OPERATING RISK ANALYSIS OF WIND INTEGRATED GENERATION SYSTEMS

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ABSTRACT

Wind power installations are growing rapidly throughout the world due to environmental concerns associated with electric power generation from conventional generating units. Wind power is highly variable and its uncertainty creates considerable difficulties in system operation. Reliable operation of an electric power system with significant wind power requires quantifying the uncertainty associated with wind power and assessing the capacity value of wind power that will be available in the operating lead time. This thesis presents probabilistic techniques that utilize time series models and a conditional probability approach to quantify the uncertainty associated with wind power in a short future time, such as one or two hours. The presented models are applied to evaluate the risk of committing electric power from a wind farm to a power system. The impacts of initial wind conditions, rising and falling wind trends, and different operating lead times are also assessed using the developed methods. An appropriate model for day-ahead wind power commitment is also presented. Wind power commitment for the short future time is commonly made equal to, or a certain percentage, of the wind power available at the present time. The risk in meeting the commitment made in this way is different at various operating conditions, and unknown to the operator. A simplified risk based method has been developed in this thesis to assist the operator in making wind power commitments at a consistent level of risk that is acceptable to the system.

This thesis presents a methodology to integrate the developed short-term wind models with the conventional power generation models to evaluate the overall operational reliability of a wind integrated power system. The area risk concept has been extended to incorporate wind power, evaluate the unit commitment risk and the well- being indices of a power system for a specified operating lead time. The method presented in this thesis will assist the operator to determine the generator units and the operating reserve required to integrate wind power and meet the forecast load for a short future time while maintaining an acceptable reliability criterion. System operators also face challenges in load dispatch while integrating wind power since it cannot be dispatched in a conventional sense, and is accepted as and when present in current operational practices. The thesis presents a method to evaluate the response risk and determine the unit schedule while satisfying a specified response risk criterion incorporating wind power. Energy storage is regarded as an effective resource for mitigating the uncertainty of wind power. New methods to incorporate energy storage with wind models, and with wind-integrated power system models to evaluate the wind power commitment risk and unit commitment risk are presented in this thesis. The developed methods and the research findings should prove useful in evaluating the operating risks to wind farm operators and system operators in wind integrated power systems.

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Dedicated to:

BIJAYA and Asutosh

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LIST OF ABBREVIATIONS

ARMA	Auto-Regressive and Moving Average
CAES	compressed air energy storage
COPT	Capacity Outage Probability Table
ELD	Economic Load Dispatch
ESS	Energy Storage System
FOR	Forced Outage Rate
HL	Hierarchical Levels
HL-I	Hierarchical Level-I
HL-II	Hierarchical Level-II
HL-III	Hierarchical Level-III
IEEE	Institute of Electrical and Electronic Engineers
IEEE-RTS	IEEE Reliability Test System
ILCC	Increase in Load Carrying Capability
km/h	Kilometer per hour
kW	Kilowatt
MT	Margin Time
MW	Megawatt
MWSR	Mean Wind Speed Ratio
NWP	Numerical Weather Prediction
OCC	Operating Capacity Credit
ORR	Outage Replacement Rate
PR	Pick-up Rate (ramp rate)

RM	Regulating Margin
RPS	Renewable Portfolio Standard
RR	Response Risk
RRC	Response Risk Criterion
RRCD	Response Risk Constrained Dispatch
RRM	Required Regulating Margin
RTS	Reliability Test System
SOC	State of Charge
SD	Standard Deviation
SR	Spinning Reserve
UCR	Unit Commitment Risk
UCRC	Unit Commitment Risk Criterion
WP	Wind Power
WPCR	Wind Power Commitment Risk
WSSDR	Wind Speed Standard Deviation Ratio
WTG	Wind Turbine Generator

LIST OF VARIABLES

μ	Mean
σ	Standard deviation
t	Time
y_t	Time series value
α	Random number
x_t	Wind speed at time ‘t’
P_t	Wind power at time ‘t’
Vci	Cut-in wind speed
V_r	Rated wind speed
V_{co}	Cut-out wind speed
P_r	Rated power

CHAPTER 1

INTRODUCTION

1.1. Power System Reliability

An electric power system is a large and complex configuration composed of equipment and circuits that are dispersed geographically for the generation, transmission and distribution of electrical energy [1]. The primary function of an electric power system is to satisfy the demand and energy requirements of its customers as economically as possible and with an acceptable degree of continuity and quality [2]. The loss of continuity of electricity service can have significant economic and social impacts on customers as well as on the utility supplying the electricity. Reliability is therefore considered to be an important attribute of a modern electric power system. The continuity of electric energy supply can be made very high with increased redundancy but this is always accompanied by excessive investment costs that are ultimately reflected in the price. The conflict between the cost and reliability of electric supply can be optimized by making suitable decisions in the planning and operating phases.

Power system reliability in a broader sense is a measure of the overall ability of the system to perform its prime function. It can be divided into the two basic concepts of system adequacy and system security [2]. System adequacy is related to the ability of the generation, transmission and distribution facilities in the system to satisfy the consumer load demand. System security is related to the ability of the system to respond to disturbances arising within the system. A disturbance may be local or wide-spread and include the loss of major generation and transmission facilities. The power system operator is responsible for preserving the security of the system by making appropriate dispatch decisions. An overall power system can be subdivided into the three basic functional zones of generation, transmission and distribution. The reliability assessment of an overall power system becomes very complicated and assessments are usually performed in each functional zone and at each hierarchical level [1], [2]. Reliability assessment performed at hierarchical level I (HL- I) considers only the generation facilities and quantifies the ability of the generation system to satisfy the total demand. Hierarchical level II (HL- II) considers both generation and transmission facilities and hierarchical level III (HL- III) covers all three functional zones. The research work presented in this thesis is focused on HL-I reliability evaluation in the operating domain.

1.2. Power System Reliability in the Operating Domain

Under normal operating conditions, load forecasts are carried out for short future periods and the generating units are scheduled such that the operating capacity is sufficient to meet the predicted load. The power system operator also schedules sufficient reserve in the committed generating units to cover the probable outages of any generating units and accommodate the load forecast uncertainty. The unloaded capacity that is synchronized and ready to take up load is known as spinning reserve [1]. The effective spinning reserve can be enhanced by considering factors such as rapid start units, hot reserves, assistance from the interconnected systems, interruptible loads and voltage and or frequency reduction [1]. The spinning reserve and the factors that add to it are known as operating reserves.

The conventional practice to determine the required operating reserve is to use a rule- of-thumb approach that considers one or more of the largest operating units as the operating reserve. Unit failures and load changes are stochastic in nature and are not specifically considered in a deterministic method. This practice can lead to over scheduling where the system is more reliable but the operating cost is excessive, or under scheduling where the cost is lower but the system is unreliable [2]. Assessment methods based upon probabilistic techniques can provide realistic and consistent risk evaluation. In addition, such methods are well suited for relative reliability assessments of different alternatives [3]. A wide range of methods have been developed for reliability evaluation of power systems based on probabilistic techniques [2, 4-12]. The terms Unit Commitment Risk (UCR) and Response Risk (RR) are two risk indices based on probabilistic assessment that have been used in this area[1] [4]. Unit commitment risk is related to the assessment of which units to commit in any given period, while the response risk is associated with dispatch decisions on the committed units [1].

The unit commitment risk (UCR) is the probability that the committed generating units are capable of just carrying or failing to carry the expected load during the period in the future in which generation cannot be replaced [1]. The stated period is called the lead time and is the time taken by a unit to start-up and reach a specified output level after being synchronized to the power system. A hydro unit or a gas turbine can be brought up to its full capacity within a few minutes while a thermal plant needs an appreciable amount of time for the steam pressure to be sufficient to provide the rated power. In case of a unit failure, assistance from another unit will be available only after the lead time of the unit delegated to replace the failed unit.

The technique presented in [13] is regarded as the pioneer approach in probabilistic spinning reserve evaluation and is known as the Pennsylvania-New Jersey-Maryland (PJM) method. The PJM method is illustrated by application in a practical hydro-thermal power system study in [14]. The basic generating unit statistic in an adequacy assessment is the unit unavailability, which is designated as the forced outage rate (FOR) [1]. Generating failures during system operation cannot be repaired during short lead times and a failed unit must be replaced by other capacity. The FOR used in an adequacy study is replaced in [14] by an outage replacement rate (ORR) in operating risk assessment. The ORR is the probability that a unit fails and is not replaced during the lead time T [1]. Unlike FOR which is a limiting state probability, ORR is a time dependent probability [15] where the initial condition is important. For a short future time duration, the ORR can be expressed as the failure rate multiplied by the lead time given that the unit is successfully operating at the initial time [1]. The basic method [13, 14] is enhanced to consider the impact of rapid start units and hot reserves in [16, 17]. The impact of load forecast uncertainty in system risk is also presented in [14]. Inclusion of assistance from the interconnected systems [18-20], interruptible loads [21], partial output states [22] and postponable outages [23] are further improvements in operating system reliability evaluation. A general security function approach was created for evaluating spinning reserve [24] and composite generation and transmission security [25-27].

Power system reliability evaluation at HL- I, based upon the loss of load technique, involves creating a system generating capacity model [28] containing all the existing capacity states and their probabilities of the system being considered. This array of capacity states and their probabilities is called a capacity outage probability table (COPT) [4]. Reference [1] presents the detailed algorithms for creating a COPT by adding one generating unit at a time. Each unit is represented by a two-state model in (1.1), and by multi-state models in (1.2)

$$P(X) = (1 - U)P'(X) + (U)P'(X - C) \tag{1.1}$$

Where:

C is the capacity (MW) of a unit being added,

U is the unavailability or the FOR of the added unit, P(X) and P'(X) are the cumulative probabilities of the capacity outage state of X MW after and before the unit is added.

$$P(X) = \sum_{i=1}^{n} p_i P'(X - C_i) \tag{1.2}$$

Where:

n is the number of unit states

C_i is the capacity outage of state i for the unit being added

p_i is the probability of the unit being in state i

The expressions (1.1) and (1.2) are initialized by setting

P' (X) = 1.0 for $X \leq 0$ and P' (X) = 0 otherwise

The COPT of all the operable generating units is convolved with a load model to evaluate system inadequacy.

The generation capacity models presented in (1.1) and (1.2) used in adequacy assessments can also be used in unit commitment risk evaluation. An operating COPT can be created using (1.1) or (1.2) where the FOR is replaced by ORR and a COPT is created for the actual number of units committed. Unit commitment assessment is performed for a specific load level and is updated on a continuous basis.

Tables A1 and A2 in Appendix A show the COPTs of eight committed units from the priority loading order of the IEEE Reliability Test System (RTS) [29] for one and four hour lead times respectively. The total committed capacity is 1547 MW. Capacity states with cumulative probabilities lower than 10^{-8} are not shown in the COPT in the appendices. As noted earlier, the unit commitment risk is the cumulative probability of the capacity state in service which is equal to or less than the load. The uncertainty in system behavior increases with the lead time, and therefore, the UCR increases with an increase in the lead time. If the UCR has to be maintained within a specified level, the COPT can be used to indicate the maximum load that can be satisfied. A risk criterion of 0.0001 can be considered for an instance. Assuming a UCR criterion of 0.0001, the eight committed units can satisfy a maximum load of 1146.9 MW if the assistance is available after one hour. The load carrying capability reduces to 1096.9 MW if the assistance is available only after four hours.

UCR analysis assists the system operator to decide which and how many units should be committed at the decision point to satisfy the forecast load over the lead time and meet a specified operating risk criterion. This analysis sets the framework but does not provide any information on the dispatch levels of these committed units [1]. The second aspect of operating reserve assessment is response risk analysis and involves spinning reserve allocation [30] and the response capability of the committed units. The units acting as spinning reserve must respond within a certain period of time to system changes such as failures of the committed units, sudden increases in load or other disturbances. The regulating margin is the change in the system generation level that can be achieved within a specified time period [31]. This period is the margin time within which the generation level must be raised to protect the system from a sudden unit failure [30]. There are two time periods of interests; a response within one minute after a contingency is required to maintain system frequency and tie line regulation while a response within 5-15 minutes is required to save load loss against the capacity loss caused by the contingency. The ability to respond to system changes can be variable and depends on the types of operating capacity and their pick up rates. The allocation of spinning reserve depends upon the type of units carrying the spinning reserve and their location [30]. Response risk is assessed by evaluating the probability of achieving a certain response or regulating margin within the required response time, which is obtained by creating a COPT of the regulating margin [30].

Spinning reserve estimation based upon probabilistic methods of unit commitment and dispatch considers the stochastic failures of the committed unit and provide a valid risk of system operation. Deterministic method such as the “N-1” criterion allocates the capacity of the largest committed unit as the spinning reserve and is widely used in system operation despite being unable to quantify the associated risk. System operators find the probabilistic risk indices relatively difficult to understand compared to a deterministic method which serves as a rule of thumb. The well- being concept [32] is a hybrid method that incorporates a deterministic criterion into a probabilistic framework and provides the probability of the system being in a “healthy”, “marginal” or “at risk” state on the basis of an accepted deterministic criterion. A system is in the “healthy” state if it has a sufficient amount of reserve required by the specified deterministic criterion. It enters into the “marginal” state if the system is operating without trouble but the reserve is not sufficient to meet the criterion. The system is in the “at risk” state when it has a capacity deficit such that the operating capacity is just equal to or less than the load. The well- being concept can be used in both long term capacity reserve [33-36] and operating reserve evaluation [37-41] to reflect the well- being of the system.

1.3. Wind Power Growth and its Impact on Power System Operation

1.3.1. Growth of wind power in power systems

Electricity generation has been largely dominated by hydro, nuclear and fossil fuel fired thermal units, which provide specified capacity and are known as conventional capacity. One of the major drawbacks of most conventional units is their negative impacts on the environment such as the emission of greenhouse and other harmful gases. Another important concern is that fossil fuel reserves are depleting and their costs are volatile. Policy makers and utilities around the world are moving to generate more electric power from renewable and alternative energy resources to reduce the environmental impact of electricity production and to diversify energy supplies. Many countries have agreed to an energy policy known as a Renewable Portfolio Standard (RPS) which is a commitment to supply a certain percentage of the total electricity consumption from renewable energy resources. As a commitment to the RPS, several states in the USA have posted their commitment to produce 10 to 25% of their total electricity consumption from renewable resources [42]. Wind turbine generators (WTG) have gone through remarkable advancements in design and operation in the past few decades that have led to increased efficiency and capacity at reduced cost. At the present time, wind power is the most preferable renewable energy technology for bulk power production and is being installed all over the world. Worldwide wind power installation reached close to 282.5 GW by the end of 2012 and it has been estimated that wind could meet 12% of the world's power demand by 2020 and more than 20% by 2030 [43]. The current installed wind power capacity of Canada is 6927 MW which is capable of supplying electricity to more than 2 million households [44]. It is expected that wind could meet 20% of Canada's total electricity demand by 2025 [44].

1.3.2. Power system operation with significant wind power

Wind power generation at a specific time depends upon the instantaneous wind speed at the wind site and the wind speed is governed by the local atmospheric condition. A system operator cannot control the wind resource and wind power cannot be dispatched in a conventional sense. Wind speed varies continuously in a random fashion and the variability is site specific. A power system with significant wind power experiences considerable variations in generation level which may increase the uncertainty of the system operation.

A power system operator has the responsibility of ensuring that adequate operating reserve exists to maintain the electric supply reliability. As stated earlier, generating units are committed in order to meet the load forecast for the lead time considered and to satisfy the reliability criterion. In conventional systems without wind power, system operators strive to balance the variations in the system demand by ensuring adequate response from the committed units through a suitable allocation of adequate generation reserves to each of these units. The addition of wind power adds variability in the power generation and makes the system operation more complex. It may be necessary to allocate additional reserves to account for the uncertainty and the variability associated with wind power generation for reliable operation of a power system.

The ratio of the installed wind power capacity to the total installed generating capacity is termed as wind power penetration. Power systems with large wind power penetrations are subjected to large and random variations in power supply, and therefore, the system operators face considerable challenges in continuously satisfying the load and maintaining the system reliability. When operating a power system with wind, knowledge of the wind speed one day ahead is required to schedule the conventional units. Short term prediction, such as 1-2 hours ahead, helps the system operator to appropriately allocate and optimize the regulating capacity [45]. It is therefore necessary to predict the wind power accurately for the time period considered in order to evaluate the operating risk incorporating wind power. Wind power prediction methods have evolved significantly but the errors of prediction are still substantial indicating that accurate wind power prediction is not an easy task. An overview of spinning reserve evaluation incorporating wind power and short term wind power predictions is presented in the following sections.

1.4. Literature Review

1.4.1. Literature review on adequacy analysis of power systems with wind power

Conventional generating units can normally produce electric power continuously at their rated capacity. The reliability of the generation system is mainly governed by the failure and repair rates of the generating units. There are established techniques to evaluate the reliability of generation systems consisting of conventional units [1]. In an analytical method, reliability evaluation is performed by developing a capacity model designated as a COPT and convolving it with a suitable load model. The output power from a wind farm fluctuates randomly with time depending upon the variability of the wind speed at the wind site. The reliability evaluation of a wind integrated power system is therefore relatively complex and requires accurate models to forecast wind speed variations at the wind farm locations and to create appropriate wind turbine generator (WTG) models. In order to obtain the complete capacity model of the wind integrated system, a combined COPT has to be developed that includes the conventional units and the WTG at each geographic wind site. Considerable work has been done on system adequacy analysis of power systems with wind power [46-65]. References [46-55] present the models for adequacy evaluation of a power system with wind power. The impacts of wind site correlation are presented in [62-65]. The adequacy of composite generation and transmission systems with wind power are presented in [59, 63, 65]. A simplified approximate wind speed model for reliability evaluation has been presented in [56] to obtain the probability distribution of wind speed from the knowledge of the annual mean and standard deviation of the wind speed data.

Wind has been conventionally considered as an energy resource and has been treated as an energy limited unit in reliability assessment [66]. There has, however, been increasing consideration to the argument that wind farms can also contribute to the system adequacy as a capacity resource. Considerable work has been done on assessing the capacity credit of wind farms [67-72]. The capacity credit for relatively small wind power penetrations approximates to the average wind power output which tends to limit as the penetration grows [67]. The contribution of WTG in system adequacy improvement has been quantitatively assessed in terms of a Load Capacity Benefit Ratio in [68]. Reference [69] presents a chronological method where certain hourly capacity factors of a wind farm are selected in chronology with the load for the capacity credit evaluation. A posterior capacity credit is evaluated from actual capacity factors of a wind farm while a priori capacity credit is evaluated by using probabilistic simulation of wind power in the proposed chronological method [69]. Reference [70] presents an analytical formula to assess the wind power capacity credit based upon wind power penetration, the annual use of the wind farm and the reliability of the conventional units. Reference [71] reviews the existing methods of evaluating wind power capacity credit. The capacity credits associated with one or more wind farms in system planning and operation are evaluated in [72] using the basic probabilistic indices of Loss of Load Expectation, Loss of Energy Expectation and UCR.

1.4.2. Literature review on security analysis of power systems with wind power

The impacts of significant wind integration on reserve demand are studied in [73-91]. Reference [73]considers wind speed and load forecast errors and the ramping rate of the conventional units for determining the reserve margins in the wind-hydro-thermal interconnected system of Sweden. The impacts of high wind penetration in secondary reserves including regulating reserve and contingency reserve are investigated for the German power system in [74]. The impacts of wind power fluctuations are considerable compared to load fluctuations causing increased area control errors and therefore, demands an increase in the regulating reserve [74]. Reference [74] also notes that the increase in the area control errors may be compensated by suitable system configurations such as pumped storage plants. A short –term prediction of wind power allows a wind farm to be considered as a capacity resource rather than just an energy resource and may replace an equivalent conventional capacity [76]. This, however, demands increase in the reserve level to cover the uncertainty of wind power generation for maintaining the operating reliability [76]. Reference [77] presents a probabilistic method to quantify reserve demand considering generator outages and wind and load forecasting errors. The reserve requirement is determined by considering the “N-1” criterion and the maximum forecast error of wind power in [78]. The case study conducted in [78]considers five European nations and shows that the market price of electricity reduces during high wind power. Reference [79] investigates the impact of wind power on unit commitment and dispatch of the Dutch power system dominated by combined heat and power thermal units. At high wind penetrations, significant wind power could be wasted because of minimum load problems at combined heat and power units [79]. A particle swarm optimization technique is used in [80] for spinning reserve evaluation and finds that increase in wind power penetration will reduce the overall cost of operation even though it requires increased regulating reserve. The ramping capabilities of the committed conventional units are considered in spinning reserve evaluation at different levels of wind power penetration in [81] and a re-dispatch of the conventional units are conducted for any imbalances caused by the wind power forecasts errors or transmission network violations. Unit commitment of a thermal-wind system considering the ramping limits of thermal units and wind curtailment is presented in [82]. The numerical results show that wind power integration, despite demanding increases in spinning reserve, reduces the overall cost [82]. Reference [83] considers net load forecast errors in presenting a future scenario tree for unit commitment and hydro-thermal scheduling. Reference [84] also utilizes a stochastic method based upon event trees, to estimate future probable wind and load scenarios for spinning reserve evaluation, which is conducted frequently on a rolling basis. Unit commitment and load dispatch is solved using a particle swarm optimization technique in [85] and finds that the spinning reserve requirement limits wind power penetration above 35% in the case considered. The net demand forecast is utilized, assuming normally distributed wind and load forecast errors, for reserve planning considering the value of load loss [86]. Reference [90] uses scenario generation for wind power based upon an autoregressive time series model for wind data to solve unit commitment and economic load dispatch for a power system model of the California ISO. Reference [91] combines the capacity outage probability table of the conventional units with the wind power model comprising the forecast error and outages of wind turbines to evaluate loss of load indices for a day ahead unit commitment. Most of the tasks relate to economics and lack risk evaluation of power system operation with significant wind power.

As noted earlier, operating risk analysis of a wind integrated power system involves the development of a combined COPT for the lead time considered. The wind power output is largely dominated by the wind regime and the failure of a WTG has relatively negligible impact on the overall reliability. Accurate wind power forecasting is a vital tool to overcome the complexities caused by the intermittency of the wind power output. An appreciation of the wind speed one day ahead is required to schedule the conventional units while short term predictions, such as 1-2 hours ahead, are used by system operators to optimize the regulating capacity [45].

1.4.3. Literature review on short term wind power prediction

A number of papers have been published regarding methods for short term wind power prediction. References [92-94] present detailed literature overviews of short term wind power prediction models. The models can be broadly categorized into the physical approach and the statistical approach. The physical approach makes use of Numerical Weather Prediction (NWP) which simulates the flow in the atmospheric condition by integrating a large number of non-linear equations governing the weather, starting with the current observations and measurements carried out by meteorological and weather stations, satellites etc. [95]. The weather and the wind speed forecast given by the NWP model is processed to predict the wind power at the wind site using either or a combination of physical, statistical and learning methods [96]. The physical method takes into account the influences caused by the local weather conditions, conversion characteristics of the WTG and other influences that affect the wind power output at the wind site. The statistical method analyzes the connection between the weather forecast given by NWP and the power output at the wind site from the historic time series in the past and uses the connection to predict the wind power in the future. The learning method makes use of artificial intelligence techniques to determine the relationship between the forecasted wind and the power output from the time series of the past [96]. Ensemble forecasting [94] is a relatively new method which gives the probability density of the future forecast instead of a point forecast. This is obtained by running multiple NWP models or running NWP with different initial conditions and parameters [97]. Ensemble forecasting gives the uncertainty of a forecast which makes it different from a traditional NWP. It has been found in [97] that the ensemble forecasting can produce more accurate forecasting, up to ten days ahead, compared to that from a single NWP based model or times series based model. The spread of the forecasts given by the ensemble predictions is used to evaluate the prediction risk in [98] to estimate the uncertainty.

Statistical methods that make use of time series models such as Auto Regressive Moving Average (ARMA) [99, 100] are also used for short term wind speed prediction. The persistence model [101] is also a type of time series model which assumes that the wind power at the future time horizon will not vary from what is available at present. Though the model is very simple and direct, the persistence model can be very effective for some short times in the future. Research models are usually compared with the persistence model to determine the relative accuracy of the developed model. The persistence model has been found to be more accurate than physical models employing NWP for prediction horizons of 4-6 hours [102, 103]. Most of the commercial wind power forecasting tools use a physical approach employing NWP models. Pure statistical methods may not compete with the more sophisticated physical models for long forecasting horizons. However, the statistical methods despite their low cost and simplicity can give accurate forecasts for short time horizons up to 6 hours and can be very useful for small to middle size wind farm owners who cannot afford the commercial forecasting tools [104]. It has been found that wind power prediction using an ARMA model can outperform the physical model employing NWP for time horizons up to 4-6 hours [105]. An ARMA model can also outperform a persistence model for time horizons greater than one hour and give accurate forecasts up to 10 hours [106].The ARMA model can be used to simulate wind speeds for large numbers of sample years and create conditional wind speed probability distributions for the next hour(s) considering the wind speed initially available. The wind power probability distribution during the lead time can be used as a multi-state generating unit and combined with the committed conventional units in a unit commitment risk analysis [107].

1.4.4. Literature review on energy storage applications to wind integrated power systems

Wind power varies in a wide range of time scales ranging from seconds, minutes, hours and seasons. Wind power variability is one of the fundamental concerns in grid integration. A suitable energy storage system is considered to be very useful in suppressing the variations caused by wind power. Energy storage with a wide range of capacity and discharge times are commercially available. References [108, 109] present different aspects of energy storage applicable to grid integration of wind power. The discharge time and module size are the main characteristics used to select a storage technology for a particular application and usually vary inversely for any storage technology. Superconducting magnetic energy storage, flywheels, nickel-metal hydride battery have discharge times at rated power from a few seconds to less than a minute and their module sizes vary from 1 kW to less than 1 MW [109]. These types of storage can be used, in conjunction with power electronic devices, for mitigating short term wind power fluctuations [110] and improving power quality.

Lead- acid batteries have discharge times in minutes and their capacities can vary over kW to MW ranges which make them suitable for power quality as well as load shifting applications. Other battery technologies such as lithium-ion, sodium sulfur, zinc-chloride, zinc-air, zinc-bromine, vanadium-redox battery and polysulfide bromide batteries have discharge times varying from minutes to hours and capacities varying from 100 kW to 10 MW. These battery technologies are suitable for grid support and load shifting. Pumped hydro and compressed air energy storage (CAES) have their discharge times in hours and the capacity can vary from 10 MW to greater than 100 MW [109]. Reference [111] presents different operating strategies of a wind – driven pumped storage power system and a general model to optimize the size and the costs of such a system. The models developed in [111] are applied in [112] to optimize the size of wind turbines, reservoirs and the pumps to utilize wind power, combined with other generation systems, in a small isolated power system. The successful operation of the first CAES plant in Germany [113] has proved to be useful in increasing the operating flexibility in a power system required to integrate significant wind power. The economic value of a CAES in a German power system with substantial wind power penetration is studied in

[114]. The economic aspect of most energy storage is justified from the benefit of the difference in electricity prices at different times of a day and the environmental incentives from the reduction in carbon emission. Various aspects of the application of energy storage with high wind power penetrations are presented in [115] stressing that high increased wind power penetration increases the significance of energy storage. It is noted in [115] that the benefit of the storage is justified from the system's perspective regarding economics and reliability including environmental factors.

The role of energy storage in mitigating the long term wind power fluctuation, occurring in a few minutes to a few hours, is presented in [116]. The stored energy can be used to cover a deficit in generation and decrease the risk of wind power commitment caused by prediction errors [116]. Depending upon the characteristics and size of an energy storage system (ESS), the main purpose of an ESS in a wind integrated power system is to manage the wind power variability, avoid wind power curtailment due to transmission congestion, overcome sort-term fluctuations and support system stability [115]. Reference [117] presents a method for operating a wind and energy storage in order to maximize the value of the wind power in a spot market system. The economics of implementing energy storage in power systems with significant wind power penetration are studied in [118] considering a power system scenario in Alberta, Canada. Reference [119] presents a time series simulation technique to evaluate the system adequacy of a small stand-alone wind energy conversion system with battery storage. Reference [120] presents a simulation technique to assess the long term reliability benefits of energy storage considering a scenario where a limitation is imposed on wind power absorption for stability reasons. There has been relatively little significant work done in assessing the operating reliability of a power system with wind and energy storage.

From the perspective of a wind farm operator, an ESS may be used to reduce the risk of wind power commitment due to the wind power variability. A study carried out by several western countries suggests that the average hourly wind power generation from a distributed wind energy system varies by a maximum of 20% of the total capacity of the wind farm [121] indicating the approximate storage capacity to start with. The variability of a single wind farm however is appreciably higher than that of the overall distributed wind farms. The fluctuations of the entire wind power in the case of western Denmark reduced by a factor of approximately 3 compared to that from a single wind farm [121].

1.5. Research Motivation

The rapid growth in wind power penetration has made it important to assess its impact on system reliability, and to consider wind power not just as an energy resource but also as a capacity resource. Reliability of the power system has always been a prime concern when integrating wind power due to the uncertain and fluctuating nature of wind. Different methods to assess the value of wind power as a capacity resource in long term reliability perspective have been developed in [47][67, 69-71]. A main challenge, however, has always been to assign the capacity credit to a wind farm in the operating time domain. Most of the work has focused on the economic aspects and relatively little work has been done to evaluate the operating risk in wind integrated power systems [107]. There is a need to develop methods that can appropriately integrate wind power in operating risk evaluation and assess wind power operating capacity credit. There is also a need to study the impact of wind power on spinning reserve allocation.

A main issue in operating a wind integrated power system is to assess the wind power contribution in the next hour or the next few hours. Different utilities or wind farm developers use different practices to estimate the wind power commitment over the next hour or the next few hours. A typical example is to commit a certain percentage of the current wind power output for the next hour(s). Such a deterministic commitment is easy to use for both system operators and wind farm owners. The risk associated with such a commitment, however, is not known using these methods. There is, therefore, a need for a method that can quantify the risk associated with such a deterministic commitment so that an appropriate amount of wind power can be allocated to satisfy an acceptable risk criterion.

Variations in wind speed can have appreciable impact on the operating reserve in a power system. System operators find it difficult to maintain the balance between the supply and demand with increasing wind power in the system due to the fluctuating nature of wind power. Due to the variability associated with wind power, it cannot be solely relied upon to supply the load continuously. When the wind penetration is very low, wind power can be absorbed as and when present. In a power system with high wind power penetration, the existing flexibility of the committed units may not be sufficient and can reach the minimum generation levels of one or more conventional units. Any further increase in wind power could force shutting down such units to absorb all the wind power during such high wind speeds. At low load and high wind periods, this could involve shutting down base load units such as coal fired plants, which is not practical. A system operator could, under such conditions, refuse to accept all the wind power available, which results in underutilization of wind power. There is therefore a need to develop methods to study the impact of energy storage on power system operating risk.

1.6. Research Objective

The major concerns regarding system operating reliability in a wind integrated power system are discussed in the previous section. The risk associated with a deterministic method of wind power commitment should be quantified and used to assess the amount of wind power to be committed in the next hour(s) while satisfying an acceptable risk criterion. An operating reserve analysis in a wind integrated power system can assist the system operator to schedule generation and maintain adequate operating reserves incorporating the wind power variations in the next hour(s). Employing suitable energy storage could aid in reducing the fluctuations in wind farm output. This could also add some controllability such that wind power could be absorbed when required by the system. A study of the operating reliability of a wind integrated power system with energy storage could help the system operator and the wind farm owner to implement suitable operating strategies to increasing the value of wind power. The proposed research work presented in this thesis is mainly focussed on evaluating the short term reliability of wind integrated power systems. The research work consists of the following four major tasks in order to meet its objectives.

1. Wind power modeling for short- term reliability evaluation
2. Quantifying the risk associated with committing wind power
3. Operating reserve analysis incorporating wind power
4. Operating risk evaluation incorporating wind power and storage

1.6.1. Wind power modeling for short- term reliability evaluation

The reliable operation of a power system incorporating wind power requires accurate wind speed forecasts for the wind farm site in order to commit the appropriate combination of wind and conventional power to meet the forecast load over the next few hours. The probability distribution of the wind speed for a short time in the future depends on the initial wind speed. Knowledge of the initial wind speed can be used in a conditional probability approach to create wind speed probability distributions for short times in the future and used to assess the risks of committing power from a wind site. An ARMA time series model can be used [35, 47, 48, 50, 52-54, 57, 60, 61, 63, 65, 72, 107, 119, 120, 122-126] to accurately simulate wind speed data for a particular wind site, and generate sufficient synthetic data to create the wind speed distributions. A new method for short term wind power modeling was developed by the Power Systems Research Group at the University of Saskatchewan [126] using a conditional probability approach based on the initial wind speed condition. An objective of this research was to further develop the wind power models for evaluating the risk of wind power commitment considering the impacts of wind farm location, diurnal and seasonal wind trends and correlated wind farms. The models will also be used to investigate their application to day ahead wind power commitment.

1.6.2. Quantifying the risk associated with committing wind power

Another objective of the research was to develop a method to quantify the risk associated with the present deterministic method of wind power commitment, which would then allow the system operator to commit an appropriate amount of wind power while satisfying a specified risk criterion. Wind power commitment is affected by the risk criterion, lead time, and wind site location. Investigation of such impacts including the influences of diurnal and seasonal wind trends and wind speed correlations between multiple wind farms in wind power commitment is a major objective of this research. Development of a simplified risk based method for wind power commitment at a short future time is considered as another research goal under this task. An assessment of the application of time series models in day ahead wind power commitment was also a set task under this research area.

1.6.3. Operating reserve analysis incorporating wind power

A reliable power system operation requires careful decisions on unit commitment as well as on the dispatch of the committed units. Such decisions should be based on risk evaluation in order to ascertain that the system is operating under an acceptable level of uncertainty. Operating reserve analysis consists of both unit commitment risk evaluation and response risk evaluation. Unit commitment risk analysis incorporating wind power and evaluation of the wind power operating capacity credit at specified UCR criterion were the major research objectives under this topic. This required development of appropriate methods to combine wind power in the unit commitment risk evaluation. A study of the impacts on UCR of rising and falling wind speed trends and adding statistically correlated wind farms were the other goals within this research topic. Another objective under this topic was to develop an approximate method to simplify the wind power modeling applied to UCR evaluation. An assessment of the unit commitment well-being incorporating wind power and wind power operating capacity credit evaluation considering dual criterion of health and risk were other important objectives. An assessment of the impact of wind power in spinning reserve allocation considering a response risk criterion was a further objective under the operating reserve analysis task. This requires the development of a method to incorporate wind power in response risk evaluation.

1.6.4. Operating risk analysis incorporating wind power and storage

An appreciation of the effects of storage in wind power commitment and on the operating risk can be useful for the system operator or/and wind farm operator in order to implement and operate the storage facility with WTG. An examination of the impact of energy storage on

WPCR by implementing a suitable operating strategy was therefore a research objective. This topic also included an assessment of the UCR considering wind power and energy storage.

1.7. Organization of the Thesis

The thesis is organized into eight chapters. Chapter 1 prepares the background and presents a basic overview of probabilistic risk assessment in power system operation, literature reviews and research objectives.

Chapter 2 presents the concept of short term wind power modeling for operating risk evaluation. This involves the wind speed model and the wind turbine generator model. The wind speed model is based upon a time series ARMA model and a conditional probability approach. The chapter considers the diurnal and seasonal wind speed trends, impact of lead time and multiple wind farms with wind speed correlation. This chapter provides the basic probabilistic model of wind power for short term risk evaluation.

Chapter 3 evaluates the wind power commitment based on specified risk criteria. The term "wind power commitment risk" is introduced and evaluated considering the impacts of initial conditions, wind site locations and lead time. It also considers the impact of diurnal and seasonal wind variations and wind speed correlations in evaluating the wind power commitment risk. A simplified risk based method for short term wind power commitment is also presented in this chapter. Chapter 3 also presents the application of a time series model in day ahead wind power commitment.

Chapter 4 presents the unit commitment risk and well-being analysis in a power system incorporating wind power. A new appropriate method based upon the area risk concept is developed in this chapter to integrate wind power in system risk evaluation.

Chapter 5 presents a simplified method for incorporating wind power in unit commitment risk evaluation. The method considers the initial conditions and the basic statistics of the historic wind speed to estimate future wind speed and power variability.

Chapter 6 presents the concept of response risk analysis considering wind power. The impact of integrating wind power on the response risk and the economics of system operation is studied in this chapter.

Chapter 7 considers energy storage in conjunction with a wind farm in assessing wind power commitment risk and unit commitment risk. Energy storage with different rated capacities and discharge times are considered to minimize the uncertainty of wind power in a short term.

Chapter 8 highlights the research findings and concludes the thesis.

1.8. Summary

The basic objective of this research work is to study the impacts of wind power integration on the operating reliability of power systems, and develop methodologies to quantify the risks associated with operating decisions. The risk based method for wind power commitment can help system operators and wind farm owners to assign appropriate amounts of wind power based on knowledge of the associated risks [127-132]. The extended area risk based method for unit commitment risk evaluation proposed in this thesis [133, 134] integrates the variability of wind power with the uncertainty of the committed units more appropriately than the available reference method. The response risk analysis considering wind power gives a combined insight of the economic and reliability impact of wind power in system operation. The studies conducted in this research work also include the impact of energy storage in association with wind farms. The developed concepts can assist wind farm owners and system operators to implement energy storage and maximize the utilization of wind energy while providing acceptable system reliability. The simplified models developed in the proposed research should prove useful in the practical incorporation of wind power in operating risk analysis.

CHAPTER 2

DEVELOPMENT OF SHORT-TERM WIND POWER MODELS FOR OPERATING RISK EVALUATION

2.1. Introduction

A power system is very dynamic as load is continuously changing and the system may be subjected to disturbances, such as adverse weather and equipment failures. Adequate operating reserves are therefore instrumental in maintaining the reliability of power supply, and generating units are committed to satisfy specified reliability criteria. The variation in demand is balanced by obtaining adequate response from the committed units when operating reserves are appropriately allocated. The uncertainty associated with wind power generation can substantially increase the overall variability making it more challenging for the system operator to maintain the required reliability. Accurate prediction of wind power for a future lead time is therefore very important when assessing the operating risk of a power system incorporating a significant amount of wind power. The appropriate allocation and optimization of the regulating capacity [45] during system operation requires short term wind power predictions such as one or two hours ahead.

Wind power is site specific and depends upon the instantaneous wind speed at the wind site. A wide range of statistical [104] and physical [102] approaches have been applied to short term wind prediction. Statistical methods that make use of time series models such as Auto Regressive Moving Average (ARMA) [99] have also been used for short term wind speed prediction. The persistence model [135] is a type of time series model that has been widely used in short term prediction due to its simple application. This model assumes that the wind power at any short time in the future will be the same as that at the present time. Reference [107] used the conditional probability approach to quantify the probable variation of wind speed at a short time in the future based on knowledge of the initial wind speed. This study utilizes an ARMA model for the wind sites considered to simulate wind speed data for a large number of years and the conditional probability distributions of the wind speed in the next hour or next few hours for operating risk assessment.

A short term wind power model is required to evaluate the system operating risks considering the wind farm connected to the system. The short term wind power model is required to integrate with the short term reliability model of the committed units to evaluate the operating risk of the system. Such a model is created in this chapter in two steps. The first step is focused on creating the short term wind speed model and the wind speed model is converted to a wind power model by utilizing a suitable wind turbine generator characteristic.

2.2. Wind Speed Model

Wind speed is site specific and varies randomly at each geographic location. An ARMA time series model can be used [47, 56, 107] to simulate wind speed for a particular wind site, and generate the sufficient synthetic data required to create the wind speed distributions. Reference [47] presents a method for developing a suitable ARMA model for a wind site. The hourly simulated wind speed values are obtained using (2.1)

$$x_t = \mu_t + \sigma_t \times y_t \tag{2. 1}$$

Where:

μ_t and σ_t are the observed mean and standard deviation of the wind speed at time t respectively. y_t is the time series value obtained sequentially using (2.2).

$$y_t = \phi_1 y_{t-1} + \phi_2 y_{t-2} + \phi_3 y_{t-3} + \ldots\ldots + \phi_n y_{t-n} + \alpha_t - \alpha_{t-1}\theta_1 - \alpha_{t-2}\theta_2 - \ldots\ldots - \alpha_{t-m}\theta_m \qquad (2.2)$$

Where:

$\O_i$ (i = 1,…,n) and Θ_j (j = 1,…,m) are the auto regressive and the moving average parameters of the model respectively. $\{\alpha_t\}$ is a normal white noise process with zero mean and a variance of σ^2 i.e. $\alpha_t \in NID(0, \sigma^2)$, where NID denotes normally independently distributed.

The wind speed at any hour is correlated with the wind speeds in the previous hours. The probability distribution of the wind speed in the next hour or hours conditional to a set of selected initial wind speeds is obtained from the simulated hourly wind speeds. The conditional wind speed distribution is combined with the power curve of the wind turbine generator (WTG) to obtain the probability distribution of the power output from the WTG.

2.1.1. Wind data and hour ahead wind models

Three different wind sites in Canada have been used in the studies described in this chapter. Wind speed models and data for wind sites located at Regina and Saskatoon in Saskatchewan and at Toronto in Ontario have been utilized. Regina is located in the southern part of the prairies and is considered to have a good wind resource. It lies in the same geographical region of Western Canada where many large wind farms exist. The average wind speed at Regina is 19.52 km per hour while the standard deviation is 10.99 km per hour. The Saskatoon site, however, is in the central part of the province has a relatively poor wind resource. The mean and standard deviation of the wind speed at Saskatoon are 16.78 km/h and 9.23 km/h respectively. Toronto is near the Great Lakes with very diverse wind resources compared to the prairies. The mean and standard deviation of the wind speed at Toronto are 17.23 km/h and 9.35 km/h respectively. The ARMA models for Regina and Saskatoon are published in [47] and the one for Toronto is published in [56].

The ARMA models for the Regina, Saskatoon and Toronto are presented in (2.3), (2.4) and (2.5) respectively.

$$\begin{aligned} y_t &= 0.9336\, y_{t-1} + 0.4506\, y_{t-2} - 0.5545\, y_{t-3} + 0.111\, y_{t-4} \\ &+ \alpha_t - 0.2033\, \alpha_{t-1} - 0.4684\, \alpha_{t-2} + 0.2301\, \alpha_{t-3} \\ \alpha_t &\in NID(0, 0.409423^2) \end{aligned} \tag{2.3}$$

$$\begin{aligned} y_t &= 1.5047\, y_{t-1} - 0.6635\, y_{t-2} + 0.1150\, y_{t-3} \\ &+ \alpha_t - 0.8263\, \alpha_{t-1} + 0.2250\, \alpha_{t-2} \\ \alpha_t &\in NID(0, 0.447423^2) \end{aligned} \tag{2.4}$$

$$\begin{aligned} y_t &= 0.4709\, y_{t-1} + 0.5017\, y_{t-2} - 0.0822\, y_{t-3} \\ &+ \alpha_t + 0.1876\, \alpha_{t-1} - 0.2274\, \alpha_{t-2} \\ \alpha_t &\in NID(0, 0.5508^2) \end{aligned} \tag{2.5}$$

The mean and standard deviations of the hourly wind speed data observed over twenty years for the Toronto site are presented in Tables B1 and B2 respectively in Appendix B. The observed mean and standard deviation of the hourly wind speed are used for wind speed simulation using (2.1). A series of one hour-ahead wind speed distributions were developed in this research work for the Toronto site for different initial wind speed conditions. The wind data simulation was conducted for 300 replication years in this study. The obtained wind speeds were grouped into 1 km/h classes, and the probability of each class was estimated from the frequency of occurrence of each class. The probability distributions of the wind speeds in the next hour conditional upon different initial wind speed values for a wind farm located in Toronto were are shown in Figure 2.1. The initial wind speeds are expressed in terms of the mean and the standard deviation of the annual wind speed characteristics at the Toronto site. The corresponding values in km/h are shown in Table 2.1. The abscissa in Figure 2.1 shows the wind speed and the ordinate gives the probability. The probability distributions of the wind speeds in the next hour in each case are close to a normal distribution. As the initial wind speed increases, the curves move to the right i.e. towards higher wind speeds. The basic statistics of the distributions are presented in Table 2.1. Table 2.1 shows that the mean wind speed in the next hour is approximately equal to the initial speed and the standard deviation increases as the initial speed increases.

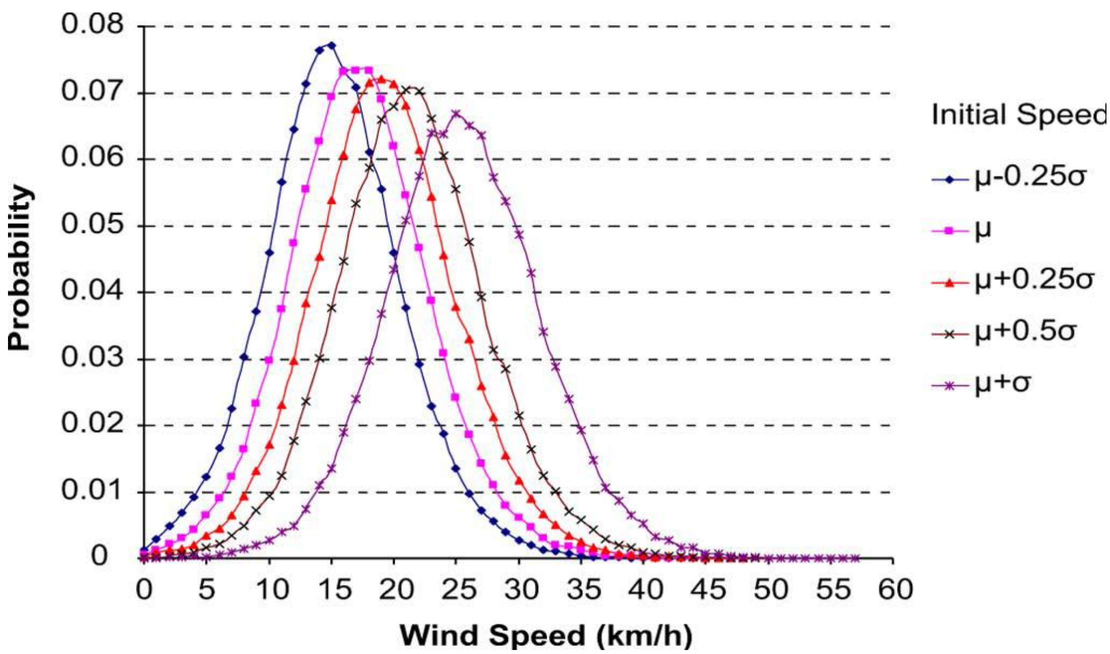

Figure 2.1: Probability distribution of wind speed in the next hour for five initial speeds

It can be seen from Figure 2.1 that the dispersion of the probability distributions increase as the initial wind speed increases. Table 2.1 shows the increase in standard deviation which indicates that the uncertainty in wind speed prediction increases with increasing initial wind speed. It can be seen that the error in the persistence model increases as the initial wind speed is increased.

Table 2.1: Basic statistics of the wind speed distributions in Figure 2.1

Initial wind speed		Statistics of wind speed in the next hour	
In μ, σ	(km/h)	Mean (μ) (km/h)	Std. dev (σ) (km/h)
μ-0.25σ	15	15.17	5.49
μ	17	17.25	5.67
μ+0.25σ	20	19.31	5.82
μ+0.5σ	22	21.42	5.96
μ+σ	27	25.56	6.25
μ+1.5σ	31	29.75	6.52

The conditional cumulative wind speed distribution obtained from the ARMA model is compared with the distribution obtained from actual data collected over 30 years, and shown in Figure 2.2. It can be seen that the conditional probability distribution of the simulated data obtained from the ARMA model is very close to the distributions obtained from actual site wind speed data. The difference in the two sets of distributions in Figure 2.2 is that the simulated data provide a continuous probability distribution, whereas the distribution of the actual data is discontinuous. This is because the actual site data has a limited number of data points compared to the simulated data obtained using 300 replicated years. The number of simulated data points for the next hour distribution in Figure 2.2 is 15 times more than the number of actual data points for an initial wind speed of 31 km/hr.

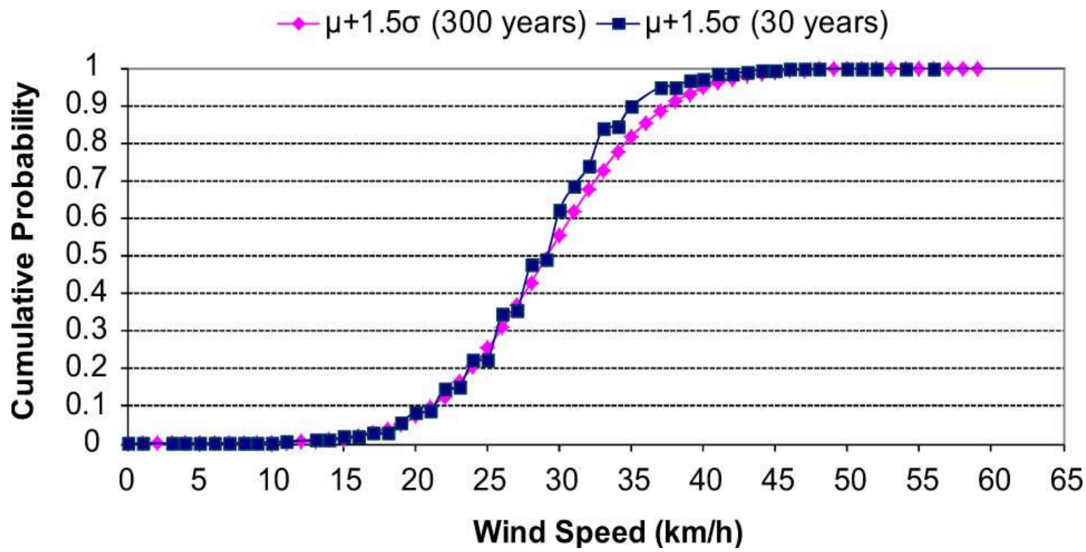

Figure 2.2: Comparison of cumulative probability distributions of wind speed in the next hour obtained using the ARMA model with actual site data of Toronto for the initial wind speed of 31 km/h (or μ+1.5σ).

Further studies were conducted in this work to analyze the characteristics of the wind speeds after one and two hours conditional upon two different initial wind speed cases for the Toronto wind site. The resulting probability distributions are shown in Figure 2.3. Case 1 considers an initial wind speed of 17 km/h, which is equal to the annual historic mean (μ). Case 2 considers an initial wind speed of 27 km/h, which is one standard deviation above the mean (μ+σ) for this wind site.

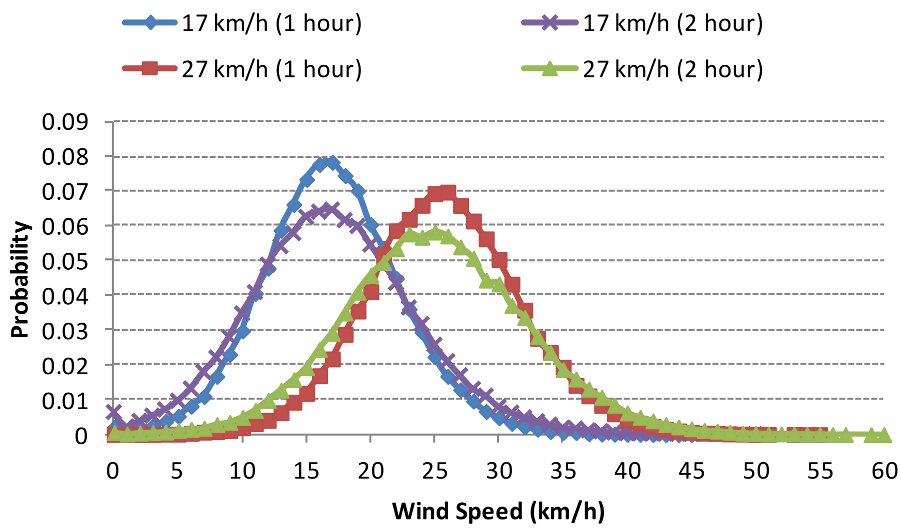

Figure 2.3: Probability distributions of wind speed for lead times of one and two hours for two initial speeds (Case 1: μ or 17 km/h and Case 2: μ+σ or 27 km/h)

The basic statistics of the distributions are presented in Table 2.2. It can be observed from Figure 2.3 that the magnitude of the wind speed distributions after two hours are lower while the dispersions are higher than those after one hour. This indicates that the uncertainty in the wind speed increases as the lead time is increased. Tables 2.1 and 2.2 show that the expected value of the wind speed in the next hour is almost equal to the initial wind speed which is in agreement with the persistence model. It is, however, important to note that the probability of not meeting the persistence model based forecast is almost 0.5, which is an appreciable risk.

Table 2.2: Basic statistics of the wind speed distributions in Figure 2.2

Initial wind speed			At wind speed (1 hr)		At wind speed (2 hrs)	
Case	In μ, σ	(km/h)	Mean (μ) (km/h)	Std. dev (σ) (km/h)	Mean (μ) (km/h)	Std. dev (σ) (km/h)
Case 1	μ	17	17.01	5.41	17.02	6.49
Case 2	μ+σ	27	25.72	6	25.09	7.17

As noted above, Figure 2.3 shows that the dispersion of the probability distribution increases as the initial wind speed increases. It can also be seen from Figure 2.3 that the dispersion of the probability distribution increases with the prediction lead time. The prediction uncertainty after two hours is higher than that after one hour and therefore the error in the persistence model increases as the initial wind speed increases or the prediction lead time is increased.

2.1.2. Diurnal and seasonal wind trend

Wind regimes may be characterized by their seasonal and diurnal wind trends [136-142]. The variations of wind speed from day to night are commonly observed in sea shores and are caused by the thermal effect [143]. The wind behaviors of the regimes investigated in [136, 137, 140, 142] show that wind speed is higher in day time compared to night showing a rising wind trend from the mornings to early afternoons where it levels off and it shows a falling wind trend in the evenings. Figure 2.4 shows the average hourly wind speed variations on the day of January 1 (Day-1) and June 10 (Day-161) at the wind site considered in Toronto using 20 years of historical data. The mean wind speed over the day is 22.85 km/h while the mean standard deviation is 11.39 km/h. Hour 8 shows a rising wind trend while Hour 20 shows a falling wind trend in the next few hours. Day-161 is a summer day where the mean wind speed varies between 9.18 to 21.89 km/h. The average wind speed over the day is 15.69 km/h and the standard deviation (SD) is 7.73 km/h both of which are relatively low compared to the Day-1 values. The diurnal wind speed variation on Day-1 and Day-161 are similar with a rising wind trend in the morning from 8 AM to 12 PM and a falling trend in the evening from 8 PM to 12 AM.

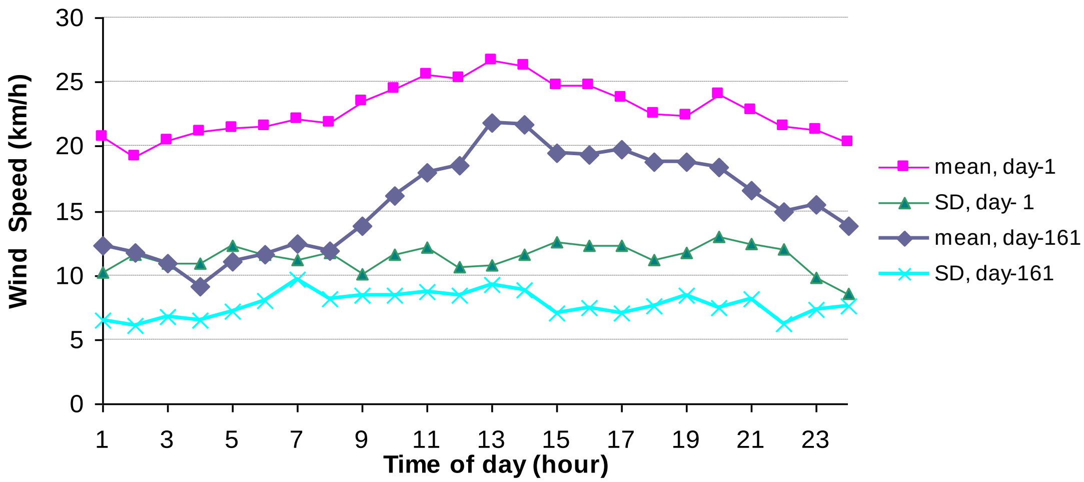

Figure 2.4: Average hourly wind speed variations on Day-1 and Day-161 at Toronto

The ARMA model given in (2.5) was used in the following study conducted in this work to simulate the wind speed data required to create the conditional wind speed probability distribution for the lead times of 1 to 3 hours for a known initial wind speed. Figure 2.5 shows the wind speed probability distributions at Hour 9 for three different initial wind speeds of 20, 25 and 30 km/h at Hour 8. Figure 2.6 shows the wind speed probability distribution at Hour 21 for the same initial condition at Hour 20. It can be seen from Figure 2.5 and Figure 2.6 that the probability distributions are similar and they move towards the direction of higher or lower wind speed as the initial wind speed is increased or decreased respectively.

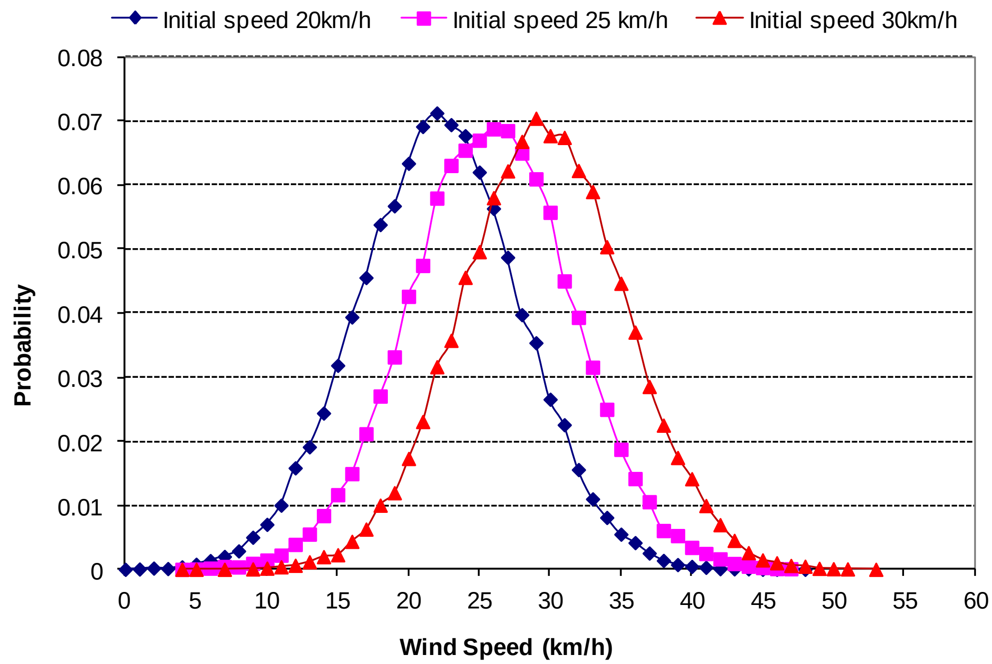

Figure 2.5: Wind speed probability distributions at Hour 9 conditional on wind speed at Hour 8

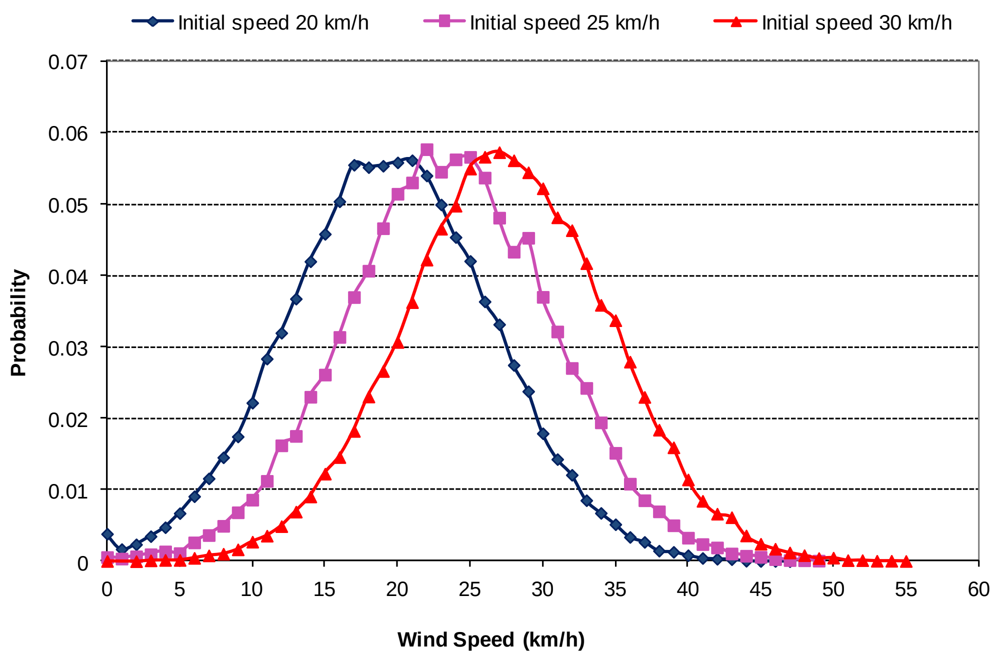

Figure 2.6: Wind speed probability distributions at Hour 21 conditional on the wind speed at Hour 20

Tables 2.3 and 2.4 show the basic statistics of the wind speed distributions for three different initial wind speeds at the rising and falling wind trend respectively on Day 1. The mean wind speed in the next hour increases with increase in the initial wind speed as expected and the distribution moves toward a higher or lower wind speed when the initial wind speed increases or decreases respectively. It is also evident that the mean wind speed at Hour 9 is almost equal to the initial wind speed at Hour 8 while the mean wind speed at Hour 21 is less than the initial wind speed at Hour 20. The diurnal rising and falling wind trends are important factors in wind speed or wind power prediction.

Table 2.3: Basic statistics of wind speed in the next hours (Day 1, Morning)

Initial wind speed at Hour 8 (km/h)	Mean wind speed (km/h)			Standard deviation (km/h)		
	Hr 9	Hr 10	Hr 11	Hr 9	Hr 10	Hr 11
20	22.24	23.19	24.31	5.70	7.75	8.97
25	25.87	26.87	27.88	5.72	7.80	9.05
30	29.54	30.71	31.53	5.72	7.76	9.02

Table 2.4: Basic statistics of wind speed in the next hours (Day 1, Evening)

Initial wind speed at Hour 20 (km/h)	Mean wind speed (km/h)			Standard deviation (km/h)		
	Hr 21	Hr 22	Hr 23	Hr 21	Hr 22	Hr 23
20	19.54	18.77	19.23	7.01	7.95	7.24
25	23.56	22.24	21.80	7.08	8.02	7.22
30	27.60	25.84	24.47	7.00	8.00	7.23

Figure 2.7 shows the conditional wind speed distribution for the rising trend at the selected summer day (Day 161) while Figure 2.8 presents the conditional wind speed distribution for the falling wind trend. The considered initial wind speed is 25 km/h in both Figures 2.7 and 2.8. It can be seen that the wind speed distribution moves slightly towards a higher wind speed in

case of the rising wind trend as the lead time increases. The movement is in the opposite direction for the falling wind trend. The basic statistics of the wind speed distribution conditional on three initial wind speeds of 20 km/h, 25 km/h and 30 km/h, for the rising and the falling wind trend, are presented in Table 2.5 and Table 2.6 respectively. It is noted from Figure 2.4 that the historic hourly standard deviations on the summer day are lower compared to those on the winter day. This is reflected in the standard deviation of the conditional wind speed distribution as well.

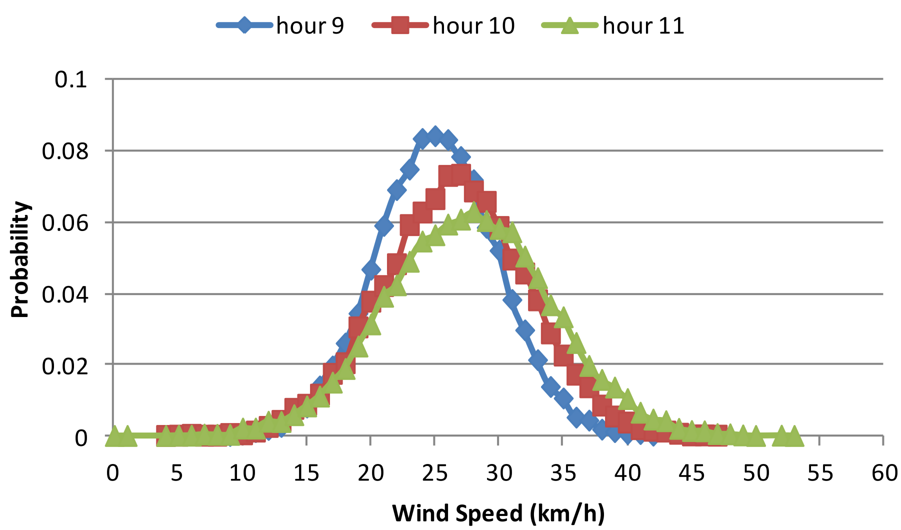

Figure 2.7: Wind speed probability distributions at Hours 9-11 conditional on the wind speed at Hour 8 on Day 161

Table 2.5: Basic statistics of wind speed in the next hours (Day 161, Morning)

Initial wind speed at Hour 8 (km/h)	Mean wind speed (km/h)			Standard deviation (km/h)		
	Hour 9	Hour 10	Hour 11	Hour 9	Hour 10	Hour 11
20	20.78	22.56	23.96	4.77	5.67	6.44
25	25.14	26.52	27.64	4.72	5.63	6.39
30	29.43	30.39	31.29	4.74	5.68	6.4

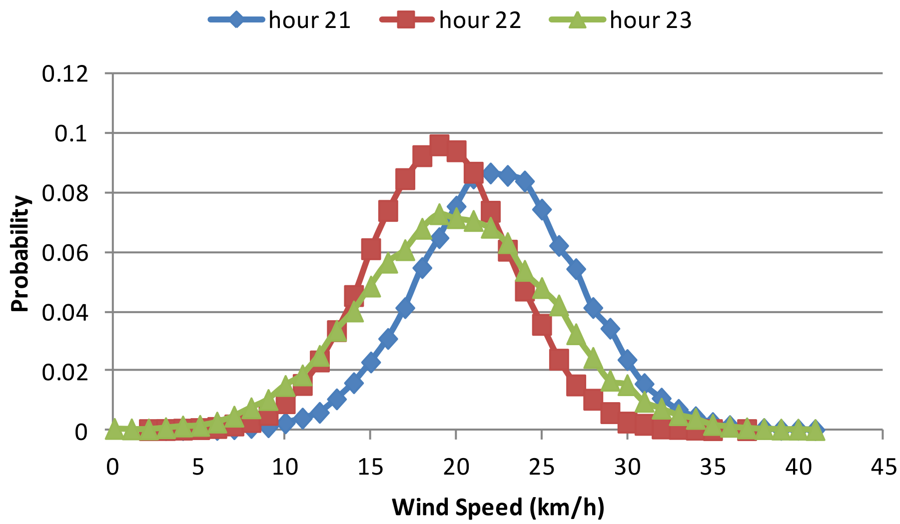

Figure 2.8: Wind speed probability distributions at Hours 21-23 conditional on the wind speed at Hour 20 on Day 161

Table 2.6: Basic statistics of wind speed in the next hours (Day 161, Evening)

Initial wind speed at Hour 20 (km/h)	Mean wind speed (km/h)			Standard deviation (km/h)		
	Hour 21	Hour 22	Hour 23	Hour 21	Hour 22	Hour 23
20	18.05	15.98	16.6	4.61	4.11	5.44
25	22.53	19.04	19.99	4.61	4.17	5.55
30	27.16	22.23	23.44	4.61	4.15	5.47

2.1.3. Wind speed correlation between wind farms

Capacity expansion of wind power can be done by installing wind turbine generators (WTG) at the same location or at different sites. If the wind turbine generators (WTG) are added in the same wind farm, the additional power output will be governed by the same wind regime. Such wind capacity additions can be designated as dependent additions. If the wind sites are significantly removed from each other and are in different terrains such wind sites could be independent. The wind speeds and hence the wind power output from multiple wind farms, located in the same geographical terrain, could have significant degrees of statistical correlation depending upon the distance between the sites [45].

The short term wind power models of individual wind farms can be easily integrated with the generation model of the conventional units for the dependent and independent cases. A large amount of wind speed or wind power data measured simultaneously would be required to build a short term wind power model for the correlated wind sites. In situations where sufficient time series data of the multiple wind sites are not available, studies have been carried out by simulating the statistically correlated time series data using different methods. Reference [144] uses a time shifting technique to obtain the statistically correlated wind speed data while [63, 124, 145] generate the correlated random numbers used in the moving average component of the ARMA model to simulate the correlated wind speed data. Cholesky decomposition of the covariance matrix can be used for generating conditional simulation of random functions [146]. References [147][148] use the Cholesky decomposition of the correlation matrix to simulate correlated random numbers. This method is also utilized in this study to create correlated random numbers and use them in the ARMA model to simulate correlated wind speed data. The covariance matrix of two normal random varaites with a correlation coefficient of ρ is given by:

$$P = \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix} \tag{2.6}$$

The Cholesky decomposition can be applied to the covariance matrix *P* to find an upper or a lower triangular matrix. The lower triangular matrix is given by:

$$L = \begin{bmatrix} 1 & 0 \\ \rho & \sqrt{1-\rho^2} \end{bmatrix} \tag{2.7}$$

If α_A and α_B are two series of independent random numbers, the correlated random number series α_C and α_D can be conditioned as:

$$\begin{bmatrix} \alpha_C \\ \alpha_D \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \rho & \sqrt{1-\rho^2} \end{bmatrix} \begin{bmatrix} \alpha_A \\ \alpha_B \end{bmatrix} \tag{2.8}$$

$$\alpha_C = \alpha_A \tag{2.9}$$

$$\alpha_D = \rho \times \alpha_A + \sqrt{1-\rho^2} \times \alpha_B \tag{2.10}$$

The correlated random variates, α_C and α_D, are then used to simulate the correlated wind speed data as shown in (2.1) and (2.2). Table 2.7 shows the wind speed correlation for the annual hourly wind speed data simulated using the same ARMA model of the Toronto wind site but with correlated random numbers. The listed wind speed correlation coefficient is the average of the annual hourly wind speed correlation based upon 1000 replicated years. It is worth noting that even with independent random numbers, the wind speed correlation is not zero mainly because of the same ARMA model and the historic wind data used in the wind speed simulation. The short term wind power model for correlated wind sites is created from the knowledge of the initial conditions at both the sites and applying a conditional probability approach.

Table 2.7: Wind speed correlation obtained by using correlated random numbers

Correlation coefficient, Random Number	Correlation coefficient, Wind Speed
0.7	0.75
0.3	0.42
0.0	0.18

2.3. Wind Power Model

The speed-power relationship for a wind turbine generator (WTG) is non-linear and is described in [149]. The speed-power characteristics of a typical WTG are presented in Figure 2.9 where the output power is zero when the wind speed is less than the cut-in speed V_{ci}. The power output is maximum or the rated capacity P_r when the wind speed is equal to or greater than the rated speed V_r. The WTG is shut down for safety reasons when the wind speed is equal to or higher than the cut-out speed V_{co}. The wind power curve is expressed in (2.11).

$$\begin{aligned} P_t &= 0 && \text{for } V_{ci} > x_t \geq V_{co} \\ &= A + Bx_t + Cx_t^2 && \text{for } V_{ci} < x_t < V_r \\ &= P_r && \text{for } V_r \leq x_t < V_{co} \end{aligned} \tag{2.11}$$

The A, B and C parameters of (1) are obtained from (2.12)-(2.14)

$$A = \frac{1}{(V_{ci} - V_r)^2}\left[V_{ci}(V_{ci} + V_r) - 4V_{ci}V_r\left(\frac{V_{ci} + V_r}{2V_r}\right)^3 \right] \tag{2.12}$$

$$B = \frac{1}{(V_{ci} - V_r)^2}\left[4(V_{ci} + V_r)\left(\frac{V_{ci} + V_r}{2V_r}\right)^3 - (3V_{ci} + V_r) \right] \tag{2.13}$$

$$C = \frac{1}{(V_{ci} - V_r)^2}\left[2 - 4\left(\frac{V_{ci} + V_r}{2V_r}\right)^3 \right] \tag{2.14}$$

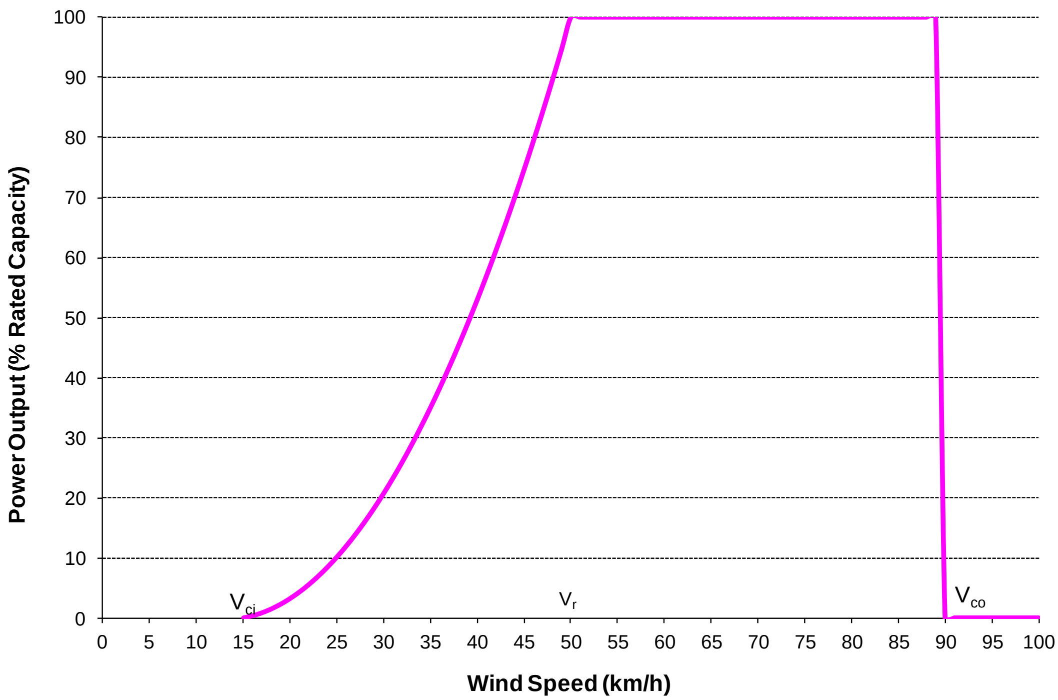

Figure 2.9: Speed-power characteristics of a typical wind turbine generator

The set of conditional wind speed data obtained from the ARMA model can be converted to a set of wind power outputs using the power curve and plotted to create a conditional wind power distribution. Figure 2.10 presents the probability distributions of the wind power in the next hour obtained in this work by passing the wind speed distributions in Figure 2.3 through the WTG power curve shown in Figure 2.9. The cut-in, rated and cut-out wind speeds are 15, 50 and 90 km/h respectively for the wind turbine and the rated capacity is 2 MW. The initial wind power outputs corresponding to the initial speeds of 17 km/h (Case 1) and 27 km/h (Case 2) are 1% and 13% respectively of the rated capacity. The probability of having zero power output is the cumulative probability of the wind speeds less than or equal to the cut-in speed and greater than the cut-out speed. The probability distribution in Figure 2.10 therefore shows a sharp decline after the cut-in wind speed. The probability of having zero output in the next hour is 0.39 and

0.04 for Cases 1 and 2 respectively. Probability values greater than 0.2 are not shown in Figure 2.10 in order to clearly show the distributions of wind power greater than zero.

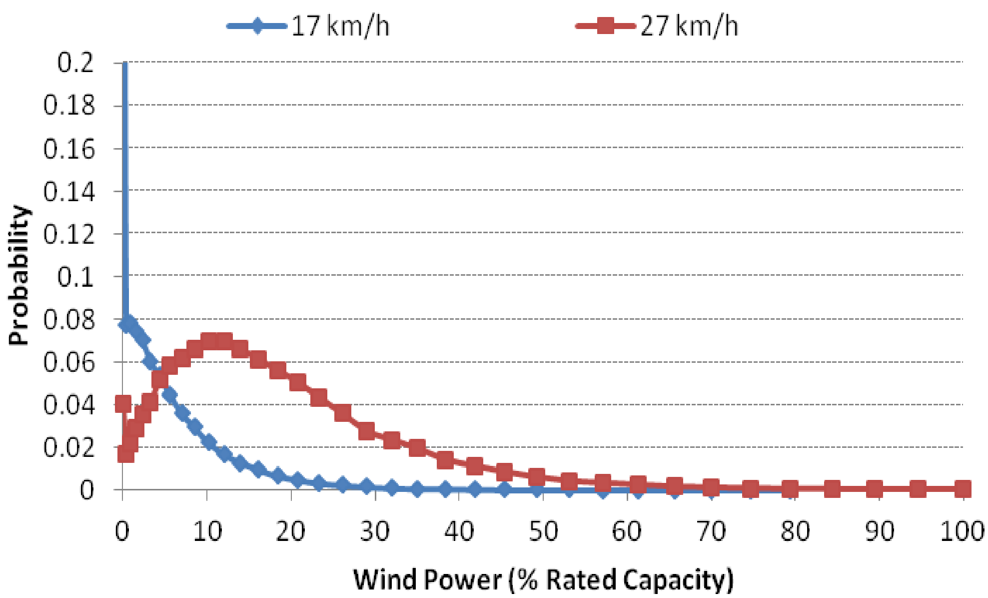

Figure 2.10: Probability distributions of wind power for a lead time of one hour for the two initial wind speed cases

The probability of having full capacity is quite low in both cases. The probability of having wind power greater than 60% of rated capacity is 0.0001 in Case 1 while it is 0.007 in Case 2. It can be seen that the two probability curves intersect at about 4% of the rated capacity. The probability of wind power being above 4% of the rated capacity in the next hour in Case 2 is always higher than in Case 1. The probabilities of wind power outputs being less than 4% of the rated capacity are however higher for Case 1 than for Case 2. The expected wind power in Case 1 and Case 2 is 1% and 11.47% of the rated capacity respectively.

Any conditional wind speed distribution presented in this chapter can be converted into the wind power distribution. The wind power distributions thus created will be used in the following chapters for risk evaluations. Table 2.8 shows the discrete capacity states and the associated probabilities for a 100 MW wind farm for case shown in Figure 2.10 at the initial wind speed of 27 km/h. The number of class interval required to present the wind speed distribution is is determined by using (2.15) [150].

$$N_{class} = 1 + 3.3 \times \log_{10}(N_{data}) \tag{2. 11}$$

Where:

N_{data} = Number of data in the conditional wind speed distribution

Table 2.8: Discrete wind power capacity states and the associated probabilities

Wind Power (Rated Capacity = 100 MW)	Probability
0	0.0924
2	0.1177
4	0.1804
8	0.2050
14	0.1760
21	0.1178
29	0.0637
38	0.0297
49	0.0115
61	0.0040
75	0.0013
89	0.000462

2.4. Summary

The wind speed and therefore the wind power output at a short time in the future are related to the wind speed at the present time. A conditional probability approach has been utilized in this study to quantify wind speed uncertainty in a short time in the future. The conditional wind speed distribution is converted into the conditional wind power distribution by using suitable wind turbine generator characteristics. The resulting capacity states and the associated probabilities quantify the uncertainty associated with wind power generation. The models have been developed to consider the impact of lead times, diurnal and seasonal wind variations, and wind speed correlations. These models are used in the following chapters for the risks evaluation from the perspective of operating wind farms as well as operating a system as a whole. This chapter presents basic wind models that are further developed and incorporated in the evaluation techniques presented in sub-sequent chapters.

CHAPTER 3

WIND POWER COMMITMENT EVALUATION IN SYSTEM OPERATION

3.1. Evaluation of Wind Power Commitment Risk

It is an important task to estimate the wind power that will be available at a short time in future when operating a power system with significant wind power penetration. Different methods are used by different utilities to estimate the wind power contribution. Methods based upon the persistence model are normally used for short term wind predictions in system operation. The uncertainty of an estimate made under such a deterministic model is not known. This chapter presents a probabilistic technique that utilizes the short term wind power models developed in the previous chapter to quantify risks and evaluates wind power commitment based upon a specified risk of such a commitment.

Figure 3.1 shows the probability distribution of an hour ahead wind speed for an initial wind speed of 27 km/h using the Toronto wind data. The wind power curve presented in Figure 2.9 is also shown and indicates that the initial wind power is 14% of the rated capacity. An operator operating a 100 MW wind farm, using a persistence model, would then make a commitment to provide 14 MW wind power to the system in the next hour. The probability of not meeting the commitment is given by the shaded area in Figure 3.1 and is 0.56 in this case. As shown in Figure 3.1, there is considerable uncertainty associated with the predicted values, and therefore, the system is exposed to an observable risk when wind power commitment is made based on a prediction. The system operator could encounter a difficult situation if the actual wind power is significantly different from the predicted value. Wind power commitment risk (WPCR) is the term used in this study to describe the probability that the actual wind power is less than the committed value. It is important for the system operator to know the risk associated with wind power commitments made during system operation.

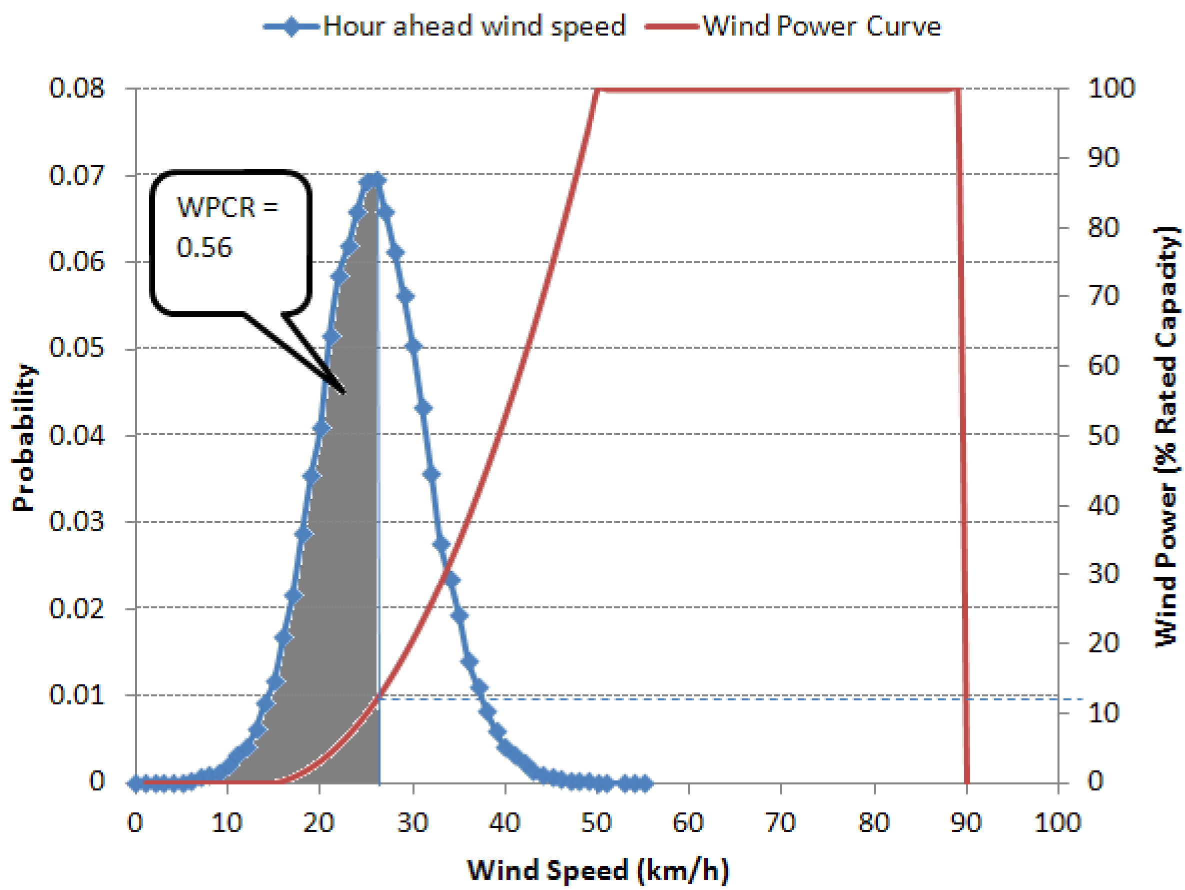

Figure 3.1: Wind power commitment risk evaluation, initial wind speed = 27 km/h

Figure 3.2 shows two wind speed probability distributions in the next hour considering Toronto wind data. The initial wind speed is $\mu+0.5\sigma$ (i.e. 22 km/h) for the distribution shown with a solid line in Figure 3.2. The initial wind power is 5.45% of the rated WTG capacity. This value of wind power is committed for the next hour if the persistence model is applied. The associated wind speed is 22 km/h, and is shown by a solid vertical line in Figure 3.2. The probability that the wind power will be less than the commitment is given by the total area to the left of this vertical line, and is equal to 0.51. This WPCR value is quite high. If the initial wind speed is 31 km/h (i.e. $\mu+1.5\sigma$) or one standard deviation higher than the previous case, the wind power commitment based on the persistence model will be 24% of the rated capacity. The WPCR will be 0.55 from the area to the left of the dashed vertical line. The distribution of the next hour wind speeds for this case is shown by the dashed line in Figure 3.2.

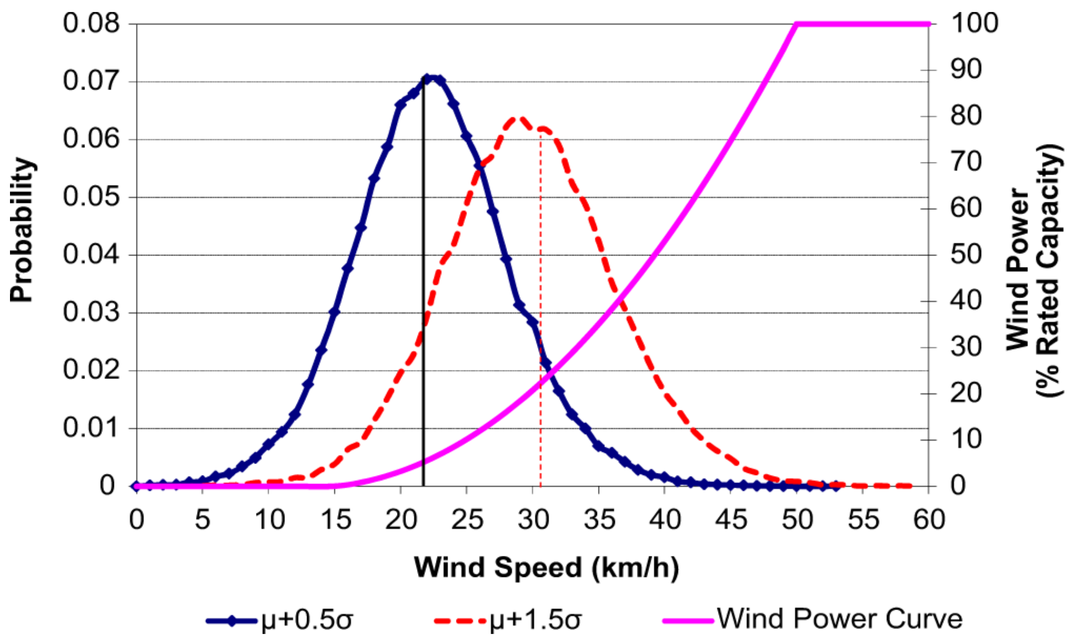

Figure 3.2: Risk of wind power commitment

The risk area can be directly obtained from the ordinate of a cumulative distribution in which the cumulative conditional probability is plotted against the wind speed. Figure 3.3 shows the cumulative distributions of the plots in Figure 3.2. The two different risk values at the two initial wind speeds can be directly obtained as shown in Figure 3.3. Wind power commitment based on the persistence model is not consistence as the risk profile varies with the initial wind speed. If the risk has to be maintained at a specified level, the wind power commitment should be varied as the initial power varies. It is therefore necessary to utilize a risk based method to appropriately commit wind power and maintain system reliability. Some utilities commit a lower percentage such as 80% of the initial power in order to reduce the risk. In the example shown, the capacity value committed is therefore 4.36% (i.e. 0.8 x 5.45%) of the rated capacity. This corresponds to a wind speed of 21 km/h, and can be calculated from the power curve equations. This value of wind speed is shown by a vertical line in Figure 3.4 and labeled as "80%". It can be seen that the WPCR of the 80% commitment is 0.44. In this case, the WPCR is decreased from 55% to 44% by committing only 80% of the initial wind power in the next hour. This risk is still quite high and may not be readily appreciated by the system operator. System operators require appropriate wind data and a readily applicable methodology in order to be able to assess the risk.

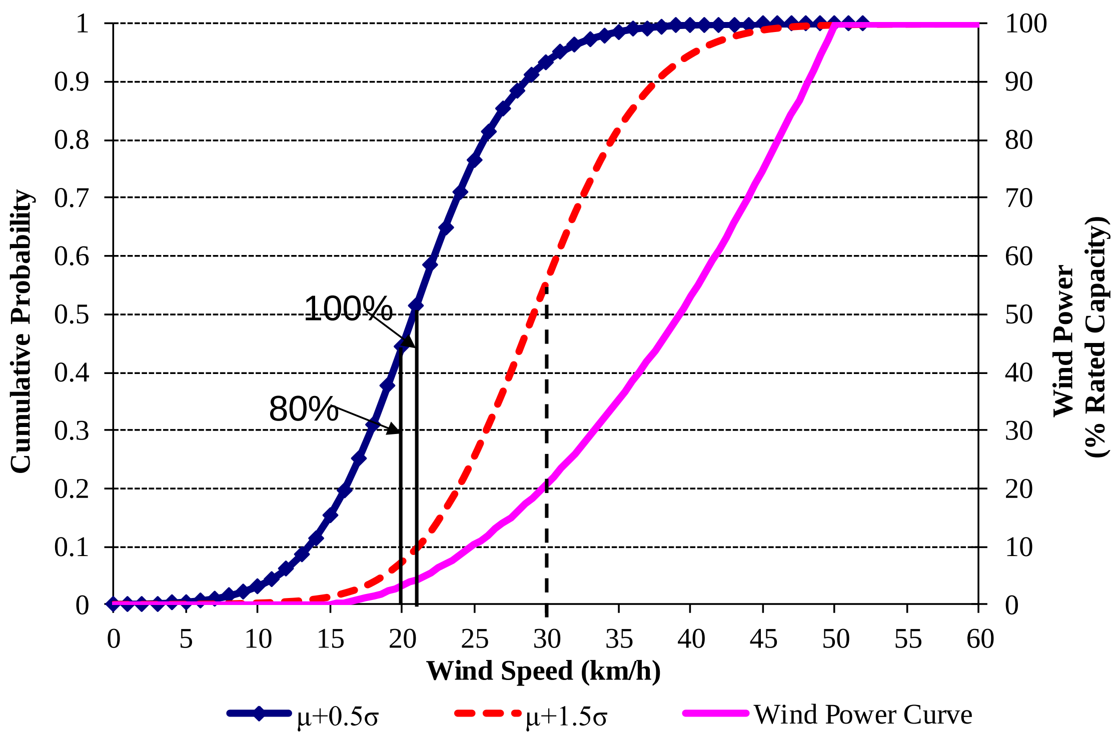

Figure 3.3: Evaluation of wind power commitment risk

3.2. Impact of the Prediction Lead Time on WPCR

The conditional wind speed distributions presented in Figure 2.3, Figure 2. 7 and Figure 2. 8 show that the variability of the predicted wind speed increases as the lead time increases. This section further illustrates the impact of lead time on WPCR. Figure 3.4 presents the cumulative wind speed probability distributions after one, two and three hours conditional on the initial wind speeds of 22 km/h and 32 km/h. The risk of committing 100% of the initial power after one, two and three hours is 0.51, 0.52 and 0.54 respectively at the initial speed of 22 km/h. The risk of committing 80% of the initial power is 0.44, 0.46, and 0.49 and is 0.30, 0.34 and 0.39 when committing 50% of the initial power after one, two and three hours respectively. The risk of committing 100% of the initial power at the initial wind speed of 32 km/h after one, two and three hours is 0.57, 0.60 and 0.63 respectively. The risk is lowered to 0.45, 0.49 and 0.55 after one, two and three hours respectively when 80% of the initial power is committed for these hours. The risk is further reduced to 0.27, 0.33, and 0.41 if 50% of the initial power is committed after one, two and three hours respectively. The risk of committing a certain percentage of the initial power increases as the prediction lead time increases. It can also be observed that the differences in the risks at different lead times is more significant at a lower commitment level such as 50% than at a higher commitment level of 80% of the initial power.

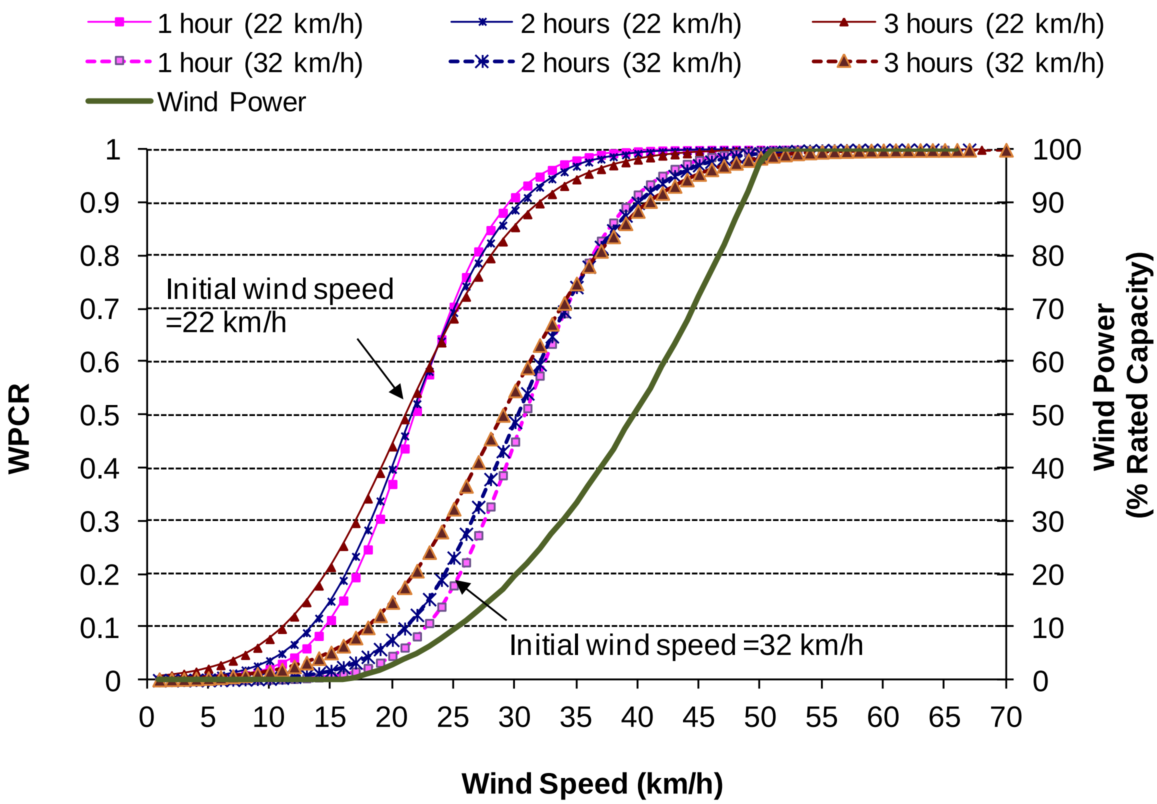

Figure 3.4: Risk of wind power commitment at different lead times

3.3. Impact of Wind Farm Location on WPCR

A second wind data set for a wind site located in Regina in Saskatchewan, Canada, is utilized in this section to illustrate the impact of wind farm location. An ARMA model developed for Regina has been used to simulate the wind data for 300 years and produce the required conditional wind speed distributions as explained earlier. The ARMA model for the Regina site published in [47] is presented in (2.3).

Figure 3.5 shows the cumulative wind speed probability distributions in the next hour for the Regina and Toronto sites at the initial wind speed of 22 km/h. The wind power curve is also shown. The WPCR is 0.51 at Toronto while it is 0.49 at Regina if wind power is committed using the persistence model. The risk of committing 80% of the initial power is 0.44 and 0.40 at Toronto and Regina respectively. The risk reduces to 0.30 and 0.25 respectively at Toronto and

Regina if 50% of the initial wind power is committed in the next hour. It is therefore, evident from Figure 3.5 that the WPCR can also vary with the wind regime at the designated site.

Table 3.1 presents the WPCR associated with committing 100%, 80% and 50% of the initial power at four different initial wind speeds for the two wind sites. A quantitative comparison of the risks for the different situations can be conducted from the values in Table 2.2. It can clearly be seen that the risk decreases as the amount of committed wind power is reduced. What is an acceptable risk is a management decision. Table 3.1 also shows that the risk in each situation tends to reach a relatively constant value above a certain high wind speed.

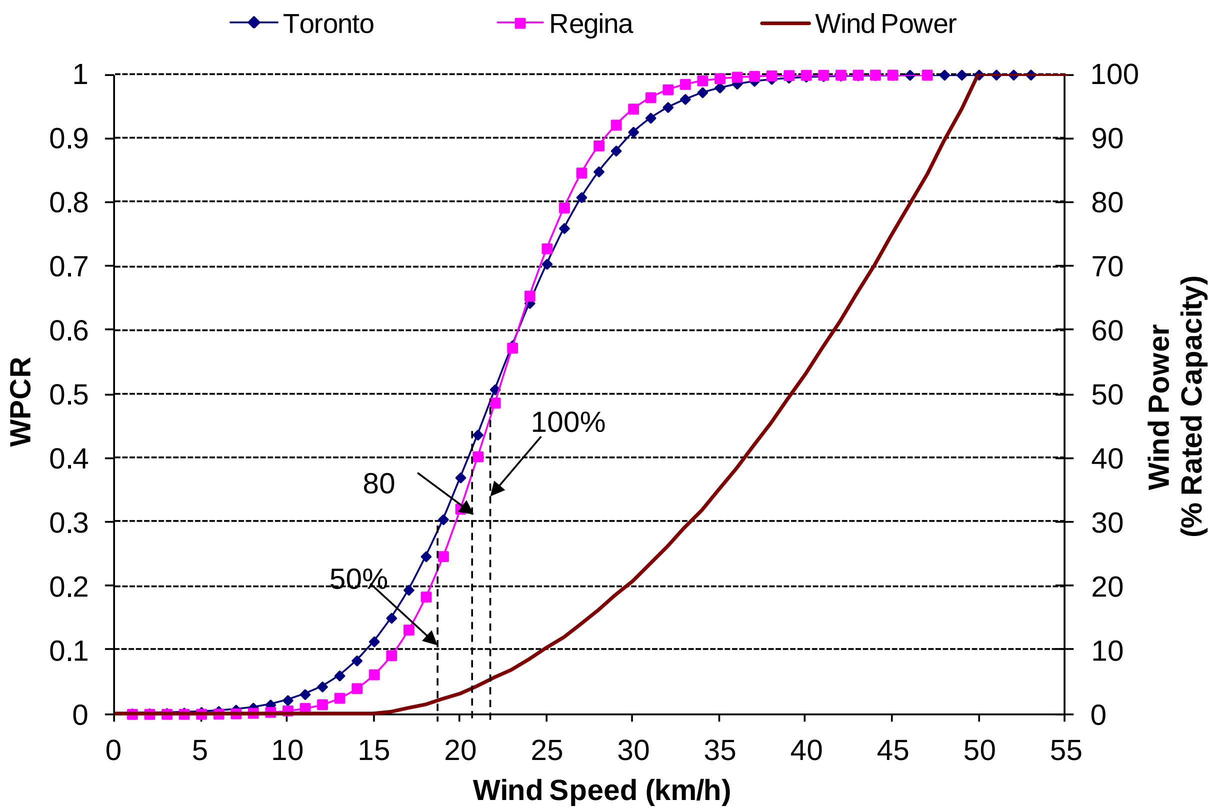

Figure 3.5: Risk of wind power commitment for wind sites located at Toronto and Regina (Initial wind speed = 22km/h)

Table 3.1: Wind power commitment risk

Initial Wind Speed (km/h)	WPCR					
	100% commitment		80% commitment		50% commitment	
	Toronto	Regina	Toronto	Regina	Toronto	Regina
22	0.51	0.49	0.44	0.40	0.30	0.25
25	0.54	0.52	0.47	0.44	0.33	0.28
30	0.58	0.58	0.45	0.42	0.27	0.21
32	0.57	0.58	0.45	0.43	0.27	0.22

3.4. Impact of Diurnal Wind Trend on WPCR

The conditional wind speed distributions at a diurnal rising and a falling wind trend was presented in Section 2.1.2 which are used in this section for WPCR evaluation. Figure 3.6 presents the cumulative wind speed probability distribution at Hours 9, 10 and 11 when the wind speed at Hour 8 is 30 km/h. The corresponding wind power output is 20% of the rated capacity. The left ordinate on Figure 3.6 gives the WPCR of committing wind power corresponding to the value given by a wind speed in the abscissa. If the wind power commitment is made on the basis of a pure persistence model, the WPCR at Hour 9 is 0.52 as shown by the 100% vertical line in Figure 3.6. It can be further observed that the WPCR drops to 0.47 and 0.43 at Hour 10 and Hour 11 respectively due to the rising wind trend at these hours. It may be desirable to lower the WPCR by reducing the committed value of wind power.

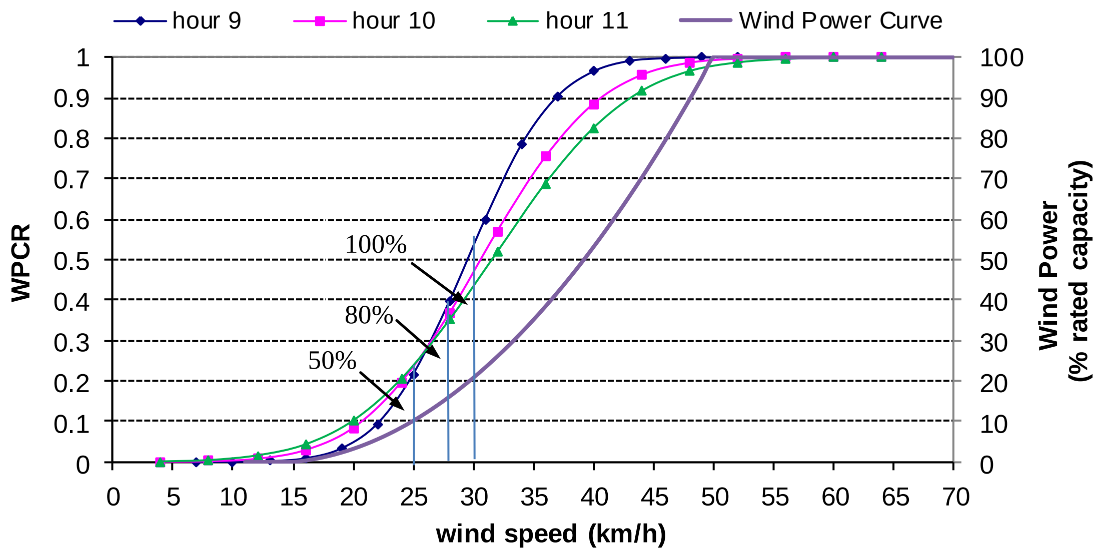

Figure 3.6: WPCR analysis for a rising wind trend (initial wind speed = 30 km/h)

Table 3.2 shows the WPCR values associated with committing 100%, 80% and 50% of the wind power available at Hour 8 for the lead times shown. The WPCR values at the 100% and 80% commitment level decrease as the lead time increases. Table 3.2 also shows that the WPCR of 50% commitment rise as the lead time is increased. The distributions crossover at a wind speed of 26 km/h and the WPCR of committing wind power below the crossover exhibit an opposite behavior to that of the ones above it.

Table 3.2: WPCR for a rising wind trend (initial wind speed = 30 km/h at Hour 8)

Hours	WPCR		
	100% commitment	80% commitment	50% commitment
9	0.52	0.36	0.19
10	0.47	0.34	0.21
11	0.43	0.33	0.22

Figure 3.7 shows the cumulative wind speed probability distributions at Hours 21, 22 and 23 for a falling wind trend. The initial wind speed at Hour 20 is 30 km/h. The distributions in

Figure 3.7 shift to the left from Hour 21 to 23, whereas the distributions in Figure 3.6 shift to the right. This indicates that the WPCR associated with committing a certain value of wind power increases as the lead time increases. The uncertainty increases as the lead time increases and is further augmented when the wind site experiences a falling wind trend. The WPCR values for the three commitments of 100%, 80% and 50% of the initial power are shown in Table 3.3.

The impact of rising and falling trends can be appreciated from the WPCR values given in Tables 3.2 and 3.3. The WPCR at any lead time in the future will be higher in situations where the wind site is experiencing a falling wind trend than in the situations when the wind site is experiencing a rising wind trend. A higher wind power value can therefore be committed during a diurnal rising trend compared that for a falling trend.

Table 3.3: WPCR for a falling wind trend (initial wind speed = 30 km/h at Hour 20)

Hours	WPCR		
	100% commitment	80% commitment	50% commitment
21	0.61	0.50	0.33
22	0.68	0.59	0.44
23	0.76	0.66	0.50

3.5. Impact of Seasonality on WPCR

Figure 3.8 presents the WPCR analysis for a lead time of two hours on a winter day represented by Day-1 and a summer day represented by Day-161 for a rising wind trend. The initial time is Hour 8 and the initial wind speeds considered are 20 km/h and 25 km/h. The distributions for the two days cross each other at 22 km/h (WPCR = 0.42) and 27 km/h (WPCR = 0.49) for the initial wind speed of 20 km/h and 25 km/h respectively. It can be seen that Day-1 lies to the left of Day-161 for wind speeds equal to or less than the initial value. This indicates that the WPCR of committing the same amount of power will be higher on a winter day than on a summer day. This is mainly because a summer day has lower variability compared to a winter day. This can be seen from the plot of the historic hourly standard deviations in Figure 2.4.

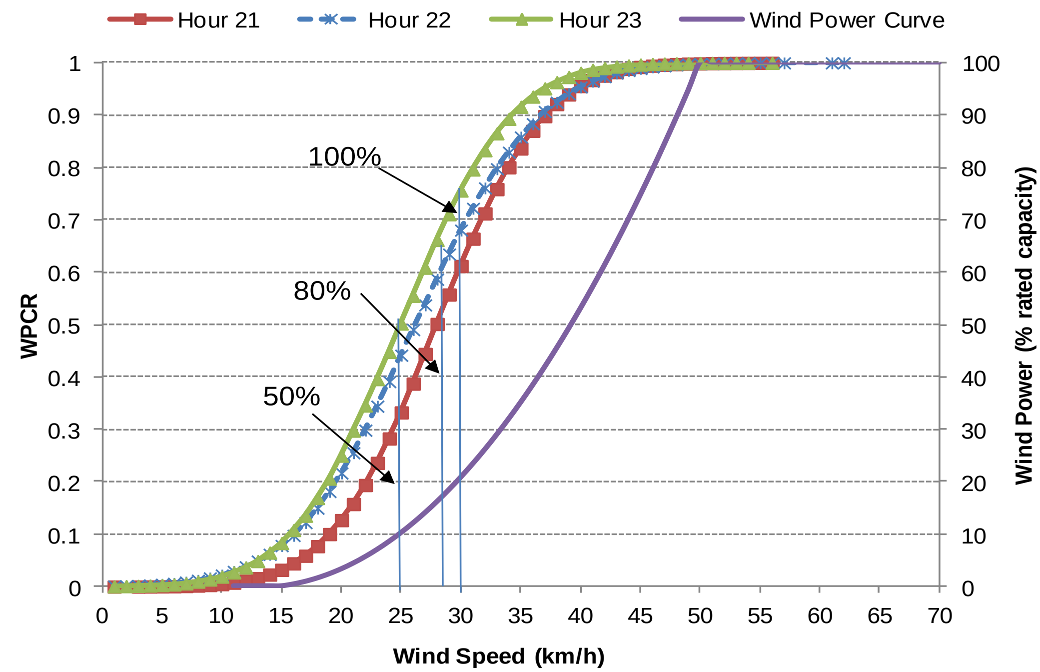

Figure 3.7: WPCR analysis for a falling wind trend (initial wind speed = 30 km/h)

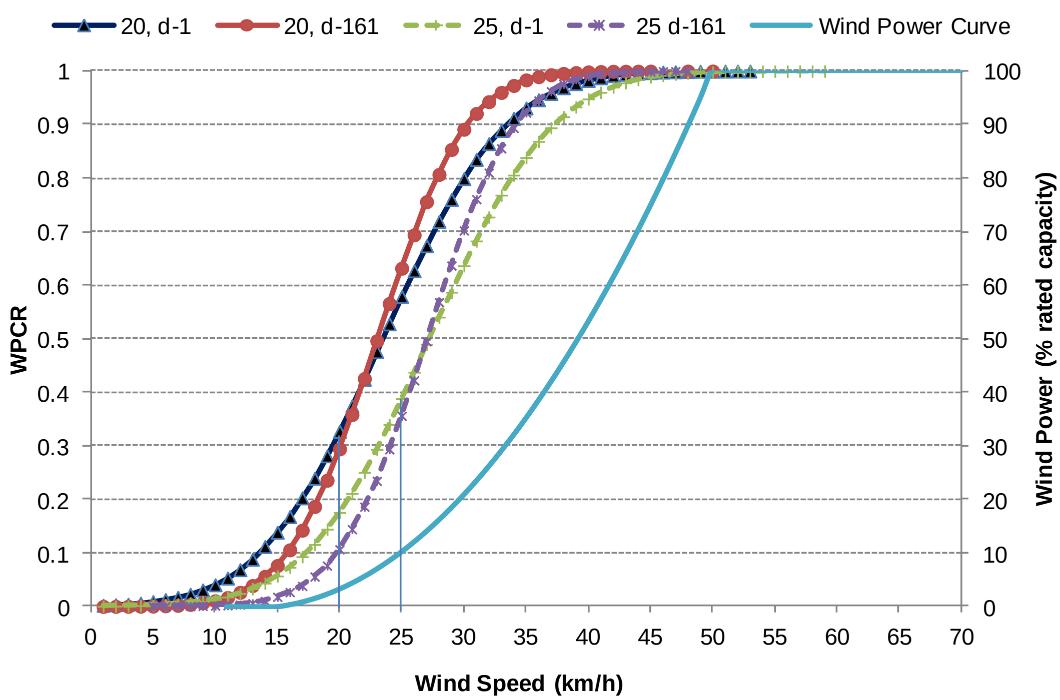

Figure 3.8: WPCR analysis during rising wind trends on Day-1 (winter) and Day-161 (summer)

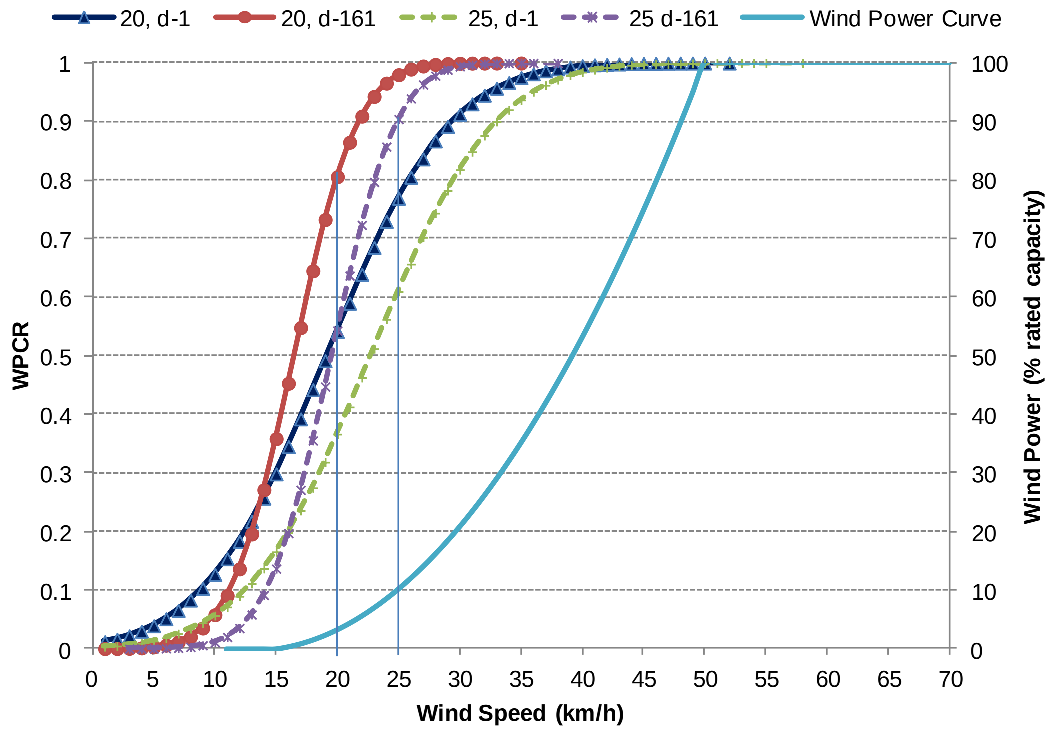

Figure 3.9: WPCR analysis during falling wind trends on Day-1 (winter) and Day-161 (summer)

Figure 3.9 similarly presents the wind speed cumulative probability distributions for Hour 22 conditional on the wind speeds at Hour 20, which occurs during the falling wind trend. The crossovers between the two respective distributions take place at 14 km/h (WPCR = 0.25) and 16 km/h (WPCR = 0.2) for initial wind speeds of 20 km/h and 25 km/h respectively. The WPCR values for the committed wind power below the crossovers are again lower on the summer day (Day-161) than on the winter day (Day-1).

Table 3.4 gives the WPCR values when committing 100%, 80% and 50% of the initial power on the two days (Day-1 and Day-161) for the different lead times when the wind site is experiencing a rising wind trend. It can be seen from the table that the WPCR in Hours 10 and 11 are lower in Day-161 than in Day-1. Table 3.5 shows the WPCR values on the two days (Day-1 and Day-161) for the different lead times when the wind site is experiencing a falling wind trend. The WPCR variations in the falling trend however are the opposite, and the WPCR are greater in

Day-161 than in Day-1. The WPCR during the falling wind trend are relatively high, and it may be desirable to lower the commitment to less than 50% in order to reduce the WPCR.

Table 3.4: WPCR for a rising wind trend (initial wind speed = 25 km/h at Hour 8)

Hours	WPCR for Day-1 (winter) at			WPCR for Day-161 (summer) at		
	100% commitment	80% commitment	50% commitment	100% commitment	80% commitment	50% commitment
9	0.41	0.34	0.22	0.45	0.36	0.22
10	0.39	0.34	0.25	0.36	0.29	0.19
11	0.36	0.32	0.25	0.31	0.26	0.17

Table 3.5: WPCR for a falling wind trend (initial wind speed = 25 km/h at Hour 20)

Hours	WPCR for Day-1 at			WPCR for Day-161 at		
	100% commitment	80% commitment	50% commitment	100% commitment	80% commitment	50% commitment
21	0.56	0.50	0.39	0.67	0.59	0.41
22	0.61	0.56	0.46	0.90	0.86	0.72
23	0.64	0.59	0.48	0.79	0.74	0.61

3.6. Impact of Wind Speed Correlation on WPCR

This study utilizes the wind data from the Toronto site to examine the effect of adding two dependent, independent and correlated wind farms of the same capacity. Simulation of correlated wind speed data was explained in Section 2.1.3 which is applied for WPCR evaluation in this section. The wind power model of dependent wind farms will have the same probability distribution but with increased capacity states. Figure 3.10 presents the cumulative probability distributions of wind power in the next hour considering four different cases of wind speed correlation. The cumulative distributions on the right, with the dotted lines, are for the initial condition of 20% rated power at each of the wind farms. The distributions on the left, with the solid lines, are for the 60% of the rated capacity as the initial power at each wind farm. The initial time is Hour 20 and the ordinate in Figure 3.10 gives the risk of committing the wind power capacity shown on the abscissa. It can be seen from Figure 3.10 that the risks of committing 100% of initial power are close for the four considered degrees of correlation. Table 3.6 shows the WPCR for 100% commitment where it can be seen that the risk is significant for all four correlation coefficients, and possibly indicates the need to commit a lower capacity. The WPCR for 50% commitment is shown in Figure 3.11 and clearly shows the reduction in risk as the correlation coefficient is reduced.

The impact of wind speed correlation can be further observed from Figure 3.12 where the distributions shown in Figure 3.11 are enlarged for a closer look at commitments less than 100% of the initial power. The risk curve or the cumulative wind power distribution of the dependent wind farms lie on the left and the curves move towards the right as the correlation coefficient decreases. This indicates that the risk of committing wind power is the highest for the dependent wind farms and the risk decreases with increase in the correlation coefficient.

Table 3.6: Hour ahead WPCR of correlated wind farms

Initial wind power (% Rated Capacity)	WPCR of 100% commitment			
	Correlation Coefficient			
	1	0.75	0.42	0.18
20%	0.618	0.613	0.612	0.603
30%	0.645	0.647	0.644	0.652
60%	0.700	0.692	0.708	0.707

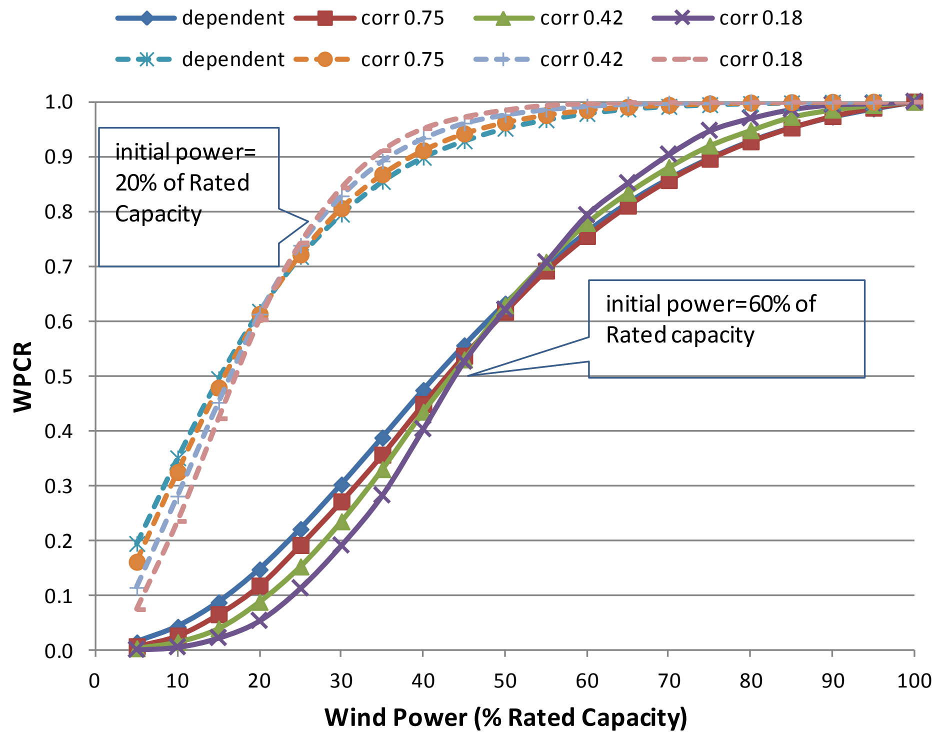

Figure 3.10: Cumulative one hour ahead wind power distribution for correlated wind farms

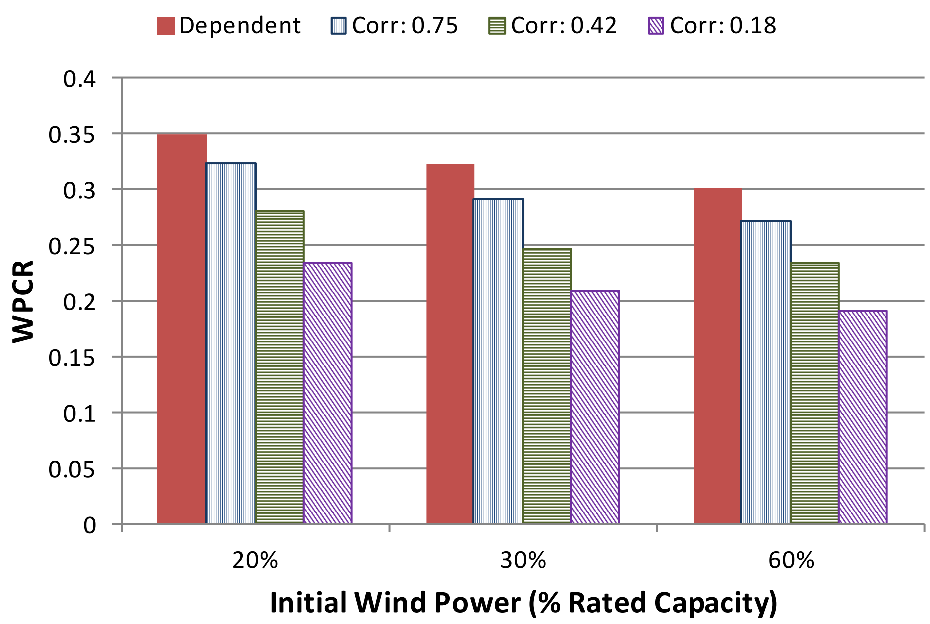

Figure 3.11: WPCR when committing 50% of the initial power from the correlated wind farms

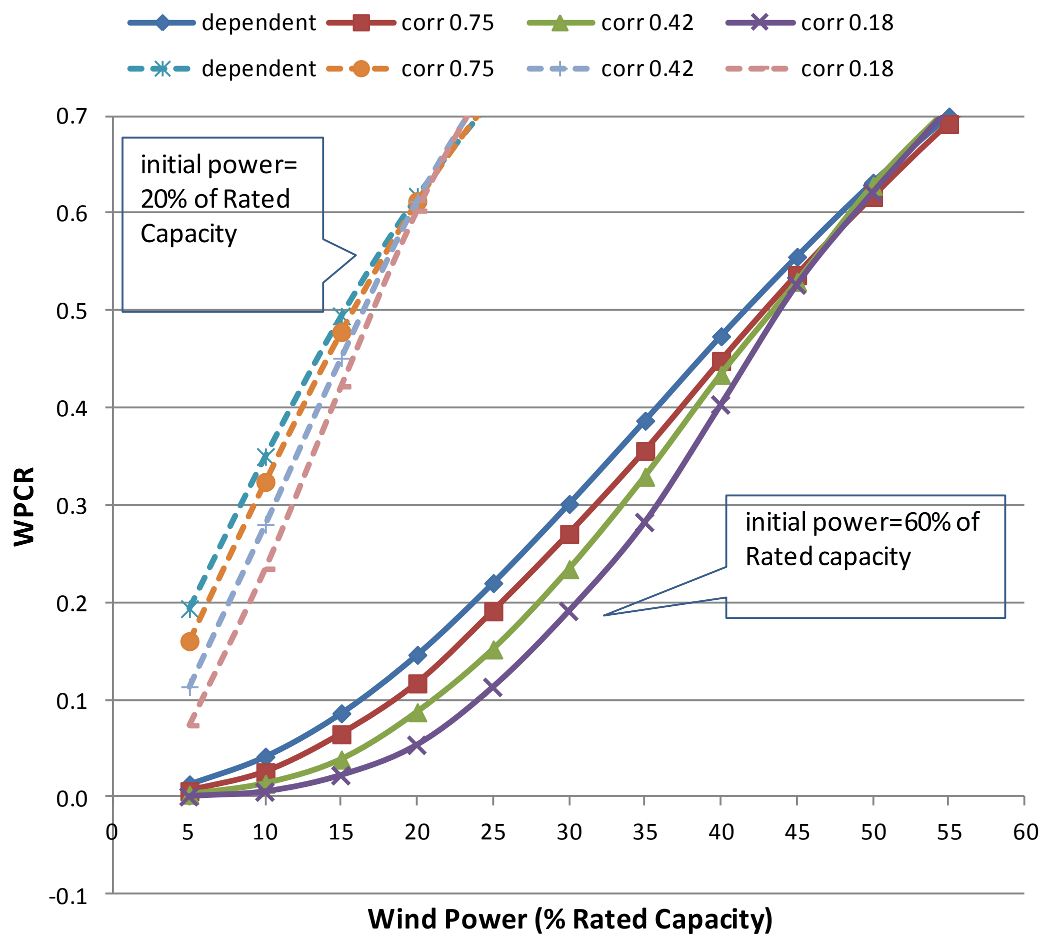

Figure 3.12: WPCR analysis for correlated wind farms (initial time = Hour 20)

3.7. Wind Power Commitment Constrained by WPCR

It may be desirable to have the risk associated with the committed wind power maintained at an acceptable value as determined by management. If the risk is maintained at a specified level, the amount of committed wind power will vary as the initial power changes. If this is the case, it is necessary to utilize a risk based method to appropriately commit the wind power and to maintain the system reliability. Evaluation of the WPCR associated with committing 100% and 80% of the initial wind power in the next hour is illustrated in Figure 3.3 and described in the previous section. The cumulative probability distribution of Figure 3.3 is presented in Figure

3.13 to evaluate wind power commitment for a specified WPCR criterion given by the horizontal line labeled “X”.

The amount of wind power that should be committed in the next hour to meet a criterion WPCR can similarly be evaluated. A specified WPCR of 0.25 is taken as an example. The initial wind speed is 22 km/h, and the initial power output is therefore 5.5% of the WTG rating. The horizontal line labeled “X” in Figure 3.13 indicates a cumulative probability of 0.25 and the vertical line, that meets the horizontal line on the risk curve, corresponds to a wind speed of 18 km/h on the abscissa which means that a wind speed of 18 km/h can be predicted at the WPCR of 0.25. It can be found using the power curve equations that the power output at this wind speed is 1.8% of the rated wind farm capacity, which is 30% of the initial wind power. This capacity value of wind power should therefore be committed to meet the WPCR of 0.25. It can similarly be observed from Figure 3.3 that the wind power commitment should be increased to 51% of the initial wind power if the initial wind speed was 31 km/h.

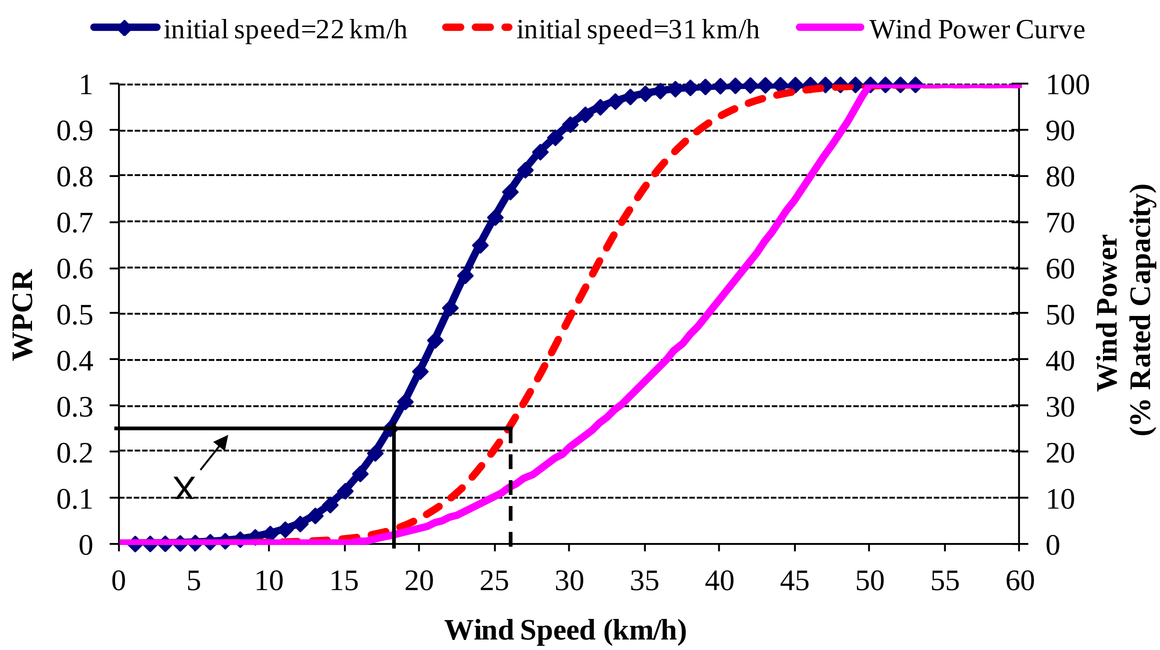

Figure 3.13: Wind power commitment based upon a WPCR criterion

Figure 3.14 presents the wind power commitment in the next hour to satisfy a WPCR criterion of 0.25. When the initial power varies from 5% to 90% of the wind farm capacity the wind power commitment varies from 23% to 61% of the initial power for a wind site located in Regina, while it varies from 12% to 57% in Toronto. The wind power commitment for the WPCR criterion increases with increase in the initial power. The increase in wind power commitment is more appreciable when the initial power is in the range of 5% to 25% of the rated capacity with corresponding wind speeds of 21 km/h to 32 km/h. Further increases in initial wind power provide a slow increase in the wind power commitment in the next hour.

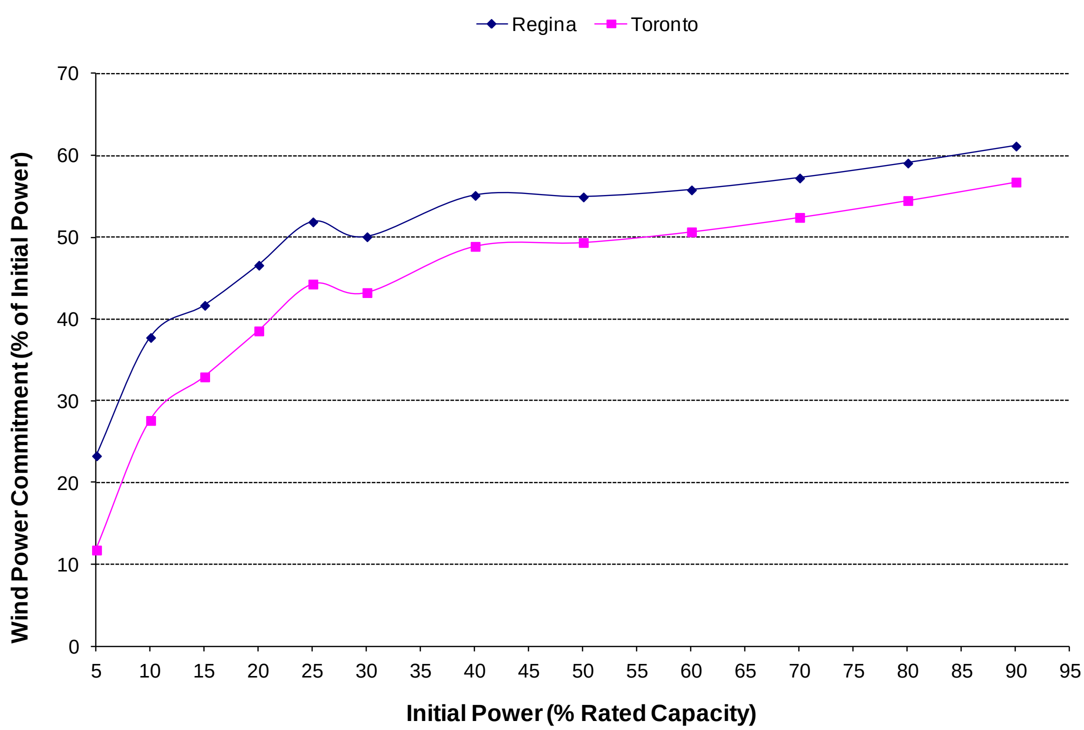

Figure 3.14: Wind power commitment for a lead time of one hour at the WPCR criterion of 0.25

The wind power commitment for a lead time of two hours to satisfy the WPCR criterion of 0.25 is presented in Figure 3.15. The wind power commitment increases from 12% to 45% of the initial power for a wind site located in Regina as the initial wind power increases from 5% to 90% of the wind farm capacity while it increases from 12% to 48% of the initial power for a

Toronto site. It shows noticeable increases in wind power commitment as the initial power is increased from 5% to 25% of the wind farm capacity in both wind sites similar to the wind power commitment for a lead time of one hour shown in Figure 3.14. The capacity value of wind power for a lead time of two hours is higher for the wind site located in Regina than the wind site located in Toronto when the initial power is in the range of 5% to 20% of the wind farm capacity. The wind power commitment is however, almost the same for both wind sites when the initial power is in the range of 25% to 90% of the wind farm capacity for the selected WPCR criterion of 0.25.

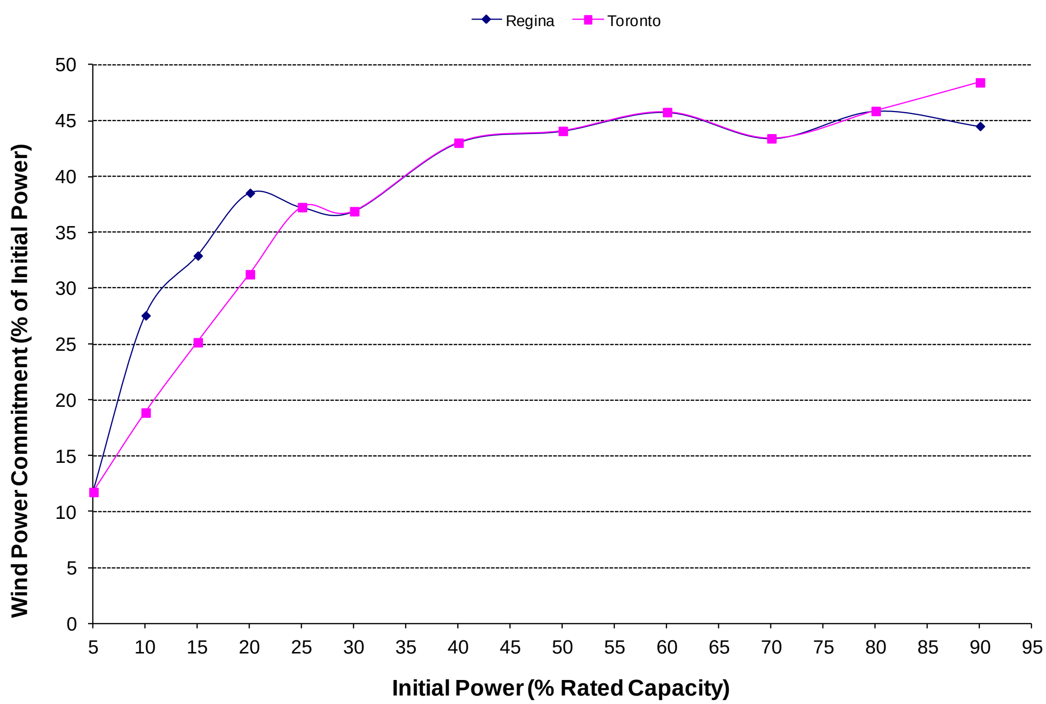

Figure 3.15: Wind power commitment for a lead time of two hours at 0.25 WPCR criterion

3.8. Development of an Approximate Risk Based Method for Short-Term Wind Power Commitment

Most system operators have real time access to wind power information, but do not usually possess actual wind speed data. Wind power prediction for the next hour(s) is therefore usually done based on information available on the initial wind power. A simple method to quantify the

WPCR associated with wind power commitments for different initial conditions could prove to be very useful to a system operator. This section presents a simplified method for wind power commitment that provides information on the associated risks, and can be easily applied in system operation. The method is based on wind power prediction, and only requires information on the initial wind power.

Three different WPCR criteria of 0.3, 0.2 and 0.1 are considered in this study. The initial power was varied between 5% to 90% of the rated capacity and the wind power commitments in the next one and two hours satisfying a WPCR criterion were calculated using the Toronto wind site. The results for the WPCR criterion of 0.3 are presented in Figure 3.16 for a lead time of one hour The wind power commitment is expressed as a percentage of the initial wind power in Figure 3.16. It can be observed that the wind power commitment in the next hour generally increases with increase in the initial power. In other words, the WPCR will decrease as the initial power increases if the wind power commitment is held at a fixed percentage. It can be seen from Figure 3.16 that the wind power commitment in the next hour varies from 30% to 63% of the initial wind power as the initial wind power increases from 5% to 90% of the wind farm rating. The wind power commitments for the Toronto site were similarly calculated for WPCR of 0.2 and 0.1 and are expressed as the percentage of the wind farm capacity in Figure 3.17. The plots in Figure 3.17 show that the wind power commitment in the next hour, for the selected WPCR criterion, has a linear trend with the initial power.

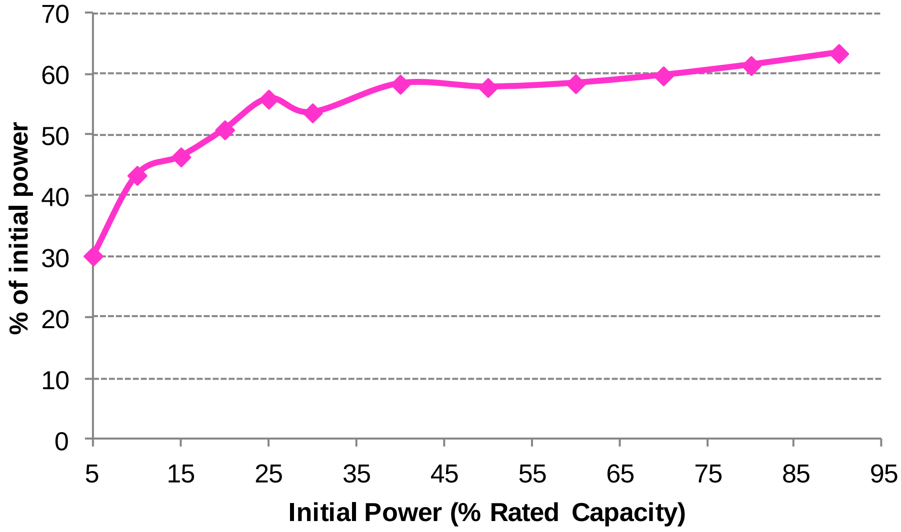

Figure 3.16: Wind power commitment in the next hour for WPCR = 0.3

The wind power commitment values for a lead time of two hours were also calculated and presented in Figure 3.18. It can be seen from Figures 3.17 and 3.18 that the wind power commitment for the next one and two hours could be increased almost linearly with increase in the initial wind power for a selected risk criterion. It can also be observed that the slope of the linear trend lines decreases as the risk criterion is lowered or becomes more stringent. This means that the next hour wind power commitment should be decreased to meet a lower risk criterion for the same initial wind power. The basic concept in the derivation of the simplified method is the approximate linear relation described in Figures 3.17 and 3.18 obtained from the conditional probability distributions of the future wind speeds.

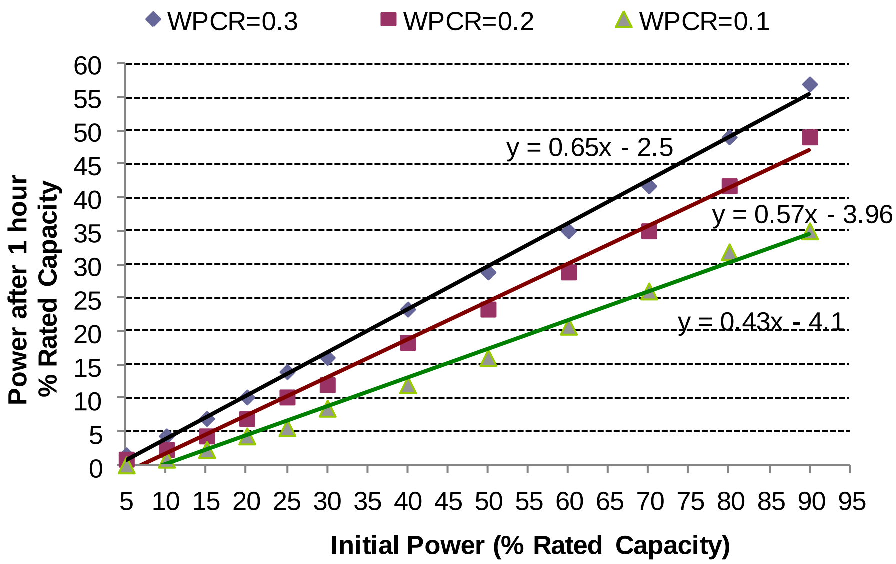

Figure 3.17: Wind power commitment in the next hour

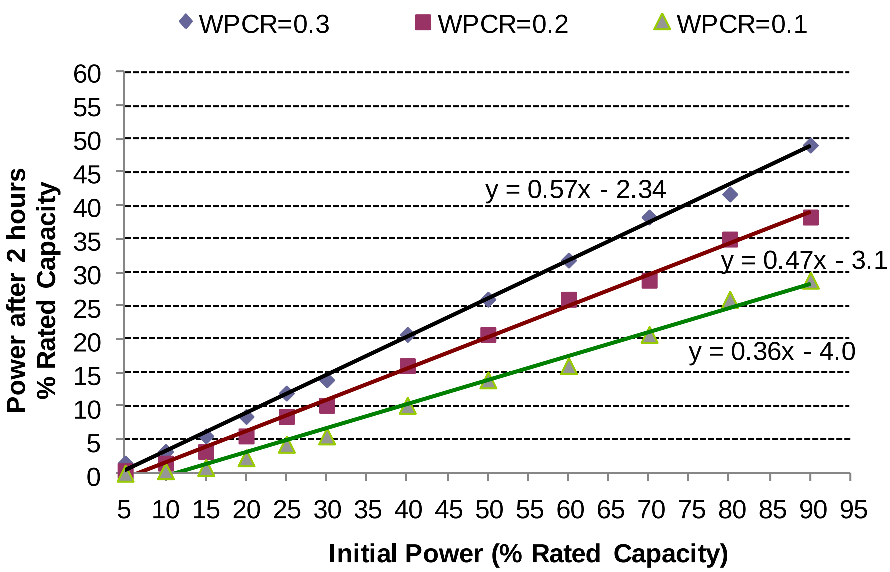

Figure 3.18: Wind power commitment for a lead time of two hours.

The risk-based simplified method for short term wind power commitment using the linear approximation shown in Figures 3.17 and 3.18 is generalized by deducing similar relationships considering the three different wind sites with diverse wind regimes. The wind power commitments for a WPCR of 0.3 were calculated for the Regina and Saskatoon sites, and expressed as a percentage of the rated wind farm capacity. The results for the three sites are shown in Figure 3.19. The average wind power commitment for the three sites increases almost linearly with the initial wind power, and can be approximated by the linear trend shown in Figure 3.19. It can be seen that the average wind power commitment varies from 2% to 56% of the rated capacity in the next hour as the initial power is varied from 5% to 90% of the rated capacity.

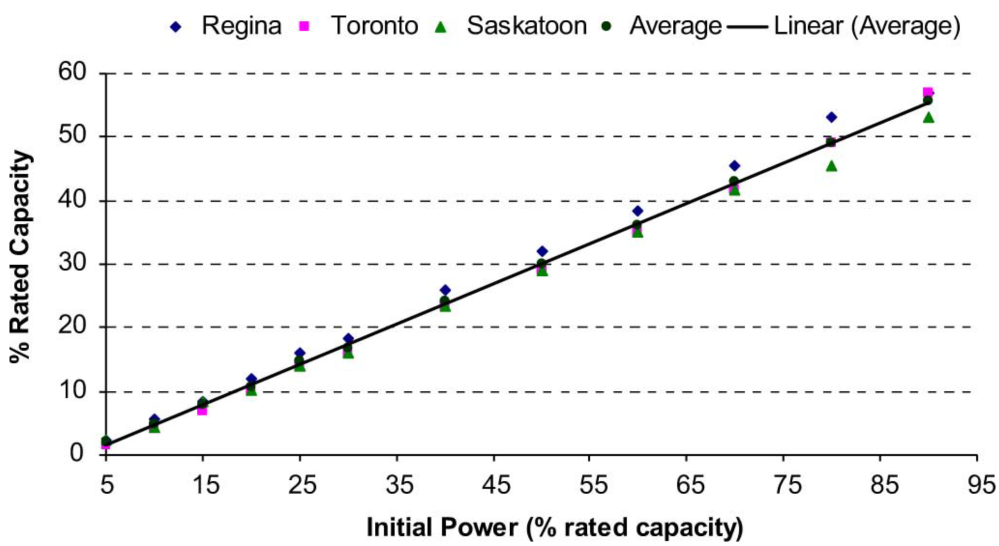

Figure 3.19: Wind power commitment in the next hour for WPCR = 0.3

Figure 3.20 shows the wind power commitment as a percentage of the wind farm rating for a WPCR of 0.2. The average wind power commitment approximated using a linear trend varies from 1.3% to 52% of the rated capacity. Similarly, Figure 3.21 shows the wind power commitment for a WPCR of 0.1 where it varies from 0.35% to 40 % as the initial wind power is varied from 5% to 90% of the wind farm rating.

Figure 3.22 presents the approximate wind power commitment considering the average of the three wind sites to satisfy the three WPCR criteria of 0.3, 0.2 and 0.1. The wind power commitment in the next hour can be estimated using the linear equations (3.1)-(3.3), which are also shown in Figure 3.22.

$$y = 0.63x - 1.55 \mid \text{WPCR} = 0.3 \tag{3. 1}$$

$$y = 0.6x - 2.5 \quad \mid \text{WPCR} = 0.2 \tag{3. 2}$$

$$y = 0.47x - 3.97 \mid \text{WPCR} = 0.1 \tag{3. 3}$$

Where:

x and y are the initial power and wind power commitment in the next hour respectively expressed as a percentage of wind farm capacity.

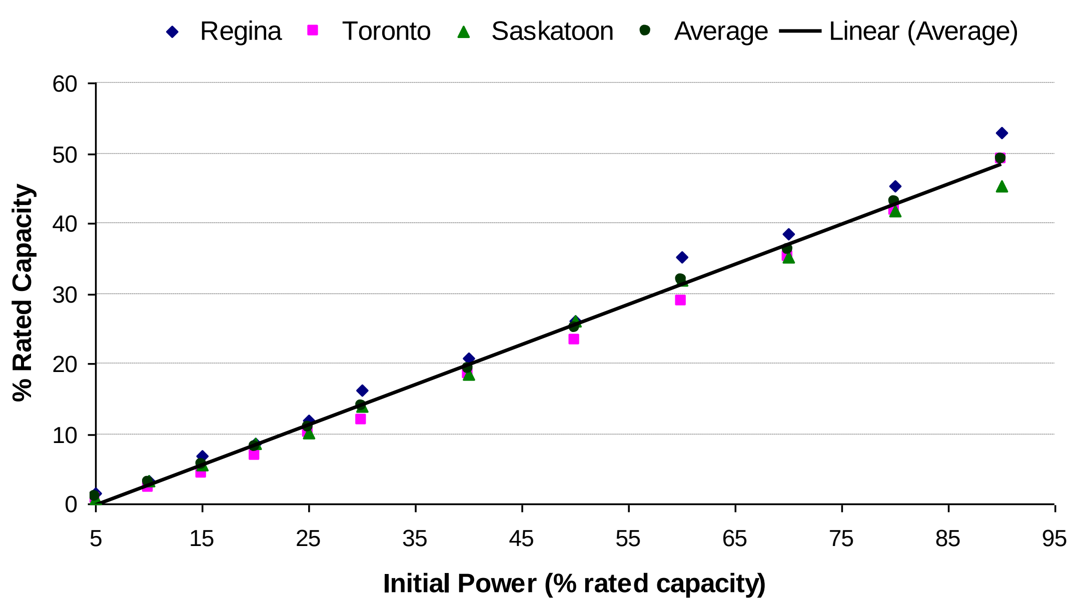

Figure 3.20: Wind power commitment in the next hour, WPCR = 0.2

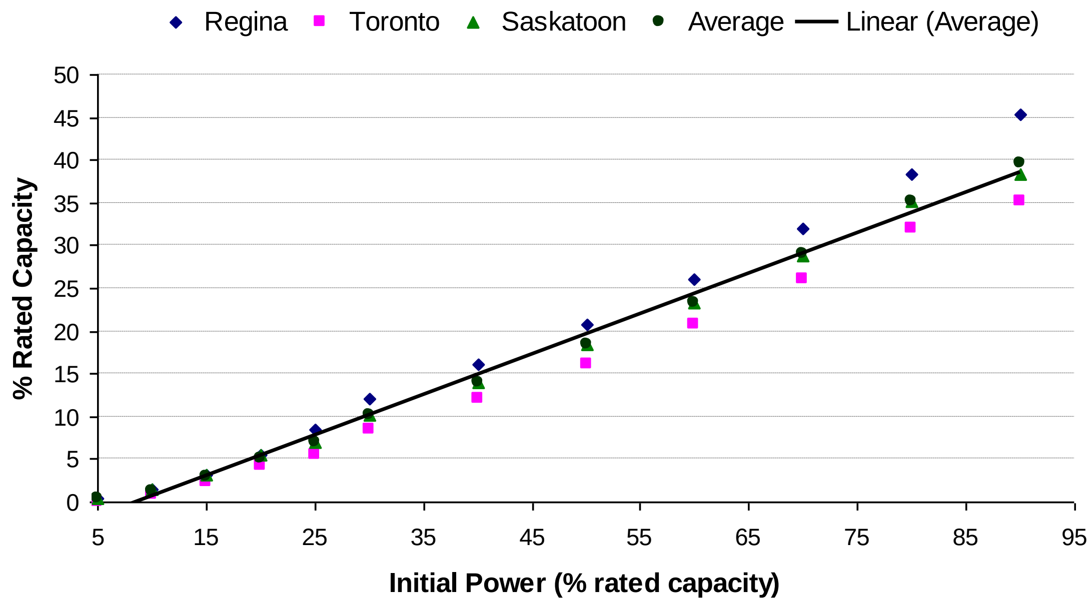

Figure 3.21: Wind power commitment in the next hour, WPCR = 0.1

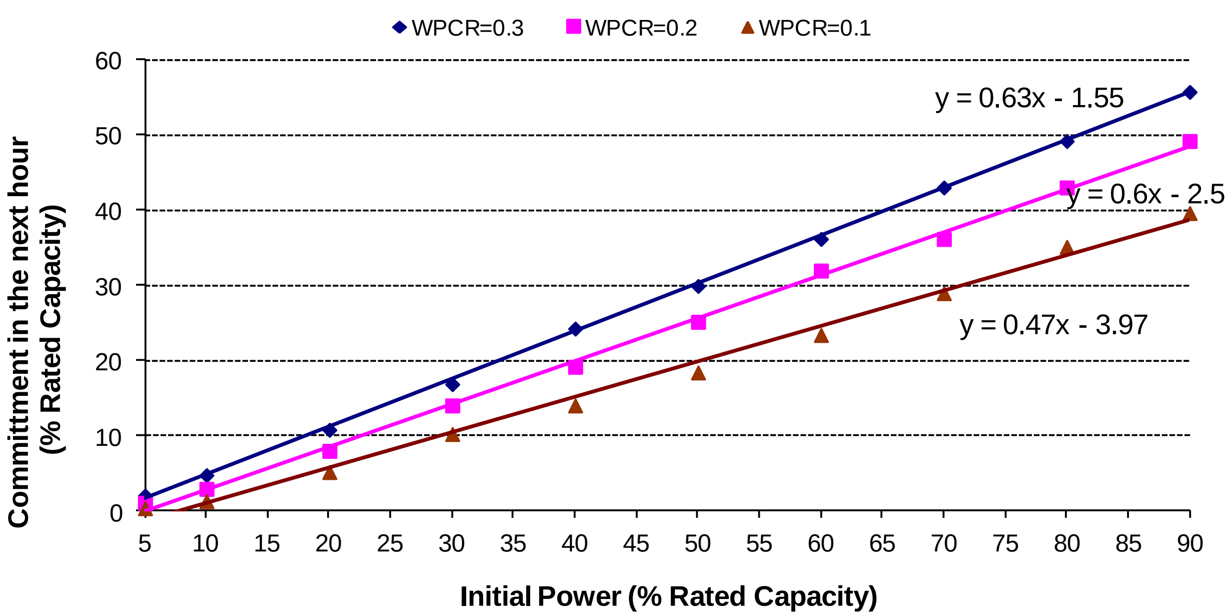

Figure 3.22: Approximate wind power commitment in the next hour

It can be seen from Figure 3.22 that the wind power commitment for the next hour increases almost linearly with increase in the initial wind power for the selected risk criteria. It can also be observed that the slope of the linear trend lines decrease as the risk criteria is lowered or becomes more stringent. This indicates that the next hour wind power commitment should be decreased to meet a lower risk criterion for the same initial wind power.

The wind power commitment for a lead time of two hours is presented in Figure 3.23. The wind power commitment for a lead time of two hours can be estimated using (3.4)-(3.6).

$$y = 0.53x\text{-}1.52 \quad | \text{ WPCR} = 0.3 \tag{3. 4}$$

$$y = 0.44x\text{-}2.55 \quad | \text{ WPCR} = 0.2 \tag{3. 5}$$

$$y = 0.34x\text{-}3.46 \quad | \text{ WPCR} = 0.1 \tag{3. 6}$$

It can be seen from Figures 3.12 and 3.13 that the slopes of the linear trend lines representing the approximate wind power commitment decrease as the lead time increases for a selected WPCR criterion. This is because of the increasing uncertainty under which the risk of wind power commitment increases as the lead time increases requiring the wind power commitment to be reduced to maintain the specified risk criterion.

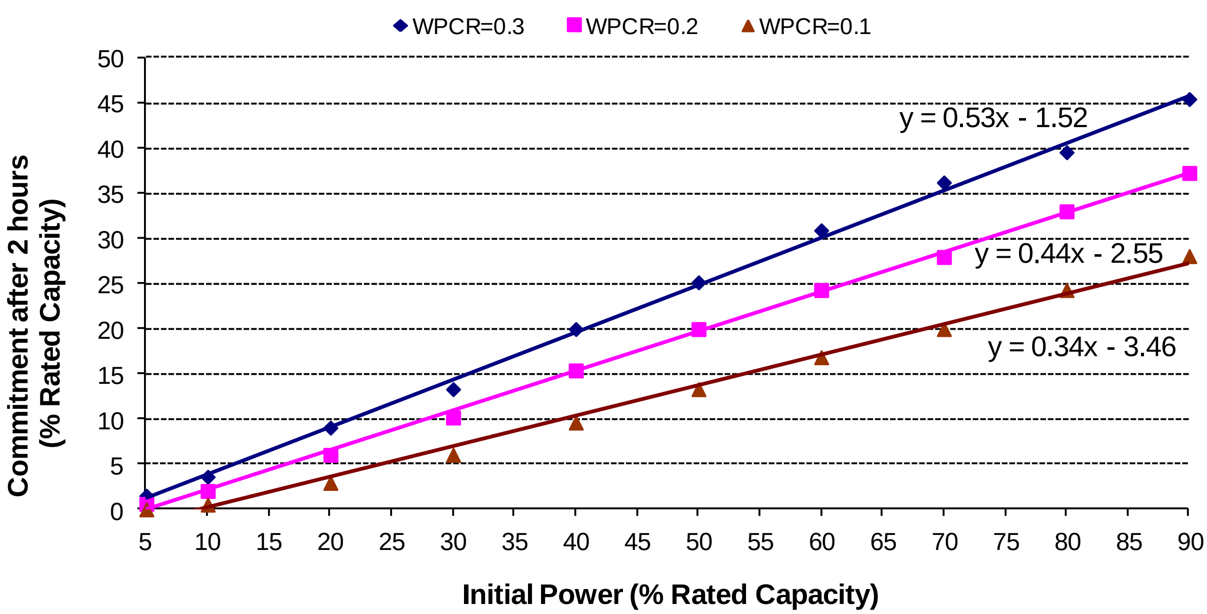

Figure 3.23: Approximate Wind power commitment for a lead time of two hours

The linear relation plots in Figure 3.22 or (3.1)-(3.3) can be used by system operators to estimate the wind power commitment for the next hour for a selected WPCR for a wind site, with wind profile similar to the wind models considered in this study, based on knowledge of the initial wind power. The approximate linear equations shown in Figure 3.23 or (3.4)-(3.6) can also be applied for wind power commitment for a lead time of two hours.

3.9. Application of the Developed Approximate Method

This section illustrates the application of the approximate method developed in the preceding section to assist in wind power commitment decisions in the next hour while meeting a specified risk criterion. It is assumed that a 100 MW wind farm is located at Swift Current in Saskatchewan, Canada. Swift Current has a good wind resource with a mean wind speed of 19.67 km/h and a standard deviation of 9.62 km/h, and is close to SaskPower's 150 MW Centennial wind farm. It is assumed that this 100 MW wind farm consists of 50 WTG units, each rated at 2 MW. The cut-in, rated, and cut-out wind speeds are 15 km/h, 50km/h and 90km/h respectively. The system operator has to commit appropriate wind power for the next hour and meet a WPCR of 0.1. The operator has knowledge of the current power being generated from the wind farm, which is 25 MW. The evaluation steps using the approximate method are described in the following sub-section.

The initial wind power is 25 MW or 25% of the wind farm rating. Equation (3.3) can be used to directly calculate the power commitment for the next hour for a WPCR of 0.1 in this example. The required wind power commitment in the next hour is therefore 7.78% of the rated capacity which is 7.78 MW. The operator should then commit 7.78 MW of wind power for the next hour knowing that there is a 10% chance that the wind power may be less than the committed value. If the operator is willing to accept a higher risk, i.e. a WPCR of 0.2, then using

(3.2), a wind power commitment of 12.5 MW is determined for the next hour. In this case, an additional 4.72 MW of wind power can be committed with the knowledge that there is a 20% risk that the wind power will be less than the committed value. Table 3.7 presents estimates of the wind power commitment in the next one and two hours using the approximate method for WPCR of 0.3, 0.2 and 0.1. The wind power commitment has also been evaluated from the probability distribution of the wind speed using the 24 years of actual data for the Swift Current site and is also presented in Table 3.7.

Table 3.7: Wind power commitment in the next hour using the approximate method and actual data of the Swift Current site

Initial Power, (MW)	Wind power commitment in the next hour(s), (MW)				WPCR
	Approximate method		Actual Data		
	1 Hour	2 Hour	1 Hour	2 Hour	
15	3.08	1.64	3.25	2.3	0.1
25	7.78	5.04	5.58	4.34	
35	12.48	8.44	12	5.58	
45	17.18	11.84	16.1	10.17	
15	6.5	4.05	5.58	4.34	0.2
25	12.5	8.45	12	8.49	
35	18.5	12.85	20.78	12	
45	24.5	17.25	23.34	16.1	
15	7.9	6.43	8.49	6.96	0.3
25	14.2	11.73	13.97	12	
35	20.5	17.03	23.34	20.78	
45	26.8	22.33	28.91	23.34	

It can be observed from Table 3.7 that the values of wind power commitment obtained using the approximate method are fairly close to the values obtained from the probability distributions of actual hourly wind speed data. One of the reasons for this difference is the lack of sufficient actual site data to create the probability distributions to calculate the associated risks.

The numerical results shown in this study are based on a particular set of WTG parameters. The developed method is not dependent on these parameters and can be applied to other WTG designs. The primary concept is the utilization of conditional wind speed distributions based on known initial wind speeds.

3.10. Day- Ahead Wind Power Commitment

3.10.1. Impact of extended lead time on wind power commitment risk

The studies presented in the previous sections consider short future times such as 1 and 2 hours. It has been established that the short term wind power commitment is dependent upon the initial condition. This section presents a study of wind power commitment and the associated WPCR as the lead time is extended to 24 hours. The wind site considered in this study is represented by the Regina wind speed data using the ARMA model noted in (2.3). Figure 3.24 shows the basic statistics of the wind speed distribution with lead times ranging from 1- 24 hours for two initial conditions at Hour 10 designated as Case A and Case B. The two cases, Case A and Case B, respectively have initial wind speeds of 25 km/h and 30 km/h giving 10% and 20% of the rated capacity as the initial wind power. It can be seen in Figure 3.24 that the mean value of the wind speed decreases while the standard deviation increases as the lead time is increased. The increase in the standard deviation is an indication that the variability and therefore the uncertainty will increase moving into the future. The mean wind speed varies from 24.37 km/h to 20.43 km/h for Case A while it varies from 28.78 km/h to 21.37 km/h for Case B as the lead time increases from 1 to 24 hours.

The plots of the mean wind speed show a sharp decline up to a lead time of about 10 hours and then settle down as the lead time further increases. The standard deviation on the other hand increases from 4.86 km/h to 9.9 km/h for Case A and 5.03 km/h to 10.07 km/h for Case B as the lead time increases from 1 hour to 24 hours. The plots of the standard deviation rise sharply up to about 10 hours and become almost constant as the lead time is further increased. The two plots of mean values for the two initial conditions start some distant apart, gradually tend to converge up to a lead time of about 6 hours, and then maintain a spread of about 1km/h as the lead time is further increased. The plots of the standard deviations are relatively close to each other for both cases at all the lead times. This indicates that the variability is quite independent of the initial conditions and is mainly dependent on the lead time.

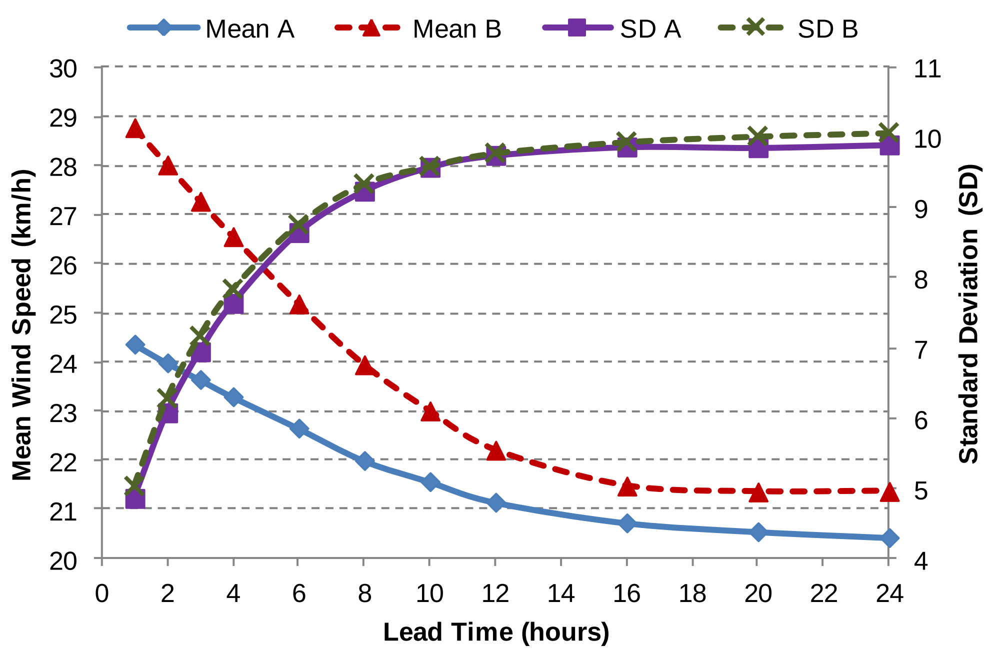

Figure 3.24: Basic statistics of conditional wind speed distributions

The capacity value of wind power at a WPCR criterion of 0.4 is presented in Figure 3.25 for the three initial conditions in Case A, Case B and Case C at Hour 10. Case C has an initial wind speed of 34 km/h giving an initial power of 30% of the rated capacity. The lead times considered are from 1to 24 hours. The capacity value of wind power obtained from the conditional wind speed distribution varies from 10.17% to 1.5% of the rated capacity for Case A. The capacity values vary from 20.78% to 2.3% and 31.91% to 2.3% of the rated capacity for Case B and Case C respectively. The WPCR constrained wind capacity value decreases significantly with lead time for any initial condition, and reaches a relatively small value when the lead time is greater than 12 hours. Figure 3.25 shows that the three curves for the initial conditions are significantly apart at small lead times (e.g. 1 to 6 hours), but become close to each other at a relatively small capacity value as the lead time increases beyond 12 hours. This suggests that the impact of the initial condition on a future wind capacity value decreases significantly as the lead time is increased beyond 12 hours, and the impact is insignificant in day-ahead wind power commitment analysis.

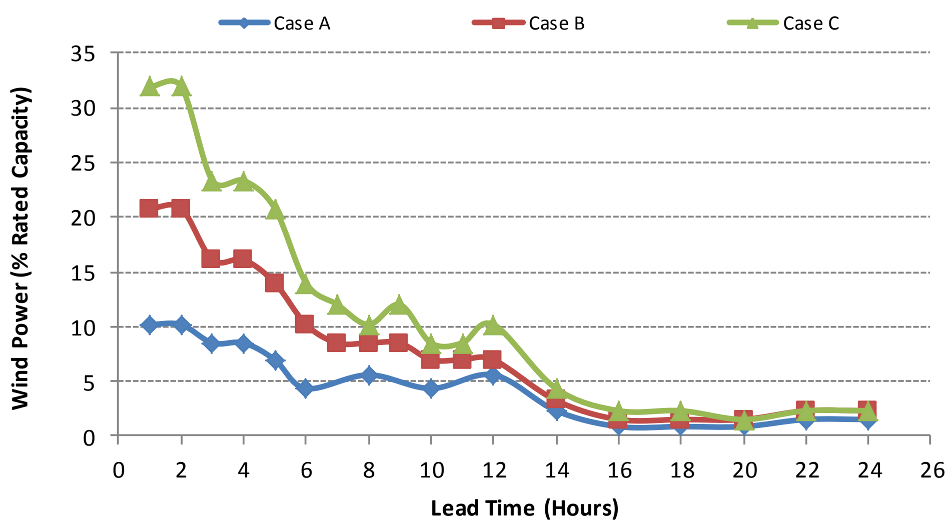

Figure 3.25: Wind power commitment for three initial conditions (WPCR = 0.4)

3.10.2. Impact of WPCR criteria

The selection of a suitable WPCR criterion is a management decision that should consider the operating strategy, types and sizes of the conventional units and the reserves that can be made available during fluctuations in wind power generation. Figure 3.26 presents the wind power commitment in a short future time period ranging from 1 hour to 24 hours constrained by three WPCR criteria of 0.4, 0.3 and 0.2 given that the wind power at the initial time is 20% of the rated capacity (Case B). The capacity value varies from 20.78% to 2.3%, 18.36% to 0.86% and 13.97% to 0% of the rated capacity respectively at the WPCR criteria of 0.4, 0.3 and 0.2 as the lead time increases from 1 hour to 24 hours. Figure 3.26 shows how the wind power profile rises as the risk criterion increases allowing a higher capacity value of the wind power to be committed in the lead time considered. It can also be seen that the day-ahead capacity value assigned to the wind power is essentially zero at WPCR criteria of 0.3 and 0.2. A higher risk criterion of 0.4 or higher could be applied for such long horizons as there is time for the system operators to employ available means to mitigate unfavorable consequences due to low wind situations by making operating adjustments a few hours ahead.

3.11. Day-Ahead WPCR

As noted earlier, knowledge of short term wind power can assist the system to optimize the required regulating capacity. It is also necessary to assess the day- ahead wind power to schedule the conventional units. Physical methods employing numerical weather prediction are often used to predict wind power over a long horizon. The physical methods however also contain forecasting errors, and hourly models are used to mitigate the errors of wind power prediction and determine the spinning reserve requirements. It has been observed from the preceding section that the impact of initial wind conditions on future wind capacity values decrease significantly beyond lead times of 10-12 hours, and have negligible impact in day-ahead wind capacity assessment. Conditional probability considerations used for short term (i.e. 1 to 4 hours) wind power commitment are not required for day-ahead wind power assessments, and historic wind speed statistics at the particular hour without consideration of initial wind conditions can be used to provide a probabilistic day-ahead wind capacity value.

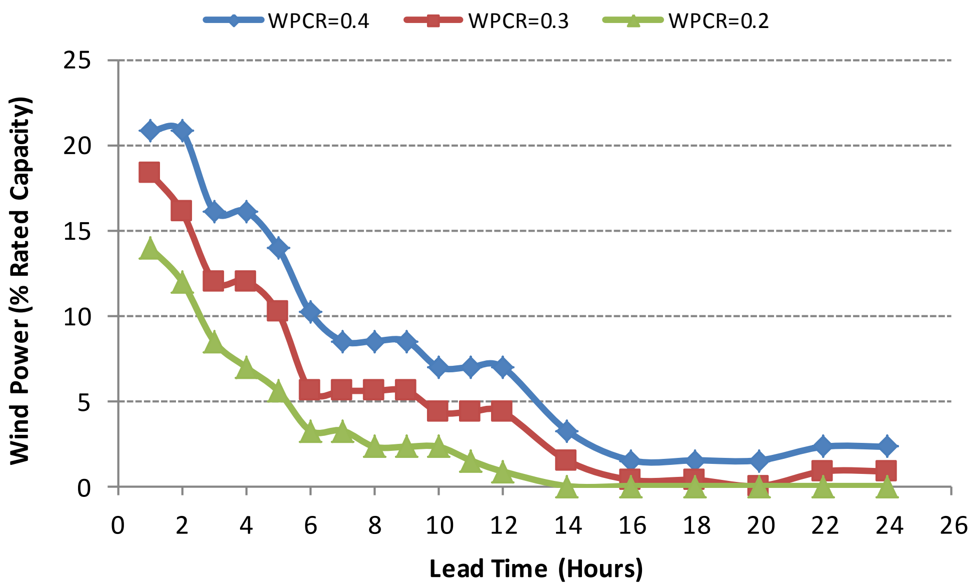

Figure 3.26: Wind power commitment for three WPCR criteria (Initial power = 20% of the rated capacity)

Figure 3.27 presents the probability distributions of the wind speed at Hour 11 and Hour 14 which represent lead times of 1 hour and 4 hours respectively. The left end of the figure has distributions shown by the solid lines without markers obtained from the hourly wind speed distributions in the ARMA model. These distributions are designated as unconditional distributions in Figure 3.27 as they do not depend upon any initial conditions. The figure also shows the conditional wind speed distributions at the two lead times for the Case B and Case C conditions. The conditional wind speed distributions for lead times of 1 hour and 4 hours move distinctively towards higher wind speeds as the initial conditions change from lower to higher wind speeds. The distributions for a lead time of 24 hours are similarly presented in Figure 3.28. Contrary to the distributions shown in Figure 3.27, the probability distributions for a lead time of 24 hours are very similar for both initial conditions and are close to the one obtained from the hourly wind speed probability distribution. This further illustrates that the initial conditions are significant in short term wind power commitment but not in longer horizons such as those for day-ahead commitment. More importantly it also indicates that the historic wind speed statistics can be directly used to assess approximate day-ahead capacity values for wind power.

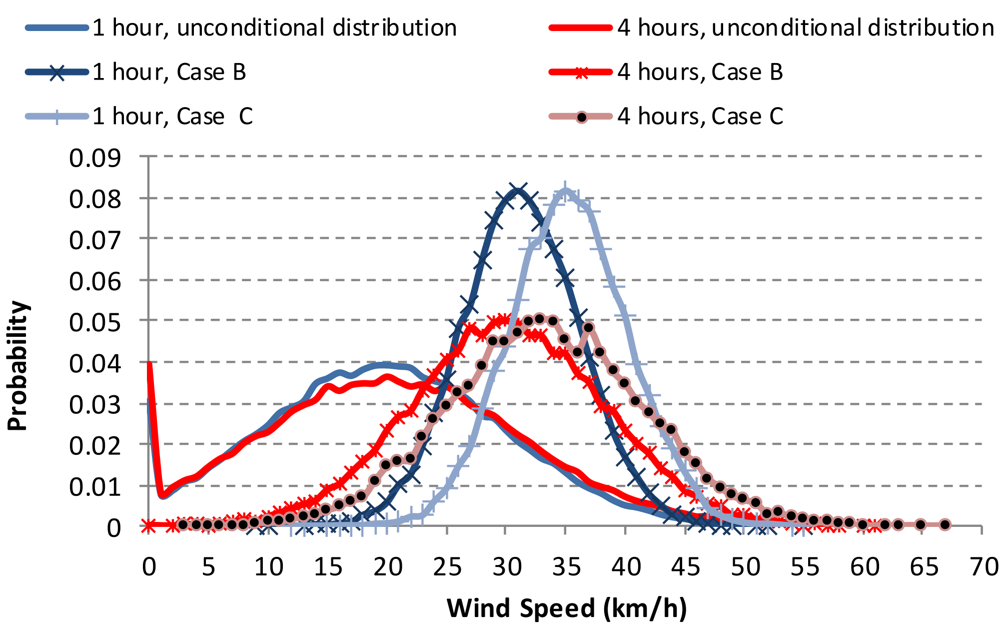

Figure 3.27: Wind speed probability distributions (conditional and unconditional) for 1 hour and 4 hour lead times

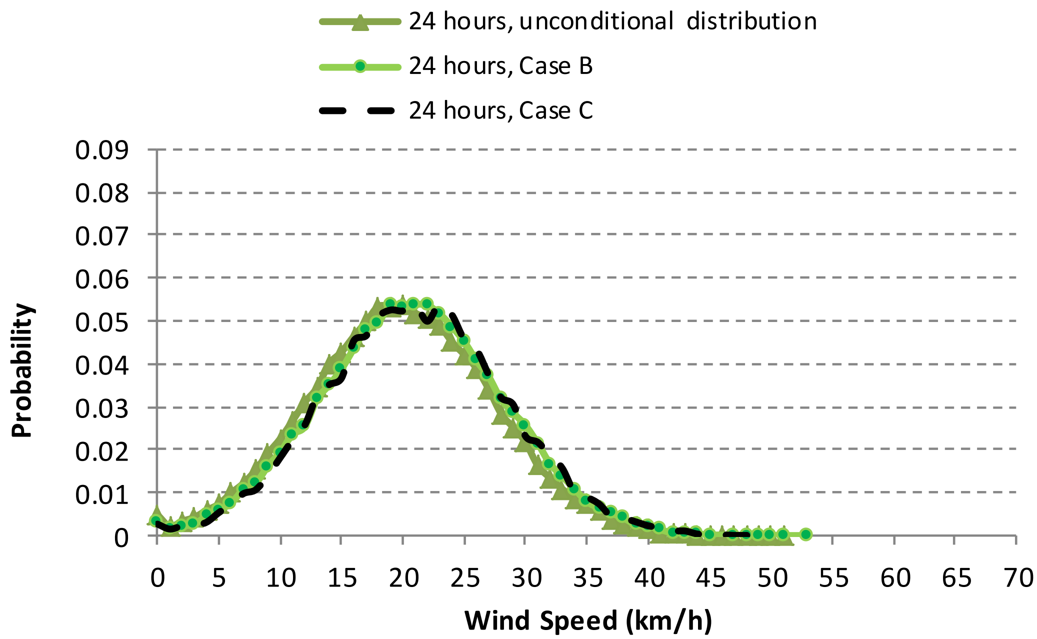

Figure 3.28: Wind speed probability distributions (conditional and unconditional) for a 24 hour lead time

3.12. Approximate Day-Ahead Wind Power Commitment

The previous section illustrates that historic wind speed statistics can be directly used to assess the day-ahead capacity value of wind power in day-ahead generation planning. Sufficient historic wind speed data are usually not available to most system operators. A simplified method requiring limited data for day-ahead wind capacity assessments could therefore prove very useful to system operators. The probability distribution obtained from historic wind speed data collected over a large number of years, or obtained from simulated data using the appropriate ARMA model as shown in Figure 3.28 can be approximated by a normal distribution based on the mean wind speed and the standard deviation for the particular hour. Figure 3.29 shows the cumulative wind speed probability distributions obtained from the wind speed data simulations using the ARMA model and the normal distribution using the hourly wind speed statistics at Hour 34 which is 24 hours of lead time with Hour 10 as the initial time. In both methods the negative values of the wind speed are converted to zeroes. The ordinate in Figure 3.29 gives the WPCR which is the probability that the wind speed will be less than the value given in the abscissa. It can be seen that the WPCR evaluated using the two methods are approximately equal.

The simplicity of the approximate normal distribution method makes it easy to apply in system operation, and the method only requires the mean wind speed and the standard deviation for a particular hour. The method can be used to assess the day-ahead capacity value of wind power for a selected WPCR criterion. The simplified normal distribution method has been applied to assess the capacity values of wind power for each hour of the next day. Figure 3.30 presents the wind power commitment for Hour 24 to Hour 48 for WPCR criteria of 0.5, 0.4 and 0.3. The capacity value varies from 1.5% to 8.5 % of the rated capacity at the WPCR of 0.5 during the hours considered with an average of approximately 5 % of the rated capacity over the time considered. The capacity value decreases to 0% to 4.3% with an average of approximately 2% of the rated capacity at the WPCR of 0.4 over the same period. It follows the same hourly trend as that shown in Figure 3.31. It is also noticeable that the wind has almost no capacity value when the WPCR criterion is reduced to 0.3. It should be noted that a low WPCR criterion such as 0.3 does not totally negate the capacity value of wind power while making a day- ahead commitment. The system operator should consider accepting a higher WPCR such as 0.5 while scheduling the units, and adjust the regulating capacity later in the day employing the hourly models using the conditional probability method.

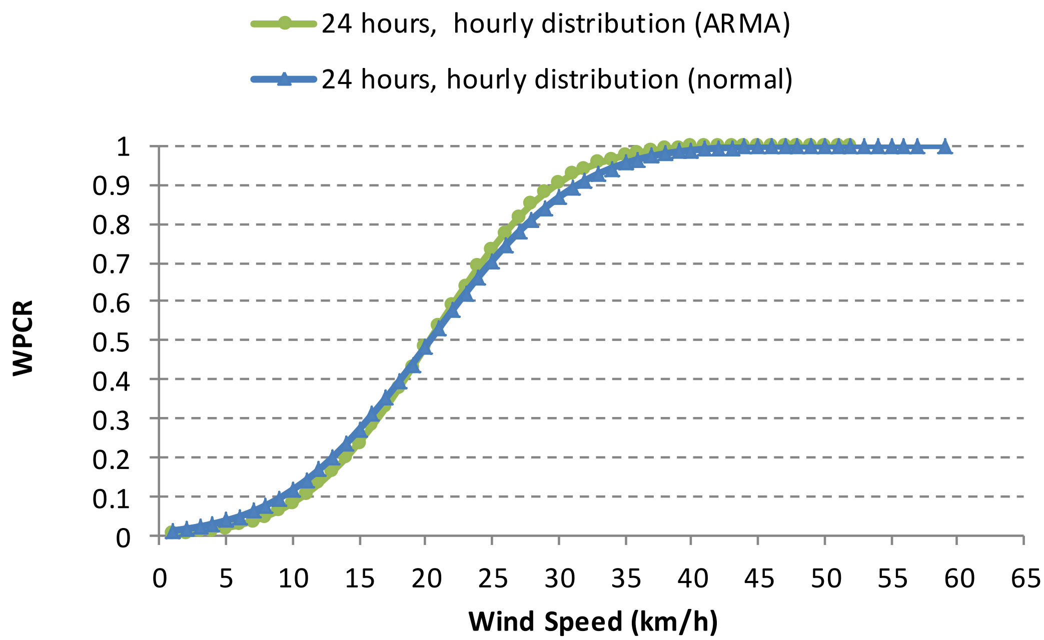

Figure 3.29: Wind speed cumulative probability distributions (ARMA and normal) for a 24 hour lead time

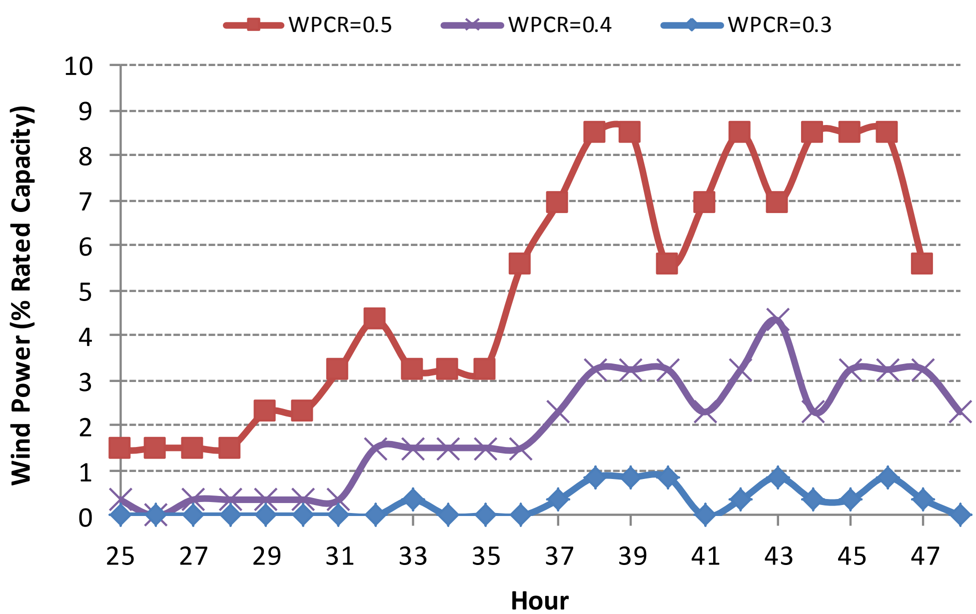

Figure 3.30: Wind power commitment using the normal distribution of the historic wind speed statistic

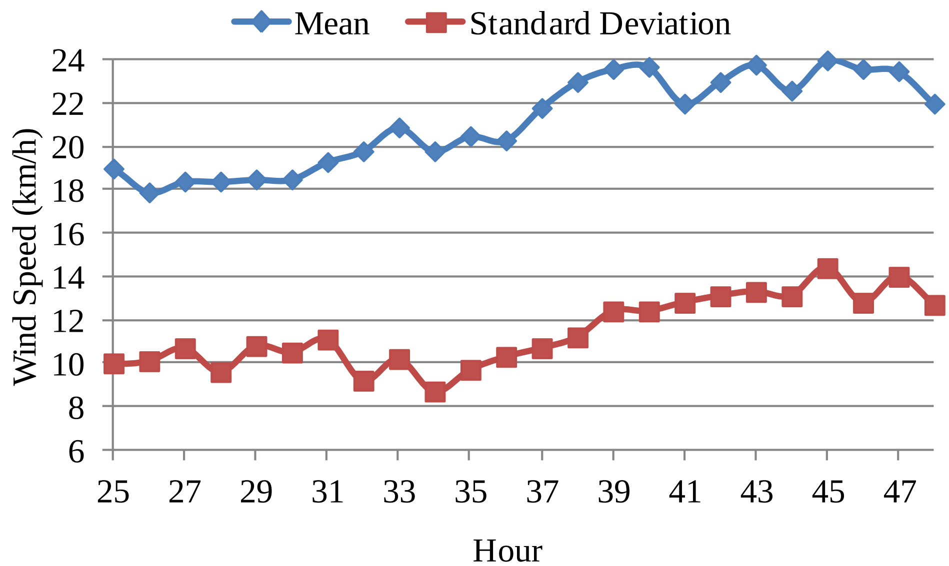

Figure 3.31: Hourly mean and standard deviation (SD) of the wind speed at Regina (Hour 1 - Hour 48)

3.13. Summary

It is necessary to estimate the amount of wind power that will be available at a short time in the future, such as one or two hours in order that the reliability of the power system is not degraded when utilizing wind power. The variability associated with wind power generation is quantified in this thesis in the form of conditional probability distributions based on the initial wind speed.

The probability distribution of the wind speed/wind power conditional upon the initial wind speed can be used to evaluate the risk associated with committing a specified level of wind power in the next hour(s). The wind power commitment of a wind farm for a short future time is usually specified as a percentage of its initial power output. It is necessary to quantify the appropriate amount of wind power available at a short time in the future in order to maintain the generating system reliability in a wind integrated power system. The risk of not meeting a pre-specified commitment can be reduced by lowering the wind power committed capacity for the next hour or the next few hours. System operators should vary the relative amount of wind power commitment in the next hour or next few hours as the initial power varies in accordance with an acceptable WPCR in order to meet the designated level of system reliability. The results show that appropriate wind power commitment is highly dependent on the risk criterion deemed acceptable to the system.

Diurnal variations are important factors to consider when estimating short term wind power. Wind power variability increases with increase in the lead time. It has been found that rising or falling wind trends can respectively offset or intensify this increase in variability. System operators may therefore need to adjust their wind farms commitments based upon acceptable WPCR values. The seasonal impact of diurnal variations has been presented using two particular days to represent winter and summer conditions.

The evaluation of wind power commitment risk for correlated wind farms indicates that wind speed correlation can have significant impacts on the perceived risk. The results show that the WPCR reduces as the correlation coefficient decreases. Wind speed correlation should therefore be incorporated in the evaluation when committing wind power from multiple wind farms.

The simplified approximate method based upon wind power commitment risk can assist the system operator and wind farm owner to commit wind power in the next few hour(s) based on knowledge of the initial available power. Risk based wind power commitment provides utilities and wind farm operators with an appreciation of the risks associated with wind capacity commitments in the next few hours, and helps them determine commensurate levels of wind power commitment at acceptable WPCR. The method outlined in this thesis is general and can be applied to a wide range of wind power situations and systems. The simplified method can be used with minimum system information and is expected to be a helpful tool for wind power commitment in the next few hour(s).

The capacity value of wind power in a short future time is driven by the initial conditions. The conditional probability approach can be used to quantify uncertainties associated with short term wind power commitment. Risk based methods are useful in assessing the capacity value of wind power as they allow the system operator to appropriately manage the short and long term system reserves. The conditional probability approach is useful in assessing the WPCR and the capacity value constrained by the WPCR criteria for short future times such as 1-4 hours. The studies presented show that the impacts of the initial conditions weaken as the lead time increases and initial conditions are not the driving factor when long lead times such as 24 hours are considered. The historic hourly wind speed probability distributions without any consideration of the initial conditions can be used to assign day-ahead capacity values to a wind farm based on a suitable WPCR criterion. The method can be simplified using a normal wind speed probability distribution of for the particular hour based on the mean wind speed and the standard deviation for the given hour. Sophisticated and complex methods of wind power prediction are not readily applied in practice. The approximate normal distribution method presented in this study can be easily applied and should prove useful for day-ahead unit scheduling. The conditional hourly models can be used for shorter term wind power commitment considering appropriate WPCR criteria.

CHAPTER 4

OPERATING RESERVE ANALYSIS OF A WIND INTEGRATED POWER SYSTEM

4.1. Introduction

One of the major tasks in electric power system operation consists of short term load forecasting and making a decision on which units to commit to serve the forecast load. Uncertainty, mainly due to the unit failures and the load fluctuations, creates power system operating risk. Power system operators prepare by committing units with a total operating capacity higher than the forecast load. The excess capacity is called operating reserve and may be spinning or non-spinning in different forms such as rapid start units, hot reserves, assistance from other interconnected systems and interruptible loads. It is an important task to determine the appropriate operating reserve.

Deterministic criteria such as “N-1” or “percentage reserve margin” are widely used by utilities. The “N-1” criterion specifies a capacity equal to the largest committed unit as the operating reserve so that the load is satisfied even when the largest committed unit fails. The “percent reserve margin” criterion provides a specified percentage of the peak load as the reserve, which is determined by the experience using the system capacity compositions and may be different in different utilities. The unit failures and load fluctuations, which cause the system risk, are probabilistic in nature and are not incorporated in deterministic methods. The integration of variable power generation such as wind power accentuates the significance of applying probabilistic methods in determining the operating reserve. Unit commitment risk (UCR) analysis is a probabilistic approach to determine a consistent and acceptable spinning reserve requirement. The Pennsylvania-New Jersey-Maryland (PJM) interconnected system initially created and applied a probabilistic method to quantify system operating risk [13] and determine the spinning reserve required to satisfy a specified risk criterion. The PJM method has evolved in the past to include various factors such as interruptible loads, load forecast uncertainty, hot reserves and rapid start units and assistance from interconnected systems [7, 14, 17, 18, 151]. Literature review relevant to spinning reserve evaluation is presented in Chapter 1. The literatures on probabilistic methods applied to power system operation are outlined in [12]. This chapter focuses on developing and applying an appropriate method to integrate wind power in probabilistic risk assessment of unit commitment.

If a conventional generating unit is modeled as a two state system, it resides in the operable and inoperable states over a short lead time with probabilities determined mainly by the unit failure rate, assuming that repair is not possible in a short lead time such as several hours [1]. Wind power generation, on the other hand, increases and decreases due to the variability of the wind speed at the wind site. The spinning capacity held on the committed units is responsible for responding to any variations in the load and/or generation. The previous chapter focuses on risk analysis of committing power from the perspective of an operator or owner of a wind farm. The previous analysis therefore did not consider the system configuration and load. This chapter is focused on the system operating risk incorporating wind power. This requires integration of the risk model of the conventional units with the appropriate wind model. The reliability contribution of wind power in the lead time is quantified in terms of the increase in load carrying capability and the operating capacity credit at a selected risk criterion.

Unit commitment is done to satisfy a specified load level and the operating criteria as determined by managerial decisions. Unit commitment decisions are updated as required to satisfy the expected changes in load on a continuous basis. If a committed unit fails while in operation, a decision is made to replace the failed unit and takes a certain lead time to bring the new unit into operation. The committed units are therefore responsible for satisfying the load and the operating criteria under any uncertainties arising within the lead time. Advances in short time load forecasting and discussions with experienced system operators indicate that the uncertainty associated with unit failures and wind power variability have a much more significant impact on the system risk compared to that associated with load forecast uncertainty. Inclusion of load forecast uncertainty in UCR analysis is illustrated in [1]. Once the units are committed, the next step is to determine how the reserves are distributed over the units so that adequate response is available to satisfy the operating criteria during load changes or component failures. This task is related to load dispatch and considers the operating cost and ramping capabilities of the committed units. This is discussed in Chapter 6.

The work described in this chapter utilizes the conditional probability approach [107] and extends the area risk concept presented in [151] to incorporate the impact of wind power in unit commitment risk and health analysis [32]. The results obtained using the extended area risk concept to incorporate wind power are compared to the method presented in [107] in this study.

4.2. Unit Commitment Risk

Unit Commitment Risk (UCR) is the probability that the committed units are capable of just satisfying or failing to satisfy the forecast load in the lead time [1]. The load is assumed to be constant in the lead time considered. The initial conditions of the committed units are known and hence the failure or success of the system is known at the initial time. The unit commitment risk analysis quantifies the uncertainty associated with the failure of the committed units within the lead time to supply the load. Repair of the failed unit is not considered possible within the short future lead time. Assuming that the unit failure rate (λ) is constant, the probability that a unit will fail in time t, given that it was operating successfully at time t = 0 is given by (4.1) [1].

$$P(\mathit{failure}) = 1 - e^{-\lambda t} \tag{4. 1}$$

For a short lead time T, which may be several hours, the probability of failure can be approximated by (4.2).

$$P(\mathit{failure}) \cong \lambda T \tag{4. 2}$$

The time dependent probability λT is designated as the outage replacement rate (ORR), which is defined as the probability that a unit fails and is not replaced in the given time. The ORR is used in creating the capacity outage probability table (COPT) of the committed units for UCR evaluation. The cumulative probability in the COPT that corresponds to the capacity state which is equal to or less than the load is the UCR [1].

4.3. Area Risk Method

The concept of area risk is presented in [1] and [151]. The failure density function for a single unit with the outage replacement rate of λt is shown in Figure 4.1 [151]. The probability that the unit will fail in the time interval [0, T] is given by (4.3). The risk for the single unit commitment is shown by the shaded area under the curve in Figure 4.1 which increases as the future lead time increases.

$$P(0,T) = \int_0^T f(R)\, dt = \int_0^T \lambda e^{-\lambda t}\, dt \tag{4. 3}$$

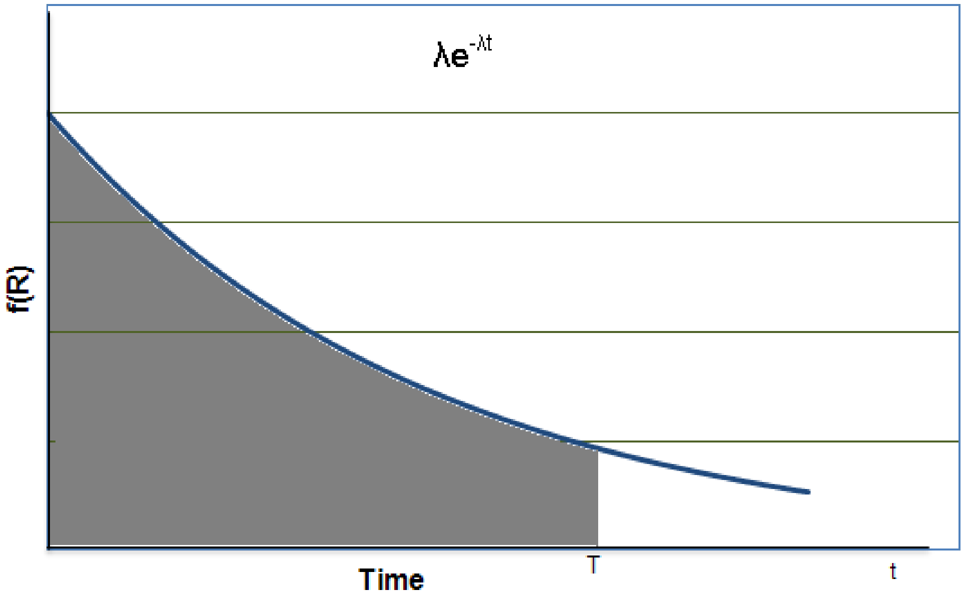

Figure 4.1: Single unit failure density function

Figure 4.2 shows the risk function for a system where multiple units are committed to satisfy a forecast load for a lead time of T hours. The basic PJM method does not include rapid start units such as gas turbines, hydro or hot reserve units, which can come on-line later within the lead time based upon decisions made at the initial time to bring them into operation. The area risk method is a modification of the basic PJM method that evaluates the total area under the curve, which is reduced due to the units being available later to support the system.

Figure 4.2 shows the case where a decision to put two additional units: a rapid start unit and a hot reserve unit into operation is made at the initial time so that they come online at times T1 and T2 respectively. The risk curve is modified by the addition of these units, and the total risk is reduced by the amount represented by the shaded area in Figure 4.2. The total risk is evaluated by summing up the risks for different periods within the lead time. The periods considered in this illustration are: (0, T1), (T1, T2) and (T2, T). The mathematical representation of the system unreliability is given in (4.4).

$$P(failure) = \int_{0}^{T1} f(R1)dt + \int_{T1}^{T2} f(R2)dt + \int_{T2}^{T} f(R3)dt \tag{4. 4}$$

Where:

f(R1) is the risk profile for the interval (0, T1)

f(R2) is the risk profile considering a rapid start unit for the interval (T1, T2) and

f(R3) is the risk profile considering a rapid start unit and a hot reserve for the interval (T2, T)

The risk at any interval is defined as the probability that the operating capacity is just equal to or less than the load.

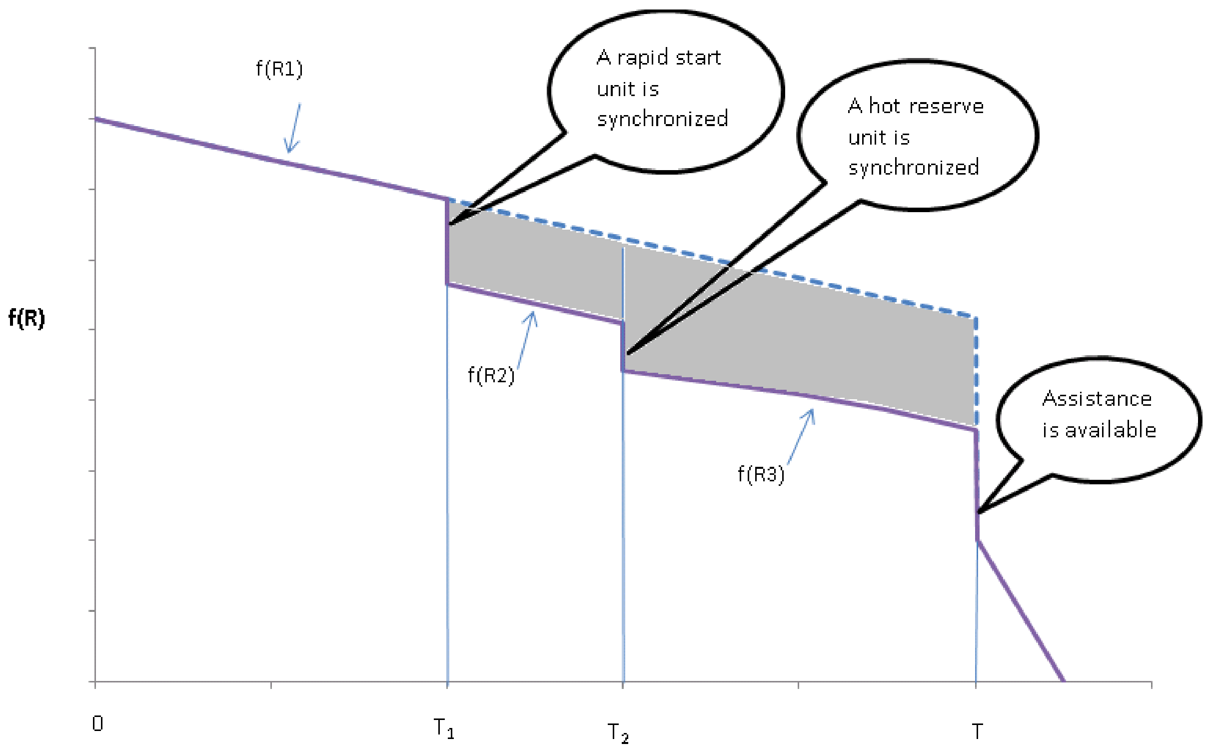

Figure 4.2: Area risk concept including the units that come into operation after t = 0+

The mathematical representation in (4.4) and the pictorial representation in Figure 4.2 are meant for a visual explanation of the area risk concept. The risk profile may not be a continuous function as shown in Figure 4.2 and the area is not actually measured to evaluate the unreliability. The risk for the first period is evaluated as in the PJM method where units committed at t = 0 are considered. Two partial risks, one at the beginning and the other at the end of each period, are however required to evaluate the second and the third period risks. The partial risk at the beginning of the second period is evaluated from the COPT created with the units on-line at t = 0 with ORR evaluated at T_1 and the rapid start unit considering its probability of failure to start as its outage probability at T_1. The partial risk evaluated at the end of the second period consists of the initial units with ORR evaluated at T2 and the rapid start unit with its state probabilities evaluated for the period (T_2-T_1). The difference of these two partial risks gives the risk for the period (T_2-T_1). The risk for the period (T_2-T) can be similarly evaluated [1]. The area risk method is a concept to evaluate the reduction in the risk as a result of the units which are brought into the system later in the lead time. This concept has been extended in the following section to evaluate UCR incorporating wind power.

4.4. Extension of the Area Risk Concept to Incorporate Wind Power in UCR Evaluation

The short term variability of wind power is quantified by the discrete capacity states and their probabilities obtained from the conditional probability distributions. The hourly time series model and the conditional probability approach, as discussed in Chapter 2, model the variability over a future lead time. The basic procedure presented in [107] is considered as the reference method in this study. In this approach the COPT of the conventional units is combined with the conditional wind power distribution created at the end of the lead time. Wind power can however vary within a lead time in different ways. The wind speed can rise or fall within the lead time depending on the diurnal wind characteristic of the site. The conditional wind power distribution obtained at the final hour may not accurately portray the wind power contribution over the entire lead time.

Figure 4.3 presents the discrete capacity states and their probabilities for a 300 MW wind farm at Hour 9 considering Toronto wind data, given that the wind speed at Hour 8 is 30 km/h and the wind power output is 60 MW. This is derived from the short term wind speed model and the wind turbine characteristics presented in Chapter 2. The wind power models for the next hours are similarly created. The proposed area risk method combines the wind power variability obtained for each sub period within the lead time with the capacity model of the committed conventional units at the appropriate times.

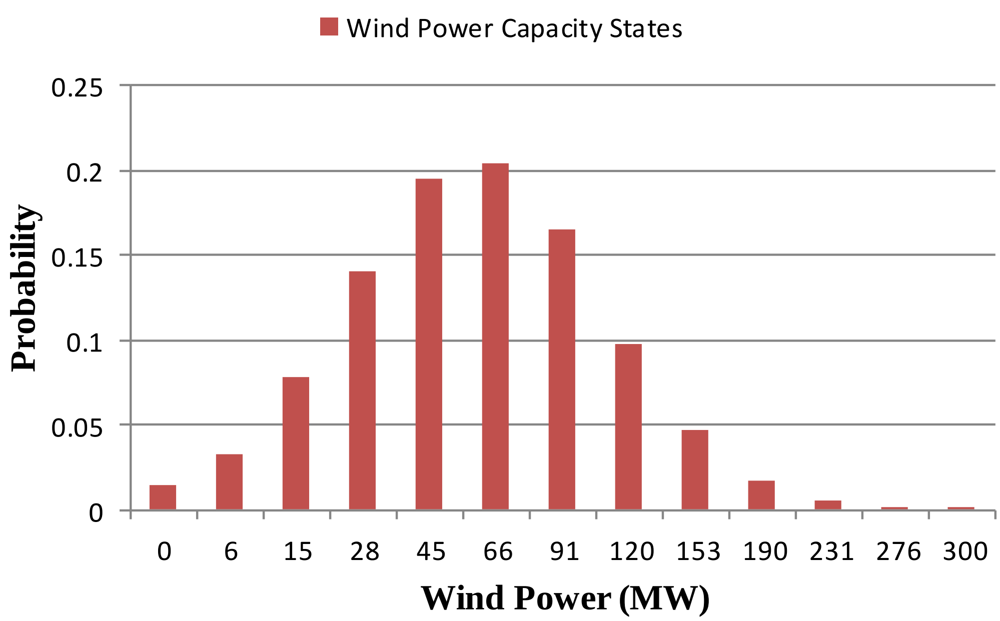

Figure 4.3: One hour ahead wind power capacity states and probabilities: the initial wind power is 60 MW from a 300 MW wind farm

The method is pictorially illustrated in Figure 4.4 where four hourly wind power distributions obtained for a known initial condition are combined with the risk function of the conventional units committed for a lead time of four hours. The wind power in a short future time is dependent upon the initial condition, and the persistence model is very effective for short term wind power prediction such as for 30 minutes [20]. The initial wind power is assumed to persist in the first 30 minutes in this study. The wind power model is based upon the historic hourly wind speed data. The hourly conditional wind power model is used to quantify the wind power variability for the interval of one hour spanning 30 minutes before and after the hour. If, for instance, wind data for each 15 minutes was available, the interval would be reduced to 15 minutes spanning 7.5 minutes before and after the time, and the initial wind power would be assumed to persist only for the first 7.5 minutes using the persistence model. In this case, the number of risk intervals will increase from 5 to17. The method is, however, the same. The risk function of the committed units is therefore modified for the first half hour period by including the initial wind capacity as shown in Figure 4.4. The risk functions for the subsequent hourly periods are modified by convolving the conditional wind power distributions obtained at each of the hourly intervals within the lead time. For example, the one-hour ahead wind power distribution includes the wind power variability for the period between 0.5 hour to 1.5 hour. The load is assumed to be constant for the entire lead time and the UCR evaluated for each period is given by (4.3)-(4.8). The risk for the first 30 minutes period, in which the initial power (WP_0) is assumed to persist, is presented in (4.5). The risk for each of the other hourly sub-periods are given in (4.6)- (4.9) and the total risk for the period is given in (4.10).

$$A_{(0-0.5)} = R_{0.5,\ WP_0} \qquad (4.5)$$

$$A_{(0.5-1.5)} = R_{1.5,\ WP_{0.5-1.5}} - R_{0.5,\ WP_{0.5-1.5}} \qquad (4.6)$$

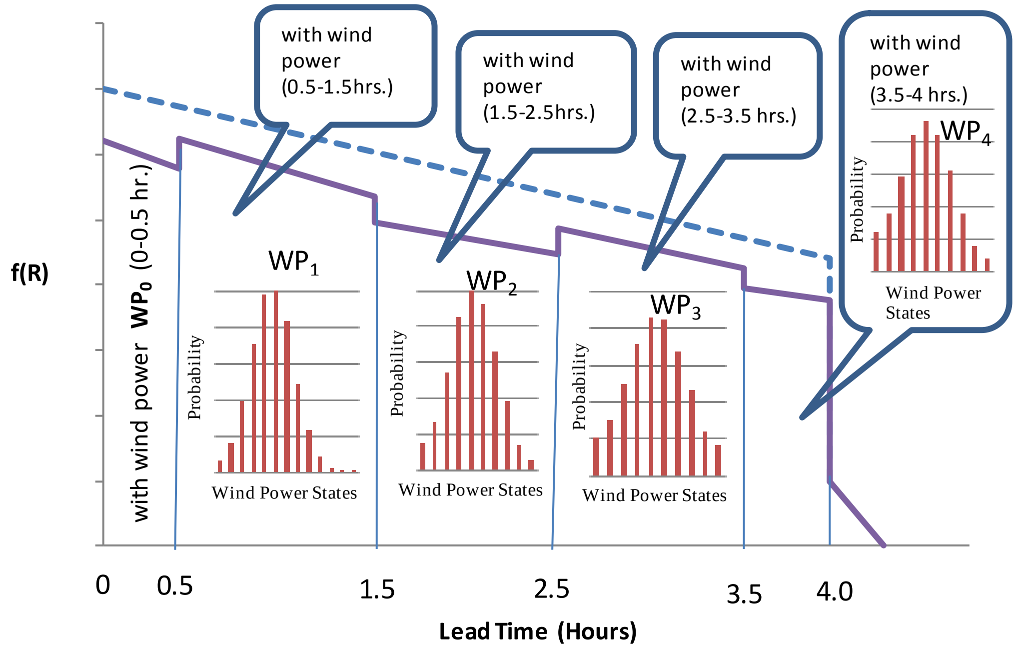

Figure 4.4: Area risk concept to incorporate wind power

$$A_{(1.5\ -\ 2.5)} = R_{(2.5,\ WP_{1.5-2.5})} - R_{(1.5,\ WP_{1.5-2.5})} \tag{4. 7}$$

$$A_{(2.5\ -\ 3.5)} = R_{(3.5,\ WP_{2.5-3.5})} - R_{(2.5,\ WP_{2.5-3.5})} \tag{4. 8}$$

$$A_{(3.5\ -\ 4.0)} = R_{(4.5,\ WP_{3.5-4.0})} - R_{(3.5,\ WP_{3.5-4.0})} \tag{4. 9}$$

$$UCR_{(0\ -\ 4)} = A_{(0\ -\ 0.5)} + A_{(0.5\ -\ 1.5)} + A_{(1.5\ -\ 2.5)} + A_{(2.5\ -\ 3.0)} + A_{(3.5\ -\ 4.0)} \tag{4. 10}$$

The risk evaluated for each sub period can be expressed in general using (4.11).

$$A_{(t\ -\ t+\Delta t)} = R_{(t+\Delta t,\ WP_{t\ -\ t+\Delta t})} - R_{(t,\ WP_{t\ -\ t+\Delta t})} \tag{4. 11}$$

Where:

R_t is the partial risk obtained from the COPT of the committed units developed for the mission time t;

$WP_{t-t+\Delta t}$ is the wind power variability for the time interval of (t, t+Δt).

$R_{(t,\, WP\, t-t+\Delta t)}$ is the partial risk obtained from the COPT modified by combining the hourly wind capacity model obtained for the period (t, t+Δt) for the given initial condition and;

$A_{(t-\, t+\Delta t)}$ is the area risk for the period t.

As shown in Figure 4.4, WP_1, WP_2, WP3 and WP_4 are the wind power variability for the time intervals of (0.5, 1.5) hour, (1.5, 2.5) hour, (2.5, 3.5) hour and (3.5, 4.5) respectively. Each of the partial risks in (4.5)- (4.9) is obtained from a combined COPT which is obtained by combining the committed COPT for the mission time with the hourly wind power capacity states and their probabilities for the specific period. For instance, the partial risk $R_{(2.5,\, WP1.5\text{-}2.5\,)}$ in (4.7) is obtained from the combination of the two- hours ahead wind power distribution and the COPT of the conventional units for which the ORR is evaluated for a mission of 2.5 hours.

4.5. UCR Analysis

The generation system considered in this study utilizes the data of the IEEE Reliability Test System (RTS) [29] and a 300 MW wind farm located at a site represented by the Toronto wind data. The short term wind speed variability considering diurnal rising and falling wind trend was presented in Chapter 2. Figure 4.5 presents the basic statistics of the historic hourly wind speed, for a span of Hour 6 to Hour 54, considering Toronto wind data. A rising wind trend occurs between Hour 8 and Hour 12, a falling wind trend between Hour 20 and Hour 24 and a relatively flat wind speed between Hour 44 and Hour 48. These three characteristics of rising, falling and flat wind speed are investigated using different wind scenario studies in the following sections. Two different scenarios of load and conventional generation are considered. The first scenario has a total of 22 units of IEEE RTS in its priority loading order committed with a total capacity of 3177 MW and is designated as the high load scenario. The second scenario represents a relatively low load condition with only 11 units committed with a total capacity of 2096 MW and is designated as the low load scenario. In both cases there are two 400 MW units as the largest units. The capacity of the smallest unit is 12 MW in the high load scenario and is 50 MW in the second scenario. A lead time of 4 hours is considered in the UCR analysis. The initial wind capacity condition at the start of the lead time is known and two initial wind power cases of 90 MW and 180 MW are considered.

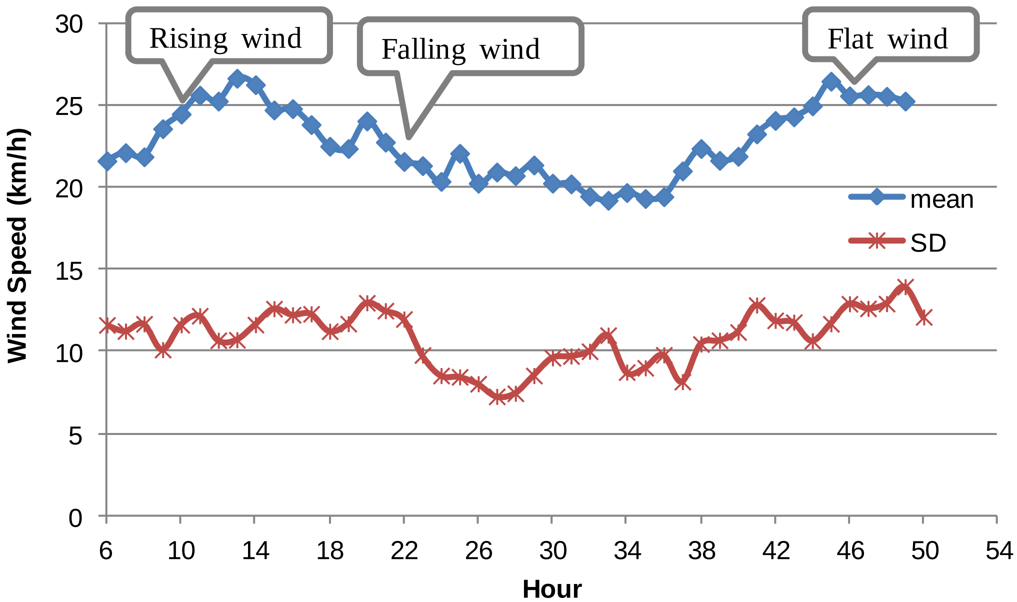

Figure 4.5: Historic hourly wind speed (mean and standard deviation) showing diurnal variation trends

Figure 4.6 shows the reduction in the period risks due to wind power when the conventional units are committed for the high load scenario at a load level of 2770 MW. The initial wind power is 90 MW which is 30% of the rated capacity. The study evaluates the impacts of the rising and falling wind trends observed in Figure 4.5. The reduction in risk is due to the additional capacity available from the wind in the different periods. The period risk evaluations show that the contribution from wind power varies in different periods and is mainly governed by the wind power variability in the period. This is the essence of utilizing the area risk method.

It can be seen from Figure 4.6 that the reductions in risk due to the rising and falling wind trends are significantly different in the third period and in succeeding periods.

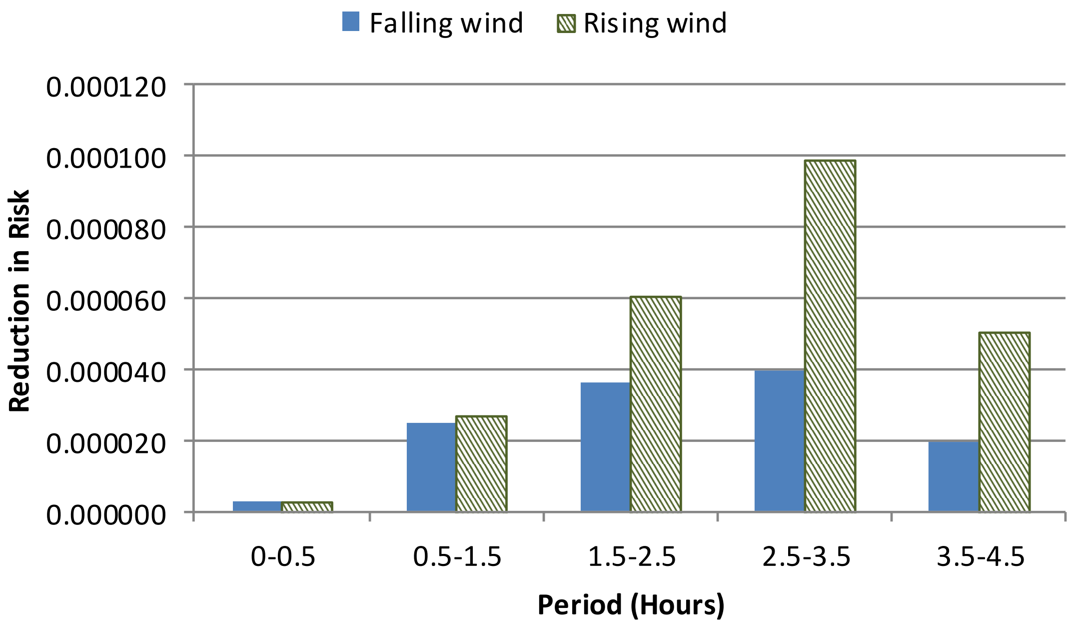

Figure 4.6: Contribution of wind power in period risks

The UCR evaluated from the area risk method is compared with the UCR evaluated using the reference method presented in [107]. In the reference method, a single conditional wind power probability distribution is created from the wind data simulated at the end of the lead time. The COPT created for the conventional units is then combined with the wind power probability distribution to create the combined COPT for the entire lead time. The unit commitment risk is given by the cumulative probability of the capacity-in state which is just equal to or less than the expected load. Table 4.1 presents the UCR evaluated for the three wind trends and for a range of loads between 2600 MW and 2850 MW. It can be observed that the UCR evaluated using the area risk method is lower than the UCR evaluated using the reference method for all three wind trends and the load levels considered. The dispersion of the conditional wind power distribution increases as the lead time is increased indicating the increased variability. The risk calculated is increased if the dispersion of the wind power distribution increases. The reference method uses the 4 hour ahead wind power distribution for the entire 4 hour lead time while the proposed method uses each hourly wind power distribution and incorporates them at the appropriate intervals. The 4 hour ahead wind power distribution is considered for the interval of 3.5-4.0 hours only. This is the main reason why the risks evaluated using the proposed method are lower than those from the reference method.

Table 4.1: UCR evaluated using the two methods: initial wind power = 90 MW and conventional capacity = 3177 MW

Load, MW	Rising Wind		Falling Wind		Flat Wind	
	Method1 $x10^3$	Method2 $x10^3$	Method1 $x10^3$	Method2 $x10^3$	Method1 $x10^3$	Method2 $x10^3$
2600	0.04854	0.05257	0.07379	0.08820	0.04933	0.05424
2625	0.07005	0.07256	0.10362	0.12341	0.06877	0.07588
2650	0.09868	0.11164	0.16406	0.19200	0.10354	0.11555
2675	0.13650	0.13861	0.20832	0.25489	0.12918	0.14132
2700	0.18053	0.20229	0.28558	0.32855	0.18806	0.20181
2725	0.25097	0.26433	0.35197	0.39928	0.24331	0.25943
2750	0.29484	0.31838	0.42603	0.47134	0.29867	0.31188
2775	0.38234	0.40207	0.49503	0.53295	0.37561	0.38821
2800	1.06001	1.53546	2.37883	3.78860	1.18795	1.80684
2825	2.13987	2.60044	3.60026	4.99953	2.00305	2.74252
2850	3.30889	4.41335	5.70560	7.64530	3.44412	4.47276

Method 1: Proposed area risk method Method 2: Reference method

Table 4.2 shows the UCR evaluated when the initial wind power is 180 MW, which is 60 % of the wind farm capacity. It can be seen from the two tables that the differences in the results for the two methods are higher for 180 MW of initial power compared to those for 90 MW of initial power. It is also worth noting that the difference is higher during a falling wind trend compared to the other trends for both initial cases of wind power. It can be seen from Figure 4.6

that the reductions in risk due to the added wind power are higher at the front and middle part of the lead time compared to the end part of the lead time for the falling trend. This cannot be accurately incorporated by the single four hour ahead wind power distribution and such a method results in a higher risk compared to that from the proposed method. The analysis is also performed for the low load scenario where 11 RTS units totaling a capacity of 2096 MW are committed for a four hour lead time. Table 4.2 shows the UCR evaluated considering the same wind trends for the initial power of 180 MW from the wind farm. The UCR evaluated from the area risk method is lower compared to that for the reference method in this case as well.

Table 4.2: UCR evaluated using the two methods: (initial wind power = 180 MW and conventional capacity = 3177 MW)

Load, MW	Rising wind		Falling wind		Flat wind	
	Method1 $x10^3$	Method2 $x10^3$	Method1 $x10^3$	Method2 $x10^3$	Method1 $x10^3$	Method2 $x10^3$
2600	0.03113	0.03631	0.05142	0.07006	0.02977	0.03479
2625	0.04196	0.05062	0.07403	0.09436	0.04179	0.05349
2650	0.05727	0.07218	0.11415	0.14649	0.05317	0.06584
2675	0.07466	0.08943	0.15562	0.21072	0.07839	0.09935
2700	0.10852	0.13459	0.21920	0.27670	0.09802	0.11442
2725	0.14290	0.18572	0.26452	0.34865	0.14110	0.17481
2750	0.18935	0.22263	0.33987	0.41163	0.16854	0.19303
2775	0.24338	0.30451	0.39441	0.48221	0.22634	0.27147
2800	0.46154	0.76979	1.22304	2.44202	0.44851	0.88121
2825	0.78528	1.40329	2.22007	3.64432	0.89908	1.68822
2850	1.42711	2.54016	3.32827	5.80656	1.08560	2.02257

4.6. Operating Capacity Credit of Wind Power

The load carrying capability (LCC) increases as wind power is added to the conventional units. This section evaluates, using both methods, the increase in load carrying capability (ILCC) due to the wind power for two UCR criteria (UCRC) of 0.0001 and 0.001. These criteria are designated as low and high UCRC respectively in the following discussion. The LCC of the 22 committed units in the high load scenario without considering wind power is 2579 MW and 2776 MW for the low and high UCRC respectively.

Table 4.3: UCR evaluated using the two methods: initial wind power = 180 MW and conventional capacity = 2096 MW

Load, MW	Rising wind		Falling wind		Flat wind	
	Method1 $x10^3$	Method2 $x10^3$	Method1 $x10^3$	Method2 $x10^3$	Method1 $x10^3$	Method2 $x10^3$
1500	0.02451	0.02807	0.03629	0.04118	0.02139	0.02511
1525	0.03005	0.03329	0.05333	0.06470	0.02786	0.03208
1550	0.03984	0.04711	0.07147	0.08604	0.03947	0.04782
1575	0.05332	0.06251	0.09949	0.13447	0.04614	0.05463
1600	0.07049	0.08847	0.12062	0.15897	0.06614	0.07881
1625	0.08735	0.09857	0.14834	0.18158	0.07751	0.08778
1650	0.11280	0.13183	0.16681	0.20216	0.10160	0.11852
1675	0.12910	0.13702	0.19086	0.22132	0.11848	0.12950
1700	0.17754	0.21983	0.37387	0.74842	0.20633	0.39231
1725	0.35478	0.63860	1.19803	2.24924	0.40045	0.78146
1750	0.74731	1.26351	2.26415	4.89115	0.85756	1.67916

Figure 4.7 and Figure 4.8 show the ILCC for different wind conditions for the high load scenario satisfying the high and low UCRC. The ILCC of the committed units is obviously lower at the low UCRC compared to that for the high UCRC. The ILCC due to wind power is, however, higher for the low UCRC compared to the high UCRC for all of the wind power cases considered. The ILCC varies with the wind trends and initial wind power conditions for a selected UCRC. The ILCC during a rising wind trend is higher compared to that for a falling

wind trend. The ILCC evaluated from the area risk method is consistently higher when compared to the ILCC from the reference method for all the cases considered in this study.

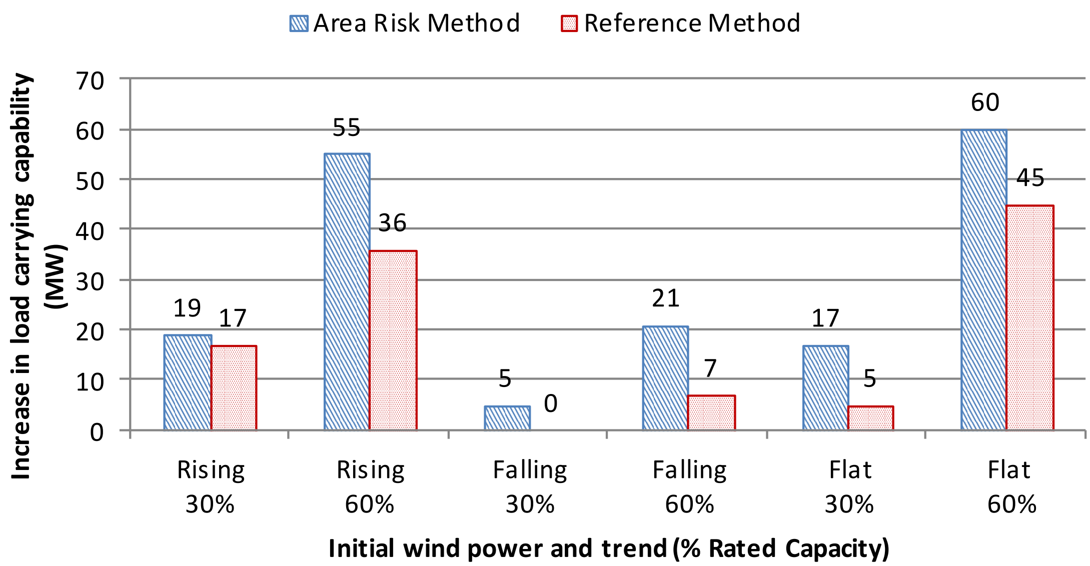

Figure 4.7: ILCC due to wind power: Conventional capacity = 3177 MW, UCRC = 0.001.

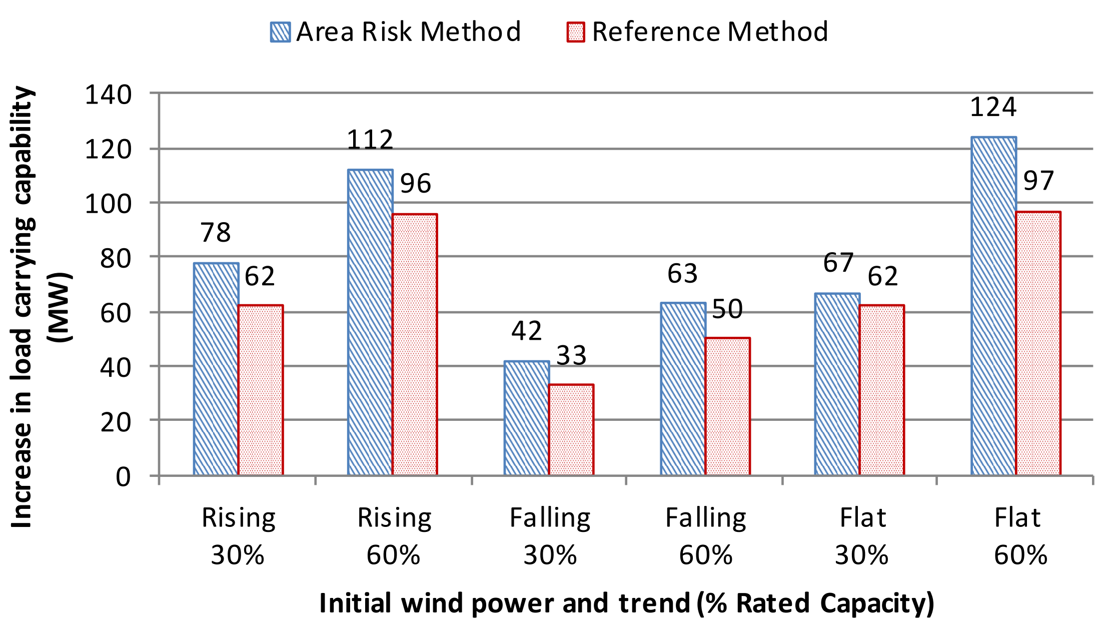

Figure 4.8: ILCC due to wind power: conventional capacity = 3177 MW, UCRC = 0.0001.

The capacity benefit of wind power is expressed as a percentage of the wind farm capacity and is designated as the operating capacity credit (OCC) [72]. Figure 4.9 presents the wind power OCC evaluated for an initial wind capacity of 180 MW added to 11 units of the RTS (i.e. low load scenario) for a lead time of four hours. The committed conventional units without considering wind power can carry a maximum load of 1497 MW and 1695 MW for the low and high UCRC respectively. It can be seen that the OCC of the wind farm depends upon the selected UCRC. The OCC evaluated using the area risk method is consistently higher than the OCC assessed using the reference method.

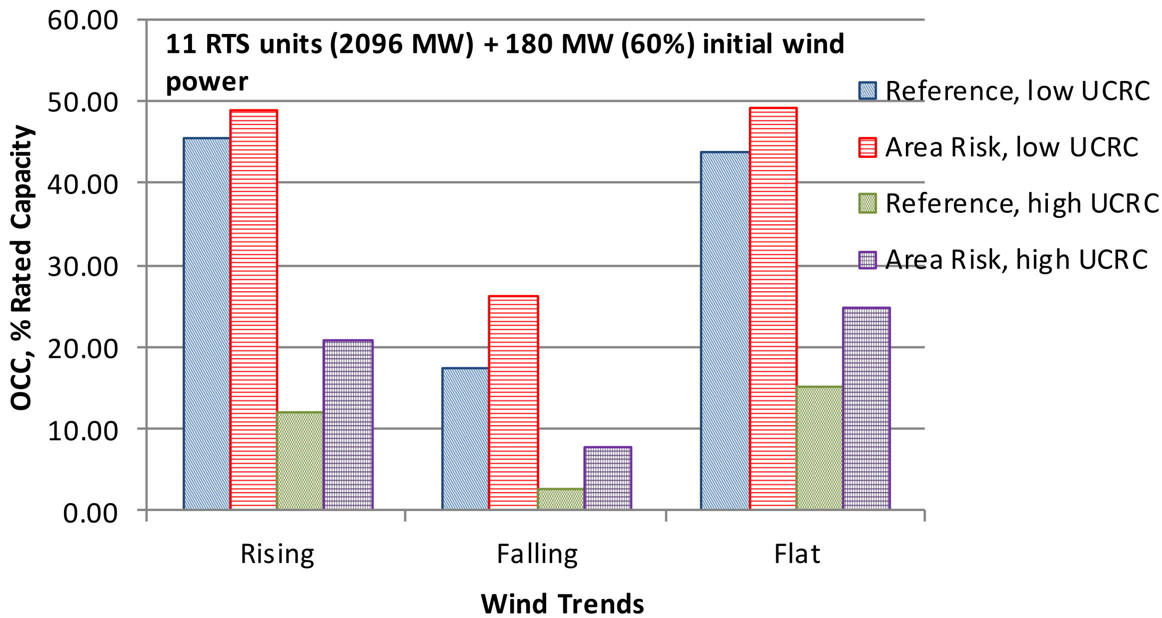

Figure 4.9: OCC of wind power: Conventional capacity = 2096 MW

The reliability of a conventional unit decreases as the lead time is increased. Repairs within the short lead times are considered to be not possible and the uncertainty associated with the committed conventional unit is quantified by the outage replacement rate (ORR) which is the probability that the unit fails in the lead time. A single COPT created for the entire lead time can be used to evaluate the UCR if there is no wind power or any decision made to bring rapid start units or hot reserve units into operation. The UCR, however, of a wind integrated power system cannot be evaluated accurately using a single COPT as the wind power can increase and /or decrease within the lead time. The area risk approach utilized in this study includes wind power variability within the lead time and therefore provides a more accurate quantification of the reliability impact of wind power than the reference method. The area risk method is therefore applied in the following sections.

4.7. Unit Commitment Risk Analysis Considering Correlated Wind Farms

4.7.1. Wind power model of multiple wind farms

A study of generating correlated wind speed data was presented in Section 2.1.3 and risk analysis of committing wind power from correlated wind farms was presented in Section 3.6. The same wind power model is used for the unit commitment risk analysis in this section. The wind speeds for wind farms located in a same geographical terrain show some degree of correlation depending upon the distance between them. The two simulated wind data series using the correlated random numbers represent the two wind farms being studied. The wind power model of dependent wind farms will have the same probability distribution as that of the single wind farm but with increased capacity states. The initial condition at both the correlated wind sites are known and the combined wind power probability distribution for a short future time is obtained by adding wind power, at the future lead time, at each site conditional on the initial wind power. One hour ahead conditional probability distributions of the total power from the two wind sites are presented in Figure 4.10 for dependent and correlated wind sites with a correlation coefficient of 0.75. The initial wind power from each wind farm at Hour 20, considered in Figure

4.10, is 30% of the rated capacity. The discrete probability distributions show that the probabilities of the capacity states associated with 0% and 5% are lower for the correlated case compared to that of the dependent wind sites. The probabilities associated with the capacity states from 10% to 50% are higher for the correlated wind sites compared to that of the dependent wind farm. The dependent wind site has higher probabilities associated with the capacity states above 50% compared to that of the correlated wind sites. The capacity states greater than 50% have relatively low probability of occurrence and will have a less significant impact upon the results.

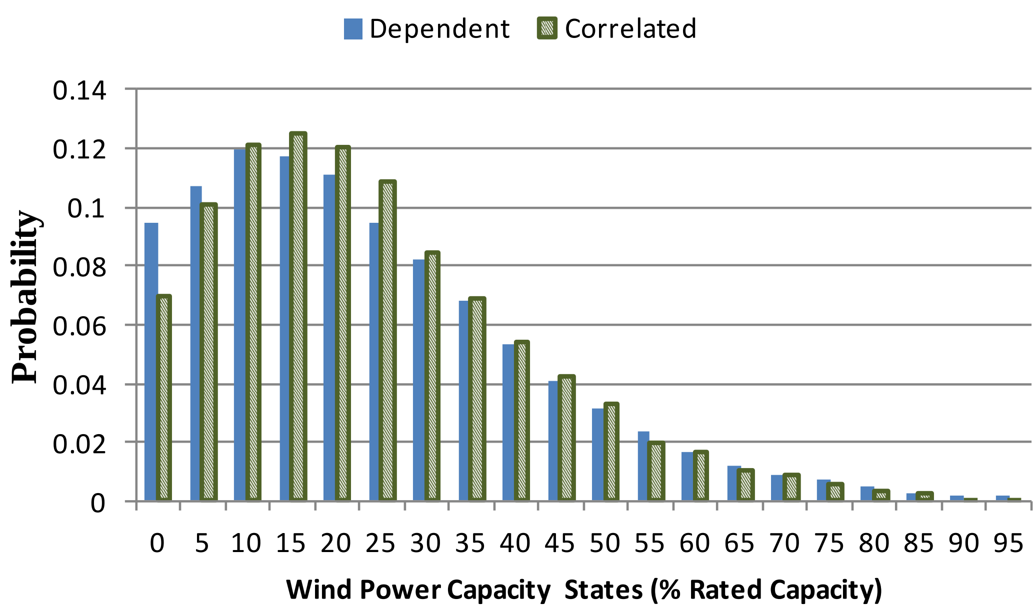

Figure 4.10: One hour ahead capacity states and their probabilities conditional to an initial power of 30% of the rated capacity at Hour 20

The following study utilizes the same wind site data to examine the effect of adding two dependent, independent and correlated wind farms each rated at 150 MW. The short term wind power models of the independent wind farms are individually combined with the COPT of the conventional units.

4.7.2. Results of Unit Commitment Risk Analysis

The IEEE Reliability Test System (RTS) [29] is used in this study to illustrate the impact on unit commitment risk of adding independent, dependent and correlated wind farms. A high load scenario (HLS) and a low load scenario (LLS) are considered where the number of the committed units from the priority loading order of IEEE RTS is 22 and 11 respectively. The rated capacity of the largest and the smallest committed units in the HLS are 400 MW and 12 MW respectively with a total conventional capacity of 3177 MW. The LLS has the same largest unit as the HLS but the rated capacity of the smallest committed units unit is 50 MW giving a total conventional capacity of 2096 MW. The rated capacity of each wind farm added to the RTS is 150 MW.

The wind speed data for Toronto is used for both wind farms. The initial condition is a 45 MW wind power output from each of the wind farm at Hour 20. The historic wind speed data at Toronto shows a falling wind trend at Hour 20 and system operators are usually more concerned during a falling wind trend compared to different conditions at other times of the day. Wind farms are modeled as multi-state capacity units and their probabilities. The state probabilities are the same for the two dependent wind farms but the capacity states are simply multiplied by a wind power expansion factor of two in this case. The capacity states and the probabilities of the two individual wind farms are combined separately with the COPT of the conventional units to obtain the independent combination. As discussed in Section 2.8 and Section 4.7.1, wind speed data for the two wind sites with a wind speed correlation coefficient of 0.75 are created. The combined wind power probability distribution at for the lead times of 1, 2, 3 and 4 hours are created for an initial power of 45 MW (30%) at Hour 20 from each wind farm.

Table 4.4 presents the UCR evaluated for the lead time of 4 hours when 22 RTS units are committed and two wind farms each rated at 150 MW are added to the system considering an initial wind power of 45 MW (30 %) each at Hour 20. Table 4.5 presents the UCR for load levels ranging from 2650 MW to 2790 MW using the step of 20 MW. It can be seen that for all load levels, the UCR is reduced due to the added wind power. The UCR is the lowest for the independent wind farms while it is the highest for the dependent wind farm. The risks for the correlated wind farms lie between the two boundaries of the dependent and independent wind farms. Table 4.5 presents the UCR evaluated for the low load scenario (LLS) for the same wind power conditions. It can be seen that the risk follows the same pattern as that in the high load scenario (HLS).

Table 4.4: Unit commitment risk for the high load scenario

Load	UCR $\times 10^3$			
	TABLE I. Dependent	TABLE II. Correlated	TABLE III. Independent	TABLE IV. No Wind
2650	0.16406	0.15979	0.14726	0.29522
2670	0.19862	0.19818	0.19812	0.29736
2690	0.25932	0.25766	0.24758	0.42152
2710	0.30922	0.31593	0.30503	0.47795
2730	0.37040	0.36683	0.36968	0.56833
2750	0.42603	0.43060	0.42402	0.57041
2770	0.48407	0.48137	0.47979	0.60744
2790	1.46842	1.48933	0.88070	7.5039

The load carrying capability (LCC) of the committed units while satisfying the UCR criteria of 0.0001 and 0.001 are presented in Table 4.6. These risk criteria are designated as UCRC1 and UCRC2 respectively. It can be seen that if the system is prepared to accept a high risk of 0.001, it can carry a higher load compared to that at a low risk criterion of 0.0001. As can be seen from Tables 4.4 and 4.5, the reduction in risk due to the wind power increases the load carrying capability in most of the cases. The LCC of independent wind farms are the highest and those of the dependent wind farms are the lowest. The LCC due to the correlated wind farms lies between that due the dependent and independent wind farms. The increase in load carrying capability (ILCC) due to wind power is usually expressed as a percentage of the total installed wind capacity and is called the operating capacity credit (OCC). The OCC of the wind power are presented in Figure 4.11. It can be seen that the OCC is higher at the low UCR criterion compared to that at a high UCR criterion. The OCC is also seen to be higher for the low load scenario compared to that for the high load scenario.

Table 4.5: Unit commitment risk for the low load scenario

Load,MW	UCR $\times 10^3$			
	Dependent	Correlated	Independent	No Wind
1550	0.11129	0.09536	0.08710	0.20275
1570	0.13411	0.12946	0.12250	0.20275
1590	0.15969	0.15608	0.14882	0.20275
1610	0.17803	0.1752	0.17063	0.21703
1630	0.18921	0.19141	0.19090	0.21703
1650	0.21472	0.20225	0.20216	0.27407
1670	0.23027	0.23017	0.21872	0.27449
1690	0.24279	0.24342	0.24171	0.27449

Table 4.6: Load carrying capability incorporating wind power

Load Level and UCRC	Load Carrying Capability (LCC), MW			
	Dependent	Correlated	Independent	No Wind
HLS, UCRC1	2621.9	2628.9	2629.9	2579.9
HLS, UCRC2	2781.9	2783.9	2793.9	2776.9
LLS, UCRC1	1543.9	1550.9	1557.9	1498.9
LLS, UCRC2	1695.9	1702.9	1712.9	1695.9

UCRC1 = 0.0001, UCRC2 = 0.001

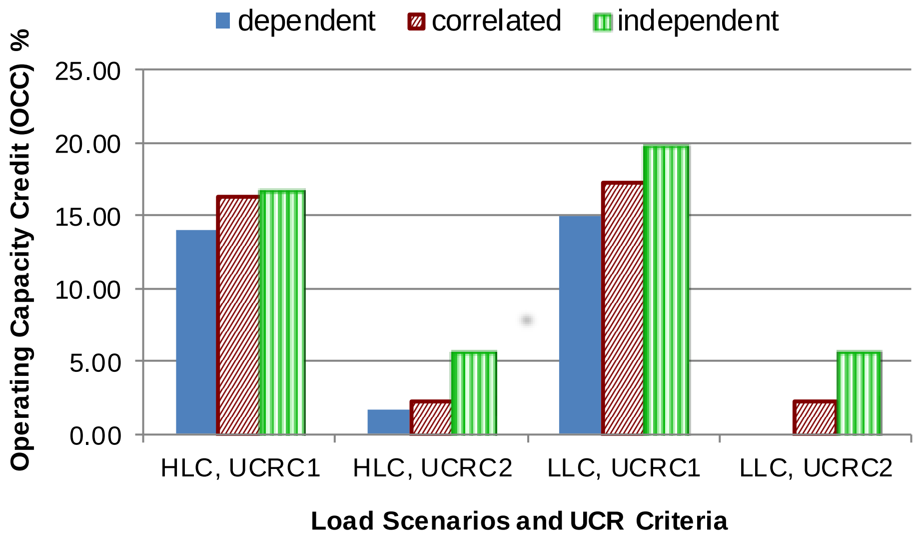

Figure 4.11. Operating capacity credit (OCC) of wind power

4.8. Unit Commitment Well-Being Analysis Incorporating Wind Power

Power system operation can be described by the five operating states designated as normal, alert, emergency, extreme emergency and restorative [152] as shown in Figure 4.12. In the normal operating state, the system generation has adequate spinning reserve and any single contingency can be tolerated. The system enters into an alert state when there is a generation outage or a load change such that the system will still be able to serve the demand but lacks adequate spinning reserve to withstand a further contingency. In an emergency state, the generation is exactly equal to the load, and in the absence of the appropriate corrective action, the system can enter into an extreme emergency state where an operating constraint is violated and some portion of the load is curtailed.

The most common deterministic criterion is to utilize a reserve margin equal to the largest operating generating unit or a certain percentage of the peak load. Despite the fact that deterministic methods can not consider the stochastic nature of component failures and load changes, utilities have been reluctant to use a probabilistic approach in assessing the operating reserve mainly due to difficulty in interpreting the resulting numerical indices. The PJM method [13] considers only two states; the comfort state where the operating capacity is greater than the load and the at risk state where the operating capacity is equal to or less than the load. System well-being analysis [32] extends the two states in the PJM method to three states designated as healthy, marginal and at risk as shown in Figure 4.13.

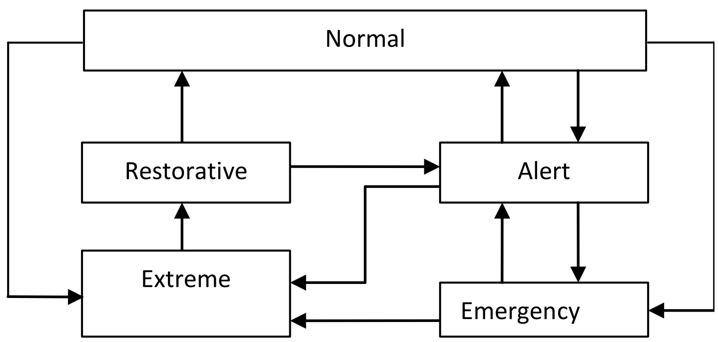

Figure 4.12: Power system operating state diagram

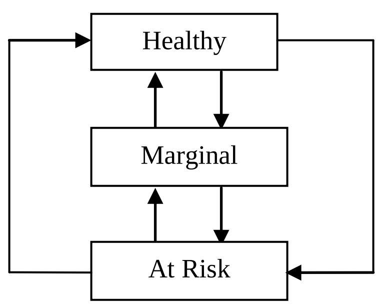

Figure 4.13: System wellbeing analysis model

The system operates in the healthy zone if it has sufficient reserve to satisfy the deterministic criterion and is identical to the normal state. In the marginal zone, there is insufficient reserve to satisfy the deterministic criterion and is identical to the alert state. The system is in the "at risk" state (emergency and extreme emergency) when the operating capacity is just equal to or less than the load. The healthy and marginal states both lie in the comfort zone of the PJM method [32] whereas the probability of the "at risk" state is identical to the UCR. This method is superior to the PJM method for two main reasons; firstly, it provides a probabilistic measure of system wellbeing based upon the accepted deterministic criterion and secondly, it gives a warning to the system operator to start up additional units if the system is operating with a high probability in the marginal state [32]. The operating health analysis of a generating system is extended to consider stand-by units, interruptible load and postponable outages in [37]. Unit commitment health analysis and composite system health analysis is presented in [153]. Reference [38] presents the system well-being approach for spinning reserve allocation.

Reference [33] presents a simplified method based on conditional probability that reduces the computation time. An approximate method is illustrated in [154] where the COPT is modified by excluding the single largest unit to evaluate the system health index. The concept outlined in [154] is further modified in this study to evaluate the system well-being indices using the area risk approach. If the load is greater or just equal to the operating capacity without considering the largest committed unit, the system loses its state of health. The probability of just satisfying or failing to satisfy the load without its largest committed unit is, therefore, designated as the loss of health probability (LOHP) and is given in (4.11). The area risk concept presented in the previous section is used to evaluate the partial LOHP and period LOHP considering wind power. The LOHP for the entire lead time is the sum of the period LOHP. The compliment of this value is the probability of health, P(h), as shown in (4.12). The area risk method provides an accurate assessment of the impact of wind power on the LOHP by incorporating the hourly variability of the wind in the appropriate periods.

The probability of risk is the UCR evaluated using the combined COPT with all the committed units and the wind power. The sum of the probabilities of being in the healthy, marginal and at risk states is unity. The health and risk probabilities are used to determine the margin state probability. The probability of system risk and margin are presented in (4.13) and (4.14) respectively.

$$LOHP = CP_{i-\mathrm{mod}\,ified} \;:\; Cap_{i-\mathrm{mod}\,ified} \le Load \tag{4. 12}$$

Where:

$CP_{i\text{-modified}}$ is the cumulative probability and $Cap_{i\text{-modified}}$ is the corresponding capacity-in of the ith state of the modified COPT.

$$P(h) = 1 - LOHP \tag{4. 13}$$

$$UCR = CP_i \;:\; Cap_i \le Load \tag{4. 14}$$

$$P(m) = LOHP - UCR = 1 - P(h) - UCR \tag{4. 15}$$

The above well-being indices were evaluated for the high load scenario (i.e. 22 RTS units committed) considering a flat wind trend using two initial wind power conditions of 90 MW and 180 MW. The health index is evaluated using the COPT modified by excluding the largest committed unit. Figure 4.14 presents plots of the health index on the primary ordinate and risk index on the secondary ordinate evaluated for a lead time of 4 hours at different load levels. As expected, the probability of health decreases while the probability of risk increases with increase in load. The impact of the initial wind power can be observed from the separation of the plots where the health index plot with higher initial wind power lies above the plot with lower initial wind power. The risk plot with higher initial wind power lies below the one with lower initial wind power. The well-being indices were also evaluated for rising and falling wind trends, and are presented in Table 4.7 and Table 4.8 for the initial wind power conditions of 90 MW and 180 MW respectively.

Table 4.7 and Table 4.8 show that the health indices are higher in the case of the rising wind trend compared to that in the case of the falling wind trend. The health index decreases while the margin and risk indices increase as load is increased. The system well-being improves with the addition of wind power and the contribution of wind power during the rising trend is higher than that during the falling wind trend.

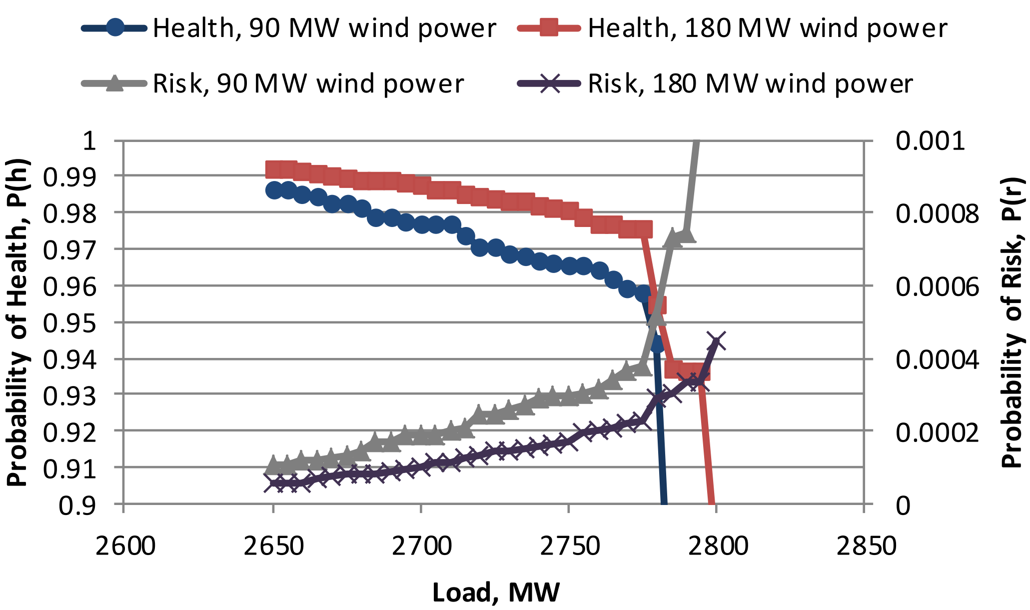

Figure 4.14: Health and risk indices incorporating wind power: flat wind trend

4.9. Dual Criteria Analysis

A system well-being is recognized by a high probability of being in the "healthy" state and a low probability of being in the "at risk" state. When a single operating criterion, such as the UCR is considered, unit commitment is continuously performed to satisfy the specified risk criterion. As the load level or wind power generation changes, the system may be exposed to a relatively low health probability while it continues to satisfy the specified risk criterion. On the other hand, if the units are committed based on a specified health probability criterion, it may be exposed to a relatively high risk at a particular operating condition. Unit commitment based on a single well-being criterion may not ensure system comfort at all operating conditions. A dual criterion involves satisfying both the health and the risk criteria simultaneously, and ensures a required comfort level of system operation.

Table 4.7: Health, margin and risk indices incorporating wind power: 3177 MW conventional capacity +90 MW initial wind power (30%)

Load, MW	Rising Trend			Falling Trend		
	Ph	Pm	Pr	Ph	Pm	Pr
2650	0.98628	0.01362	0.00010	0.98043	0.01941	0.00016
2660	0.98545	0.01444	0.00011	0.97837	0.02144	0.00019
2670	0.98190	0.01797	0.00013	0.97593	0.02387	0.00020
2680	0.98013	0.01972	0.00015	0.97412	0.02565	0.00023
2690	0.97802	0.02181	0.00017	0.97127	0.02847	0.00026
2700	0.97694	0.02288	0.00018	0.96893	0.03078	0.00029
2710	0.97560	0.02420	0.00020	0.96766	0.03203	0.00031
2720	0.96932	0.03044	0.00024	0.96176	0.03789	0.00035
2730	0.96817	0.03157	0.00026	0.95963	0.03999	0.00037
2740	0.96673	0.03299	0.00028	0.95723	0.04237	0.00040
2750	0.96454	0.03517	0.00029	0.95528	0.04429	0.00043
2760	0.96211	0.03756	0.00033	0.95417	0.04538	0.00044
2770	0.95799	0.04164	0.00037	0.95025	0.04927	0.00048
2780	0.93169	0.06779	0.00052	0.82750	0.17160	0.00090
2790	0.90266	0.09670	0.00064	0.66025	0.33828	0.00147
2800	0.80282	0.19612	0.00106	0.50133	0.49629	0.00238

Table 4.9 shows the LCC of the two unit commitment scenarios with 180 MW of initial wind power when using a dual criteria UCR and P(h) of 0.0001 and 0.99 respectively. It can be seen that the LCC satisfying the health criterion is close to the LCC satisfying the low UCRC.

The LCC and the wind power OCC for the single UCR criterion of 0.0001 are presented in Figure 4.8 and Figure 4.9 in the previous section. It can be seen from Table 4.9 that the LCC and the wind power OCC will change when the dual criteria is applied.

Table 4.8: Health, margin and risk indices incorporating wind power: 3177 MW conventional capacity +180 MW initial wind power (60%)

Load, MW	Rising Trend			Falling Trend		
	Ph	Pm	Pr	Ph	Pm	Pr
2650	0.99139	0.00855	0.00006	0.98598	0.00011	0.00011
2660	0.99052	0.00942	0.00006	0.98485	0.00013	0.00013
2670	0.98967	0.01026	0.00007	0.98289	0.00015	0.00015
2680	0.98786	0.01205	0.00009	0.98206	0.00016	0.00016
2690	0.98709	0.01281	0.00010	0.98020	0.00018	0.00018
2700	0.98610	0.01379	0.00011	0.97721	0.00022	0.00022
2710	0.98475	0.01513	0.00012	0.97588	0.00024	0.00024
2720	0.98308	0.01678	0.00014	0.97498	0.00025	0.00025
2730	0.98083	0.01901	0.00016	0.97199	0.00028	0.00028
2740	0.98023	0.01960	0.00017	0.96933	0.00031	0.00031
2750	0.97841	0.02140	0.00019	0.96774	0.00034	0.00034
2760	0.97443	0.02535	0.00021	0.96398	0.00036	0.00036
2770	0.97287	0.02689	0.00024	0.96240	0.00038	0.00038
2780	0.96906	0.03066	0.00028	0.88546	0.00056	0.00056
2790	0.94261	0.05704	0.00035	0.80034	0.00085	0.00085
2800	0.90261	0.09693	0.00046	0.68255	0.00122	0.00122

It can be seen from Table 4.9, for the high load scenario, that the LCC is dominated by the risk criterion for the rising wind trend. The health criterion seems more stringent for the falling and flat wind trend compared to the risk criterion. The LCC satisfying the dual criteria during the low load scenario is dominated by the health criterion for the rising and the flat wind trend while it is dominated by the risk criterion for the falling wind trend as shown in Table 4.9. The system resides with a high margin state probability at the load levels in which the committed units meet the risk criterion but fail to satisfy the health criterion. This prompts the system operator to take necessary action such as starting up additional units in order to maintain the wellbeing of the supply.

Table 4.9: LCC of the RTS considering the wind power for a 4 hour lead time considering different single criteria

Initial condition		Maximum load carrying capability (MW)	
IEEE RTS	Initial wind power	UCRC = 0.0001	P(h) = 0.99
3177 MW	No Wind	2579	2577
	180MW Rising	2675	2676
	180 MW Falling	2642	2632
	180 MW Flat	2703	2694
2096 MW	No Wind	1497	1496
	180MW Rising	1634	1623
	180 MW Falling	1553	1568
	180 MW Flat	1645	1629

The wind power operating capacity credit (OCC) considering the dual criteria is presented in Figure 4.15. It can be seen that the wind power OCC tends to decrease while satisfying both the health and risk criteria compared to satisfying a single health or risk criterion. Dual criteria ensure an acceptable comfort level in system operation, but may often require increased operating capacity when compared to a single criterion method.

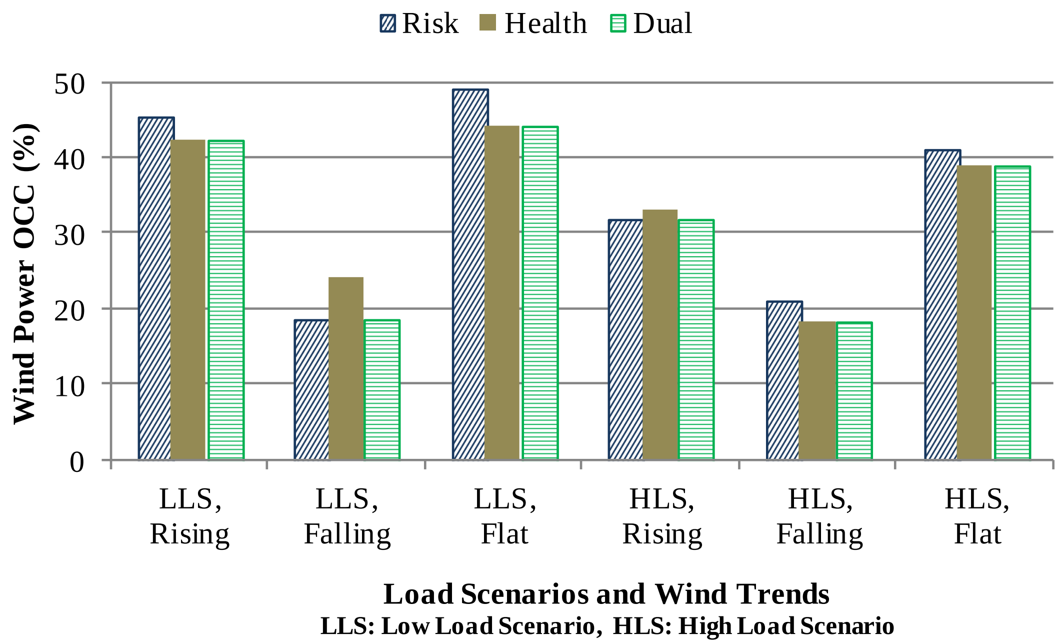

Figure 4.15: Wind power operating capacity credit considering the single and dual criteria: initial wind power = 180 MW

4.10. Summary

Wind power is a variable resource and the reliability contribution of wind power should be assessed recognizing this variability. The area risk method developed to consider rapid start units in UCR evaluation has been extended to consider wind power. The existing reference method considers only the wind power variability at the end of the lead time. The UCR evaluated using the area risk method is consistently different from that obtained using the reference method and the assessed OCC are also different. The PJM method for determining the spinning reserve requirement quantifies the uncertainties associated with the committed conventional units within the lead time. The area risk method as applied in this study combines the uncertainties of wind power at different periods within a lead time and is a more appropriate method. The method is more capable, compared to the reference method, of assessing the wind power contribution when wind power varies within a lead time with a rising or falling wind trend or when random wind power variations within the lead time are significant.

The reliability contribution of wind power is expressed as the increase in load carrying capability and it has been shown that the contribution is affected by the initial conditions and the reliability criterion selected. The contribution of wind power for a known initial condition and a selected reliability criterion is also affected by diurnal wind trends as a rising wind trend can offer a significantly higher OCC compared to a falling wind trend.

This study utilizes the area risk concept to evaluate the impact of adding wind farms. As expected, the operating capacity credit of wind power increases if the added wind farms are independent of each other. A method to incorporate correlated wind farms has been presented to quantify the reliability benefit of statistically correlated wind farms in system operation. The study shows that correlation is an important factor and needs to be considered rather than just assuming the wind sites to be independent or dependent. The study also shows that if capacity expansion of a wind farm is required, locating the added wind capacity at some distance reduces the dependency and diminish the degree of correlation resulting a higher operating capacity credit compared to that of dependent wind farms.

The area risk method developed for UCR evaluation has also been applied to evaluate the health index of a system incorporating wind power. The OCC considering the dual criteria of health and risk have also been evaluated and the results show that wind power OCC may be lower when satisfying a dual criterion and that the system well-being is affected by wind power diurnal trends.

The extension of the area risk method to incorporate wind power in unit commitment risk and health evaluations is the prime contribution of this chapter and the method is proposed as an appropriate technique to evaluate the reliability contribution of wind power in system operation.

CHAPTER 5

DEVELOPMENT OF A SIMPLIFIED METHOD TO INCORPORATE WIND POWER IN UNIT COMMITMENT RISK EVALUATION

5.1. Introduction

The short term wind power model developed and utilized in the previous chapters is based upon a conditional probability approach in which a conditional wind speed/wind power distribution is used to quantify uncertainties associated with wind power for short lead times of 1-4 hours. Chapter 4 presents the unit commitment risk analysis incorporating wind power in which an hourly wind power distribution conditional on the initial wind power is created for each hourly period within the considered lead time. An ARMA time series model was used to simulate the hourly wind speed data in this study. Development of an accurate ARMA model requires a significant amount of relevant historic wind speed data. This chapter focuses on the development of an approximate method, which utilizes the basic wind speed statistics in unit commitment risk evaluation incorporating wind power. The method looks into the relationship between the basic statistics of the hourly conditional wind speed distribution and the initial condition.

5.2. Study of the Sensitivity of the Basic Statistics of Short Term Wind Power with Initial Wind Speed

Figure 5.1 presents one hour ahead conditional wind speed distributions for four different initial wind speeds at the initial time (Hour 125). The wind speed data for Regina, Saskatchewan is used in this case and wind speed data simulation is conducted using the ARMA model presented in (2.8) and the annual hourly wind speed data obtained from Environment Canada. As the initial wind speed increases from 22 km/h to 38 km/h, the hour ahead distribution moves in Figure 5.1 from the left to the right in the direction of lower to higher wind speed. Table 5.1 shows the basic statistics of the wind speed probability distributions presented in Figure 5.1. It can be observed that the mean wind speed closely follows the initial wind speed. The standard deviation however is almost constant at all the initial conditions.

Figure 5.2 shows the wind speed probability distributions at four different lead times ranging from 1 to 4 hours conditional on the initial wind speed of 32 km/h at Hour 125. The shape of the distributions shown in Figure 5.1 and Figure 5.2 resemble normal distributions. The distributions peak at wind speeds close to the initial values but the dispersions increase as the lead time increases. This is also evident from the basic statistics presented in Table 5. 2. This work investigates the basic statistics of the hourly conditional wind speed distributions in order to observe their relationship with the initial speed.

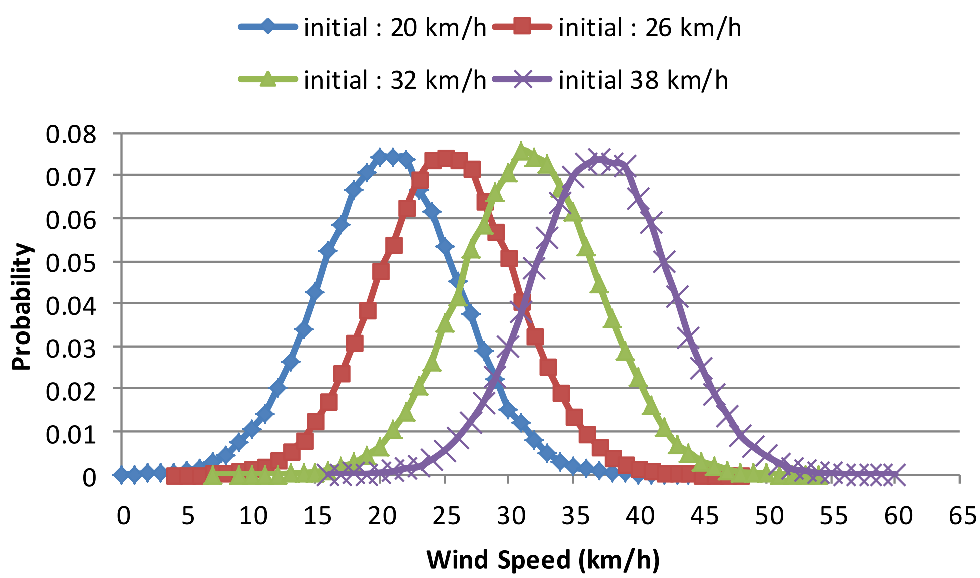

Figure 5.1: One hour ahead wind speed distributions for different wind speeds at hour 125, Regina Data

Table 5. 1: Basic wind speed statistics of Figure 5.1

Initial Speed (km/h)	Mean (km/h)	Std. Dev. (km/h)
20	20.64	5.34
26	26.15	5.33
32	31.60	5.34
38	37.12	5.30

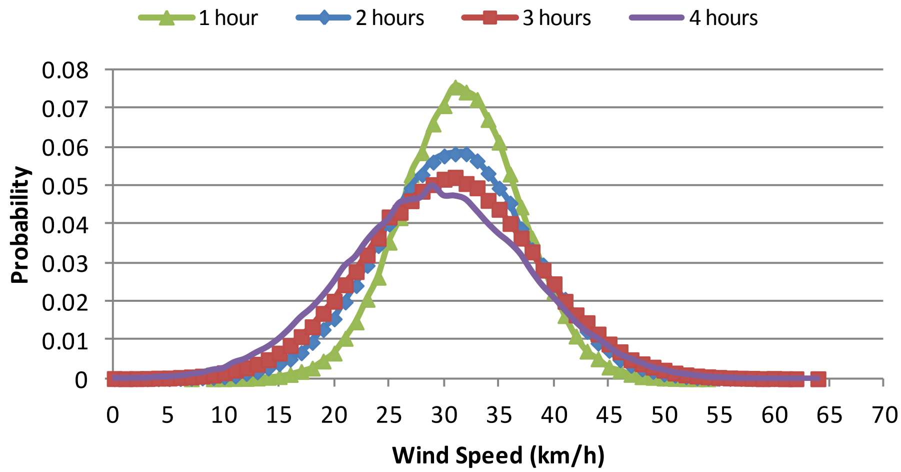

Figure 5.2: Wind speed distribution for different lead times for an initial wind speed of 32 km/h at Hour 125, Regina Data

Table 5.2: Basic wind speed statistics of Figure 5.2

Lead Time (hours)	Mean (km/h)	Std. Dev. (km/h)
1	31.60	5.34
2	31.00	6.84
3	30.54	7.69
4	29.44	8.26

The impacts of the diurnal wind trends on system risks are discussed in Chapter 3 and Chapter 4. The studies showed that the wind power variability at a short future time is influenced by the diurnal wind trends. The basic statistics of the conditional wind speed distribution is therefore different during a rising wind trend compared to that during a falling wind trend. The basic wind speed statistics of the conditional wind speed distribution for lead times of 1, 2, 3 and 4 hours are noted for different initial wind speeds. The mean wind speed of the conditional distribution is divided by the initial wind speed and the ratio is designated as the mean wind speed ratio (MWSR) in this study. Figure 5.3 shows the plots of the mean wind speed ratio for lead times of 1, 2 and 4 hours, considering Toronto wind data and the initial time as Hour 8. This time span represents a diurnal rising wind trend. The historic mean wind speed and the standard deviation at Hour 8 are 21.86 km/h and 11.69 km/h respectively. The initial wind speed is varied from a one -half standard deviation below the mean to two standard deviations above the mean. The initial wind speed on the abscissa is, therefore, expressed in terms of the historic mean and standard deviation at Hour 8. It can be seen from Figure 5.3 that the MWSR is close to unity and it decreases as the initial wind speed increases. Figure 5.4 shows the plot of the MWSR for the Regina wind site where the considered initial time is Hour 58 which also has a rising wind trend. The plots for the three different lead times are spread apart and indicate that the MWSR increases as the lead time increases. The spread of the plots shown in Figure 5.3 and Figure 5.4 illustrate the site specific nature of wind power variability.

The following analysis is focused on the wind speed relationships for a site located in Regina. The basic statistics of the historic hourly wind speed for the considered rising, falling and the flat wind trends are presented in Table 5. 3.

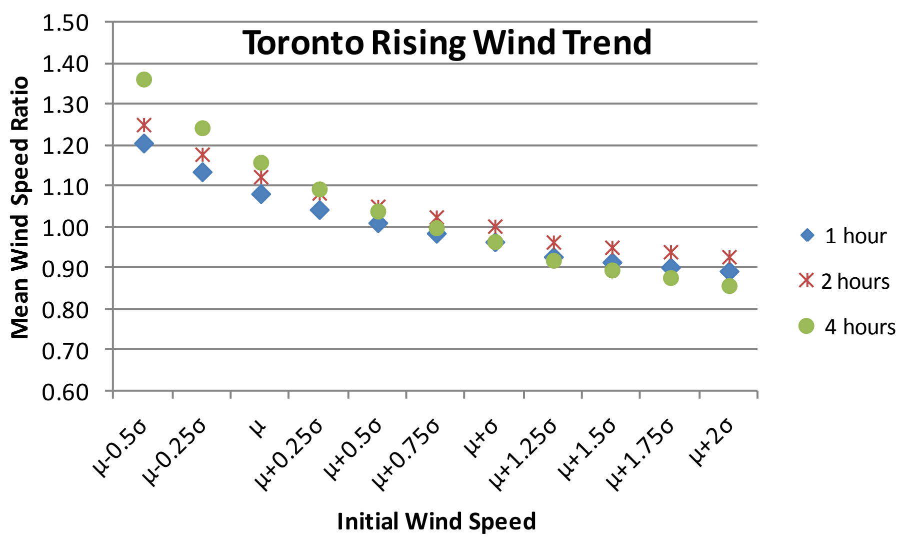

Figure 5.3: Variation of the mean wind speed ratio (MWSR) during a rising wind trend, Toronto data

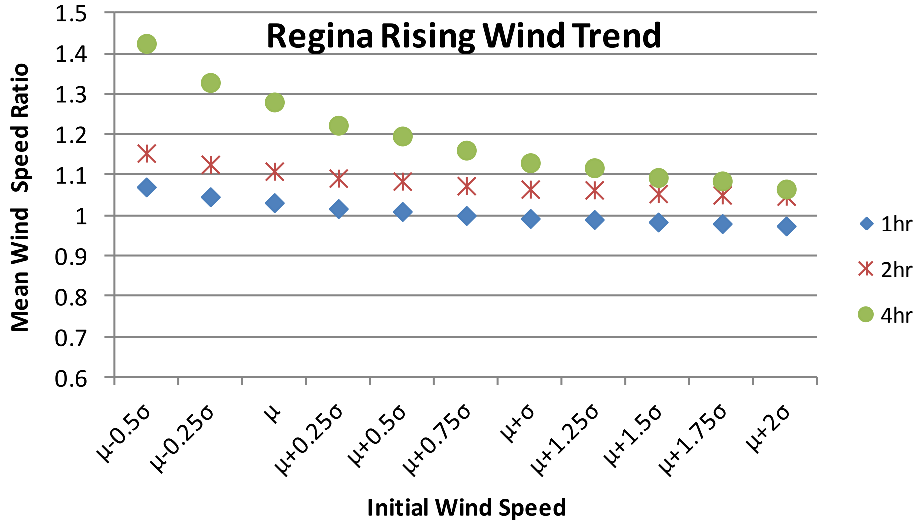

Figure 5.4: Variation of the mean wind speed ratio (MWSR) during a rising wind trend, Regina data

Table 5.3: Basic statistics of historic Regina wind speed data considering rising, falling and flat wind trends

Rising			Falling			Flat		
Hour	μ (km/h)	σ (km/h)	Hour	μ (km/h)	σ (km/h)	Hour	μ (km/h)	σ (km/h)
58	19.30	10.2	22	21.7	12.3	125	22.6	12.2
59	19.90	10.8	23	20.2	9.3	126	23	12.8
60	21.40	12.6	24	19.2	10.4	127	22.8	13.3
61	23.30	12.9	25	19	10	128	23	13.3
62	24.60	13.2	26	17.9	10.1	129	22.6	13.1

Figure 5.5 shows the plot of MWSR during a falling wind trend at the Regina wind site where the considered initial time is Hour 22. Opposite to that of Figure 5.4, the mean wind speed decreases as the lead time increases due to the falling wind trend and the ratio is mainly significantly less than unity. The variation in the MWSR at the three different lead times for a relatively flat wind trend considering the initial time of Hour 125 is presented in Figure 5.6. The

MWSR observed for the 1 and 2 hour lead times are close to unity. The ratio, however, decreases with increase in the initial wind speed for a lead time of 4 hours.

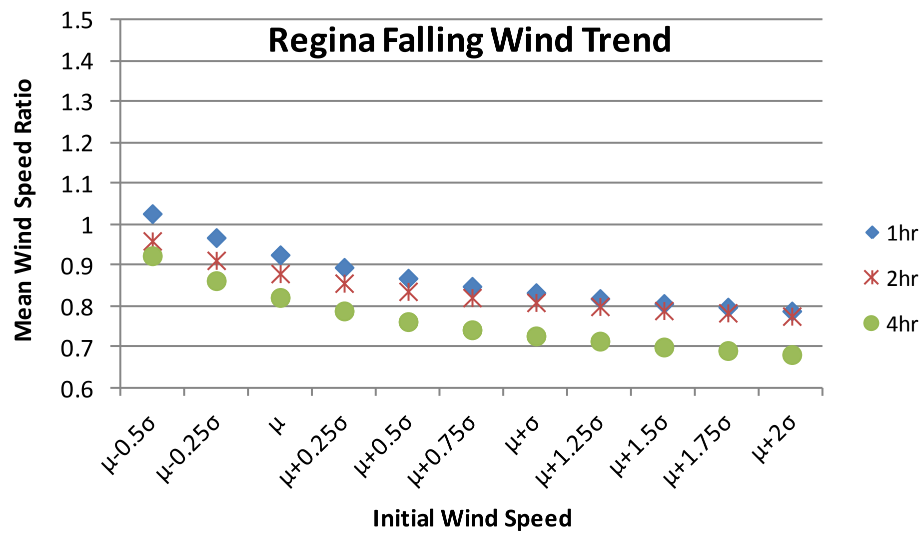

Figure 5.5: Variation of the mean wind speed ratio (MWSR) during a falling wind trend, Regina data

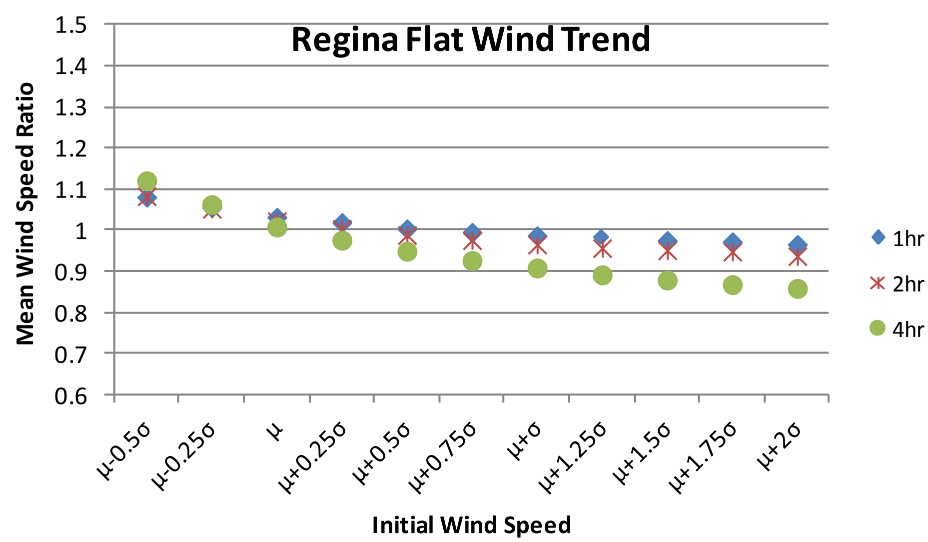

Figure 5.6: Variation of the mean wind speed ratio (MWSR) during a flat wind trend, Regina data

The standard deviation of the wind speed distribution at Hour 59 conditional on the wind speed of 19 km/h at Hour 58 is found to be 4.48 km/h. The historic wind speed standard deviation at Hour 59 is 10.2 km/h. The standard deviation of the conditional wind speed distribution is divided by the historic standard deviation and the resulting ratio is termed as the wind speed standard deviation ratio (WSSDR) which is 0.415 in this case. Figure 5.7 shows the WSSDR for lead times of 1, 2 and 4 hours for the rising wind speed trend. Figure 5.8 and Figure 5.9 show the WSSDR for the falling and flat wind trends respectively. It can be noted that the ratio increases with increase in the lead time but is fairly constant for all three lead times at the initial wind speeds considered.

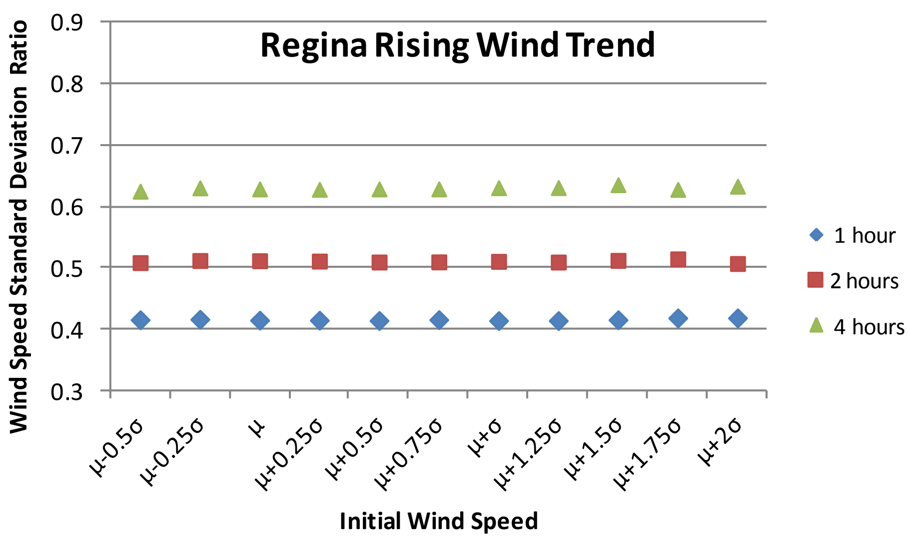

Figure 5.7: Variation of the wind speed standard deviation ratio (WSSDR) during a rising wind, Regina data

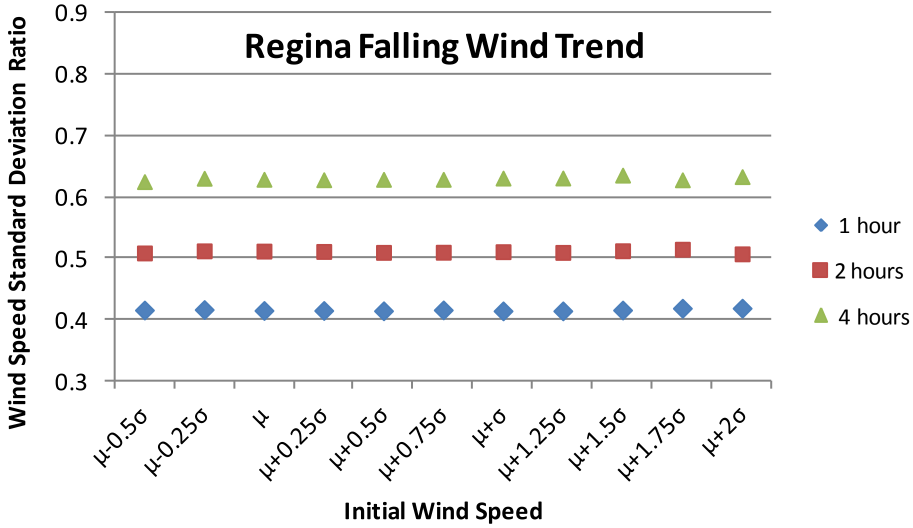

Figure 5.8: Variation of the wind speed standard deviation ratio (WSSDR) during a falling wind, Regina data

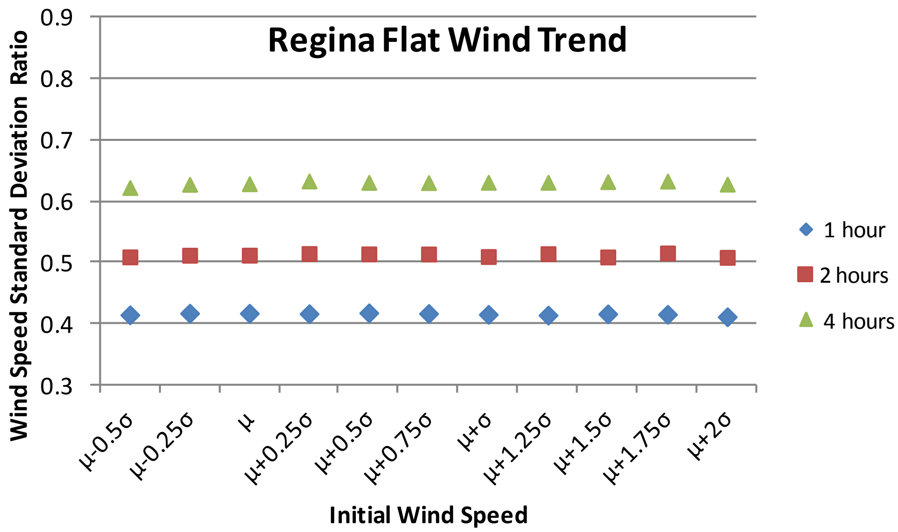

Figure 5.9: Variation of the wind speed standard deviation ratio (WSSDR) during a flat wind, Regina data

These basic studies were further investigated considering additional wind trends for the considered wind site in order to obtain an approximate relationship between the statistics of wind speed at a given lead time and the initial wind speed.

5.3. Development of an Approximate Method for UCR Evaluation

The capacity value of a wind farm in a short future time depends upon the initial conditions and the diurnal trend. This study illustrates the development of an approximate method to establish the statistics of conditional wind speed distributions during a rising wind trend for a wind site where the basic historic hourly wind speed statistics are known. The approximate methods applicable to falling and flat wind trends can be developed using a similar approach. The previous studies showed that the wind speed distribution for 1-4 hours ahead is similar to a normal distribution with the mean value controlled by the initial wind speed. A wind site represented by the Regina wind data was considered in this study. The following hour spans are two rising wind trends observed from the historic wind speed data in addition to the trend (Hour 58-Hour 62) considered in Table 5. 3. Hour 58 is 10 AM on the 3rd day of the year. Hour 634 and Hour 1044 are 10 AM on the 27th day and 12 PM on the 44th day of a year respectively. The basic statistics of the historic hourly wind speed for the two additional rising wind trend time spans are presented in Table 5. 4.

Table 5.4: Basic historical wind speed statistics of the considered rising wind trends

Hour 634- Hour 638			Hour 1044- Hour 1048		
Hour	μ (km/h)	σ (km/h)	Hour	μ (km/h)	σ (km/h)
634	20.7	12.1	1044	19.7	11.9
635	21.1	13.5	1045	21	11.8
636	21.9	13.1	1046	21.9	11.1
637	23.4	12.6	1047	22.4	10.5
638	24.7	13.5	1048	23.5	11.0

The variations in the mean wind speed ratio (MWSR) with changes in the initial conditions for a lead time of 1 hour are presented in Figure 5.10. The average of the three plots is also shown in Figure 5.10. The average plot of the MWSR exhibits a linear trend and its equation is presented in Figure 5.10. Figures 5.11, 5.12 and 5.13 present the plots of the mean wind speed ratio for the lead times of 2, 3 and 4 hours respectively.

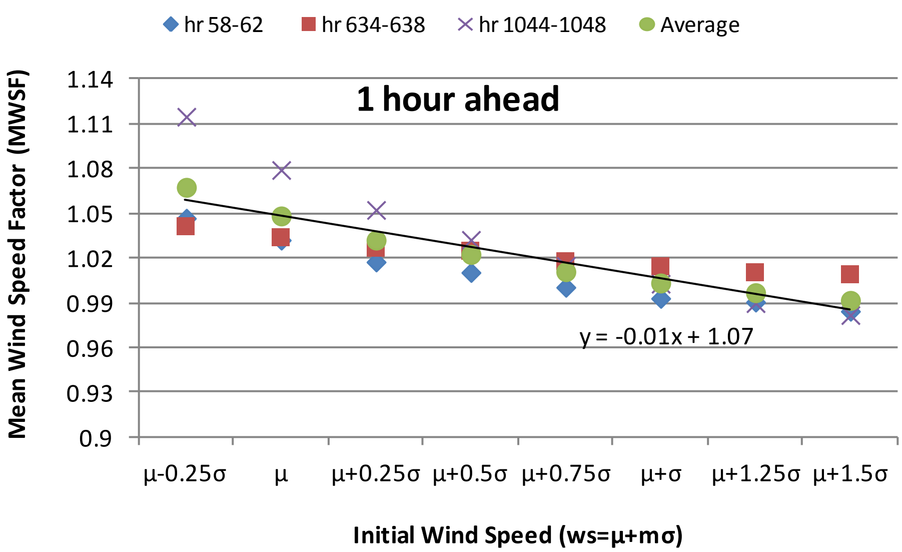

Figure 5.10: Variation of the one hour ahead MWSR during a rising wind trend

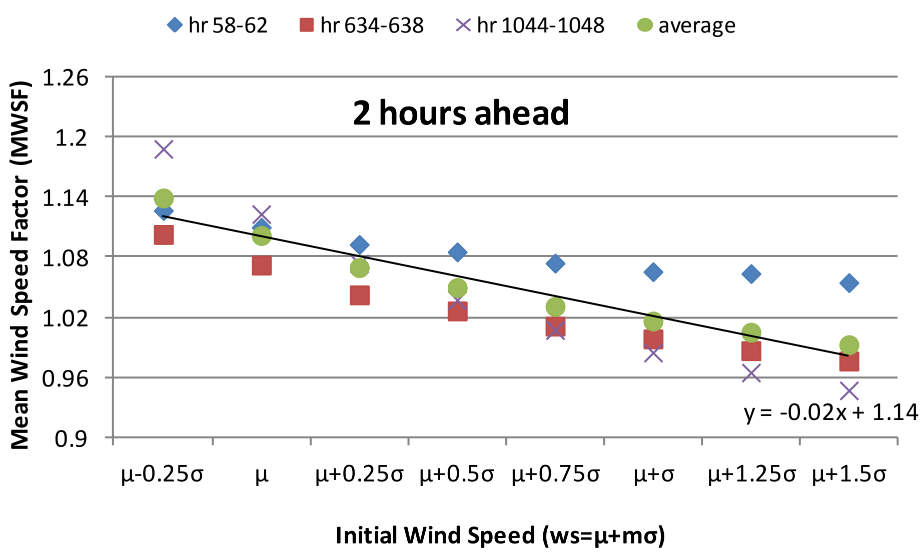

Figure 5.11: Variation of the two hours ahead MWSR during a rising wind trend

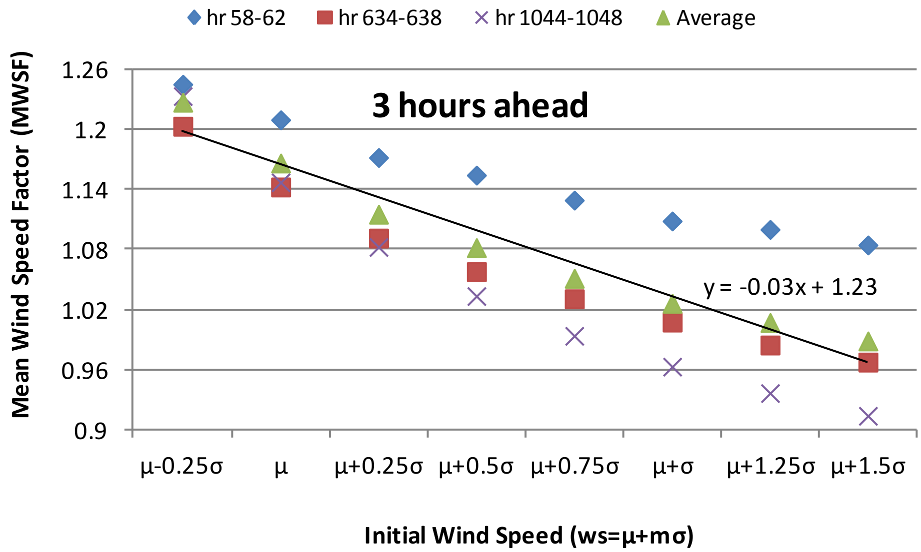

Figure 5.12: Variation of the three hours ahead MWSR during a rising wind trend

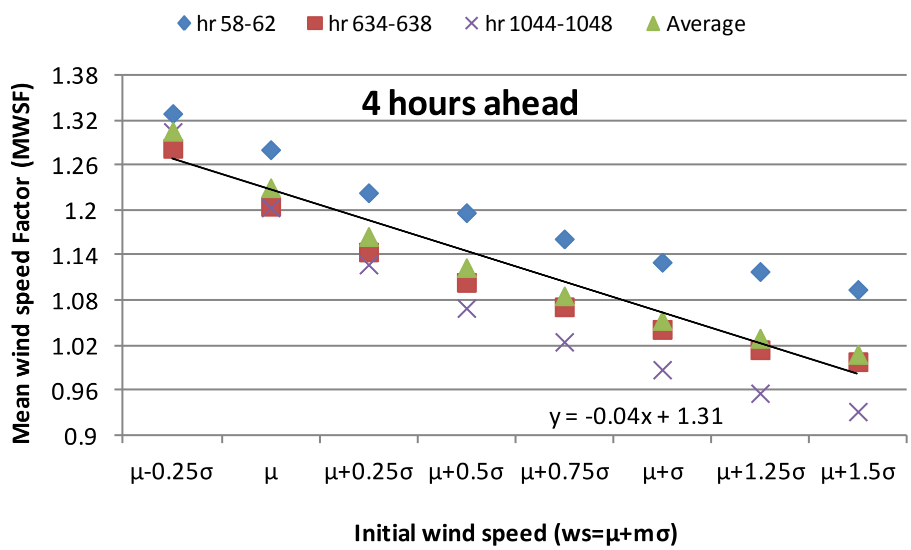

Figure 5.13: Variation of the 4 hours ahead MWSR during a rising wind trend

The equations of the linear trends for lead times of 1, 2, 3 and 4 hours are given in (5. 1)-(5. 4) respectively. It can be seen that the slope of the straight line is negative indicating that the mean wind speed ratio decreases with increase in the initial wind speed. The negative slope also increases with increase in the lead time.

$$y_1 = 1.07\text{-}0.01x \tag{5. 1}$$

$$y_2 = 1.14\text{-}0.02x \tag{5. 2}$$

$$y_3 = 1.23\text{-}0.03x \tag{5. 3}$$

$$y_4 = 1.31\text{-}0.04x \tag{5. 4}$$

Where y_1, y_2, y_3 and y_4 are the mean wind speed after 1, 2, 3,and 4 hours of the initial time as a factor of the initial wind speed. The "x" term in (5.1)-(5.4) represents the initial wind speed such that:

$$x = 2 + \frac{ws - \mu}{0.25\,\sigma} \tag{5. 5}$$

Where 'ws' is the wind speed observed at the initial time and μ and σ are the historic mean and standard deviation at the initial time.

The wind speed standard deviation ratio (WSSDR) at different lead times conditional on the wind speed at Hour 58 are presented in Figure 5.7. Tables 5.5 and 5.6 show the standard deviations and WSSDR for the two other rising trends; Hour 634 - Hour 638 and Hour 1044 - Hour 1048 respectively. The standard deviation is approximately constant for a given lead time irrespective of the initial condition and understandably, it increases as the lead time increases. The standard deviations are different for the three different rising trends. It is however observed that WSSDR is approximately constant for a lead time irrespective of the different time periods considered. The WSSDR on average is 0.42, 0.51, 0.58 and 0.63 respectively for lead times of 1, 2, 3 and 4 hours respectively. The mean value and the standard deviation thus obtained from the approximate method incorporate both the impact of the initial wind speed as well as the inherent hourly wind speed variation of the wind site. These approximate basic statistics are used to quantify the uncertainty of the wind speed or wind power and applied in Unit Commitment Risk (UCR) analysis.

Table 5.5: Standard deviations of the conditional wind speed distributions (Hour 634-Hour 638)

Hour 634		Hour 635		Hour 636		Hour 637		Hour 638	
Initial Speed		HSD	13.5	HSD	13.1	HSD	12.60	HSD	13.5
μ+mσ	km/h	SD	WSSDR (SD:HSD)	SD	WSSDR (SD:HSD)	SD	WSSDR (SD:HSD)	SD	WSSDR (SD:HSD)
μ-0.25σ	18	5.61	0.42	6.69	0.51	7.27	0.58	8.50	0.63
μ	21	5.63	0.42	6.71	0.51	7.27	0.58	8.44	0.63
μ+0.25σ	24	5.59	0.41	6.70	0.51	7.30	0.58	8.56	0.63
μ+0.5σ	27	5.64	0.42	6.73	0.51	7.39	0.59	8.60	0.64
μ+0.75σ	30	5.62	0.42	6.67	0.51	7.27	0.58	8.51	0.63
μ+σ	33	5.67	0.42	6.73	0.51	7.36	0.58	8.58	0.64
μ+1.25σ	36	5.67	0.42	6.70	0.51	7.30	0.58	8.51	0.63
μ+1.5σ	39	5.53	0.41	6.64	0.51	7.26	0.58	8.63	0.64
μ+1.75σ	42	5.67	0.42	6.64	0.51	7.27	0.58	8.48	0.63
μ+2σ	45	5.63	0.42	6.84	0.52	7.59	0.60	8.70	0.64

Table 5.6: Standard deviations of the conditional wind speed distribution (Hour 1044-Hour 1048)

Hour 1044		Hour 1045		Hour 1046		Hour 1047		Hour 1048	
Initial Speed		HSD	11.8	HSD	11.1	HSD	10.5	HSD	11
μ+mσ	km/h	SD	SD:HSD	SD	SD:HSD	SD	SD:HSD	SD	SD:HSD
μ-0.25σ	17	4.90	0.42	5.68	0.51	6.07	0.58	6.91	0.63
μ	20	4.88	0.41	5.67	0.51	6.04	0.58	6.96	0.63
μ+0.25σ	23	4.89	0.41	5.69	0.51	6.08	0.58	6.92	0.63
μ+0.5σ	26	4.91	0.42	5.70	0.51	6.06	0.58	6.94	0.63
μ+0.75σ	29	4.88	0.41	5.65	0.51	6.06	0.58	6.93	0.63
μ+σ	32	4.88	0.41	5.68	0.51	6.08	0.58	6.94	0.63
μ+1.25σ	35	4.92	0.42	5.70	0.51	6.09	0.58	6.95	0.63
μ+1.5σ	38	4.94	0.42	5.71	0.51	6.11	0.58	6.95	0.63
μ+1.75σ	41	4.94	0.42	5.72	0.52	6.05	0.58	6.94	0.63
μ+2σ	44	5.03	0.43	5.78	0.52	6.17	0.59	6.94	0.63

HSD: Historic Hourly Standard Deviation
SD: Standard deviation of the conditional wind speed distribution

5.4. Application of the Approximate Method for UCR Evaluation

The approximate method developed in the earlier section has been applied to evaluate the UCR for a rising wind trend at Hour 490-Hour 494, which is 10 AM to 2 PM of the 21st day of a year. The historic mean wind speed shows a rising wind trend over the considered hours. The conditional mean wind speed for each hour is obtained using (5.1)-(5.5) while the standard deviation is estimated from the constant WSSDR observed in the previous section. The ratio was found to be 0.42, 0.51, 0.58 and 0.63 respectively for lead times of 1, 2, 3 and 4 hours. A normal distribution based on the approximate mean and standard deviation is used to represent the wind speed probability distribution conditional to a known initial wind speed. The wind speed distribution is converted to a wind power distribution using the Wind Turbine Generator (WTG) power curve, presented in Section 2.3, with cut-in, rated and cut-out wind speeds of 15 km/h, 50 km/h and 90 km/h respectively. The added wind farm has a rated capacity of 300 MW. Hourly wind power models in the form of multiple capacity states and their probabilities were created using the basic and the approximate methods.

The approximate method uses the basic statistics derived in the earlier sections of this chapter to create the required normal distributions. For the initial wind speed of 30 km/h at Hour 491, equation (5.1) shown in the approximate method gives the mean wind speed and the standard deviation of the one hour ahead wind speed distribution as 30.62 km/h and 4.58 km/h respectively. Figure 5.14 shows the discrete capacity states and the associated probabilities obtained from the normal distribution of the hour ahead wind speed distribution and the WTG power curve. The two, three and four hour ahead approximate conditional wind power distributions can be obtained following (5.2)-(5.5). The basic method utilizes an ARMA model in the wind speed simulation of the wind site and a conditional probability for each initial condition. The wind power models, obtained from both methods, were combined with the COPT of the committed units of the IEEE RTS system employing the area risk concept presented in Chapter 4 for UCR evaluation.

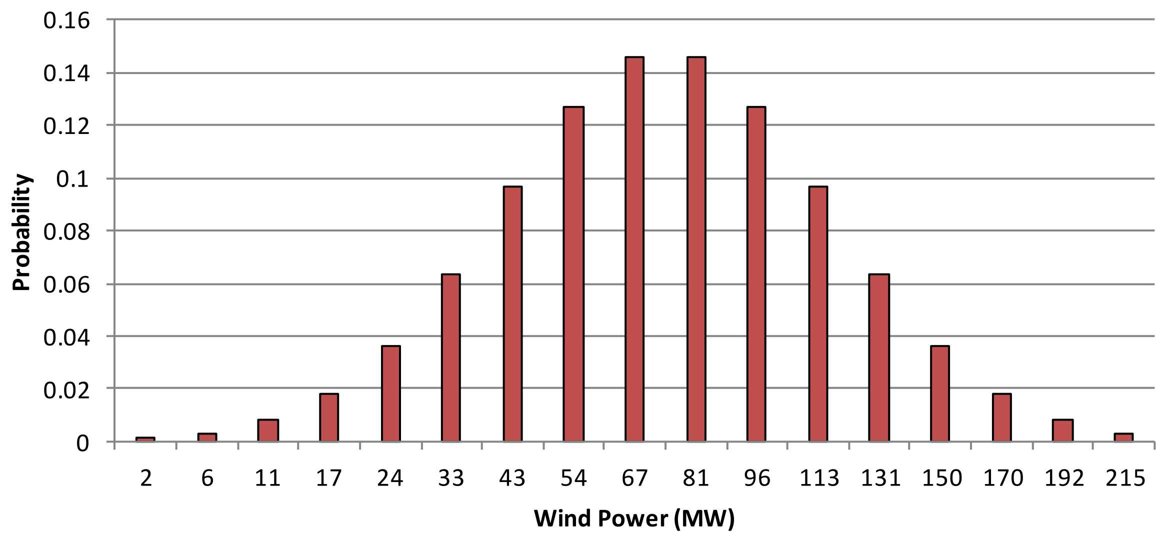

Figure 5.14: One hour ahead conditional wind power distribution obtained using the approximate method, initial wind power at Hour 491 = 60 MW using Toronto data

Figure 5 -15 presents the UCR evaluated using the approximate and the basic method where 11 units of the IEEE RTS with a total capacity of 2096 MW are committed at a low load level of 1660 MW. The initial wind speed is 30 km/h providing a power output of 60 MW, (20% of the rated capacity). The UCR was evaluated for a lead time of 4 hours. It can be seen from Figure 5.15 that the evaluated UCR for the two methods lie very close to each other up to a load level of 1700 MW where the risk is 0.0002. The UCR values start to spread out slightly with further increases in load.

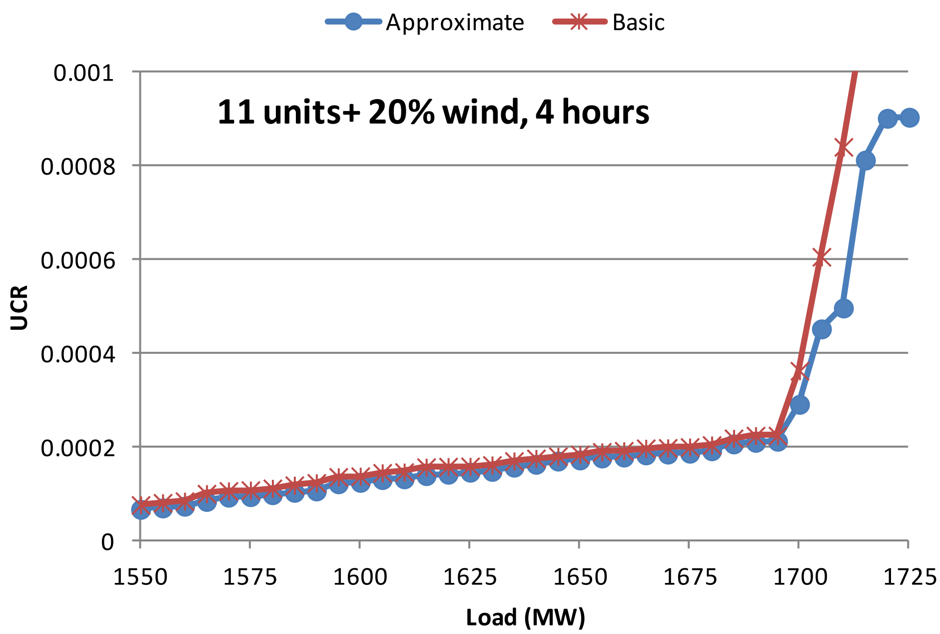

Figure 5.15: UCR evaluation for a low load level using the basic and approximate hourly wind models (lead time = 4 hours, initial wind power = 20% of the rated capacity)

Figure 5.16 shows the UCR evaluated for a high initial wind power of 60% of the rated capacity. The UCR values evaluated from the two different methods stay close to each other but are relatively more spread out compared to the previous case of 20% initial wind power. The UCR evaluated using the basic method is slightly higher than that using the approximate method. This shows that the differences between the UCR results using the approximate and the basic method increase with the increase in the initial wind speed.

Figure 5.17 and Figure 5.18 present the UCR evaluated for the initial wind powers of 20% and 60 % of the rated capacity respectively for a high load level of 2760 MW. The 22 units in the priority loading order of the IEEE RTS are committed with a total capacity of 3177 MW. It can be observed that the UCR values evaluated from the approximate method are close to the ones evaluated using the basic method. Similar to the case with the low load scenario, the differences in the UCR results are higher at the 60% initial wind power compared to those at

20% initial wind power. It can be seen from Figure 5.15 and Figure 5.18 that the difference in the UCR results are lower at the high load scenario compared to the low load scenario. This is due to the fact that the proportion of wind power to the total operating capacity is relatively low at the high load scenario and the approximate method does not cause significant difference on the UCR results.

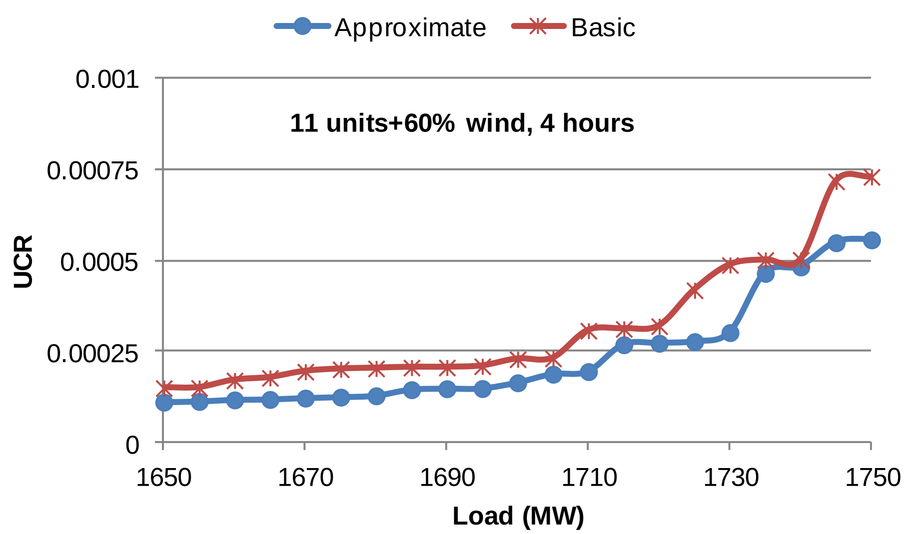

Figure 5.16: UCR evaluation during a low load level using the basic and approximate hourly wind models (lead time = 4 hours, initial wind power = 60% of the rated capacity)

The load carrying capability of the committed wind power considering the added wind power is evaluated at the UCR criteria of 0.001 and 0.0001. The load carrying capability of the 11 committed units without wind power is 1498 MW at the UCR criterion of 0.0001 and 1695 MW at the UCR criterion of 0.001. The load carrying capabilities measured from the approximate and the basic methods are presented in Figure 5.19 for the low load scenario. The load carrying capability is increased due to the added wind power. The load carrying capabilities evaluated using the two methods are relatively close and remain close at the UCR criterion of 0.001. The load carrying capabilities are presented for the high load scenarion in Figure 5.20.

The two figures show that the developed method can be used to incorporate wind power in UCR evaluation without a major compromise in the results. The same process can be applied to develop an approximate method for other wind site diurnal trends.

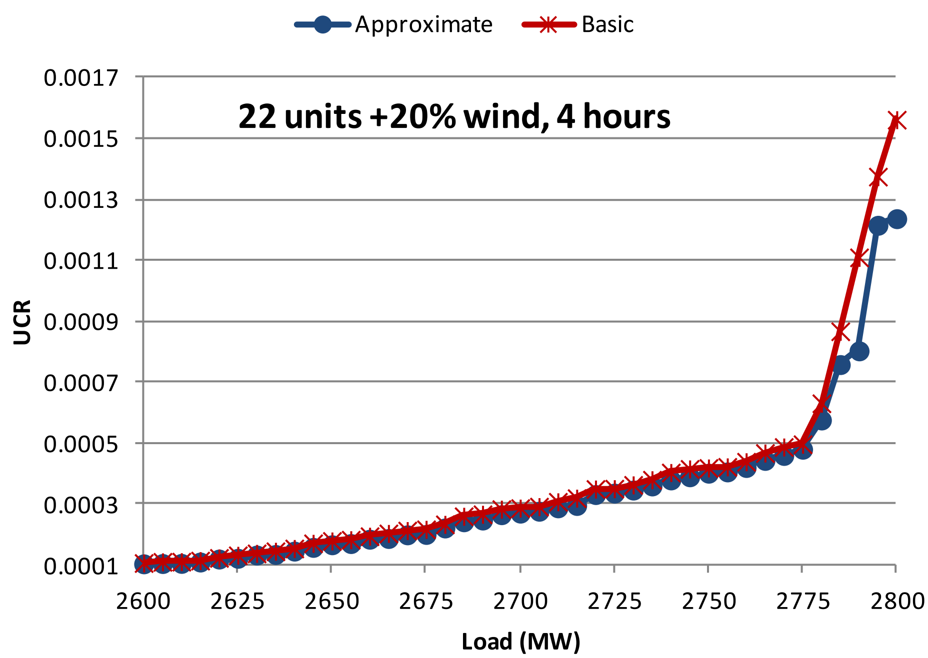

Figure 5.17: UCR evaluation during a high load level using the basic and approximate hourly wind models (lead time = 4 hours, initial wind power = 20% of the rated capacity)

5.5. Summary

Wind power variability at a short time in the future is determined by the initial conditions and the inherent characteristics of the wind regime. A diurnal trend can have an appreciable impact on the operating capacity value of the wind power. The operating capacity credit (OCC) evaluated by determining the increase in load carrying capability due to the wind power under a specified UCR criterion. The UCR evaluation is conducted by combining the short term capacity model of the added wind power with the capacity model (COPT) of the committed conventional units. The basic short term wind power model applied in this approach is created using a conditional probability approach. This requires a large number of wind speed data simulations and the model run for each initial condition, which is time consuming. The approximate method described in this chapter can be used with minimum information, using the initial wind speed and the basic statistics of the historic wind speed. The method was developed for diurnal rising wind trends and can be extended to falling wind trends using a similar approach. The results obtained from the approximate method were compared to those obtained using the ARMA model approach and were found to be very close. The method should prove useful to electric power system operators in determining the capacity value of wind power at a short time in the future.

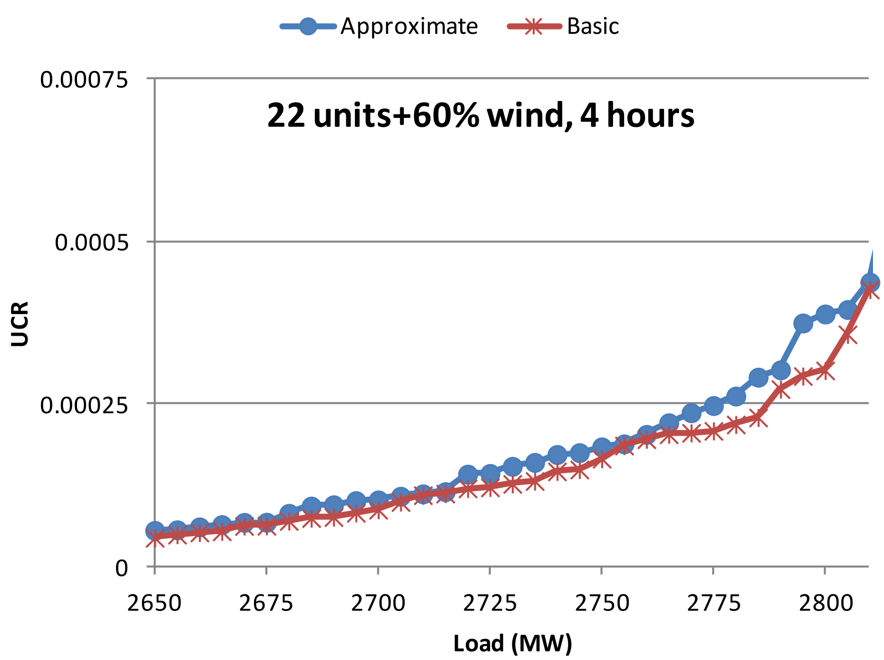

Figure 5.18: UCR evaluation during a high load level using actual and approximate hourly wind models (lead time = 4 hours, initial wind power = 60% of the rated capacity)

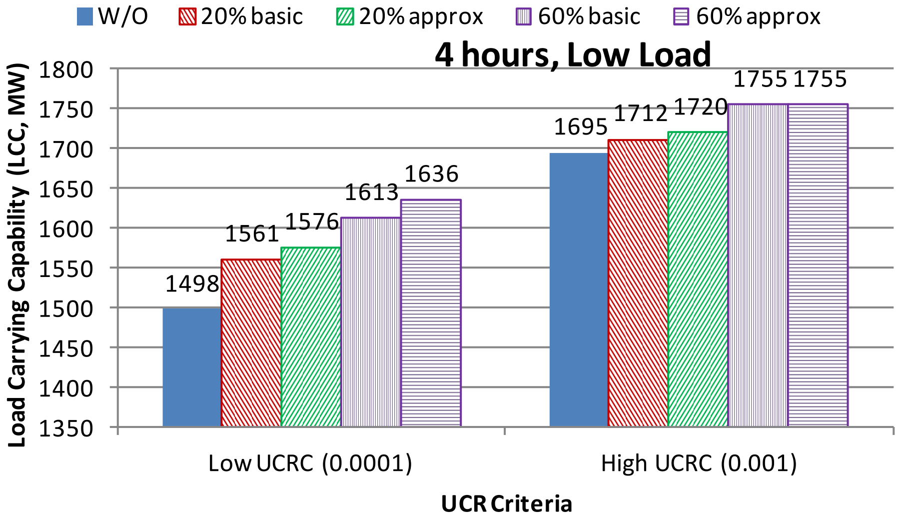

Figure 5.19: Load Carrying Capabilities (LCC) evaluated at the two UCR criteria for a lead time of 4 hours during low load conditions

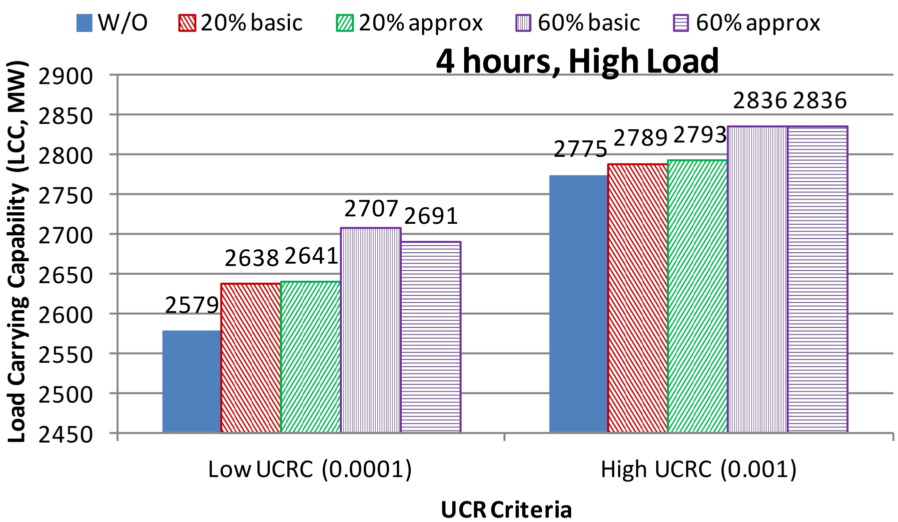

Figure 5.20: Load Carrying Capabilities (LCC) evaluated at the two UCR criteria for a lead time of 4 hours during high load conditions

CHAPTER 6

RESPONSE RISK ANALYSIS INCORPORATING WIND POWER

6.1. Introduction

Unit commitment risk (UCR) analysis allows an electric power system operator to decide which units to commit and to determine the reserve capacity required for maintaining the system risk within an acceptable criterion. The concept and application of UCR analysis incorporating wind power is presented in Chapters 4 and 5. This analysis, however, does not consider the actual allocation of the spinning reserve to the committed units. This allocation is directly related to dispatch decisions determined by the economics of system operation. The least cost generating unit dispatch does not necessarily ensure that the reserve units can respond with adequate capacity within a specified margin time after a contingency. An adequate response within the specified time is required to save the system from undesirable consequences when a major contingency occurs. Response risk analysis can be conducted to determine the appropriate allocation of the spinning reserve in order to keep the probability of not obtaining the required response at an acceptable risk level. Response risk is defined as the probability that the actual response obtained within the margin time is just equal to or less than the required capacity [30]. Response risk evaluation takes into consideration the probability that the unit that is responsible to respond may fail within the margin time [1].

There are generally two time periods of interest, a short time and a long time, where response is required from the spinning reserves [1]. A short time response within 1 minute is required to preserve the system frequency and tie line regulation. A longer time response of a few minutes (5-15 minutes) is required to avoid emergency actions such as disconnecting some load. The reliability standards of the North American Electric Reliability Corporation (NERC) [155] state that an immediate response should be obtained through automatic generation control (AGC) in order to maintain frequency and tie line regulation and the contingency reserve should be able to restore the system to its pre-disturbance state within 15 minutes of a disturbance in order to avoid any undesired load curtailment. The following studies consider the long term response where the margin time is taken as 10 minutes.

This chapter presents a method to incorporate wind power in unit scheduling and perform response risk analysis. The method utilizes the persistence model with a normal distribution to quantify the variability of wind speed for the 10 minute margin time.

6.2. Methodology for Generating Unit Scheduling and Response Risk Evaluation

6.2.1. Unit scheduling

The number of units placed in service to satisfy a forecast load is determined based upon unit commitment decisions as stated in the previous section. The next step is to distribute the load and the total spinning reserve within the committed units. This section presents a method to evaluate the response risk incorporating wind power. Wind power cannot be dispatched in a conventional sense and is considered as negative load in this study. The unit scheduling is fundamentally governed by the operating cost so that units with lower operating costs are loaded before the ones with higher operating costs. The proposed method is illustrated using the IEEE Reliability Test System (RTS) [29]. The priority loading order for the IEEE RTS is specified [156] and a UCR analysis is used to determine which units to commit.

The forecast load is distributed over the committed units using an economic load dispatch and the response risk is evaluated for the margin time of 10 minutes. The load is then redistributed over the committed units if the specified response risk criterion is not satisfied while minimizing the cost [38]. A response risk criterion of 0.001 is considered in this study. The first order gradient method [157] is employed for the economic load dispatch. This method starts with a feasible solution where the sum of the power generated by each committed units is equal to the load neglecting the losses. The cost function of each thermal unit is represented by a second order cost equation and is presented in (6. 1). The cost parameters ‘a’ and ‘c’ in (6. 1) are zero for hydro units. The objective function of the economic load dispatch is given in (6.2) followed by the constraints related to the load and the allowable upper and lower limit of each generating unit. The incremental cost of a unit is given in (6. 3). The unit with the highest incremental cost is unloaded and the incremental load is shifted to the unit with the lowest incremental cost. The cost parameters, pick-up rates, failure rates and the minimum and maximum generation level of the IEEE RTS units are presented in Table 6.1 in the priority loading order.

$$F_i(G_i) = a_i * G_i^2 + b_i * G_i + c_i \tag{6. 1}$$

Where a_i, b_i and c_i are the operating cost parameters for a unit i, G_i is the power output or load on the unit.

$$Minimize \quad F_T = \sum_{i=1}^{n} F_i(G_i) \tag{6. 2}$$

Subjected to:

$G_{i\,\min} < G_i \le G_{i\,\max}$ and

$$\sum_{i=1}^{n} G_i = Load$$

Where:

G_{imin} and G_{imax} are the minimum and maximum allowable output levels. The 'i' and 'T' in the equations denote the individual and total respectively.

$$\frac{dF_i(G_i)}{dG_i} = 2 * a_i * G_i + b_i \tag{6. 3}$$

Table 6.1: IEEE RTS Priority loading order and generation data

Unit	Type	Pgmax (MW)	Pgmin (MW)	Ramp /Pick-up rate (MW/min)	Failure rate (occ/yr)	Cost Parameters ($/hr)		
						a $(MW)^{-2}$	b $(MW)^{-1}$	c
1-4	Hydro	50	0	10	4.42	0	0.5	0
5-6	Nuclear Steam	400	200	0	7.96	216.576	5.345	0.00028
7	Coal Steam	350	150	9	7.62	388.25	8.919	0.00392
8-10	Oil Steam	197	80	6	9.22	301.223	20.023	0.003
11-14	Coal Steam	155	60	5	9.13	206.703	9.2706	0.00667
15-17	Oil Steam	100	40	3	7.3	286.241	17.924	0.0022
18-21	Coal Steam	76	25	2	4.47	100.349	12.145	0.01131
22-26	Oil Steam	12	5	1	2.98	30.396	23.278	0.13733
27-30	Oil Combustion	20	6	4	19.47	40.000	37.554	0.18256
31-32	Hydro	50	0	10	4.47	0	0.5	0

6.2.2. Response risk analysis

Response risk analysis deals with how the spinning reserves are distributed over the committed units once a unit commitment decision is made. The economic load dispatch discussed in the previous section determines the least cost unit schedule, but does not consider the ramp rates (or the pick-up rates) of the generating units. The responding capability or the regulating margin RM_i of the ith generating unit within the margin time MT_i is determined by the pick-up rate PR_i and the spinning reserve held in the unit SR_i, as expressed in (6.4). The spinning reserve in a generating unit, and in the total system are given by (6.5) and (6.6) respectively. The total regulating margin RM_T of the system within the margin time is obtained using (6.7).

$$RM_i = Minimum\ (PR_i \times MT\ ,\ SR_i) \tag{6. 4}$$

$$SR_i = G_{i\max} - G_i \tag{6. 5}$$

$$SR_T = \sum_i SR_i \tag{6. 6}$$

$$RM_T = \sum_i RM_i \tag{6. 7}$$

A system is usually operated with a required amount of regulating margin expressed as a fixed percentage of the spinning reserve [38]. The selected amount is a managerial decision. Required regulating margins (RRM) of 40% and 50% of the spinning reserve are considered in the response risk studies presented in Section 6.3.

The response risk (RR) of a particular load dispatch, as defined in [30], is expressed in (6.8).

$$RR = \sum_{j=1}^{NC} P_j \times Q_j \tag{6. 8}$$

Where,

NC is the total number of contingencies from ‘n’ number of generating units, such that

$NC = 2^n$

P_j is the probability of contingency ‘j’ occurring within the margin time.

$$Q_j = \begin{cases} 0 & when \quad (Load + RRM) < Ge_j \\ 1 & when \quad (Load + RRM) \geq Ge_j \end{cases} \tag{6. 9}$$

Where:

Ge_j in (6.9) correspond to the total effective operating capacity for contingency 'j' and is obtained from (6.10).

$$Ge_j = \sum_{\substack{i=1 \\ i \neq j}}^{NC} G_i + RM_i \tag{6.10}$$

An illustration of response risk evaluation conducted in this work is presented considering an economic load dispatch for a forecast load of 1660 MW in the IEEE-RTS. Eleven generating units selected from the priority loading order are committed to meet the load with a total of 436 MW of spinning reserve. Table 6.2 shows the unit schedule of the committed units together with their individual spinning reserve and the regulating margin for the specified margin time of 10 minutes. The sum of the unit scheduled capacity and the regulating margin gives the total effective capacity that is available within the margin time, which is also shown in Table 6.2. It can be seen from Table 6.2 that the first six units are fully loaded and reserves are carried by the last five units. From the perspective of regulating margin, units U7 and U11 are overloaded as their spinning reserve is less than their 10 minute response capabilities. Units U8- U10, on the other hand, are under loaded as their individual spinning reserves are more than their 10 minute response capabilities.

The maximum capacity that can be available in the margin time is designated as the effective capacity. The COPT algorithm presented in (1.1) can be used to create the generation model for the response risk evaluation. Table 6.3 presents the COPT considering the effective operating capacity and the outage replacement rates (ORR) of the individual units presented in Table 6.2. The capacity- in states less than 1500 MW are not shown in this table. As expressed in (6.8) and (6.9), the response risk is the probability of the total effective capacity being just equal to or less than the sum of load and the required regulating margin (RRM). If the required regulating margin (RRM) is specified as 40% of the spinning reserve, the RRM is 174.4 MW. The response risk, for the load of 1660 MW and the RRM of 174.4 MW, is the cumulative probability associated with the capacity-in state of 1825 MW in Table 6.3 and in this case is 0.00114731. If the specified RRM is increased to 50% of the spinning reserve, the response risk is 0.00148322 which illustrates that the response risk increases as the RRM specification is increased. The operating cost of this dispatch is $15,495 per hour.

Table 6.2: Unit Schedule for a load level of 1660 MW

Units	Rated capacity, MW	(A) Schedule, MW	SR, MW	10 minute response capacity, MW	(B) RM, MW	Effective Capacity (A + B) MW	ORR $\times 10^3$
U1	50	50	0	50	0	50	0.08409
U2	50	50	0	50	0	50	0.08409
U3	50	50	0	50	0	50	0.08409
U4	50	50	0	50	0	50	0.08409
U5	400	400	0	0	0	400	0.15145
U6	400	400	0	0	0	400	0.15145
U7	350	285	65	90	65	350	0.14498
U8	197	80	117	60	60	140	0.17542
U9	197	80	117	60	60	140	0.17542
U10	197	80	117	60	60	140	0.17542
U11	155	135	20	50	20	155	0.17371
Total	2096	1660	436	520	265	1925	

Table 6.3: COPT of the effective operating capacity considering the regulating margin for the economic load dispatch

Capacity out, MW	Capacity in, MW	Cumulative probability x10^3
0	1925	1000.00000
50	1875	1.48322
100	1825	1.14731
140	1785	1.14727
150	1775	0.62170
155	1770	0.62170
190	1735	0.44822
200	1725	0.44804
205	1720	0.44804
240	1685	0.44799
255	1670	0.44799
280	1645	0.44799
290	1635	0.44789
295	1630	0.44789
305	1620	0.44780
330	1595	0.44780
340	1585	0.44780
345	1580	0.44780
350	1575	0.44780
355	1570	0.30302
380	1545	0.30302
395	1530	0.30302
400	1525	0.30302
420	1505	0.00048

Table 6.4 shows the unit schedule obtained from the economic load dispatch for the relatively high load condition of 2760 MW where 21 units from the priority loading order of the IEEE RTS are committed. The total spinning reserve is 405 MW while the regulating margin is

234 MW. The response risk of this dispatch for a specified RRM of 40% of the SR is 0.0024258 and the response risk for a RRM of 50% of the SR is 0.0027622. These values are different from the response risks evaluated for the same RRM specifications at the low load condition of 1660 MW. This indicates that although the RRM is the same, the risk level may not be consistent for different unit schedules. It may therefore be desirable to utilize a risk constrained dispatch where the economic load dispatch is modified, if necessary, to maintain the response risk at a level deemed acceptable to the system. The following section presents a risk constrained load dispatch method for redistributing the spinning reserve with minimum increase in the cost to attain this objective.

6.2.3. Load dispatch with response risk criterion

Response risk analysis takes into consideration the responding capability and the probability of the unit failing while responding. The response risk of an economic load dispatch (ELD) is first evaluated and is subjected to modifications in the dispatch if the risk is higher than the specified response risk criterion (RRC). The method proposed in [38] reloads the committed units by dividing the units into three different groups from the perspective of the regulating margin and the dispatch.

i. Group I (ideally loaded): where $SR_i = RM_i$

ii. Group II (over loaded): where $SR_i < RM_i$

iii. Group III (under loaded): where $SR_i > RM_i$

The units in Group I are ideally loaded and do not need any modification. The load from the units in Group II is transferred to the units in Group III in small steps. In order to keep the essence of the economic dispatch, the load transfer is done starting from the unit in Group II with the highest incremental cost to the unit in Group II with the lowest incremental cost. Response risk is evaluated for each load transfer and the process is stopped when the RRC is satisfied. The process is repeated until the RRC is satisfied. It may even require increasing the operating reserve by committing additional units to satisfy the risk criterion.

Table 6.4: Unit Schedule for a load level of 2760 MW

Unit #	Rated Capacity, MW	Unit Schedule, MW	SR, MW	RM, MW
U1	50	50	0	0
U2	50	50	0	0
U3	50	50	0	0
U4	50	50	0	0
U5	400	400	0	0
U6	400	400	0	0
U7	350	350	0	0
U8	197	80	117	60
U9	197	80	117	60
U10	197	80	117	60
U11	155	155	0	0
U12	155	155	0	0
U13	155	155	0	0
U14	155	155	0	0
U15	100	82	18	18
U16	100	82	18	18
U17	100	82	18	18
U18	76	76	0	0
U19	76	76	0	0
U20	76	76	0	0
U21	76	76	0	0
Total	3165	2760	405	234

In the illustration presented in Section 6.2.2, the response risk for the RRM specification of 40% of the spinning reserve is 0.00114731. If the response risk criterion is 0.001, for instance, the unit schedule obtained from the economic load dispatch (ELD) needs modification. Table 6.4 presents the modified unit schedule for the same load and unit commitment case where the 11 units are committed to serve a load of 1660 MW. It can be seen from Tables 6.2 and 6.4 that the loads from units U7 and U11 are transferred to unit U8. The total effective capacity has increased from 1925 MW to 1975 MW as the load schedule of the ELD is modified using the method presented in this section. The response risk is 0.00079689 which is within the risk criterion of 0.001, for 40% of the spinning reserve as the RRM specification. The operating cost is $15,979 per hour for this response risk constrained dispatch.

Table 6.5: Modified unit schedule for a load level of 1660 MW

Units	U1	U2	U3	U4	U5	U6	U7	U8	U9	U10	U11	Total
Schedule, MW	50	50	50	50	400	400	265	130	80	80	105	1660
SR, MW	0	0	0	0	0	0	85	67	117	117	50	436
RM, MW	0	0	0	0	0	0	85	60	60	60	50	315
Effective Capacity, MW	50	50	50	50	400	400	350	190	140	140	155	1975

6.3. Development of a Wind Power Model for Response Risk Analysis

Wind power generation is generally characterized by its random variability and cannot be dispatched like a conventional generating unit. The persistence model is a practical tool for predicting wind power for a short time horizon such as 10 minutes [158]. Such a prediction can be accompanied by forecast uncertainty which needs to be quantified for response risk assessment incorporating wind power. Reference [158] uses a truncated normal distribution

(TND) to simulate the forecast error by setting three standard deviations above and below the hourly forecast error as the limit for hour ahead load scheduling.

The hourly wind data obtained from Environment Canada for different Canadian wind sites are not readily applicable in response risk analysis. This requires wind speed data over relatively short time such as 10 minutes, which is the margin time selected in this study. In the absence of such wind data, simulated ten minute wind speed data from the National Renewable Energy Laboratory (NREL) [159] have been analyzed. Reference [159] shows the simulated wind data for several on and offshore geographical wind sites in the United States for three years. The Regina wind site, located in Saskatchewan, Canada is close to the state of North Dakota of the USA. A study conducted on the ten minute wind data of the wind site numbered 02641 located in North Dakota, whose geographical coordinates (latitude: 48.17 and longitude: -101.23) are close to the coordinates of Regina (latitude: 50.43 and longitude: -104.67), showed that the standard deviation of the forecast error is 9.21% of the mean value or the initial wind speed. Based upon this analysis, a value of 10% of the mean wind speed has been assumed as the standard deviation of the wind speed in the margin time of ten minutes. The wind speed variability is then represented as a discrete distribution with seven states spaced one standard deviation apart. The corresponding wind speed is converted into wind power states to obtain the ten minute wind power distribution.

Figure 6.1 shows the 10 minute ahead wind power variability for an initial wind speed of 25 km/h at the Regina wind site. The power output corresponding to this initial wind speed is 30 MW from a 300 MW wind farm. The WTG characteristic presented in Chapter 2 with 15, 50 and 90 km/h as the cut-in, rated and cut-out speeds respectively is used in this study. Table 6.6 presents the wind speed and wind power variability for initial wind speeds of 30 km/h, 34 km/h and 42 km/h. These wind power outputs are 20%, 30% and 60 % of the rated capacity respectively.

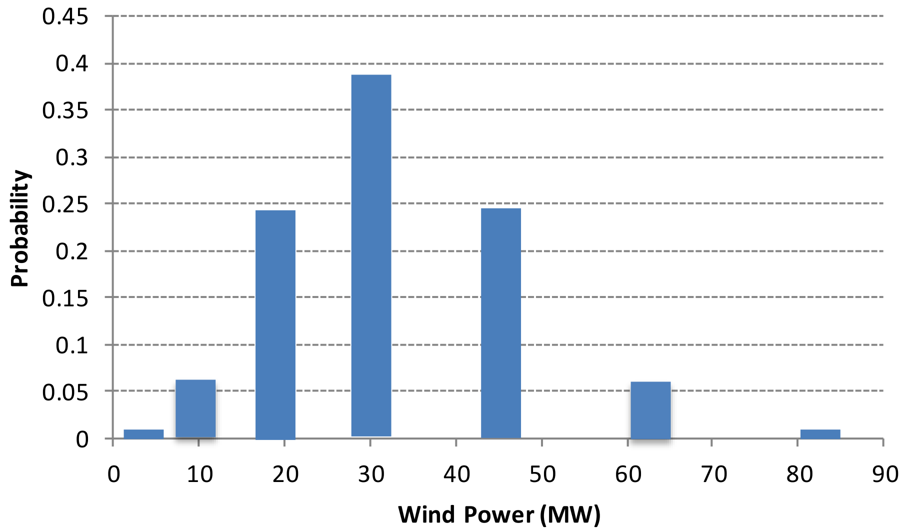

Figure 6.1: 10 minute ahead wind power variability for an initial wind speed of 25 km/h (30 MW)

Table 6.6: Ten minute variability of wind speed (WS) and wind power (WP) for different initial wind speeds

Probability	For initial speed of 30 km/h		For initial speed of 34 km/h		For initial speed of 42 km/h	
	WS (km/h)	WP (MW)	WS (km/h)	WP (MW)	WS (km/h)	WP (MW)
0.006	21	13	23.8	25	29.4	58
0.061	24	25	27.2	43	33.6	92
0.242	27	42	30.6	67	39.5	153
0.382	30	60	34	90	42	180
0.242	33	87	37.4	130	44.5	217
0.061	36	115	40.8	169	47	253
0.006	39	147	44.2	213	49.5	292

Wind power is considered as must- take power. Net load is the term designated as the difference between the load and the wind power. The knowledge of the initial wind power is used to determine the economic load schedule for the net load. The response risk is evaluated for each wind power state at the margin time by modifying the effective operating capacity (6.11) with the capacity of the wind power state presented in Table 6.3. The response risk evaluated for each wind power state is weighted by the corresponding probability to obtain the response risk incorporating wind power as shown in (6.11). The net load schedule of the conventional units is modified using the method described in Section 6.2.3 if the response risk does not meet the risk criterion.

$$Ge_{k,j} = \sum_{\substack{i=1 \\ i \neq j}}^{NC} (G_i + RM_i + WP_k - WP_{ini}) \tag{6.11}$$

Where:

$Ge_{k,j}$ is effective operating capacity for the k^{th} wind power capacity state WP_k

WP_{ini} is the initial wind power

$$RR_{with\ wind\ power} = \sum_{i}^{7} (RR_i \times P_{wp_i}) \tag{6.12}$$

Where RR_i and P_{wp_i} are the response risk and probability pertaining to the i^{th} wind power state. A WTG is assumed to have no responding capability in the presented studies.

6.4. Results and Analysis

This section presents an application of the proposed method to evaluate the response risk for the IEEE RTS. The operating costs associated with both the economic load dispatch and the response risk constrained dispatch are evaluated in this study. The cost functions of the conventional units of the IEEE RTS are known. The cost of wind power is assumed to be the system marginal cost, which is the incremental cost of the last committed unit in the priority loading order. The marginal cost can therefore vary at different load levels where the committed units are different. Two load scenarios are considered in this study with load levels of 2760 MW as the high load scenario and 1660 MW as the low load scenario. The conventional units are committed from the priority loading order shown in Table 6.1 to satisfy a unit commitment risk criterion (UCRC) of 0.001. The low load scenario has 11 conventional units with a total of 2096 MW of operating capacity. The high load scenario is supplied by 21 units with a total operating capacity of 3165 MW. The economic load schedule is determined using the first gradient method explained earlier in Section 6.2.1. Multiple contingencies are not considered as the probability of more than one unit failing in the margin time of 10 minutes is very low.

Table 6.7 shows the operating cost per hour for the economic load dispatch and the response risk constrained dispatch for a RRM specification at 40% of the spinning reserve. The specified response risk criterion is 0.001 in this study. Table 6.8 shows the operating cost and the response risk for the RRM specification at 50% of the spinning reserve. It can be seen that the response risk of the economic load dispatch is higher than the specified risk criterion of 0.001 in all the wind power cases studied. The dispatch is modified to satisfy the risk criterion as explained in Section 6.2.3. This increases the regulating margin and also increases the operating cost. Addition of wind power reduces the net load and this causes the spinning reserve to increase as shown in Table 6.7 and Table 6.8. Wind power is not capable of providing any regulating margin but the wind power variability causes the net load to change within the margin time. The addition of the wind power increases the spinning reserve. It should be noted that the RRM also increases to meet the 40% or 50% spinning reserve requirement as the wind generation increases. The additional regulating margin must be provided by the conventional units since wind is considered to have no responding capability.

Table 6.7: Response risk analysis incorporating wind power (RRM = 40%, High Load Scenario)

Wind Power, MW	Op cap, MW	SR, MW	RRM, MW	Economic Load Dispatch			Response Risk Constrained Dispatch		
				RR $x10^3$	Cost ($/hr)	RM, MW	RR $x10^3$	Cost ($/hr)	RM, MW
0	3165	405	162	2.4258	31538	234	0.7987	32124	321
30	3195	435	174	1.9797	31406	246	0.9029	31883	313
60	3225	465	186	2.0810	31276	222	0.9346	31774	294
90	3255	495	198	2.2956	31146	192	0.9592	31761	279
180	3345	585	234	1.8999	30944	156	0.9625	31721	234

Table 6.8: Response risk analysis incorporating wind power (RRM = 50%, High Load Scenario)

Wind Power, MW	Op cap, MW	SR, MW	RRM, MW	Economic Load Dispatch			Response Risk Constrained Dispatch		
				RR $x10^3$	Cost ($/hr)	RM, MW	RR $x10^3$	Cost ($/hr)	RM, MW
0	3165	405	202.5	2.7622	31538	234	0.9741	32351	359
30	3195	435	217.5	2.6392	31406	246	0.8335	32376	366
60	3225	465	232.5	2.6467	31276	222	0.9592	32388	356
90	3255	495	247.5	2.6573	31146	192	0.8581	32486	351
180	3345	585	292.5	2.6611	30944	156	0.8801	32545	311

As expected, the response risk increases when the regulating margin requirement is increased from 40% to 50% of the spinning reserve. As a result, the operating cost also increases to satisfy the response risk criterion of 0.001 when the RRM is increased from 40% to 50% of the spinning reserve. The marginal cost of wind power for the high load scenario, associated with the 21st unit, is $13.86/MW/hr. It can be seen that the operating cost of the economic load dispatch (ELD) is reduced with the increase in wind power.

Figure 6.2 presents the reduction in the operating cost per MW of wind power. The reduction in cost is caused by the difference between the marginal cost applied to wind power as a first order cost function and the actual cost that would incur without wind power from the second order cost function of the conventional unit. The cost reductions for the ELD and risk constrained dispatch RRCD1 (RRM = 40% of SR) are positive for all the four cases of wind power considered and can be considered as the cost benefit of wind power. The cost benefit for the ELD is relatively constant but it decreases as the wind power is increased. The cost reduction of risk constrained dispatch RRCD1 (RRM = 50% of SR) is negative indicating that addition of wind power puts a cost burden on the system operation if the required regulating margin is increased. Table 6.9 presents the load schedule of the units U8-U21 for the high load scenario. The first seven units (U1-U7) of the priority loading order are fully loaded giving a sub- total operating capacity of 1350 MW and are not shown in Table 6.9. The last column on the unit schedule shows the wind power (WP). The load schedule shows that RRCD2 requires backing up unit 14 which is less expensive but has a higher ramp rate compared to units 15-17 or units 18-21 and is the reason for the increased cost compared to that for RRCD1.

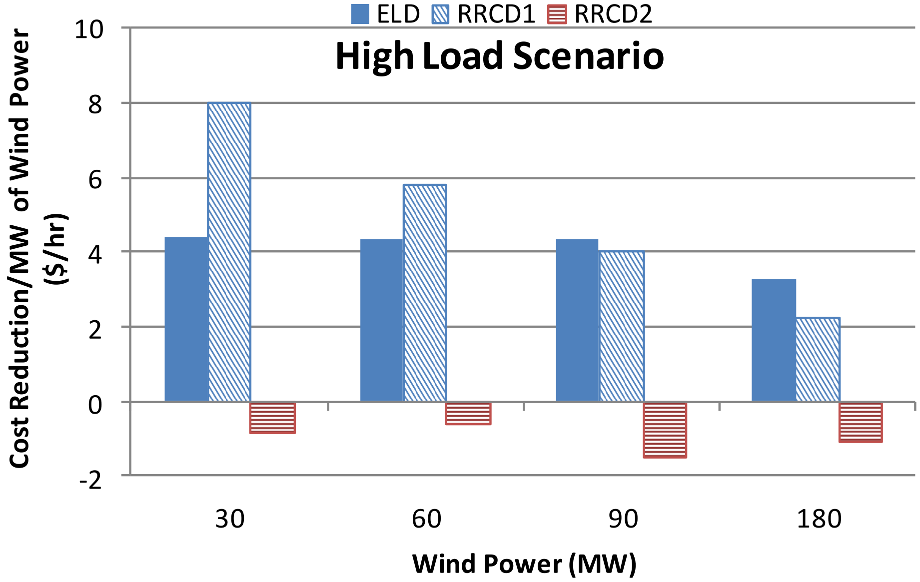

Figure 6.2: Operating cost reduction due to wind power at the high load scenario. (ELD: economic load dispatch, RRCD1: RRM = 40% of Spinning Reserve and RRCD2: RRM = 50% of Spinning Reserve)

Table 6.10 and Table 6.11 show the response risk analysis for the low load scenario where 11 RTS units, with a total operating capacity of 2096 MW, are committed to serve a load of 1660 MW. The wind power cases considered are the same as with the high load scenario. The response risk of the ELD is higher than the risk criterion of 0.001 and needs to be modified causing an increase in the cost. The marginal cost of wind power is $10.60/MW/hr associated with the incremental cost of the 11th unit. Figure 6.3 shows the cost benefit of wind power where the cost benefit is relatively lower for the ELD compared to the high load scenario. The cost benefit for the risk constrained dispatch is higher compared to the ELD and increases with the increase in wind power. The benefit is however negative at the high wind power of 180 MW indicating that wind power is a cost burden on the system. Table 6.12 shows the load schedule of the committed units for ELD, RRCD1 and RRCD2. It can be observed that the least expensive hydro units U2-U4 are forced to reduce their output level to provide the required regulating margin during the high wind power of 180 MW increasing the total operating cost.

Table 6.9: Load Schedule of ELD, RRCD1 and RRCD2 for the high load scenario (the first seven units, U1-U7, are fully loaded)

Units	U8	U9	U10	U11	U12	U13	U14	U15	U16	U17	U18	U19	U20	U21	WP
ELD	80	80	80	155	155	155	155	82	82	82	76	76	76	76	
RRCD1	140	110	80	155	155	155	155	82	82	72	56	56	56	56	
RRCD2	140	140	100	155	155	155	140	67	67	67	56	56	56	56	
ELD	80	80	80	155	155	155	155	72	72	72	76	76	76	76	30
RRCD1	140	90	80	155	155	155	155	72	72	72	66	56	56	56	30
RRCD2	140	140	95	155	155	155	115	67	67	67	56	56	56	56	30
ELD	80	80	80	155	155	155	155	62	62	62	76	76	76	76	60
RRCD1	140	95	80	155	155	155	155	62	62	62	61	56	56	56	60
RRCD2	140	140	100	155	155	145	105	62	62	62	56	56	56	56	60
ELD	80	80	80	155	155	155	155	52	52	52	76	76	76	76	90
RRCD1	140	110	80	155	155	155	145	52	52	52	56	56	56	56	90
RRCD2	140	140	125	155	155	120	105	52	52	52	56	56	56	56	90
ELD	80	80	80	155	155	155	155	40	40	40	62	63	63	62	180
RRCD1	140	115	80	155	155	150	105	40	40	40	52	53	53	52	180
RRCD2	140	140	135	155	120	105	105	40	40	40	52	53	53	52	180

Table 6.10: Response risk analysis incorporating wind power (RRM = 40%, Low Load Scenario)

Wind Power, MW	Op cap, MW	SR, MW	RRM, MW	Economic Load Dispatch			Response Risk Constrained Dispatch		
				RR $x10^3$	Cost ($/hr)	RM, MW	RR $x10^3$	Cost ($/hr)	RM, MW
0	2096	436	174.4	1.1478	15495	265	0.797	15979	315
30	2126	466	186.4	1.1447	15490	285	0.84	15928	330
60	2156	496	198.4	0.9736	15474	320	0.9736	15474	320
90	2186	526	210.4	1.1772	15493	305	0.9872	15524	330
180	2276	616	246.4	1.3355	15386	295	0.8532	16278	364

Table 6.11: Response risk analysis incorporating wind power (RRM = 50%, Low Load Scenario)

Wind Power, MW	Op cap, MW	SR, MW	RRM, MW	Economic Load Dispatch			Response Risk Constrained Dispatch		
				RR $x10^3$	Cost ($/hr)	RM, MW	RR $x10^3$	Cost ($/hr)	RM, MW
0	2096	436	218	1.4842	15495	265	0.9724	16883	360
30	2126	466	233	1.1704	15490	285	0.9612	16815	379
60	2156	496	248	1.2198	15474	320	0.9847	16615	379
90	2186	526	263	1.5381	15493	305	0.9856	16315	379
180	2276	616	308	1.6343	15386	295	0.9206	17568	433

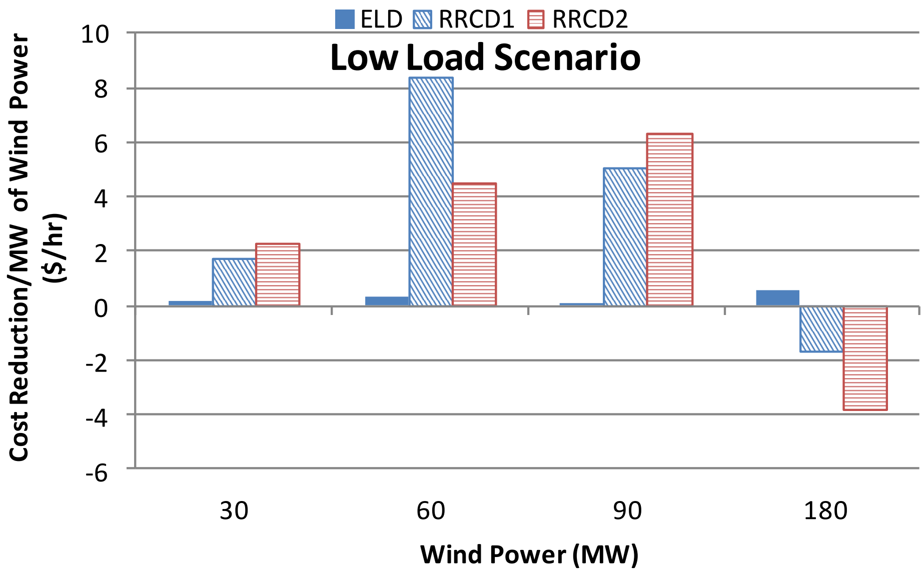

Figure 6.3: Operating cost reduction due to wind power at the low load scenario: RRC = 0.001

6.5. Response Risk Analysis of Economic Load Dispatch

Figures 6.4 and 6.5 show the response risk of the ELD at the high and low load scenarios considering the four wind power cases. As expected, the risk is higher for the case when the regulating reserve requirement is 50% of the spinning reserve compared to that at 40%. It can be seen that the risk is reduced for the wind power cases of 30 MW and 60 MW and then starts to rise as the wind power is increased further. This shows that the conventional units can absorb the wind power to a certain extend but as the wind power penetration increases i.e. to 20% of the rated capacity in this case, the response risk will rise. This is due to the fact that wind power does not provide any response capability causing the effective regulating margin of the ELD to decrease.

Table 6.12: Load Schedule of ELD, RRCD1 and RRCD2 for the low load scenario

Units	U1	U2	U3	U4	U5	U6	U7	U8	U9	U10	U11	WP
ELD	50	50	50	50	400	400	285	80	80	80	135	
RRCD1	50	50	50	50	400	400	265	130	80	80	105	
RRCD2	50	50	50	10	400	400	260	135	120	80	105	
ELD	50	50	50	50	400	400	250	80	80	80	140	30
RRCD1	50	50	50	40	400	400	260	115	80	80	105	30
RRCD2	50	50	45	1	400	400	260	134	105	80	105	30
ELD	50	50	50	50	400	400	260	80	80	80	100	60
RRCD1	50	50	50	50	400	400	260	80	80	80	100	60
RRCD2	50	50	45	1	400	400	260	129	85	80	100	60
ELD	50	50	50	50	400	400	210	80	80	80	120	90
RRCD1	50	50	50	45	400	400	230	80	80	80	105	90
RRCD2	50	50	40	1	400	400	260	94	90	80	105	90
ELD	50	50	50	50	400	400	180	80	80	80	60	180
RRCD1	50	50	30	1	400	400	229	100	80	80	60	180
RRCD2	50	15	1	1	400	400	229	129	115	80	60	180

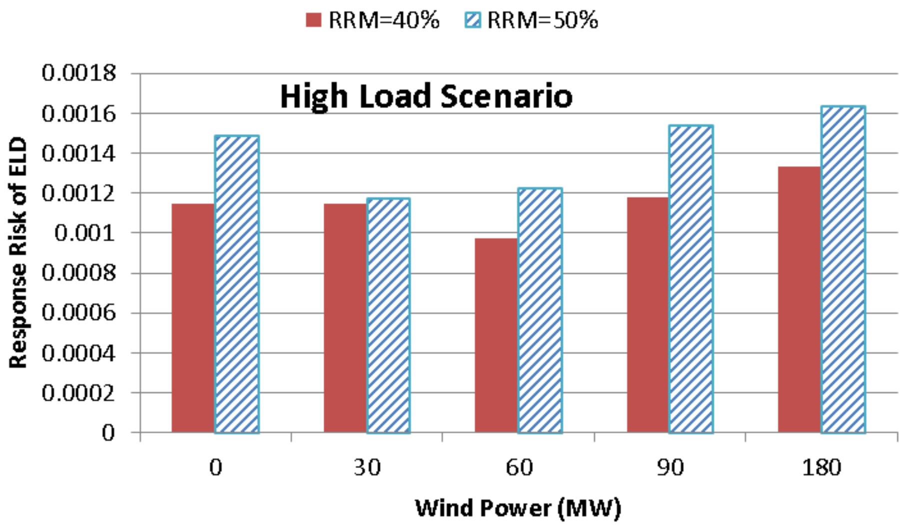

Figure 6.4: Response risk of economic load dispatches (high load scenario)

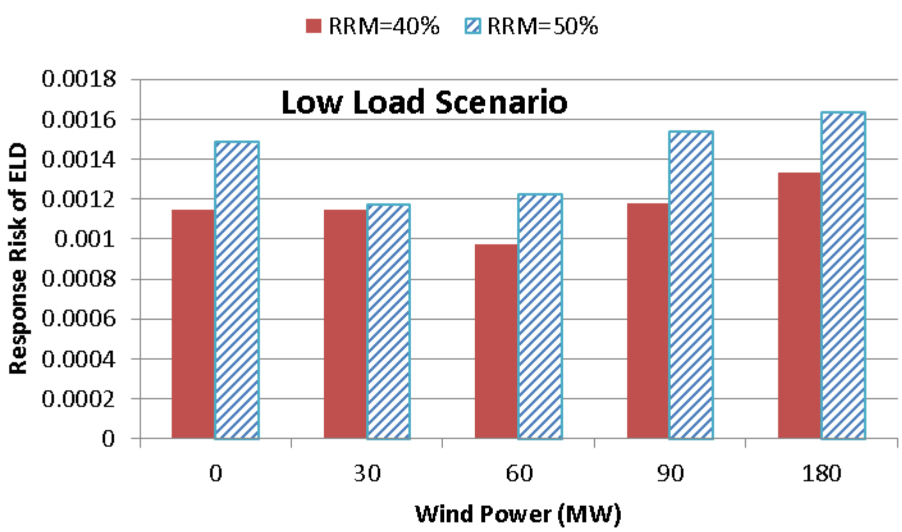

Figure 6.5: Response risk of economic load dispatches (low load scenario)

6.6. Summary

Response risk assessment is a significant component of operating reserve analysis as it assists in the determination of the actual dispatch of the committed units required for reliable system operation. The risk factors associated with wind power are significantly different than those of conventional units. Generating unit dispatch schedules involving conventional units need to be modified when the system includes wind power. This study considers two types of dispatch: economic load dispatch (ELD) and response risk (RR) constrained dispatch considering wind power. The ten minute ahead wind power variability is represented by a persistence model and a normally distributed prediction error with mean zero and a 10% standard deviation. Ten minute wind speed data from the National Renewable Energy Laboratory was used to estimate the standard deviation of 10 minute ahead wind speed prediction error. The wind power and its variability were combined with the forecast load and the dispatch optimized to satisfy the net load requirements.

The studies show that the operating cost increases when the ELD has to be modified to satisfy the RRC. The marginal cost of the last committed unit is considered as the cost of wind power. The operating cost of the ELD is reduced due to the difference between the constant marginal cost of wind power and the actual cost that would have occurred. The cost benefit of the response risk constrained dispatch depends upon the risk criterion and the wind power level. An increase in the regulating margin requirement, which in this study is from 40% to 50%, increases the cost of operation with wind power. The response risk and the operating cost both increase when the wind power increases beyond a certain value. This is caused by the necessity to increase the regulating margin by transferring load from the conventional units with high responding capability and low incremental cost to the ones with low responding capability and high incremental cost at such high wind conditions. This chapter presents a method to include wind power in the load dispatch and to evaluate response risk incorporating wind power. The method and the analysis presented in this chapter should prove useful for the system operator in determining operating strategies incorporating wind power.

CHAPTER 7

EVALUATION OF OPERATING RISKS INCORPORATING WIND POWER AND STORAGE

7.1. Introduction

Energy storage is considered to be a useful resource to mitigate the uncertainty associated with wind power. Energy storage can also help to reduce the possible wastage of wind energy during low load and high wind periods and during the times of transmission congestion when wind power cannot be utilized. A review of the relevant literature and background information on different energy storage technologies together with their application and limitations, with regard to wind power, is noted in Chapter 1. The studies presented in Chapter 3 deal with evaluating the risk of committing electric power from a wind farm. Chapters 4 and 5 evaluate the system risk by integrating the risk model of wind power with the existing conventional committed units. This chapter is focused on wind power commitment risk (WPCR) and unit commitment risk (UCR) evaluations considering electric energy storage in conjunction with wind power. The impacts of energy storage on the WPCR and the UCR are evaluated in the studies presented in this chapter.

The main purpose of an energy storage system (ESS) in a wind integrated power system could be to manage the wind power variability, avoid wind power curtailment due to transmission congestion, overcome sort-term fluctuations and support system stability [115]. The ability to perform these functions depends on the characteristics and size of the ESS. In an open market system, a wind farm has to participate in power bidding 2-3 hours ahead [115]. The application of an ESS can be significant in a future competitive market where penalties are enforced on wind farm operators for failing to supply a committed capacity. Probabilistic risk analysis associated with committing wind power is presented in [127] and a simplified method for wind power commitment based on a risk criterion is illustrated in [130]. An energy storage system (ESS) can be applied for the purpose of reducing the wind power commitment risk and considered as a form of forecast hedging.

7.2. Development of Wind Power Models Incorporating Storage

Conditional probability distributions of wind speed and wind power, created from the simulated wind speed data using ARMA models, are used to quantify the uncertainties of wind speed and wind power in the next one and two hours in Chapter 2. A combined wind and storage model can also be created from the initial wind and energy storage conditions using a conditional probability approach. The total power P_t considering wind and storage in the future is given by (7.1) where the wind power P_{wp} is obtained from the wind data series simulated using an ARMA model. If the actual wind power in the next hour is less than the committed value P_{com}, the ESS will supply the deficit power P_{ess}. The initial state of charge (SOC) of the energy storage is known and the new SOC after discharging occurs is given in (7.2). The ESS is assumed to be located in close proximity to the wind farm facility and operated by the wind power producer to minimize the WPCR.

$$P_t = P_{wp} + P_{ess} \tag{7.1}$$

Where,

$$P_{ess} = P_{com} - P_{wp} \quad if \quad ((P_{wp} < P_{com}) \; AND \; (P_{com} - P_{wp}) < SOC_{ess})$$

$$= SOC_{ess} \quad if \quad ((P_{wp} < P_{com}) \; AND \; (P_{com} - P_{wp}) \geq SOC_{ess})$$

$$= 0 \quad if \quad (P_{wp} \geq P_{com}) \tag{7.2}$$

$$SOC_{ess_{new}} = SOC_{ess} - P_{ess} \tag{7.3}$$

$$0 \leq SOC_{ess_{new}} \leq 100 \; \%$$

And,

SOC_{ess}: state of charge of the energy storage system

$SOC_{ess\ new}$: new state of charge after being discharged (or charged)

Hour-ahead capacity models obtained using this method of incorporating wind and energy storage are illustrated for a 300 MW wind farm with Toronto site data, which includes a 30 MWh energy storage facility. The initial conditions considered are those at Hour 8 with a rising wind trend. An initial wind speed of 30 km/h is assumed, which corresponds to 60 MW of wind power based on the WTG characteristics provided in Section 2.3. Based on the persistence model, the wind farm producer commits 60 MW for the next hour, and operates the ESS to assist in meeting the wind power commitment. Two cases are considered with different initial SOC, 100% and 50%, of the ESS. The discharge times for both cases are one hour, whereas the discharge capacity is assumed to be proportional to the SOC. The maximum capacity that the ESS can provide is 30 MW and 15 MW for SOC of 100% and 50% respectively. Figure 7.1 presents the wind power probability distributions without the ESS and with the ESS for the two cases. The wind power distributions with the ESS show a peak close to the committed value, which in this case is 60 MW. The ESS with 50% initial SOC can supply only 15 MW and hence the minimum capacity state of the total power is 15 MW. The capacity states higher than 60 MW have the same probabilities for all three cases in Figure 7.1. This is because the ESS is not used if the actual wind power is higher or equal to the committed power.

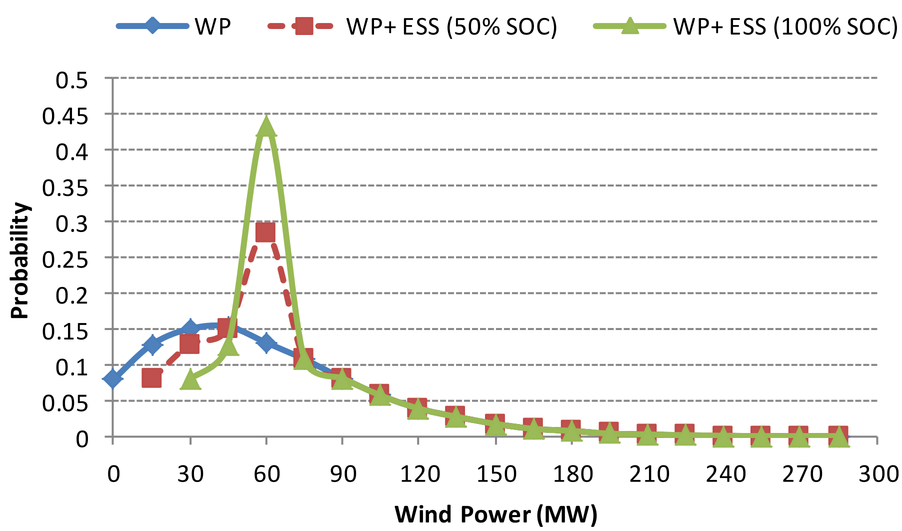

Figure 7.1: Conditional wind power probability distribution in the next hour, initial power = 60 MW

When the wind power penetration is very low, wind power can be easily absorbed by the system as and when present. Wind penetration has increased significantly in many jurisdictions around the world and these systems cannot absorb all the wind power at all times and especially under low load and high wind conditions. In the future scenarios, a system operator may have to put a limit on the power delivered by a wind farm based on the wind power commitment made by the wind farm operator. Energy storage can assist in two ways in such a scenario. It can support the wind farm when the actual wind power is less than the committed value and store the wind energy if it is higher than the committed value. In this case, the P_{ess} shown in (7.4) can have both positive and negative values. A positive value indicates deficit power, whereas, a negative value indicates surplus wind power available for storage. The combined wind and storage model for this wind operating strategy is different from the model shown in Fig. 7.1, since (7.4) is used instead of (7.2) to evaluate P_{ess} in developing the model.

$$\begin{aligned} P_{ess} &= P_{com} - P_{wp} \quad if \quad (P_{com} - P_{wp}) < SOC_{ess} \\ &= SOC_{ess} \quad if \quad (P_{com} - P_{wp}) \geq SOC_{ess} \end{aligned} \tag{7.4}$$

The wind power commitment is 60 MW for the case illustrated in Figure 7.1. The wind power distribution is truncated at 60 MW in the present case since wind power in excess of 60 MW will be stored and not provided to the system. The wind power model for this case without and with storage is presented in Table 7.1. The cumulative probabilities associated with the capacity states including the ESS are also shown in Table 7.1. It can be seen that the probability of having 60 MW in the next hour is 0.488367 without the ESS. The probability increases to 0.641766 with a 50% charged ESS and 0.792049 with a fully charged ESS.

The available wind capacity that is not supplied to the power system is designated as excess wind power in this study. The probability distribution of the excess wind power under this operational paradigm is presented in Table 7.2. With the application of energy storage, the excess wind energy can be stored. The new SOC of the energy storage at the beginning of the next hour can be either higher or lower than the initial SOC depending on either excess or deficit wind power is available in the next hour. The SOC at the beginning of the next hour can be modeled by a probability distribution. Table 7.3 presents the probability distribution of the SOC at the beginning of Hour 9. The initial discharge capacity of the ESS was 15 MW which corresponds to a 50% SOC at Hour 8. It can be seen from Table 7.3 that there are capacity states higher than the discharging capacity corresponding to the initial SOC. The sum of the probabilities associated with all those capacity states is 0.48 which is the probability that the new SOC at Hour 9 will be higher than the initial SOC of 50% at Hour 8. This is due to the excess wind energy diverted to the ESS.

Table 7.1: Conditional wind power probability distributions when the wind power is limited to the committed value

WP			WP +ESS (50% SOC)			WP+ ESS (100%SOC)		
Cap., MW	Prob.	Cum. Prob.	Cap., MW	Prob.	Cum. Prob.	Cap., MW	Prob.	Cum. Prob.
0	0.019389	0.019389	15	0.015749	0.015749	30	0.012514	0.012514
4	0.019223	0.038612	18	0.012419	0.028168	32	0.006875	0.019389
8	0.021340	0.059951	21	0.015606	0.043774	34	0.008779	0.028168
12	0.026645	0.086597	24	0.016177	0.059951	36	0.010444	0.038612
16	0.031784	0.118380	27	0.019794	0.079745	38	0.010087	0.048699
20	0.034520	0.152900	30	0.022291	0.102036	40	0.011253	0.059951
24	0.037018	0.189918	33	0.024980	0.127016	42	0.013013	0.072965
28	0.037446	0.227364	36	0.025884	0.152900	44	0.013632	0.086597
32	0.038850	0.266213	39	0.027454	0.180354	46	0.015440	0.102036
36	0.041609	0.307822	42	0.027597	0.207951	48	0.016344	0.118380
40	0.040467	0.348289	45	0.029167	0.237118	50	0.017795	0.136175
44	0.039397	0.387686	48	0.029095	0.266213	52	0.016725	0.152900
48	0.042918	0.430604	51	0.031379	0.297592	54	0.018033	0.170933
52	0.040753	0.471357	54	0.030499	0.328092	56	0.018985	0.189918
56	0.040277	0.511633	57	0.030142	0.358234	58	0.018033	0.207951
60	0.488367	1.000000	60	0.641766	1.000000	60	0.792049	1.000000

7.3. Evaluation of Wind Power Commitment Risk Considering Energy Storage

The supplied wind power will be lower than the committed value if the power supported by the ESS is not sufficient to overcome the deficit associated with the wind power commitment and the actual wind power. The cumulative probability distributions presented in Table 7.1 can be used to directly obtain the WPCR incorporating the ESS. It can be seen from Table 7.1 that the probability of the total power being less than the committed value of 60 MW is 0.511633 without the ESS, and is the WPCR as explained in Chapter 3. The WPCR reduces to 0.358234 and 0.207951 respectively when the ESS is considered with 50% and 100% initial SOC. It can be seen that the WPCR can be significantly lowered by including an ESS in wind power commitment. The reduction in WPCR, however, depends on the initial SOC of the ESS.

Table 7.2: Probability distribution of the excess wind capacity at Hour 9 when the initial wind power is 60 MW at Hour 8

WP, MW	Probability
0	0.1271901
16	0.1014966
32	0.0694624
48	0.0484297
64	0.0319631
80	0.0207015
96	0.0121133
112	0.0083279
128	0.0041640
144	0.0026971
160	0.0016325
176	0.0006624
192	0.0003076
208	0.0003312
224	0.0005205

Table 7.3: Probability distribution of the storage SOC at the beginning of the next hour given that the initial SOC was 50%

SOC_{new}, MW	Probability
0	0.378408
2	0.019794
4	0.022196
6	0.020222
8	0.020840
10	0.019532
12	0.019889
14	0.020008
16	0.018319
18	0.019389
20	0.019032
22	0.015630
24	0.017557
26	0.015487
28	0.016249
30	0.357449

Previous chapters showed that diurnal wind trends have significant impacts on the WPCR and the UCR. It is observed in Chapter 2 that the historic wind speed data for the

Toronto site shows a rising wind trend at Hour 8 and a falling wind trend at Hour 20. The impact of an ESS for rising and falling wind trends is assessed for these two different periods of the day. Table 7.4 presents the WPCR of committing 100% of the initial power at Hour 9 and Hour 21 for an initial wind power of 60 MW at Hour 8 and Hour 20. The WPCR is reduced due to the ESS. The reduction in the WPCR is also shown in Table 7.4 as a percentage of the WPCR without the ESS. It is obvious that the reduction in the WPCR will be higher when the initial SOC of the ESS is 100% compared to that when the initial SOC is only 50%. It can also be seen that the reduction in the WPCR is higher at Hour 9 which has a rising wind trend compared to that at Hour 21 which has a falling wind trend. Table 7.5 shows the WPCR at Hour 9 and Hour 21 when the initial power at Hour 8 and Hour 20 is 90 MW or the 30% of the rated capacity. It can be seen from Table 7.5 that the reductions in the WPCR due to the ESS are lower at the initial power of 90 MW compared to those at the initial power of 60 MW. This is caused by the ESS capacity limit which is fixed and becomes smaller relative to the initial wind power of 90 MW.

Table 7.4: WPCR for the lead time of one hour considering the ESS, initial wind power = 60 MW

Initial condition of ESS	Hour 9		Hour 21	
	WPCR	WPCR Reduction (%)	WPCR	WPCR Reduction (%)
No ESS	0.511633	-	0.617211	-
ESS with 50 % SOC	0.358234	29.98	0.494377	19.90
ESS with 100 % SOC	0.207951	59.36	0.349906	43.31

7.4. Wind Power Commitment Risk Evaluation for a Lead Time of More Than One Hour

The analysis presented in the previous section assumes that the ESS can be discharged at the rated capacity for one hour. The initial SOC is known and its contribution to the total power is considered accordingly for a one hour lead time. In order to evaluate the WPCR for a lead time of two hours, it is necessary to estimate the available state of charge at the end of the first hour. The SOC at the beginning of the next hour can be modeled by a conditional probability distribution as described in Section 7.2. Table 7.3 presents the probability distribution of the SOC at the beginning of Hour 9 given that the initial discharge capacity of the ESS at Hour 8 is 15 MW. The conditional probability distribution model for the ESS SOC is combined with the two hour wind capacity model to obtain the combined wind and energy storage capacity model for the two-hour lead time.

Table 7.5: WPCR for the lead time of one hour considering the ESS, initial wind power = 90 MW

Initial condition of ESS	Hour 9		Hour 21	
	WPCR	WPCR Reduction (%)	WPCR	WPCR Reduction (%)
No ESS	0.568250	-	0.643199	-
ESS with 50 % SOC	0.444483	21.78	0.549086	14.63
ESS with 100 % SOC	0.315691	44.45	0.438268	31.86

The maximum support available from the ESS for the second hour therefore depends upon the initial wind power and the initial SOC for the specified operating strategy. The operating strategy, as explained earlier, is to commit 100% of the initial wind power and obtain support from the ESS if the actual wind power becomes less than the committed value. In order to compare different types of ESS, this study considers two different rated capacities and discharge times but with the same energy storage capability in order to provide a comparative analysis. The first storage facility ESS1 has a rated capacity of 30 MW and a discharge time of 1 hour, and the second ESS2 has a rated capacity of 15 MW and a discharge time of 2 hours. The two facilities are considered separately in conjunction with a 300 MW wind farm with the Toronto wind site data. The energy storage capabilities of both ESS1 and ESS2 are 300 MWh.

Figure 7.2 presents the probability distribution of the SOC for ESS1 at the beginning of Hour 9 for an initial wind power of 90 MW at Hour 8. Figure 7.2 shows the probability distribution of the new SOC for 50% and 100% of the initial SOC for ESS1. It can be seen that the probabilities of the ESS being at zero or at the rated capacity SOC are higher than those at the other intermediate SOC states. The probabilities of having rated SOC in the next hour for the 50% initial SOC are 0.357449 and 0.323680 for initial wind powers of 60 MW and 90 MW respectively and are due to surplus wind power in the first hour.

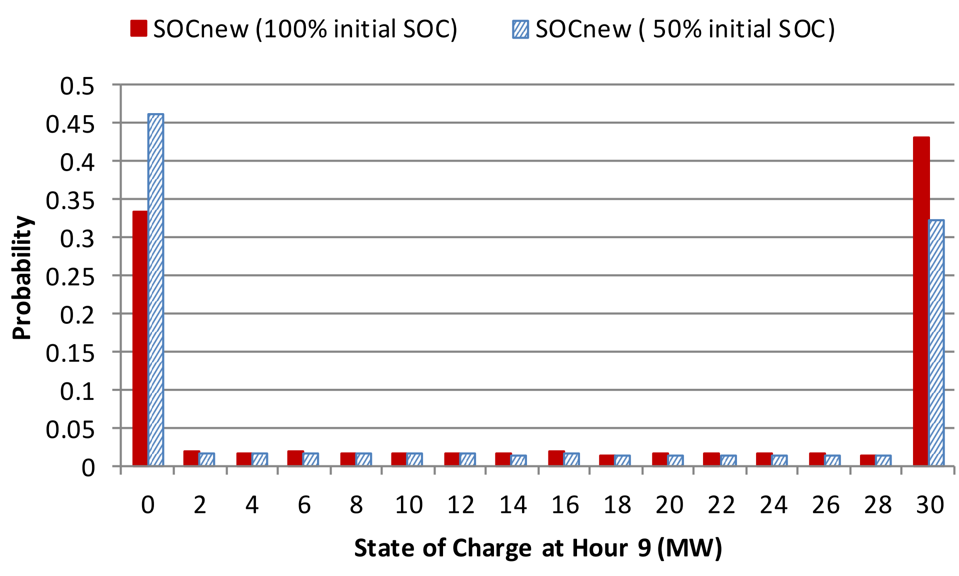

Figure 7.2: Probability distribution of the SOC in the next hour, initial power = 90 MW at Hour 8, ESS1 with 100% and 50% initial SOC

The two hour ahead wind capacity model is modified by each capacity state of the available SOC of the ESS and the resulting frequency distribution of the total power P_t is weighted by the probability associated with the capacity state of the available SOC. The weighted sum of the frequency distribution considering all the capacity state of the available SOC is used to build the probability distribution for the two hours ahead conditional wind power probability distribution including the ESS. The second storage option, ESS2 has a discharge time of two hours and can support the wind power with a capacity corresponding to its initial SOC for two hours. The WPCR for lead times of 1 and 2 hours are presented in Figure 7.3 considering both ESS1 and ESS2 at 100% SOC at the initial time. It can be seen, as expected, that the WPCR is reduced due to the storage. The WPCR without any storage is lower at Hour 10 compared to that at Hour 9 due to the rising wind trend starting at Hour 8. It can be seen that the reduction in WPCR is higher for ESS1compared to that for ESS2 for one hour lead time and is the opposite for the two hour lead time. It can also be seen that the WPCR increases when the initial power is increased from 60 MW to 90 MW for all the cases.

Figure 7.4 presents the WPCR when the wind power commitment is made for the initial condition at Hour 20. The falling wind trend is the reason that the WPCR at hour 21 and Hour 22 are higher than that at Hour 9 and Hour 10 which have a rising wind trend. The general impacts on WPCR of the different storage options are similar for both the rising and the falling wind trends.

7.5. Unit Commitment Risk Analysis Incorporating Wind Power and Storage

The studies conducted in the previous sections are focused on the impact of the uncertainty associated with committing wind power considering an ESS. An assessment of the impact on the overall operational power system reliability of adding an ESS to a wind farm can be conducted by evaluating the UCR. Unit commitment risk evaluation, as presented in Chapter 4, is conducted by creating a generation model containing the conventional units and the wind power. In this section, the total wind power model, containing the wind power and the energy storage facilities, presented in the previous sections is combined with the committed COPT to evaluate the UCR.

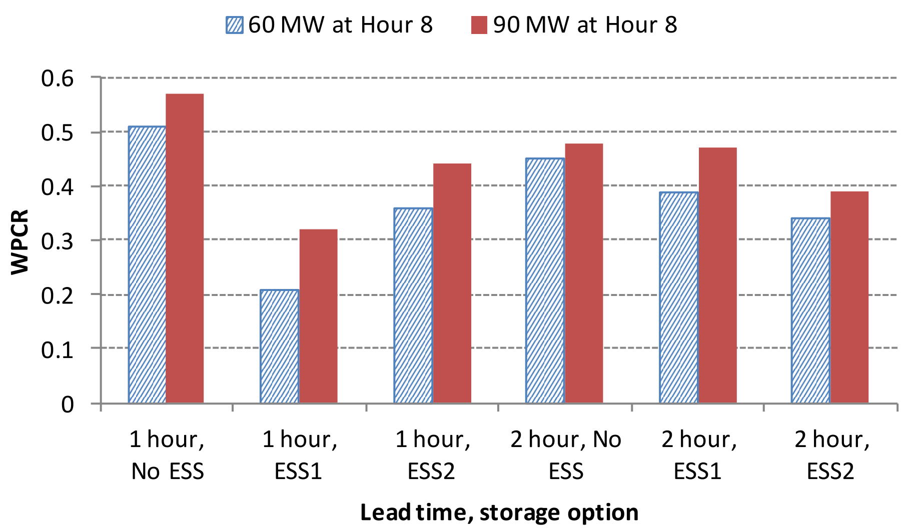

Figure 7.3: WPCR for lead times of 1 and 2 hours, initial time: Hour 8

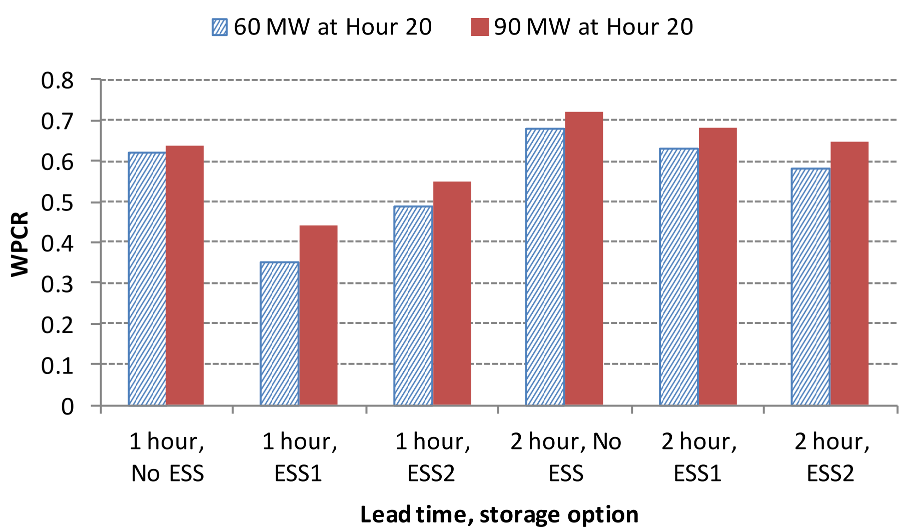

Figure 7.4: WPCR for lead times of 1 and 2 hours, initial time: Hour 20

The wind and energy storage system models presented in Sections 7.2 and 7.4 are combined with the generation model of the IEEE RTS. Table 7.6 presents the wind power capacity states and their probabilities considering the two energy storage options. This is based upon an operating strategy in which the energy storage system supplies power to the system if the wind power in the next one and two hours become less than the commitment level based on the persistence model and the system accepts wind power without any curtailment.

Table 7.6: One and two hour ahead wind power states and probabilities considering ESS1 and ESS2 for 60 MW initial wind power at Hour 8

Hour 9, ESS1		Hour 10, ESS1		Hour 9, ESS2		Hour 10, ESS2	
Capacity, (MW)	Probability	Capacity, (MW)	Probability	Capacity, (MW)	Probability	Capacity, (MW)	Probability
30	0.094117	0	0.045862	15	0.102066	15	0.139649
47	0.455166	19	0.095814	33	0.164224	33	0.132283
64	0.138975	38	0.206640	51	0.327138	51	0.237866
81	0.109538	57	0.201890	69	0.138546	69	0.109254
98	0.073247	76	0.108386	87	0.099662	87	0.089206
115	0.049807	95	0.088901	105	0.066013	105	0.074259
132	0.032507	114	0.071589	123	0.042049	123	0.057428
149	0.019942	133	0.053345	141	0.026343	141	0.045032
166	0.011113	152	0.040055	159	0.014897	159	0.033613
183	0.007401	171	0.029976	177	0.009400	177	0.026318
200	0.003665	190	0.022147	195	0.004617	195	0.018690
217	0.002261	209	0.013828	213	0.002404	213	0.011467
234	0.001118	228	0.009902	231	0.001428	231	0.009154
251	0.000452	247	0.006365	249	0.000500	249	0.005292
268	0.000357	266	0.004189	267	0.000381	267	0.003528
285	0.000333	285	0.001110	285	0.000333	285	0.006961

It can be seen from Table 7.6 that the minimum capacity states with ESS1 at Hour 9 is 30 MW, which is the rated discharge capacity of ESS1. The smallest capacity state in the second hour (Hour 10) is however zero MW and represents the state where both the wind power and the new SOC of ESS1 are zero. The minimum capacity state considering ESS2 is 15 MW in both Hour 9 and Hour 10 as ESS2 has a rated discharge capacity of 15 MW and can discharge at the rated capacity for two hours given that it was fully charged at the initial time. The area risk concept described in Chapter 4 is utilized to evaluate the UCR incorporating wind power and the ESS. A lead time of 2 hours is considered in this assessment.

Table 7.7 presents the UCR when 21 units in the priority loading order of the IEEE-RTS are committed and the initial wind power is 60 MW at Hour 8. The last column in Table 7.6 presents the UCR without energy storage. It can be seen from Table 7.7 that the UCR is reduced due to the ESS. The load carrying capability of the committed units and the wind power without storage for the UCR criterion of 0.001 is 2802.9 MW. The load carrying capability increases to 2811.9 MW and 2815.9 MW respectively due to ESS1and ESS2.

Table 7.7: Unit commitment risk analysis considering wind power and storage for a 2 hour lead time

Load	UCR x10^3, Initial wind power = 60 MW at Hour 8		
	ESS1	ESS2	No ESS
2750	0.104780	0.107140	0.103890
2760	0.110830	0.112550	0.111440
2770	0.158950	0.121710	0.534510
2780	0.167020	0.426300	0.543760
2790	0.253590	0.434450	0.880080
2800	0.424840	0.840100	0.886070
2810	0.607560	0.846170	1.210240
2820	1.436190	1.631390	1.420500

The following sections present the UCR assessments considering a limit imposed on wind power based on the wind power commitment made by the wind farm owner. The wind power model based on this strategy is explained in Section 7.2 where the ESS operated by the wind farm owner is applied to minimize the WPCR. The wind power model obtained using this operating policy, expressed in (7.4), is again used in this section for the UCR evaluation. The two cases of initial wind power of 60 MW and 90 MW at both Hour 8 and at Hour 20 are used to evaluate the UCR. Both ESS1 and ESS2 are considered to be at 100% SOC at the initial time in these case studies. The UCR is examined at two load levels. The high load scenario (HLS) has 21 units from the priority loading order of the IEEE RTS committed with a total conventional capacity of 3165 MW and the low load scenario (LLS) has 11 units from the priority loading order with a total capacity of 2096 MW.

Table 7.8 presents the discrete wind power capacity states and their probabilities including ESS1, for the lead times of one and two hours for an initial wind power of 60 MW. The SOC of the energy storage is 100%. The probability distribution of the wind power is modified by the state of charge of the storage system. It can be seen from Table 7.8 that the smallest capacity state for the one hour ahead distribution is 30 MW and the highest state is truncated at 60 MW, which is the committed power. The probabilities associated with the 60 MW capacity state are 0.792049 and 0.609199 for the one hour and two hour lead times respectively for the rising wind trend. The probabilities are 0.650094 and 0.372496 for one hour and two hour lead times respectively for the falling wind trend.

Table 7.9 shows the wind power states and their probabilities considering ESS2 for the initial wind power of 60 MW. The energy storage system can discharge the rated capacity of 15 MW for 2 hours for a 100% SOC at the initial time. The minimum capacity state is therefore 15

MW in both the one hour and two hour lead times. The combined (wind and storage) wind power distributions for an initial power of 90 MW are developed for both initial times of Hour 8 and Hour 20.

Table 7.8: One and two hour ahead wind power states and probabilities considering energy storage, ESS1 (Initial wind power = 60 MW)

Capacity States, MW (wind and Storage)	Probability, 1 hour ahead		Capacity States, MW (wind and Storage)	Probability, 2 hours ahead,	
	Initial Time: Hour 8	Initial Time: Hour 20		Initial Time: Hour 8	Initial Time: Hour 20
30	0.012514	0.058623	0	0.010805	0.059524
32	0.006875	0.020615	4	0.007169	0.026125
34	0.008779	0.021752	8	0.008441	0.027047
36	0.010444	0.019897	12	0.009626	0.026949
38	0.010087	0.021010	16	0.010783	0.028125
40	0.011253	0.020755	20	0.012169	0.028169
42	0.013013	0.020801	24	0.012995	0.028222
44	0.013632	0.020847	28	0.022302	0.051303
46	0.015440	0.020407	32	0.030739	0.065883
48	0.016344	0.022100	36	0.025365	0.043999
50	0.017795	0.020546	40	0.025958	0.040161
52	0.016725	0.020755	44	0.026762	0.037787
54	0.018033	0.019572	48	0.030073	0.038897
56	0.018985	0.021636	52	0.069270	0.058353
58	0.018033	0.020592	56	0.088343	0.066960
60	0.792049	0.650094	60	0.609199	0.372496

Table 7.10 presents the UCR evaluated using the area risk concept for the high load scenario (HLS) when the initial wind power is 60 MW and both ESS1 and ESS2 are at the 100% SOC. It can be seen from Table 7.10 that the UCR is reduced due to both the ESS1 and ESS2 but in most of the cases the UCR due to ESS1 is lower than that due to ESS2. It can also

be observed that the UCR including the ESS is higher at the falling wind trend, starting at Hour 20, compared to that at the rising wind trend, starting at Hour 8, at most load levels. Table 7.11 shows the UCR evaluated for the low load scenario (LLS) where the impacts of the ESS1 and ESS2 are similar to those at the HLS.

Table 7.9: One and two hour ahead wind power states and probabilities considering energy storage, ESS2

Initial 60 MW, ESS2					
Capacity States, MW (wind and Storage)	Probability, 1 hour ahead		Capacity States, MW (wind and Storage)	Probability, 2 hours ahead,	
	Initial Time: HR 8	Initial Time: HR 20		Initial Time: HR 8	Initial Time: HR 20
15	0.015749	0.068989	15	0.042466	0.142152
18	0.012419	0.032001	18	0.018128	0.041370
21	0.015606	0.030285	21	0.018104	0.039445
24	0.016177	0.031375	24	0.020103	0.037358
27	0.019794	0.031028	27	0.020031	0.034065
30	0.022291	0.031028	30	0.020531	0.033903
33	0.024980	0.032349	33	0.020935	0.033787
36	0.025884	0.031051	36	0.022220	0.030981
39	0.027454	0.030448	39	0.021768	0.029520
42	0.027597	0.031352	42	0.021340	0.028987
45	0.029167	0.029752	45	0.022743	0.027920
48	0.029095	0.028755	48	0.023005	0.027572
51	0.031379	0.029961	51	0.022981	0.027317
54	0.030499	0.029010	54	0.022149	0.025439
57	0.030142	0.026993	57	0.021911	0.022703
60	0.641766	0.505623	60	0.661583	0.417480

The increase in load carrying capability at a specified UCR criterion can be used to quantify the contribution of the ESS. The maximum load carrying capability (LCC) for a lead time of 2 hours during the HLS without wind power is 2763.9 MW and 2664.9 MW respectively for the UCR criteria of 0.001 and 0.0001. The LCC for the LLS is 1694.9 MW for both the UCR criteria of 0.001 and 0.0001 without wind power.

Table 7.10: Unit commitment risk considering wind and storage, HLS, initial wind power = 60 MW

UCR ($x10^3$), Lead time = 2 hours						
Load	Initial time: Hour 8			Initial time: Hour 20		
	ESS1	ESS2	No ESS	ESS1	ESS2	No ESS
2650	0.03730	0.03997	0.04560	0.03923	0.04638	0.05267
2660	0.03977	0.04310	0.04877	0.04494	0.05008	0.05678
2670	0.06552	0.06599	0.06948	0.06674	0.06791	0.07480
2680	0.07450	0.07491	0.07784	0.07386	0.07745	0.08257
2690	0.07516	0.07676	0.08077	0.07466	0.08099	0.08728
2700	0.07685	0.07886	0.08400	0.07808	0.08392	0.09135
2710	0.07907	0.08236	0.08793	0.08244	0.08943	0.09574
2720	0.09289	0.09516	0.09995	0.09560	0.10036	0.10766
2730	0.10782	0.10912	0.11280	0.10834	0.11313	0.11852
2740	0.10917	0.11166	0.11607	0.11057	0.11699	0.12290
2750	0.12206	0.12347	0.12673	0.12270	0.12701	0.13162
2760	0.12368	0.12608	0.12967	0.12559	0.13080	0.13479
2770	0.14132	0.12823	0.26266	0.12943	0.13330	0.53117
2780	0.17545	0.20916	0.40802	0.14463	0.39032	0.77403
2790	0.20708	0.33702	0.66478	0.14575	0.66008	1.10551
2800	0.30871	0.51425	0.85741	0.42605	0.91960	1.31398
2810	0.47985	0.78568	1.15002	0.75071	1.23882	1.59870
2820	0.72322	1.00606	1.41353	1.13076	1.46410	1.96060

Table 7.12 presents the LCC for the lead times of two hours when wind power is added to the IEEE RTS without any storage. The contribution of wind power can be observed from Table 7.12 and noted that the wind power contribution to the LCC at the rising trend starting at

Hour 8 is higher compared to that at the falling wind starting at Hour 20. Figure 7.5 presents the increase in load carrying capability (ILCC) due to the ESS for the two UCR criteria of 0.001 and 0.0001. The ILCC due to ESS1 and ESS2 are the same for the UCR criterion of 0.0001 for the rising wind trend. The ILCC due to ESS1 is higher than that due to ESS2 at the falling wind trend at the UCR criterion of 0.0001. The ILCC at the UCR criterion of 0.001 is however different at different times and storage options. At the initial wind power level of 60 MW, the ILCC considering ESS1 is higher than that with ESS2 for both the rising and the falling wind trends. The ILCC with ESS1 is found to be higher during a falling wind trend compared to a rising wind trend in this case.

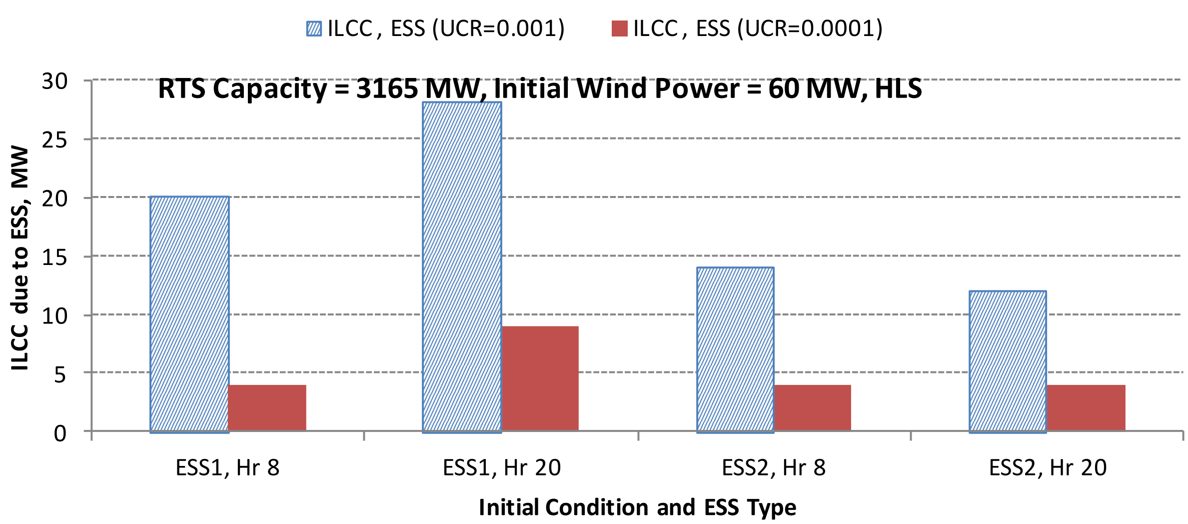

Figure 7.5: ILCC due to energy storage at the HLS, initial wind power 60 MW for a two hour lead time

The ILCC due to ESS1 and ESS2 for the initial wind power of 90 MW is presented in Figure 7.6. The ILCC with ESS1 is higher than that with ESS2 in this case as well. It can also be seen that the ILCC with ESS2 is higher with the falling wind trend compared to that with the rising wind trend and that the ILCC with ESS2 at the 0.0001 UCR criterion is zero.

Table 7.11: Unit commitment risk considering wind and storage, LLS, initial wind power = 60 MW

UCR ($x10^3$), Lead time = 2 hours						
Load	Initial time: Hour 8			Initial time: Hour 20		
	ESS1	ESS2	No ESS	ESS1	ESS2	No ESS
1600	0.03541	0.03804	0.04040	0.03941	0.04124	0.04386
1610	0.05092	0.05106	0.05137	0.05092	0.05138	0.05191
1620	0.05092	0.05124	0.05170	0.05092	0.05171	0.05231
1630	0.05101	0.05156	0.05195	0.05144	0.05215	0.05258
1640	0.05110	0.05182	0.05230	0.05196	0.05244	0.05291
1650	0.05122	0.05209	0.05327	0.05253	0.05272	0.05530
1660	0.05251	0.05451	0.05594	0.05451	0.05451	0.05793
1670	0.05251	0.05558	0.05723	0.05451	0.05734	0.05964
1680	0.05267	0.05645	0.05817	0.05595	0.05868	0.06069
1690	0.05298	0.05745	0.05962	0.05774	0.05991	0.06213
1700	0.05343	0.05890	0.19436	0.05993	0.06145	0.46573
1710	0.05882	0.06900	0.33663	0.06900	0.06902	0.70887
1720	0.05884	0.26458	0.59773	0.06903	0.59298	1.04632
1730	0.05830	0.43995	0.78952	0.34990	0.85361	1.25552
1740	0.05743	0.64373	1.08756	0.68122	1.10051	1.54560
1750	0.05618	0.94083	1.35398	1.06939	1.40843	1.90870
1760	0.94638	3.65761	3.78785	3.65887	3.66081	3.96878
1770	0.95108	3.76015	3.92015	3.66610	3.91792	4.13550

Table 7.12: Load carrying capability considering wind power, lead time = 2 hours

Initial Wind Power	UCRC	HLS, LCC, MW		LLS, LCC, MW	
		Hour 8	Hour 20	Hour 8	Hour 20
60 MW	UCR = 0.001	2804.9	2788.9	1739.9	1719.9
	UCR = 0.0001	2720.9	2715.9	1695.9	1695.9
90 MW	UCR = 0.001	2820.9	2800.9	1755.9	1737.9
	UCR = 0.0001	2749.9	2730.9	1701.9	1695.9

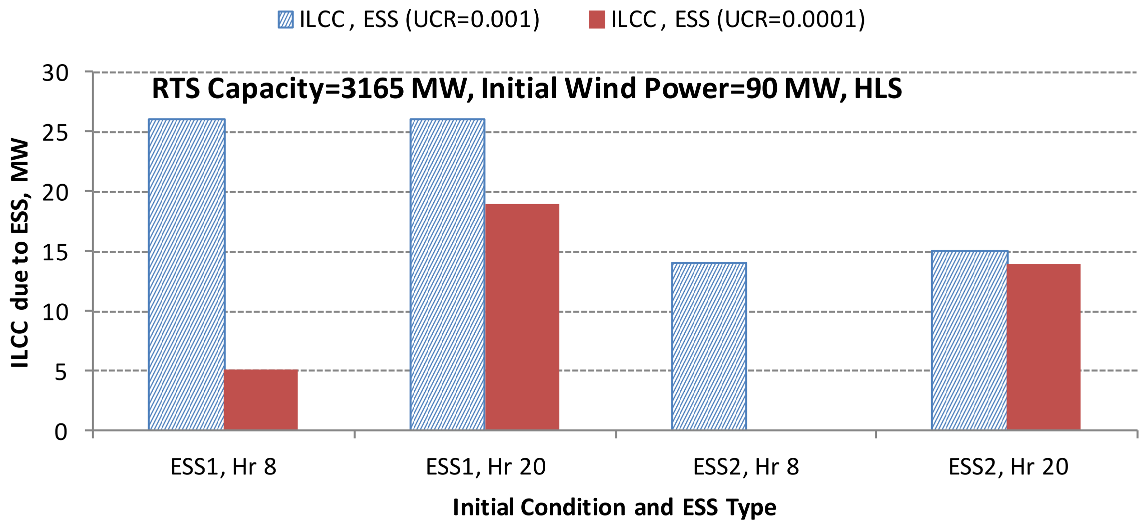

Figure 7.6: ILCC due to energy storage at the HLS and initial wind power of 90 MW for a two hour lead time

Figure 7.7 shows the ILCC due to the ESS for the LLS and 60 MW initial wind power. Similar to the ILCC shown in Figure 7.5 for the HLS, the ILCC due to ESS1 at the LLS is relatively high at the falling wind trend when the initial wind power is 60 MW. The ILCC with ESS2 is almost equal to the capacity associated with the initial SOC. Figure 7.8 shows the ILCC due to the ESS1 and ESS2 at the initial wind power of 90 MW for the LLS. It can be seen that there is no capacity benefit due to the ESS1 at the falling wind trend for the 0.0001 UCR criterion. It can be observed from Figures 7.5 to 7.8 that the ILCC due to ESS1 is higher than that of ESS2 but the ILCC due to ESS2 is relatively consistent at different initial wind power conditions, wind trends and load levels.

7.6. Summary

Energy storage can be employed in conjunction with wind power to reduce the uncertainty associated with wind power commitment. The storage facility is operated by the wind farm for the explicit purpose of minimizing the WPCR in this study. The short term wind power model is combined with the energy storage to create a combined model for one and two hour lead times based on a conditional probability approach. The contribution of energy storage is evaluated for two energy storage options with different rated capacities and discharge times but with the same energy storage capabilities.

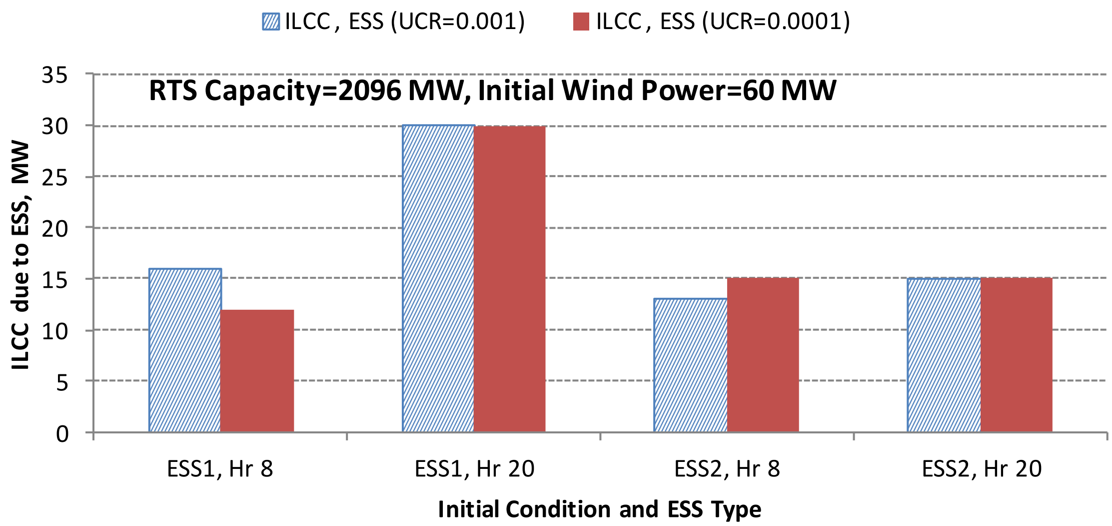

Figure 7.7: ILCC due to energy storage at the LLS and initial wind power of 60 MW for a two hour lead time

This study can be summarized in two parts. The first part presents a method to evaluate the WPCR considering storage. The ESS is aimed at minimizing the WPCR by adding stored energy to the wind power and keeping the total power close to the committed value. The support provided by the storage is limited by the state of charge and the rated capacity. It can be seen that the WPCR is notably reduced by the ESS and the reduction in the WPCR is different for different lead times and energy storage options. Based on the case study considered in this chapter, ESS1 with a higher discharge capacity was better option in regard to reducing the

WPCR for the lead time of one hour while the ESS2 having lower discharge capacity was better option for a lead time of two hours.

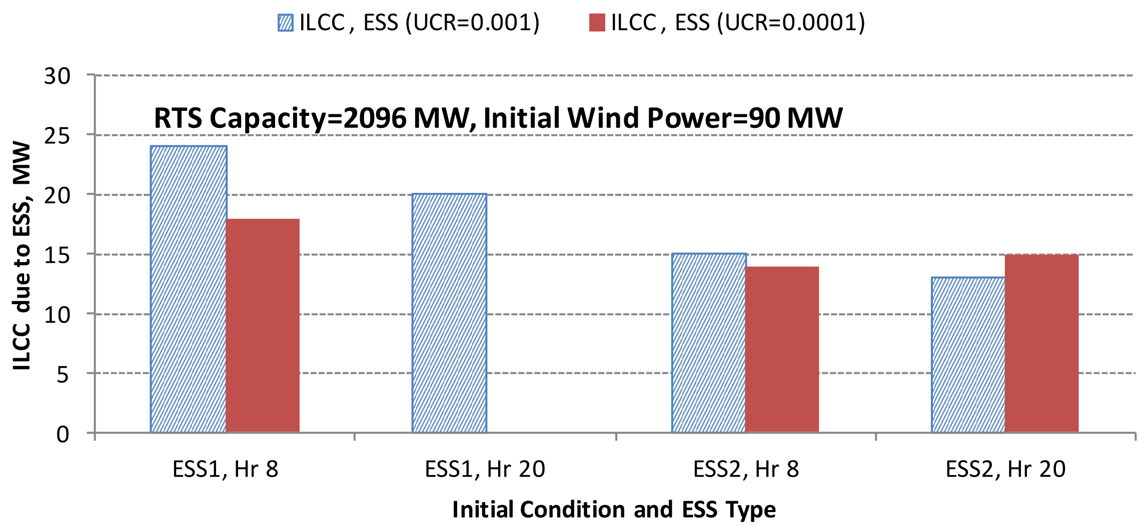

Figure 7.8: ILCC due to energy storage at the LLS and initial wind power of 90 MW for a two hour lead time

The impact of storage on the UCR was evaluated by combining the one and two hour ahead models of wind and storage with the capacity model of the IEEE RTS. A method based on the area risk concept was employed for the UCR evaluation. As expected, the unit commitment risk was reduced for both the energy storage options. The increases in load carrying capability (ILCC) of the system due to the two energy storage options were evaluated. The increase in load carrying capability while satisfying the specified UCR criterion due to the energy storage varied with the rated capacity of the storage, initial wind conditions and the load level. The energy storage option with the higher discharge capacity had in general the higher capacity contribution compared to that with the lower discharge capacity. The study indicates that energy storage has the potential to significantly offset the uncertainties of wind power but is limited by the rated capacity and the discharge time. An ESS can also mitigate wasting wind power in those situations where the operating policy allows wind power to be curtailed at high wind conditions or due to transmission congestions. The method presented in this study can prove useful to the wind farm operators and the power system operators in evaluating the operating risk of wind power considering energy storage. The numerical results provide an insight on the reliability benefits of employing an appropriate energy storage system in conjunction with a wind farm.

CHAPTER 8

SUMMARY AND CONCLUSION

The rapid growth of wind power, stimulated by public awareness and government policies towards mitigating the environmental impacts of electric power generation, has increased the need to evaluate the reliability impacts of wind power in electric power systems. Wind power is highly variable and the actual amount of wind power that will be available in the near future is not known. A power system with high wind penetration, therefore, experiences significant fluctuations in system generation that can adversely impact the operating system reliability. There is therefore a need to evaluate the risks in power system operation incorporating wind power and to evaluate the load carrying capability of the committed generating units including wind power while satisfying a specified reliability criterion.

The basic objective of this research work was to study the impacts of wind power integration on the operating reliability of electric power systems, and to develop methodologies to quantify the risks associated with operating decisions. Operating risk can be assessed from two perspectives. The first one is from the perspective of a wind farm operator, who is concerned with the risk in committing a certain wind power capacity to the system in a short future time. The second is from the perspective of a power system operator, where the total system risk with the integration of wind power is the prime concern. Risk evaluation of wind power commitment is directly related to quantifying wind power uncertainty. Risk evaluation in power system operation involves combining the uncertainty of wind power with the residual uncertainty of the conventional generating units in the power system.

The evaluation of operating risk considering wind power requires the development of suitable short term wind power models. The wind speed and therefore the wind power output at a short time in the future depends upon the wind speed at the present time. A conditional probability approach based on the initial wind speed has been applied to the time series wind speed data to create probability distributions that quantify the wind speed uncertainty for a short time in the future. The conditional wind speed distributions are converted to conditional wind power distributions using the wind turbine generator characteristics. The resulting capacity states and the associated probabilities are used to quantify the uncertainty associated with wind power generation. Appropriate models have been developed to consider the impact of lead times, diurnal and seasonal wind variations, and wind speed correlation. These models are used in risk evaluation from the perspective of operating a wind farm and from the perspective of operating a power system as a integrated entity.

It is necessary to estimate the amount of wind power that will be available at a short time in the future, such as one or two hours, in order to determine the operating reserve required to maintain the generating system reliability in a wind integrated power system. The wind power commitment from a wind farm for a short future time is usually specified as a percentage of its initial power output. The actual wind power can vary which depends on the characteristics of the wind site at the particular time of the day. The risk of committing a fixed capacity from a wind farm is designated as the wind power commitment risk (WPCR) in this thesis. It has been found that WPCR increases with the increase in the lead time. It also varies with the initial conditions and with the diurnal and seasonal wind trends. For the cases considered in this thesis, WPCR was lower during a rising wind trend than that during a falling wind trend. It was also observed, from the case studies conducted for the similar diurnal trend, that WPCR was lower during the summer time than that during the winter time. It was therefore noted that the wind power committed capacity should be modified or adjusted to maintain a WPCR acceptable to the system. Risk based wind power commitment therefore assesses the capacity value of wind power based on risk criteria deemed acceptable to the system.

Studies of the impacts of wind speed correlation on the WPCR have been conducted by simulating correlated wind speed data and found that wind speed correlation can have significant impacts on the perceived risk. The results show that the WPCR reduces as the correlation coefficient decreases. Wind speed correlation is an important factor that should therefore be incorporated in the evaluation when committing wind power from multiple wind farms.

A simplified approximate method based upon wind power commitment risk has been developed in this thesis which can assist the system operator and wind farm owner to commit wind power in the next few hour(s) based on knowledge of the initial available power. Risk based wind power commitment provides utilities and wind farm operators with an appreciation of the risks associated with wind capacity commitments in the next few hours, and helps them determine appropriate levels of wind power commitment at acceptable WPCR. The method outlined in this thesis is general and can be applied to a wide range of wind power situations and systems. The simplified method can be used with minimum system information and should be a helpful tool for wind power commitment in the next few hour(s).

As noted throughout the thesis, the capacity value of wind power in a short future time is driven by the initial conditions. Risk based methods are useful in assessing the capacity value of wind power as they allow the system operator to appropriately manage the short and long term system reserves. The conditional probability approach is applied to assess the WPCR and the capacity value constrained by the WPCR criteria for short future times such as 1-4 hours. The studies presented show that the impact of the initial conditions weaken as the lead time increases and the initial conditions are not the driving factor when long lead times such as 24 hours are considered. The historic hourly wind speed probability distributions without any consideration of the initial conditions can be used to assign day-ahead capacity values to a wind farm based on a suitable WPCR criterion. The method can be simplified using a normal wind speed probability distribution for the particular hour based on the mean wind speed and the standard deviation for the given hour. Sophisticated and complex methods of wind power prediction are not readily applied in practice. The approximate normal distribution method presented in this thesis can be easily applied and should prove useful for day-ahead unit scheduling. The conditional hourly models can be used for shorter term wind power commitment considering appropriate WPCR criteria.

Wind power is a variable resource and the reliability contribution of wind power should be assessed recognizing this variability. The area risk method developed to consider rapid start generating units in unit commitment risk (UCR) evaluation has been extended to consider wind power in this research. The existing reference method considers only the wind power variability at the end of the lead time. The UCR evaluated using the area risk method is consistently different from that obtained using the reference method and the assessed operating capacity credit (OCC) are also different. The Pennsylvania-New Jersey-Maryland (PJM) method for determining the spinning reserve requirement quantifies the uncertainty associated with the committed conventional units within the lead time. The area risk method developed in this research combines the uncertainties of wind power at different periods within a lead time and is a more appropriate method. The method is more capable, compared to the reference method, of assessing the wind power contribution when wind power varies within a lead time with a rising or falling wind trend or when random wind power variations within the lead time are significant.

The capacity contribution of wind power is often expressed as the increase in load carrying capability at a specified reliability criterion. It has been shown that the contribution is affected by the initial wind conditions and the reliability criterion selected. The contribution of wind power for a known initial condition and a selected reliability criterion is also affected by diurnal wind trends as a rising wind trend can offer a significantly high OCC compared to a falling wind trend.

The extended area risk method developed in this thesis is utilized to evaluate the impact on the UCR of adding wind farms. As expected, the operating capacity credit of wind power increases if the added wind farms are independent of each other. A method to incorporate correlated wind farms has been presented to quantify the system operating reliability benefit of statistically correlated wind farms. The study shows that correlation is an important factor and should be specifically considered rather than just assuming the wind sites to be either independent or dependent. The study also shows that if capacity expansion of a wind farm is considered, locating the added wind capacity at a geographic site with low wind correlation with respect to the existing wind installation will yield a higher operating capacity credit compared to that of a site close to or within the existing wind farm.

The area risk method developed for UCR evaluation has also been applied to evaluate the health index of a system incorporating wind power. The OCC considering the dual criteria of health and risk have been evaluated, and the results show that the wind power OCC decreases when satisfying a dual criterion and that the system well-being is affected by wind power diurnal trends.

The extension of the area risk method to incorporate wind power in unit commitment risk and health evaluation is one of the important contributions of this thesis, and the method is proposed as an appropriate technique to evaluate the reliability contribution of wind power in system operation. An approximate method was also developed to create conditional wind power distributions with minimum information using the basic statistics of historic wind speed data and the initial wind speed. The approximate method avoids applying wind speed simulation using an actual ARMA model and having the model run for each initial condition. The method should prove useful to electric power system operators in determining the capacity value of wind power at a short time in the future.

Response risk analysis is an important part of operating reserve analysis as it can be used to determine the actual dispatch of the committed units required for reliable system operation. A method to incorporate wind power in evaluating response risk is presented in this thesis. Due to its inherent uncertainty, wind power cannot be dispatched like a conventional generating unit. Wind power can be utilized as and when available or be wasted. In order to utilize wind power, the dispatch of the conventional units must be modified in a manner that allows the system to tolerate the additional variability created by the addition of wind power. The research described in this thesis considers two types of dispatch: economic load dispatch (ELD) and response risk (RR) constrained dispatch considering wind power. The ten minute ahead wind power variability is represented by a persistence model with a normally distributed prediction error with mean zero and a 10% standard deviation. Ten minute wind speed data from the US National Renewable Energy Laboratory was used to estimate the standard deviation of the 10 minute ahead wind speed prediction error. The wind power and its variability, quantified by the multi- state wind power states and their corresponding probabilities, is combined with the forecast load, and the dispatch optimized to satisfy the net load requirements.

It has been found that the operating cost increase when the ELD has to be modified to satisfy the response risk criterion. The marginal cost of the last committed unit is considered as the cost of wind power. The addition of wind power causes one or more conventional units to reduce their output and the cost of the ELD reduces as a result of the difference between the cost of wind power and the cost that would have occurred without wind power. The cost benefit of the response risk constrained dispatch depends upon the operating criterion and the wind power level. An increase in the regulating margin requirement, which in this study is from 40% to 50% of the spinning reserve (SR), increases the cost of operation considering wind power. Based on the case studies conducted, the cost of the response risk constrained dispatch also increases if the wind power increases to a high level as one or more less expensive units will be forced to reduce their output in order to provide the required regulating margin. The numerical results also show that the response risk of the economic load dispatch is reduced at low wind power but the risk increases as the wind power increases beyond a certain value.

Utilization of energy storage in combination with wind power can reduce the uncertainty associated with wind power commitment. A study has been conducted in this research involving energy storage for the explicit purpose of minimizing the wind power commitment risk (WPCR). A short term wind power model incorporating energy storage has been developed in this research for different lead times based upon a conditional probability approach. The reliability contribution of energy storage has been evaluated for energy storage options with different rated capacities and discharge times but with the same energy storage capabilities. The energy storage is utilized to support the wind farm output and reduce the risk associated with persistence model based wind power commitment. The support provided by the storage is limited by the initial state of charge and the rated capacity. It has been observed that the wind power commitment risk (WPCR) is notably reduced by utilizing energy storage, and the reduction in the WPCR is different at different lead times and energy storage options. An energy storage facility with higher discharge capacity is a better option with regard to reducing the WPCR for a lead time of one hour while one with lower discharge capacity is a better option for a lead time of two hours.

The impact of storage on the unit commitment risk (UCR) was evaluated by integrating the combined one and two hour ahead models of wind and storage with the capacity model of the conventional units. A method based upon the area risk concept was developed in this research for UCR evaluation incorporating wind and energy storage. Two different energy storage options were separately considered. The first option has a higher discharge capacity and lower discharging time but the second option has a lower discharge capacity and a higher discharging time and both have the same energy storage capacity. As expected, UCR was reduced for both energy storage options. The increases in load carrying capability (ILCC) of the system for the two energy storage options were evaluated. The ILCC due to the energy storage while satisfying the specified UCR criterion varied with the rated capacity of the storage, initial wind conditions and the load level. The energy storage option with the higher discharge capacity had in general a higher capacity contribution compared to that of a lower discharge capacity. The study indicates that energy storage provides the ability to offset the residual uncertainty associated with wind power. The benefits however, are limited by the rated capacity and discharge time of the storage facility. Energy storage can also serve to store unutilized wind power if operating policies limit the utilization of wind power. The method presented in this thesis should prove useful to wind farm operators and power system operators in evaluating the operating risks of wind power considering energy storage. The numerical results also provide insight on the reliability and economic benefits of employing energy storage in conjunction with a wind farm.

In conclusion, the research described in this thesis has resulted in methods to evaluate various system operating reliability metrics; namely wind power commitment risk, unit commitment risk, operating health, margin and risk, and response risk indices incorporating wind power. The extended area risk concept, developed in this research, to incorporate wind power in unit commitment risk and health evaluation is a significant achievement of this research. The simplified risk based method for wind power commitment and the approximate methods for unit commitment risk evaluation are other valuable outcomes of this research. The response risk analysis presented in this thesis provides a method to assess the reliability and economic impacts of wind power in actual unit dispatch and is another creditable contribution of this research. A method to integrate energy storage, in operating risk evaluation, in combination with a wind farm has also been developed. The noted methods should prove useful to system operators in making decisions related to unit commitment and dispatch in a wind integrated power system based upon specified risk criteria. The specific conclusions related to the case studies described in this thesis should also prove to be valuable to researchers and system operators working in this area.

LIST OF REFERENCES

[1] R. Billinton and R. N. Allan, *Reliability Evaluation of Power Systems.* New York: Plenum, 1996.

[2] R. Billinton and R. N. Allan, "Power-system reliability in perspective," *Electronics and Power,* vol. 30, pp. 231-236, 1984.

[3] G. Calabrese, "Generating reserve capacity determined by the probability method," *American Institute of Electrical Engineers, Transactions of the,* vol. 66, pp. 1439-1450, 1947.

[4] R. Billinton, *Power System Reliability Evaluation.* Gordon and Breach Science Publishers, 1970.

[5] R. Billinton, "Bibliography on the Application of Probability Methods In Power System Reliability Evaluation," *IEEE Transactions on Power Apparatus and Systems,* vol. PAS-91, pp. 649-660, 1972.

[6] Power System Engineering Committee, "Bibliography on the Application of Probability Methods in Power System Reliability Evaluation 1971-1977," *IEEE Transactions on Power Apparatus and Systems,* vol. PAS-97, pp. 2235-2242, 1978.

[7] R. Billinton and R. N. Allan, *Reliability Evaluation of Power Systems.* Pitman Books, 1984.

[8] R. N. Allan, R. Billinton, S. M. Shahidehpour and C. Singh, "Bibliography on the application of probability methods in power system reliability evaluation: 1982-7," *IEEE Transactions on Power Systems,* vol. 3, pp. 1555-1564, 1988.

[9] R. Billinton, R. N. Allan and L. Salvaderi, "Applied reliability assessment in electric power systems," 1991.

[10] R. N. Allan, R. Billinton, A. M. Breipohl and C. H. Grigg, "Bibliography on the application of probability methods in power system reliability evaluation: 1987-1991," *IEEE Transactions on Power Systems,* vol. 9, pp. 41-49, 1994.

[11] R. Billinton, M. Fotuhi-Firuzabad and L. Bertling, "Bibliography on the application of probability methods in power system reliability evaluation 1996-1999," *IEEE Transactions on Power Systems,* vol. 16, pp. 595-602, 2001.

[12] M. T. Schilling, R. Billinton and M. G. dos Santos, "Bibliography on power systems probablistic security analysis 1968-2008," *International Journal of Emerging Electric Power Systems,* vol. 10, May 2009.

[13] L. T. Anstine, R. E. Burke, J. E. Casey, R. Holgate, R. S. John and H. G. Stewart, "Application of Probability Methods to the Determination of Spinning Reserve Requirements for the Pennsylvania-New Jersey-Maryland Interconnection," *IEEE Transactions on Power Apparatus and Systems,* vol. 82, pp. 726-735, 1963.

[14] R. Billinton and M. P. Musick, "Spinning Reserve Criteria in a Hydro Thermal System by the Application of Probability Mathematics," *The Engineering Journal, the Journal of the Engineering Institute of Canada*, pp. 1-6, 1965.

[15] R. Billinton and R. N. Allan, *Reliability Evaluation of Engineering Systems.* Plenum Press New York, 1983.

[16] R. Billinton and A. V. Jain, "The Effect of Rapid Start and Hot Reserve Units In Spinning Reserve Studies," *IEEE Transactions on Power Apparatus and Systems,* vol. PAS-91, pp. 511-516, 1972.

[17] A. V. Jain and R. Billinton, "Unit committment reliability in a hydro-thermal system," in *IEEE PES Winter Meeting,* New York, N.Y., 1973, pp. 1-8.

[18] R. Billinton and A. V. Jain, "Interconnected System Spinning Reserve Requirements," *IEEE Transactions on Power Apparatus and Systems,* vol. PAS-91, pp. 517-525, 1972.

[19] R. Billinton and A. V. Jain, "Power System Spinning Reserve Determination in a Multi System Configuration," *IEEE Transactions on Power Apparatus and Systems,* vol. PAS-92, pp. 433-441, 1973.

[20] M. E. Khan and R. Billinton, "Composite system spinning reserve assessment in interconnected systems," *IEE Proceedings- Generation, Transmission and Distribution,* vol. 142, pp. 305-309, 1995.

[21] M. Fotuhi-Firuzabad and R. Billinton, "Impact of load management on composite system reliability evaluation short-term operating benefits," *IEEE Transactions on Power Systems,* vol. 15, pp. 858-864, 2000.

[22] R. Billinton and A. V. Jain, "Unit Derating Levels in Spinning Reserve Studies," *IEEE Transactions on Power Apparatus and Systems,* vol. PAS-90, pp. 1677-1687, 1971.

[23] R. Billinton and M. Alam, "Outage postponability effects in operating capacity reliability studies," in *IEEE PES Winter Meeting, New York, NY,* 1978, .

[24] A. D. Patton, "Short-Term Reliability Calculation," *IEEE Transactions on Power Apparatus and Systems,* vol. PAS-89, pp. 509-513, 1970.

[25] A. D. Patton, "A Probability Method for Bulk Power System Security Assessment, I-Basic Concepts," *IEEE Transactions on Power Apparatus and Systems,* vol. PAS-91, pp. 54-61, 1972.

[26] A. D. Patton, "A Probability Method for Bulk Power System Security Assessment, II - Development of Probability Models for Normally-Operating Components," *IEEE Transactions on Power Apparatus and Systems,* vol. PAS-91, pp. 2480-2485, 1972.

[27] A. D. Patton, "A Probability Method for Bulk Power System Security Assessment - III - Models for Stand-by Generators and Field Data Collection and Analysis," *IEEE Transactions on Power Apparatus and Systems,* vol. PAS-91, pp. 2486-2493, 1972.

[28] J. Hall, R. Ringlee and A. Wood, "Frequency and duration methods for power system reliability calculations: I-Generation system model," *IEEE Transactions on Power Apparatus and Systems,* pp. 1787-1796, 1968.

[29] P. M. Subcommittee, "IEEE Reliability Test System," *IEEE Transactions on Power Apparatus and Systems,* vol. PAS-98, pp. 2047-2054, 1979.

[30] A.V.Jain and R.Billinton, "Spinning reserve allocation in a complex power system," in *IEEE Winter Power Meeting,* New York, N.Y., 1973, C 73 097-3, pp. 1-8.

[31] W. Stadlin, "Economic allocation of regulating margin," *IEEE Transactions on Power Apparatus and Systems,* pp. 1776-1781, 1971.

[32] R. Billinton and M. Fotuhi-Firuzabad, "A basic framework for generating system operating health analysis," *IEEE Transactions on Power Systems,* vol. 9, pp. 1610-1617, 1994.

[33] R. Billinton and R. Karki, "Capacity reserve assessment using system well-being analysis," *IEEE Transactions on Power Systems,* vol. 14, pp. 433-438, 1999.

[34] A. L. da Silva, L. C. de Resende, da Fonseca Manso, Luiz Antonio and R. Billinton, "Well-being analysis for composite generation and transmission systems," *IEEE Transactions on Power Systems,* vol. 19, pp. 1763-1770, 2004.

[35] Bagen and R. Billinton, "Incorporating well-being considerations in generating systems using energy storage," *IEEE Transactions on Energy Conversion,* vol. 20, pp. 225-230, 2005.

[36] W. Wangdee and R. Billinton, "Bulk electric system well-being analysis using sequential Monte Carlo simulation," *IEEE Transactions on Power Systems,* vol. 21, pp. 188-193, 2006.

[37] R. Billinton and M. Fotuhi-Firuzabad, "Generating system operating health analysis considering stand-by units, interruptible load and postponable outages," *IEEE Transactions on Power Systems,* vol. 9, pp. 1618-1625, 1994.

[38] M. Fotuhi-Firuzabad, R. Billinton and S. Aboreshaid, "Spinning reserve allocation using response health analysis," *IEE Proceedings- Generation, Transmission and Distribution,* vol. 143, pp. 337-343, 1996.

[39] R. Billinton, M. Fotuhi-Firuzabad and S. Aboreshaid, "Power system health analysis," *Reliab. Eng. Syst. Saf.,* vol. 55, pp. 1-8, 1, 1997.

[40] R. Billinton, M. Fotuhi-Firuzabad and S. Aboreshaid, "Unit commitment health analysis for interconnected systems," *IEEE Transactions on Power Systems,* vol. 12, pp. 1194-1201, 1997.

[41] M. Fotuhi-Firuzabad and R. Billinton, "Unit commitment health analysis in composite generation and transmission systems considering stand-by units," *IEE Proceedings- Generation, Transmission and Distribution,* vol. 146, pp. 164-168, 1999.

[42] U.S. Environment Protection Agency. (April, 2009). *Renewable Portfolio Standards Fact Sheet*. Available: http: // www.epa.gov/chp/stat-policy/renewable_fs.html.

[43] *Global Wind Energy Outlook 2012*. Available: http://www.gwec.net/publications/global-wind-energy-outlook/.

[44] *Wind Vision 2025, powering Canada's Future*. Available: http://canwea.ca/windvision_e.php.

[45] T. Ackermann, *Wind Power in Power Systems.* John Wiley &Sons, Ltd., 2005.

[46] R. Billinton and A. A. Chowdhury, "Incorporation of wind energy conversion systems in conventional generating capacity adequacy assessment," *IEE Proceedings on Generation, Transmission and Distribution,* vol. 139, pp. 47-56, 1992.

[47] R. Billinton, H. Chen and R. Ghajar, "Time-series models for reliability evaluation of power systems including wind energy," *Microelectron. Reliab.,* vol. 36, pp. 1253-1261, 9, 1996.

[48] R. Billinton, Hua Chen and R. Ghajar, "A sequential simulation technique for adequacy evaluation of generating systems including wind energy," *IEEE Transactions on Energy Conversion,* vol. 11, pp. 728-734, 1996.

[49] F. Castro Sayas and R. N. Allan, "Generation availability assessment of wind farms," *IEE Proceedings- Generation, Transmission and Distribution,* vol. 143, pp. 507-518, 1996.

[50] R. Billinton and Guang Bai, "Generating capacity adequacy associated with wind energy," *IEEE Transactions on Energy Conversion,* vol. 19, pp. 641-646, 2004.

[51] R. Billinton and Y. Li, "Incorporating multistate unit models in composite system adequacy assessment," in *International Conference on Probabilistic Methods Applied to Power Systems, 2004,* 2004, pp. 70-75.

[52] R. Karki and R. Billinton, "Cost-effective wind energy utilization for reliable power supply," *IEEE Transactions on Energy Conversion,* vol. 19, pp. 435-440, 2004.

[53] A. A. Chowdhury, "Reliability models for large wind farms in generation system planning," in *IEEE Power Engineering Society General Meeting, 2005.* 2005, pp. 1926-1933 Vol. 2.

[54] R. Karki and Po Hu, "Wind power simulation model for reliability evaluation," in *Canadian Conference on Electrical and Computer Engineering, 2005.* 2005, pp. 541-544.

[55] M. Haghifam and M. Omidvar, "Wind farm modeling in reliability assessment of power system," in *International Conference on Probabilistic Methods Applied to Power Systems, PMAPS 2006.* 2006, pp. 1-5.

[56] R. Karki, Po Hu and R. Billinton, "A simplified wind power generation model for reliability evaluation," *IEEE Transactions on Energy Conversion,* vol. 21, pp. 533-540, 2006.

[57] R. Billinton and W. Wangdee, "Reliability-Based Transmission Reinforcement Planning Associated With Large-Scale Wind Farms," *IEEE Transactions on Power Systems,* vol. 22, pp. 34-41, 2007.

[58] N. B. Negra, O. Holmstrom, B. Bak-Jensen and P. Sorensen, "Aspects of Relevance in Offshore Wind Farm Reliability Assessment," *IEEE Transactions on Energy Conversion,* vol. 22, pp. 159-166, 2007.

[59] W. Wangdee and R. Billinton, "Reliability assessment of bulk electric systems containing large wind farms," *International Journal of Electrical Power & Energy Systems,* vol. 29, pp. 759-766, 12, 2007.

[60] R. Billinton and Yi Gao, "Multistate Wind Energy Conversion System Models for Adequacy Assessment of Generating Systems Incorporating Wind Energy," *IEEE Transactions on Energy Conversion,* vol. 23, pp. 163-170, 2008.

[61] R. Karki, P. Hu and R. Billinton, "Reliability assessment of a wind integrated hydro-thermal power system," in *2010 IEEE 11th International Conference on Probabilistic Methods Applied to Power Systems (PMAPS),* 2010, pp. 265-270.

[62] S. H. Karaki, B. A. Salim and R. B. Chedid, "Probabilistic model of a two-site wind energy conversion system," *IEEE Transactions on Energy Conversion,* vol. 17, pp. 530-536, 2002.

[63] W. Wangdee and R. Billinton, "Considering load-carrying capability and wind speed correlation of WECS in generation adequacy assessment," *IEEE Transactions on Energy Conversion,* vol. 21, pp. 734-741, 2006.

[64] M. S. Miranda and R. W. Dunn, "Spatially correlated wind speed modelling for generation adequacy studies in the UK," in *IEEE Power Engineering Society General Meeting, 2007.* 2007, pp. 1-6.

[65] R. Billinton, Yi Gao and R. Karki, "Composite System Adequacy Assessment Incorporating Large-Scale Wind Energy Conversion Systems Considering Wind Speed Correlation," *IEEE Transactions on Power Systems,* vol. 24, pp. 1375-1382, 2009.

[66] C. Singh and A. Lago-Gonzalez, "Reliability Modeling of Generation Systems Including Unconventional Energy Sources," *IEEE Power Engineering Review,* vol. PER-5, pp. 33-33, 1985.

[67] J. Haslett and M. Diesendorf, "The capacity credit of wind power: A theoretical analysis," *Solar Energy,* vol. 26, pp. 391-401, 1981.

[68] R. Billinton and H. Chen, "Assessment of risk-based capacity benefit factors associated with wind energy conversion systems," *IEEE Transactions on Power Systems,* vol. 13, pp. 1191-1196, 1998.

[69] R. M. G. Castro and L. A. F. M. Ferreira, "A comparison between chronological and probabilistic methods to estimate wind power capacity credit," *IEEE Transactions on Power Systems,* vol. 16, pp. 904-909, 2001.

[70] K. R. Voorspools and W. D. D'haeseleer, "An analytical formula for the capacity credit of wind power," *Renewable Energy,* vol. 31, pp. 45-54, 1, 2006.

[71] C. Ensslin, M. Milligan, H. Holttinen, M. O'Malley and A. Keane, "Current methods to calculate capacity credit of wind power, IEA collaboration," in *IEEE Power and Energy Society General Meeting - Conversion and Delivery of Electrical Energy in the 21st Century, 2008,* 2008, pp. 1-3.

[72] R. Billinton, D. Huang and B. Karki, "Wind power planning and operating capacity credit assessment," in *IEEE 11th International Conference on Probabilistic Methods Applied to Power Systems (PMAPS), 2010,* 2010, pp. 814-819.

[73] L. Soder, "Reserve margin planning in a wind-hydro-thermal power system," *IEEE Transactions on Power Systems,* vol. 8, pp. 564-571, 1993.

[74] G. Dany, "Power reserve in interconnected systems with high wind power production," in *IEEE Power Tech Proceedings, Porto 2001,* 2001, pp. 6 pp. vol.4.

[75] E. M. Gouveia and M. A. Matos, "Operational reserve of a power system with a large amount of wind power," in *Probabilistic Methods Applied to Power Systems, 2004 International Conference on*, 2004, pp. 717-722.

[76] R. Doherty, E. Denny and M. O'Malley, "System operation with a significant wind power penetration," in *Power Engineering Society General Meeting, 2004. IEEE,* 2004, pp. 1002-1007 Vol.1.

[77] R. Doherty and M. O'Malley, "A new approach to quantify reserve demand in systems with significant installed wind capacity," *IEEE Transactions on Power Systems,* vol. 20, pp. 587-595, 2005.

[78] R. Barth, H. Brand, P. Meibom and C. Weber, "A stochastic unit-commitment model for the evaluation of the impacts of integration of large amounts of intermittent wind power," in *Probabilistic Methods Applied to Power Systems, 2006. PMAPS 2006. International Conference on,* 2006, pp. 1-8.

[79] B. C. Ummels, M. Gibescu, E. Pelgrum, W. L. Kling and A. J. Brand, "Impacts of Wind Power on Thermal Generation Unit Commitment and Dispatch," *IEEE Transactions on Energy Conversion,* vol. 22, pp. 44-51, 2007.

[80] T. Lee, "Optimal spinning reserve for a wind-thermal power system using EIPSO," *IEEE Transactions on Power Systems,* vol. 22, pp. 1612-1621, 2007.

[81] Jianhui Wang, M. Shahidehpour and Zuyi Li, "Security-Constrained Unit Commitment With Volatile Wind Power Generation," *IEEE Transactions on Power Systems,* vol. 23, pp. 1319-1327, 2008.

[82] Chun-Lung Chen, "Optimal Wind–Thermal Generating Unit Commitment," *IEEE Transactions on Energy Conversion,* vol. 23, pp. 273-280, 2008.

[83] F. Bouffard and F. D. Galiana, "Stochastic security for operations planning with significant wind power generation," in *Power and Energy Society General Meeting-Conversion and Delivery of Electrical Energy in the 21st Century, 2008 IEEE,* 2008, pp. 1-11.

[84] A. Tuohy, P. Meibom, E. Denny and M. O'Malley, "Unit Commitment for Systems With Significant Wind Penetration," *IEEE Transactions on Power Systems,* vol. 24, pp. 592-601, 2009.

[85] V. S. Pappala, I. Erlich, K. Rohrig and J. Dobschinski, "A stochastic model for the optimal operation of a wind-thermal power system," *IEEE Transactions on Power Systems,* vol. 24, pp. 940-950, 2009.

[86] M. A. Ortega-Vazquez and D. S. Kirschen, "Estimating the spinning reserve requirements in systems with significant wind power generation penetration," *IEEE Transactions on Power Systems,* vol. 24, pp. 114-124, 2009.

[87] M. Milligan, P. Donohoo, D. Lew, E. Ela, B. Kirby, H. Holttinen, E. Lannoye, D. Flynn, M. O'Malley and N. Miller, "Operating reserves and wind power integration: An international comparison," in *Proc. 9th International Workshop on Large-Scale Integration of Wind Power into Power Systems,* 2010, pp. 18-29.

[88] E. Ela, B. Kirby, E. Lannoye, M. Milligan, D. Flynn, B. Zavadil and M. O'Malley, "Evolution of operating reserve determination in wind power integration studies," in *Power and Energy Society General Meeting, 2010 IEEE,* 2010, pp. 1-8.

[89] J. Xiao, B. S. Hodge, J. F. Pekny and G. V. Reklaitis, "Operating reserve policies with high wind power penetration," *Comput. Chem. Eng.,* vol. 35, pp. 1876-1885, 2011.

[90] A. Papavasiliou, S. S. Oren and R. P. O'Neill, "Reserve requirements for wind power integration: A scenario-based stochastic programming framework," *IEEE Transactions on Power Systems,* vol. 26, pp. 2197-2206, 2011.

[91] M. A. Matos and R. J. Bessa, "Setting the operating reserve using probabilistic wind power forecasts," *IEEE Transactions on Power Systems,* vol. 26, pp. 594-603, 2011.

[92] G. Geibel, R. Brownsword and G. Kariniotakis. The state-of-the art in short-term prediction of wind power, A literature overview. Project ANEMOS. August 2003Available: http://anemos.cma.fr/download/ANEMOS_D1.1_StateOfTheArt_v1.1.pdf.

[93] G. Giebel et al. The state-of-the-art in short-term prediction of wind power A literature overview, 2nd edition. ANEMOS.plus and SafeWind projects. 2011Available: http://orbit.dtu.dk/getResource?recordId=274635&objectId=1&versionId=1.

[94] A. M. Foley, P. G. Leahy, A. Marvuglia and E. J. McKeogh, "Current methods and advances in forecasting of wind power generation," *Renewable Energy,* vol. 37, pp. 1-8, 1, 2012.

[95] M. Lange, "Physical Approach to Short-Term Wind Power Prediction," 2006.

[96] B. Ernst, B. Oakleaf, M. L. Ahlstrom, M. Lange, C. Moehrlen, B. Lange, U. Focken and K. Rohrig, "Predicting the Wind," *IEEE Power and Energy Magazine,* vol. 5, pp. 78-89, 2007.

[97] J. W. Taylor, P. E. McSharry and R. Buizza, "Wind Power Density Forecasting Using Ensemble Predictions and Time Series Models," *IEEE Transactions on Energy Conversion*, vol. 24, pp. 775-782, 2009.

[98] P. Pinson, H. A. Nielsen, H. Madsen and G. Kariniotakis, "Skill forecasting from ensemble predictions of wind power," *Appl. Energy*, vol. 86, pp. 1326-1334, 2009.

[99] G.E.P.Box and G.M. Jenkins, *Time Series Analysis;Forecasting and Control.* San Francisco: Holden-Day, 1970.

[100] S.M. Pandit and S.M. Wu, *Time Series and System Analysis, with Applications.* New York: John Wiley, 1983.

[101] T. S. Nielsen, A. Joensen, H. Madsen, L. Landberg and G. Giebel, "A new reference for wind power forecasting. ," *Wind Energ.*, vol. 1, pp. 29-34, 1998.

[102] L. Landberg, "Short-term prediction of the power production from wind farms," *J. Wind Eng. Ind. Aerodyn.*, vol. 80, pp. 207-220, 3/1, 1999.

[103] L. Landberg, "Short-term prediction of local wind conditions," *J. Wind Eng. Ind. Aerodyn.*, vol. 89, pp. 235-245, 3, 2001.

[104] M. Milligan, M. Schwartz and Y. Wan. Statistical wind power forecasting models: Results for U.S. wind farms. National Renewable Energy Laboratory. May 2003Available: http://www.nrel.gov/docs/fy03osti/33956.pdf.

[105] A. Boone. Simulation of short-term wind speed forecast errors using a multi-variate ARMA(1,1) time-series model. 2005.

[106] J. L. Torres, A. García, M. De Blas and A. De Francisco, "Forecast of hourly average wind speed with ARMA models in Navarre (Spain)," *Solar Energy*, vol. 79, pp. 65-77, 7, 2005.

[107] R. Billinton, B. Karki, R. Karki and G. Ramakrishna, "Unit Commitment Risk Analysis of Wind Integrated Power Systems," *IEEE Transactions on Power Systems*, vol. 24, pp. 930-939, 2009.

[108] EPRI-DOE, "*Handbook supplement of energy storage for grid connected wind Generation Applications*," EPRI, Palo Alto, CA, and the U.S. Department of Energy, Washington, DC, Tech. Rep. 1008703., 2004.

[109] Electric Power Research Institute, "*Electricity energy storage technology Options: A white paper primer on applications, costs and benefits.* ," EPRI, Paulo Alto, CA, Tech. Rep. 1020676, 2010.

[110] Electric Power Research Institute, "*Wind power integration: Smoothing short-term power fluctuations*," EPRI, Palo Alto, CA, Tech. Rep. 1008852, 2005.

[111] C. Bueno and J. A. Carta, "Technical–economic analysis of wind-powered pumped hydrostorage systems. Part I: model development," *Solar Energy*, vol. 78, pp. 382-395, 3, 2005.

[112] C. Bueno and J. A. Carta, "Technical–economic analysis of wind-powered pumped hydrostorage systems. Part II: model application to the island of El Hierro," *Solar Energy,* vol. 78, pp. 396-405, 3, 2005.

[113] F. Crotogino, K. U. Mohmeyer and R. Scharf, "Huntorf CAES:More than 20 years of successful operation," in *Solution Mining Res. Inst. Spring 2001 Meeting* , Orlando, Florida, USA, 2001, .

[114] Derk J. Swider, "Compressed Air Energy Storage in an Electricity System With Significant Wind Power Generation," *IEEE Transactions on Energy Conversion,* vol. 22, pp. 95-102, 2007.

[115] A. Tuohy and M. O'Malley, "Wind power and storage," in *Wind Power in Power Systems,* 2nd ed., T. Ackermann, Ed. Chichester, UK: John Wiley & Sons, Ltd, 2012, pp. 465-487.

[116] Electric Power Research Institute, "*Wind power integration: Energy storage for firming and shaping.*," EPRI, Palo Alto, California, USA, Tech. Rep. 1008388, 2005.

[117] M. Korpaas, A. T. Holen and R. Hildrum, "Operation and sizing of energy storage for wind power plants in a market system," *International Journal of Electrical Power & Energy Systems,* vol. 25, pp. 599-606, 10, 2003.

[118] L. E. Benitez, P. C. Benitez and G. C. van Kooten, "The economics of wind power with energy storage," *Energy Econ,* vol. 30, pp. 1973-1989, 7, 2008.

[119] R. Billinton, Bagen and Y. Cui, "Reliability evaluation of small stand-alone wind energy conversion systems using a time series simulation model," *IEE Proceedings- Generation, Transmission and Distribution,* vol. 150, pp. 96-100, 2003.

[120] P. Hu, R. Karki and R. Billinton, "Reliability evaluation of generating systems containing wind power and energy storage," *IET Generation, Transmission & Distribution,* vol. 3, pp. 783-791, 2009.

[121] D. Milborrow and L. Harrison, "The real cost of integrating wind," *Windpower Monthly,* vol. 20, pp. 35-39, 2004.

[122] L. Kamal and Y. Z. Jafri, "Time series models to simulate and forecast hourly averaged wind speed in Quetta, Pakistan," *Solar Energy,* vol. 61, pp. 23-32, 7, 1997.

[123] R. Karki, P. Hu and R. Billinton, "A simplified wind power generation model for reliability evaluation," *IEEE Transactions on Energy Conversion,* vol. 21, pp. 533-540, 2006.

[124] Y. Gao and R. Billinton, "Adequacy assessment of generating systems containing wind power considering wind speed correlation," *Renewable Power Generation, IET,* vol. 3, pp. 217-226, 2009.

[125] P. Hu, R. Karki and R. Billinton, "Development of an interactive reliability model for wind and hydro power systems," in *IEEE Electrical Power & Energy Conference (EPEC), 2009,* 2009, pp. 1-6.

[126] B. Karki, "Operating Reserve Assessment of Wind Integrated Power System," *A Ph. D. Thesis Submitted to the University of Saskatchewan,* 2010.

[127] S. Thapa, R. Karki and R. Billinton, "Evaluation of wind power commitment risk in system operation," in *Electrical Power and Energy Conference (EPEC), 2011 IEEE,* 2011, pp. 284-289.

[128] S. Thapa, R. Karki and R. Billinton, "Quantifying risk in power system operation incorporating wind power," in *5th International Conference on Quality, Reliability and Information Technology,* Kathmandu, Nepal, 2011, .

[129] R. Karki, S. Thapa and R. Billinton, "Operating Risk Analysis of Wind-integrated Power Systems," *Electric Power Components and Systems,* vol. 40, pp. 399-413, 01/30; 2012/06, 2012.

[130] R. Karki, S. Thapa and R. Billinton, "A Simplified Risk-Based Method for Short-Term Wind Power Commitment," *IEEE Transactions on Sustainable Energy,* vol. PP, pp. 1-1, 2012.

[131] S. Thapa, R. Karki and R. Billinton, "Probabilistic risk analysis in wind-integrated electric power system operation," in *Proceedings of the International Symposium on Engineering Under Uncertainty: Safety Assessment and Management (ISEUSAM-2012),* 2013, pp. 593-602.

[132] S. Thapa, R. Karki and R. Billinton, "Application of hourly time series models in day-ahead wind power commitment," in *Reliability and Risk Evaluation of Wind Integrated Power Systems*Anonymous Springer, 2013, pp. 45-58.

[133] S. Thapa, R. Karki, R. Billinton and H. Kile, "Extension of area risk concepts to incorporate wind power in unit commitment risk evaluation," in *Electrical and Computer Engineering (CCECE), 2013 26th Annual IEEE Canadian Conference on,* 2013, pp. 1-4.

[134] S. Thapa, R. Karki and R. Billinton, "Utilization of the Area Risk Concept for Operational Reliability Evaluation of a Wind-Integrated Power System," *IEEE Transactions on Power Systems,* vol. 28, pp. 4771-4779, 2013.

[135] T.S. Nielsen, A. Joensen, H. Madsen, L. Landberg and G. Giebel. *A New Reference for Wind Power Forecasting* 1999.

[136] S. Rehman, "Wind energy resources assessment for Yanbo, Saudi Arabia," *Energy Conversion and Management,* vol. 45, pp. 2019-2032, 8, 2004.

[137] M. Li and X. Li, "Investigation of wind characteristics and assessment of wind energy potential for Waterloo region, Canada," *Energy Conversion and Management,* vol. 46, pp. 3014-3033, 11, 2005.

[138] A. Keyhani, M. Ghasemi-Varnamkhasti, M. Khanali and R. Abbaszadeh, "An assessment of wind energy potential as a power generation source in the capital of Iran, Tehran," *Energy,* vol. 35, pp. 188-201, 1, 2010.

[139] I. Fyrippis, P. J. Axaopoulos and G. Panayiotou, "Wind energy potential assessment in Naxos Island, Greece," *Appl. Energy*, vol. 87, pp. 577-586, 2, 2010.

[140] R. Soler-Bientz, S. Watson and D. Infield, "Wind characteristics on the Yucatán Peninsula based on short term data from meteorological stations," *Energy Conversion and Management*, vol. 51, pp. 754-764, 4, 2010.

[141] D. Yu, J. Liang, X. Han and J. Zhao, "Profiling the regional wind power fluctuation in China," *Energy Policy*, vol. 39, pp. 299-306, 1, 2011.

[142] M. R. Islam, R. Saidur and N. A. Rahim, "Assessment of wind energy potentiality at Kudat and Labuan, Malaysia using Weibull distribution function," *Energy*, vol. 36, pp. 985-992, 2, 2011.

[143] E. Hau, *The Wind Resource.* Springer, 2013.

[144] K. Xie and R. Billinton, "Considering wind speed correlation of WECS in reliability evaluation using the time-shifting technique," *Electr. Power Syst. Res.*, vol. 79, pp. 687-693, 4, 2009.

[145] J. M. Morales, R. Mínguez and A. J. Conejo, "A methodology to generate statistically dependent wind speed scenarios," *Appl. Energy*, vol. 87, pp. 843-855, 3, 2010.

[146] M. Davis. Production of conditional simulations via the LU triangular decomposition of the covariance matrix. *Mathematical Geology* [Online]. *19(2)*, pp. 91-98. 1987. Available: http://link.springer.com/article/10.1007/BF00898189. DOI: 10.1007/BF00898189.

[147] T. V. D. Berg. *Generating Correlated Random Numbers*. Available: http://www.sitmo.com/article/generating-correlated-random-numbers/.

[148] T. R. Mayes. Generating correlated normally distributed random numbers in excel. *The Journal of Financial Modeling and Educational Technology* [Online]. *1(1)*, pp. 59-66. 2010. Available: http://www.afmet-online.org/jfmet_files/winter2010/Generating_Normally_Distributed_Random_Numbers_in_Excel.pdf.

[149] P. Giorsetto and K. F. Utsurogi, "Development of a New Procedure for Reliability Modeling of Wind Turbine Generators," *IEEE Transactions on Power Apparatus and Systems*, vol. PAS-102, pp. 134-143, 1983.

[150] H. A. Sturges, "The choice of a class interval," *Journal of the American Statistical Association*, vol. 21, pp. 65-66, 1926.

[151] R. Billinton and A. V. Jain, "The Effect of Rapid Start and Hot Reserve Units In Spinning Reserve Studies," *Power Apparatus and Systems, IEEE Transactions on*, vol. PAS-91, pp. 511-516, 1972.

[152] P. Kundur, *Power System Stability and Control.* MCGraw-Hill, 1994.

[153] R. Billinton and M. Fotuhi-Firuzabad, "A reliability framework for generating unit commitment," *Electr. Power Syst. Res.*, vol. 56, pp. 81-88, 10/5, 2000.

[154] R. Billinton and B. Karki, "Well-Being Analysis of Wind Integrated Power Systems," *IEEE Transactions on Power Systems,* vol. 26, pp. 2101-2108, 2011.

[155] North American Electric Reliability Corporation, "Reliability standards for the bulk electric systems of north america," NERC, Atlanta, GA, 2013.

[156] R. N. Allan, R. Billinton and N. Abdel-Gawad, "The IEEE reliability test system-extensions to and evaluation of the generating system," *IEEE Transactions on Power Systems,* vol. 1, pp. 1-7, 1986.

[157] R. R. Shoults and D. T. Sun, "Optimal Power Flow Based Upon P-Q Decomposition," *Power Apparatus and Systems, IEEE Transactions on,* vol. PAS-101, pp. 397-405, 1982.

[158] Y. V. Makarov, C. Loutan, Jian Ma and P. de Mello, "Operational Impacts of Wind Generation on California Power Systems," *Power Systems, IEEE Transactions on,* vol. 24, pp. 1039-1050, 2009.

[159] (06/2012). *Transmission Grid Interconnection*. Available: http://www.nrel.gov/electricity/transmission/eastern_wind_dataset.html.

APPENDIX A
OPERATING CAPACITY MODEL OF THE IEEE-RTS

The COPT of the IEEE RTS for an operating condition where the first eight generating units in the priority loading order are committed for a lead time of one hour is presented in Table A1. The generating unit data of the test system is presented in Table 6.1. The total operating capacity of the eight committed units is 1547 MW. Table A2 presents the operating COPT of the same units for a lead time of four hours. The capacity states with probabilities less than 10^{-8} are not shown.

Table A1: COPT of 8 IEEE RTS units for 1 hour lead time

Capacity Out, MW	Capacity In, MW	Cumulative Probability
0	1547	1.00000000
50	1497	0.00574370
100	1447	0.00373602
150	1397	0.00373449
197	1350	0.00373449
200	1347	0.00268693
247	1300	0.00268693
297	1250	0.00268481
347	1200	0.00268481
350	1197	0.00268481
397	1150	0.00181919
400	1147	0.00181919
450	1097	0.00000888
500	1047	0.00000523
547	1000	0.00000523
550	997	0.00000432
597	950	0.00000432
600	947	0.00000241
647	900	0.00000241
697	850	0.00000241
747	800	0.00000241
750	797	0.00000241
797	750	0.00000083
800	747	0.00000083

Table A2: COPT of 8 IEEE RTS units for 4 hour lead time

Capacity Out, MW	Capacity In, MW	Cumulative Probability
0	1547	1.00000000
50	1497	0.02280428
100	1447	0.01489937
150	1397	0.01487539
197	1350	0.01487535
200	1347	0.01074392
247	1300	0.01074392
297	1250	0.0107105
347	1200	0.0107104
350	1197	0.0107104
397	1150	0.00729842
400	1147	0.00729842
450	1097	0.00014127
500	1047	0.00008352
547	1000	0.00008334
550	997	0.00006892
597	950	0.00006892
600	947	0.00003866
647	900	0.00003866
697	850	0.00003841
747	800	0.00003841
750	797	0.00003841
797	750	0.00001352
800	747	0.00001352
850	697	0.00000031
900	647	0.00000021
947	600	0.00000021
950	597	0.0000001
997	550	0.0000001
1000	547	0.00000005
1047	500	0.00000005
1097	450	0.00000005
1147	400	0.00000005
1150	397	0.00000005

APPENDIX B
WIND SPEED DATA

The hourly mean wind speed at the Toronto wind site for a year considering twenty years of observed wind speed is presented in Table B1. The hourly wind speed standard deviations are presented in Table B2. The first row contains data for Day 1 (January 1st) starting at 1:00 AM (Hour 1), the second row for Day 2, and so on. These hourly wind speed statistics are used to simulate the wind speed data using the ARMA model of the wind site.

Table B1: Observed hourly mean wind speed at Toronto (km/hr)

Hr 1	Hr 2	Hr 3	Hr 4	Hr 5	Hr 6	Hr 7	Hr 8	Hr 9	Hr 10	Hr 11	Hr 12	Hr 13	Hr 14	Hr 15	Hr 16	Hr 17	Hr 18	Hr 19	Hr 20	Hr 21	Hr 22	Hr 23	Hr 24
20.75	19.21	20.46	21.21	21.39	21.61	22.11	21.86	23.57	24.46	25.61	25.25	26.64	26.25	24.71	24.79	23.82	22.5	22.36	24.04	22.75	21.57	21.32	20.36
22.07	20.25	20.93	20.71	21.36	20.25	20.21	19.46	19.21	19.68	19.32	19.43	21	22.36	21.64	21.89	23.25	24.07	24.29	24.96	26.46	25.57	25.64	25.54
25.25	25.29	25.79	25.82	25.86	25.86	24.89	22.93	23.71	22.86	23.57	22.25	24.5	24.46	25.04	25.21	24	23.29	23.18	22.11	22.32	22.75	22.04	21.96
24.36	26.46	25.21	24.86	24.29	24.71	22.5	21.71	22	24.86	26.36	26.61	28.21	28.64	28.71	26.5	25.89	26.79	25.89	25.32	24.46	26.71	24.39	22.61
21.86	20.89	20.29	19.07	21.54	21.61	21.61	20.79	20.75	20.86	20.82	20.25	22.86	22.18	22.5	23.32	22.71	22.46	22.18	21.89	23.79	23.25	23.89	23.39
22.43	22.32	20.39	22.18	21.68	22.79	23.96	23.64	24.46	23.54	24.21	24.71	25.46	26.11	24.71	24.5	25.86	24.96	26.11	26.89	26.39	24.89	24.21	23.25
24.07	24.32	23.43	23	24.57	24.71	24.32	23.64	24.29	25.57	27.86	26.57	25.64	27.04	26.61	24.46	26.32	26.04	24.64	25.82	23.32	22	20.43	21.36
19.14	18.36	16.61	18.36	18.25	18.93	18.5	18.18	19.36	21.86	22.21	20.57	21.18	22.21	23.57	24.64	24.32	26.18	27.39	26.57	26.86	24.64	24.93	25.89
25.71	27.61	28.14	28.89	27.14	25.86	26.39	26.07	25.18	23.64	23.75	24.75	24.32	25.18	25.46	24.57	24.5	24.11	23.71	24.68	24.29	21.61	21.75	23.11
24.36	23.25	22.61	23.11	22.79	23.18	21.79	20.5	20.36	21.96	23.79	23.36	24.32	25.93	27.11	27.04	27.18	26.64	26.61	26.04	27.36	26.82	25.89	25.46
25.89	25.04	24.79	24.79	25.54	25.04	26.64	25.82	25.93	26.18	26.54	27.46	26.57	26	28	27.79	28.04	27.79	27.43	26.96	25.64	26.11	24.86	24.96
23.82	22.71	23.68	22.04	21.29	20.11	20.11	20.68	19.86	20.39	22.07	20.57	21.46	22.14	20.5	21.32	20.79	20.32	19.5	20.43	22.75	23.61	22.86	22.36
20.57	21.54	22.93	22.39	24.36	27.75	26.71	26.39	25.82	26.75	27.04	25.36	26.64	23.75	23.96	23.54	23.75	23.46	21.43	19.93	20.68	21.5	21.14	20.89
19.39	19.36	18.96	18.57	18.5	19.07	18.29	20.57	20.5	21.75	22.07	22.64	20.5	20.82	22.79	22.68	22.93	23.5	22.64	21.36	22.71	20.96	20.32	22.39
23.32	22.46	20.89	22.04	21.64	21.21	22.68	23.21	24.11	24.07	23.89	24.71	24.18	23.21	23.18	22.57	21.86	21.11	23.61	23.96	22.43	22.39	21.57	20.79
21.54	20.5	20.18	22.61	22.14	22.39	24.57	23.75	23.75	23.93	25.46	26.04	26.04	26.82	26.61	25.75	24.25	24.21	23.32	24	24.75	24.39	22.54	20.57
19.96	20.46	21.71	22.11	22.25	21.86	20.96	20.93	21.75	23.82	22.93	23.68	24.14	24	23.07	22.36	23.64	21.5	21.46	22.71	24.07	23.04	21.82	23.04
24.11	20.75	20.82	21.04	20.93	21.5	21.71	21.11	21.14	21.96	21.5	21.04	21.57	22	21.14	21.18	22.25	21.82	20.21	20.29	17.36	18.5	18.11	18.32
17.96	18.11	18.11	17.96	17.57	19.18	17.82	19.25	19.46	19.93	20.75	20.5	21.04	21.71	24.29	24.61	27.25	27.39	28.39	27.89	27.21	27.32	25.25	26.04
25.61	25.29	24.18	23.96	24.36	23.61	23.71	24.04	24.86	25.04	24.32	24.39	25.39	25.29	24.96	23.89	22.64	23.11	21.54	23.04	20.57	20.21	20.21	19.71
19.57	19.39	19.61	20.36	19.68	19	18.61	17.93	18.04	18.25	17.04	20.57	22.14	21.14	22.32	21.93	22.79	22.61	21.39	22.61	24.18	23.93	24.36	25.54
25.32	23.61	23.14	23.75	24.79	25	24.96	26.04	25.14	26.96	26.61	26.61	26.64	28.18	26.93	26.04	25.82	24.54	24.36	25.5	24.57	23.79	25.21	23.14
23.75	22.29	22.93	22.86	21.86	22.46	22.5	21	20.54	20.04	20.5	19.39	20.07	19.39	20.86	20.82	20.89	22.68	19.93	19.86	22.04	21.18	19.86	20.14
19.82	19.79	21.04	21	21.93	19.79	21.25	20.75	20.32	21.57	22.71	22.68	23.61	23.64	23	23.54	24.39	24.25	24.43	23.79	23.86	22.36	22.04	21.25
21.68	19.5	19.43	17.64	17.04	16.75	17.82	18.04	19.29	21.04	20.25	21.36	22.46	22.89	21.96	22.46	21.25	21.25	22.68	25.25	25.82	24.5	22.5	22.43
23.79	23.25	23.57	22.68	22.07	22.14	21.82	22.07	22.32	22.96	22.43	22.75	24.07	24.36	24.39	25.18	25.96	26.5	25.89	26.82	26.89	25.43	23.86	22.82
23.93	22.11	23	22.32	22.57	22.79	21.57	23.54	23.82	24.96	25.32	25.46	24.89	25	26.18	27.5	24.07	24.61	20.46	20.82	21.71	22.96	22.75	22.11
21.21	20.79	17.14	17.75	17.64	17.54	18.5	17.57	19.46	21.5	20.46	21.43	22.29	22.36	24.54	24	24.39	22.25	20.25	20.57	22.18	21.86	22.04	20.82
20.89	19.14	19.54	19.21	18.04	18.96	18.71	17	17.07	19.11	18.04	19.07	17.43	17.07	18.71	21.96	21.14	21.04	21.32	20.25	19.21	18.54	18.07	18.39
19.43	19.61	21.14	19.82	20.18	18.71	20	19.5	19.5	20.11	19.21	19.54	21.32	21.82	20.43	19.79	18.96	17.86	18.14	17.5	18.61	18.29	18.32	18.25
19.11	19.25	21.29	20.46	20.61	20.39	21.79	21.43	23.04	24.32	24.07	25.89	25.36	27.39	26.75	27.11	27.68	26.68	24.68	24.04	23.11	23.11	20.64	20.04
20.96	20.68	22	21.5	21.07	21.46	19.32	17.57	18.18	18.36	19.89	20.32	22.36	23.68	23.68	22.11	22.71	23.07	20.57	21.75	21.29	22.14	20.71	20.32
18.61	18.43	17.68	17.18	17.71	16.07	16.46	17.75	17.14	17.14	19.07	20.25	22.29	21.79	20.71	22	22.43	22.04	22.11	23.32	21.64	20.32	21.93	20.14
18.61	18.11	16.89	18.07	17.5	18.21	17.5	17.68	18.75	18.93	19.46	21	21.14	22.29	22.75	23.18	23.75	23.39	23.21	24.07	24.18	23.43	22.54	24.89
25	23.04	21.46	20.96	22.32	20.07	20.96	20.75	22.86	23.75	21.64	24.36	24.14	23.29	25.61	23.5	22.93	23.57	23.18	24	22.57	22.86	19.57	18.57
19.64	18.07	17.46	16.57	15.89	16.79	18.82	19.36	19.43	20.93	22.07	22.39	23.75	23.61	26.11	24.5	24.25	24.32	24.14	23.57	22.89	22.43	20.25	20.89
21.07	19.25	21.39	18.75	20.61	20.14	19.32	19.82	19.43	19.46	20.21	19.89	22	22.36	23.11	24.32	25.64	24.11	24.61	24.32	23.32	24.29	25.04	23.82
22.39	21.89	22.14	22.5	23	22.43	22.11	22.96	23.5	24.07	23.79	24.43	26.61	28	27.89	27.75	27.18	27.25	25.61	26.68	25.5	26.29	25.39	23.32
24.14	21.64	20.93	21.5	20.21	21	20.86	20.75	19.96	20.79	22.25	22.11	22.39	23.54	23.61	23.43	23.57	23	22.86	22.32	23.07	23.36	22.71	23.25
22.57	22.32	21.5	20.86	20.39	19.71	20.57	18.04	18.54	18.68	18.75	19.57	19.71	19.5	20.64	21.61	22.93	22.64	22.86	22.43	23.71	24.21	22.32	24
24.25	22.79	19.68	18.43	18	16.86	15.75	17.04	18.25	18.86	18.68	20.18	19.93	21.43	21.79	22.29	21.57	21.57	21.79	20.46	21.04	21.5	20.96	19.96
21.39	20.21	19.54	18.93	18.93	19.64	19.25	18.93	18.14	21.29	21.14	21.89	22.86	21.54	23.32	24.21	25.25	23.75	24.89	26.39	25.04	24.11	24.61	23.36
21.82	19.79	19.79	19.71	18.79	19.89	21.43	22.86	23.32	23.36	23.75	24.29	24.75	24	25.11	26.71	26.54	25.25	25.07	25.11	24.21	23.14	23.25	23.07
21.5	19.07	19.61	19.36	20.29	20.25	20.96	20.29	20.29	20.11	22.54	24.07	24.75	24.86	24.46	23.18	24.75	24.57	24.21	24.64	25.21	24.82	25.18	25.43
24	23.18	21.07	20.5	21.36	20.04	19.64	19.71	19.29	18.96	18.79	16.93	17.39	18.25	17.04	17.75	16.39	17.43	16.82	15.43	15.64	14.64	14.21	15.11

15.79	14.25	14.86	16.46	14.86	14.93	16.07	17.36	16.54	17.04	18.93	19.89	18.96	19.11	20.32	21.64	20.5	19.64	19.64	18.5	16.29	17.82	16.43	17.07
16.07	15.96	16.5	16.5	15.82	15.82	15.11	16.25	15.75	15.21	17.07	16.25	17.89	19.39	20.75	23.14	21.43	20.86	19.61	20.11	21.32	21.04	22.18	23.46
21.71	21.68	21.61	21.68	20.5	20.36	21.79	21.61	22.04	21.89	21.07	22.93	24.43	24.36	24.18	23.46	24.39	23.5	21.57	22.32	23	21.86	21.43	20.36
21.93	20.29	19.64	19.36	19.04	18.82	19.29	19.68	20.18	20.18	20.5	20	21.29	21.71	21.86	22.79	21.57	22.82	20.68	22.07	21.57	21.57	20.79	19.96
19.39	18.54	18.89	18.93	19.46	18.32	17.32	17.61	18.25	18	17.71	18.79	18.11	17.43	19.61	17.54	18.14	17.46	18	17.93	17.89	16.79	16.14	15.36
15.82	14.21	14.25	15.25	16.82	16.68	17.36	17.75	17.25	18.5	17.75	18.93	18.5	18.46	18.79	19.25	19.71	18.64	19.5	18.14	17.86	16.5	15.14	16.5
16.82	14.68	15.79	16.57	17.64	18.39	16.71	16.68	18.21	19.61	21.82	20.18	20.32	21.68	20.29	22.14	22.46	21.96	21.75	19.36	19.61	20.29	20.36	20.54
22.14	20.11	18.5	19.04	18.04	18.89	18.07	19.5	19.11	21.29	20.07	19.71	20.04	19.43	20.64	19.96	19	18.11	17.93	18.29	18.54	17.36	18.82	18.04
19.43	19.25	18.54	20.04	20.29	20.82	20.5	20.93	20.71	20.61	23.04	23.18	23.21	23.93	24.96	25.43	25.21	25	24.32	22.61	20.5	21.04	20.79	19.86
18.43	18.36	17.71	16.32	17.86	19.36	19.54	19.43	20.36	21.39	21.61	22.93	22.68	23.93	23.07	22.75	23.11	23.04	23.64	22.36	21.07	21.71	20.39	20.25
20.04	18.96	16.89	17.89	15.43	15.86	15.36	15.75	16.82	17.5	18.61	19.79	21.89	22.79	21.82	24.07	24.29	23.61	23.29	21.96	21.29	20.46	19.93	20.46
19.86	19.93	18.64	18.68	17.61	18.36	18.36	17.93	18.36	19.32	20.14	21.61	20.07	21.5	21.57	22.46	21.14	19.71	18.04	17.25	17.14	16.54	17.75	18.57
18.75	18.18	18.61	18.07	19.18	18.32	19.14	19	19.5	22.14	22.86	23.18	22.89	24.07	24.75	25.89	25.93	24.61	22.96	20.79	19	19.39	18.07	16.96
16.36	15.21	15.75	14.96	14.71	14.71	16.25	16.71	15.57	15.11	15.21	17.79	21.18	21.96	22.61	22.64	22.11	21.11	21.64	22.46	21.21	20.39	20.64	21.25
19.5	19.11	21.39	21.39	20.96	21.93	21.25	20.04	19.61	21.25	21.14	21.93	22.75	23.14	23.11	23.68	22.71	22.57	20.96	18.75	17.82	18.14	17.11	17.04
15.61	15.89	16.79	16.14	17.14	18.93	19.14	19.96	19.75	21.54	21.25	22.36	23.25	24.89	24.57	24.36	22.46	22.93	20.11	19.25	19.54	19.21	18.54	17.64
17.14	16.82	16.36	16.25	18	18.54	16.82	17.54	18.29	19.21	21.5	21.36	21.14	22.14	21.75	22.96	23.14	23.79	22.36	20.54	20.86	20.5	20.29	20.61
22.18	22.46	22.68	22.54	22.79	24.39	24.04	25.36	24.32	26.14	25.21	27.25	26.14	25	27.18	26.61	25.54	24.5	22.29	21.89	20.61	18.75	16.21	16.07
16.39	18	18.79	18.68	18.18	17.82	19.68	19.5	20.07	22.29	23.64	24.21	24.96	24.61	24.79	25.46	23.57	21.36	21.32	20.07	19.93	19.04	18.89	18.11
19.25	17.43	17.75	18.21	17.46	16.82	15.96	16.07	15.21	17.14	18.14	17.36	18.79	17.93	18.07	17.96	19.39	19.64	19	18.89	18.75	18.43	18.21	17.93
17.79	16.82	17.82	17.11	17.43	17.61	17.36	17.96	18.18	19.39	19.5	20.25	21.86	21.36	21.82	20.5	21.46	20.64	19.43	19.82	18.14	17.89	18.29	18.14
17.86	19.07	18.14	18.5	17.29	17.64	16.39	17.57	18.68	20.5	21.29	21.54	22.71	23.82	23.96	24	23.5	19.96	20.75	18.36	17.21	17.89	17.64	17.11
17.39	16.14	16.11	16.04	16.14	16.64	15.86	15.96	17	19.71	19.14	20.54	21.11	21.5	20.39	21.89	20.89	22.75	22.29	21.32	21.79	19.93	20	18.14
19.68	18.86	17.18	16.5	16.79	16.25	15.71	15.36	16.07	18.14	19.07	20.21	21.25	22.5	23.43	22.54	22.75	22.57	21.29	19.21	18.93	17.93	17.11	15.82
15.89	16.18	14.64	15.64	14.14	14.89	15.21	16.39	16.96	17.54	18.25	20.04	22.32	23.18	23.89	22.5	21.61	21.68	20.96	21.39	21.75	20.18	21.43	22.04
20.46	19.43	19.25	18.82	19.07	18.5	20.07	18.64	20.32	21.07	21.21	22.64	23.36	22.43	22.14	23	23.96	24.11	21.46	20.39	19.04	17.25	17.82	18.21
17.25	18.96	19.07	19.46	19.82	18.82	19.14	19.46	20.82	23.68	22.29	22.54	21.21	23.5	23.39	23	21.96	21.93	21.61	22.57	22.29	19.57	19.25	20.71
19.18	19.39	18.64	20.75	21.75	20.89	21.04	21.29	22.39	23.82	22.93	22.89	22.43	24.61	25.61	24.79	24.32	23.46	21.32	21.25	20.07	20.04	18.46	17.18
15.43	15.79	16.68	17.46	16.86	16.07	16.04	15.46	17.21	18.86	20.04	21.96	21.68	21.5	23.25	22.43	23.04	23.29	23.64	21	21.25	21.18	20.25	20.32
19.75	18.39	17.46	18.89	19.71	19.96	20.43	20.46	20.46	21.18	21.79	21.96	22.86	21.79	21.64	22.07	22.36	22.25	19.86	19.79	19.68	17.61	18.18	17
19.54	19.39	19.82	19.29	21.04	21.46	21.57	20.5	22.29	22.96	23.82	24.07	24.32	24.29	24.54	25.46	23.29	22.5	23.61	22.32	21.61	21.68	20.79	19.46
20.25	19.57	18.5	18.57	18.82	19.89	17.07	18.29	19.75	20.32	20.75	21	22.96	21.75	21.93	20.43	20.39	19.93	19.64	18.79	19	20.39	18.57	18.64
20.68	18.57	19.32	19.57	18.86	19	20.25	19.86	20.79	21.11	22.18	24.04	24.5	23.93	24.43	24.54	23.32	22.14	22.36	20.32	19.21	18.61	18.07	17.86
16.89	15.86	14.54	13.89	13.71	14.39	14.96	15.36	16.43	17.79	19.82	19.46	20.86	21.21	21.04	22.68	21.82	22.07	21.71	20.04	20.25	19.93	18.11	16.89
16.07	15.57	16.46	17.75	17.89	16.39	17.07	18.71	19.39	19.54	20.93	23.04	23.14	22.79	23.07	23.93	22.54	22.18	20.68	20.25	20.89	19.61	19.29	20.86
18.82	21	20.07	21.32	21.46	22.5	22.39	22.75	23.11	23.11	23.11	22.93	25.57	26.18	24.82	24.68	25.21	25.11	23.5	22.61	20.14	20.79	19.29	18.18
19.29	19.29	19.07	16.86	17.5	18.39	16.18	16.93	18.5	20.25	19.93	21.75	21.75	20.96	22.89	22.29	21.96	22.93	21.61	21.54	19.82	18.39	19.68	18.43
18	18.64	19.43	18.82	20.32	21.86	20.82	21.36	21.43	21.43	21.32	21.11	21.25	22.39	23.46	23.14	23.11	23.04	21.07	20.64	17.82	18.25	17.61	16.39
16.64	16.68	15.32	16.25	15.32	15.54	14.57	15.79	16.86	16.54	16.71	18.14	17.43	19.86	20.82	21.64	20.21	19.75	19.39	18.18	18.14	19.82	18.46	18.46
17.82	18.64	19.14	18.39	17.93	17.18	17.25	16.36	19.04	18.54	20.57	20.93	22.36	22.68	24.32	25.04	23.75	24.54	23.82	19.86	18.79	18.61	19.43	16.96
17.79	15.57	14.36	13.5	14	13.86	14.54	16.36	15.36	17.89	17.82	18.54	19.25	20.79	20.11	22.29	20.75	19.04	16.86	16.68	14.39	14.61	14.11	13.32
14.75	13.54	13.25	12.64	11.71	12.29	14.04	14.79	16.07	15.57	17.46	18.07	16.54	18.79	18.96	19.11	20.25	20.64	18.68	17.89	17.5	17.93	15.39	17.43
16.71	16.75	17.71	17.29	18.36	18.11	18.86	19.18	19.21	19.25	20.61	20.64	21.54	22.64	22.29	24.36	23.64	22.86	22.14	17.57	18.07	17.43	17.04	17.39
19.18	19.21	19.5	20.14	20.04	19.61	20.21	19.32	20.07	21.32	22.14	23.93	23.57	22.04	23.36	23.07	22.07	19.04	20.61	19.5	18.79	18.21	17.57	16.89
17.82	17.39	16.71	15.89	17.54	17.18	16.39	17.64	18.46	18.57	21.11	21.57	21.18	22.5	21.96	24	23.96	23.32	22.36	21.25	19.61	18.54	18.54	18.57
16.82	16.89	17.32	17.89	18.36	18.36	16.5	18.36	18.68	18.64	21.43	20.89	23.14	22.14	21.75	22.93	21.86	22.46	21.64	19.64	19.14	17.79	17.11	17.93
18.07	18.93	18.71	17.04	18.07	18.11	18	17.75	17.46	18.43	20.32	20.29	19.71	19.79	20.96	20.57	20.32	18.96	17.5	18.43	17.93	18.21	18.61	16.93
18.57	17.46	18.89	19.57	18.79	18.79	17.46	20.79	20.39	21.36	21.36	20.43	21.04	21.93	20.46	23.18	22	20.86	18.89	18.82	16.43	17.86	17.61	16.68
17.54	16.36	17.46	17.07	18.36	19.79	21.21	20.68	21.32	24.07	23.64	25.75	25.39	24.11	25.75	25.86	26.61	25.68	23.93	23.75	23.93	24.68	24.36	22.57
22.54	23.18	21.21	22.39	23.43	23.89	21.68	20.86	22.61	22.75	25.5	24.82	26.21	27.04	26.61	26.25	25.86	25.86	27	26.25	23.46	23.89	24.79	24.82
25.29	24.64	25.5	24.57	24.64	25.93	23.93	24.18	24.18	26.5	27.21	26.43	27.21	27.18	27.79	28.36	27.32	25.82	24.29	22.93	21.32	19.96	19.32	20.18
19.54	19.61	19.5	19.5	20.57	20.86	18.93	20.43	22.39	23.71	22.89	23.82	25.25	24.68	25.71	25.61	25.64	25	23.71	23.14	20.54	22.32	19.68	17.79
17.5	18.14	18.18	18.25	18.43	19.39	19.32	19.39	20.57	21.07	20.96	20.96	23.21	22.61	22.57	22.93	23.18	22.64	22.21	21.07	18.39	20.39	18.57	19
17.89	17.39	16.61	17.39	16.46	17.54	17.21	19.57	18.82	20.36	20.79	22.18	24.18	25.14	24.93	23.61	22.21	21.21	20.32	17.57	17.82	17.29	17.46	14.5
14.36	15.54	15.96	14.61	16.36	15.93	14.54	16.43	17.64	19.93	18.61	21.39	21.89	20.93	21.86	21.32	21.36	20.5	19.82	17.93	17.07	17.29	16.11	16.46
15.64	13.82	14.71	14.5	14.54	15.36	16.82	17.64	20.32	20.68	20.5	21.04	22.11	23.32	23.11	22.39	21.61	21.07	19.18	18.32	16.71	17.79	16.86	15.04
14.11	14.14	15.71	15.89	16.86	17.54	18.04	17.14	18.96	19	21.11	20.04	21.5	23.25	21.04	19.61	19.25	19.57	18.18	19.29	17.5	15.68	17.36	15.71
15.61	14.75	15.86	16.46	15.29	14.96	17.14	18.46	19.39	20.21	22.5	22.93	23.25	22.89	25.04	24	24.18	22.57	22.57	20.29	17.86	17.32	17.21	15.71
17.07	14.46	16.04	15	15.5	17.07	16.93	17.32	19.21	22.21	22.21	23.43	22.32	21.82	23.43	22.79	21.39	20.64	19.18	17.29	16.25	15.5	14.61	14.96
13.96	13.82	13.71	15.14	14.61	14.5	14.29	15.79	18.5	17.89	19	20.43	21.71	21.04	20.64	20.57	21.11	22.21	20.25	18.64	16.11	15.96	15.29	14.61
16.93	16.04	15.75	15.29	15.25	15.04	15	16.61	17.36	19.04	20.29	22.43	22.93	22.61	23.89	23.82	22.93	21.61	20.64	18.75	16.39	14.21	15.36	17.07
16.11	15.64	14.25	14	18.36	16.79	16.61	16.14	19.07	18.68	19.04	20.5	22.39	22	22.89	23.04	21.68	21.71	21.71	21.61	19.64	16.5	17.46	16.07
16.07	17.21	17.25	17.64	18.07	17.11	17.21	17.36	19.21	19.61	20.11	20.32	19.43	20.71	22.36	22.57	20.18	21.39	19.21	18.36	16.04	16.39	16.46	14.61
13.21	14.57	13.21	11.96	11.54	14.89	15.04	16.5	18.57	19.07	19.07	19.89	21.43	21.86	20.75	20.93	21.71	20.21	19.46	18.36	17.57	17.75	17.14	14.71
14.04	14.79	13.21	11.57	13.54	14.04	13.96	15.82	16.5	16.57	18.36	17.86	18.93	20.86	21.36	19.79	21.11	20.32	19.36	18	14.86	14.07	14.11	13.96
14.82	13.46	13.86	13.54	13.68	13.32	14.32	14.29	15.82	16.29	17.93	18.39	18.32	19.32	18.79	18.43	17.86	18.61	18.14	18.07	16.14	16.21	17.71	17.43
19.57	19.43	17.39	16.14	16.75	16.21	16.89	17.04	17.79	17.79	18.18	18.11	18.64	16.54	15.64	17.21	16.54	18.21	15.86	15.93	14.04	14.04	13.86	15.14
13.93	14.86	15.64	14.43	14.07	15.32	16.11	17.64	19.71	20.11	21.46	21.32	22.14	20.79	20.68	19.82	19.61	17.18	17.14	14.18	14.43	15.54	16.39	14.39
14.68	16.04	15.21	17.36	16.21	16.57	16.46	18.21	19.96	19.18	19.18	20.25	20.86	20.39	20.04	21.11	21.71	21.61	19.5	17.5	17.61	15.43	13.96	13.93
12.64	11.96	14.54	12.57	12.39	12.11	13.07	13.43	15.36	16.57	16.93	19.07	20.46	19.54	20.25	19.64	19.64	17.18	19.04	16.89	15.29	13.64	12.71	12.5
12.86	12.93	13.36	13.57	15.21	13.5	14.04	16.25	17.61	18.25	19.57	18.61	17.89	19.71	18.43	17.54	15.25	14.32	14.46	14.43	14.36	12.57	11.93	13.25

10.46	9.43	10.32	12.11	11.93	11.21	13.5	12.57	15.39	16.39	16.68	18.18	18.46	18.68	19.79	18.14	19.54	18.93	20.14	19.07	19.43	18.11	18	16.5
18.18	17.29	18.75	18.11	18.04	18.71	17.14	20	19.07	22	23.36	25.39	22.57	25.57	24.43	23.5	20.61	18.82	18.54	16.75	16.14	16.68	15.5	16.29
15.21	14.68	14.46	13.93	13.75	14.46	13.75	16	14.89	15.89	16.68	16.54	17.79	19.57	19.25	20.07	18.18	17.57	17.39	16.86	17.29	16.29	15.5	14.93
15.96	12.96	12.32	13.04	11.25	12.36	11.89	13.04	14.29	16.68	17.79	18.57	17.57	16.96	18.07	17.54	18.07	18.96	18.64	18.96	17.18	16.71	16.04	16.79
16.29	15.36	16.07	15.96	16.04	14.14	14.21	15.43	15.36	15.11	17.07	18.32	20	23.29	21.64	23.07	22.75	20.14	17.39	17.04	16.07	17.18	16.04	15.86
15.04	15.21	16.5	16.18	16.04	15.57	16.11	16.21	16.96	16.93	19.25	20.11	20.39	19.32	18.07	19.86	17.61	18.79	16	18.18	15.57	14.79	14.04	13.82
12.89	14.86	16.39	15.46	16.11	13.71	15.07	15.5	14.96	15.43	15.43	16.04	16.18	16.46	14.93	15.64	17.64	19.11	17.07	16.14	15.04	13.61	12.29	13
11.39	11.11	12.79	13.18	13.04	13.82	13.36	14.75	15.11	18.11	18.29	19.04	19.21	19.79	19.25	19.32	20.68	19.18	17.82	16.64	14.18	13.82	13.96	12.86
13.25	12.5	12.14	11.54	12.18	12.29	13.07	14.86	17.21	16.79	16.32	15.54	16.32	16	16.5	16.79	18.29	16.86	17.18	16.07	18	16.11	15.36	15
12.89	13.57	13.46	14.68	15.46	14.86	15	16.43	16.54	18.89	18.21	20.57	21.54	22.82	22.89	23	21.46	20.39	20.07	18.79	17.14	16.64	15.75	15.93
14.46	14.86	13.71	12.64	13.61	14.04	12.82	13.71	15.39	16.18	17.39	17.14	18.39	19.07	17.96	17.21	17.64	18.71	18.25	17.96	17.93	15.57	13.96	14.46
15.39	14.96	13.29	12.75	12.64	13.61	13.79	15.61	18.64	17.36	16.57	19.36	20.29	20.29	20.36	18.82	17.93	16.89	18.14	16.82	15.61	13.93	13.14	12.96
11.93	13.93	14.14	14.39	13.79	13.11	14.21	13.5	15.5	15.93	16.39	18.82	18.89	19.46	21.25	20.14	22.29	22.57	22.04	19.25	17.82	17.36	14.04	15.04
13.18	14.39	13.89	13.93	14.25	14.57	15.11	15.64	18.32	17.07	18.43	18.75	19.43	18.79	19.21	20.54	20.64	19.14	16.86	16.36	14.29	12.86	12.21	13.11
13.75	12.43	11.57	12.89	11.43	10.82	11.57	11.57	12.61	13.64	14.46	17.75	16.54	18.57	19.18	19.75	22.61	22.11	19.57	17.75	17.54	15.68	14.43	15.29
14.46	15.04	14.57	14.96	15.21	16.61	13.46	16.39	17.86	18.86	17.21	19.32	19.46	20.82	20	18.93	19.75	19.21	16.89	17.46	17.57	16	15.46	14.36
15.29	14.89	12.96	15.07	13.68	14.32	14.71	16.68	17.14	19.18	21.21	19.75	21.36	22.04	22.57	21.54	20.18	20.11	18.07	17.11	16.93	17.14	16.39	14.43
12.04	11.71	12.11	12.21	12.25	13.11	14.11	14.43	15.04	16.36	17.29	16.71	17.14	17.32	16.71	16.71	17.39	18.96	17.71	15.11	14.36	14.25	13.46	12.57
12.07	12.93	12.96	14.82	14.61	14.86	14.86	15.46	16.54	18.43	18.43	19.18	20.14	19.79	18.64	17.93	18.43	18.21	16.79	15.75	16.64	14.07	14.75	13.93
12.64	13.11	13.14	13.43	12.64	14.11	15.04	15.68	16.36	17.75	18.25	19.54	19.04	19.86	19.75	20.25	19.71	20.79	19.86	19.36	18.5	17.86	15.43	15.5
12.5	13.61	13.04	12.64	12.43	12.04	11.93	12.79	13.04	13.61	15.29	16.89	16.68	17.46	16.89	16.68	18.32	18.07	16.68	15.75	13	13.04	12.79	12.61
11.96	11.89	8.54	9.75	10.71	9.54	10.82	10.07	11.93	13.21	13.46	15.57	15.43	16.25	15.21	15.04	15.43	15.64	14.71	17.18	15.89	15.64	14.89	14.32
13.75	12	13	12.68	13.5	13.61	12.64	15.25	14.96	14.32	13.29	15.14	14.29	14.79	16.68	17.25	16.04	14.29	16.64	14.61	15.32	13.43	12.86	13.29
11.36	11.07	11.93	11.89	11.71	12.75	12.36	13.71	13.5	16.54	16.43	17.5	17.36	17.82	20.86	17.93	19.32	18.25	16.93	16.5	14.96	14.79	13.39	12.96
10.89	12.29	12.79	13.21	14.07	14.32	13.82	16	15.07	15.64	17.86	15.5	15.29	18.46	15.14	17.11	17.46	16.39	14.89	16.71	13.96	13.07	11.64	10.64
10.18	10.57	8.71	11.11	10.5	12.04	11.68	13.29	13.89	13.79	15.07	15.07	15.5	14.82	13.96	13.54	13.86	15.04	13.96	13.57	11.32	11.46	11.93	11.75
12.14	12	10.36	10.36	10.43	11.36	11.46	11.64	13.46	14.21	14	16.11	17.18	18.79	19.36	18.32	18.04	15.68	13.64	15.21	12.64	9.93	12.46	10.25
10.71	11.82	11.71	11.39	9.82	9.64	11	9.68	10.04	11.75	13.36	15.25	15.18	16.07	15.61	15.32	16	15.04	16.54	13.93	12.93	13.61	13.82	13.86
12.32	12	12.36	10.96	10.36	11.04	12.21	12.32	13.86	15.46	15.11	15.89	14.54	16.5	17.68	17.5	18.71	17.21	14.93	13.61	14.57	14.21	14.79	14.96
13.5	13.39	14.07	13.68	13.54	14.32	12.96	13.36	14.14	15.89	16.25	17.96	17.18	16.93	18.14	17.04	15.18	14.29	15.14	13.54	11.57	12	12.71	12
11.79	10.93	11.5	12.32	11.71	10.07	11.07	11.93	12.93	13.5	15.61	14.46	16.25	16.25	16.82	17.57	17.18	16	14.18	13.82	13.89	12.29	9.57	9.46
8.61	9.79	9.39	10.32	9.79	9.25	10.25	11.18	10.71	11.82	11.96	14.43	15.14	14.39	16.71	15.96	15.36	16	15	13.96	11.04	10.75	10.57	10.5
9.79	7.82	8.07	8.5	10.29	10.54	11.89	13.04	14.04	14.29	15.29	15.07	16.04	16.68	15.46	17.46	14.96	17.11	14.5	15.71	14.86	12.57	11.68	10.93
11.11	12.11	11.07	12.68	10.93	10.46	11.46	10.75	11.43	13.11	12.57	13.21	14.43	15.54	15.5	15.46	16.46	14.86	14.75	16	13.5	11.96	11.71	12.43
14.11	11.75	10.5	12.25	9.36	9.14	11.21	11.71	11.61	15.11	17.14	16.75	16.89	15.71	19.75	19.54	19.11	18.04	18.36	16.25	14.57	15.82	13.43	13.57
12.07	10.46	13.36	12.93	14.71	16.25	16.21	16.29	18.39	16.79	18.07	19.07	21.14	20.18	20.68	21.25	19.96	17.68	17.89	16.14	13.79	13.14	12.21	9.82
9.82	11.89	12.11	11.39	11.39	11.21	10.82	11.89	13.5	14.21	13.82	13.96	14.82	16.82	18.54	20.29	19.36	18.61	17.96	17.75	15.61	13.75	12.25	12
13.36	11.68	12.57	12.25	10.18	11.43	10.21	10.64	12.5	12.46	13.14	13.79	15.64	15.68	14.18	14.68	13.36	13.61	13.86	12.32	12.18	12.29	10.32	10.82
10.79	9.25	10.43	10.39	9.25	10.64	12.18	12.54	11.86	12.61	13.96	14.11	16.21	16.71	16.64	16.25	15.57	15.68	14.36	11.46	11.29	11.89	10.89	10.18
11	10.36	9.54	11.43	9.68	11.25	11.82	13.21	14.25	13.96	14.75	15.54	15.71	16.43	16.14	16.25	16	16.04	15.54	14.21	12.89	12.75	11.61	11.21
10.68	11.43	11.82	12.25	11.25	10.79	12.04	12.75	13.89	16	16.57	17.46	16.54	17.64	18.86	18.68	20.14	18.36	17.14	15.29	13.93	13.57	14.07	13.18
12.57	11.61	12.96	12.36	12.61	12.43	13.14	14.04	14.61	14.75	15.5	15.43	16.5	17.89	18	18.32	17.79	16.64	16.57	14.61	13.54	12.89	12.07	11.07
11.21	10.43	9.64	10.04	9.89	10.5	10.43	12.25	12.18	12.18	14.96	15.36	16.14	15.89	18.25	19.18	19.14	17.82	16.29	14.68	11.61	10.64	10.32	12.36
9.54	9.64	11.36	11	10.25	11.5	11	11.82	12.79	14.71	16.43	17.29	18.43	19.68	19.18	18.93	19.07	17.75	18.11	16.64	14.21	13.46	12.46	10.39
12.25	11.75	10.96	9.18	11.11	11.64	12.39	11.93	13.79	16.21	17.96	18.46	21.89	21.64	19.54	19.29	19.75	18.82	18.79	18.39	16.57	14.96	15.5	13.82
13	13	11.57	11.79	11.71	11.43	13.61	14.43	13.96	14.5	16.54	17.29	17.61	19.57	21.18	20.79	19.5	18.71	17.71	16	12.29	10.79	10.36	11.29
11.32	11.43	11.54	10.43	11.32	12.32	11.36	13.82	13.04	13.36	16.5	17.68	18.46	19.64	20.43	19.86	18.5	16.14	14.71	14.18	13.36	10.79	12.14	9.86
11.14	11.39	11.68	10.71	10.07	11.04	10.54	12.46	13.29	13.21	14.46	14.18	14.21	16.64	16.86	16.04	15.86	15.75	14.14	14.64	13.71	12.32	10.5	9.21
9.96	9.11	9.82	10.07	9.68	9.57	11.82	11.82	12.5	12.93	13.64	14.86	17.61	17.36	18.14	18	17	17.21	14.71	14.04	13.79	12.82	10.32	9.46
10.61	9.82	9.5	10.11	9.32	9.07	9.61	11.11	11.54	12.5	13.04	14.64	15.96	17.07	16.79	17.79	19.32	17.29	17.79	15.89	12.5	9.21	10.68	9.75
9.57	9.07	9.75	9.18	9.89	7.25	8.11	10.54	12.54	11.61	14.18	15.43	15.61	17.18	18.89	18.68	17.14	15.68	15.11	13.75	14.25	12.07	10	10
8.86	9.5	8.36	11.04	10.25	11.96	11.36	12.82	12.89	14.71	14.25	15.14	16.07	16.5	16.07	16.29	15.71	16.46	15.89	14.07	11.71	9.93	10.25	8.89
10.71	10	8.57	10.32	10.82	8.57	8.61	9.82	9.68	11.29	12.46	14	14.07	14.04	14.14	15.07	15.71	15	15.25	12.25	11.32	9.21	8.71	7.14
7.25	6.75	7.64	8.21	10.07	8.21	9.18	10.39	11.57	11.68	13.07	13.86	16.18	14.68	15.11	14.71	16.54	16.46	15.54	15.54	14.89	14.68	11.68	11.29
10.32	11	11.07	9.68	10.39	11.5	13.21	13.07	13.07	14.07	14.04	14.5	15.32	15.86	15.11	15.68	16.5	15.54	15.29	14.71	12.29	11.96	10.82	9.36
9.04	8.25	9.07	9.43	10.36	10.93	10.21	12.57	14.14	14.36	15.75	16.93	16.43	17.61	18.25	16.64	17.43	17.14	16.46	14.82	13.61	13.68	13.75	12.96
11.21	11	10.71	10.82	9.96	10.43	9.61	11.43	13.04	11.64	13.04	14.32	15.21	17.21	17.54	17	18.07	17.29	16.04	14.21	13.71	11.29	10.39	10.64
9.79	10.61	9.43	11.04	12.14	11.29	10.14	11	13.29	13	12.89	15.36	16.43	17.96	17.07	17.43	17.18	16.68	14.04	14.5	11.82	10.46	10.5	10
9.57	9.21	8.75	8.39	8.71	9.39	9.71	11.96	14.36	15.04	14.93	15.82	15.93	15.57	16.46	16.54	19.36	17.46	18.04	16.11	14.32	12.36	11.04	8.89
8.96	9.32	9.39	7.93	8.14	9.54	10.18	10.61	11.07	12.14	13.36	14.14	17.36	17.5	17.57	16.57	16.29	15.61	14.25	13.32	10.86	10.64	10.93	8.93
11.57	10.71	11.11	10.79	12.04	11.04	12.57	12.82	13	13.96	14.39	16.75	17.54	18.86	20.61	18.82	19.36	18.25	16.29	15.64	13.36	10.68	10.04	10.93
11.11	9.25	8.96	9.54	8.21	9.32	10.14	12.04	13.18	14.5	13.64	15.32	16.61	16.71	17.14	17.18	17.25	16.43	17.57	15.14	11.43	12.14	9.39	10.14
10.18	10.39	10.57	10.75	11.86	9.14	9.21	11.93	13.75	13.86	15.54	16.43	17.57	17.75	18.36	16.21	16.61	17.18	16.54	15.64	12.68	13.75	11.68	9.57
7.89	8.61	8.79	9.5	10.5	10.54	10.86	11.18	12.18	14.07	16.32	17.68	19.07	18.18	18.21	16.29	18	17.96	16.96	14.86	14.64	11.79	11.07	8.89
10.04	10.89	11.71	10.25	10.43	9.46	10.79	11.82	12.36	12.71	14.07	13.82	14.86	15.89	15.96	16.39	16	17.79	16	14.21	13.43	11.21	10.18	9.68
10.93	10.25	10.79	10.86	9.5	9.64	9.96	12.71	13.75	16.96	16.79	18.21	18.29	19.5	19.64	19.61	18.07	16.75	14.93	15.86	15.04	14.54	11.43	11.5
11.96	10.18	11.11	10.57	10.86	10.61	11.25	13.11	14.36	16	16.82	17	18.11	19.11	18.21	19.86	19.68	20.18	17.57	14.93	13.43	11.89	10.79	8.71
9.21	8.71	8.39	8.64	8.68	8.39	9.43	10.29	11.93	13.43	13.61	14.36	16.36	15.43	15.21	15.04	16.75	15.57	14.5	12.5	13.18	9.68	8.32	8.96
7.25	7.82	7.82	7.89	8.25	8.86	8	9.36	11.32	11.75	12.86	15.39	17.14	17	17.07	18.11	16.43	15.21	14.82	14.61	13.68	13.29	11.5	11.96
10.32	10.75	11.14	9.61	9.82	9.86	9.82	10.32	11.43	12.96	14.64	15.57	16.11	18.29	16.46	17.14	16.71	17.71	14.29	13.79	14.79	12.04	10.64	9.75
7.96	10.11	9.18	9.18	9.07	10.21	9.61	9.61	10.32	11.71	14.29	14.5	15.32	16.39	15.07	16.43	16.64	15.29	15.21	15.54	13.75	11.18	10.82	10.93

9.82	8.79	11.39	10.54	8.86	8.21	9.64	11	10.68	12.25	13.29	14.39	15.93	14.86	15.71	16.29	16	16.04	14.82	13.39	11.75	12.61	12	10.32
9.11	9.29	8.96	10.18	9.25	8.75	9.46	10.54	11.86	12.64	13.36	14.64	15.79	17	17.46	18.14	18.5	16.93	14.61	13.68	10.39	10.46	7.75	7.5
7.04	6.75	7.86	8.86	9.5	9.25	8.86	10.86	11.54	11.11	13.11	14.14	15.39	15.61	17.21	17.46	18.07	17.5	17.64	15.79	15.07	13.39	12.79	11.21
11.96	12	12.18	11.86	13.11	12.71	12.36	14.21	15	14.21	15.21	16.25	15.75	17.64	17.07	15.89	15	15.61	14.82	14.43	13.43	11.21	10.54	9.64
9.75	9.25	8.68	9.04	10.71	9.86	10.89	12.18	12.14	13.96	16.25	17	16.57	15.71	17.14	18.75	17.46	15.04	14.68	15.11	11.93	10.93	12.36	10.32
10.18	11.21	10.39	10.18	10	9.68	9.04	10.68	10.96	11.04	13.11	14.25	14.93	18.46	16.07	18.57	17.68	17.5	16.39	15.29	10.71	10.57	9.54	10.11
9.71	9.07	9.86	9.07	8.93	8.36	9.64	11.82	11.68	13.11	14.39	14.39	14.25	15.86	16.04	16.64	17.14	17	15.89	15	12.89	13.36	11.25	10.82
10.11	10.14	10.25	11.96	11.82	12.11	12.14	12.46	13.5	13.96	14.68	14.39	15.82	17.11	17.68	17.54	16.68	16.75	16.07	14.64	14.18	11.71	10.32	9.5
8.07	8.54	8.36	9.21	6.96	7.39	7.64	8.54	10.43	10.96	12.14	14.43	14.68	15.11	14.61	15.39	12.64	14.75	14.39	12.18	12.54	10.32	9.89	9.32
8.14	8.21	8.46	7.64	8.82	8.46	8.64	9.57	10.39	12.64	13.39	14.79	13.54	13.46	16.29	15.29	14.71	14.96	13.89	12.5	11.39	10.71	10.43	10
8.93	9.21	8.39	7.71	8.07	7.82	8.54	9.64	10	11.14	12.64	13.21	14.29	16.39	15.61	14.96	16.54	15.68	15.43	13.75	12.21	10.43	8.82	9.68
7.5	8.61	8.68	8.29	8.86	7.68	8.07	9.18	10.75	11.29	12.18	13.11	14.79	15.25	16.07	17.21	15.82	16.14	16.18	16.04	14.04	11.11	9.93	8.54
8.93	9.21	9.46	8	10.14	10.14	9.43	10.61	11.54	12.14	13.21	15.68	16.39	16.68	16.25	17.04	16.14	15.75	14.11	13.86	11.71	10.32	9.93	10.25
8.75	7.96	8.11	8.57	7.11	7.89	8.21	9.43	10.93	12.89	14	13.36	14.54	16.07	15.57	16.29	14.46	14.61	15.07	11.79	11.11	9.64	9.57	9.39
9.32	9.86	9.36	9.18	8.86	7.61	8.14	10.14	11.96	13.14	13.68	15	14.93	15.39	15.46	14.61	13.68	13.46	14.68	11.04	9.71	9.11	8.14	7.61
10.21	10.07	8.89	7.29	7.25	8.18	7.57	8.18	10.14	12.43	12.86	13.82	14.89	15.64	15.32	14.61	13.75	14.61	12.57	11.82	10.5	10.25	9.86	9.21
8.11	8.89	8	8.04	7.75	7.36	9.29	9.39	10.75	12.79	13.93	15.29	17.14	16.29	16.54	17.21	15.79	15.36	12.11	11.79	10.64	9.14	9.18	8.64
9.54	9.29	9.64	9.46	8.61	10	9.68	10.5	12.11	13.46	13.93	14.75	14.96	14.32	15.14	16.43	15.43	15.82	14.82	14.46	12.39	11.43	10.57	9.82
10.04	10.36	9.46	9.11	7.36	8.68	10.36	10.57	12.96	13.18	14.43	15.75	17.46	18.39	17.93	17	16.54	15.68	15.07	13.89	12.46	10.14	11.32	9.64
8.36	8.36	8.54	9.5	9.96	8.89	10.14	10.11	13.07	13.39	15.68	15.86	15.36	16.07	16.04	15.5	14.43	14.39	11.57	12	12.07	10.07	10.39	11.36
8.39	9.43	8.11	8.18	9.29	8.43	8.21	9.43	12.82	12.96	13.25	13.86	15.25	16.71	16.36	17.36	16.96	16.5	14.18	12.64	12	11.36	11.14	10.82
10.57	10.61	10.21	9.29	9.89	9.61	10.68	12.54	12.61	14.68	14.86	16.29	18.46	19.54	20.71	18.75	18.43	18.14	17.21	16.18	14.64	13.07	12.82	11.93
10.54	10.39	10.21	10.5	9.96	8.89	9.5	9.79	10.71	12.96	11.86	14.61	15.39	15.96	15.57	15.14	16.11	14.96	14.07	12.25	11.36	11.61	8.18	8.61
8.68	9.46	10.14	9.36	9.32	9.57	9.82	8.79	9.18	11	12	13.18	12.86	14.75	12.5	12.93	13.54	13	11.29	10.29	8.71	10.5	9.89	8.71
8.64	8.54	8.11	8.54	9.36	9.57	9.46	11	11.29	12.07	12.89	13.64	14.18	14.25	14.82	14.68	17.21	13.93	10.68	12.14	9.75	8.29	7.93	6.96
6.61	7.71	7.57	7.61	7.61	6.71	6.11	6.43	8.64	9.57	10.04	13.71	13.54	15.54	15.11	15.75	14.79	15.93	15.5	13.68	12.07	9.21	9.25	10.39
9.5	9.21	8.04	8.68	7.75	7.29	8.36	10.07	11.04	11.21	12.86	15.25	14.46	17.82	15.04	16.82	16.64	15	13.04	11.14	10.11	9.43	8.04	7.71
8.46	7.89	8.61	7.89	8.11	8.75	7.21	8.5	10.64	11.11	11.14	11.32	12.43	13.71	14.96	14.71	13.46	12.82	12.29	10.93	9.61	9.93	9.82	7.75
6.68	6.96	7.04	6.04	7.96	7.82	8.86	7.61	8.39	9.07	11.68	14.25	14.57	13.36	12.64	14.39	15.36	13.61	13.79	12.11	11.61	11.18	10.46	11.43
9.89	11.07	10.18	9.5	8.79	9.43	9.68	10.18	11.29	11	11.04	12.43	13.89	14.57	14.07	13.68	13.43	12.64	11.39	10.79	9.61	10.64	9.64	10.71
9.75	9.54	9.61	9.32	8.71	9	8.89	9.36	9.89	11.43	13.07	14.54	15.14	16.32	15.21	15.46	14.39	14.39	12.68	13.86	12	13.11	11.21	10.89
10.64	11.36	11.04	8.61	10.04	10.29	10.57	11.21	10.18	11.5	12.46	15.14	15.86	16.07	15.18	16.32	15.39	13.75	14.04	13.18	12.32	11.25	9.89	9.39
10.36	9.54	8.79	8.57	8.11	8.5	8.36	9.11	11.04	12.82	13.36	15.39	16.54	16.25	17.25	16.36	16.46	15.86	14.46	11.68	12.39	11.54	9.43	9.89
9.71	9.79	8.11	7.29	7.5	8.11	9.5	10.57	10.89	12.43	14.68	15.75	15.64	15.57	16.57	15.89	16.07	15.86	15.57	15.14	12.54	12.25	10.54	10.36
11.57	10.75	10.43	11.61	11.29	11.07	9.79	10.68	10.21	9.89	12.61	13.39	15.21	14.61	16.5	16.64	16.71	16.54	15.14	13.5	12.79	10.36	11.75	11.39
10.04	9.79	9.79	9.68	9.29	9.11	9.86	10.64	13.71	14.46	14.71	15.25	16	15.96	15.96	16.14	15.07	13.75	12.96	11.71	11.29	11.43	11.04	11.57
11.68	12	11.21	9.86	10.29	10.57	10.46	11.32	11.04	13.39	14.29	14.75	15.46	15.82	15.46	14.75	13.43	13.68	14.11	11.86	10.04	8.68	8.96	8.86
7.43	7.64	7.5	7.29	6.18	6.54	5.64	7	7.71	9.36	10.25	11.64	13.93	14.11	15.21	15.11	14.96	14.07	12.54	11.79	12.25	11.21	11.32	10.75
12.18	11.68	9.68	11.07	12	10.82	12.43	11.82	12.89	14.18	13.36	16	15.64	15.96	16.39	15.86	16.46	14.64	13.46	11.79	10.68	9.46	7.93	7.18
8.04	8.43	8.5	6.79	7.57	6.93	8.07	9.25	9.93	10.75	10.89	13.11	14.61	15.68	15.68	13.54	16.64	14.57	16.21	13.39	12.29	10.43	9	9.57
7.86	8.71	9.82	9.39	10.11	9.61	8.57	9.32	11.11	13.25	14	14.25	16.43	17.61	18.54	17.5	18.93	17.14	15.57	15	12.82	11.07	10.25	10.21
10.18	11.04	10.86	11.32	10.25	10.68	10.43	12.07	13.25	13.93	14.79	15.36	15.64	16.21	17.96	17.32	18.11	16.79	16.79	16.25	14.39	14.36	13.29	13.43
12.64	11.36	10.93	10.64	11.07	10.43	10.25	11.75	12.14	12.29	12.96	13.5	15.29	15.68	16.18	14.32	15.64	15.04	13.68	11.25	9.57	9.86	10.36	10.36
10.89	10.82	10.46	9.29	8.18	8.89	8.43	8.71	10.04	11.21	14.04	13.93	15.89	16.14	16.29	16.82	16.18	15.29	13.79	13.96	15.14	11.82	10.36	11.11
10.04	10.11	9.82	9.86	9.04	9.68	8.14	10.89	11.11	13.25	14.29	14.82	15.39	16.57	17.64	18.04	16.86	17.07	15.89	14.86	13.21	12.14	11.75	11.11
11.25	10.75	8.36	7.71	7.61	7.79	7.25	8.68	9.93	11.14	11.86	12.04	12.25	13.93	14.71	16.07	14.39	14.18	14.75	11.96	12	11.46	12	11.79
10.54	11.43	11.96	10.68	11.68	11.96	12.14	12.29	14.96	14.36	14.79	17.32	16.54	16.75	17.79	16.29	16.54	15.71	12.71	13	13.29	10.86	10.18	9.64
9.04	8.61	8.71	9.18	9.46	8.82	7.96	9	10.43	11.79	12.36	13.75	14.68	16	16.75	15.46	16.5	16.36	15.18	16	13.93	12.71	11.39	11.86
11.07	11.25	10.79	13.57	13.29	12.18	11.61	12.36	13.89	12.75	15.75	16.32	17.71	17.89	18.57	17.86	17.32	18.36	17.25	16.25	13.5	13.11	12.61	12.5
13.54	13.21	13	12.96	14.64	13.04	14.29	14.21	12.14	13.93	15.79	16.93	17.04	16.54	16.68	15.86	13.79	13.07	12.07	10	9.93	8.89	9.39	7.89
9.64	10.5	9.82	8.71	9.11	9.96	9.46	9	11.36	11.11	12.57	14.04	14.93	16.36	17.46	16.43	15.71	14.64	14.36	13.61	12.86	12.29	10.21	9.5
8.54	9.75	10.29	10.46	11.89	11.43	10.32	11.04	13.04	14.29	14.79	16.36	16.46	17.32	17.68	18.43	17.04	16.79	15.5	15.75	13.04	10.89	9.93	9.25
10.43	9.79	10.64	10.64	10.18	12	11.75	11.71	12.93	13.89	15.54	17.07	19.43	20.71	18.54	18.57	19.68	18.46	17.21	15.93	13.79	13.86	14.5	12.89
13.07	13.11	12.79	13.18	11.57	11.68	11.43	12.11	12.04	12	14.04	14.43	15.64	18.14	17.11	17.89	17.39	16.54	14.89	14.79	13.82	12.71	13.46	12.96
12.07	10.64	11.36	9.5	10.25	10.39	10.71	12.07	12.86	12.14	14.18	15.29	17.11	16.64	16.75	16.79	15.36	14.93	13.61	12.68	11.43	11.89	10.71	10.82
10.39	10.54	10.07	9.64	9.32	10.25	10.43	11	11.71	12.75	15.04	14.46	15.54	15.36	14.86	15.46	14.93	15.14	13.86	13.29	13.54	12.75	11.29	11.18
10.29	9.71	10.18	10.96	10.75	9.11	9.75	10.43	10.86	11.29	12.89	13.32	13.89	15.79	15.75	16.96	15.57	15.07	14.86	13.86	13.11	10.71	8.61	7.82
6.61	8.18	7.54	8.25	7.79	8.21	8.93	9.11	8.89	11.79	12.43	12.5	13.32	14.43	15.21	14.11	13.39	14.18	15.36	14.07	13	12.07	10.39	9.43
10.46	10.64	10.21	10.82	10.89	11.04	11.14	11.64	11.89	13.61	14.25	16.29	16.68	16.64	15.54	17.46	16.36	17.39	15.36	14.25	13.18	14	11.43	11.89
11.29	11.04	9.39	9.86	8.93	8.79	8.46	9.86	9.64	11.64	13.39	14.46	15.5	17.43	18.43	18.18	15.75	16.86	16.14	14.57	13.32	13.14	11.14	10.07
9.82	10.21	9.79	10.43	9.04	9.21	9.5	9.5	11.21	12.89	13.61	15.46	14.68	15.71	16.32	16.64	18.11	16.14	16.07	13.82	13.89	12.11	11.43	11.79
11.46	9.43	10.96	10.39	11.64	12.82	12.96	13.25	15	15.36	15.86	15.46	17.43	18.29	18.54	17.82	18.46	17.04	15.93	15.11	12.89	11.68	11.36	9.25
8.54	8.75	10.14	10.18	10.89	11.71	12	12.46	13.75	15.18	15.36	15.89	16.46	16.64	17.79	17.89	17.5	15.86	15.79	13.93	14.36	13.93	13.36	13.25
12.71	12.5	12.14	10.89	11	12.86	13.04	13.36	13.89	13.75	15.61	16.68	17.57	17.14	18.36	18.5	17.46	17.96	15.61	14.18	12.89	12.46	11.04	10.04
9.86	8.82	8.25	9.79	10.46	10.04	11.64	11.29	12.43	15.57	16.21	17.39	17.96	19.89	19.89	19.71	20.21	18.14	18.68	15.21	14.21	16.21	15.11	14.29
14	14.04	13.14	13.11	13.36	12.75	13.46	13.39	13.57	14.14	16.32	16.25	16.89	17	16.93	17.75	18.18	16.25	16.82	14.93	14.43	12.57	12.89	12.07
11.96	12.57	11.32	11.32	11.5	9.61	10.89	11.82	12.29	14.11	14.71	16.71	18.21	18.57	18.93	19.93	18.32	16.36	15.21	13.25	11.57	9.89	10.89	10.68
8.32	8.07	8.43	8.25	9	10.79	9.5	10.39	11.71	12.21	14.54	15.39	16.61	18	18.86	18.61	16.07	16.29	15.25	14.21	14.61	14.93	12.61	11.82
13.71	12.25	11.39	10.61	11.79	11.64	11.93	13.5	14	16.21	16.43	17.14	18	18.54	19.36	18.96	20.68	18.75	18.61	15.36	15.71	16.07	14.36	13
14.07	12.89	11.68	11.14	11.43	11.93	12	11.82	13.39	14.54	15.21	15.89	18.25	17.11	18.07	17.75	17.79	17.46	14.43	13.93	16.04	13.46	13.36	13.25
12.18	11.96	11	10.96	10.46	9.5	11.32	11.93	12.39	12.64	13.5	15.46	16.75	17.32	17.32	17.57	16.86	17.11	16.29	15.68	14.68	14.43	11.46	12.5

11.75	12.11	11.11	12.54	11.89	10.21	11.29	12.18	13.64	13.25	14.79	15.61	15.82	17.18	17.32	17.82	17.36	15.14	13.86	13.64	12.11	12.29	12.86	11.57
11.29	10.11	11.43	9	9.82	8.96	11.79	11.96	13.5	12.04	14.79	16.86	18.89	18.61	18.64	18	18.68	18.18	17.11	15.79	14.75	14.14	14.04	12.32
12.5	10.89	10.75	13.07	13.07	11.61	13.07	14.07	14.14	14.04	15.71	17.79	17.39	18.07	18.07	18.46	20	18.57	17.07	16.07	14.18	12.68	12.64	12.46
12.86	13.43	12.29	12.89	11.71	11.71	10.96	11.71	13.21	12.32	14	15.61	16.14	15.39	15.5	15.18	15.07	15.61	14.89	13.11	13.21	14.07	12.14	12.39
10.39	10.79	10.32	11.43	11.39	11.75	11.79	14.04	12.39	13.96	14.96	15.07	16.86	16.82	17.61	17.57	17.14	16.54	14.61	15.29	13.54	13.46	12.25	10.11
11.54	11.54	10.93	11.43	11.14	12.14	10.54	10.86	11.11	11.04	14.04	14.54	16.39	17.29	16.11	17.43	18.46	15.32	15.11	15.29	15.21	14.86	14	14.21
13.04	14.32	13.54	14	14	13.54	13.75	14	15.46	16.18	16.43	17.54	17.64	19	18.86	19.04	17.25	18.36	17.96	16.14	15.21	14.46	15.5	15.82
14	15.5	14.61	15.43	15.29	13.29	13.18	13.07	14.75	16.68	16.68	18.46	20.21	20.57	20.68	20.5	20.64	18.86	18.86	15.89	14.61	12.32	12.89	11.46
12.57	11.21	11.11	11.04	10.29	10.79	11.04	11.46	12.21	13.68	14.25	17.89	19.11	18.82	18.25	19.29	19.14	17.89	17.57	14.68	15.36	14.5	15.36	15.25
14.71	13.25	11.86	12.93	10.96	10.11	10.96	10.79	12.46	14.14	14.46	14.36	15.89	15.18	16.57	17	18.46	17.57	16.04	14.43	13.89	13.89	13.43	12.75
13.14	13.61	12.04	13.5	12.93	10.25	10.71	12.32	12.96	12.46	12.64	12.68	14.86	16.04	17.61	18.96	18.86	18.21	16.32	13.64	13.43	14.25	13.64	12.29
11.68	14.71	12.96	14	13.36	14.5	14.86	17.04	16.46	16.57	17.39	18.96	18.43	19.11	20.39	21.79	21.36	18.36	16.96	14.82	16.07	17.43	15.82	15.07
13.21	11.79	11.82	12.39	12.71	12.18	12.57	11.79	13.89	15.54	15.36	17.5	17.79	18.96	18.71	17.86	17.71	17.25	15.46	14.36	12.75	11.96	11.39	11.04
11.04	10.89	10.54	11.43	10.43	11.96	10.25	10	11.57	12	13.32	15.18	16.25	17.46	18	16.75	17.46	18.86	16.68	15.36	14.93	14.86	14.04	13.71
12.5	12.32	12.89	13.68	12.11	11.43	11.75	12.11	12.82	13.21	15.29	16.82	17.18	17.75	16.14	16.89	16.86	15.18	13.54	14.32	12.61	12.5	12.14	12.43
12.39	11.64	11.93	12	11.79	12.71	14.25	13.21	13.43	14.71	14.11	17.07	17.18	16.14	17.14	17.29	17	15.54	13.14	12.36	11.43	11.71	14.11	11.79
12.79	12.68	12.29	13.36	12.25	13.14	12.43	13.14	14	16.71	16.14	16	17.11	16.21	17.25	20.39	19.07	19.07	15.46	12.71	12.71	12.07	12.89	12.71
11.86	14.39	13.11	13.57	12.07	11.54	13.82	13.54	15.68	16	18.71	19.82	20.86	20.93	21.68	21.32	20.14	21.57	20.36	17.54	19.86	16.18	15	15.93
16.32	16.11	15.36	15.43	15.46	14.11	14.36	13.71	16.25	13.82	15.32	17.14	18.93	20.82	19.21	21.18	19.04	18.93	16.21	15.46	16.07	13.04	12.57	12.04
12.21	12.04	10.64	13	14.61	14.82	13.75	13.54	14.79	15.61	17.89	18	19.43	20.07	19.68	20.68	20.32	19.75	18.64	17.75	16.71	18.21	15.32	15.04
16.75	17.07	16.64	16.79	16.93	14.93	14.54	15.57	18.89	17.75	19.14	19.82	20.21	22.68	23.29	24.36	24.36	21.29	20.25	19.25	18.43	17.75	16.04	15.93
15.43	15.82	15.32	15.46	15.82	16.64	15.93	15.96	17.5	17.79	19.36	19.36	20.36	21.54	23.04	21.36	20.61	20.54	18.18	16.5	15.39	15.11	13.79	14
13.68	13.75	12.71	12.43	12.86	13	12.18	11.04	12.21	13.32	15.07	15.75	14.89	15.18	18.14	16.96	15.43	14.54	14.64	13.71	15.71	15.14	14.25	13.21
11.61	10.89	10.54	11.82	10.68	11.5	11.18	13.29	14.11	14.68	15.39	17.39	17.39	18.14	17.11	16.86	16.64	15.32	14.75	12.32	13.14	12.68	12.39	11.5
12.39	11.68	12.68	13.14	14.07	13.68	13.68	14.71	16.04	16.57	17.18	18.18	19	19.68	18.86	17.39	17.25	17.21	15.96	14.79	14.46	13.75	11.75	11.57
11.39	10.32	10.93	11.96	12.43	13.07	13.57	12.61	13.96	15.14	16.25	16.89	18.36	18.79	17.79	18.46	18.11	18.25	17.82	16.57	15.96	15.21	14.21	13.46
13.57	12.96	12.61	13.14	13.79	13.61	13.71	12.93	14.64	15.79	16.96	19.54	19.11	19.14	19.93	19.71	19.93	18.46	17.71	16.64	14.82	13	12.32	12.93
11.18	10.89	11	9.57	9.61	9.43	9.25	10.61	12.07	13.39	14.04	17.68	18.25	18.14	18.93	18.64	18.46	17.86	16.64	15.79	15.11	15.29	13.18	12.86
11.86	12.21	12.32	11.96	13	12.5	11.61	11.36	13.57	16.29	18.14	19.89	20.39	21.18	21.5	22.43	22.64	21.64	21.39	18.86	17.93	16.57	16.68	15.93
15.32	15.86	15.89	15.82	16.68	15.96	14.57	15.96	16.11	18.11	18.79	20.36	18.79	20.64	19.86	19.86	19.57	17.61	14.29	12.64	13.89	13.61	11.25	11.64
13.29	12.29	12.61	12.89	12.25	13.86	12.75	14.07	15.46	17.57	19.61	21.39	20.39	20.21	19.39	20.96	20.96	20.29	17.07	16.25	15.5	15.11	15.39	15.21
14.71	13.96	13.93	13.46	14.14	14.96	16.21	15.14	15.5	15.21	16.04	16.46	17.04	18.04	17.68	19.11	16	17.21	15.54	15.75	15.57	15.64	14.07	14.11
13.11	13.61	12.14	12.71	13.54	12.68	12.68	12.61	16.04	16.29	18.04	18.75	18.93	20.43	19.32	18.21	18.64	18.04	17.11	16.93	16.32	15.32	15.36	14.75
16.96	15.54	15.5	16.07	15.68	16.29	16.43	14.68	16.18	16.64	16.82	17.25	17.89	17.46	17.57	19.93	18.86	17.39	17.14	16.32	16.75	17	15.68	15.93
14.61	15	14.54	15.96	17.21	15.86	15.82	17	17.46	19.29	20.04	21.36	22.89	23.82	21.96	24.5	24.43	23.39	21.04	18.14	18.5	19.32	19.25	18.75
17.71	18.82	17.64	16.82	15.75	15.57	16.07	16.5	17.04	17.82	19	19	21.07	19.79	20.64	19.61	21.11	21.79	20.25	17.89	17.04	15.5	16	17.04
16.75	15.46	16.36	16.54	15.36	16.18	17.18	17.46	17.04	18.64	19	18.18	18.36	21.11	20.61	20.61	20.18	19.36	18.61	17.93	16.32	16.93	17.29	16.86
15.43	15.82	15.04	16.61	16.14	15.54	17	16.68	16.68	17.11	18.07	19.14	19.04	19.04	20.75	21.04	19.68	21.79	18.21	16.86	16.57	15.14	15.82	15.21
13.93	13.96	14.25	14.18	13.79	14.21	14.5	13.86	15.25	15.57	16	17.64	17.43	19.11	19.75	19.14	18.86	18.39	18.14	17	16.54	16.68	16.07	15.57
16.71	15.68	15.46	17.39	16.32	15.71	16.21	17.36	18.18	19.32	20.71	20.29	21.54	21.18	21.89	21.57	21.64	21.86	20.39	19.32	19.39	18.43	19.21	16.79
16.46	16.04	15.79	15.89	15.5	14.14	15.57	14.89	16.14	16.18	16.54	16.61	17.54	18.54	18.11	19.11	18.89	17.82	16.29	15.71	15.36	14.64	16.32	15.43
15.29	13.79	15.11	15.29	14.57	14.93	13.46	13.43	15.29	15.14	16.25	15.54	16.04	16.32	14.61	17.68	17.71	16.93	15.54	15.39	14.21	14.39	15	14.46
16.07	15.11	13.43	14	13.14	11.71	12.43	14.57	13.96	15.61	17.14	17.32	18.29	18.43	18.57	20	20.46	20.11	18.96	19.04	17.79	18.11	16.68	15.79
15.5	16.86	14.04	14.5	16.21	15.11	15.86	18.11	18.43	20	20.36	20.86	21.61	22.07	21.79	21.57	19.57	20.96	20.25	18.86	17.75	16.61	17.96	16.32
16.71	16.29	15.61	16.04	15.11	14.68	16.18	15.64	14.79	15.61	14.64	15.43	16.14	16.79	16.32	15.68	16.71	15.25	15.46	15.18	14.46	13.82	13.5	14
13.93	14.25	13.68	13.39	12.5	13.04	12.93	12.21	13.79	14.96	16.18	17.75	18.68	17.57	17.61	18.79	17.54	16.93	17.11	16.54	17.79	17.86	17.79	17.89
17	16.93	17.43	17	16.68	17.68	18.39	18.29	19.36	20.25	20.14	22.07	21.68	22.07	23.5	23.79	23.43	22.5	19.39	19.18	19.36	18.14	18.04	17.25
16.36	16.79	16.11	16.39	16.07	16.61	16.86	16.79	17.11	19.04	17.5	19.75	19.64	19.25	20.04	20.07	20.07	19.04	18.14	17.96	18.14	17.79	15.82	15.07
14.61	12.96	14.36	14.75	14.64	17.36	16.54	15.89	15.11	17.57	18.36	18.43	19.14	19.46	20.25	20.68	21.32	22.32	20.71	20.11	19.25	17.68	17.43	16.93
18.29	15.86	15.93	16.5	17.5	17.75	15.07	15.5	15.54	17.89	18.71	20.39	20.64	20.86	22.57	20.21	18.46	19.79	17.29	16.64	15.79	14.71	15.18	16.14
17.25	16.04	16.32	15.71	16.21	16.39	16.54	17.71	18.64	19.32	20.32	19.57	20.32	20.36	20.18	21.14	22.71	23	18.68	20.04	18.86	17.75	17.43	16.32
16.46	17.39	16.11	16.79	17.96	16.79	17.54	16.82	16.21	17.93	19.39	21.75	21.46	22.61	23.18	22.14	22.14	22.29	20.57	19.68	19.39	19.5	19.79	21.07
21.18	21.82	19.64	20.64	21.71	18.96	18.39	18.43	19.61	20.39	21.18	21.57	22.54	22.25	22.57	21.82	21.79	20.18	19.57	19.46	18.25	18.07	17.5	16.11
16.07	17.36	17.11	16.89	17.86	17.57	16.71	17.5	17.93	18.29	18.5	19.5	21.21	23.5	23.82	23.32	22.14	21.21	19.54	18.46	16.79	15.89	16	13.21
14.5	14.61	15.25	14.43	15.29	13.96	14.36	14.18	15.46	17.18	16.93	17.82	18.96	19.82	20.21	19.61	19.25	19.29	20.46	20.04	19.61	18.43	16.57	16.29
14.89	15.57	15.21	14.96	15.21	14.93	14.75	15.07	15.46	16.04	16.21	15.82	19.18	20.64	22.39	22.89	21.57	21.29	19.39	19.21	19.79	20.29	19.89	18.68
19.57	20.07	18.36	18.75	18.93	20.54	19.68	20.5	19.71	21.18	22.07	22.86	24.79	24.11	23.86	24.54	23.11	23.36	22.54	20.96	19.54	20.29	19.29	19.32
19.68	18.75	20.61	20.82	19.43	18.36	17.68	17.57	17.68	19.04	18.61	19.04	19.25	18.86	20.29	20.07	20	18.54	18.14	18.18	16.68	14.86	15.86	16.32
17.29	16.96	16.25	16.46	16.36	16.64	16.82	16.21	16.96	19.04	19.46	19.39	19.68	19.86	19.36	20	20.75	18.79	17.46	17.29	17.25	18.57	17.89	18.54
17.82	18.11	16.89	15.79	15.93	15.75	16.61	18.93	18.71	18.46	20.21	22.36	22.32	23.21	23.14	21.89	22.36	19.18	18.32	16.89	17.29	17.79	18.04	16.82
16.79	17.39	18.43	19.14	18.75	17.64	17.75	18.36	18.5	19.21	21.54	21.04	20.79	21.32	20.61	22.21	23.07	21.61	21.96	20.43	20.14	19.86	20.79	19.68
19.96	19.39	18.43	18.32	18.07	17.57	17.18	17.96	20.36	19.43	19.93	21.54	22	23.32	24.75	24.18	23.29	23.11	22.25	22.18	20.93	20.29	23.39	23.14
22.11	23.57	23.25	21.86	20	19.25	18.82	17.68	17.71	18.79	18.14	19.5	19.61	21.86	21.21	21.86	21.61	19.36	20.29	19.39	20	18.36	18.07	18.11
18.64	19.46	19.43	19.36	19.79	18.89	17.71	17.96	18.04	19.54	20.29	21.79	23	23.5	22.68	23.96	22.96	22.29	22.86	21.07	21.79	19.61	20.46	19.54
20.86	20.14	19.86	20.29	19.18	18.11	19.29	18.54	16.5	17.61	17.18	18.57	20.25	20.82	21.54	22.18	20.64	19.75	20.14	19.14	19	18.75	19.04	18.54
19.61	18.64	17.71	18.71	19.29	20.71	19.96	20.82	20.79	21.71	23.04	21.54	20.75	23.93	23.43	23.68	23.71	23.86	22.43	21.5	19.71	18.54	18.57	20.14
21.39	21.21	18.86	19.79	21	20.43	21.14	20.71	19.57	19.14	20.36	20.71	22.64	23.57	23.32	24.82	24.29	22.79	22	21.14	22	21.71	21.21	19.86
20.54	20.5	18.29	18.32	18.21	19.04	19.07	18.75	19.54	20.21	21.68	20.43	21.14	21.54	20.75	20.93	18.71	19.29	18.86	17.36	17.46	18.32	18.36	18.68
17.79	17.11	18.07	17.5	17.86	17.07	17.89	18.71	19.82	21.89	20.68	20.75	23.04	23.36	22.89	23.29	22.96	22.14	21.61	21.39	21.14	21.39	19.89	19.39
19.11	19.07	19.29	18.5	18.14	21.82	21.64	20.04	21.32	21.71	23.25	22.75	23.89	25.29	25.18	24.57	23.79	22.43	21.07	20.14	20	20.21	19.93	20.18
19.32	18.25	18.79	16.82	17.89	18.36	18.39	19	19.96	21.18	21.25	21.82	22.86	21.71	21.79	23.5	23.04	22.71	22.79	22.11	20.89	21.04	20.89	20.39

19.57	19.79	19.68	19	19.64	19.82	19	19.64	19.46	18.39	18.75	18.96	20.36	20.86	21.96	23	21.79	20.57	20.71	20.07	19.32	19	17.46	18.57
18.75	17.61	17.89	17.86	20.04	19.25	20.43	19.54	20.11	22	21.14	22.43	22.89	22.79	23.71	24.32	23.43	21.89	22.46	21.39	22.18	20.11	19.07	17.71
16.32	17.07	18.36	17.25	19.25	20.57	21.5	21.25	21.71	24.21	24.54	27.07	26.71	27.54	27.11	27	27	25.89	24.07	25.29	24.29	22.68	21.75	21.29
20.82	20.82	21.64	21.5	20.29	20.25	19.46	20.36	21.71	21.64	21.86	22.46	23.68	24.32	23.36	25.64	23.07	23.86	22.79	23.07	23.93	21.71	22.68	21.89
21.75	22.39	23.54	23.25	23.57	23.96	23.64	22.75	23.89	24.57	24.71	24.07	23.5	23.39	23.89	23.79	21.14	20.96	22.43	23.71	26.14	24.46	24.96	23.14
21.68	22.36	21.21	22.54	21	22.25	22.71	23.07	22.68	23.46	23.04	22.89	23.39	25.14	26.39	27.21	25.39	25.14	22.61	24.04	23.04	24	24.57	24.96
24.86	25.29	24.93	25	23.39	23.79	24.43	24.46	23.18	23.29	23.82	25.43	24.96	24.36	25	24.5	22.89	21.14	21.43	20.93	20.5	19.39	21	20.71
22.39	20.93	21.5	21.25	20.43	21.5	19.79	18.39	18.25	17.68	20.14	17.86	18.61	21.68	23.14	22.43	20.82	20.43	20.32	19.07	18.68	18.5	20.32	20.68
20.04	20.86	20.39	20.64	19.18	19.43	19.68	19.46	18.82	19.21	21.46	22.46	24.18	23.39	23.32	20.93	20.89	19.14	17.39	17	16.96	17.36	17.25	16.61
16.89	16.39	17.89	19.18	19.21	21.43	20.29	20.54	21.89	21.5	22.11	22.54	24.14	24.07	23	22.36	22.11	21.04	20.75	20.29	20.96	18.89	20.36	21
19.79	20.32	20.68	19.25	20.43	21.29	20.46	21.79	22.79	21.43	22.25	23.75	23.82	20.86	21.32	20.39	21.39	19.93	20.32	19.5	20.39	19.29	19.32	17.14
19.29	19.68	17.96	19.36	17.86	19.43	18.96	19.32	19.21	20.75	23.39	23.57	23.64	23.39	24.68	25.14	24.79	22.75	23.39	24.25	22.79	23.64	22.36	20.36
19.82	18.82	17.79	17.86	15.64	15.57	14.64	14.11	16.96	17.93	17.43	17.39	19	19.79	19.36	20.04	19.29	20.18	19.46	19.5	19.39	21.18	20.46	19.43
20.07	17.86	18.25	17.14	18.96	18.61	17.04	18.54	18.96	20.43	21.89	22.11	23.04	23.71	24.89	24.07	22.43	22.61	20.89	20.89	20	19.18	20.71	20.11
20.5	19.54	18.64	19.57	20.57	19.82	21.21	20.96	19.79	21.39	22.39	22.14	23.61	24.14	24.46	25.04	24.36	24.96	23.54	23.82	23.25	23.18	23.32	23.93
24.18	22.54	21.68	21.79	20.79	20.29	19.5	20.32	20.93	19.71	22.54	22.64	22.61	22.82	23.93	22.54	24.5	22.93	20.54	21.93	21.86	21.36	20.11	19.61
19.54	18	18.14	19.04	16.68	16.43	15.82	16.36	17.07	18.68	19.71	17.39	18.75	19.29	20.46	21.14	21.64	19.93	21.25	21.86	23.04	22.32	21.68	20.04
18.68	19.43	18.64	20.46	18.82	18.57	18.64	17.79	17.36	18.43	18.96	18.36	19.36	18.71	19.36	19.82	21.71	20.93	22.04	22.5	22.04	23	22.29	22.64
21.71	19.89	21.82	22.57	21.75	20.64	21.57	20.61	22.46	21.29	20.96	23.25	24.43	21.68	22.18	21.71	22.75	21.32	22.14	21.43	19.96	19.54	20.14	19.5
19.89	19.68	22.36	22.71	22.04	20.89	21.07	20.89	23.54	24.54	25.5	25.96	26.82	26.61	26.5	24.96	23.18	23.71	21.93	22.32	23.82	22.89	22.18	21.36
22.61	21.21	21.29	21.46	20.68	21.75	22.64	20.14	20.5	19.36	20.43	19.82	18.46	20.46	19.14	21.11	19.89	21.36	20.18	20.39	20.71	20.96	20.61	20.57
19.18	19.04	18.89	19.18	20.04	21.39	20.18	19.89	21.79	22.32	21.29	21.96	21.75	22.39	21.61	22.82	20.82	21.64	21.39	22.14	22.79	22.46	22.32	22.57
21.68	22.86	21.54	20.64	20.71	20.57	19.04	18.96	18.75	18.25	20.07	19.61	20.79	21.61	22.11	22.96	22.71	22.68	23.5	22.86	22.21	23.36	23.25	22.29
21.25	21.75	20.71	20.25	22.21	21.57	22.64	21.96	21.86	21.79	21.43	23.29	22.61	22.79	22.14	21.04	21.25	20.39	20.39	19.89	19.32	19.93	20.29	19.96
17.89	17.71	17.39	17.93	19.39	21.32	20.57	20.54	20.43	20.21	19.89	20.14	22.21	23.04	21.96	22.18	21.54	22.82	21.82	22.11	22.29	22.25	23.43	21.32
22.25	21.21	20.82	20.5	22.86	22.07	23	21.29	21.93	22.32	22.32	22.68	23.25	23.18	24.14	23.89	23.36	22.86	22.07	21.36	22.54	20.46	21.14	22.29
21.75	20.57	20.68	20.39	20.29	19.96	20.14	22.96	24.25	25.36	25	25.29	27.25	27.96	26.64	28.89	27.89	26.68	25.46	24.89	24.86	23.32	23.36	22.5
23.25	22	21.04	20.61	22.32	20.86	21.57	21.89	21.29	21.36	21.61	23.46	23.68	23.36	22.79	23.54	23.71	22.29	23.11	21.43	21.93	20.43	20.75	19.5
18.14	18.89	17.86	17.89	19.54	20.5	22.57	20.5	21.64	21.36	22.46	21.36	22.29	21.21	22.11	22.14	20.75	22.46	23.75	23.64	24.29	24.68	22.43	21.71
22.14	21.04	22.07	21.5	20.82	21.71	20.57	19.57	20.39	22.46	24.11	22.71	25.21	27.18	26.89	24.93	25.54	27.93	28.61	28.11	29.39	29.75	27.89	27.68
26.54	26.5	24.61	26.29	22.89	22.89	23.46	25.79	23.25	24.25	23.29	24.46	25.11	26.04	25.86	24.89	24.79	23.29	22.96	22.96	24.25	22.75	21.96	21.75
22.75	21.93	21.64	21.18	20.25	20.75	20.61	20.79	20.86	23.21	24.11	23.07	23.71	23.46	23.46	21.57	21.11	20.96	18.96	19.5	18.96	20	18.46	19.11
17.43	19.14	18.93	20.64	20.5	20.43	19	20.11	19.04	20.21	21.11	20.82	20.96	21.29	23.54	21.82	22.96	22.68	23.25	21.04	21.71	20.46	21.75	21.54
20.89	19.93	18.71	20.64	20.68	20.96	20.21	21.5	20.89	20.75	20.25	22.75	22.21	22.32	22.64	22.79	24.07	22.96	22.93	22.43	24.25	23.89	23.79	23.21
20.86	21.68	20.86	20.89	20.46	20.64	21.86	21.57	22.61	22.75	22.64	22.54	22.64	22.14	21.93	23.75	22.82	21.89	22.21	22.68	22.54	21.82	20.71	21.61
19.93	21.25	19.93	19.29	19.07	18.39	18.07	19.79	21.57	19.75	20.61	20.46	19.89	19.39	18.68	19.61	19.18	19.79	20.32	21.07	20.64	21.29	20.71	20.61

Table B2: Observed hourly wind speed standard deviation at Toronto (km/hr)

Hr 1	Hr 2	Hr 3	Hr 4	Hr 5	Hr 6	Hr 7	Hr 8	Hr 9	Hr 10	Hr 11	Hr 12	Hr 13	Hr 14	Hr 15	Hr 16	Hr 17	Hr 18	Hr 19	Hr 20	Hr 21	Hr 22	Hr 23	Hr 24
10.17	11.66	10.91	10.92	12.25	11.62	11.25	11.69	10.12	11.63	12.18	10.69	10.72	11.64	12.61	12.24	12.3	11.24	11.7	12.97	12.49	11.98	9.79	8.54
8.46	8.04	7.27	7.46	8.54	9.62	9.73	10.02	11	8.75	9.01	9.8	8.17	10.46	10.69	11.19	12.83	11.9	11.79	10.66	11.69	12.91	12.63	12.92
13.95	12.11	11.7	10.61	11.77	10.74	10.36	10.45	9.85	10.15	10.7	11.86	12.37	13.5	13.07	12.95	12.39	13.39	13.59	13.49	13.88	13.52	11.44	11.61
10.27	11.61	12.57	13.19	13.22	12.86	11.94	11.46	12.19	11.88	13	14.51	11.79	13.77	12.93	11.23	13.14	12.77	13.42	13.73	14.23	14.31	13.91	13.14
14.13	12.96	13.28	13.74	14.09	13.01	13.61	13.61	12.59	13.47	13.61	12.86	12.04	11.14	13.17	13.13	12.41	13.47	14.97	14.57	15.55	14.75	15.57	15.72
16.48	14.68	15.89	16.77	14.81	14.41	13.68	13.54	11.84	12.64	12.71	13.22	13.04	12.74	12.64	12.51	12.14	11.28	11.83	11.95	12.45	12.46	11.84	11.52
9.66	10.94	9.76	9.61	11.77	11.21	12.48	11.21	11.48	13.83	13.23	13.53	12.64	13.06	14.39	12.59	12.97	12.51	11.75	13.45	10.69	10.61	9.55	10.52
8.86	8.35	9.52	8.22	9.6	10.01	11.45	11.06	10.6	11.33	13.41	13.42	13.9	13.2	11.68	13.24	10.47	11.58	11.69	11.54	11.11	10.82	10.96	9.49
10.12	10.6	11.86	10.58	11.29	10.59	11.91	12.18	12.07	11.41	12.22	13.28	12.29	12.79	12.95	12.09	10.87	10.76	11.37	10.9	11.16	9.98	9.73	10.62
10.65	10.01	11.27	11.9	10.54	11.12	9.9	10.25	13.04	13.28	11.96	10.92	11.03	12.17	13.49	15.37	13.23	12.91	13.28	14.42	14.62	14.87	15.03	15.12
16.35	15.16	15.73	15.68	15.16	14.4	13.01	11.79	11.34	12.39	12.15	12.64	11.8	10.27	11.94	11.32	11.61	9.98	11.43	13.11	12.73	12.79	13.25	12.89
14.53	14.29	15.07	13.76	12.9	13.25	13.19	12.56	12.45	12.42	12.02	12.26	12.26	12.94	14.06	10.78	11.89	10.46	10.96	12.2	12.63	12.92	12.82	12.52
12.27	11.78	11.9	10.71	13.46	17.27	13.43	10.18	9.8	9.94	10.99	10.08	12.45	11.23	11.99	11.47	10.19	9.54	8.6	8.84	9.21	9.63	8.99	9.16
10.2	9.88	10.23	11.38	11.89	10.54	10.72	11.1	10.76	12.5	11.62	13.7	12.64	13.34	14.76	15.15	14.03	14.08	12.56	11.48	10.67	9.23	9.17	9.7
9.09	11.13	11.51	11.31	11.3	11.6	10.59	11.99	11.19	9.26	9.99	8.97	9.61	8.58	8.66	9.55	8.86	8.99	9.44	10.01	8.62	8.75	8.43	9.42
9.81	9.67	11.54	10.69	11.97	13.49	11.11	11.7	12.3	12.33	12.29	12.16	12.38	12.67	11.69	12.19	10.91	11.11	11.63	11.7	11.7	10.82	10.54	9.62
10.76	10.44	11.52	11.73	12.39	11.88	12.55	9.79	11.11	12.06	12.91	12.36	11.97	12.54	12.33	13.61	12.7	8.76	9.64	10.31	10.26	10.14	9.45	11.49
9.75	9.7	7.75	7.37	9.02	9.02	9.44	8.65	10.1	10.98	11.75	10.9	11.97	11.49	10	11.76	10.56	11.36	9.81	11.31	10.88	10.19	10.37	9.65
10.72	11.46	11.44	12.4	12.36	12.98	12.13	12.08	11.05	12.22	12.53	13.66	12.43	12.66	11.86	9.03	12.11	12.4	12.51	13.95	13.28	13.63	12.26	13.66
12.33	11.19	10.89	11.29	12.54	11.48	11.38	11.11	12.11	11.61	11.98	10.93	11.68	8.65	9.83	8.9	8.88	9.45	9.95	11.35	10.57	9.88	10.91	10.12
11.4	11.4	12.32	10.89	14.34	12.57	12.6	12.04	9.91	10.27	11.45	10.44	11.12	9.62	11.22	10.8	9.75	9.69	9.12	10.51	10.38	10.43	9.06	10.96
10.89	11.8	10.55	10.52	10.7	10.94	9.99	8.18	8.32	9.23	10.76	10.29	8.78	9.97	10.63	12.28	12	12.54	12.07	11.54	12.54	13.32	13.95	13.58
13.83	13.02	11.54	12.92	13.76	12.62	12.91	11.43	11.86	9.92	9.04	8.26	8.62	8.79	12.39	12.38	13.63	12.87	12.66	12.47	12.2	11.51	11.58	12.87
11.19	11.78	11.77	11.17	11.23	10.7	11.13	11.51	11.65	10.99	12.47	12.11	11.21	12.53	10.8	11.96	11.55	11.03	10.8	10.57	10.79	11.11	11.02	10.87
12.25	9.8	9.68	9.02	9.1	8.85	9.6	11.67	12.1	12.06	12.99	13.85	13.33	13.52	12.66	12.97	12.34	11.62	11.7	11.83	12.35	11.7	10.11	11.45
11.3	10.41	11.42	10.57	11.84	12.49	10.24	12.42	12.8	16.62	15.91	16.02	15.27	15.84	16.57	18.17	16.28	17.51	15.85	15.92	17.67	14.02	12.46	13.94
16.01	15.4	14.33	12.8	13.43	14.64	13.67	14.24	13.2	12.82	13.32	12.59	12.94	13.66	14.12	11.91	11.48	12.01	10.72	11.82	12.26	9.98	11.28	12.58
10.99	11.03	7.8	9.54	9.45	9.92	10.34	7.72	7.89	8.9	9.5	9.39	10.55	13.88	13.7	12.25	13.77	11.06	10.44	10.51	13.26	12.23	12.41	12.14
12.96	12.47	12.69	10.84	11.82	11.09	11.05	9.62	10.46	10.85	11.76	11.89	10.09	10.16	9.92	14.95	14.42	14.27	15.61	12.86	12.36	11.81	12.6	10.91

11.97	11.12	10.31	10.16	12.54	10.6	11.59	10.84	12.06	10.16	11.02	10.64	10.49	11.13	10.25	10.51	9.79	9.16	10.08	9.64	9.25	9.47	8.72	8.76
7.64	9.26	10.87	11.72	10.75	10.94	12.34	10.77	11.1	11.44	11.73	13.23	12.16	11.81	11.52	10.54	10.69	10.16	10.01	9.99	10.1	10.02	10.01	10.93
11.19	11.68	11.58	10.9	9.71	10.61	10.11	9.01	8.22	7.64	8.2	8.71	10.37	11.62	12.99	12.47	13.45	12.5	9.96	10.44	9.47	9.22	9.32	10.57
9.1	10.81	10.82	9.44	9.37	9.44	9.31	9.78	9.37	9.61	10.26	12.11	11.68	11.32	10.79	11.59	13.15	13.03	10.69	11.43	11.02	11.08	12.02	9.18
9.6	9.78	10.01	9.97	11.31	10.43	11.73	10.98	11.26	9.67	10.38	10.45	9.29	9.4	10.28	10.47	10.6	9.51	9.59	10.4	10.57	9.99	8.34	9.8
9.86	10.12	9.15	8.3	7.4	8.47	8.36	9.6	10.43	11.57	9.86	11.57	12.24	12.21	12.71	10.68	11.47	12.53	11	11.24	10.27	10.31	9.74	9.56
9.45	9.71	9.01	10.05	10	9.15	10.89	10.64	10.99	11.31	10.56	11.84	12.35	12.8	12.34	10.38	9.7	10.27	9.89	8.74	9	8.06	9.93	8.35
10.66	9.5	11.15	10.52	10.65	9.51	9.1	10.38	10.4	10.37	10.26	9.56	10.05	9.51	10.04	8.93	9.31	7.95	8.9	9.18	9.02	9.37	10.7	11.38
11.65	10.03	10.75	11.07	12.2	11.05	11.71	11.93	14.18	11.85	12.47	11.43	12.4	12.71	12.18	13.01	12.12	10.61	11.13	10.76	11	12.24	11.93	10.82
10.6	11.01	11.38	11.09	10.25	12.67	11.75	11.45	10.56	9.45	9.77	10.3	10.59	11.45	11.79	11.81	11.19	11.93	12.61	11.48	12.62	13.22	12.02	11.21
12.42	12.01	13.91	12.27	12.44	12.96	14.45	12.21	11.82	12.01	12.11	11.5	11.91	10.04	9.8	9.73	11.56	10.54	11.48	11.32	12.18	11.07	10.08	9.24
11.78	13.05	9.74	11.69	11.68	11.57	11.26	8.58	10.47	10.13	10.99	10.95	10.67	11.11	10.6	9.79	11.09	10.46	10.89	11.44	10.35	11.15	11.07	11.69
14.12	13.26	12.84	10.67	10.35	9.81	9.93	9.2	8.65	8.67	8.39	8.86	10.02	9.25	11.41	11.68	12.11	12.68	13.1	13.37	12.83	12.92	14.32	12.77
10.6	10.09	12.69	12.65	13.16	13.42	14.1	14.82	12.96	12.5	13.87	12.65	14.33	14.21	14	15.42	14.28	14.82	12.05	12.14	11.68	11.81	11.36	10.67
12.34	11.56	10.46	10.37	10.37	10.12	9.53	10.67	10.85	11.83	10.98	10.45	9.39	10.23	8.85	9.14	9.25	9.2	9.76	9.86	10.11	10.27	9.64	10.11
8.53	9.56	9.31	8.7	10.33	10.56	10.03	10.49	10.93	10.92	8.93	8.36	8.04	7.41	9.7	9.54	8.51	9.3	9.98	8.78	8.36	8.64	8.03	7.02
7.04	6.34	7.48	9.01	8.11	8.24	8.25	9.32	8.33	8.91	9.48	10.61	9.37	10.77	11.19	11.77	10.38	11.21	10.28	9.98	8.72	8.89	10.37	9.71
9.72	8.06	6.83	7.62	6.2	7.22	7.1	8.03	7.32	8.16	8.36	8.29	8.42	9.61	10.61	10.89	9.44	9.57	8.21	8.53	8.04	8.43	6.93	8.9
9.59	10.21	10.93	10.39	8.41	9.91	10.83	10.35	8.83	11.06	10.04	11.03	9.78	10.03	9.37	9.87	11.14	11.44	10.79	11.43	12.92	12.42	10.77	11.13
12.16	11.48	11.19	11.27	11.79	11.7	12.08	12.26	12.44	10.86	10.52	10.68	11.16	11.31	10.58	11.26	10.49	10.7	10.17	10.77	10.27	11.06	10.7	10.11
12.48	11.82	11.96	11.32	12.55	14.52	12.26	14.39	12.42	12.82	12.04	10.75	10.78	10.85	9.67	9.76	10.4	10.72	9.26	9.1	8.43	8.1	8.77	9.26
8.05	7.58	7.45	7.09	9.21	9.34	9.4	8.44	9.51	9.71	10.54	11.09	11.11	10.02	9.9	11.11	12.85	10.66	12.58	12.08	11.49	10.9	9.88	9.96
10.06	9.77	9.3	10.92	10.23	10.11	9.58	10.59	11.55	12.76	13.04	11.32	11.98	11.38	11.32	10.82	10.96	12.07	11.88	11.45	13.16	13.62	13.72	14.26
13.53	13.77	13.25	12.41	12.53	11.43	11.98	11.81	12.91	12.57	12.61	11.71	12.47	11.3	9.57	11.81	10.72	10.01	10.18	10.86	11.05	11.56	13.38	12.3
13.96	12.04	11.87	11.85	11.71	11.74	11.72	10.57	10.51	11.08	11.4	11.29	10.39	11.18	9.36	11.3	10.81	11.86	10.62	12.05	11.71	11.1	11.86	10.83
10.55	9.75	9.68	9.64	9.62	11.97	11.41	11.62	12.71	10.26	10.6	11.29	13.19	12.08	13.21	11.17	9.67	9.95	7.69	8.78	8.63	7.98	8.4	8.86
8.6	10.02	8.83	8.94	7.46	7.76	6.83	8.6	7.8	9.2	10.27	10.11	10.78	11.11	11.46	11	11.52	10.85	12.56	11.63	10.31	11.55	11.58	11.19
11.49	11.05	11.42	10.66	11.49	10.95	11.55	9.41	10.4	10.41	10.76	11.84	11.61	11.47	11.54	10.72	10.89	10.58	10.78	11.27	11.71	10.78	11.51	12.11
11.54	11.38	11.87	11.03	12.65	10.16	10.91	11.15	11.95	11.33	10.34	10.3	10.76	10.45	9.85	10.65	11.14	11.75	10.74	9.24	9.62	9.94	8.48	8.72
8.5	8.08	6.66	6.8	10.15	11.86	10.16	9.78	10.06	10.21	9.75	9.33	10.83	11.62	12.62	12.76	12.93	12.24	12.53	11.75	11.64	10.5	10.34	11.74
11.61	11.69	12.39	12.69	11.97	11.48	11.88	11.97	11.15	11.07	11.14	10.92	11.41	11.05	9.97	10.33	10.34	9.87	9.84	9.36	10.06	9.71	8.65	9.4
8.73	8.1	8.86	9.03	8.93	8.48	10.37	9.6	8.59	8.79	7.76	10	8.82	9.7	9.59	8.5	8.43	10.3	10.84	9.06	9.17	7.75	9.95	10.05
9.93	9.64	10.03	9.71	10.42	10.52	10.47	9.93	9.26	11.08	10.86	12.46	11.27	12	11.09	12.21	10.41	11.03	11.91	11.14	9.24	9.78	9.74	9.73
11.88	13.25	13.31	13.1	14.26	14.69	14.5	15.98	15.82	16.45	16.48	15.1	14.98	14.25	14.72	15.48	13.52	12.3	10.62	9.58	11.47	10.16	10.48	11.18
11.43	11.53	11.42	11.68	12.51	10.95	10.67	10.13	10.53	9.96	11.16	11.85	9.98	9.69	10	9.38	8.66	8.59	10.3	9.17	9.35	9.56	8.97	8.78
10.73	10.33	11.24	11.85	11.88	11.91	13.41	11.45	10.51	9.14	9.76	8.95	9.51	8.45	9.11	10.19	10.64	9.81	10.28	10.08	10.41	11.1	8.95	10.4
10.41	9.96	9.13	9.49	8.34	9.64	9.35	9.65	9.12	8.39	8.12	9.12	8.69	7.19	7.6	7.75	9.2	8.25	8.55	9.02	8.41	7.45	10.16	8.79
9.89	10.11	8.31	10.65	9.47	10.09	9.71	9.11	9.62	9.69	11.37	10.85	11.45	12.31	12.24	11.09	11.17	10.15	12.11	12.2	10.88	10.84	9.66	10.21
10.82	10.14	9.57	10.02	10.11	11.14	11.3	11.24	11.21	11.11	10.85	10.52	10.02	8.61	8.45	9.26	8.23	10.32	9.36	9.22	12.89	12.49	14.71	12.26
13.49	13.31	11.68	12.5	12.49	13.09	12.66	11.93	12.91	13.6	11.23	12.89	10.43	10.32	10.84	11.66	10.66	10.75	10.73	10.35	11.21	11.63	11.53	10.09
10.25	10.23	9.95	10.32	11.25	9.89	10.31	10.61	9.84	9.09	9.66	8.91	7.92	7.91	8.13	8.36	8.17	8.35	10.01	8.99	9.46	9.44	10.36	10.99
10.55	9.71	10.2	10.4	9.92	9.66	11	10.74	13.32	12.46	12.57	12.92	13.3	13.2	13.42	12.59	12.03	10.99	11.15	12.45	11.3	10.26	10.66	9.81
8.74	10.72	11.83	12.11	12.36	12.41	13.4	13.62	13.01	14.05	13.31	12.75	12.74	13.36	13.44	12.33	13.54	13.54	13.27	14	12.45	11.97	12.69	12.68
11.76	12.5	11.63	12.83	13.46	15.92	15.91	16.3	16.65	15.73	15.04	12.55	13.12	12.34	12.16	11.38	12.31	10.88	10.5	9.05	10.27	9.95	12.22	12.47
13.14	13.54	11.6	11.5	11.25	10.45	9.21	8.53	10.15	11.57	10.98	11.09	10.49	10.48	11.12	10.33	9.68	10.2	9.13	8.44	7.86	9.92	8.37	9.55
10.26	9.97	10.15	11.42	11.66	10.84	10.44	10.65	10.59	11.62	10.65	10.32	10.7	11.13	10.41	10.81	10.43	10.22	9.22	9.87	9.38	9.86	8.16	9.43
9.51	8.83	9.08	9.74	10.04	9.58	9.26	9.26	10.47	11.67	12.63	11.58	13.53	12.27	12.26	13.69	12.4	11.82	12.57	11.6	10.07	11.84	10.27	9.47
9.38	8.37	7.34	8.16	9.77	12.63	10.28	10.88	12.81	12.56	11.22	9.44	10.63	11.72	12.15	11.32	11.38	13.61	12.54	12.13	11.74	10.13	9.93	10.21
9.84	8.19	9.17	10.55	9.9	11.58	11.85	10.51	12.23	12.9	11.17	11.86	11.58	12.19	12.57	13.69	12.37	11.11	11.09	11.84	10.32	9.61	9.57	8.7
8.87	8.98	7.42	8.15	7.35	8.1	8.6	9.86	8.71	9.23	8.98	9.26	9.98	10.31	10.63	9.83	10.07	8.4	9.83	10.85	10.75	9.73	9.54	10.37
8.5	8.65	10.28	10.07	9.05	9.36	10.2	10.07	10.75	11.35	9.75	9.79	9.39	8.75	8.82	9	9.83	10.05	10.39	11.31	9.95	9.66	7.66	7.74
8.67	8.63	9.6	10.06	9.83	11.02	11.9	12.3	11.95	11.92	11.62	11.37	11.74	11.4	12.39	10.99	12.08	14.03	13.28	13.82	11.99	10.71	9.31	11.18
10.45	11.21	12.46	10.7	11.65	12.69	11.33	11.47	10.4	11.62	11.97	12.16	9.98	8.94	8.81	8.96	8.58	7.82	8.89	7.83	7.85	8.97	10.51	9.83
10.49	11.12	11.01	10.94	10.1	11.77	12.56	12.56	12.39	11.92	10.8	10.55	10.24	10.36	10.69	11.35	11.77	10.97	10.85	9.13	8.16	9.67	9.04	9.6
7.29	7.45	8.46	10.02	7.82	7.99	8.49	9.19	8.85	7.64	8.9	9.24	9.53	9.88	10.75	9.82	8.85	9.73	11.04	9.91	10.3	10.98	10.67	12.11
11.86	13.23	12.64	12.71	12.68	12.06	11.48	12.87	11.99	12.22	10.79	10.36	9.2	10.85	11.42	11.65	10.75	11.79	11.19	11.78	10.88	11.16	12.99	10.4
12.13	11.45	11.13	10.74	11.7	10.81	11.73	11.02	9.16	10.91	8.73	10.89	9.01	9.54	9.24	9.97	11.09	12.23	9.58	9.7	8.62	9.2	7.79	7.58
6.88	6.68	8.38	9.17	8.61	7.9	8.43	8	9.02	8.49	8.45	7.74	8.25	7.24	8.47	7.85	8.6	10.4	8.88	10.99	11.13	11.23	11.78	11.6
11.97	11.05	11.46	10.98	10.33	10.86	9.22	9.82	10.44	10.04	11.79	10.23	9.97	10.95	9.59	9.15	10.22	10.29	11.26	9.81	11.49	11.49	11.2	12.11
11.78	10.23	10.82	11.73	11.35	11.41	11.77	11.21	10.54	11.41	10.91	9.59	8.94	8.89	10.41	8.84	9.95	11.2	11.16	10.9	10.02	10.35	10.56	9.99
10	9.49	8.84	8.55	10.87	9.89	10.06	11.66	11.39	11.72	10.35	10.95	9.82	10.98	9.1	9.07	9.54	10.09	10.77	11.03	11	8.93	9.54	10.92
9.96	11.47	11.12	10.48	10.17	10.02	10.7	10.78	12.01	11.44	12.85	12.57	11.08	11.34	10.83	12.66	11.65	10.47	12.3	12.88	12.09	11.39	10.8	12.13
10.55	10.77	11.87	11.26	9.88	9.77	7.63	8.75	8.47	8.55	10.05	9.3	8.5	10.38	10.82	9.62	9.25	11.64	10.48	11.37	11.32	12.26	14.94	15.09
14.12	14.66	14.35	13.92	11.47	11.63	9.61	9.6	8.69	8.78	8.11	8.48	9.7	9.63	9.88	11.12	10.97	13.24	12.94	12.45	10.73	11.42	10.65	11.06
11.61	12.1	10.55	9.49	9.84	11.04	12.88	12.93	13.21	12.8	11.79	12.32	11.81	13.02	14.35	12.56	14.19	14.32	14.56	17.82	16.2	16.84	17.09	16.16
12.94	13	12.65	12.18	11.46	11.1	10.14	9.79	10.85	9.37	10.05	11.63	10.75	10.98	10.3	12.06	12.03	10.89	10.63	12.56	11.24	11.01	12.08	11.07
10.43	10.65	12.55	13.85	14.55	14.37	12.78	14.23	13.77	14.83	13.25	15.47	14.76	13.3	13.29	13.23	14.14	13.05	11.56	14.47	10.9	11.3	11.1	9.67
10.12	10.57	9.71	12.63	10.81	10.92	9.72	9.12	10.03	10.73	8.12	10.53	11.01	11.95	12.75	13.31	12.54	12.58	12.66	13.31	13.25	11.91	9.38	9.96
9.66	9.45	7.16	10.14	9.42	11.6	9.65	9.64	9.62	9.64	10.73	9.72	9.3	7.97	10.55	10.64	9.12	9.89	10.7	8.24	8.19	7.35	6.85	9.17
8.85	7.67	8.17	8.42	8.14	8.64	10.33	9.76	9.58	11.42	12.02	12.1	10	9.8	9.92	10.4	8.61	7.41	9.56	9.21	9.78	9.55	9.26	9.24
8.71	7.8	7.85	7.72	6.87	5.96	6.55	6.57	8.05	8.57	8.09	9.47	8.64	8.4	9.89	8.84	8.95	9.73	8.35	8.81	7.17	7	7.51	7.47
6.41	7.04	7.26	7.97	9.34	10.11	9.7	11.29	11.06	11.39	12.37	10.89	12.16	13.38	10.63	11.36	10.75	12.55	12.27	12.02	11.62	9.16	8.39	7.85

7.13	7.77	9.37	8.56	9.19	8.44	9.48	9.91	9.9	11.41	10.3	10.79	9.12	9.05	9.87	9.94	12.2	14.74	14.94	13.79	11.12	8.5	9.59	9.79
9.36	10.95	12.3	11.32	10.33	9.73	10.27	9.44	9.73	10.22	10.76	9.95	10.67	9.44	11.53	11.6	12.18	10.43	12.13	9.64	8.68	7.47	9.01	10.34
10.8	9.98	8.88	9.1	10.49	10.49	9.78	10.84	10.4	11.28	10.54	10.84	11.75	9.51	11.64	11.08	10.59	11.91	11.98	10.97	9.53	10.8	9.99	10.95
10.55	11.58	9.81	10.11	11.1	9.97	10.96	10.27	11.22	10.67	10.9	11.42	12.22	10.77	12.07	11.23	11.18	10.19	10.43	9.85	9.41	8.45	7.66	8.15
10.63	10.5	11.05	10.01	10.49	10.39	10.53	10.42	10.92	11.05	9.55	10.05	9.98	9.9	10.46	10.8	11.96	10.8	10.77	10.09	9.48	8.57	8.09	8.66
9.98	10.68	10.3	9.84	10.05	9.46	10.28	10.97	11.54	10.98	11.48	11.09	10.8	10.31	9.62	10.82	11.93	11.68	13.85	13.45	13.3	11.07	11.31	10.3
9.3	10.02	8.42	9.02	11.18	9.8	10.73	8.92	11.43	10.42	10.44	12.05	11.3	10.43	11.45	12.4	12.08	13.26	12.91	12.54	11.31	11.33	10.29	10.32
9.88	9.54	10.39	10.3	9.81	11.75	9.96	11.96	13.82	11.83	11.59	13.26	11.87	11.86	11.09	10.59	11.34	10.93	12.68	10.33	12.64	11.92	11.83	11.16
9.84	10.95	9.8	11.83	11.27	11.19	12	10.98	10.34	8.95	10.06	8.97	9.06	8.45	10.07	8.27	10.42	9.81	10.58	10.12	8.53	9.35	9.66	10.7
9.49	9.35	10.8	11.24	10.81	9.64	9.74	9.37	9.65	10.3	9.21	10.52	8.95	8.67	9.04	7.57	10.11	9.83	10.85	11.94	11.91	12.11	11.93	13.09
14.92	14.85	12.84	11.81	11.25	10.79	10.43	10.26	8.95	8.8	8.42	7.76	8.68	8.11	7.78	9.44	10.1	7.63	8.51	8.06	7.31	7	7.31	8.17
7.38	9.92	9.38	8.33	9.33	9.42	10.43	10.16	9.33	8.63	8.05	8.3	10.28	10.82	9.46	10.03	11.25	10.73	11.32	9.55	9.21	9.07	8.39	7.86
9.13	11.77	11.52	8.15	8.57	8.39	9.06	8.56	11.39	8.15	8.26	9.27	8.85	8.78	9.17	9.39	10.31	10.92	10.9	11.07	8.39	9.18	8.38	8.57
8.59	8.46	8.26	8.05	8.37	9.75	10.26	9.92	8.38	10.02	9.33	7.68	9.13	8.53	9.4	9.89	9.33	8.51	11.79	9.02	8.79	7.96	6.17	6.21
7.7	8.71	7.43	7.8	8.13	8.2	9	10.96	11.03	10.27	9.77	10.3	10.8	11.08	11.16	10.48	10.76	8.44	9.88	10.8	9.34	7.93	7.57	8.4
7	6.49	7.03	7.32	8.61	8.97	11.23	8.67	10.08	8.33	8.04	8.83	7.26	8.71	8.74	9.43	8.89	11.4	9.95	7.68	8.79	9.15	8.87	8.56
9.42	9.93	11.97	10.4	12.64	12.51	10.93	11.97	12.31	9.25	9.75	10.46	9.7	10.76	10.28	10.08	8.47	8.84	8.12	8.07	8.1	8.71	9.34	10.68
9.84	9.45	8.88	8.85	9.49	9.09	8.79	9.29	7.69	7.82	7.59	8.55	8.96	9.65	10.7	10.5	9.4	11.97	9.37	8.83	8.26	9.04	8.07	8.5
8.87	8.2	8.82	9.45	8.72	10.21	9.51	9.11	9.82	10.63	10.59	11.15	11.54	10.89	10.58	11.29	11.31	10.57	8.78	9.03	8.62	8.97	8.42	9.53
10.47	9.29	10.75	10.52	10.65	10.71	11.09	9.3	9.71	11.12	11.53	11.78	11.82	13.7	14.92	15.21	15.65	14.42	13.66	13.62	11.76	11.7	11.52	11.42
11.62	11.74	11.5	10.44	10.85	11.26	10.2	11.96	10.33	9.08	8.57	8.33	9.45	9.74	10.21	9.39	10.2	10.59	9.82	10.01	8.67	7.99	9.74	9.25
8.21	8.92	10.08	9.2	9.72	9.19	9.23	9.59	10.26	9.68	7.35	8.02	6.37	7.2	7.16	7.82	8.96	11.31	10.05	8.77	8.8	7.6	7.74	8.61
8.7	7.79	7.68	8.09	8.65	8.51	8.5	8.42	8.33	9.19	8.36	9.48	9.44	10.39	10.25	9.69	10.6	9.86	10.44	10	8.36	9.28	9.03	8.97
10.16	9.41	9.16	7.96	7.96	9.65	8.73	9.28	7.98	7.71	7.72	6.99	8.43	8.2	9.56	9.79	9.97	10.56	10.48	9.02	9.33	10.08	9.47	9.97
9.7	10.07	8.8	7.66	7.37	8.55	8.74	9.17	8.15	8.15	8.97	7.72	7.69	8.84	8.61	8.42	8.99	9.32	9.2	9.36	10.14	10.97	10.05	10.23
9.05	10.22	7.9	7.33	6.92	6.95	7.17	8.12	8.7	8.54	9.5	8.79	9.18	8.11	8.18	8.41	8.31	8.66	9.47	10.01	9.55	7.52	7.27	9.01
8.12	8.49	7.55	7.47	8.28	9.49	9.69	10.65	9.5	9.16	9.48	9.33	10.25	10.32	10.15	9.45	11.3	11.66	11.33	11.59	12.03	9.72	10.22	10.62
12.07	11.75	10.69	10.23	8.76	7.03	8.03	8.47	9.46	8.46	7.35	7.27	7.31	8.41	9.88	9.71	12.54	9.93	10.19	9.28	9.03	9.95	8.01	8.1
7.56	9.12	8.56	8.66	9.18	8.96	9.14	10.01	11.46	10.68	10.68	10.55	12.61	12.03	11.22	11.97	12.58	11.31	10.13	9.28	8.34	7.06	8.14	9.64
10.16	10.45	10.26	10.88	9.94	8.93	9.37	9.37	8.8	9.05	9.04	9.09	9.12	8.97	7.78	10.27	9.98	9.9	10.25	10.18	9.8	7.76	8.46	9.62
9.88	11.52	9.97	9.54	8.98	9.03	9.6	8.94	9.02	8.47	10.3	10.87	11.67	9.71	9.1	8.88	10.7	9.46	10.33	11.19	9.71	8.61	8.49	9
8.17	8.94	9.98	9.37	8.48	9.04	8.4	9.11	7.59	7.6	7.01	7.66	6.31	6.78	7.88	8.68	8.29	8.93	8.61	9.06	7.36	8.38	7.7	8.51
6.9	7.26	7.36	8.58	8.74	10.1	9.99	10.96	9.22	9.95	8.62	8.95	8.22	8.04	8.2	10.51	9.46	11.08	11	9.61	10.03	9.23	9.97	9.63
9.07	8.71	9.44	9.42	9.14	9.71	9.05	10.63	10.42	10.25	9.79	10.65	10.13	11.8	10.34	9.38	9.48	10.55	9.46	9.65	8.84	8.46	8.4	8.21
9.08	7.73	7.71	8.5	8.58	9.18	9.07	9.78	9.32	9.57	9.38	9.9	9.45	9.65	7.72	10.18	10.37	9.45	8.37	9.16	8.93	9.78	8.99	7.84
7.89	7.95	9.34	7.54	6.88	7.86	7.56	7.43	6.89	6.65	6.98	7.35	8.6	8.15	6.79	7.52	7.65	9.15	10.04	8.3	9	7.35	6.5	9.92
8.7	9.01	5.46	7.32	6.74	5.29	5.54	6.09	6.6	6.45	7.86	6.36	7.34	7.67	8.24	8.72	9.17	8.01	8.93	8.82	7.11	6.95	8.01	9.18
9.33	8.6	7.59	9.56	8.15	8.11	7.97	9.36	9.05	8.13	8.48	9.5	7.6	7.28	8.41	9.44	9.08	8.21	8.18	8.93	8.43	8.73	8.46	9.44
7.25	8.22	9.2	8.87	7.58	7.51	8.47	9.05	9.08	8.7	9.21	8.61	9.52	10.31	10.28	9.29	9.43	9.31	8.84	9.65	9.98	10.68	9.36	7.71
8.99	7.03	7.97	8.67	9.85	8.8	9.43	8.34	7.99	10.11	8.97	7.83	7.78	8.69	10.53	11.17	10.85	10.43	8.13	7.8	7.48	7.87	7.01	7.54
6.42	6.21	6.57	7.62	8.05	8	9.98	9.87	10.26	9.8	8.85	8.68	9.17	8.33	8.64	7.91	8.95	9	8.91	9.94	8.4	8.02	7.38	6.78
8.87	7.49	7.21	7	8.95	7.25	8.79	7.42	7.92	7.29	7.95	8.96	7.34	9.06	9.93	10.48	11.73	9.98	10.46	10.18	8.53	8.34	8.71	7.66
8.08	8.39	8.67	8.75	7.87	6.83	7.22	7.28	7.6	6.69	8.2	7.68	7.64	6.96	8.61	7.06	9.09	10.29	11.29	10.31	8.8	10.11	8.93	8.93
8.94	8.68	7.99	8.13	7.49	8.13	8.7	8.44	8.61	9.5	11.02	9.32	10.18	10.03	10.92	11.06	12.63	9.96	10.26	9.05	9.63	9.59	8.24	9.65
9.11	10.83	10.28	8.32	8.09	9.35	8.42	9.29	7.51	8.33	6.4	5.74	5.69	7.49	7.99	8.01	9.27	9.24	10.22	8.97	7.94	6.31	6.53	7.52
6.49	6.18	6.53	6.38	7.49	6.55	7.65	9	8.56	7.85	6.77	6.59	7.73	8.31	9.02	9.01	9	9.49	7.56	9.22	7.03	7.75	6.6	7.23
7.51	8.1	7.03	6.32	7.99	8.65	7.95	6.46	5.88	5.97	5.47	6.27	5.26	6.66	7.53	7.17	6.76	9.66	8.79	9.24	8.64	7.98	8.4	9.34
5.78	6.42	5.96	5.72	6.97	7.29	6.83	8.42	9.13	7.78	8.38	9.28	7.45	7.76	8.28	8.27	7.46	7.34	7.34	8.94	7.46	7.48	6.97	7.49
6.99	7.42	9.32	9.46	7.52	7.75	8.34	7.72	7.85	7.97	9.47	7.16	6.85	7.09	7.12	6.89	7.23	7.63	8.31	9.27	8.37	7.81	8.46	8.6
8.6	9.06	7.77	9.02	7.86	7.42	7.82	7.24	7.32	10.17	8.97	10.99	7.86	7.01	10.19	9.69	9.97	10.48	10.66	8.49	8.18	8.92	8.65	8.13
9.18	7.61	8.1	8.22	8.31	9.3	9.35	10.33	11.39	12.25	9.88	10.38	8.59	11.45	10.52	10.65	10.27	9.93	9.4	9.61	10.31	9.3	9.07	7.67
8.12	8.89	7.56	8.11	8.88	8.86	9.61	8.35	9.12	9.3	9.6	9.43	8.61	7.99	7.49	7.8	9.5	9.99	7.98	7.84	7.72	9.26	7.53	8.38
8.71	8.5	7.36	6.93	6.53	7.72	5.95	6.42	6.51	5.77	7.52	6.34	7.34	6.88	7.26	7.96	9.58	10.53	9.92	9.46	7.5	7.68	7.63	7.39
7.2	7.14	6.69	6.11	6.9	8.98	10.21	10.21	9.3	9.82	9.76	9.14	9.4	8.85	8.7	8.82	7.28	9.41	9.64	9.09	8.32	8.26	8.57	8.59
7.9	6.77	6.61	7.06	6.96	6.31	6.97	8.55	8.79	8.59	8.76	8.5	8.04	7.66	7.46	7.21	7.6	7.22	7.53	7.16	7.68	7.91	6.7	6.91
6.69	6.76	6.35	6.76	7.08	7.61	6.97	7.94	8.46	8.86	9.97	8.58	8.55	10.67	10.35	9.99	9.78	10.29	9.83	9.87	10.32	8.33	8.67	8.86
7.37	7.49	9.14	7.41	8.18	8.33	7.01	8.72	8.41	7.46	8.1	6.82	6.71	8	8.61	8.17	7.91	8.8	8.81	7.63	7.98	5.34	6.87	6.83
7.31	8.16	7.13	8.1	6.92	7.45	6.3	6.17	6.51	7.05	7	7.86	6.45	7.36	8.48	8.61	7.65	8.02	8.94	8.85	6.86	7.14	7.36	6.49
6.22	7.09	7.17	7.76	7.89	7.41	7.01	8.04	10.34	11.5	9.9	11.46	9.92	9.28	10	11.14	11.9	10.81	11.23	8.22	7.31	6.71	6.23	6.13
6.43	6.1	6.77	6.46	7.19	7.97	9.72	8.2	8.46	8.43	8.66	8.5	9.31	8.91	7.07	7.49	7.04	7.55	8.39	7.53	8.18	6.17	7.37	7.65
7.45	7.36	7.63	8.04	9.9	9.41	9.77	9.93	9.79	8.74	8.41	8.32	8.28	8.14	8.84	8.31	8.85	8.53	8.47	7.72	7.5	8.84	6.94	7.03
7	7.52	7.29	6.86	7.97	7.61	8.05	9.08	8.94	8.52	8.93	10.11	10.18	8.38	7.5	8.56	10.23	10.3	9.68	7.81	8.11	7.88	10.04	9.36
10.7	9.81	10.91	9.5	9.3	9.7	9.51	9.24	7.39	6.97	7.71	6.78	6.55	7.11	7.34	7.98	8.81	8.62	8.81	8.36	6.89	6.66	6.57	6.67
6.55	6.3	8.13	7.14	6.54	7.59	9.06	8.11	8.7	7.18	8	8.44	8.16	7.88	8.93	7.85	8.55	10.29	8.86	7.04	7.22	7.32	6.66	5.35
6.69	6.56	6.97	7.33	6.32	5.23	6.24	6.39	6.18	6.6	6.29	7.01	6.51	6.39	6.62	8.62	7.17	6.94	7.22	8.23	6.86	6.24	8.09	7.19
7.74	7.45	6.38	7.04	6.09	5.78	6.28	6.52	7.28	7.21	7.22	6.57	6.97	9.48	8.55	10.94	9.86	8.54	9.75	8.52	8.53	7.35	7.76	8.32
7.44	7.27	6.7	6.74	7.76	9	6.96	8.22	7.79	7.3	5.64	4.63	5.31	5.37	6.97	5.78	6.12	6.39	6.85	6.49	7.73	6.12	6.49	5.61
7.81	6.46	6.68	7.02	5.17	5.32	6.5	6.35	5.83	5.26	7.08	7.23	6.79	7.06	8.22	9.88	8.35	8.21	8.53	7.54	8.04	6.42	6.17	6.41
5.28	5.4	6.22	5.95	7.51	6.16	6.83	6.77	7.7	7.55	9.48	9.49	8.75	9.24	9.42	7.99	9.47	9.39	7.99	8.34	8.16	6.16	5.55	6.93
5.64	6.1	5.73	6.16	6.78	7.5	6.36	7.32	8.27	6.81	7.02	8.1	7.9	7.22	7.85	8.1	6.97	7.32	9.31	8.43	8.3	8.05	7.35	8.21
6.5	6.51	7.05	8.92	7.02	6.75	7.98	7.92	9.58	9.22	8.79	9.36	7.88	8.34	8.97	9.58	10.14	9.44	7.92	7.76	8.48	6.95	6.72	6.21
5.54	6.19	6.16	6.12	5.68	6.69	6.13	5.8	6.48	6.1	5.99	5.56	6.15	7.68	8.47	9.74	10.57	8.43	10.01	9.65	7.67	6.81	7.33	6.84

7.29	6.9	6.52	6.86	5.87	6.18	6.59	7.47	7.71	7.32	7.49	8.51	6.73	8.67	10.27	10.73	8.15	8.69	8.37	9.36	8.1	7.24	6.17	6.88
6.97	5.31	5.15	4.78	5.06	5.02	6.51	7.18	7.11	6.37	7.62	8.14	6.56	6.86	8.64	8.14	7.16	8.51	9.77	9.97	9.28	8.91	7.43	7.23
7.38	7.66	7.97	6.81	6.06	6.53	8.21	7.1	5.81	6.14	6.24	6.14	7.81	8.45	9.49	8.66	9.09	9.26	10.16	8.71	8.66	7.78	9.28	5.59
7.72	6.24	6.14	5.52	8.7	5.56	7.42	9.86	7.07	7.59	6.73	6.93	6.99	6.86	7.53	6.94	8.09	8.92	8.07	8.48	8.51	7.09	8.85	9.42
8.5	7.73	8.18	8.63	6.92	5.84	6.88	6.21	6.12	6.83	5.29	5.18	5.78	6.46	8.82	8.66	8.18	7.73	8.27	8.96	6.12	5.84	6.2	7.46
5.57	6.48	7.95	6.97	7.33	6.08	6.04	7.79	6.66	7.36	6.86	6.29	5.77	7.3	6.91	7.31	7.73	8.34	6.97	7.71	7.55	7.57	7.31	7.31
5.16	6.9	7.02	6.89	6.58	6.46	6.42	6.64	7.75	9.4	10.51	10.09	10.78	9.53	11.27	10.3	10.68	8.81	9.08	7.76	9.5	6.61	5.93	5.65
5.45	4.87	5.29	5.79	5.33	4.74	7.53	6.75	8.17	7.95	6.81	6.77	5.92	6.25	7.23	7.65	8.85	8.72	8.31	7.39	9.4	7.09	5.95	6.64
6.88	5.94	6.43	6.54	8.13	7.78	7.46	9.15	7.97	9.64	9.6	8.34	8.71	9.6	10.25	9.03	8.64	9.2	8.26	10.49	9.5	9.52	7.56	7.09
6.67	7.25	8.37	7.61	8.1	8.25	6.85	6.88	8.45	8.59	8.28	7.82	7.95	6.72	6.72	7.63	7.98	6.29	6.53	7.04	6.04	5.97	6.44	6.2
7.58	6.41	5.7	5.87	5.41	6.32	5.57	6.29	6.7	8.38	7.95	7.66	6.67	7	7.7	8.38	6.9	7.57	8.2	6.63	7.31	6.57	6.76	4.99
6	5.41	6.76	5.69	6.33	7.37	6.23	6.82	7.6	6.35	5.84	6.77	8.75	6.87	6.59	7.98	9.07	8.42	9.11	8.35	8.59	6.56	6.66	7.21
7.05	7.46	7.04	6.76	6.05	5.76	5.68	6.85	6.4	6.2	5.95	7.07	7.24	8.58	8.01	7.9	9.16	10.68	9.38	8.44	11.64	7.89	6.86	6.31
6.14	6.69	7.29	6.29	5.5	5.25	6	6.29	6.63	7.26	8.3	8	7.44	7.66	8.53	9.54	9.9	8.43	9.97	10.18	6.8	7.09	8.02	6.47
6.64	6.62	6.73	7.41	6.07	6.34	7.5	7.45	7.25	8.42	7.36	8.72	8.21	6.55	7.44	7.42	7.67	9.17	8.42	8.79	7.06	5.85	6.56	5.76
6.42	5.98	5.88	7.14	6.66	6.28	6.4	7.35	7.06	5.81	6.9	7.37	7.69	6	5.56	8.14	9.41	8.25	7.69	7.31	6.2	5.31	5.74	5.38
4.07	3.92	4.5	5.25	7.38	5.19	4.98	6.11	7.37	7.33	6.43	6.38	6.8	7.2	7.83	6.53	8.18	8.95	7.82	9.32	7.82	7.2	6.46	6.1
6.33	6.83	7.18	7.72	6.79	6.71	6.71	7.85	8.5	8.42	8.02	7.54	6.64	6.76	7.37	6.41	7.18	6.74	6.41	8.74	7.29	5.95	6.01	6.82
7.47	6.48	5.6	5.14	5.95	6.11	6.41	8.32	7.84	6.17	5.85	5.74	6.46	6.84	5.36	6	5.82	5.77	7.11	9.25	6.47	5.87	7.46	6.59
6.86	7.9	8.33	7.8	7.17	6.86	8.24	9.15	10.09	7.7	9.22	7.79	7.95	8.27	8.22	7.77	8.81	10.49	9.88	9.13	7.69	6.14	5.43	5.22
6.64	6.71	6.38	6.73	5.09	6.61	6.61	8.48	6.78	7.53	7.41	7.16	6.7	5.93	6.16	6.99	7.78	8.28	9.07	8.92	8.18	7.21	6.69	6.89
7.01	7.22	8.08	8.12	8.2	7.86	7.44	7.87	7.85	7.57	7.98	7.77	7.55	7.58	7.39	7.03	6.97	6.64	7.53	7.3	5.74	5.59	5.55	4.96
5.59	4.9	6.66	5.65	4.77	4.41	5.27	5.72	4.64	5.75	5.39	5.35	5.55	6.67	7.42	8.72	8.28	7.2	7.24	7.78	7.47	5.64	5.31	6.07
5.72	5.23	6.95	7	6.79	7.79	6.3	8.02	7.48	5.88	5.61	7.44	6.58	6.99	7.58	7.08	7.88	7.42	9.14	7.43	6.67	6.17	6.11	6.63
5.99	6.18	5.01	4.59	7.44	4.48	6.5	5.75	5.38	5.37	6.27	6.34	6.95	7.71	7.97	7.87	10.02	8.29	8.92	8.39	7.21	6.16	5.23	5.29
5.43	4.72	5.57	5.72	6.11	6.13	6.42	6.34	7.13	7.09	6.74	6.68	7.36	6.64	8.11	7.33	7.75	8.44	7.17	8.27	6.77	5.94	5.7	4.88
5.43	5.33	7.08	7.61	8.1	8.35	8.23	7.64	6.46	6.92	6.69	7.07	5.77	5.68	5.75	6.98	7.87	5.89	7.01	6.37	5.67	5.49	5.66	5.42
4.95	5.88	5.55	6.78	5.02	6.18	6.83	6.21	6.02	6.64	7.89	6.81	6.88	6.16	7.98	7.87	9.01	8.87	9.49	8.81	6.97	5.41	5.31	6.05
5.74	5.95	7.06	5.46	6.28	6.12	5.57	7.56	8.26	7.16	6.87	7.44	8.3	7.88	7.87	7.52	7.85	7.11	7.38	7.68	7.32	6.54	5.8	4.74
8.76	7.45	8.26	6.27	5.42	6.23	5.98	6.01	5.72	5.77	5.79	6.72	5.82	5.8	5.83	6.31	6.04	8.21	7.43	7.57	7.27	5.74	4.96	6.56
5.39	5.6	5.55	5.62	5.12	5.95	5.67	6.74	7.07	6.85	6.71	7.43	9.9	7.59	8.35	8.51	8.23	7.85	7.56	8.25	7.41	7.65	5.71	5.31
5.27	5.82	5.56	6.64	6.13	6.9	5.84	6.78	7.55	6.37	6.69	6.15	6.75	7	7.65	8.5	6.81	8.08	8.34	10.13	7.5	6.09	5.99	5.5
5.28	6.17	5.72	6.06	5.59	5.89	6.62	5.84	7.07	7.13	7.75	7.59	8.9	7.18	8.17	6.83	8.48	8.17	8.42	8.25	8.61	7.16	6.5	6.44
6.9	5.76	5.93	6.72	6.11	5.18	5.76	6.04	7.86	5.53	6.65	6.29	8.26	6.91	6.14	7.05	5.27	6.15	6.39	7.52	9.06	6.45	7.1	6.77
6.13	5.96	5.29	6.46	6.55	6.8	6.35	6.5	5.88	6.47	6.37	5.91	7.24	8.86	8.37	8.83	8.37	8.75	8.54	9.37	7.27	6.33	6.52	5.84
6.32	6.46	6.66	6.18	7.25	7.33	7.67	6.92	7.1	8.39	8.22	7.68	8.42	8.59	9.92	9.13	9.74	8.75	8.71	8.85	7.36	7.16	7.31	6.62
7.79	6.3	5.97	5.49	6.53	6.21	6.73	6.08	6.71	6.95	6.62	5.77	6.82	6.24	7.49	6.94	7.67	7.23	7.73	7.76	6.61	6.9	6.04	6.08
5.58	5.96	5.96	5.16	7.33	6.75	7.55	7.03	7.72	7.84	7.5	6.63	7.2	6.21	6.38	6.41	6.64	6.85	7.36	6.96	5.5	5.77	5.19	4.72
5.08	4.82	5.26	5.73	5.28	6.53	6.78	7.09	6.56	5.73	6.13	6.99	6.69	6.41	4.71	6.69	12.85	6.93	7.38	7.37	6.47	7.46	6.21	5.93
5.33	5.39	5.71	5.97	5.95	4.85	5.33	3.71	4.85	4.81	4.61	5.1	6.04	5.87	4.78	5.1	6.09	5.54	6.84	7.18	6.49	5.83	6.3	6.16
6.07	5.71	4.72	5.89	5.3	4.32	5.16	6.65	6.89	6.5	6.86	7.73	6.82	7.15	7.42	7.27	6	6.54	6.16	5.93	5.69	5.25	4.83	4.17
4.89	4.97	6.02	4.7	5.63	6.04	6.04	6.08	6.38	6.02	6.25	6.32	4.89	5.18	5.34	5.57	5.19	5.83	6.26	6.77	7.28	6.11	8.11	6.48
6.61	6.67	6.46	6.16	6.07	6.32	5.36	4.89	5.29	4.71	5.14	6.28	4.81	4.86	5.49	6	6.29	7.24	7.02	7.56	6.61	7.24	6.07	6.86
6.01	7.01	5.81	5.8	4.94	5.17	6.8	6.85	6.19	6.23	5.18	6.28	5.85	5.51	5.62	5.54	5.26	5.62	8.23	8.71	7	7.6	6.72	7.06
6.73	8.06	7.71	7.04	6.7	6.49	6.15	6.05	5.33	7.33	6.4	6.35	6.44	6.58	6.95	6.84	8.5	6.95	8.63	8.25	8.91	10.07	8.7	8.87
7.46	7.33	7.5	6.11	6.79	7.76	7.79	8.17	7.48	5.2	4.75	5.8	6.31	7.04	5.13	6.79	6.62	6.35	7.81	7.09	7.86	8.33	6.92	6.1
5.57	5.03	5.06	4.63	5.01	4.57	5.25	5.87	5.6	6.86	6.53	7.72	8.37	6.63	7.22	7.79	9.23	7.98	8.36	8.12	8.18	6.97	4.92	5.65
6	5.45	5.04	5.38	5.51	5.18	5.91	6.16	5.66	5.47	6.32	4.95	6.42	6.76	5.82	6.74	7.27	8.14	7.06	7.09	6.52	6.84	4.84	5.67
7.27	6.07	7.51	8.2	7.08	5.8	7.11	6.6	7.88	6.62	6.09	6.28	6.29	5.58	6	5.83	6.67	6.16	6.95	7.42	7.73	6.4	6.16	6.82
7.21	5.51	6	4.96	6.5	6.77	6.46	6.01	6.86	6.4	7.43	7.27	7.56	6.23	7.67	6.99	7.46	6.91	7.35	8.32	7.98	6.68	6.61	7.15
6.41	6.62	5.21	5.1	6.79	6.48	6.54	6.3	5.34	6.38	6.61	4.94	5.74	4.68	6.01	5.62	6	5.97	7.1	7.73	8.07	7.09	6.61	6.2
5.33	4.08	4.89	4.37	5.52	4.63	4.19	5.14	5.3	4.12	4.28	5.23	5.89	5.77	6.32	6.48	7.11	7.64	7.37	7.7	8	7.84	8.32	7.67
6.56	7.96	7.21	8.08	7.76	6.29	6.01	6.12	6.24	6.91	6.58	7.96	7.74	6.76	6.73	6.15	7.43	6.79	7.93	6.68	5.77	6.33	5.84	7.1
7.1	6.4	6.12	6.08	5.21	4.46	5.68	6.39	6.05	6.85	4.97	6.1	6.43	6.62	5.37	5.49	7.04	6.73	7.88	7.97	7.65	7.37	4.8	4.86
3.34	5.47	7.82	7.36	6.59	7.06	5.33	6.52	7.85	7.99	8.23	6.45	6.9	8.57	8.82	8.67	10.17	8.45	8.62	7.44	6.83	6.58	6.28	7.66
7.74	8.77	8.99	8.25	7.91	7.29	8.23	8.85	9.09	9.55	9.86	8.02	7.94	8.07	8.96	9.07	10.34	8.38	7.29	6.56	6.26	5.11	5.48	5.63
5.7	5.18	5.95	5.39	5.06	5.78	6.11	6.3	6.19	7.18	6.34	6.49	6.14	6.59	7.18	7.77	6.83	7.73	7.69	7.59	7.21	8.06	8.76	6.74
6.35	6.23	5.42	6.47	5.28	5.69	5.81	4.95	6.35	6.3	6.44	5.08	6.9	5.73	6.17	6.52	6.72	7.74	6.73	8.17	8.3	7.24	6.4	5.44
4.65	5.3	5.42	5.77	6.01	6.22	7.01	6.04	5.77	6.7	6.2	6.13	8	7.04	7.1	6.47	6.98	7.31	7.61	7.34	6.98	6.63	6.58	7.78
8.25	7.95	6.19	5.68	5.85	6.43	6.24	6.82	6.5	5.28	5.05	5.13	5.16	5.86	7.04	7.44	7.97	7.42	8.14	8.1	7.69	6.98	8.59	6.9
6.43	6.57	7.47	7	7.38	7.38	7.94	7.32	8.13	7.73	7.08	6.91	6.72	6.73	8.12	6.86	6.63	8	7.17	7.48	7.75	6.28	6.55	6.33
5.67	5.33	6.19	7.05	5.88	5.69	5.62	6.58	7.33	7.09	6.72	8.42	6.29	7.24	7.89	8.28	7.81	11.11	9.06	7.66	7.76	6.83	6.61	6.25
5.13	5.82	7.29	8.07	6.9	6.08	5.89	7.23	7.93	7.1	7.17	6.68	6.7	7.51	7.51	6.92	8.01	8.06	7.87	7.2	7.15	7.13	5.9	7.47
8.24	7.8	8.18	7.18	6.62	6.65	7.07	6.49	7.35	7.44	7.83	5.15	5.39	6.57	6.83	5.26	4.72	7.71	7.27	7.69	6.98	6.42	5.34	6.1
6.85	7.5	6.95	6.7	7.94	8.37	6.24	7.48	7.35	6.97	6.72	6.65	8.11	7.51	8.5	7.52	6.62	6.61	6.97	6.6	5.67	5.6	5.77	4.92
6.6	7.79	8.7	8.7	8.15	8.29	8.28	8.83	7.72	8.66	7.82	6.21	6.69	5.88	6.89	7.37	6.14	6.58	7.82	7.79	7.42	7.26	6.18	6.35
6.5	7.04	7.05	7.33	6.39	6.39	5.7	6.7	7.24	7.93	7.5	8.12	8.1	7.88	7.74	7.47	7.3	8.21	6.97	6.03	6.05	5.21	6.71	5.94
6.91	6.63	7.44	6.24	6.8	6.26	6.69	6.64	6.5	6.18	7.25	6.21	6.08	6.42	6.96	8.14	6.4	7.82	8.46	7.53	7.19	6.51	6.97	7.47
7.57	7	9.64	7.23	7.58	7.77	7.16	6.92	8.45	7	6.71	6.01	7.53	7.53	6.62	8.05	6.5	7.24	7.52	7.15	6.01	6.11	6.58	6.27
6.11	7	7.58	7.6	7.8	7.97	7.58	8.12	7.35	7.65	8.63	6.83	6.74	6.48	7.83	7.4	7.87	8.15	8.48	6.41	7.28	6.3	6.29	6.96
7.82	6.76	6.38	6.71	6.73	5.23	6.21	5.49	5.66	5.67	5.7	5.62	5.08	6.25	6.64	6.03	7.58	8.21	7.76	7.78	7.92	6.22	6.18	5.29
5.65	4.18	4.81	5.01	4.1	5.07	4.46	5.38	4.72	4.76	4.31	5.18	5.45	5.63	7.17	5.88	6.69	8.57	9.29	9.26	8.51	7.69	5.46	6.81

6.51	5.35	4.99	5.21	5.07	6.51	6.54	7.17	8.24	8.13	7.36	6.5	6.79	6.73	7.7	8.97	8.02	9.24	9.08	9.47	8.52	9.22	8.15	7.24
7.77	5.5	6.16	6.19	6.24	6.07	5.91	5.57	5.87	6.11	6.37	6.24	5.38	6.28	4.95	6.92	6.43	7.14	6.6	6.61	5.28	6.02	5.69	5.73
6.5	7.13	5.55	6.23	6.6	6.53	6.85	6.24	7.48	5.63	6.09	5.12	6.5	4.92	6.09	7.53	8.56	7	8.5	6.89	7.55	7.39	8.1	6.97
6.3	6.62	7.96	6.91	7.64	8.7	8.68	8.97	9.13	8.36	8.53	8.04	7.79	8.59	9.46	8.87	8.26	8.26	8.1	7.61	7.03	6.14	7.86	6.06
5.93	5.58	5.24	5.99	6	5.2	4.54	5.67	8.51	7.74	7.45	7.76	6.47	7.73	6.87	7.42	6.38	6.65	7.45	7.25	7.4	8.79	8.84	8.95
8.9	7.38	8.32	8.22	8.67	7.68	7.51	8.17	8.76	8.6	9.18	10.23	9.45	9.15	8.86	8.15	8.93	11.5	8.92	9.2	7.12	6.05	5.88	4.86
5.05	5.49	6.3	7.76	8.07	8.83	10.08	7.95	8.65	9.11	8.74	7.21	7.54	6.83	6.33	6.38	7.19	7.42	8.42	7.47	8.49	9.06	8.6	10
9.58	9.5	8.14	8.5	10.21	9.73	10.17	9.21	7.78	8.71	9.19	9.58	9.91	9.36	8.43	9.14	8.61	8.23	9.14	6.71	7.27	6.43	8.19	8.9
8.17	8.97	8.96	9.27	7.3	7.08	7.03	7.25	6.57	9.71	8.76	8.52	7.97	8.06	9.06	7.86	8.92	7.57	8.56	7.64	7.01	7.37	7.05	8.77
7.42	7.99	9.08	7.58	7.09	8.77	7.14	8.04	7.87	7.66	7.21	8.37	9.36	9.39	7.42	8.73	8.42	7.77	7.59	7.23	6.39	8.02	6.24	8.31
7.55	7.33	6.33	6.49	7.56	7.96	6.83	7.22	7.7	8.27	8.33	6.77	7.25	7.47	9.1	8.92	8.45	8.53	8.32	8.23	6.73	7.32	7.45	7.06
8.35	7.1	8.24	8.07	8.31	7.4	7.86	7.62	8.02	9.38	7.81	6.97	6.4	6.79	7.17	6.92	6.17	6.07	7.24	7.25	7.2	8.19	7.92	7.67
6.64	6.73	6.87	7.14	7.11	6.17	7.83	6.55	6.09	7.29	7.3	7.72	7.42	8.94	7.7	7.73	7.72	8.34	7.21	6.7	5.71	6	5.82	5.85
6.07	7.15	7.25	7.92	8.44	7.11	8.56	7.92	8.03	7.93	7.35	6.54	6.66	7.95	7.3	8.05	8.4	9.01	8.87	9.56	8.28	7.97	8.64	7.92
7.97	8.3	8.62	7.26	7.21	7.01	7.61	8.17	8.26	6.37	7.21	6.32	6.84	7.85	7.85	7.76	8.41	7.26	6.94	6.37	6.93	7.74	7.77	5.99
6.67	6.02	7.97	8.93	9.27	8.67	7.22	7.78	8.14	7.44	8.92	9.56	7.93	8.13	8.28	7.39	8.2	9.48	9.72	6.93	7.05	7.44	7.09	6.9
8.48	9.76	9.96	9.78	9.86	10.52	10.71	11.38	10.84	8.34	9.25	7.71	7.81	6.66	5.86	6.4	7.77	7.3	8.63	8.09	8.57	6.86	6.69	8.29
8.05	8.41	9.25	9.99	9.6	9.33	10.18	9.14	7.26	8.08	6.89	6.32	7.42	6.99	5.94	5.75	5.97	8.27	6.59	6.46	6.85	6.52	6.65	6.65
8.01	8.22	9.32	8.82	7.85	9.2	8.51	8.78	6.55	6.65	6.61	6.8	7.34	7.24	6.11	6.21	7.54	6.81	7.61	7.19	7.65	5.75	6.56	8.03
5.79	6.35	6.38	6.87	7.2	7.29	6.32	6.89	10.22	9.29	8.1	9.44	8.78	7.67	8.94	8.34	6.43	7.64	10.22	8.52	8.57	7.74	8.09	10.97
8.6	10.23	9.4	10.21	11.32	9.83	10.27	8.84	9.45	8.04	8.66	7.93	7.46	8.75	9.28	10.14	10.4	10.33	9.62	8.48	7.85	6.46	7.21	6.37
7.14	9.39	8.12	8.31	8.82	9.11	7.92	8.52	8.83	9.22	10.25	10.36	10.75	9.92	8.91	10.26	9.78	8.85	9.34	9.95	9.12	8.24	8.43	8.85
10.53	8.6	8.51	9.91	7.39	7.92	7.62	7.08	8.14	8.2	8.63	6.89	5.98	5.91	6.68	6.75	7.09	7.93	8.81	9.83	8.53	7.31	6.9	6.53
6.86	8.23	8.38	8.1	8.5	8.58	8.51	6.27	9.57	7.84	6.01	6.41	7.07	8.78	8.93	8.62	10.19	10.3	10.09	7.54	6.05	6.29	6.75	8.15
7.53	11.05	10.96	10.45	10.49	10	8.62	10.13	10.14	10.4	8.92	8.85	8.89	7.44	6.28	6.68	9.15	8.46	9.65	7.92	7.36	9.05	7.66	7.58
9.25	9.07	7.27	8.21	6.66	6.66	5.98	7.16	8.36	8.74	9.34	8.33	8.79	9.36	9.44	7.77	9.2	9.76	8.62	7.75	7.16	7.01	7.67	9.05
6.85	6.48	6.74	6.8	6.87	7.04	7.55	7.34	8.03	8.26	7.09	8.04	7.33	11.19	9.42	11.01	9.93	11.55	10.72	9.18	11.33	9.61	7.8	6.91
7.7	8.7	9.27	9.84	8.58	8.12	7.6	7.16	8.1	7.24	8.4	8.82	8.61	9.35	8.66	8.49	9.43	7.73	7.38	8.62	8.18	8.46	8.45	8.51
8.68	8.78	8.78	9.48	8.41	9.3	11.16	10.93	9.4	10.04	9.22	10.08	9.06	8.21	7.66	8.69	8.27	10	7.86	7.61	6.28	7.24	8.24	8.07
8.68	10.12	9.45	9.31	9.35	8.45	7.95	9.44	8.84	7.45	10.17	9.52	7.25	7.59	8.32	7.41	7.14	6.99	6.1	6.48	8.68	8.05	7.81	9.14
8.97	10.73	8.69	7.89	7.95	8.28	7.48	8.79	8.67	9.17	8.29	8.69	10.79	10.79	10.73	9.98	10.36	9.97	10.42	8.62	11.64	9.78	9.04	10.53
10.39	8.69	6.81	7.81	8.45	7.64	7.47	7.65	8.12	8	8.39	7.55	8.91	9.44	9.34	10	10.24	11.66	10.75	10.32	12.49	9.33	11.18	9.75
9.33	8.7	8.62	10.32	11.09	10.53	11.66	10.86	9.85	9.5	9.95	10.6	11.07	9.8	10.73	11.77	9.25	10.28	10.28	11.1	10.86	15.69	10.75	11.13
12.1	12.08	11.2	9.71	9.78	8.98	8.94	9.54	11.11	11.06	10.27	9.81	9.38	10.96	11.04	10.81	12.14	9.6	8.97	9.23	10.02	9.65	10.25	11.95
10.55	9.98	11.93	11.31	10.22	9.31	8.84	8.4	8.91	10.5	11.17	10.12	10.09	8.45	8.99	8.63	8.92	9.11	7.31	7.23	7.39	9.45	8.17	8.47
8.36	8.8	8.7	7.57	7.34	8.15	8.48	7.68	8.67	8.15	8.07	6.83	7.14	7.75	7.23	8.6	8.37	8.7	8.91	7.96	6.79	8.51	7.5	8.13
6.7	6.42	8.5	7.72	7.9	7.53	7.29	7.63	6.98	7.46	6.77	8.33	8.98	8.87	8.29	8.52	8.77	8.47	8.5	8	8.55	9.41	8.8	8.83
8.38	8.47	8.78	9.29	7.2	8.59	9.32	10.8	8.75	7.43	8.71	8.02	8.71	7.76	8.57	8.81	9.1	9.49	8.82	9.39	9.49	7.91	7.53	7.26
7.7	7.1	6	6.57	6.32	6.23	7.32	6.62	6.68	6.78	7.53	8.48	8.14	6.84	7.34	8.36	8.84	9.43	8.48	8.07	9.07	7.45	7.61	7.06
9.4	8.86	8.83	8.47	9.54	9.09	8.61	8.3	9.48	7.69	8.27	8.68	8.55	8.22	8.38	8.03	7.51	7.83	6.99	7.36	8.28	7.11	6.81	6.49
6.27	5.8	6.94	7.32	7.2	7.18	7.57	7.26	7.65	8.79	7.41	10.18	7.66	8.34	7.05	8.07	7.3	8.36	8.12	9.43	9.21	10.52	9.67	11.25
8.22	8.39	7.98	7.76	10.03	9.48	9.87	9.81	11.08	11.35	11.68	9.46	9.94	11.02	10.38	11.23	12.29	12.15	13.16	10.96	11.09	10.44	10.6	9.3
9.77	9.71	10.78	10.18	12.08	11.43	11.03	10.4	9.97	11.1	11.69	11.59	12.88	11.85	12.48	12.3	11.48	10.53	7.85	7.55	6.71	5.81	6.75	6.64
7.16	7.86	8	9.02	8.49	8.12	9.65	8.45	7.58	9.19	9.62	10.08	9.16	9.65	10.37	10.4	9.3	8.4	8.68	8.7	8.33	9.21	8.48	9.94
8.77	8	7.95	8.17	8.27	9.14	9.95	10.02	9.53	9.91	9.45	8.95	8.7	10.6	9.83	9.8	8.51	9.12	9	6.56	10.04	9.06	8.43	7.96
8.75	9	9.96	9.55	9.08	9.08	8.38	9.39	10.44	10.23	8.18	10.38	9.69	10.12	10.05	10.11	9.04	10.3	10.34	11.21	11.24	10.22	10.92	10.06
11.81	13.15	11.56	12.47	11.85	12.38	12.44	11.04	10.41	9.23	10.25	9.52	9.47	10.01	8.46	10.88	12.46	7.97	9.07	10.21	9.33	10.11	10.03	9.54
10.76	10.47	10.89	10.46	11.09	10.3	8.74	9.32	10.53	9.74	10.15	10.76	10.14	8.71	7.17	10.6	9.72	11.21	12.85	11.68	10.86	12.22	12.79	12.45
12.86	12.63	12.03	11.54	11.56	10.84	12.65	9.74	10.02	9.11	10.7	10.23	10.25	10.47	9.24	8.36	8.09	8.63	8.11	8.18	8.45	9.33	9.52	8.34
8.97	10.22	9.97	9.4	9.81	10	9.48	9.16	8.89	9.64	8.71	8.89	9.19	9.08	10.78	9.42	9.15	8.21	7.92	6.63	6.55	7.13	6.5	5.66
6.68	6.64	6.59	7.74	6.91	8.28	7.67	8.19	8.3	7.88	8.15	8.53	8.36	8.47	8.63	9.16	7.97	8.79	6.85	7.36	6.95	5.96	6.54	5.58
6.61	6.53	5.62	6.87	8.2	8.73	9.7	8.26	8.11	8.2	6.78	9.08	7.93	10.02	10.58	10.04	9.36	10.28	10.01	9.18	9.8	8.81	9.75	10.53
10.14	9.73	9.61	9.35	8.18	9.57	8.96	10.58	10.11	12.03	12.61	10.82	11.82	12.42	11	10.35	11.47	12.31	10.81	10.21	11.36	10.59	11.02	10.31
12.15	11.38	12.25	12.14	11.94	11.37	11.97	11.42	12.16	11.81	11.82	9.51	9.88	9.5	9.41	9.43	12.38	12.25	11.44	10.87	9.12	10.06	12.39	10.95
11.81	11.19	10.62	10.02	11.4	9.47	8.57	9.37	11.79	10.38	10.44	10.35	11.34	12.83	11.14	11.9	12.95	9.78	8.72	8.86	9.54	8.91	8.46	9.5
8.56	7.92	6.86	7.81	8.94	8.53	8.93	8.63	9.27	9.36	9.23	10.59	10.18	8.33	8.38	8.49	9.27	7.93	9.05	11.39	10.6	9.44	9.26	9.66
9.98	10.24	7.14	6.89	7.05	7.17	8.13	8.59	9.22	8.44	8.12	8.33	8.35	7.77	7.03	7.82	7.9	7.21	7.17	7.38	9.43	8.69	8	9.09
8.89	9.22	8.73	9.03	8.5	9.11	8.07	8.26	7.85	7.69	7.2	7.86	7.04	8	6.57	6.71	7.72	6.97	7.87	7.42	7.81	8.85	8.32	8.54
8.32	7.53	8.12	8.74	7.26	6.85	6.75	6.08	7.72	7.88	9.14	9.76	11.63	9.73	9.82	8.17	10.66	11.45	10.65	10.06	9.64	10.54	10.13	11.02
11.91	12.31	10.97	11.28	11.34	12.17	12.43	12.62	12.8	11.96	11.3	11.75	10.97	12.1	11.18	11.38	10.47	12.35	10.41	10.66	10.56	11.38	10.91	11.19
11.23	10.58	9.59	8.35	9.54	9.24	10.94	9.73	10.23	9.19	9.17	9.79	11.04	10.92	7.58	8.05	9.11	8.51	9.17	9.13	8.47	8.36	10.51	10.39
9.4	9.98	10.08	11.17	9.85	9.85	10.7	10.56	10.97	11.77	11.46	9.54	10.62	9.73	11.36	9.54	9.61	10.41	9.74	9.5	9.92	10.98	9.44	10.33
11.61	10.38	10.93	11.45	10.06	11.48	10.42	10.86	10.83	10.49	9.93	10.67	10.09	10.95	11.75	11.04	10.78	9.32	9.87	10.49	8.5	9	8.47	9.42
9.73	8.16	7.44	7.69	7.67	8.54	9.23	9.5	10.73	11.34	10.19	9.41	9.39	10.24	10.34	11.24	12.24	10.66	9.27	9.99	10.69	10.19	8.94	10.85
11.27	11.82	12.51	11.07	11.51	10.92	9.91	9.98	8.86	10.07	11.23	10.33	10.39	9.84	11.14	11.35	9.04	8.62	8.55	7.78	8.72	8.63	8.5	9.28
9.04	10.27	9.19	9.89	10.64	9.47	9.86	11.12	11.04	10.3	10.75	10.28	11.22	9.75	11.51	10.2	10.93	9.95	10.26	9.8	9.38	10.92	11.01	10.62
10.12	9.78	10.04	10.37	10.65	12.15	10.3	12.13	11.35	11.49	10.44	10.09	9.11	10.54	10.67	11.1	11.39	12.84	10.24	10.26	7.83	7.38	7.72	8
9.42	8.49	7.6	8.45	7.93	8.42	7.96	6.64	6.94	8.02	9.06	10.52	10.38	9.73	9.78	9.28	9.37	10.62	9.25	8.07	8.01	9.24	8.23	9.45
8.04	7.54	7.62	7.52	7.21	7.85	7.69	7.57	7.51	7.24	6.45	8.31	8.6	10.67	10.99	10.48	9.97	9.59	12.4	9.6	9.44	10.4	10.87	9.6
11.25	11.42	11.52	11.79	12.33	12.11	11.27	11.77	12.07	13.42	12.63	12.22	13.36	13.76	13.08	12.78	11.67	10.45	11.95	12.61	11.12	10.69	10.27	10.62
9.88	8.24	9.77	9.73	9.26	8.16	9.86	9.8	9.15	9.65	10.06	11.23	11.38	10.31	10.97	12.25	12	11.16	10.65	11.05	10.62	8.67	8.62	9.22
8.84	10.71	10.19	10.84	10	10.59	13.5	12.65	10.3	11.97	10.21	9.81	8.55	8.21	8.91	8.45	8.98	10.81	10.06	9.05	9.93	9.65	11.83	10.25

10.67	12.02	10.71	9.57	9.18	10.9	10.33	13.68	11.48	12.27	12.23	12.13	12.85	11.38	10.74	12.47	11.85	11.82	11.09	10.83	9.7	10.36	10.88	10.36
10.69	10.4	10.38	8.56	9.38	10.09	8.74	8.8	8.82	8.57	9.05	10.98	9.74	9.54	8.99	8.69	9.84	10.97	12.08	10.48	9.73	11.27	11.02	10.39
10.2	9.88	9.9	10.48	10.17	10.54	10.13	10.14	11.65	10.91	10.67	11.58	11.76	12.57	10.84	10.03	10.77	10.3	9.99	10.71	11.32	10.43	11.6	11.47
12.21	13.8	12.77	13.34	12.75	12.81	12.8	10.68	11.2	11.78	12.07	11.16	12.27	9.95	10.31	10.51	11.01	9.57	11.39	10.28	13.56	11.5	12.4	10.79
12.5	12.93	12.69	13.39	13.79	13.89	13.16	12.66	12.28	13.46	13.33	11.84	12.21	11.96	11.09	10.35	9.36	8.44	9.23	7.98	8.52	6.66	9.51	9.85
9.82	8.86	8.05	10	9.35	8.47	9.59	11.02	10.66	9.7	10.71	9.66	11.21	10.1	9.8	11.61	11.86	11.36	10.63	11.85	10.35	9.96	10.75	10.23
11.55	9.4	9.48	9.42	9.56	11.32	10.02	9.77	9.53	7.94	8.47	8.38	9.43	8.66	8.46	7.11	6.9	11.65	7.89	9.57	8.7	8.79	10.21	8.59
10.04	8.8	7.59	8.85	7.72	8.5	9.72	9.24	9.06	10.2	10.21	9.02	9.42	10.13	9.12	9.7	9.43	10.68	9.22	8.61	9.35	9.42	8.02	9.16
8.43	7.77	8.76	6.32	8.62	9.31	9.27	9	11.46	11.25	14.26	10.23	9.56	9.92	9.95	8.9	10.74	8.78	9.2	8.11	9.79	9.03	9.97	9.92
10.41	10.39	10.23	8.85	8.61	8.9	8.87	9.94	10.56	9.63	10.63	10.42	10.12	10.17	10.16	11.98	10.58	10.65	11.88	9.95	9.35	10.41	10.91	12.92
11.81	10.38	10.83	10.94	9.56	11.22	11.02	10.23	8.97	9.29	9	10.77	9.98	12.07	10.89	10.13	10.32	9.64	10.63	10.83	10.16	10.61	11.37	10.95
9.28	10.92	11.19	10.44	9.74	9.92	10	9.88	10.87	11.81	10.36	12.05	11.19	11.15	10.64	11.04	9.91	8.44	9.01	8.39	9.08	7.84	7.78	7.4
7.54	7.83	9.51	9.72	9.32	9.61	10.11	8.84	9.08	9.58	9.97	9.62	8.2	9.14	11.81	11.03	11.94	11.75	12.36	10.09	12.91	11.38	11.27	11.81
11.73	10.39	10.73	11.75	12.79	12.89	12.33	12.23	13.69	12.31	12.53	13.7	11.86	11.33	11.16	12.01	12.55	11.99	11.49	11.99	11.86	11.56	10.39	9.44
11.28	11.11	9.57	9.95	10.49	10.07	11.22	12.74	11.82	13.08	13.3	13.55	14.3	13.03	10.79	12.78	11.08	9.67	9.16	9.42	9.11	11.49	10.65	12.1
11.14	10.02	9.05	9.48	10.4	12.01	12.75	12.15	12.52	11.94	11.92	11.48	11.19	11.21	11.59	13.01	10.69	11.48	11.97	11.83	12.71	11.55	12.2	12.19
13.1	13.11	13.25	12.04	13.11	12.28	12.18	11.02	11.07	10.94	11.78	11.53	11.73	11.48	9.73	10.97	10.59	10.87	10.1	10.19	13.42	10.36	14.51	11.02
9.98	12.4	11.1	13.3	12.73	10.7	10.97	10.5	10.76	11.33	13.49	12.78	12.65	12.66	13.62	13.12	12.9	13.12	11.58	13.39	11.45	10.23	10.41	10.4
9.6	10.85	10.72	12.48	12.99	12.66	12.83	12.82	11.15	10.59	10.49	10.94	11.84	12.27	10.82	12.25	11.45	12.06	11.07	8.94	9.94	10.16	10.77	10.69
10.7	9.07	10.92	10.36	11.19	13.31	12.09	11.01	10.48	13.28	13.16	11.98	10.68	11.91	11.92	13.41	10.42	10.54	11.82	10.84	11.2	10.83	13.99	12.32
14.32	12.3	14.28	13.28	14.75	14.06	13.13	13.09	12.81	13.14	13.64	13.19	14.32	13.92	13.1	11.21	12.57	11.41	11.47	9.26	10.78	10.73	10.81	10.69
10.54	11.63	13.36	12.09	11.66	11.13	11.66	11.88	12.79	14.03	13.58	14.05	14.15	14.63	13.39	14.13	13.17	12.14	10.96	11.32	9.3	10.89	11.41	10.48
9.92	11.49	11.81	11.41	11.82	13.82	11.68	12.38	12.15	11.08	10.41	10.61	11.98	11.05	10.57	11.35	12.2	8.98	8.34	9.55	11.06	9.75	9.93	9.38
10.9	11.67	12.69	12.83	12.7	13.45	12.75	11.77	11.71	11.46	10.34	12.5	12.13	10.3	11.62	11.14	10.72	9.87	10.62	9.53	10.81	11.99	12.31	12.49
10.59	10.91	10.55	10.75	11.29	10.52	10.3	9.88	11.78	10.98	12.03	11.97	10.56	10.38	10.49	10.5	11	12.38	11.63	11.18	12.34	15.43	16.29	14.96
15.22	12.73	12.43	11.54	12.06	10.87	9.21	10.49	11.08	12.1	12.33	13.49	12.43	14.44	12.36	13.86	11.74	11.18	10.35	12.46	11.83	10.22	10.88	11.42
11.9	10.98	10.72	10.75	12.16	11.34	11.99	11.55	10.94	10.43	11.45	11.75	11.28	12.26	11.12	11.32	11.04	10.67	9.56	10.01	9.58	10.16	10.39	9.64
11.82	11.45	10.27	12.38	10.43	10.03	10.06	10.31	9.65	9.37	10.4	10.91	12.61	12.6	13.3	13.13	12.75	13.86	13.77	14.27	13.39	11.81	12.72	11.89
10.88	10.86	10.63	9.56	9.36	8.8	9.7	7.92	8.88	8.61	8.35	8.43	7.67	6.76	7.41	7.57	8.1	6.06	7.33	7.58	9.75	7.7	8.88	9.9
10.1	8.41	9.26	8.36	9.57	9.67	10.77	12.33	11.69	12.77	12.3	13.36	12.35	9.3	11.15	11.21	13.08	12.93	12.89	11.53	12.2	11.15	10.21	8.78
10.3	10.54	12.09	11.86	12.22	12.06	11.84	10.22	12.71	11.46	11.37	12.1	12.2	11.81	11.33	11.42	12.86	9.45	9.89	10.94	11.23	11.11	11.47	10.8
10.71	11.41	11.22	13.42	13.31	11.6	12.67	14.25	16.6	17.25	15.39	14.78	14.98	14.25	13.35	12.32	12.13	13.05	12.57	12.6	12.84	14.47	13.09	10.98
12.58	12	11.44	9.99	11.33	10.71	10.82	10.38	11.41	11.61	10.96	10.65	9.79	11.38	11.85	11.54	10.7	12.2	11.63	13.68	12.37	11.55	11.48	11.46
11.58	9.72	9.04	9.35	11.09	12.11	10.98	12.2	12.42	11.4	12.66	11.5	12.34	11.96	9.93	11.76	12.17	11.32	11.26	10.78	10.59	11.04	11.03	10.64
9.77	10.49	10.9	10.79	9.95	10.3	10.43	12.04	11.51	9.64	9.08	8.82	10.15	11.47	11.58	12.94	11.12	11.98	11.26	12.89	10.86	13.19	13.61	12.49
13.57	14.11	14.26	13.46	11.65	10.66	10.68	10.18	9.21	9.75	8.51	10.82	9.4	8.22	9.05	8.48	8.92	8.7	7.8	8.3	8.27	9.66	8.52	9.08
7.62	7.41	7.86	7.93	7.75	9.62	10.61	12.03	12.18	11.43	12.57	12.91	13.12	11.99	11.83	11.71	11.42	11.32	12.36	10.35	11.93	10.44	10.54	11.19
11.78	11.14	10.67	11.48	11.83	11.99	13.38	12.82	14.03	13.67	13.12	13.37	14.31	12.21	12.43	11.75	12.73	12.77	11.76	10.72	9.53	8.3	9.47	11.18
10.99	10.54	9.4	10.68	9.95	9.98	8.91	10.31	9.61	8.94	10.36	12.14	14.14	12.36	12.07	11.54	11.57	12.29	13.35	13.05	13.32	11.84	11.35	12.1
12.77	11.97	11.22	10.39	11.72	9.96	10.12	10.83	10.1	11.08	10.01	10.05	10.04	10.84	10.53	10.37	12.54	10.78	11.57	10.68	12.16	12.28	12.87	13.53
12.03	11.26	10.01	10.27	9.72	11.46	11.79	11.7	12.29	11.8	10.84	12.58	13.6	12.59	12.99	12.17	11.54	10.46	9.35	9.78	9.86	9.81	10.55	9.39
8.77	9.43	8.57	10.53	8.14	8.81	11.52	10.29	9.65	10.41	10.52	10.95	12.65	13.04	11.97	10.82	11.12	11.59	13	11.34	14.54	13.84	12.77	11.95
11.83	11.11	10.61	11.98	11.09	10.48	12.4	12.96	11.71	13.07	11.74	12.98	13.88	12.38	12.36	11.25	12.28	11.59	10.73	12.73	13.11	11.51	11.85	11.21
11.8	12.45	11.34	10.6	10.61	10.33	11.83	10.91	10.69	9.04	7.78	7.96	8.24	7.75	8.55	8.38	8.69	7.79	8.45	7.11	8.79	9.4	8.7	9.15
9.07	8.28	9.77	10.62	10.98	10.92	10.08	10.59	11.56	10.66	11.49	11.6	11.1	11.7	11.79	12.3	13.17	13.56	14.26	13.87	14.14	13.34	12.24	11.52
12.76	10.85	9.87	12.28	12.46	10.92	12.37	11.93	11.27	10.33	9.52	9.8	8.41	9.2	9.99	10.11	8.44	9.96	11.6	11.07	12.24	13.52	13.09	10.7
11.92	13.64	11.57	12.6	10.51	9.94	9.72	9.67	10.84	12.62	11.28	11.14	11.92	10.92	10.67	11.59	10.93	11.89	11.26	11.55	11.51	12.32	12.24	12.56
11.42	13.74	12.71	11.14	10.31	11.24	12.63	12.5	12.94	12.48	14.04	11.64	12.16	13.58	12.23	11.7	12.19	11.55	10.11	9.37	9.35	8.78	8.97	8.47