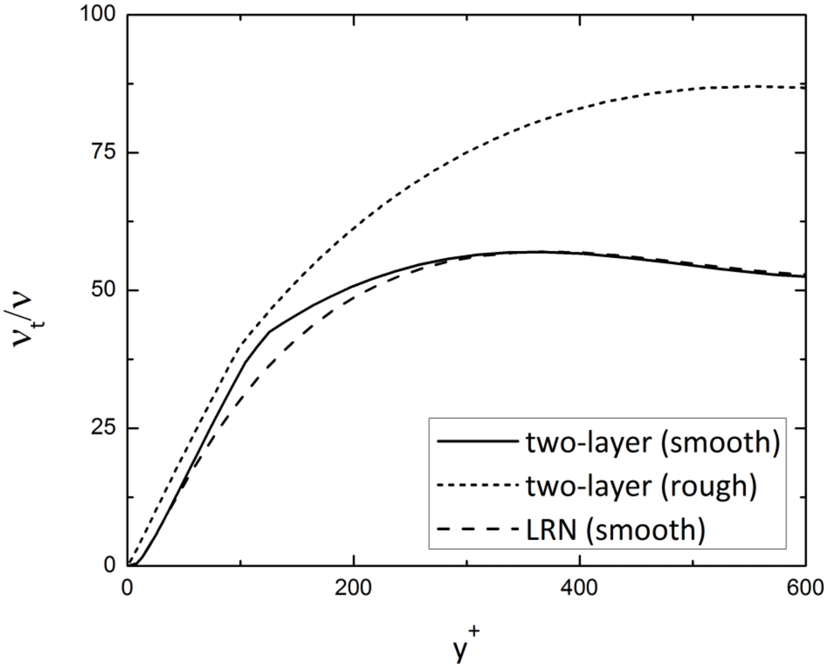


Figure 3.3 Prediction of the eddy viscosity

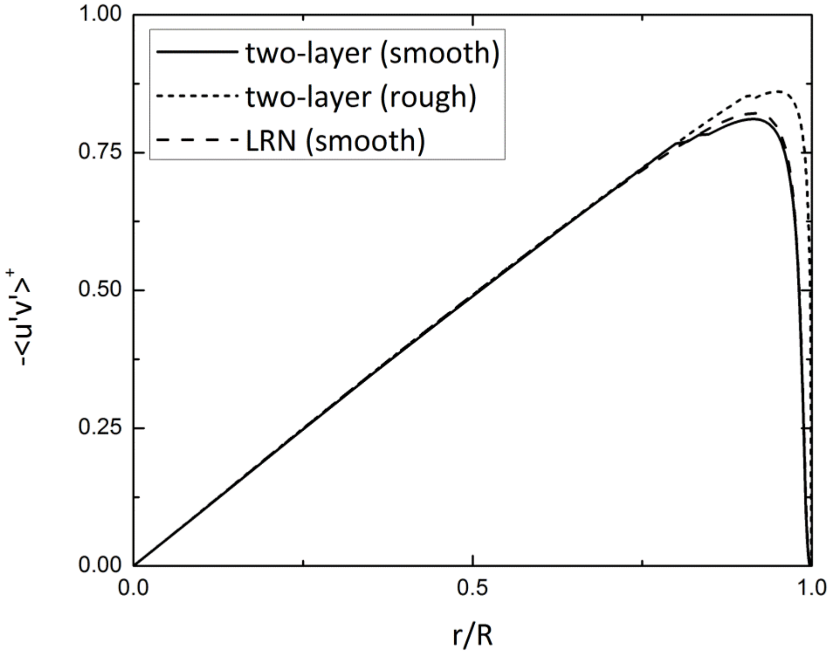


Figure 3.4 Prediction of the Reynolds shear stress

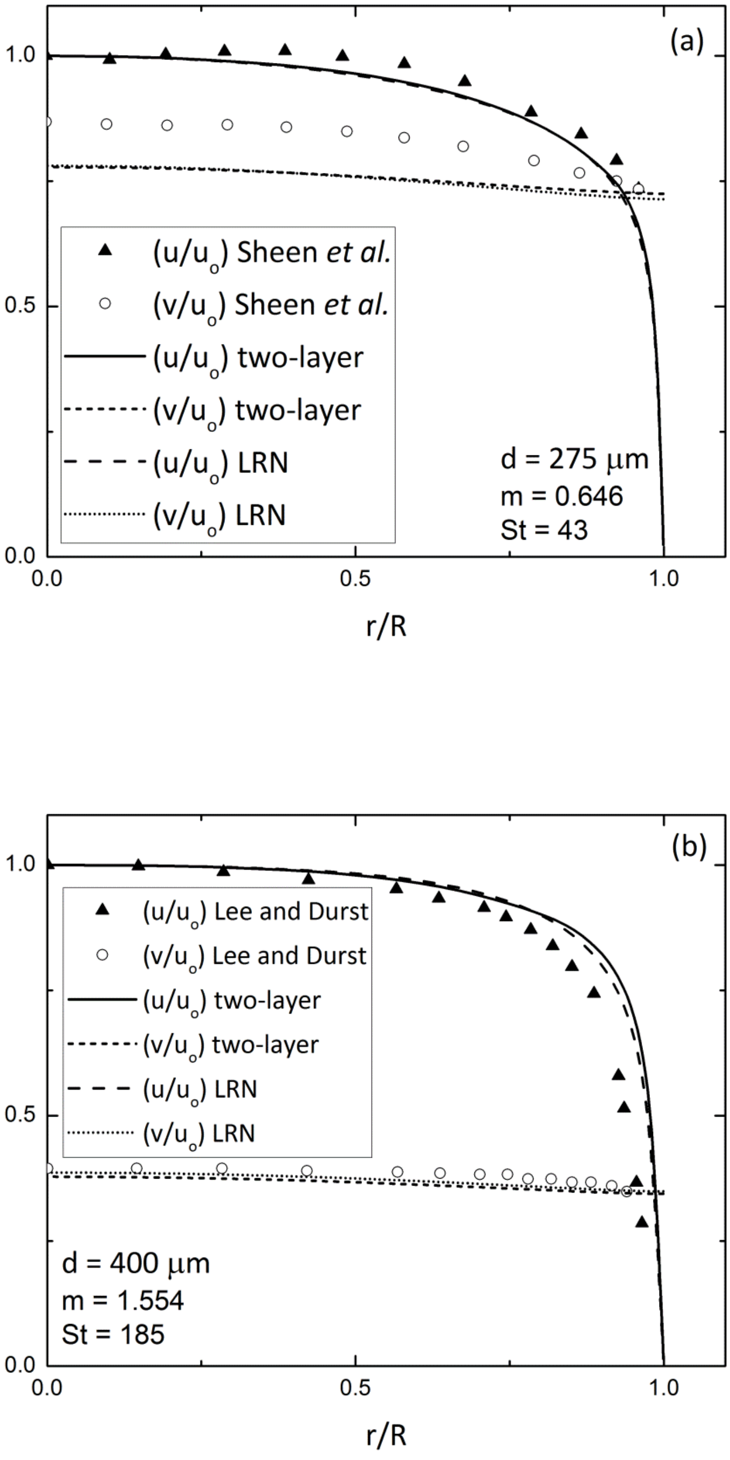


Figure 3.5 Prediction for the phasic mean velocities

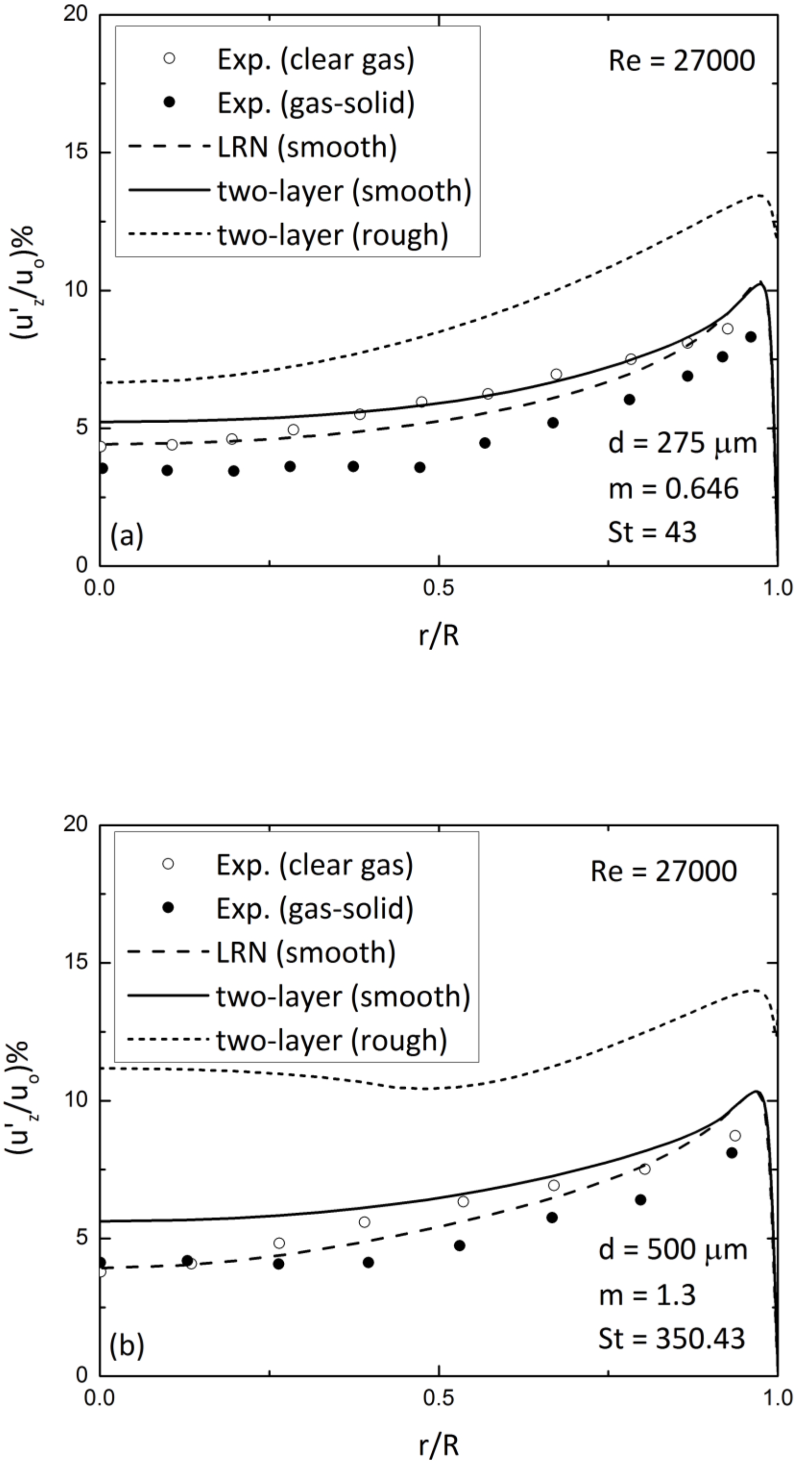
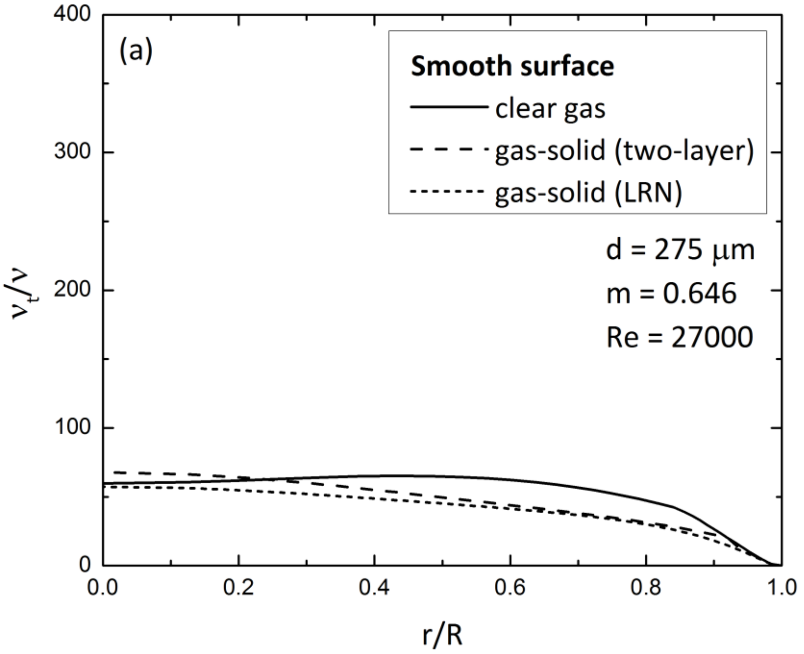
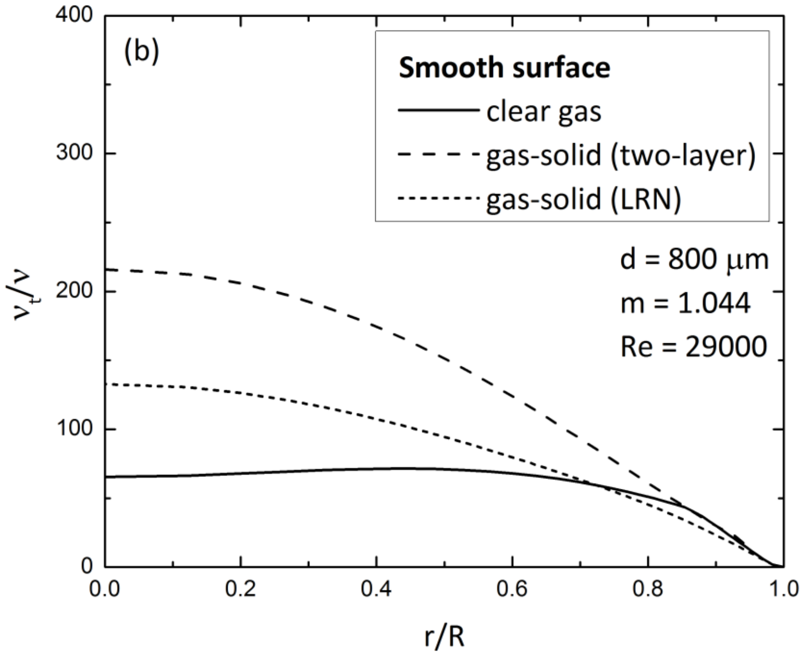


Figure 3.6 Prediction for the gas-phase fluctuating velocity


(a)
Smooth surface
clear gas
gas-solid (two-layer)
gas-solid (LRN)
d = 275 μm
m = 0.646
Re = 27000
ν_t/ν
0
100
200
300
400
0.0
0.2
0.4
0.6
0.8
1.0
r/R


(b)
Smooth surface
clear gas
gas-solid (two-layer)
gas-solid (LRN)
d = 800 μm
m = 1.044
Re = 29000
ν_t/ν
0
100
200
300
400
0.0
0.2
0.4
0.6
0.8
1.0
r/R

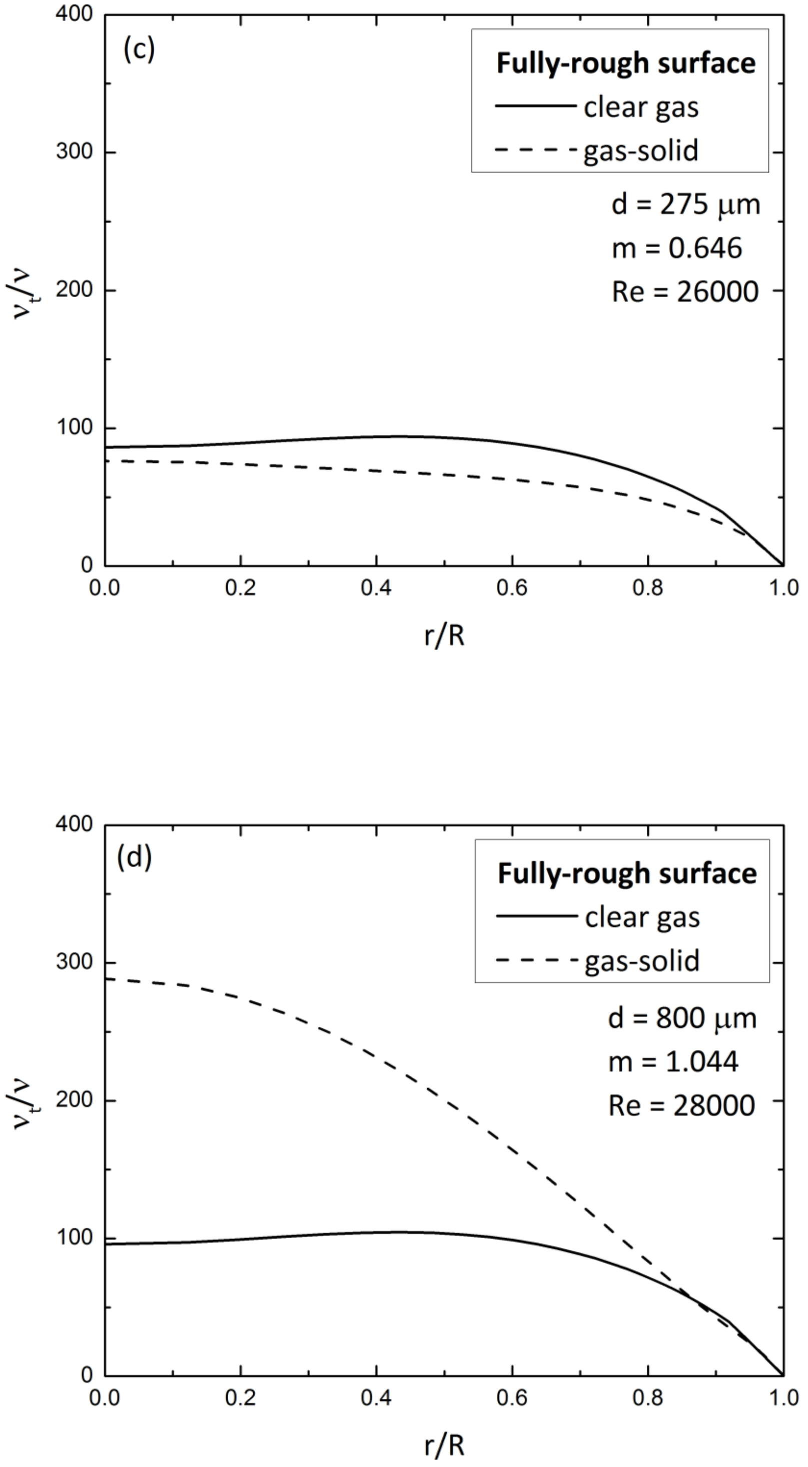


Figure 3.7 Prediction for the turbulent viscosity

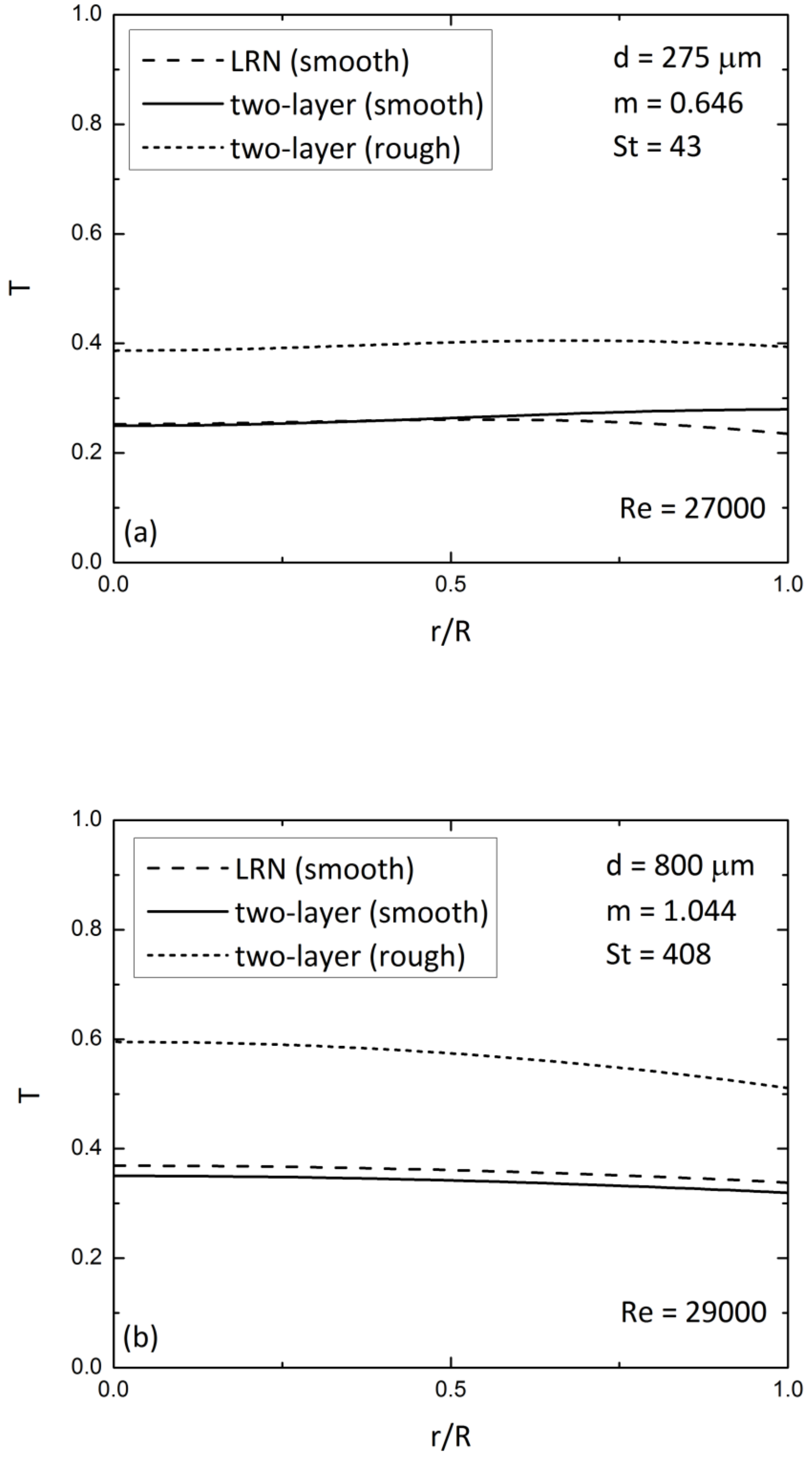
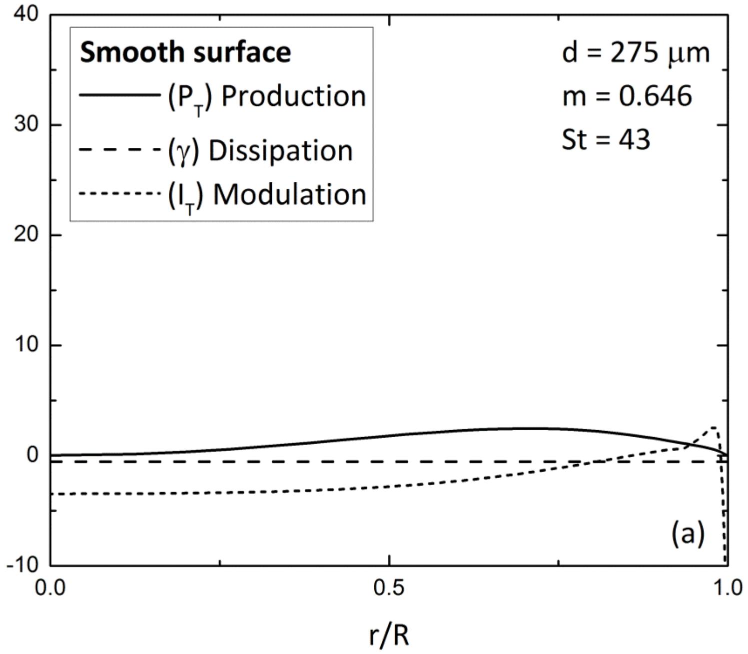
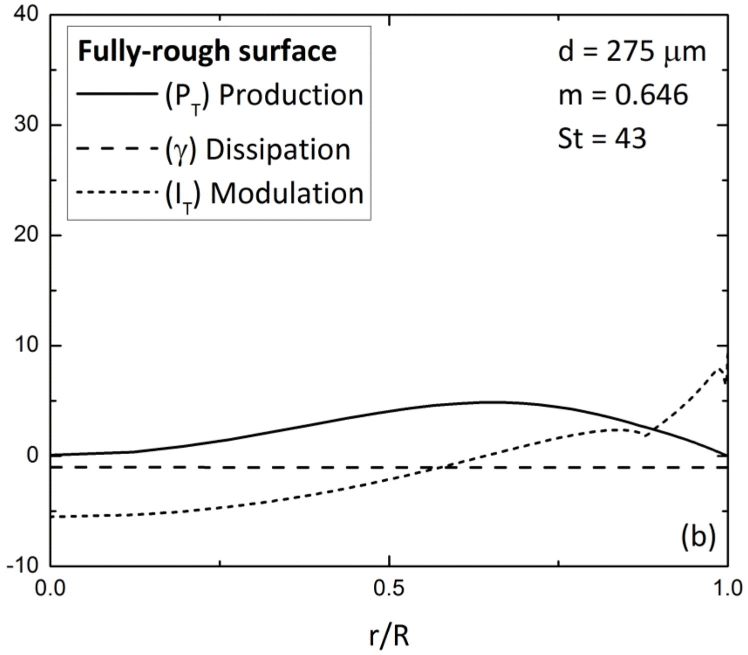


Figure 3.8 Prediction for the granular temperature


Smooth surface
(P_T) Production
(γ) Dissipation
(I_T) Modulation
d = 275 μm
m = 0.646
St = 43
(a)
40
30
20
10
0
-10
0.0
0.5
1.0
r/R


Fully-rough surface
(P_T) Production
(γ) Dissipation
(I_T) Modulation
d = 275 μm
m = 0.646
St = 43
(b)
40
30
20
10
0
-10
0.0
0.5
1.0
r/R

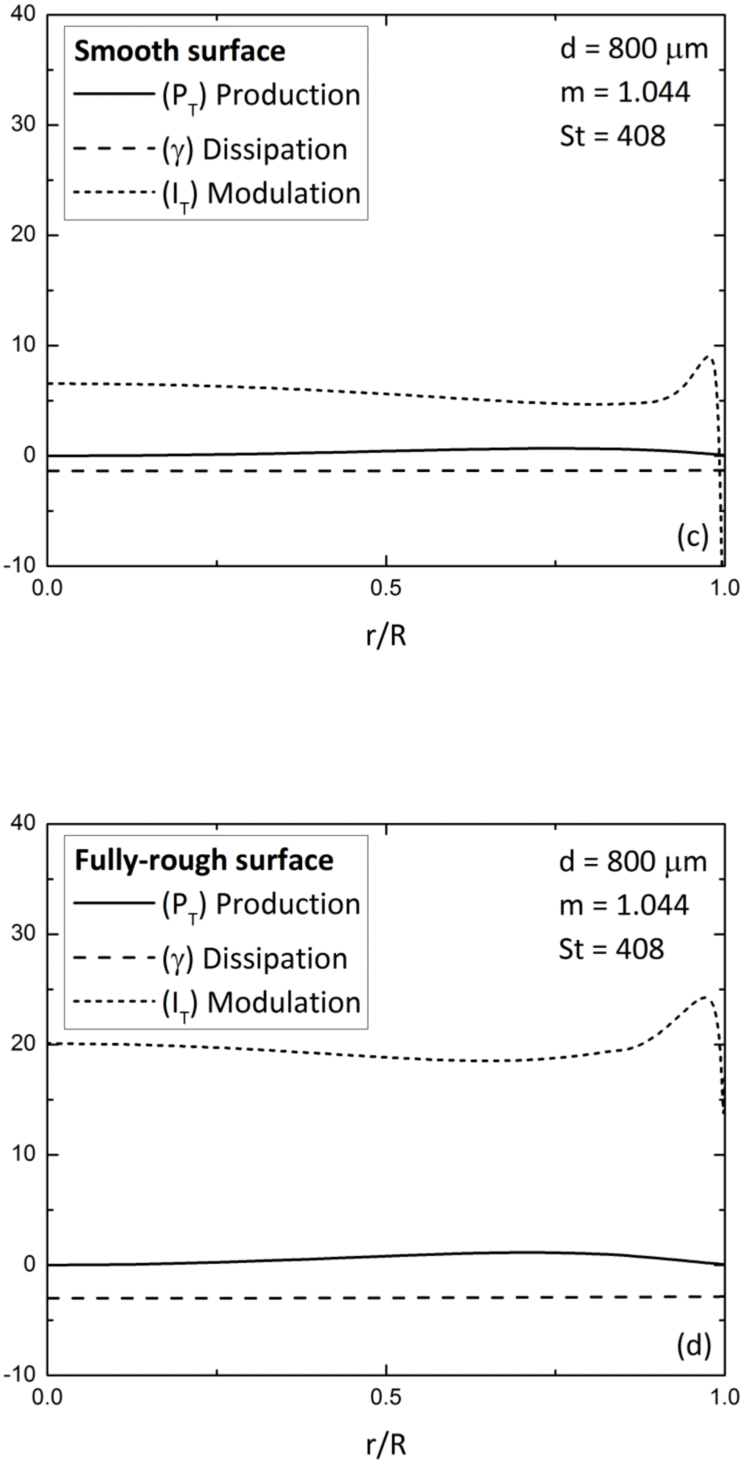
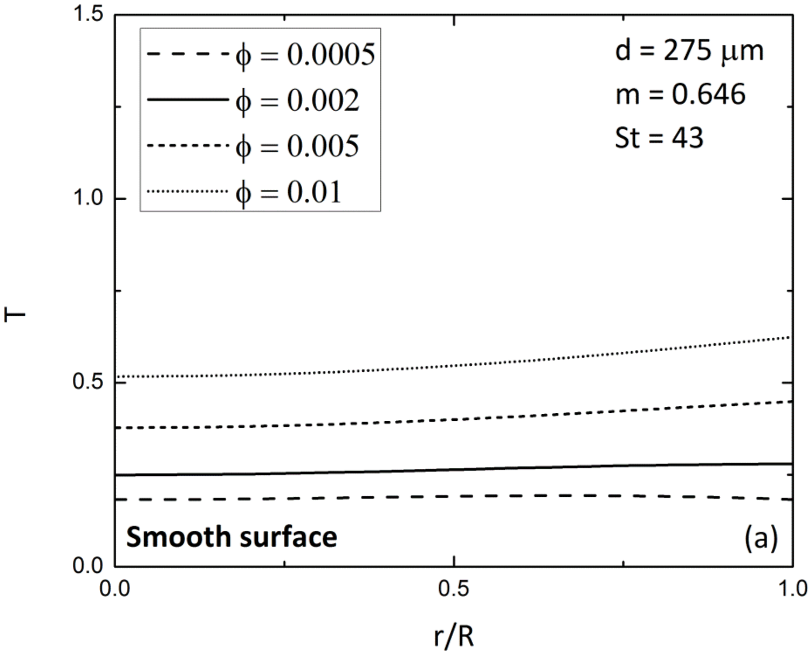
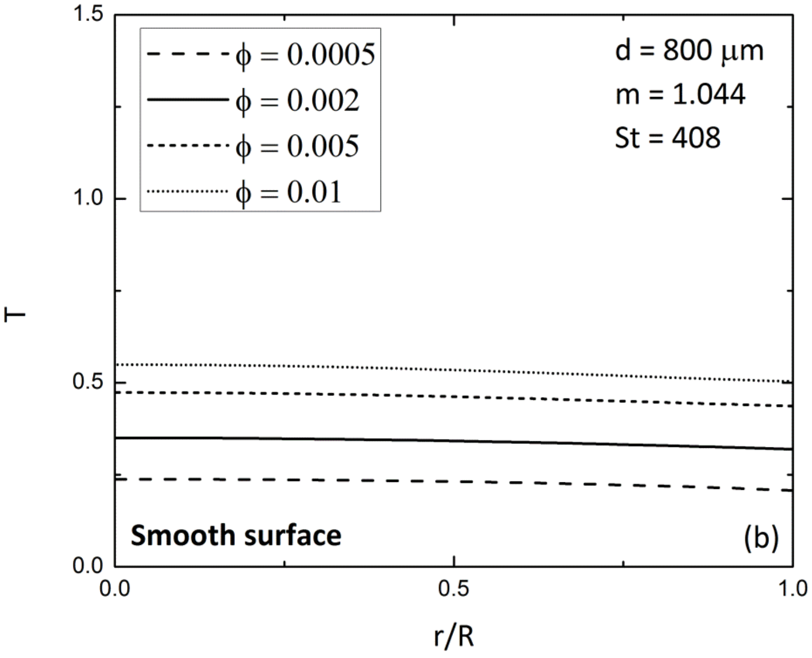


Figure 3.9 Prediction for the source terms in granular temperature equation


1.5
1.0
0.5
0.0
T
0.0
0.5
1.0
r/R
- - - ϕ = 0.0005
—— ϕ = 0.002
------- ϕ = 0.005
.... ϕ = 0.01
d = 275 μm
m = 0.646
St = 43
Smooth surface
(a)


1.5
1.0
0.5
0.0
T
0.0
0.5
1.0
r/R
- - - ϕ = 0.0005
—— ϕ = 0.002
------- ϕ = 0.005
.... ϕ = 0.01
d = 800 μm
m = 1.044
St = 408
Smooth surface
(b)

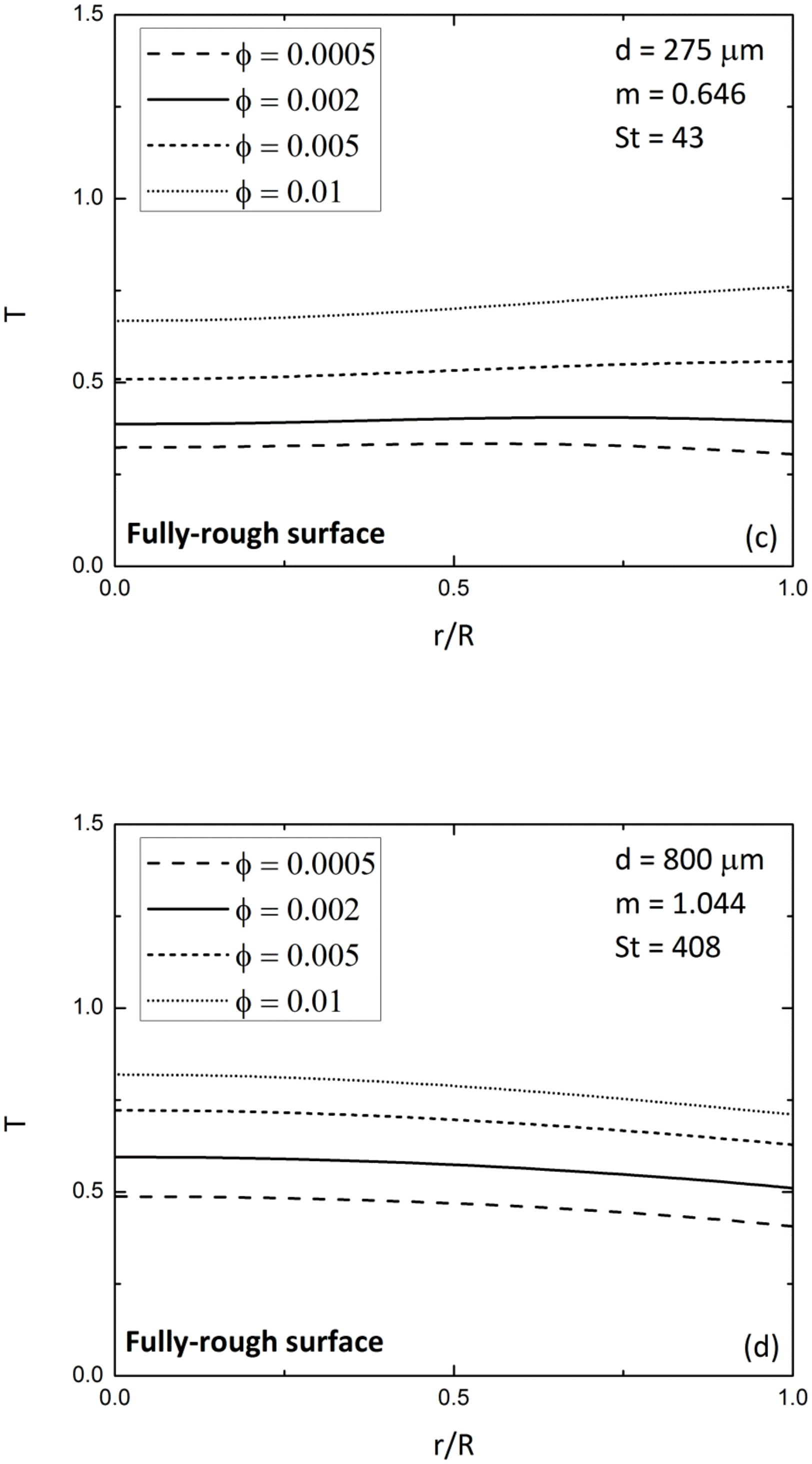


Figure 3.10 Effect of specularity coefficient for flows in both smooth and rough pipes

CHAPTER FOUR

CONCLUDING REMARKS AND FUTURE WORK

4.1 Summary and Conclusion

This dissertation presents a comprehensive numerical analysis of gas-solid turbulent flow in a vertical pipe using a two-fluid model. The in-house code was overhauled to implement current two-fluid models for turbulent gas-solid flows in a vertical pipe for both smooth and rough surfaces. The performance of the state-of-the-art two-fluid models was examined by comparing the results with the available experimental data. Next, the effect of surface roughness on the fluctuating velocity field of both the gas and solids phases was documented. The accomplishments and findings of the current research work are summarized below:

[1]. The manuscript documented in chapter 2 presents a thorough analysis of the state-of-the-art two-fluid models of Rao *et al.* (2011) and Crowe (2000). Although the mean velocity fields predicted by both models were relatively similar, the model of Rao *et al.* (2011) showed better agreement than the model of Crowe (2000) with the experimental data for the gas-phase velocity fluctuation. However, the model of Rao *et al.* (2011) adopts an ad hoc wake model of Lun (2000) that implements different expressions as the particle Reynolds number changes across the pipe and therefore is not smooth. The model of Rao *et al.* (2011) also predicts an erroneous fluctuating energy transfer from the fluid-phase near the wall in the granular temperature equation, whereas the prediction of the modulation term in the turbulence kinetic energy equation suggests that the fluid-phase fluctuating energy diminishes to zero at the wall. The solids volume fraction prediction

of both turbulence modulation models does not follow the experimental data of Sheen *et al.* (1993) especially near the wall. The manuscript also provides a detailed analysis of the behavior of the eddy viscosity and Reynolds shear stress for flows with both small and large particles, which gives a more complete understanding of the turbulent transport of the gas-phase. However, a more extensive and comprehensive database, including measurements of the Reynolds shear stress, solids velocity fluctuation and volume fraction would be useful for validation of the numerical models.

[2]. An analysis of the performance of the near-wall turbulence model of Durbin *et al.* (2001) was carried out for the prediction of gas-solid flow in a vertical pipe and the results were documented in the manuscript presented in chapter 3. The two-layer model generally predicts the mean velocities of both phases to be similar to the predictions of a conventional low Reynolds number model. However, the phenomenon that small particles suppress the gas-phase turbulence is better captured by the low-Reynolds-number model. The manuscript also documents a prediction of turbulent flow with small and large particles in a fully-rough pipe. The prediction of the model of Durbin *et al.* (2001) showed that the additional friction imposed by surface roughness causes an enhancement in the turbulence kinetic energy of the gas-phase and the granular temperature of the particle-phase. The effects of specularity coefficient on the prediction of the granular temperature were also investigated and the results showed that both surface roughness and particle size have significant impact on the prediction for the granular temperature. The use of a constant value for the specularity coefficient based on empirical correlations constrains the flexibility of the model.

4.2 Future Work

The current numerical study of turbulent gas-solid flow in a vertical pipe using the two-fluid model has identified the scope for further improvements. Some future work intended to provide a better assessment of the numerical model is summarized below:

a) The solids volume fraction is calculated from the radial momentum equation of the solids-phase. The kinetic theory that is used to develop the constitutive relations of the radial normal stresses in the solids-phase should be thoroughly reviewed. The prediction of the current model should also be compared with the prediction of a two-dimensional model where the solids volume fraction would be calculated from the mass conservation equation.

b) Although the turbulence modulation model of Rao *et al.* (2011) shows good agreement overall with the experimental data, the wake term is not smooth or well behaved. Therefore, improved wake models should be developed (or identified in the literature) to capture the turbulence enhancement in flows with large particles.

c) The modulation term that represents the fluctuating energy transfer between the two phases in the granular kinetic energy equation acts as a strong sink term at the wall. As the turbulence kinetic energy diminishes at the wall, the model of Rao *et al.* (2011) should be modified so that the modulation term in the granular temperature equation also reduces to zero at the wall.

d) The cross-correlation of the two fluctuating velocities is developed following the approach of Sinclair and Mallo (1998). Although the hypothetical modulation model of

Rao *et al.* (2011) advocates the use of this cross-correlation expression which uses a geometric mean of the two fluctuating velocities, further modeling work is needed to develop a more rigorous cross-correlation formulation.

e) The two-layer model shows some deviations in the predictions for low Reynolds number turbulent flows in the near wall region. Therefore, the model should be modified to include appropriate wall-damping as well as turbulence modulation in the so-called inner-layer algebraic expressions for both the eddy viscosity and dissipation rate.

f) A constant value (almost arbitrarily) for the specularity coefficient of the particle-wall collision is typically used in the boundary conditions for the particle-phase. Extensive studies should be performed to develop a comprehensive expression for the specularity coefficient that considers important factors such as: surface roughness, particle size, particle concentration, direction of gravity, etc.

g) A wide range of experimental data is still not available for granular temperature and solids volume fraction to reach a general conclusion. Therefore, more experiments should be performed for fully-developed gas-solid flows in vertical pipes over both smooth and rough surfaces.