DETERMINATION OF RESONANCE FREQUENCIES AND VIBRATION CHARACTERISTICS OF STATORS HAVING ENCASED CONSTRUCTION

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by

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ABSTRACT

An accurate assessment of the resonance frequencies of stators is essential for designing a quiet electrical machine. Also, detailed information about the vibration characteristics of the stator is of much interest.

In the newer construction of stators, the stator-core is encased tightly in the frame. This construction is considered to be simpler and more economical as it eliminates the use of ribs between the core and the frame. An analysis, of a general nature, for the determination of the various resonance frequencies and mode shapes of stators having encased construction is presented in the thesis. As the general frequency equation is derived on a three-dimensional basis, it delivers information about not only the radial vibrations of the stator but also about torsional and axial vibrations. In addition, the flexural vibrations along the machine-length are taken into account in the analysis. The validity of the analysis is confirmed by experimental results which were obtained from the literature for stators of small as well as of large machines.

A detailed study of the vibration characteristics of a representative stator is carried out and the results are presented in a generalized form. The advantages of the encased construction of stators over the conventional construction from the vibration considerations are discussed.

Finally, the validity of the simplified frequency equations, which are derived from the general frequency equation, is checked.

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LIST OF PRINCIPAL SYMBOLS

Symbol	Parameter
E	modulus of elasticity
G	shear modulus of elasticity = $\frac{E}{2(1+\mu)}$
h	frame-thickness
I_n, K_n	modified Bessel functions of the first and second kinds; respectively.
i	$\sqrt{-1}$
J_n, Y_n	Bessel functions of the first and second kinds; respectively.
L	length of the stator-stack
m	number of half waves along a generatrix of the stator
M	longitudinal wave-number = $\frac{m\pi R}{L}$
n	number of waves around the circumference of the stator
r,θ,z	radial, circumferential and axial coordinate variables in the polar cylinderical coordinate system
r_1	inner radius of the stator-core
r_2	inner radius of the stator-yoke
r_3	inner radius of the frame
R	mean radius of the frame
t	time
w,v,u	radial, tangential and axial components of displacement
μ	poisson's ratio
ω	circular frequency, rad./sec.
ω_o	reference frequency = $\frac{1}{r_3}\sqrt{\frac{E_c}{\rho_c}}$

ρ mass-density

$\bar{\rho}_c$ equivalent mass-density of the stator-yoke taking into account the mass of teeth and windings

$\bar{\rho}_f$ equivalent mass-density of the frame taking into account the mass of cooling ribs

σ_r radial component of the normal stress

$\tau_{r\theta}$, τ_{rz} shear stresses in planes $r\theta$ and rz; respectively.

∇^2 laplacian differential operator

$$= \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2} + \frac{\partial^2}{\partial z^2}$$

Δ $= \frac{\partial w}{\partial r} + \frac{w}{r} + \frac{1}{r}\frac{\partial v}{\partial \theta} + \frac{\partial u}{\partial z}$

ϕ, ψ, χ displacement potentials functions

Subscripts

c core

f frame

tw teeth and windings

Derivatives

$(\ )' = R\frac{\partial(\ )}{\partial z}$

$(\ )^{\cdot} = \frac{\partial(\ )}{\partial \theta}$

$\dot{f}_{(x)} = \frac{\partial f_{(x)}}{\partial x}$

Marks

A quantity raised to an asterik (*) means that this quantity is evaluated at the interface between the stator-core and the frame.

$|F|$ = modulus of the quantity F .

1. INTRODUCTION

1.1 General Review

Noise has become a serious threat to the well-being of everyone of us. It is with us everywhere and all the time. Although there is a conspicuous disagreement among investigators on the psychological and physiological effects of noise[1], there seems to be, nevertheless, substantial evidence that noise reduction has an overall beneficial effect[2-7]. In this regard, the reduction of vibration and noise has become very important in rotating machines for both commercial and military applications[2]. In the case of naval vessels, audible noise limits the effective range of sonar and increases the danger of detection. In industrial applications, low-vibration motors are essential for machine tools to produce finer tolerances of finished products. In air-conditioning and refrigeration equipment , great efforts are put to minimize the noise transmitted by structure or by air. Also, industry is becoming aware of the risks to the ear and hearing of workers and of the time wasted through interruption of communication which may occur in an environment of high noise-levels[3,4].

Due to the innumerable applications of electrical machines, the study of noise produced by them forms an important consideration. Electrical machines of all sizes are frequently sources of unpleasant noises of mechanical, aerodynamical and electromagnetic origins[5-7]. These noises are produced by bearings, ventilating fans and magnetic field present in the air-gap of the electrical machine. The continuous increase in the applications of electrical machines, these days, has resulted in a growing awareness of noise problems in electrical machines. It is now becoming necessary for machine manufacturers to make their machines quieter than those of their competitors. For a designer, it has become essential to be able to predict accurately the noise production of the machine under design[6]. This is especially important in the case of large machines, where it is impracticable to build a prototype model, and it is expensive to apply means of external noise control. Furthermore, under the economic pressures which characterize production nowadays, manufacturers try by all means to keep the volume of the electrical machine as small as possible for a specified rating[5]. This means a maximum exploitation of the materials involved, for example, using higher flux and current densities, etc. As a result, the amplitudes of the exciting magnetic forces exerted on the different machine members increase considerably and, therefore, the electrical machine tends to be considerably noisier.

It is a well-known fact that a small impressed force causes excessive vibrations in a structure if this structure is nearly in resonance with the impulses. Accordingly, the various exciting forces acting on the stator and rotor of the electrical machine may produce objectionable noise and excessive vibrations when the frequencies of the exciting forces are equal to, or near, the natural frequencies of the members of the machine concerned. The vibrations and consequently the noise-level produced by an electrical machine can, therefore, be reduced to a large extent by modifying the construction and dimensions of the members of the machine in such a way that their mechanical response to the exciting forces is diminished. The determination of the resonance froquencies and vibration characteristics of the different parts of an electrical machine is, thus, of much importance in relation to noise reduction[6-20].

In the newer construction of the stator, which is used for machines of small and medium power capacities these days, the stator-core is encased tightly in the frame. This construction eliminates the use of ribs between the stator-core and frame and is, therefore, considered to be simpler and more economical. A detailed study of the vibration characteristics of such a stator is, therefore, very much desired. At the same time, this stator model is suitable for the initiation of a rigorous analysis for the calculation of the different resonance frequencies of stators of electrical machines. It is the aim of this thesis to contribute towards the effort in reduction of noise in electrical machines, in general, by evolving an analytical method for the determination of the resonance frequencies of such stators and presenting an extensive study of their vibration characteristics.

1.2 Comments on Published Literature

Although the importance of the mechanical response of stators of electrical machines in the problem of noise was realized a long time ago[8,9], little attention has been paid to it by investigators. Probably the first one to investigate the problem of mechanical response of stators of electrical machines was Den Hartog[10,11]. He considered only the frame and neglected the stator-core on the basis that the frame usually emits a larger portion of the total noise. Whereas, this can be partially true when dealing with the problem of calculating the electromagnetic acoustic noise emitted by the electrical machine, it is not the case when one attempts to calculate the resonance frequencies of the stator.

On the other hand, several authors[12,16,17,19] considered the vibrations of the stator-core alone because it is subjected to electromagnetic forces. Stators of electrical machines, in general, are more complicated than a single ring. An outer frame is usually provided to support the stator-core. In the modern construction of electrical machines, the thickness of the frame is, generally, kept small. In recent investigations[15,18,20], it has been observed that the frame can play a very important role in the problem of noise production in electrical machines. Consideration of the frame is, therefore, very important in any investigation related to the problem of noise in electrical machines.

Although the importance of the effects of the frame was realized a long time ago, no published work dealt with the actual construction of stators of electrical machines until 1955. Erdelyi[13-15] treated the stator as a two-ring system with radial and tangential ribs in-between For the sake of simplicity, Erdelyi assumed modes of inextensional nature (which means that the length of the midsurface of the vibrating rings does not change by deformation). He considered the effects of the stator teeth and winding by only adding their masses to the mass of the yoke. In addition, he treated both core and frame as thin rings. As a matter of fact, the stator-core has been treated in the same manner in most of the analyses available in the literature[12-17]. As the ratio of the thickness of the stator-core to its mean radius may well exceed the value 0.2 in the case of turbogenerators, special low-noise machines, machines of small capacities, 2-pole medium-sized machines and several other types of machines, the assumption of a thin-shell may lead to considerable errors in the results[18].

Recently, as an extension of Erdelyi's work, Ellison and Yang[18] calculated the natural frequencies of a stator having a thin frame coupled solidly through key bars to a thick ring loaded with teeth and windings, taking into account bending, shear, extension and rotary inertia.

It may be mentioned that the studies of Ellison and Yang as well as those of Erdelyi and all others are confined to the lowest resonance frequency of a mode of deformation. Moreover, the analyses are limited to plane vibrations only (involving radial and tangential components of displacement). In other words, the presence of axial vibrations has been ignored. The stator of an electrical machine is subjected not only to radial and torsional vibrations but also to axial ones[21,22]. All these vibration components may be excited simultaneously during machine operation.

Further, in the analyses available[10-20] in the literature, the authors have invariably assumed that the amplitudes of vibrations of the stator are uniform along the machine-axis. However, this may not be the case in machines having considerable length, e.g. in turbogenerators[8,9]. Moreover, in the presence of skewing, the distribution of the magnetic forces along a generatrix is not uniform[13,14]. For these considerations, flexural vibrations of the stator along a generatrix are of much interest.

In the case of the conventional construction of stators where the stator-core is supported by the frame through ribs, it has been noticed[13-15,18] that the frame creates additional resonance frequencies. In addition, the presence of the frame may lead to an increase in the noise-level of the machine by 20-30 decibels.[20] Although these disadvantages from the noise considerations have been known for a long time, the conventional construction of stators is still widely used. Unfortunately, very little information on the vibration characteristics and influence of the frame in the case of an alternative construction is available in the published literature. There is, therefore, a need for investigating the vibration behaviour of stators of alternative construction.

For this and for the reasons mentioned before, it is felt that an analysis of a general nature based on three-dimensional considerations is required for the determination of the various resonance frequencies of a stator of encased construction. In the published work, very little information about the vibration behaviour of such a construction is available.

1.3 The Problem

To design a quiet electrical machine, one requires complete information about the vibration characteristics of the stator of the machine under consideration. The object of this thesis is to develop a suitable analysis for the determination of the various resonance frequencies of a stator having encased construction, in which a thick stator -core, which is loaded with teeth and windings, is encased in a thin frame. The analysis should take into account the variations that may be present in the amplitudes of vibrations along the machine-length, and also provide information not only about the lowest resonance frequency for each mode but also about all the possible resonance frequencies of the stator. In contrast to the previous work[10-20], the stator vibrations should not be considered as plane vibrations only, but also axial vibrations should be taken into account. For the investigations of the vibration behaviour of the stator, the solutions presented should be general and applicable to any arbitrary values of the physical parameters and dimensions of the stator.

1.4 Brief Description of Contents

The thesis consists mainly of two sections. The first section deals with the development of the analysis for the free vibrations of a stator having encased construction. The second section contains a detailed study of the vibration characteristics and mechanical response of such a stator.

Before actually developing the rigorous analysis for the determination of the resonance frequencies and vibration characteristics, it is advisable that the different aspects of the problem of electromagnetic noise in electrical machines be considered closely. In this connection, it is essential to discuss the nature of electromagnetic force excitations which are usually present during the operation of an electrical machine. In the study of the noise problem, the response of the human ear is a crucial consideration.

After discussing the different aspects of the electromagnetic noise problem in electrical machines, an analysis for the calculation of the various resonance frequencies and mode shapes of a stator having encased construction is developed. The exact equations of motion are used to describe the motion of both core and frame and the effect of teeth and windings is taken into account in a general manner. The approach used in the present analysis is different from the various approaches adopted in the published literature. In spite of the rigor of the present analysis, an effort has been made to derive the solutions in a simple and straightforward way.

After deriving the general frequency equation, special frequency equations are derived from the general equation for the cases of equal amplitude vibrations along the machine-length and/or along the machine-circumference. For the purpose of reducing the calculation effort, three different types of simplification are separately introduced in the general frequency equation and the validity of these simplifications is checked on the stator of a typical induction motor.

For confirmation of the validity of the analysis contained in this thesis, comparisons between the computed and measured values of the resonance frequencies of some typical stators of electrical machines, having different constructions and power capacities, are drawn. It is felt that the usual practice of testing theories on a single model,[13-15,18] or on models of nearly the same power rating, is inadequate[16,19]. In addition, a detailed discussion of the agreements and errors existing between the calculated and measured values is included.

In order to obtain the vibration characteristics of stators of encased construction, many frequency calculations have been done for a representative model. The parameters of the chosen model are realistic for small, as well as for medium and large electrical machines. The frequency spectrum, mechanical response and mode shapes are given and discussed in detail. As a matter of fact, such studies have not been considered in the published literature even for stators of the conventional construction.

Finally, the advantages of stators of encased construction over those of conventional construction from the vibration point of view are pointed out.

Four appendices are given at the end of this thesis. Appendix-A describes the solution for the general frequency equation while Appendix-B describes the technique of calculating the various resonance frequencies of the stator from the frequency equation derived in this thesis. Appendix-C gives the derivation of the frequency equations for the special cases, and Appendix-D gives the derivation of the factor which takes into account the effects of teeth, windings and cooling ribs.

It may be added that a major part of the work reported in this thesis has been accepted for publication[63-65].

2. THE MAIN ASPECTS OF THE PROBLEM OF ELECTROMAGNETIC NOISE IN ELECTRICAL MACHINES

In general, an electrical machine is a source of noise because the main members of the machine are subjected to periodically varying forces. Among the three different types of noise produced in electrical machines (ventilation noise, bearing noise and electromagnetic noise[2,5,7]), the electromagnetic noise is the one which is produced by the forces created by the magnetic field present in the machine. These forces cause vibrations in the various parts of the machine. The amplitude of such vibrations does not only depend on the amplitude of the exciting forces, but also depends on the mechanical response of the various machine parts to the exciting forces[6,7]. With large amplitudes of vibration , the noise emitted from the machine is high. In addition, the loudness of the machine noise does not only depend on the amplitude of vibrations but also depends on the physical response of the human ear[3,4,7]. The problem of electromagnetic noise and vibrations in electrical machines involves, therefore, the complex inter-relationships between electromagnetic forces produced in the machine, mechanical response of the machine parts and physical response of the human ear[2]. In the following, the three main aspects of the problem are discussed.

2.1 Electromagnetic Exciting Forces

Noises of electromagnetic origin appear at different frequencies depending on the exciting forces. Here, the mechanism of the production of the main electromagnetic forces in the machine, together with the mode shapes and frequencies of the vibrations caused by these forces, are discussed. Although the discussion will be restricted to the three-phase induction motors, the electromagnetic noise problem in the other types of electrical machines is not very much different.

The electromagnetic forces created in electrical machines be classified into two major categories. Those forces which are generally present in an electrical machine are considered as one category, while the forces which are created only in the presence of asymmetries (magnetic, electrical or mechanical) fall into another category.

2.1.1 Forces produced in the absence of asymmetries

2.1.1.1 The fundamental force-wave produced by the air-gap magnetic field

Due to the presence of the magnetic field in the air-gap of the machine, a radial force proportional to the square of the field density is created[12]. This force has the tendency to mutually attract stator and rotor. To illustrate this, Fig. 2.1 is given which shows the developed air-gap of an induction motor with the flux-wave over two poles.

As the flux is sinusoidal, the distribution of the force in the air-gap has a constant component and a sinusoidal component having twice as many cycles as the original field. The first component is uniform around the periphery and so it produces a tangential compression in the stator laminations and remains constant at all times and, therefore, has no effect on noise production. On the other hand, as the flux rotates in the air-gap at synchronous speed, the sinusoidal component of the force rotates with the same speed. As a result, every point on the stator and rotor will be subjected to a periodic force with 2p cycles

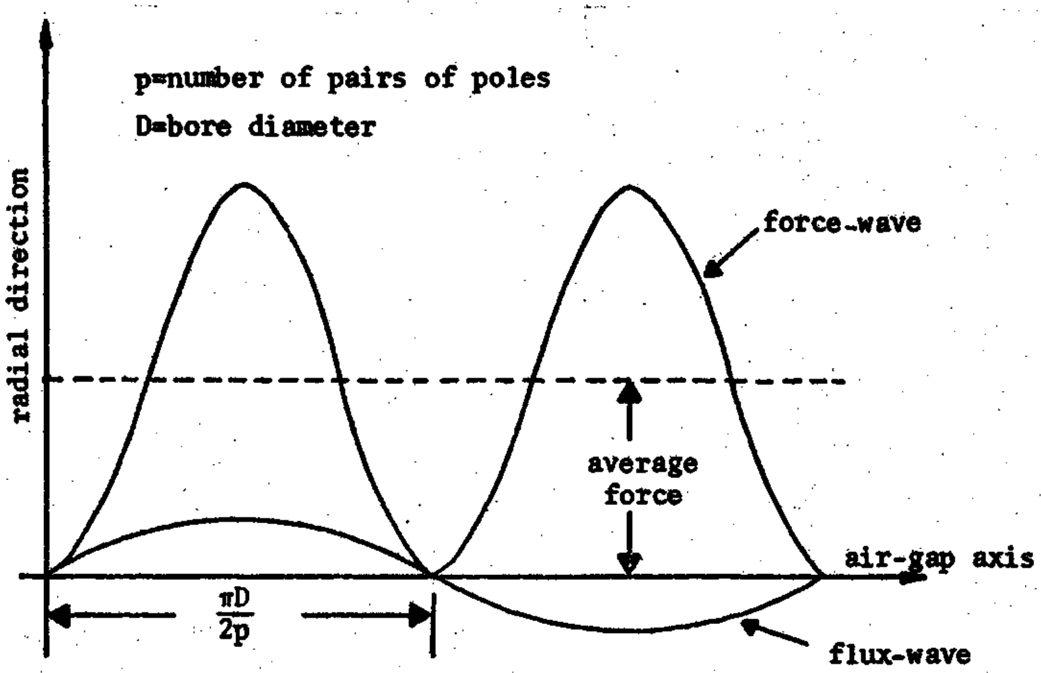


Fig. 2.1 Radial magnetic flux and force-waves

per revolution for a 2p-pole machine. Under the effect of such a force-wave, the stator is deformed into the shape of a rotating pattern having 2p lobes. In this case, it can be easily seen that every point of the stator will vibrate with double the supply frequency. On the other hand , the rotor does not, usually, show any appreciable response to such a force because of its highly stiff construction.

2.1.1.2 The effect of magnetostriction

It is well-known that when the steel used in electrical machines is magnetized it expands very slightly along the axis of magnetization. Hence, around the periphery of the stator-core, the length of an arc increases and decreases as the flux-density rises and falls. As the magnetostrictive effect does not depend upon the direction of the flux, the pattern of the magnetostrictive expansion will have 2p cycles around the periphery of the stator-core in a 2p-pole machine. This pattern rotates with the field, causing very small vibrations in both radial and tangential directions. Consequently, a point of the stator-core will vibrate with a frequency equal to double the supply frequency[12]. As the resulting vibrations are extremely small, the magnetostriction-effect does not play an important role in the production of noise in rotating electrical machines. In contrast, magnetostriction-effect is one of the main considerations in the study of noise produced in transformers[6].

2.1.1.3 Force-waves produced by the harmonic fields in the air-gap

As it is well-known, the air-gap field is at no instant distributed according to a simple sinusoidal wave. Also, the distribution changes as the field rotates[23,24]. This is equivalent to saying that, in addition to the fundamental field,there are other fields with other than the fundamental number of poles and some with other than the fundamental frequency.

With all these harmonics, the radial component of the air-gap flux-density can be expressed in the form of a series of trigonometric functions as follows[25]:

$$B = \Sigma B_k \cos (p_k x - \omega_k t + \theta_k) \quad \ldots. \ (2.1)$$

where; p_k is the number of pairs of poles of a given sinusoidal component, ω_k is the corresponding frequency and θ_k is the general phase difference angle. The summation is made for the several necessary values of p_k and ω_k. In equation (2.1), x is in mechanical degrees.

As the radial force resulting from the presence of such a field is proportional to the square of the field density B at a given point, then the expression for the radial force between the stator and rotor will have the following form:

$$F = C\,\{\Sigma B_k \operatorname{Cos}(p_k x - \omega_k t + \theta_k)\}^2 \qquad \ldots. (2.2)$$

where; F is the radial force and C is a constant which differs according to the system of units used.

The expression of the force F , equation (2.2), can be expanded into a series of sinusoids, the general term of which has the form:

$$\begin{aligned} F_{mn} &= CB_mB_n \operatorname{Cos}(p_m x-\omega_m t+\theta_m)\operatorname{Cos}(p_n x-\omega_n t+\theta_n) \\ &= \frac{CB_mB_n}{2}[\operatorname{Cos}\{(p_m-p_n)x-(\omega_m-\omega_n)t+(\theta_m-\theta_n)\} \\ &\qquad + \operatorname{Cos}\{(p_m+p_n)x-(\omega_m+\omega_n)t+(\theta_m+\theta_n)\}] \qquad \ldots. (2.3) \end{aligned}$$

From this general expression of the force (which results from the presence of two fields having p_m and p_n pairs of poles and frequencies of ω_m and ω_n, respectively) it can be found that[25]:

(i) Any two fields having same number of poles and different frequencies give rise to a radial force with no poles, which means uniform force around the periphery. The force produced in this case tends to distort both stator and rotor in a zero-mode, which is generally termed a breathing mode, with frequency equal to the difference between the frequencies of the two fields. The effect of such a force is, usually, of negligible importance with reference to the rotor due to its very stiff construction.

(ii) Any two fields differing by two poles give rise to a force of two poles tending to displace the rotor and stator as a whole in a beam-type motion. This type of vibration is usually very important for the consideration of the noise produced by the rotor[26].

(iii) Any two fields differing by four poles result in forces, balanced across any diameter, which tend to distort the stator into a rotating ellipse and whose frequency is equal to the difference between the frequencies of the two fields.

(iv) Any two fields having poles whose sum or difference equals any given number $g>4$ result in a force which tends to distort the stator or rotor into a rotating pattern having $g/2$ lobes. Due to the fact that the mechanical response of the stator decreases with the increase of the complexity of the mode of vibrations, limited attention is usually given to modes of high orders.

(v) A single field having g poles tends to distort the stator or rotor into a rotating pattern having g lobes. The noise component produced in this case has a frequency equal to double the frequency of the field producing it.

Effects of the slot-combination

In order to show the relation between the slot-combination of the machine and the noise behaviour, the air-gap in an induction motor will be considered smooth for the sake of simplicity. Also, it will be assumed that only the fundamental and the slot harmonics of both stator and rotor fields are present in the air-gap field. Accordingly, the air-gap field will have the following form[12]:

$$B = B_1 \text{Cos}(px-\omega_s t)+B_2 \text{Cos}[(S-p)x+\omega_s t]+B_3 \text{Cos}[(S+p)x-\omega_s t]$$

$$+B_4 \text{Cos}[(R-p)x+\omega_s(1-\frac{R(1-s)}{p})t]+B_5 \text{Cos}[(R+p)x-\omega_s(1+\frac{R(1-s)}{p})t] \quad .... (2.4)$$

where; B is the air-gap flux-density, B_1, B_2, B_3, B_4 and B_5 are the amplitudes of the various field components, S is the number of stator slots, R is the number of rotor slots, p is the number of pairs of poles, s is the slip, ω_s is the supply frequency and t is the time.

The force-wave resulting from this field is:

$$F \propto \frac{B_1^2}{2}(1+\mathrm{Cos}(2px-2\omega_s t))+B_3B_4\mathrm{Cos}\{(S-R+2p)x+\omega_s(\frac{R(1-s)}{p}-2)t\}$$

$$+(B_2B_4+B_3B_5)\mathrm{Cos}\{(S-R)x+\omega_s(\frac{R(1-s)}{p})t\}+B_2B_5\mathrm{Cos}\{(S-R-2p)x$$

$$+\omega_s(\frac{R(1-s)}{p}+2)t\}+ \text{terms of minor importance} \quad .... (2.5)$$

The $\frac{B_1^2}{2}$ term is the fundamental force-wave having 2p pairs of poles and double the supply frequency. The other three terms are force-waves having (S-R+2p), (S-R) and (S-R-2p) pairs of poles and frequencies of $(\frac{R(1-s)}{p}-2)\omega_s$, $\frac{R(1-s)}{p}\omega_s$ and $(\frac{R(1-s)}{p}+2)\omega_s$; respectively. It is, therefore, advisable that (R-S) and/or (R-S±2p) are kept as large as possible in order to achieve a quieter operation of the machine[12,27-29].

2.1.2 Forces produced due to asymmetries

The presence of electrical, mechanical or magnetic asymmetries produces several noise components in addition to those already present. The most important cases of asymmetries will be discussed in the following:

2.1.2.1 Force-waves produced by eccentricity

In designing electrical machines, it is necessary to allow a dimensional tolerance on every part required for the manufacturing. Generally speaking, the greater the tolerance the lower is the manufacturing cost[30]. In electrical machines with a small air-gap, as is the case with induction motors, slight variations in the dimensionsof the stator,rotor, end shields and bearings may lead to a considerable variation of the air-gap at different angular positions between stator and rotor. This non-uniform air-gap will give rise to unbalanced magnetic pull and consequently additional noise[30-36]. As a matter of fact, there are several types of eccentricities, which will be discussed in the following.

2.1.2.1.1 Case 1 - rotor is not exactly centered in the stator-bore and is stiff

Any rotating machine in which the rotor is not exactly centered in the stator-bore will have an unbalanced magnetic pull between the stator and the rotor, tending to pull them together on the side that has the smaller air-gap having greater flux-density, Fig. 2.2. As the

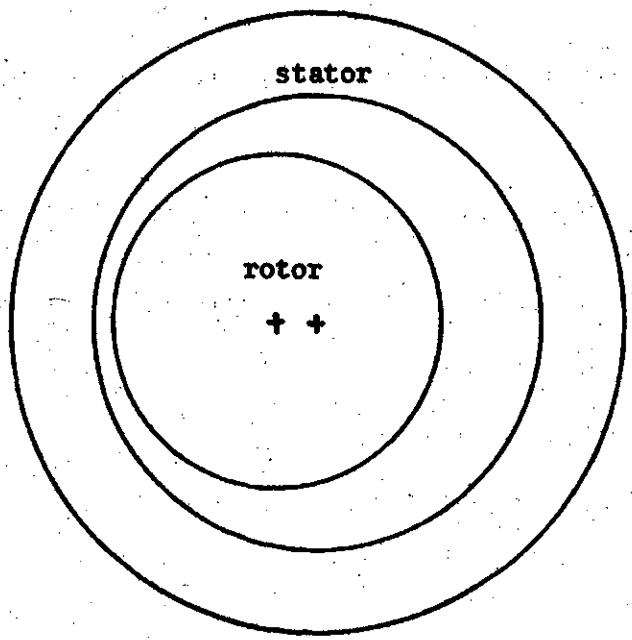


Fig. 2.2 Machine with eccentric rotor

magnitude of the pull will be different for different positions of the poles of the fluz-wave, the side-force will thus go through cycles of variations. It has been proved in several publications[33,36] that when the rotor is stiff, the stator as well as the rotor will vibrate as a whole with a frequency equal to twice the frequency of the supply. In $2\omega_s$ this case, the inequality of the air-gap stays constant, that is, the stator and rotor do not move sideways relative to each other under the influence of the force.

2.1.2.1.2 Case 2 - rotor is not exactly centered in the stator-bore and is not stiff

If it is supposed that the rotor moves to the right under the influence of the elastic restoring forces of the shaft, as the unbalanced magnetic pull to the left on the rotor diminishes to a minimum then, side-to-side vibrations of both stator and rotor of a frequency equal to the ω_s supply frequency will be produced[36].

2.1.2.1.3 Case 3 - rotor is eccentric with respect to the shaft, and bearings are concentric with respect to the stator

This is usually called the rotating eccentricity because in this case the rotor starts whirling and a rotating eccentricity is formed. The frequency of the noise produced due to such eccentricity is equal $2s\omega_s$ to twice the slip frequency and the associated vibrations are characterized by a side-to-side motion[33].

2.1.2.2 Force-wave produced by dynamic unbalance

In some cases, the rotor is dynamically unbalanced and at the same time it is not perfectly centered in the stator-bore. Accordingly, two forces are present, one is electromagnetic (unbalanced magnetic pull) and the other is mechanical (dynamic unbalance-force). The direction of the first is always towards the point of the minimum air -gap, while the direction of the second force rotates with the rotor and is in the direction of the mechanical unbalance. The combination of these two forces produces a noise of slip-frequency[37].

2.1.2.3 Force-waves produced by electrical asymmetries in the rotor

Electrical asymmetries in the rotor, such as a defective bar, are a source of additional electromagnetic noise components. The presence of such an asymmetry results in an asymmetrical field distribution in the air-gap. As the defective bar rotates with the rotor, a noise very similar to that produced as a result of rotating eccentricity will be produced[37].

2.1.2.4 Pulsating Torques

The presence of electrical or magnetic asymmetries creates various pulsating torques of different amplitudes and frequencies[38,39]. These torques may cause structure-borne noise of considerable level[25]. The pulsating torques may also play an important role in the production of air-borne noise.

2.1.2.5 Axial forces

In induction motors with skewed slots, the flux density is not uniform along the machine length. This results in a non-uniform distribution of magnetic energy along a generatrix. Consequently, the rotor as well as the stator experience axial vibrations[22]. Also, in the presence of skewing, the values of the permeance at different points along the machine length are different and, accordingly, the stored magnetic energy in the air-gap of the machine is non-uniform along a generatrix. Thus, additional axial vibrations are produced[22].

Moreover, an axial displacement of the rotor due to any reason (like axial forces produced by skewing, imperfect levelling of the shaft, the pull of the ventilating fan, etc.) results in changing the alignment of the ends of the stator with the corresponding ends of the rotor, as well as the alignment of the edges of the ducts of rotor and stator. Consequently, both rotor and stator experience two additional axial forces[21].

Since the torques produced in electrical machines are due to tangential forces perpendicular to the stator and rotor slots, therefore, in a machine with skewed slots, these forces will have a component in the axial direction. The resulting axial force, in this case, is proportional to the torque and angle of skewing[21].

From the above discussions, it appears that the rotor and stator may be subjected to considerable axial vibrations during the operation of the machine, especially in the presence of skewing. It may also be mentioned that the frequencies of axial vibrations of a machine could lie within a broad range of frequency.

2.1.4 Numerical example

It is now opportune to determine, in a general way, the range of frequency of the exciting forces encountered in a typical electrical machine. To illustrate this, a 40-kW, 4-pole, three phase squirrel -cage induction motor, with 36 slots in the stator and 30 slots in the rotor, is chosen. In this machine, the main force-waves produced by the magnetic field harmonics (equation (2.5)) are those having 20, 12 and 4 force-poles. The frequencies of these force-waves are $[15(1-s)-2]\omega_s$, $15(1-s)\omega_s$ and $[15(1-s)+2]\omega_s$; respectively. As the response of both stator and rotor to the force-waves having higher modes is usually very small, only the second and third force components will be considered important for the production of noise in the machine under consideration. Besides, the consideration of additional field harmonics shows the presence of other force-waves of modes having 0,2,4, etc. force-poles.

With reference to the previous discussion in connection with the different noise components produced in electrical machines, in the absence as well as in the presence of asymmetries, the tone-frequency lines of the noise emitted by the motor under consideration are obtained as shown in Figs. 2.3 and 2.4, in which the variation of the frequency of the various noise components, together with the mode shape associated with each component, is given against the slip of the motor.

With the help of Figs. 2.3 and 2.4, the frequencies of the important exciting electromagnetic forces, which may be experienced by the rotor and stator of the motor under consideration at a slip of 2.5% can be found. The frequency-spectrum is shown in Fig. 2.5. In this example, the supply frequency is considered to be 60 Hz. It should be noted that the frequencies of the pulsating torques, the axial forces, forces having more than six pairs of force-poles and forces produced by the air-gap field harmonics due to the variations in reluctance of the air-gap of the machine, are not included in Fig. 2.5 for the sake of brevity.

It is observed that the frequency-spectrum of the noise produced by an

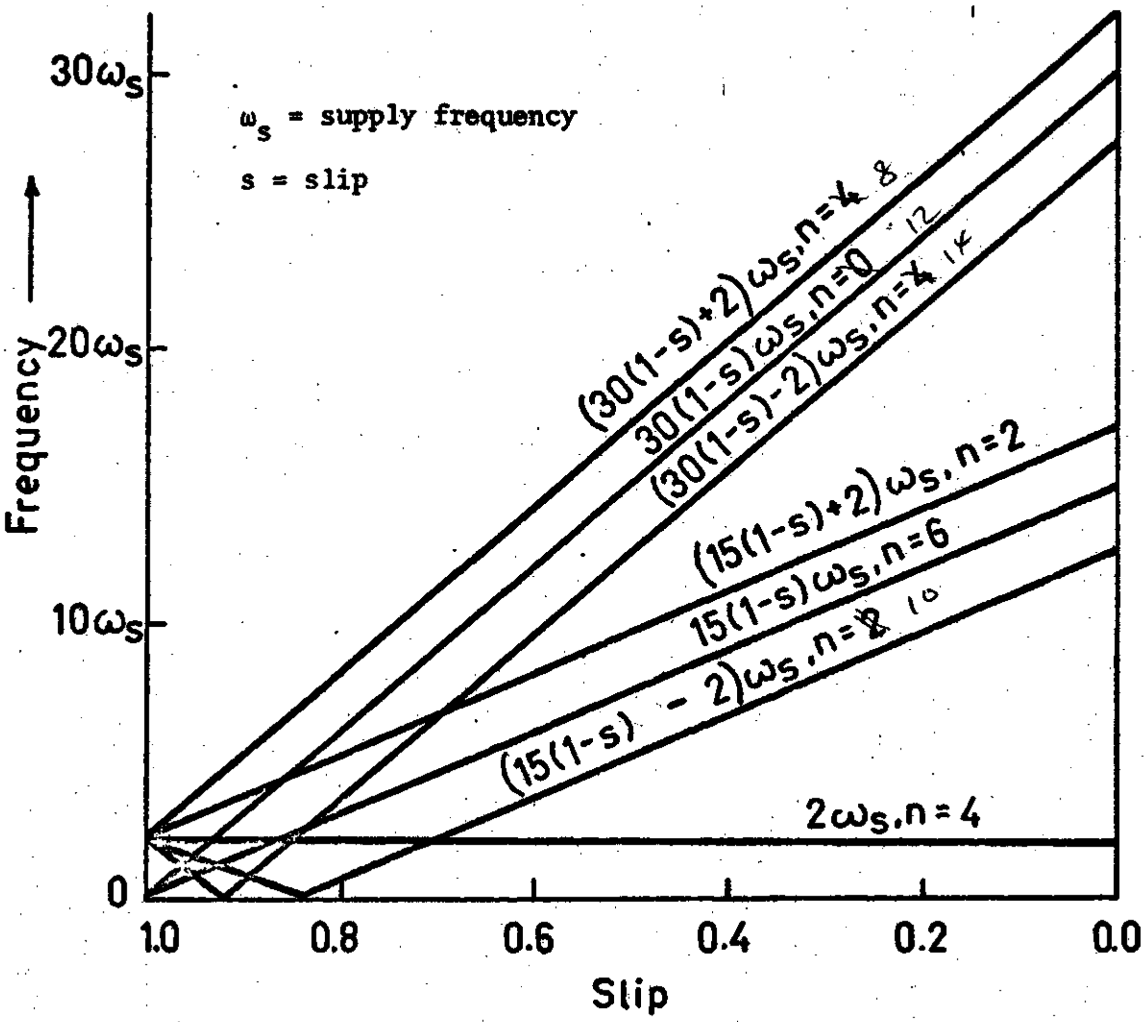


Fig. 2.3 Tone-frequency lines in the absence of asymmetries

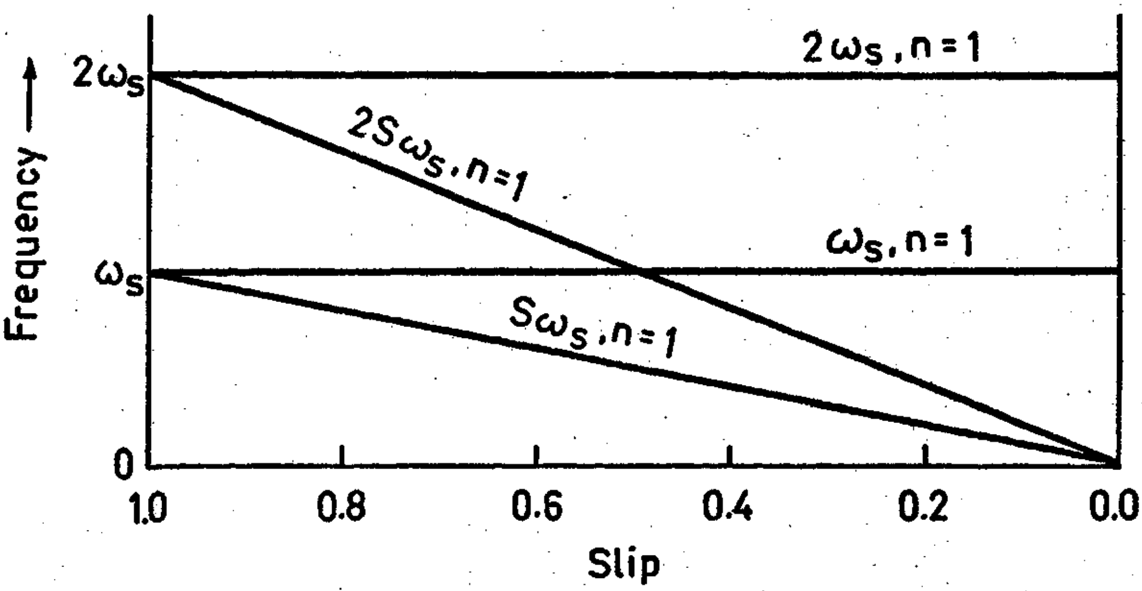


Fig. 2.4 Tone-frequency lines due to asymmetries

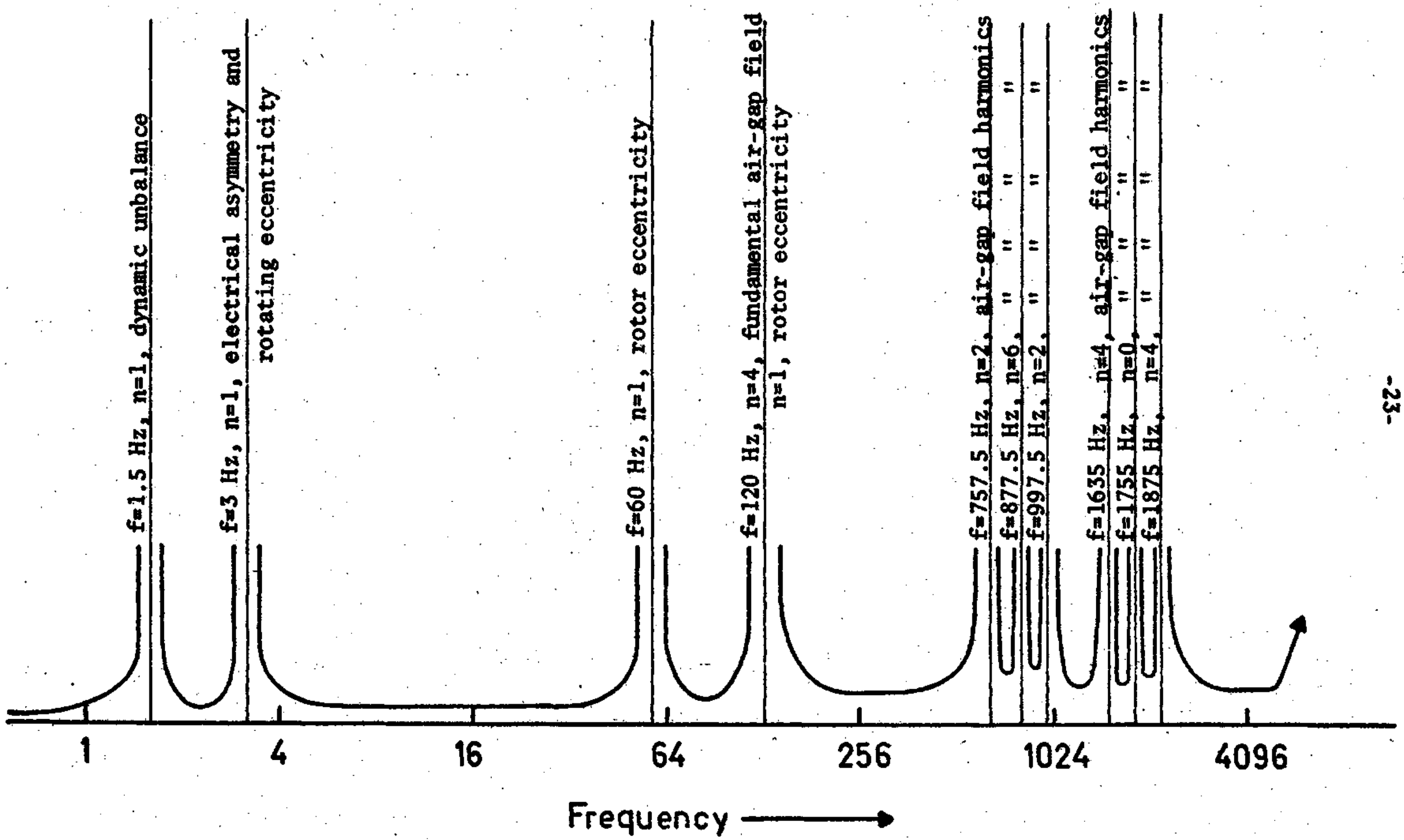


Fig. 2.5 Frequency spectrum of the important electromagnetic forces experienced by a 40-kW,4-pole, 3 phase squirrel-cage induction motor at slip of 2.5% and supply frequency of 60 Hz.

electrical machine spreads over a fairly wide range. In addition, both stator and rotor are subjected to various modes of vibrations.

2.2 Mechanical Response

In order to explain the important role which the mechanical response of the machine structure plays in the problem of electromagnetic noise in electrical machines, a simple vibrating system as shown in Fig. 2.6 will be considered. The system consists of a mass m, a spring of stiffness k and a viscous damper of damping coefficient c.

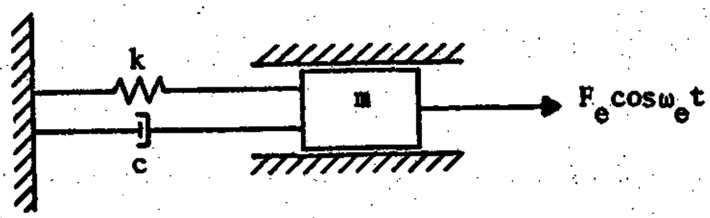


Fig. 2.6 A simple vibrating system

The system is considered to be subjected to an external sinusoidally varying force of amplitude F_e and frequency ω_e.

Applying the law of motion for a rigid body in translation, the equation of motion can be written as:

$$m\ddot{x} + c\dot{x} + kx = F_e \text{Cos } \omega_e t$$

From this equation, the forced vibrations (steady-state vibrations) in the system in response to the exciting force $F_e \text{Cos}\omega_e t$ will be also sinusoidal in nature with frequency equal to the exciting frequency. The amplitude of the vibrations can be expressed as[49,50]:

$$X = \frac{X_e}{\sqrt{\left(1-\frac{\omega_e^2}{\omega_n^2}\right)^2 + 4\eta_d^2\frac{\omega_e^2}{\omega_n^2}}}$$

where; ω_n = natural frequency of the system $= \sqrt{\frac{k}{m}}$

η_d = damping factor = $\frac{c}{2\sqrt{km}}$

X_e = amplitude of the vibrations of the same system corresponding to static application of the force = $\frac{F_e}{k}$

In Fig. 2.7, the amplitude X of the steady-state vibrations is shown as a function of the frequency ratio $\frac{\omega_e}{\omega_n}$ for selected values of the damping ratio η_d.

The figure shows the well-known fact that a small impressed force causes excessive vibrations in a structure if this structure is nearly in resonance with the impulses. Accordingly, the magnetic -forces acting on the iron of the stator and rotor of an electric machine, and the mechanical vibratory forces acting on the bearings may produce excessive noise and vibrations, especially when the frequencies of the exciting forces are equal to, or near, the natural frequencies of the parts of the machine concerned. On the other hand, a relatively large exciting force may cause insignificant vibrations if the response of the structure at the forcing frequency of the particular mode of vibration is small. Also, it is worthwhile mentioning that those force -waves having a smaller number of force-poles are more important in the production of noise and vibrations.

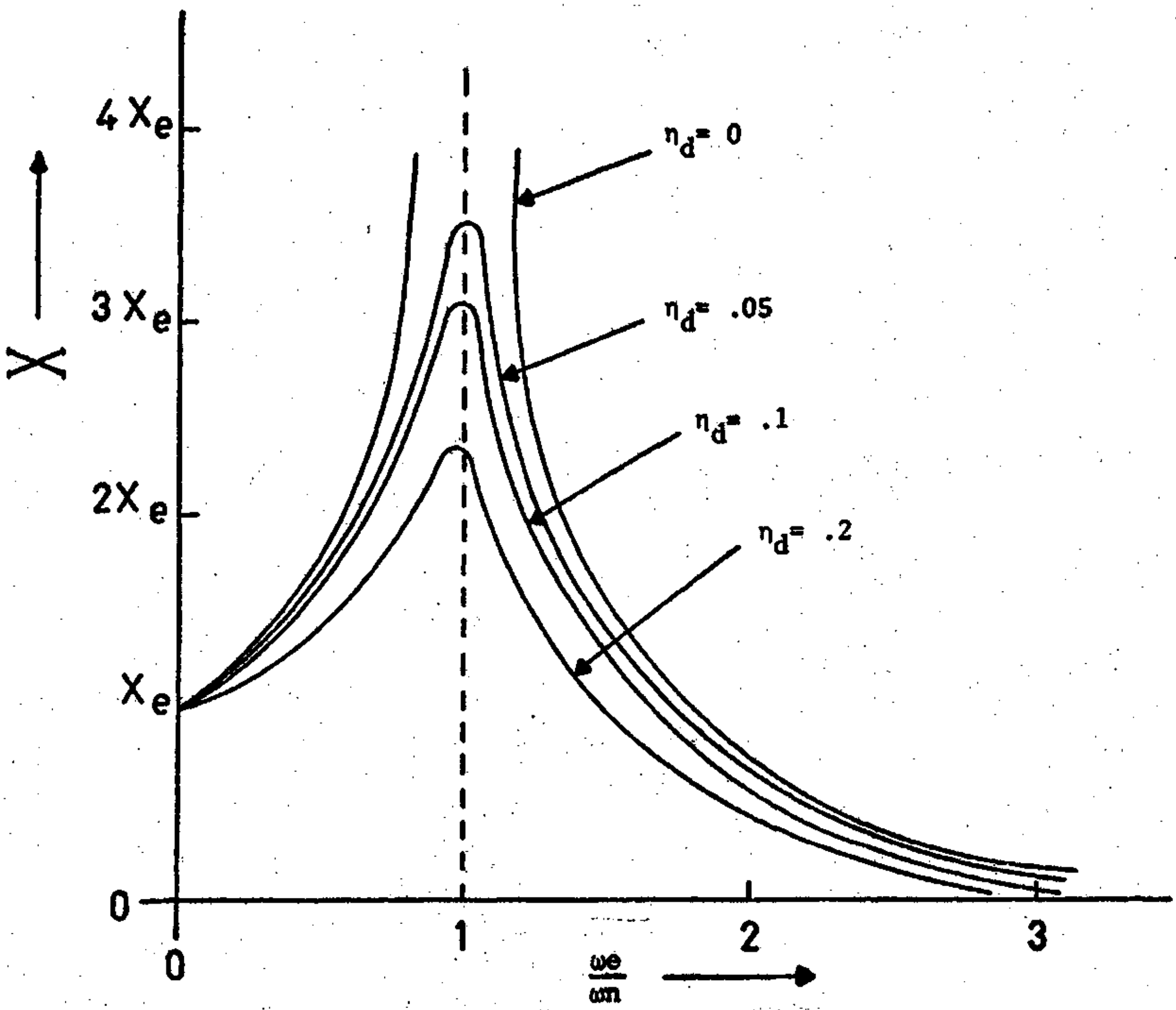


Fig. 2.7 Amplitude of forced vibrations of a simple vibrating system

2.3 Ear Response

The human ear is capable of responding to frequencies up to 20000 Hz. However, the sensitivity of the ear varies with frequency as shown in Fig. 2.8[1,4,51]. The lower curve shows the minimum noise -level which is audible to the human ear.The upper curve shows the threshold where the ears start feeling pain. However it is to be mentioned that the ear starts feeling discomfort at about 20 decibels[1] below the pain threshold. From the figure, it is evident that the human ear is less sensitive to sounds of low frequencies, whereas it is

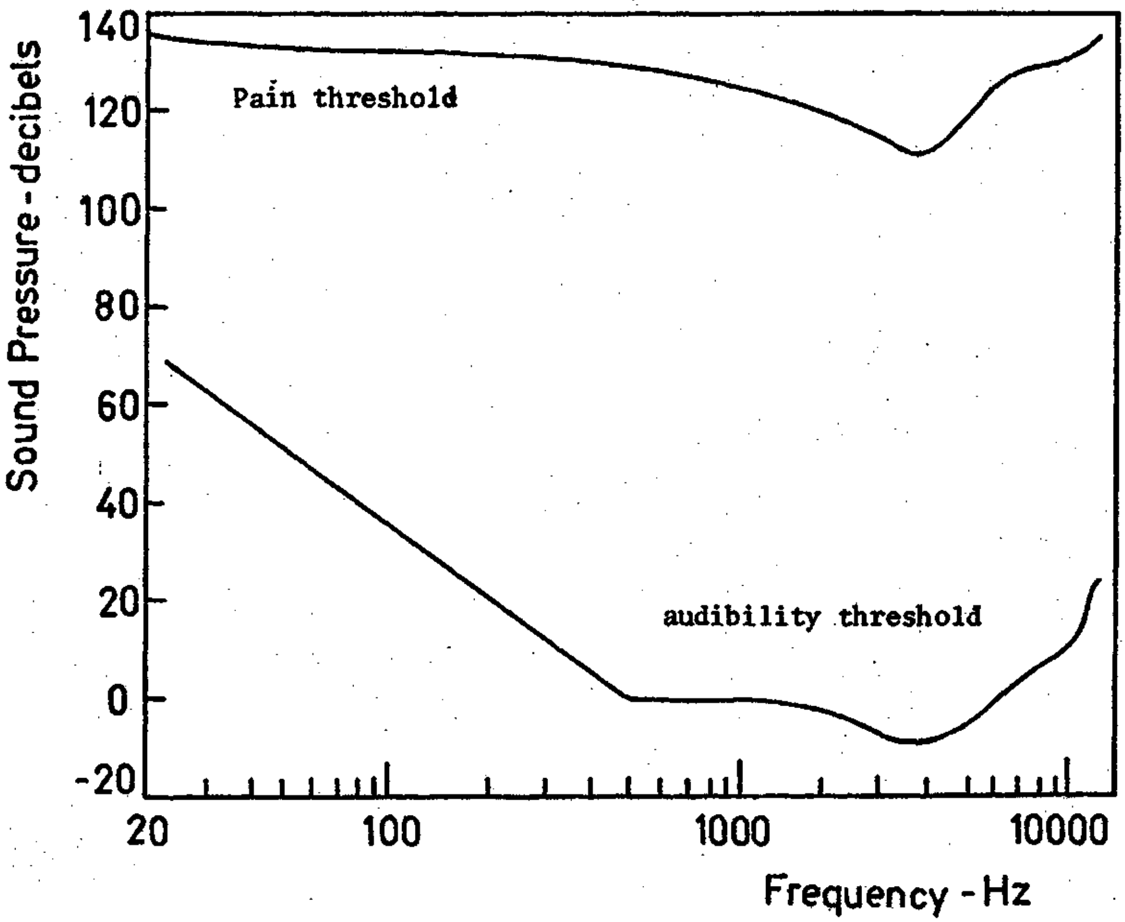


Fig. 2.8 The response of an average human ear

much more sensitive at frequencies in the range of 500 to 5000 Hz. Because of this, sound levels in the 710 to 1400 Hz, 1400 to 2800 Hz and 2800 to 5600 Hz octave-bands are used in industry as a measure of whether or not reliable speech communication is possible[7]. Also, another characteristic of the human ear is that noise becomes more annoying if it includes pure tones[1].

2.4 Important Aspects for the Reduction of Noise

In the light of the above discussions of the three main aspects of electromagnetic noise problem in electrical machines, the following considerations are of great importance:

i) The force-waves produced by the rotating magnetic field constitute the main source of the electromagnetic noise in electrical machines. Since the magnetic field is the means by which power is transferred between stator and rotor, to some degree this noise is unavoidable. Also, the problem is accentuated by the harmonic content of the field. This content is very much dependent on the number of stator and rotor slots. The slot-combination is determined by considerations of speed and the size of the machine, stray losses, synchronous and asynchronous torque -dips and the noise produced. Usually, the choice of a slot -combination may prove advantageous for some of these considerations, but detrimental to the remaining. The choice of the slot-combination is, therefore, inevitably a compromise which may not necessarily result in quiet operation of the machine. Moreover, the presence of any kind of asymmetries in the electrical machine is almost unavoidable and, accordingly, there are always several noise components produced as a result of these asymmetries.

ii) According to the physiological characteristics of the human ear, electromagnetic noise of electrical machines is most troublesome as it is characterized by pure tones. In this connection, the specifications - BS4142 (British standards of noise-levels permitted in mixed residential and industrial areas) state that the noise-level permitted in a residential area is to be reduced by about 5 decibels if a pure tone is present in the noise[3]. However, even if the total noise-level is reduced, the fact remains that a person can still distinguish the tone and may still complain. It is essential, therefore, to reduce the pure tones produced by electromagnetic vibrations in electrical machines.

iii) To reduce noise, it is usually better to reduce vibrations produced within the machine by adopting an appropriate design, rather than by providing noise-reducing enclosures (sound absorbers) and mountings which increase the overall size of the machine and are expensive to install[5].

iv) There are only two means by which the noise produced by electrical machines can be reduced. One method is reduction of the amplitudes of the main magnetic forces by reducing the flux density in the air-gap. Obviously, this increases the size and the cost of the machine considerably. This is in direct opposition to increasing efforts which are directed towards the use of lighter and more economical machines. The second method is to arrange the resonance frequencies of the stator and rotor in such a way that they lie well away from the frequencies of the main exciting forces. This is considered to be the most effective means of achieving a quiet electrical machine.

v) The vibrations of the stator of electrical machines are generated in several mode-shapes ranging from the Zero-mode to mode-shapes having high number of nodes around the periphery. Moreover, electromagnetic vibrations are excited not only in the radial direction but also in tangential and axial directions. The vibrations are generated at frequencies ranging from a few cycles per second to several thousand cycles per second. Although the ear is not sensitive to noises of low frequencies, structure-borne vibrations at low frequencies could be very detrimental[5].

In conclusion, the study of the vibration characteristics and mechanical response of the various members of an electrical machine is of great importance in the design and manufacture of quieter machines. The study should be carried out on a three-dimensional basis over a wide range of frequency extending from zero to at least 5000 Hz for the different modes of vibration.

3. THE GENERAL FREQUENCY EQUATION FOR STATORS HAVING ENCASED CONSTRUCTION

For an accurate assessment of the mechanical response of a vibrating system, accurate information about the natural frequencies of the system is essential. The frequency equation from which the various resonance frequencies of a stator having encased construction can be obtained is derived here. The construction of the stator under consideration is illustrated in Fig. 3.1. It consists of a thick cylindrical core (loaded with teeth and winding), which is encased by a thin cylindrical frame.

3.1 Method of Analysis

Initially, the core and the frame are considered separately. To solve the problem of the composite structure of the stator, the following two boundary conditions should be satisfied:

1- The forces existing at the inner surface of the stator-yoke are equal and opposite to those exerted by the stator teeth and winding.

2- Equilibrium and compatibility at the interface of the core and the frame.

By satisfying the conditions mentioned above, six homogenous equations are obtained. The coefficient determinant of these equations yields the frequency equation of the system.

The following assumptions have been made in the analysis presented here;

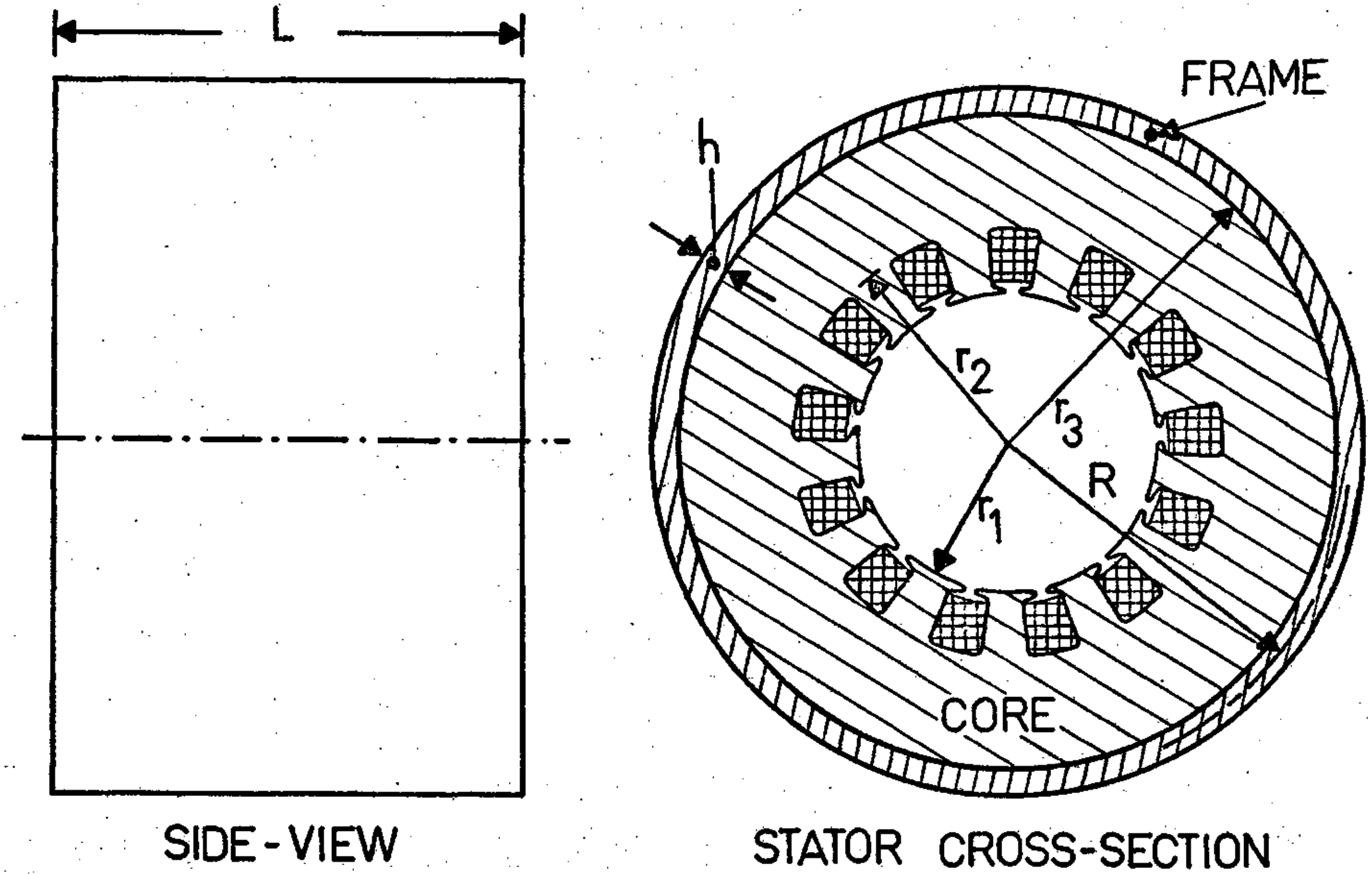


Fig. 3.1 Construction of the stator under consideration

i) All displacements are extremely small in comparison with the dimensions of the stator.

ii) Materials for both core and frame are homogenous, isotropic, linearly elastic and compressible.

iii) The bonding at the interface of core and frame is perfect.

iv) While considering the general form of the flexural vibrations, the stator is treated as if it is freely supported at both ends without axial constraint[40,41].

v) The frame is considered as a complete cylinderical shell; the effects of connecting box, any ventilation holes and other irregularities are neglected.

vi) The masses of cooling ribs, feet and any other similar parts are assumed to add to the mass of the frame.

On the basis of experimental evidence, the assumptions mentioned above have been found appropriate by several authors[8,13-19] in connection with vibrations and noise investigations in electrical machines.

In the present analysis, the three-dimensional elasticity theory[42,43] is used to describe the vibrations of the stator-core. For the frame, the original Flügge's theory[44] of thin shells is used. Further, no simplifications or approximations are introduced in deriving the frequency equation. In spite of the rigorousness of the present analysis, an effort has been made to derive the solutions in a simple way.

3.2 The General Frequency Equation

3.2.1 Equations of the core

The field equations for vibrations of a compressible elastic continua have the following form in the polar cylinderical co-ordinates[45]:

$$\nabla^2 w_c - \frac{w_c}{r^2} - \frac{2}{r^2}\frac{\partial v_c}{\partial \theta} + \frac{1}{1-2\mu_c}\frac{\partial \Delta_c}{\partial r} = \frac{\rho_c}{G_c}\frac{\partial^2 w_c}{\partial t^2} \qquad .... (3.1\ a)$$

$$\nabla^2 v_c - \frac{v_c}{r^2} + \frac{2}{r^2}\frac{\partial w_c}{\partial \theta} + \frac{1}{1-2\mu_c}\frac{\partial \Delta_c}{r\partial \theta} = \frac{\rho_c}{G_c}\frac{\partial^2 v_c}{\partial t^2} \qquad .... (3.1\ b)$$

$$\nabla^2 u_c + \frac{1}{1-2\mu_c}\frac{\partial \Delta_c}{\partial z} = \frac{\rho_c}{G_c}\frac{\partial^2 u_c}{\partial t^2} \quad . \qquad .... (3.1\ c)$$

Solutions of these equations of motion are readily obtained[45] with the help of displacement potentials ϕ, ψ and χ, as:

$$w_c = \frac{\partial \phi}{\partial r} + \frac{\partial^2 \psi}{\partial r \partial z} + \frac{\partial \chi}{r \partial \theta} \qquad .... (3.2\ a)$$

$$v_c = \frac{\partial \phi}{r\partial \theta} + \frac{\partial^2 \psi}{r\partial \theta \partial z} - \frac{\partial \chi}{\partial r} \qquad .... (3.2\ b)$$

$$u_c = \frac{\partial \phi}{\partial z} - \left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}\right)\psi \quad . \qquad .... (3.2\ c)$$

It may be verified by direct substitutions that equations (3.1) are satisfied by these components of displacement, provided that ϕ, ψ and χ are taken as solutions of the following differential equations:

$$\nabla^2 \phi = \frac{1}{c_1^2}\frac{\partial^2 \phi}{\partial t^2} \qquad .... (3.3\ a)$$

$$\nabla^2 \psi = \frac{1}{c_2^2}\frac{\partial^2 \psi}{\partial t^2} \qquad .... (3.3\ b)$$

$$\nabla^2 \chi = \frac{1}{c_2^2}\frac{\partial^2 \chi}{\partial t^2} \quad . \qquad .... (3.3\ c)$$

where, $c_1^2 = \xi^{-2}\frac{G_c}{\rho_c}$, $c_2^2 = \frac{G_c}{\rho_c}$ and $\xi^2 = \frac{1-2\mu_c}{2(1-\mu_c)}$.

Equations (3.3) are actually wave-equations, whose solutions are well-known[45,46]. The solutions of these equations will, therefore, be periodic in θ and z with periods of $\frac{2\pi}{n}$ and $\frac{2L}{m}$, respectively. In addition, the solution must also satisfy the end conditions (zero radial and tangential displacements at z = 0, L and non-zero axial displacement at the same points). The solution is found to be:

$$\phi = \phi_{n(r)} \text{Sin } m \frac{\pi}{L} z \text{ Cos } n\theta \cdot e^{i\omega t} \qquad \text{.... (3.4 a)}$$

$$\psi = \psi_{n(r)} \text{Cos } m \frac{\pi}{L} z \text{ Cos } n\theta \cdot e^{i\omega t} \qquad \text{.... (3.4 b)}$$

$$\chi = \chi_{n(r)} \text{Sin } m \frac{\pi}{L} z \text{ Sin } n\theta \cdot e^{i\omega t} \qquad \text{.... (3.4 c)}$$

In order to determine the expressions of ϕ_n, ψ_n and χ_n, equations (3.4) are substituted in equations (3.3), which gives:

$$\ddot{\phi}_{n(r)} + \frac{1}{r}\dot{\phi}_{n(r)} + [(\frac{\omega^2}{c_1^2} - \frac{m^2\pi^2}{L^2}) - \frac{n^2}{r^2}]\ \phi_{n(r)} = 0 \qquad \text{.... (3.5 a)}$$

$$\ddot{\psi}_{n(r)} + \frac{1}{r}\dot{\psi}_{n(r)} + [(\frac{\omega^2}{c_2^2} - \frac{m^2\pi^2}{L^2}) - \frac{n^2}{r^2}]\ \psi_{n(r)} = 0 \qquad \text{.... (3.5 b)}$$

$$\ddot{\chi}_{n(r)} + \frac{1}{r}\dot{\chi}_{n(r)} + [(\frac{\omega^2}{c_2^2} - \frac{m^2\pi^2}{L^2}) - \frac{n^2}{r^2}]\ \chi_{n(r)} = 0 \qquad \text{.... (3.5 c)}$$

The dots (•) in the above equations denote differentiation with respect to "r". Equations (3.5) are nothing but $n^{\underline{th}}$ order Bessel's differential equations[47,48], the general solution of which is given in terms of the Bessel functions J and Y, or the modified Bessel functions I and K, depending on the relative magnitudes of the longitudinal half -wave length $\frac{L}{m}$, the circular frequency ω and the physical parameters of the core material represented by the constants c_1 and c_2. Table 3.1 gives the solution of equations (3.5) for the various intervals of the frequency ω.

Substituting equations (3.4) in equations (3.2), the general solution for the displacement components of the stator-core is obtained in the following form:

$$w_c = [\dot{\phi}_{n(r)} - \frac{m\pi}{L}\dot{\psi}_{n(r)} + \frac{n}{r}\chi_{n(r)}] \text{Sin} m\frac{\pi}{L}z \ \text{Cos} n\theta \cdot e^{i\omega t} \quad \text{.... (3.6 a)}$$

$$v_c = [-\frac{n}{r}\phi_{n(r)} + \frac{mn\pi}{Lr}\psi_{n(r)} - \dot{\chi}_{n(r)}] \text{Sin} m\frac{\pi}{L}z \ \text{Sin} n\theta \cdot e^{i\omega t} \quad \text{.... (3.6 b)}$$

$$u_c = [\frac{m\pi}{L}\phi_{n(r)} + \omega_2^{\ 2}\psi_{n(r)}] \ \text{Cos} m\frac{\pi}{L}z \ \text{Cos} n\theta \cdot e^{i\omega t} \quad , \quad \text{.... (3.6 c)}$$

where; n and m can assume any positive integer value including zero. Each of the three components of displacement have 2n nodes around the circumference of the stator and m half-waves along the machine-length, for values of n and m other than zero. In this case, the radial and tangential displacements are equal to zero at both ends of the stator, i.e. at z=0 and z=L, while the axial displacement has a non-zero value at the ends. It is evident that, for the cases of n=0 and m=0, the expressions of equations (3.6) result in uniform vibrations around the circumference of the stator and along the machine length, respectively.

3.2.2 Equations of the frame

The frame has a mean radius R and thickness h. As h is usually small compared to R, the original Flügge's equations of motion, which are based on the assumptions of the linear thin-shell theory, have been used for the description of the motion of the frame in the present analysis. Since the effects of shear distortion and rotatory inertia

Case No.	Interval	Solution
I	$\frac{m\pi}{L} c_1 < \omega > 0$	$\phi_n(r) = A_{n_1} J_n(\omega_1 r) + A_{n_2} Y_n(\omega_1 r)$ $\psi_n(r) = A_{n_3} J_n(\omega_2 r) + A_{n_4} Y_n(\omega_2 r)$ $\chi_n(r) = A_{n_5} J_n(\omega_2 r) + A_{n_6} Y_n(\omega_2 r)$
II	$\frac{m\pi}{L} c_2 < \omega < \frac{m\pi}{L} c_1$	$\phi_n(r) = A_{n_1} I_n(\omega_1 r) + A_{n_2} K_n(\omega_1 r)$ $\psi_n(r) = A_{n_3} J_n(\omega_2 r) + A_{n_4} Y_n(\omega_2 r)$ $\chi_n(r) = A_{n_5} J_n(\omega_2 r) + A_{n_6} Y_n(\omega_2 r)$
III	$o < \omega < \frac{m\pi}{L} c_2$	$\phi_n(r) = A_{n_1} I_n(\omega_1 r) + A_{n_2} K_n(\omega_1 r)$ $\psi_n(r) = A_{n_3} I_n(\omega_2 r) + A_{n_4} K_n(\omega_2 r)$ $\chi_n(r) = A_{n_5} I_n(\omega_2 r) + A_{n_6} K_n(\omega_2 r)$

Table 3.1 Expressions of ϕ_n, ψ_n and χ_n for various intervals of ω,

$$\omega_1^2 = \left|\frac{\omega^2}{c_1^2} - \frac{m^2\pi^2}{L^2}\right| \qquad \text{and } \omega_2^2 = \left|\frac{\omega^2}{c_2^2} - \frac{m^2\pi^2}{L^2}\right| .$$

of the shell walls have been neglected in deriving Flügge's equations, the results apply only when the half-wave length of the mode shape is more than ten times the shell wall thickness[40], i.e., $\frac{L}{m}$ and $\frac{\pi R}{n} > 10h$. These limitations correspond to values of m and n of less than $0.1\,\frac{L}{h}$ and $0.314\,\frac{R}{h}$, respectively. In electrical machines, values of $\frac{L}{h}$ and $\frac{R}{h}$ may well exceed the value 10 and, accordingly, the use of Flügge's equations for the frame is only valid for modes m=0 & 1 and n=0, 1, 2, 3 and 4. However, these modes are the most interesting ones for the study of the mechanical response of stators of electrical machines to the exciting forces[7,12]. Nevertheless, even for the higher modes where the use of Flügge's equations seems to be not very accurate, the accuracy of the calculations will not be greatly affected because the frame is the weaker member in the vibrating system, and as a result, its contribution to the vibration behaviour of the whole stator-assembly may be little for those modes.

According to Flügge[44], the differential equations governing the motion of the frame can be written (in terms of the displacement components of its midsurface w_f, v_f and u_f) in the following form:

$$- ku_f''' + \frac{1-\mu_f}{2} ku_f^{\prime\cdot\cdot} + \mu_f u_f' - \frac{3-\mu_f}{2} kv_f^{\prime\prime\cdot} + \dot{v}_f + w_f +$$

$$k(w_f'''' + 2w_f^{\prime\prime\cdot\cdot} + w_f^{\cdot\cdot\cdot\cdot} + 2\ddot{w}_f + w_f) + \eta^2 \frac{\partial^2 w_f}{\partial t^2} - \frac{R^2}{D_f} \overset{*}{p}_r = 0 \qquad \ldots. \ (3.7\ a)$$

$$\frac{1+\mu_f}{2} \dot{u}_f' + \ddot{v}_f + \frac{1-\mu_f}{2} (1+3k) v_f'' - \frac{3-\mu_f}{2} kw_f^{\prime\prime\cdot} + \dot{w}_f -$$

$$\eta^2 \frac{\partial^2 v_f}{\partial t^2} + \frac{R^2}{D_f} \overset{*}{p}_\theta = 0 \qquad \text{.... (3.7 b)}$$

$$u_f'' + \frac{1-\mu_f}{2}(1+k)\ddot{u}_f + \frac{1+\mu_f}{2} \dot{v}_f' - k w_f''' + \frac{1-\mu_f}{2} k \ddot{w}_f' +$$

$$\mu_f w_f' - \eta^2 \frac{\partial^2 u_f}{\partial t^2} + \frac{R^2}{D_f} \overset{*}{p}_z = 0 \qquad \text{.... (3.7 c)}$$

where, $k = \frac{1}{12}(\frac{h}{R})^2$, $D_f = \frac{E_f h}{1-\mu_f^2}$ and $\eta^2 = \frac{\rho_f R^2 h}{D_f}$

In the above equations, $\overset{*}{p}_r$, $\overset{*}{p}_\theta$ and $\overset{*}{p}_z$ are the forces per unit area acting on the inner surface of the frame.

3.2.3 Solution for the stator-assembly

As mentioned before, the stresses σ_r, $\tau_{r\theta}$ and τ_{rz}, evaluated at the inner surface of the stator-yoke (radius r_2), are equal and opposite to the stresses p_{tw_r}, p_{tw_θ} and p_{tw_z} exerted by teeth and windings. The mathematical expressions of the stresses σ_r, $\tau_{r\theta}$ and τ_{rz} in terms of the displacement components w, v and u are well-known and have the following form[45]:

$$\sigma_r = 2G \frac{\partial w}{\partial r} + q\,\Delta \qquad \text{.... (3.8 a)}$$

$$\tau_{r\theta} = G\left(\frac{1}{r}\frac{\partial w}{\partial \theta} - \frac{v}{r} + \frac{\partial v}{\partial r} \right) \qquad \text{.... (3.8 b)}$$

$$\tau_{rz} = G\left(\frac{\partial w}{\partial z} + \frac{\partial u}{\partial r} \right) \qquad \text{.... (3.8 c)}$$

where, $q = \frac{2G\mu}{1-2\mu}$

By substituting the expressions of w_c, v_c and u_c, equations (3.6), in the above equations, and equating the resulting stresses (evaluated at $r = r_2$) to p_{tw_r}, p_{tw_θ} and p_{tw_z} the following relations are obtained:

$$[2+\frac{q_c}{G_c}]\ddot{\phi}_{n(r_2)}+[\frac{1}{r_2}\frac{q_c}{G_c}]\dot{\phi}_{n(r_2)}+[-\frac{q_c}{G_c}(\frac{n^2}{r_2{}^2}+\frac{M^2}{R^2})]\phi_{n(r_2)}+$$

$$[-(2+\frac{q_c}{G_c})\frac{M}{R}]\ddot{\psi}_{n(r_2)}+[-\frac{M}{r_2 R}\frac{q_c}{G_c}]\dot{\psi}_{n(r_2)}+[\frac{q_c}{G_c}\cdot\frac{M}{R}(\frac{n^2}{r_2{}^2}-\omega_2{}^2)]$$

$$\psi_{n(r_2)}+[\frac{2n}{r_2}]\dot{\chi}_{n(r_2)}+[-\frac{2n}{r_2{}^2}]\chi_{n(r_2)} = -p_{tw_r}/G_c \qquad .... (3.9\ a)$$

$$[-\frac{2n}{r_2}]\dot{\phi}_{n(r_2)}+[\frac{2n}{r_2{}^2}]\phi_{n(r_2)}+[\frac{2nM}{r_2R}]\dot{\psi}_{n(r_2)}+[-\frac{2nM}{r_2{}^2R}]\psi_{n(r_2)}+$$

$$[-1]\ddot{\chi}_{n(r_2)}+[\frac{1}{r_2}]\dot{\chi}_{n(r_2)}+[-\frac{n^2}{r_2{}^2}]\chi_{n(r_2)} = -p_{tw_\theta}/G_c \qquad .... (3.9\ b)$$

$$[\frac{2M}{R}]\dot{\phi}_{n(r_2)}+[\omega_2{}^2-\frac{M^2}{R^2}]\dot{\psi}_{n(r_2)}+[\frac{nM}{r_2R}]\chi_{n(r_2)} = -p_{tw_z}/G_c\ . \qquad .... (3.9\ c)$$

Now, considering the second boundary condition, the displacements $\overset{*}{w}_c$, $\overset{*}{v}_c$ and $\overset{*}{u}_c$ of the outer surface of the core (radius r_3) must be equal to those of a corresponding point on the inner surface of the frame. Next, it is now required to find the proper expressions for the displacements w_f, v_f and u_f of the middle surface of the frame. It is proved in reference (44) that the relation between the displacement components w_A, v_A, u_A of a point A lying on the middle surface of a thin cylindrical shell, and the displacement components of another point B lying on the same radial line of A, Fig. 3.2, has the form:

$$w_B = w_A \qquad .... (3.10\ a)$$

$$v_B = (1+\frac{\delta r}{R})\ v_A - \frac{\delta r}{R}\dot{w}_A \qquad .... (3.10\ b)$$

$$u_B = u_A - \frac{\delta r}{R}w'_A \qquad .... (3.10\ c)$$

where, δR = distance between points A and B with the proper sign.

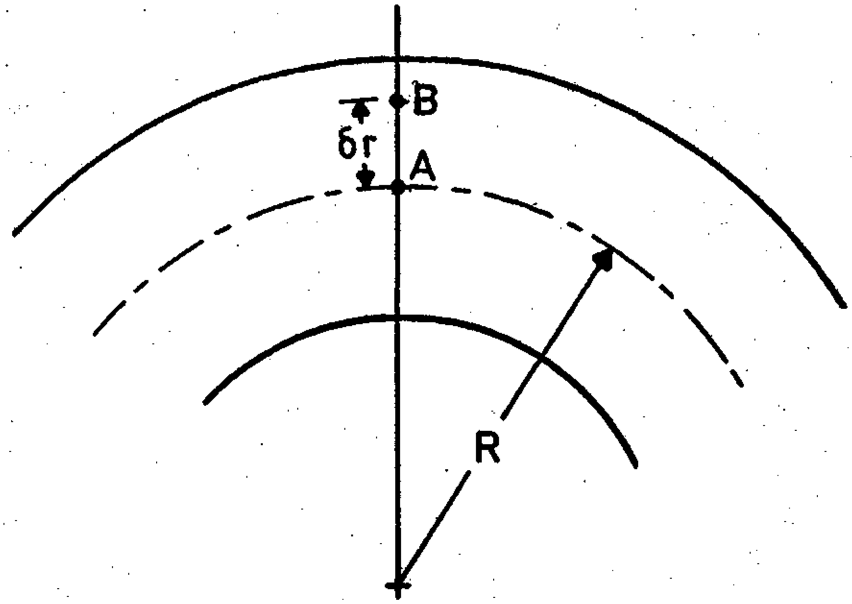


Fig. 3.2 Displacements of two points of a thin-shell

Accordingly, the relation between the displacement components of the middle surface of the frame (w_f, v_f and u_f) and those of the inner surface of the frame($\overset{*}{w}_c$, $\overset{*}{v}_c$ and $\overset{*}{u}_c$) will take the form:

$$\overset{*}{w}_c = w_f \qquad \text{.... (3.11 a)}$$

$$\overset{*}{v}_c = (1- \frac{h}{2R}) v_f + \frac{h}{2R} \dot{w}_f \qquad \text{.... (3.11 b)}$$

$$\overset{*}{u}_c = u_f + \frac{h}{2R} w_f' \quad . \qquad \text{.... (3.11 c)}$$

From these equations,

$$w_f = \overset{*}{w}_c \qquad \text{.... (3.12 a)}$$

$$v_f = \frac{1}{(1- \frac{h}{2R})} \{\overset{*}{v}_c - \frac{h}{2R} \dot{\overset{*}{w}}_c\} \qquad \text{.... (3.12 b)}$$

$$u_f = \overset{*}{u}_c - \frac{h}{2R} \overset{*}{w}_c' \qquad \text{.... (3.12 c)}$$

With the help of equations (3.6) and (3.12), the displacement components of the middle surface of the frame are obtained as:

$$w_f=[\dot{\phi}_{n(r_3)}-\frac{m\pi}{L}\dot{\psi}_{n(r_3)}+\frac{n}{r_3}\chi_{n(r_3)}]\mathrm{Sin}\,m\frac{\pi}{L}z\ \mathrm{Cos}\,n\theta\cdot e^{i\omega t} \quad \ldots.\ (3.13\ a)$$

$$v_f=\frac{1}{r_3}[-\frac{nR}{r_3}\phi_{n(r_3)}+\frac{Mn}{r_3}\psi_{n(r_3)}-R\dot{\chi}_{n(r_3)}+\frac{nh}{2}\{\dot{\phi}_{n(r_3)}-$$

$$\frac{M}{R}\dot{\psi}_{n(r_3)}+\frac{n}{r_3}\chi_{n(r_3)}\}]\mathrm{Sin}\,m\frac{\pi}{L}z\ \mathrm{Sin}\,n\theta\cdot e^{i\omega t} \quad \ldots.\ (3.13\ b)$$

$$u_f=[\frac{M}{R}\phi_{n(r_3)}+\omega_2{}^2\psi_{n(r_3)}-\frac{hM}{2R}\{\dot{\phi}_{n(r_3)}-\frac{M}{R}\dot{\psi}_{n(r_3)}+$$

$$\frac{n}{r_3}\chi_{n(r_3)}\}]\mathrm{Cos}\,m\frac{\pi}{L}z\ \mathrm{Cos}\,n\theta\cdot e^{i\omega t} \quad \ldots.\ (3.13\ c)$$

To satisfy the equilibrium conditions at the interface, $\overset{*}{P}_r$, $\overset{*}{p}_\theta$ and $\overset{*}{p}_z$ in equations (3.7) are to be replaced by the stresses $(-\overset{*}{\sigma}_r)$, $(-\overset{*}{\tau}_{r\theta})$ and $(-\overset{*}{\tau}_{rz})$, respectively. These stresses have to be evaluated at the outer surface of the stator-core ($r=r_3$) using equations (3.8). Then, equations (3.13) and (3.7) deliver the following set of equations:

$$[-\frac{R^2G_c}{D_f}(2+\frac{q_c}{G_c})]\ddot{\phi}_{n(r_3)}+[\frac{M^2h}{2R}(kM^2-\frac{1-\mu_f}{2}kn^2+\mu_f)+\frac{n^2h}{2r_3}(M^2k\cdot$$

$$\frac{3-\mu_f}{2}+1)+1+k(M^4+2M^2n^2+n^4-2n^2+1)-$$

$$\eta^2\omega^2+\frac{R^2q_c}{D_f r_3}]\dot{\phi}_{n(r_3)}+[\frac{M^2}{R}(-kM^2+\frac{1-\mu_f}{2}kn^2-\mu_f)-\frac{n^2R}{r_3{}^2}(M^2k\frac{3-\mu_f}{2}$$

$$+1)-\frac{q_c}{D_f}(\frac{n^2R^2}{r_3{}^2}+M^2)]\phi_{n(r_3)}+[-M(2+\frac{q_c}{G_c})\frac{RG_c}{D_f}]\ddot{\psi}_{n(r_3)}+[\frac{M^3h}{2R^2}(-kM^2$$

$$+\frac{1-\mu_f}{2}kn^2-\mu_f)-\frac{n^2hM}{2r_3R}(M^2k\frac{3-\mu_f}{2}+1)-\frac{M}{R}(1+k(M^4+2M^2n^2+n^4-2n^2$$

$$+1)-\eta^2\omega^2)-\frac{Mq_cR}{D_f r_3}]\dot{\psi}_{n(r_3)}+[\omega_2{}^2M(-kM^2+\frac{1-\mu_f}{2}kn^2-\mu_f)+\frac{Mn^2}{r_3{}^2}(1+$$

$$M^2k\frac{3-\mu_f}{2}) + \frac{q_c MR}{D_f}(\frac{n^2}{r_3^2} - \omega_2{}^2)]\psi_{n(r_3)} + [-\frac{nR}{r_3}(M^2k\frac{3-\mu_f}{2}+1) +$$

$$\frac{2nR^2G_c}{D_f r_3}]\dot{\chi}_{n(r_3)} + [-\frac{nM^2h}{2r_3R}(-kM^2+kn^2\frac{1-\mu_f}{2}-\mu_f) + \frac{n^3h}{2r_3^2}(M^2k\frac{3-\mu_f}{2}$$

$$+1) + \frac{n}{r_3}(1+k(M^4+2n^2M^2+n^4-2n^2+1)-\eta^2\omega^2) - \frac{2G_c nR^2}{D_f r_3^2}]\chi_{n(r_3)} = 0 \quad \ldots. \; (3.14\ a)$$

$$[-\frac{1+\mu_f}{4}\cdot\frac{M^2hn}{R} + \frac{nh}{2r_3}(\eta^2\omega^2-n^2-M^2(1+3k)\frac{1-\mu_f}{2})-n-\frac{3-\mu_f}{2}kM^2n+$$

$$\frac{2nG_cR^2}{D_f r_3}]\dot{\phi}_{n(r_3)} + [\frac{M^2n}{R}\cdot\frac{1+\mu_f}{2} + \frac{nR}{r_3^2}(n^2+M^2\frac{(1-\mu_f)(1+3k)}{2}-\eta^2\omega^2) -$$

$$\frac{2G_c nR^2}{D_f r_3^2}]\phi_{n(r_3)} + [\frac{1+\mu_f}{4R^2}M^3nh + \frac{nhM}{2Rr_3}(n^2+M^2\cdot\frac{1-\mu_f}{2}(1+3k)-\eta^2\omega^2) +$$

$$\frac{M}{R}(n+\frac{3-\mu_f}{2}kM^2n) - \frac{G_c}{D_f}\cdot\frac{2nMR}{r_3}]\dot{\psi}_{n(r_3)} + [\frac{1+\mu_f}{2}Mn\omega_2{}^2 - \frac{Mn}{r_3^2}(n^2+$$

$$M^2\frac{1-\mu_f}{2}(1+3k)-\eta^2\omega^2) + \frac{2G_c MnR}{D_f r_3^2}]\psi_{n(r_3)} + [\frac{R^2G_c}{D_f}]\ddot{\chi}_{n(r_3)} + [\frac{R}{r_3}(n^2+$$

$$M^2\frac{(1-\mu_f)(1+3k)}{2}-\eta^2\omega^2) - \frac{R^2G_c}{D_f r_3}]\dot{\chi}_{n(r_3)} + [-\frac{M^2hn^2}{4Rr_3}(1+\mu_f) -$$

$$\frac{n^2h}{2r_3^2}(n^2+M^2\frac{(1-\mu_f)(1+3k)}{2}-\eta^2\omega^2) - \frac{n}{r_3}(n+\frac{3-\mu_f}{2}kM^2n) +$$

$$\frac{n^2G_cR^2}{D_f r_3^2}]\chi_{n(r_3)} = 0 \quad \ldots. \; (3.14\ b)$$

$$[\frac{Mh}{2R}(M^2+\frac{1-\mu_f}{2}(1+k)n^2-\eta^2\omega^2) + \frac{n^2hM}{4r_3}(1+\mu_f) + M(M^2k-\frac{1-\mu_f}{2}kn^2+$$

$$\mu_f) - \frac{2MRG_c}{D_f}]\dot{\phi}_{n(r_3)} + [-\frac{M}{R}(M^2 + \frac{1-\mu_f}{2}(1+k)n^2 - \eta^2\omega^2) -$$

$$\frac{n^2M(1+\mu_f)R}{2r_3{}^2}]\phi_{n(r_3)} + [-\frac{M^2h}{2R^2}(M^2 + \frac{1-\mu_f}{2}(1+k)n^2 - \eta^2\omega^2) -$$

$$\frac{n^2hM^2(1+\mu_f)}{4r_3R} - \frac{M^2}{R}(M^2k - \frac{1-\mu_f}{2}kn^2 + \mu_f) - \frac{G_c}{D_f}(\omega_2{}^2R^2 - M^2)]\dot{\psi}_{n(r_3)}$$

$$+[-\omega_2{}^2(M^2 + \frac{1-\mu_f}{2}(1+k)n^2 - \eta^2\omega^2) + \frac{M^2n^2(1+\mu_f)}{2r_3{}^2}]\psi_{n(r_3)} + [-$$

$$\frac{nM(1+\mu_f)R}{2r_3}]\dot{\chi}_{n(r_3)} + [\frac{Mhn}{2Rr_3}(M^2 + \frac{1-\mu_f}{2}(1+k)n^2 - \eta^2\omega^2) + \frac{n^3Mh(1+\mu_f)}{4r_3{}^2}$$

$$+ \frac{nM}{r_3}(M^2k - \frac{1-\mu_f}{2}kn^2 + \mu_f) - \frac{G_c}{D_f}\frac{MnR}{r_3}]\chi_{n(r_3)} = 0 \qquad \ldots.\ (3.14\ c)\checkmark$$

Equations (3.9) and (3.14) constitute the complete solution to the problem. It is advisable, now, to rearrange these equations in a matrix form in such a way that the elements of the matrix become dimensionless quantities, as given in equation (3.15). This matrix presents the general form of the solution. To take into account the effect of teeth and windings in detail, expressions for p_{tw_r}, p_{tw_θ} and p_{tw_z} should be developed and introduced in equation (3.15). However, in the solution for the general frequency equation in the present analysis, the presence of the teeth and windings will be considered only by adding their mass to that of the stator-yoke. Accordingly, p_{tw_r}, p_{tw_θ} and p_{tw_z} are equated to zero and, at the same time, the value of the density of the core ρ_c has to be modified to the value:

$$\bar{\rho}_c = \rho_c\ \{\ 1 + \frac{\text{mass of teeth and windings}}{\text{mass of stator-yoke}}\}$$

$2+\frac{q_c}{G_c}$	$\frac{q_c}{G_c}\cdot\frac{R}{r_2}$	$-\frac{q_c}{G_c}(M^2+\frac{R^2}{r_2^2}n^2)$	$-(2+\frac{q_c}{G_c})M$	$-\frac{q_c}{G_c}\cdot M\cdot\frac{R}{r_2}$
0	$-2n\frac{R}{r_2}$	$2n(\frac{R}{r_2})^2$	0	$2nM\frac{R}{r_2}$
0	2M	0	0	$\omega_2^2R^2-M^2$
$(2+\frac{q_c}{G_c})\frac{RG_c}{D_f}$	$\frac{M^2h}{2R}(kM^2-\frac{1-\mu_f}{2}kn^2+\mu_f)+1+\frac{n^2h}{2Ra}(1+\frac{M^2k(3-\mu_f)}{2})$ $+\frac{Rq_c}{D_fa}+k(M^4+2M^2n^2+n^4-2n^2+1)-\eta^2\omega^2$	$M^2(-kM^2+\frac{kn^2(1-\mu_f)}{2}-\mu_f)$ $-\frac{n^2}{a^2}(1+\frac{M^2k(3-\mu_f)}{2})-\frac{q_cR}{D_f}(\frac{n^2}{a^2}+M^2)$	$-\frac{MRG_c}{D_f}(2+\frac{q_c}{G_c})$	$\frac{M^3h}{2R}(-kM^2+\frac{kn^2(1-\mu_f)}{2}-\mu_f)-\frac{n^2hM}{2Ra}(1+\frac{M^2k(3-\mu_f)}{2})$ $-\frac{MRq_c}{D_fa}-M[1+k(M^4+2M^2n^2+n^4-2n^2+1)-\eta^2\omega^2]$
0	$\frac{nh}{2Ra}[\eta^2\omega^2-n^2-M^2\frac{(1-\mu_f)(1+3k)}{2}]-\frac{M^2nh(1+\mu_f)}{4R}$ $+2n\frac{RG_c}{D_fa}-n(1+\frac{kM^2(3-\mu_f)}{2})$	$\frac{n}{a^2}(n^2+\frac{M^2(1+3k)(1-\mu_f)}{2}-\eta^2\omega^2)$ $+\frac{M^2n(1+\mu_f)}{2}-\frac{2nRG_c}{D_fa^2}$	0	$\frac{nhM}{2Ra}(n^2+\frac{M^2(1-\mu_f)(1+3k)}{2}-\eta^2\omega^2)+\frac{M^3nh(1+\mu_f)}{4R}$ $-\frac{2nMRG_c}{D_fa}+Mn(1+\frac{kM^2(3-\mu_f)}{2})$
0	$\frac{Mh}{2R}(M^2+\frac{1-\mu_f}{2}(1+k)n^2-\eta^2\omega^2)+\frac{n^2hM(1+\mu_f)}{4Ra}$ $-2M\frac{RG_c}{D_f}+M(M^2k+\mu_f-\frac{1-\mu_f}{2}kn^2)$	$-M(M^2+\frac{1-\mu_f}{2}(1+k)n^2-\eta^2\omega^2)$ $-\frac{n^2M(1+\mu_f)}{2a^2}$	0	$-\frac{M^2h}{2R}(M^2+\frac{1-\mu_f}{2}(1+k)n^2-\eta^2\omega^2)-\frac{n^2hM^2}{4Ra}(1+\mu_f)$ $-\frac{RG_c}{D_f}(\omega_2^2R^2-M^2)-M^2(M^2k+\mu_f-\frac{1-\mu_f}{2}kn^2)$

$$
\begin{bmatrix}
\cdots & \frac{Mq_c}{G_c}\left(n^2\frac{R^2}{r_2^2}-\omega_2^2R^2\right) & 0 & 2n\frac{R}{r_2} & -2n\frac{R^2}{r_2^2} \\
\cdots & -2Mn\frac{R^2}{r_2^2} & -1 & \frac{R}{r_2} & -n^2\frac{R^2}{r_2^2} \\
\cdots & 0 & 0 & 0 & nM\frac{R}{r_2} \\
\cdots & \begin{matrix}\omega_2^2R^2M\left(-kM^2+\frac{kn^2(1-\mu_f)}{2}-\mu_f\right) \\ +\frac{Mn^2}{a^2}\left(\frac{kM^2(3-\mu_f)}{2}+1\right)+\frac{q_cRM}{D_f}\left(\frac{n^2}{a^2}-\omega_2^2R^2\right)\end{matrix} & 0 & \frac{n}{a}\left[\frac{2RG_c}{D_f}-1-M^2k\frac{3-\mu_f}{2}\right] & \begin{matrix}\frac{nM^2h}{2Ra}\left(-kM^2-\mu_f+\frac{kn^2(1-\mu_f)}{2}\right)+\frac{n^3h}{2Ra^2}\left(1+M^2k\frac{3-\mu_f}{2}\right) \\ -\frac{2nRG_c}{D_fa^2}+\frac{n}{a}\left[1+k(M^4+2M^2n^2+n^4-2n^2+1)-\eta^2\omega^2\right]\end{matrix} \\
\cdots & \begin{matrix}\frac{Mn\omega_2^2R^2(1+\mu_f)}{2}+\frac{2MnRG_c}{D_fa^2} \\ -\frac{Mn}{a^2}\left(n^2+M^2\frac{(1-\mu_f)(1+3k)}{2}-\eta^2\omega^2\right)\end{matrix} & \frac{RG_c}{D_f} & \begin{matrix}\frac{1}{a}\Big[n^2+\frac{M^2(1+3k)(1-\mu_f)}{2} \\ -\eta^2\omega^2-\frac{RG_c}{D_f}\Big]\end{matrix} & \begin{matrix}-\frac{n^2}{a}\left[\frac{M^2h(1+\mu_f)}{4R}+1-\frac{RG_c}{D_fa}\right. \\ \left.+\frac{h}{2Ra}\left(n^2+M^2\frac{(1+3k)(1-\mu_f)}{2}-\eta^2\omega^2\right)+\frac{3-\mu_f}{2}kM^2\right]\end{matrix} \\
\cdots & \begin{matrix}-\omega_2^2R^2\left(M^2+\frac{n^2(1+k)(1-\mu_f)}{2}-\eta^2\omega^2\right) \\ +\frac{M^2n^2(1+\mu_f)}{2a^2}\end{matrix} & 0 & -\frac{nM(1+\mu_f)}{2a} & \begin{matrix}\frac{Mhn}{2Ra}\left(M^2+\frac{(1-\mu_f)(1+k)n^2}{2}-\eta^2\omega^2\right)-\frac{G_cMR_n}{D_fa} \\ +\frac{nM}{a}\left(M^2k-\frac{kn^2(1-\mu_f)}{2}+\mu_f\right)+\frac{n^3hM(1+\mu_f)}{4a^2R}\end{matrix}
\end{bmatrix}
\cdot
\begin{bmatrix}
\ddot{\phi}_n(r_{2,3}) \\
\frac{\dot{\phi}_n(r_{2,3})}{R} \\
\frac{\phi_n(r_{2,3})}{R^2} \\
\frac{\ddot{\psi}_n(r_{2,3})}{R} \\
\frac{\dot{\psi}_n(r_{2,3})}{R^2} \\
\frac{\psi_n(r_{2,3})}{R^3} \\
\ddot{\chi}_n(r_{2,3}) \\
\frac{\dot{\chi}_n(r_{2,3})}{R} \\
\frac{\chi_n(r_{2,3})}{R^2}
\end{bmatrix}
=
\begin{bmatrix}
-\frac{P_{tw_r}}{G_c} \\
-\frac{P_{tw_\theta}}{G_c} \\
-\frac{P_{tw_z}}{G_c} \\
0 \\
0 \\
0
\end{bmatrix}
\quad \ldots.(3.15)
$$

where; $a = 1-\frac{h}{2R}$

In appendix-A, the frequency equation has been derived from equation (3.15) and it has the form:

$$|P| = 0 \quad , \qquad \ldots . \ (3.16)$$

where, $|P|$ is the determinant of the coefficient matrix $[P]$

It does not appear feasible to obtain an analytical expression for the resonance frequency ω, which satisfies the transcendental equation (3.16). Thus, one has to resort to a numerical evaluation of the resonance frequencies, Appendix-B.

3.3 Frequency Equations for Special Cases

3.3.1 The case of uniform vibrations along the machine-length ($m=0$)

In several electrical machines, especially those having small length compared to the bore-radius, the vibrations of the stator are almost uniform along a generatrix[12-19]. In this case, the value of m is zero due to the absence of nodes in the axial direction. In such a case, two uncoupled frequency equations are obtained, which are:

$$|P_1| = 0 \qquad \ldots . \ (3.17)$$

$$|P_2| = 0 \qquad \ldots . \ (3.18)$$

as explained in Appendix-C. Equation (3.17) involves four equations in terms of the four unknowns A_{n_1}, A_{n_2}, A_{n_5} and A_{n_6}. Accordingly, as it is evident from equations (9.2) and (3.4), equation (3.17) corresponds to a mode involving ϕ and χ only. With the help of equation (3.2), it can be seen that equation (3.17) corresponds to plane vibrations, involving radial and tangential displacements only. On the other hand, equation (3.18) corresponds to a mode involving χ only and, accordingly, it corresponds to pure axial vibrations.

It is worthwhile to mention that these two types of vibrations are coupled in the general case of flexural vibrations. ?

3.3.2 The case of uniform vibrations along the machine-circumference (n=0)

In this case, the following frequency equations (Appendix-C) are obtained:

$$|P_3| = 0 \qquad \ldots\ (3.19)$$

$$|P_4| = 0 \qquad \ldots\ (3.20)$$

Following similar steps as those used in the previous case, it can be proved that equation (3.19) corresponds to coupled radial and axial vibrations, while equation (3.20) corresponds to pure torsional vibrations. ?

3.3.3 The case of uniform vibrations along both machine-length and machine-circumference (m=0, n=0)

For this special case, the following frequency equations, as obtained in Appendix-C, hold good:

$$|P_5| = 0 \qquad \ldots\ (3.21)$$

$$|P_4| = 0 \qquad \ldots\ (3.22)$$

$$|P_2| = 0 \qquad \ldots\ (3.23)$$

These equations correspond to pure radial, pure torsional and pure axial vibrations; respectively. ?

3.4 Determination of the Mode Shapes

For the determination of the mode shape associated with a given resonance frequency of the stator, the relative amplitudes of the various displacement components must be calculated at this frequency. For this purpose, the relative magnitudes of the constants A_{n_1}, A_{n_2}, ... and A_{n_6} should be calculated at first from equation (9.3). The substitution of the values so obtained in equations (9.2) delivers expressions for ϕ_n, ψ_n and χ_n. The substitution of ϕ_n, ψ_n and χ_n in equations (3.6) and (3.13) gives the required mode shapes for the core and frame; respectively.

4. EXPERIMENTAL VERIFICATION OF THE ANALYSIS

To verify the validity of the analysis, the resonance frequencies of five different experimental models of stators of electrical machines were computed. The calculated results are then compared with the corresponding measured values. The data as well as the measured values for the first four models were obtained from references (16) and (19). The results of the experimental measurements on the fifth model, which were carried out by Dr. S. P. Verma at the Institute of Electrical machines of the Technical University of Hannover (W. Germany), were made available for the verification of the theory in the case of stators having encased construction.

4.1 Model-I

Model-I[16], whose dimensions are given in Fig. 4.1, represents the stator-core of an induction motor of small power capacity. The weights of the yoke and the teeth are 7.7 kg and 4.65 kg; respectively.

The resonance frequencies of this model were calculated in two steps as described below:

1. The resonance frequencies of the stator-yoke only were calculated according to the general frequency equation derived in the previous chapter, equation (3.16), by equating the thickness of the frame h to zero. Due to the existence of free-body vibrations of the stator during experimentation, the vibrations along the machine-length are found to be uniform[16]. So a value of zero is assigned to the longitudinal wave-number M.

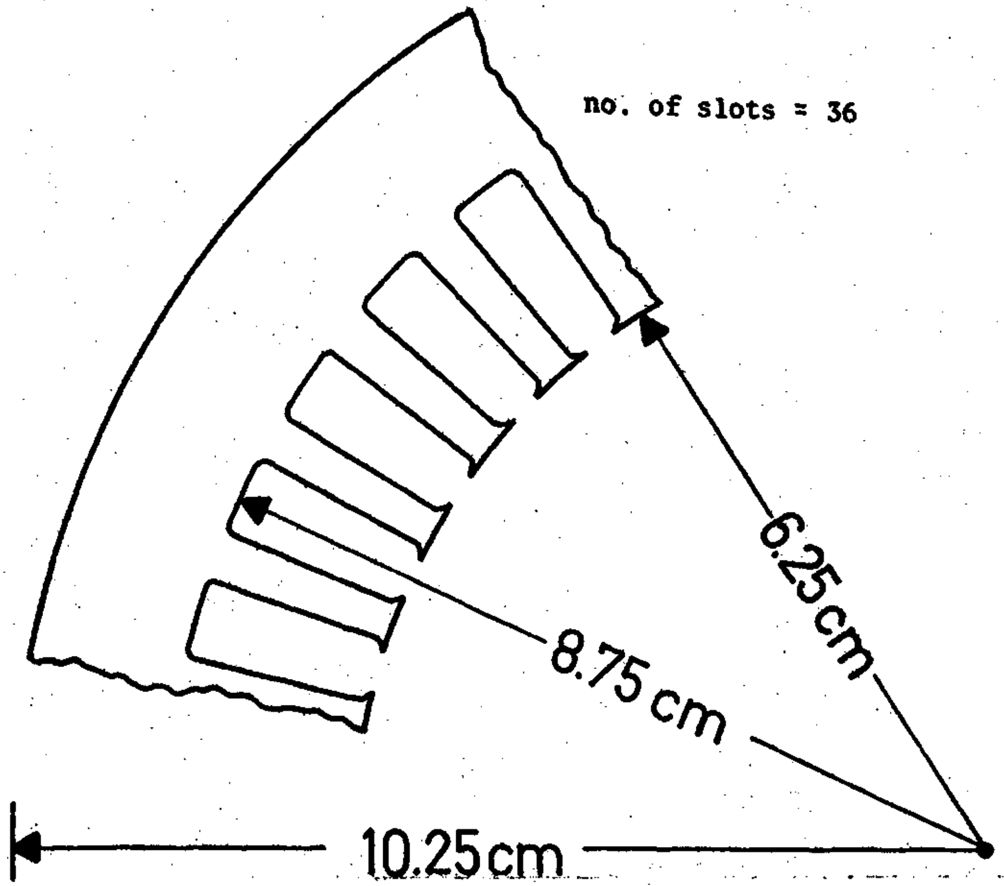


Fig. 4.1 Dimensions of Model-I

2. The effect of teeth is then introduced as a factor. This factor is derived in Appendix-D according to Frohne's expressions[16,17] and calculated here for the present model.

Frohne's expressions have been derived on the basis of simplified analysis; neglecting axial vibrations, flexural vibrations along the machine length, displacement gradients across the yoke thickness, coupling between the different displacement components, etc. Moreover, it is stated in reference (17) that Frohne's expressions become erroneous at higher frequencies because of the resonant excitation of the teeth.

The calculated frequencies according to both reference (16) and the present analysis, together with the experimental values are given in Table 4.1. The table also includes values of the teeth-factor for

n		0	2	3	4	5	6	7	8
$f_{meas.}$		6620	770	2025	3550	4800	5630	6050	6250
According to present analysis	η_t	0.7900	—	0.7432	0.6960	0.6620	0.5235	0.4283	0.3520
	$f_{calc.}$	6870	—	2156	3766	5275	6283	6820	7111
	error (%)	+3.78	—	+6.48	+6.1	+9.9	+11.6	+12.7	+13.8
According to reference (16)	$f_{calc.}$	6630	784	2080	3640	5070	6070	6660	6970
	error (%)	+0.15	+1.9	+2.7	+2.5	+5.6	+7.8	+10.0	+11.5

Table 4.1 Calculated and measured values of the resonance frequencies of Model-I

$f_{meas.}$ = measured resonance frequency.

$f_{calc.}$ = calculated resonance frequency.

η_t = teeth-factor

the different modes of vibration.

Table 4.1 shows a reasonable agreement between the calculated values of the resonance frequencies, obtained by using the present analysis, and the measured values. The errors in the values of the resonance frequencies by using the present analysis as well as those resulting by using the theory given in reference (16) increase with the increase in the complexity of the mode of vibration. The theory given in reference (16) is based on the assumption of a thin-ring and is, therefore, likely to yield results of less accuracy, especially when the wave-length of the circumferential mode becomes comparable with the thickness of the stator-yoke, which is the case for higher modes. In contrast, as the present analysis is based on the rigorous equations of elastic deformation, it should be expected that the results will have nearly the same accuracy for all the modes. However, this is not the case in Table 4.1. Here, higher errors are associated with higher frequencies. This contradiction can, therefore, be attributed to the followings:

i- The factor, for taking the effects of teeth into account, is not based on rigorous considerations. Therefore, at frequencies where the effect of teeth becomes pronounced, the accuracy of the results will be severely affected. This is the case at higher modes.

ii- There is always a certain amount of discrepancy between the actual values of the physical parameters of materials (modulus of elasticity, density, Poisson's ratio) and the values used in calculations.

iii- Inherent errors, which are always present in the measurements of resonance frequencies, especially the higher ones.

4.2 Model-II

For further confirmation of the causes of error mentioned in the foregoing, the calculated and measured values of frequencies for Model-II, whose dimensions are given in Fig. 4.2, are presented in Table 4.2 .

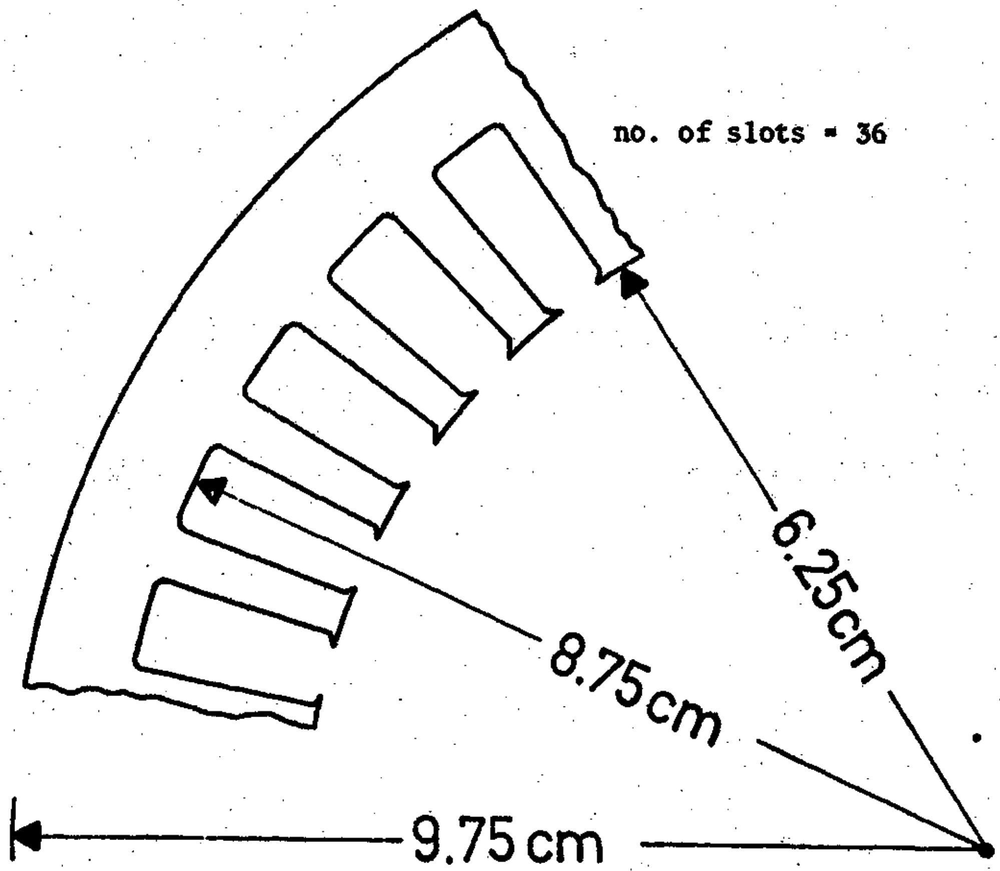


Fig. 4.2 Dimensions of Model-II

It is noticed from Table 4.2 that almost similar observations, as those made for Model-I in connection with errors, can also be made for this model. Further, it may also be mentioned that due to computational difficulties, it was not possible to trace the natural frequency associated with the mode n=2, for both models. These difficulties generally arise when the yoke-thickness to radius ratio of a stator is

n		0	2	3	4	5	6	7	8	9
$f_{meas.}$		6170	530	1400	2410	3450	4450	5180	5590	5850
According to present Analysis	n_t	0.7200	—	0.6741	0.6347	0.5850	0.5242	0.4576	0.3942	0.3368
	$f_{calc.}$	6350	—	1369	2469	3628	4703	5524	6087	6496
	error (%)	+2.92	—	-2.2	+2.45	+5.15	+5.7	+6.6	+8.9	+11.0
According to reference (16)	$f_{calc.}$	6190	502	1340	2380	3490	4490	5300	5900	6280
	error (%)	+0.3	-5.3	-4.3	-1.2	+1.2	+0.9	+2.3	+5.5	+7.3

Table 4.2 Calculated and measured values of the resonance frequencies of Model-II

very small. In such cases, the resonance frequency for mode n=2 is quite small. In the range of such low frequencies, the magnitudes of the Y and K Bessel functions become so high that the digital computer cannot calculate and locate the resonance frequency. However, Model-I and Model-II were especially built to suit the assumptions of the analysis based on thin-ring considerations as described in reference (16). In general, stators of electrical machines have much larger yoke-thickness to radius ratio.

4.3 Model-III

Now, it is of great importance[52] to test the analysis on a model which represents a large machine. Fig. 4.3 shows an experimental model[19] of an unslotted stator-core of a 400-MVA turbogenerator. The dimensions of this model are related to the actual dimensions of the turbogenerator by the ratio 1:2.36 . The length of the model was kept small (21 Cm) to avoid any flexural-vibrations along the machine length. Moreover, the model possesses a solid construction.

For this model, the measured values of the resonance frequencies for only two modes, n=0 and n=2, are available. The values of the resonance frequencies calculated according to Frohne's analysis and those calculated by using the present analysis, together with the measured values, are given in Table 4.3 .

It is interesting to note that a striking agreement exists between the calculated values according to the present analysis and the measured values, whereas the analysis according to reference (16) gives rise to considerable errors. Such a good agreement obtained by using the present analysis for an unslotted stator-core leads to the conclusion

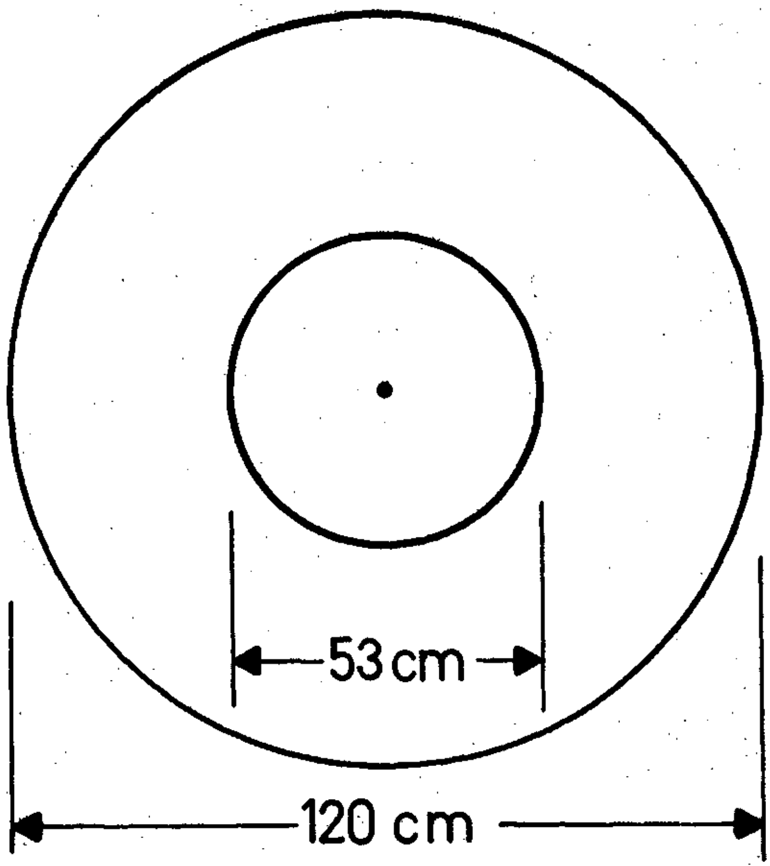


Fig. 4.3 Dimensions of an experimental stator-yoke of a turbogenerator-Model III

n	$f_{meas.}$	According to present analysis		According to reference (16)	
		$f_{calc.}$	error(%)	$f_{calc.}$	error(%)
0	2009	2012.6	+0.18	1890	-5.9
2	943	935.4	-0.80	870	-7.8

Table 4.3 Calculated and measured values of the resonance frequencies of Model-III

that the deviations noticed in Tables 4.1 and 4.2, between the calculated values according to the present analysis and the measured values, are those involved in the factor used for the consideration of the teeth effects. On the other hand, the considerable negative errors obtained by using Frohne's analysis, as noticed in Table 4.3, can be attributed to the fact that only the membrane-stiffness of the walls of the stator is considered in the analysis (neglecting the bending-stiffness).

4.4 Model-IV

Model-III was then provided with slots, as shown in Fig. 4.4, in order to make it representative of the stator-core of a large turbo-generator.

As in the case of Model-I and Model-II, the resonance frequencies of Model-IV were calculated by considering first the stator-yoke alone and then the teeth-factor was applied to obtain the required frequencies. For this model, the experimental and the calculated values of the resonance frequencies are given in Table 4.4 .

By comparing Table 4.4 with Table 4.3, it is revealed that the relatively increased errors in the calculated values of the resonance frequencies of the slotted stator-core, according to the present analysis, are caused by the factor used for the consideration of teeth effects. Also, the comparison between Table 4.4 and Table 4.3 shows that the excellent agreement obtained between the measured resonance frequencies and those calculated[19] by using Frohne's theory for Models I, II and IV results, in the opinion of the author, from the compensation of negative errors caused by considering the yoke as a thin ring and positive errors caused by the inaccurate consideration of teeth effects. Accordingly,

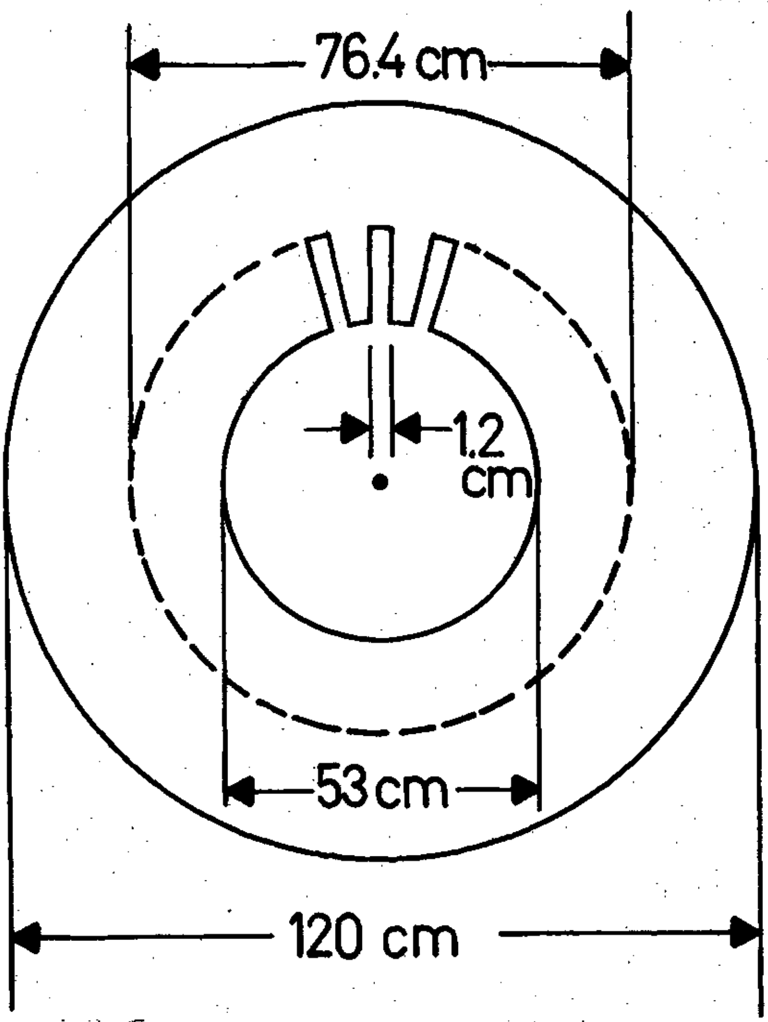


Fig. 4.4 Dimensions of an experimental stator-core of a turbogenerator-Model V

n	$f_{meas.}$	According to present analysis			According to reference(16)	
		η_t	$f_{calc.}$	error(%)	$f_{calc.}$	error(%)
2	465	0.882	481	+3.44	458	-1.5
3	1099	0.790	1092	-0.63	1095	-0.37

Table 4.4 Calculated and measured values of the resonance frequencies of Model-IV

such a good agreement may not be obtained in the case of other models.

4.5 Model-V

In contrast to the models treated in the foregoing, Model-V is a stator, having encased construction, of a typical 11-kW induction motor. The details of the stator are shown in Fig. 4.5 . The weights of stator -yoke, teeth, windings, frame and cooling ribs are 16.6, 5.9, 3.1, 9.25 and 7.9 kg; respectively. The stator-core is made of steel laminations, while the frame is of cast iron. The values of the physical parameters of these two materials are given in Table 4.5 .

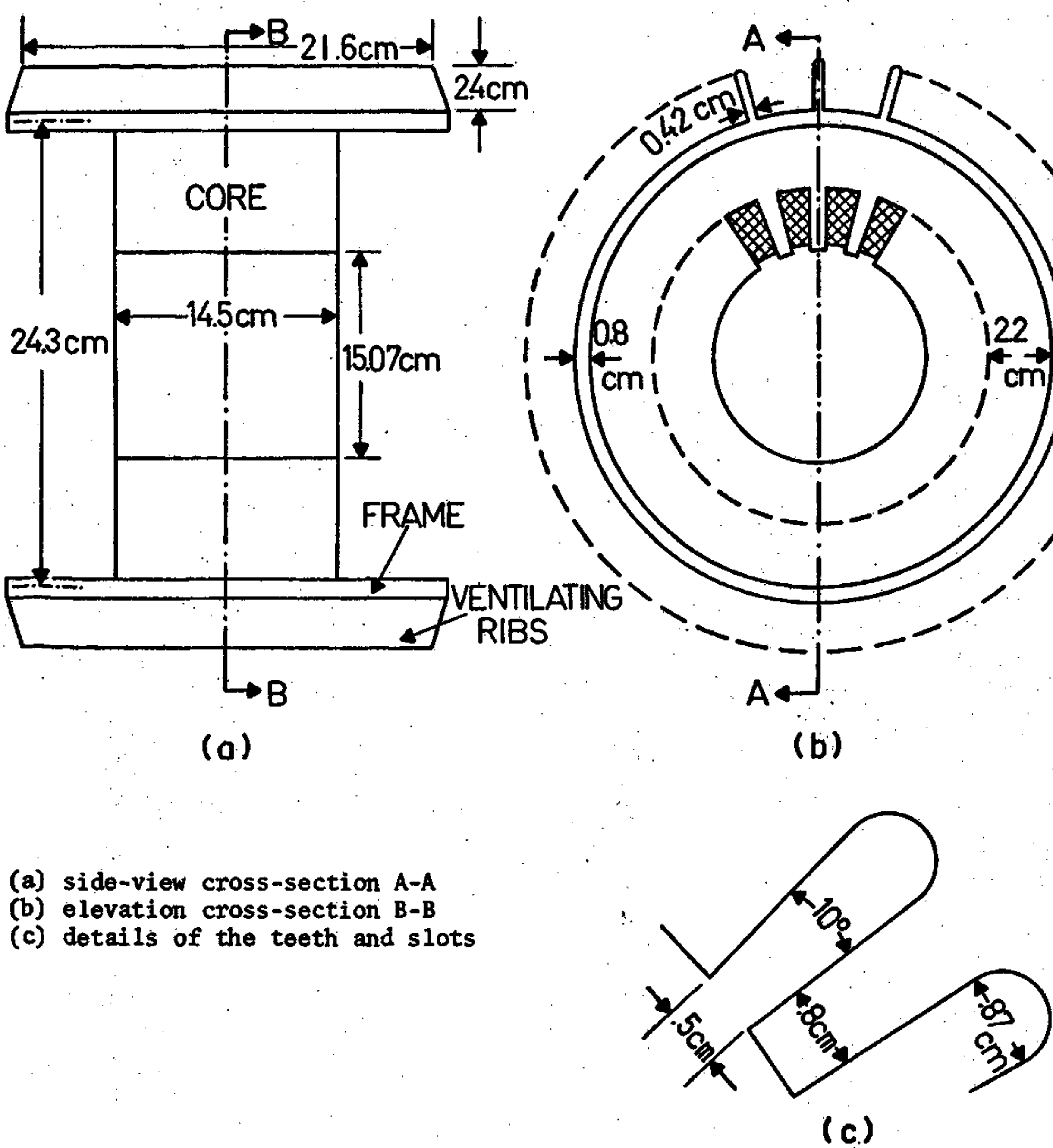
Material	Steel	Cast Iron
Poisson's Ratio	0.28	0.25
Modulus of Elasticity	2.1×10^6 kg/cm^2	1.05×10^6 kg/cm^2
Density	7.8 gm/cm^3	7.0 gm/cm^3

Table 4.5 Constants of steel and cast iron

In the calculations of the resonance frequencies of the stator under consideration, the whole mass of the frame is assumed to be concentrated in the portion of the frame which encases the stator-stack.

For this model, the measured and the calculated values of the resonance frequencies are given in Table 4.6 for the mode n=2, which is usually the most important mode of vibration in noise studies of electrical machines. The frequency is calculated by using the present analysis.

The effect of teeth, windings and cooling ribs is considered according to the factor given in equation (9.16). As this factor is



(a) side-view cross-section A-A
(b) elevation cross-section B-B
(c) details of the teeth and slots

Fig. 4.5 Dimensions of a stator of an 11-kW induction motor—Model-V derived on the basis of a single-ring stator, the application of it in the present case requires some modifications. It is appropriate, therefore, to consider the teeth, windings and cooling ribs as acting on a single stator-shell which is enlarged to include the frame.

$f_{meas.}$	η_{twR}	$f_{calc.}$	error(%)
920	0.7435	930	+ 1.1

Table 4.6 Calculated and measured values of the resonance frequency of Model-V for mode n=2

η_{twR}= Factor for taking into account effects of teeth, windings and cooling ribs

It can be seen from Table 4.6 that the present analysis, together with the factor for taking into account the effects of teeth, windings and cooling ribs, delivers striking agreement between the calculated and the measured value of the most important resonance frequency even with the approximation involved in considering the stator as a single ring in calculating that factor. In the light of this agreement and the conclusions reached in the foregoing comparisons, it is expected that similar agreement will be obtained for the higher modes $n \geq 3$ for this model.

4.6 Approximation Involved in Considering the Mass only of Teeth, Windings and Cooling Ribs

In order to examine the degree of approximation involved in considering the mass only of teeth, windings and cooling ribs in the present analysis, the resonance frequencies of Models I, II, IV and V were calculated using the present analysis by considering the mass of teeth and windings added to that of the stator-yoke and the mass of the cooling ribs added to that of the frame. The calculated values were compared with the measured values of the resonance frequencies and the percentage errors involved are presented in Table 4.7 .

	n								
Model	0	2	3	4	5	6	7	8	9
I	+3.8	—	+12.6	+20.0	+38.9	+67.5	+108	+155	*
II	+2.9	—	+4.5	+15.5	+29.0	+44.5	+67	+100	+138
IV	*	+5.5	+12.1	*	*	*	*	*	*
V	*	+5.55	*	*	*	*	*	*	*

Table 4.7 Percentage errors involved in considering the mass only of teeth, windings and cooling ribs

* : Measured value of the frequency is not available
— : Theoretical value of the frequency could not be traced.

Table 4.7 shows that the aforesaid approximation involves considerable errors, especially for higher modes. This shows the importance of taking into consideration the various effects of teeth, windings and cooling ribs in a rigorous way.

4.7 General Comments

The experimental verification on various models and the discussions lead to the following :

(i) The present analysis, together with an approximate factor for taking the effects of teeth, windings and cooling ribs into account delivers reasonable accuracy in calculating the values of the resonance frequencies especially with reference to lower modes.

(ii) The effects of teeth, windings and cooling ribs should be considered rigorously along with the present analysis for a more accurate assessment of the resonance frequencies of stators.

(iii) In the calculation of the resonance frequencies of stators of both small and large machines, the consideration of the mass only of teeth, windings and cooling ribs in the present analysis leads to substantial errors, especially for the higher modes.

5. VIBRATION CHARACTERISTICS OF STATORS HAVING ENCASED CONSTRUCTION

In order to find the vibration characteristics of stators having encased construction in a general way so that the results may be applicable to both small and large size electrical machines, it is preferable to consider the parameters involved in terms of dimensionless quantities. For this purpose, the elements of the frequency equation, derived in chapter(3) of this thesis, have been so arranged that they are functions of the dimensionless quantities:

$$\frac{E_f}{E_c}\ ,\ \frac{\bar{\rho}_f}{\rho_c},\ \frac{\bar{\rho}_c}{\rho_c},\ \mu_f\ ,\ \mu_c\ ,\ \frac{h}{R},\ \frac{r_2}{R},\ \frac{L}{R},$$

and the frequency factor $\frac{\omega}{\omega_o}$. The reference frequency ω_o is indicative of the lowest extensional natural frequency[40] of the core in plane-strain.

The dimensionless parameters of the stator shown in Fig. 4.5 are found to be realistic for most machine sizes. Therefore, the values of these parameters have been used in the following investigations. The values of these parameters are:

$$\frac{r_2}{R} = 0.788 \quad , \quad \frac{h}{R} = 0.066 \quad , \quad \frac{L}{R} = 1.19$$

$$\frac{E_f}{E_c} = 0.5 \quad , \quad \frac{\bar{\rho}_c}{\rho_c} = 1.542 \quad , \quad \frac{\bar{\rho}_f}{\rho_f} = 2.75$$

$$\mu_c = 0.28 \quad , \quad \mu_f = 0.25$$

5.1 Frequency Spectrum

In connection with the vibration and noise studies in electrical machines, one requires complete information about the spectrum of the resonance frequencies of the stator. Moreover, to understand the physical significance of encasing the stator-core in the frame, it is essential to determine the frequency spectrum of the composite stator as well as the frequency spectra of both members of the stator separately.

The frequency spectra, which have been obtained by using the present analysis, are given in Fig. 5.1 and Fig. 5.2 for the two cases of m=0 and m=1; respectively. Each figure contains the various frequency spectrums of the free frame, the free core and the composite stator for different circumferential nodal patterns n. Both figures include all the possible resonance frequencies which exist within the frequency range under consideration. This frequency range is generally sufficient, even when a large machine is considered. For the usual steel stator-cores, $\omega_o \cong \frac{5130}{r_3}$; r_3 being in meters. So, for a fairly large-sized machine having, for example, a radius $r_3 \cong 70$ cm, ω_o will have a value of 7350 rad./sec. . Accordingly, the frequency range considered here ($\frac{\omega}{\omega_o}$ =0→5) corresponds to frequencies up to about 6000 Hz for such machines. This frequency range is a fairly broad range from noise considerations.

Figs. 5.1 and 5.2 show that the composite stator has more than one resonance frequency for every modal pattern. In fact, it is well -known[45,53-59] that a thick cylinderical shell possesses an infinite number of resonance frequencies for a given nodal arrangement of m and n. As shown in Figs. 5.1 and 5.2, three resonance frequencies of the composite stator lie within the frequency range considered here. These frequencies correspond to different amplitude ratios of the three components of displacement w, v and u[40], as will be explained later in this chapter.

In contrast, the previous investigations available in the published literature[10-20] in connection with the determination of the res-

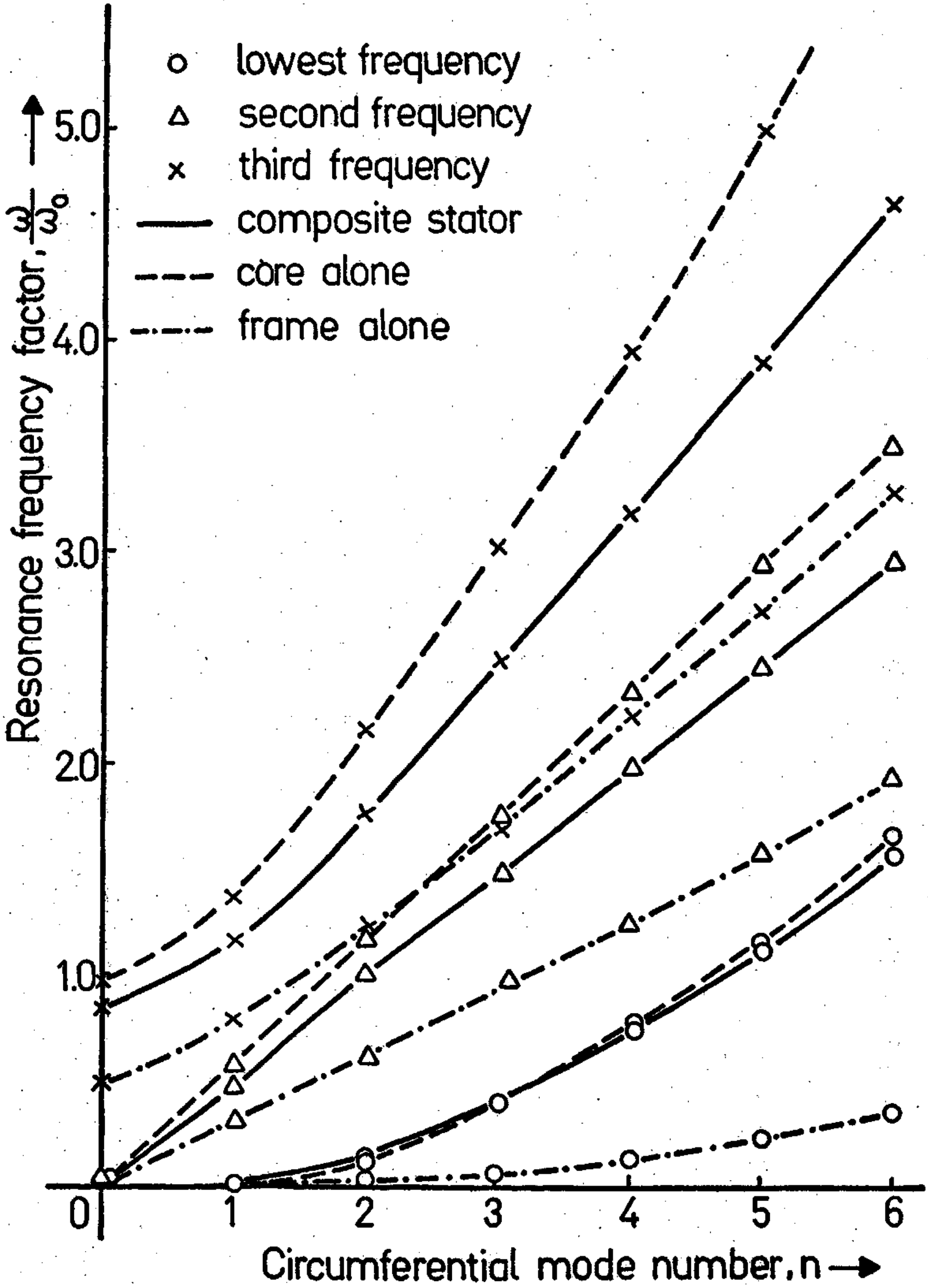


Fig. 5.1 Frequency spectrums for the case m=0

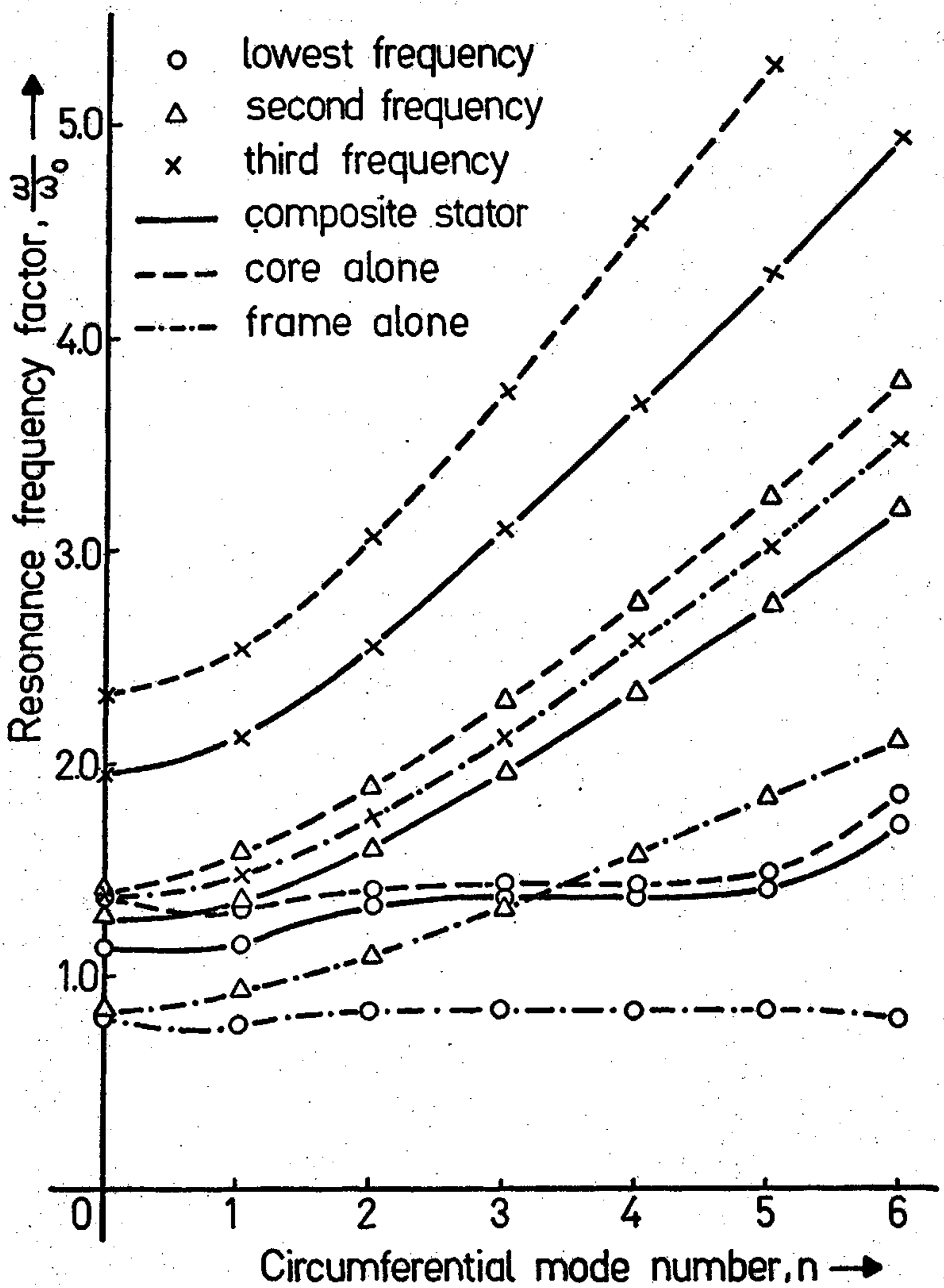


Fig. 5.2 Frequency spectrums for the case m=1

onance frequencies of stators, in general, are invariably confined to only the lowest resonance frequency associated with a mode. Such confinement can be partially attributed to the fact that most of the theories used for these investigations in the literature have the drawback of being incapable of delivering information about frequencies other than the lowest one. Apart from this, previous investigations have considered the lowest frequency the only significant resonance frequency of the stator with reference to vibration problems in electrical machines. Although this can be true for small machines, it is not the case with large machines where several frequencies for each important modal pattern may lie within the dangerous frequency range of noise production. The presence of these frequencies may be one of the causes of noticeable high noise-levels,as it increases the possibility of coincidence between these frequencies and the exciting frequencies. For example, with the help of Fig. 5.1 it can be shown that a stator having dimensions ten times that of the stator shown in Fig. 4.5 (such a stator could be suitable for a large power machine, for example, a high speed machine of about 300 to 400-MVA capacity) will have resonance frequencies of about 94 Hz, 682 Hz and 1180 Hz for mode n=2 and m=0.

5.2 Mode Shapes and Mechanical Response

For the purpose of acquiring a physical understanding of the phenomenon of the presence of more than one resonance frequency for each modal pattern, information about the amplitudes of the different displacement components, w,v and u of the core and the frame at the lowest three resonance frequencies of the stator under consideration when it is vibrating in a mode identified by four circumferential nodes, are given in Table 5.1 for the two cases of m=0 and m=1. As there is no unique basis for comparing the magnitudes of the various displacement components, the maximum displacement at each frequency is taken as the normalizing factor for that frequency.

As a matter of fact, the various vibration components are associated with each other in each of the resonance frequencies but with different ratios as exemplified in Table 5.1 . It is shown in Table 5.1a that the lowest resonance frequency is associated with only radial and tangential vibrations. At this frequency, the radial vibrations are predominant and hence they characterize the mode of vibration. The slight difference between the amplitude of the tangential component of the core vibrations and that of the frame, at this frequency, shows a relative rotation of the frame about the core. At the second frequency, the vibrations are purely axial. With respect to the third frequency, the vibrations are again plane vibrations, involving radial and tangential vibrations, with predominance of the tangential vibrations. It is interesting to note that the ratio of predominance of the radial and the tangential vibrations in the first and third frequencies is 2.6 and 2.8; respectively, as compared to the values of 2.0 and 2.0 in the case of vibrations of thin rings[40].

The most striking result of Table 5.1a is that, for instance, a purely radial electromagnetic force having frequency equal to, or near, the first resonance frequency of the stator, excites significant torsional vibrations in addition to the radial vibrations. On the other hand, a tangential force having the same frequency will excite radial vibrations of much greater amplitude (in this case 2.6 times) than that of the torsional

	core displacements			frame displacements		
frequency order	radial	tangential	axial	radial	tangential	axial
first frequency	1	0.387	0	1	0.332	0
second frequency	0	0	1	0	0	1
third frequency	0.33	0.945	0	0.33	1	0

(a) For the case m = 0

	core displacements			frame displacements		
frequency order	radial	tangential	axial	radial	tangential	axial
first frequency	0.655	1.000	0.419	0.655	0.990	0.476
second frequency	0.414	0.940	0.738	0.414	1.000	0.700
third frequency	0.031	0.895	0.997	0.031	0.927	1.000

(b) For the case m = 1

Table 5.1 Relative displacements of the core and frame for mode n=2 vibrations excited by the same force. Similar observations can be made for the third frequency, Table 5.1a, as well as for the three frequencies given in Table 5.1b. In the presence of flexural vibrations along the machine-length, it can be noticed from Table 5.1b that all the three displacement components are associated with each other at all the resonance frequencies of the stator regardless of their order. In this case, a purely radial, tangential or axial exciting force will give rise to significant responses not only in its own direction but also in the other two directions. Such coupling between the different displacement components are due to Poisson's effect in materials.

5.3 Effect of Encasing the Stator-Core in the Frame

From Figs. 5.1 and 5.2, it can be observed that the presence of the frame has practically insignificant effect on the lowest resonance frequency of the stator, for a given modal pattern. This reveals the presence of a weak coupling between the stator-core and the frame at this frequency. Such a faint coupling results in a predominance of the stator-core to the extent that this frequency can be calculated without appreciable error by totally ignoring the presence of the frame and considering the vibrations of the stator-core alone. This is in agreement with the observations made by Jordan and Lorenzen[60].

However, the frame has a considerable effect on the higher resonance frequencies as shown in Figs. 5.1 and 5.2. This results from the substantial coupling between the stator-core and the frame at these frequencies. The presence of the strong coupling at higher frequencies is revealed by the fact that the resonance frequency of the composite stator lies considerably below the corresponding resonance frequency of the stator-core alone.

The observations mentioned above show that the coupling between the stator-core and the frame depends on the order of the frequency and on the associated mode of vibration.

As exemplified in Table 5.1a, the lowest frequency of the stator for $n \geq 1$ and $m=0$ is associated with a mode characterized by a predominance of the radial vibrations, while the second and third frequencies are associated with pure axial and predominant tangential vibrations; respectively. Also, it is found that the only non-zero frequency for the case of $n=0$ and $m=0$, Fig. 5.1, is associated with pure radial vibrations. Similar observations have been made in several publications[40,45,53,59].

Referring to Fig. 5.1, the coupling between the core and the frame is negligible for modes characterized by predominance of radial vibrations. In the case of modes characterized by predominance of axial or tangential vibrations, the coupling is substantial. This is to be expected on the basis of physical considerations.

By considering Fig. 5.2 and Table 5.1b, similar observations can be made for the case of $m=1$. The vibrations associated with the lowest as well as the other resonance frequencies for $n \geq 1$ are characterized by coupled radial, tengential and axial displacements. Therefore, a stronger coupling between the stator-core and the frame exists in the case of $m=1$ as compared to the case of $m=0$ for the different frequencies. Also, it is found that the mode associated with the lowest resonance frequency of the stator for the case $n=0$ and $m=1$ is purely torsional and this is the reason of the relatively strong coupling at this frequency, Fig. 5.2. Moreover, Fig. 5.2 shows that the third frequency for the case $m=1$ is associated with a relatively strong coupling between core and frame. This can be perhaps attributed to what has been noticed in Table 5.1b of negligible radial vibrations associated with this frequency.

Finally, Figs. 5.1 and 5.2 show that the coupling between the stator-core and the frame does not actually depend on the complexity of the mode around the machine circumference, especially for modes $n \geq 2$.

5.4 Effect of the Circumferential Wave-Number on the Resonance Frequencies of the Stator

It can be, further, noticed from Figs. 5.1 and 5.2 that not only the lowest resonance frequency increases with the increase in the complexity of the circumferential mode n, but also the higher resonance frequencies follow the same trend in the range of mode numbers under consideration.

In the case of uniform amplitude of vibrations along the machine length, i.e. when m=0, the total stiffness of the stator walls is almost entirely composed of bending stiffness[41]. As n increases, the bending stiffness of the stator walls increases[40,41,53] and consequently the resonance frequencies of the stator will show a consistent increase in their values with the increase of number of waves about the machine circumference, as shown in Figs.5.1. On the contrary, in the case of m=1 the stretching stiffness will be more prominent than the bending stiffness[41] and that is why the amount of increase in the values of frequencies with the increase of n, in general, is less for the case of m=1 than for the case of m=0, Figs. 5.1 and 5.2, particularly at the lower values of n. Further, according to reference (41) the stretching energy decreases rapidly with the increase of the number of the circumferential nodes, while the bending energy varies in the reverse manner. It is not surprising, therefore, that there is a n-range within which these two effects balance each other in such a manner that the frequency remains essentially constant as is the case at the lowest resonance frequency of the stator under consideration, Fig. 5.2.

5.5 Advantages of the Encased Construction of Stators from Vibration Considerations

In order to show the advantages of the encased construction of stators over the conventional construction from the vibration point of view, the model investigated by Erdelyi and Horvay[15] is considered here. This model is a stator of a 30-hp, 4-pole induction motor. In this model, the stator-core is supported by the frame with the help of radial and tangential ribs as shown in Fig. 5.3.

The calculated values of the resonance frequencies of this stator, according to reference (15), are given in Table 5.2.

n	2	3	4	5	6	7
Resonance frequencies	460	1170	1295	2240	3660	5200
	490	1225	2060	2940	4240	5580
	1140	1580	2560	3950	6240	9780
	10430	8200	2940	4750	6980	11650

Table 5.2 The calculated values of the resonance frequencies of the stator model investigated in reference (15)

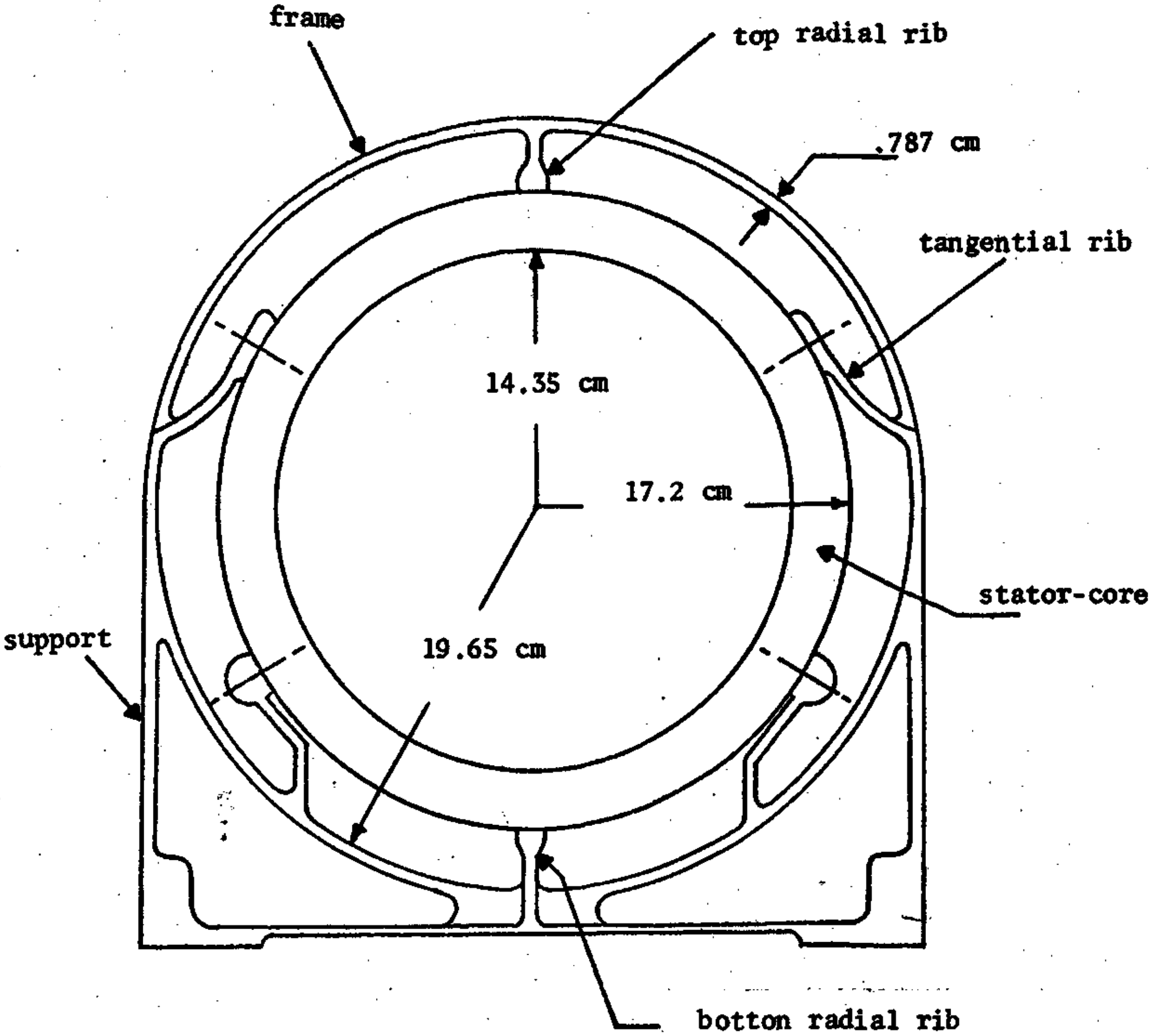


Fig. 5.3 Stator investigated in reference (15)

It has been pointed out by Erdelyi and Horvay that, in the case of such a construction, there are four resonance frequencies for every mode, as shown in Table 5.2. These frequencies correspond to whether the antinodes or the nodes of the mode lie on a radial rib and whether core and frame move in phase or in antiphase.

For the purpose of investigating the advantages of the encased construction, the stator-core of this model will now be considered to be encased in the frame (ribs removed), as shown in Fig. 5.4.

The resonance frequencies of this stator, Fig. 5.4, were calculated

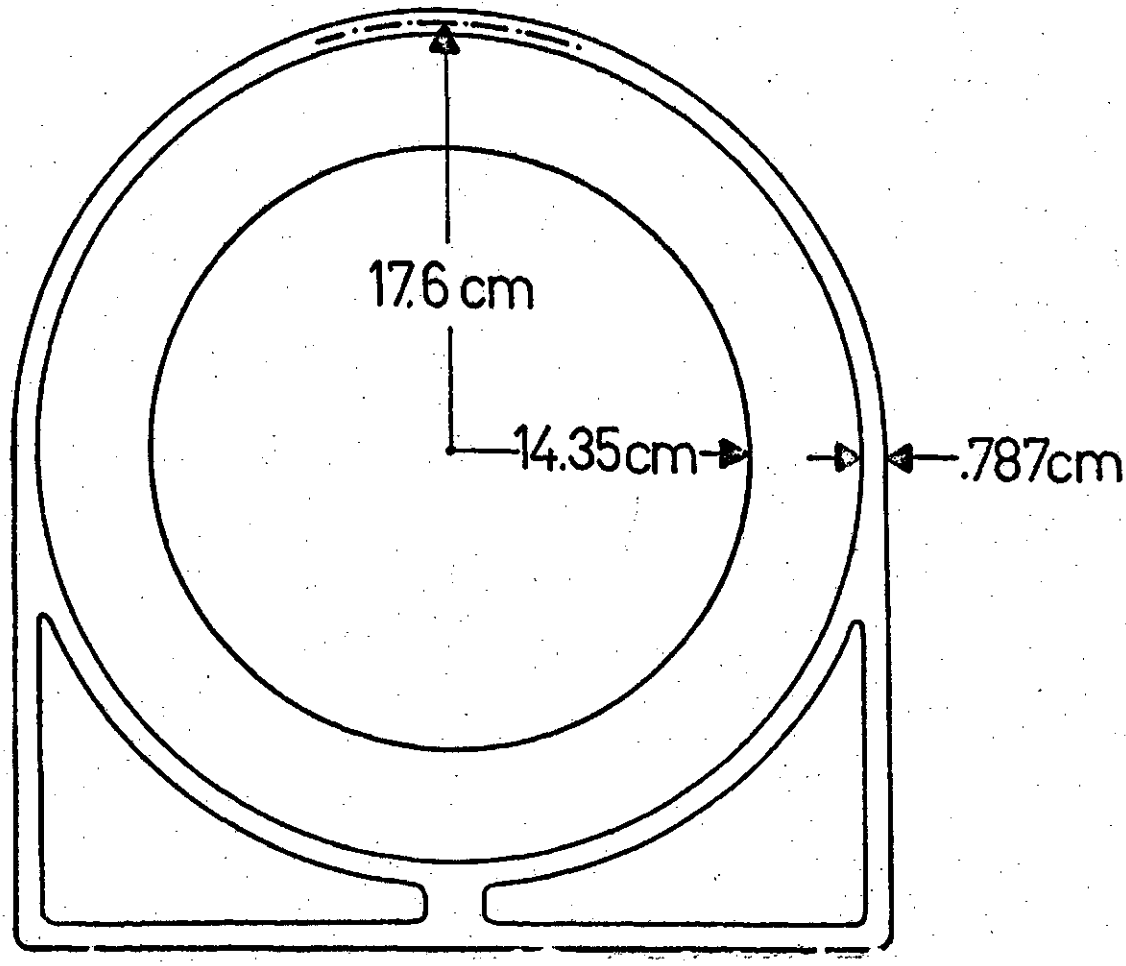


Fig. 5.4 The encased version of the stator investigated in reference (15)

by using the present analysis, considering the same values of mass-density and other material constants as those used for the calculation of the frequencies given in Table 5.2. Moreover, the effect of the teeth was considered by adding the mass of the teeth to the mass of the yoke, as done in reference (15). The calculated values of the resonance frequencies for the encased version of the stator are given in Table 5.3.

n	2	3	4	5	6	7
Resonance frequencies	560	1572	2897	4469	6227	8123

Table 5.3 The calculated values of the resonance frequencies for the stator of Fig. 5.4

A comparison between Tables 5.2 and 5.3 shows two distinct advantages of the encased construction over the conventional one with ribs from the vibration considerations. These advantages are:

i) The conventional construction with ribs delivers four resonance frequencies (or even more depending on the number of ribs) for each modal pattern, whereas the encased construction delivers, correspondingly, only one frequency. This is to be expected as the frame is forced to follow the motion of the core due to the absence of discrete points of support between the core and the frame in the encased construction.

ii) For each mode, the only existing resonance frequency in the case of the encased construction is higher than two or three of the corresponding resonance frequencies of the conventional construction.

From the above, it is evident that the encased construction is very advantageous from vibration and noise considerations.

6. SIMPLIFIED FREQUENCY EQUATIONS

In order to reduce the amount of calculations and simplify the analysis to a considerable extent, simplified frequency equations are often desired. The modified equations should be developed in such a way that results with reasonable accuracy are obtained. Three types of simplifications are considered here. These simplifications are introduced on the basis of the actual construction of the electrical machine, where the frame-thickness to frame mean radius ratio is in the range of 1/12 or even less. It is obvious, therefore, that only the part of the general frequency equation which involves the parameters of the frame, has to be modified. The simplifications are described in the following and the validity of each of the simplified frequency equations is discussed.

6.1 Modified Frequency Equation According to Donnell's Simplifications

The simplified Flügge's equations for the motion of the frame, which are obtained by considering Donnell's simplifications[44], are:

$$\mu_f u_f' + \dot{v}_f + w_f + k\left(w_f'''' + 2\ddot{w}_f'' + w_f''\right) - \frac{R^2}{D_f}\overset{*}{p}_r + \eta^2 \frac{\partial^2 w_f}{\partial t^2} = 0 \qquad \text{.... (6.1 a)}$$

$$\frac{1+\mu_f}{2}\dot{u}_f' + \ddot{v}_f + \frac{1-\mu_f}{2} v_f'' + \dot{w}_f + \frac{R^2}{D_f}\overset{*}{p}_\theta - \eta^2 \frac{\partial^2 v_f}{\partial t^2} = 0 \qquad \text{.... (6.1 b)}$$

$$u_f'' + \frac{1-\mu_f}{2}\ddot{u}_f + \frac{1+\mu_f}{2}\dot{v}_f' + \mu_f w_f' + \frac{R^2}{D_f}\overset{*}{p}_z - \eta^2 \frac{\partial^2 u_f}{\partial t^2} = 0 \qquad \text{.... (6.1 c)}$$

When equations (3.7) are replaced by the above equations, the last three rows of the matrix [T], given in equation (3.15), will be considerably simplified and will assume the form given in equation (6.2).

$(2+\frac{q_c}{G_c})\frac{RG_c}{D_f}$	$\frac{M^2h\mu_f}{2R}+1+\frac{n^2h}{2Ra}+\frac{Rq_c}{D_fa}$ $+k(M^2+n^2)^2-\eta^2\omega^2$	$-M^2\mu_f-\frac{n^2}{a^2}$ $-\frac{q_cR}{D_f}(\frac{n^2}{a^2}+M^2)$	$-\frac{MRG_c}{D_f}(2+\frac{q_c}{G_c})$	$-\frac{M^3h\mu_f}{2R}-\frac{n^2hM}{2Ra}-\frac{MRq_c}{aD_f}$ $-M(1+k(M^2+n^2)^2-\eta^2\omega^2)$
0	$\frac{nh}{2Ra}(\eta^2\omega^2-n^2-\frac{(1-\mu_f)M^2}{2}$ $-\frac{nhM^2(1+\mu_f)}{4R}+n(\frac{2RG_c}{D_fa}-1)$	$\frac{n}{a^2}(n^2+\frac{M^2(1+\mu_f)}{2}-\eta^2\omega^2)$ $+\frac{M^2n}{2}(1+\mu_f)-\frac{2nRG_c}{D_fa^2}$	0	$\frac{nhM}{2Ra}(n^2+\frac{M^2(1-\mu_f)}{2}-\eta^2\omega^2)$ $+\frac{M^3nh(1+\mu_f)}{4R}-\frac{2nMRG_c}{D_fa}+Mn$
0	$\frac{Mh}{2R}(M^2+\frac{1-\mu_f}{2}n^2-\eta^2\omega^2)$ $+\frac{n^2hM}{4ar_2}(1+\mu_f)-\frac{2MRG_c}{D_f}+M\mu_f$	$-M(M^2+\frac{n^2(1-\mu_f)}{2}-\eta^2\omega^2)$ $-\frac{n^2M(1+\mu_f)}{2a^2}$	0	$-\frac{M^2h}{2R}(M^2+\frac{n^2(1-\mu_f)}{2}-\eta^2\omega^2)-M^2\mu_f$ $-\frac{n^2M^2h}{4aR}(1+\mu_f)-\frac{RG_c}{D_f}(\omega_2^2R^2-M^2)$

$\frac{q_c RM}{D_f}(\frac{n^2}{a^2} - \omega_2^2 R^2) + \frac{Mn^2}{a^2}$ $-M\mu_f \omega_2^2 R^2$	0	$\frac{n}{a}(\frac{2RG_c}{D_f} - 1)$	$-\frac{n}{a}[\frac{M^2 h \mu_f}{2R} - \frac{n^2 h}{2Ra} + \frac{2RG_c}{D_f a}$ $+ n^2\omega^2 - 1 - k(M^2 n^2)^2]$
$\frac{Mn}{a^2}(\frac{2RG_c}{D_f} + n^2\omega^2 - n^2 - \frac{1-\mu_f}{2} M^2)$ $+ \frac{1+\mu_f}{2} Mn\omega_2^2 R^2$	$\frac{RG_c}{D_f}$	$\frac{1}{a}[n^2 + \frac{M^2(1-\mu_f)}{2} - n^2\omega^2]$ $- \frac{RG_c}{aD_f}$	$-\frac{n^2}{a}[\frac{M^2 h(1+\mu_f)}{4R} + 1 - \frac{RG_c}{D_f a}$ $+ \frac{h}{2aR}(n^2 + \frac{M^2(1-\mu_f)}{2} - n^2\omega^2)]$
$-\frac{2}{2}R^2(M^2 + \frac{1-\mu_f}{2} n^2 - n^2\omega^2)$ $+ \frac{M^2 n^2(1+\mu_f)}{2a^2}$	0	$-\frac{nM(1+\mu_f)}{2a}$	$\frac{Mhn}{2Ra}(M^2 + \frac{1-\mu_f}{2} n^2 - n^2\omega^2)$ $+ \frac{nM}{a}\mu_f + \frac{n^3 hM(1+\mu_f)}{4a^2 R}$

....(6.2)

6.2 Further Modification of Frequency Equation by Neglecting Displacement-Gradients Across the Frame-Thickness

In this case, the displacements of a point situated on the mid-surface of the frame and the corresponding displacements of a point on the inner surface of the frame (both the points lying on the same radial line) are assumed to be equal. Accordingly, equations (3.11) are to be replaced by the followings:

$$\overset{*}{w}_c = w_f \qquad \text{.... (6.3 a)}$$

$$\overset{*}{v}_c = v_f \qquad \text{.... (6.3 b)}$$

$$\overset{*}{u}_c = u_f \qquad \text{.... (6.3 c)}$$

Introducing the above relations in the analysis, in which Donnell's simplifications have been included, the last three rows of the matrix [T] will now reduce to the form given in equation (6.4)

6.3 Modified Frequency Equation by Neglecting the Presence of the Frame

In the absence of the frame, the stator-assembly can be considered to have a frame of zero thickness. By introducing "h=0" in equation (3.15), the last three rows of the matrix [T] will be considerably simplified to the form given in equation (6.5)

It may be noted that these rows are similar to the first three rows of the general matrix [T], equation (3.15). This similarity is to be expected due to the fact that in the absence of the frame, the boundary conditions at both inner and outer surfaces of the core are the same. Further, it is worthwhile to mention that these rows can also be obtained by assigning a value of zero to the thickness of the frame in the original equations of motion of the frame [equations (3.7)], and then seeking the solution for

$(2+\frac{q_c}{G_c})\frac{RG_c}{D_f}$	$1+k(M^2+n^2)^2-\eta^2\omega^2+\frac{Rq_c}{D_f a}$	$-\mu_f M^2-\frac{n^2}{a}-\frac{q_c R}{D_f}(\frac{n^2}{a^2}+M^2)$	$-\frac{MRG_c}{D_f}(2+\frac{q_c}{G_c})$	$-M(1+(M^2+n^2)^2-\eta^2\omega^2-\frac{Rq_c}{D_{fa}})$
0	$n(\frac{2RG_c}{D_f a}-1)$	$\frac{n}{a}(n^2-\eta^2\omega^2-\frac{2RG_c}{D_f a})+\frac{M^2 n}{2}(1+\mu_d+\frac{1-\mu_f}{2a})$	0	$nM(1-\frac{2RG_c}{D_{fa}})$
0	$M(\mu_f-\frac{2RG_c}{D_f})$	$-M[M^2-n^2\omega^2+n^2(\frac{1-\mu_f}{2}+\frac{1+\mu_f}{2a})]$	0	$-M^2\mu_f-\frac{RG_c}{D_f}(\omega_2^2R^2-M^2)$

$\frac{q_c MR}{D_f}(\frac{n^2}{a^2}-\omega_2^2R^2)-\omega_2^2R^2M\mu_f+\frac{Mn^2}{a}$	0	$n(\frac{2RG_c}{D_f a}-1)$	$\frac{n}{a}[1+k(M^2+n^2)^2-\eta^2\omega^2-\frac{2RG_c}{D_f a}]$
$\frac{1+\mu_f}{2}Mn\omega_2^2R^2+\frac{2MnRG_c}{D_f a^2}-\frac{Mn}{a}(n^2+\frac{M^2(1-\mu_f)}{2}-\eta^2\omega^2)$	$\frac{RG_c}{D_f}$	$n^2+\frac{M^2(1-\mu_f)}{2}-\eta^2\omega^2-\frac{RG_c}{D_f a}$	$\frac{n^2}{a}(1-\frac{RG_c}{D_f a})$
$-\omega_2^2R^2(M^2+\frac{1-\mu_f}{2}n^2-\eta^2\omega^2)+\frac{M^2n^2(1+\mu_f)}{2a}$	0	$-\frac{Mn(1+\mu_f)}{2}$	$\frac{Mn}{a}(\mu_f-\frac{RG_c}{D_f})$

....(6.4)

$2+\frac{q_c}{G_c}$	$\frac{q_c}{G_c}$	$-\frac{q_c}{G_c}(M^2+n^2)$	$-M(2+\frac{q_c}{G_c})$	$-\frac{q_c}{G_c}M$	$\frac{q_c}{G_c}M(n^2-\omega_2{}^2R^2)$	0	2n	-2n
0	-2n	2n	0	2nM	-2nM	-1	1	$-n^2$
0	2M	0	0	$\omega_2{}^2R^2-M^2$	0	0	0	nM

....(6.5)

the stator-assembly. In this connection, equations (3.7) reduce to the following:

$$\sigma_r \Big|_{r=r_3} = \tau_{r\theta} \Big|_{r=r_3} = \tau_{rz} \Big|_{r=r_3} = 0 ,$$

which are the boundary conditions at the outer surface of a free stator -core.

6.4 Validity of the Simplified Frequency Equations

To check the validity of the three different simplifications introduced in the general frequency equation, the model used for obtaining the results of Figs. 5.1 and 5.2 is considered. The various frequencies have been calculated by considering these simplifications for the two cases of m=0 and m=1. These frequencies are the lowest three resonance frequencies for the modes n=0, 1, 2 and 4. Thus, the validity of the simplified frequency equations can be checked for the most important modes of vibrations.

The various resonance frequencies of the stator calculated according to the general frequency equation were compared with those calculated by using the three simplified frequency equations and the errors involved are given in Table 6.1 .

In Table 6.1, the first approximation corresponds to Donnell's simplifications, the second corresponds to neglecting the displacement -gradients across the thickness of the frame together with Donnell's simplifications and the third corresponds to completely ignoring the presence of the frame. From the table, it can be observed that:

(i) Introducing Donnell's simplifications in the general frequency equation has practically no effect on the values of the computed frequencies in general.

(ii) The second simplification delivers results with good accuracy for the resonance frequencies other than the lowest order resonance frequency associated with modes $n \geq 2$. Even for this lowest order resonance frequency, the accuracy is adequate. In this connection, it can be seen from equations(3.11) that the second simplification will involve errors of higher magnitudes in the calculation of the resonance frequencies associated with modes characterized by predominance of radial vibrations and having a higher value of n.

(iii) The third simplification results, in general, in a considerable error in the calculation of the resonance frequencies. The error,

frequency order	n	first simplification		second simplification		third simplification	
		m = 0	m = 1	m = 0	m = 1	m = 0	m = 1
1	0	--	*	--	-0.33	--	+12.9
	1	--	*	--	+0.11	--	+12.5
	2	*	*	-3.64	-<3.0	-5.43	-<3.0
	4	+0.11	*	-3.9	-3.8	-2.63	-2.0
2	0	--	*	--	-0.11	--	+9.05
	1	*	*	*	*	+9.47	+10.1
	2	*	*	*	*	+9.55	+10.0
	4	*	*	*	*	+9.92	+10.3
3	0	*	*	*	+0.11	+9.15	+11.5
	1	*	*	+0.32	+0.27	+11.2	+12.0
	2	*	*	+0.46	+0.37	+12.64	+12.2
	4	*	*	+0.37	+0.31	+13.18	+13.4

-- : The value of the resonance frequency is zero.

* : The percentage error is less than 0.1.

Table 6.1 Errors in the values of the resonance frequencies caused by the various simplifications however, is considerably less in the case of the lowest frequency associated with modes $n \geq 2$. This is to be attributed to what has been observed from Figs. 5.1 and 5.2, that the contribution of the frame towards the resonance frequencies associated with aforesaid modes is of minor importance.

7. CONCLUSIONS

An analysis of general nature for the calculation of the various natural frequencies and mode shapes of a stator having encased construction has been developed. The stator considered consists of a thick core, loaded with teeth and windings, which is encased in a thin frame. In the analysis, the three-dimensional theory of elasticity has been used for the core, and Flügge's theory of thin-shells has been applied to the frame. The analysis has been developed in such a way that the effect of the teeth and windings can be considered either rigorously or in a simple manner by taking into account only the mass of the teeth and windings. The effect of the frame has been treated rigorously assuming that the encasing is ideal. In addition, the flexural vibrations along the machine-length have been considered in the analysis.

The frequency equation obtained is applicable to any mode of vibration including the zero and beam-type modes. Moreover, the frequency equation provides information about not only radial vibrations, but also about torsional and axial vibrations. Besides, the analysis delivers complete information about all the possible natural frequencies associated with a mode of vibration of the stator.

In the course of investigating the nature of the vibrations associated with some special modes, it has been found that the general frequency equation for such modes degenerates into two or three uncoupled frequency equations. In this connection, it has been found that the resonance frequencies become associated with either pure torsional or coupled radial and axial vibrations when the vibrations are uniform along the machine-circumference. On the other hand, when the vibrations are uniform along the machine-length, the modes associated with the various resonance frequencies become either pure axial vibrations or plane vibrations involving coupled radial and tangential vibrations. In the very special case which comprises both the above cases simultaneously, the vibrations become either pure axial, pure torsional or pure radial.

The numerous comparisons drawn between the measured values of the resonance frequencies and those calculated according to the present analysis lead to the following:

(i) The present analysis, together with an approximate factor for taking the effects of teeth, windings and cooling ribs into account, delivers reasonable accuracy in calculating the values of the resonance frequencies especially with reference to lower modes.

(ii) The effects of teeth, windings and cooling ribs should be considered rigorously along with the present analysis for a highly accurate assessment of the resonance frequencies of stators.

(iii) In the calculation of the resonance frequencies of stators of both small and large machines, consideration of the mass only of teeth, windings and cooling ribs in the present analysis leads to substantial errors, especially for the higher modes.

The detailed investigations done in this thesis, for the study of the vibration characteristics of stators having encased construction, have led, further, to the followings:

(iv) The stator has several resonance frequencies for a given modal pattern, and in the case of large-sized machines at least two resonance frequencies, other than the lowest one, may lie within the dangerous frequency range of noise production.

(v) A purely electromagnetic radial, tangential or axial force having an exciting frequency equal to, or near, any of the resonance frequencies of the stator, of the same mode, may give rise to significant responses not only in its own direction but also in the other two directions.

(vi) Whereas the presence of the frame has practically insignificant effect on the lowest resonance frequency of the stator for a given modal pattern, the higher frequencies are considerably affected and they are lower than the corresponding frequencies of the stator-core alone.

(vii) The presence of flexural vibrations along the machine-length results, in general, in a considerable increase in the values of the resonance frequencies of the stator.

(viii) The amount of coupling between the stator-core and the frame is not determined by the complexity of the mode around the circumference but by the order of the resonance frequency and the mode shape along the machine-length.

(ix) Stators having encased construction possess several advantages from vibration and noise considerations over the stators having the conventional construction (with ribs).

With a view to reducing the calculation effort and simplifying the analysis to a large extent, three different simplifications have been introduced separately in the general frequency equation. These simplifications have been suggested on the basis of the actual construction of electrical machines. Regarding the validity of the simplified frequency equations, it has been found that:

1- The frequency equation with Donnell's simplifications delivers excellent accuracy with considerably less calculation effort.

2- Neglecting the displacements-gradients across the frame-thickness in the analysis retains good accuracy especially in calculating the resonance frequencies other than the lowest order frequencies associated with modes $n \geq 2$.

3- The calculation of the resonance frequencies of the composite stator considering the vibrations of the stator-core alone (i.e. ignoring the presence of the frame), results in a considerable error which may well exceed 10%. The error is, however, less in calculating the lowest resonance frequency for modes having four or more circumferential nodes.

Finally, although the various frequency equations have been derived for the stator-assembly consisting of the core and the frame, they can also be used for the determination of the vibration behaviour of the stator-core alone; in the absence of the frame,by assigning a value of zero to the frame height.

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65) Verma, S.P. and Girgis, R.S., "On the Stator-Vibrations of Electrical Machines", Presented at the 1972 E.I.C. CONGRESS, October 1972.

9. APPENDICES

Appendix A. Solution for the General Frequency Equation

Equation (3.15) can be written in the simple form:

$$[T] \cdot [S] = 0 \qquad \text{.... (9.1)}$$

where, [T] is a six by nine matrix, while [S] is a nine elements vector. Further, the solution of equations (3.5), according to Table 3.1, has the general form:

$$\phi_{n(r)} = A_{n_1} Z_n(\omega_1 r) + A_{n_2} Q_n(\omega_1 r) \;, \qquad \text{.... (9.2 a)}$$

$$\psi_{n(r)} = A_{n_3} Z_n(\omega_2 r) + A_{n_4} Q_n(\omega_2 r) \;, \qquad \text{.... (9.2 b)}$$

and $$\chi_{n(r)} = A_{n_5} Z_n(\omega_2 r) + A_{n_6} Q_n(\omega_2 r) \;. \qquad \text{.... (9.2 c)}$$

Here, Z denotes a J or I Bessel function and Q denotes a Y or K Bessel function.

By substituting equations (9.2) in equation (3.15), a system of six homogenous algebraic equations is obtained. These equations can be written in the following matrix-form:

$$[P] \cdot [A_n] = 0 \quad , \qquad \text{.... (9.3)}$$

where; [P] is a six by six square matrix whose elements are:

$$P_{11} = \omega_1{}^2 R^2 T_{11} \ddot{Z}_{n(\omega_1 r_2)} + \omega_1 R T_{12} \dot{Z}_{n(\omega_1 r_2)} + T_{13} Z_{n(\omega_1 r_2)} \qquad \text{.... (9.4 a)}$$

$$P_{13} = \omega_2{}^2 R^2 T_{14} \ddot{Z}_{n(\omega_2 r_2)} + \omega_2 R T_{15} \dot{Z}_{n(\omega_2 r_2)} + T_{16} Z_{n(\omega_2 r_2)} \qquad \text{.... (9.4 b)}$$

$$P_{15} = \omega_2{}^2 R^2 T_{17} \ddot{Z}_{n(\omega_2 r_2)} + \omega_2 R T_{18} \dot{Z}_{n(\omega_2 r_2)} + T_{19} Z_{n(\omega_2 r_2)} \qquad \text{.... (9.4 c)}$$

$$P_{21} = \omega_1{}^2 R^2 T_{21} \ddot{Z}_{n(\omega_1 r_2)} + \omega_1 R T_{22} \dot{Z}_{n(\omega_1 r_2)} + T_{23} Z_{n(\omega_1 r_2)} \qquad \text{.... (9.4 d)}$$

$$P_{23}=\omega_2^{\,2}R^2T_{24}\ddot{Z}_{n(\omega_2 r_2)}+\omega_2 RT_{25}\dot{Z}_{n(\omega_2 r_2)}+T_{26}Z_{n(\omega_2 r_2)} \qquad \text{.... (9.4 e)}$$

$$P_{25}=\omega_2^{\,2}R^2T_{27}\ddot{Z}_{n(\omega_2 r_2)}+\omega_2 RT_{28}\dot{Z}_{n(\omega_2 r_2)}+T_{29}Z_{n(\omega_2 r_2)} \qquad \text{.... (9.4 f)}$$

$$P_{31}=\omega_1^{\,2}R^2T_{31}\ddot{Z}_{n(\omega_1 r_2)}+\omega_1 RT_{32}\dot{Z}_{n(\omega_1 r_2)}+T_{33}Z_{n(\omega_1 r_2)} \qquad \text{.... (9.4 g)}$$

$$P_{33}=\omega_2^{\,2}R^2T_{34}\ddot{Z}_{n(\omega_2 r_2)}+\omega_2 RT_{35}\dot{Z}_{n(\omega_2 r_2)}+T_{36}Z_{n(\omega_2 r_2)} \qquad \text{.... (9.4 h)}$$

$$P_{35}=\omega_2^{\,2}R^2T_{37}\ddot{Z}_{n(\omega_2 r_2)}+\omega_2 RT_{38}\dot{Z}_{n(\omega_2 r_2)}+T_{39}Z_{n(\omega_2 r_2)} \qquad \text{.... (9.4 i)}$$

$$P_{41}=\omega_1^{\,2}R^2T_{41}\ddot{Z}_{n(\omega_1 r_3)}+\omega_1 RT_{42}\dot{Z}_{n(\omega_1 r_3)}+T_{43}Z_{n(\omega_1 r_3)} \qquad \text{.... (9.4 j)}$$

$$P_{43}=\omega_2^{\,2}R^2T_{44}\ddot{Z}_{n(\omega_2 r_3)}+\omega_2 RT_{45}\dot{Z}_{n(\omega_2 r_3)}+T_{46}Z_{n(\omega_2 r_3)} \qquad \text{.... (9.4 k)}$$

$$P_{45}=\omega_2^{\,2}R^2T_{47}\ddot{Z}_{n(\omega_2 r_3)}+\omega_2 RT_{48}\dot{Z}_{n(\omega_2 r_3)}+T_{49}Z_{n(\omega_2 r_3)} \qquad \text{.... (9.4 l)}$$

$$P_{51}=\omega_1^{\,2}R^2T_{51}\ddot{Z}_{n(\omega_1 r_3)}+\omega_1 RT_{52}\dot{Z}_{n(\omega_1 r_3)}+T_{53}Z_{n(\omega_1 r_3)} \qquad \text{.... (9.4 m)}$$

$$P_{53}=\omega_2^{\,2}R^2T_{54}\ddot{Z}_{n(\omega_2 r_3)}+\omega_2 RT_{55}\dot{Z}_{n(\omega_2 r_3)}+T_{56}Z_{n(\omega_2 r_3)} \qquad \text{.... (9.4 n)}$$

$$P_{55}=\omega_2^{\,2}R^2T_{57}\ddot{Z}_{n(\omega_2 r_3)}+\omega_2 RT_{58}\dot{Z}_{n(\omega_2 r_3)}+T_{59}Z_{n(\omega_2 r_3)} \qquad \text{.... (9.4 o)}$$

$$P_{61}=\omega_1^{\,2}R^2T_{61}\ddot{Z}_{n(\omega_1 r_3)}+\omega_1 RT_{62}\dot{Z}_{n(\omega_1 r_3)}+T_{63}Z_{n(\omega_1 r_3)} \qquad \text{.... (9.4 p)}$$

$$P_{63}=\omega_2^{\,2}R^2T_{64}\ddot{Z}_{n(\omega_2 r_3)}+\omega_2 RT_{65}\dot{Z}_{n(\omega_2 r_3)}+T_{66}Z_{n(\omega_2 r_3)} \qquad \text{.... (9.4 q)}$$

$$P_{65}=\omega_2^{\,2}R^2T_{67}\ddot{Z}_{n(\omega_2 r_3)}+\omega_2 RT_{68}\dot{Z}_{n(\omega_2 r_3)}+T_{69}Z_{n(\omega_2 r_3)} \qquad \text{.... (9.4 r)}$$

where; P_{ij} and T_{ij} are the elements $P_{(i,j)}$ and $T_{(i,j)}$; respectively. The missing elements $P_{(i,j+1)}$ of the matrix [P] can be found without difficulty by replacing Z_n in the above expressions of $P_{(i,j)}$ by Q_n. The elements of the vector $[A_n]$ are A_{n_1}, A_{n_2}, $A_{n_{3/R}}$, $A_{n_{4/R}}$, A_{n_5} and A_{n_6}.

Equation (9.3) can have a non-trivial solution if, and only if, the determinant of the coefficient matrix [P] vanishes. Hence, the frequency equation takes the form:

$$| P | = 0 \qquad .... (9.5)$$

Appendix B. The Calculation Procedure for Determining the Values of the Resonance Frequencies of the Stator

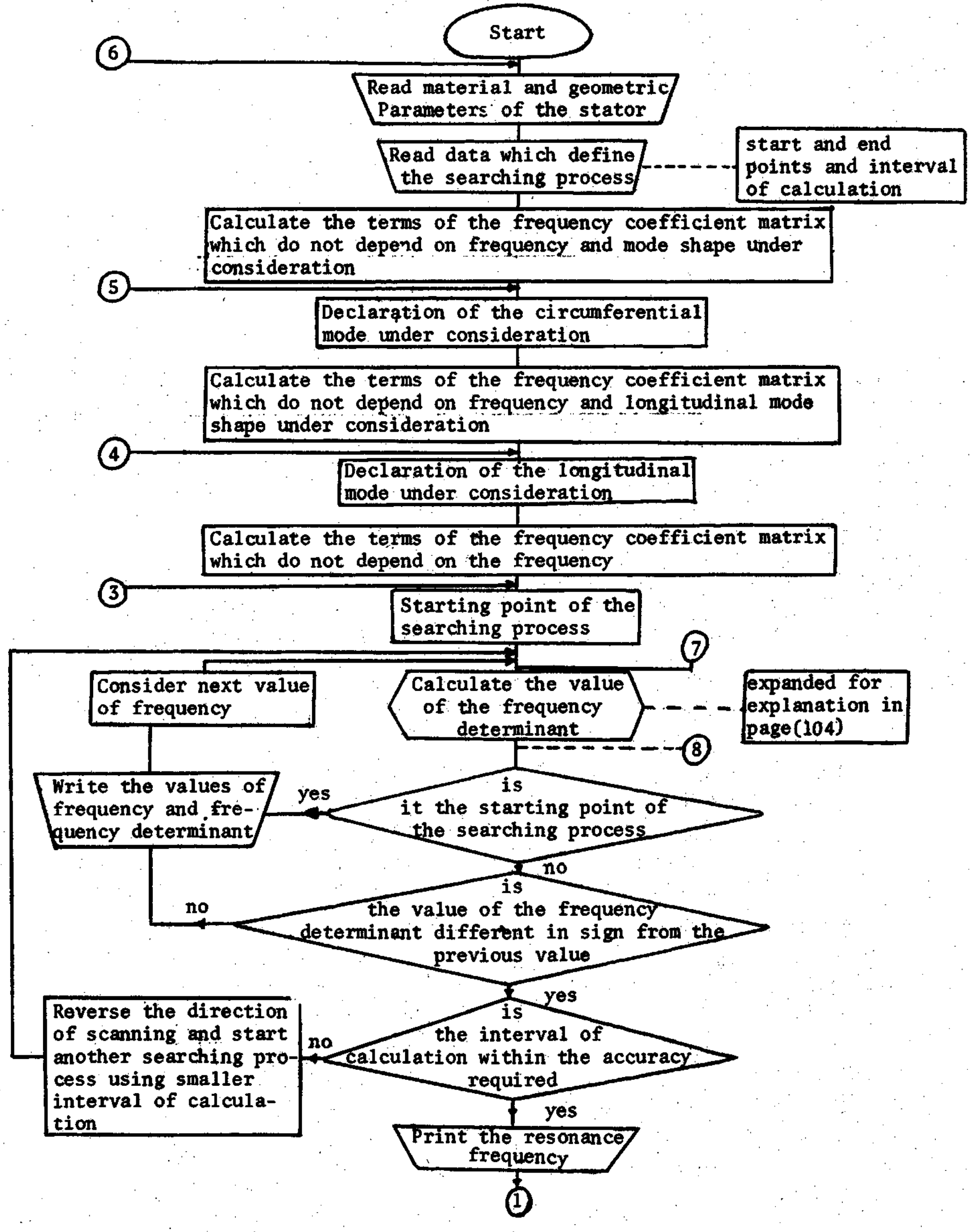
The frequency equation given by equation (3.16) is a very complicated transcendental equation involving a large number of material and geometric parameters. The solution of equation (3.16) is a fairly difficult numerical problem due to the non-linear change of resonance frequencies with any of these parameters. However, once values are assigned to these parameters the frequency determinant $|P|$ becomes a function of the frequency factor $\frac{\omega}{\omega_o}$ alone, and accordingly the zeros of the determinant can be evaluated. As it is clear from equation (3.16), these zeros correspond to the resonance frequencies of the stator under consideration.

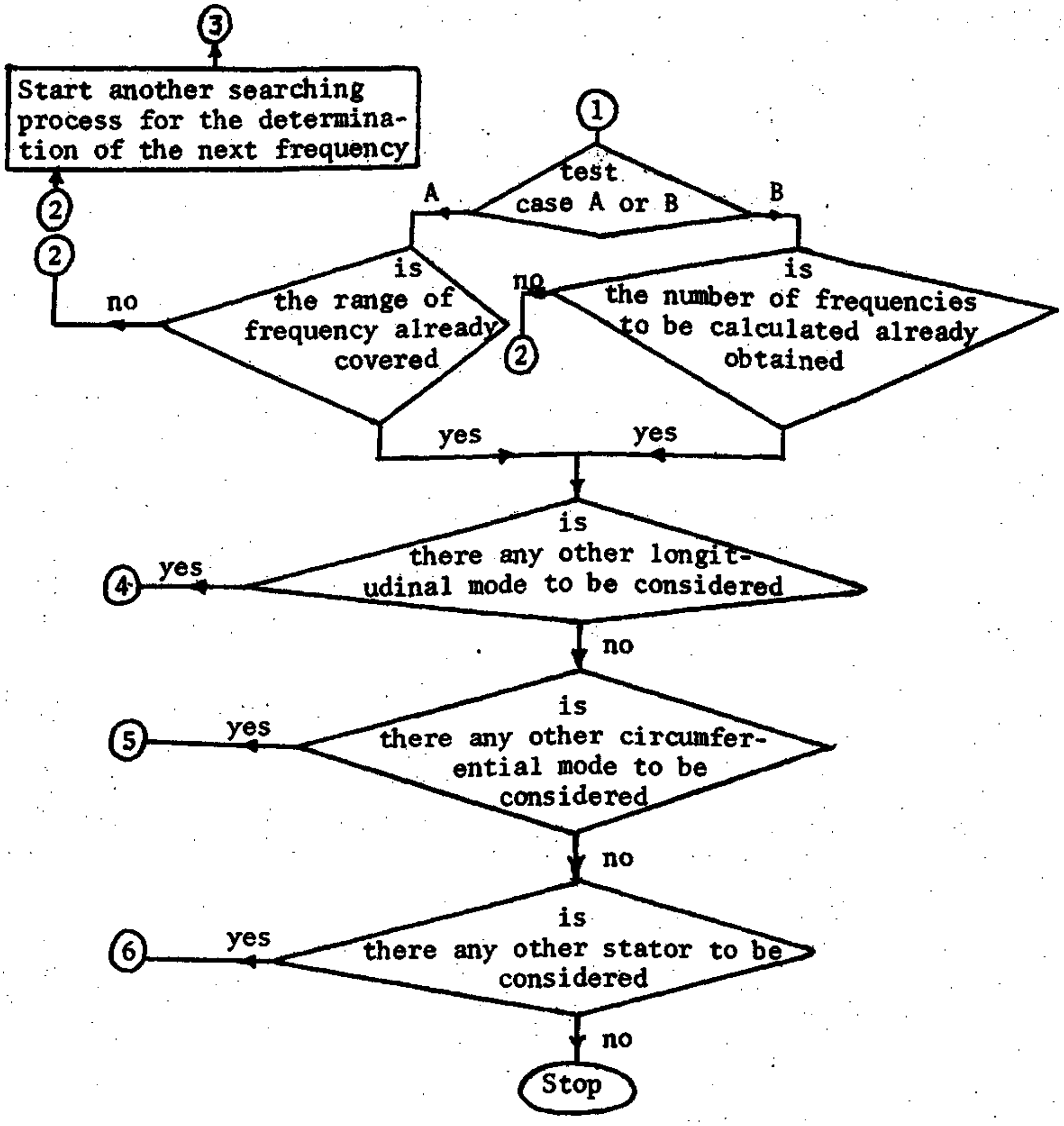
A computer programme was developed using a standard computer subroutine for the evaluation of determinants. In contrast to the earlier graphical method[54] of determining the zeros of the frequency determinant, by plotting its values versus frequency, the computer is ordered to do this searching process without plotting and to deliver the required values of the frequency with an accuracy of about 0.1%.

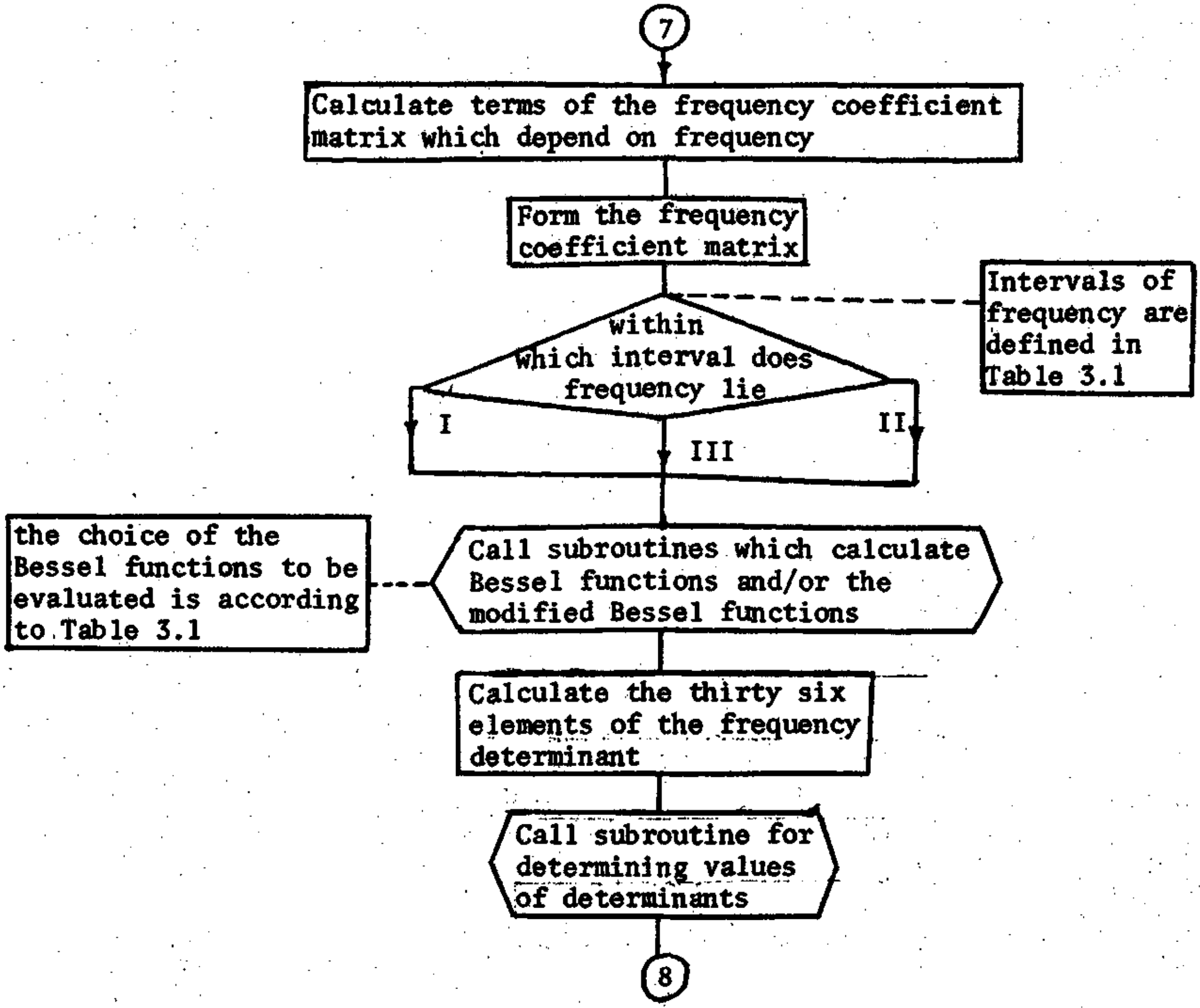
First of all, the value of the determinant is evaluated at a prescribed starting point and at intervals of specified width thereafter, up to and including a prescribed end point. A change of sign of the determinant across a certain frequency interval indicates a root in that interval. The interval of calculation is then made much smaller and the direction of scanning is reversed. The process is repeated until a root is obtained within a prescribed accuracy. Additional roots are obtained by re-entering successively into the iteration subroutine with a new starting point closely beyond the root obtained just previously until the entire region of calculations is covered or until a prescribed number of roots is obtained.

It is worthwhile to mention that switching from the Bessel functions J and Y to the modified Bessel functions I and K across the boundary values of $\omega = c_1 \frac{m\pi}{L}$ and $\omega = c_2 \frac{m\pi}{L}$, according to Table 3.1, will give rise to spurious roots[45]. This simply means that undesirable change of sign has been registered across these lines during the iteration process.

The flow chart of the computer programme prepared for the calculation of the resonance frequencies is given in the following pages. The symbols used in the flow chart are consistent with those developed by the ASAX3.6 committee on flow chart symbols for information processing[61,62].


Start
6
Read material and geometric Parameters of the stator
Read data which define the searching process
start and end points and interval of calculation
Calculate the terms of the frequency coefficient matrix which do not depend on frequency and mode shape under consideration
5
Declaration of the circumferential mode under consideration
Calculate the terms of the frequency coefficient matrix which do not depend on frequency and longitudinal mode shape under consideration
4
Declaration of the longitudinal mode under consideration
Calculate the terms of the frequency coefficient matrix which do not depend on the frequency
3
Starting point of the searching process
7
Consider next value of frequency
Calculate the value of the frequency determinant
expanded for explanation in page(104)
8
Write the values of frequency and frequency determinant
yes
is it the starting point of the searching process
no
no
is the value of the frequency determinant different in sign from the previous value
yes
Reverse the direction of scanning and start another searching process using smaller interval of calculation
no
is the interval of calculation within the accuracy required
yes
Print the resonance frequency
1


3
Start another searching process for the determination of the next frequency
1
test case A or B
A
B
2
2
no
is the range of frequency already covered
no
2
is the number of frequencies to be calculated already obtained
yes
yes
4
yes
is there any other longitudinal mode to be considered
no
5
yes
is there any other circumferential mode to be considered
no
6
yes
is there any other stator to be considered
no
Stop


7
Calculate terms of the frequency coefficient matrix which depend on frequency
Form the frequency coefficient matrix
Intervals of frequency are defined in Table 3.1
within which interval does frequency lie
I
II
III
the choice of the Bessel functions to be evaluated is according to Table 3.1
Call subroutines which calculate Bessel functions and/or the modified Bessel functions
Calculate the thirty six elements of the frequency determinant
Call subroutine for determining values of determinants
8

Appendix C. Derivation of the Frequency Equations for the Special Cases

a) The case m=0

When m=0, the longitudinal wave-number M vanishes. Under such a condition, certain elements of the matrix [T], equation (9.1), vanish and accordingly certain elements of the matrix [P], equation (9.3), will vanish which reduces the matrix [P] into two subdeterminants, $[P_1]$ and $[P_2]$, such that

$$|P_1| \cdot |P_2| = 0 \quad , \qquad \ldots. \; (9.6)$$

where;

$$[P_1] = \begin{vmatrix} P_{11} & P_{12} & P_{15} & P_{16} \\ P_{21} & P_{22} & P_{25} & P_{26} \\ P_{41} & P_{42} & P_{45} & P_{46} \\ P_{51} & P_{52} & P_{55} & P_{56} \end{vmatrix} \qquad \ldots. \; (9.7)$$

and

$$[P_2] = \begin{vmatrix} P_{33} & P_{34} \\ P_{63} & P_{64} \end{vmatrix} \qquad \ldots. \; (9.8)$$

The elements P_{ij} in equations (9.7) and (9.8) can be evaluated from equations (9.4) by putting M=0. For this case, of course, only interval-I of Table 3.1 will be required in the calculations.

b) The case of n=0

Similarly, for this case, the frequency equation becomes:

$$|P_3| \cdot |P_4| = 0 \qquad \ldots. \; (9.9)$$

where,

$$[P_3] = \begin{vmatrix} P_{11} & P_{12} & P_{13} & P_{14} \\ P_{31} & P_{32} & P_{33} & P_{34} \\ P_{41} & P_{42} & P_{43} & P_{44} \\ P_{61} & P_{62} & P_{63} & P_{64} \end{vmatrix} \qquad \text{.... (9.10)}$$

$$\text{and } [P_4] = \begin{vmatrix} P_{25} & P_{26} \\ P_{55} & P_{56} \end{vmatrix} . \qquad \text{.... (9.11)}$$

The elements P_{ij} in the above equations are to be obtained from equations (9.4) by substituting n=0.

c) The case of m=0 and n=0

In this particular case, the frequency equation reduces to the form:

$$|P_2| \cdot |P_4| \cdot |P_5| = 0 \qquad \text{.... (9.12)}$$

where;

$$[P_5] = \begin{vmatrix} P_{11} & P_{12} \\ P_{41} & P_{42} \end{vmatrix} \qquad \text{.... (9.13)}$$

Where the elements P_{ij} of matrices $[P_2]$, $[P_4]$ and $[P_5]$ are to be obtained from equations (9.4) by substituting a zero value for m and n.

Appendix D. Derivation of the Factor for Taking the Effects of Teeth, Windings and Cooling Ribs into Account

According to reference (16) and (17), the resonance frequency f_n of a single-ring stator, Fig. 9.1, for mode n can be calculated from the following formula:

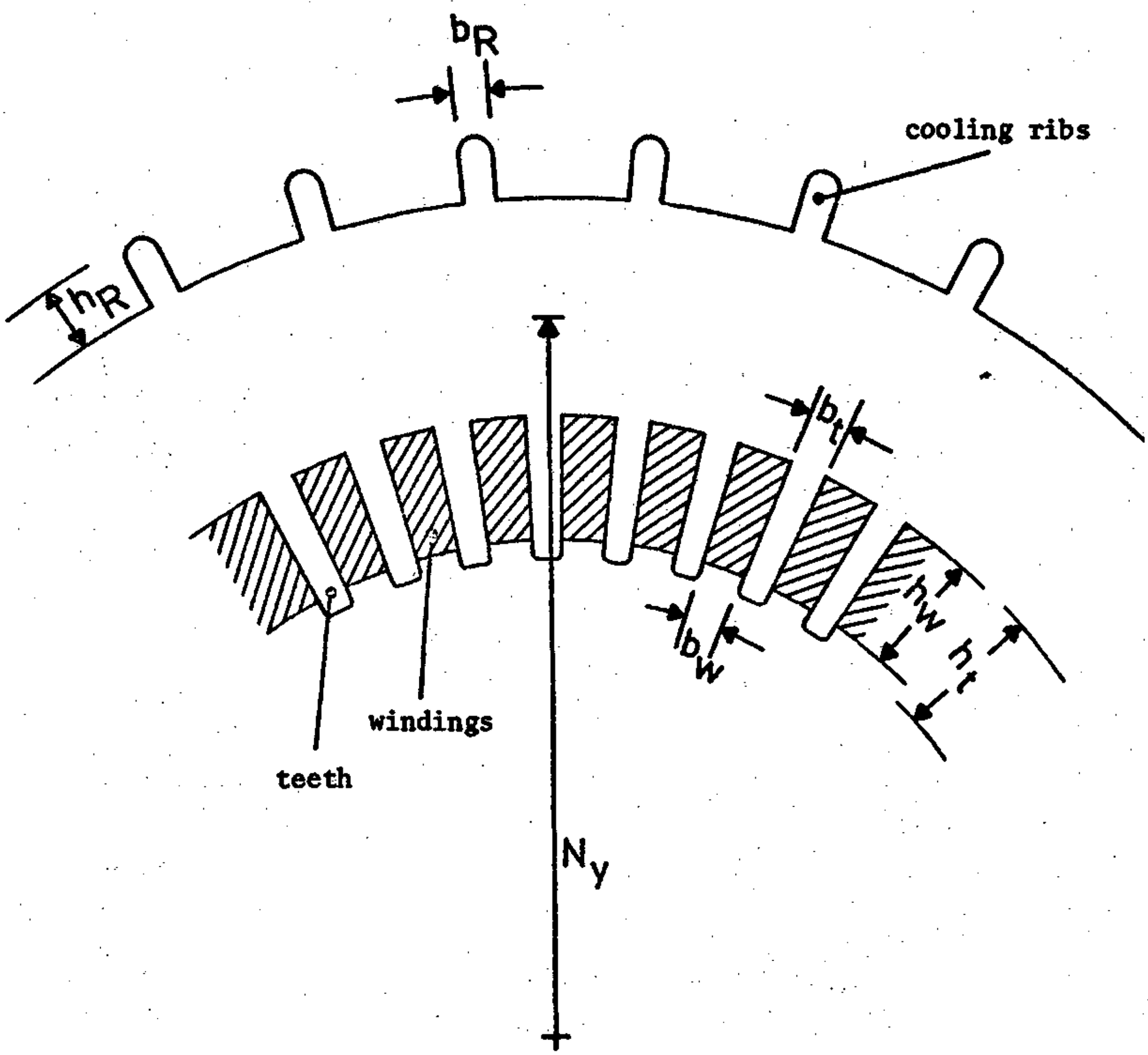


Fig. 9.1 Details of the stator studied in reference (17)

$$f_n = \frac{C_y I_y \, n(n^2-1)}{N_y \sqrt{n^2 \Delta_r + \Delta_\theta}} \cdot \frac{1}{\sqrt{1 + I_y^2 \dfrac{(n^2-1)\,[n^2(\Delta_{m/\Delta_r} + \Delta_{\theta/\Delta_r} + 3) + 3\Delta_{\theta/\Delta_r}]}{(n^2 + \Delta_{\theta/\Delta_r})}}} \qquad \text{.... (9.14)}$$

where;

Δ_r = factor representing the effect of normal forces produced by teeth, windings and cooling ribs.

$$= 1 + \frac{1}{W_y}(W_t \eta_{r_t} + W_R \eta_{r_R} + W_w \eta_{r_w}) \quad ,$$

Δ_θ = factor representing the effect of shear forces produced by teeth, windings, and cooling ribs.

$$= 1 + \frac{1}{W_y}(W_t \eta_{\theta_\tau} + W_R \eta_{\theta_R} + W_w \eta_{\theta_w}) \quad ,$$

Δ_m = factor representing the effect of bending moments produced by teeth, windings, and cooling ribs.

$$= 1 + \frac{1}{W_y}(W_t \beta_t \eta_{m_t} + W_w \beta_w \eta_{m_w} + W_R \beta_R \eta_{m_R}) \quad ,$$

W_i = weight ,

$$\beta_i = 4\left(\frac{h_i}{h_y}\right)^2 + 6\left(\frac{h_i}{h_y}\right) + 3 \quad ,$$

$$\eta_{r_i} = \frac{\tan(\Omega_i h_i)}{\Omega_i h_i} \quad ,$$

$$\eta_{\theta_i} = \frac{\gamma_i c_i + \mathcal{L}_i s_i}{\bar{\Omega}_i h_i (\mathcal{L}_i c_i + 1)} \quad ,$$

$$\eta_{m_i} = \frac{\left[\dfrac{s_i \mathcal{L}_i - \gamma_i c_i}{(\bar{\Omega}_i h_i)^3}\right] + \dfrac{h_y}{h_i}\left[\dfrac{\gamma_i s_i}{(\bar{\Omega}_i h_i)^2}\right] + \left(\dfrac{h_y}{2h_i}\right)^2 \left[\dfrac{\gamma_i c_i + s_i \mathcal{L}_i}{(\bar{\Omega}_i h_i)^2}\right]}{\left[\dfrac{1}{3} + \dfrac{h_y}{2h_i} + \left(\dfrac{h_y}{2h_i}\right)^2\right](\mathcal{L}_i c_i + 1)} \quad ,$$

$$\Omega_i = \sqrt{\frac{\rho_i}{E_i}}\,\omega_n \quad , \qquad \bar{\Omega}_i = \left\{\frac{12\rho_i}{E_i b_i^2}\omega_n^2\right\}^{1/4} \quad ,$$

$s_i = \sin(\overline{\Omega}_i h_i)$, $c_i = \cos(\overline{\Omega}_i h_i)$,

$\gamma_i = \sinh(\overline{\Omega}_i h_i)$, $\ell_i = \cosh(\overline{\Omega}_i h_i)$,

$c_y = \sqrt{\dfrac{E_y}{\rho_y}}$, $I_y = \dfrac{1}{2\sqrt{3}} \dfrac{h_y}{N_y}$,

N_y = mean radius of the stator-yoke, Fig. 9.1,

h_i = height in the radial direction, Fig. 9.1,

b_i = width in the tangential direction, Fig. 9.1,

E_i = modulus of elasticity.

ρ_i = density.

i = suffix refers to stator-yoke (y) or teeth (t) or windings (w) or cooling ribs (R),

and $\omega_n = 2\pi f_n$.

In the case when the teeth, windings and cooling ribs are absent, the resonance frequency of the uniform stator-yoke can be calculated from equation (9.14) by substituting Δ_r=1, Δ_θ=1 and Δ_m=1. Accordingly, equation (9.14) will reduce to the form:

$\overset{*}{f}_n$ = resonance frequency of the uniform stator-yoke.

$$= \frac{C_y I_y \, n(n^2-1)}{N_y\sqrt{n^2+1}} \cdot \frac{1}{\sqrt{1+I_y^2 \dfrac{(n^2-1)[5n^2+3]}{(n^2+1)}}} \qquad \ldots. (9.15)$$

From equations (9.14) and (9.15), the factor which takes into account the effects of teeth, windings and cooling ribs in the calculation of the resonance frequency of the stator will have the following form:

$$\alpha = \sqrt{\frac{n^2+1+I_y(n^2-1)(5n^2+3)}{n^2\Delta_r+\Delta_\theta+I_y^2(n^2-1)(n^2(\Delta_m+\Delta_\theta+3\Delta_r)+3\Delta_\theta)}} \qquad \text{.... (9.16)}$$