

# **A Quantitative Study of Alliance Structures in the Warring States of Ancient China, 453-221 B.C.**

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## ABSTRACT

This study makes a unique contribution to applied game theory and to the studies of *Shiji* (*Records of Grand Historian*) and the Warring States Period of ancient China (453-221B.C.) by constructing and analyzing the annual series of alliance structures or partitions of the seven states during the period of two hundred thirty three years. It shows that twenty six of the eight hundred seventy seven possible partitions were observed, and that the three most frequent partitions were the finest partition (146 years), partitions with four singletons and one three-member coalition (63 years), and partitions with five singletons and one two-member coalition (33 years). Such quantitative results have future applications in alliance studies, game theory, and international economics. They also provide a list of future research topics such as the unknown statistical properties of a series of partition of seven elements with 233 observations.

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## 1. INTRODUCTION

At the beginning of the Warring States period of ancient China (453-221B.C.), there were seven powerful states or regimes whose rulers were in contention to conquer one another and eventually unify the country<sup>1</sup>. These seven states were Qin, Qi, Chu, Yan, Han, Wei, Zhao, which were, as shown in the figure below, geographically located as Qin in the west, Qi in the east, Chu in the south, Yan in the north, and Han-Wei-Zhao in the central region, where Han was roughly in the south part, Wei in the middle part, and Zhao in the north part of the central region.



**Figure 1: The Location of the Seven Powerful States**

During the 233 year period, almost all states, including those small states whose rulers were non-contenders, needed to develop alliances to achieve their individual goals. I define an alliance as a group of states that was created for its members' mutual benefits or the interests of

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<sup>1</sup> For overviews or comments about the Warring States Period, see Crump (1970), Lewis (1999), Loewe and Shaughnessy (1999), Needham (1954), Ting (1985), Yang (1984, 1985), and Zhang et al (1987). For recent studies, see Chao (2013), Schaberg (2005), and Yang (2001, 2003).



an individual member. Most of the alliances' goals were centered on eliminating current or future external territorial threats. I also classify all alliances during this period as defensive or offensive or both. The defensive or block-oriented alliance was developed mainly to defend against attacks by invaders and did not have the ability to invade back into an opponent's territory. A large alliance or an alliance with several others was not necessary for preventing attacks as long as a smaller military alliance could successfully defend its member states against enemies. Additionally, an offensive or attack-oriented alliance was formed to actively attack another regime to carve up the occupied land into spheres of influence. The involvement of a large number of states augmented the probability that the offensive alliance could win the war, but sometimes weakened its combat capability or even made it collapse if its members acted uncooperatively or their actions were poorly coordinated.

When facing a situation in which weaker states could help each other deal with invasions, powerful states would employ either military action or diplomatic pressure to prevent the formation of a defensive alliance or to compel at least some of the states to abandon their intended objectives. That is to say, some of the weaker states were forced to either align themselves militarily with a powerful one or to take no action and remain neutral in conflicts. If weaker states didn't capitulate to the demands of the powerful state, it often created a risk of being beaten in the field of war or being left out a stronger alliances headed by the powerful states. There was the possibility that some states could team up with a powerful state; either to derive benefits from the conflict or to compete against a common enemy.

Our main task here is to construct the annual series of alliance structures or partitions of the seven states for each of the two hundred thirty three years, which was reported in Tables 3-6 in Chapter 2. Our main source for constructing the tables is Appendix III: Chronological Table of

Six States, 476-207 B.C. (Liuguo Nianbiao) in *Shiji (Records of Grand Historian)* by Qian Sima (145 or 135 - 86 B.C.), acclaimed the greatest historian in Chinese history, which was available online. While the alliance structure was uniquely defined throughout most of the 233 years, there were many years in which the alliance structure changed once, twice, or more times. When two or more partitions of the seven states can be observed in one year, I record the alliance structure by the end of the year, marked by \* or by \* in Tables 3-6. The details about such changes were discussed in the next chapter.

Our first main result is that only twenty six of the eight hundred seventy seven possible partitions<sup>2</sup> were observed. These twenty six partitions were defined in Tables 1 and 2 in Chapter 2, which were arranged by the order of their first observation. Column 4 of the tables records the frequency or number of observations for each of the twenty six partitions. Note that the sum of such observations exceeds the number of years 233, because in some years I can observe two or more alliance structures.

Our second main result is that the three most frequent observable partitions are: 1) the finest partition with 146 observations, 2) partitions with four singletons and one three-member coalition with 63 observations (this case had five configurations), and 3) partitions with five singletons and one two-member coalition with 33 observations (this case had seven configurations).

Our quantitative results have future applications in alliance studies, international relations and game theory. They also provide a list of other future research topics such as the unknown statistical properties of our series of partition of seven elements with 233 observations.

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<sup>2</sup> This was the Bell number or the number of partitions of  $n$  subjects. For  $n = 1, \dots, 10$ , the Bell numbers were: 1, 2, 5, 15, 52, 203, 877, 4140, 21147, and 115975.

The rest of the paper was organized as follows. Chapter 2 constructs the annual series of alliance structures and provides quantitative analysis, Chapter 3 provides detailed accounts of main historical events in the period, Chapter 4 discusses the turning points or main changes in alliance structures, and Chapter 5 provides conclusions. Tables and figures were provided in Chapter 1 and in the ends of Chapters 2 and 4.

## 2. QUANTITATIVE ANALYSES

For simplicity, let  $N = \{Q_n, Q_i, C, Y, H, W, Z\} = \{Qin, Qi, Chu, Yan, Han, Wei, Zhao\}$  denote the set of our seven states.<sup>3</sup> Let an alliance structure or a partition of  $N$  be denoted by  $\rho = \{S_1, S_2, \dots, S_t\}$ , where  $t$  is an integer between 1 and 7, each  $S_i \subseteq N$  is a subset of  $N$ , called a coalition or alliance, satisfying  $S_i \cap S_j = \emptyset$  for all  $i \neq j$  and  $\bigcup_{i=1}^t S_i = N$ . Two special types of alliance structures are the single-member alliance structure (or finest partition) and the grand-coalition alliance structure (or the coarsest partition), denoted by  $\rho^0 = \{Q_n, Q_i, C, Y, H, W, Z\}$  and  $\rho^* = \{N\}$ , respectively. The single-member structure  $\rho^0$  is made up of seven unilateral alliances or seven singletons, and the grand-coalition structure  $\rho^*$  contains only the grand coalition or one unified state called China. Here and in the rest of the paper, I further simplify coalitions  $(H, W, Z)$  as  $HWZ$ ,  $(Q_i)$  as  $Q_i$ . Similar simplifications hold for all other coalitions.

I am now ready to construct the sequence of alliance structures during the Warring States Period, denoted by  $\rho_m$  for  $m = 453 \text{ B.C.}, 452 \text{ B.C.}, \dots, 221 \text{ B.C.}$  Later on, I analyze how the alliance structure changed from  $\rho_{453 \text{ B.C.}} = \rho^0$  to  $\rho_{221 \text{ B.C.}} = \rho^*$ .

An offensive alliance was established in 453 B.C. among three noble Jin families (namely, Han, Wei, and Zhao) as a mechanism to protect against extinction from each of them. Eventually, they won conflicts and pledged not to fight wars with one another. This alliance was obviously a winning alliance. Later that year, the three families divided their spheres of influence within the Jin State and the alliance dissolved. So  $\rho_{453 \text{ B.C.}}^{\text{Initial}} = \rho^1 = \{S_1, S_2, S_3, S_4, S_5\} \rightarrow \rho_{453 \text{ B.C.}}^{\text{Final}} = \rho^0$ , where  $S_1 = HWZ$ ,  $S_2 = C$ ,  $S_3 = Qi$ ,  $S_4 = Qn$  and  $S_5 = Y$ .

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<sup>3</sup> I adopt the same order introduced earlier for the finest partition. For all other partitions reported in the rest of the paper, I list the members of each proper coalition alphabetically.

Then, during the next thirty-three years (452-420 B.C.), no offensive alliances were created, largely due to the regimes' lack of military buildups, and the alliance structures remained the same. In other words,  $\rho_m = \rho^0$  for  $m = 452 \text{ B.C.}, \dots, 420 \text{ B.C.}$

In the period between 419 and 405 B.C., the Qi, the Han, and the Wei States each had armed conflicts with one or more other regimes. Almost all of the intentions of the invaders were to expand their borders by acquiring some land in part of the targeted regimes. Therefore, there was no need for either regime to create a military alliance (defensive or offensive). That is to say,  $\rho_m = \rho^0$  for  $m = 419, \dots, 405 \text{ B.C.}$ <sup>4</sup>

In 404, 400, and 391 B.C., respectively, the three noble Jin families worked together to improve the rankings of their heads and to remove threats posed by the Chu State. Meanwhile, the four other states, Qin, Qi, Chu and Yan did not had any kind of military alliance, whether it was attack or block oriented. Thereupon,  $\rho_m^{\text{Initial}} = \rho^0 \rightarrow \rho_m^{\text{Final}} = \rho^1$  for  $m = 404, 400$  and  $391 \text{ B.C.}$  However, in each of three following years, alliance structure switched back from  $\rho^1$  to  $\rho^0$ , or precisely,  $\rho_m^{\text{Initial}} = \rho^1 \rightarrow \rho_m^{\text{Final}} = \rho^0$  for  $m = 403, 399$  and  $390 \text{ B.C.}$  This was due to the fact that the alliance ended after fulfilling its stated objective. And, no doubt, the alliance (attack-oriented) between the three families was a winning alliance.

During 389-367 B.C., no military alliances existed between states because there were no large-scale hostilities between any of them. Then  $\rho_m = \rho^0$  for  $m = 389, \dots, 367 \text{ B.C.}$  But the Zhao State incurred the enmity of the Wei State by invading the small state Wey (衛國), a vassal of the Wei State in 383 B.C. Hence, the Wei State abandoned the promise made to the Zhao State and provided direct military assistance to the Wey State. Furthermore, in 370 B.C., the Han and Zhao States intervened in the internal conflict of the Wei State aiming to split it into two

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<sup>4</sup> This is shown in Column 3 of Table 3, where the number of alliances remains a constant 7 until 405 BC.

parts, but they failed to accomplish this goal. In consequence, a seed of hatred was planted in the Wei State. Later the Wei State was in a disadvantageous situation because its capital city, Anyi, faced potential military threats from the Qin, the Han, and the Zhao States. To prevent invasions by multiple opponents, the Wei State formed a military alliance with the Han State (both offensive and defensive) in 366 B.C. Other states, including Qin, Qi, Chu, Yan and Zhao, did not align themselves with each other militarily. But later that year, the alliance lost the fight against the Qin State, and was soon disbanded because the Wei State chose to move its capital to Daliang (modern-day Kaifeng City in Henan Province) and to improve its strength through annexations of other states' territory (which did not include the territory of the Qin State). Hence,  $\rho_{366 \text{ B.C.}}^{\text{Initial}} = \rho^0 \rightarrow \rho^2 = \{S_1, S_2, S_3, S_4, S_5, S_6\} \rightarrow \rho_{366 \text{ B.C.}}^{\text{Final}} = \rho^0$ , where  $S_1 = \text{HW}$ ,  $S_2 = \text{C}$ ,  $S_3 = \text{Qi}$ ,  $S_4 = \text{Qn}$ ,  $S_5 = \text{Y}$  and  $S_6 = \text{Z}$ .

In 362 B.C., due to the possible security threat the Wei State posed to the capital city of Handan, the Zhao State allied itself with the Han State to resist all possible future attacks. Meanwhile, the remaining five states, Qin, Qi, Chu, Yan and Wei, did not engage in any type of bilateral, trilateral, quadrilateral or multilateral alliance activities. Therefore,  $\rho_{362 \text{ B.C.}}^{\text{Initial}} = \rho^0 \rightarrow \rho_{362 \text{ B.C.}}^{\text{Final}} = \rho^3 = \{S_1, S_2, S_3, S_4, S_5, S_6\}$ , where  $S_1 = \text{HZ}$ ,  $S_2 = \text{C}$ ,  $S_3 = \text{Qi}$ ,  $S_4 = \text{Qn}$ ,  $S_5 = \text{W}$  and  $S_6 = \text{Y}$ . The Zhao-Han alliance (both offensive and defensive) only lasted until 357 B.C., after which the Wei State coerced the Han State into capitulating to its demand by force. Similarly, only one military alliance was formed this year and thus  $\rho_{357 \text{ B.C.}}^{\text{Initial}} = \rho^3 \rightarrow \rho_{357 \text{ B.C.}}^{\text{Final}} = \rho^2$ . Six years later, in 351 B.C., a military alliance agreement was reached between the Wei and Zhao States that led the Wei State to halt a punitive expedition against the Zhao State's second military invasion of the Wei State. As a result, the Wei-led alliance expanded from two to three members. In that year, no other military alliance existed, so  $\rho_{351 \text{ B.C.}}^{\text{Initial}} = \rho^2 \rightarrow \rho_{351 \text{ B.C.}}^{\text{Final}} = \rho^1$ .

Later in 344 B.C., Marquis Hui declared himself King of Wei, which made the Han and Zhao States so furious that they both terminated their alliance with the Wei State. After that point,  $\rho_{344 \text{ B.C.}}^{\text{Initial}} = \rho^1 \rightarrow \rho_{344 \text{ B.C.}}^{\text{Final}} = \rho^0$ . In the period between 343 and 340 B.C., the Wei State had suffered losses of fighting invaders, including being soundly defeated by the Qin State in the region east of the Yellow River and by the Qi State at Maling. To prevent future attacks from the Qin State, the Wei State initially aligned itself with the Han State and then used the flattery strategy to establish a military alliance with the Qi State in 334 B.C. There were no other two-member, three-member, or multi-member alliances. So  $\rho_{334 \text{ B.C.}}^{\text{Initial}} = \rho^0 \rightarrow \rho_{334 \text{ B.C.}}^{\text{Final}} = \rho^4 = \{S_1, S_2, S_3, S_4, S_5\}$ , where  $S_1 = \text{HQiW}$ ,  $S_2 = \text{C}$ ,  $S_3 = \text{Qn}$ ,  $S_4 = \text{Y}$  and  $S_5 = \text{Z}$ .

The Qi-Han-Wei alliance (both offensive and defensive) lasted for nearly twelve years, until 323 B.C. At that time, the Qin State aligned itself actively with the Qi and Chu States (an offensive alliance) to forestall potential future Qi-Chu-Han-Wei invasions. In the face of such overt Qin action, the Wei State soon formed a multilateral defensive alliance in dealing with the threat to survive in the near future. Thus,  $\rho_{323 \text{ B.C.}}^{\text{Initial}} = \rho^4 \rightarrow \rho^5 = \{S_1, S_2, S_3, S_4\} \rightarrow \rho^6 = \{S_1^*, S_2^*\} \rightarrow \rho^0 = \rho_{323 \text{ B.C.}}^{\text{Final}}$ , where  $S_1 = \text{CQiQn}$ ,  $S_2 = \text{HW}$ ,  $S_3 = \text{Y}$ ,  $S_4 = \text{Z}$ ,  $S_1^* = S_1$ , and  $S_2^* = \text{HWYZ}$ .

The existence of the four-state alliance had no deterrent effect on the Qin or the Chu State. Still, they made wars on the Wei State. The accumulated war losses prompted the Wei State to better fulfill its mission by building a trilateral alliance (both offensive and defensive) with the Qin and Han States in 322 B.C. As a result,  $\rho_{322 \text{ B.C.}}^{\text{Initial}} = \rho^0 \rightarrow \rho_{322 \text{ B.C.}}^{\text{Final}} = \rho^7 = \{S_1, S_2, S_3, S_4, S_5\}$ , where  $S_1 = \text{HQnW}$ ,  $S_2 = \text{C}$ ,  $S_3 = \text{Qi}$ ,  $S_4 = \text{Y}$ , and  $S_5 = \text{Z}$ . However, both the Wei and Han States' acquiescence to the Qin State's operation posed an immediate danger to the territory of Qi and constituted a future threat against the Chu State's territory. Hence, under pressure from both

states, the Wei and Han States stayed away from the alliance few years later in 319 B.C.

Accordingly,  $\rho_{319\text{B.C.}}^{\text{Initial}} = \rho^7 \rightarrow \rho_{319\text{B.C.}}^{\text{Final}} = \rho^0$ .

In the next year, the Chu, Yan, Han, Wei and Zhao States formed a military alliance of five states (attack-oriented) and launched a military operation against the Qin State. Due to uncooperative actions of the Chu and Yan armies, the alliance failed to win the war. This was, no doubt, a losing alliance. Hence,  $\rho_{318\text{B.C.}}^{\text{Initial}} = \rho^0 \rightarrow \rho^8 = \{S_1^*, S_2^*, S_3^*\} \rightarrow \rho_{318\text{B.C.}}^{\text{Final}} = \rho^1$ , where  $S_1^* = \text{CHWYZ}$ ,  $S_2^* = \text{Qi}$ , and  $S_3^* = \text{Qn}$ . After the war, the defeated Wei State tried to win over the Qi State as an ally for the later conflicts. By 316 B.C., a military alliance (both offensive and defensive) between the Qi, Han, and Wei States was formed against the Qin State by Yan Gongsun, so  $\rho_{316\text{B.C.}}^{\text{Initial}} = \rho^1 \rightarrow \rho_{316\text{B.C.}}^{\text{Final}} = \rho^4$ . Soon after, in 315 B.C., King of Han changed his mind and attempted to make an alliance with the Qin State to avoid a costly war and to rely on the Qin military to weaken the Chu State. However, this movement was eventually stopped by countermeasure developed by the Chu courtier Zen Chen. Thereupon,  $\rho_{315\text{B.C.}}^{\text{Initial}} = \rho^4 \rightarrow \rho_{315\text{B.C.}}^{\text{Final}} = \rho^9 = \{S_1, S_2, S_3, S_4, S_5, S_6\}$ , where  $S_1 = \text{QiW}$ ,  $S_2 = \text{C}$ ,  $S_3 = \text{H}$ ,  $S_4 = \text{Qn}$ ,  $S_5 = \text{Y}$  and  $S_6 = \text{Z}$ .

Qin's military victories, in 314 B.C., reflected the greater strength of Qin's army and unwillingness of Qi to take costly actions in helping the Wei and Han States. At first the Han State became Qin's only military ally, so  $\rho_{314\text{B.C.}}^{\text{Initial}} = \rho^9 \rightarrow \rho_{314\text{B.C.}}^{\text{Final}} = \rho^{10} = \{S_1, S_2, S_3, S_4, S_5\}$ , where  $S_1 = \text{QiW}$ ,  $S_2 = \text{HQn}$ ,  $S_3 = \text{C}$ ,  $S_4 = \text{Y}$  and  $S_5 = \text{Z}$ . Then in 313 B.C., the Wei State became another Qin ally. And later in the same year, another military alliance (both offensive and defensive) was formed between the Qi and Chu States to forestall potential future Qin-led invasion of the Chu State. The Qi-Chu alliance lasted only a short time, and its collapse as a



result of disruption strategy by Yi Zhang. Thereby  $\rho_{313 \text{ B.C.}}^{\text{Initial}} = \rho^{10} \rightarrow \rho^7 \rightarrow \rho^{11} = \{S_1, S_2, S_3, S_4\} \rightarrow \rho_{313 \text{ B.C.}}^{\text{Final}} = \rho^7$ , where  $S_1 = \text{CQi}$ ,  $S_2 = \text{HQnW}$ ,  $S_3 = \text{Y}$  and  $S_4 = \text{Z}$ .

The Qin State changed its diplomatic practices toward the Wei and Han States in 310 B.C., the year that the new king, King Wu of Qin<sup>5</sup> (the eldest son of King Hui) chose to expand territorial sphere of influence into the central plains. Hence,  $\rho_{310 \text{ B.C.}}^{\text{Initial}} = \rho^7 \rightarrow \rho_{310 \text{ B.C.}}^{\text{Final}} = \rho^0$ . Four years later in 306 B.C., the Chu State conditionally promised the Qi State to help create a multilateral military alliance (attack-oriented) that would defeat the Qin State. Therefore, initially,  $\rho_{306 \text{ B.C.}}^{\text{Initial}} = \rho^0 \rightarrow \rho_{306 \text{ B.C.}}^{\text{Final}} = \rho^{12} = \{S_1, S_2, S_3, S_4, S_5, S_6\}$ , where  $S_1 = \text{CQi}$ ,  $S_2 = \text{H}$ ,  $S_3 = \text{Qn}$ ,  $S_4 = \text{W}$ ,  $S_5 = \text{Y}$  and  $S_6 = \text{Z}$ . But, the Qi-Chu alliance lasted up until 304 B.C., when the Qin State won over the Chu State through a marriage alliance. At this point in time,  $\rho_{304 \text{ B.C.}}^{\text{Initial}} = \rho^{12} \rightarrow \rho_{304 \text{ B.C.}}^{\text{Final}} = \rho^{13} = \{S_1, S_2, S_3, S_4, S_5, S_6\}$ , where  $S_1 = \text{CQn}$ ,  $S_2 = \text{H}$ ,  $S_3 = \text{Qi}$ ,  $S_4 = \text{W}$ ,  $S_5 = \text{Y}$  and  $S_6 = \text{Z}$ .

Both the Wei and Han States suffered the loss of lands due to invasions from the Qin State in 303 B.C., and this led them to ally with the Qi State in order to resist future Qin military invasions. Thus  $\rho_{303 \text{ B.C.}}^{\text{Initial}} = \rho^{13} \rightarrow \rho_{303 \text{ B.C.}}^{\text{Final}} = \rho^{14} = \{S_1, S_2, S_3, S_4\}$ , where  $S_1 = \text{HQiW}$ ,  $S_2 = \text{CQn}$ ,  $S_3 = \text{Y}$  and  $S_4 = \text{Z}$ . Later, due to the military intervention by the Qin State, the Qi-led invasion of the Chu State failed. For this reason, the Qi-Han-Wei alliance (both offensive and defensive) could be considered as a losing alliance. The crown prince of the Chu State, a diplomatic hostage in the Qin State, absconded to his home state in 302 B.C., after killing the advisor to the Qin king. And yet Chu's unwillingness to send the crown prince back undermined the Qin-Chu alliance. Further, the Wei and Han States rallied to the side of the Qin State, hoping to benefit from participating in the Qin-led military actions (the Qin-Han-Wei alliance was offensively oriented).

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<sup>5</sup> He died in 307 B.C. and was succeeded in 305 B.C. by his younger brother who was crowned King Zhao of Qin.

Therefore,  $\rho_{302 \text{ B.C.}}^{\text{Initial}} = \rho^{14} \rightarrow \rho^4 \rightarrow \rho_{302 \text{ B.C.}}^{\text{Final}} = \rho^7$ . Interestingly, in 301 B.C. the Wei and Han States once again rallied around the Qi State against the Chu State. Then  $\rho_{301 \text{ B.C.}}^{\text{Initial}} = \rho^7 \rightarrow \rho_{301 \text{ B.C.}}^{\text{Final}} = \rho^4$ .

The Qi-Qin relations improved in 300 B.C. Sometimes after that, the Zhao State was aware of the possibility of future interventions by the Qin State in the small state of Zhongshan, the target of the Zhao State's offensive action. To eliminate this possibility, the Zhao State took an active step to fuel a military conflict between the Qin State and the Qi-led alliance. The Qin reaction to this unfavorable situation was the establishment of the Qin-Zhao alliance (both offensive and defensive). So  $\rho_{298 \text{ B.C.}}^{\text{Initial}} = \rho^4 \rightarrow \rho_{298 \text{ B.C.}}^{\text{Final}} = \rho^{15} = \{S_1, S_2, S_3, S_4\}$ , where  $S_1 = \text{HQiW}$ ,  $S_2 = \text{QnZ}$ ,  $S_3 = \text{C}$ , and  $S_4 = \text{Y}$ . The Qin-led alliance lasted only about three years, dissolving in 295 B.C. due to the Qin State's military defeat and the Zhao State's strategy for preventing the future acts of the Qi-led alliance. Thus,  $\rho_{295 \text{ B.C.}}^{\text{Initial}} = \rho^{15} \rightarrow \rho_{295 \text{ B.C.}}^{\text{Final}} = \rho^4$ . For the Qin State, the Qin-Zhao alliance was a losing alliance.

In 294 B.C., the Qi State terminated its military alliance with the Wei and Han States as a means of winning over the Qin State as its ally, and securing the acquiescence of the Qin State in its attacks on the Song State. Meanwhile, the Yan State aligned itself militarily with the Qi State in an effort to weaken the Qi State's strength in the war against the Song State. As a consequence,  $\rho_{294 \text{ B.C.}}^{\text{Initial}} = \rho^4 \rightarrow \rho_{294 \text{ B.C.}}^{\text{Final}} = \rho^{16} = \{S_1, S_2, S_3, S_4, S_5\}$ , where  $S_1 = \text{QiQnY}$ ,  $S_2 = \text{C}$ ,  $S_3 = \text{H}$ ,  $S_4 = \text{W}$  and  $S_5 = \text{Z}$ .

During the five years between 293 and 289 B.C., both the Wei and Han States suffered severe losses of territory to the Qin armies. To prevent more territorial losses, they sought protection from the Zhao State, whose military capability grew rapidly and would match Qi's capability. In addition, the Zhao State also relied on their military support to make its invasion of

Song less costly. In late 288 B.C., both the Qi and Qin States chose to undertake a joint military action against the Zhao State to break up the trilateral alliance (both offensive and defensive). Yet soon after Qin Su's exhortation, the Qi State chose to terminate its alliance with the Qin State, and later initiated an anti-Qin military alliance (attack-oriented) with the Yan, Han, Wei, and Zhao States. Accordingly  $\rho_{288 \text{ B.C.}}^{\text{Initial}} = \rho^{16} \rightarrow \rho^{17} = \{S_1, S_2, S_3\} \rightarrow \rho_{288 \text{ B.C.}}^{\text{Final}} = \rho^{18} = \{S_1^*, S_2^*, S_3^*\}$ , where  $S_1 = \text{QiQnY}$ ,  $S_2 = \text{HWZ}$  and  $S_3 = \text{C}$ , and  $S_1^* = \text{HQiWYZ}$ ,  $S_2^* = \text{C}$ , and  $S_3^* = \text{Qn}$ .

However, the five-state alliance was considered a losing alliance because the allied forces were disbanded while marching on the Qin State. This alliance ended in late 287 B.C., when once again the Qi State won over the Qin State as an ally. So therefore  $\rho_{287 \text{ B.C.}}^{\text{Initial}} = \rho^{18} \rightarrow \rho_{287 \text{ B.C.}}^{\text{Final}} = \rho^{17}$ . The Qi State's annexation of the Song State in 286 B.C. greatly enhanced the power of the state, but that change made itself the common enemy between the Qin and Zhao States. The following year in 285 B.C., the Qin State allied itself with the Zhao State to create a multilateral offensive alliance against the Qi State. Afterward  $\rho_{285 \text{ B.C.}}^{\text{Initial}} = \rho^{17} \rightarrow \rho_{285 \text{ B.C.}}^{\text{Final}} = \rho^{19} = \{S_1, S_2, S_3, S_4, S_5, S_6\}$ , where  $S_1 = \text{QnZ}$ ,  $S_2 = \text{C}$ ,  $S_3 = \text{Qi}$ ,  $S_4 = \text{H}$ ,  $S_5 = \text{W}$  and  $S_6 = \text{Y}$ .

Later in 284 B.C., another five-state alliance was created by the Qin, Yan, Han, Wei, and Zhao States, with the objective of weakening the Qi State's strength. This led to  $\rho_{284 \text{ B.C.}}^{\text{Initial}} = \rho^{19} \rightarrow \rho_{284 \text{ B.C.}}^{\text{Final}} = \rho^{20} = \{S_1^*, S_2^*, S_3^*\}$ , where  $S_1^* = \text{HQnWYZ}$ ,  $S_2^* = \text{C}$ , and  $S_3^* = \text{Qi}$ . The alliance's members cooperated well with each other and achieved their stated goal. The alliance was thus viewed as a winning alliance.

However, this successful five-state alliance was short lived. Next year, the Han State was forced into alliance with the Qin State to attack the Wei State, so  $\rho_{283 \text{ B.C.}}^{\text{Initial}} = \rho^{20} \rightarrow \rho_{283 \text{ B.C.}}^{\text{Final}} = \rho^{21} = \{S_1, S_2, S_3, S_4, S_5, S_6\}$ , where  $S_1 = \text{HQn}$ ,  $S_2 = \text{C}$ ,  $S_3 = \text{Qi}$ ,  $S_4 = \text{W}$ ,  $S_5 = \text{Y}$  and  $S_6 = \text{Z}$ . Later, the assistances of the Yan and Zhao militaries helped the Wei State prevent the Qin State from

seizing the capital city of Daliang and, based on these losses, the Qin-Han alliance was regarded as a losing alliance. It lasted until 276 B.C. when the Han State provided military supports for the Wei State. During the period from 279-277 B.C., no alliance formations among eastern vassals (i.e., those states that were located east of the Yao mountains and the Hangu pass), largely because the Qin State allowed the Zhao State to launch attacks on the Wei State in exchange for a promise not to invade its territory at any time during the Qin State's fight against the Chu State.

Thus,  $\rho_{276 \text{ B.C.}}^{\text{Initial}} = \rho^{21} \rightarrow \rho_{276 \text{ B.C.}}^{\text{Final}} = \rho^0$ .

Neither trilateral nor quadrilateral nor multilateral alliances were created during the period between 265 B.C. and 258 B.C., which was largely a result of the Qin State's friendly stances toward the distant states of Zhao and Yan.

Early in 257 B.C., the Zhao State took over of the territory ceded to the Qin State by the Han State. In retaliation, the Qin State attacked the Zhao State with strong and large-scale military actions. The Zhao State suffered a crushing defeat and reluctantly accepted an armistice. Later in the same year, the Qin State launched a second attack on the Zhao State, who refused to honor its obligations under the armistice (such as cession of lands). Such threat of extinction drove the Zhao State to form a military alliance with the Chu and Wei States, so  $\rho_{257 \text{ B.C.}}^{\text{Initial}} = \rho^0 \rightarrow \rho_{257 \text{ B.C.}}^{\text{Final}} = \rho^{23} = \{S_1, S_2, S_3, S_4, S_5\}$ , where  $S_1 = \text{CWZ}$ ,  $S_2 = \text{H}$ ,  $S_3 = \text{Qi}$ ,  $S_4 = \text{Qn}$  and  $S_5 = \text{Y}$ . Their victories over the Qin forces during 257-256 B.C. demonstrated that the trilateral defense alliance was a winning alliance.

This successful alliance collapsed in 254 B.C, mainly because Qin's military coercion caused the Wei State to capitulate to the demand of the Qin State, so  $\rho_{254 \text{ B.C.}}^{\text{Initial}} = \rho^{23} \rightarrow \rho_{254 \text{ B.C.}}^{\text{Final}} = \rho^0$ . Breaking up the trilateral alliance helped the Qin State achieve its goal of avoiding future conflicts with the possible five-state alliance by the Chu, Yan, Han, Wei and

Zhao States. The Qin State maintained this goal in the next four years by resting its forces. As a result, there was no alliance activity in the period, so  $\rho_m = \rho^0$  for  $m = 253, \dots, 250$  B.C. However, the Qin State's appetite for expansion grew so strong in 249 B.C. that it began to take aggressive actions against the Wei State. This forced the Wei State to seek military supports from the Chu, Yan, Han and Zhao States. In less than three years, the five-state alliance of the Chu, Yan, Han, Wei and Zhao States, first formed in 318 B.C., was formed again to counter security threat from the Qin State. Hence,  $\rho_{247 \text{ B.C.}}^{\text{Initial}} = \rho^0 \rightarrow \rho_{247 \text{ B.C.}}^{\text{Final}} = \rho^8$ . Late in 247 B.C., the allied armies pursued the invading Qin soldiers and repelled them back to the Hangu pass. And the five-state alliance (both offensive and defensive) was, therefore, regarded by the Wei State as a winning alliance.

The alliance partially collapsed the next year, after the Wei State's invasion and occupation of the Han territory of Guan (southeast of the Ying River) and its subsequent conflict with the Chu State over competition for the Qi territories. The Wei State's selfish actions were the root cause of the alliance's complete collapse in the following year, so  $\rho_{246 \text{ B.C.}}^{\text{Initial}} = \rho^8 \rightarrow \rho_{246 \text{ B.C.}}^{\text{Final}} = \rho^{24} = \{S_1, S_2, S_3, S_4\}$ , and  $\rho_{245 \text{ B.C.}}^{\text{Initial}} = \rho^{24} \rightarrow \rho_{245 \text{ B.C.}}^{\text{Final}} = \rho^0$ , where  $S_1 = \text{CWYZ}$ ,  $S_2 = \text{H}$ ,  $S_3 = \text{Qi}$ ,  $S_4 = \text{Qn}$ .

The Qin State nibbling away at the Wei and Han States for the next two years (244-243 B.C.) didn't pose major threat to their existence, and hence, no anti-Qin alliance was created between the eastern vassals. In other words,  $\rho_m = \rho^0$  for  $m = 244$  B.C. and  $m = 243$  B.C. Yet the Qin State's continuous occupation of the northwest territory of Wei during 242 B.C. would soon create a geographic isolation between the states of upper division, Yan and Zhao, and the states of central division, Wei and Han. Consequently, the five-state offensive alliance of the Chu, Yan, Han, Wei and Zhao States was formed for the third time in the next year, to counter

the Qin State's territorial ambitions. Hence,  $\rho_{241 \text{ B.C.}}^{\text{Initial}} = \rho^0 \rightarrow \rho_{241 \text{ B.C.}}^{\text{Final}} = \rho^8$ . However, the Qin State's repulsion of allied troops in late 241 B.C. turned the multilateral alliance into a losing alliance. In 240 B.C., the alliance dissolved owing in large part to the fact that the Chu State opted to quit to prevent more loss of military power, resulting in  $\rho_{240 \text{ B.C.}}^{\text{Initial}} = \rho^8 \rightarrow \rho_{240 \text{ B.C.}}^{\text{Final}} = \rho^0$ .

Two years later, in 238 B.C., the states of central division, Wei and Han were geographic isolated from the states of upper division, Yan and Zhao, proper by the Qin-occupied areas of the Wei State, making it impossible the formation of any future anti-Qin alliance (block or attack-oriented). Thus,  $\rho_{238 \text{ B.C.}} = \rho^0$ . In the next six years (237-232 B.C.), no bilateral alliances were formed among the Chu, Yan, Zhao and Wei States, with the aim of enhancing the state's strength through conquering either the Qi or the Han State, who were both protected by the Qin State. More precisely, the Qin State would take military actions against states that engaged in this type of alliance activity. Thus,  $\rho_m = \rho^0$  for  $m = 237, \dots, 232 \text{ B.C.}$

In the next ten years, the Qin State successively conquered the other six states, so the number of states decreased from seven in 231 B.C. to one in 221 B.C. Precisely,

$$\rho_{230 \text{ B.C.}}^{\text{Initial}} = \rho^0 \rightarrow \rho_{230 \text{ B.C.}}^{\text{Final}} = \rho^h = \{S_1, S_2, S_3, S_4, S_5, S_6\},$$

where  $S_1 = \text{Qn-H}$ ,  $S_2 = \text{C}$ ,  $S_3 = \text{Qi}$ ,  $S_4 = \text{W}$ ,  $S_5 = \text{Y}$  and  $S_6 = \text{Z}$ ;

$$\rho_{225 \text{ B.C.}}^{\text{Initial}} = \rho^h \rightarrow \rho_{225 \text{ B.C.}}^{\text{Final}} = \rho^{\text{wh}} = \{S_1, S_2, S_3, S_4, S_5\},$$

where  $S_1 = \text{Qn-W-H}$ ,  $S_2 = \text{C}$ ,  $S_3 = \text{Qi}$ ,  $S_4 = \text{Y}$  and  $S_5 = \text{Z}$ ;

$$\rho_{223 \text{ B.C.}}^{\text{Initial}} = \rho^{\text{wh}} \rightarrow \rho_{223 \text{ B.C.}}^{\text{Final}} = \rho^{\text{cwh}} = \{S_1, S_2, S_3, S_4\},$$

where  $S_1 = \text{Qn-C-W-H}$ ,  $S_2 = \text{Qi}$ ,  $S_3 = \text{Y}$  and  $S_4 = \text{Z}$ ;

$$\rho_{222 \text{ B.C.}}^{\text{Initial}} = \rho^{\text{cwh}} \rightarrow \rho_{222 \text{ B.C.}}^{\text{Final}} = \rho^{\text{cyzwh}} = \{S_1, S_2\},$$

where  $S_1 = \text{Qn-C-Y-Z-W-H}$   $S_2 = \text{Qi}$ ; and finally,

$$\rho_{221 \text{ B.C.}}^{\text{Initial}} = \rho^{\text{cyzwh}} \rightarrow \rho_{221 \text{ B.C.}}^{\text{Final}} = \rho^* = \{S^*\},$$

where  $S^* = \text{Qn-Qi-C-Y-Z-W-H}$  or the unified China.

The observational result of partitions was shown in Tables 1 and 2, which shows the frequencies of twenty-six partitions. The first thing to note was that the most frequently occurring partition was the finest partition, noted  $\rho^0$ , with 146 observations. Secondly, a partition that consists of four singletons and one three-member alliance was the second most frequent partition, with 63 observations. Furthermore, a partition characterized by five singletons and one two-member alliance was the third most commonly occurring partition, with 33 observations.

During the early period of Warring States, the Wei State played mostly a leadership role within its alliances, and these alliances were generally beneficial to all members. During the late period, the Wei State played mostly a subordinate role within its alliances. And, over the entire period, the Wei State participated more in the anti-Qin alliances than in the Qin-led alliances.

In the early period, the Han State played either a subordinate role or other non-leadership role within its alliances. Toward the end of this period, the Han State played mostly a subordinate role in its alliances. Similar to the Wei State, the Han State participated more in the anti-Qin alliances than in the Qin-led alliances over the entire period.

During the early and middle periods, the Qi State played either a leadership role or other non-subordinate roles within its alliances. In the late period, the Qi State did not involve in any military alliance largely because it became the target of foreign aggressions. Similar to the Han and Wei States, the Qi State also participated more in the anti-Qin alliances than in the Qin-led alliances over the entire period.

During the entire period, the Chu State played mostly an average role (i.e., either non-leadership or non-subordinate role) within its alliances; however, an exception occurred when the Chu State was invited by the Zhao State to play a leadership role within the military alliance against the Qin State. Similarly, the alliances in which the Chu State joined were mostly the anti-Qin alliances.

During the early and middle period, the Zhao State played mostly an average role within its alliances. During the late period, the roles played by the Zhao State depended heavily on its military power. It played mostly a leadership role within its alliances when its military power was stronger and mostly a subordinate role when its military power was weaker. Over the entire period, the Zhao State similarly participated more in the anti-Qin alliances than in the Qin-led alliances.

Over the entire period, the Yan State played mostly an average role within its alliances, with one exception at the end of middle period, when it played a subordinate role in the alliance with the Qi and Qin States, created by itself, because its goal was to wear down the Qi State's strength in the war against the Song State. During the entire period, the Yan State participated in alliance activities less frequently than the other six states and its alliances were also mostly the anti-Qin alliances.

The above results showed that the Qin State was seen as the most threatening to eastern vassals. Over the entire period, the Qin State played mostly both the leadership and average roles within the alliances it created and mostly the leadership roles within the alliances it joined. During the middle to mid-late period, its alliances were mostly the anti-Qi alliances and, in the late period, its alliances were mostly the anti-Wei alliances. This result indicated that the Qi State



was the major obstacle to hindering invasions of the Qin State, and that the Qin State wanted to geographically isolate eastern vassals by conquering the entire Wei State.

The above formations and disruptions of alliances were documented in Tables 3-6, which displays the annual series of alliance structures over the entire period. As mentioned earlier, the tables record the alliance structure at the end of each year. Those years in which there were at least two alliance structures or two configurations of the partition were marked by \* or \*\*, where the symbol \* denotes years in which the alliance structure changed twice (i.e., the year observed two partitions), and symbol \*\* denotes years in which the alliance structure changed at least twice (i.e., the year observed three or more partitions).

Finally, the mean and variance of proper alliances in the whole period were low because there were no alliance activities during 63% of the period (i.e. 146 out of 233 years). More precisely, the Qin State, the most powerful state during the middle to late period, regularly forced the neighboring state of Han, Wei, or Zhao (or any two of them) to take a neutral stance toward its intended targets, which lowered overall level of military tension. Consequently, the mean number of alliances in each partition (i.e. the final count of alliances in tables 3-6) was high, which was caused by brief durations of a two-trilateral-one-singleton partition, or a one-quadrilateral-one-trilateral partition, or a one-trilateral-one-bilateral-two-singleton partition.

**Table 1: List of Observed Coalition Structures ( $\rho^0$ - $\rho^{14}$ )**

$\rho$	Singletons	Proper Alliances	Frequency or Years Observed	Notes
$\rho^0$	Qn, Qi, C, Y, H, W, Z	None	146	
$\rho^1$	C, Qi, Qn, Y	HWZ	18	
$\rho^2$	C, Qi, Qn, Y, Z	HW	8	
$\rho^3$	C, Qi, Qn, W, Y	HZ	6	
$\rho^4$	C, Qn, Y, Z	HQiW	21	
$\rho^5$	Y, Z	CQiQn, HW	1	
$\rho^6$	None	CQiQn, HWYZ	2	
$\rho^7$	C, Qi, Y, Z	HQnW	10	
$\rho^8$	Qi, Qn	CHWYZ	4	
$\rho^9$	C, H, Qn, Y, Z	QiW	2	
$\rho^{10}$	C, Y, Z	QiW, HQn	2	
$\rho^{11}$	Y, Z	CQi, HWQn	1	
$\rho^{12}$	H, Qn, W, Y, Z	CQi	3	
$\rho^{13}$	H, Qi, W, Y, Z	CQn	2	
$\rho^{14}$	Y, Z	HQiW, CQn	2	

**Table 2: List of Observed Coalition Structures ( $\rho^{15}$ - $\rho^*$ )**

$\rho$	Singletons	Proper Alliances	Frequency or Years Observed	Notes
$\rho^{15}$	C, Y	HQiW, QnZ	4	
$\rho^{16}$	C, H, W, Z	QiQnY	10	
$\rho^{17}$	C	QiQnY, HWZ	1	
$\rho^{18}$	C, Qn	HQiWYZ	2	
$\rho^{19}$	C, H, Qi, W, Y	QnZ	2	
$\rho^{20}$	Qi, C	HQnWYZ	2	
$\rho^{21}$	Qi, C, W, Y, Z	HQn	10	
$\rho^{22}$	C, Qi, Y	HQn, WZ	8	
$\rho^{23}$	H, Qi, Qn, Y	CWZ	4	
$\rho^{24}$	H, Qi, Qn	CWYZ	2	
$\rho^h$	C, Qi, W, Y, Z	QnH*	6	
$\rho^{wh}$	C, Qi, Y, Z	QnHW*	3	
$\rho^{chw}$	Qi, Y, Z	QnCHW*	2	
$\rho^{chwyz}$	Qi	QnCHWYZ*	2	
$\rho^*$	QnQiCYHZW*	QnQiCYHZW*	1	

**Table 3: Annual Series of Coalition Structures (453-386 B.C.)**

<b>Year (B.C.)</b>	<b>Final Count of Proper Alliances</b>	<b>Final Count of Alliances</b>	<b>Year (B.C.)</b>	<b>Final Count of Proper Alliances</b>	<b>Final Count of Alliances</b>
453	0	7	419	0	7
452	0	7	418	0	7
451	0	7	417	0	7
450	0	7	416	0	7
449	0	7	415	0	7
448	0	7	414	0	7
447	0	7	413	0	7
446	0	7	412	0	7
445	0	7	411	0	7
444	0	7	410	0	7
443	0	7	409	0	7
442	0	7	408	0	7
441	0	7	407	0	7
440	0	7	406	0	7
439	0	7	405	0	7
438	0	7	404	1	5
437	0	7	403	0	7
436	0	7	402	0	7
435	0	7	401	0	7
434	0	7	400	1	5
433	0	7	399	0	7
432	0	7	398	0	7
431	0	7	397	0	7
430	0	7	396	0	7
429	0	7	395	0	7
428	0	7	394	0	7
427	0	7	393	0	7
426	0	7	392	0	7
425	0	7	391	1	5
424	0	7	390	0	7
423	0	7	389	0	7
422	0	7	388	0	7
421	0	7	387	0	7
420	0	7	386	0	7

**Table 4: Annual Series of Coalition Structures (385-318 B.C.)**

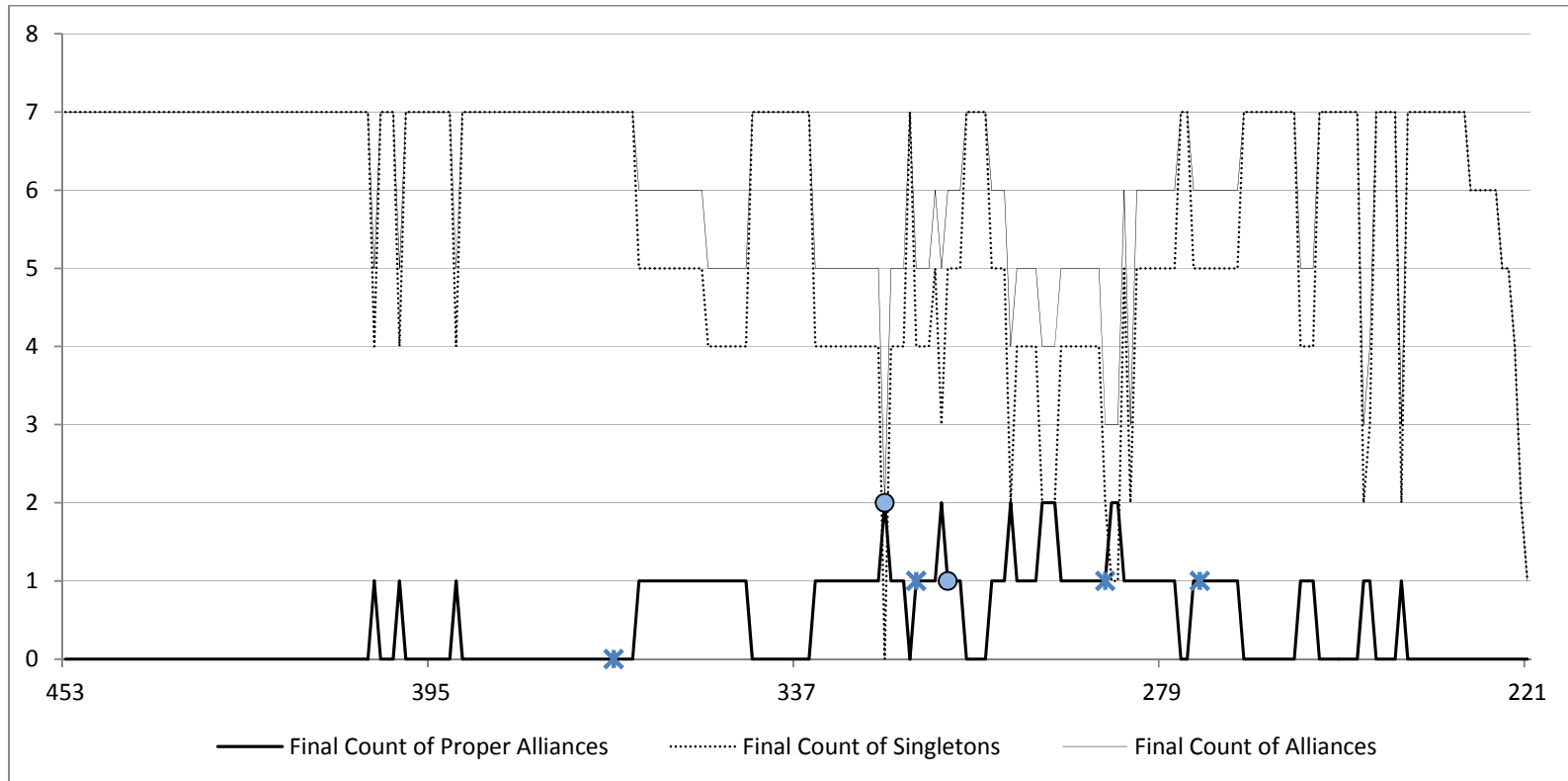
<b>Year (B.C.)</b>	<b>Final Count of Proper Alliances</b>	<b>Final Count of Alliances</b>	<b>Year (B.C.)</b>	<b>Final Count of Proper Alliances</b>	<b>Final Count of Alliances</b>
385	0	7	351	1	5
384	0	7	350	1	5
383	0	7	349	1	5
382	0	7	348	1	5
381	0	7	347	1	5
380	0	7	346	1	5
379	0	7	345	1	5
378	0	7	344	0	7
377	0	7	343	0	7
376	0	7	342	0	7
375	0	7	341	0	7
374	0	7	340	0	7
373	0	7	339	0	7
372	0	7	338	0	7
371	0	7	337	0	7
370	0	7	336	0	7
369	0	7	335	0	7
368	0	7	334	1	5
367	0	7	333	1	5
366	0	7	332	1	5
365	0	7	331	1	5
364	0	7	330	1	5
363	0	7	329	1	5
362	1	6	328	1	5
361	1	6	327	1	5
360	1	6	326	1	5
359	1	6	325	1	5
358	1	6	324	1	5
357	1	6	323	2	2
356	1	6	322	1	5
355	1	6	321	1	5
354	1	6	320	1	5
353	1	6	319	0	7
352	1	6	318	1	5

**Table 5: Annual Series of Coalition Structures (317-250 B.C.)**

<b>Year (B.C.)</b>	<b>Final Count of Proper Alliances</b>	<b>Final Count of Alliances</b>	<b>Year (B.C.)</b>	<b>Final Count of Proper Alliances</b>	<b>Final Count of Alliances</b>
317	1	5	283	1	6
316	1	5	282	1	6
315	1	6	281	1	6
314	2	5	280	1	6
313	1	6	279	1	6
312	1	6	278	1	6
311	1	6	277	1	6
310	0	7	276	0	7
309	0	7	275	0	7
308	0	7	274	1	6
307	0	7	273	1	6
306	1	6	272	1	6
305	1	6	271	1	6
304	1	6	270	1	6
303	2	4	269	1	6
302	1	5	268	1	6
301	1	5	267	1	6
300	1	5	266	0	7
299	1	5	265	0	7
298	2	4	264	0	7
297	2	4	263	0	7
296	2	4	262	0	7
295	1	5	261	0	7
294	1	5	260	0	7
293	1	5	259	0	7
292	1	5	258	0	7
291	1	5	257	1	5
290	1	5	256	1	5
289	1	5	255	1	5
288	1	3	254	0	7
287	2	3	253	0	7
286	2	3	252	0	7
285	1	6	251	0	7
284	1	3	250	0	7

**Table 6: Annual Series of Coalition Structures (249-221 B.C.) with the Mean Values and Variances for 233 years**

<b>Year (B.C.)</b>	<b>Final Count of Proper Alliances</b>	<b>Final Count of Alliances</b>
249	0	7
248	0	7
247	1	3
246	1	4
245	0	7
244	0	7
243	0	7
242	0	7
241	1	3
240	0	7
239	0	7
238	0	7
237	0	7
236	0	7
235	0	7
234	0	7
233	0	7
232	0	7
231	0	7
230	0	6
229	0	6
228	0	6
227	0	6
226	0	6
225	0	5
224	0	5
223	0	4
222	0	2
221	0	1
<b>Mean Value (Round off to 2 decimal places)</b>	<b>0.44</b>	<b>6.24</b>
<b>Variance (Round off to 2 decimal places)</b>	<b>0.4</b>	<b>1.24</b>



**Figure 2: The Number of Proper Alliances as a Function of Years**

\* indicated years (366 B.C., 318 B.C., 288 B.C., and 273 B.C.) in which there were 2 configurations of the alliance structure.

• indicated that in 323 and 313 B.C. there were 3 configurations of the alliance structure.



### **3. COMPENDIUMS OF THE SIGNIFICANT EVENTS**

Details of the following events can be founded in chapters 1, 8, and 10 of Yang (1984), chapters 1, 2, and 5 of Yang (1985), chapters 1, 3, 4, and 7-20 of Yang (2001), chapters 7-9 of Yang (2003), chapters 4, 5, 14, 39-40, 45-46, 70, 75, and 77 of Ting, et al. (1985), and chapters 60, 78, and 106 of Zhang, et al. (1987).

#### **3.1. The Wars of Annexation Between Six Noble Families**

After the time of Duke Ping, the six powerful noble families, namely the Zhao, the Wei, the Han, the Zhi, the Fan, and the Zhonghang, in reality controlled the central plains state Jin. Among them, the Zhi family was the most powerful, with the ability and ambition of eliminating the other five families. Initially (458 B.C.) it teamed up with the Zhao, Wei, and Han families to accomplish its goals of conquering the Fan and Zhonghang families.

Later (455 B.C.), the Zhi family attempted to eliminate the Zhao family in one fell swoop by forging a military alliance with the Wei and Han families, and yet they failed to win the war. At the same time, the ambition of the Zhi family also constituted a major threat to both the Wei and Han families, and, in order to avoid being the next targets, they turned to team up with the Zhao family to conquer the Zhi family.

#### **3.2. Three-Jin's Invasion of the Qi State and Its Punitive Attack on the Chu State**

When the Zhi family was eliminated in 453 B.C., the Jin State was partitioned into spheres of influence and was primarily controlled by the Zhao, Wei, and Han families (Three-Jin). The Wei family took the lead in undertaking institutional reforms and became relatively more powerful. Later on, Wei Si, the head of the Wei family led troops from three

families (Three-Jin) to besiege the Qi territory of Pingyang (modern-day Northeastern Pingyang County, Shandong Province). This accomplished Wei Si's goal of pushing the Zhou king to make the heads of the three families eligible for the rank of Marquis, instead of annexing lands.

The Chu State's continuous occupations of the Zheng territories posed territorial threats to both the Wei and Han State. To eliminate that threat, Marquis Wen of Wei (Wei Si) led Three-Jin armies southward to the Chu State. By 400 B.C., they defeated the Chu army in combat and forced the Chu State to return the permanent sovereignty of the Yu pass to the Zheng State where it was an important frontier pass that connected north and south, and a contested target for the Chu State and Three-Jin.

### **3.3. Proclamation of Marquis Hui as King**

The strength of the Wei State enabled its ruler to compel both the Zhao and Han State to attend the meeting on the military invasion of the Qin State. Duke Xiao, the Qin ruler, was very concerned about it, and thus asked his courtier Yang Wei to find a way out of the unfavorable situation. Yang Wei suggested using a means of enticing Marquis Hui to proclaim himself King of Wei, to disrupt the alliance between the Han, Wei, and Zhao States. Duke Xiao accepted Yang Wei's suggestion and dispatched him as his personal representative to the Wei State. And Yang Wei also successfully enticed Marquis Hui to not only prepare a proclamation to declare himself King, but also to win over the Yan State to help the Wei State in coercing both the Qi and Chu States to capitulate by force (And, later, forcing the Zhao and Han States to capitulate by diplomatic pressure). Thus, in 344 B.C., Marquis Hui held a proclamation ceremony in Pangze (modern-day Southern Kaifeng City). The success of Yang Wei's lobbying not only made both

the Zhao and Han States angry enough to leave the alliance, but also made the Qi or the Chu State the target of the Wei State's offensive action.

### **3.4. The Mutual Recognition of the Title of King at Xuzhou**

In 341, 340, 339, 338 B.C. respectively, the Wei State suffered several military defeats at the hands of the Qi and Qin States, King Hui of Wei was despondent and therefore asked Prime Minister Shi Hui for an answer. He suggested that the Qi-Wei-Han alliance would deter Qin military aggression. He believed also that made the ruler of Qi as the top leader of all vassal states could induce the Chu State to attack the Qi State, therefore, the Wei State must succumb servilely to the Qi State.

In 334 B.C., the rulers of the Wei and Han States personally came to Xuzhou to call Duke Wei King of Qi and the ruler of all rulers very respectfully. He (King Wei of Qi) would be unwilling to make himself over-prominent by these titles and thus agreed to recognize the Wei ruler as king. The success of Shi Hui's strategy not only facilitated alliance formation between the Qi, Wei, and Han States, but also gave rise to a military conflict between the Qi and Chu States.

### **3.5. Yi Zhang's Attempt to Break Up the Qi-Han-Wei alliance**

After Yang Wei's 350 B.C. reforms, the Qin State became the most powerful State; and this situation gave rise to two policies: Hezong and Lianheng. The so-called Hezong policy was designed to unite the weak states to resist the most powerful one. The so-called Lianheng policy was designed to win weak allies against weaker opponents, in order to achieve the objective of territorial expansion.

Yi Zhang, the primary Lianheng advocate who suggested Duke Hui of Qin to help the Wei State fight against the invading Chu forces during 329 B.C. and asked the duke to return the permanent sovereignty of Puyang when the Qin State captured it from the Wei State in 328 B.C. and also sent Prince You to the Wei State as a diplomatic hostage, so as to improve the relationship.

In 325 B.C., Duke Hui staged a ceremonial proclamation of King, and according to precedent, he invited the rulers of the Wei and Han States to join him in the ceremony. Both rulers esteemed him as King of Qin, at the same time, he agreed to recognize their King titles. The idea of the ceremony proposed by Yi Zhang was attempting to win over the Wei and Han States as Qin's allies.

### **3.6. The Mutual Recognition of the Title of King of the Four States**

Soon after Yi Zhang carried out a policy of winning the Wei and Han States against either the Qi or the Chu State in the Qin State, Yan Gongsun, who has been an advocate for elimination of the Wei State for achieving the goal of completely occupying the central plains, left the Qin State for the Wei State and became a Field Marshal there. Due to the large decline in strength of the Wei State, it alone was unable to win the annexation war, for this reason, the Wei field marshal Yan Gongsun pushed the Wei State to work militarily with the Qi State to achieve the goal of annexing the territory. In 325 B.C., he and the Qi general Pan Tian jointly launched a war against the Zhao State designed to expand the Wei territory. But, the Qi State was the only state in the alliance that captured Pingyi (modern-day Northwestern Nanle County, Henan Province) and Xincheng from the Zhao State, which showed that Yan Gongsun's strategy of win did not help the Wei State to annex new lands.

To prevent the Wei State having a motive to forge an anti-Qin alliance with the Qi, Chu, and Han States, Yi Zhang met the courtiers of the Qi and Chu States, respectively to make these two states into the Qin State's allies (they agreed on the division of China into three spheres of influence).

In order to compete successfully against the Qi-Qin-Chu alliance, the Wei field marshal Yan Gongsun held a meeting about the mutual recognition of the title of King of four states: Yan, Wei, Zhao, and Han. But, unfortunately, this military alliance did not help the Wei State much.

### **3.7. The Short-Lived Alliance Between the Qin, Han, and Wei States**

The strategic activities adopted by both Shi Hui and Yan Gongsun did not bring the Wei State any benefits, which pushed King Hui into a trilateral military alliance with both the Qin and Han States. To achieve his goals of giving his troops a rest and weakening the Qi State's strength, King Hui named Yi Zhang as prime minister in 322 B.C. However, due to the fact that Yi Zhang's plan was to further the Qin incursions into the Qi and Chu States, he could certainly take the Wei king's request and still held his prime minister position at the Qin State.

In 320 B.C., with acquiescence from the Wei and Han States, the Qin State initiated a transnational attack on the Qi State but failed to win the war. The next year, in 319 B.C., both the Qi and Chu States brought pressures to bear on the Wei and Han States and pushed them to leave the alliance so that future Qin aggressions could be prevented. The collapse of the Qin-led alliance resulted in the failure of Yi Zhang's Lianheng policy.

### **3.8. The Five-State Alliance by Yan Gongsun Against the Qin State**

After expelling Yi Zhang from the Wei State, King Hui of Wei named Field Marshal Yan Gongsun as prime minister in 319 B.C. and then pursued a different diplomatic strategy involving an alliance between the Wei State and some eastern vassals. In 318 B.C., the five-state alliance was formed between the Chu, Yan, Han, Wei, and Zhao States, to fight against the Qin State. But, the lack of unity in both the Yan and Chu States created the opportunity for the Qin State to inflict heavy losses on Three-Jin's forces.

### **3.9. The Three-State Alliance by Yan Gongsun and the Failure of Hezong**

Under varying circumstances, states replaced their prime minister more frequently. In order to win over the Qi State as an ally of the Wei State, Xu Tian, a friend of Shi Hui, was named to replace Yan Gongsun as prime minister of the Wei State. But such a personnel change created discord between Xu Tian and Yan Gongsun, because recognizing the Qi king as ruler of all rulers would create conflicts between the Wei State and other eastern vassals. Thus, to successfully forge an anti-Qin alliance with the Qi and Han States, Yan Gongsun suggested King Xiang to name Wen Tian as prime minister of the Wei State, which because Ying Tian, the father of Wen Tian, was trusted by King of Qi. In 316 B.C., the Wei king made Wen Tian prime minister. Meanwhile, Yan Gongsun assumed duties as the Han prime minister.

When the Qin State invaded the Han State in 315 B.C., the Han courtier Peng Gongzhong asked the king to turn the Qin State from foe to ally through offering it a walled city, as he believed that have an alliance tie with the Qin State would avoid greater losses, and enabled the Han State to weaken the Chu State's strength jointly with the Qin State. Peng Gongzhong was then sent to the Qin State to establish alliance tie. King Huai of Chu was afraid of the alliance of

the Qin and Han States and asked the courtier Zhen Chen to think of remedy as a way to save the Chu State. Zhen Chen suggested the king to convey exaggerated information about the military aid the Han State would receive and thus strengthened its resolve to resist attack by the Qin State. In 315 B.C., the Han king was persuaded by the Chu envoy not to capitulate to the Qin State's demand as the Chu State pledged to make every effort to help the Han State fight against the Qin State, which resulted in the Qin resentment towards the Han State. The following year (314 B.C.), the Qin State decisively defeated the Han State at Anmen and later took the advantage of the victory to conquer the Wei territories of Jiao and Quwo, which induced compliance by both the Wei and Han States. The erroneous decisions of the Han king made the Hezong policy even more difficult for Yan Gongsun to implement.

### **3.10.A Military Stalemate Between the Qin-led Alliance and the Qi-Chu Alliance**

In 313 B.C., both the Wei and Han States were forced to ally with the Qin State and assisted in initiating a transnational attack the Chu State. For the sake of the state security, Chu dispatched Field Marshal Cui Jin to deploy the armed forces, and with the help of the Qi State, he conquered the Qin territory of Quwo and would take the advantage of the victory to attack Shangyu. Yet at the moment the Qin State set itself ambitious goals of defeating the Chu invasion and occupying the Chu territory of Hanzhong. For that reason, the Qin State voluntarily agreed to cede large portions of the Shangyu region to the Chu State in exchange for its withdrawal from the alliance. The Qin State later refused to fulfill its promise, and King Huai of Chu then ordered Cui Jin and his men to attack Shangyu. In 312 B.C., Qin repulsed the invading Chu army and, subsequently, conquered the 600 miles of the Hanzhong region, which was a big strategic victory for the Qin State.

### **3.11. Implementation of the Hezong Action by Lord Mengchang**

After returning home to the Qi State, Lord Mengchang Wen Tain, the second well-known advocate of Hezong policy, took up the position of the prime minister and prepared to forge a military alliance against the Qin State. He advised the king to win over the Chu State as an ally of the Qi State, since the union of the Qi and Chu States would be able to fascinate other eastern vassals to join the alliance. In 306 B.C., the Chu State made a conditional promise to work with the Qi State to create a multilateral alliance against the Qin State but, in view of the Qin State's efforts to improve the relationship with the Chu State (through the marriage of King Zhao to the daughter of the Chu royal house.), the Chu king forged an anti-Qi alliance in 304 B.C. with the Qin State. In 303 B.C., the Qin State tried to force both the Wei and Han States to capitulate to its demand by forces. The invasions posed serious threats to their state securities and made them want to seek the support of the Qi State, in order to prevent themselves from getting attacked a second time by the Qin State. This gave Lord Mengchang the opportunity to create a military alliance between the Qi, Wei and Han States to punish the Chu State for deviating from the established alliance relationship.

### **3.12. A Military Conflict Between the Qin-led Alliance and the Qi-led Alliance**

Owing to the widespread recognition that Lord Mengchang was the best diplomatic strategist of the mid-late Warring States period, he was appointed in 299 B.C. as prime minister of the Qin State. This designation was made to prevent future acts of aggression by the Qi-led alliance (the Qi-Han-Wei) and to adopt a friendly stance toward Qi.



But the close relationship of the Qi and Qin States would give the Qin State time to hinder the expansion of the Zhao State in Zhongshan. To capture more Zhongshan land, the best thing the Zhao State could do was to change the current situation and created a stalemate between these two great powers. In 298 B.C., King WuLing of Zhao dispatched the courtier Tou Jin to the Qin State to damage the relationship between King Zhao and Lord Mengchang. In consequence, King Zhao of Qin wanted to kill Lord Mengchang just because Tou Jin persuaded him that Lord Mengchang made strategic plans mainly for the Qi State, which would bring disaster to the Qin State. Lord Mengchang detested King Zhao for chasing and hunting him down, and when he fled his home, he asked King of Qi to fight the Qin State jointly with the Wei and Han States on the grounds of the victory of Qi-led alliance against the Chu State. In response to the formation of the Qi-Han-Wei alliance taking place at this time, a military alliance was formed between the Qin and Zhao States. The intention of the Zhao State here was to strengthen the resolve of the Qin State to resist aggressors. In late 298 B.C., the Qi-led alliance launched a massive attack on the Qin State, engaging in combat with single enemy (only Qin troops were engaged in warfare.), spending almost three years to breach the Hangu Pass, and finally forced the Qin State to sue for peace. Without valuable support from the Zhao State, the Qin State had no chance of winning such asymmetric conflict.

Both Yi Zhang's East-West and Lord Mengchang's North-South were capable of defeating their opponents in war, but only the East-West alliance, whose leader occupied considerable land within the opponents' territory.

### **3.13. Qin Su's Motives for Forming the Qi-Yan Alliance**

The Yan State was the weakest among the seven warring states, when it experienced an internal conflict in 315 B.C., the Qi State almost succeeded in conquering it by taking advantage of that opportunity. After Zhi Ji (the son of King Kuai), later King Zhao, succeeded to the throne of the Yan State, he spent twenty years improving state strength for the purpose of retaliation against the Qi State. In 295 B.C., Qin Su suggested King Zhao of Yan that, under the circumstances in which the Qi, Qin, and Zhao States had roughly equal power; the Yan State must rely on the military capabilities of the Qin and Zhao States to smash the Qi State. And he would be willing to engage in activities in the Qi State designed to encourage King Min to attack the Song State, and simultaneously to damage the Qi State's relations with the Qin and Zhao States.

Later, the Yan State sent Qin Su to the Qi State to create a military alliance. He convinced King Min to attack the Song State instead of invading the Yan State, which because the Song State was a rich central plain state, and it's more valuable to conquer ten miles from the Song State rather than occupying the hundred miles of the Yan territory. Thus, the Qi State, together with the Yan State, launched an attack on the Song State. King Min also dispatched his courtier YuDan Gong to win the Zhao State as an ally to its cause by promising to give Prime Minister Dui Li the Song territory of Meng for his own fief. Qin Su did not want to see that the Qi State defeated the Song State easily, thus he suggested King Min not to rely on military support from the Zhao State, which because Dui Li's ambitions not only of taking advantage to obtain Meng, but also to conquer the resource-rich lands in the Song State in order to enhance the overall strength of the Zhao State. Furthermore, solidarity within the Song State had made it more costly for the Qi State to conquer it. Hence, to eliminate the Song State in one fell swoop,

King has to wait for the right moment. In consequence, King Min of Qi retracted his promise to forge an alliance with the Zhao State and stopped attacks by the alliance of the Qi and Yan States on the Song State.

### **3.14. The Strategic Alliance Between the Qin and Qi States**

In early 294 B.C., the Qi courtier Fu Zhu suggested King Min of Qi, through letting the Qin State invade both the Wei and Han States, to win over the Qin State as an ally of its cause (that is, the occupation of the Song State) King Min of Qi took his suggestion and expelled the pro-Wei courtier Zui Zhou and appointed Li Lu, an advisor to the Qin king, to the position of prime minister. At this time, Lord Mengchang had fled to the Wei State, as King Min of Qi believed that Lord Mengchang schemed to abduct him.

After the Qi State succeeded in establishing alliance with the Qin State in late 294 B.C., they respectively implemented their own program of territorial expansion (the Qi State would like to annex the Song State, and the Qin State wanted to annex the former Wei capital of Anyi).

### **3.15. The Five-State Alliance by Qin Su Against the Qin State**

A few years later the Qin State realized that the rise of the Zhao State would not only create obstacles to its territorial expansion, but would constitute a threat to its territory. Thus, in order to straighten out the problem in one fell swoop, the Qin State had to get additional military aid from the Qi State. In 288 B.C., King Zhao of Qin proclaimed himself Emperor of West at Yiyang, and, following that, he dispatched Prime Minister Ran Wei to elicit the Qi State's support by acknowledging King Min as Emperor of East. King Min adopted the title of emperor and decided to help the Qin State attack the Zhao State.

After the promise was made, Emperor of East summoned Qin Su and listened to his views about the military action. Still, Qin Su firmly believed that the Qi State will benefit more from conquering the Song territory, and he also suggested to Emperor that the abolishment of his imperial title would enable the Qi State to call other eastern vassals to punish the Qin ruler for declaring himself emperor, which gave the Qi State a chance to invade the Song State without any military disturbances. The emperor accepted this suggestion and announced his decision to abolish the title of emperor. Qin Su's real purpose in instigating a war was to damage the relationship between the Qi and Qin States. In 287 B.C., the Qi State allied itself with the Yan State and Three-Jin to attack the Qin State but did not reach its territory, which because King Min of Qi recalled his armies after he received information that the Zhao general XuWei Han had the intention to conspire with the Wei Prime Minister Lord Mengchang to launch a sneak attack on the Qi State. Although the alliance failed to attack the Qin State, its tremendous momentum still forced Emperor of West to revoke his title and returned the territories that once belonged to the Zhao and the Wei States.

### **3.16. The Give-and-Take Negotiation Between the Qin and Qi States**

To prevent future Qin interference with the Qi invasion of the Song State, King Min of Qi, through appointing Min Han, a boon companion to King Zhao of Qin to the position of the prime minister to once again win over the Qin State as an ally of its cause. The Qi-Qin alliance was established in late 287 B.C. allowing them to pursue their respective annexation goals. Not long after that, Qin dispatched troops to attack the Wei State and conquered the Wei territories of Xingyuan and Quyang (modern-day Western Jiayuan City, Henan Province) on the north bank of

the Yellow River. In 286 B.C., the Qin State again sent an army against the river's north bank in the Wei State, which forced its cession of Anyi to the Qin State, in exchange for peace.

Although King Zhao of Qin had earlier agreed to let the Qi State conquer the Song State, he was still very angry with his good friend Min Han taking charge of planning a military operation that aimed to eliminate the Song State.

In order to make invasion of the Song State work, Qin Su came to the Qin State to convince King Zhao that all Min Han did was to make the Qi State a major threat to the territories of both the Wei and Han States in a manner to make them capitulate to the demands of the Qin State for securing its help against potential Qi aggressions. Thus, King Zhao of Qin kept his promises to the Qi State that rendered it capable of eliminating the Song State.

### **3.17. The Five-State Alliance by the Qin and Zhao States Against the Qi State**

After its conquest of the Song State in 286 B.C., the Qi State greatly improved the overall state strength that not only constituted a threat to the territory of the Zhao State, but also created an obstacle to the Qin State's annexation of the central plains. Thus, in 285 B.C., the Qin and Zhao States chose to develop an anti-Qi alliance in the name of punishing the Qi State for its conquest of the Song State. In the meantime, the Qin general Ao Meng was dispatched to lead an army to attack the Qi State, which showed the Qin State's determination to fight.

During the next year, in 284 B.C., the five-state alliance was created between the Qin, Yan, Han, Zhao, and Wei States. The Qin State recommended the Yan Field Marshal Yi Yue, a courtier to the former king WuLing of Zhao, as joint prime minister for the Yan and Zhao States and as commander-in-chief of the alliance forces in order to prevent the Qi State from trying to win over the Zhao State to its side. King Min of Qi had asked Qin Su to stop this invasion and,

subsequently, Qin Su tried to convince King HuiWen of Zhao that the Qin State did in fact worry about the Qi State's intervention in its invasions of the Han and Eastern Zhou States that it must rely on other vassals to weaken the Qi State's strength. Previously, the Qi State had helped defend the Zhao State against Qin aggression, and hence should consider the Qi State the best ally of the Zhao State. But, it's unfortunate that King HuiWen of Zhao did not take Qin Su's advice and insist on attacking the Qi State.

### **3.18. The Qin State's Victory Over the Wei-Zhao Alliance in Huayang**

In 274 B.C., the Qin State forced the Han State to become its ally and to let it initiate a transnational attack on the Wei State. Later, the Qin general Yang Hu conquered several territories of the Wei State near its common border with the Han State, including Quan (modern-day Western Yuanyang County, Henan Province), Cai (modern-day Southwestern Shangcai County, Henan Province), Zhongyang (modern-day Eastern Zhengzhou, Henan Province), and Changshe (modern-day Northeastern Changge City, Henan Province). To prevent future losses of land, the Wei State aligned itself militarily with the Zhao State and used the power of the alliance as leverage to compel the Han State to leave its alliance with the Qin State. The next year, in 273 B.C., the Wei-Zhao alliance conquered the Han territory of Huayang (modern-day Northern Xinzheng City, Henan Province), and because the Han State along will not defeat them, its Prime Minister Peng Gongzhong petitioned the Qin State for a military assistance. The Qin Prime Minister Ran Wei and General Yang Hu thus were dispatched to lead the Qin armies to assist the Han State; they defeated the Wei-Zhao alliance force in Huayang and went one step further to besiege Daliang. Under such circumstances, the Wei king's advisor Gu Xu was dispatched to convince Ran Wei to withdraw troops from Daliang, which because King

AnLi of Wei would employ the entire military forces to defend Daliang against attack. On the other hand, the Wei courtier GanChong Duan was sent to offer the Qin State the territory of Nanyang, in exchange for peace. The following year (272 B.C.), the Qin State established the Nanyang Prefecture, a military district covering the former Wei territory of Nanyang and the former Chu territory of Yuan.

### **3.19. The Five-State Alliance by Lord Xinling Against the Qin State**

The crushing defeat of the Qin State at Handan in 257 B.C. did not inflict too much damage on state strength. Thus, after 1 year of recovery, the Qin State continued its attacks on neighboring states. In 247 B.C., the Qin State's continuous occupation of Three-Jin's territories posed a significant threat to the security of Daliang. So, in order to drive the Qin forces back, King AnLi of Wei sought to recall his younger brother, Lord Xinling, to serve as command-in-chief of the armed forces. Up to the present time, Lord Xinling had lived in the Zhao State for approximately 10 years (257-248 B.C.). He was still very concerned about King AnLi's resentment towards him at his unauthorized deployment of military forces, and at first refused to accept that request; but afterwards, he was convinced by anchorites in the Zhao State that his high popularity was built as a result of his earlier action on behalf of the Wei State gave military aid to the Zhao State, his reckless disregard for the survival of the Wei State would make him despised by others. Thereupon, he went back to the Wei State to prepare for launching a counterattack that would stop the Qin aggression. He petitioned for military assistance, and states such as Chu, Zhao, and Han were eager to forge a military alliance with the Wei State on the basis that they have a higher chance to weaken the Qin State's strength. Sometime in 247 B.C. Lord Xinling created an alliance between the Chu State and Three-Jin against the Qin State.

Besides that, a Yan forces was sent to assist Lord Xinling as the Yan State had high regard for him. Afterward, he gathered all the five forces together, to attack the army of Qin led by Field Marshal Ao Meng. This five-State alliance defeated invading armies south of the Yellow River and eventually forced a retreat back to the Hangu Pass.

This was the last of the strategic alliances worth of mention here which because of the lack of historical information for the military alliance led by the Zhao Field Marshal Nuan Pang in 241 B.C., the details of the last real alliance can only be made if there were new archeological findings on that.



#### **4. TURNING POINTS OF CRITICAL BATTLES DURING THE WARRING STATES PERIOD**

Similar to the events discussed in the previous chapter, details about the following battles are available in many publications such as Yang (1984, 1985, 2001, and 2003), Ting, et al. (1985), and Zhang, et al. (1987).

##### **4.1. The Qi State's Military Victory in Maling**

In 342 B.C., the Zhao State suffered military invasion from the Wei State, and it, in the midst of crisis, petitioned the Qi State for a military assistance. King Wei of Qi accepted its request, but he took up Field Marshal Ji Tian's suggestion to commit troops to war only when both the Wei and Zhao States fought each other to the brink of exhaustion.

The next year, in 341 B.C., the Qi king ordered Ji Tian and military advisor Bin Sun to stage an attack on Daliang. To defend against this attack, the Wei State withdrew the remaining troops from the Zhao State and sent Field Marshal Juan Pang and Crown Prince Shen to lead an army of 100,000 men against the invading Qi forces. Bin Sun wanted to mislead the Wei military commanders into believing that Qi troops were afraid to fight the Wei army; thus he required the military to retreat and reduce the number of stoves from 100,000 to 20,000 so as to generate the phenomenon where large numbers of Qi soldiers were deserted before the war. The Wei commanders were fooled by this strategy and led an elite unit against the retreating Qi army. They chased the Qi State's men to Maling but ran into an ambush laid by Bin Sun that completely destroyed the Wei army. Crown Prince Shen was caught alive and military depletion drove Juan Pang to commit suicide. And, never again would the Wei State be a powerful state.

#### **4.2. Qi Bai's Military Victory in Yique**

By the year 293 B.C. the Qin courtier Shou Xiang was dispatched to lead an army to attack the Han frontier fortress of Yique. Because it's difficult for the Han State to defeat the invading Qin forces, it petitioned the Wei State to intervene and stop the attack. King Zhao of Wei promised aid and issued instructions to Field Marshal Xi Gongsun telling him to lead an army into the Han State. To defeat both the Wei and Han forces, the Qin State replaced Shou Xiang with Qi Bai, an outstanding officer and military strategist. Both the Wei and Han forces were unwilling to play the vanguard role in the war and took only actions that were effortless, which gave Qi Bai the opportunity to defeat the army more than twice the size of his army. He initially sent small units to lure Han soldiers away. He then selected crack troops to attack Xi Gongsun's forces. These elites not only annihilated Xi Gongsun's main units, but also killed him in battle. That military defeat forced both the remnants of the Wei and Han armies to retreat. Qi Bai's forces proceeded to chase and hunt them down, and later they decapitated 240,000 of them. Additionally, they also conquered 5 walled cities from the Han State, and consequently the Han State was greatly weakened.

#### **4.3. Yi Yue's Defeat of the Qi State**

In 284 B.C., Yi Yue led troops from four states (Yan, Han, Wei, and Zhao), and they moved southeastward from the Zhao State towards the Ji River before launching attacks on the river's west bank in the Qi State. Initially, they occupied Lingqiu (modern-day Southern Gaotang County, Shandong Province) as a logistic base and then proceeded to march toward Linzi. In the meantime, the Qin brigadier general Li Si was dispatched to lead an army to join forces with them.

In order to protect his land, King Min of Qi named Zi Chu as field marshal to command the Qi main forces and required him to repulse invaders; otherwise his family would be put to death and his ancestors' graves would be destroyed. Zi Chu was upset because it's hard to win when the soldiers were exhausted from fighting wars against the Song State, and he ordered a withdrawal just after the war began. The alliance's forces chased the Qi forces to the shores of the Ji River and killed many of them there; Zi Chu went missing in action during the retreat. And Yi Yue then led Yan troops toward Linzi and faced the remnants of the Qi forces led by General Zi Da in Qinzhou (the western gateway to Linzi). Later, the Yan forces annihilated the demoralized Qi troops and killed Zi Da in combat. Also, it conquered Linzi and compelled King Min to flee to Ju (modern-day Ju County, Shandong Province). Consequently, the Qi State fell sharply and did not rise again thereafter.

#### **4.4. Daliang's Two Crises**

The defeat of the Qi State resulted in a significant loss of state strength and thus it was no longer a threatening opponent to the Qin State. Because of that, the Qin State would wantonly invade its neighbors. In 283 B.C., in order to connect its mainland to newly acquired Qi territory of Dingtao, the Qin State initiated a military campaign against the Wei capital of Daliang aimed to eliminate the Wei State in one fell swoop. To keep the Wei State from becoming extinct, the only thing Prime Minister Lord Mengchang could do was by seeking military aid from the other vassals. Then he came to the Zhao State to ask King HuiWen to send Wei soldiers. Initially, the Zhao king hesitated to assist the Wei State in defending the invasion by the Qin State. But, when the king had been told that the best thing that the Wei State now can do was to surrender to the Qin State quickly and later serve its interests in invading the Zhao State, he immediately

dispatched 100,000 troops to the Wei State. And afterwards Lord Mengchang met with the Yan king and asked him to help the Wei State fight against the Qin State. Similarly, he initially refused to send his troops on the basis of the two consecutive years of poor harvests. But, when the king had been told that the only thing the Wei State can do now was to offer the Qin State a great deal of lands, in exchange for peace, and would later invade the Yan State, together with the Qin and Han State, he immediately sent 80,000 troops out to save the Wei State.

To avoid a costly battle, the Qin State withdrew its troops from the Wei State. But still, King Zhao of Qin had a strong desire to conquer the Wei State. Thus, for the sake of avoiding the Chu State's military rescue of the Wei State, the Qin State promised the Chu State not to intervene militarily in the event of the Chu State's attacks on the Han territory of Nanyang. Besides, the Qin State also terrorized the Zhao State into the acceptance of non-intervention in its invasion of the Wei State through military actions. After that, the Qin State laid siege to Daliang but failure to seize it which, because the Zhao State broke its promise to the Qin State and, together with the Yan State, launched a military operation to rescue the Wei State. With their help, the army of Qin was enclosed within the woods, which made the Qin State felt obliged to conclude the peace treaty with the Yan and Zhao States, by allowing the Yan State to completely conquer the Shandong peninsula and the Zhao State to conquer the whole of the region west of the Ji River. Providing military supports via the Yan and Zhao States helped the Wei State to avoid extinction.

#### **4.5. The Qin State Considered the Chu State a New Target for Its Invasion**

In fact, either the subjugation or the conquest of the Wei State would make the Qin State a major threat to every other state on its east sides. Therefore, states like Zhao and Yan would

save the Wei State every time the Qin State attacked it, which made it difficult for the Qin State to conquer the Wei State. In 280 B.C., someone suggested King Zhao of Qin to attack the Chu State, which because it's now weaker (soundly defeated by the Qi-Han-Wei alliance in 301 B.C.) and that nobody would be willing to lend it military support during the conflict, and thus the Qin State could increase state strength from occupying its territory. King Zhao of Qin took the suggestion and had preparation of military attacks against the Chu territories of Yan (modern-day Southeastern Yichang City, Hubei Province) and Ying (modern-day Northwestern Jiangling County, Hubei Province). In order to focus the Qin State's attacks at the Chu State, King Zhao improved relations with the Zhao State through having a meeting with King Huiwen in Mianchi (modern-day Western Mianchi County, Henan Province). The next year, in 279 B.C., the Qin Field Marshal Qi Bai was dispatched to lead an army to attack the Chu State. He initially used flood waters as a means to defeat the Chu State's defense at Yan and later took advantage of the victory to conquer Ying and its surrounding wealthy areas with a radius of several hundred miles. Consequently, the Chu State was weakened even more.

#### **4.6. The Core Ideas of Ju Fan's Annexation Strategy, and the Zhao State's Defeat at Changping**

After one year staying in the Qin State (269-268 B.C.), Ju Fan finally had a chance to secure an audience with King Zhao and gave him advice on the program of territorial expansion he was implementing. Ju Fan strongly suggested that the idea of befriending the distant states and attacking the nearby ones would help the Qin State to retain control over the territory it had just occupied. Besides, the idea was not only to prevent the neighboring state from being saved but also to avoid a direct confrontation between the Qin State and a military alliance. He also suggested King Zhao to conquer the Han State, rather than seized the territory of the Qi State, as

the Han territories were interlaced with the Qin State's and can become a military threat to the Qin State in such complex circumstances. The purpose of Ju Fan's current strategic plan was to isolate the Shangdang Prefecture completely from the rest of the Han State so that the Qin State could capture it all. In 265 B.C., the Qin State conquered the Han territories of Shaoqu (modern-day Southeastern Jiyuan City, Henan Province) and Gaoping (modern-day Western Mengzhou City, Henan Province) where two strategic locations were located between Shangdang and the Han capital of Xinzheng. The following year, in 264 B.C., Qi Bai was dispatched to lead an army to attack the Han State, and he conquered several territories in three consecutive years, including Xingcheng (modern-day Northeastern Xinjiang County, Shanxi Province) in 264 B.C., Nanyang, south of the Taihang mountains, in 263 B.C., and Yewang (modern-day Qinyang City, Henan Province) in 262 B.C. When Qin laid siege to Xingcheng, Ju Fan had advised King Zhao not only on land occupation, but also on the subornation of officials which, because it would lead to internal strife, the pro-Qin group could drive opposition through political struggle that enabled Qin to extract more benefits from the Han State. After Qin isolated Shangdang in 262 B.C., it also dispatched troops to attack the Han territory of Xingyang, which prompted the Han State to offer the Qin State the entire Shangdang Prefecture in avoiding greater losses. But, the current governor Zhong Jin and his successor Ting Feng were extremely unwilling to hand over Shangdang to the Qin State that Ting Feng eventually offered it to the Zhao State, for the purpose of relying on the Zhao State's military capability to confront the Qin State.

In 261 B.C., the Qin general He Wang was dispatched to lead an army to attack Shangdang. He later successfully invaded it, and he then led the head-on attack towards the reinforcements led by the Zhao Field Marshal Po Lian in Changping (modern-day Gaoping City,

Shanxi Province). Yet Qin's army was unable to defeat the opponent after three years of fighting. In the process of military stalemate, the Qin State tried to avoid the Wei State's military rescue of the Zhao State by allowing it to take over the Han territory of Yuanyong (modern-day Western Yuanyang County, Henan Province) that was bordered by the Ying River to the south that could constitute a major threat to Daliang. In order to break through Po Lian's strong defense, the Qin agents were dispatched to the Zhao State to spread rumors that it's relatively easy for the Qin State to deal with Po Lian and he will soon be obliged to surrender himself to the Qin State's hands, and Kuo Zhao (the son of deceased Field Marshal She Zhao) was the only person who was able to make the fighting forces feared by He Wang's army. In fact, King XiaoCheng of Zhao was unhappy with Po Lian's poor performance that led to numerous losses. When he heard it, he was even angry with Po Lian, and replaced him with Kuo Zhao, who was engaged primarily in armchair theorizing about warfare. In response to this personnel change, the Qin State secretly let Qi Bai take over command of Qin troops in Changping. Later on, Qi Bai left the enemy with nowhere to retreat, which forced Commander-in-chief Kuo Zhao to lead his men to break through the encirclement. But, unfortunately, they were unable to do so that eventually did Qin soldiers kill Kuo Zhao. Besides, more than 400,000 Zhao troops were buried while alive.

After the war, in early 259 B.C., the Qin State forced the Han State to surrender its territory of Yuanyong, so as to threaten the Wei State into submission.

#### **4.7. The Successful Rescue of Handan by the Chu and Wei States**

After Qi Bai's defeat of the Zhao State in Changping, he took a further step in besieging the Zhao capital city of Handan for the purpose of achieving his objective to the conquest of the

Zhao State. At the same time, there was someone in the Qin State who suggested Ju Fan not to let Qi Bai eliminate the Zhao State which, because King Zhao will bestow him with the position of the ducal minister for his years of significant contributions to the Qin State's war effort that he will have more influence than Ju Fan. In order to prevent Bai Qi from proceeding to a higher position, Ju Fan suggested King Zhao to call off the attack since Qin troops out there had become extremely fatigued, and to exert diplomatic pressure on the Zhao State to cede portions of its territory. King Zhao of Qin took his advice and promised to lift the siege of Handan once the Zhao State expressed its commitment to the cession of land. Initially, King Xiaocheng of Zhao would sue for peace through offering the Qin State 6 walled cities. Soon, however, the courtier Qing Yu convinced him that it's impossible to guarantee a complete cessation of hostility from the Qin State, thus rather than making it stronger, the Zhao State should actually win the aid of other eastern vassals in the struggle against the Qin State. Later on, the Qin State was angered by the refusal of the Zhao State to fulfill its promise to cede ground.

In September 259 B.C., the king's advisor Ling Wang was sent to the Zhao State to besiege Handan. He failed to conquer it and lost at least 40,000 men killed in action. To win this battle, King Zhao of Qin again summoned Qi Bai and instructed him to supersede Ling Wang. Qi Bai, however, used his illness as an excuse to avoid the new task because he firmly believed that the Qin State missed a great opportunity for eliminating the Zhao State in one fell swoop. Now, for the sake of survival, the Zhao State was not only strengthening its defense, but winning aids from the Chu, Yan, and Wei States. If the Qin State spent too long there without defeating the Zhao forces, they would come to the rescue of the Zhao State. No wonder therefore that Qi Bai was dissatisfied with Ju Fan for ending the war. In the face of successive military setbacks and



Qi Bai's repeated refusal to conduct operations in Handan, the Qin general AnPing Zheng was appointed to take over Ling Wang's duties.

After nearly three years of fighting, the Qin State could not conquer the city of Handan. To prevent the Qin State from driving the Zhao State to the brink of extinction, Lord Pingyuan of Zhao came to the Chu State in 257 B.C. to request King KaoLie to play a leadership role in establishing an alliance to help the Zhao State defeat the Qin army at Handan. At first, the Chu king hesitated to give the Zhao State military aid. But later Lord Pingyuan's advisor, Sui Mao, convinced him that the alliance not only helped the Zhao State but also enabled the Chu State to take revenge for Qin's early occupation of its several territories in the years 279 B.C. and 278 B.C., respectively. Thereupon the Chu king made a promise to attack cooperatively. And yet the Zhao State was completely separated from the Chu State by the entire Wei territory. It therefore must also rely on the participation of the Wei State for its survival. Lord Pingyuan had repeatedly requested the Wei State's aid through the help of his wife, who was the elder sister of Lord Xinling of Wei. And the Wei State finally sent General Bi Jin to lead an army to help the Zhao State fight against the Qin State. But after some time, King AnLi of Wei received a verbal warning that the Qin State will soon finish the conquest of the Zhao State and will mobilize its troops to attack any vassal that was involved in the rescue of the Zhao State. The Wei king thereupon instructed Bi Jin to station troops in Ye (modern-day Southern Ci County, Hebei Province) so as to observe the trend in the war's progress. Lord Xinling did not want to see the Zhao State disappear, therefore proceeded directly to Ye to kill Bi Jin and took direct control of the military. Then he led armies of 80,000 elite troops to the Zhao State. The Chu Field Marshal Yang Jing was also dispatched to lead an army to rescue Handan.

Under a pincer attack from the Chu, Zhao and Wei forces, the army of Qin suffered a great defeat that made AnPing Zheng felt obliged to surrender himself together with 20,000 troops into the Zhao State's hands. Armies of Wei and Chu later took advantage of a Qin retreat to attack its territory on the east bank of the Yellow River, and the Wei State recaptured its lost lands there. With the help of the Chu and Wei States, the Zhao State extricated itself from the crisis.

#### **4.8. The Qin State's Unification**

Prior to submitting himself to the Qin State, Teng served as the acting administrator of the Han territory of Nanyang, but was bought off by the secret agent of the Qin State, and give it that territory as its domain in September of 231 B.C. Before that time, King JingMin of Wei had given away parts of his territory to avoid letting the Wei State itself become a primary target of the future Qin military action. Teng was later ordered to attack the Han State because of his deeper understanding of the Han State's condition. In 230 B.C., the Qin State conquered the whole of the Han State and established a prefecture there called Yingchuan and, consequently, the Han State became extinct.

In the meantime, in 230 B.C., the severe drought condition in the Zhao State immediately caught the Qin State's eye: a chance to occupy all of the Zhao State. The following year, then the Qin State dispatched three columns into the Zhao State. The main column, led by Field Marshal Jian Wang, moved out of the Shangdang Prefecture and passed through the Taihang mountains further to the north to attack Jingxing (modern-day western Jingxing County, Hebei Province), another led by General DuanHe Yang, moved out of the south bank of the Yellow River and laid siege to Handan, and the third column, led by General Xin Li, proceeded to attack and gain

control of Taiyuan and the Prefecture of Yunzhong. The armies of Zhao, under the command of Field Marshal Mu Li and General Shang Sima, tenaciously resisted the Qin State's attack and thus, made it more difficult for the Qin State to achieve operational objectives. To overcome this, the Qin State sent a secret agent to bribe King Qian's principal favorite, Kai Guo, who later accused Mu Li and Shang Sima of colluding with the Qin State to eliminate the Zhao State in one fell swoop so as to acquire fiefs. The Zhao king Qian really believed that it had happened, therefore, replaced Mu Li and Shang Sima with Cong Zhao and Ju Yan. Mu Li, however, refused to hand over his command because he believed that Cong Zhao and Ju Yan were totally incapable of fighting off the Qin forces. Mu Li's refusal to obey the command aroused King Qian's suspicion, and he secretly sent men to kill Mu Li. Three months later, in 228 B.C., the Qin State conquered most of the Zhao State: it killed Cong Zhao in war and also caught King Qian alive. But the Qin State would not catch Prince Jia and his clansmen escaping so they fled to the Prefecture of Dai and created a new state there called Dai. That same year the Qin State established the Prefecture of Handan, a district that had jurisdiction over Handan and its surrounding areas.

After the war, the armies of Qin assembled on the banks of the Ji River and prepared themselves to invade the Yan State. Crown Prince Dan of Yan felt considerable anxiety about the fate of the state because the Yan State could not win against the Qin State even if it mobilized the entire military. Therefore, he sought the help of the anchorite Ke Jing in pursuing his plan of action in way that make use of a formal audience with the Qin king (King Yin Zheng of Qin) to kidnap him and compelled him to recall his troops. Ke Jing accepted his request and later appeared before the Qin king to show the obedience of the Yan State by giving him a map of its fertile land and the severed head of Yi Huan, a former general of Qin, who defected to the Yan

State after losing the 233 B.C. war with the Zhao State. In order to abduct the Qin king, Ke Jing sprang a surprise attack on him when he fully spread out the map. Ke Jing failed, however, to complete the catch, and this failure led to his death. The Qin king was mad at Crown Prince Dan at the time for planning the abduction, and therefore sent more troops against the Yan State. In 227 B.C., the Qin armies, led by Jian Wang and Sheng Xin crossed the Ji River into the Yan State and soundly defeated the forces of Yan and Dai west of the Ji River. The next year, in 226 B.C., Jian Wang and Sheng Xin launched large-scale campaigns to conquer the Yan capital of Jicheng, which forced the Yan king (King Xi of Yan) to flee to the Prefecture of Liaodong. For the survival purpose, King Xi of Yan followed the Dai king's advice in giving the Qin State Crown Prince Dan's severed head that the Yan State was able to avoid extinction.

In 225 B.C., the Qin general Ben Wang (the son of Field Marshal Jian Wang) was dispatched to lead an army to conquer the Wei State. The Qin military forces diverted the Ying River to flood Daliang, and its ruins drove King Jia of Wei (the son of King JingMin) to surrender. The Wei State therefore became extinct.

That same year (225 B.C.), Qin was prepared to invade against the Chu State. King Yin Zheng of Qin had asked Jian Wang his opinion of the number of soldiers required to conquer the Chu State but had not yet accepted his suggestion that 600,000 troops would be needed to conquer the Chu State, since the Qin king believed that Jian Wang was too timid and cautious. The Qin king later accepted Xin Li's proposal and dispatched the two columns, 200,000 strong, toward the Chu State: one led by Xin Li marched towards Pingyu (modern-day Northern Pingyu County, Henan Province) and the other directed by Wu Meng (the son of General Ao Meng) advanced against Qinqiu (modern-day Southeastern Shenqiu County, Henan Province). By this time, Jian Wang had already used the excuse of old age to resign his post and return home to

Pinyang (modern-day Northeastern Fuping County, Shaanxi Province), to retire. The Qin forces first seized control of Pingyu and Qinqiu and assembled in Chengfu (modern-day Southeastern Bozhou City, Anhui Province) to start another war. Sometime later, an anti-Qin revolt broke out in the former Chu territory of Ying (modern-day Northwestern Jiangling County, Hubei Province) under the leadership of Lord Changping, son of King KaoLie, who was a diplomatic hostage in the Qin State. This new situation forced Xin Li's army to pull back and they quell the unrest that had cut off their supply lines. But one surprise attack by Lord Changping's army inflicted heavy casualties on Xin Li's forces. This unit then also regained control over Pingyu and proceeded to engage in attacks on the South Prefecture of Qin. Earlier, the defeat forced the Chu king to make peace with the Qin State, but Lord Changping's military actions sabotaged the peace between the Qin and Chu States. The following year, in 224 B.C., the Chu general Yan Xiang hailed Lord Changping as king.

King Yin Zheng of Qin, infuriated by Lord Changping's invasion of 225 B.C., re-appointed Jian Wang as field marshal in 224 B.C. and asked him to lead a large army (600,000 soldiers) to recapture lost ground. Qin troops moved in two columns: one column proceeded to lay siege to Ying, the other led by Jian Wang advanced to attack Pingyu. Soon after they regained territory and also defeated Lord Changping's army at Southern Qi (modern-day Southeastern Suzhou City, Anhui Province). Lord Changping was killed in battle and the depletion of soldiers led Yan Xiang to commit suicide. The next year, in 223 B.C., they attacked and captured Shouchun, the capital city of the Chu State and captured alive the Chu king. Subsequently (in 222 B.C.), the armies of Qin went on to conquer the entire region south of the Yangtze River and then establish the Prefecture of Kuaiji there. Consequently, the Chu State became extinct.

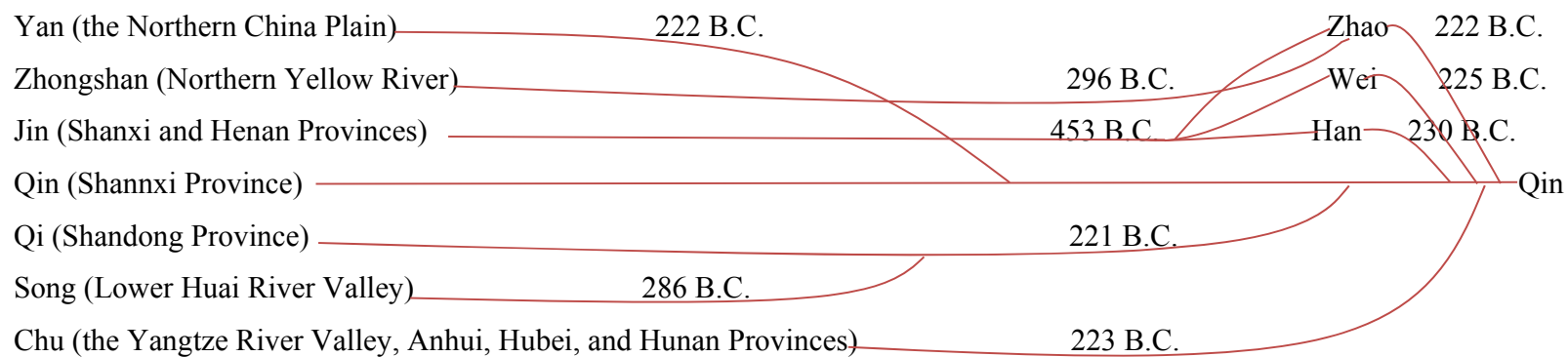
During the year 222 B.C., the Qin State dispatched General Ben Wang to lead an army to attack the Yan State. It eventually conquered the whole of Liaodong, captured alive the Yan king, and established there the Liaodong Prefecture. It also launched an attack against the Dai State. The attack eliminated it, and its territory was made into the Dai Prefecture. Consequently, the Yan and Zhao States became extinct.

The Qi Prime Minister Sheng Hou had been bribed to influence Qi's foreign policy. He suggested King Jian to adopt non-confrontation stance vis-à-vis the Qin State because the Qin State was now so powerful that it could beat others on all. Thus when the Qin State attacked other eastern vassals, the king of Qi did not send any troops out to aid them. Furthermore, the Qi State lacked sufficient defensive capabilities, which gave Ben Wang's army an opportunity to eliminate it in 221 B.C. in one fell-swoop: the army encountered no resistance and captured alive the Qi king. After that, the Qin State divided the former Qi State into two prefectures, namely the Prefecture of Qi and the Prefecture of Langya, and, consequently, the Qi State became extinct.

It took King Yin Zheng of Qin 10 years (231-221 B.C.) to unify China, and he created the first highly centralized, non-hereditary bureaucratic empire in Chinese history. He also claimed the title of emperor (China's first august emperor) to emphasize his extraordinary achievement.

**Table 7: Extinction Dates of the Feudal States**

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## 5. CONCLUSION AND DISCUSSION

I have quantified the alliance structures during the Warring States Period of ancient China as the annual series of partitions of the seven states given in Tables 3-6. I have also discussed the formation and disruption of some major alliances documented in the tables. The military alliance was initially built for the purpose of struggling for power and was afterwards used to enlarge and defend the territory. A member's decision to leave its alliance was either forced (unnatural) or unforced (natural) by outside forces. Precisely, "unforced" means that one or more alliance members left voluntarily because the expected benefits were low or zero in the future, and "forced" means that one or more alliance members left involuntarily due to outside temptations and coercions.

Many conclusions about the Warring States had been drawn by scholars in the past two thousand two hundred thirty years. My quantitative results allow one to revisit these known conclusions by quantifying some conclusions or representing them precisely in mathematical terms. Here, I briefly summarize the precise representations of two major conclusions.

One major conclusion is that the Qin State succeeded in unifying China because it was often able to wear down the strength of its chief enemy by utilizing its enemy's threats to the other states and forming alliances with the threatened states. The Qin State's chief enemy changed over time, from the Wei State in the early-to-middle time period to the Qi State in the mid-late period to the Zhao State in the late period. The decline in the power of the foresaid three states was partially due to their greed in territorial acquisition, and the Qin State's strategy was to utilize such opportunity to wear down their strength by forming alliances with the threatened states. One prominent example of this key strategy was represented by the successful formation



of the five member alliance  $HQ_nWYZ$  in wearing down the Qi State's strength or the following change in alliance structure observed in 284 B.C:

$$\rho^{19} = \{Q_nZ, C, H, Q_i, W, Y\} \rightarrow \rho^{20} = \{HQ_nWYZ, C, Q_i\}.$$

Conversely, neither the Qi nor the Zhao State succeeded in unifying China because they failed to fully utilize various opportunities to weaken the Qin State by forming alliances with those weaker states threatened by the Qin State.

Another major conclusion is that the Qin State succeeded in unifying China because it duly disrupted major enemy alliances or weakened the solidarity between members of hostile alliances. Four of such disruptions were represented by the following four changes in the alliance structure:

$$\rho^4 = \{HQ_iW, C, Q_n, Y, Z\} \rightarrow \rho^5 = \{CQ_iQ_n, HW, Y, Z\} \text{ in } 323 \text{ B.C.},$$

$$\rho^8 = \{CHWYZ, Q_i, Q_n\} \rightarrow \rho^1 = \{HWZ, C, Q_i, Q_n, Y\} \text{ in } 318 \text{ B.C.},$$

$$\rho^{23} = \{CWZ, H, Q_i, Q_n, Y\} \rightarrow \rho^0 = \{Q_n, Q_i, C, Y, H, W, Z\} \text{ in } 254 \text{ B.C.}, \text{ and}$$

$$\rho^8 = \{CHWYZ, Q_i, Q_n\} \rightarrow \rho^{24} = \{CWYZ, H, Q_i, Q_n\} \text{ in } 246 \text{ B.C.}$$

Finally, my quantitative results generate a long list of future research topics, four of which are briefly discussed below. First, it would be useful to expand my annual series of partitions of the seven states to a quarterly or even monthly series of partitions or to correct and revise my annual series of partitions of the seven states, using available results that have refined the results in or corrected the errors in *Shiji*. This is feasible because new archaeological and historical discoveries about the Warring States are reported almost every month. For example, one immediate future research problem is to correct and revise my Tables 3-6, using the revised Chronological Table of the Warring States at end of Yang (2001). I have focused on the

Chronological Table of Six States in *Shiji* because my purpose was not to summarize the improvements and corrections of the materials reported in *Shiji*.

Second, my annual series of partitions of the seven states can be used to test the theory on the formation and stability of coalition structures in alliance studies, game theory, and international economics.

Third, it was useful to study the unknown empirical as well as theoretical statistical properties of my series of partition of the seven states with a large number of observations.

Finally, it was possible to obtain new conclusions and understandings about the Warring States by further analyzing my quantitative results.

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