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Properties of ρ- and ω-mesons in dense and hot nuclear matter near the critical pion mode softening

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Abtract:

The propagators of ρ- and ω-mesons in nuclear matter are analyzed. Due to the strong coupling to pions in $\rho\pi\pi$, $\omega\pi\pi\pi$, $\rho\rho\pi\pi$ etc. vertices, the ρ, ω-meson properties depend sensitively on the behaviour of the pionic mode. Relying on previous investigations of the pion propagator in nuclear matter, we elucidate the main features of the ρ- and ω-meson behaviour in dense and hot nuclear matter in the asymptotic case near the critical pion mode softening. We find that under such conditions the ρ-meson mode becomes stiffer, while the ω-meson mode softens. The widths of both the ρ, ω-mesons increase significantly and become essentially greater then ones of the free mesons.

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1. Introduction

One of the most interesting problems in present nuclear physics is: how much does a hadron inside nuclear matter, at various densities and temperatures, differ in its properties from a free hadron? Even at normal nuclear matter density n_0 and temperature $T = 0$, i.e., in nuclei near ground state, changes of hadron properties are observable. The medium influence is expected to become stronger with increasing density and temperature of nuclear matter. Accordingly, many efforts have been devoted last years to investigate the hadron properties in compressed and heated nuclear matter which may be obtained in intermediate-energy heavy-ions collisions.

Since some time the properties of pions, as the lightest of the strongly interacting particles, have been considered by many authors (cf. Ref. [1] for reviews). At present it is commonly accepted that the pionic mode in nuclear matter suffers a softening which increases with increasing nucleon density. Since the pion in turn couples strongly to the other hadrons, their properties are altered in nuclear matter also, which is expected to be most important at high density and temperature. As the nucleon is surrounded by a pion cloud, the nucleon properties change simultaneously with a change of the pion mode in nuclear matter [2]. To describe nuclear matter at large density and temperature one needs to include not only the nucleon and pion degrees of freedom, but also one has to take into account the heavier mesons, in particular the ρ, ω-mesons. A knowledge of the properties of ρ, ω-mesons in compressed and heated nuclear matter is necessary in order to investigate the equation of state under such extreme conditions. Also the study of several processes, such as meson, photon, and di-lepton production [3], in the course of heavy-ion collisions is aimed to learn more about mesonic degrees of freedom in excited nuclear matter. The theoretical works [4, 5] are devoted to these goals. In these investigations the ρ-meson propagator is considered at vanishing temperature and within special approximations for the pions in nuclear medium. The relationship between the ρ-meson propagator and the dilepton production rate is analyzed in the quoted works [4, 5]. We also mention the Refs. [6] in which the ρ-meson properties in nuclear matter have been analyzed by different methods, like QCD sum rules or the Nambu - Jona-Lasinio model.

In the present paper we investigate the properties of the ρ, ω-meson propagators in dense nuclear matter at finite temperature. In doing so we employ for the pion propagator in medium an asymptotic expression, which includes interactions between baryons and pions, and is valid near the critical pion mode softening. We investigate the asymptotic behaviour of the ρ, ω-mesons in hot and strongly compressed nuclear matter. We take our results as some fix point and focus on the debated problems of heavy meson mass and width in medium.

2. The π, ρ, ω-meson propagators

We investigate the properties of mesons in nuclear matter at finite temperatures using Matsubara's Greens functions (cf. Ref. [7]). The meson propagators are defined as usual by

$$\mathcal{D}^{\mu\nu}_{ik}(\tau, \vec{r}) = - < T_\tau \left(\varphi^\mu_i(\tau_1, \vec{r}_1)\varphi^\nu_k(\tau_2, \vec{r}_2)\right) >= \delta^{\mu\nu}\mathcal{D}^\nu_{ik}(\tau, \vec{p}), \quad \frac{-1}{T} \leq \tau \leq \frac{1}{T}, \qquad (1)$$

$$\tau = \tau_1 - \tau_2, \quad \vec{r} = \vec{r}_1 - \vec{r}_2,$$

where $\mathcal{T}_\tau$ is a "time" ordering operator, and $\varphi_i^\mu(\tau, \vec{r})$ denotes the field operators of the considered π, ρ, ω-mesons. The indices i and μ are Lorentz and isospin indices, respectively. We consider here an isospin symmetrical medium, therefore the isospin indices can be omitted. In case of the pseudo-scalar pions the Lorentz indices have to be skipped too. The pion propagator takes then the form

$$\mathcal{D}(\tau, \vec{r}) = T \sum_n \exp\{-\omega_n \tau\} \mathcal{D}(\omega_n, \vec{r}) = \tag{2}$$

$$T \sum_n \exp\{-\omega_n \tau\} \int \frac{d\vec{k}}{(2\pi)^3} \exp\{i\vec{k}\vec{r}\} \mathcal{D}(\omega_n, \vec{k}), \quad \omega_n = 2\pi n i T.$$

According to usual transformations [7] one obtains

$$\mathcal{D}(\tau, \vec{r}) = -\int_C \frac{d\xi}{2\pi i} \exp\{\xi|\tau|\} \mathcal{D}(\xi, \vec{r}) \chi(\xi), \tag{3}$$

$$\mathcal{D}(\xi, \vec{r}) = \int \frac{d\vec{k}}{(2\pi)^3} \exp\{i\vec{k}\vec{r}\} \mathcal{D}(\xi, \vec{k}), \quad \chi(\xi) = \left(\exp\{\frac{\xi}{T}\} - 1\right)^{-1}.$$

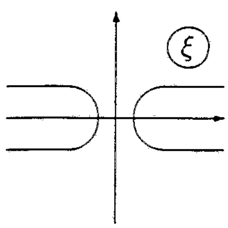


Fig. 1. *The contour C in the complex ξ plane.*

The integration contour is shown in Fig. 1. All the formulae here apply also for the vector mesons. The vector meson propagator $\mathcal{D}_{ik}(\xi, \vec{r})$ has, of course, the Lorentz indices. $\mathcal{D}(\xi, \vec{k})$ is the analytical continuation of the Matsubara Greens function from discrete values ω_n into the full complex plane. Due to the isospin symmetry of the considered medium one has

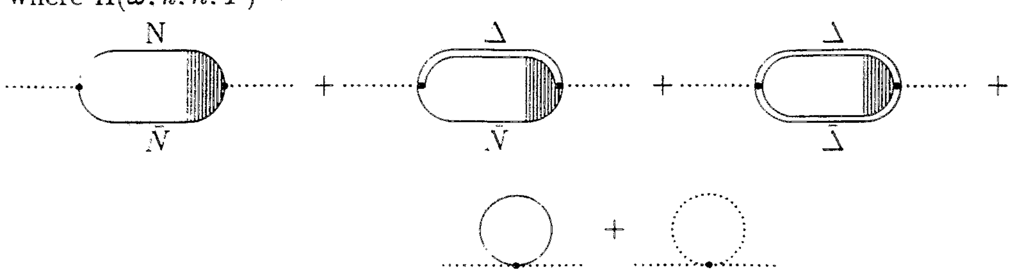
$$\mathcal{D}(\xi + i0) = \mathcal{D}(-\xi - i0), \quad \mathrm{Im}\mathcal{D}(\mathrm{Re}\xi = 0) = 0$$

which has been used in deriving eq. (3).

The pion propagator can be cast in the form [1]

$$\mathcal{D}(\xi, \vec{k}, n, T) = \left(\xi^2 - \vec{k}^2 - 1 - \Pi(\xi, \vec{k}, n, T)\right)^{-1}, \tag{4}$$

where $\Pi(\omega, \vec{k}, n, T) =$



is the polarization operator which is determined by the πNN, $\pi\Delta N$ and $\pi\Delta\Delta$ interactions.

(We use as units $m_\pi = \hbar = c = 1$.) The solid lines indicate nucleon and nucleon-hole propagators, the double line stands for the Δ-isobar propagator, the dotted line represents the pion propagator. The dot denotes the irreducible πNN, $\pi\Delta N$ and $\pi\pi$NN interactions. The hatched area is the full vertex in nuclear matter. The polarization operator in eq. (4) has been analyzed thoroughly in many works, the results of them we rely on. The quantity Π includes various excited states of an interacting nucleon - nucleon-hole, delta - nucleon-hole and delta - delta-hole in nuclear matter. At larger density and temperature also the $\pi\pi$-interaction (last term) needs to be included. The polarization operator in medium has a logarithmic singularity. $\Pi(\omega)$ and $\mathcal{D}(\omega)$ are defined in the complex plane with cuts along the real axis [1] (see Fig. 1), and the relation

$$\mathcal{D}(\xi + i0) - \mathcal{D}(\xi - i0) = 2i\mathrm{Im}\mathcal{D}(\xi + i0)$$

holds, which is used for calculations of the integrals of type as in eq. (3).

The previous analysis of the pion propagator [1, 8] has shown that the propagator $\mathcal{D}(\omega, \vec{k})$ has a sharp maximum at baryon density $n \to n_c$ and pion momentum $k \approx k_c$, where it can be approximated by

$$\begin{aligned}
\mathcal{D}^{-1}(\omega, \vec{k}, n, T) &= \omega^2 - 1 - \vec{k}^2 - \mathrm{Re}\Pi(\omega, \vec{k}, n, T) - i\mathrm{Im}\Pi(\omega, \vec{k}, n, T) \qquad (5)\\
&\approx \omega^2 a(T) + i\omega\gamma(T)\mathrm{sign}(\mathrm{Im}\omega) - X(\omega, \vec{k}, n, T)\\
X = b(k - k_0)^2 &+ \tilde{\omega}^2(\omega, \vec{k}, n, T), \quad \tilde{\omega}^2 = C(n_c(T) - n) + \frac{5\lambda T k_0^2}{8\pi\sqrt{b\tilde{\omega}}},\\
k_0^2(n, T) &\approx k_c^2 + \kappa(n - n_c(T)), \quad b = 0.8, \quad \gamma(T) = 0.5 + 0.3T, \quad C = 1.1,\\
a(T) = 1.6 - 0.1T, &\quad \kappa = 1.3, \quad k_c = 3.0, \quad n_c(T) = 1.5 + 0.1T + 0.025, \quad \lambda \sim 0.1 - 0.5.
\end{aligned}$$

From these expressions it becomes obvious that at $\tilde{\omega} \to 0$, i.e., when the density approachs the critical value, $n \to n_c$, the pion mode becomes very soft and $\mathcal{D}^{-1}$ gets the sharp minimum at $k = k_0$. We exploit this comparative simplicity of the pion propagator at $\tilde{\omega} \to 0$ and analyze the asymptotics of the ρ- and ω- propagators just in this limit.

For the ρ, ω-vector mesons the propagators take the form

$$\mathcal{D}^{\mu}_{kl}(\omega, \vec{k}) = \delta_{kl}\mathcal{D}^{\mu}(\omega, \vec{k}), \qquad (6)$$

$$\mathcal{D}^{\mu}(\omega, \vec{k}) = \left(\omega^2 - \vec{k}^2 - m_V^2 - \mathcal{P}^{\mu}(\omega, \vec{k})\right)^{-1},$$

where we have introduced the quantity

$$\mathcal{P}^{\mu} = \frac{1}{3}\mathrm{Tr}_{kl}\mathcal{P}^{\mu}_{kl},$$

and $V = \rho$ or ω. In what follows we consider only $\mathcal{P}$. For the isospin singlet ω-meson the isospin index has to be dropped.

Our goal now is to investigate the change of $\mathcal{D}$ and $\mathcal{P}$ in nuclear matter in comparison with their corresponding vacuum values. The propagator is represented as follows

$$\mathcal{D}^{\mu}(\omega, \vec{k}, n, T) = \left(\omega^2 - \vec{k}^2 - m_V^2 - \delta\mathcal{P}^{\mu} - i\Gamma^{\mu}_V\right)^{-1}, \qquad (7)$$

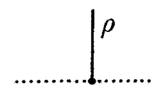
$$\delta\mathcal{P}^{\mu} = \mathcal{P}^{\mu}(\omega,\vec{k},n,T) - \mathcal{P}^{\mu}(\omega,\vec{k},0,0),$$

where m_V and Γ_V^{μ} denote, respectively, the observed physical mass and width of the corresponding meson in vacuum. Hence there do not appear problems with singularities because they cancel each other in $\mathcal{P}(n,T) - \mathcal{P}(0,0)$.

3. Interactions of π,ω,ρ-mesons and the ρ,ω-meson polarization operators

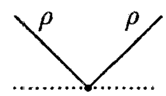
The quantity $\mathcal{P}$ contains certainly an infinite set of different interactions, including also the coupling to the pion field. In our context such processes turn out to be of most importance where pions participate. This is due to the strong modifications of pions in nuclear matter at large density and temperature. To obtain the contributions to ω- and ρ-meson polarization operators from the pionic degrees of freedom one needs the different vertices of the vector-meson pion couplings. We list the interaction Lagrangians which we exploit [9]

$$\mathcal{L}_{\rho\pi\pi} = -\frac{g}{\sqrt{2}} V_{\mu}^{l} \pi^{j} \partial_{\mu} \pi^{k} \epsilon_{jkl} = \tag{8}$$

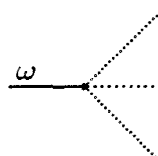


where $g = \sqrt{2} f_{\rho\pi\pi}$, $f_{\rho\pi\pi}^2/4\pi = 3.1$ and ϵ_{jkl} is the Levi-Civita tensor.

$$\mathcal{L}_{\rho\rho\pi\pi} = \frac{g^2}{2}\left(\vec{V}_{\mu}^2 \vec{\pi}^2 - (\vec{\pi}\vec{V})(\vec{\pi}\vec{V})\right) =$$



$$\mathcal{L}_{\omega 3\pi} = -3hU_{\alpha}\epsilon_{\alpha\mu\nu\lambda}\pi_{\mu}^{0}\pi_{\nu}^{-}\pi_{\lambda}^{+} =$$



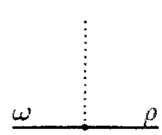
$$\pi_{\mu} \equiv \partial_{\mu}\pi, \quad h = h_0 - \frac{g^3}{8\pi \tilde{F}_{\pi}} \sum_{a=0,\pm} \left((\vec{p}_{\omega} - \vec{p}_{\pi}{}^{a})^2 + m_{\rho}^2\right)^{-1},$$

$$h_0 = -\frac{g}{2\pi^2 \tilde{F}_{\pi}^3}\{1 - \frac{3\tilde{F}_{\pi}^2}{4m_{\rho}^2} + \frac{3}{32}\left(\frac{g^2\tilde{F}_{\pi}^2}{m_{\rho}^2}\right)^2\}, \quad \tilde{F}_{\pi} = m_{\pi}/1{,}05.$$

Here $\vec{\pi}, V_i^{\mu}, U_i$ denote the field operators of the π,ρ- and ω-mesons. Heavy lines stand for vector mesons.

In addition to these most important interaction vertices there are also others, e.g.,

$$\mathcal{L}_{\omega\rho\pi} = -\frac{3g^2}{32\pi^2 \tilde{F}_{\pi}} \epsilon_{\mu\nu\alpha\beta}\partial^{\mu}V^{\nu}\partial^{\alpha}\vec{U}^{\beta}\vec{\pi} = \tag{9}$$



which corresponds to the radiation or absorption of a pion by a the transition between ρ- and ω-mesons.

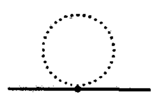
We also need a description of the $\pi\pi$-interaction in nuclear matter. For this we adopt the following form

$$\mathcal{L}_{\pi\pi} = -\frac{\lambda}{2}\vec{\pi}^2\left(-\partial_{\mu}\vec{\pi}\partial^{\mu}\vec{\pi} + \frac{1}{2}\vec{\pi}^2\right) = \tag{10}$$

which coincides in its form with the Weinberg vacuum $\pi\pi$-Lagrangian, but can include another amplitude Λ. It should be emphasized that in our approach the pions interact not only by ρ-meson exchange but also due to other process according to eq. (10). The attractive $\pi\pi$-interaction which causes the ρ-meson formation, i.e., a bound $\pi\pi$-state with quantum numbers of the ρ-meson, must be excluded from eq. (10) because the ρ-meson is directly taken into account. Accordingly we have to use only such effective $\pi\pi$-interaction which does not result in a bound state in the ρ-meson channel. For later estimates we assume $\Lambda \approx f_\pi^{-1}$, $f_\pi = 93$ MeV.

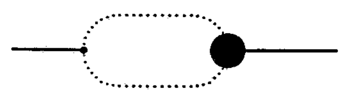
It is clear, that the quantity $\delta\mathcal{P}$ contains, besides pion-including processes, many others. Among them are the excitations as displayed in eq. (4). However, due to the large mass of the vector mesons, such contributions are of minor importance. The processes of the types as displayed in eq. (4) do not result in a noticeable softening of the ρ- and ω-modes. Due to the essential softening of the pion mode with increasing nuclear density the dominating contribution to $\delta\mathcal{P}$ comes from processes which include just the pions. Therefore, we focus on such processes and consider their asymptotic behaviour in the limit $\tilde{\omega} \to 0$ in eq. (5). Naturally, one expects the largest contributions from processes with most internal pion lines. The interactions (8) result in the following contributions to the self-energies defined in eqs. (6)

$$\mathcal{P}_\rho^{1\pi} = \frac{4}{3} g \mathcal{N}(n,T) = \quad (11)$$



$$\mathcal{N}(n,T) = \int \frac{d\vec{k}}{(2\pi)^3} \int_0^\infty \frac{d\xi}{\pi} \mathrm{Im}\mathcal{D}(\xi - i0, \vec{k}, n, T)(1 + 2\chi(\xi)),$$

$$\mathcal{P}_\rho^{2\pi}(\omega \pm i0, \vec{p}) = -\sqrt{2} g \mathcal{A}(\omega \pm i0, \vec{p}) \mathcal{T}(\omega, \vec{p}) = \quad (12)$$



with

$$\mathcal{A} = -\frac{1}{6} \sum_{n=1,4} \int \frac{d\vec{k}_1}{(2\pi)^3} \int \frac{d\vec{k}_2}{(2\pi)^3} \int_C \frac{d\xi_1}{2\pi i} \int_C \frac{d\xi_2}{2\pi i} \chi(\xi_1)\chi(\xi_2) \times \quad (13)$$

$$\mathcal{D}(\xi_1, \vec{k}_1)\mathcal{D}(\xi_2, \vec{k}_2)\delta(\vec{k}_1 + \vec{k}_2 - \vec{p})(k_{1n} + k_{2n})^2 t_2 Z_2 \mathcal{T}(\omega, \vec{p}),$$

where $k_n \equiv (\xi, \vec{k})$ and

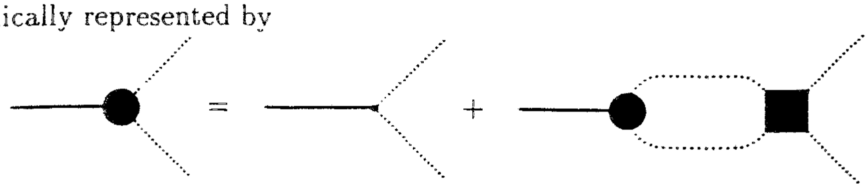
$$t_2 = \exp\{\frac{\xi_1 + \xi_2}{T}\} - 1,$$

$$Z_2 = (\xi_1 + \xi_2 + \omega \pm i0)^{-1} + (\xi_1 + \xi_2 - \omega \mp i0)^{-1}.$$

The full reducible $\rho\pi\pi$-vertex $\mathcal{T}$ (indicated by a heavy dot) is expressed via the effective $\pi\pi$-interaction.

$$\mathcal{T}(\omega, \vec{p}) = \hat{\tau} + 2\hat{\Lambda}\mathcal{A}(\omega, \vec{p})\mathcal{T}(\omega, \vec{p}), \quad (14)$$

which is graphically represented by

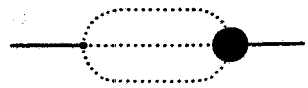


($\hat{\tau} = -g/\sqrt{2}$) and hence

$$\mathcal{T}(\omega, \vec{p}) = \frac{\hat{\tau}}{1 - 2\hat{\Lambda}\mathcal{A}(\omega, \vec{p})}. \quad (15)$$

$$\mathcal{P}^{3\pi}_{\omega} = -\frac{3}{2} h \epsilon^{amnl} \epsilon_{abcd} \int \frac{d\vec{k}_1}{(2\pi)^3} \frac{d\vec{k}_2}{(2\pi)^3} \frac{d\vec{k}_3}{(2\pi)^3} \delta(\vec{k}_1 + \vec{k}_2 + \vec{k}_3 - \vec{k}) \times \tag{16}$$

$$\int_C d\xi_1 \int_C d\xi_2 \int_C d\xi_3 \mathcal{D}(\xi_1, \vec{k}_1) \mathcal{D}(\xi_2, \vec{k}_2) \mathcal{D}(\xi_3, \vec{k}_3) \chi(\xi_1) \chi(\xi_2) \chi(\xi_3) \times$$

$$t_3 Z_3 k_{1m} k_{2n} k_{3l} k^{1b} k^{2c} k^{3d} \tilde{h}(\xi, \vec{k}) =$$  ,

$$t_3 = \exp\{\frac{\xi_1 + \xi_2 + \xi_3}{T}\} - 1,$$
$$Z_3 = (\xi_1 + \xi_2 + \xi_3 + \omega \pm i0)^{-1} + (\xi_1 + \xi_2 + \xi_3 - \omega \mp i0)^{-1}.$$

This expression contains the full $\omega \to 3\pi$ vertex $\tilde{h}$, which in turn contains various processes including $\pi\pi$-interactions, however, not such diagrams of the type as in eq. (16) which have three outgoing pion lines from one point. One should note that in our investigations there is no direct $\pi\pi\omega\omega$-interaction which would correspond to the "sea-gull" term of the $2\pi 2\omega$-interaction. The reason for disregarding such anomalous interactions is that at present there is no evidence for it. Of course, the π- and ω-mesons may interact via ρ-meson exchange due to the $\rho\pi\omega$-vertex being described in eq. (9). The corresponding contribution to the polarization operator of the ω-meson takes the form

.

In case of a essentially softened pion mode in nuclear matter such processes can be neglected in comparison to process represented by diagrams of type (11) - (16).

4. Calculation of ρ, ω-polarization operators in nuclear matter near the critical point of the pion field

All above-presented expressions in nuclear matter are different from the corresponding expressions in vacuum due to the replacements of the pion propagator, the vertices, and amplitudes. Properly one should self-consistently consider the changes of the ρ- and ω-mesons due to the influence of the surrounding nuclear medium. However, if the density approaches the critical value then, due to the pion mode softening, the asymptotical behaviour of the expressions (11) - (16) at $\omega \to 0$ is determined by the products of $\mathcal{D}$-functions, while the changes in the vertices and amplitudes are more moderate, since they do not diverge at $\tilde{\omega} \to 0$. Therefore, in this asymptotic limit we can replace the irreducible vertices and amplitudes in eqs. (11) - (16) by their vacuum values. Further, we need only the difference $\delta\mathcal{P}$ in eq. (7) and, therefore, we do not need the quantities (11) - (16) themselves but only the differences between their values in medium and in vacuum. It is possible to transform (or renormalize) these differences in such a way that in the integrals, instead of the products of two or three pion propagators, only the differences of these products in medium and in vacuum appear, i.e.,

$$\Pi^j_{i=1} \mathcal{D}(\xi_i, \vec{r}_i) \to \Pi^j_{i=1} \mathcal{D}(\xi_i, \vec{r}_i, n, T) - \Pi^j_{i=1} \mathcal{D}(\xi_i, \vec{r}_i, 0, 0), \quad j = 1, 2, 3 \tag{17}$$

In the same time the vertices and amplitudes are replaced by their full, reducible vacuum values. That is, in eq. (14) the reduced vertex $\tilde{\tau}$, which is determined by the equation

$$\tilde{\tau}(\omega,\vec{p}) = \hat{\tau} + 2\mathcal{A}(\omega,\vec{p},n=0,T=0)\tilde{\tau}(\omega,\vec{p}) \tag{18}$$

occurs instead of irreducible vertex $\hat{\tau}$. So, we pick up the most important parts of these diagrams which determine their asymptotics at $\tilde{\omega} \to 0$. In what follows we focus on the investigation of the asymptotics of these selected parts of the diagrams. Obviously, at $\tilde{\omega} \to 0$ the $\mathcal{D}$ function possesses a sharp maximum at $k \sim k_0$, $\omega \sim 0$. Under such conditions the region $k \sim k_0$, $\xi \sim 0$ gives the main contribution to the integrals (11) - (16). The vertices and amplitudes are smooth in this region, as well as the $\mathcal{D}$- function of pion in vacuum. This behaviour allows one to insert in the integrals (13) and (16) the products of the asymptotic propagators (5) without subtracting the vacuum propagators, i.e., to neglect the second term in eq. (17) for $j = 2, 3$. The vertices and amplitudes can be factorized out of the integrals and replaced by their vacuum values. At large values of k the pion propagators in nuclear matter and in vacuum are equal. This ensures a natural regularization of the integrals in eqs. (11) - (16).

Now we are in the position to calculate the asymptotic behaviour and limiting values of the quantities (11) - (16) at different temperatures and densities under the condition $\tilde{\omega} \to 0$. First of all, we obtain for the quantity $\mathcal{N}(n,T)$ in eq. (11),

$$\mathcal{N}(n,T) = \frac{k_0^2 T}{8\pi\tilde{\omega}\sqrt{b}}\left(\frac{\pi\gamma T}{6\tilde{\omega}^2}\right)^x + C, \tag{19}$$

where $x = 0$ for $\tilde{\omega}^2 \ll 2\pi T$ and $x = 1$ for $T \ll \tilde{\omega}^2 \ll 1$. C remains constant at $\tilde{\omega} \to 0$. Then one can conclude from eq. (19) that the contribution (11) to the ρ-meson self energy is real and does not change the meson width. In the same time its real part is positive and it grows essentially in the limit $\tilde{\omega} \to 0$ at finite temperatures. At $T = 0$ there is no singularity in $\mathcal{N}$ so that eq. (11) remains finite at $\tilde{\omega} \to 0$.

The quantities $\mathcal{A}(\omega,\vec{p},n,T)$ in eq. (13) and $\mathcal{P}_\omega^{3\pi}$ in eq. (16) contain the sum over the projections of a four momentum. We are interested in the asymptotic main term. One can easily verify that the dominant contribution stems from the sum over the spatial components, i.e., the indices n in eq. (13) and $b, c, \ldots$ in eq. (16) degenerate here to purely space-like indices. The reason is the sharp maximum of the integrand at $\xi \to 0$ and $\tilde{\omega} \to 0$ due to the properties of the propagator $\mathcal{D}$ and the expression within the parenthesis in eq. (13), (16). Therefore we need to consider only the spatial components in eqs. (13), (16) (for details see below). In this approximation the expressions (13) and (16) become

$$\mathcal{A}(\omega \pm i0,\vec{p}) \approx -\frac{1}{6}k_0^4 I_2(p)\int_{-\infty}^{\infty}\frac{d\xi_1}{\pi}\int_{-\infty}^{\infty}\frac{d\xi_2}{\pi}Z_2 t_2\,\Pi_{i=1,2}\int_{-\infty}^{\infty}\frac{dk_i\,\xi_i\gamma\chi(\xi_i)}{X^2(k_i)+\gamma^2\xi_i^2}, \tag{20}$$

$$I_2(p) = \frac{4}{(2\pi)^3 k_0}\int d\vec{n}_1\int d\vec{n}_2(\vec{n}_1-\vec{n}_2)^2\delta(n_1^2-1)\delta(n_2^2-1)\delta(\vec{n}_1+\vec{n}_2-\vec{p}/k_0), \tag{21}$$

$$\mathcal{P}_\omega^{3\pi}(\omega \pm i0,\vec{p}) \approx -\frac{3h^2k_0^6}{2}I_3(p)\int_{-\infty}^{\infty}\frac{d\xi_1}{\pi}\int_{-\infty}^{\infty}\frac{d\xi_2}{\pi}\int_{-\infty}^{\infty}\frac{d\xi_3}{\pi}Z_3 t_3\times \tag{22}$$

$$\times\Pi_{i=1,2,3}\int_{-\infty}^{\infty}\frac{dk_i\,\xi_i\gamma\chi(\xi_i)}{X^2(k_i)+\gamma^2\xi_i^2},$$

$$I_3(p)=\frac{8k_0^3}{(2\pi)^6k_0}\int d\vec{n}_1\int d\vec{n}_2\int d\vec{n}_3\delta(n_1^2-1)\delta(n_2^2-1)\delta(n_3^2-1)\times \tag{23}$$
$$\delta(\vec{n}_1+\vec{n}_2+\vec{n}_3-\vec{p}/k_0)\left((\vec{n}_1\times\vec{n}_2)\vec{n}_3\right)^2.$$

The integrals $I_{2,3}$ can solved in closed form

$$I_2(p)=\begin{cases}\frac{4-p^2k_0^{-2}}{4\pi^2p} & 0<\ p\le 2k_0,\\ 0 & 2k_0<\ p,\end{cases}$$

$$I_3(p)=\frac{\pi k_0^4}{(2\pi)^6p}J(p/k_0),\quad J(k)=\begin{cases}\frac{2k^3(k^4-42k^2+105)}{105} & 0\ <k\le\ 1,\\ \frac{k^7-42k^5+105k^4+105k^3-378k^2+81}{105} & 1\ \le k\le\ 3,\\ 0 & 3\ <k.\end{cases}$$

In obtaining eqs. (20), (22) we suppose that the momenta of the ρ,ω-mesons are not too small, i.e., $|\vec{p}|\gg\tilde{\omega}\sqrt{b}$, in order that the different maximum values of the various propagators do not necessarily coincide. Correspondingly, these relations are valid only in such a region of p.

In the limiting case of large temperatures, $T\gg\tilde{\omega}^2$, we can reduce the above relations (20) - (23) by using the expansion

$$\exp\{\frac{\xi}{T}\}\approx 1+\frac{\xi}{T}, \tag{24}$$

which is justified since the functions in the integrals have sharp maxima at $\xi\le\tilde{\omega}^2$. After a direct calculation we get for the real and imaginary parts of the expressions (20), (22)

$$\mathrm{Re}\mathcal{A}(\omega,\vec{p})=-\frac{\pi^2Tk_0^4}{3b\tilde{\omega}^2}I_2(p)\left(1-\frac{\omega^2\gamma^2}{(2\tilde{\omega}^2)^2+\omega^2\gamma^2}\right), \tag{25}$$

$$\mathrm{Re}\delta\mathcal{P}_\omega^{3\pi}(\omega,\vec{p})=-3h^2k_0^6T^2\left(\frac{\pi}{\sqrt{b}\tilde{\omega}}\right)^3I_3(p)\left(1-\frac{\omega^2\gamma^2}{(3\tilde{\omega}^2)^2+\omega^2\gamma^2}\right), \tag{26}$$

$$\mathrm{Im}\mathcal{A}(\omega\pm i0,\vec{p})=\mp\frac{\pi^2Tk_0^4\omega\gamma}{3b(\omega^2\gamma^2+\tilde{\omega}^4)}I_2(p), \tag{27}$$

$$\mathrm{Im}\delta\mathcal{P}_\omega^{3\pi}(\omega\pm i0,\vec{p})=\mp3h^2T^2k_0^6\left(\frac{\pi}{\sqrt{b}}\right)^3I_3(p)\frac{\sin(2\alpha/3)}{(\omega^2\gamma^2+\tilde{\omega}^4)^{3/4}} \tag{28}$$

with $\alpha=\arctan(\omega\gamma\tilde{\omega}^{-2})$. The presented imaginary parts change their sign at $\omega=0$ and are proportional to ω at small values of ω in accordance with general analytical properties of the quantities $\delta\mathcal{P}$ and $\mathcal{A}$. For the expression (28) this becomes obvious by considering the limit $\omega\to 0$.

Here we comment briefly the reason to neglect contributions of the time like terms in eqs. (13), (16). Such terms would give contributions similar to eqs. (20) and (22), however with replaced integrals over ξ by

$$\int_{-\infty}^{\infty}d\xi_i\frac{\xi^{1+m}\chi(\xi_i)}{X^2(k_i)+\gamma^2\xi_i^2},\quad m=1,2. \tag{29}$$

The integrands here have not so sharp maxima at $k\sim k_0,\tilde{\omega}\to 0$, i.e. $X\sim 0$ as in case $m=0$ which appears in the spatial components.

5. Results of calculations and discussion

The relations (25) - (28) allow the explicit determination of the properties of the ρ, ω-propagators in the limit of an essentially softened pionic mode. The quantities (25) - (28) increase strongly in the limit $\tilde{\omega} \to 0$ if the value of ω is not too large, i.e., $\omega \leq \tilde{\omega}$. The quantity $\mathcal{P}^{2\pi}(\omega \pm i0, \vec{p})$ behaves regularly in this limit, because the function $\mathcal{A}$ appears in both the nominator and denominator, i.e.,

$$\delta\mathcal{P}_{\rho}^{2\pi}(\omega \pm i0, \vec{p}) = 2\frac{\tau\mathcal{A}(\omega \pm i0, \vec{p})\tau}{1 - 2\tilde{\Lambda}\mathcal{A}(\omega \pm i0, \vec{p})}, \quad \mathrm{Re}\mathcal{A} < 0. \tag{30}$$

This quantity occurs in the propagator $\mathcal{D}(\omega, \vec{p}, n, T)$ of the ρ-meson in nuclear matter and determines its properties. Since the quantities $\delta\mathcal{P}_{\rho}^{2\pi}, \delta\mathcal{P}_{\omega}^{3\pi}$ possess imaginary parts, the propagators of the ρ, ω-mesons do not have poles, but are defined on the complex plane with cuts along the real axis, as the pion propagator (4). In order to investigate the change of the ρ-meson in nuclear matter we determine the energies ω at which the real part of the denominator of the ρ-meson propagator vanishes. Simultaneously we calculate the imaginary part of $\delta\mathcal{P}_{\rho}^{2\pi}(\omega - i0)$ at these energies. This imaginary part determines the modification of the width of the ρ-meson in medium. That is, we solve the equation

$$\mathcal{D}^{-1}(\omega, \vec{p}, n, T) = \omega^2 - p^2 - m_{\rho}^2 - \delta\mathrm{Re}\delta\mathcal{P}_{\rho}^{2\pi}(\omega, p) - \delta\mathcal{P}^{1\pi} = 0, \tag{31}$$

and obtain the quantity $\omega(p)$ in the region $p \sim k_0$ at various temperatures and values of $\tilde{\omega} < 1$, i.e., at nuclear matter density $n \sim n_c$ in eq. (5). The dependence $\omega(p)$ is displayed in Figs. 2 and 3 at $T = 1$ and $T = 0.5$. For comparison the spectrum of the free ρ-meson is displayed too. One observes that the energy $\omega(p, \tilde{\omega})$ increases with deceasing values of $\tilde{\omega}$. This indicates that there is no softening of the ρ-meson mode near the critical point of the pion mode. On the contrary, the ρ-meson mode becomes stiffer at decreasing values of $\tilde{\omega}$ and increasing temperature. And the width of the ρ-meson increases because the value of the quantity $\mathrm{Im}\delta\mathcal{P}_{\omega}^{2\pi}$ increases with increasing temperatures and decreasing values of $\tilde{\omega}$, as, e.g., seen in Fig. 4., where the values of $\mathrm{Im}\mathcal{P}_{\rho}^{2\pi}$ are displayed at characteristic momenta p of the ρ-meson. One can see that the change of the width is remarkably larger (namely $\approx$ 1400 MeV) than the width of the free ρ-meson. It means that in nuclear matter at large temperature and density the ρ-meson does not exist as stable particle.

Considering the properties of the ω-meson in nuclear matter, we start with the investigation of the propagator $\mathcal{D}_{\omega}(\omega, \vec{p})$ at $\omega = 0$. In this case $\mathrm{Im}\delta\mathcal{P}(0) = 0$. We calculate this quantity

$$\mathrm{Re}\mathcal{D}^{-1}(\omega, \vec{p}) = \omega^2 - \omega_p^2 + \frac{B}{\tilde{\omega}^3}\frac{(3\tilde{\omega}^2)^2}{((3\tilde{\omega}^2)^2 + \gamma^2\omega^2}, \tag{32}$$

$$B = 3h^2k_0^6T^2\left(\frac{\pi}{\sqrt{b}}\right)^3 I_3(p) > 0, \quad \omega_p^2 = m_{\omega}^2 + p^2$$

at $\omega = 0$ as function of p at various values of $\tilde{\omega}$. A representative example of such calculations is displayed in Fig. 5. For the free ρ-meson one has the curve 1. One observes (see curves 2, 3) that the quantity $-\mathcal{D}_{\omega}^{-1}(0, \vec{p})$ decreases and shows a minimum when the value of $\tilde{\omega}$ decreases. In the limit $\tilde{\omega} \to 0$, i.e., near the critical point, the ω mode becomes unstable (cf. dashed line). This instability has the same meaning as the pion mode instability which has been considered in many previous works (see, for example, [1, 8]) In this case the energies ω of ω-mesons in certain momentum intervals would have a negative imaginary part, and consequently the ω-meson field would increase unlimited. For a self-consistent description one needs to take into account the processes, which are non-linear in the meson field. Here we focus on the pre-critical behaviour of the ω-meson propagator by means of eqs. (6, 26, 28, 30) and postpone the investigations of these non-linear effects to later work. To elucidate the behaviour of the ω-meson propagator we calculate the quantity (32) as function of ω at various values of T, $\tilde{\omega}$, p. Results of these calculations are displayed in Fig. 6, which reflect the same properties of the ω-meson propagator as shown in Fig. 5. At the relatively large values of $\tilde{\omega}$ and p outside of the interval, where the dashed line in Fig. 5 dives into the negative area (curve 1 in Fig. 6), the equation

$$\mathrm{Re}\mathcal{D}_\omega^{-1}(\omega) = 0 \tag{33}$$

with $\mathrm{Re}\mathcal{D}_\omega^{-1}(\omega)$ from eq. (32) has only a single solution, ω_1, not being significantly smaller than ω_p. If $\tilde{\omega}$ decreases, a second solution ω_2 of eq. (33) appears, being much smaller than ω_p. It is not difficult to trace back immediately the rise of the second solution from eqs. (32, 33) if one takes care that at small $\tilde{\omega}$ the last term in eq. (32) grows essentially at small $\omega < \omega_p$. If ω becomes very small and p resides in the above mentioned interval, then eq. (33) has no real, positive solution. Of course, a imaginary solution exists, which just reflects the fact, that the ω-meson field under such conditions grows unlimited, as already demonstrated in Fig. 5. The dependence of the solutions $\omega_{1,2}$ on $\tilde{\omega}$ is displayed in Fig. 7. One observes that, in agreement with results displayed in Figs. 5 and 6, there is no real positive solution of eq. (33) at very small $\tilde{\omega}$. In some interval of $\tilde{\omega}$-values both solutions $\omega_{1,2}$ do exist. At larger values of $\tilde{\omega}$ only one solution ω_1 survives, which approaches the free ω-meson energy ω_p. From eq. (33) we also obtain the solutions $\omega_{1,2}(p)$ as function of momentum p for certain values of $\tilde{\omega} < 1$ and $T = 1.5$. The results of the calculations are displayed in Fig. 8. If $\tilde{\omega}$ is not too small then two solutions exist in some interval of the momentum p, and at large p only one solution remains, which finally coincides with ω_p. This behaviour is illustrated by the solid and long-dashed lines. At very small values of $\tilde{\omega}$ (see short-dashed line) both solutions disappear in some interval of p. That means, as mentioned above, that there is no real positive solution at such values p, $\tilde{\omega}$, T. In Fig. 9 the imaginary part of $\delta\mathcal{P}_\omega^{3\pi}$ is displayed as function of p with $\omega(p)$ as shown in Fig. 8. It can be seen that the width of ω-meson in nuclear matter at large density and temperature can be considerably larger than in vacuum.

6. Conclusions

According to our investigations the behaviour of ρ,ω-mesons in compressed and heated nuclear matter are quite different. The ρ-meson propagator does not show a very particular behaviour. The ρ-meson degrees of freedom becomes stiffer. (one may say that the mass increases), and the width broadens essentially. In contrast, the ω-meson degrees of freedom becomes softer, if the temperature and density increase. In extreme case, when $\tilde{\omega} \ll 1$ and the temperature is sufficiently large, the unlimited amplification of the ω-meson field sets on. As we have seen, the ω-meson width broadens too with increasing temperature and density. We are speaking about the density and temperature at which the ω-meson field does not yet increase infinitely. The expressions for ρ, ω-meson propagators, which we derive here, are to be used in future investigations of various phenomena, occurring in dense and hot nuclear matter.

7. Summary

In summary, we analyze the properties of ρ- and ω-mesons in dense and hot nuclear matter. Near the critical pion mode softening one obtains a comparatively clear picture of the ρ, ω-mesons due to their strong coupling to the pions. We find indications for shifting the ρ-meson mass to considerably larger values (in comparison with the vacuum value), and its width is strongly increased. Under such extreme conditions one cannot expect to observe distinguished ρ-meson peak, e.g., in di-lepton signals. Also the ω-meson width is increased, however the mass is reduced in the same time.

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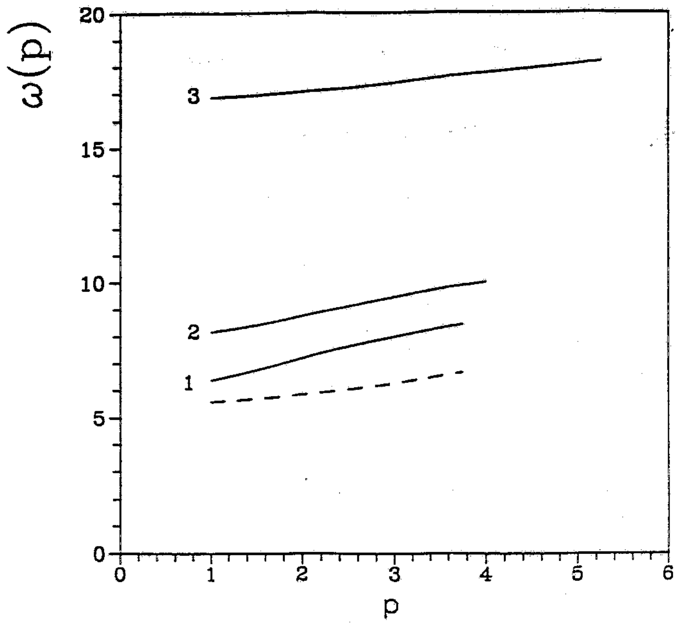

Figure 2. Solutions $\omega(p)$ of eq. (31) for the ρ-meson at T=1 and various values of the quantity $\tilde{\omega}$ (solid lines, $\tilde{\omega} = 0.5$ (1), 0.3 (2), 0.1 (3)). The dashed line depict $\omega(\vec{p})$ for the free ρ-meson for comparison.

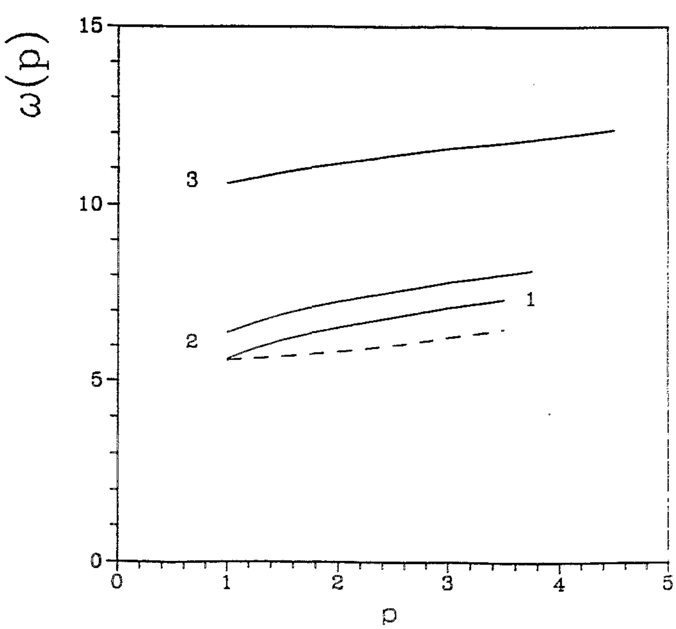

Figure 3. The same as in Fig. 2. but at $T = 0.5$

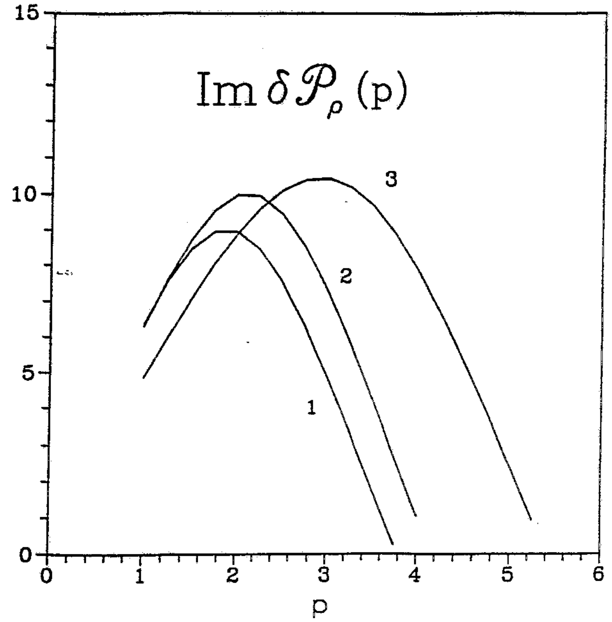

Figure 4. The imaginary part of the modification of the ρ-meson polarization operator, $\mathrm{Im}\delta\mathcal{P}_\rho^{2\pi}(\omega(\vec{p}),p)$, in medium as function on p at $T = 1.0$ with $\omega(p)$ being just the energy displayed in Fig 2.

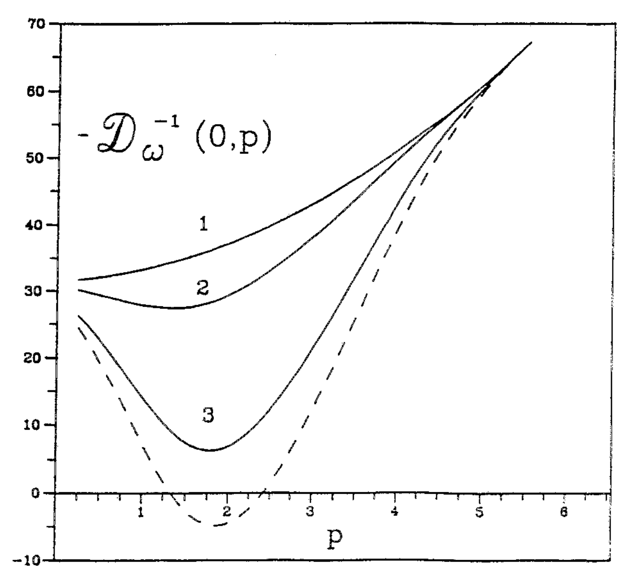

Figure 5. The quantity $-\mathcal{D}_\omega^{-1}(\omega,\vec{p})$, eq. (32), for the ω-meson as function on p at $\omega = 0$ (that is $\mathrm{Im}\mathcal{D}_\omega^{-1} = 0$). The curves 2, 3 and the dashed one are obtained at $T = 1$ and $\omega = 0.5$, 0.375 and 0.35, respectively. The quantity $-\mathcal{D}_\omega^{-1}(0,\vec{p})$ of the free ω-meson is also displayed (curve 1) for comparison.

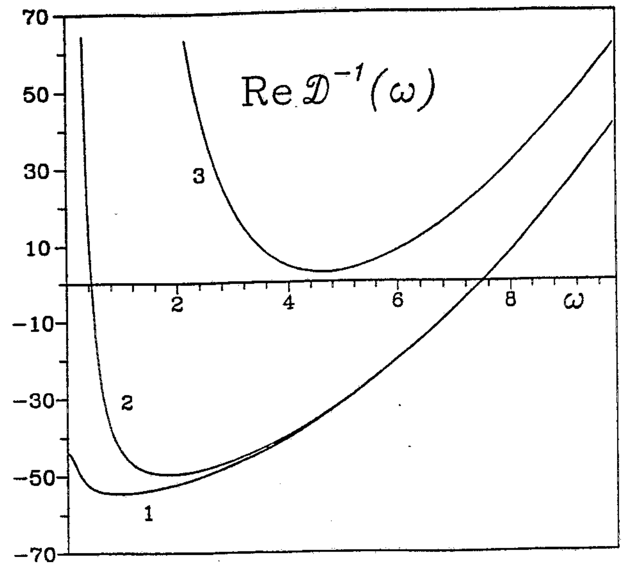

Figure 6. The dependence of the quantity Re$\mathcal{D}_{\omega}^{-1}(0,\vec{p})$ for the ω-meson on ω for various values of T, p and $\tilde{\omega}$. The curves correspond to the following parameters sets: 1 - T = 1, p = 5, $\tilde{\omega}$ = 0.3; 2 - T = 1, p = 5, $\tilde{\omega}$ = 0.1; 3 - T = 1.5, p = 3, $\tilde{\omega}$ = 0.05.

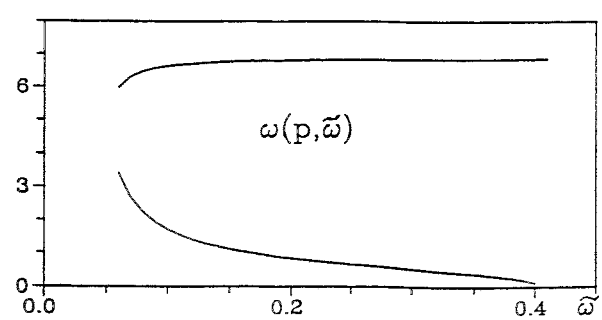

Figure 7. The two solutions, $\omega_1(\tilde{\omega})$ and $\omega_2(\tilde{\omega})$, of eq. (33) as function on $\tilde{\omega}$ at T = 1.5, p = 4. The upper (lower) curve represents $\omega_1(\tilde{\omega})$ $(\omega_2(\tilde{\omega}))$.

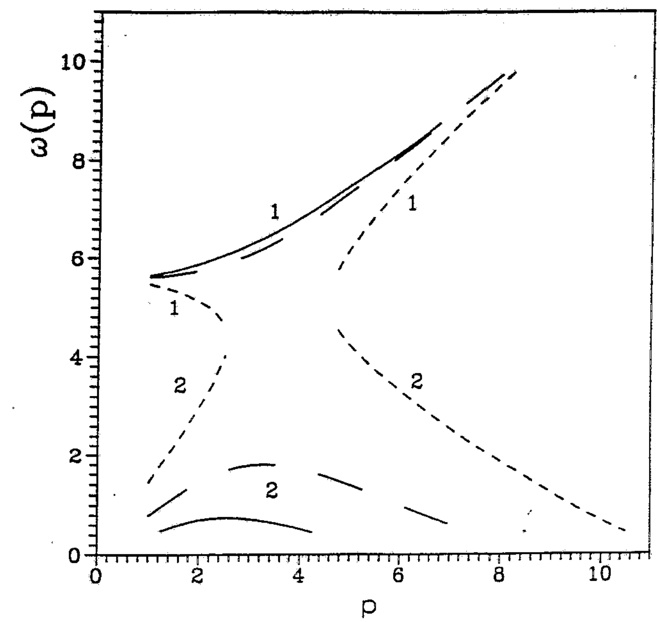

Figure 8. The solution $\omega(p)$ of eq. (33) as function on p at $T = 1.5$. The solid, short-dashed and long-dashed curves correspond to the values of $\tilde{\omega} = 0.3$, 0.05 and 0.1. The two solutions, $\omega_1(p)$ and $\omega_2(p)$, existing in this interval of p for given p, are marked in as "1" and "2". In the region $2.4 < p < 4.8$ there are no solution in case of $\tilde{\omega} = 0.05$.

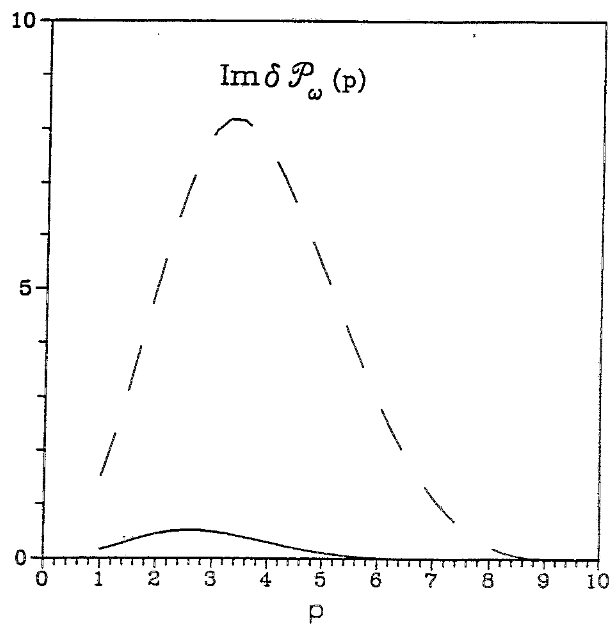

Figure 9. The imaginary part of the modification of the ω-meson polarization operator $\mathrm{Im}\delta\mathcal{P}^{3\pi}_{\omega}(\omega(\vec{p}), p)$ at $T = 1.5$, $\tilde{\omega} = 0.3$ (solid curve) and $\tilde{\omega} = 0.1$ (dashed curve), as function on p and with $\omega = \omega(p)$ as represented in Fig. 8.