# IS THE VIX A RELIABLE INDICATOR OF STOCK MARKET VOLATILITY? 

A Thesis Submitted to the<br>College of Graduate and Postdoctoral Studies<br>In Partial Fulfillment of the Requirements<br>For the Degree of Master of Science in Finance<br>In the Department of Finance and Management Science<br>Edwards School of Business,<br>University of Saskatchewan<br>Saskatoon

By

Arinze Ezeonyeka
© Copyright Arinze Ezeonyeka, December, 2018. All rights reserved.

## PERMISSION TO USE

This thesis is presented in partial fulfillment of the requirements for a graduate degree from the University of Saskatchewan. I agree that the Libraries of this University may make it freely available for inspection. I also agree that permission for copying of this thesis in any way, in whole or in part, for scholarly purposes may be granted by the professor(s) who supervised my thesis work or, in their absence, by the head of the department or the dean of the college in which my thesis work was done. It is understood that any copying or publication or use of this thesis or parts thereof for financial gain shall not be allowed without my written permission.

It is also understood that due recognition shall be given to me and to the University of Saskatchewan in any scholarly use which may be made of any material in my thesis.

Requests for permission to copy or to make other use of the material in this thesis in whole or part should be addressed to:

Head of the Department of Finance and Management Science, Edwards School of Business,
25 Campus Drive
University of Saskatchewan.
Saskatoon, Saskatchewan S7N 5A7 Canada
OR

Dean
College of Graduate and Postdoctoral Studies
University of Saskatchewan
116 Thorvaldson Building, 110 Science Place
Saskatoon, Saskatchewan S7N 5C9 Canada


#### Abstract

This thesis examines the reliability of the Chicago Board Options Exchange Volatility Index (VIX) as an indicator of realized stock market volatility. The VIX is published daily by the Chicago Board Options Exchange (CBOE) and is widely referred to as the 'fear gauge' in the market. The VIX is computed from the relevant S\&P 500 call options prices, put options prices and the forward S\&P 500 index. The VIX is our measure of implied volatility in this thesis. We proxy realized market volatility using the daily range of the S\&P 500 index.

Using a GARCH $(1,1)$ model, we find a positive, statistically significant, contemporaneous relationship between the daily closing values of the VIX and the range of the market index. Our results also support a stronger magnitude of relationship between increases in the value of the VIX and realized market volatility, compared to the relationship between decreases in the value of the VIX and realized market volatility. This finding confirms the VIX as a reliable measure of market volatility for market participants.

Additionally, in a major contribution to existing literature, we separately model changes in the VIX that occur during non-trading hours vis-à-vis changes in the VIX that occur during trading hours. We find that changes in the VIX during non-trading hours predict realized market volatility; up to four days. Additionally, changes in the VIX during trading hours predict realized market volatility; up to five days. Notably, the magnitude of this relationship is still positive and mostly diminishes as the time lag increases.

Our analyses of the separate changes in the VIX during non-trading hours and changes in the VIX during trading hours highlight the need for market participants to isolate changes in the VIX that occur during these two different times. These isolated changes in the VIX are more informative for predicting realized volatility compared to analyses that only consider changes in the closing values of the VIX.

Furthermore, we find that the scale of the relationship between the VIX and realized market volatility is strongest on Fridays compared to other trading days of the week. This increase in the magnitude of the relationship between the VIX and realized volatility around the 'weekend' is likely


due to uncertainties that are associated with news including major changes in economic or monetary policy, company earnings etc. that are typically released around the end of the week. These uncertainties are reasonably captured in the VIX.

In sum, the evidence provided by our thesis supports the VIX as a reliable tool for predicting realized market volatility for up to five days ahead, with positive changes in the VIX on Fridays being very informative.

## ACKNOWLEDGEMENTS

I am ever grateful to my supervisors; Professor Fan Yang and Prof George Tannous. Their patience, detailed reviews, priceless support and coaching throughout the entire research and writing process cannot be overemphasized. I also acknowledge all staff and faculties at the Edwards School of Business, University of Saskatchewan who provided some form of support and encouragement for this research.

I am thankful to my siblings, friends and fellow students at the MSc Finance program in the University of Saskatchewan; their support and motivation were of immense help. Special thanks to the University of Saskatchewan for providing a conducive environment for researchers to learn and explore new fields.

## TABLE OF CONTENTS

PERMISSION TO USE ..... i
ABSTRACT ..... ii
ACKNOWLEDGEMENTS ..... iv
TABLE OF CONTENTS ..... V
LIST OF FIGURES ..... vi
LIST OF TABLES ..... vii

1. INTRODUCTION .....  1
2. LITERATURE REVIEW AND LITERATURE GAP ..... 5
2.1. Literature Review ..... 5
2.2. Literature Gap ..... 15
3. HYPOTHESES AND MODEL SPECIFICATION ..... 16
3.1. Hypotheses Statements ..... 17
4. VARIABLES AND METHODOLOGY ..... 24
4.1. Variable Description ..... 25
4.2. Model Specification ..... 29
5. DATA ..... 35
5.1. Data Source and Data Preparation ..... 35
5.2. Note on VIX and VXO Comparison ..... 35
6. RESULTS, INTERPRETATION, AND IMPLICATIONS ..... 38
6.1. Descriptive Statistics and Correlation Matrices ..... 38
6.2. Presentation of Results ..... 39
6.2.1. Different Lags and Directions of Changes in the VIX. ..... 39
6.2.2. Changes in the VIX During Non-trading Hours ..... 45
6.2.3. Changes in the VIX on Specific Trading Days ..... 49
6.3. Robustness Check ..... 52
6.4. Implication of Findings ..... 57
7. SUMMARY, CONCLUSION, AND AREAS FOR FUTURE RESEARCH. ..... 59
7.1. Summary ..... 59
7.2. Conclusion and Areas for Future Research ..... 60
REFERENCES ..... 62
APPENDIX. ..... 66

## LIST OF FIGURES

Graph 2.1. Changes in the VIX and Changes in the S\&P 500 Index.................................... 6
Graph 2.2. Changes in the VIX and Changes in the S\&P 500 Index................................... 6
Graph 5.1. VIX vs VXO................................................................................... 36
Graph 5.2. VIX vs VXO......................................................................................... 37

## LIST OF TABLES

Table 4.1. Correlation Coefficients (Range and Returns) ..... 27
Table 5.1. Correlation Coefficients (VIX and VXO) ..... 36
Table 6.1. Summary Statistics ..... 38
Table 6.2. Correlation Coefficients (Key Variables) ..... 39
Table 6.3. 1st Hypothesis, Support Table ..... 40
Table 6.4. 2nd Hypothesis, Support Table. ..... 43
Table 6.5. 3rd Hypothesis, Support Table ..... 46
Table 6.6. 4th Hypothesis, Support Table ..... 50
Table 6.7. Robustness Check, 1st Support Table ..... 53
Table 6.8. Robustness Check, 2nd Support Table ..... 55

## 1. INTRODUCTION

In 1993, the Chicago Board Options Exchange (CBOE) launched the Volatility Index (VIX). According to the CBOE, the VIX is a measure of expected volatility for the next 30 days in the US equity market. The VIX is derived from the forward S\&P500 Index (SPX) and relevant option prices ('The Cboe Volatility Index - VIX;' 2015). The VIX expectedly serves as a tool for forecasting market volatility as the S\&P500 index covers a significant portion of equities in the US stock markets; the prices of these SPX options are reflective of the buyers' and sellers' expectation of future stock prices and in turn upcoming 30-day market volatility. The expiration dates of the options used in the VIX calculation fall between 24-days ('near-term' options) and 37-days ('next-term' options). Thus, the VIX is expected to be a measure of implied volatility in the US stock market for approximately the next thirty days.

A substantial portion of existing studies on the VIX mostly models the relationship between the daily values of the VIX and market returns or volatility. Some of these studies include Fleming et al. (1995), Whaley (2000), Giot (2005), Bekaert \& Hoerova (2014), Fernandes et al. (2014), and Smales (2016) which investigate the relationship between the VIX and market returns. Fleming et al. (1995) finds a contemporaneous, negative significant relationship between the VIX and stock market returns. Similar results are seen in Bekaert and Hoerova (2014) and Fernandes et al. (2014). However, Smales (2016) and Giot (2005) note that the VIX has a negative statistically significant relationship with future returns; high levels of the VIX indicate taking a short position in the index. Similar studies by Banerjee et al. (2007) compare the relationship between the VIX, thirty-day market returns (and sixty-days market returns). They find that the VIX has a stronger relationship with the sixty-day market returns compared to the thirty-day market returns.

Some other studies model the characteristics of stock returns during non-trading hours. These studies mostly focus on stock characteristics during non-trading hours compared to stock characteristics during trading hours. Key among these studies are Branch \& Ma (2012), Kelly \& Clark (2011), and Stoll \& Whaley (1990) which highlight major differences in the characteristics of stock returns or volatility during the day (trading hours) and these returns or volatility overnight (non-trading hours). Branch \& Ma
(2012), Kelly \& Clark (2011) note that overnight stock returns are notably higher than stock returns during the day while Stoll \& Whaley (1990) note that volatility is higher during trading hours compared to non-trading hours. Similar works by Berument \& Kiymaz (2001), Mehdian \& Perry (2001), Keim \& Stambaugh (1984), Gibbons \& Hess (1981), French (1980) study the characteristics of volatility or returns for different trading days of the week. On average, they find that stock returns on Mondays are statistically different and negative compared to stock returns for other trading days of the week. Berument \& Kiymaz (2001) notes that market volatility is lowest on Wednesdays compared to other trading days of the week.

Although findings by previous studies are very insightful, these studies mostly focus on daily observations of the VIX without any distinction between changes in the VIX during non-trading hours, and trading hours. A significant portion of these previous studies also model the relationship between the VIX and stock market returns but not necessarily the relationship between the VIX and the daily range of the market index. Additionally, to the best of our knowledge, no previous study is yet to investigate the information content of the VIX for distinct trading days of the week.

These gaps necessitate the need for our thesis. Notably, this thesis synthesizes previous study areas by modeling specific characteristics of the VIX during non-trading hours, different days of the week, and also studying the information content of different directions and time of changes in the VIX. We model these specific characteristics and information content of the VIX in relation to the daily range of the market index. Precisely, we explore how changes in the VIX during non-trading hours and changes in the VIX during trading hours may separately impact the range of the market index. We study how different directions of changes in the VIX impact the range of the market index. Also, we investigate how the relationship between the VIX, and the range of the market index may differ depending on the trading day of the week.

Similar to Martens \& Van Dijk (2007), Brandt and Diebold (2006), and Parkinson (1980), we utilize the range of the market index as our measure of realized market volatility. The VIX is our measure of implied volatility.

This thesis is based on the following research questions:
i. Does the VIX have predictive power regarding the daily range of the market index?
ii. Does the relationship between the range of the market index and the VIX differ depending on the direction of changes in the VIX?
iii. Does the overnight change in the VIX (during non-trading hours) have any predictive power regarding the range of the upcoming market session?
iv. Does the relationship between the VIX and the range of the market index differ depending on the trading day of the week?

In the first question, we seek to explore how the range of the market index reacts to same-day changes and previous day changes in the VIX. We explore whether the changes in VIX and the range of the market index are contemporaneous or whether the VIX has a leading effect on the range of the market index. The second question is also essential as it builds on the first question to understand what direction of change in the VIX is more informative for the range of the market index. We investigate how positive or negative changes in the VIX comparatively impact the range of the market index.

In the third research question, we separately investigate the predictive power changes in the VIX during non-trading hours, and changes in the VIX during trading hours may have on the upcoming range of the market index. In the fourth question, we investigate if the relationship between the VIX and the range of the market index varies depending on each trading day of the week. We examine if the impact of the VIX on the range of the market index changes depending on the trading day of the week.

We analyze various observations of the VIX from 1993 through 2017; we find a positive, contemporaneous and statistically significant relationship between all daily closing values of the VIX and the daily range of the market index. We also find evidence that positive changes in the VIX have a larger impact on the range of the market index, compared to negative changes in the VIX.

Notably, we also find some predictive abilities in overnight changes in the VIX for up to four business days. Our results show that changes in the values of the VIX during non-trading hours have a
positive and statistically significant relationship with the range of the market index for up to four days ahead. Similar results are seen between changes in the values of the VIX during trading hours and the range of the market index for up to five days ahead.

Additionally, results from our analyses of data for each trading day of the week shows a positive and statistically significant relationship between the VIX and the range of the market index for each trading day. The scale of this relationship is strongest on Fridays compared to other trading days.

The rest of this thesis is organized as follows. Chapter 2 reviews the existing literature relating to the VIX, market volatility, and discusses the gap in the literature this thesis intends to fill. Chapter 3 provides information on our research hypotheses and the methodology used in this thesis. Chapter 4 considers the data used in this thesis and how the variables are computed. Chapter 5 presents the results, the interpretations, and the implications of these results. Chapter 6 concludes the thesis.

## 2. LITERATURE REVIEW AND LITERATURE GAP

### 2.1. Literature Review

Different studies have been carried out to understand the characteristics of returns and volatility in the market. There are several studies that analyze the information content of the VIX with respect to market returns and the practical implications for market participants. These studies relating to the VIX and the market can be divided into different strands.

The first strand of literature relates to studies that investigate the relationship between the daily values of the VIX and market returns. The second strand of literature compares the characteristics of returns or volatility during non-trading hours and trading hours. Another important strand of literature relates to studies that focus on how implied volatility relates to market returns when different stock characteristics are considered. The last strand of literature analyzes the characteristics of market returns or volatility for different trading days of the week. We provide more details on the key strands of these various studies below.

We also perform graphical analyses of the relationship between daily changes in the VIX and daily changes in the market index. The results for some of these analyses are shown in Graphs 2.1 and 2.2.

Graph 2.1. Changes in the VIX and Changes in the S\&P 500 Index (Dec, 2004 - Jan, 2005).


Graph 2.2. Changes in the VIX and Changes in the S\&P 500 Index (August, 2015 - Oct, 2015)


Based on Graphs 2.1 and 2.2, the magnitude of changes in the VIX appears to differ when compared to the magnitude of changes in the index for the same period. Particularly, these magnitudes seem to differ depending on whether there is an increase or decrease in the VIX. We proceed to review the existing literature to provide more insight into the relationship among the VIX, market returns, and market volatility.

## i. Some Negative Relationship Between the VIX and Stock Market Returns

Some studies in this strand of literature note a negative relationship between the VIX and market returns. Key among these studies are Smales (2016) and Fernandes et al. (2014). Smales (2016) is one of the most recent studies that investigate the relationship between the VIX and stock market returns. This study confirms the presence of a negative, contemporaneous relationship between changes in the S\&P 500 index returns and changes in the VIX. The relationship is also shown to be unidirectional as stock market declines or negative news sentiments precede larger changes in the VIX.

Fernandes et al. (2014) also confirm a statistically significant negative relationship between daily returns on the stock market and the VIX. The relationship between the VIX and volume of trading on the stock market is also found to be positive and statistically significant with changes in the market returns also preceding changes in the VIX. This study also investigates the relationship between the VIX and the term spread (interest rate spread); it defines the term spread as the difference between the 10 -Year and 3month treasury constant maturity rates. It finds that in the long run, the term spread has a negative impact on the volatility index.

Earlier studies by Bekaert \& Hoerova (2014) focus on the power of the VIX in predicting stock market-related activities. Precisely, it studies the relationship between the VIX and future returns on the stock markets, future economic activities, and financial stability. This study splits the VIX into the conditional variance of returns on equities and the variance premium on equities. It finds that the conditional variance within the VIX plays a significant role in predicting activities in the economy as well as financial instability. The variance premium aspect is found to have strong predictive power for stock
returns as well as financial instability; this predictive power for financial stability is however weaker compared to the predictive power in the conditional variance component.

Similar results to Bekaert \& Hoerova (2014) are seen in Giot (2005) who investigates the relationship between relative changes in VIX, and present returns as well as future returns on the stock market. Results from this study show a statistically significant and negative contemporaneous relationship between returns on the stock markets in the US and the VIX. A further investigation of these results shows that the strength of this contemporaneous relationship varies depending on the trading environment; specifically, during periods of low trading volatility, the relationship between VIX and stock returns is shown to be stronger compared to periods of high trading volatility.

Fleming et al. (1995) is among the earliest studies to investigate the predictive power of the VIX. They find that the VIX and stock market returns are strongly related. Particularly, a contemporaneous strong negative correlation is seen to exist between returns on the S\&P 100 index and the value of the VIX; thus, increases in the VIX and the present returns on the S\&P 100 index typically move in different directions. The VIX is also seen as a useful volatility forecast tool; a positive and statistically significant relationship is seen between the present values of the VIX and future stock market volatility. This study also argues that the predictive power of the VIX is significantly more accurate when compared with a first-order autoregressive volatility model.

## ii. Mixed Findings on Characteristics of Returns, Volatility During Trading Hours and NonTrading Hours

There are few studies that model the specific characteristics of returns or volatility during nontrading hours (overnight returns) compared to these characteristics during trading hours (returns during the day). Branch \& Ma (2012), Kelly \& Clark (2011), and Hong \& Wang (2000) are key among these few studies that specifically compare the characteristics of returns during the day and overnight returns. Most of these studies find that stock returns during the day are statistically different from overnight stock returns.

In Branch and Ma (2012), returns during the day and returns overnight have a strong negative correlation. This study samples US stock returns from January 1994 through December 2010 using a Fama-MacBeth two-stage regression. This study also emphasizes that the negative relationship between market returns during the day and overnight is not in tandem with the efficient market hypothesis (weak form).

Kelly \& Clark (2011) similarly compares the risk-adjusted returns of stocks during non-trading hours and trading hours in the US market. This paper studies returns for various indices including the S\&P 500, Nasdaq 100, Dow 30, etc. This study finds that on average, risk-adjusted returns during nontrading hours are significantly higher than risk-adjusted returns during trading hours. This result is also robust across all indices studied from 1996 through 2006.

Earlier studies by Wang et al. (2009) investigate the relationship between daytime returns, overnight returns and total returns in the market. Total return is defined as the sum of daytime return and overnight return. On average, they find that returns during the daytime and returns overnight are anticorrelated. Notably, daytime returns are higher than returns overnight returns for the period 1988 to 2007. Additionally, compared to overnight returns, daytime returns are more correlated with total returns. This study also notes that a greater component of total returns is made up of daytime returns, compared to overnight returns.

A different finding is seen in Hong \& Wang (2000). This study models how periodic closures in the market affect returns. This study notes that stock returns during trading hours are notably higher than stock returns during non-trading hours. Additionally, stock returns during trading hours are more volatile than stock returns during non-trading hours. Patterns of returns during trading hours are notably Ushaped.

These findings by Hong \& Wang (2000) are similar to findings by Stoll \& Whaley (1990) who document a negative serial correlation between stock returns during the day and stock returns overnight. Additionally, this study notes that volatility during the day is over five times greater than overnight volatility. They attribute this difference in volatility to the greater amount of public information available during the day compared to the amount of public information available at night. Additionally, stock with
low trading volume has a greater amount of overnight volatility compared to stock with high trading volume.

## iii. The Impact of the VIX on Stock Returns Depends on the Stock Characteristics

This other strand of studies explores the relationship between the VIX and several classes of stocks. Some of these studies also analyze trading strategies built on changes in the VIX. Popular studies in this area include Kozyra \& Lento (2011), Banerjee et al. (2007), and Copeland and Copeland (1999) which model how different classes of stocks react to changes in the VIX.

Kozyra \& Lento (2011) analyzes the returns of trading strategies that are constructed based on changes in the VIX compared to changes in stock prices. They find that returns for portfolio trading strategies based on changes in the VIX are significantly higher than returns on portfolio trading strategies built on only changes in stock prices. Precisely, the adjusted excess returns (AERs) from the filter rule, the moving average cross-over (MACO) rule, and trading-range breakout rule are higher when implemented based on changes in the stock prices and the VIX together, rather than changes in stock prices alone. These returns are also seen to be significantly higher during periods of increased volatility. Similar uses of changes in the VIX as trading strategies are seen in Copeland \& Copeland (1999).

Similar studies by Banerjee et al. (2007) find that variables related to the VIX display significant powers in predicting stock market returns. This paper studies portfolios which are grouped based on specific criteria including; book-to-market ratios, size, and beta. Among these criteria, the predictive power of the VIX for stock market returns is more significant in portfolios with high-beta stocks compared to portfolios with low-beta stocks.

Comparably, Copeland \& Copeland (1999) studies the usefulness of the VIX in asset allocation decisions. This study shows that timing portfolio allocation decision in line with changes in the VIX has the potential to bring about excess returns in portfolios. Precisely, changes in the value of the VIX from its 75-day moving average value is found to be beneficial when used as a criterion for asset allocation decision for a portfolio that allocates resources between assets that are based on 'size' and 'style'. When there is an upward change in the value of the VIX, portfolios built with large capitalization stocks show
superior performance compared to portfolios built with small capitalization stocks while portfolios built with value-based stocks also show superior performance compared to portfolios that are built with growth-based stocks. Similarly, when there is a downward change in the VIX, portfolios built with small capitalization stocks show superior performance compared to portfolios built with large capitalization stocks while portfolios built with growth-based stocks also show superior performance compared to portfolios that are built with value-based stocks.

## iv. Mixed Evidence on Specific Characteristics of Stock Market Returns for Different Trading Days of the Week

Another major strand of literature examines the characteristics of stocks for different trading days in the week. Some of the major studies within this strand of literature include Zhang et al. (2017), Morey \& Rosenberg (2012), Olson et al. (2015), and Mehdian \& Perry (2001).

Zhang et al. (2017) finds some mixed evidence of 'Monday effects' and 'Tuesday effects' in US markets for some sub-periods within their study. This study examines market returns on different trading days of the week using some rolling sample intervals with 500,1000 , and 1500 days. Using returns data for the DOW and SPX from 1993 through 2016, this paper finds some statistically significant positive returns for Mondays and Tuesdays compared to other trading days; with these returns highest on Mondays. However, these results do not hold for every sub-period in the sample. Precisely, average returns are positive, statistically significant and higher on Mondays than other trading days for the period 2012 through 2016. Additionally, average returns on Tuesdays are positive, statistically significant and higher than returns for other trading days except for Mondays for the period 1993 through 1996 and 2010 through 2014.

However, earlier studies including Morey \& Rosenberg (2012) and Olson et al. (2015) present some evidence to support the non-existence of 'Monday effects' in stock returns in US markets for study periods after the mid-1990s. Precisely, Morey \& Rosenberg (2012) group stocks in the US markets (NYSE and NASDAQ) into deciles and test for some statistically significant difference in returns on Mondays compared to other trading days in the week for these stocks in the period 1966 through 2007.

Monday returns on average are seen to be statistically significant and different from other trading days in the week from 1966 through the mid-1990s. However, after the mid-1990s, the statistically significant Monday returns are seen to be non-existent for the rest of the study period. This conclusion holds for all firm capitalization sizes considered.

Similarly, Olson et al. (2015) using data from 1973 through 2013 confirms that there are no abnormal returns on Mondays in the long run. Building on studies by Cross (1973), Olson et al. (2015) tests the presence of 'weekend effect' for stock market returns in the US markets. Specifically, they study how average returns on Mondays differ from the average returns for other days in the week using seven US stock market indices (S\&P 500, AMEX, S\&P Mid CAP 400, S\&P Small CAP 400, NASDAQ, DOW, and NASDAQ 100). US stock market returns are seen to be negative on Mondays in the year 1973 through 1974. Irregular weekend effect on Monday returns in the US stock markets was also noted for some periods during the Global Financial Crisis of 2007 through 2008. However, through all other observations in the study period, this weekend effect is absent in the long run. The stock returns on Mondays are barely different from returns on other days of the week in the long run.

Berument \& Dogan (2012) investigates the relationship between stock market returns and volatility for different days of the weeks. This paper does not find any evidence that the relationship between stock market returns and volatility is the same for each day of the week in the US markets for the period 1952 through 2006. Using an EGARCH model specification with a varying conditional risk; the return-volatility relationship is seen to be significant but not the same for each trading day of the week. Similar to Brusa \& Liu (2004), Berument \& Dogan (2012) also notes that on average, the most informed trading is carried on Mondays compared to other days of the week. There are more individual investors trading stocks on Monday compared to institutional investors. These classes of investors also have different preferences; thus, risk is priced differently across different trading days of the week.

A different finding is seen in Doyle \& Chen (2009). They investigate fixed seasonality effects for each trading day of the week in the period 1993 through 2007 and find that there are wandering weekday effects in the US stock markets that change over time. Notably, the size of the wandering weekday effect showed no reduction in size within the study period. This study shows that average returns from the
previous week do not always significantly affect the returns for trading days in the present week. Thus, there are no fixed 'weekend effect' or 'Monday effects.' This finding is contrary to earlier findings including Mehdian \& Perry (2001).

In a different twist to previous findings, Brusa and Liu (2004) notes that Monday returns in the US markets are statistically significant and positive only in the first and the third week of each month examined. This study examines returns from the S\&P 500, DJIA, NASDAQ, NYSE composite and CRSP VW. The monthly data examined spans from 1988 through 1998. This study also links these positive Monday returns to a boost in the volume of stock trading activities of institutional investors. This study notes that Monday returns are also adversely affected by the volume of stock trading activities of individual investors. This increase in the volume of institutional investor activities on Mondays compared to other trading days of the week is highlighted as a reason for the positive Monday returns identified.

Mehdian \& Perry (2001) analyzes data from 1964 through 1998 and highlight the presence of 'weekend effect' in stock returns. They note changes in the sign of returns on Mondays in the US markets for different periods in their studies. However, from 1987 to 1998, these Monday returns are seen to be statistically significant and positive on average especially for large capitalized firms. These average returns are although still negative for low capitalized firms from 1987 to 1998.

Berument \& Kiymaz (2001) investigates the 'day of the week effect' on stock market returns and volatility in the US markets using a $\operatorname{GARCH}(1,1)$ model. They find that the 'day of the week effect' is present in both volatility and returns in the US market. Precisely, market volatility is lowest on Wednesdays and highest on Fridays. Additionally, average market returns are highest on Wednesdays and lowest on Mondays.

However, Kamara (1997) finds in earlier study periods from 1962 through 1993, that there are no statistically significant negative returns on average for Mondays in the later part of the sample period. This study examines returns for low capitalized firms using the bottom decile of the NYSE securities, and also large capitalized firms using the S\&P 500. Returns for both the large capitalized firms and low capitalized are seen to be positive on Mondays in contrast to the negative returns seen in earlier study periods.

Comparable findings are seen in Brusa et al. (2000) who study returns for stocks in the New York Stock Exchange (NYSE) in the period 1990 through 1994. Brusa et al. (2000) note that the average returns on Mondays are positive and higher than returns on other days of the week. This study, however, notes that the sign of market returns on Mondays may vary depending on the size of firms considered. They study the returns for low capitalized (the lowest decile of firms in the NYSE), and large capitalized firms (the top decile of firms in the NYSE). On average, low capitalized firms are found to have negative returns on Monday while large capitalized firms are seen to have positive returns on Monday.

Several early studies including French (1980), Gibbons and Hess (1981), Keim and Stambaugh (1984) all study returns on the S\&P 500 index. These authors document statistically significant negative returns for stocks on average for Mondays compared to other trading days while also noting that returns across different trading days in the week also differ in some sample periods. Similar findings are also seen in Lakonishok and Smidt (1988) who perform analogous studies using the Dow Jones Industrial Average (DJIA) for the period 1897 to 1986 . On average, they note statistically significant negative returns on Mondays for most of the sample periods analyzed. Siegel (1998) also confirms these findings of low returns on Mondays compared to other trading days for the DJIA using data from the year 1885 through 1997.

Cross (1973) is one of the earliest and widely cited paper which studies the difference in stock market returns for distinctive trading days of the week. Using data from 1953 through 1970, this study finds that on average, price changes on Mondays following positive returns on Fridays are significantly distinctive when compared to other trading days of the week. This finding also holds during a study of price changes on Mondays following a negative return in the previous Friday. Precisely, Cross (1973) finds that on average, following positive returns the previous Friday, investors make gains on Monday only $48.8 \%$ of the time but make gains $63.9 \%$ of the time for other preceding trading days of the week. Similarly, when there is a negative return on the previous Friday, investors on average make gains only $24 \%$ of the time on Monday compared to $63.9 \%$ for other succeeding days of the week. Thus, Mondays are seen to be mostly characterized by average negative returns.

### 2.2. Literature Gap

Findings from these previous studies are very insightful for understanding the characteristics of the VIX and the market. These previous studies have largely explored the relationship between the VIX and market returns or volatility. Some of these studies also investigate the characteristics of returns or volatility during non-trading hours (overnight) and for different trading days of the week.

However, these previous studies are yet to synthesize the relationship between changes in the VIX and the range of the market index. Previous studies are also yet to uncover the information content of changes in the VIX during non-trading hours compared to trading hours. There are also no studies on the information content of changes in the VIX for different trading days of the week. Thus, these gaps necessitate the need for our thesis.

This thesis bridges the following gaps in the existing literature. Firstly, we model the relationship between the VIX and the range of the index; with the latter as our proxy for realized market volatility. This is vital as previous literature have extensively modeled the relationship between the VIX and market returns.

We also uncover how realized market volatility reacts to different directions of changes in VIX. We consider positive and negative changes in the daily closing values of the VIX. This part of this thesis is important as it provides more information on what specific direction of changes in the VIX is more informative for realized market volatility.

Notably, we also model the usefulness of changes in the VIX during non-trading hours and changes in the VIX during trading hours for market participants. This is crucial as results from these analyses provide information on the practical implications of overnight changes in the VIX compared to changes in the VIX during the day.

Lastly, we investigate how the relationship between the range of the market index and the VIX may differ depending on the trading day of the week. This is critical to understand how the information content and the usefulness of the VIX differ for each trading day of the week.

## 3. HYPOTHESES AND MODEL SPECIFICATION

The Chicago Board Options Exchange proposes that "the VIX calculation measures 30-day expected volatility of the S\&P 500 Index" (The CBOE Volatility Index - VIX; 2015: Pg. 5). Several studies including Fleming et al. (1995), Whaley (2000), Bekaert and Hoerova (2014) show that the VIX has a statistically significant relationship with present market returns or volatility. The VIX is also shown to have a statistically significant relationship with future volatility; up to 30 days.

Particularly, Fleming et al. (1995), Giot (2005), Banerjee et al. (2007) and Bekaert and Hoerova (2014), view the VIX as a useful tool for forecasting market returns or volatility; up to 30 days. Based on these studies, the VIX is expected to serve as a tool for predicting future events based on market participants' present behavior. However, some studies including Smales (2016) provide some evidence of a contemporaneous relationship between the VIX and market returns while Kozyra and Lento (2011) notes that the relationship between the VIX and market returns are more pronounced during periods of increased volatility.

Findings from these previous studies are very insightful and provide evidence of some relationship between the VIX and market activities. Building on these findings, we change the response variable in our analyses to further explore the relationship between the VIX and the daily range of the market index.

Thus, we test the relationship between the current values of the VIX and the current range of the market index. Furthermore, we test the relationship between past values of the VIX and the current range of the market index. We also evaluate if the relationship between the VIX and the range of the market index differs depending on whether the changes in the VIX are positive or negative.

Other studies additionally provide some evidence of special characteristics for market returns or volatility during non-trading hours or on specific trading days of the week. Branch \& Ma (2012), Kelly \& Clark (2011), and Hong \& Wang (2000) note significant differences in average returns during non-trading hours and average returns during trading hours. Stoll \& Whaley (1990) notes that on average, weekly market volatility is at its lowest point on Wednesdays. Cross (1973), Gibbons and Hess (1981), Keim and Stambaugh (1984) note that average returns in US markets are negative on Mondays. Later studies
including Brusa et al. (2000), Mehdian and Perry (2001) document positive average returns in US markets for Mondays. Berument \& Dogan (2012) also note that on average, more informed trading is carried out on Mondays compared to other trading days of the week. However, Morey and Rosenberg (2012) and Olsen et al. (2015) provide some evidence to the non-existence of special return characteristics.

Findings from these previous studies are very enlightening and point towards different characteristics for market returns or volatility during non-trading hours and certain trading days of the week. We build on these findings to model how the VIX relates to market activities during non-trading hours and each trading day in the week. Thus, we model the relationship between the VIX and the range of the market index during non-trading hours compared to trading hours. We also test if any identified relationship between the VIX and the range of the market index differs depending on the trading day of the week.

### 3.1. Hypotheses Statements

This thesis examines the proposition that daily changes in the VIX contain some beneficial information that market participants can use in predicting realized volatility in upcoming market sessions. Several studies investigate this proposition and find a statistically significant relationship between past values of the VIX and realized market volatility or returns (Bekaert \& Hoerova, 2014; Banerjee et al., 2007; Giot, 2005; Fleming et al. 1995). Other studies also note that the daily range of prices in the market is an efficient measure of realized volatility in the market (Martens \& Van Dijk, 2007; Brandt \& Diebold, 2006; Parkinson, 1980).

Fleming et al. (1995) emphasize that the VIX contains information related to expectations of market participants. This information content of the VIX is advanced by Fleming et al. (1995) as a possible explanation for the significant relationship seen between the VIX and future volatility in the market in their studies. This significant relationship is due to changes in the bid and ask prices of the options used in the computation of the VIX. These changes in option prices represent market participants’ expectation for future market activities. These price changes in turn affect the value of the VIX. Fleming et al. (1995) notably recommends the VIX as a suitable tool for predicting market volatility up to 30 days.

Similarly, Banerjee et al. (2007) notes that the positive relationship between future market returns and the current value of the VIX in their study is due to a negative market price of volatility risk. This negative volatility risk premium is also consistent with the negative relationship between contemporaneous market returns and the current values of the VIX. This is possibly due to the unwillingness of risk-averse investors to hold too volatile assets in their portfolios. Risk-averse investors are typically willing to trade off more volatile assets in exchange for less volatile assets.

Although findings from these previous studies are very insightful, a significant portion of these previous studies has majorly modeled the relationship between the VIX and market returns without exploring the relationship between changes in the daily values of the VIX and daily realized market volatility. Building on these studies, we change the response variable by examining the relationship between daily changes in the VIX and the daily range of the market index. We utilize the daily market index range as our proxy for daily realized market volatility. We investigate if daily changes in the VIX up to 30 days ago have any impact on the current range of the market index. Therefore, our first hypothesis is stated as follows.

Hypothesis 1: Ceteris paribus, changes in the VIX have an impact on the daily range of the market index

Additionally, this thesis investigates the proposition that an increase in the VIX provides a greater signal for an upcoming increase in the range of the market index, compared to a decrease in the VIX. An early study by Kahneman and Tversky (1979) provides some motivation for this hypothesis; this study explores the behavior of market participants from the realm of behavioral finance by developing what is widely referred to as the 'prospect theory.'

The prospect theory relates the decision making of market participants to behavior biases which cause these participants to make irrational choices, in stark contrast to expected utility theory which posits that market participants are rational. This prospect theory notes that market participants evaluate their decision making by the probable profit or probable loss for each trading activity, and not on the possible
joint outcome of the trading activity. Notably, the value function for losses is convex-shaped while the value function for profits are concave-shaped, with the former having a steeper slope. Thus, market participants have a significantly different attitude to the potential profit or potential loss within any investment decision.

Kahneman and Tversky (1979) further emphasize that this isolated evaluation of profit or loss brings about cognitive biases in market participants when considering potential gains compared to potential losses; thus, causing market participants to make irrational investment decisions. These findings are very revealing and provide the main motivation for our second hypothesis. We conduct similar analyses using an updated data set and methodology to uncover possible key differences in market participants' interpretation of an increase in the VIX compared to a decrease in the VIX. We evaluate how market participants' behavior differ regarding an increase in the VIX compared to a decrease in the VIX.

More studies including Kozyra and Lento (2011) notably find that the relationship between the VIX and market returns are significantly higher during periods of increased volatility compared to periods of reduced volatility. On average, the VIX has significant increases during periods of increased volatility compared to periods or reduced volatility. This finding points to some possible distinction in the way the market reacts to increments in the VIX compared to decrements in the VIX. This distinction in the nature of the relationship between the VIX and market returns are majorly due to the significantly larger changes seen in the VIX during periods of increased volatility compared to periods of reduced volatility.

Other studies also highlight contagion among different stocks during periods of increased volatility (Mollah et al.; 2016, Bekaert et al.; 2014, and Capiello et al.; 2006). These studies note significant increments in correlations between different markets during periods of increased volatility, compared to periods of decreased volatilities. Increased volatility is seen to have a more significant impact on synchronized stock performances compared to reduced volatility. Building on this finding, we employ an updated dataset to investigate how increases in implied volatility have a different impact on realized market volatility compared to the impact decreases in the VIX have on realized market volatility.

Although, findings from these earlier studies are very remarkable; these studies mostly model the relationship between the VIX and market returns. We extend these analyses to study how the signals
between positive changes in the VIX and negative changes in the VIX significantly differ concerning the daily range of the market index. This analysis is essential to understand what directions of changes in the VIX is critical for realized volatility in the market. We investigate if positive changes in the VIX provide a clear signal of an increase in the range of the market index, compared to negative changes in the VIX.

Thus, our second hypothesis is stated as follows.

Hypothesis 2: Ceteris paribus, the magnitude of the relationship between positive changes in the daily closing value of the VIX and changes in the daily range of the market index differs from the magnitude of the relationship between negative changes in the daily closing value of the VIX and changes in the daily range of the market index.

Thirdly, in a major contribution to existing literature, this thesis explores the predictive power changes in the VIX during non-trading hours (overnight changes in the VIX) may have regarding upcoming market volatility. We investigate differences in the relationship between the VIX and the range of the market index during non-trading hours and this relationship during trading hours.

Notably, Branch \& Ma (2012), Kelly \& Clark (2011), and Hong \& Wang (2000) all find statistically significant differences in market returns during trading hours and market returns during nontrading hours. Branch and Ma (2012) notes a strong negative correlation between overnight returns (returns during non-trading hours) and returns during the day (returns during trading hours).

Branch and Ma (2012) attributes the negative autocorrelation between returns during non-trading hours and returns during trading hours to some factors including; the behavior of market-makers. During periods of imbalances in orders to buy and sell in the market, market makers have profitable reasons to set the opening price of a stock above or below its closing price for the previous trading day. Branch and Ma (2012) note that such moves by market makers often pushes the stock price away from its equilibrium price during the opening hours of trading. However, as trading volume increases later in the day, the price of the stock usually returns to its intrinsic worth. These series of changes in the market price are
responsible for the negative relationship between returns during non-trading hours and returns during trading hours.

Similarly, Kelly \& Clark (2011) attributes the negative relationship between returns during nontrading hours and trading hours to the activities of day traders ('semiprofessional' traders) in the market. This study notes that trades by day traders make up a significant portion of the total trading volume in the US markets. These traders tend to have undiversified portfolios and are usually concerned about negative news that may affect their portfolio overnight. Thus, these day traders frequently liquidate a significant portion of their undiversified portfolio during the closing hours of trading and restore their positions at the start of the next trading period, to manage volatility in their portfolios. This increased selling and buying during the market closing and opening hours are potential explanations for the differences noted in returns during non-trading hours compared to trading hours.

Also, earlier studies by Stoll \& Whaley (1990) provide some reasons for the differences in the characteristics of returns during non-trading hours compared to trading hours. This study links the greater volatility observed during opening periods in the US markets to the presence of private information acted upon by traders during this opening trading hours. This private information can prompt increased buying and selling by traders during the opening period. Thus, market prices are more likely to show a reversal around the opening period of trading compared to the closing period of trading. Additionally, Stoll \& Whaley (1990) note that market volatility during the day is significantly higher than market volatility at night due to the large amount of public information made available during trading hours compared to the reduced public information available during non-trading hours.

While these findings are interesting, it is very useful to conduct similar analyses on the relationship between the VIX and the range of the market index. We closely explore if overnight changes in the VIX have any predictive power regarding the upcoming range of the market index. We also compare the predictive ability of overnight changes in the VIX (non-trading hours) and the predictive ability of day changes in the VIX (trading hours) concerning the upcoming range of the market index. Thus, our third hypothesis is as follows.

Hypothesis 3: Ceteris paribus, changes in the VIX during non-trading hours have predictive power regarding the upcoming range of the market index.

Lastly, this thesis examines the relationship between the VIX and the range of the market index for different trading days of the week. Cross (1973), French (1980), Gibbons and Hess (1981), Lakonishok and Smidt (1988) find that average returns are negative on Mondays compared to other trading days of the week while Brusa et al. (2000) find that these average returns on Mondays are positive and higher than other days of the week. These studies note that the negative returns on Mondays are largely due to the 'weekend effect' in stock markets. On average, market participants have amassed information over the weekend and tend to act on this large amount of information when trading resumes on Mondays. This action by market participants tends to drive opening market prices on Mondays away from previous Friday closing prices.

In contrast, Berument \& Kiymaz (2001) notes that mean returns are highest on Wednesdays while market volatility is also lowest on Wednesdays. On average, Fridays have the highest level of uncertainty in the market. This study attributes the high level of uncertainties on Fridays to the high inclination for 'bad news' about companies to be released around the weekend. Typically, investors trade based on this 'bad news' around the weekend, so they tend to take this 'expected bad news' into consideration when trading on Fridays. Additionally, economic news releases are typically on Thursdays and Fridays; this adds further uncertainties to trades carried out around Fridays.

Berument \& Kiymaz (2001) interestingly notes that the low volatility seen in the market on Wednesdays is possibly due to the fact that Wednesdays falls perfectly in the middle of the trading week. Thus, market participants on Wednesdays are acting on information accumulated in the last two trading days (Monday and Tuesday) and information forecasted for the next two trading days (Thursdays and Fridays). Thus, on Wednesdays, market participants have a large amount of information and reasonable time to make decisions based on this information. Other studies including Morey and Rosenberg (2012) and Olson et al. (2015) however find the non-existence of specific return characteristics for different trading days of the week.

While these findings are remarkable, results from some of these studies are conflicting. Hence, we carry out further tests with a different response variable. We test if the relationship between the VIX and the range of the market index differ depending on the trading day of the week. The trading days considered are Mondays, Tuesdays, Wednesdays, Thursdays, and Fridays. The motivation is to study if changes in the VIX are more informative on some trading days. Therefore, our fourth hypothesis is as follows.

Hypothesis 4: Ceteris paribus, the relationship between the VIX and the range of the market index is not the same for each trading day of the week.

## 4. Variables and Methodology

Kanas (2014), Kelly \& Clark (2011) and Becker et al. (2007), utilize GARCH models in studying market volatility or returns. In line with these studies, we employ a $\operatorname{GARCH}(1,1)$ model to examine the relationship between the VIX and the range of the market index.

Fleming et al. (1995) documents a statistically significant relationship between values of the VIX and market returns. In this study, the VIX is also viewed as a useful tool for forecasting realized market volatility. Similar use of the VIX for forecasting realized market returns and volatility is proposed by Bekaert and Hoerova (2014). Based on these studies, we include present and past values of the VIX to model its relationship with the present-day range of the market index.

Additionally, previous studies like Fleming (1998), Fernandes et al. (2014), and Bekaert and Haerova (2014) document the effect of key variables including interest rate and trading volume on the macroeconomy and overall market performance. In line with these studies, we include interest rate and trading volume as control variables in our model to control for the effects these variables may have on returns and volatility within the study period.

Further works by Berument \& Kiymaz (2001) note that market volatility in the US is lowest on Wednesdays. Other similar studies including Zhang et al. (2017), Olson et al. (2015) note special return characteristics for specific trading days of the week. Thus, we model the relationship between the VIX and the range of the market index for different trading days of the week.

Additionally, Olson et al. (2015) notes special characteristics for returns during financial crises. There are notable crises times within our study period. Following these studies by Olson et al. (2015), we create dummy variables for these crises' periods within our study: the Asian Financial Crisis, Russian Financial Crisis, the Dot.com Crash, and the 2007-2009 Global Financial Crisis.

The VIX is our measure of implied volatility. We model the relationship between the daily closing values of the VIX and the daily range of the market index. We likewise model the different impact positive and negative changes in the VIX may have on the daily range of the market. We also split observations of the VIX to separately model the observations of the VIX during non-trading hours and observations of the VIX during trading hours, and their impact on the range of the market index. Lastly,
we model how the relationship between the VIX and the range of the market index may differ for different trading days of the week. We fit a $\operatorname{GARCH}(1,1)$ to the mean regression of the range of the market index on past and present values of VIX while also controlling for interest rate, trading volume, and different crises periods.

### 4.1. Variable Description

In our models, we utilize the percentage change for each variable in the regression analyses. We compute percentage change in each variable by taking the difference between the value of the variable at 'time t ' and 'time $\mathrm{t}-1$ ', divided by the value of the variable at 'time $\mathrm{t}-1$ '.

The Range of the Market Index: This is the main dependent variable in our models. The range of the market index is a daily series. Similar to Andersen et al. (2005), we calculate the range of the market index as the daily difference between the highest point of the S\&P 500 index and the lowest point of the S\&P 500 index. The range of the market index is our proxy for realized market volatility. Previous works including Bekaert and Haerova (2014), Fernandes et al. (2014), Smales (2016) utilized S\&P 500 index returns in their studies. Our thesis builds on this by utilizing the daily range of the S\&P 500 index.

Our use of the range as a proxy for realized market volatility is largely built on earlier findings by Martens \& Van Dijk (2007), Brandt and Diebold (2006), and Parkinson (1980). Martens \& Van Dijk (2007) analyses the characteristics of the daily price of stocks in the S\&P 100. This study compares the efficiency in the use of the daily range of stock prices (low-frequency data) as a measure of realized market volatility vis-à-vis the use of intra-day realized price variance (high-frequency data) as a measure of realized market volatility. They find that due to popular microstructure noise in market data, the low frequency data (realized daily range) is more efficient as a measure for realized market volatility compared to high frequency data (intra-day realized price variance). This finding also holds when market frictions such as discontinuous trading are considered in the analyses. These results are notably more pronounced for actively traded stocks.

Parkinson (1980) earlier noted that the daily range of stock prices is a more efficient estimator of daily volatility compared to daily squared returns. This daily range of the market is an unbiased estimator of daily volatility in the market. These findings also hold when the daily range is compared to the squared value of intra-day returns. Similar findings are also seen in Brandt and Diebold (2004) who carries out comparable studies using an updated dataset.

Based on these findings by previous studies and the absence of intra-day data in our thesis, we employ the daily range of the S\&P 500 index as our measure of realized volatility.

We also analyze the relationship between the range and the daily variance of stock market returns in the market. The variance is defined as the square of the standard deviation of daily returns in the market. We calculate variance from the one-day return and the ten-day return in the US markets. We compute the variance of the one-day return as the square of the difference between today's return, and the average return for the last one day. Similarly, we calculate the variance of the ten-day return as the square of the difference between today's return, and the average return for the last ten days.

We find a strong, positive, and statistically significant correlation between the daily range and the daily variance of stock market one-day returns throughout our study period. This result is shown in Table 4.1. This positive correlation is also seen in our analyses of the relationship between the daily range of the market index and the ten-day variance of stock market returns. These results are shown in Table 4.1.

## Table 4.1. CORRELATION COEFFICIENTS (RANGE AND RETURNS)

## Pearson Correlation Coefficients Between the Daily Range of the S\&P 500 Index and Daily Variance of Returns for the S\&P 500 Index

This table presents results for correlation between the Daily Range of the S\&P 500 Index and Daily Variance of Returns for the S\&P 500 Index. The daily range is the difference between the daily highest value and daily lowest value of the S\&P 500 index. We compute the variance of the one-day return as the square of the difference between today's return, and the average return for the last one day. Similarly, we calculate the variance of the ten-day return as the square of the difference between today's return, and the average return for the last ten days.

We show the correlation between the daily range of the S\&P 500 Index and variance of one-day returns for the S\&P Index. We also show the correlation between the daily range of the S\&P 500 Index and variance of ten-day returns for the S\&P Index. ${ }^{* * *}$, ${ }^{* *}$, and * indicate $1 \%, 5 \%$ and $10 \%$ significance levels respectively.

|  | Pearson Correlation Coefficients |  |
| :--- | :---: | :---: |
|  | Variance of One-day Returns for the S\&P <br> $\mathbf{5 0 0}$ | Variance of Ten-day Returns for the <br> S\&P 500 |
| Range | $0.49^{* * *}$ | $0.55^{* * *}$ |

Based on the statistical significance of the strong positive correlation between the daily range of the index and the daily variance of the stock market returns, the former is a reasonable proxy for realized volatility in the US market.

Volatility Index (VIX): This is the main variable of interest in our models. The VIX is a daily series. We use the Chicago Board Options Exchange Volatility Index as our measure of implied volatility. We also split these changes in the VIX into changes that occur during non-trading hours (overnight changes; i.e., the current day opening value of the VIX less the previous day closing value of the VIX) and changes that occur during trading hours of the day (the current day closing value of the VIX less current day opening value of the VIX).

Interest Rate: This is a control variable in our models. The interest rate is a daily series. Similar to Fleming (1998), we control for the impact of interest rates on market activities. We use the US Effective Federal Funds Rate to represent interest rate in our study.

Trading Volume: The total daily trading volume in the market is another control variable in our model. This variable is the daily trading volume of all stocks that make up the S\&P 500 index. The inclusion of the trading volume in our models in line with previous findings by Fernandes et al. (2014).

Information for other key variables in our analyses as follows:
Kozyra and Lento (2011) note that the relationship between the VIX and market returns are more pronounced during periods of increased financial or economic crises. Based on these, we adopt different dummy variables as shown below to control for notable crises times during our study periods. We include these variables in our regression to control for differences in the relationship between the VIX and the range of the market index during periods of financial or economic crises.

Dummy variable 1 ( $\mathbf{d}_{\mathbf{1}}$ ): observations from 02 July 1997-31 October 1998; both dates inclusive. This represents the Asian Financial Crisis.

Dummy variable 2 ( $\mathbf{d}_{\mathbf{2}}$ ): observations from 01 November 1998-31 December 1998; both dates inclusive. This represents the Russian Financial Crisis.

Dummy variable 3 (d $\mathbf{d}_{3}$ ): observations from 01 March 2001 - 30 November 2001; both dates inclusive. This represents the "Dot.com" crash.

Dummy variable 4 ( $\mathbf{d}_{4}$ ): observations from 01 December 2007 - 30 June 2009; both dates inclusive. This represents "the 2007-2009 Global Financial Crisis".

Non-recessionary Period (d0): observations between 04 January 1993 and 01 July 1997, 01 January 1999 and 28 February 2001, 01 December 2001 and 30 November 2007, 01 July 2009 and 29 December 2017.

Several studies including Zhang et al. (2017), Mehdian and Perry (2001) note the existence of some special return characteristics for some trading days of the week. Thus, we adopt dummy variables as shown below to account for different trading days of the week, and directions of changes in the VIX.

Dummy Variable 11 ( $\mathbf{d}_{11}$ ): Dummy variable representing positive changes in the daily closing value of the VIX.

Dummy Variables 21-24 (d $\mathbf{d}_{21-24}$ ): Dummy variable representing the changes in the VIX that occur respectively on Mondays, Tuesdays, Thursdays, and Fridays.

### 4.2. Model Specification

We run $\operatorname{GARCH}(1,1)$ models to study the relationship between realized volatility and the VIX. These models incorporate the effect past daily changes in the VIX may have on present-day realized volatility. These models also help incorporate the effect of present, past changes in the VIX during nontrading hours, trading hours and specific trading days of the week have on realized market volatility.

Similar to previous works by Kanas (2014), Kelly \& Clark (2011), we utilize the generalized autoregressive conditional heteroskedasticity (GARCH) model instead of the ordinary least square (OLS) model due to the absence of some fundamental OLS assumptions in our daily data set. Particularly, for our data set, an OLS model would not account for possible autocorrelations in the error terms, as well as dependence in the variance of the error terms in our model. Thus, the OLS estimators for our model would be misleading since our dataset does not satisfy all the required OLS assumptions. Thus, in line with previous studies, we utilize the GARCH model for our hypotheses testing. The GARCH $(1,1)$ model is useful here as it helps integrate the autoregressive term and the conditional variance of the error term in our model. The GARCH model is essential to control for autocorrelations in our error terms which would make inferences carried out based on ordinary least square estimations misleading. The GARCH model also accounts for dependence in the variance of the error terms.

The GARCH $(1,1)$ models for testing our different hypotheses are specified below.

First Model Specification - Equation (4.1):

$$
\begin{align*}
\Delta \text { Range }_{\mathrm{t}}= & \text { Constant }+\sum_{i=0}^{-5} \alpha_{i} \Delta V I X_{i}+\alpha_{-10} \Delta V I X_{-10}+\alpha_{-20} \Delta V I X_{-20}+\alpha_{-30} \Delta V I X_{-30}  \tag{4.1}\\
& + \text { CONTROL VARIABLES }+\mu_{\mathrm{t}} .
\end{align*}
$$

where;
$\Delta$ Range $=$ Percentage change in the daily range of the S\&P 500 index
$\Delta$ VIX $=$ Percentage change in the daily closing value of the Volatility Index
control variables:
$\Delta \mathrm{INT}=$ Percentage change in interest rate
$\Delta$ VOLUME $=$ Percentage change in trading volume
$d_{1}, d_{2}, d_{3}$, and $d_{4}=$ Dummy variables representing the Asian Financial Crisis (1997-1998), Russian Financial Crisis (1998), Dot.com Crash, and 2007-2009 Global Financial Crisis respectively
$\mu_{t}=\sqrt{ } \psi_{t}, \varepsilon_{t}$
where:
$\varepsilon_{t}$ is a random variable which is i.i.d with zero mean and unit variance, and
$\psi_{\mathrm{t}}=\alpha_{0}+\sum_{i=1}^{P} \alpha_{i} \mu_{t-i}^{2}+\sum_{j=1}^{q} \beta_{j} \psi_{t-j}$
$\alpha_{0}>0, \alpha_{i} \geq 0, \beta_{j} \geq 0$

Using equation (4.1), we test our first hypothesis to ascertain if current and past changes in values of the VIX have any significant relationship with changes in the current range of the market index. We examine previous values of the VIX as far back as thirty trading days. Present realized market volatility is proxied by the range of the index.

In all our models, we control for the effect of changes in interest rate and trading volume. We also control for these key financial or economic crises: 'the 1997-1998 Asian Financial Crisis', 'the 1998 Russian Financial Crisis', 'the dot.com crisis' and 'the 2007-2009 Global Financial Crisis'.

In our models, we anticipate a positive relationship between current changes in the VIX and realized volatility (the range of the market index) since Giot (2005) notes a similar relationship between contemporaneous changes in the VIX and market returns. Thus, the coefficient for contemporaneous
changes in the VIX is expected to be positive in equation 4.1. Also, similar to findings by Fleming et al. (1995), we expect a positive relationship between current changes in the range of the market index and past changes in the VIX in our models. Thus, $\alpha_{i}$ is expected to be positive in equation 4.1.

Additionally, based on findings by Fleming et al. (1995) and Marc (2000), we anticipate a positive relationship between changes in the control variables in our models (interest rate, trading volume) and changes in the range of the market index in our models.

Kozyra and Lento (2011) documents a more pronounced relationship between the VIX and market returns during crises period. Thus, based on this finding, we expect a positive relationship between the categorical variables for crises periods in our study and changes in the range of the market index in our models. We anticipate a positive sign for the coefficients of the dummy variables for crises periods in our studies.

Second Model Specification - Equation (4.2):
$\Delta$ Range $_{\mathrm{t}}=$ Constant $+\alpha_{1} \Delta \mathrm{VIX}_{0}+\alpha_{2} \mathrm{POS}+\alpha_{3} \Delta \mathrm{VIX}_{0} * \mathrm{POS}+\mathrm{CONTROL}$ VARIABLES $+\mu_{\mathrm{t}}$
where;
POS = Dummy variable representing positive changes in the daily closing value of the VIX

Other variables are as defined in Equation 4.1.

We utilize equation (4.2) to test our second hypothesis. We investigate the relationship among positive changes in the daily closing values of the VIX, negative changes in the daily closing values of the VIX and changes in the range of the market index. Changes in the daily closing value of the VIX is a binary dummy variable; negative changes in the daily closing value of the VIX is the base category. We
interact these positive changes in the daily closing value of the VIX with changes in the daily closing values of the VIX in our model. We also utilize the same control variables explained in equation (4.1).

Similar to Banerjee et al. (2007), we expect a positive relationship between positive changes in the VIX and changes in the range of the market index. We expect a positive coefficient for the interaction term (positive changes in the VIX and changes in the daily value of the VIX). Thus, the value for $\alpha_{3}$ is expected to be positive. $\alpha_{2}$ may be positive or negative since it is the intercept for the dummy variable representing positive changes in the closing value of the VIX. $\alpha_{1}$ is expected to be negative since it is the slope for negative changes in the daily closing values of the VIX. The expected signs for other variables are as specified for equation 4.1.

Third Model Specification - Equation (4.3):

$$
\begin{align*}
& \Delta \text { Range }_{\mathrm{t}}=\text { Constant } \beta_{1} \Delta \text { VIXoc }_{0}+\beta_{2} \text { VIXoc }_{-1}+\beta_{3} \Delta \text { VIXoc }_{-2}+\beta_{4} \Delta \text { VIXoc }_{-3}+\beta_{5} \Delta \text { VIXoc }_{-4} \\
& +\beta_{6} \Delta \text { VIXoc }_{-5}+\beta_{7} \Delta \text { VIXoc }_{-6}+\beta_{8} \Delta \text { VIXoc }_{10}+\beta_{9} \Delta \text { VIXoc }_{-20}+\beta_{10} \Delta \text { VIXoc }_{-30} \\
& +\beta_{11} \Delta \text { VIXco }_{0}+\beta_{12} \Delta \text { VIXco }_{-1}+\beta_{13} \Delta \text { VIXco }_{-2}+\beta_{14} \Delta \text { VIXco }_{-3}+\beta_{15} \Delta \text { VIXco-4 }_{-4} \\
& +\beta_{16} \Delta \text { VIXco }_{-5}+\beta_{17} \Delta \text { VIXco }_{-10}+\beta_{18} \Delta \text { VIXco }_{-20}+\beta_{19} \Delta \text { VIXco }_{-30} \\
& + \text { CONTROL VARIABLES }+\mu_{\mathrm{t}} \tag{4.3}
\end{align*}
$$

where;
$\Delta \mathrm{VIXco}=$ Change in the difference between the daily closing and opening value of the VIX (change in the VIX during trading hours)
$\Delta$ VIXoc $=$ Change in the difference between the current day opening value and the previous day closing value of the VIX (change in the VIX during non-trading hours)

Other variables are as defined in Equation 4.1.

Our third hypothesis is tested using equation (4.3) above. We test the predictive power of changes in the VIX during non-trading hours (overnight changes in the VIX) on the range of the market index. Likewise, we also examine the predictive power of changes in the VIX during trading hours on the range of the market index. We test changes in the VIX during non-trading hours as far as 30 days ago. We also carry out the same tests on changes in the VIX during trading hours. We include the same control variables used in equation 4.1 to account for the effect macroeconomic factors and financial crises periods may have on our results.

Similar to Fleming et al. (1995), we expect a positive relationship between past changes (and current changes) in the VIX during non-trading hours and changes in the current range of the market index. Similarly, we anticipate a positive relationship between previous changes (and current changes) in the VIX that occur during trading hours and changes in the current range of the market index. Thus, $\beta_{1}$ through $\beta_{19}$ are expected to be positive. The expected signs for other variables are as specified in equation 4.1.

Fourth Model Specification - Equation (4.4):

$$
\begin{align*}
\Delta \text { Range }_{\mathrm{t}}= & \text { Constant } \alpha_{1} \Delta \mathrm{VIX}_{0}+\alpha_{2} \mathrm{MON}+\alpha_{3} \text { TUES }+\alpha_{4} \text { THURS }+\alpha_{5} \text { FRI } \\
& +\alpha_{6} \mathrm{MON}^{*} \Delta \mathrm{VIX}_{0}+\alpha_{7} \text { TUES }^{*} \Delta \text { VIX }_{0}+\alpha_{8} \text { THURS }^{*} \Delta \mathrm{VIX}_{0}+\alpha_{9} \mathrm{FRI}^{*} \Delta \mathrm{VIX}_{0} \\
& + \text { CONTROL VARIABLES }+\mu_{\mathrm{t}} \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \tag{4.4}
\end{align*}
$$

where;

MON, TUES, THURS, and FRI = Dummy variable representing changes in the VIX that occur on Mondays, Tuesdays, Thursdays, and Fridays respectively (Base category: changes in the VIX that occur on Wednesdays)

Other variables are as defined in Equation 4.1.

We make use of equation (4.4) in the test of our fourth hypothesis. We test if the relationship between the VIX and the range of the market index is the same for each trading day of the week.

Following studies by Berument \& Kiymaz (2001) which note that market volatility is lowest on Wednesdays, we add dummy variables to account for different trading days of the week with Wednesday as the base category.

In equation 4.4, we compare the relationship between changes in the VIX and changes in the range of the market index for each trading day with this relationship on Wednesdays. The goal here is to ascertain if the relationship between the VIX and the range of the market index on each trading day of the week (Mondays, Tuesdays, Thursdays, and Fridays) is statistically different from this relationship on Wednesdays. The control variables in equation (4.4) are as specified in equation (4.1).

In line with Olson et al. (2015), we expect a positive relationship between changes in the VIX and changes in the range of the market index for each trading of the week. We also expect volatility on other trading days to be statistically different from volatility on Wednesdays. Therefore, $\alpha_{6}$ through $\alpha_{9}$ are expected to be positive. $\alpha_{2}$ through $\alpha_{5}$ may be positive or negative since these are the intercept terms. The anticipated signs for other variables are as specified in equation 4.1.

## 5. DATA

### 5.1. Data Source and Data Preparation

We obtain daily data for the VIX from the Chicago Board Options Exchange (CBOE) for the study period: 1993 through 2017. We obtain the daily value of The Standard \& Poor's 500 (S\&P 500) index and trading volume from Wharton Research Data Services.

We get data for our control variables (interest rates and trading volume) for the study period from the Federal Reserve Bank of St. Louis.

### 5.2. Note on VIX and VXO Comparison

For the sake of robustness, we also perform a comparison of the information content of the Chicago Board Options Exchange S\&P 100 Volatility Index (VXO) and the Chicago Board Options Exchange S\&P 500 Volatility Index (VIX). The VXO is the original CBOE volatility index for the U.S. stock market, first created in 1993. In 2003, the CBOE replaced the VXO with the VIX as the primary yardstick for US stock market volatility. The VIX is often featured in prominent business news outlets and financial publications and referred to as the "fear gauge." The CBOE still publishes data for the VIX and the VXO daily on its website (http://www.cboe.com/products).

The VIX and the VXO are expected to be measures of implied market volatility for the next 30 days. However, the VXO is computed using the near-term options of the S\&P 100 stocks while the VIX is computed using the near-term options of the S\&P 500 stocks. We compare the daily time series data for the VIX and the VXO to ascertain if there are major differences between both series.

We run correlation analysis between the VIX and VXO throughout our study period. This result is shown in Table 5.1. We find a strong positive statistically significant correlation between the VIX and the VXO. The numerical value of this correlation is approximately 1.0. Based on this perfect correlation, both the VIX and VXO series are barely different.

## Table 5.1. CORRELATION COEFFICIENTS (VIX AND VXO)

## Pearson Correlation Coefficients Between the Daily VIX and Daily VXO

This table presents results for correlation between the Daily VIX and VXO. The VIX and the VXO are volatility indices for the US market. The VIX is computed is based on options of the S\&P 500 while the VXO is computed based on options of the S\&P 100.
***, **, and $*$ indicate $1 \%, 5 \%$ and $10 \%$ significance levels respectively.

|  | Pearson Correlation Coefficients |  |
| :--- | :---: | :---: |
|  | VIX | VXO |
| VIX | 1 |  |
| VXO | $0.99^{* * *}$ | 1 |

We also carry out further analyses between the VIX and VXO by rescaling the VXO data at the start of the study period, so the first observations of the VXO and VIX are the same. We then compare changes in the VXO and VIX overtime. We show some of these changes graphically in Graphs 5.1 and 5.2. The changes captured by the VXO are also captured by the VIX throughout our study period.

Graph 5.1. VIX vs VXO (January 1995 - December 1999)


Graph 5.2. VIX vs VXO (January 2015 - December 2017)


This comparison shows that the information contained in the VIX and VXO are very similar. Changes in expected market volatility as observed in the VIX are also captured by the VXO and vice versa. Graph 5.1 and Graph 5.2 show the level of the VIX and VXO at selected times in our study period. The VIX and VXO are shown to have very similar information about market volatility.

## 6. RESULTS, INTERPRETATION AND IMPLICATIONS

### 6.1. Descriptive Statistics and Correlation Matrices

Table 6.1 presents the descriptive statistics for all variables in our model. There is a total of 6602 daily observations in the study period. The mean value of percentage changes in the daily range of the market index is about $16.73 \%$. The mean value of percentage changes in the daily value of the VIX is about $0.21 \%$. The mean value of percentage changes in the interest rate and trading volume is about $0.33 \%$ and $2.26 \%$ respectively.

## Table 6.1. SUMMARY STATISTICS

## Summary Statistics for Key Variables in the Model

This table presents the number of observations, mean, standard deviation, minimum and maximum for key variables in our model. The mean values are shown in percent.

| Summary Statistics |  |  |  |  | Mean |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Std Dev | Min | Max |  |  |  |
| Variable | $\mathbf{N}$ | Market | 6602 | 16.73 | 0.70 |
| Percentage Change in Daily Range of the Mark <br> Index | -0.81 | 9.09 |  |  |  |
| Percentage Change in Daily Closing Value of the <br> VIX | 6602 | 0.21 | 0.07 | -0.29 | 1.15 |
| Percentage Change in Trading Volume | 6602 | 2.26 | 0.39 | -0.96 | 26.52 |
| Percentage Change in Interest Rate | 6602 | 0.33 | 0.09 | -0.55 | 1.47 |

We also consider the correlation between the main variables in our model. The correlation coefficients among key variables are shown in Table 6.2. Correlation coefficients among key variables in our model are below 0.3 . Thus, there is no strong correlation between any variables in our models. We do not expect any bias arising from perfect multicollinearity in our estimated results.

Table 6.2. CORRELATION COEFFICIENTS (KEY VARIABLES)

## Pearson Correlation Coefficients for Key Variables in the Model

This table presents results for correlation among key variables in our model. We test for correlation among percentage changes in the: daily range of the market index, daily closing values of the VIX, trading volume, and interest rate. ${ }^{* * *},{ }^{* *}$, and $*^{\text {indicate }} 1 \%, 5 \%$ and $10 \%$ significance levels respectively.

| Pearson Correlation Coefficients |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: |
|  | $(1)$ | $(2)$ | $(3)$ | $(4)$ |
| (1) Percentage <br> Change in the Daily <br> Range of the <br> Market Index | 1 |  |  |  |
| (2) Percentage <br> Change in the Daily <br> Closing Value of the <br> VIX | $0.25^{* * *}$ | 1 |  |  |
| (3) Percentage <br> Change in the <br> Trading Volume | $0.20^{* * *}$ | $0.06^{* * *}$ | 1 |  |
| (4) Percentage <br> Change in the <br> Interest Rate | $0.03^{* * *}$ | $0.03^{* *}$ | 0.02 | 1 |

Changes in the current range of the market index are positively correlated with changes in the current value of the VIX. There is also a positive correlation between the range and the main control variables in our model.

### 6.2. Presentation of Results

### 6.2.1. Different Lags and Directions of Changes in the VIX

Firstly, we run equation (4.1) to ascertain the relationship between past and current changes in the daily closing value of VIX and current changes in the range of the market index. This result is shown in column (1) of Table 6.3. Similar to Fleming et al. (1995), Whaley (2000), Bekaert and Hoerova (2014), Tsai et al. (2015) and Kambouroudis et al. (2016); we find a statistically significant relationship between contemporaneous percentage changes in the daily closing value of the VIX and percentage changes in the range of the market index. On average, 100 percentage increase in the daily closing value of the VIX leads to a 246 percentage increase in the daily market range, holding other factors constant.

Table 6.3. (1st Hypothesis, Support Table):
GARCH $(1,1)$ Regression for Percentage Changes in the Daily Range of the Market Index on Percentage Changes in the Daily Closing Values of the VIX
This table presents results for the regressions of percentage changes in the daily range of the market index on percentage changes in the daily closing values of the VIX. The dependent variable in all columns is percentage change in the range of the daily market index. The independent variables are percentage changes in the daily closing value of the VIX at lags: $0,1,2,3,4,5,10,20$, and 30.
Column 1 shows the relationship among percentage changes in current and past values of the VIX, and percentage changes in the daily range of the market index. Our control variables are percentage changes in: interest rates and trading volume. We also include dummy variables to control for these crises' periods: Asian Financial Crisis, Russian Financial Crises, Dot.com Crash, and 2007-2009 Global Financial Crisis. Column 2 shows the contemporaneous relationship between percentage change in the range of the market index and percentage change in the VIX.
T-statistics are shown in the brackets. ${ }^{* * *}$, ${ }^{* *}$, and $*$ indicate $1 \%, 5 \%$ and $10 \%$ significance levels respectively.

|  | (1) | (2) |
| :---: | :---: | :---: |
| Percentage change in Daily Closing Value of the VIX | $\begin{aligned} & 2.460 * * * \\ & (31.47) \end{aligned}$ | $\begin{aligned} & \text { 2.484*** } \\ & (32.36) \end{aligned}$ |
| Percentage change in Daily Closing Value of the VIX at $\mathrm{t}_{-1}$ | $\begin{aligned} & -0.164 \\ & (-1.21) \end{aligned}$ |  |
| Percentage change in Daily Closing Value of the VIX at $\mathrm{t}_{-2}$ | $\begin{aligned} & -0.168 \\ & (-1.40) \end{aligned}$ |  |
| Percentage change in Daily Closing Value of the VIX at $\mathrm{t}_{-3}$ | $\begin{aligned} & -0.132 \\ & (-1.14) \end{aligned}$ |  |
| Percentage change in Daily Closing Value of the VIX at $\mathrm{t}_{-4}$ | $\begin{aligned} & -0.188 \\ & (-1.51) \end{aligned}$ |  |
| Percentage change in Daily Closing Value of the VIX at $\mathrm{t}_{\text {- }}$ | $\begin{aligned} & -0.129 \\ & (-1.02) \end{aligned}$ |  |
| Percentage change in Daily Closing Value of the VIX at $\mathbf{t}_{-10}$ | $\begin{array}{r} 0-0.166 \\ (-1.41) \end{array}$ |  |
| Percentage change in Daily Closing Value of the VIX at $\mathbf{t}_{-20}$ | $\begin{gathered} 0.094 \\ (0.88) \end{gathered}$ |  |
| Percentage change in Daily Closing Value of the VIX at $\mathrm{t}_{30}$ | $\begin{gathered} 0.073 \\ (0.61) \end{gathered}$ |  |
| Percentage change in Interest Rate | $\begin{aligned} & 0.183 * \\ & (1.91) \end{aligned}$ | $\begin{aligned} & 0.178^{*} \\ & (1.85) \end{aligned}$ |
| Percentage change in Trading Volume | $\begin{aligned} & 0.270 * * * \\ & (81.99) \end{aligned}$ | $\begin{aligned} & 0.276 * * * \\ & (83.27) \end{aligned}$ |
| Asian Financial Crisis | $\begin{aligned} & 0.008 \\ & (0.27) \end{aligned}$ | $\begin{aligned} & 0.006 \\ & (0.21) \end{aligned}$ |
| Russian Financial Crisis | $\begin{aligned} & 0.022 \\ & (0.31) \end{aligned}$ | $\begin{aligned} & 0.025 \\ & (0.34) \end{aligned}$ |


| Dot.com Crash | -0.015 | -0.018 |
| :--- | :--- | :--- |
|  | $(-0.39)$ | $(-0.45)$ |
| 2007 - 2009 Global Financial Crisis | -0.033 | -0.035 |
|  | $(-1.05)$ | $(-1.10)$ |
| Constant |  |  |
|  | $0.198^{* * * *}$ | $0.197 * * *$ |
| Autoregressive Term | $(19.29)$ | $(19.22)$ |
|  | $0.266 * * *$ | $0.248 * * *$ |
| Conditional Variance Term | $(14.32)$ | $14.45)$ |
|  | 0 | 0 |
|  | $(0)$ | $(0)$ |
| Observations |  |  |
| Adjusted R-squared | 6572 | 6602 |
|  | 0.09 | 0.09 |

This relationship is also positive as expected; on average, an increase in the VIX is associated with an increase in the range of the market index. This positive relationship is likely due to rapid changes in the value of the underlying options used in the computation of the VIX. The prices of these call options and put options are indications of market participants' expectations for market activities.

We also find a statistically significant relationship between the trading volume and the range of the market index in our first model. On average, 100 percentage increase in the daily trading volume leads to a 27-percentage increase in the daily range of the market index, holding other factors constant. This relationship is also positive as expected, with increases in the VIX and trading volume moving simultaneously. This positive relationship between trading volume and the range of the index can be attributed to increased market activities during periods of increased volatility. During these periods of increased volatility, market participants are more involved in buying and selling of stocks to help rebalance their portfolios and better manage portfolio risk. Similarly, the relationship between the interest rate and the range of the market index is positive and statistically significant. On average, 100 percentage
change in the daily value of the interest rate leads to 18.3 percentage increase in the daily range of the market index, holding other factors constant.

Additionally, we run a univariate regression from equation (4.1) to confirm this contemporaneous relationship between current changes in the daily closing value of VIX and current changes in the range of the market index. The result is shown in Column (2) of Table 6.3. On average, 100 percentage increase in the daily closing value of the VIX leads to a 248 percentage increase in the daily range of the market index, holding other factors constant. This result is in line with the main result obtained from equation (4.1). These results are consistent with our first hypothesis which states that changes in the VIX do have an impact on changes in the daily range of the market index.

Secondly, we extend these analyses to consider how positive changes in the daily closing values of the VIX, and negative changes in the daily closing value of the VIX comparatively impacts the range of the market index. We utilize a dummy variable to distinguish between positive and negative changes in the VIX. We interact this dummy variable with the change in the daily closing value of the VIX. This result is shown in column (1) of Table 6.4.

Table 6.4. (2nd Hypothesis, Support Table)
GARCH $(1,1)$ Regression for Percentage changes in the Daily Range of the Market Index on Positive, Negative Percentage changes in the Daily Closing Values of the VIX.
This table presents results for the regression of percentage changes in the daily range of the daily market index on positive, and negative percentage changes in the daily closing values of the VIX. The dependent variable is percentage changes in the daily range of the market index.
The independent variables are positive percentage changes, negative percentage changes in the current daily closing values of the VIX; a dummy variable that equals one for positive percentage change in the daily closing value of the VIX. We interact daily closing values of the VIX and positive percentage changes in these closing values. Our control variables are percentage changes in: interest rates and trading volume. We also include dummy variables to control for these crises' periods: Asian Financial Crisis, Russian Financial Crises, Dot.com Crash, and 2007 - 2009 Global Financial Crisis.
T-statistics are shown in the brackets. ${ }^{* * *}$, ${ }^{* *}$, and ${ }^{*}$ indicate $1 \%, 5 \%$ and $10 \%$ significance levels respectively.

|  | $(1)$ |
| :--- | :--- |
| Percentage change in Daily Closing Value of the VIX | $-2.361 * * *$ |
|  | $(-7.75)$ |
| Positive Percentage change in the VIX | 0.034 |
|  | $(1.49)$ |
| Percentage change in Daily Closing Value of the VIX * | $7.398^{* * *}$ |
| Positive Percentage change in the VIX | $(23.52)$ |
|  |  |
| Percentage change in Interest Rate | $0.150^{*}$ |
|  | $(1.69)$ |
| Percentage change in Trading Volume | $0.308^{* * *}$ |
|  | $(103.66)$ |
| Asian Financial Crisis | 0.010 |
|  | $(0.27)$ |
| Russian Financial Crisis | 0.010 |
|  | $(0.09)$ |
| Dot.com Crash | -0.004 |
|  | $(-0.08)$ |
| 2007-2009 Global Financial Crisis | -0.049 |
|  | $(-1.29)$ |
| Constant | -0.022 |
|  | $(-1.08)$ |
| Autoregressive Term | 0.022 |
| Conditional Variance Term | $(1.62)$ |
|  | 0 |
| Observations | $(0)$ |
| Adjusted R-squared |  |
|  | 6602 |
|  | 0.15 |

We find that positive changes in the VIX have a statistically different and higher impact on the range of the market index compared to negative changes in the VIX. On average, 100 percentage increase in the daily closing value of the VIX leads to an increase in the daily range of the market index that is 504 percentage higher than the effect 100 percentage decrease in the daily closing value of the VIX has on the daily closing value of the VIX, ceteris paribus. Notably, the average effect of 100 percentage decrease in the daily closing value of the VIX on the daily closing value of the market index is about -236 percentage, ceteris paribus.

These findings support the prospect theory postulated by Kahneman and Tversky (1979) which emphasize that market participants consider probable profit or probable loss from a trading activity in an isolated manner, and do not necessarily consider the probable overall outcome of an investing activity. Based on our results, market participants clearly assign more considerations to positive changes in the VIX compared to negative changes. Thus, the market reacts very differently to increases in the VIX compared to decreases in the VIX. Based on findings by Giot (2005), increases in the VIX provide attractive opportunities for profits in the market. Thus, market participants are more concerned about increases in the value of the VIX, compared to reductions in the VIX. This possible isolated consideration of increases in the VIX and decreases in the VIX in the market is a plausible explanation for the significant different changes in realized volatility during periods of VIX increases and VIX decreases.

These findings also support findings by previous studies which note significant 'contagion' among equities during periods of increased volatility (Mollah et al.; 2016, Bekaert et al.; 2014, and Capiello et al.; 2006). Periods of increased market turmoil are seen to have more pronounced ripple effects on realized volatility across various equities compared to the impact decreases in the VIX has on the realized market volatility.

This result is consistent with our second hypothesis which posits that the magnitude of the relationship between positive changes in VIX and the range of the market index differs from the magnitude of the relationship between negative changes in the VIX and the range of the market index. Our result in table 6.4 notably shows that the average impact of positive percentage changes in the VIX on the range of the market index is over $500 \%$ higher than the average impact negative percentage
changes in the VIX have on the range of the market index. These findings remarkably provide evidence that upward movements in the VIX are more informative for the range of the market index, compared to downward movements in the VIX.

### 6.2.2. Changes in the VIX During Non-trading Hours

Thirdly, we present results for changes in the VIX during non-trading hours. We run a regression to jointly analyze how changes in the current range of the market index react to current and past changes in the VIX during non-trading hours (and these changes during trading hours). This regression result is shown in column (5) of Table 6.5.

Table 6.5. (3rd Hypothesis, Support Table)
GARCH $(1,1)$ Regression for Percentage changes in the Daily Range of the Market Index on Percentage changes in the VIX during non-trading hours and Percentage changes in the VIX during trading hours
This table presents results for the regression of percentage changes in the daily range of the market index on percentage changes in the values of the VIX during non-trading hours and percentage changes in the VIX during trading hours. The dependent variable is percentage changes in the daily range of the daily market index.
The first set of independent variables are percentage changes in the values of the VIX during non-trading hours for lags: $0,1,2,3,4,5,10,20$ and 30 . The second set of independent variables are percentage changes in the values of the VIX during trading hours for lags: $0,1,2,3,4,5,10,20$ and 30 . Our control variables are percentage changes in: interest rates and trading volume. We also include dummy variables to control for these crises' periods: Asian Financial Crisis, Russian Financial Crises, Dot.com Crash, and 2007 - 2009 Global Financial Crisis. Additionally, we incorporate the autoregressive and the conditional variance terms. In Panel A (columns 1 and 2), we model the relationship between percentage changes in the values of the VIX during trading hours and Percentage changes in the Daily Range of the market index. In Panel B (columns 3 and 4), we model the relationship between percentage changes in the values of the VIX during non-trading hours and percentage change in the range of the market index. In Panel C (column 5), we model the relationship among both percentage changes in the values of the VIX during non-trading hours, percentage changes in the VIX during trading hours, and percentage changes in the daily range of the market index.
T-statistics are shown in the brackets. ${ }^{* * *}$, **, and * indicate $1 \%, 5 \%$ and $10 \%$ significance levels respectively.
(1)
(2)
(3)
(4) (5)

Percentage change in the difference between the opening and closing value of the VIX during non-trading hours
Percentage change in the difference between the opening and closing value of the VIX during non-trading hours at $t_{-1}$
Percentage change in the difference between the opening and closing value of the VIX during non-trading hours at $t-2$
Percentage change in the difference between the opening and closing value of the VIX during non-trading hours at $\mathbf{t}-3$
Percentage change in the difference between the opening and closing value of the VIX during non-trading hours at $t-4$

Percentage change in the difference between the opening and closing value of the VIX during non-trading hours at $t-5$

Percentage change in the difference between the opening and closing value of the VIX during non-trading hours at $\mathbf{t}$-10

Percentage change in the difference between the opening and closing value of the VIX during non-trading hours at t-20

Percentage change in the difference between the opening and closing value of the VIX during non-trading hours at t-30

Percentage change in the difference between the closing and opening value of the VIX during trading hours
Percentage change in the difference between the closing and opening value of the VIX during trading hours at $t_{-1}$

| $\begin{aligned} & 0.026^{* * *} \\ & (3.87) \end{aligned}$ | $\begin{aligned} & 0.055^{* * *} \\ & (7.19) \end{aligned}$ | $\begin{aligned} & 0.060^{* * *} \\ & (7.86) \end{aligned}$ |
| :---: | :---: | :---: |
|  | $\begin{aligned} & 0.057^{* * *} \\ & (5.13) \end{aligned}$ | $\begin{aligned} & 0.066^{* * *} \\ & (6.46) \end{aligned}$ |
|  | $\begin{aligned} & 0.052^{* * *} \\ & (4.18) \end{aligned}$ | $\begin{aligned} & 0.052 * * * \\ & (4.69) \end{aligned}$ |
|  | $\begin{aligned} & 0.037 * * * \\ & (3.03) \end{aligned}$ | $\begin{aligned} & 0.036^{* * *} \\ & (3.13) \end{aligned}$ |
|  | $\begin{aligned} & 0.022^{*} \\ & (1.94) \end{aligned}$ | $\begin{aligned} & 0.018^{*} \\ & (1.70) \end{aligned}$ |
|  | $\begin{aligned} & -0.003 \\ & (-0.34) \end{aligned}$ | $\begin{aligned} & -0.004 \\ & (-0.49) \end{aligned}$ |
|  | $\begin{aligned} & 0.002 \\ & (0.3) \end{aligned}$ |  |
|  | $\begin{aligned} & 0.008 \\ & (1.06) \end{aligned}$ |  |
|  | $\begin{aligned} & 0.017 * * \\ & (2.23) \end{aligned}$ |  |


| $0.051 * * *$ | $0.086^{* * *}$ |  |
| ---: | :--- | :--- |
| $(12.82)$ | $(19.85)$ | $0.088^{* * *}$ |
|  | $0.072^{* * *}$ | $(20.10)$ |
|  | $(8.77)$ | $0.074^{* * *}$ |
|  |  | $(9.30)$ |

Percentage change in the difference between the closing and opening value of the VIX during trading hours at $\mathbf{t}$-2
Percentage change in the difference between the closing and opening value of the VIX during trading hours at $t$ - 3
Percentage change in the difference between the closing and opening value of the VIX during trading hours at $t-4$
Percentage change in the difference between the closing and opening value of the VIX during trading hours at $t$ - 5
Percentage change in the difference between the closing and opening value of the VIX during trading hours at $\mathbf{t}-10$
Percentage change in the difference between the closing and opening value of the VIX during trading hours at $t_{-20}$
Percentage change in the difference between the closing and opening value of the VIX during trading hours at $\mathbf{t}-30$

Percentage change in Interest Rate

Percentage change in Trading Volume

Asian Financial Crisis

Russian Financial Crisis

Dot.com Crash

2007-2009 Global Financial Crisis

## Constant

Autoregressive Term

## Conditional Variance Term

| Observations | 6540 | 6511 | 6540 | 6510 | 6535 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Adjusted R-squared | 0.05 | 0.07 | 0.04 | 0.04 | 0.07 |

We find a positive and statistically significant relationship between changes in the current range of the market index, and current and past daily changes in the VIX during non-trading hours; up to 4th lag. Notably, the impact of changes in the VIX during non-trading hours on changes in the current range
of the market index declines as the time lag increases. On average, 100 percentage increase in the difference between the opening and closing values of the VIX during non-trading hours leads to a six percentage increase in the current daily range of the market, holding other factors constant. The magnitude of this significant relationship continually diminishes for most past changes in the VIX during non-trading hours until the 4th lag where the magnitude is 1.8 percentage. This is shown in column (5) of Table 6.5.

Changes in the VIX during trading hours also have similar effects on changes in the daily range of the market index. On average, 100 percentage increase in the difference between the opening and closing values of the VIX during trading hours leads to an 8.8 percentage increase in the current daily range of the market index, holding other factors constant. This significant relationship is also present in the relationship between current changes in the daily range of the market index and past changes in the range of the market index; up to the 5th lag. The magnitude of this relationship notably diminishes as the lag length increases up to the 5th lag where this magnitude is about 1.8 percentage. These results are in line with our third hypothesis which states that changes in the VIX during non-trading hours do have predictive power regarding the upcoming range of the market index.

We also confirm these results using separate regression models. We separately analyze the relationship between current changes in the daily range of the index, current and past changes in the difference between the opening and closing values of the VIX during non-trading hours; up to 30th lag. This result is shown in column (4) of Table 6.5. We also separately analyze the relationship between current changes in the daily range of the market index, current and past changes in the difference between the opening and closing values of the VIX during trading hours; up to 30th lag. This result is shown in column (2) of Table 6.5.

Our results from these separate regression models are in line with the results of the joint regression model. We see a positive and statistically significant relationship between percentage changes in the current range of the market index, and current and past percentage changes in the VIX during both non-trading hours and trading hours. The magnitude of this relationship mostly diminishes as the time lag length increases as shown in the previous joint analyses.

This significant relationship between changes in the daily range of the market index and changes in the VIX during non-trading hours can be attributed to the value of information market participants have amassed during non-trading hours as highlighted in Stoll \& Whaley (1990). Market participants cannot take significant trading actions concerning this information when markets are closed. Thus, they tend to take these actions in the next trading sessions. Notably, the impact of this information from the most recent non-trading sessions has the greatest influence on the present trading sessions. Market participants make more decisions based on the most recent changes in the VIX. This information content of these changes in the VIX is also weakened as time goes by - both for changes in the VIX during trading hours and non-trading hours.

These results highlight the essence of studying changes in the VIX during non-trading sessions separately from changes in the VIX during trading session. The predictive ability and the information content of the VIX are more pronounced when we separate changes in the VIX that occur during nontrading hours from changes in the VIX that occur during trading hours.

### 6.2.3. Changes in the VIX on Specific Trading Days

Fourthly, we present results for the relationship between changes in the range of the market index and changes in the daily closing values of the VIX for specific trading days of the week. We compare the relationship for each trading day in the week with this relationship on Wednesdays. We still find a positive, statistically significant relationship between changes in the VIX and changes in the range of the market index for each trading day of the week considered. The magnitude of this relationship for each day of the week considered (Mondays, Tuesdays, Thursdays, and Fridays) is statistically different from the relationship on Wednesdays. These results are shown in Table 6.6.

Table 6.6. (4th Hypothesis, Support Table)
GARCH $(1,1)$ Regression for Percentage changes in the Daily Range of The Market Index on Percentage changes in The Daily Closing Values of The VIX for Different Trading Days of The Week.
This table presents results for the regression of Percentage changes in the Daily Range of the market index on percentage changes in the daily closing values of the VIX for different trading days of the week. The dependent variable is percentage change in the daily range of the market index. We represent trading days of the week using dummy variables. The main independent variable in column 1, 2, 3 and 4 is day of the week dummy variable for Mondays, Tuesdays, Thursdays, and Fridays respectively. Dummy variable for Wednesday is the base category in each column.
We interact the dummy variable for each trading day of the week with daily closing values of the VIX. Our control variables are percentage changes in: interest rates and trading volume. We also include dummy variables to control for these crises' periods: Asian Financial Crisis, Russian Financial Crises, Dot.com Crash, and 2007 - 2009 Global Financial Crisis. Additionally, we incorporate the autoregressive and the conditional variance terms.
T-statistics are shown in the brackets. ${ }^{*},{ }^{* *}$, and ${ }^{* * *}$ indicate $1 \%, 5 \%$ and $10 \%$ significance levels respectively.

|  | (1) | (2) | (3) | (4) |
| :---: | :---: | :---: | :---: | :---: |
| Percentage change in Daily Closing Value of the VIX | $\begin{aligned} & 1.419^{* * *} \\ & (5.36) \end{aligned}$ | $\begin{aligned} & 1.168 * * * \\ & (5.20) \end{aligned}$ | $\begin{aligned} & 1.190^{* * *} \\ & (5.04) \end{aligned}$ | $\begin{aligned} & 1.258 * * * \\ & (5.19) \end{aligned}$ |
| Day of the Week Dummy | $\begin{aligned} & -0.076^{* * *} \\ & (-2.67) \end{aligned}$ | $\begin{gathered} -0.011 \\ (-0.41) \end{gathered}$ | $\begin{aligned} & 0.036 \\ & (1.45) \end{aligned}$ | $\begin{aligned} & 0.047 * \\ & (1.87) \end{aligned}$ |
| Percentage change in Daily Closing Value of the VIX * Day of the Week Dummy | $\begin{aligned} & 1.382 * * * \\ & (3.81) \end{aligned}$ | $\begin{aligned} & 1.522 * * * \\ & (5.53) \end{aligned}$ | $\begin{aligned} & 0.897 * * * \\ & (2.96) \end{aligned}$ | $\begin{aligned} & 1.601^{* * *} \\ & (5.39) \end{aligned}$ |
| Percentage change in Interest Rate | $\begin{aligned} & 0.379^{* * *} \\ & (2.69) \end{aligned}$ | $\begin{aligned} & 0.373 * * * \\ & (3.00) \end{aligned}$ | $\begin{aligned} & -0.056 \\ & (-0.38) \end{aligned}$ | $\begin{aligned} & 0.177 \\ & (1.15) \end{aligned}$ |
| Percentage change in Trading Volume | $\begin{aligned} & 0.180^{* * *} \\ & (42.67) \end{aligned}$ | $\begin{aligned} & 1.800 * * * \\ & (40.64) \end{aligned}$ | $\begin{aligned} & 1.21^{* * *} \\ & (47.99) \end{aligned}$ | $\begin{aligned} & 0.999 * * * \\ & (30.00) \end{aligned}$ |
| Asian Financial Crisis | $\begin{aligned} & -0.045 \\ & (-0.71) \end{aligned}$ | $\begin{aligned} & -0.031 \\ & (-0.56) \end{aligned}$ | $\begin{aligned} & 0.032 \\ & (0.66) \end{aligned}$ | $\begin{aligned} & 0.045 \\ & (0.79) \end{aligned}$ |
| Russian Financial Crisis | $\begin{aligned} & 0.269 * * \\ & (2.23) \end{aligned}$ | $\begin{aligned} & -0.086 \\ & (-0.47) \end{aligned}$ | $\begin{aligned} & -0.057 \\ & (-0.33) \end{aligned}$ | $\begin{aligned} & -0.140 \\ & (-0.59) \end{aligned}$ |
| Dot.com Crash | $\begin{aligned} & -0.036 \\ & (-0.47) \end{aligned}$ | $\begin{aligned} & -0.015 \\ & (-0.24) \end{aligned}$ | $\begin{aligned} & -0.027 \\ & (-0.42) \end{aligned}$ | $\begin{aligned} & 0.063 \\ & (0.98) \end{aligned}$ |
| 2007-2009 Financial Crisis | $\begin{aligned} & 0.020 \\ & (0.32) \end{aligned}$ | $\begin{aligned} & 0.030 \\ & (0.62) \end{aligned}$ | $\begin{aligned} & -0.039 \\ & (-0.71) \end{aligned}$ | $\begin{aligned} & -0.120^{*} \\ & (-1.93) \end{aligned}$ |
| Constant | $\begin{aligned} & \begin{array}{l} 0.153 * * * \\ (7.23) \end{array} \end{aligned}$ | $\begin{aligned} & 0.100 * * * \\ & (5.20) \end{aligned}$ | $\begin{aligned} & 0.128^{* * *} \\ & (6.10) \end{aligned}$ | $\begin{aligned} & 0.124^{* * *} \\ & (5.76) \end{aligned}$ |
| Autoregressive Term | $\begin{gathered} 0 \\ (0) \end{gathered}$ | $\begin{aligned} & 0.284 * * * \\ & (9.57) \end{aligned}$ | $\begin{aligned} & 0.077 * * * \\ & (3.16) \end{aligned}$ | $\begin{gathered} 0 \\ (0) \end{gathered}$ |
| Conditional Variance Term | $2.0$ <br> (0) | $\begin{gathered} 0 \\ (0) \end{gathered}$ | $\begin{aligned} & 0.751 * * * \\ & (10.40) \end{aligned}$ | $\begin{gathered} 0 \\ (0) \end{gathered}$ |


| Observations | 2576 | 2685 | 2660 | 2649 |
| :--- | ---: | ---: | ---: | ---: |
| Adjusted R-squared | 0.07 | 0.20 | 0.13. | 0.13 |

On average, 100 percentage increase in the change in daily closing value of the VIX on Fridays leads to a 160 percentage increase in the change in the daily range of the market index, holding other factors constant. The magnitude of this effect is also 92.3 percentage higher than the effect percentage changes in the daily closing value of the VIX on Wednesdays have on the changes in the daily closing range of the market index. This result is shown in column (4) of Table 6.6.

A similar positive relationship between percentage changes in the VIX and percentage changes in the range of the market index are also seen on Mondays, Tuesdays, Wednesdays, and Thursdays. On average, the magnitude of the relationship between the VIX and the range of the index is notably highest on Fridays and lowest around Wednesdays and Thursdays. The results for changes in the VIX on Mondays, Tuesdays, and Thursdays are respectively shown in column (1), column (2), and column (3) of Table 6.6.

These results are similar to findings by Berument \& Kiymaz (2001) which note that market volatility is lowest around Wednesdays. These results provide support for our fourth hypothesis which states that the relationship between the VIX and the range of the market index is not the same for each trading day of the week.

Similar to results seen in Berument \& Kiymaz (2001), the relationship between the VIX and realized volatility is rather more pronounced towards the end and the start of the week. This is due to the uncertainties that are typically associated with trading on Fridays and towards the weekend. Market participants typically expect 'bad' news about companies towards the end of the week. Also, major economic policy or financial announcements are typically made around the weekend. Major changes in any financial or economic policies can have significant effects on the performance of the market. Thus,
the timing of all these uncertainties is plausible explanations for the more pronounced relationship between the VIX and the range of the market index seen on Fridays in our results.

Our findings are a little different from some previous works including Morey and Rosenberg (2012) and Olson et al. (2015) who emphasize the non-existence of specific return characteristics for different trading days of the week. This difference is possibly due to the disparity in the response variable in our thesis compared to these previous studies. These previous studies mostly utilize market returns as their response variable while our thesis utilizes daily realized market volatility as our response variable.

Overall, our results provide substantial evidence that changes in the value of the VIX during nontrading hours and trading hours are informative for the range of the upcoming market index; for the next four to five trading days. Furthermore, our results notably highlight the need to separately study changes in the VIX during non-trading hours from changes in the VIX during trading hours. However, we do not find any evidence that the VX predicts the range of the market index as far as the next thirty days.

### 6.3. Robustness Check

We confirm our findings using modified models to test our hypotheses. Firstly, we test our first hypothesis using a GARCH $(1,1)$ model; a modified version of equation (4.1). In this modified version, we replace the dummy variables for Asian Financial Crisis (1997-1998), Russian Financial Crisis (1998), Dot.com Crash (2001), and Global Financial Crisis (2007-2009) with different dummy variables that incorporate the year effect for each year in our study; the year 1993 is the base category. The result of this first robustness check is shown in Table 6.7.

Table 6.7. (Robustness Check, 1 st Support Table): GARCH $(1,1)$ Regression for Percentage changes in the Daily Range of the Market Index on Percentage changes in the Daily Closing Values of the VIX
This table presents the first result for our robustness tests; the regression of percentage changes in the daily range of the market index on percentage changes in the daily closing values of the VIX. We notably control for the year effect in this regression throughout the period of our study in order to support the results from the test of our first hypothesis.
Our results show the relationship among the percentage change in current and past values of the VIX, and the range of the market index. The dependent variable in all columns is percentage change in the range of the daily market index. The independent variables are percentage change in the daily closing value of the VIX at lags: $0,1,2,3,4,5,10,20$, and 30 . Our control variables are percentage changes in: interest rates and trading volume.
T-statistics are shown in the brackets. ${ }^{* * *}$, ${ }^{* *}$, and * indicate $1 \%, 5 \%$ and $10 \%$ significance levels respectively.

|  | (1) |
| :---: | :---: |
| Percentage change in Daily Closing Value of the VIX | $\begin{aligned} & 2.433 * * * \\ & (30.64) \end{aligned}$ |
| Percentage change in Daily Closing Value of the VIX at $\mathrm{t}_{-1}$ | $\begin{aligned} & -0.171 \\ & (-1.30) \end{aligned}$ |
| Percentage change in Daily Closing Value of the VIX at $\mathrm{t}_{-2}$ | $\begin{aligned} & -0.137 \\ & (-1.17) \end{aligned}$ |
| Percentage change in Daily Closing Value of the VIX at $\mathrm{t}_{\text {- }}$ | $\begin{aligned} & -0.123 \\ & (-1.06) \end{aligned}$ |
| Percentage change in Daily Closing Value of the VIX at $t_{-4}$ | $\begin{aligned} & \text { 0.217* } \\ & \text { (1.80) } \end{aligned}$ |
| Percentage change in Daily Closing Value of the VIX at $\mathrm{t}-5^{\text {- }}$ | $\begin{aligned} & -0.143 \\ & (-1.16) \end{aligned}$ |
| Percentage change in Daily Closing Value of the VIX at $\mathrm{t}_{10}$ | $\begin{aligned} & -0.197^{*} \\ & (-1.69) \end{aligned}$ |
| Percentage change in Daily Closing Value of the VIX at $\mathrm{t}_{-20}$ | $\begin{aligned} & 0.090 \\ & (0.84) \end{aligned}$ |
| Percentage change in Daily Closing Value of the VIX at $t_{\text {-30 }}$ | $\begin{aligned} & 0.056 \\ & (0.48) \end{aligned}$ |
| Percentage Change in Interest Rate | $\begin{aligned} & 0.188^{* *} \\ & (2.00) \end{aligned}$ |
| Percentage Change in Trading Volume | $\begin{aligned} & 0.262 * * * \\ & (85.46) \end{aligned}$ |
| Constant | $\begin{aligned} & 0.250^{* * *} \\ & (12.22) \end{aligned}$ |
| Autoregressive Term | $\begin{aligned} & 0.336 * * * \\ & (15.68) \end{aligned}$ |
| Conditional Variance Term | $\begin{gathered} 0 \\ (0) \end{gathered}$ |

Year Effect Yes
Observations 6572
Adjusted R-squared 0.09

This result still confirms the presence of a mostly positive, contemporaneous and statistically significant between changes in the daily range of the market index and changes in the current and past closing values of the VIX. On average, a 100 percentage increase in the daily closing value of the VIX leads to a 243.3 percentage increase in the daily range of the market index.

We also still find a statistically significant and positive relationship between percentage changes in the daily range of the market index and percentage changes in interest rate as seen in our main hypothesis test. 100 percentage increase in interest rate on average leads to an 18.8 percentage increase in the daily range of the market index, ceteris paribus. Similar results are also seen in the relationship between trading volume and the range of the market index. On average, 100 percentage increase in trading volume leads to a 26.2 percentage increase in the range of the market index. The result of this robustness test further supports our first hypothesis which postulates that there is a statistically significant relationship between changes in the daily closing values of the VIX and changes in the daily range of the market index.

Secondly, we modify equation (4.3) to test our third hypothesis. This modified equation is a GARCH $(1,1)$ model that corporates the year effect in our study period; the year 1993 is the base category. We replace the dummy variables for Asian Financial Crisis (1997-1998), Russian Financial Crisis (1998), Dot.com Crash (2001), and Global Financial Crisis (2007-2009) with dummy variables for each year of observation in our model. This modified model jointly tests the impact current and past changes in the VIX during non-trading hours (and trading hours) have on current changes in the range of the market index. The result of this model is shown in Table 6.8.

Table 6.8. (Robustness Check, 2nd Support Table):
GARCH (1, 1) Regression for Percentage changes in the Daily Range of the Market Index on Percentage changes in the VIX during non-trading hours and Percentage changes in the VIX during trading hours
This table presents the second result for our robustness tests; the regression of percentage changes in the daily range of the market index on percentage changes in the values of the VIX during non-trading hours and percentage changes in the VIX during trading hours. Notably, in order to buttress the results from the test of our third hypothesis, we control for the year effect in this regression throughout the period of our study.
We model the relationship among percentage changes in the values of the VIX during non-trading hours, percentage changes in the VIX during trading hours, and percentage change in the range of the market index. The dependent variable is percentage change in the range of the daily market index. The first set of independent variables are percentage changes in the values of the VIX during non-trading hours for lags: $0,1,2,3,4$, and 5 . The second set of independent variables are percentage changes in the values of the VIX during trading hours for lags: $0,1,2,3,4$, and 5 . Our control variables are percentage changes in: interest rates and trading volume. Additionally, we incorporate the autoregressive and the conditional variance terms.
T-statistics are shown in the brackets. ${ }^{* * *}$, ${ }^{* *}$, and ${ }^{*}$ indicate $1 \%, 5 \%$ and $10 \%$ significance levels respectively.

|  | (1) |
| :---: | :---: |
| Percentage change in the difference between the opening and | 0.088*** |
| Closing value of the VIX during non-trading hours | (14.52) |
| Percentage change in the difference between the opening and closing value of the VIX during non-trading hours at $t_{-1}$ | $\begin{aligned} & 0.078 * * * \\ & (9.73) \end{aligned}$ |
| Percentage change in the difference between the opening and closing value of the VIX during non-trading hours at $t_{-2}$ | $\begin{aligned} & 0.070^{* * *} \\ & (7.09) \end{aligned}$ |
| Percentage change in the difference between the opening and closing value of the VIX during non-trading hours at $t_{-3}$ | $\begin{aligned} & 0.046 * * * \\ & (4.72) \end{aligned}$ |
| Percentage change in the difference between the opening and closing value of the VIX during non-trading hours at $t_{-4}$ | $\begin{aligned} & 0.034 * * * \\ & (3.62) \end{aligned}$ |
| Percentage change in the difference between the opening and closing value of the VIX during non-trading hours at $t-5$ | $\begin{aligned} & -0.003 \\ & (-0.34) \end{aligned}$ |
| Percentage change in the difference between the closing and opening value of the VIX during trading hours | $\begin{aligned} & 0.099 * * * \\ & (25.32) \end{aligned}$ |
| Percentage change in the difference between the closing and opening value of the VIX during trading hours at $t_{-1}$ | $\begin{aligned} & 0.084^{* * *} \\ & (14.79) \end{aligned}$ |
| Percentage change in the difference between the closing and opening value of the VIX during trading hours at $t_{-2}$ | $\begin{aligned} & 0.063 * * * \\ & (10.28) \end{aligned}$ |
| Percentage change in the difference between the closing and opening value of the VIX during trading hours at $t_{-3}$ | $\begin{aligned} & 0.050 * * * \\ & (7.32) \end{aligned}$ |
| Percentage change in the difference between the closing and opening value of the VIX during trading hours at $t_{-4}$ | $\begin{aligned} & 0.036^{* * *} \\ & (5.19) \end{aligned}$ |
| Percentage change in the difference between the closing and opening value of the VIX during trading hours at t-5 | $\begin{aligned} & 0.012 * * \\ & (2.33) \end{aligned}$ |
| Percentage change in Interest Rate | 0.02 |

## Percentage change in Trading Volume

0.036***
Constant ..... 0.034
Autoregressive Term 0.220***
Conditional Variance Term ..... 0.763***
YEAR EFFECT ..... Yes
Observations ..... 6596
Adjusted R-squared ..... 0.12

100 percentage increase in the difference between the opening and closing value of the VIX during non-trading hours on average leads to an 8.8 percentage increase in the current change in the daily range of the market index, holding other factors constant. As seen in our third main hypothesis test, this significant relationship is present and diminishes up to the 4th lag; where the magnitude of the relationship is lowest at 3.4 percentage. This result confirms the findings seen in our previous main test of the third hypothesis and further supports our third hypothesis which posits that changes in the VIX during non-trading hours predict changes in the range of the market index.

Similar results are seen in the analyses of changes in the VIX during trading hours. On average, 100 percentage increase in the difference between the opening and closing value of the VIX during trading hours brings about a 9.9 percentage increase in the current change in the daily range of the market index, ceteris paribus. This relationship is also positive, statistically significant and diminishes through the 5 th lag where the magnitude of the relationship is about 1.2 percentage.

### 6.4. Implications of Findings

The results of our data analyses confirm some predictive abilities of the VIX concerning the range of the upcoming market index, for up to five days. The information content of overnight changes is also critical for the upcoming market session. Compared to past changes in the VIX during non-trading sessions, changes in the VIX in the last non-trading session in the market have the strongest link with the range of the upcoming market index.

Based on our findings, a market participant making use of the VIX as a risk analyses tool would be better off isolating changes in the VIX to changes that occur during non-trading hours from changes in the VIX that occur during trading hours. More attention should also be paid to recent changes in the VIX compared to older changes in the VIX. This is essential as the most recent changes in the VIX have the strongest impact on current realized market volatility. Furthermore, our results show the VIX as a reliable risk analyses tool during periods of volatility in the market. Thus, market participants can rely on the VIX for reasonable information on realized volatility in the market.

Additionally, an increase in the VIX provides more information concerning an increase in realized volatility, compared to a decrease in the VIX. Investors need to pay closer attention to increases in the value of the VIX as these provide more information on realized volatility in the market. An increase in the value of the VIX is a useful signal for increased volatility in the market. Investors can plausibly make profits from rebalancing their portfolios based on these increases in the value of the volatility index.

This relationship between the VIX and realized market volatility is also strongest on Fridays. Changes in the VIX on Fridays are most informative for making decisions on trading. Changes in the VIX on Fridays have a stronger impact in the markets, compared to changes that occur on other trading days of the week. Investors may make profits when portfolios are rebalanced based on changes in the VIX that occurs notably on Fridays. Thus, more attention should also be paid to changes in the VIX that occurs on Fridays compared to other trading days.

Additionally, risk-averse investors may be better off conducting trades around Wednesdays and Thursdays compared to other trading days of the week. This is due to the relatively lower market volatility seen on Wednesdays and Thursdays compared to other trading days.

Based on these findings, changes in the VIX during non-trading hours and changes in the VIX during trading hours are reliable tools for forecasting volatility in market activities in the short-run with increases in the VIX on Fridays being very critical.

## 7. SUMMARY, CONCLUSION, AND AREAS FOR FUTURE RESEARCH

### 7.1. Summary

This thesis investigates the information content of the Chicago Board Options Exchange Volatility Index (VIX) as it relates to the range of the market index in the United States. The S\&P 500 index is our proxy for the US market. We employ the VIX as a measure of implied volatility in the US market. We proxy realized market volatility using the range of the S\&P 500 index. We utilize a GARCH $(1,1)$ model to study critical information about the relationship between the VIX and realized market volatility; with the latter as the response variable in our model.

Some unique contributions of this thesis involve examining key impacts changes in the VIX during non-trading hours have on the daily range of the market index. We isolate changes in the VIX that occur overnight from changes in the VIX that occur during the day. We separately model the impact changes in the VIX during non-trading hours (overnight changes in the VIX) and changes in the VIX during trading hours (day changes in the VIX) have on the range of the market index.

Notably, we also investigate how positive changes in the VIX and negative changes in the VIX comparatively impact the range of the market index. We uncover if the range of the market index reacts better to positive changes in the VIX compared to negative changes in the VIX.

In another major contribution to existing literature, this thesis also studies the changes in the VIX on each trading day of the week to understand how changes on each of these days separately impact the range of the market index. Specifically, we separately study changes in the VIX that occur on each of these days: Mondays, Tuesdays, Wednesdays, Thursdays, and Fridays.

Our daily data set spans from January 1993 through December 2017. We also account for the performance of the VIX during different economic or financial crises within the study period including the Asian Financial Crisis, Russian Financial Crisis, Dot.com Crash, and the 2007-2009 Global Financial Crisis. We also control for key variables in our models including interest rate and trading volume.

### 7.2. Conclusion and Areas for Future Research

Similar to Smales (2016), Giot (2005), and Fleming et al. (1995), we uncover a statistically significant contemporaneous relationship between daily changes in the closing values of the VIX and the range of the market index. The range of the market index is our proxy for realized market volatility.

Furthermore, there is a positive, statistically significant relationship between positive changes in the daily closing values of the VIX and the current range of the market index. When compared to negative changes in the VIX, the impact of positive changes in the VIX on the range of the market index is significantly higher and statistically different. This provides reasonable evidence that an increase in the VIX has a greater impact on the range of the market index, compared to a decrease in the VIX. Investors who are skeptical about market volatility would find the VIX as a useful risk analytical tool during periods of increased volatility.

Additionally, there is a positive and statistically significant relationship between the current day range of the market index and lagged changes in the VIX during non-trading hours; up to four nights before. Similarly, there is evidence of a statistically significant and positive relationship between the current range of the market index and lagged changes in the VIX during trading hours; up to five days before. Notably, the magnitude of this relationship also diminishes as the lag length increases. These results underline the usefulness of the most recent changes in the VIX on realized market volatility. Market participants need to pay closer attention to more recent changes in the value of the volatility index when making trading decisions.

Furthermore, our results emphasize the need for market participants to separately observe changes in the VIX that occur during non-trading hours and changes in the VIX that occur during trading hours. This isolated analysis is critical for understanding the information content of the VIX concerning realized volatility. An observation of only the end-of-day value of the VIX would not sufficiently help market participants predict realized volatility in the market.

Also, our analyses of weekly observations show that all daily changes in the VIX values have a positive and statistically significant relationship with change in the values of the index for each trading
day of the week. The relationship between the VIX and the range of the market index is notably strongest on Fridays. Changes in the VIX on Fridays are very critical for realized volatility in the market. Investors can make better trading decisions by paying close attention to changes in the VIX that occur on Fridays. Thursdays and Wednesdays are also good days for risk-averse investors to trade since on average, realized market volatility is comparably low on Thursdays and Wednesdays.

Our findings are important as they provide evidence of the predictive ability of the VIX for as much as five days. It also provides proof on what direction and days of changes in the VIX are more informative for realized market volatility. These findings provide clarity for investors on how best to use the VIX and what kind of information the VIX can provide concerning market volatility.

While these findings are interesting, further research can improve on this thesis by the use of intra-day data to provide more insight into the information content of intra-day changes in the VIX and the market index. Studies around the intraday spread of the VIX, changes in the VIX at different times in the day, changes in the VIX at various times of the year, changes in the VIX at different points in the economic cycle, changes in realized market volatility at different times in the year etc. may reveal more on the relationship between the VIX and realized volatility in the market. Further research may also explore what levels of changes in the VIX are very significant for investors and how best investors can interprete the information content of these changes.

## REFERENCES

Andersen, T.G.; Bollerslev, T.; Diebold, F.X.; \& Ebens, H. (2001). The distribution of realized stock return volatility. Journal of Financial Economics 61, 43-76.

Andersen, T. G.; Bollerslev, T.; \& Meddahi, N. (2005). Correcting the errors: volatility forecast evaluation using high-frequency data and realized volatilities. Econometrica, Vol.73(1), 279-296.

Banerjee, P. S.; Doran, J. S.; \& Peterson D. R. (2007). Implied volatility and future portfolio return. Journal of Banking \& Finance 31, 3183-3199.

Becker, R.; Clements, A. E.; \& White, S. I. (2007). Does implied volatility provide any information beyond that captured in model-based volatility forecasts? Journal of Banking \& Finance 31, 2535-2549.

Bekaert, G.; \& Hoerova, M. (2014). The VIX, the variance premium, and stock market volatility. Journal of Econometrics. Volume 183, Issue 2, 181-192.

Berument, M.; \& Dogan, N. (2012). Stock market return and volatility: day-of-the-week effect. Journal of Economics and Finance, Vol.36(2), pp.282-302.

Berument, H.; \& Kiymaz, H. (2001). The Day of the Week Effect on Stock Market Volatility. Journal of Economics and Finance, Vol.25(2), pp.181-193.

Bekaert, G.; Ehrmann, M.; Fratzscher, M.; \& Mehl, A. (2014). The Global Crisis and Equity Market Contagion. Journal of Finance, Vol.69(6), pp.2597-2649.

Branch, B.; \& Ma, A. (2012). Overnight Return, the Invisible Hand Behind Intraday Returns? Journal of Applied Finance; Vol. 22, Iss. 2, 90-100.

Brandt, M.W. and F.X. Diebold (2006), A No-arbitrage approach to range-based estimation of return covariances and correlations. The Journal of Business, Vol.79(1), pp.61-73

Blair, B. J.; Poon, S.; \& Taylor, S. J. (2001). Forecasting S\&P 100 volatility: the incremental information content of implied volatilities and high-frequency index returns. Journal of Econometrics 105, 5-26.

Brusa, J.; \& Liu P. (2004). The Day-of-the-Week and the Week-of-the-Month Effects: An Analysis of Investors' Trading Activities. Review of Quantitative Finance and Accounting, 23: 19-30.

Brusa, J., P. Liu, and C. Schulman (2000). "The Weekend Effect, 'Reverse' Weekend Effect, and Firm Size," Journal of Business Finance and Accounting (June/July 2000), pp. 555-574.

Capiello, L., Engle, R., \& Sheppard, K. (2006). Asymmetric Dynamics in The Correlations Of Global Equity and Bond Returns. Journal of Financial Econometrics 4, 537-572.

Chen, S. (2009). Predicting the bear stock market: Macroeconomic variables as leading indicators. Journal of Banking \& Finance 33, 211-223.

Copeland, M. M.; \& Copeland, T. E. (1999). Market Timing: Style and Size Rotation Using the VIX. Financial Analysts Journal, Vol. 55 (2), p.73(9).

Cross, F. (1973). The behavior of stock price on Fridays and Mondays. Financial Analysts Journal, 29, 67-69.

Doyle, J.R.; \& Chen, C.H. (2009). The Wandering Weekday Effect in Major Stock Markets. Journal of Banking \& Finance 33, 1388-1399.

Fernandes, M.; Medeiro M.; \& Scharth M. (2014). Modeling and predicting the CBOE market volatility index. Journal of Banking \& Finance 40, 1-10.

Fleming, J.; Ostdiek, B.; \& Whaley, R. (1995). Predicting Stock Market Volatility: A New Measure. Journal of Futures Markets, Vol. 15, no. 3 (May): 265-302.

Fleming, J. (1998). The quality of market volatility forecasts implied by S\&P 100 index option prices. Journal of Empirical Finance 5, 317-345.

Francisco, J.; Román, F.; \& Stanislava, M. (2016). US Stock Market Sensitivity to Interest and Inflation Rates: A Quantile Regression Approach. Applied Economics, 06 January, p.1-13.

Franses, PH; \& Van Dijk, D. (1996). Forecasting stock market volatility using (non-linear) GARCH models. Journal of Forecasting Vol. 15, Iss. 3, 229-235.

French, K. (1980). Stock Returns and the Weekend Effect. Journal of Financial Economics, 55-69.
George, D.; \& Kanago, B. (1996). On Measuring the Effect of Inflation Uncertainty on Real GNP Growth. Oxford Economic Papers 48, 163-75.

Gibbons, M.; \& Hess, H. (1981). "Day of the Week Effects and Asset Returns," Journal of Business, 579-596.

Giot (2005). Relationships Between Implied Volatility Indexes and Stock Index Returns. Journal of Portfolio Management, Vol.31, p. 92.

Hong, H.; \& Wang, J. (2000). Trading and Returns under Periodic Market Closures. The Journal of Finance, Vol. 55, No. 1, 297-354.

Kahneman, D.; \& Tversky, A. (1979). Prospect Theory: An Analysis of Decision Under Risk. Econometrica (pre-1986), Vol.47(2), p. 263.

Kamara, A. (1997). New Evidence on the Monday Seasonal in Stock Returns. Journal of Business, 63-84.

Kambouroudis, D. S.; \& McMillan, D. G. (2016). Does VIX or volume improve GARCH volatility forecasts? Applied Economics, 48:13, 1210-1228.

Kanas, A. (2014). Uncovering a positive risk-return relation: the role of implied volatility index. Review of Quantitative Finance and Accounting, Volume 42, Issue 1, 159-170.

Keim, D.; \& Stambaugh, R. (1984). A Further Investigation of the Weekend Effect in Stock Returns. Journal of Finance, 819-835.

Kelly, M. A; \& Clark, S. P. (2011). Returns in trading versus non-trading hours: The difference is day and night. Journal of Asset Management; London Vol. 12, Iss. 2, 132-145.

Kozyra, J.; \& Lento, C. (2011). Using VIX data to enhance technical trading signals. Applied Economics Letters, 18, 1367-1370.

Lakonishok, J.; \& Smidt, S. (1988). Are Seasonal Anomalies Real? A Ninety-Year Perspective. Review of Financial Studies, 10 (Winter), pp. 403-425.

Laurence, B. (1992). Why does high inflation raise Inflation uncertainty? Journal of Monetary Economics 29 (June 1992): 371-88.

Marc, H. (2000). Inflation uncertainty, unemployment uncertainty, and economic activity. Journal of Macroeconomics, Volume 22, Issue 2, Pages 315-329.

Martens, M.; \& Van Dijk, D. (2007). Measuring Volatility with The Realized Range. Journal of Econometrics, Vol. 138 (1), pp.181-207

Mehdian, S.; \& Perry, M. (2001). The Reversal of the Monday Effect: New Evidence from US Equity Markets. Journal of Business Finance and Accounting, 1043-1065.

Mollah, S.; Quoreshi, A.; \& Zafirov, G. (2016). Equity Market Contagion During Global Financial and Eurozone Crises: Evidence from a Dynamic Correlation Analysis. Journal of International Financial Markets, Institutions \& Money, Vol.41, p. 151

Morey, M. R.; \& Rosenberg, M. (2012). Using Annual Panel Data to Examine the Monday Effect. Journal of Applied Business Research; 28, 4; ABI/INFORM Collection pg. 595.

Olson, D.; Mossman, C.; \& Chou, N. (2015). The evolution of the weekend effect in US markets. The Quarterly Review of Economics and Finance 58, 56-63.

Parkinson, M. (1980), The extreme value method for estimating the variance of the rate of return, Journal of Business 53, 61-65.

Poon, S. H.; \& Granger, C. (2003). Forecasting Volatility in Financial Markets: A Review. Journal of Economic Literature, Vol 41, 478-539.

SAS Institute Inc. (2012). SAS/ETS® ${ }^{\circledR}$ 12.1 User's Guide. Cary, NC: SAS Institute Inc. http://support.sas.com/documentation/cdl/en/etsug/65545/PDF/default/etsug.pdf

Siegel, J. (1998). Stocks for the Long Run. New York: McGraw Hill.
Smales, L. (2016). Time-varying relationship of news sentiment, implied volatility and stock returns. Applied Economics, 48:51, 4942-4960.

Stoll, H. R.; \& Whaley, R. E. (1990). Stock Market Structure and Volatility. The Review of Financial Studies, Vol.3(1), pp.37-71.

S\&P 500® Ticker: SPX https://ca.spindices.com/indices/equity/sp-500.
'The Cboe Volatility Index - VIX' (2015). Cboe White Paper, Revised. http://www.cboe.com/micro/vix/vixwhite.pdf

Tsai, W.; Chiu, Y.; \& Wang, Y. (2015). Journal of Futures Markets, Vol.35(8), pp.715-737
US Business Cycle Expansions and Contractions - The National Bureau of Economic Research.
https://www.nber.org/cycles/cyclesmain.html
Wang, F.; Shieh, S.; Havlin, S.; \& Stanley, H.E. (2009). Statistical analysis of the overnight and daytime return. Physical review. E, Statistical, nonlinear, and soft matter physics, Vol. 79 (5 Pt 2), pp. 056109 .

Whaley, R. (2000). The investor fear gauge. Journal of Portfolio Management 26, 12-17
Yan, L.; David, N.; \& Bhaskaran, S. (2013). Predicting market returns using aggregate implied cost of capital. Journal of Financial Economics 110, 419-436.

Zhang, J.; Lai, Y.; Lin, J. (2017). The day-of-the-Week effects of stock markets in different countries. Finance Research Letters, February 2017, Vol.20, pp.47-62

Zhou, Y. (2014). Modeling the joint dynamics of risk-neutral stock index and bond yield volatilities. Journal of Banking \& Finance 38, 216-228.

## APPENDIX

How the VIX is Constructed ('The Cboe Volatility Index - VIX', 2015)

The VIX is computed using this formula:
$\sigma^{2}=\frac{2}{\mathrm{~T}} \sum_{i} \frac{\Delta K i}{K_{i}^{2}} e^{R T} \mathrm{Q}\left(\mathrm{k}_{\mathrm{i}}\right)-\frac{1}{\mathrm{~T}}\left[\frac{F}{K_{0}}-1\right]^{2}$
where;
VIX $=\sigma * 100$
$\mathrm{T}=$ time to expiration
F = forward S\&P 500 Index level
$\mathrm{Ko}=$ the strike price immediately below ' $F$ '
$\mathrm{Ki}=$ the strike price of the ith out-of-the money option
$\Delta k i=$ the difference between the last strike price before ki and the next strike price after ki; divided by 2
$\mathrm{R}=$ the risk-free interest rate to expiration
$\mathrm{Q}(\mathrm{Ki})=$ Mid-point of the bid-ask price for each option with strike Ki.

## Steps in Applying this Formula:

## Note in these steps:

There are strike prices, call option prices, put option prices.

## Step 1:

- Obtain the relevant options for computation of the VIX
$\checkmark$ These options (out-of-the money SPX calls and puts) are concentrated around 'ko' (at-the-money strike price)
$>$ Ascertain ' F ' (forward S\&P500 Index) - at which the absolute discrepancies in the strike prices between the relevant option prices (calls and puts) are at its lowest point
- (Note: ' $F$ ' could fall within the column for the 'near term options' or the next term options')
> Ascertain 'Ko' (the strike below ' F ') - for both the near-term option and next-term option
> Below 'Ko', select all successive put options with non-zero strike prices. However, when any put options have two sequential zero strike prices, stop selecting any put options thereafter
> Above ' $K$ ', select all successive call options with non-zero strike prices. However, when any call options have two sequential zero strike prices, stop selecting any call options thereafter
$>$ Next obtain both the call options and the put options for the strike price directly below ' Ko '
- Here, for the near-term option, we utilize the average of the call and put option prices. Same applies to the next-term option.


## Step 2:

- Compute a value for volatility of the near-term option and next-term option under consideration
$>$ Here, the VIX formula is applied to the next-term options and the near-term options selected in 'Step 1' above with expiration date T 1 and T 2 respectively
$>$ Begin by calculating $\Delta k i$; recall $k i$ is the strike price
- $\Delta k i$ is obtained by taking the difference between the last strike price before $k i$ and the next strike price after $k i$; then divide by 2
- However, if $k i$ is the last strike price on any selected strip of options, simply take the difference between $k i$ and the next strike price in the selected strip of options
- Repeat this for all the strike prices in the next term and near-term option strips
> Then obtain the 'contribution' for each option strip by computing this:

$$
\frac{\Delta K i}{K_{i}^{2}} e^{R T} \mathrm{Q}\left(\mathrm{k}_{\mathrm{i}}\right)
$$

> Sum up these contributions - different sums for the near-term and next-term options
> Multiply the sum by $\frac{2}{T_{1}}$ and $\frac{2}{T_{2}}$ for near-term and next-term options respectively
$>$ Then Calculate $\frac{1}{\mathrm{~T}}\left[\frac{F}{K_{0}}-1\right]^{2}$ for the near-term and next-term options
$>$ Next compute $\sigma_{1}^{2}$ and $\sigma_{2}^{2}$ with the formula below:

$$
\begin{aligned}
& \sigma_{1}^{2}=\frac{2}{T_{1}} \sum_{i} \frac{\Delta K i}{K_{i}^{2}} e^{R_{1} T_{1}} \mathrm{Q}\left(\mathrm{k}_{\mathrm{i}}\right)-\frac{1}{T_{1}}\left[\frac{F_{1}}{K_{0}}-1\right]^{2} \\
& \sigma_{2}^{2}=\frac{2}{T_{2}} \sum_{i} \frac{\Delta K i}{K_{i}^{2}} e^{R_{2} T_{2}} \mathrm{Q}\left(\mathrm{k}_{\mathrm{i}}\right)-\frac{1}{T_{2}}\left[\frac{F_{2}}{K_{0}}-1\right]^{2}
\end{aligned}
$$

## Step 3:

- Compute the weighted average values of $\sigma_{1}^{2}$ and $\sigma_{2}^{2}$ obtained in Step 2 above
$>$ Use ' 30 days' as the weight since the VIX is a measure of ' 30 -day' implied volatility.

$$
\left(\left\{T_{1} \sigma_{1}^{2}\left[\frac{N_{T_{2}}-N_{30}}{N_{T_{2}}-N_{T_{1}}}\right]+T_{2} \sigma_{2}^{2}\left[\frac{N_{30}-N_{T_{1}}}{N_{T_{2}}-N_{T_{1}}}\right]\right\} * \frac{N_{365}}{N_{30}}\right)
$$

- Take the square root

$$
\sqrt{ }\left(\left\{T_{1} \sigma_{1}^{2}\left[\frac{N_{T_{2}}-N_{30}}{N_{T_{2}}-N_{T_{1}}}\right]+T_{2} \sigma_{2}^{2}\left[\frac{N_{30}-N_{T_{1}}}{N_{T_{2}}-N_{T_{1}}}\right]\right\} * \frac{N_{365}}{N_{30}}\right)
$$

- Multiply the value of the square root by 100 to get the VIX
$V I X=100 * \sqrt{ }\left(\left\{T_{1} \sigma_{1}^{2}\left[\frac{N_{T_{2}}-N_{30}}{N_{T_{2}}-N_{T_{1}}}\right]+T_{2} \sigma_{2}^{2}\left[\frac{N_{30}-N_{T_{1}}}{N_{T_{2}}-N_{T_{1}}}\right]\right\} * \frac{N_{365}}{N_{30}}\right)$

See more information at https://www.cboe.com/micro/vix/vixwhite.pdf

