## ACOUSTIC MEASUREMENT OF SNOW

A Thesis Submitted to the College of Graduate Studies and Research<br>In Partial Fulfillment of the Requirements For the Degree of Doctor of Philosophy In the Department of Geography \& Planning (Centre for Hydrology) University of Saskatchewan Saskatoon

## By

NICHOLAS JOHN STANISLAUS KINAR
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#### Abstract

Instrumentation commonly used to measure snowpack stratigraphy, snow density, Snow Water Equivalent (SWE), temperature and liquid water content is usually invasive and requires disruption of the snowpack. Most measurement techniques modify the snow medium and more than one sample cannot be taken at the same location. This does not permit continuous monitoring of these parameters using a single measurement instrument. An acoustic wave sent into the snowpack was used to measure snow. To provide the theory required to make acoustic measurements, the Biot-Stoll model of sound wave propagation in porous media was modified using a mixture theory so that it was applicable to a multiphase porous medium. The combined model is called the Unified Thermoacoustic Model (UTAM) for snow. An acoustic measurement device, the System for the Acoustic Sensing of Snow (SAS2), was designed to send sound waves into snow and to receive the reflected sound waves using a loudspeaker and a microphone array. A stationary version of the SAS2 was deployed on a met station and a portable version of the SAS2 was placed on a roving ski-based platform. The systems were deployed at field sites in the Canadian Rocky Mountains, Alberta. The results showed that the SAS2 was able to measure snow density, temperature, and liquid water content and serve as a replacement technology for snowtube and snowpit measurements. Snow density was estimated more accurately by the SAS2 than from commonly-used snow tube techniques.


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## NOMENCLATURE

## Acronyms

| CE | Common Era of historical time |
| :--- | :--- |
| RMSD | Root Mean Squared Difference |
| MB | Mean Bias |
| LWC | Liquid Water Content (\%) |
| MLS | Maximum Length Sequence |
| SNR | Signal-to-Noise Ratio |
| DAC | Digital-to-Analog Converter (DAC) |
| ADC | Analog-to-Digital Converter (ADC) |
| LTI | Linear Time-Invariant (LTI) |
| FFT | Fast Fourier Transform |
| IFFT | Inverse Fast Fourier Transform |
| LFSR | Linear Feedback Shift Register |
| SOS | Second Order Sections |
| IIR | Infinite Impulse Response |
| FIR | Finite Impulse Response |
| STFT | Short Time Fourier Transform |
| ISTFT | Inverse Short Time Fourier Transform |
| OLA | Overlap-Add |
| FBS | Filter-Bank Summation |
| PCB | Printed Circuit Board |
| SAS2 | System for the Acoustic Sensing of Snow |
| FET | Field -Effect Transistor |
| DC | Direct Current |
| IC | Integrated Circuit |
| V | Volt |
| A | Amp |
| LED | Light Emitting Diode |
| USB | Universal Serial Bus |
| FPGA | Field-Programmable Gate Array |
| I2C | Inter-Integrated Circuit bus |
| SPL | Sound Pressure Level |

## Roman

| SWE | Snow Water Equivalent ( mm or $\mathrm{kg} \mathrm{m}^{-2}$ ) |
| :--- | :--- |
| $\mathbf{u}$ | Vector that describes movement of the porous frame |
| $\mathbf{U}$ | Vector that describes movement of the fluid in the porous frame |
| $\operatorname{div}()$ | Divergence |
| $t$ | Time (s) |
| $k_{p}$ | Permeability of porous medium $\left(\mathrm{m}^{2}\right)$ |
| $\kappa$ | Effective thermal conductivity of porous medium $\left(\mathrm{W} \mathrm{m}^{-1} \mathrm{~K}^{-1}\right)$ |


| $q_{F}$ | Heat flux ( $\mathrm{W} \mathrm{m}^{-1}$ ) |
| :---: | :---: |
| $A_{s}$ | Area of snow pillow ( $\mathrm{m}^{2}$ ) |
| $K_{r}$ | Bulk modulus of snow particles comprising the frame ( Pa ) |
| $K_{b}$ | Bulk modulus of the porous medium frame ( Pa ) |
| $f$ | Frequency (Hz) |
| $m_{f}$ | Inverse fluid mobility ( $\mathrm{Pa} \mathrm{s} \mathrm{m}^{-2}$ ) |
| $m_{s}$ | Mass of snow ( $\mathrm{kg} \mathrm{m}^{-3}$ ) |
| $F(\kappa)$ | Dimensionless correction factor as function of $\kappa \in \mathbb{R}$ |
| $\operatorname{Ber}(\kappa)$ | Real part of Kelvin function |
| $\operatorname{Bei}(\kappa)$ | Imaginary part of the Kelvin function |
| $i=\sqrt{-1}$ | Imaginary number |
| E | Young's modulus of snow ( Pa ) |
| $G$ | Shear modulus of snow ( Pa ) |
| $E_{0}$ | Young's modulus of material comprising the frame ( Pa ) |
| $G_{0}$ | Shear modulus of material comprising the frame ( Pa ) |
| $l_{k}$ | Complex root of Biot solution equation |
| $c_{s, k}$ | Speed of sound in snow ( $\mathrm{m} \mathrm{s}^{-1}$ ) corresponding to complex root $k$ |
| $f_{c}$ | Percolation theory exponent (dimensionless) |
| $T$ | Temperature ( ${ }^{\circ} \mathrm{C}$ or Kelvin, as noted in text) |
| $\mathrm{F}_{\mathrm{t}}$ | Surface tension force vector |
| $L_{s}$ | Distance over which surface tension force acts (m) |
| C | Curve over which the surface tension force acts |
| $\left\|\mathbf{F}_{\text {t, }}\right\|$ | Magnitude of surface tension force acting in the vertical direction (N) |
| $\left\|\mathbf{F}_{\text {ac }}\right\|$ | Magnitude of the force vector holding the snow particles together (N) |
| $\left\|\mathbf{F}_{\mathbf{p}}\right\|$ | Magnitude of the downward-acting force of the snow particles (N) |
| $V_{a}$ | Volume of air occupying the pore spaces of the control volume ( $\mathrm{m}^{3}$ ) |
| $V_{i}$ | Volume of ice occupying the pore spaces of the control volume ( $\mathrm{m}^{3}$ ) |
| $V_{w}$ | Volume of water occupying the pore spaces of the control volume ( $\mathrm{m}^{3}$ ) |
| $V_{T}$ | Total control volume of porous medium ( $\mathrm{m}^{3}$ ) |
| $S_{w}$ | Water saturation (dimensionless) in control volume |
| $g$ | Acceleration due to gravity ( $\mathrm{m} \mathrm{s}^{-2}$ ) |
| $R_{g}$ | Ideal gas constant for air ( $\mathrm{J} \mathrm{kg}^{-1} \mathrm{~K}^{-1}$ ) |
| $R$ | Radius at air-water interface (m) |
| $K_{a}$ | Bulk modulus of air (Pa) |
| $K_{f}$ | Bulk modulus (Pa) of pore space fluid |


| $P$ | Air pressure (Pa) |
| :---: | :---: |
| $K_{w}$ | Bulk modulus (Pa) of water |
| $c_{w}$ | Speed of sound in water ( $\mathrm{m} \mathrm{s}^{-1}$ ) |
| $c_{s h}$ | HFA speed of the second Biot P-wave in the pore space of the medium ( $\mathrm{m} \mathrm{s}^{-1}$ ) |
| $c_{s}$ | Speed ( $\mathrm{m} \mathrm{s}^{-1}$ ) of the second Biot P-wave at frequency $\omega$ (rad) |
| $c_{0}$ | Speed of sound in air ( $\mathrm{m} \mathrm{s}^{-1}$ ) |
| $a_{p}$ | Mean diameter of pore spaces in snowpack (m) |
| $b_{k}$ | Cumulative sum of terms (real number) |
| $n_{T}$ | Number of terms added during summation |
| $\left\{f_{1}, \ldots, f_{N}\right\}$ | Frequencies (Hz) |
| $\left\{c_{s, 1}, \ldots, c_{s, N}\right\}$ | Speeds of sound in snow ( $\mathrm{m} \mathrm{s}^{-1}$ ) corresponding to known frequencies |
| $\left\{c_{s, 1}^{\prime}, \ldots, c_{s, N}^{\prime}\right\}$ | Speed outputs of the forward model ( $\mathrm{m} \mathrm{s}^{-1}$ ) |
| $s[t]$ | Digital (discrete) source signal as a function of discrete timestep $t$. |
| $r_{k}[t]$ | Signal received by microphone $k$ |
| $n[t]$ | Measurement noise signal |
| $m_{k}[t]$ | Microphone $k$ frequency response |
| $p[t]$ | Frequency response of the loudspeaker |
| $e[t]$ | Electronic circuit frequency response |
| $d[t]$ | Direct air-coupled wave travelling from loudspeaker to microphones |
| $c[t]$ | Enclosure case effect on the acoustic signal |
| $L_{M}$ | Length of the MLS (integer number) |
| $L_{\text {min }}$ | Minimum length of MLS |
| $f_{d s}$ | DAC sampling rate (Hz) |
| $t_{m l s}$ | Time duration of the MLS (s) |
| $P_{s}$ | Signal power (W) |
| $P_{\text {MLS }}$ | MLS signal power (W) |
| B | MLS Bandwidth (Hz) |
| F() | Discrete Fourier Transform operator |
| IF ( ) | Inverse Discrete Fourier Transform operator |
| $\operatorname{Rev}()$ | Operator that reverses a discrete signal |
| $L_{l a g}$ | Lag (number of samples) |
| conj( ) | Complex conjugate transform operator |
| $x_{w}[t], y_{w}[t]$ | Discrete signal buffers |
| $B_{k}^{*}[t]$ | Raw bitstream from a microphone |
| $B_{k}[t]$ | Binary signal from microphone |


| $f_{s}$ | Microphone sampling rate (Hz) |
| :---: | :---: |
| $Q$ | Coefficient representative of attenuation in medium (dimensionless) |
| $Q_{0}$ | Zero-offset $Q$ value |
| $a$ | Attenuation coefficient ( Nepers $\mathrm{m}^{-1}$ ) |
| $A[t, \omega]$ | Discrete attenuated signal spectrum at discrete time $t$ and frequency $\omega$ |
| $A_{0}[t, \omega]$ | Source signal spectrum |
| $M_{Q}[t, \omega]$ | Function relating $Q, \omega$ and $c$ variables |
| $G[t, \omega]$ | Discrete Gabor transform |
| $g\left[t, t^{\prime}\right]$ | Inverted Gabor transform |
| $w[t]$ | Time-domain windowed trace |
| $W[f]$ | Frequency-domain windowed trace |
| $h[t]$ | Input signal |
| $t_{k}$ | Discrete timestep at element $k$ |
| $x[t]$ | Discrete signal in time domain |
| $x^{\prime}[t]$ | Reconstructed estimate of discrete signal |
| $w_{f}[t]$ | Gaussian window function |
| $G^{\prime}[t, \omega]$ | Regularized Gabor transform |
| $G_{\text {lim }}$ | Gain limit (dB) |
| $Q[t]$ | Discrete Q-values as a function of time |
| $\mathbf{x o s}_{\text {os }}$ | Discrete vector of microphone offset distances (m) |
| $m_{s}$ | Slope of offset line |
| $H_{k}[t, \omega]$ | Gabor transform of attenuation-corrected traces for microphone $k$ |
| $U\left[Z_{t}, Z_{x}\right]$ | Z-transform of $u[t, x]$ signal |
| $\left\{Z_{t}, Z_{x}\right\}$ | Time-shift and spatial-shift operators |
| $v[t, x]$ | Moveout velocity as a function of time $t$ and offset $x$ |
| $t_{0}$ | Offset time (s) |
| $S_{w}$ | Water saturation (dimensionless or \%) |
| $\overline{\theta_{w}}$ | Depth-averaged water content (dimensionless or \%) |
| $\bar{T}$ | Depth-averaged snow temperature ( ${ }^{\circ} \mathrm{C}$ or Kelvin) |
| $\bar{y}$ | Snow depth measured using a depth rod ( cm or m) |
| $\bar{\rho}_{s}$ | Depth-averaged snow density ( $\mathrm{kg} \mathrm{m}^{-3}$ ) |
| $\rho_{s}$ | Snow density ( $\mathrm{kg} \mathrm{m}^{-3}$ ) |
| $L_{n}$ | Input noise level for microphone (dB) |
| $a_{c}$ | Attenuation coefficient at maximum MLS frequency ( Nepers $\mathrm{m}^{-1}$ ) |
| $P_{0}$ | Reference RMS pressure ( Pa ) |
| P | Sound power of source (W) |


| $S_{0}$ | Directionality factor ( $\mathrm{m}^{2}$ ) |
| :---: | :---: |
| $c_{a}$ | Speed of sound in air ( $\mathrm{m} \mathrm{s}^{-1}$ ) |
| $y_{a}$ | Distance from the SAS2 microphones to the snow surface (m) |
| $y_{s}$ | Snow depth used in SAS2 design calculations (m) |
| $p_{a}$ | Barometric air pressure ( Pa ) |
| $\left\{p_{1}, p_{2}, p_{3}\right\}$ | Pressure at interfaces (Pa) |
| $\left\{L_{1}, L_{2}, L_{3}\right\}$ | SPL at interfaces (dB) |
| $p_{\text {ref }}$ | Reference pressure (Pa) |
| $L_{n}$ | Microphone noise level (dB) |
| $f_{\text {max }}$ | Maximum frequency component in signal (Hz) |
| $d$ | Diameter of loudspeaker (cm or m) |
| $x_{a}$ | Inter-element microphone array spacing ( cm or m) |
| $f_{a}$ | Frequency of wave incident on array (Hz) |
| $D_{d B}(\theta)$ | Beam pattern magnitude (dB) |
| $x_{s}$ | Footprint of sound wave on snow surface ( cm or m) |
| $\Delta y$ | Minimum resolvable acoustic layer width ( cm or m) |
| $a_{s}$ | Sound source dimension (m) |
| $d_{r}$ | Longest room dimension (m) |
| $l_{w}$ | Length of sound-absorbing wedges in anechoic chamber (m) |
| $T$ | Period of sound wave (s) |
| $d H / d t$ | Internal energy change of snowpack ( $\mathrm{W} \mathrm{m}^{-2}$ ) |
| $K \downarrow$ | Downward shortwave radiation flux ( $\mathrm{W} \mathrm{m}^{-2}$ ) |
| $K \uparrow$ | Upward shortwave radiation flux ( $\mathrm{W} \mathrm{m}^{-2}$ ) |
| $L \downarrow$ | Downward longwave radiation flux ( $\mathrm{W} \mathrm{m}^{-2}$ ) |
| $L \uparrow$ | Upward longwave radiation flux ( $\mathrm{W} \mathrm{m}^{-2}$ ) |
| $H_{s}$ | Sensible heat flux ( $\mathrm{W} \mathrm{m}^{-2}$ ) |
| $H_{L}$ | Latent heat flux ( $\mathrm{Wm}^{-2}$ ) |
| $H_{P}$ | Heat flux due to blowing snow or precipitation ( $\mathrm{W} \mathrm{m}^{-2}$ ) |
| $G$ | Ground heat flux ( $\mathrm{W} \mathrm{m}^{-2}$ ) |
| $K^{*}$ | Net shortwave radiation flux ( $\mathrm{W} \mathrm{m}^{-2}$ ) |
| $L^{*}$ | Net longwave radiation flux ( $\mathrm{W} \mathrm{m}^{-2}$ ) |
| $Q^{*}$ | Net radiation flux ( $\mathrm{W} \mathrm{m}^{-2}$ ) |

## Greek

| $\xi$ | Effective fluid content increment (dimensionless) |
| :---: | :---: |
| $\nabla^{2}$ | Laplacian |
| $\rho$ | Total density of the control volume ( $\mathrm{kg} \mathrm{m}^{-3}$ ) |
| $\rho_{f}$ | Fluid density in the pore spaces of the medium ( $\mathrm{kg} \mathrm{m}^{-3}$ ) |
| $m_{f}$ | Inverse fluid mobility ( $\mathrm{Pa} \mathrm{s} \mathrm{m}^{-2}$ ) |
| $\eta$ | Dynamic viscosity of pore space fluid ( Pa s ) |
| $\mu$ | Shear modulus of the porous medium ( Pa ) |
| $\phi$ | Porosity (dimensionless) |
| $\alpha$ | Tortuosity (dimensionless) |
| $\omega=2 \pi f$ | Angular frequency of sound wave ( $\mathrm{rad} \mathrm{s}^{-1}$ ) |
| $\phi_{c}$ | Critical porosity (dimensionless) used in percolation theory |
| $\sigma_{s}$ | Surface tension coefficient for water ( $\mathrm{Nm}^{-1}$ ) |
| $\theta_{F}$ | Angle over which surface tension force acts (rad) |
| $\phi_{i}$ | Ice fraction (dimensionless) in control volume |
| $\theta_{\text {c }}$ | Contact angle (rad) |
| $\sigma_{i w}$ | Surface tension between snow particle and water ( $\mathrm{Nm}^{-1}$ ) |
| $\sigma_{w a}$ | Surface tension between water and air ( $\mathrm{Nm}^{-1}$ ) |
| $\rho_{i}$ | Ice density ( $\mathrm{kg} \mathrm{m}^{-3}$ ) |
| $\rho_{w}$ | Water density ( $\mathrm{kg} \mathrm{m}^{-3}$ ) |
| $\rho_{s}$ | Snow density ( $\mathrm{kg} \mathrm{m}^{-3}$ ) |
| $\varepsilon_{a}$ | Volume fraction (dimensionless) of air in the pore space of the medium |
| $\varepsilon_{w}$ | Volume fraction (dimensionless) of water in the pore space of the medium |
| $\chi_{g}$ | Dimensionless gas constant; heat capacity ratio of air (1.402) |
| $\eta_{a}$ | Dynamic viscosity of air ( Pa s ) |
| $\eta_{w}$ | Dynamic viscosity of water ( Pa s ) |
| $\delta$ | Dimensionless loss tangent |
| $\gamma_{s}$ | Berryman coefficient (dimensionless) |
| $\tilde{\omega}$ | Angular frequency above which speed of the P-wave is constant in snow (rad) |
| $\Psi=\psi_{G}=\psi_{G}$ | Dimensionless scaling coefficient |
| $\varepsilon_{T}=1.0 \mathrm{e}-10$ | Small number used in loop iteration |
| $\varphi[t]$ | Filter kernel of the snowpack |
| $\mu_{k}[t]$ | Total measurement response of the system for microphone $k$ |


| $\odot^{*}$ | Hadamard multiplication operator |
| :--- | :--- |
| $\varphi_{k}^{*}[f]$ | Truncated filter kernel in Fourier domain |
| $\varphi_{k}[t]$ | Truncated filter kernel in time domain |
| $\Upsilon$ | Datatype (container) that holds the LFSR |
| $\pi$ | Pi constant (3.1415...) |
| $\Delta[t, \omega]$ | Filter kernel |
| $\sigma_{p}$ | Stabilization parameter (dimensionless) |
| $\omega_{h}$ | Highcut frequency (Hz) |
| $\beta_{1}[t, \omega]$ | Gabor transform of attenuated signal passed through snowpack |
| $\Gamma\left[t, x, \omega_{\ell}\right]$ | Discrete matrix used for speed analysis |
| $\Omega$ | Parameter used in the application of the plane wave destruction filter |
| $\Gamma_{M}[t, x]$ | Time-remapped matrix |
| $\bar{\rho}$ | Depth-averaged snow density (kg m${ }^{-3}$ ) |
| $\left\{\Lambda_{1}, \Lambda_{2}\right\}$ | Transmission coefficients (dimensionless) |
| $\Gamma_{r}$ | Reflection coefficient (dimensionless) |
| $\varsigma$ | Reflection/transmission coefficient parameter (dimensionless) |
| $\lambda$ | Wavelength of sound pressure wave incident on array (m) |
| $\theta$ | Sound wave incidence angle (rad or degrees) |
| $\theta_{m}$ | Half-width angle (rad) |
| $\lambda_{s r}$ | Longest wavelength that can be measured in an anechoic chamber (m) |

## CHAPTER 1 INTRODUCTION

### 1.1. Importance of Snow

Snow has long been recognized as an important component of the hydrologic cycle (King et al., 2008; McKay and Adams, 1981). The production of water from snowpack runoff and the influences of snow on regional and global climates have always been of extreme importance to the stability of human societies. For example, in the Middle East, the Tigris and Euphrates rivers are dominated by snowmelt from the Taurus and Zagros Mountains (Beaumont, 1998; Özdoğan, 2011; Şensoy et al., 2005). To ancient societies situated near these mountains, floods and the effects of climate change may have been associated with snowfall. Perhaps the earliest mention of this relationship can be found in the Avesta, a sacred Zoroastrian religious text recounting the story of Yima, a shepherd instructed to create an enclosure of land to protect humans, plants and animals from a great flood. The story portrays the flood occurring due to snowmelt (Darmesteter, 1880, pp. 15-16):

And Ahura Mazda spake onto Yima, saying: 'O fair Yima, son of Vîvanghat! Upon the material world the fatal winters are going to fall, that shall bring the fierce, foul frost; upon the material world the fatal winters are going to fall, that shall make snow-flakes fall thick, even [fourteen fingers] deep on the highest tops of mountains. And all the three sorts of beasts shall perish, those that live in the wilderness, and those that live on the tops of the mountains, and those that live in the bosom of the dale, under the shelter of stables. Now with floods that stream, with snows that melt, it will seem a happy land in the world, the land wherein footprints even of sheep may still be seen.'

Snow may have been mentioned in an Egyptian temple inscription attributed to Ramesses II that is associated with travelling difficulties in cold regions (New York Times, 18 October 1906). The Greek historian Herodotus wrote on snowmelt contributions and climate to the Danube River in Central Europe. Considering that snowmelt would contribute negligible amounts of water to the discharge of the Nile River due to the climate of Egypt and Africa (Beloe 1812, Euterpe XXII, pg. 75), Herodotus later reasoned in a similar fashion that snowmelt comprised the flow of the Danube River in Central Europe (Beloe 1812, Melpomene, L, pg. 203). During Roman rule of Asia Minor, contemporary writers recounted that large amounts of
snowfall would often disrupt the life of people living on the Cappadocian plateau (Van Dam, 2002, p. 14). Snowmelt caused flooding in this region, endangering homes and livelihoods. An example is the Lycus ("Wolf") River that often raged with snowmelt (Van Dam, 2002, p. 42).

Snow had major economic importance in the past. As recounted by Needham (1959), quantitative measurements of snow were made as early as 1247 CE by Chinese investigators, who trapped snow in containers to estimate the amount of water entering rivers as runoff (Peck, 1972). Since Medieval times up to the invention of refrigeration, "snow merchants" would collect snow from mountains bordering the Mediterranean Sea. The snow would lucratively be sold for consumption in drinks and deserts or for the preservation of imported food during transport (McNeill, 1992, pp. 124-125). In 1836, the chief engineer of Chanago Canal in the state of New York, John B. Jervis, may have conducted one of the first snow surveys in the United States. Jervis was concerned that not enough snow would fall to provide water for canal transportation (Harrington, 1953). From 1850 to 1900, settlers in the Sacramento Valley of California realized that much of the water in the region was from snowmelt. Snow depth was also measured in the nearby Sierra Nevada mountains to ascertain the amount of water available from runoff (Powell, 1987). Historical accounts may suggest that runoff from snowmelt contributed to the great flood of 1862 in the Sacramento Valley, indicating that snow has long affected the hydrology and agriculture of this region (Kattelmann, 1996). J. E. Church popularized snow surveying in California and the United States during the early part of the $20^{\text {th }}$ century (Church, 1933). Welsh settlements in Wales and North America from 1170 CE to 1902 CE were dependent on snowmelt for farming and agriculture (Pomeroy and Jones, 1996).

In a modern context, snow is a major contribution to the water balance, climate and economy of many geographic regions. Approximately 33\% of precipitation in Canada originates as snow (Brown, 1995; Pomeroy and Goodison, 1997) and "is a primary source of Canadian water supplies" (McKay and Findlay, 1971). Snow contributes more than $80 \%$ of water to streams located in the Prairies (Barnaby, 1980; Gray and Landine, 1988; Gray et al., 1970; McKay and Blackwell, 1961), Arctic (Marsh, 1990), and western United States (Daly et al., 2000; Marks et al., 2001; Rice et al., 2007).

Nearly all streamflow in the High Arctic is derived from snowmelt (Prowse, 1990; Woo, 1980). Snow is present in the High Arctic for more than 8 months per year. The hydrology of the High Arctic is completely dependent on snow (Woo, 1982a) and climate change in this
region will affect vegetation, animals, and well as biogeochemical cycles, influencing human lifestyles and forestry operations. Because snow covered area influences energy exchange with the atmosphere, global climates will also be affected by changes in the Arctic snowpack (Callaghan et al., 2011).

The Arctic is characterized by snowpacks that are dense, extremely variable, and cold. Snow temperatures can remain less than $-15{ }^{\circ} \mathrm{C}$ until the month of June, thereby depressing snowmelt and the production of liquid water for most of the winter season (Woo and Sauriol, 1980). Moreover, ground heat flux contributes negligible energy for snowmelt in the Arctic due to the presence of extensive permafrost underlying ice layers at the base of the snowpack (Woo et al., 1982). When the snowpack becomes isothermal, reaching $0^{\circ} \mathrm{C}$ due to higher levels of net radiation and increasing day length, snowmelt occurs rapidly. Hilltop areas with lower snow depth become snow-free before valleys with greater snow depth that retain snow until late in the ablation season. Preferential flow channels occur through the snowpack, diverting water to lakes and streams (Woo, 1980; Woo and Sauriol, 1980). The high density and hardness of snowpacks in valleys contribute to the formation of these flow channels (Woo and Sauriol, 1980), creating "snow dams" that impede runoff and rupture due to seepage of liquid water through the snow (Xia and Woo, 1992). The movement of liquid water through snow-covered catchments in the Arctic is thereby dependent on the density, liquid water content, and temperature of the snowpack in these environments. Because the magnitude and timing of snowmelt governs the hydrology of the Arctic, understanding of these processes is important to predict flooding and adequate apportionment of water supplies (Marsh and Woo, 1985, 1984a, 1984a, 1984b; Woo and Marsh, 1978).

Snowmelt is a major contribution to discharge in rivers situated within the United States. Examples include Oregon (Harr, 1981); Washington (Koch and Fisher, 2000; Pelto, 2004); Colorado (Fassnacht et al., 2001); and California (Davis et al., 1999). Snowfall from the Catskill Mountains provides water to the city of New York (Matonse et al., 2010; Pierson and Kick, 1995). The contribution of snow to the water supply of this city has been monitored for decades (Porter and Horton, 2010). Snowmelt contributions from the San Juan Mountains and Sangre de Cristo mountains contribute runoff to the Rio Grande Basin of New Mexico that provides water to the city of El Paso, Texas and parts of the country of Mexico (Rango et al., 2010). Snow provides more than $50 \%$ of runoff in California (Mount, 1995) and more than $75 \%$ of water for
agricultural use in this state (Rosenthal and Dozier, 1996). Reservoirs on streams draining from the Sierra Nevada Mountains of California do not hold significantly more water than annual snowmelt runoff (Rittger et al., 2011). Much of the water supply of agriculture-producing state Idaho is derived from snowmelt, leading Abramovich (2007) to refer to snow as "Idaho's frozen liquid gold." In a similar fashion, snow is considered to be "white gold" in California (Roos, 1991).

Population increase in the western United States will place increasing demands on water use in the near future. Accurate knowledge of the amount of water available from snow and the timing of runoff is required to manage water in this region (Bryant and Painter, 2010; Harshburger et al., 2005). Barnett et al. (2008) have indicated that due to a reduction in snowfall, an increase in precipitation falling as rain, and snowmelt occurring earlier in the spring, smaller amounts of water will be available during the summer in the western United States. This foretells water shortages for the region and implies that careful planning is necessary to mitigate the effects of these changes.

Further south in the Sierra Nevada of California, higher air temperatures during the winter months have resulted in an increase in snowmelt and an increase in streamflow during the winter season (Johnson et al., 1997). From April to July, snowmelt from the Sierra Nevada contributes 18 billion cubic meters of water. This represents $35 \%$ of annual flow used for urban and agricultural water use (Roos, 2006). Hydroelectric power generation using water from 25 rivers in California is also dependent on snowmelt. Adequate apportionment, use and competitive pricing of electricity is dependent on measuring the amount of water that enters these rivers in the form of snowmelt (Freeman, 1997). Reservoirs providing water to the City of San Francisco are dependent on snowmelt for normal operation (Mazurkiewicz et al., 2011).

More than $90 \%$ of electricity from hydroelectric power generation in British Columbia is derived from snowmelt (Assaf et al., 1995). A depth of 1 mm runoff from snowmelt produces $\$ 1$ million worth of hydroelectric power in Quebec (Brown and Tapsoba, 2007). In the Mount Megantic area of the Quebec Eastern Townships, snow is a vital resource for tourism involving skiing and fishing and also provides water for human consumption (Fourtier, 2009). Snow provides water for hydroelectric power generation in the Grand Lake watershed of Newfoundland. Snow surveys are conducted in this watershed to assess the amount of water available for hydroelectric power generation and the potential for flooding (Banfield, 1991).

Hydroelectric power generation is also dependent on snowmelt in the mountains of France (Duband and Lallement, 1992) and the Swiss Alps (Martinec and Rango, 1995).

In areas where snowmelt is a major contribution to the hydrological cycle, large amounts of runoff can result in flooding. Flooding has occurred in watersheds draining into the Peace River of Alberta (Sampson, 1961) and the St. Lawrence River of eastern Canada (Bruce, 1962). In 1967 flooding due to snowmelt occurred along the Chena River near Fairbanks, Alaska (Gray, 1968), leading the United States government to establish a regional early flood warning system. In 1969 excessive snowmelt contributions to the Souris River caused downstream flooding in the city of Minot, North Dakota (Pomeroy et al., 2007a). Extensive flooding also occurred during 2011 in the same area. This led government officials in both Canada and the United States to begin cross-boundary border talks in March 2013 following the ablation of large amounts of snow that increased water levels in the Alameda and Rafferty reservoirs situated near Estevan, Saskatchewan (The Star Phoenix, 23 March 2013). Flooding from snowmelt is an environmental hazard in New Brunswick, where forest thinning may influence the timing and amount of runoff (Daugharty and Dickison, 1982).

The southern part of the state of Wyoming has been subjected to flooding from snowmelt originating in the Sierra Madres and Snowy Range mountains (Dietrich and Rieman, 2001). The rapid melting of large amounts of snow deposited in Arizona during the Labor Day storm of 1970 caused extensive flooding, the destruction of roads and infrastructure by debris, and the loss of 22 lives (Thorud and Ffolliott, 1972). In the western Cascades of Oregon (Christner and Harr, 1982) and the Rocky Mountains of British Columbia (Toews, 1991), landslides were triggered by snowmelt flooding. Landslides influenced by snowmelt were found to damage fish habitat by the addition of sediment to Quesnel Lake, British Columbia (Cheng, 1987). Despite the presence of the Folsom Dam, flooding due to snowmelt occurred in 1995 within the American River basin near the city of Sacramento, California (Kattelmann, 1996). Extensive flooding occurred throughout most of the northeastern United States due to rapid snowmelt during January 1996. The flooding caused the deaths of 30 people and $\$ 1.5$ billion in damages (Anderson and Larson, 1996).

Rain-on-snow events increase snowmelt and the potential for flooding (Osterhuber, 1999) Harr (1981) identified rain-on-snow events as the major contributing factor to flooding caused by snowmelt in northern Oregon. Rain-on-snow events commonly occur in the Pacific Northwest
from British Columbia to Idaho and south to northern California (Brunengo, 2007), and are often observed during the winter in the mountains of California (Kattelmann, 1997; McGurk et al., 1993), in the Coquitlam River Basin of the British Columbia Coast Mountains (Solano, 2008), and in the Alps of Switzerland (Collins, 1988). Most severe floods in United Kingdom are caused by rain-on-snow events (Blyth et al., 1974). Rain-on-snow was the causal factor of the January 2005 flood in the Santa Clara River basin of Utah that destroyed 20 homes (Bardsley and Julander, 2005).

In the Sierra Nevada, high flow rates are significantly increased by mid-winter rain-onsnow events that flush nutrients and pollutants into Lake Tahoe, thereby influencing the ecology of the lake. The floods also transport sediment into the lake due to stream bank erosion that occurs during high-magnitude discharge rates (Kattelmann, 1999). Large amounts of snowmelt that enter rivers are often passed through spillways instead of dams, causing supersaturation of water with air (Speers, 1972). This can cause death of fish from gas bubble disease, and has been an issue in the Columbia River Basin (McGrath et al., 2006) where water from snow is used for hydroelectric power generation (Speers and Versteg, 1982).

Large amounts of snowmelt due to heat transport by pyroclastic flows can also cause excessive flooding, necessitating the monitoring of snow on Mount St. Helens in Washington State (Lea, 2007). Flooding occurred along the Toutle River when Mount St. Helens erupted in 1980 (Lea and Reid, 2006). Volcanic ash from the volcano reduced the albedo of snowpacks in the vicinity of the mountain and increased the rate of snowmelt (Crook et al., 1981; Tangborn and Lettenmaier, 1981).

The 2013 Alberta floods that affected the city of Calgary, the Kananaskis Valley and the town of Canmore were exacerbated by rain-on-snow events. Due to large amounts of sediment transported during the flood, as well as landslides and stream bank erosion, the hydrology and ecology of the Rocky Mountains was irrevocably changed. The cross-sectional area of stream channels in the region must be re-measured to accurately determine discharge and fish species such as trout that spawn in gravel river bottoms may be affected by sediment transported during the flood (Davison and Powers, 2013; Martel, 2013; Weber, 2013). The flooding in Alberta displaced more than 10000 people and will likely cost more than $\$ 256$ million to rebuild infrastructure (CBC, 2013). The flooding may eventually cost the city of Calgary more than a billion dollars (Kaufmann, 2013).

On the Canadian Prairies, frozen soils during the time of ablation and fast melt rates ensure that most water from snowmelt enters the prairie hydrological cycle as surface runoff (Gray and Landine, 1988; Gray et al., 1985; Shook and Pomeroy, 2012). Snowmelt on the prairies contributes runoff to areas of internal drainage, such as sloughs and wetlands, where high rates of evapotranspiration due to vegetation exceed rates of infiltration to groundwater reservoirs (van der Kamp and Hayashi, 1998). Snowmelt in this region provides water for irrigation, farming, hydroelectric power and human consumption (Steppuhn et al., 2003). The rate and timing of snowmelt is important for the proper design of hydrologic structures in this region (Gray et al., 1970).

Along with low rainfall rates, the occurrence of drought is an aspect of the prairie environment influenced by these runoff processes. The Prairies have shown significant increases in temperature and decreases in precipitation and streamflow since the 1950s (Gan et al., 1997). The reduction of snowfall during drought can reduce snowmelt contributions to a wetland up to $79 \%$ (Fang and Pomeroy, 2008). The depletion of snow from wetlands consequently reduces runoff to rivers. During the recent drought of 1999-2005, snowmelt contributions to prairie rivers from both mountain snowpacks and prairie wetlands remained low, along with lower than average precipitation (Pomeroy et al., 2011). Due to reduced water availability for agriculture, the recent drought caused the loss of 41000 jobs in 2001-2002 and was also associated with a drop in national agricultural production of $\$ 3.6$ billion (Stewart et al., 2011).

Climate change and drought in California also necessitates the need for efficient snow management systems to ensure adequate snowmelt water supply (Davis et al., 1999). During the 1976-1977 drought in California (Hannaford and Howard, 1979), 8000 wells were drilled to provide adequate water for agricultural use, fish spawning was affected due to low water levels, the reuse of irrigation water exposed ecosystems to toxic chemicals, a drop in hydroelectric power generation led to an increase in fossil fuel consumption, aquatic recreational businesses were pushed close to bankruptcy, and forests were affected by disease and fire (Brown, 1978). Another drought affected California for six years from 1987 to 1992 (Roos, 1995).

The amount of snowfall on the prairies decreased by $17.2 \%$ from the 1960 s to the early 2000s (Steppuhn et al., 2003). Climate change models for the Canadian Prairies predict future increase in surface temperature, a decrease in available soil moisture during the summer months,
a decrease in the water available from snowmelt due to a smaller amount of precipitation falling as snow, and an earlier snow ablation season (Barnett et al., 2005).

Gan (2000) suggests that on the prairies, snow trapping and water storage management techniques should be used to locally mitigate the effects of climate change to increase the amount of water from snowmelt. The trapping of snow is accomplished by snow fences (Cooley et al., 1981; Pomeroy and Gray, 1995; Tabler, 1971), controlling the height of stubble left on a field during the winter season (Steppuhn et al., 2009), planting trees, and by moving snow or soil using earth-moving equipment (Stepphun, 1980). Stubble height control and the trapping of snow allows for farmland with saline soil to increase grain yield due to dilution of the salt by runoff (Stepphun, 2005). Approximately 30 million hectares of farmland on the North American Great Plains are affected by saline soils (Steppuhn and Miller, 1998). Stepphun and Wall (1996) found that trapping snow on alfalfa fields increased the water content in the top 1.2 m of the soil by $78 \%$ and increased forage yield by $40 \%$. Snow can increase the survival of winter wheat by providing insulation against colder temperatures during the winter (Stepphun and Nicholaichuk, 1986). Trapping of snow also prevents snowdrifts from forming on roads and highways (Tabler and Jairell, 1980) and can be used to provide water for animals (Jairell and Schmidt, 1995; Sturges, 1989). "Snow harvesting" involves the trapping of blowing snow by pits situated in the lee of obstructions. This can be significantly cheaper than drilling wells to provide water for agriculture (Jairell and Schmidt, 1992).

Snowmelt in the Rocky Mountains provides most of the water in major rivers on the Prairies due to extensive non-contributing areas in Alberta, Saskatchewan and Manitoba, making water use in this region reliant on mountain hydrology (Harlan, 1969; Pomeroy et al., 2007a). An example is the Saskatchewan (Curry and Mann, 1965) and Athabasca River (Pomeroy et al., 2009) systems that provide water to major cities in the provinces of Alberta and Saskatchewan as well as oil sand development regions.

As the "world's natural water towers," mountain chains are estimated to globally provide more than $80 \%$ of fresh water to human populations (Viviroli et al., 2003). Due to sensitivity of hydrologic systems in the mountains, monitoring of snow and climate in these regions is similar to "canaries in the coal mine" (Diaz, 2007). The mountains thus provide early warning of changes in natural resources. Rather than using empirical relationships, physically-based models driven by measurements of snow in mountain catchments are required to properly assess changes
in water resources (Cline et al., 1997). However, as argued by Viviroli et al. (2003), there is a lack of research on hydrological processes in the mountains, making mountain hydrology a scientific priority.

In the Rocky Mountains, a decrease in river discharge due to a decrease in snow accumulation, an earlier snow ablation season, and an increase in water consumption by stakeholders in Alberta will contribute to reductions in future water availability (Bonsal et al., 2003; Byrne et al., 2006; Lapp et al., 2005; St. Jacques et al., 2010). Decreases in snow accumulation have been observed for the past two decades over the entire region of the Rocky Mountains (Ingersoll et al., 2009). The understanding of climate change in and near mountain regions is hampered by the relative sparcity of meteorological observations (Freeman, 2009; Lundquist and Rochford, 2007; Smith, 2009).

To estimate the rate, timing and volume of snowmelt in the Rocky Mountains, various researchers have contributed to an understanding of the processes in this region. The Cold Regions Hydrological Model (CRHM) was tested to ascertain the applicability of this model for prediction of hydrological processes in ungauged basins (Fang et al., 2013). Turbulent energy transport in the near-surface boundary layer at meteorological observation sites situated in valley bottoms are affected by wind gusts, necessitating the use of modelling techniques beyond simple mass and energy-balance equations (Helgason and Pomeroy, 2012a). Due to the complexity of terrain in the mountains, shading of solar radiation can influence snowmelt, necessitating the development of snowmelt energetics models using algorithms from computer graphics (Marsh et al., 2012). Reba et al. (2012) estimated sublimation from snowpacks using eddy covariance techniques. Debeer and Pomeroy (2009) modelled snowmelt and snowcover ablation in an alpine cirque.

Forest-gap thinning treatments will increase the amount of runoff from snow, but this is influenced by energy balance and microclimate in complex terrain (Ellis et al., 2013). Thinning of forests reduces interception of snow, decreases evaporation and sublimation, and increases the amount of water available from snowmelt (Ffolliott and Gottfried, 2003). Forest thinning experiments have been conducted to increase snowpack runoff for human use (Bernier and Swanson, 1986; Swanson and Stevenson, 1971). At the Marmot Creek Research Basin (MCRB) in Alberta, Canada, Swanson and Golding (1982) created a thinned forest in a "honeycombed"
pattern, showing that this treatment increased runoff by $20 \%$ to $30 \%$. However, Chinook winds in Alberta can sublimate snow in clearings, reducing snowmelt runoff (Swanson, 1980).

Reduction of forest canopy by fire can increase overland flow from snowmelt and cause flooding. An example of this was the 2009 Mill Flat Fire near Harmony, Utah (Julander et al., 2011). Skidmore et al. (1994) found that burned areas of a forest had $9 \%$ more runoff from snowmelt and an ablation rate $57 \%$ greater than unburned areas. Burning of vegetation by wildfire can reduce forest canopy cover and change snow accumulation patterns, influencing the amount of snowmelt produced during ablation (Drake et al., 2008).

The mountain pine beetle epidemic in western Canada has affected the hydrology of the Rocky Mountains (Pomeroy et al., 2012). Infestation of Lodgepole pine trees by the mountain pine beetle causes tree morality. This is due to infection of trees by blue-strain fungi transmitted by the beetle as an epidemiological vector (Solheim, 1995). The death of trees in a forest reduces canopy cover, decreases the amount of water available from snow (Winkler and Boon, 2009) and increases the rate of snowmelt (Teti, 2009). Hydrological modelling of mountain catchments affected by the mountain pine beetle has been conducted to assess the influence of attacks on water availability from mountain catchments (Borden et al., 2010; Pomeroy et al., 2012).

Snow acts as an insulator for tree species such as Yellow Cedar (Chamaecyparis nootkatensis) by protecting the roots from freezing and moderating soil warming during the spring. Changes in snowpack melt rate and distribution due to climate change has led to the decline of yellow cedar in Alaska. This decline has commercial and ecological ramifications (Hennon, 2007). Snowcover affects the distribution of mountain hemlock (Tsuga mertensiana) since this plant thrives on the additional moisture from snowmelt (Nachlinger and Berg, 1988). Melting snow affects the composition and spatial distribution of plants in mountain communities (Billings and Bliss, 1959).

Measurement of snow depth and hardness has been used to assess the ability for reindeer to forage for food during the winter in western Alaska (Clagett, 1978). The amount of snow on the ground is associated with the movement patterns of animals. For example, it has been observed that elk are found in areas with less than 70 mm SWE from snowmelt (Farnes and Romme, 1993). Snow also provides a home for algae (Hoham et al., 1993; Takeuchi, 2001), bacteria and fungi (Brooks et al., 1993) that help to comprise a snow ecosystem (Hoham, 1989).

Snow protects animals from cold temperatures during the winter since it is a thermal insulator (Kalliomaki et al., 1984).

Snow deposited on lake ice influences the timing and rate of ice development and meltout. The formation of lake ice shows spatial patterns of thickness over the entire water body. Thicker ice formation as "black ice" initially occurs near the lake edges. Wind-redistributed snow accumulates near the lake edges, insulating the ice surface and reducing ice growth. Over time, ice growth becomes thicker near the lake center, but snow deposited at the lake edges increases in mass. The heavier snow near the edges can crack the ice, causing water to seep into the bottom of the snowpack. Refreezing of the water produces "white ice" on top of the black ice, thereby increasing the strength of the black ice areas (Adams, 1982). Understanding the interactions between snow, ice formation and melt on lakes can be of importance to establishing the rate and timing of nutrient release from the ice into the lake (Adams and Lasenby, 1982). Physical measurements of snow are required for models of ice formation on lakes (Ménard et al., 2002).

Snow accumulation affects the mass (Pelto and Brown, 2011), energy balance (Sloan, 1986) and chemical composition (Winski et al., 2011) of glaciers, thereby influencing the magnitude of runoff (Krimmel and Tangborn, 1974), the timing of the spring freshet during ablation, and the rate of glacial retreat. Meltwater from glaciers was found to contribute up to $50 \%$ of the discharge to the Bow River above Banff during the month of August (Hopkinson and Young, 1997). Due to a rise in global temperatures over the past 150 years, the area of glaciers has been shrinking. The meltwater from glacier wastage increases discharge in rivers situated within the glacial basin watershed (Hopkinson, 1997). In the Hood River basin of Oregon, glacier wastage provides $\sim 75 \%$ of water to the Upper Middle Fork River, with a $31 \%$ decrease in discharge occurring over the next 50 years due to climate change and glacier retreat (Nolin and Phillippe, 2008). Most of the flow in the Yukon River above Whitehorse is derived from snowmelt. This river provides hydroelectric power and water for the City of Whitehorse. Climate change may affect the future availability of water from this river (Janowicz and Ford, 1994). Glacier melt contributions to runoff will change stream temperatures and influence fish spawning (Nolin and Phillippe, 2008).

Avalanches develop due to the formation of a weak layer of snow with low shear strength that causes structural instability in the snowpack, thereby resulting in the movement and
transport of snow under the influence of gravity (Perla, 1981). The development of a layer of weak snow in the snowpack is coincident with a temperature gradient and the presence of facetted crystals of depth hoar (Scheler et al., 2003), low density, brittle forms of snow particles with large grains (Pfeffer and Mrugala, 2002). Avalanches are a major threat to human life in mountainous terrain (McClung and Schaerer, 2006). For example, avalanches killed 24 people in western Canada and the United States in 2008 (CBC, 2008). Avalanches can also destroy buildings and are hazardous to roads that travel through mountainous terrain (Armstrong, 1981, 1973), necessitating the development of early warning systems (Judson, 1977). These warning systems are effective when dependent on physically-based models of snowpack evolution that require accurate measurement of snowpack parameters (Bartelt and Lehning, 2002; Lehning et al., 2002a, 2002b, 1999).

Avalanches play a significant role in mountain hydrology. The redistribution of snow by avalanches can contribute between $19 \%$ and $57 \%$ of snow to the mass balance of a glacier, increasing the amount of snowmelt during ablation. Because snow deposited at higher elevations is redistributed to lower elevations, this permits glaciers to develop up to 1600 m below the regional snowline. The redistributed snow may have a quicker melt rate at lower elevations due to higher temperatures (de Scally, 1996). Alternately, snow redistributed to lower elevations can accumulate in areas where topographic shading influences snowmelt, thereby changing the timing and duration of streamflow in mountain catchments (de Scally, 1995). Sediment deposited by the avalanche can also affect the energy balance of the glacier by reducing the albedo (de Scally, 1996).

Avalanches can dam rivers, reducing discharge and increasing the potential for flooding during ablation. The mass of snow from an avalanche deposited on an ice-covered water body can either break up the ice or depress the ice enough to cause the development of a flood wave at the lake outlet. Sediment and other organic debris transported by the avalanche will be deposited in the lake, reducing water depth and affecting fish that spawn in gravel beds. The sediment is also a source of gravel required by these fish species, indicating a long-term relationship between avalanche activity and fish spawning (de Scally, 1996). Williams and Clow (1990) reported that an avalanche on the ice of Emerald Lake, California, produced a flood wave that lowered the survival rate of brook trout embryos from $90 \%$ to $2.5 \%$. Avalanches also remove soil from mountain regions, influencing the hydrologic response of watersheds (de Scally, 1996).

The geographic presence or absence of snow affects the location and operations of ski resorts (Crowe and Baker, 1983; Mote et al., 2007). For example, projected climate change in Ontario (Harrison et al., 1986), Colorado (Lazar et al., 2006) and the Columbia River basin (Mote and Hamlet, 2009) will increase regional temperatures and decrease the number of days available for skiing. Birkeland (1996) found that snow density and SWE of snow was greater on artificial ski runs cleared from forest than under the forest canopy due to the presence of artificial snow and the mechanical packing of the snowpack by skis, thereby influencing the amount of water available for runoff. Compaction of snow will influence metamorphism and snow thermal properties affecting the timing of snowmelt runoff (Kattelmann, 1986). Extensive runoff from snow can occur in ski areas due to deep snowpacks resulting from the removal of vegetation and the creation of buildings and roads that increase overland flow (Osterhuber et al., 2007).

### 1.2. Snow Measurement Parameters

Mathematical models of snowpack evolution require energy and mass flux inputs. From these inputs, energy and mass balance equations are used to calculate internal state variables. Internal state variables of some snowpack evolution models are listed in Table 1.1.

Not all of these models have a "detailed" development of snowpack physics. The internal state variables of the models are updated at model timesteps and show temporal changes in snowpack properties. The internal state variables are not directly measured by instrumentation due to a lack of deployed observation technologies and the inability to measure these variables at all locations where the model is applied.

The models of snowpack evolution are used to predict climate, the amount of water available for runoff during the time of ablation, or the potential for avalanche activity in a particular area. The importance of these processes has been discussed in the previous section. The predictive ability of the model is related to the accuracy of the inputs and internal state variables. Table 1.1 shows that six variables are often used in models of snowpack evolution.

The snow depth $\bar{y}$ is a measure of the vertical length of snow determined with respect to a reference datum normally taken as the ground. The density $\rho_{s}$ of snow is the mass of snow $m_{s}$ in a reference volume $V_{T}$. Together these variables are used to define Snow Water Equivalent (SWE), a point measurement of the depth of water created by the melting of the snow.

Assuming that water spread over an area of $1 \mathrm{~m}^{2}$ has a depth of 1 mm and a mass of 1 kg and that snow depth $\bar{y}$ is reported in centimeters, snow density and SWE is defined by (Pomeroy and Gray, 1995):

$$
\begin{gather*}
\rho_{s}=m_{s} / V_{T}  \tag{1.1}\\
\text { SWE }=\left(0.01 \mathrm{~cm}^{-1} \mathrm{~m}^{3} \mathrm{~kg}^{-1} \mathrm{~mm}\right) \bar{y} \bar{\rho} \tag{1.2}
\end{gather*}
$$

SWE is the mathematical product of snow depth and density, and is commonly reported in units of mm or $\mathrm{kg} \mathrm{m}^{-2}$. Using Equation (1.2) to obtain up-scaled estimates of SWE over an area requires the addition of a term that models the covariance between depth $\bar{y}$ and density $\bar{\rho}$ (Pomeroy and Gray, 1995; Stepphun, 1975). For prairie snowpacks with depth less than 60 cm , the covariance term is small (Shook et al., 1993). As given by Pomeroy and Gray (1995), Equation (1.2) uses a unit conversion factor for historical purposes, since snow depth is often reported in centimeters rather than meters.

The measurement of depth alone is not sufficient to obtain SWE since density can vary over an area (Gray et al., 1970) and must be measured rather than assumed to be a particular value such as $100 \mathrm{~kg} \mathrm{~m}^{-3}$ for freshly-fallen snow (Pomeroy and Gray, 1995). Judson and Doesken (2000) found that new snow density ranges between $10 \mathrm{~kg} \mathrm{~m}^{-3}$ and $350 \mathrm{~kg} \mathrm{~m}^{-3}$ at sites in the United States. Nominal snow densities are between $80 \mathrm{~kg} \mathrm{~m}^{-3} \leq \rho \leq 600 \mathrm{~kg} \mathrm{~m}^{-3}$ (Pomeroy and Gray, 1995).

| Model | $\rho_{s}$ | $\phi$ | $\theta_{w}$ | $\alpha$ | $\kappa$ | $T$ | Note | Reference |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2LM |  |  |  |  |  |  | a | (Yamazaki, 2001; Yamazaki et al., 2004) |
| CLASS |  |  |  |  |  |  | b | (Bartlett et al., 2006; Verseghy, 2000, 1996, 1991; Verseghy et al., 1993) |
| CLIM2-TOP |  |  |  |  |  |  | c | (Bonan et al., 2002; Yang and Niu, 2003) |
| CLM3 |  |  |  |  |  |  | d | (Oleson et al., 2004) |
| COLA-SSib |  |  |  |  |  |  | e | (Mocko and Sud, 2001; Mocko et al., 1999; Sud and Mocko, 1999, 1999) |
| CoupModel |  |  |  |  |  |  | f | (Gustafsson et al., 2001) |
| CRHM |  |  |  |  |  |  | g | (Pomeroy et al., 2007b) |
| CROCUS |  |  |  |  |  |  | h | (Brun et al., 1992, 1989) |
| ISBA |  |  |  |  |  |  | i | (Douville et al., 1995; Noilhan and Mahfouf, 1996) |
| MATSIRO |  |  |  |  |  |  | j | (Takata et al., 2003) |
| MOSES |  |  |  |  |  |  | k | (Cox et al., 1999) |
| SAST |  |  |  |  |  |  | 1 | (Jin et al., 1999) |
| SNOWPACK |  |  |  |  |  |  | m | (Bartelt and Lehning, 2002; Fierz and Lehning, 2001; Lehning et al., 2001, 1999) |
| SNTHERM |  |  |  |  |  |  | n | (Gustafsson et al., 2001; Hardy et al., 2000; Jordan, 1991) |

Table 1.1. Internal parameters explicitly used in snowpack models. The snow density is given by $\rho_{s}$, the porosity by $\phi$, the water content by $\theta_{w}$, the tortuosity by $\alpha$, the thermal conductivity by $\kappa$ and the temperature by $T$.
a. Explicitly includes the presence of depth hoar
b. Snow density is initially considered to be $100 \mathrm{~kg} \mathrm{~m}^{-3}$ and increases with time
c. The porosity is used in calculating the mass balance of liquid water
d. Implicitly includes the water saturation as changes in the mass of snow and liquid water
e. Thermal conductivity is noted as being one of the most sensitive of all parameters
f. The model does not parameterize the movement of liquid water through the snowpack
g. CRHM provides a selection of algorithms for modeling snowpack evolution and uses SWE as an input
h. Snow metamorphism affects albedo and heat fluxes through the snowpack
i. Models heat transport between vegetation and the snowpack
j. Directly calculates for SWE using a balance equation
k. Snow modifies the thermal conductivity, temperature and saturation of the soil layer

1. Includes the water saturation in the effective snow density
m . Includes the effects of snow metamorphism and vapor exchange in snow pore spaces
n . Uses a mixture to model the water saturation of snow

Snow is a porous medium consisting of pore spaces of air and a frame comprised of ice from snow particles sintered together by metamorphic processes (Buser and Good, 1986). Due to thermal gradients in the snowpack created due to mass and energy fluxes and the production of water by melting snow, a snowpack is an uneven mixture of air, ice and water (Colbeck, 1976a). Assuming that the volumes of air $\left(V_{a}\right)$, ice $\left(V_{i}\right)$ and water $\left(V_{w}\right)$ are fractional constituents of a medium with control volume $V_{T}$, the air porosity $\theta_{a}$, liquid water content $\theta_{w}$ and ice fraction $\theta_{i}$ are given by (Bartelt and Lehning, 2002):

$$
\begin{gather*}
\theta_{a}=\frac{V_{a}}{V_{T}}, \theta_{w}=\frac{V_{w}}{V_{T}}, \theta_{i}=\frac{V_{i}}{V_{T}}  \tag{1.3}\\
\theta_{a}+\theta_{w}+\theta_{i}=1
\end{gather*}
$$

Equation (1.3) is subjected to a continuity constraint since the fractional constituents of the medium cannot exceed unity. Liquid water content $\theta_{w}$ is a measure of the fractional amount of liquid water in the pore spaces of the snowpack. This is distinct from Snow Water Equivalent (SWE), which is the resulting depth of water after the snowpack has completely melted.

The density of snow $\rho_{s}$ is related to the density of air $\rho_{a}$, water $\rho_{w}$ and ice $\rho_{i}$ by (Bartelt and Lehning, 2002):

$$
\begin{equation*}
\rho=\rho_{s}=\rho_{a} \theta_{a}+\rho_{w} \theta_{w}+\rho_{i} \theta_{i} \tag{1.4}
\end{equation*}
$$

The porosity is the volume fraction of pore spaces in the medium:

$$
\begin{equation*}
\phi=\frac{V_{a}+V_{w}}{V_{T}} \tag{1.5}
\end{equation*}
$$

The water content $\theta_{w}$ and water saturation $S_{w}$ are measurements of snow wetness and are often reported as a percentage calculated from a volume fraction. Whereas the water content $\theta_{w}$ is the volume fraction of water per total volume $V_{T}$, the water saturation $S_{w}$ as used by authors
such as Colbeck (1976) is the volume fraction of water in the pore space of the snowpack (Marsh, 1991):

$$
S_{w}=\frac{\theta_{w}}{\phi}
$$

The irreducible water content $\theta_{e}$ is the residual fraction of water per unit volume that cannot be removed from the pore spaces of the snowpack (Marsh, 1991). This is also expressed as the irreducible water saturation $S_{e}$ (Colbeck, 1976a). Let $V_{e}$ be the volume of irreducible water in a control volume. Then,

$$
\begin{gathered}
\theta_{e}=\frac{V_{e}}{V_{T}} \\
S_{e}=\frac{\theta_{e}}{\phi}
\end{gathered}
$$

From the definition of irreducible water saturation, the effective water saturation $S^{*}$ as the total amount of water held by the pore spaces of the snowpack and the effective porosity $\phi_{e}$ is defined as (Colbeck and Anderson, 1982):

$$
\begin{aligned}
& S^{*}=\frac{S_{w}-S_{i}}{1-S_{i}} \\
& \phi_{e}=\phi\left(1-S_{e}\right)
\end{aligned}
$$

The diffusion of gas into a snowcover in the absence of wind is dependent on the tortuosity $\alpha$, which is a dimensionless number quantifying the straight-line deviation of the pore spaces in a medium. The tortuosity is a measure of the convoluted nature of the pore spaces. The tortuosity is always $\alpha \geq 1$, with $\alpha=1$ for cylindrical pores oriented parallel to the direction of fluid motion (Stoll, 1989). Tortuosity is one of the parameters used to model gas diffusion in snow using Fick's Law (Jones et al., 1999). It can be measured using x-ray tomography (Kaempfer et al., 2005).

Since tortuosity cannot be easily measured in the field, Sommerfeld et al. (1996) assumed that $\alpha=1$. For a model of $\mathrm{CO}_{2}$ transport through snow, Massman et al. (1995) assumed that $\alpha \approx 0.7$ and that $\alpha$ varies weakly with density. Assuming that the range of tortuosity in snow is $0.75 \leq \alpha \leq 0.92$, Mast et al. (1998) related tortuosity to porosity by $\alpha=\phi^{1 / 3}$ using the theory of Millington (1959). However, according to Bear (1972, pg. 111), the assertion that $\alpha<1$ for all porous media is incorrect, since this neglects the effect of tortuosity on the speed of flow through the porous medium. Moreover, the $\alpha \geq 1$ assumption has been used in the Buser (1986) model of sound propagation through snow, where $1.0 \leq \alpha \leq 1.25$ (Arons and Colbeck, 1995). In this thesis, it is assumed that $\alpha \geq 1$ in accordance with the definition given by Bear (1972).

The temperature $T$ is a measure of heat present in a substance. For a snowpack, temperature measures the effective heat of the air, ice and water that comprises the porous medium. Assuming that the effective thermal conductivity of the porous snow medium is $\kappa$ and the temperature gradient over a depth of snow is $\partial T / \partial y$, the heat flux $q_{F}$ for steady-state heat flow is given by Fourier's Law (Langham, 1981):

$$
\begin{equation*}
q_{F}=-\kappa \frac{\partial T}{\partial y} \tag{1.6}
\end{equation*}
$$

Snow density has been related to thermal conductivity by an empirical equation developed from gravimetric and heat pulse probe measurements of snow (Sturm et al., 1997; Usowicz et al., 2008).

### 1.3. Near-Surface Snowpack Measurement Instrumentation

The following review discusses snowpack measurement instrumentation deployed near the earth's surface. Although snowpack properties have been measured by sensors mounted on satellites (Frei et al., 2012; König et al., 2001; Lucas and Harrison, 1990) and airplanes (Rees, 2006), these applications are outside of the scope of this review.

The purpose of the review is to describe measurement instrumentation that determines similar snowpack properties to the sensing system developed in this thesis. This includes commercial devices used for snowpack measurement. These devices obtain snowpack
measurement parameters that were described in the previous sections. The purpose of these measurement devices is to obtain data for input into models of snowpack evolution and for validation of the internal state variables.

### 1.3.1. Depth Rod

Measurements of snow depth along a transect are commonly made using a snow ruler. The ruler is a graduated aluminum rod or stick with markings indicating a measurement scale (Goodison et al., 1981). On a 1 m snow depth rod (Snowmetrics Inc., Fort Collins, Colorado), markings are engraved every 1 cm , with a number scale given in 10 cm increments. Screw threads on each rod permit multiple rods to be coupled together so that snow depths greater than 1 m can be measured. The rod is pointed on the measurement end to ensure that the ground surface can be contacted and the measurement rod remains upright when snow depth is determined. Snow sampling rod design is discussed by Woo (1997).

Snow depth rulers are also installed in permanent locations along a snow course. The numbers on the ruler are manually read by snow surveyors. In geographic areas that are not easily accessible during the winter season, the numbers are large enough to be read from an airplane using binoculars. Snowmelt around the base of the ruler can cause inaccuracies in the measured snow depth (Pomeroy and Gray, 1995). Some early snow course markers did not have numbers but markers spaced every two feet along a twenty-foot long post (Henderson, 1953).

Because measurement of snow depth is time-consuming, Sturm (1999) devised a selfrecording snow depth probe (Snow-Hydro Inc., Fairbanks, Alaska). Similar to a ski pole, the device uses a sliding electromechanical attachment that coincides with the snow surface when the tip of the pole contacts the ground surface. A switch is closed when a depth measurement is to be made, and a circuit logs the distance between the tip of the pole and the ground surface.

Changes in snow depth were determined by Elder et al. (1999) using GPS measurements of elevation. A GPS receiver was placed on the snow surface next to a marker and the elevation recorded. The measurement was repeated at the maker at various times over the snow accumulation season and changes in snow depth were determined. A similar measurement principle was used to determine changes in snow depth when the GPS receiver was placed on a moving snow machine.

Determining snow depth using a measurement ruler can be inaccurate if vegetation, logs or ice layers are situated beneath the snow surface and the ruler cannot easily contact the ground. If the soil under the snowpack is unfrozen and consists of organic matter, the rod can penetrate the soil and an inaccurate measurement of snow depth is obtained (Elder et al., 1999). During each measurement, care must be taken to ensure that the rod is inserted parallel to the ground surface. This can produce inaccurate measurements of snow depth if the ground surface under the snowpack is not flat due to the presence of undulating topography or bedrock near the snow surface. The snow ruler creates a hole in the snowpack at every sampling location. This does not permit multiple samples to be taken at the same sampling point. Blyth et al. (1974) indicate that snow depth measured using a conventional measurement rod is often inadequate for forecasting snowmelt runoff due to the inability and labour required to sample at a large number of points over a landscape.

### 1.3.2. Snow Sampler Tubes

A snow sampling tube ("snowtube") is used to extract gravimetric samples of snow to measure Snow Water Equivalent (SWE) and depth-integrated snow density (Goodison et al., 1981). Snowtubes and snow surveying were initially popularized by Church (1933) in the early 1900s. The snowtube is made of metal or non-opaque plastic with a serrated cutting end (Crook and Freeman, 1973; McKay and Blackwell, 1961). The plastic tube allows the snow surveyor to visibly see the snow sample after it has been extracted. A sample is extracted by pushing the tube through the snowpack until the serrated cutting end reaches the ground surface. Handles on the top side of the tube allow the sampler to be turned so that the serrated cutting end extracts a plug of soil. The sampler is carefully removed from the snowpack and placed on a calibrated scale to measure SWE. Depth measurements taken with a measurement ruler permit the calculation of depth-averaged snow density from SWE measurements made by the tube (Goodison et al., 1981). The snow sampling tube accuracy can show high correlations with snowpit measurements of SWE (Fassnacht et al., 2010). Snow sampling tubes are used along snowcourses by snow surveyors to measure areal estimates of SWE for streamflow forecasting and runoff modelling (Goodison, 1975; Tabler, 1982).

Farnes et al. $(1980,1982)$ tested 22 different snowtube designs against gravimetric snowpit measurements of SWE. Ensuring that snow was not lost from the tube, they found that
smaller-diameter snowtubes without a sharpened cutting end overestimated SWE by $9 \%$ to $10 \%$, whereas snowtubes with sharper cutting ends had an overestimation of $3 \%$ to $6 \%$. This finding coincides with Peterson and Brown (1975), who compared "Federal" snow sampling tubes to samples of snow carefully extracted from the snowpack, finding that the snow tube overestimated SWE. For shallow snowpacks with less than 1 m depth, the error in measurement was found to be $1 \%$ to $3 \%$ (Goodison, 1978). Farnes et al. $(1980,1982)$ found that increasing the snowtube diameter decreased the overestimation of SWE. Snowtubes with a sharpened cutting end area $>20 \mathrm{~cm}^{2}$ had the least error and the ESC-30 snowtube with a cutting end area of $30 \mathrm{~cm}^{2}$ showed the least measurement error when used to sample snowpacks with a depth of less than 1 m . Turčan and Loijens (1975) present a simple model of snow tube insertion into snow, showing that compression of the snow as the snowtube moves through the snowpack can be an source of error in the application of this technique.

A complete mechanical design and blueprint for the ESC-30 snowtube and calibrated weighting scale is presented by Farnes et al. (1982). The total length of the snowtube is 126 cm and the tube body is made out of clear plastic. This allows for the snow sample contents to be visible to the snow surveyor so that the presence of vegetation in the extracted sample can be determined. The inner diameter of the snowtube is 69.85 mm and markings are placed on the tube in centimeter increments. The cutting end is comprised of 16 slanted teeth. The spring scale has calibrated markings to report the amount of SWE in centimeter increments.

The snow sampling tube is invasive since it destroys the snowpack at each sampling location (Smith et al., 1967). Davis (1973) refers to snow sampling as "archaic, inefficient, and hard work." If a plug of soil is not extracted to cap the bottom of the tube, the extracted snow sample can fall out and the measurement will have to be repeated. When the snow temperature is colder than the air temperature, snow can freeze to the inside of the tube (Goodison et al., 1981). The snow will have to be extracted before another measurement is made. The presence of buried vegetation and logs beneath the snow surface will often interfere with the collection of a snow sample. Ice crust formation inside of the snowpack can cause difficulties with the extraction of a sample. (Powell, 1987). Due to the relative size of the snowtube, it can be difficult to carry over long distances. To allow for easier transportation of the snowtube, Gluns and Rose (1992) designed a special backpack referred to as the "Kootenay Cruiser."

### 1.3.3. Snowpits

A snowpit is created when measurements of snow stratigraphy are required, invasive instruments (as discussed in the following sections) are used to measure the snowpack, or when visual observations of snowpack properties or processes are being made. The snowpit is simply a hole in the snowpack that is dug using a shovel down to the ground surface. The ground surface is taken as the origin of the depth reference datum, with a ruler used to measure the distance of each layer from the bottom. Layers are classically identified by an observer in the field using visual interpretation (Fierz et al., 1999).

To reduce the subjectivity of layer identification, infrared imaging and image processing algorithms has been used to identify layers after the snowpit is created (Shea et al., 2012; Tape et al., 2010). The size of snow particles in the snowpack can be measured by microscopy or spectroscopy (Painter et al., 2007). By photographing the snow surface, the snow surface profile can also be determined (Fassnacht et al., 2009; Kinar and Pomeroy, 2008a; Rees, 1998).

Marsh and Woo (1984) measured the propagation of water in a melting snowpack by injecting a red-colored dye tracer into the side of a snowpit. Photographs of the snowpit side revealed preferential flow paths for meltwater. A similar experiment was conducted by Peitzsch et al. (2008) at a field site using red food coloring as the dye tracer.

McGurk and Kattelman (1988) and McGurk and Marsh (1995) used a "cutter box" to extract horizontal slices out of a snowpit wall. The slices were photographed and then observed for melt channel formation.

Williams et al. (2010) injected a blue dye tracer into a snowpack and then used a large "snow guillotine" to progressively shave away vertical sections from the snowpit wall. The instrument created by Williams et al. (2010) is similar to but much larger than a microtome utilized to prepare biological specimens for mounting on a microscope slide (Hodge et al., 1954). The Visible Human Project also created a large microtome (referred to as a "cryomactrotom") for thin-sectioning a frozen human cadaver so that medical imaging techniques could be tested (Sachse, 2004). In a similar fashion to the Visible Human Project, Williams et al. (2010) photographed the side of the snowpit after each snow slice was removed and then used commercial software to recombine the camera images into a 3D image of meltwater flow in snow.

Fortin et al. (2002) injected artificial ice layers with a florescent dye tracer and inserted the ice layers into the side of a snowpit. Interaction of water with the ice layers was revealed by movement of the dye tracer.

The density of each layer in the snowpit is normally estimated using a measurement scoop with a known volume. Samples are extracted from a pit wall not facing the sun to mitigate the effects of solar insolation on changes in snowpack properties. A sample from each snowpack layer is extracted using the measurement scoop. The mass of the measurement scoop is determined using a weight scale and the density of each layer calculated by dividing mass by volume (Fierz et al., 1999).

Triangular measurement scoops (RIP Sampler - from Ronald Ivan Perla, a Canadian snow scientist) are used to determine snow density. The triangular measurement scoop (Snowmetrics Inc., Fort Collins, Colorado) is ideal for density measurement since a metal cutter fits into a slide on the top of each scoop. The triangular scoop is inserted into a snowpack layer and the cutter is used to extract a snow sample. Two types of triangular scoops are available: a small scoop with a volume of $250 \mathrm{~cm}^{3}$ and a large scoop with a volume of $1000 \mathrm{~cm}^{3}$.

The "CRREL" snow sampler is a $500 \mathrm{~cm}^{3}$ cylinder with a serrated metal edge that is inserted into the side of the snowpit (Crook and Freeman, 1973). The length of the CRREL sampler is 20 cm long. Alternately, the "Canadian" snow sampling scoop has a volume of 250 $\mathrm{cm}^{3}$, is only 19 cm in length and may contain serrated cutting teeth for sampling high-density snow. Both types of snow tubes can be capped for sample extraction. Snow density samples collected using both types of tubes show high correlations between datasets, indicating that the samplers perform in a similar fashion (Gary, 1967).

Snowpit sampling is time-consuming and the creation of a snowpit is labour-intensive (Pomeroy and Gray, 1995). In a similar fashion to the other measurements discussed in this section, the digging of a snowpit is an invasive procedure that destroys the snowpack structure. After the snowpit is created, heat transport from the surrounding landscape and insolation from the sun will cause melting of the snow. This is the reason why the snowpit is covered with a tarp (Marsh and Woo, 1985) if further experiments are required hours after the snowpit is dug. If the snowpit is not backfilled with snow after experiments are completed the snowpit creates a safety issue at backcountry locations and presents a hazard to skiers.

### 1.3.4. Pressure Sensors

The most common pressure sensor used to measure SWE is a snow pillow. The classic snow pillow (i.e. SP 3 from Sommer Messtechnik GmbH, Austria) is an oval (Beaumont, 1965) or rectangular (Molnau, 1971) pressure transducer buried in the ground surface under the snowpack. The pressure transducer is filled with an antifreeze chemical. A manometer senses the mass of snow on top of the pillow by measuring changes in fluid pressure (Beaumont, 1965). The mass of the snow is related to SWE by:

$$
\begin{equation*}
\mathrm{SWE}=\frac{m_{s}}{A_{s}} \tag{1.7}
\end{equation*}
$$

In the above Equation (1.7), $A_{s}$ is the area of the snow pillow computed using simple geometry.
Snow pillows are only suitable for permanent installation at a field site and cannot be moved. This adds logistical and cost considerations to the selection of a snow pillow site (Farnes, 1967). Moreover, because the pressure transducer is filled with a liquid, mechanical abrasion can cause leakage (Ord, 1968), damage can be inflicted by animals (Davis, 1973; Shannon, 1968) and air bubbles can develop in transducer lines (Cox et al., 1978). Frost-heave processes and the presence of bears in high-latitude environments necessitates the need for snow pillows in Alaska and the Yukon Territory to be installed on top of a specially-designed frame (Clagett and McClure, 1994). Because the antifreeze chemical mixture commonly used in the snow pillow is hazardous to animals, some research has been conducted to find suitable mixtures that are less toxic (McGurk, 1992).

The size of the pillow is related to the maximum SWE that can be measured. Pillows with larger area are required for sites with greater snow depth and SWE (Beaumont, 1965). A recommended minimum diameter for the snow pillow is 3 m (Farnes and Rompel, 1969). The development of ice layers between the pillow and the ground surface and uneven melt rates between the ground and the top of the snow pillow (Johnson and Schaefer, 2002) can cause measurement errors. Julander (2007) noted that snow pillows can be manufactured from steel or Hypalon, a black synthetic rubber compound. Julander (2007) found that the steel pillows accumulated more SWE than the Hypalon pillows, suggesting that the black color of the Hypalon rubber influenced snowmelt rates. The snow pillow is often painted white to reduce heating by solar insolation (Beaumont, 1965).

Snow pillows are tested in the field by a laborious procedure of removing all of the snow from the top of the pillow. The mass of removed snow is then carefully determined by bagging and weighing (Davis, 1973). Smith and Boyne (1981) installed a snow pillow in a laboratory test chamber, finding that the response of the transducer did not show a linear relationship with applied pressure. They also presented a mathematical model of changes in snow pillow pressure with environmental temperature. Johnson (2004) realized that most snow pillow designs rely on "rule-of-thumb" engineering estimates. To quantify errors in snow pillow measurements, Johnson (2004) used mathematical modelling to show that SWE pressure sensor errors occur due to: (1) a shear stress exerted from the side of the snow pillow caused by a difference in compressibility between the adjacent snow and the pillow; (2) differences between thermal properties of the pillow and the surrounding snowpack causing an uneven melt rate; and (3) sensors with a high compressibility that influence measurement of the snowpack's mass. A mathematical procedure for correction of these errors has been proposed by Johnson and Marks (2004).

Modern snow measurement devices that are similar to snow pillows use load cells in lieu of antifreeze. This reduces the possibility of mechanical breakdown. The load cell is a transducer that changes in resistance with a change in applied pressure. Moffitt (1995) designed a load-cell device referred to as a "snow plate." A circular plate with 1.1 m diameter was suspended on three load cells. Due to the smaller area of the sensor relative to most snow pillow designs, ice-bridging effects were observed to be a major influence on sensor results. Johnson et al. (2007) describe a load-cell based SWE measurement device that consists of eight aluminum plates with dimensions of 3.2 m by 3.0 m (i.e. SSG from Sommer Messtechnik GmbH, Austria). In a similar fashion to a snow pillow, the plates are installed during the snow-free season in a frame flush with the ground surface to ensure that measurement errors do not occur due to uneven settling of the snow. The plates are perforated with holes to reduce the ponding of meltwater on the surface of the device. Load cells are situated on each corner of the center plate to measure the mass of the snow on each of the surrounding plates. The idea is similar to a much older design that arranges triangular plates around a weighing scale (Beaumont, 1966).

### 1.3.5. Dielectric

Dielectric measurement techniques relate the complex dielectric constant (relative permittivity) of snow to the porosity $\phi$ and/or volumetric water content $\theta_{w}$. For most of these techniques, the relationship between the quantities is empirical. Dielectric instruments generally measure snowpack wetness as volumetric water content $\theta_{w}$ instead of water saturation $S_{w}$.

The Finnish snow fork consists of two parallel stainless-steel prongs with sharpened ends that act as a transmission line (Sihvola and Tiuri, 1986; Tiuri and Sihvola, 1986; Toikka, 1992). Although popularized by Finnish investigators, similar methods involving three prongs (Gerdel, 1954) and four prongs (Kendra et al., 1994) have also been proposed.

A triangle wave generator connected to a Voltage Controlled Oscillator (VCO) sends an RF wave into one of the prongs of the Finnish snow fork via a coaxial cable (Denoth et al., 1984). After inserting the prongs into the snowpack, the snow medium acts as a load that terminates the transmission line (Sihvola and Tiuri, 1986). Adjusting the frequency of the VCO from 500 MHz to 900 MHz (Toikka, 2010) enables measurement of the real and imaginary parts of the snow dielectric constant by determining the resonant frequency of the snow. The resonant frequency of the air medium must also be determined since the ratio of these frequencies is related to the dielectric constant (Sihvola and Tiuri, 1986). Assuming that the snow has a neutral pH level of 7, an empirical relationship developed for snow at a frequency of 1 GHz is then used to calculate porosity and water content. All snow samples used to develop this relationship were collected in Finland. For snow with impurities and a known pH level, another empirical relationship can be used to calculate these quantities (Tiuri et al., 1984).

The Finnish snow fork is normally used in a sampling configuration where the prongs are inserted horizontally into the side of a snowpit. However, the prongs can be placed on the snow surface for a dielectric half-space measurement, or inserted into the snow surface at a $30^{\circ}$ or $45^{\circ}$ angle. This requires the snow fork measurements to be multiplied by a correction factor (Moldestad, 2005).

The effective measurement volume of the Finnish snow fork has been estimated as 47 $\mathrm{cm}^{3}$ (Techel and Pielmeier, 2011). Alternately, assuming that the volume of measurement is comprised of a cylinder with 1 cm radius and 7.5 cm height, the volume is $24 \mathrm{~cm}^{3}$ (Moldestad, 2005). The effective sampling area has been estimated as $15 \mathrm{~cm}^{2}$ (Techel and Pielmeier, 2011).

The Finnish snow fork can measure snow with a maximum liquid water content of $10 \%$ and a maximum snow density of $600 \mathrm{~kg} \mathrm{~m}^{-3}$ (Toikka, 2010), with a minimum of $\sim 0 \%$ liquid water content (Techel and Pielmeier, 2011). The water content measurement error of the Finnish snow fork is reported as $\pm 0.5 \%$ (Sihvola and Tiuri, 1986) but has been claimed to be as low as $\pm 0.3 \%$ (Moldestad, 2005). Mechanical compression of the snow as the snowfork is inserted into the snowpack will cause snow density to increase by $1 \%$ to $2 \%$, adding another source of error to the measurement (Techel and Pielmeier, 2011). The error in determination of density is $\pm 5 \mathrm{~kg} \mathrm{~m}^{-3}$ (Moldestad, 2005). The Finnish snow fork was estimated by Harper and Bradford (2003) to have a density measurement error of 5\%. A possible disadvantage of the Finnish snow fork is that the empirical relationship developed for snow at a frequency of 1 GHz may not be valid for snow at tuning frequencies of 500 MHz to 900 MHz .

Two other resonant-frequency sensors have been proposed. The saw resonator consists of a saw blade with a parallel wire. Designed for hard snow such as melt-crusts and ice layers, the sensor operates using a sensing principle where the RF wave creates a mirror image of the saw blade in the snowpack. In a similar fashion to the snow fork, the ratio of resonant frequencies is used to calculate the dielectric constant. The coaxial resonator sensor is placed on the snow surface. The sensor is pushed slightly into the snow surface to ensure proper coupling of the sensor to the snow medium. A dielectric half-space calculation is used to determine the complex dielectric constant of the snow surface layer (Denoth et al., 1984).

Free-space measurement sensors require the extraction of a snow sample from the snowpack (Denoth, 1989). The sample is placed in a Styrofoam holder between a transmitting and a receiving horn antenna. An electromagnetic wave at frequencies greater than 10 GHz is sent between the two horns. Measurements made in the air medium without a sample present allow for determination of the complex transmission and reflection coefficients of the snow sample. The reflection coefficients are related to the dielectric constant of the snow (Hallikainen et al., 1986). An alternate "ellipsometric" measurement places the two horns at an angle to the surface of the snow sample. Measurement of the reflection coefficient is related to the dielectric constant by an empirical relationship (Denoth, 1999).

Capacitor cell sensors are comprised of two parallel metal plates, between which is placed a snow sample. Similar to the measurement of an unknown capacitor on a circuit board, an impedance bridge is used to measure the capacitance of the snow using frequencies in the 10
kHz to 10 MHz range. The ratio of snow capacitance to the empty-cell capacitance is used to calculate the dielectric constant of the snow (Perla, 1991). An alternate design uses an aluminum pipe. The snow sample is placed inside of the pipe (Camp and LaBrecque, 1992; Perla and Banner, 1988). A parallel plate capacitor sensor can be placed on the end of a rod and the rod used to penetrate the snow surface to obtain snow profile measurements (Louge et al., 1998).

The Denoth capacitive sensor (referred to in this thesis as a "Denoth meter") consists of a flat plate antenna that is inserted into the snowpack (Denoth, 1994). The volume of measurement of a flat plate antenna sensor is $900 \mathrm{~cm}^{2}$ corresponding to a 4 cm by 15 cm by 15 cm cuboid (Boyne and Fisk, 1990). The area of measurement has been estimated as $176 \mathrm{~cm}^{2}$ (Techel and Pielmeier, 2011). This is a larger measurement area than the Finnish snow fork.

The Denoth sensor is similar to a printed circuit board (PCB) antenna. Three types of antennas can be used. Two of these antennas have an effective measurement depth of 1.5 cm and 1 cm respectively, whereas another antenna is used for snow surface measurements in the top 5 mm of the snowpack (Denoth and Wilhelmy, 1988). The first two sensors can also be placed on top of the snow surface to make half-space measurements of snow surface wetness.

A 20 MHz oscillator and the flat-plate Denoth sensor are connected to a twin-T bridge. The bridge is balanced by turning a potentiometer or can be automatically balanced by an electronic circuit connected to the bridge. The current across the balanced bridge is determined and related to the dielectric constant of snow by an empirical equation. Gravimetric measurements of snow density and the dielectric constant of snow are used to calculate the liquid water content of snow by this empirical equation. The balancing of a twin-T bridge is also used as the operating principle of a seven-plate air-gap condenser sensor (Denoth et al., 1984).

The flat plate sensor can be buried in the snowpack and used to measure diurnal cycles in snowpack wetness (Denoth, 1994). Techel and Pielmeier (2011) have shown that when compared to snow wetness measurements made with the Denoth capacitive sensor, the Finnish snow fork overpredicted water content by $1 \%$. Although Denoth et al. (1984) found that the Finnish snow fork had a slightly higher standard deviation of measurements as compared to Denoth meter measurements, Williams et al. (1999) showed that there is no statistically significant difference between Denoth meter and Finnish snow fork measurements.

The Snow Pack Analyzer (SPA) system (Sommer Messtechnik GmbH, Austria) uses a flat coaxial cable antenna installed at an angle to the ground surface. The antenna is used by the
system to measure density and liquid water content of snow. A measurement of snow depth using an acoustic ranging sensor permits the calculation of SWE. A number of ribbon sensors situated parallel to the ground surface can be used to measure snow densities and liquid water contents throughout the snowpack. The sensor assembly needs to be installed during the snowfree season so that snowfall can accumulate over the antenna, which is buried in the snowpack in a similar fashion to a thermocouple assembly.

The theory of operation of the SPA sensor is given by Niang et al. (2003). It is assumed that the snowpack is comprised of a mixture of air, ice and water. Three equations (one serving as a constraint function) are used to solve for the volume fractions of the constituents. Substitution of the constraint function equation into one of the other equations provides two equations relating the dielectric constant of the snow to the dielectric constants of the air, ice and water comprising the snowpack. This does not require the use of empirical equations directly relating the snow dielectic constant to snow density.

The SPA sensor measures the dielectic constant of snow using Time Domain Reflectometry (TDR). This relates the speed of the pulse in the antenna to the snow dielectic constant (Jones et al., 2002). Determination of the snow dielectic constant using 6 kHz and 50 kHz electromagnetic waves permits two equations to be solved for two unknowns. The error in the measurement associated with a change in dielectric constant at the air-snow interface is corrected using a mathematical model. Niang et al. (2003) determined that the sensor could measure snow density with a maximum RMSD of $5 \%$ between gravimetric and dielectic measurements. The RMSD was $3 \%$ during the ablation period when the snowpack was isothermal.

### 1.3.6. Radar

Radar snow measurement systems operate by sending electromagnetic waves into snow and receiving the reflections. Two different types of radar measurements have been used for snowpack sensing applications.

Pulse radar sends a short impulse into the snowpack and the time difference between the sent and received pulses is used to determine snowpack stratigraphy (Harper and Bradford, 2003). The disadvantage of pulse radar is that the resolution is limited by the duration of the pulse. Pulses of smaller duration will give better resolution of layered media (Klauder et al.,
1960). However, the minimum pulse duration is limited by the transmitter and receiver circuits at frequencies in the GHz range. This in turn limits the operating resolution of pulse radars (Meikle, 2008). A unique application of pulse-based radars is tomographic imaging of a snowpack (Fortin and Fortier, 2001).

Frequency-Modulated Continuous Wave (FMCW) radar uses a frequency-swept pulse (chirp) to determine the distance to a reflector. Two horn antennas are situated at an offset distance. One horn produces an electromagnetic wave, whereas the other receives reflections (Stove, 1992). This is a bistatic radar system (Willis, 1991). The sent and received waves are homodyned by a time-domain analog mixer, creating a waveform that is low-pass filtered and digitized by an Analog-to-Digjtal Conveter (ADC). The difference waveform frequency spectrum is related to the snowpack layer distances and snow stratigraphy using an assumed snow density or dielectric constant of snow (Yankielun et al., 2004). The bandwidth of the chirp is commonly situated in the $2-6 \mathrm{GHz}, 8-12 \mathrm{GHz}$, and $14-18 \mathrm{GHz}$ ranges. Higher frequencies have better ability to resolve layers in the snowpack, but greater attenuation and less depth penetration (Koh et al., 2002). Lower frequencies are more appropriate for wet snowpacks due to attenuation created by high liquid water contents (Marshall et al., 2004).

FMCW radars are deployed above the snow surface and the antennas are moved over a short distance (less than 3 m ) to obtain images of snowpack stratigraphy (Marshall et al., 2004). Moverment of the radar system enables a measurement of SWE to be made over a large area (Marshall et al., 2005). The FMCW radar can also be set up to measure snowpack stratigraphy at a point location (Ellerbruch and Boyne, 1980). FMCW radars have been mounted on a sled towed behind a snow machine to measure snowpack stratigraphy along a transect (Gubler and Hiller, 1984; Holmgren et al., 1998; Pomeroy and Gray, 1995). Mounted on a gondola situated $\sim 30 \mathrm{~m}$ above the snow surface, the FMCW system has measured snowpack stratigraphy along a transect ranging over $\sim 500 \mathrm{~m}$ (Yankielun et al., 2004). Upward-looking FMCW radar has been used to measure snowpack stratigraphy in avalanche-prone areas by burying the system in the ground (Gubler and Hiller, 1984). Snowpack density and SWE can be determined from FMCW radar using an empirical relationship between the radar-measured dielectric constant of snow and these two parameters (Lundberg et al., 2006). Further applications and a review of FMCW radars are given by Marshall and Koh (2008).

The high cost of the measurement system and the attenuation of electromagnetic waves by liquid water in snow are two disadvantages of the FMCW method. The cost of the measurement system could be mitigated by building the system using coffee-can antennas and simple electronics (Charvat et al., 2013).

Using radar, overestimation of SWE by $20 \%$ can occur if the liquid water content of the snowpack is $5 \%$ by volume (Granlund et al., 2007). An upward-looking FMCW radar system can be used to determine liquid water content by measuring the attenuation of radar waves in snow and using an empirical relationship to relate attenuation to water content (Mitterer et al., 2011). The attenuation of electromagnetic radiation in snow was recently investigated by Sundström et al. (2013) using Ground-Penetrating Radar (GPR) and a path-dependent attenuation function. Sundström et al. (2013) found that liquid water content was underestimated by $\sim 50 \%$ but the mean error in determining SWE was reduced from $34 \%$ for a dry snowpack to $16 \%$ when the snowpack was wet.

Williams and Knoll (2002) used tomographic imaging of radar waves sent between two antennas to determine the dielectric constant of snow and measure snow wetness. One antenna was placed at the bottom of the snowpack, whereas the other antenna was placed at the top. Assuming that the snowpack had a constant density of $500 \mathrm{~kg} \mathrm{~m}^{-3}$, a mixture theory equation was used to determine liquid water content. The 250 MHz frequency used for the radar waves enabled penetration of the snowpack with a depth of 1.5 m .

### 1.3.7. Laser-Ranging

The use of laser-ranging equipment to measure snow depth relies on two observations. One observation is made during the snow-free season, whereas the other observation is made when snowcover is present. The laser range-finder is placed at a fixed location that is the same for all measurements. An observation is made by sending a short laser pulse toward the ground or snow surface. The pulse is reflected, and the time-of-flight between the sent and received pulse is used to measure distance from the range-finder to the ground or snow surface. The difference between the snow-free and snow-covered measurements is used to determine snow depth. Due to the similarity of laser-ranging to monopulse sonar and radar, the same limitations related to pulse width is also present in the application of this method.

Osterhuber et al. (2008) used a ground-based LiDAR system (Optech Inc., Ontario, Canada) with a depth measurement accuracy of $\pm 4 \mathrm{~mm}$ to determine snow depth at the Central Sierra Snow Laboratory in California. The LiDAR scanner was able to obtain a threedimensional volume of the snowcover by moving the laser beam over the snow surface using a computer-controlled mirror. Gravimetric measurement of snow density permitted computation of SWE. The $1.5 \mu \mathrm{~m}$ laser wavelength did not penetrate the snowpack and was suitable for use with wet snow. Hood and Hayashi (2010) used a hand-held laser rangefinder (Lasercraft Contour XLRIC, Earth Vector Systems) with a $0.905 \mu \mathrm{~m}$ wavelength, finding that snow depth in complex mountain terrain could be measured with an accuracy of $12 \%$ compared to manual depth measurements.

### 1.3.8. Near-Surface Photography

Photogrammetry is the use of two cameras situated at a known offset distance to determine the depth of objects in a scene. This method has been applied to measure snow depth using cameras situated near the ground surface. Although having similar principles of operation, near-surface photogrammetry is to be distinguished from conventional photogrammetry (Mikhail et al., 2001) conducted using downward-looking cameras mounted on airplanes (Cline, 1993; Smith et al., 1967). In a similar fashion to the laser-ranging method, images of the landscape before and after snow accumulation are required. The cameras are situated so that there is overlap between the images. Depth in the image pair is determined using linear algebra equations that relate points on each image to the camera positions.

Blyth et al. (1974) took photographs at two locations facing the side of a hillslope in an upland area near Plynlimon, Wales. The camera locations were situated 30 m apart on a vertical line and 2.5 m high markers were used to delineate the corners of a trapezoidal area spanning 4 hectares. A stereo-plotter normally used to measure depth from air-photo pairs was used to determine distances to the snow surface. Comparison with depth rod measurements revealed that this method could determine snow depth within $10 \%$ error.

Vallet (2001) and Skaloud et al. (2005) described a single-camera system used to measure snow depth. The system was mounted on the side of a helicopter for deployment near the ground and in Alpine areas. Placed near the single camera was a high-accuracy differential GPS system, inertial measurement sensors and a laser-ranging device. The position of the
camera relative to a Digital Elevation Model (DEM) was used to obtain estimates of the volume of snow for an avalanche control application.

Essery et al. (2008) used a digital camera fitted with a hemispherical (fisheye) lens to calculate the fraction of the sky (expressed as the "sky-view factor") not obscured by a forest canopy so that longwave radiation incident on snow beneath the canopy could be calculated. Debeer and Pomeroy (2009) used near-surface photography to measure snow covered area (SCA). The investigators set up a single camera facing a cirque on the side of Mount Allen in the Rocky Mountains. Using a DEM of the area and the known position of the camera, a software tool developed by Corripio (2004) was used to extract SCA from the single camera image.

### 1.3.9. Radioactivity

Measurement of snow density or SWE by radioactivity can be classified by sensor type: active or passive. The classification scheme is similar to that of sonar (Lurton, 2002). An active sonar device sends and receives acoustic waves, whereas a passive device only detects acoustic waves. Devices that use radioisotopes to measure snowpack properties either send and detect radioactive particles (active) or only detect radioactive particles (passive).

Active measurements often use a radioisotope such as ${ }^{60} \mathrm{Co}$ or ${ }^{137} \mathrm{Ce}$ to produce gamma radiation (Goodison et al., 1981). The advantage of using ${ }^{60} \mathrm{Co}$ over other isotopes such as ${ }^{65} \mathrm{Zn}$ is that this isotope has a greater penetration of snow (Gerdel et al., 1950), thereby allowing for a larger sampling area to be sensed (Armstrong, 1976). The ${ }^{60} \mathrm{Co}$ isotope also emits a narrow wavelength spectrum of radiation that can be empirically related to snow density with less error than an isotope with a broader wavelength spectrum (Gerdel et al., 1950). The source is collimated so that the radiation escapes from a slit in a lead box. A complete design for the collimator is presented by Gerdel (1952).

Two common active designs exist: either the source and a detector are situated above and below the snow surface (Gerdel et al., 1950), or in two parallel metal tubes installed perpendicular to the ground so that snow density can be measured by moving the source and detector vertically (Armstrong, 1976; Smith and Halverson, 1969). Movement of the source and detector is accomplished using an electric motor and mechanical assembly (Paulin, 1978; Smith and Willen, 1966). The disadvantage of using the above-and-below configuration is that snow
profiles cannot be recorded (Smith et al., 1967). Davis (1973) noted that the above-and-below configuration was used at a field site in California with a radioactive source buried under the snowpack and a detector situated on an airplane passing over the site.

Portable versions of active sampling gauges were once utilized by the Division of Hydrology at the University of Saskatchewan (Pomeroy, 1988), and the parallel tube device was deployed at the Bad Lake research site near Rosetown, Saskatchewan (Gray et al., 1970). Laboratory testing and calibration of the parallel tube device at the University of Saskatchewan was reported by Olfert (1970) and Gray et al. (1970). The spacing between the parallel tubes can range from 30.5 cm to 61 cm center-to-center (Gray et al., 1970).

An increase in the SWE and density of snow results in a decrease of the number of particles per unit time transmitted through the snowpack from the radioactive isotope to the detector. Empirical equations are used to relate particle counts to SWE or snow density. Gerdel et al. (1950) used Beer's Law with an empirically-determined extinction coefficient. Smith et al. (1965) used a simple logarithmic relationship.

Geiger-Muller tube (Armstrong, 1976) and sodium iodide tube (Smith et al., 1965) detectors have been used to detect the particles. The advantage of using a Geiger-Muller tube is operational stability (Blincow and Dominey, 1974) due to temperature shifts in sodium iodide tubes. Cosmic background radiation particle counts are required to completely correct for errors in received counts (Gerdel, 1952). Empirical calibration may be necessary before each run to correct for temporal changes in the system (Howe and Houghton, 1968). Gray et al. (1970) found that calibration is essential at least once every five readings. An empirical calibration procedure to correct for temperature changes in the metal tubes of the active gauge was proposed by Gray et al. (1970).

The accuracy of such active gauges is within $1 \%$ to $7 \%$ of SWE measured using gravimetric techniques (Blincow and Dominey, 1974; Pyper, 1962; Smith et al., 1965). Inherent difficulties in the deployment of the system arise due to temperature shifts in the electronics and snowmelt occurring around tubes buried in the snow (Smith et al., 1967). Uneven snow drifts can also form around the tubes, influencing the accuracy of the measurement and making detection of the snow surface more difficult. For an active gauge with two tubes, misalignment of the source and receiver can also occur (Kattelmann et al., 1983). Compared to snow pillows, daily estimates of SWE using active radioisotope gauges show greater variability in
measurements (Ord, 1968). Due to environmental and human health hazards associated with radioisotopes, active gauges are presently not used in an operational context (Pomeroy and Gray, 1995) despite "acceptable-risk" arguments once used to rationalize the deployment of such instrumentation (Gerdel et al., 1950; Morrison, 1976).

The backscattering method is an active measurement method that uses only one tube containing a source and receiver (Anderson et al., 1965; Morris, 2008a). A gauge of similar design was once available commercially as the Digiray Portable Profiling Snow Gauge PSG-1 (Blincow and Dominey, 1974). Initially, a core of snow is extracted from the snowpack (Young, 1976). The single-tube device is then inserted into the hole created by removal of the snow sample. A source containing ${ }^{85} \mathrm{Kr}$ (Blincow and Dominey, 1974), ${ }^{187} \mathrm{Cs}$ (Anderson et al., 1965) or ${ }^{241} \mathrm{Am}$ (Hawley and Morris, 2006; Morris and Cooper, 2003) produces radioactive particles that escape through slits in a lead collimator into the snowpack (Blincow and Dominey, 1974). The backscattered particles from the snowpack are sensed by a Geiger-Muller tube receiver. For the PSG-1, the sampling volume is a torus situated around the tube with a vertical extent of 5 cm and a horizontal extent of 30 cm (Young, 1976). Insertion of the single-tube device into a provided "calibration can" containing a substance of known density allows for counts of backscattered particles to be related to snow density (Blincow and Dominey, 1974) or empirical calibration is performed using gravimetric samples at a field site (Schytt et al., 1962). Recently, Morris (2008a; b) presented a non-empirical model for determining snow density using the back scattering method. The model is based on the physics of backscattered particles from snow and does not require empirical relationships.

The back-scattering method was tested in the mountains of California (Anderson et al., 1965; Gay, 1962) and on glaciers in Sweden (Schytt et al., 1962). The investigators found that errors in snow density determination were greatest near air-snow (Anderson et al., 1965; Schytt et al., 1962) and ground-snow (Anderson et al., 1965) interfaces. Young (1976) tested the PSG-1 device on Peyto Glacier in the Canadian Rockies, finding that depth-averaged densities agreed to within $5 \%$ of gravimetric measurements. The densities of individual layers in the snowpack did not necessarily coincide with gravimetric measurements due to the sampling volume of the detector and lags in particle counts associated with vertical movement of the sensing apparatus. Gerland et al. (1999) used the back-scattering method to measure snow density in Antarctica, finding an error of $2 \%$ comparison with gravimetric estimates of snow density. Measurements of
snow density on the Devon Ice Cap (Morris and Cooper, 2003) and the Greenland Ice Sheet (Hawley and Morris, 2006) in Nunavut, Canada showed that a disadvantage of the backscattering method was "smoothing" of the density profile due to the size of the back-scattered sampling volume.

Passive radioactivity gauges work in a similar fashion to the active gauges, but rely on the natural background radiation in the soil to provide a source of gamma radiation. The passive radioactivity gauge is situated in the air above the snow surface. Although this technique is very safe, background radiation counts at the same field site during the snow-free season are required and the gauge must count the number of particles accumulated over daily time periods. This does not permit "on-demand" measurements of SWE. As the SWE and snow depth increases, particle attenuation increases and the signal-to-noise ratio decreases. The gamma radiation particles are often highly attenuated by the snow, thereby limiting the snow depth and SWE that can be determined by this measurement technique (Bissell and Peck, 1973). Morrison (1976) briefly describes a passive radioactivity gauge being developed for prairie hydrology applications. The gauge was able to measure snow with SWE less than 50 mm and a depth less than 150 cm .

The CS725 SWE gauge (also referred to as the "GMON," from Campbell Scientific Canada, Edmonton, Alberta) can measure up to 600 mm of SWE using the passive radioactivity method. The SWE measurement is limited due to attenuation of radioactive particles by the snow and the sensitivity of the detector. The CS725 was initially developed by Hydro Québec. When suspended 3 m above the ground, the effective measurement area of the gauge is $500-100 \mathrm{~m}^{2}$ due to collimation of the detector. Aside from particle count rates during the snowfree season, the empirical equation used to determine SWE also requires measurements of soil moisture prior to winter freeze-up (Wright et al., 2011).

### 1.3.10. Chemical

Chemical measurement of liquid water content $\theta_{w}$ relies on physical changes in the concentration of ions in a snow sample. These methods involve the extraction of a sample of snow from a snowpit or from the snow surface.

Davis and Dozier (1984) added a florescent dye (Rhodamine WT) to a snow sample and then measured the florescent intensity of the mixture using a chemical fluorometer. The
intensity of the sample was related to water saturation of snow using an empirical curve and chemical mixing formulae.

Perla and LaChapelle (1984) and Davis et al. (1985) conducted experiments in the Canadian Rockies to develop an acid dilution technique. A solution of $0.01-\mathrm{N}$ hydrochloric acid was added to a snow sample and an electrolytic conductivity meter was used to measure the conductivity of the resulting solution. The conductivity was then related to snow wetness. Because the acid dilution technique does not involve the use of empirical curves, it can be used to calibrate capacitor cells (Perla and Banner, 1988).

The error in dilution methods is $\pm 1.5 \%$ liquid water content (Davis et al., 1985). Although these methods can be used to measure snow wetness in a snow pit at a field site, the preparation and mixing of chemicals at a remote location can be tedious and time-consuming.

### 1.3.11. Calorimetric and Centrifugal

Calorimetry is used to measure the liquid water content $\theta_{w}$ of snow. A calorimeter consists of an insulated container into which is placed a snow sample. The insulated container is fitted with a temperature sensor to measure internal temperature. The mass of the snow sample is determined before a liquid is added to the insulated container. Stirring of the sample occurs over the time of measurement to ensure that heat is evenly distributed between the liquid and the snow sample. A change in heat content due to a change in phase of the mixture is related to the liquid water content of the snow sample using a heat-balance equation. Empirical calibration coefficients in the heat balance equation can correct for heat loss by the insulated container. Freezing, melting and chemical calorimeters have been used to measure snow (Boyne and Fisk, 1990; Jones et al., 1983). In a similar fashion to the chemical methods, the time taken for measurement and the equipment used creates logistical problems at field sites.

Freezing calorimetry requires a liquid freezing agent that is held at a temperature between $-40^{\circ} \mathrm{C}$ and $-50^{\circ} \mathrm{C}$. This is accomplished by keeping the freezing agent in a portable cooler that is packed with dry ice and taken to the field site. Toluene has been used as a freezing agent, but this is a hazardous chemical and so silicone oil can be used instead. After addition of the freezing agent to the insulated container, the container is shaken by hand and a change in temperature is recorded in 30 second intervals for approximately 10 minutes. The error in determination of liquid water content $\theta_{w}$ using freezing calorimetry is $\pm 1 \%$ to $\pm 2 \%$ (Jones et al.,
1983), but the method can be inaccurate due to uneven transfer of heat between the snow sample and the freezing agent (Fisk, 1986a). Moreover, the specific heat of the silicone oil will often change between purchased bottles and must be measured before use in the calorimeter (Boyne and Fisk, 1990). The presence of snow chunks in the sample can affect the measurement due to liquid water contained inside the snow chunk (Boyne and Fisk, 1987).

Melting calorimetry involves the addition of hot water to the snow sample. The hot water serves as a melting agent. The hot water can be directly added to the snow sample (Akitaya, 1985; Halliday, 1950; Yosida, 1967), or the water can be heated using an electric coil, which has an advantage of controlling the rise in temperature of the sample (Brun, 1989; Halliday, 1950). Brun et al. (1989) describe a melting calorimeter that uses a combination of calorimetry and dielectric techniques to measure liquid water content. The melting calorimetry method has better transfer of heat between the snow sample and the melting agent and is quicker than freezing calorimetry (Fisk, 1986a). The absolute measurement error in liquid water content ranges from $\sim 1 \%$ to ~5\% (Akitaya, 1985; Halliday, 1950; Morris, 1981).

Chemical methods involve the addition of a chemical to a snow sample. The chemical breaks intermolecular bonds holding the snow particles together and disperses ions and molecules into the solution, causing a resulting change in temperature. The use of methanol (alcohol) as a chemical agent has an absolute liquid water content measurement error of $\sim 1 \%$ (Fisk, 1986b). Because methanol is hygroscopic, a drying agent must be used to prevent water contamination during storage of the methanol solution (Boyne and Fisk, 1987). Morris (1981) used a NaCl (salt) solution, finding that the measurement error in snow liquid water content was $\pm 4 \%$ when the salt dissolved in solution did not exceed $5 \%$ by volume. Bader (1950) reported on the use of a NaOH (sodium hydroxide) solution as the chemical agent. Boyne and Fisk (1990) compared the alcohol and freezing calorimetry methods to dielectric techniques, finding that all methods had a maximum $\pm 2 \%$ measurement error.

Centrifugal methods involve the spinning of a snow sample to separate out liquid water from the snow. Drawbacks of this method include melting that occurs due to friction and the incomplete extraction of all liquid water from the sample (Colbeck, 1978; Jones et al., 1983). The irreducible liquid water content $\theta_{e}$ can be obtained by soaking a snow sample in water, draining the snow sample in an insulated chamber that is kept at an isothermal temperature by an ice bath, and then measuring $\theta_{e}$ using a freezing calorimeter (Coléou and Lesaffre, 1998).

### 1.3.12. Acoustic Measurement of Snow Depth

Most acoustic measurements of snow depth have been made using ultrasonic devices that determine the distance to the snow surface. The measurement device is fixed to a mast of known height situated above the snowpack. An ultrasonic sensor produces an inaudible ( $>20 \mathrm{kHz}$ ) acoustic pulse that is reflected from the snow surface. Measuring the time between the sent and received pulses enables calculation of snow depth using kinematics. This is a "time of flight" measurement. Concomitant measurements of air temperature are used to calculate the speed of sound in air. The measurement system principle is similar to pulse sonar and radar systems, and the range is limited by the pulse duration.

Gubler (1981) designed a snow depth measurement system using separate sending and receiving piezoelectric transducers. The transducers were situated next to each other in a bistatic sonar configuration. The sending transducer was driven at its resonant frequency of 40 kHz . Drifting and blowing snow caused scattering of the acoustic wave due to the small signal-tonoise ratio of the pulse. Gubler (1981) also observed that soft surface layers of snow would not cause strong reflections of the pulse, implying that some penetration of the snowpack by the ultrasonic wave may have occurred. This was recognized by Caillet et al. (1979) as occurring due to changes in the porosity and roughness of the snowpack surface.

Goodison et al. (1984) adapted an acoustic rangefinder from the Polaroid Corporation to measure the distance to the snow surface. This approach was later followed by Earl et al. (1985).

An improved sensing system was proposed by Goodison et al. (1988). Only a single transducer was used in this system operating at a frequency between $50-60 \mathrm{kHz}$ with a range of 27 cm to 10.7 m , an accuracy of $\sim 3 \mathrm{~mm}$ for distances ranging up to 3 m , and an overall accuracy of $\pm 1 \%$ (Mims, 2000). Increasing the speed of the microcontroller increased the time resolution and permitted the measurement of snow depth to within 1 mm depth error (Goodison et al., 1988). The air temperature was automatically measured at the same time that the acoustic pulse was sent toward the snowpack. The ultrasonic sensor device was able to detect drifting and blowing snow events. The system was observed to underestimate snow depth by 6 cm relative to snow ruler measurements (Goodison, 1985). Goodison et al. (1984) remarked that this bias may be due to partial penetration of the snow surface by the acoustic wave. This may occur if the snow is of relatively low density.

Acoustic scattering of the sound wave at the snow surface and temperature gradients near the air-snow interface were also considered by Goodison et al. (1984) to influence the accuracy of the gauge. Measurement test details of the gauge are given by Edey (1985) and Metcalfe et al. (1987). Osterhuber et al. (1994) present an empirical calibration procedure for removing the influence of temperature changes on spurious fluctuations in determined snow depth.

Snow depth was measured by Lahtinen (2011) using an ultrasonic sensor situated on the top of a post inserted into the snowpack. Since the length of the post was known, the distance to the snow surface could be determined using a time-of-flight measurement in a similar fashion to the measurement sensors previously discussed. The portability of the system enables it to be considered as being similar to the mechanical measurement system developed by Sturm (1999).

Acoustic measurement gauges are available commercially from several companies. Ryan et al. (2008) tested commercially-available ultrasonic snow depth gauges, finding that the maximum RMSD between ultrasonic and snow ruler measurements was less than 3 cm for 17 sites in the USA. Anderson and Wirt (2008) examined the factors that contribute to errors in ultrasonic snow depth measurement, finding that contamination of the transducer by dirt or insects, tears in the transducer surface, and obstruction of the transducer or snow surface under the transducer were major influences. Bergman (1989) found that heating of the thermocouple used to measure air temperature could affect the resolution of the depth measurement by up to $\pm 3$ cm , indicating the need to ensure adequate shielding of the thermocouple.

### 1.3.13. Acoustic Measurement of Snow

Smith (1965) and Smith (1969) sent sound waves through cylindrical snow samples extracted from a snowpack and related the speed of the sound wave to density using semiempirical equations. Buser (1986) and Buser and Good (1986) placed snow in an impedance tube and used the Zarek porous media model (Zarek, 1978) to explicitly determine snow density. Lee and Rogers (1985) set up a loudspeaker and microphone above a snow surface. The loudspeaker was used to send a 30 ms impulse into a 2 cm snow layer that had accumulated on a roof. Selecting the parameters of the Zarek porous media model allowed for generation of a synthetic waveform. The synthetic waveform was compared with the waveform recorded by the microphone. The measurement method used by Lee and Rogers (1985) was not systematic and was not applied to a seasonal snowpack.

In a similar fashion to Vertical Seismic Profiling (VSP) in seismology, Moore et al. (1991) created a probe microphone that was inserted beneath the snow surface on a rod. A loudspeaker situated above the snow surface produced white noise. The porous media model proposed by Attenborough (1983) was used to predict porosity from the transfer function measurement within < $20 \%$ error.

Using a seismic array and acoustic models of snow, Albert (1998) used an inversion procedure (Albert, 2001) to determine snow depth and permeability. The acoustic inversion procedure was similar to Lee and Rogers (1985) but fully automated.

SWE has been measured using a loudspeaker and microphone situated above the snow surface in a similar fashion to bistatic radar or sonar. In an analogous fashion to FMCW radar, the measurement system sent a frequency-swept sound wave into snow. The frequency-sweep was between 20 Hz and 20 kHz . The reflection was homodyned and related to SWE using a recursive acoustic model (Kinar and Pomeroy, 2007). Later work used a Maximum Length Sequence (MLS) as the source signal and another recursive acoustic model relating geometric spreading and reflection of the sound wave beneath the snow surface (Kinar and Pomeroy, 2008a, 2008b).

### 1.3.14. Heat Pulse Probes

A heat-pulse probe measurement apparatus consists of a number of needles inserted into a medium such as a soil to measure the thermal conductivity and density. There are two major types of heat pulse probes that have been used to measure snow. These can be classified using the number of needles: single-needle or multiple-needle heat pulse probes.

The heat pulse probe measurement begins by applying an electrical current to a heater wire looped inside of one of the needles. The heater needle warms up, and a rise in temperature is sensed by a thermocouple. The thermocouple is either placed inside of the heater needle (single-needle), or inside another needle situated at a radial distance to the heater needle (multiple-needle) (Mori et al., 2003). The change in temperature during heating or cooling of the needle is related to material properties by a mathematical model (Carslaw and Jaeger, 1959; Kluitenberg et al., 1993; Liu and Si, 2010; Morin et al., 2010).

Most heat pulse probes used to measure the thermal conductivity of snow utilized a single needle (Jaafar and Picot, 1970; Riche and Schneebeli, 2013; Sturm and Johnson, 1992; Sturm et
al., 1997). Fowler (1974) and Sturm et al. (1997) measured the thermal conductivity of snow using a single needle heat pulse probe and related this measurement to density using an empirical relationship. Single-needle probes have been buried in a snowpack and used to measure effective thermal conductivity of snow over a three-month period (Morin et al., 2010). Heating of cylinders, spheres, plates and wires have been used since 1886 to measure the thermal conductivity of snow (Sturm et al., 1997).

Liu and Si (2008) showed that both the density and thermal conductivity of snow could be measured using a multiple-needle heat pulse probe. Care must be taken to ensure that the strength of the heat pulse is not high enough to melt snow and that the radial distance between the heater needle and temperature sensing needle is calibrated in a medium with a known thermal conductivity. Although the multiple-needle heat pulse probe is an invasive measurement apparatus, the investigators realized that the probe could measure snow density and thermal conductivity in a snowpit. Alternately, the probe could be buried in the snowpack to make continuous measurements (Liu and Si, 2008).

Ross and Picot (1975) describe a heat-pulse probe where the surface responsible for thermal transfer of heat to the snowpack was a circular plate with a diameter of 15 cm . The entire probe was buried in the snowpack. The temperature change of the plate in contact with the snow was related to Fourier's Law of heat transport and used to measure the thermal conductivity of snow. Comparisons between measurements made with a single-needle heat pulse probe, a heat flux plate and a model related to snow microstructure have been reported by Riche and Schneebeli (2013). Microtomography images of snow structure and a mathematical model of heat transport have been used to measure snow thermal conductivity (Calonne et al., 2011).

### 1.3.15. Temperature Sensors

Temperature sensors are used to record snow temperature. The temperature sensor most often used by snow surveyors is a dial thermometer (Fierz et al., 1999; McGurk, 1983) or a thermocouple with a digital readout (digital thermometer) (Koh and Jordan, 1995). The thermometer is inserted into the snowpit wall at successive depths beneath the snow surface (Crook and Freeman, 1973). The temperature of the thermometer sensing surface is allowed to equalize with snowpit temperature before a measurement is read from the dial.

Brandt and Warren (1997) inserted a tube with 3 mm diameter into the side of a snowpit and used the tube to extract a small core of snow. The tube was removed and a thermocouple was placed into the hole created by the tube. Snow was then placed by hand over the thermocouple leads to retain the thermocouple in the snowpack. The process was repeated for layers at various depth in the snowpack to obtain a temperature profile. Back-shovelling of snow into the snowpit allowed for thermocouples to be buried under snow and temperature profiles to be determined over an extended period of time.

Infrared thermometers and IR cameras have been used to record the spatial distribution of snow temperature in a snowpit (Shea et al., 2012); to record the spatial distribution of temperature in snow and shrubs (Pomeroy et al., 2006); and to measure the temperatures of snow and tree trunks in a forest (Pomeroy et al., 2009). IR thermometers and cameras measure longwave radiation emitted by a surface (Bates and Gerard, 1989). Thermal imaging devices can have an absolute measurement error of $2{ }^{\circ} \mathrm{C}$, but a measurement error of only $0.2{ }^{\circ} \mathrm{C}$ between pixels in an image (Pomeroy et al., 2006). Correction and calibration of IR imagers for measuring snow temperature has been discussed by Shea et al. (2012). Snowpit-based measurement of snow temperature is laborious, invasive and time-consuming due to the creation of a snowpit (Sauter and Tanner, 1992).

Swanson (1968) described a system for measuring snow surface temperature using a thermocouple. The thermocouple was mounted on a plastic frame suspended above the snow surface on a vertical metal wire fastened to the ground. The plastic frame was held motionless by a nylon cord. To drop the frame onto the snow surface, a circuit was used to warm up a section of Nichrome heating wire that severed the nylon cord, thereby allowing the frame to drop under the influence of gravity. The output signal of a 2 kHz oscillator was applied to the top and bottom of the wire. A reservoir filled with mercury was mounted on the plastic frame. The reservoir made contact with the metal wire and a grounding point, modifying the oscillation frequency. The change in the oscillation frequency was then related to the position of the frame along the wire and the snow depth. The disadvantage of this system was that more than one plastic frame was required to make measurements of the snow surface over time. Modern implementations of this system would not use mercury due to the environmental hazards associated with its use.

Thermocouple arrays have been buried in the snowpack to measure temperature distribution throughout the snow. The basic setup of this apparatus is a series of thermocouples situated over a vertical distance. Installation of the stationary version of this apparatus occurs during the snow-free season so that snow can accumulate around the thermocouples. The vertical extent is selected to be higher than the maximum depth of snow over which temperature is to be measured.

Bowles (1967) suspended thermocouples on a post supported by a triangular frame. The post was 3 m high and the spacing between thermocouples was 10 cm . McGurk (1983) suspended thermocouples on a wire stretched between the ground and a boom arm situated at a distance of 3.4 m above the ground. The spacing between thermocouples was 30.5 cm .

Desrochers and Granberg (1986) used a system of thermocouples installed on a vertical post to measure snow temperatures in a woodland area. They describe a portable version of this system where a single thermocouple situated on the end of the vertical post is used to measure snow temperature profiles throughout the snowpack. The observer pushes the post through the top of the snow surface. Desrochers and Granberg (1986) recount that a disadvantage of the portable version of the system is that the temperature measurement is invasive, destroying the snowpack. A similar system was used by Metcalfe and Buttle (1998), where the thermocouples were placed along a wooden stick situated vertical to the ground surface. The stick was painted white and installed at a stationary location to measure vertical temperature distributions throughout the snowpack.

Tanner and Gaza (1990) suspended a rod between two pipe tripods. The rod was placed at a distance of 160 cm above the ground and was used to suspend thermocouples at 10 cm intervals along a wire. Tanner and Gaza (1990) found that heat conduction along the wires suspending the thermocouples in the upper 35 cm of the snowpack caused recorded temperatures to be significantly greater than $0^{\circ} \mathrm{C}$ during ablation, despite the use of low-thermal conductivity wire intended to minimize these effects. Tanner and Gaza (1990) also used chromel-constantan thermocouples instead of copper-constantan thermocouples to reduce thermocouple self-heating. Moreover, snow settling around the thermocouple supports may have influenced temperature measurements.

To minimize these effects, Sauter and Tanner (1992) created a ladder-type structure to suspend the thermocouples throughout the snowpack. The ladder was comprised of nylon
fishing wire. The vertical height of the wire was 2 m above the ground and thermocouples were spaced at distances of 10 cm along the ladder. Despite these precautions, Sauter and Tanner (1992) found that temperature conduction errors affected measurements at distances up to 20 cm beneath the surface of the snowpack, but snow temperatures measured during ablation were closer to $0^{\circ} \mathrm{C}$ than the measurements made by Tanner and Gaza (1990).

Sturm and Johnson (1991) suspended a two-dimensional network of thermocouples on strings between two posts before the snow accumulation season. Four rows of vertical thermocouples over an 80 cm horizontal distance were used to record temperature gradients throughout the snowpack. The vertical thermocouple spacing was every 5 cm over a 60 cm distance. The thermocouple array setup was able to detect convection plumes under the snowpack surface. Conway and Benedict (1994) later used a similar setup to measure temperature changes caused by meltwater propagation through snow. The network of thermistors spanned a 2 m vertical distance and a 1.5 width. The spacing between thermocouples was 15 cm .

Albert and McGilvary (1991) progressively laid down rows of thermocouples on the snow surface before a snow deposition event. Over the winter season, this caused the thermocouples to be buried inside of the snowpack without the need for a supporting structure. Although this method would have minimized the effects associated with heat conduction apparent in application of the other methods, the thermocouples could only be buried at depths related to snow deposition events. Albert and McGilvary (1992) later suspended thermocouples on nylon fishing wire between wooden dowels. Arranging thermocouple arrays in two separate directions allowed for a three-dimensional measurement of snowpack temperature.

Luce and Tarboton (2001) used nylon fishing wire to place thermocouples vertically along a line at distances of $5,12.5,27.5$ and 35 cm from the ground surface. Temperature measurements made by the thermocouples were used to test a model of heat transport through snow.

### 1.4. Automation of Snow Property Measurements

Although many environmental scientists and hydrologists use commercially-available instrumentation such as dataloggers, some investigators have measured snowpack properties
with custom-built automated systems. As is apparent from the previous section of this thesis, hydrologists have a long-standing tradition of building instrumentation to make observations.

Researchers at the University of Saskatchewan's Division of Hydrology pioneered automated measurement systems in the $20^{\text {th }}$ century, building dataloggers that recorded hydrologic observations of snow to cassette tapes (Gray, 1972) and measuring radiation fluxes to snow using instrumentation situated on the ground (O'Neill and Gray, 1973a) and on aircraft (O'Neill and Gray, 1973b). The mass flux of blowing snow was autonomously measured using particle detectors developed at the same research institute (Brown and Pomeroy, 1989). These detectors have been utilized to measure blowing snow in prairie (Pomeroy, 1988; Pomeroy et al., 1993), sub-Arctic (Pomeroy et al., 1999), Arctic (Savelyev et al., 2006) and Antarctic (Leonard and Cullather, 2008) environments. An improved version of the system with a collimating lens and on-board signal processor was reported by Sato et al. (1993).

Active profiling radioactivity gauges have been remotely controlled at field sites over telephone communications (Armstrong, 1976) or the pulses from such gauges have been transmitted to a base station using radio communications (Gerdel et al., 1950). Meteorological and hydrological instrumentation have been remotely controlled and data collected at various field sites since the 1960s (Bowles, 1967; Fletcher, 1967; Gray, 1967; Maxwell et al., 1960; Shannon, 1968; Weaver, 1967). In the United States, more than 600 SNOTEL (SNOwTelemetry) sites automatically measure SWE with an average error of $\sim 3 \mathrm{~mm}$ using snow pillows (Cooley and Palmer, 1997).

Painter et al. (2000) describe a system to automatically measure the angular distribution of snow reflectance using a spectroradiometer mounted on the end of a robotic arm controlled in the field using a program running on a portable computer. Further tests and field deployment of the system was reported by Painter and Dozier (2002).

Lundquist et al. (2003) describe the development of a metrological sensor network deployed in Yosemite National Park consisting of instrumentation specifically designed for the measurement application. The network utilized a custom-designed electronic circuit serving as a datalogger that conveyed environmental measurements over wireless distribution networks. The purpose of this device was to provide high-resolution measurements of snow properties for predictive hydrological applications.

Hubbart et al. (2005) suggested that iButtons (Maxim Integrated Circuits Inc., California) could be used for environmental temperature measurements at locations where dataloggers are not available or are too difficult to deploy. The iButton is an environmentally-sealed module measuring $\sim 16 \mathrm{~mm}$ in diameter. It contains an integrated circuit, battery, and data storage for observations. The iButton can be programmed to automatically take temperature measurements on a set time schedule. Lundquist and Rochford (2007) used iButtons to automatically log temperature measurements at different locations in the mountains of California and Colorado. They suggested that temperature differences between sensors could be empirically related to the surface albedo of snow. The iButton sensors were also deployed in the Andes mountains of Peru and used to monitor temperatures for glaciological studies (Hellström and Mark, 2006). PVC pipe shielded the iButton temperature sensors from sunlight.

Rice and Bales (2008) discuss the development of an embedded sensor network to measure snow depth and meteorological parameters. Skalka et al. (2009) and Moeser et al. (2011) report the development of the "Snowcloud" sensor network. The sensor network was comprised of nodes linked together using mesh radio communications. This permitted transmission of data to a base station using inter-node communication. The network was "selfhealing" so that failure of one node would not disrupt the flow of communications. The loss of a node would cause the network to intelligently route communications to another node, eliminating the need for a high-powered radio transmitter or antenna towers. Each node was used to obtain data from an ultrasonic sonar receiver/transmitter used to measure snow depth. Processing of sonar data was not performed in the field. The sensor network was deployed at the Sagehen Creek Field Station near Truckee, California.

Koh et al. (2010) mounted an FMCW radar system on a sled and used an autonomous robot to pull the sled over a 150 m transect in Antarctica. The reflection data from the FMCW radar was used to detect crevasses. FMCW radar systems have been designed by investigators who have used these devices to measure snow (Marshall et al., 2007).

Lahtinen (2011) described preliminary tests of an instrument by which profiles of light extinction, reflectance and temperature in snow were determined using sensors mounted on the end of a pole. In a similar fashion to the backscattering active radioisotope method (Section 1.3.9) or the dielectric instrument described by Louge et al. (1998), the pole was pushed beneath
the snow surface and profiles automatically logged. An ultrasonic depth sensor situated on the top of the pole enabled measurement of snow depth.

### 1.5. Problem

The previous sections of this thesis have shown that snowpack processes are of major environmental and human importance. Models used to understand these processes and predict runoff from snowmelt require quantitative measurements of snow as model inputs. Internal state variables commonly used by these models are not directly measured and are computed by mass and energy flux inputs. These models are validated using quantitative measurements of snow.

Researchers have long recognized that snow surveying using snow sampling tubes and the creation of snowpits to measure snow properties is a laborious process that is timeconsuming and prone to human error. This fact has been explicitly stated by many authors (Davis, 1973; Marsh, 1991; Pomeroy and Gray, 1995; Sturm, 1999).

Any device that requires the creation of a hole in the snowpack is invasive and does not allow for multiple measurements to be performed at the same location. Moreover, these devices require the presence of a human observer. The skill of the human observer affects the measurement error. The human observer cannot be present at locations that are difficult to access or where dangerous conditions such as avalanches are present during the snow accumulation and ablation season.

Electronic instrumentation can be deployed at a field site and set up to make automatic measurements on a time schedule. This has been attempted using some of the electronic devices described in the review section of this thesis. Devices that require installation during the snowfree season are not portable and cannot be ubiquitously deployed without prior preparation of a field site.

Devices that rely on empirical relationships between instrumentation outputs and snowpack variables may not work at field sites where environmental conditions are different than the conditions used to derive the relationships. However, some of the devices described in the previous sections do not rely on empirical relationships. Models used to process data from these devices have been proposed to reduce the reliance on empirical relationships.

Variables commonly used in models of snowpack evolution (Table 1.1) are estimated using multiple measurement devices. There is not a single device currently available that can
simultaneously measure snow density $\rho_{s}$, water content $\theta_{w}$ as a measurement of snow wetness, and temperature $T$. Different devices can be used to obtain measurements of these variables.

Non-destructive measurements of snow wetness would be useful to fully understand processes of water transport through snow (Marsh, 1991). A few decades ago, Colbeck (1978) presented a review identifying snow density and wetness as being two crucially important variables measured in snow hydrology. These two variables govern the rate, timing and amount of meltwater produced by snow. Colbeck (1978) suggested that acoustics might be used to obtain measurements of these quantities. Techel and Pielmeier (2011) suggested that a portable device would be useful for field measurements of snow wetness.

### 1.6. Objectives

The previous sections suggest objectives for the research described in this thesis. An overall objective is to use a non-invasive measurement system to determine snow (1) density, (2) liquid water content, and (3) temperature by the use of acoustics, which can be used to obtain all of these variables simultaneously in a non-invasive fashion. Under this overall objective, the following sub-objectives are:

1. To develop and test a model of sound wave propagation in snow as the Unified Thermoacoustic Model (UTAM);
2. To develop novel signal processing techniques and use these to design an electronic system-the System for the Acoustic Sensing of Snow (SAS2) - that sends sound waves into snow and receives the reflected waves; and
3. To deploy the electronic device at field locations in order to show how the electronic device can report data indicative of snowpack measurements.

## CHAPTER 2 <br> LITERATURE

### 2.1. Introduction

This chapter of the thesis provides an overview of the literature required to provide a necessary background for the following chapters. The review is limited in scope to provide an understanding of developments presented in this thesis. A review of snow acoustics research and models of sound propagation through snow places the research reported in this thesis in the context of experiments conducted by other investigators. Because the System for the Acoustic Sensing of Snow (SAS2) is similar to sonar, these acoustic sensing systems are also reviewed. Signal processing techniques required for the development of the theory are discussed.

### 2.2. Overview of Processes

### 2.2.1. Precipitation

In this thesis, a "snow particle" is considered to be the smallest visible unit of snow that comprises the snowpack. This is similar to the definition of a "snow grain" given by Colbeck (1986) and follows the terminology used by Pomeroy and Schmidt (1993).

The formation of a snow particle begins in the atmosphere (Pruppacher and Klett, 2010). The particle falls as precipitation on the land surface. Successive snowfall events create a snowpack that is a layered snow medium. However, layers can be created and destroyed by metamorphic processes (Colbeck, 1991). Metamorphic processes are discussed in Sections 2.2.8 and 2.2.10 below.

The amount of snow deposited as precipitation is strongly influenced by global atmospheric circulation as well as by the land surface and oceans (Intihar and Stewart, 2005). Fitzharris (1987) studied the climatology of the Rogers Pass area in the Rocky Mountains, finding that cold and dry Arctic air masses depressed temperatures. Frontal systems from the Pacific Ocean were responsible for raising temperatures and bringing moisture resulting in snowfall. Cullen and Marshall (2011) state that continental air masses from the southwestern United States also influence the climate of this area of the Rocky Mountains. The direction of travel of these three air masses is influenced by the polar jet stream and topography. Claus et al. (1984) note that snowfall is higher on the western side of the Rocky Mountains but lower in
semi-arid valleys. As air masses move further east over the Rocky Mountains, they lose moisture and precipitation rates are lower. As noted by Cullen and Marshall (2011), Chinook winds will develop in the lee of the Rocky Mountains of Canada. This makes climate on the eastern side of the mountains more arid and reduces precipitation.

### 2.2.2. Wind Transport

Falling snow or snow that is deposited on the ground can be subjected to transport by the wind. Snow particles that have not contacted the snow surface can be entrained into preferential flow paths due to changes in surface topography that constrain air flow (i.e. valleys or gullies). Preferential flow paths are also created by turbulent eddies resulting due to surface topography. The eddies cause local acceleration and de-acceleration of air flow (Kind, 1981). When snow particles reach the snow surface, wind transport occurs due to processes of creep, saltation and suspension. The snow particles that fall on the ground comprise a "bed" of particles that are subjected to these three processes (Pomeroy and Gray, 1995).

For snow particles to be transported, mechanical and cohesive forces between snow particles at the bed must be overcome. Moreover, rougher surfaces and surfaces with vegetation stalks will have greater shear strength, requiring shear stresses of higher magnitude to dislodge particles. This is why wind transport over a hard snow surface and denser snow is more efficient than wind transport over a freshly-deposited snow layer. Vegetation with greater height and exposed stalk area will be more efficient at increasing the strength of the snow surface and reducing the mobility of snow particles, thereby trapping snow and increasing snow depth. Snow particles that are subjected to saltation and suspension are mechanically disaggregated, reducing snow particle size (Pomeroy and Gray, 1995). Sublimation will occur during transport of snow particles in the air. This causes loss of mass from the snow particles and the production of latent heat that is advected by the airflow (Pomeroy and Essery, 1999).

In areas of complex topography such as the mountains, wind will remove snow from ridges and transport snow by turbulent eddies occurring due to disruption of airflow by mountain peaks. Snow mass transport is constrained by topography and mountain barriers (Mott et al., 2010). Wind-scouring of snow can occur on a ridge and transport snow into forests situated on the slope leading up to the ridge (Pomeroy et al., 2009). Despite the interception of snow by a forest canopy (Pomeroy et al., 2002), wind can transport snow into forests, increasing snow
depth in the forest relative to an open clearing. The forest can be considered as a porous medium that resists air flow (Miller, 1964). Turbulent eddies in forest clearings can redistribute snow in both the clearing and the surrounding forest (Golding and Swanson, 1978).

### 2.2.3. Interception

Snow particles are retained by the canopy due to the collection efficiency of the tree branches. This is dependent on the tree species and canopy structure (Pomeroy et al., 1998), with an increase in Leaf Area Index (LAI) and canopy density coincident with an increase in interception efficiency (Pomeroy et al., 2002). Coniferous trees have greater collection efficiencies than deciduous trees due to the greater area spanned by the branches during the winter season (Pomeroy and Schmidt, 1993).

Interception influences the distribution of snow in a forest. Due to snow interception by the canopy, "tree-wells" form around the base of coniferous trees. This is distinct from a "snow cone" that forms at the base of a deciduous tree. Whereas the tree-well is a depression with an increase in snow depth away from the tree trunk, the snow cone shows an increase in snow depth at distances closer to the tree trunk. Snow cone formation occurs because the deciduous tree has less interception efficiency than the coniferous tree (Sturm, 1992). Snow depth as a function of distance from the tree trunk can be determined by empirical models (Hardy and Albert, 1995; Sturm, 1992; Woo and Steer, 1986).

### 2.2.4. Radiation

Electromagnetic shortwave radiation with wavelengths between $0.1 \mu \mathrm{~m}$ and $4 \mu \mathrm{~m}$ is provided to the land surface by the sun. The sun provides the majority of downward shortwave radiation flux $(K \downarrow)$ in the net shortwave radiation balance ( $K^{*}$ ). These wavelengths of radiation are important for providing energy for photosynthesis and for warming the land surface (Oke, 1992; Petty, 2006). Shortwave radiation is a dominant term in the snowpack energy balance and contributes large amounts of energy required for snowmelt (Link and Marks, 1999; Male and Gray, 1975). Topographic shading in mountain regions reduces the magnitude of shortwave radiation incident on the snowpack (Marsh et al., 2012). South-facing slopes are oriented toward the sun and tend to receive more shortwave radiation than north-facing slopes
(Pomeroy et al., 2003), but this is moderated by trees providing canopy cover that attenuates shortwave radiation and reduces air temperatures under the canopy (Ellis and Pomeroy, 2007; Ellis et al., 2011; Essery et al., 2008; Metcalfe and Buttle, 1998).

Outgoing shortwave radiation ( $K \uparrow$ ) is radiation that is reflected from the snowpack. The reflection of a land surface is modelled by the albedo. The albedo is the ratio $K \uparrow / K \downarrow$. Higher albedos imply that a greater magnitude of shortwave radiation is being reflected, whereas lower albedos imply that more shortwave radiation is being absorbed. The albedo ratio is dependent on the magnitude of incoming shortwave radiation. Vegetated surfaces and bare ground have a lower albedo than snow (Oke, 1992). The albedo of snow has a high magnitude ( $>0.80$ ) over shortwave radiation wavelengths (Wiscombe and Warren, 1981) for most of the snow accumulation season, decreasing during the time of ablation (Gray and Landine, 1988) due to metamorphism (Warren, 1982) and the presence of dust from aerosol particles (Conway et al., 1996; Painter et al., 2007; Warren and Wiscombe, 1981). An increase in snow particle size over time due to snowpack metamorphism is the primarily mechanism for reducing the albedo of snow. For wet snow, the presence of water will have little influence on the albedo, but water can accelerate the growth of snow particles by metamorphism. An increase in snow particle size will increase the probability that a photon will collide with the snow particle, thereby causing scattering, absorption of photons by ice comprising the snow particle, and a consequent reduction in the albedo (Warren, 1982; Wiscombe and Warren, 1981).

Longwave radiation has wavelengths between $4 \mu \mathrm{~m}$ to $100 \mu \mathrm{~m}$ (Oke, 1992; Petty, 2006). Radiation fluxes with these wavelengths are emitted from the sky, clouds, snow, vegetation and the land surface as a consequence of heating by shortwave radiation. Direct longwave radiation from the sky can be modelled using the Stefan-Boltzmann law, treating the sky as a grey body emitter with an emissivity coefficient dependent on temperature and humidity (Sicart et al., 2004). Semi-empirical relationships take into consideration geographic location and an increase in the magnitude of longwave radiation by cloud cover (Granger and Gray, 1990; Granger et al., 1978). Longwave radiation is also produced by the snow surface. This is modelled using the Stefan-Boltzmann law. The assumption can be made that snow without the presence of surface impurities is a nearly-perfect blackbody emitter (Geiger et al., 2003).

Snowmelt is strongly influenced by the net radiation flux $Q^{*}$ (Ellis et al., 2011; Male and Granger, 1981; Oke, 1992; Pomeroy et al., 2003; Sicart et al., 2006). Net radiation often shows a
strong correlation with the net shortwave radiation flux. This is because net shortwave radiation is a dominant contribution to the net energy balance (Granger and Gray, 1990). When the net radiation balance is positive, the snowpack is warming, whereas when the net radiation balance is negative, heat is being lost from the snowpack to its surroundings and the snowpack is cooling. The net radiation flux is negative during the night, when shortwave radiation contributions to the energy balance are negligible. The negative energy balance will increase the cold content of the snowpack. The cold content is defined as the energy required to raise the temperature of snow to the melting point of $0{ }^{\circ} \mathrm{C}$. Because snow is a thermal insulator (Sturm et al., 1997), it has low thermal conductivity and retains energy. This results in a time lag between rising levels of net shortwave radiation, an increase in the temperature of the snowpack, and a decrease in the cold content (Bengtsson, 1982; Gray and Landine, 1988; Marks et al., 2008).

### 2.2.5. Snow Surface Heat Fluxes

Due to uneven spatial distribution of radiation over natural land surfaces, transfer of heat occurs from areas with an excess of thermal energy to areas with a deficiency of thermal energy. In the energy balance taken at the snow surface, sensible heat is heat that causes a change in temperature, whereas latent heat occurs due to a phase change associated with the melting or sublimation of snow. In high-latitude environments where shortwave radiation inputs are of small magnitude during the pre-ablation winter season, energy required for snowmelt can be mostly provided by sensible heat from warm air masses (Male and Granger, 1981). However, low magnitudes of sensible heat flux can contribute negligible energy to snowmelt, causing ablation of snow to be primarily driven by net radiation fluxes (Pomeroy et al., 1998). Ablation of snow by high-magnitude net radiation fluxes tends to occur earlier in the snow accumulation season before snowmelt, when advection of heat between snow patches is a dominant contribution to the energy balance (Marsh and Pomeroy, 1996). When the snow temperature reaches $0{ }^{\circ} \mathrm{C}$, further energy inputs produce additional latent heat as a result of phase changes (Rowlands et al., 2002).

In the mountains, turbulent fluxes of sensible and latent heat are strongly influenced by forests and complex topography (Helgason and Pomeroy, 2005). At a clearing site in the Rocky Mountains, intermittent wind gusts were found to be responsible for half of the sensible and latent heat fluxes to the snowpack (Helgason and Pomeroy, 2012).

### 2.2.6. Heat and Vapour Transport at the Snow-Soil Interface

During the snow-free season, heat is transferred to the land surface. Some of the heat is retained by the soil. This is due to the soil's heat capacity (Ronda and Bosveld, 2009). When snow is deposited on the ground, the low thermal conductivity of the snow medium ensures that the snowpack acts as a thermal insulator (Sturm et al., 1997). This traps heat in the soil. When the soil loses heat during the winter season and the air temperature above the snow surface is colder than the soil, there is heat flux from the ground into the snowpack. Most of the heat flux is due to latent heat produced due to gradual freezing of the soil and sublimation of water vapour from the soil to the basal layer of the snowpack (Sturm and Benson, 1997). Some of the heat can be from geothermal sources (Powers et al., 1985). Vapour transport from the soil into the snowpack causes depth hoar formation in the basal layer (Sturm and Benson, 1997; Woo, 1982b).

Due to a difference in temperature between the ground and the snow surface, a temperature gradient occurs in snow. This temperature gradient is responsible for driving snow metamorphic processes. By the end of the winter season, the presence of a temperature gradient destroys much of the snowpack layering due to the development of extensive depth hoar throughout the snowpack (Sturm and Benson, 1997). This extensive depth hoar is also observed in snow deposited on an ice-covered lake, where the water beneath the ice has a higher temperature than the snow surface (Adams, 1981). This temperature gradient may not always be of high magnitude. Temperature gradients of lower magnitude occur in snowpacks that form at lower latitudes due to differences in regional energy balances.

The heat flux magnitude is moderated by frozen ground. At locations where the soil contains permafrost, the perennially-frozen ground reduces soil temperatures and heat is transferred from the snowpack into the ground. However, when the ground is incompletely frozen, soil temperatures are higher and heat is transferred from the ground into the snowpack (Marsh and Woo, 1984b). Diurnal cycles in heat flux are observed before ablation. During the time of ablation, heat cycles are strongly suppressed (Granger and Male, 1978).

| Heat Flux $G$ ( $\mathrm{W} \mathrm{m}^{-2}$ ) | Reference | Description |
| :---: | :---: | :---: |
| -5 (early snow season) to -1 (late snow season) | (Marks et al., 2008) | Measurements made in a sub- <br> alpine watershed in Colorado <br> from early to late snowmelt |
| -10 | (Gold, 1957) | Measurement made during the <br> time of early ablation at a site <br> near Ottawa, Ontario |
| -5.3 (minimum) to 4.4 <br> (maximum) | (Male and Granger, 1979) | Three years of observations at <br> a prairie site (Bad Lake <br> Research Basin near <br> Rosetown, Saskatchewan) |
| -20 (early snowmelt) to 80 (late snowmelt) | (Pomeroy et al., 2003) | Measurements made at a <br> subarctic site (Wolf Creek <br> Research Basin, near <br> Whitehorse, Yukon Territory) <br> during the time of ablation |
| 4 | (Sturm and Benson, 1997) | Average value of <br> measurements made over the <br> winter season before ablation <br> at a site near Fairbanks, <br> Alaska |
| 80 | (Marsh and Woo, 1984a) | Meltwater flux at the bottom <br> of a snowpack measured at a <br> site in the High Arctic |

Table 2.1. Measured heat fluxes between the ground and snowpack.

Table 2.1 shows example ground-snow heat fluxes measured before and during the time of ablation at field sites situated in the Arctic to mid-continental United States. Using the convention of this thesis, heat fluxes are negative when there is transfer of heat toward the snow surface and positive when there is transfer of heat toward the ground.

During ablation, when meltwater is transported through the snowpack and reaches the ground, the higher temperature of the water transitions the ground heat flux toward positive values. This indicates that heat is being transferred to the ground from the snowpack. Heat is also transported into the ground as the soil thaws and meltwater infiltrates into the soil. Latent heat is released in the snowpack due to re-freezing of the meltwater in the snowpack or at the base of the snowpack, and refreezing of meltwater in the soil is the soil is permeable. These processes warm the snowpack and the ground (Marsh and Woo, 1984a, 1984b).

### 2.2.7. Heat and Vapour Transport in Dry Snow

Heat fluxes at the snow surface act along with ground heat fluxes to transport heat in dry snow. Heat transport within the snowpack occurs due to five processes: radiation (Brandt and Warren, 1993), conduction (Mellor, 1977), free convection, forced convection (Albert and McGilvary, 1992b; Albert, 1993) and vapour transport (Adams and Sato, 1993; Mellor, 1977).

The absorption of light causes heating in the near-surface layer of the snowpack when the rate of warming exceeds the rate of cooling (Colbeck, 1989a). This causes a temperature maximum to occur beneath the snow surface. Heat flow occurs toward the snow surface because the snow surface is cooler than the temperature maximum (Brandt and Warren, 1993). Heat is transported in and away from this near-surface layer (Town et al., 2008). The development of the near-surface layer commonly occurs in natural snowcovers (Koh and Jordan, 1995; Shea et al., 2012).

Conduction of heat takes place in the ice matrix frame of the snowpack and in the pore spaces of the frame. Heat transport by conduction occurs primarily in the frame of the snowpack because the ice has a higher thermal conductivity than air in the pore spaces and the snow particles are in contact (Singh, 1999). The rate of conduction increases with snow density and is dependent on the bond contact area and geometry of snow particles (Arons et al., 1998). This makes a temperature gradient less pronounced in a dense layer of snow within a snowpack (Colbeck, 1991). Bonds with larger surface area conduct greater amounts of heat than bonds with less surface area due to more efficient transfer of heat over a larger area (Adams and Sato, 1993). When the bonds between snow particles have smaller area, water vapour becomes a dominant aspect of heat transport (Mellor, 1977), making the thermal conductivity of snow more dependent on temperature (Adams and Sato, 1993). Conduction is a significant heat transfer process in cold and dry environments such as the Antarctic ice sheet where water vapour fluxes are negligible and air transport is limited to surface snow of higher permeability (Brandt and Warren, 1997, 1993; Town et al., 2008).

Free convection (Albert and McGilvary, 1992b) is heat transport due to thermally-driven movement of air in the snowpack. This process has been observed in sub-Arctic snowcovers, where convection plumes develop. Heating of the snowpack due to ground heat flux and solar radiation causes warm air to rise to the snow surface whereas cold air is transported into the snowpack. The development of a convection plume causes "warm spots" to develop in the
snowpack at locations where air is rising (Sturm and Johnson, 1991). When thermal diffusivity in the snowpack is greater than mass diffusivity from vapour fluxes, free convection of heat will be accelerated in the snowpack. When vapour fluxes have greater magnitude, free convection of heat will be suppressed (Powers et al., 1985). Small trees buried beneath the snow can also serve as conduits for heat transport, where warm air rises in a "chimney" surrounding the tree. Ground temperatures are colder in the area surrounding the tree (Sturm, 1992).

Forced convection of heat occurs when wind pushes air into the porous snowpack. This is referred to as "windpumping" and occurs due to variations in pressure caused by surface relief (i.e. hills and gullies) or turbulence-induced transfer over a rough snow surface (Colbeck, 1997; Colbeck, 1989b). Ice layers and changes in snow permeability disrupt the flow of air, causing most of the induced pressure flow to travel horizontally along layer boundaries (Colbeck, 1997). Dissipation of pressure gradients by the porous snow medium can act as a significant source of heat near the surface of the snowpack (Clarke and Waddington, 1991). Alternately, temperature gradients that are already present in the snowpack can be modified by windpumping when colder air is forced into the pore spaces. Higher air flow rates will disrupt temperature gradients at lower depths in the snowpack. High-magnitude temperature gradients will be less disrupted by windpumping than low-magnitude temperature gradients (Albert and McGilvary, 1992b). The rate of water vapor transport in snow will be significantly enhanced by windpumping, causing regions of sublimation and condensation to develop in the snowpack (Albert, 2002).

Vapour transport in snow occurs due to diffusion of water vapour (Mellor, 1977). A source of water vapour is near the basal layer of the snowpack due to the presence of a ground heat flux. In the absence of windpumping, vapour diffusion occurs from the bottom to the top of the snowpack in the direction of the temperature gradient from warmer to colder areas (Sturm and Benson, 1997). The temperature gradient induces a corresponding gradient in saturation vapour pressure (Mellor, 1977). Vapour fluxes diffuse in a "hand-over-hand" fashion within this gradient (Yosida, 1955) and can be modelled using a mass-balance approach (Colbeck, 1991). The diffusion process begins by condensation of water vapour on the bottom of a snow particle. Due to conservation of energy, sublimation concomitantly occurs on top of the snow particle. Water vapour is progressively transferred in this fashion throughout the snowpack (Sturm and Benson, 1997). The rate of diffusion is dependent on the size of the pores in the snowpack. Greater rates of diffusion are predicted for smaller pore sizes (Colbeck, 1993)

### 2.2.8. Dry Snow Metamorphism

Metamorphic processes create and modify the structure of the porous media frame. Snow crystals that are situated close together undergo sintering, where "necks" or bonds form between adjacent crystals in the snowpack due to vapor transport (Brown and Edens, 1991) caused by vapour pressure differences between surfaces (Colbeck, 1997). This results in the linking together of particles to form a frame (Gubler, 1978). As the size of the bonds increase, the bond growth rate is a self-limiting process (Johnson and Hopkins, 2005), but when the snow is placed under compression, there is no relationship between the compressive force applied and the length of the bonds (Edens and Brown, 1991). However, the rate of bond formation can be increased by application of an external pressure force to the snow (Mahajan and Brown, 1993).

Snow particles deposited at lower depth in the snowpack are subjected to pressure from overlying snow layers. The mechanical compression increases the density. As snow metamorphism proceeds over time, the number of bonds between adjacent particles increases, strengthening the snow structure, reducing porosity and increasing density (Anderson and Benson, 1963). Where snow ablation does not occur due to cold conditions, the increase in density will transition the snow to glacial firn and then ice (Kawashima et al., 1993).

Colbeck (1986) proposed a process-based classification of snowpack metamorphism that is physically accurate. The scheme does not classify snow particles based solely on morphological properties but also takes into consideration the physics describing changes in particles. This supersedes other classification schemes (Sommerfeld and LaChapelle, 1970) due to a better understanding of the processes that operate over time in the snowpack. In the following paragraphs and sections of this thesis, the various stages of this classification scheme are given in brackets.

Surface hoar (Colbeck IVA) develops on the surface of the snowpack when longwave radiative cooling establishes a temperature difference at the air-snow interface, causing mass transport of water vapour from the air to the snow. Growth of the surface hoar occurs in a parasitic fashion from a particle already on the snow surface. Rapid growth of the surface hoar in a direction away from the snow surface causes facetted striations to develop and an expansion of particle area. When particles are redistributed by the wind, mechanical disaggregation will break the particle up, reducing the surface area and causing the formation of a windcrust as a hardened layer of snow (Colbeck IVB). The smaller particles are subjected to rapid
metamorphism that causes the particles to become rounded and bonded together. The bonding process is referred to as "sintering." A sun-crust (Colbeck IVD) develops when radiation from the sun melts snow, causing the production of meltwater. The meltwater then refreezes, creating a hardened layer on the snow surface. When water from rain refreezes on the surface of the snowpack, a freezing rain crust is created (Colbeck IVe).

Snow particles can develop into low density, brittle forms (Colbeck II3C) with large grains (Pfeffer and Mrugala, 2002). When these types of crystals form beneath the snow surface due to an upward flux of heat and vapor, the snow layer is referred to as "depth hoar." The bonding of depth hoar occurs in the preferential direction of the temperature gradient, causing the depth-hoar to migrate upwards toward the surface of the snowpack (Armstrong, 1980). It is possible for extensive depth-hoar formation to occur anywhere in the snowpack due to local temperature gradients (Fukuzawa and Akitaya, 1993). The vapor fluxes move in a layer-to-layer fashion (Sturm and Benson, 1997). The presence of a strong temperature gradient within an area of a snowpack is indicative of the formation of a weak layer of depth hoar (Adams and Brown, 1982). Weak layers that develop in snow are often the genesis of slab avalanches (Arons et al., 1998; Marbouty, 1980; Pfeffer and Mrugala, 2002). However, when depth hoar snow particles are bonded together, the snowpack becomes more structurally cohesive and less prone to avalanche failure (Pfeffer and Mrugala, 2002). The strength of depth hoar is dependent on the initial snow density before temperature-gradient formation. Snow with low density ( $\sim 100 \mathrm{~kg} \mathrm{~m}^{-3}$ ) will form weakly-cohesive depth hoar, whereas higher-density snow ( $\sim 300 \mathrm{~kg} \mathrm{~m}^{-3}$ ) will form a hard-crusted layer of depth hoar (Perla and Ommanney, 1985). Due to intense temperature gradients in most seasonal snowcovers, a large volume of the snowpack can be classified as depth hoar by midwinter (Sturm, 1992).

### 2.2.9. Heat and Water Transport in Wet Snow

Warming of the snow surface will decrease the cold content of the snowpack and cause the production of meltwater. In the near-surface layer of the snowpack, where warming occurs due to extinction of radiation, the liquid water content and temperature of the snowpack will initially increase near the surface. The meltwater can either re-freeze when the snowpack becomes colder and the cold content increases, thereby forming an ice layer, or the meltwater can percolate through the snowpack under the influence of gravity (Das and Alley, 2005; Koh
and Jordan, 1995; Techel and Pielmeier, 2011). Another source of heat and water transport through snow is rain (Fortin et al., 2002) that is at a greater temperature ( $>0^{\circ} \mathrm{C}$ ) than the snow. The rain transfers sensible heat to the snowpack and will also transfer latent heat when the water re-freezes (Conway and Benedict, 1994; Marks et al., 2001).

The production of meltwater in a snowpack can be divided into three major temporal stages: the warming phase, the ripening phase, and the melting phase (Dingman, 2002). During the warming phase, the snowpack increases in temperature. The end of this phase occurs when the melting point of snow $\left(0^{\circ} \mathrm{C}\right)$ is reached and the cold content is numerically close to zero. In the ripening phase, the snowpack is isothermal and "ripe" but water transport does not occur until the output phase. At the output phase, the snowpack cannot hold any more water. Energy that is added to the snowpack during the output phase will drive snowmelt processes (Dingman, 2002; Marks et al., 2001).

Melting snowpacks can be spatially divided into three separate regions before meltwater reaches the ground surface. The uppermost region consists of an isothermal layer of wet snow. The lower boundary of the uppermost region is referred to as a "wetting front." Under the wetting front is a transition layer of mixed wet and dry snow. Following this transition layer is a layer of dry snow (Marsh and Woo, 1984a).

Net movement of the wetting front occurs due to the amount of meltwater available for transport and the microstructure of the snowpack. The meltwater produced in the region of wet snow must exceed the irreducible water saturation $S_{e}$ of the snow immediately ahead of the wetting front before movement occurs. Moreover, enough water must be frozen at the wetting front to release latent heat sufficient to raise the temperature to $0^{\circ} \mathrm{C}$ so that additional meltwater is produced (Colbeck, 1976a). This causes temperatures in the uppermost region to rise and heat to be transported away from the wetting front into the colder snow at greater depth in the snowpack (Conway and Benedict, 1994).

The re-freezing of meltwater at the wetting front increases the mass of snow particles but will not create an ice layer unless the wetting front is stationary and there is sufficient heat flux for water freezing (Marsh and Woo, 1984a). Ice layer growth can also occur in the absence of melting and freezing due to temperature differences caused by sensible and latent heat fluxes. Ice layers can be wetted by meltwater and will gradually disappear (Colbeck, 1991). The
formation of ice layers is a major contribution to the heat balance of the snowpack (Marsh and Woo, 1984a).

The wetting front arrival at the base of the snowpack results in the development of a layer of wet snow overlying a layer of wet and dry snow (Marsh and Woo, 1984a). Freezing of the water at the base of the snowpack creates a basal ice layer. The basal ice layer will only develop when the ground is frozen. Because the basal ice layer has a low permeability, additional meltwater will be impeded by the ice layer and frozen ground. This causes ponding of the meltwater (Woo and Heron, 1981). On a sloping land surface, movement of the meltwater will occur in a downhill direction without infiltration into the soil (Colbeck, 1974a).

On an ice-covered lake, water from a melting snowcover will interact with water rising by capillary action through cracks in the ice, forming a slush layer at the snow-ice interface. When the ice cover melts, water from snowmelt will enter the lake (Adams and Lasenby, 1982; Adams, 1981).

The spatial variability of water transport through snow ensures that liquid water content $\theta_{w}$ is highly variable in both time and space over distances spanning 50 cm (Techel and Pielmeier, 2011) and over areas of $7 \mathrm{~m}^{2}$ (Kattelmann, 1989). Water transport through snow follows three-dimensional flow paths, with more continuity between flow fingers in a snowpack layer than flow fingers in vertically lower layers (McGurk and Marsh, 1995; Williams et al., 2010). A larger number of meltwater drains have been found on south-facing slopes than northfacing slopes due to greater magnitude of shortwave radiation from the sun (Williams et al., 2000).

In the Colorado Rocky Mountains, Williams et al. (1999) took dielectric measurements of liquid water content at the snow surface. Using semivariogram analysis, they found at distances less than 6 m that liquid water contents were strongly correlated due to preferential flow of meltwater beneath the snow surface. Moldestad (2005) measured the liquid water content of snow on a cross-country ski track, finding higher liquid water contents in the top 5 cm of the snowpack due to friction heating by skiing. The liquid water content on the ski track decreased with ambient temperature.

Over the winter season, climate and geographic location causes distinctive differences in the liquid water content $\theta_{w}$ of snow due to variation in mass and energy balances (Jarosch and Denoth, 2004). At a study site in the Sierra Nevada, Kattelman (1990) measured the liquid water
content using a Denoth meter and averaged the values over snowpit depth. The liquid water content was found to be negligible when snow temperatures were less than $-2^{\circ} \mathrm{C}$ earlier in the winter season. However, the liquid water content increased to an average value of $3 \%$ to $5 \%$ for snowpits sampled in mid-May. Fierz and Fohn (1994) also found higher values of liquid water contents at later times in the winter season, with a diurnal increase in water content close to solar noon owing to higher magnitude of shortwave radiation from the sun.

Colbeck (1974) states that the irreducible water saturation $S_{i}$ of snow is approximately $7 \%\left(S_{i}=0.07\right)$. However, values of $S_{i}$ measured from a ripe snowpack have been reported between $2 \%$ to $4 \%$, which corresponds to an irreducible water content $\theta_{e}$ of $1 \%$ to $2 \%$ (Marsh, 1991).

Dielectric measurements of snow have shown that the liquid water content $\theta_{w}$ can range from $\sim 0 \%$ for dry snow to $\sim 10 \%$ for a wet snowpack (Denoth and Wilhelmy, 1988; Denoth, 1994; Denoth et al., 1984; Sihvola and Tiuri, 1986).

For dielectric measurements, Techel and Pielmeier (2011) proposed a wetness classification scheme. In the context of this classification scheme, a water content $\theta_{w}$ of $<1.3 \%$ by volume is dry or "barely moist" snow and $>1.3 \%$ by volume is wet snow.

Another alternate classification scheme is given by Techel and Pielmeier (2011) and referred to as the "Martinec" classification scheme after the author of an unpublished report. The scheme shows a water content of $<0.5 \%$ as corresponding to dry snow, $0.5 \%$ to $2 \%$ as corresponding to moist snow, $2 \%$ to $4 \%$ as corresponding to wet snow, $4 \%$ to $5 \%$ as corresponding to very wet snow, and $>5 \%$ as wet snow.

The classification scheme proposed by Fierz et al. (1999) defines dry snow as having a water content of $\sim 0 \%$, moist snow as having a water content of $0 \%$ to $3 \%$, wet snow as having a water content of $3 \%$ to $8 \%$, very wet snow has having a water content of $8 \%$ to $15 \%$, and "soaked" snow as having a water content of $>15 \%$.

Using the Colbeck classification scheme, the degree of water saturation $S_{w}$ in the snowpack is classified by two different stages: the pendular and funicular regimes. In the pendular regime of saturation ( $<7 \%$ to $14 \%$ of water saturation $S_{w}$ ), air has a continuous pathway through the pore space surrounding the snow particles (Colbeck, 1982a,b). The water is contained in a film around each particle (Denoth, 1982). The transition between the pendular
and funicular regimes of saturation occurs above $14 \%$ of water saturation $S_{w}$. This transition region was measured using dielectric techniques by Denoth (1980) as occurring from $11 \%$ to $15 \%$ (Colbeck, 1982a,b). The funicular regime of saturation is from $14 \%$ to $25 \%$ of water saturation $S_{w}$ (Denoth, 1980), where the water is continuous throughout the pore space of the snowpack and the air is only found as bubbles in localized areas of the liquid (Colbeck, 1982a,b).

### 2.2.10. Wet Snow Metamorphism

The addition of water to snow causes wet snow metamorphism. The processes of wet snow metamorphism are significantly different from dry snow metamorphism (Colbeck, 1997).

Grain clusters (Colbeck IIIA) develop when multiple snow particles coalesce in the presence of small amounts of liquid water. The grain clusters develop due to the minimization of surface free energy. The growth of snow particle grains in the presence of water is referred to as "grain-coarsening." Smaller snow particles tend to form small spherical grains. The smaller grains are then merged to form larger grains (Colbeck IIIA). Merging of the grains occurs at a constant rate until all of the particles are approximately the same size. Although some studies have found that the mean grain size increases rapidly after the start of metamorphism and that a decline in the growth rate occurs shortly thereafter, it is possible for grain growth rate to be approximately linear over time (Marsh, 1987). When the water is found in a continuous layer without air inclusions around the snow particle (Colbeck IIIC), grain clusters cease to exist and the snow particles are found as isolated grains (Denoth, 1982) that have a spherical shape and are cohesionless (Colbeck, 1986b; Colbeck, 1982a,b). When ambient environmental temperatures drop below freezing and freeze-melt cycles occur, clusters of snow crystals are produced (Colbeck IIIB). These clusters are attached together with large bonds (Colbeck, 1986a). Refreezing of melt water in the snowpack forms ice layers (Colbeck IVC).

### 2.2.11. Approximation of Snow Particles as Spheres

In lieu of measuring snow particle morphology using photogrammetry and visual inspection (Jamieson and Schweizer, 2000; Perla, 1985, 1982), snow particles comprising the snowpack are approximated as spheres in the UTAM theory presented in this thesis. However, snow particles have a wide variety of shapes and are facetted (LaChapelle, 1977; Libbrecht,
2005). Since the assumption that snow crystals are spheres may not always be valid, all snow crystals in a control volume over which the UTAM model is applied are assumed to have an effective radius that is approximated by a sphere.

Other investigators have assumed that snow particles are spheres. Partially to explain the cohesion of snow particles in a snowball, Michael Faraday (Faraday, 1859) conducted a series of experiments to pry apart spherical adhering ice particles (Hobbs, 1974; Kingery, 1960). Kuroiwa (1961) set up an experiment with two ice spheres to study sintering of snow particles during metamorphism. Colbeck and Parssinen (1978) proposed a model of sintering of wet snow using two adjacent spherical grains. This model was later elaborated on by Morris and Kelly (1990). Raymond and Tusima (1979) used an effective snow particle radius to describe the growth of snow particles in water-saturated snow. Brown (1980) proposed a constitutive model for snow with particles having effective radii. The model predicted sintering and the growth of bonds between adjacent snow particles. Similar models were also described by Edens and Brown (1991) and Mahajan and Brown (1993). The Watanabe (1993) and Dent (1993) models assumed that snow crystals in a snowpack are spheres packed in a "net-like" configuration within 3D space. Pomeroy and Schmidt (1993) assumed that snow particles were spheres to calculate sublimation rates of intercepted snow. Gay et al. (2002) used an effective radius when measuring snow crystal size with image analysis techniques.

The assumption of spherical snow particles can be eliminated by using Finite Element Methods (FEM) to model changes in the mechanical properties of snow (Dandekar and Brown, 1986), but this requires additional inputs that cannot be measured during application of the UTAM model proposed in this thesis. As noted by Bear (1972, pp. 445-446, cf. Figure 9.2.4) an "idealized model" of spherical grains is used to reduce the complexity of porous media so that it can be successfully modelled.

### 2.3. Acoustics

Acoustics is the science of sound. Sound waves are mechanical waves that propagate in matter. The sound wave has a phase velocity that is also referred to as a speed. The speed $c$ as a scalar quantity is the rate at which a peak or a trough of the wave with a frequency $f$ travels in a medium (Raichel, 2006). For porous media comprised of void spaces filled with a fluid, the
speed of a sound wave propagating in a medium is a function of frequency (Allard and Atalla, 2009).

The frequency $f$ is the number of peaks or troughs of a waveform that occurs in a certain amount of time. The angular frequency $\omega$ is defined as $\omega=2 \pi f$. The period $T=1 / f$ is the distance between successive peaks or troughs (Raichel, 2006).

Sound waves are often linear combinations of waveforms with different frequencies. The magnitude of each frequency can be expressed as a spectral distribution of waveforms, with some frequencies having greater power. A broadband sound source is comprised of waves with a larger number of frequencies than a narrowband source (Raichel, 2006).

Power $P$ is a measure of the acoustic energy carried by a wave per unit time. This is to be distinguished from the instantaneous root-mean-square pressure $p$ of an acoustic wave, which is a measure of the force per area of the sound wave at an instance of time at a position within the medium (Raichel, 2006).

The Sound Pressure Level (SPL) is a measure of the instantaneous pressure of an acoustic wave expressed in units of decibels (Raichel, 2006):

$$
\begin{equation*}
L_{p}=20 \log _{10}\left(\frac{p_{\text {rms }}}{p_{\text {ref }}}\right) \mathrm{dB} \tag{2.1}
\end{equation*}
$$

In Equation (2.1), the reference pressure is $p_{\text {ref }}=20 \mu \mathrm{~Pa}$. The SPL is reported in decibels since this is a logarithmic scale that allows for wide-ranging values to be more easily represented (Waite 2001).

Waves that travel in a medium by compressions and rarefactions are P-waves. Alternately, transverse waves that travel in an elastic medium are referred to as S -waves. Because a gas such as air is incapable of shear, S-waves can only travel in a solid (Gadallah and Fisher, 2005).

Audible sound waves are considered to have frequencies between 20 Hz and 20 kHz and can be heard by human beings with normal hearing, although this is dependent on age, location to a source, room acoustics and noise. Infrasound is sound waves with frequencies $<20 \mathrm{~Hz}$. Ultrasound is sound waves with frequencies > 20 kHz (Raichel, 2006).

The attenuation coefficient $a_{c}$ of a medium is a measure of the rate at which sound is attenuated. Attenuation occurs when sound waves are gradually reduced in amplitude passing through a medium (Meyer and Neumann, 1972).

The attenuation of sound in porous media is exponential over the distance of wave travel and is a function $a_{c}(f)$ or $a_{c}(\omega)$ of the sound wave frequency, with greater attenuation present at higher frequencies (Allard and Atalla, 2009). The attenuation of sound in air is negligible at audible frequencies of sound, with a reduction in wave amplitude only occurring due to spherical spreading (Rossing and Fletcher, 1995).

Attenuation is distinguished from spherical spreading, a reduction in amplitude that occurs when sound waves spatially move outward from a source. When sound waves are produced by a hemispherical source, the waves propagate over a $180^{\circ}$ sector of a sphere (Raichel, 2006).

Assuming that air is an ideal gas, the speed of sound in air is related to the air temperature $T_{a}$ (Kelvin) by (Raichel, 2006):

$$
\begin{equation*}
c_{0}=\left(\chi_{g} R_{g} T_{a}\right)^{1 / 2} \tag{2.2}
\end{equation*}
$$

In Equation (2.2), the dimensionless ratio of the specific heats for air is $\chi_{g}$ and the ideal gas constant for air is $R_{g}$.

### 2.4. Mechanical Properties

The mechanical properties of a medium are referred to as the "elastic constants" (Schreiber et al., 1974). The propagation of sound in snow is dependent on these constants (Johnson, 1982).

By definition, the Young's modulus $E$ of an isotropic material is the ratio of tensile stress $\sigma_{\text {stress }}$ to tensile strain $\sigma_{\text {strain }}$ and the shear modulus $G$ is the ratio of shear stress $\gamma_{\text {stress }}$ to shear strain $\gamma_{\text {strain }}$, as shown by Equations (2.3) and (2.4) below.

$$
\begin{equation*}
E=\frac{\sigma_{\text {stress }}}{\sigma_{\text {strain }}} \tag{2.3}
\end{equation*}
$$

$$
\begin{equation*}
G=\frac{\gamma_{\text {stress }}}{\gamma_{\text {strain }}} \tag{2.4}
\end{equation*}
$$

The tensile stress $\sigma_{\text {stress }}$ is the force exerted over an initial cross-sectional area, and the tensile strain $\sigma_{\text {strain }}$ is the fractional change in length of the medium resulting from the applied force. Qualitatively, the Young's modulus $E$ is a measure of how easily a material can be stretched or compressed. As $\sigma_{\text {strain }} \rightarrow \infty, E \rightarrow 0 \mathrm{~Pa}$. The material can be more easily stretched or compressed for a lower value of $E$ as compared to higher values of $E$.

The shear stress $\gamma_{\text {stress }}$ is the force exerted over a cross-sectional area of the material, and $\gamma_{\text {strain }}$ is the fractional lateral displacement of the medium resulting from the applied force. The shear modulus can be considered as a measure how easily a material can be deformed by a shearing force. In a similar fashion to the Young's modulus, as $\gamma_{\text {strain }} \rightarrow \infty, G \rightarrow 0 \mathrm{~Pa}$. For materials where $G \rightarrow 0 \mathrm{~Pa}$, a shearing force can more easily deform the material.

The bulk modulus $K$ of an isotropic material is calculated from the Young's modulus and the shear modulus as:

$$
\begin{equation*}
K_{b}=\frac{E G}{3(3 G-E)} \tag{2.5}
\end{equation*}
$$

### 2.5. Snow Acoustics Research

The propagation of sound in snow has been studied by investigators so that physics of sound waves can be related to snow material properties or acoustic emissions from snow can be related to avalanche processes (Sommerfeld, 1982). Using the same terminology as sonar (Lurton, 2002), devices that send sound waves into snow are classified in this thesis as active acoustic devices, whereas devices that only detect sound waves from snow are classified as passive acoustic devices. The classification is similar to the scheme given in Section 1.3 .9 of this thesis for radioactive snow measurement devices. The scope of the following review is limited to acoustic research performed using active devices since these are similar to the device
described in this thesis. Passive devices are described and reviewed by Sommerfeld (1982), O'Neel et al. (2007), and Lacroix et al. (2012).

Invasive snow acoustics experiments can be classified as sending sound waves through a snow sample (Section 2.5.1), or the sending of sound waves through a snowpack (Section 2.5.2). Non-invasive experiments involve the sending of sound waves above the snow surface (Section 2.5.3) in a similar fashion to the experimental apparatus reported in this thesis. This review excludes research conducted on the propagation of shock waves through snow with amplitudes high enough to cause permanent deformation of the snowpack (Albert, 1983; Brown and Hansen, 1987; Brown, 1979; Johnson, 1990).

### 2.5.1. Invasive Snow Sample Extraction Experiments

Nakaya (1959) extracted snow samples from the Greenland ice sheet and measured the Young's modulus and attenuation of sound in snow using flexural vibrations. Lee (1961) showed that the speed of sound in snow samples increased with the density. Smith (1965) sent sound bursts with microsecond duration into a cylindrical snow sample, using a time-of-flight measurement between two transducers to determine the speed of the pulse. The speed of sound in the snow sample was then related to the density and elastic constants of the snow using empirical and semi-empirical relationships. Smith (1969) conducted a similar experiment, but used a continuous-wave source signal and the theory proposed by Lee (1963) to measure the attenuation of sound in snow and elastic constants of snow by finding the resonant frequency of a snow sample. The range of frequencies used in the experiment was less than 5 kHz .

Bogorodskii et al. (1974) measured the speed of ultrasound pulses in Arctic snow samples placed between two transducers (Bogorodskii and Dobrotin, 1963), finding that the speed of sound was independent of frequency but changed with snow density in the 25 kHz to 800 kHz frequency range. This was a repeat of an experiment initially conducted by Kuz'min (1957) at lower latitudes. Bogorodskii et al. (1974) used the Zwikker and Kosten (1949) rigidframe theory of sound propagation in porous media to qualitatively explain these results, indicating that coupling of the motion of the fluid and the frame was important at higher frequencies. This finding can be interpreted using the high frequency approximation of Johnson and Plona (1980) and Johnson et al. (1982), and the Biot theory of sound propagation in porous
media (Biot, 1956a). These theories predict that the speed of sound is independent of frequency above a high-frequency threshold.

Impedance tubes have been the most common apparatus used to measure the impedance of sound in snow samples. The impedance tube determines the characteristic acoustic impedance of a material, which is a measure of flow resistance to air pressure waves. This measurement can then be related to the speed and attenuation of sound in snow. An impedance tube measurement is made by placing the material inside of the tube. A loudspeaker is placed at one end of the tube, whereas the other end of the tube is closed.

Two methods are used to measure the acoustic impedance of a material placed in the impedance tube. (1) The standing wave method uses the loudspeaker to produce a sound wave with a single frequency. This sets up a standing wave in the tube. The standing wave is spatially sampled using a probe microphone that is translated down the length of the tube. To obtain the acoustic impedance at a different frequency, the frequency of the source must be changed. Alternately, the length of the tube or the sound wave frequency can be changed until acoustic resonance occurs. (2) The two-microphone method uses two microphones placed on the walls of the impedance tube. A broadband sound wave is sent from the loudspeaker. Signal processing and spatial differences between the microphones are used to obtain the impedance of the material as a function of frequency.

Oura (1952a, b) used the standing wave method to measure snow impedance at frequencies from 500 Hz to 6 kHz . The speed of sound in snow was found to be dependent on the air temperature inside of the tube, with a decrease in the speed of sound in snow apparent with a decrease in temperature. Since the speed of sound in ice will increase at lower temperatures, Oura reasoned that most of the sound wave propagated in the pore spaces of the snowpack medium. Oura also found that the speed of sound in snow was less than the speed of sound in air. This finding is in agreement with Tillotson (1966), who made impedance tube measurements on a snow sample with density $185 \mathrm{~kg} \mathrm{~m}^{-3}$, finding that the speed of sound in the sample was $60 \%$ of the speed of sound in air.

Buser (1986) and Buser and Good (1986) also used a standing wave method with source frequencies ranging from 100 Hz to 4 kHz . They quantitatively applied the Zwikker and Kosten (1949) rigid-frame theory, showing that their experimental results agreed with the theory within a $10 \%$ error. Attenborough and Buser (1988) later used this data to apply Attenborough's rigid-
frame model (Attenborough, 1987, 1983) based on the Zwikker and Kosten (1949) rigid-frame theory. In agreement with Oura (1952a, b), these investigators found that sound pressure waves were mostly propagated in the pore spaces of the snow.

Marco et al. (1996) and Marco et al. (1998) used the two-microphone method to measure snow density using a broadband source with frequencies between 100 Hz and 4 kHz . They recognized that snow density may control sound wave propagation in the pore spaces of snow or the ice matrix of the frame. Wave propagation in the ice matrix was postulated to become more important for snow with densities $>500 \mathrm{~kg} \mathrm{~m}^{-3}$. The results obtained by these investigators showed a deviation from rigid-frame theory for snow with densities $>500 \mathrm{~kg} \mathrm{~m}^{-3}$ and frequencies $<2.5 \mathrm{kHz}$.

### 2.5.2. Invasive Snowpack Experiments

Oura (1952) buried two microphones within a snowpack. The microphones were separated by an offset distance. An explosive sound source provided an impulse that allowed Oura to determine that the speed of sound in snow observed during the experiment ranged between $190 \mathrm{~m} \mathrm{~s}^{-1}$ and $240 \mathrm{~m} \mathrm{~s}^{-1}$. Oura also noted that the explosion created an air wave in the air medium above the snowpack and passed through an ice layer in the snowpack. The speed of sound in the ice layer was from $700 \mathrm{~m} \mathrm{~s}^{-1}$ to $1000 \mathrm{~m} \mathrm{~s}^{-1}$.

Ishida (1965) created a reverberant "room" out of snow by digging into the snowpack. A loudspeaker was installed in one of the walls of this room and a snow sample was placed at the opening of the room. A white noise generator was attached to the loudspeaker. The generator was used to produce broadband white noise with a frequency distribution between 100 Hz and 8 kHz . This allowed for the transmission loss of the snow sample to be determined using the reverberant room method (Mariner, 1961). Transmission loss is the reduction in amplitude of the sound wave passing through the snow sample. The transmission loss increases with an increase in the distance travelled by the sound wave. Cummings and Holliday (1983) performed a similar experiment, where tones with frequencies ranging from 105 Hz to 10.510 KHz were used to determine attenuation of sound through a model seal layer created out of snow on the ice near Prudhoe Bay, Alaska.

Lang (1976) created a snowpit and placed a loudspeaker facing the snowpit wall. Two microphones pushed through the side of the snowpit measured sinusoidal tones sent from the loudspeaker at frequencies between 100 Hz to 30 kHz . Lang found that sound with frequencies less than 3 kHz could propagate in snow for at least 3 m , with greater attenuation of sound at higher frequencies. Sound waves with frequencies from 100 Hz to 200 Hz showed the least attenuation.

Iwase et al. (2001) suspended a loudspeaker in the air above the snow surface and buried microphones in the snowpack at depths of $30 \mathrm{~cm}, 40 \mathrm{~cm}, 55 \mathrm{~cm}$ and 78 cm . A broadband sound wave was sent from the loudspeaker. Sound waves at frequencies between 100 Hz and 10 KHz were able to penetrate snow to be detected at these depths.

Moore et al. (1991) placed a loudspeaker on the snow surface and pushed a microphone mounted on a probe beneath the snow surface. Although the method was invasive, Moore et al. (1991) were able to determine snow porosity by application of the Attenborough model.

### 2.5.3. Non-invasive Experiments

Reynolds (1873) recognized that audible sound is attenuated by snow. Since the time of this early paper, researchers have measured the attenuation of sound propagating horizontally over snow between a loudspeaker and microphone pair. Kaye and Evans (1940) measured the attenuation of sound in snow within a reverberation chamber. Watson (1948) sent sound waves with frequencies ranging from 100 Hz to 2 kHz above snow, finding that the wave attenuation was greater for higher frequencies. This attenuation was also apparent in the experiments reported by Takada et al. (1954). Tillotson (1966) found that a source with SPL 100 dB to 105 dB could be detected by a microphone placed at horizontal distances greater than 10 m from the source. Tillotson (1966) used the theory proposed by Rudnick (1947) to calculate the characteristic impedance of a ground surface with a snow layer.

Hess et al. (1990) used curve-fitting to fit the semi-empirical Delany and Bazley (1970) model as well as the Attenborough model to attenuation data of sound waves propagating horizontally over snow. The curve-fitting procedure was not unique for all experimental data. Iwase et al. (2001) showed that attenuation of sound waves passing over snow was greater for fresh snow than for compressed snow.

Impulsive acoustic sources have been used to send sound waves into snow. These sound sources were provided by: military vehicles (Albert, 1987), explosives (Albert and Hole, 2001), as well as handguns and sledgehammers (Albert and Orcutt, 1990, 1989; Albert, 2003, 2001, 1998; Albert, 1993a; Boeckx et al., 2004; Moran and Albert, 1996). Experiments performed with these sound sources showed that acoustic waves originating in the air medium can be transferred to the snowpack by acoustic-to-seismic coupling. The air-coupled waves were detected by geophones placed on the snow surface and microphones placed near the surface of the snowpack. Snow depth and permeability were determined using a rigid-frame model (Albert, 2001, 1998). Seismic surveys with geophones and sledgehammers have been used to determine snow porosity using geophysical methods (Booth et al., 2013). Using impulsive sources, Albert (2003) showed that waves originating in the air medium can also propagate along the snow surface.

Some investigators have sent sound waves directly into snow using loudspeaker and single microphone pairs situated parallel to the snow surface. The loudspeaker was situated at an offset distance to the microphone. Lee and Rogers (1985) sent impulsive sound waves into a 2 cm snow layer that had accumulated on a roof using a loudspeaker and microphone pair. The source signal was impulsive and the rigid-frame Zarek porous media model (Zarek, 1978) was used to create a synthetic waveform that was compared to the measured waveform. Kinar and Pomeroy (2007; 2008a, b; 2009) used non-impulsive source signals sent from loudspeaker and microphone pairs to determine SWE.

### 2.5.4. Biot-Stoll Model

In a series of papers, Biot (1956a; b; c, 1962) established a theory of sound propagation through porous media. The theory considers the porous medium to be comprised of a fluid that completely saturates a sedimentary frame with pore spaces. The theory was later modified by Stoll (1989) to make the mathematics physically reasonable. This Biot-Stoll model was experimentally tested by Holland and Brunson (1988). Biot theory was first applied to snow by Johnson (1982) and Albert (1993), assuming that the snow sedimentary frame is comprised of snow particles and the pore spaces are filled with air.

Despite the presence of small amounts of liquid water as an irreducible water saturation (Colbeck, 1972), the Biot theory provides an approximate description of dry snow, with the fluid
as air and the frame comprised of ice (Buser and Good, 1986). Petrovic (2003) further classifies snow as a cellular solid form of ice. Although the densities of individual snow particles can differ from the density of ice (Ishizaka, 1993; Pruppacher and Klett, 2010), in the absence of any additional information the assumption that the snow particles are comprised of ice is used to apply the Biot theory to snow in this thesis. Scaling coefficients are later introduced to model differences between ice and snow particles.

The classical Biot theory predicts that three different waves propagate through a porous medium. The first wave is a pressure wave (P-wave) that travels through the porous frame. This is referred to as the "fast wave." The second wave is a P-wave that propagates through the pore spaces of the medium. This is referred to as the "slow wave." The speed of the second wave is often slower relative to the fast wave. The third type of wave is a shear wave (S-wave) that propagates only through the frame. The speed of sound changes with the frequency of the wave. The attenuation of the sound wave is dependent on the pore space geometry, grain size and permeability. Due to viscous damping forces, porous media comprised of coarse granular materials can have higher rates of sound attenuation than fine granular materials (Stoll, 1989).

Investigators have directly applied the Biot theory to model acoustic properties of snow (Albert, 1993b; Johnson, 1985). Since the frame is rigid an assumption can be made that it does not move during propagation of the sound wave. Simplified theories based on this assumption have been tested (Albert, 2001; Attenborough and Buser, 1988; Marco et al., 1998, 1996), but may not be applicable for snow due to a deviation from rigid-frame theory for densities $>500 \mathrm{~kg} \mathrm{~m}^{-3}$ and frequencies $<2.5 \mathrm{kHz}$ when small movements of the frame may occur (Marco et al., 1998, 1996).

Because of this consideration, the full Biot theory is being applied in this thesis to measure snowpack properties of interest to hydrologists. However, the full Biot theory does not take into consideration the presence of a mixture of air and water in the pore spaces of snow. This necessitates the development of the Unified Thermoacoustic Model (UTAM) presented in this thesis (Section 3.2).

The High Frequency Approximation (HFA) model presented in a later section of this thesis (Section 3.3) uses the rigid-frame approximation for sound with frequencies $>2.5 \mathrm{kHz}$. The HFA model is then coupled with the UTAM (Section 3.4.4). This reconciles the full Biot theory with the rigid-frame models.

Sound pressure waves that travel from air into snow are conveyed in the pore spaces of the medium (Boeckx et al., 2004). The acoustic-to-seismic coupling of air waves into snow (Albert and Orcutt, 1989; Albert, 2001) is important in the context of UTAM theory since this physical process is being used to determine the snowpack properties. Because air as a gas is incapable of shear wave propagation, the $S$-wave that propagates in the frame will not be detected in the air medium. Moreover, the P-wave that travels through the frame will be of relatively low amplitude due to the frame stiffness. This implies that sound propagation into the snowpack and reflections of sound waves travelling from the snowpack back into the air medium will be conveyed by the second P-wave (Boeckx et al., 2004).

The pore spaces of snow are mostly interconnected (Buser, 1986), indicating that propagation of the second P-wave as described by the Biot model will occur. As noted by Stoll (1989) and Albert (1993), the second P-wave propagation mode cannot be neglected in media where the pore space fluid is air.

### 2.6. Mixture Theory Models

The UTAM model presented in this thesis (Section 3.2) is a modified version of the BiotStoll model. To modify the Biot-Stoll model for snow, an approach is taken using "effective" parameters. The effective parameter approach replaces model parameters by substitution of mathematical expressions. These expressions represent changes in the parameters that model sound wave propagation in multiphase media. Similar ideas have been used in other models of wave propagation in unsaturated media. These are mixture theory models (Corapcioglu and Tuncay, 1996).

For example, modelling the propagation of water waves over a porous bed using Biot theory, Yamamoto et al. (1978) define an effective volume compressibility coefficient that includes a mixture of air and water in the bed. Colbeck and Anderson (1982) define an effective porosity to take into consideration the irreducible water saturation of snow. Corapcioglu and Tuncay (1996) mention that closed pores in a medium can be modelled by an effective porosity. Carcione et al. (2004), in the derivation of a model for partially-saturated porous media, introduce an effective permeability that takes into consideration the solid, liquid and gas phases of the medium.

### 2.7. Sonar and Seismology

Sonar is an acronym for "SOund Navigation and Ranging" (Chevalier, 2002). The principles of sonar were used to design ultrasonic snow depth sensors (Section 1.3.12). Sonar devices are commonly deployed in ocean environments and are used to obtain images of bathymetry as distance to the bottom of an ocean, lake or river. Models of the speed of sound in saline water are used for the calculation of depth. Sonar has been used to detect underwater objects including submarines, the hull of ships, mines and fish. Sonar can obtain estimates of underwater sediment transport (Lurton, 2002). The physical properties of sediment have been determined using empirical relationships (LeBlanc et al., 1991). The Biot-Stoll theory has been applied to model sound wave propagation through sediment at the bottom of an ocean (Stoll, 1989).

Reflection of sound will only occur when there is a change in the acoustic impedance of a medium (Lurton, 2002). Changes in the acoustic impedance of sediment will cause these reflections to occur (Ewing et al., 2013).

Sonar devices can be active or passive, either sending sound waves that are bounced off of objects or receiving sound waves from propellers or underwater animals (Waite, 2001). Although sonar devices are commonly designed with a single piezoelectric transducer that sends and receives sound waves at a single frequency, bistatic sonar devices have also been created with a separate source and receiver (Cox, 1989) or many multiple sources and receivers in an array (Wilson, 1989). The device described in this thesis is similar to bistatic sonar.

Seismology is the use of sound waves to obtain images of layered rocks (Claerbout, 1992). A seismic survey uses a number of transducers (geophones) (Burger et al., 2006). Seismology is often performed on the land surface and has been used to measure snow (Section 2.5.3). However, seismic surveys can also be performed in water using an airgun (implosive) sound source and a number of transducers situated at an offset distance from the source. The offset distances between transducers are on the order of meters (Claerbout, 1992).

### 2.8. Acoustic Scattering and Mode Conversion

When sound from acoustic sources such as sonar contacts a rough surface such as a seabed with facets, acoustic scattering occurs. Volume backscattering also occurs within the sediment of the seabed. Scattering is a diffusion effect where sound is reflected in a number of
directions from an acoustic reflector, reducing the amplitude of the detected sound wave. The scattering is most prevalent for a surface with larger-amplitude facets and vegetation (Lurton, 2002). Attenuation and scattering will occur within porous media when the medium is incompletely saturated (Dutta and Odé, 1979). Scattering of sound waves has been observed in the atmosphere due to atmospheric turbulence (Wilson et al., 1999) and is also present in medical ultrasound (Insana et al., 1992). Scattering of sound waves will occur in suspended sediment in the ocean (Richards et al., 1997).

As shown by Kinar and Pomeroy (2008a,b), acoustic scattering of sound waves from snow can be modelled using the V-cavity paradigm (Torrance and Sparrow, 1967) and the Rayleigh parameter from wave-scattering theory (Brekhovskikh and Lysanov, 2003), but an estimate of surface slope angles is required. Acoustic scattering by snow occurs due to uneven snow surface topography and buried vegetation that causes diffusion of sound waves. The refraction of sound by tree wells will also occur and acts similar to acoustic scattering when sound waves travel directly though the snowpack into the tree well space. The effect of acoustic scattering and refraction causes higher errors to occur between acoustic and measured (gravimetric) snowpack properties (Kinar and Pomeroy, 2009, 2008a, 2008b).

Mode conversion occurs when a sound wave is incident at an angle on an interface between materials with different acoustic impedances and there is a resulting conversion from P waves into S-waves. This causes a change in the energy conveyed by the P -waves and is a potential source of error in acoustic measurements (Furukawa and Date, 1995). Mode conversion occurs in unsaturated porous media when sound waves interact with a two-phase fluid (Chen et al., 2012).

### 2.9. Signal Processing

Signal processing is mathematical manipulation of sampled data. Data is sampled in the time domain and stored as a sequence of numbers. Let $s[t] \in \mathbb{R}$ denote a discrete time-domain signal as a function of time $t \in \mathbb{Z}$. In a computer program, the values of $s[t]$ are normally stored in an array. Digital signals are represented by square brackets [ ]. When square brackets are used, $t \in \mathbb{Z}$ is a counting number that is used to identify a discrete element $(t=0,1,2,3, \ldots)$ of the
signal. This is to be distinguished from a continuous function $s(t) \in \mathbb{R}$ where $t \in \mathbb{R}$ (Lyons, 2004).

The convolution of two time-domain functions $s[t]$ and $g[t]$ is an operation that results from translating and adding the two functions. Continuous and discrete convolutions are represented by the following (Smith, 2002):

$$
\begin{gather*}
s(t) * g(t)=\int_{-\infty}^{\infty} s(t-u) g(u) d \tau \\
s[t] * g[t]=\operatorname{IFFT}(\operatorname{FFT}(s[t]) \odot \operatorname{FFT}(g[t])) \tag{2.6}
\end{gather*}
$$

FFT( ) represents the discrete Fourier transform operator (numerically implemented as the Fast Fourier Transform), IFFT( ) represents the inverse discrete Fourier transform (numerically implemented as the Inverse Fast Fourier Transform), and following Hansen (2002), the $\odot$ symbol represents Hadamard (point-by-point) multiplication between the complex signals in the frequency domain. To prevent circular convolution from occurring, $s[t]$ and $g[t]$ must be padded with zeros so that the length of both signals are $L=M+N-1$, where $M$ is the number of elements in $s[t]$ and $N$ is the number of elements in $g[t]$.

The convolution operation can be used to model the response of a system. For example, if $s[t]$ is an acoustic wave passing through a medium, then $g[t]$ is the effect of the medium on the wave (Claerbout, 1985). Using systems theory $g[t]$ can be represented as a black-box system, $s[t]$ as the input, and the convolution $s[t] * g[t]$ as the output (Figure 2.2).


Figure 2.2. Diagram showing the effect of a system on an input signal $s[t]$.

If $g[t]$ is a filter kernel, then $s[t]$ can be filtered by convolution. A filter kernel is a signal that is constructed to modify the frequency response of $s[t]$ by convolution. Low-pass, band-pass and high-pass filters can be used to eliminate unwanted noise or signals, but many other types of filters exist (Claerbout, 1985). A Finite Impulse Response (FIR) filter can be applied by direct multiplication in the frequency domain (Equation (2.6)), whereas an Infinite Impulse Response (IIR) filter must be applied in the time domain. Although FIR filters are normally stable, an IIR filter can often exhibit numerical instability due to an inherent feedback effect (Smith, 2002).

Deconvolution is the operation that removes the filter kernel $g[t]$ from $s[t] * g[t]$ to get $s[t]$ given an estimate of $g[t]$ (Jansson, 1996). Blind deconvolution is the operation where an estimate of $g[t]$ is not available and $s[t]$ is estimated (Campisi and Egiazarian, 2007; Kundur and Hatzinakos, 1996).

### 2.10. Maximum Length Sequences

Maximum Length Sequences are used as source signals for acoustic observations involving signal processing. Applications of MLS signals include: hearing aid testing (Schneider and Jamieson, 1993); determining the response of loudspeakers (Atkinson, 1997); measuring the acoustics of architectural spaces (Bradley, 1996; Chu, 1984; Vanderkooy, 1994); and exciting control systems for determining system response (Pintelon and Schoukens, 2012). MLS measurements have also been used to determine the electrical impedance of a biological cell (Gawad et al., 2007).

Other investigators have used the MLS for sonar applications. Farina (1998) used MLS measurements to determine the profile of an underwater object placed in a pool. Farina performed the MLS measurements using a waterproof loudspeaker and microphone set up in a bistatic sonar arrangement. Morgan and Watts (2003) placed a loudspeaker and microphone on a trailer at the back of a vehicle and used MLS sequences to measure the absorption coefficient of porous road surfaces. Xiang and Chu (2004) set up a bistatic sonar experiment to measure the backscattered waves from a copper sphere in a water tank. An ultrasonic transmitter produced the MLS at a sampling rate of 200 kHz , and the reflection from the sphere was detected by a receiver set up at an offset distance to the transmitter.

### 2.11. Imaging of Snow and Snowpits

In an extensive study conducted on a 20 m by 20 m plot on the Pika Glacier, Alaska, Harper and Bradford (2003) compared 600 point measurements of snow density and stratigraphy collected in 19 snowpits to images of snow obtained from ground penetrating radar transects. Density measurements were made using gravimetric and dielectric techniques. They concluded that images of snowpit stratigraphy recorded by human observers did not always coincide with the radar profile images and dielectric profiles. The radar images and dielectric profiles showed more fine scale variability than the snowpit observations due to changes in the dielectric constant of snow over the depth of the snowpack.

Marshall et al. (2007) compared snow micro-penetrometer data collected in snowpits to FMCW radar data. The penetrometer recorded changes in snow hardness over the depth of snow. A probability density function (PDF) was applied to the FMCW radar data to show the positions of major reflections in the radar image.

### 2.12. Comparison of Observations to Models

The Root Mean Squared Difference (RMSD) is a comparison between model outputs and observed data reported in the same physical units. The higher the RMSD, the higher the overall differences between the datasets. RMSD is defined as (Patil et al., 2010):

$$
\begin{equation*}
\mathrm{RMSD}=\sqrt{\frac{1}{N} \sum_{i=1}^{N}\left(x_{i}-\hat{x}_{i}\right)^{2}} \tag{2.7}
\end{equation*}
$$

In the above Equation (2.7), the number of observations is $N$, the index is $i \in \mathbb{Z}$, the modelled value is $x_{i}$, and the measured value is $\hat{x}_{i}$.

The Mean Bias (MB) is a measure of the under-prediction or over-prediction of a model reported in the same units of measurement as the model and data (IEC and CIE, 1987). If the MB is positive, then the model over-predicts relative to the observed data. If the MB is negative, then the model under-predicts relative to the observed data. The MB is calculated as:

$$
\begin{equation*}
\mathrm{MB}=\frac{1}{N} \sum_{i=1}^{N}\left(x_{i}-\hat{x}_{i}\right) \tag{2.8}
\end{equation*}
$$

## CHAPTER 3 THEORY

### 3.1. Introduction

The following sections of this thesis develop and apply the Unified Thermoacoustic Model for Snow (UTAM). Starting with the Biot-Stoll theory of sound wave propagation in porous media, solutions of the constituent equations were modified to take into consideration the presence of liquid water in the pore spaces of the snowpack and the mechanical and thermal properties of the snow porous medium. Consolidation theory is proposed in this thesis to predict when the snowpack will cease to act as a solid and become an equivalent fluid. A High Frequency Approximation (HFA) model was also proposed and coupled with the UTAM. To apply the UTAM and to measure snowpack properties by the use of acoustics, a signal processing procedure was developed. The vanishing constant filter (VCF) was proposed as a method of blind deconvolution to remove attenuation from the signal. Speed analysis algorithms obtained the speed of the sound wave in the snowpack. A spectrogram bandpass filter bank analysis (SBFA) was proposed to couple the signal processing with the UTAM.

### 3.2. UTAM Model

### 3.2.1. Biot-Stoll Constitutive Equations

Stoll modified Biot's theory so that it could be physically realizable (Stoll and Bryan, 1970; Stoll and Kan, 1981; Stoll, 2002, 1985, 1980, 1979). This is the Biot-Stoll theory. For a control volume of the porous media used in the UTAM model, coupled equations governing the motion of the frame (Equation (3.1)) and the fluid (Equation (3.2)) are given by (Stoll, 1985):

$$
\begin{gather*}
\nabla^{2}(H e-C \xi)=\frac{\partial^{2}}{\partial t^{2}}\left(\rho e-\rho_{f} \xi\right)  \tag{3.1}\\
\nabla^{2}(C e-M \xi)=\frac{\partial^{2}}{\partial t^{2}}\left(\rho_{f} e-m_{f} \xi\right)-m_{f} \frac{\partial \xi}{\partial t}  \tag{3.2}\\
H=\frac{\left(K_{r}-K_{b}\right)^{2}}{D-K_{b}}+K_{b}+\frac{4 \mu}{3} \tag{3.3}
\end{gather*}
$$

$$
\begin{gather*}
M=\frac{K_{r}^{2}}{D-K_{b}}  \tag{3.4}\\
C=\frac{K_{r}\left(K_{r}-K_{b}\right)}{D-K_{b}}  \tag{3.5}\\
D=K_{r}\left(1+\phi\left(\frac{K_{r}}{K_{f}}-1\right)\right)  \tag{3.6}\\
m_{f}=\frac{\alpha \rho_{f}}{\phi}  \tag{3.7}\\
\xi=\phi \operatorname{div}(\mathbf{u}-\mathbf{U}) \tag{3.8}
\end{gather*}
$$

In the above Equations (3.1) through (3.8), $\xi$ is an effective fluid content increment that occurs when the sound wave propagates through the porous medium; $\mathbf{u}$ is a vector with small magnitude that describes the motion of the frame; $\mathbf{U}$ is a vector that describes the movement of the fluid; $\operatorname{div}()$ is the divergence; $\nabla^{2}$ is the Laplacian; $t$ is a time variable; $\rho$ is the density of the control volume; $\rho_{f}$ is the density of the fluid saturating the pore spaces of the medium, $e=\operatorname{div}(\mathbf{u})$; the inverse fluid mobility (also given by Equation (3.7)) is $m_{f}=\eta / k_{p}$, where $\eta$ is the dynamic viscosity of the fluid saturating the pore spaces of the medium and $k$ is the permeability; $K_{r}$ is the bulk modulus of the sediment (snow particles) comprising the frame; $K_{b}$ is the bulk modulus of the frame; $\mu$ is the shear modulus; $\phi$ is the porosity; and $\alpha$ is the tortuosity.

For application of the UTAM, the "control volume" is an arbitrary volume of the medium over which the constituent equations are applied (Wylen et al., 1993). All of the parameters in the UTAM model are assumed to be average values that are constant within the control volume. The control volume is thereby assumed to be isotropic.

Equations (3.1) and (3.2) have solutions given by the matrix determinant (Stoll, 1989):

$$
\left|\begin{array}{cc}
H l^{2}-\rho \omega^{2} & \rho_{f} \omega^{2}-C l^{2}  \tag{3.9}\\
C l^{2}-\rho_{f} \omega^{2} & m_{f} \omega^{2}-M l^{2}-i \omega F(\kappa) m_{f}
\end{array}\right|=0
$$

$$
l_{k}=c_{s, k}+a_{s, k} i
$$

Grouping terms and re-writing Equation (3.9) as a polynomial, it follows that:

$$
\begin{align*}
& \left(H l^{2}-\rho \omega^{2}\right)\left(m_{f} \omega^{2}-M l^{2}-i \omega F(\kappa) m_{f}\right)-\left(C l^{2}-\rho_{f} \omega^{2}\right)\left(\rho_{f} \omega^{2}-C l^{2}\right)=0 \\
& \quad\left(C^{2}-H M\right) l^{4}+\left(\rho \omega^{2} M+H\left(m_{f} \omega^{2}-i \omega F(\kappa) m_{f}\right)-2 \rho_{f} \omega^{2} C\right) l^{2}  \tag{3.10}\\
& \quad-\rho \omega^{2}\left(m_{f} \omega^{2}-i \omega F(\kappa) m_{f}\right)+\rho_{f}^{2} \omega^{4}=0
\end{align*}
$$

In Equation (3.10), $\omega=2 \pi f$ is the angular frequency of the sound wave, $f$ is the frequency, $F(\kappa)$ is a correction factor that models the frequency-dependent fluid flow resistance through the pore spaces of the medium, $i=\sqrt{-1}$, and $l_{k}$ is one of the complex roots $k$ of Equation (3.9), with $1 \leq k \leq 4, k \in \mathbb{N} \backslash\{0\}$. The speed of sound in snow is $c_{s, k}$ and the attenuation is $a_{s, k}$ for one of the roots $\left\{l_{1}, l_{2}, l_{3}, l_{4}\right\}$ at a frequency $f$. In addition, $l_{1}=-l_{2}$ and $l_{3}=-l_{4}$, with the order of the roots being unimportant. This implies that the absolute values of two of the four roots are the same, giving two complex numbers corresponding to the speed and attenuation of the slow and fast P -waves travelling in the pore spaces of snow. For purposes of nomenclature, the speed of the second P -wave is denoted as $c_{s}$, whereas the attenuation of the second P -wave is $a_{s}$.

The correction factor $F(\kappa)$ is a complex number given by (Stoll, 1989):

$$
\begin{gather*}
F(\kappa)=\frac{1}{4} \frac{\kappa \Lambda(\kappa)}{1-2 \Lambda(\kappa) / i \kappa}  \tag{3.11}\\
\Lambda(\kappa)=\frac{\frac{\partial \operatorname{Ber}(\kappa)}{\partial \kappa}+i \frac{\partial \operatorname{Bei}(\kappa)}{\partial \kappa}}{\operatorname{Ber}(\kappa)+\operatorname{Bei}(\kappa)}  \tag{3.12}\\
\kappa=\alpha\left(\frac{\omega \rho_{f}}{\eta}\right)^{1 / 2} \tag{3.13}
\end{gather*}
$$

In the above Equations (3.11) to (3.13), $\operatorname{Ber}(\kappa)$ is the real part of the Kelvin function, $\partial \operatorname{Ber}(\kappa) / \partial \kappa$ is the derivative of the real part of the Kelvin function, $\operatorname{Bei}(\kappa)$ is the imaginary part of the Kelvin function, $\partial \operatorname{Bei}(\kappa) / \partial \kappa$ is the corresponding derivative, $\eta$ is the viscosity of the pore
fluid, and all other parameters are as previously defined. In addition to the Biot correction factor above, Albert (1993) has tested other correction factors proposed by Attenborough (1983) and Attenborough (1987), finding that all of these expressions give similar results. Due to this finding and for simplicity, only the Biot-Stoll correction factor given by Equation (3.11) above is used in the UTAM.

The Kelvin functions are defined as infinite series (Abramowitz and Stegun, 1964) that are only calculated for a finite number of terms required to approximate each function:

$$
\begin{align*}
& \operatorname{Ber}_{v}(x)=\left(\frac{1}{2} x\right)^{v} \sum_{k=0}^{N} \frac{\cos \left(\pi\left(\frac{3}{4} v+\frac{1}{2} k\right)\right)}{k!\Gamma(v+k+1)}\left(\frac{1}{2} x^{2}\right)^{k}  \tag{3.14}\\
& \operatorname{Bei}_{v}(x)=\left(\frac{1}{2} x\right)^{v} \sum_{k=0}^{N} \frac{\sin \left(\pi\left(\frac{3}{4} v+\frac{1}{2} k\right)\right)}{k!\Gamma(v+k+1)}\left(\frac{1}{2} x^{2}\right)^{k} \tag{3.15}
\end{align*}
$$

In the above Equations (3.14) and (3.15), $v=0$ for $\operatorname{Ber}(x)$ and $\operatorname{Bei}(x), N \in \mathbb{N} \backslash\{0\}$ is the number of terms for the series expansion, and $x \in \mathbb{R}$ is a real-valued argument. The derivatives of the Kelvin functions are given by the reflection formulae (Spanier and Oldham, 1987):

$$
\begin{align*}
& \frac{\partial \operatorname{Ber}_{v}(x)}{\partial x}=\frac{1}{\sqrt{8}}\left[\operatorname{Ber}_{v+1}(x)+\operatorname{Bei}_{v+1}(x)-\operatorname{Ber}_{v-1}(x)-\operatorname{Bei}_{v-1}(x)\right]  \tag{3.16}\\
& \frac{\partial \operatorname{Bei}_{v}(x)}{\partial x}=\frac{1}{\sqrt{8}}\left[\operatorname{Bei}_{v+1}(x)+\operatorname{Ber}_{v+1}(x)-\operatorname{Bei}_{v-1}(x)-\operatorname{Ber}_{v-1}(x)\right] \tag{3.17}
\end{align*}
$$

### 3.2.2. Multiphase Modifications

The Biot-Stoll equations do not completely describe an acoustic model of snow. Naturally-occurring snow is a multiphase porous medium comprised of a frame filled with a mixture of air as a gas and water as a fluid (Colbeck, 1997). This is a "mixture theory" approach that has been used in models of snowpack evolution. The snowpack is considered as being comprised of a fraction of these three constituents (Morland et al., 1990; Morris and Kelly, 1990).

From the definitions given in Section 1.2 of this thesis, the volume fraction of air $\varepsilon_{a}$, the volume fraction of water $\varepsilon_{w}$ and the effective fluid density $\rho_{f}$ in the pore spaces of the snow medium can be calculated using the following derivation:

$$
\begin{gather*}
\phi=\frac{V_{a}+V_{w}}{V_{T}} \\
\varepsilon_{a}+\varepsilon_{w}=1 \\
\varepsilon_{a}=\frac{V_{a}}{V_{a}+V_{w}} \\
\frac{1}{\varepsilon_{a}}=1+\frac{V_{w}}{V_{a}} \\
\varepsilon_{a}=\frac{1}{1+\frac{\theta_{w}}{\theta_{a}}}  \tag{3.18}\\
\varepsilon_{w}=\frac{V_{w}}{V_{a}+V_{w}}=S_{w} \\
\frac{1}{\varepsilon_{w}}=1+\frac{V_{a}}{V_{w}} \\
\varepsilon_{w}=\frac{1}{1+\frac{\theta_{a}}{\theta_{w}}}  \tag{3.19}\\
\rho_{f}=\varepsilon_{a} \rho_{a}+\varepsilon_{w} \rho_{w} \tag{3.20}
\end{gather*}
$$

### 3.2.3. Percolation Theory (PT) Model

Percolation theory (Grimmett, 1999) is used to relate the shear modulus and bulk modulus to the porosity of snow. Percolation theory provides a macroscopic model of
connectivity in a porous medium that is mathematically rigorous. Based on the work of Phani and Niyogi (1987), the Young's modulus $E$ and shear modulus $G$ of a porous material was related to porosity by Kováčik (2001), who proposed Equations (3.21) and (3.22).

$$
\begin{align*}
& E=E_{0}\left(\frac{\phi_{c}-\phi}{\phi_{c}}\right)^{f_{c}}  \tag{3.21}\\
& G=G_{0}\left(\frac{\phi_{c}-\phi}{\phi_{c}}\right)^{f_{c}} \tag{3.22}
\end{align*}
$$

In the above, $E_{0}$ is the Young's modulus of the solid material comprising the porous frame, $G_{0}$ is the shear modulus of the solid material comprising the porous frame, $\phi$ is the porosity of the medium, $\phi_{c}$ is a critical porosity at which $E \approx 0 \mathrm{~Pa}$ and $G \approx 0 \mathrm{~Pa}$, and $f_{c}$ is the percolation theory exponent. Equations (3.21) and (3.22) imply that if $\phi \leq \phi_{c}$, then $E=0 \mathrm{~Pa}$ and $G=0 \mathrm{~Pa}$. Kováčik and Emmer (2013) later related the speed of shear waves in the porous media frame to the porosity using Equation (3.22).

In the context of the UTAM model for snow, $E_{0}$ is the Young's modulus of the snow particles, the corresponding shear modulus for the snow particles is $G_{0}$, the shear modulus of the porous snow medium is $G$, and $\phi$ is the porosity representing the volume fraction of air in the snowpack. Following Kováčik (2001), in lieu of any additional information on the percolation theory exponent $f_{c}$ for snow, $f_{c}=2.1$ for the UTAM model. This is the theoretical value of $f_{c}$ that is assumed to be the same for all continuous porous media (Sahimi, 1994). The value is applicable because snow is a porous medium that has continuous pore spaces (Buser and Good, 1986). In addition, as proposed by Kováčik (2001), $f_{c}$ is the same exponent used to calculate the Young's modulus and the shear modulus of the porous material.

The Young's modulus and shear modulus for ice are calculated using formulae developed by investigators who have measured the speed of shear wave propagation in ice and related this measurement to the density of the ice at different temperatures. This provides an empirical equation for determining $\left\{E_{0}, G_{0}\right\}$ as a function of temperature $T$ (Celcius). The equation has to
be empirical due to the lack of available theory relating inter-molecular forces to macroscopic measurements of $\left\{E_{0}, G_{0}\right\}$ (Magnanimo et al., 2008).

Ashton (1986) reviews Young's modulus measurements made on ice samples without bubbles, ice samples collected from freshwater locations, and ice samples taken from an ice shelf. Godbout et al. (2000) provide a similar review, also adding an equation developed from measurements on ice extracted from snow. Due to the small number of measurements used to develop these equations and since the shear modulus $G_{0}$ must be calculated as well, other formulae were used to compute parameters $\left\{E_{0}, G_{0}\right\}$.

The elastic constants of ice have been measured by a number of investigators (Hobbs, 1974). Arguably the most complete dataset was the one collected by Dantl (1968), who determined elastic moduli over the range of $-140^{\circ} \mathrm{C} \leq \mathrm{T} \leq 0^{\circ} \mathrm{C}$ (Petrenko and Whitworth, 2002). Gammon et al. (1983) note that the elastic moduli data of Dantl (1968) are 5\% lower than the elastic moduli data collected by other investigators. Although Dantl (1968) explains this discrepancy as occurring due to the use of ice crystals grown in a water bath and then aged for eight months before acoustic measurements were made, Gammon et al. (1983) disagree with this claim, postulating that experimental discrepancies may be associated with ice impurities or systematic laboratory error.

Gammon et al. (1983) measured the elastic moduli of ice using Brillouin spectroscopy, but their measurements were only made at $-16^{\circ} \mathrm{C}$. Although Petrenko and Whitworth (2002) state that the Gammon et al. (1983) measurements supersede the measurements made by Dantl (1968), empirical formulae published by Dantl (1968) are used for the UTAM since they provide a way to calculate the temperature dependence of elastic moduli.

Assuming that the ice is an isotropic medium, the coefficients $\left\{S_{11}, S_{12}\right\}$ are given by Dantl (1968), and the equations for the Young's modulus and shear modulus of ice ( $E_{i}$ and $G_{i}$ ) are given by Petrenko and Whitworth (2002), with the temperature $T$ in units of Celsius:

$$
\begin{align*}
& S_{11}=10.40\left(1+\left(1.070 \times 10^{-3}\right) T+\left(1.87 \times 10^{-6}\right) T^{2}\right)  \tag{3.23}\\
& S_{12}=-4.42\left(1+\left(0.463 \times 10^{-3}\right) T-\left(2.06 \times 10^{-6}\right) T^{2}\right) \tag{3.24}
\end{align*}
$$

$$
\begin{gather*}
E_{i}=\frac{1}{S_{11}}  \tag{3.25}\\
G_{i}=\frac{1}{2\left(S_{11}-S_{12}\right)} \tag{3.26}
\end{gather*}
$$

The bulk modulus of the ice $K_{i}$ is computed from $\left\{S_{11}, S_{12}\right\}$ by (Petrenko and Whitworth, 2002):

$$
\begin{equation*}
K_{i}=\frac{1}{3\left(S_{11}+2 S_{12}\right)} \tag{3.27}
\end{equation*}
$$

To calculate the elastic constants of the snow particles from the elastic constants of ice, Equations (3.25) to (3.27) are scaled using coefficients $\left\{\psi_{E}, \psi_{G}\right\}$ :

$$
\begin{align*}
& E_{0}=E_{i} / \psi_{E}  \tag{3.28}\\
& G_{0}=G_{i} / \psi_{G} \tag{3.29}
\end{align*}
$$

The scaling coefficients model the differences between the elastic constants of ice and the elastic constants of the snow particles. Due to a lack of reported measurements on the Young's modulus and the shear modulus of individual snow particles, the scaling coefficients cannot be directly measured and must be found using an optimization procedure described in Section 3.4.5 below.

### 3.2.4. Consolidation Theory (CT) Model

For application of percolation theory, the critical porosity $\phi_{c}$ is calculated using a model referred to in this thesis as a "consolidation theory." This theory predicts when the Young's modulus and shear modulus of snow will be effectively zero, such that $E \approx 0 \mathrm{~Pa}$ and $G \approx 0 \mathrm{~Pa}$.

By definition, the Young's modulus $E$ of a material is the ratio of tensile stress $\sigma_{\text {stress }}$ to tensile strain $\sigma_{\text {strain }}$ and the shear modulus $G$ is the ratio of shear stress $\gamma_{\text {stress }}$ to shear strain $\gamma_{\text {strain }}$, as shown by Equations (2.3) and (2.4) below.

$$
\begin{align*}
& E=\frac{\sigma_{\text {stress }}}{\sigma_{\text {strain }}}  \tag{3.30}\\
& G=\frac{\gamma_{\text {stress }}}{\gamma_{\text {strain }}} \tag{3.31}
\end{align*}
$$

The tensile stress $\sigma_{\text {stress }}$ is the force exerted over an initial cross-sectional area, and the tensile strain $\sigma_{\text {strain }}$ is the fractional change in length of the medium resulting from the applied force. Qualitatively, the Young's modulus $E$ is a measure of how easily a material can be stretched or compressed. As $\sigma_{\text {strain }} \rightarrow \infty, E \rightarrow 0 \mathrm{~Pa}$. The material can be more easily stretched or compressed for a lower value of $E$ as compared to higher values of $E$.

The shear stress $\gamma_{\text {stress }}$ is the force exerted over a cross-sectional area of the material, and $\gamma_{\text {strain }}$ is the fractional lateral displacement of the medium resulting from the applied force. The shear modulus can be considered as a measure how easily a material can be deformed by a shearing force. In a similar fashion to the Young's modulus, as $\gamma_{\text {strain }} \rightarrow \infty, G \rightarrow 0 \mathrm{~Pa}$. For materials where $G \rightarrow 0 \mathrm{~Pa}$, a shearing force can more easily deform the material.

In lieu of measuring snow particle morphology using photogrammetry and visual inspection (Jamieson and Schweizer, 2000; Perla, 1985, 1982), snow particles comprising the snowpack are approximated as spheres in the context of consolidation theory (Figure 3.1).

The spheres shown in Figure 3.1 are assumed to have an effective radius, and are joined by an inter-particle contact referred to as a "neck" (Brown, 1980) created due to sintering, compaction and metamorphism (Colbeck, 1982a,b). In a similar fashion to Figure 9.2.4 of Bear (1972) and Colbeck (1982), water occupies the neck space between the two snow particles. The effective radius of this water bridge is $R$. When a P-wave travels through the pore spaces of the snowpack over a control volume of the UTAM, the snow particles are assumed to be in static equilibrium, and are not changing in shape or moving due to mechanical compression or metamorphism. Although compression and metamorphism causes changes in snow particle geometry, radii, volume, and density, the time scale of these processes are on the order of minutes to hours, whereas the time of P -wave propagation through the snowpack is on the order of seconds.


Figure 3.1. Diagram of two snow particles used for the CT model. The two particles are joined by a neck created due to sintering, compaction and metamorphism. A water bridge with radius $R$ is situated outside of the neck. The distance $L$ over which the surface tension force acts is the arc length of the water bridge circle with radius $R$. Curve $C$ is part of the water bridge circle. The surface tension force vector is $\mathbf{F}_{\mathbf{t}}$, the surface tension force vector acting in the vertical direction is $\mathbf{F}_{\mathbf{t}, \mathrm{y}}$, the angle over which the surface tension force acts is $\theta_{F}$, the adhesive and cohesive force vector is $\mathbf{F}_{\mathrm{ac}}$, and $\mathbf{F}_{\mathbf{p}}$ is the downward-acting force of the snow particles and the water.

Treating the particles as a physical system, a statics problem is written to predict when $E \approx 0 \mathrm{~Pa}$ and $G \approx 0 \mathrm{~Pa}$. In a similar fashion to Colbeck and Parssinen (1978), the statics problem resolves the forces acting on a single snow particle joined by a neck to another snow particle. The forces exerted on the sides and top of the snow particle pair by other snow particles are assumed to cancel.

The shear modulus $G \rightarrow 0 \mathrm{~Pa}$ when the medium is a fluid (Adam et al., 2006). This also occurs when $E \rightarrow 0 \mathrm{~Pa}$. For a snow particle shown in Figure 3.1 to undergo this transformation, the net sum of the inter-particle forces acting on the snow particle pair must be non-zero and consequently the snowpack medium becomes "unconsolidated" and acts as an effective fluid. This physical process is similar in principle to loose snow avalanches caused by cohesionless snow that is either dry and unsintered or wet (Perla, 1981).

In the proposed consolidation theory, four different forces are assumed to act on a snow particle shown in Figure 3.1. In the coordinate system shown, the magnitude of all forces acting in the upward direction is positive, whereas all forces acting in the downward direction have negative magnitude.
(1) The surface tension force of the water acts at the air-water interface (Potter, 2002). The forces between molecules in the water are equal in all directions and thereby cancel, but the surface tension force acts toward the air medium tangent to the snow particle. Let $\mathbf{F}_{\mathrm{t}}$ be the surface tension force vector, and $\sigma_{s}$ be the surface tension coefficient for water. The force $\mathbf{F}_{\mathbf{t}}$ acts over a distance $L$ at an angle $\theta_{F}$ measured normal to the interface of the water in contact with the air. Let $C$ be the curve over which the $\mathbf{F}_{\mathbf{t}}$ force acts. The magnitude of the force $\mathbf{F}_{\mathbf{t}}$ acting in the vertical direction is written as:

$$
\begin{equation*}
\left|\mathbf{F}_{\mathrm{t}, \mathrm{y}}\right|=L \sigma_{s} \cdot \int_{C} \frac{d \mathbf{F}_{\mathrm{t}, \mathbf{x}}}{d \ell} d \ell=2 L \sigma_{s} \sin \left(\theta_{F}\right) \tag{3.32}
\end{equation*}
$$

In the above Equation (3.32), the dot signifies the dot product, and the integral represents an integration over the curve $C$.

For consolidation theory, the angle $\theta_{F}$ over which the surface tension force acts is related to the water content $\theta_{w}$ by the following Equation (3.33). This equation assumes that as $\theta_{w}$ increases, the surface tension force angle $\theta_{F} \rightarrow(\pi / 2) \mathrm{rad}$, as the water completely fills the neck space between the two particles.

$$
\begin{equation*}
\theta_{F}=\frac{\theta_{w} \pi}{2} \tag{3.33}
\end{equation*}
$$

Combining Equations (3.33) and (3.32) results in Equation (3.34) for the surface tension force as a function of water content $\theta_{w}$.

$$
\begin{equation*}
\left|\mathbf{F}_{\mathrm{t}, \mathrm{x}}\right|=2 L \sigma_{s} \sin \left(\frac{\theta_{w} \pi}{2}\right) \tag{3.34}
\end{equation*}
$$

Using geometry, the distance $L$ over which the surface tension force acts is the arc length $L=\theta_{F} R$ of the circle with radius $R$ at the air-water interface. Substituting this expression into Equation (3.34) yields Equation (3.35).

$$
\begin{equation*}
\left|\mathbf{F}_{\mathrm{t}, \mathrm{x}}\right|=2 \theta_{F} R \sigma_{s} \sin \left(\frac{\theta_{w} \pi}{2}\right) \tag{3.35}
\end{equation*}
$$

The surface tension force coefficient $\sigma_{s}$ as a function of temperature $T_{k}$ (Kelvin) was computed using the empirical interpolating Equation (3.36) given by Vargaftik et al. (1983), developed from measurements made on water at temperatures $0^{\circ} \mathrm{C} \leq T \leq 185^{\circ} \mathrm{C}$.

$$
\begin{equation*}
\sigma_{s}=B\left[\frac{T_{k}-T}{T_{c}}\right]^{a}\left[1+b\left(\frac{T_{k}-T}{T_{c}}\right)\right] \tag{3.36}
\end{equation*}
$$

In the above Equation (3.36), $B=235.8 \times 10^{-3} \mathrm{~N} \mathrm{~m}^{-1}, b=-0.625, a=1.256$, and $T_{c}=647.15 \mathrm{~K}$ as a reference temperature. For free water in the snowpack, $T=0^{\circ} \mathrm{C}$ and $T_{k}=273.15 \mathrm{~K}$, so $\sigma_{s}=7.57 \times 10^{-2} \mathrm{~N} \mathrm{~m}^{-1}$.
(2) The adhesive force between the water and a snow particle and the cohesive force between snow particles occurs due to differences in surface energy and molecular attraction. These two forces are modelled using a version of the Young-Dupre equation that calculates the work required to keep the two surfaces together. A similar approach was used by Meuler et al. (2010) to model ice accretion on surfaces. The magnitude of the force $\mathbf{F}_{\mathrm{ac}}$ holding the particles together is given by Equation (3.37) derived in this thesis from an equation modelling surface tension between two surfaces (Tas et al., 1996). The force is assumed to be positive, and acting in an upward direction.

$$
\begin{equation*}
\left|\mathbf{F}_{\mathrm{ac}}\right|=2 L\left(\sigma_{i w}+\sigma_{w a}\left(1-\cos \left(\theta_{\mathrm{c}}\right)\right)\right)=2 \theta_{F} R\left(\sigma_{i w}+\sigma_{w a}\left(1-\cos \left(\frac{S_{w} \pi}{2}\right)\right)\right) \tag{3.37}
\end{equation*}
$$

In the above Equation (3.37), the contact angle $\theta_{c}$ is taken to be the same as the surface tension angle $\theta_{\mathrm{c}}=\theta_{F}$, the surface tension between the snow particle and the water is $\sigma_{i w}$, the surface tension between the water and the air is $\sigma_{w a}$ and all other variables are as previously defined.

Measurements of $\sigma_{i w}$ and $\sigma_{w a}$ have been reported by Van Oss et al. (1992) for water at $T=0^{\circ} \mathrm{C}$, and are given below as Equation (3.38) and (3.39).

$$
\begin{gather*}
\sigma_{\mathrm{iw}}=4.0 \times 10^{-4} \mathrm{~N} \mathrm{~m}^{-1}  \tag{3.38}\\
\sigma_{\mathrm{wa}}=7.58 \times 10^{-2} \mathrm{~N} \mathrm{~m}^{-1} \tag{3.39}
\end{gather*}
$$

(3) The net force resulting from the weight of the snow particles and the water in the snowpack is calculated by Equation (3.40) for the porosity $\phi$ expressed as a function of the snow density $\rho_{s}$ and the ice density $\rho_{i}$ comprising the particles. The specific weight of the water is given by Equation (3.41), with $\rho_{w}$ as the water density.

$$
\begin{gather*}
\phi=1-\frac{\rho_{s}}{\rho_{i}}  \tag{3.40}\\
\gamma_{w}=\theta_{w} \rho_{w} g \tag{3.41}
\end{gather*}
$$

The density of ice $\rho_{i}$ as a function of temperature $T$ is calculated using empirical equations given by Fukusako (1990), developed from ice density measurements made using Xray diffraction:

$$
\begin{array}{cc}
\rho_{i}=917\left(1-1.17 \times 10^{-4} T\right)\left(\mathrm{kg} \mathrm{~m}^{-3}\right), & -140^{\circ} \mathrm{C} \leq \mathrm{T} \leq 0^{\circ} \mathrm{C} \\
\rho_{i}=930\left(1-1.54 \times 10^{-5} \mathrm{~T}\right)\left(\mathrm{kg} \mathrm{~m}^{-3}\right), & -260^{\circ} \mathrm{C} \leq \mathrm{T}<-140^{\circ} \mathrm{C} \tag{3.43}
\end{array}
$$

Re-arranging Equation (3.40), adding Equation (3.41), multiplying by the gravitational acceleration $g=9.81 \mathrm{~m} \mathrm{~s}^{-2}$ and dividing by the total volume $V_{T}$ gives Equation (3.44) below as the specific weight of the snow particles and the water in the pore space of the snow medium. Equation (3.44) is the magnitude of the downward acting force $\left|\mathbf{F}_{\mathbf{p}}\right|$ of the snow particles per volume $V_{T}$ :

$$
\begin{equation*}
\frac{\left|\mathbf{F}_{\mathbf{p}}\right|}{V_{T}}=g\left(\phi \rho_{i}+\theta_{w} \rho_{w}\right) \tag{3.44}
\end{equation*}
$$

Summing the forces acting in Figure 3.1 and assuming that the net force is zero gives the following Equation (3.45) from Equations (3.35), (3.37) and (3.44).

$$
\begin{gather*}
\sum \mathbf{F}=\left|\mathbf{F}_{\mathbf{t}, \mathbf{y}}\right|+\left|\mathbf{F}_{\mathbf{a c}}\right|-\left|\mathbf{F}_{\mathbf{p}}\right|=0 \\
\sum \mathbf{F}=\theta_{w} \pi R\left[\sigma_{s} \sin \left(\frac{\theta_{w} \pi}{2}\right)+\left(\sigma_{i w}+\sigma_{w a}\left(1-\cos \left(\frac{\theta_{w} \pi}{2}\right)\right)\right)\right]-\left|\mathbf{F}_{\mathbf{p}}\right|=0 \tag{3.45}
\end{gather*}
$$

Dividing (3.45) by $V_{T}$, re-arranging, and substituting in Equation (3.44) gives Equation (3.46).

$$
\begin{equation*}
\frac{\theta_{w} \pi R}{V_{T}}\left[\sigma_{s} \sin \left(\frac{\theta_{w} \pi}{2}\right)+\left(\sigma_{i w}+\sigma_{w a}\left(1-\cos \left(\frac{\theta_{w} \pi}{2}\right)\right)\right)\right]-g\left(\phi \rho_{i}+\theta_{w} \rho_{w}\right)=0 \tag{3.46}
\end{equation*}
$$

Equation (3.46) can be re-written as Equation (3.47):

$$
\begin{equation*}
\frac{V_{w} \pi R}{V_{T}^{2}}\left[\sigma_{s} \sin \left(\frac{\theta_{w} \pi}{2}\right)+\left(\sigma_{i w}+\sigma_{w a}\left(1-\cos \left(\frac{\theta_{w} \pi}{2}\right)\right)\right)\right]-g\left(\phi \rho_{i}+\theta_{w} \rho_{w}\right)=0 \tag{3.47}
\end{equation*}
$$

By multiplying both sides of Equation (3.47) by Equation (3.48), the force-balancing equation becomes Equation (3.49).

$$
\begin{gather*}
\frac{1}{\theta_{w} \theta_{a}}=\frac{V_{T}^{2}}{V_{w} V_{a}}  \tag{3.48}\\
\frac{\pi R}{V_{a}}\left[\sigma_{s} \sin \left(\frac{S_{w} \pi}{2}\right)+\left(\sigma_{i w}+\sigma_{w a}\left(1-\cos \left(\frac{S_{w} \pi}{2}\right)\right)\right)\right]-\frac{g}{\theta_{w} \theta_{a}}\left(\phi \rho_{i}+\theta_{w} \rho_{w}\right)=0 \tag{3.49}
\end{gather*}
$$

Assuming that the volume of the air $V_{a}$ is the volume of a sphere, substituting an alternate expression for $\theta_{a}$ and taking $\phi=\phi_{c}$ :

$$
\begin{gathered}
V_{a}=\frac{4 \pi R^{3}}{3} \\
\phi=\frac{V_{w}+V_{a}}{V_{T}}=\theta_{w}+\theta_{a}
\end{gathered}
$$

$$
\begin{gather*}
\theta_{a}=\phi-\theta_{w} \\
\frac{3}{4 R^{2}}\left[\sigma_{s} \sin \left(\frac{S_{w} \pi}{2}\right)+\left(\sigma_{i w}+\sigma_{w a}\left(1-\cos \left(\frac{S_{w} \pi}{2}\right)\right)\right)\right]-\frac{g}{\theta_{w}\left(\phi_{c}-\theta_{w}\right)}\left(\phi_{c} \rho_{i}+\theta_{w} \rho_{w}\right) \cong 0 \tag{3.50}
\end{gather*}
$$

The two unknowns $\left\{R, \phi_{c}\right\}$ are found by taking values such that the RHS of Equation (3.50) is equal (or approximately equal) to zero using optimization techniques.

### 3.2.5. Mechanical Parameters

Assuming that $E$ and $G$ are the mean values over a control volume of the UTAM, the bulk modulus $K_{b}$ of snow is calculated using Equation (2.5).

$$
\begin{equation*}
K_{b}=\frac{E G}{3(3 G-E)} \tag{3.51}
\end{equation*}
$$

The density of water $\rho_{w}$ used in the consolidation theory calculations in the previous section and in the mixture theory approach below is computed using an empirical expression developed by Tanaka et al. (2001) for water in the temperature range $0^{\circ} \mathrm{C} \leq \mathrm{T} \leq 40^{\circ} \mathrm{C}$. Because naturally-occurring surface water found in rivers and snow contains dissolved oxygen, a correction factor can be added. Moreover, another equation is also used to correct for small changes in density due to compressibility of water at an atmospheric pressure $P$. Combining these three equations from Tanaka et al. (2001) yields the following Equation (3.52).

$$
\begin{equation*}
\rho_{w}=\left[1+\left(k_{0}+k_{1} T+k_{2} T^{2}\right)(P-101325 \mathrm{~Pa})\right]\left[a_{5}\left[1-\frac{\left(T+a_{1}\right)^{2}\left(T+a_{2}\right)}{a_{3}\left(T+a_{4}\right)}\right]+\left[s_{0}+s_{1} T\right]\right] \tag{3.52}
\end{equation*}
$$

In the above, $\left\{k_{k}, a_{k}, s_{k}\right\}$ with $k \in \mathbb{Z}$ are constants given by Tanaka et al. (2001), the temperature is $T$ (Celcius) and the atmospheric pressure is $P$. The constants are chosen to ensure that Equation (3.52) is dimensionally correct. For free water in the snowpack $T=0^{\circ} \mathrm{C}$.

The density of the fluid $\rho_{f}$ in the pore spaces of the medium is calculated using a mixture theory approach using Equations (3.18) to (3.20).

To calculate the density of air $\rho_{a}$ in the pore spaces of the medium, the air is assumed to be an ideal gas. Although water vapour and the mole fraction of gases in air will influence $\rho_{a}$, the ideal gas assumption is applied in the UTAM model since it is useful in the absence of any other information. Given the air pressure $P$ and the temperature $T$ (Kelvin), the ideal gas law equation is:

$$
\begin{equation*}
\rho_{a}=\frac{P}{R_{g} T} \tag{3.53}
\end{equation*}
$$

In Equation (3.53), the ideal gas constant is $R_{g}=287.058 \mathrm{~J} \mathrm{~kg}^{-1} \mathrm{~K}^{-1}$.
Using the same mixture theory approach, the effective density $\rho$ of the entire snowpack medium is calculated:

$$
\begin{gather*}
\rho=\rho_{s}=\rho_{a} \theta_{a}+\rho_{w} \theta_{w}+\rho_{i} \theta_{i}  \tag{3.54}\\
\phi=\theta_{w}+\theta_{i}  \tag{3.55}\\
\theta_{a}=1-\theta_{w}-\theta_{i} \tag{3.56}
\end{gather*}
$$

Taking the bulk modulus of air $K_{a}$ as the mean value in the pore spaces of the control volume and using the same ideal gas assumption as given above:

$$
\begin{equation*}
K_{a}=\chi_{g} P \tag{3.57}
\end{equation*}
$$

In the above Equation (3.57), $\chi_{g}=1.402$ as a dimensionless constant and $P$ is the air pressure. Once again using the mixture theory approach and assuming that the bulk modulus of the mixture is related in a similar fashion to parallel resistors (Rau and Chaney, 1988), the effective bulk modulus of the pore space fluid is given by:

$$
\begin{equation*}
\frac{1}{K_{f}}=\frac{1}{\varepsilon_{a}} \frac{1}{K_{a}}+\frac{1}{K_{w}} \tag{3.58}
\end{equation*}
$$

In the above Equation (3.58), $K_{w}$ is the bulk modulus of water, and all other parameters are as previously defined.

The bulk modulus of water $K_{w}$ is calculated from the Newton-Laplace equation (Raichel, 2006) that is used in acoustics to determine the speed of sound $c_{w}$ in water given the water density $\rho_{w}$. Re-arranging the equation to calculate the bulk modulus of water:

$$
\begin{equation*}
K_{w}=c_{w}^{2} \rho_{w} \tag{3.59}
\end{equation*}
$$

For water temperatures in the range $0^{\circ} \mathrm{C} \leq \mathrm{T} \leq 100^{\circ} \mathrm{C}$, the speed of sound in water is calculated using the empirical equation (Bilaniuk and Wong, 1996, 1993):

$$
\begin{equation*}
c_{w}=\sum_{k=0}^{5} \varphi_{k} T^{k} \tag{3.60}
\end{equation*}
$$

In Equation (3.60), $k \in \mathbb{Z}, 0 \leq k \leq 5$, the temperature is $T$ (Celcius), and $\varphi_{k}$ is a coefficient given in Table I, Part 2 of Bilaniuk and Wong (1993, 1996). For water in the snowpack, $T \approx 0^{\circ} \mathrm{C}$ and so $c_{w}=1402.39 \mathrm{~m} \mathrm{~s}^{-1}$. Then the bulk modulus of water $K_{w}$ is computed using Equation (3.52) and (3.59).

The viscosity $\eta$ of the pore space fluid is used to compute the correction factor $F(\kappa)$ given by Equation (3.13). Once again using the mixture theory approach:

$$
\begin{equation*}
\eta=\varepsilon_{a} \eta_{a}+\varepsilon_{w} \eta_{w} \tag{3.61}
\end{equation*}
$$

In Equation (3.61), the viscosity of the air is $\eta_{a}$ and the viscosity of the water is $\eta_{w}$.
The viscosity of air $\eta_{a}$ is computed using the Sutherland formula as a function of temperature (Anderson, 1991). Assuming that the air is an ideal gas, the Sutherland formula predicts that for air at a temperature $T$ (Kelvin):

$$
\begin{equation*}
\eta_{a}=\eta_{0}\left(\frac{T}{T_{0}}\right)^{\frac{3}{2}}\left(\frac{T_{0}+C_{c}}{T+C_{c}}\right) \tag{3.62}
\end{equation*}
$$

In Equation (3.62), the constants for air are taken as $\eta_{0}=1.7894 \times 10^{-5} \mathrm{~Pa} \mathrm{~s}, T_{0}=288.16 \mathrm{~K}$, and $C_{c}=110 \mathrm{~K}$.

The viscosity of water $\eta_{w}$ is calculated using an empirical equation derived from the paper of Huber et al. (2009). An assumption is made that the temperature $T$ of the water is much lower than the temperature at which water as a substance enters the critical region where it ceases to exist as a liquid or a gas, and that $T<900^{\circ} \mathrm{C}$ and $P<1 \mathrm{GPa}$. Then, the viscosity is calculated as:

$$
\begin{gather*}
\eta_{w}=\frac{100(\hat{T})^{1 / 2}}{\sum_{i=0}^{3} \frac{L_{i}}{\hat{T}^{i}}}+\exp \left[\hat{\rho} \sum_{i=0}^{5}\left(\frac{1}{\hat{T}}-1\right)^{i} \sum_{j=0}^{6} H_{i j}(\hat{\rho}-1)^{j}\right]  \tag{3.63}\\
\hat{T}=T / 647.096 \mathrm{~K} \\
\hat{\rho}=\rho_{w} / 322.0 \mathrm{~kg} \mathrm{~m}^{-3}
\end{gather*}
$$

In Equation (3.63), the $\hat{T}$ and $\hat{\rho}$ are the normalized temperature and water density, $L_{i}$ and $H_{i j}$ are dimensionless coefficients given by Huber et al. (2009).

To compensate for viscoelastic losses in the porous medium, the shear modulus $\mu$ is taken as a complex value, defined by Albert (1993) as:

$$
\begin{equation*}
\mu=G(1+i \delta) \tag{3.64}
\end{equation*}
$$

In Equation (3.64), the real-valued shear modulus is $G$, a complex number is represented by $i=\sqrt{-1}$, and $\delta$ is the dimensionless loss tangent that models the dissipation of acoustic waves in the porous medium (Stoll, 1989).

For application of the UTAM, the tortuosity $\alpha$ of the snowpack was estimated using the theory of Berryman (1980). To validate Biot's theory, Berryman (1980) derived a relationship between the tortuosity $\alpha$ and porosity $\phi$ of a porous medium using a theoretical dimensionless shape factor $\gamma_{s}$. Starting from the Biot constitutive equations, Berryman (1980) showed:

$$
\begin{equation*}
\alpha=1-\gamma_{s}(1-1 / \phi) \tag{3.65}
\end{equation*}
$$

Considering the particles comprising the porous medium as spheres, $\gamma_{s}=1 / 2$, whereas if the particles are randomly-oriented needles, $\gamma_{s}=2 / 3$ (Berryman, 1983).

### 3.3. High Frequency Approximation (HFA) Model

A simplified model for calculating the porosity $\phi$ and snow density $\rho_{s}$ of the snowpack medium is proposed in this section. The model uses a high-frequency approximation (HFA) for the speed of the slow P-wave. The HFA model is coupled with the UTAM model. For this purpose, the HFA model is referred to as a "primer" model in analogy to priming an engine before use.

Similar to other acoustic models applied to snow (Albert, 2001; Attenborough and Buser, 1988; Marco et al., 1998, 1996), the porous frame is assumed to be rigid. This implies that compressibility of air in the pore spaces of the snowpack is orders of magnitude greater than compressibility of the frame. Given this assumption, the speed of the P -wave $c_{s h}$ in the porous medium is approximately constant above an angular frequency $\tilde{\omega}$, and does not show large changes with an increase in frequency (Le et al., 2010). In a series of papers, it was shown that (Johnson and Plona, 1980; Johnson et al., 1982):

$$
\begin{equation*}
c_{s h}=\frac{c_{0}}{\alpha^{1 / 2}}, \quad \text { for } \omega>\tilde{\omega}=\frac{2 \eta_{a}}{\rho_{a} a_{p}^{2}} \tag{3.66}
\end{equation*}
$$

In Equation (3.66), $\tilde{\omega}$ is the theoretical angular frequency above which the expression is valid, (referred to as a cutoff frequency), $c_{0}$ is the speed of sound in air, $a_{p}$ is the mean diameter of pore spaces in the snowpack, and all other parameters are as previously defined. Equation (3.66) has been experimentally validated by Allard et al. (1994), Allard and Atalla (2009) and Le et al. (2010).

Assuming that air in the pore spaces of the snowpack is an ideal gas, the speed of sound in air is related to the air temperature $T_{a}$ (Kelvin) by (Raichel, 2006):

$$
\begin{equation*}
c_{0}=\left(\chi_{g} R_{g} T_{a}\right)^{1 / 2} \tag{3.67}
\end{equation*}
$$

In Equation (2.2), the dimensionless ratio of the specific heats for air is $\chi_{g}=1.402$ and the ideal gas constant for air is $R_{g}$ as defined in the previous section.

Combining Equations (3.65), (3.66) and (2.2) gives Equation (3.68). Assuming that $\chi_{g}$ is as defined in the previous section and that the speed of sound $c_{s h}$ in the snowpack is known above a highpass frequency $\tilde{\omega}$, the snow density and porosity $\left\{\rho_{s}, \phi\right\}$ are determined from Equation (3.68) and (3.69). The density of ice $\rho_{i}$ is calculated using Equations (3.42) and (3.43) above.

$$
\begin{gather*}
\phi=\frac{c_{s h} \gamma_{s}}{c_{s h}^{2}\left(\gamma_{s}-1\right)+\chi_{g} R_{g} T_{a}}  \tag{3.68}\\
\phi=1-\frac{\rho_{s}}{\rho_{i}} \tag{3.69}
\end{gather*}
$$

Assume that the snowpack is an effective medium and the air in the pore spaces of the snow is in thermal equilibrium with the snow particles comprising the porous frame. This assumption is also used when measuring the temperature of the snowpack using thermocouples. Then, to couple the HFA model with the UTAM model, Equation (3.68) is re-arranged with $T_{a}=T$, and Equation (3.70) is then used to calculate the temperature $T_{k}$ for the UTAM model.

$$
\begin{equation*}
T=\frac{c_{s h}^{2}}{\chi_{g} R_{g}}\left(1-\gamma_{s}\left(1-\frac{1}{\phi}\right)\right) \tag{3.70}
\end{equation*}
$$

### 3.4. Application of Forward and Inverse Models

The five models (PT, CT, HFA and UTAM) can be considered as forward models. A forward model is a set of mathematical equations that provide model outputs, given known model input parameters. The forward model is well-posed, indicating that model outputs are unique for fixed inputs. Alternately, an inverse model is a reconstructed approximation of the forward model inputs, given only the forward model outputs. The inverse model is ill-posed, indicating that further assumptions are required to reconstruct the forward model inputs.

The following sections show how the forward and inverse models are calculated using numerical methods and also present a sensitivity analysis of model parameters. The
programming language used for model calculations was $\mathrm{C}++$, with plots created in Matlab. The sensitivity analysis being used is the one-at-a-time (OAT) design, with a baseline parameter held constant while the other parameters are varied (Saltelli et al., 2009).

### 3.4.1. PT Model

The effective input parameters for the percolation theory (PT) model are the porosity of snow $\phi$, the water content $\theta_{w}$, the temperature $T$, and the scaling coefficients $\left\{\psi_{E}, \psi_{G}\right\}$ used to calculate the elastic constants of snow particles from the elastic constants of ice. The outputs of this model are the Young's modulus and the shear modulus of the porous snow medium $\{E, G\}$. A sensitivity analysis uses the following parameter ranges:

- The porosity of snow ranges between $0.35 \leq \phi \leq 0.91$. This corresponds to snow densities between $80 \mathrm{~kg} \mathrm{~m}^{-3} \leq \rho \leq 600 \mathrm{~kg} \mathrm{~m}^{-3}$ and is inclusive of the density range observed for fresh snow, shallow prairie snowcovers and deep mountain snowpacks (Pomeroy and Gray, 1995). The baseline porosity is $\phi=0.67$ corresponding to an average snow density of $300 \mathrm{~kg} \mathrm{~m}^{-3}$.
- The water content of snow ranges between $0.01 \leq \theta_{w} \leq 0.25$. This range includes the volume fraction of irreducible water held in the pore spaces of the snowpack and the volume fraction of a fully saturated wet snowpack. The baseline parameter value for the water content of snow is 0.05 for $5 \%$ water content.
- $-40^{\circ} \mathrm{C} \leq T \leq 0^{\circ} \mathrm{C}$ for the temperature range. The lowest temperature is coincident with the minimum operating temperature of the electronics used for acoustic sensing of snow in this thesis, and the highest temperature is set at the melting point of snow. The baseline temperature is $-20^{\circ} \mathrm{C}$. This temperature range also includes the range of snow temperature observations reported in this thesis.
- $1.0 \times 10^{-3} \leq \psi_{E} \leq 2.0$ and $1.0 \times 10^{-3} \leq \psi_{G} \leq 2.0$ for the scaling coefficient range. This range produces physically-reasonable outputs. The baseline parameters are $\psi_{E}=2.0$ and $\psi_{G}=2.0$.

As shown by Figure 3.2 and Figure 3.3, the PT model predicts that the porosity $\phi$ is the most sensitive parameter for calculations of the Young's modulus $E$ and shear modulus $G$.

Both $\{E, G\}$ exhibit an approximately hyperbolic decrease with an increase in porosity. Because an increase in porosity is associated with a decrease in snow density, the model predicts that the Young's modulus and shear modulus parameters are of greater magnitude for more dense snow. For the Young's Modulus $E$, this behavior is supported by measurements reported by Mellor (1974) and Keeler (1969). The measurements conducted by Perla et al. (1982) also show a similar increase in a shear strength index with an increase in density. The predictions given by the PT model are of similar order of magnitude as the Young's modulus and shear modulus values given by Albert (1993a).

The water content parameter $\theta_{w}$ is not as sensitive as porosity (Figure 3.2 and Figure 3.3), exhibiting a gradual reduction in $\{E, G\}$ with an increase in $\theta_{w}$. However, when $\theta_{w}>0.10$, both $\{E, G\}$ exhibit drops in magnitude. This indicates a lack of cohesion between snow particles for wet snow. The lack of cohesion is most apparent for $\theta_{w}>0.15$, corresponding with the $15 \%$ "soaked" snow in the Fierz et al. (1999) classification.


Figure 3.2. Sensitivity analysis for the Young's modulus $E$ calculated using the PT model. The porosity $\phi$, water content $\theta_{w}$, temperature $T$ and scaling coefficient $\Psi$ are used in the sensitivity analysis. Figure 3.2 b shows drops in magnitude for $\theta_{w}>0.10$.


Figure 3.3. Sensitivity analysis for the shear modulus $G$ calculated using the PT model. The porosity $\phi$, water content $\theta_{w}$, temperature $T$ and scaling coefficient $\Psi$ are used in the sensitivity analysis. Figure 3.3b shows drops in magnitude for $\theta_{w}>0.10$.

A gradual reduction in $\{E, G\}$ is also exhibited when the temperature increases (Figure 3.2c and Figure 3.3c). However, an increase in temperature does not cause large changes in $\{E, G\}$. For $E$, this behavior is supported by observations given by Mellor (1974) and for $G$ by Schweizer and Camponovo (2002). The scaling parameters exhibit linear increases over the parameter range (Figure 3.2d and Figure 3.3d) as is expected from Equations (3.28) and (3.29).

### 3.4.2. CT Model

The inputs for the CT model are the water content $\theta_{w}$ of snow and the temperature $T$. The parameter ranges are the same as those given for the PT model. The outputs of this model are the two unknowns $\left\{R, \phi_{c}\right\}$ in Equation (3.50) that make the LHS of the equation approximately equal to zero. To determine $\left\{R, \phi_{c}\right\}$, a Nelder-Mead optimizer (Nelder and Mead, 1965) as implemented by the NLopt library ${ }^{1}$ was used for the minimization process. C++ bindings were used for interfacing the library with the model code. Box constraints (Box, 1965)

[^0]were placed on the outputs $\left\{R, \phi_{c}\right\}$ such that $5 \times 10^{-5} \mathrm{~m} \leq \mathrm{R} \leq 2.5 \times 10^{-3} \mathrm{~m}$ and $0.1 \leq \phi_{c} \leq 1.0$. Because the water radius $R$ cannot be measured, the parameter ranges were chosen to ensure that the CT model gave physically-reasonable outputs when coupled with the other models presented in this thesis. For expository purposes, the $\phi_{c}$ parameter was converted to equivalent snow density $\rho_{c}$ when preparing the sensitivity analysis plots (Figure 3.4).

For water contents $\theta_{w}<0.15$, the snow density (Figure 3.4a) corresponding to $\phi_{c}$ is less than $\rho_{c}=100 \mathrm{~kg} \mathrm{~m}^{-3}$. This indicates that only fresh or cohesionless snow of low density that does not correspond to the "soaked" snow in the Fierz et al. (1999) classification will act as an equivalent fluid when $E \rightarrow 0 \mathrm{~Pa}$ and $G \rightarrow 0 \mathrm{~Pa}$. However, for water contents $\theta_{w} \geq 0.2$, the density at which snow acts as an equivalent fluid will approach $\rho_{c}=300 \mathrm{~kg} \mathrm{~m}^{-3}$. As predicted by the CT model, this is due to an increase in the surface tension force angle $\theta_{F}$ as the water completely surrounds the snow particles. Consequently, the forces keeping the particles together are weakened, decreasing $\phi_{c}$ and increasing $\rho_{c}$.

The snow density corresponding to $\phi_{c}$ remains under $\rho=100 \mathrm{~kg} \mathrm{~m}^{-3}$ over the range of $T$ selected for the sensitivity analysis (Figure 3.4b), indicating that $\phi_{c}$ is relatively insensitive to temperature. Although a decrease in temperature will strengthen the bonds between snow particles, the presence of liquid water will encourage snow particle growth and decrease cohesion, thereby making $\theta_{w}$ the more sensitive parameter. The insensitibity to temperature indicates that when using the CT model as a part of the UTAM model to compute an inverse solution (Section 3.4.5), the inversion is not sensitive to temperature as well. This decreases the accuracy of the temperature determined from the inverse model.

As shown in Figure 3.4c, the water bridge radius $R$ exhibits a linear increase between $2 \times 10^{-4} \mathrm{~m}$ to $8 \times 10^{-4} \mathrm{~m}$ with a corresponding increase in $\theta_{w}$. The $R$ parameter shows almost no sensitivity with temperature (Figure 3.4d). This demonstrates that changes in temperature do not influence the geometry of the water bridge between snow particles. The water bridge can still be present at low temperatures due to the presence of a quasi-liquid layer on the ice of the particles comprising the snowpack.


Figure 3.4. Sensitivity analysis for the CT model. The water content $\theta_{w}$ and the temperature $T$ are the two parameters being used in the sensitivity analysis.

### 3.4.3. HFA Model

The inputs for the HFA model are the speed of sound in snow $c_{s h}$ at a frequency greater than $\tilde{\omega}$, the temperature $T$ and the Berryman shape parameter $\gamma_{s}$. The air pressure $P$ and the mean pore space diameter $a_{p}$ are used to calculate the cutoff frequency $\tilde{\omega}$. Calculation of $\tilde{\omega}$ is only done for the sensitivity analysis performed in this section, and is not required for application of the HFA model coupled with the UTAM. The outputs of the HFA model are the porosity $\phi$ and the angular frequency $\tilde{\omega}$ above which the HFA assumption is valid. The temperature range is taken to be the same as the range used in the sensitivity analyses for the other two models above. In the same fashion as the sensitivity analysis given in the previous section, the $\phi$ parameter was converted to equivalent snow density $\rho$. The $\tilde{\omega}$ parameter was converted from angular frequency to frequency expressed in units of Hertz. The other parameter ranges were taken as the following:

- $10 \mathrm{~m} \mathrm{~s}^{-1} \leq c_{s h} \leq 325 \mathrm{~m} \mathrm{~s}^{-1}$ as the range of the P -wave in the pore spaces of the snowpack used for this sensitivity analysis. This range is based on the observations made in this thesis and by Oura (1952) and Johnson (1982). For the purposes of the HFA model sensitivity analysis, the baseline parameter is selected as $300 \mathrm{~m} \mathrm{~s}^{-1}$.
- $30 \mathrm{kPa} \leq p_{a} \leq 120 \mathrm{kPa}$ as the normal range of barometric air pressures. This range is inclusive of barometric air pressures at locations in Canada (Hare and Thomas, 1979) and the low barometric air pressures observed on mountain tops (CRC, 1981). The baseline air pressure is 100 kPa , a typical value at sea level.
- The shape parameter $\gamma_{s}$ is varied over the range of $0.5 \leq \gamma_{s} \leq 0.67$. This assumes that $\gamma_{s}=0.5$ for spherical parameters and $\gamma_{s}=0.67$ for randomlyoriented needles (Berryman, 1983). The baseline parameter is $\gamma_{s}=0.5$.
- The mean pore space diameter $a_{p}$ can be estimated from Good (1986) as ranging between 2 mm for large-grained snow such as depth hoar and 0.7 mm for finegrained snow ( $7 \times 10^{-4} \mathrm{~m} \leq a_{p} \leq 2 \times 10^{-3} \mathrm{~m}$ ). The baseline parameter is taken as being near the center of this range at 1 mm .

Figure 3.5a below shows that the speed of sound $c_{s}$ in the pore spaces of the snowpack is the most sensitive parameter of the HFA model. As $c_{s}$ increases, the calculated snow density $\rho$ shows a parabolic decrease. For speeds $c_{s} \rightarrow 10 \mathrm{~m} \mathrm{~s}^{-1}$, the calculated density of snow approaches the density of ice. As $c_{s} \rightarrow 325 \mathrm{~m} \mathrm{~s}^{-1}$, the calculated snow density decreases and is similar to the density of air. This behavior is similar to the acoustic observations on snow made by Ishida (1965), Lang (1976) and Johnson (1982).

The second most sensitive parameter of the HFA model is the snow temperature $T$. Figure 3.5 b shows a hyperbolic increase in calculated snow density that is nearly linear. This demonstrates that the HFA model incorporates the influence of temperature on the speed of the P -wave in the pore spaces of the snowpack. Changes in temperature from $-40^{\circ} \mathrm{C}$ to $0^{\circ} \mathrm{C}$ will cause a change of $210 \mathrm{~kg} \mathrm{~m}^{-3}$ in the calculated density $\rho$.

The Berryman shape parameter $\gamma_{s}$ is the least sensitive of all model parameters (Figure 3.5c). As $\gamma_{s}$ increases over the range $0.5 \leq \gamma_{s} \leq 0.67$, the calculated snow density drops by only


Figure 3.5. Sensitivity analysis for the HFA model outputs. The snow density is $\rho$, the temperature is $T$, and $\gamma_{s}$ is the Berryman shape factor.


Figure 3.6. Sensitivity analysis for the HFA model cutoff frequency $f$. The temperature is $T$, the air pressure is $P$ and $a_{p}$ is the mean pore space diameter.
$48 \mathrm{~kg} \mathrm{~m}^{-3}$. Due to the low sensitivity of $\gamma_{s}$, this analysis shows that $\gamma_{s}=0.5$ can be taken as a fixed number for all applications of the HFA model. For all applications of the UTAM model, $\gamma_{s}=0.5$. The $48 \mathrm{~kg} \mathrm{~m}^{-3}$ change in density over the range of $\gamma_{s}$ is smaller than the measurement differences between snow tube and snow pit measurements of density (Fassnacht et al., 2010), and is likely adequate for operational use of the HFA model.

Figure 3.6 shows that the theoretical cutoff frequency can be lower than 5 Hz , with the pore space diameter $a_{p}$ being the most sensitive of all parameters. The cutoff frequency (Figure 3.6a) exhibits a linear relationship with temperature. The cutoff frequency drops with an increase in air pressure (Figure 3.6b). The relationship between air pressure and cutoff frequency is slightly hyperbolic and almost linear. As the pore space diameter increases (Figure 3.6 c ), the cutoff frequency decreases, indicating that pores with larger diameter in the snowpack have a lower cutoff frequency.

### 3.4.4. UTAM Model

The PT, CT and HFA models are coupled with the UTAM model. These three models provide estimates of parameters for the UTAM model that cannot be easily measured. Consequently, the required inputs for the UTAM model are the frequency $f$ of the sound wave, the dimensionless loss tangent $\delta$, the Berryman shape factor $\gamma_{s}$, the air pressure $P$, the scaling parameters $\left\{\psi_{G}, \psi_{E}\right\}$, the high frequency approximation speed $c_{s h}$, the porosity $\phi$, and the water content $\theta_{w}$. The UTAM model outputs used for the inverse model are the speed $c_{s}$ and the attenuation $\alpha_{s}$ of the slow P-wave travelling in the pore spaces of the snowpack. Because only the speed $c_{s}$ is used in the inverse model, the sensitivity analysis presented in this section shows the $c_{s}$ parameter as an output. When the HFA model is coupled with the UTAM model, the $c_{s}$ parameter is constrained so that $c_{s} \leq c_{s h}$. Since there is a high frequency limit to the speed of the sound wave in the snowpack, this ensures that the UTAM model outputs are physically accurate.

For application of the UTAM model, the scaling parameters are assumed to be such that $\Psi=\psi_{G}=\psi_{E}$. The ranges for $\left\{\gamma_{s}, P, c_{s h}, \phi, S_{w}\right\}$ are the same as those given in the sections above, with the following parameters also used:

- $1.0 \times 10^{-3} \leq \Psi \leq 2.0$ for the scaling coefficient range. This range produces physically-reasonable outputs. In a similar fashion to the HFA sensitivity analysis given above, the baseline parameters are $\psi_{E}=2.0 \times 10^{-3}$ and $\psi_{G}=2.0 \times 10^{-3}$.
- $20 \mathrm{~Hz} \leq f_{a} \leq 10 \mathrm{kHz}$ for the range of frequencies being used. This is dependent on the range of sound wave frequencies sent into snow. Further information on the selection of this frequency range can be found in Chapter 4 of this thesis.
- $1.0 \times 10^{-3} \leq \delta \leq 2.0$ for the range of loss tangents. This range is selected with recourse to the acoustic observations on snow made in this thesis. The range includes the range of physically-reasonable loss tangents identified by Stoll (1989). The baseline parameter for this range is $\delta=0.1$.

Direct numerical evaluation of Equations (3.11) and (3.12) requires the use of multipleprecision floating-point arithmetic (Nikolaevskaya et al., 2012) due to the large number outputs of the Kelvin functions for argument $\kappa$ calculated using Equation (3.13). IEEE floating- point types native to any processor cannot be used due to numerical overflow. To directly compute these equations, the MPIR ${ }^{2}$ and MPFR ${ }^{3}$ "big number" libraries with the MPFR C++ binding ${ }^{4}$ were used to calculate Equations (3.11) through (3.17).

Because the Kelvin functions Equations (3.14) and (3.15) are infinite series, the maximum number of terms required for calculating these functions using a computer program was set at $N_{T}=100$, with the following early termination criterion. Let $b_{k}$ be the cumulative sum of terms of either Equation (3.14) or (3.15) calculated using a loop. The loop iteration number is $k \in \mathbb{N}$. Taking $\varepsilon_{T}=1.0 \mathrm{e}-10$ as a small number, the loop used for the summation is terminated if the following inequality holds for $n_{T}=10$ consecutive $b_{k}$ :

$$
\begin{equation*}
\left|b_{k}-b_{k-1}\right|<\varepsilon_{T} \tag{3.71}
\end{equation*}
$$

The $\left\{N_{T}, n_{T}, \varepsilon_{T}\right\}$ values were chosen to ensure that the Kelvin functions could be calculated for all values of argument $\kappa$ computed using Equation (3.13).

[^1]To speed up the calculation of Equation (3.11), the $\kappa$ argument was evaluated from $\kappa_{1}=150 \mathrm{~m}^{-1}$ to $\kappa_{2}=3 \times 10^{6} \mathrm{~m}^{-1}$ and non-linear curve-fitting was used to determine Equation (3.72) below. For $\kappa \leq \kappa_{1}$ or $\kappa \geq \kappa_{2}$, the direct numerical calculation using Equation (3.11) was used. For Equation (3.72), the $R^{2}$ correlation value was $R^{2}=1.0$, with the Root-Mean Squared Error $(\mathrm{RMSD})$ as $\mathrm{RMSE}=7.2 \times 10^{-3}$. The imaginary part of the expression is $\operatorname{Im}\}$ and the real part is $\operatorname{Re}\}$.

$$
\begin{array}{ll}
\operatorname{Im}\{F(\kappa)\}=0.0006281 \kappa^{2} \mathrm{~m}^{2}, & \text { for } 150 \leq \kappa \leq 3.0 \times 10^{6}  \tag{3.72}\\
\operatorname{Re}\{F(\kappa)\}=25.25, & \text { for } 150 \leq \kappa \leq 3.0 \times 10^{6}
\end{array}
$$



Figure 3.7. Sensitivity analysis for the UTAM model. The frequency of the sound wave in the pore space of the snowpack is $f$, the loss tangent is $\delta$, the Berryman shape factor is $\gamma_{s}$, and the air pressure is $P$.


Figure 3.8. Sensitivity analysis for the UTAM model. The scaling parameter is $\Psi$, the speed of the sound wave at frequencies greater than $\tilde{\omega}$ is $c_{s h}$, the porosity of snow is $\phi$, and the water content is $\theta_{w}$.

The sensitivity analysis of the UTAM model is shown by Figure 3.7 and Figure 3.8. The speed of the sound wave in the snowpack $c_{s}$ shows a linear increase with frequency (Figure 3.7a) indicating that P-waves waves with higher frequencies will travel through snow at a greater speed than waves at lower frequencies. This is physically-accurate behavior for sound wave propagation in porous media (Allard and Atalla, 2009). The loss tangent $\delta$ shows very little sensitivity (Figure 3.7b), dropping only $5 \mathrm{~m} \mathrm{~s}^{-1}$ over its range. Although the Berryman shape parameter $\gamma_{s}$ shows some small variability over its range (Figure 3.7c), the speed of sound in snow $c_{s}$ is approximately constant. This suggests that the choice of $\gamma_{s}=0.5$ is good for operational use of the UTAM model, and does not need to be varied by sampling location. The air pressure $P$ shows very little sensitivity and is approximately linear, with the speed of sound dropping by only $1 \mathrm{~m} \mathrm{~s}^{-1}$ over its range (Figure 3.7 d ). Figure 3.8 b shows that the model is not stable for frequencies greater than the highcut frequency of $c_{s h}$.

The scaling parameter $\Psi$ shows a hyperbolic decrease over $1.0 \times 10^{-3} \leq \Psi \leq 2.0$ (Figure 3.8a). The rate of change of this decrease is greatest for lower values of $\Psi$. This indicates that selection of lower values of $\Psi$ has a greater potential for introducing error into the UTAM
model output $c_{s}$ value than higher values of $\Psi$. Figure 3.8 b demonstrates that the speed of the sound wave $c_{s h}$ must be appropriately determined. This is because the UTAM model output $c_{s}$ is constant until $c_{s h}$ exceeds a threshold frequency. The sensitivity of the porosity $\phi$ is comparable to the sensitivity of the sound wave frequency $f$, with the porosity showing a linear increase over its range (Figure 3.8c). The water content $\theta_{w}$ shows a similar linear increase, with some additional variability for $S_{w}>0.18$ (Figure 3.8d).

### 3.4.5. Inverse Model

The inverse model inputs assumed to be known are the Berryman shape factor $\gamma_{s}$, the atmospheric air pressure $P$, the dimensionless loss tangent $\delta$, and the speed $c_{s h}$ measured above a highpass frequency $\tilde{\omega}$. For operational application of the inverse model, the shape factor $\gamma_{s}$ is taken as $\gamma_{s}=0.5$. The air pressure $P$ is taken as a known value at each field site, whereas the loss tangent $\delta$ and the speed $c_{s h}$ is measured using acoustics.

Although the pressure of air within the pore spaces of the snowpack is dependent on snow permeability and is less than the atmospheric air pressure (Colbeck, 1997), the sensitivity analysis (Figure 3.7d) shows that $P$ is an insensitive parameter in the UTAM model relative to the other parameters. In lieu of any additional information on the air pressure within the pore spaces of the snowpack, the pressure used for $P$ in the inverse UTAM model calculations is taken as the air pressure above the snow surface. This assumption is later justified by application of the UTAM model to make measurements of snowpack physical properties.

The four unknowns determined using the UTAM inverse model are the porosity $\phi$, the water content $\theta_{w}$, the temperature $T$ and the scaling parameter $\Psi$. These parameters are estimated using the following numerical procedure. The idea is similar to the use of the KnottZoeppritz equations to determine the speeds of P - and S -waves in a layered earth medium, given angle-dependent reflection coefficients computed from a seismic dataset (Bruin et al., 1990; Chavent, 2009). The inversion procedure uses the following steps:

1. Let $\left\{f_{1}, \ldots, f_{N}\right\}$ be known frequencies at which speeds $\left\{c_{s, 1}, \ldots, c_{s, N}\right\}$ of the slow P-wave are determined using signal processing. There are $N \in \mathbb{Z}$ frequencies in total at which speeds in the snowpack medium have been determined using signal processing.
2. The forward model is computed using known parameters at frequencies $\left\{f_{1}, \ldots, f_{N}\right\}$, with the $\left\{\phi, \theta_{w}, T, \Psi\right\}$ parameters taken to be initially unknown. The starting values of the $\left\{\phi, \theta_{w}, T, \Psi\right\}$ parameters are $\phi=0.58, \theta_{w}=0.07, T=0^{\circ} \mathrm{C}$ and $\Psi=1$. These parameters are limited to the ranges used in the sensitivity analyses using Box constraints (Box, 1965). The outputs of the forward model are the speeds $\left\{c_{s, 1}^{\prime}, \ldots, c_{s, N}^{\prime}\right\}$.
3. A Nelder-Mead optimizer (Nelder and Mead, 1965) as implemented by the NLopt library is used to iteratively evaluate the forward model for different values of $\left\{\phi, S_{w}, T, \Psi\right\}$ to minimize the square root of the sum of squared errors $e_{r}$ between the observed and calculated wave speeds:

$$
\begin{equation*}
e_{r}=\sqrt{\sum_{k=1}^{N}\left(c_{s, k}-c_{s, k}^{\prime}\right)^{2}} \tag{3.73}
\end{equation*}
$$

The $\left\{\phi, \theta_{w}, T, \Psi\right\}$ are determined by application of the inverse model. This allows for measurement of snow porosity $\phi$, density $\rho$, temperature $T$ and the structure coefficient $\Psi$ from the inverse model output.

Because the UTAM model is not very sensitive to snow temperature $T$, choosing $T$ as a fixed value exhibits minuscule effect on the solution of density and liquid water content. Therefore, $T$ can be either found from the inversion procedure given above or can be set to an approximate estimate of snow temperature at a field site. The inversion procedure is thus more useful for determining density $\rho$ and temperature $T$.

### 3.5. Acoustic Measurement Model

To collect acoustic data from snow for the purposes of (1) obtaining an image from the snowpack and (2) applying the inverse UTAM, a measurement system model is required. The measurement system must send sound waves into the snowpack and receive the sound waves. Implementation details of this system are given in Chapter 4.

The measurement system is situated in the air medium above the surface of the snowpack, and is comprised of a loudspeaker situated at an offset distance of 10.8 cm to six microphones with separation distances of 3.7 mm . Selection of these parameters is discussed in

Chapter 4 of this thesis. The setup is similar to FMCW (Gubler and Hiller, 1984), bistatic sonar (Cox, 1989) and radar (Willis, 1991) systems (Figure 3.9). The loudspeaker produces an acoustic wave in the air medium, the acoustic wave propagates into the snowpack due to acoustic-to-seismic coupling (Albert and Orcutt, 1989), and reflections from the snowpack are sensed in the air medium by the microphone. The acoustic wave is band-limited by the frequency response of the loudspeaker transducer and the sensing system. Because audible sound waves are in the frequency range of 20 Hz to 20 kHz (Raichel, 2006), the sound wave can be heard by human beings. As discussed in later sections of this thesis, the source signal is bandlimited to 10 kHz .


Figure 3.9. Conceptual diagram showing the acoustic measurement model. A loudspeaker is situated at an offset distance from six microphones. The signal sent from the loudspeaker is denoted as $s[t]$ and the signal received by a microphone is $r_{k}[t]$ for microphone number $k \in \mathbb{Z}$ where $1 \leq k \leq 6$. The measurement arrangement is referred to in this thesis as a "pseudo-CSG."

Reflections from the snow medium will occur due to changes in acoustic impedance. Changes in acoustic impedance do not always coincide with stratigraphic snowpack layers. The interfaces between the layers are taken to be "effective," and can only be detected when there is a reflection. Over a time scale of seconds as the wave passes through the snowpack, these parameters are assumed to not change. This is because snowpack evolution processes operate on time scales of hours to days. This is a reasonable assumption because snowpack evolution models (Pomeroy et al., 2007b) are often run on hourly timesteps. The snowpack is assumed to
be anisotropic, such that the acoustic parameters do not show change over direction within a control volume of the snow medium.

Similar to sonar being reflected from a rough seabed (Lurton, 2002), buried vegetation and rough interfaces between layers will cause acoustic scattering. Acoustic scattering occurs when the sound wave deviates from a straight-line reflection as it passes through the medium. Although acoustic scattering can be modelled using a Rayleigh parameter (Brekhovskikh and Lysanov, 2003), this is not considered due to the need to estimate surface roughness. The theory behind estimation of the Rayleigh parameter for acoustic sensing of snow is discussed by Kinar and Pomeroy (2008). Following Kinar and Pomeroy (2007), Figure 3.9 shows the interface between layers as being modelled by a line that approximates the average position of the rough interface.

Using seismic terminology, the uneven snowpack layers are referred to as "dipping reflectors" (Gadallah and Fisher, 2005). As recognized by Kinar and Pomeroy (2007), the speed of the sound wave may also gradually change over the vertical extent of the layer. Although sophisticated algorithms exist for seismic imaging and speed analysis in dipping layers using traveltime reflection tomography (Biondi, 2007), these algorithms are computationally intensive, and are not addressed in this thesis. The speed of the sound wave is assumed to be the average speed in each layer.

The idea of an acoustic source situated at an offset distance to receiving sensors is widely used in reflection seismology to obtain echoes from a layered earth medium (Burger et al., 2006). The receiving sensors are situated on a line collinear with the source in a survey transect. Because the source and receivers are not moved, the sensing system is similar to a common-shot gather (CSG) in seismic data acquisition, where the receivers sense a reflection from different points on a reflector. Due to spatial offset, there is a time difference between reflections arriving at different receivers (Robinson and Treitel, 2000). The novel idea being presented here is that the measurement system is similar to a CSG, but at a smaller conceptual scale. For seismic data acquisition using geophones, the distance between receivers is on the order of meters, whereas for the measurement model described in this section, the microphones are only situated millimeters apart. In this thesis, the arrangement shown in Figure 3.9 is a "pseudo-CSG."

Most seismic data acquisition surveys use a common midpoint (CMP) gather, where the reflection from a layer is situated at the midpoint between the source and the receiver. This is
accomplished by collecting a series of CSGs where the geophones and microphones have been translated along the survey transect. The CSGs are then subjected to a geophysical procedure referred to as stacking, where the digitized signals from the geophones are added to improve the signal-to-noise ratio (SNR). The disadvantage of using a CSG for data acquisition is that the reflections arriving at the microphones are of small amplitude and are not resilient to noise contamination. Although these effects cannot be completely eliminated, the traces are corrected for attenuation using signal processing and a Maximum Length Sequence (MLS) is used as the source signal to reduce the effects of noise contamination.

Let $s[t]$ be the digital source signal sent from the loudspeaker, and let $r_{k}[t]$ be the signal received by the six microphones, with $k \in \mathbb{Z}$ and $1 \leq k \leq 6$. The digital source signal $s[t]$ is a Maximum Length Sequence (MLS). The microphone selection and sampling rate is discussed in Chapter 3.

For the snowpack medium, the digital signal received by the microphones $r_{k}[t]$ is expressible as a convolution (Equation (3.74)). The convolutional model with additive noise $n[t]$ has been used in seismology to describe the effects of a layered seismic medium (Liner, 2004), and is adapted here for use with snow.

$$
\begin{gather*}
r_{k}[t]=\mu_{k}[t] * \varphi[t]+n[t]  \tag{3.74}\\
\mu_{k}[t]=m_{k}[t] * p[t] * e[t] * d[t] * c[t] * s[t]
\end{gather*}
$$

In the above Equation (3.74), $m_{k}[t]$ is the time response of each microphone, $p[t]$ is the time response of the loudspeaker, $e[t]$ is the time response of the electronic circuit used to sample the microphones and produce the source signal, $d[t]$ is the direct wave that travels from the loudspeaker to the microphones, $c[t]$ is the effect of the enclosure case on which the loudspeaker and the microphones are mounted, $\varphi[t]$ is the filter kernel of the snowpack medium, and $n[t]$ is environmental noise from wind, animals or anthropogenic sources. The $\mu_{k}[t]$ signal represents the total measurement response of the system. The loudspeaker as an electromechanical transducer does not have a flat frequency response (Atkinson, 1997).

Equation (3.74) treats the snowpack as a type of digital filter (Oppenheim and Schafer, 2010), with the $\varphi[t]$ as a filter kernel being applied to the measurement system response $\mu_{k}[t]$. This implies that the snowpack modifies the source signal $s[t]$ by a multiplication in the frequency domain.

Following Rosa and Ulrych (1991), and assuming that the snowpack is similar to a seismic earth medium, the snowpack filter kernel is considered to be minimum-phase. This is reasonable assumption for most seismic media (Pritchett, 1990).

### 3.6. Maximum Length Sequence (MLS) Measurements

### 3.6.1. Overview

The signal being sent from the loudspeaker is a Maximum Length Sequence (MLS). As produced by the loudspeaker, the MLS is a binary (two-state) pseudorandom noise signal (Borish and Angell, 1983). A pseudorandom noise signal is a self-repeating sequence that appears to be random, but is completely deterministic (Brown and Hwang, 1996; Golomb and Gong, 2005). Qualitatively, the MLS sounds similar to a burst of white noise sent from the loudspeaker. Due to the MLS similarity to white noise, a filter can be applied to the MLS sequence to produce colored noise (Borish, 1985).

The MLS is used to determine the filter kernel of a system. Treating the snowpack as a digital filter, the MLS allows for determination of the snowpack filter kernel $\varphi[t]$. This can be recognized as a form of deconvolution (Orfanidis, 1988; Smith, 2002): removing the effects of the convolution (Equation (3.74)) to determine $\varphi[t]$.

The MLS is selected for determining the kernel $\varphi[t]$ of the snowpack medium for the following reasons. Measurements of $\varphi[t]$ of a system using transient clicks (Fincham, 1985), frequency sweeps (Clarkson et al., 1985) or tones at a single frequency (Yong-Shen, 1985) have a low Signal-to-Noise Ratio (SNR) relative to the MLS. Although the MLS measurement obtains a similar estimate of $\varphi[t]$, the time-integrated signal energy of the MLS is greater than these other methods due to the length of time that the signal is produced and the greater power of the signal.

The MLS has a longer duration than a transient click so the signal energy is spread over a longer amount of time. The length $L_{M}$ of the MLS is always $\left(L_{M}=2^{m}-1\right) \in \mathbb{N} \backslash\{0\}$, with $m \in \mathbb{N} \backslash\{0\}$ as the MLS order (Vanderkooy, 1994). For a DAC sampling rate $f_{d s}$ the duration $t_{m l s}$ of the MLS sequence is (Rife and Vanderkooy, 1989):

$$
\begin{equation*}
t_{m l s}=\frac{L}{f_{d s}} \tag{3.75}
\end{equation*}
$$

For $f_{d s}$ in the audible frequency range of $20 \mathrm{~Hz} \leq f_{d s} \leq 20 \mathrm{kHz}$, the duration of the MLS is $t_{m l s}$. The duration $t_{m l s}$ is on the order of seconds, making the MLS have greater time-integrated signal energy than a click lasting for less than one second.

Let $P_{s}$ be the power of a periodic non-MLS sequence. The signal power $P_{\text {MLS }}$ of the MLS is related to the power $P_{s}$ of the periodic sequence by (Rife and Vanderkooy, 1989):

$$
\begin{equation*}
P_{\mathrm{MLS}}=(L+1) P_{s}=2^{m} P_{s} \tag{3.76}
\end{equation*}
$$

Equation (3.76) shows that the signal power $P_{\text {MLS }}$ is orders of magnitude greater than the power of a frequency sweep or a tone as $m \rightarrow \infty$.

When the MLS is used to determine the kernel $\varphi[t]$ of a system, an assumption is being made that the system is Linear Time Invariant (LTI) (Rife and Vanderkooy, 1989; Vanderkooy, 1994). An LTI system is a system where (1) the input is related to the output by a linear map, and (2) the system is assumed to not change over the duration of the measurement (Schetzen, 2002). The first assumption is satisfied by a mathematical property of convolution (Oppenheim and Schafer, 2010), whereas the second assumption is satisfied by the property that the snowpack will not change over the time of the measurement.

Although the loudspeaker may not be a perfect LTI system due to distortion (Vanderkooy, 1994) non-linearity in the electromechanical system (Dunn and Hawksford, 1993), start-up transients (Rife and Vanderkooy, 1989) or temperature changes due to environmental or self-heating, it is considered to approximate an LTI system. Some of these effects are removed by empirically determining the MLS sent from the loudspeaker and the system response $\mu_{k}[t]$. Moreover, the audio amplifier used in the sensing system described in
this thesis had a built-in filter at the audio amplifier input. This helped to reduce the distortion of the MLS sent into the amplifier by limiting the slew rate of the MLS signal (Vanderkooy, 1994)

Although SineSweep source signals can be used to remove the LTI assumption (Farina, 2000), this source signal shows a lowered immunity to noise and is more appropriate for measurements made in quiet environments (Stan et al., 2002). Signal pre-emphasis techniques can correct for distortion artifacts by modifying the MLS before the signal is sent to the loudspeaker (Cobo et al., 2007; Kovitz, 1994; Niedrist, 1993), but this application is not considered here and is suitable for future research. Immunity of the MLS to distortion is discussed by Dunn and Hawksford (1993) and Vanderkooy (1994), and effects of pre-filtering the MLS is discussed by Godfrey (1975) and Vanderkooy (1994).

The bandwidth of the MLS signal is given by (Rife and Vanderkooy, 1989):

$$
\begin{equation*}
B=\left(\frac{f_{d s}}{L_{M}}\right)\left(\frac{L_{M}-1}{2}\right) \approx \frac{f_{d s}}{2} \tag{3.77}
\end{equation*}
$$

Because the MLS bandwidth is maximally-flat (Rife and Vanderkooy, 1989) up to the frequency $B \approx f_{d s} / 2$, this makes the MLS suitable for wideband measurements. The Biot theory model in previous sections have shown that the response of the snowpack is frequency-dependent, as would be expected for a porous medium (Allard and Atalla, 2009), and the MLS as a source signal is able to measure some of this frequency dependence.

### 3.6.2. Measurement of Snowpack Filter Kernel

From the signal $r_{k}[t]$ detected by the microphones and the $\mu_{k}[t]$ system response, the kernel $\varphi[t]$ of the snowpack as sensed by a microphone $k$ is determined by cross-correlation (Borish and Angell, 1983):

$$
\begin{equation*}
\varphi_{k}[t]=\mu_{k}[t] \star r_{k}[t] \tag{3.78}
\end{equation*}
$$

Although efficient algorithms have been devised to recompose $\varphi_{k}[t]$ given the mathematical sequence of the MLS (Borish and Angell, 1983; Jeffery Borish, 1985; Cohn and Lempel, 1977; Fino and Algazi, 1976), these algorithms cannot be applied here because the system response $\mu_{k}[t]$ is measured in an anechoic chamber (cf. Chapter 4.4).

Treating Equation (3.78) as a convolution between two signals, with one of the signals reversed (Smith, 2002), Equation (3.78) is re-written as Equation (3.79). Let the discrete signal $\mu_{k}[t]$ have $M$ elements, and the discrete signal $r_{k}[t]$ have $N$ elements. Expressing the correlation as a convolution that is of length $M+N-1$ to avoid the effects of circular convolution:

$$
\begin{equation*}
\varphi_{k}[t]=\sum_{\substack{j=0 \\(i-j) \neq 0 \\(i-j) \neq(N-1)}}^{M-1} \mu_{k}[j] r_{k}[N-1-(i-j)], \quad \forall 0 \leq i \leq(M+N-1) \tag{3.79}
\end{equation*}
$$

Equation (3.79) shows that the deconvolution operation to obtain $\varphi_{k}[t]$ is a form of matched filtering, which is known to be optimal in the rejection of noise $n[t]$ uncorrelated with the source signal (Rife and Vanderkooy, 1989). The $\{i, j\} \in \mathbb{Z}$ elements are indices.

However, because Equation (3.79) is slow to calculate in the time domain due to the repeated summation of terms, the correlation process can be done more efficiently in the Fourier domain. Using Fourier transform properties (Bracewell, 1978; Hansen, 2002; Jansson, 1996), Equation (3.79) is re-written in the frequency domain as:

$$
\begin{equation*}
\mathrm{F}\left(\varphi_{k}[t]\right)=\mathrm{FFT}\left(\mu_{k}[t]\right) \odot \operatorname{Rev}\left(\operatorname{FFT}\left(r_{k}[t]\right)\right) \tag{3.80}
\end{equation*}
$$

In Equation (3.80), $\mathrm{F}($ ) represents the discrete Fourier transform operator (numerically implemented as the FFT), $\odot$ represents the Hadamard (point-by-point) multiplication between the complex signals in the frequency domain, and $\operatorname{Rev}()$ is an operator that reverses the discrete signal so that the first element is the last and the last element is the first. Equation (3.80) is well-suited for small signals, but since the cross-correlation between signals $\mu_{k}[t]$ and $r_{k}[t]$ cannot fit into processor memory, a more efficient algorithm must be used based on Equation (3.80). The algorithm is similar to the Overlap-Add (OA) signal processing procedure of convolution (Smith, 2002), and is implementable in programmable logic (Blahut, 2010).

Using terminology from sonar and radar signal processing, the maximum number of samples required of $\varphi_{k}[t]$ is referred to as a "lag" (Nielsen, 1991). Because the kernel $\varphi_{k}[t]$ is truncated at the end of the signal to remove unwanted reflections at arrival times greater than needed for acoustic imaging of snow, selection of the lag $L_{l a g}$ must be greater than the time
sample $t$ corresponding to the maximum time of travel of the sound wave in the snowpack, and $L \ll M+N-1$.

Before running the efficient algorithm for cross-correlation, $\mu_{k}[t]$ and $r_{k}[t]$ are padded with zeros to ensure that both signals are the same size and that a sequence of size $2 L_{\text {lag }}$ can evenly fit into the padded sequences. Let $x_{w}[t]$ be a discrete signal buffer extracted from $\mu_{k}[t]$, and $y_{w}[t]$ be a discrete signal buffer extracted from $r_{k}[t]$. Let $w$ represent a sub-sequence of the $\mu_{k}[t]$ and $r_{k}[t]$ signals that is the same size as the buffers. Let $\varphi_{k}^{*}[f]$ represent the truncated kernel in the Fourier domain. The $\varphi_{k}^{*}[f]$ array is also the same size as the buffers and can hold complex numbers. Then for each sub-sequence $w$, the following signal processing procedure is applied (Nielsen, 1991):

$$
\begin{gathered}
x_{w}[t]=\left\{\begin{array}{cc}
\mu_{k}\left[t+(w-1) L_{l a g}\right] & \text { for } t=0, \ldots, L_{\text {lag }}-1 \\
0 & \text { for } t=\mathrm{L}_{\text {lag }}, \ldots, 2 L_{\text {lag }}-1
\end{array}\right. \\
y_{w}[t]=y\left[t+(w-1) L_{\text {lag }}\right] \quad \text { for } t=0, \ldots, 2 L_{\text {lag }}-1
\end{gathered}
$$

Once the summation process is performed for all of the buffers, the inverse Fourier transform (numerically implemented as the IFFT) is performed on $\varphi_{k}^{*}[t]$ to obtain $\varphi_{k}[t]=\operatorname{IF}\left(\varphi_{k}^{*}[t]\right)$. The calculated $\varphi_{k}[t]$ is then truncated to have the same number of elements as $L_{l a g}$.

### 3.6.3. Production of the MLS

The mathematical MLS sequence is produced using a linear feedback shift register (LFSR). The LFSR is a signal processing procedure where bits in a datatype are successively added and shifted (Davies, 1966; Lüke, 1986; MacWilliams and Sloane, 1976; Stahnke, 1973; Vanderkooy, 1994). Figure 3.10a shows a shift register corresponding to the MLS polynomial $x^{4}+x+1$. The binary MLS output of this polynomial is shown in Figure 3.10b. The shift register can be considered as being similar to a queue datatype where elements are ordered according to a FIFO (First-in First-out) property (Cormen et al., 2009).

Figure 3.10c shows the optimal flatness of an MLS magnitude in the frequency domain. The corresponding phase of the MLS is shown as Figure 3.10d. Because the MLS magnitude is flat, it is the phase that is most important in the cross-correlation operation of Equation (3.78).


Figure 3.10. Diagram showing the Linear Feedback Shift Register (LFSR) as (a) and the output MLS as (b). The example LFSR is shown corresponding to the MLS polynomial $x^{4}+x+1$ for an MLS order of $m=4$. The $\{\Upsilon[0], \Upsilon[1], \Upsilon[2], \Upsilon[3]\}$ are the elements of the LFSR. The exclusive OR operation is represented by the $\oplus$ symbol. Dots on subfigure (a) represent connections similar to electrical schematics. Subfigure (c) shows the magnitude response of a MLS with $m=10$, whereas subfigure (d) shows the phase of the MLS.

The shift register is stored in a datatype or a container $\Upsilon$ that is at least the same length as the order $m$ of the MLS. The shift register is filled with a known initial state: each element of the container is set to binary 1. Randomization of initial state is not required because the MLS is not being used for cryptographic purposes (Paar and Pelzl, 2010). However, randomization of initial state may be useful if more than one acoustic sensing system is deployed per field site.

Because MLS sequences with randomized state are uncorrelated (Golomb and Gong, 2005), more than one system may be able to take measurements at the same time due to the ability for the cross-correlation operation (Equation (3.78)) to reject uncorrelated noise.

Using a loop to execute $L_{M}=2^{m}-1$ times, the elements in the shift register corresponding to terms in the MLS polynomial are extracted and summed using a mod-2 adder (MacWilliams and Sloane, 1976). In the C/C++ and Verilog programing languages, mod-2 addition is represented by the exclusive- $\mathrm{OR}^{\wedge}$ operator. This is also referred to as a bitwise XOR operator.

Using the shift register shown in Figure 3.10a as an example, the term of the MLS polynomial ( $x^{4}$ ) with the highest degree is the length of the shift register $m=4$ and the second and first elements of the shift register are used in the exclusive-OR operation. Accessing the container using C programming language notation, the binary output of the shift register is $\Upsilon[0]^{\wedge} \Upsilon[1]$. The output is then placed as the leftmost element in the shift register, whereas the first element is removed. The loop then repeats for the next iteration.

For different orders of $m$, the table given by Stahnke (1973) is used to obtain MLS polynomials for $m=1$ to $m=168$. Since the output of the LFSR is binary $\{0,1\}$, the output is mapped by changing $0 \rightarrow 1$ and $1 \rightarrow-1$ for a voltage output (Dunn and Hawksford, 1993).

### 3.6.4. Reconstruction of PDM Sound Wave

The signal $r_{k}[t]$ from each measurement microphone is recomposed from a Pulse Density Modulated (PDM) bitstream. As discussed in Chapter 4, this bitstream is produced by an oversampling 1-bit Delta-Sigma ADC situated inside of each microphone. The PDM bitstream is similar to an MLS in that it has only two binary states $\{0,1\}$. A greater number of pulses per unit time represent a higher amplitude sound wave than a smaller number of pulses per unit time. Lowpass filtering of the bitstream will recompose the $r_{k}[t]$ sensed waveform (Candy and Temes, 1991).

Let $B_{k}^{*}[t]$ be the raw bitstream from microphone number $k$. For a 16-bit output, the raw bitstream is scaled by mapping $0 \rightarrow 0$ and $1 \rightarrow\left(2^{16}-1=65535\right)$. This produces binary signal $B_{k}[t]$ with range $[0,65535]$. The $B_{k}[t]$ signal was then passed through four lowpass

Butterworth filters designed as series Second-Order Sections (SOS) using integer arithmetic. Integer math was used to ensure filter stability and increase the speed of the computations (Smith, 2002).

The lowpass Butterworth filter was chosen in lieu of any other digital filter due to its maximally-flat passband and minimum-phase property (Smith, 2007). Because the snowpack medium is assumed to be a minimum-phase filter, convolution of the lowpass Butterworth filter with the $B_{k}[t]$ signal will produce an output that is minimum-phase. If the lowpass filter is zerophase, then the convolution will produce an output that is mixed-phase (Misra and Chopra, 2011). For a microphone sampling rate of $f_{s}=2.1 \mathrm{MHz}$ (as discussed in Chapter 3), the lowpass cutoff of the Butterworth filter was selected as $f_{c}=44 \mathrm{kHz}$. The cutoff frequency is greater than the bandwidth of the acoustic source signal being sent from the loudspeaker.

The SOS filter design is necessary to ensure stability (Smith, 2007) since the Butterworth filter is designed with an Infinite Impulse Response (IIR) topology. The IIR filter uses feedback, and has the potential to become unstable unlike Finite Impulse Response (FIR) filters (Smith, 2002).

The SOS was implemented as three cascaded Direct II Form Digital Biquad Filters (Smith, 2007). A signal processing flow diagram (Figure 3.11) shows the difference equation calculations (Equation (3.81)).

In Equation (3.81), the filter coefficients are $b_{k}$ with $0 \leq k \leq 8$ and $a_{k}$ with $1 \leq k \leq 6$, the $B_{k}[t]$ is the raw data input from microphone number $k$ and $r_{k}[t]$ is the microphone output signal. These filter coefficients are for a $7^{\text {th }}$ order filter.

Figure 3.12 shows the cutoff frequency response of the filter. Since all of the poles and zeros are inside or on the unit circle (Figure 3.13), the filter is stable (Lyons, 2010).

$$
\begin{align*}
& y_{1}[t]=b_{0} h_{1}[t]+b_{1} h_{1}[t-1]+b_{2} h_{1}[t-2]  \tag{3.81}\\
& h_{1}[t]=B_{k}[t]-a_{1} h_{1}[t-1]-a_{2} h[t-2] \\
& y_{2}[t]=b_{3} h_{2}[t]+b_{4} h_{2}[t-1]+b_{5} h_{2}[t-2] \\
& h_{2}[t]=y_{1}[t]-a_{3} h_{2}[t-1]-a_{4} h_{2}[t-2]
\end{align*}
$$

$$
\begin{aligned}
& r_{k}[t]=b_{6} h_{3}[t]+b_{7} h_{3}[t-1]+b_{8} h_{3}[t-2] \\
& h_{3}[t]=y_{2}[t]-a_{5} h_{3}[t-1]-a_{6} h_{3}[t-2]
\end{aligned}
$$



Figure 3.11. Signal processing flow diagram for the lowpass Butterworth filter. The input signal is $B_{k}[t]$ and the output is $r_{k}[t]$. Additions are represented by $\oplus$ and multiplications by $\otimes$. All other coefficients are the filter coefficients in Equation (3.81).



Figure 3.12. Frequency response of the lowpass Butterworth filter.


Figure 3.13. Zeros and poles for the Butterworth digital filter. The circles on the plot are the zeros and the x-markers are the poles. The zero-pole plot shows that the filter is stable.

### 3.7. Attenuation Analysis

### 3.7.1. Introduction

The attenuation of a sound wave passing through a medium such as snow is expressed as a dimensionless value $Q$ that relates the energy loss $\Delta W$ of the medium to the energy $W$ stored in the medium during application of energy to the medium by a sound wave (Wang, 2008):

$$
\begin{equation*}
Q=\frac{2 \pi W}{\Delta W} \tag{3.82}
\end{equation*}
$$

From Equation (3.82), the attenuation coefficient $a$ as used by the Biot theory is related to the $Q$ value by angular frequency $\omega$ and sound speed $c_{s}$ in a layer of the snowpack for the zero-offset $Q_{0}$ value:

$$
\begin{equation*}
a=\frac{\omega}{2 c_{s} Q_{0}} \tag{3.83}
\end{equation*}
$$

The loss tangent $\delta$ is related to $Q_{0}$ by the following expression (Stoll, 1985):

$$
\begin{equation*}
\delta=\frac{\pi}{Q_{0}} \tag{3.84}
\end{equation*}
$$

As shown by the model of Equation (3.83), attenuation of the sound wave changes with respect to frequency and speed. For a broadband pulse such as a Maximum Length Sequence (MLS), attenuation of the sound wave in the medium will be different for each instantaneous frequency.

Estimation of $Q$ is used to correct for the effects of attenuation on a seismic trace. This enhances seismic resolution, and in a similar fashion to radar or sonar system processing, is a form of automatic gain control (Yilmaz, 2001) that strengthens reflections from successive depths beneath the surface of the earth medium. Correcting for attenuation allows for better interpretation of stratigraphy from reflections (Wang, 2008), and in the context of acoustic imaging of snow, enables an estimate of the distance from the top surface of the snowpack to the ground. Without correction for attenuation, reflections from greater depth beneath the surface of the snowpack appear smaller than reflections near the snowpack surface, making it difficult to detect the ground surface or snowpack stratigraphy. An extensive review on the geophysics of Q is given by Barton (2007).

A forward-Q filter is a signal processing procedure that attenuates an acoustic wave. The purpose of the forward-Q filter is to test Q -analysis algorithms. An inverse-Q filter is a filter that takes an attenuated acoustic wave and attempts to find a non-attenuated reconstruction of the waveform. An inverse-Q filter requires estimates of $Q$ values using Q -analysis techniques (Wang, 2008). In lieu of knowing $Q$ values, a reconstruction of the original waveform can be estimated by smoothing in the time-frequency domain (Margrave et al., 2011).

### 3.7.2. Q-Analysis Techniques

Determination of $Q$ directly from seismic data has been performed by a number of methods. Vertical seismic profiling (VSP) techniques of Q-analysis involve a spectral ratio method. Recorded signals from a string of geophones inserted down a borehole are subjected to spectral analysis (Stainsby and Worthington, 1985). Ratios between signal spectra at successive depth beneath the surface are used to estimate the Q-value (Raikes and White, 1984). In signal
processing, this is equivalent to determining a transfer function between each geophone as the seismic wavefield propagates from the surface into the medium (Gadallah and Fisher, 2005). Transfer function determination of snow has been performed by Moore et al. (1991), but this requires the placement of microphones beneath the snow surface in a similar fashion to VSP.

A similar spectral-ratio technique is also used for estimating $Q$ from seismic reflection data collected from geophones placed near the surface of a seismic medium (Spencer et al., 1982). However, since reflection data is of geophysical waves that have passed through the medium, the acoustic response is not determined in the same fashion as VSP surveying and it is impossible to directly compute a transfer function. The spectral ratio is therefore computed using a source signal such as the first reflection from a sea bed (Dasgupta and Clark, 1998).

Assuming that $A[t, \omega]$ is the attenuated signal spectrum at an instantaneous time $t$ and angular frequency $\omega, A_{0}[t, \omega]$ is the spectrum of the source signal, and $M_{Q}[t, \omega]$ is a function that is used to model the relationship between the $Q, \omega$ and $c$ variables, the equation used for determining $Q$ from seismic reflection data using the spectral ratio method is of the following form:

$$
\begin{equation*}
A[t, \omega] / A_{0}[t, \omega]=M_{Q}[t, \omega] \tag{3.85}
\end{equation*}
$$

Because the ratio $A[t, \omega] / A_{0}[t, \omega]$ is computed over a moving time window, an inaccurate source signal spectrum will also lead to spurious estimates of $Q$ values. Although some implementations of Q-analysis algorithms assume that the source spectrum is flat, such that $A_{0}[t, \omega]=1$ for all $t$ and $\omega$ (Crider and Gutowski, 1995), this is an approximation that may not hold when sound sources such as loudspeakers are used. The time window length for spectral analysis will also influence the determined Q values. Moreover, automated picking of the source signal spectrum $A_{0}[t, \omega]$ from the reflection data can be challenging to automate, and is better suited for offline processing with a human seismic interpreter (Dasgupta and Clark, 1998).

Equation (3.85) can be recognized as a form of deconvolution by division in the frequency domain (Smith, 2002). It is apparent that $A_{0}[t, \omega] \neq 0$ and small errors in $A_{0}[t, \omega]$ will cause the deconvolution to be inaccurate. Although regularization algorithms can be used to ensure smoothness of the deconvolution (Hansen, 2002), this does not eliminate the need to estimate the source spectrum.

Elapavuluri and Bancroft (2004) describe a technique to determine $Q$ values from synthetic seismic traces passed through a forward filter with a constant value of $Q$. The traces were cross-correlated with a number of synthetic wavelets. Each synthetic wavelet had a known Q-value. The highest cross-correlation between a given trace and wavelet was used to determine the constant Q -value of the synthetic seismic trace. This method has not been tested on real seismic data and can only be used when the seismic trace has a constant Q -value. Moreover, the signal processing is quite intensive due to the generation of a large number of synthetic wavelets. These wavelets may not model actual wavelets that propagate through an earth medium.

Wang (2008) determined $Q$ by collapsing a 2D Gabor Transform spectrum into a 1D spectrum. Non-linear curve fitting was then used to determine $Q$ over the support of the 1D spectrum. However, the model being used to estimate $Q$ via this technique is not the same as the model used by Wang (2008) to correct for attenuation. A similar procedure was used by Margrave et al. (2011) to automatically correct for $Q$ using the Gabor transform of a seismic trace. The procedure is similar to Wang (2008), but the spectral collapsing is applied by Margrave et al. (2011) as hyperbolic smoothing of the seismic trace in the time-frequency domain of a Gabor transform. However, attenuation of waves in porous media is best modelled by an exponential loss function (Allard and Atalla, 2009; Meyer and Neumann, 1972; Wang, 2008), and so the hyperbolic smoothing is only an approximation of attenuation.

### 3.7.3. Forward and Inverse Gabor Transforms

Following Wang (2008), the forward and inverse Gabor transforms are required for application of forward and inverse Q filters. In a later section of this thesis, these transforms are also used to apply the vanishing constant filter (VCF).

The Gabor transform $G[t, \omega]$ is a Short-Time Fourier Transform (Allen and Rabiner, 1977) with a window $w[t]$. The STFT produces a time-frequency representation of frequency changes in a signal over time (Burrus et al., 1997). The window $w[t]$ is a sub-sequence of a time-domain input signal $h[t]$. For acoustic sensing of snow, $h_{k}[t]=r_{k}[t]$ is one of the microphone signals.

The Gabor transform is used here in lieu of wavelet-transform signal processing methods (Burrus et al., 1997) due to historical purposes. This is because the STFT mathematics is reasonably well-established (Allen and Rabiner, 1977) and the STFT has been used for similar Q-analysis and filtering operations (Wang, 2008). The filtering operation discussed in the following sections could potentially be adapted for use with wavelet-transform signal processing. Wavelet transforms are more accurate at representing fast changes in the signal with respect to time. This is a potential improvement.

The $G[t, \omega]$ is a $m \times n$ matrix with complex elements, where the number of columns is the same length $n$ of the discrete time-domain signal. For each signal timestep $t_{k}$, where $0 \leq k<n$, the Gabor transform is computed using an odd-numbered window size of $w_{s}$ so there is an even number of points on either side of the window. The $t_{k}$ element is situated at the center of the window. To compute elements at the beginning and end of the signal vector $h[t]$, the time window is padded with zeros so that the number of rows in the Gabor-transformed matrix is the same length $n$ as the signal vector $x[t]$.

Following Quatieri (2001), the Short-Time Fourier Transform (STFT) is computed by extracting each time domain window $w[t]$ from $h[t]$, multiplying the window $w[t]$ by a Gaussian windowing function $w_{f}[t]$ (for the Gabor transform), taking each window into the frequency domain using the complex FFT as $W[f]=\mathrm{F}(w[t])$, and arranging elements of the $W[f]$ along the columns of the Gabor matrix $G[t, \omega]$. Because the complex FFT of a real signal in the time domain is symmetric in the frequency domain (Lyons, 2010), each column of the transform is symmetric around the $\left(w_{s} / 2+1\right)$ element, for a total number of $w_{s}=m$ elements. The frequency-domain variable is $f$ and is the dual of the $t$ variable.

The Gaussian window function $w_{f}[t]$ with length of $n$ is given by (Wang, 2008):

$$
\begin{equation*}
w_{f}[t]=\frac{2}{T_{h} \sqrt{\pi}} \exp \left[-\left(\frac{2 t_{w}}{T_{h}}\right)^{2}\right] \tag{3.86}
\end{equation*}
$$

In the above Equation (3.86), $T_{h}$ is the total time on either side of the center of the window and the time at the center of the window is $t_{w}=0 \mathrm{~s}$. To evaluate Equation (3.86) the range of $t_{w}$ is $-T_{h} \leq t_{w} \leq T_{h}$ and the timestep $\Delta t=1 / f_{s}$ is related to the sampling rate $f_{s}$ of the signal.

The inverse Gabor transform reconstructs an estimate of a time domain signal $x^{\prime}[t]$ from the time-frequency representation $G[t, \omega]$. This is an Inverse Short-Time Fourier Transform (ISTFT) that is referred to as a synthesis operation. There are two methods for recomposing the input signal. The Overlap-Add (OLA) method takes the IFFT of each column of $G[t, \omega]$ and adds the columns together. The Filter-Bank Summation (FBS) method (Portnoff, 1976) considers each row of $G[t, \omega]$ as a filter bank. The rows are heterodyned and then added together to recompose an estimate of $x^{\prime}[t]$ as a time-domain signal (Quatieri, 2001).

If modifications are made in the frequency domain by $\Delta[t, \omega] \odot G[t, \omega]$ where $\Delta[t, \omega]$ is a filter kernel, then the time domain signal $\operatorname{IFT}\left(\Delta\left[t_{0}, \omega\right]\right)$ as the inverse FFT of a column of $\Delta[t, \omega]$ must have a time-domain response that is less than $2 m-1$ points to prevent spectral aliasing when using the OLA method of synthesis. This is because $\Delta[t, \omega] \odot G[t, \omega]$ is a multiplication in the frequency domain (Rabiner and Schafer, 1978). Modifications made before FBS does not result in an actual convolution (Allen and Rabiner, 1977), so the OLA method is used instead of FBS.

However, because the ISTFT may not produce a signal that is mathematically valid after the modification $\Delta[t, \omega] \odot G[t, \omega]$, the OLA procedure is implemented (Quatieri, 2001) so that the signal $h_{k}[t]$ is estimated in a least-squares sense (Griffin and Lim, 1984):

$$
\begin{equation*}
h_{k}[t]=\frac{\sum_{y=-\infty}^{\infty} w_{f}[y-t] g\left[y, t^{\prime}\right]}{\sum_{y=-\infty}^{\infty}\left(w_{f}[y-t]\right)^{2}} \tag{3.87}
\end{equation*}
$$

In Equation (3.87), the IFFT of each column of $G[t, \omega]$ is $g\left[t, t^{\prime}\right]$, where $t$ is the time element corresponding to a column of $G[t, \omega]$ and the time element corresponding to a row of $G[t, \omega]$ is $t^{\prime}$.

### 3.7.4. Forward and Inverse Q Filters

Forward Q filtering is applied in the frequency domain by determining a filter kernel $\Delta[t, \omega]$ that will attenuate $G[t, \omega]$ by the scaling operation $\Delta[t, \omega] \odot G[t, \omega]$. An inverse Q filter is applied by dividing $G[t, \omega]$ by $\Delta[t, \omega]$ so that $G[t, \omega] \odot(1 / \Delta[t, \omega])$. This is a form of simple deconvolution with $\Delta[t, \omega] \neq 0$. Although Tikhonov regularization and matrix-based methods of deconvolution (Hansen, 2010) were tried when developing these algorithms, a smooth method of deconvolution (Claerbout, 1992) with a small number $\varepsilon_{s}=1.0 \times 10^{-7}$ was used. This is due to the processing time required to run the matrix inversion and the good performance of the smooth method. The addition of a small number is similar to a form of regularization (Hansen, 2002), and is given by (Claerbout, 1992):

$$
\begin{equation*}
G^{\prime}[t, \omega]=G[t, \omega] \odot \frac{\operatorname{conj}(\Delta[t, \omega])}{\Delta[t, \omega] \operatorname{conj}(\Delta[t, \omega])+\varepsilon_{s}^{2}} \tag{3.88}
\end{equation*}
$$

In Equation (3.88), conj( ) is the complex conjugate of the kernel function matrix $\Delta[t, \omega]$.
Most inverse-Q filters use a form of downward wave continuation that is an integration over time (Bickel and Natarajan, 1985; Hale, 1981; Hargreaves and Calvert, 1991; Hargreaves, 1992). This method for forward and inverse-Q filtering is re-written as the kernel $\Delta[t, \omega]$ using theory given in the monograph by Wang (2008):

$$
\begin{gathered}
\sigma_{p}=\exp \left[-\left(0.23 G_{\mathrm{lim}}+1.63\right)\right] \\
\Phi[t, \omega]=\exp \left[-\int_{0}^{t} \frac{\omega}{2 Q\left[t^{\prime}\right]}\left(\frac{\omega}{\omega_{h}}\right)^{-\frac{1}{\pi Q\left[t^{\prime}\right]}} d t^{\prime}\right] \\
\Lambda_{1}[t, \omega]=\frac{\Phi^{2}(t, \omega)+\sigma_{p}}{\Phi(t, \omega)+\sigma_{p}} \\
\Lambda_{2}[t, \omega]=\exp \left[i \omega \int_{0}^{t}\left(\left(\frac{\omega}{\omega_{h}}\right)^{\frac{1}{\pi Q\left[t^{\prime}\right]}}-1\right) d t^{\prime}\right]
\end{gathered}
$$

$$
\begin{equation*}
\Delta[t, \omega]=\Lambda_{1}[t, \omega] \Lambda_{2}[t, \omega] \tag{3.89}
\end{equation*}
$$

In Equation (3.89), $\sigma_{p}$ is a dimensionless semi-empirical stabilization parameter used to prevent numerical instability, $G_{\text {lim }}$ is a gain limit (dB), $\omega$ is the angular frequency in radians corresponding to the rows of $G[t, \omega]$ from the first to the $\left(w_{s} / 2+1\right)$ element, $\omega_{h}$ is a highcut frequency taken in this thesis to be the maximum frequency $f_{s} / 2$ at the Nyquist sampling rate corresponding to the $\left(w_{s} / 2+1\right)$ element, $i=\sqrt{-1}$ as an imaginary number, $Q[t]$ as a discrete vector of elements with $n$ columns to correspond with the $n$ columns of the $G[t, \omega]$, and the integration is a continuous integration, not a discrete integration as found in signal processing (Oppenheim and Schafer, 2010). The continuous integration is numerically implemented using the trapezoidal rule.

Taking the real part of Equation (3.89) and assuming that $Q[t]=Q$ is constant (or approximately constant) over time, it is apparent that:

$$
\begin{equation*}
\Phi[t, \omega] \approx \exp \left[-\frac{\omega}{2 Q}\right] \approx \Lambda_{1}[t, \omega] \tag{3.90}
\end{equation*}
$$

Equation (3.90) supports the idea identified by Margrave et al. (2011) that deconvolution using a constant Q value is suitable for seismic signals with approximately constant Q .

Before the scaling operation $\Delta[t, \omega] \odot G[t, \omega]$ is applied, the kernel $\Delta[t, \omega]$ must be symmetrical around the $\left(w_{s} / 2+1\right)$ element. The first half of the kernel is computed using Equation (3.89) or (3.90), whereas the second half is the complex conjugate of the first half. This is done to ensure that the kernel is similar to a FFT (Lyons, 2010).

The scaling operation $\Delta[t, \omega] \odot G[t, \omega]$ must result in a minimum-phase signal $x^{\prime}[t]$ due to the assumption that the attenuating medium acts as a minimum-phase filter (Hale, 1981). This is also an assumption made for the snowpack model. Before the scaling operation $\Delta[t, \omega] \odot G[t, \omega]$ is applied, each column of the kernel $\Delta[t, \omega]$ is passed through a homomorphic filter that computes the minimum phase equivalent (Smith, 2007).

### 3.7.5. The Vanishing Constant Filter (VCF)

As discussed in the previous sections, methods of Q analysis use spectral ratios, crosscorrelation of wavelets, or spectral collapsing. This implies that two different models are being used: one model to estimate Q values from an input signal $h[t]$, and another model to correct the input signal $h[t]$ for attenuation. The Vanishing Constant Filter (VCF) presented in this section uses the same model to estimate and correct for attenuation. This makes the filtering procedure more robust. Similar to Margrave et al. (2011), the filter automatically corrects for attenuation without measuring the value of Q . The basic idea behind the VCF is that the term with the Q variable is made to disappear by mathematical transformations. This is a blind deconvolution algorithm that does not involve smoothing of the waveforms using spectral collapsing or averaging. Robinson-Weiner deconvolution (Margrave et al., 2011) is another way of accomplishing this type of signal processing operation.

Using the relationship given by Equation (3.90), take $\beta_{1}[t, \omega]$ as the Gabor transform $G[t, \omega]$ of an attenuated signal $\varphi_{k}[t]$ that has passed through a porous medium, let $A_{a}[t, \omega]$ be the Gabor transform of the un-attenuated signal, and let $\Theta[t, \omega]$ be a function that models the phase in a similar fashion to $\Lambda_{2}[t, \omega]$ :

$$
\begin{equation*}
\beta_{1}[t, \omega]=A_{a}[t, \omega] \exp \left(\frac{-\omega t}{2 Q}\right) \exp (i \Theta[t, \omega]) \tag{3.91}
\end{equation*}
$$

Taking the absolute value of Equation (3.91) eliminates the exponential function with an imaginary argument. This is due to the multiplicative property of the absolute value and the property $\operatorname{abs}(\exp (i x))=1$ with $x \in \mathbb{R}$. Equation (3.91) becomes Equation (3.92), where it is assumed that $\exp (-\omega t /(2 Q))>0$. Also let $A[t, \omega]=\left|A_{a}[t, \omega]\right|$. Equation (3.92) takes on only real values.

$$
\begin{equation*}
\beta_{2}[t, \omega]=\left|A_{a}[t, \omega]\right| \exp \left(\frac{-\omega t}{2 Q}\right)=A[t, \omega] \exp \left(\frac{-\omega t}{2 Q}\right) \tag{3.92}
\end{equation*}
$$

Homomorphic filtering is a signal processing procedure that attempts to separate two signals multiplied together (Lim, 1990). Homomorphic filtering can be considered as a type of
blind deconvolution (Kundur and Hatzinakos, 1996), where the signals are separated without knowledge of the filter kernel. Homomorphic filtering has been used to remove multiplicative noise kernels from images (Lim, 1990); to remove the source signature and noise from seismic traces (Buhl et al., 1974; Stoffa et al., 1974); to improve the resolution of ultrasound images (Taxt, 1997, 1995); to obtain a minimum-phase signal from a mixed-phase signal (Oppenheim et al., 1976); and to separate components of speech for speech analysis and speaker recognition (Quatieri, 2001). A review of homomorphic signal processing is given by Childers et al. (1977).

The basic idea behind homomorphic filtering is that the mathematical property of logarithms is used to transform a multiplied pair of signals into an additive pair (Oppenheim et al., 1976). Linear filtering techniques are then used to suppress the unwanted noise (Lim, 1990). The VCF takes this idea further by using the differentiation and integration operations as digital filters to remove attenuation.

For expository purposes, the Vanishing Constant Filter (VCF) is described in a series of steps. Each of these steps corresponding with an equation is implemented as a signal processing procedure. The derivatives and cumulative integrals in the section below are computed using FFT methods, as discussed in the next section.

The Vanishing Constant Filter (VCF) is aptly named since the $Q$ is removed from a term in Equation (3.93). The $Q$ is said to "vanish."

Taking the logarithm of both sides of Equation (3.92),

$$
\begin{equation*}
\gamma_{1}[t, \omega]=\log (A[t, \omega])-\frac{\omega t}{2 Q} \tag{3.93}
\end{equation*}
$$

Taking the partial derivative of Equation (3.93) with respect to $t$,

$$
\begin{equation*}
\gamma_{2}[t, \omega]=\frac{\partial}{\partial t} \log (A[t, \omega])-\frac{\omega}{2 Q} \tag{3.94}
\end{equation*}
$$

Taking the partial derivative with respect to $t$ again,

$$
\begin{equation*}
\gamma_{3}[t, \omega]=\frac{\partial^{2}}{\partial t^{2}} \log (A[t, \omega]) \tag{3.95}
\end{equation*}
$$

Equation (3.95) shows that the term with the $Q$ has been removed. Integrating Equation (3.95) over time yields:

$$
\begin{equation*}
\gamma_{4}[t, \omega]=\int_{0}^{t} \gamma_{3}\left[t^{\prime}, \omega\right] \partial t^{\prime}=\frac{\partial}{\partial t} \log (A[t, \omega])+C_{1}[\omega] \tag{3.96}
\end{equation*}
$$

The time integration shown in Equation (3.96) is cumulative, and the $C_{1}[\omega] \in \mathbb{R}$ sequence is a constant of integration as a function of $\omega$. Taking the difference,

$$
\begin{equation*}
\gamma_{5}[t, \omega]=\gamma_{4}[t, \omega]-\gamma_{2}[t, \omega]=C_{1}[\omega]+\frac{\omega}{2 Q} \tag{3.97}
\end{equation*}
$$

If $C_{1}[\omega]$ is constant for all $\omega$, then by taking the partial derivative of Equation (3.97) with respect to $\omega$ will produce $1 /(2 Q)$, and it is trivial to solve for $Q$. However, for signals that have passed through an attenuating medium, $C_{1}[\omega]$ is not constant and varies with $\omega$.

Taking the partial derivative with respect to $\omega$ of $\gamma_{5}[t, \omega]$ yields:

$$
\begin{equation*}
\gamma_{6}[t, \omega]=\frac{\partial}{\partial \omega} C_{1}[\omega]+\frac{1}{2 Q} \tag{3.98}
\end{equation*}
$$

Taking the integral of $\gamma_{6}[t, \omega]$ yields:

$$
\begin{equation*}
\gamma_{7}[t, \omega]=C_{1}[\omega]+C_{2}+\frac{\omega}{2 Q} \tag{3.99}
\end{equation*}
$$

In Equation (3.99), the constant of integration is $C_{2} \in \mathbb{R}$. From Equations (3.97) and (3.99), it is apparent that:

$$
\begin{equation*}
C_{2}=\operatorname{mean}\left(\gamma_{7}[t, \omega]-\gamma_{5}[t, \omega]\right) \tag{3.100}
\end{equation*}
$$

In Equation (3.100), the mean value of all elements in the matrix is denoted by mean (), and the subtraction is performed in a point-by-point fashion.

Taking the second derivative of $\gamma_{6}[t, \omega]$ removes the term with the $Q$ variable:

$$
\begin{equation*}
\gamma_{8}[t, \omega]=\frac{\partial^{2}}{\partial \omega^{2}} C_{1}[\omega] \tag{3.101}
\end{equation*}
$$

Integrating Equation (3.101) over $\omega$,

$$
\begin{equation*}
\gamma_{9}[t, \omega]=\int_{0}^{\omega} \gamma_{8}\left[t, \omega^{\prime}\right] \partial \omega^{\prime}=\frac{\partial}{\partial \omega} C_{1}[\omega]+C_{3} \tag{3.102}
\end{equation*}
$$

In Equation (3.102), $C_{3}$ is a constant of integration. When the cumulative integration is computed using FFTs using the signal processing procedure given in the next section, the $C_{3}$ is given by the first point in the $\partial C_{1}[\omega] / \partial \omega$ signal. The $\partial C_{1}[\omega] / \partial \omega$ signal is one element less than the number of rows in $\gamma_{5}[t, \omega]$ due to the truncation created by the signal processing procedure that computes the derivatives using FFTs. For $\omega_{k}$ with $1 \leq k \leq m$,

$$
\begin{equation*}
C_{3}=-\frac{\partial}{\partial \omega} C_{1}\left[\omega_{2}\right] \tag{3.103}
\end{equation*}
$$

The $C_{1}\left[\omega_{2}\right]$ element is the first element in $\partial C_{1}[\omega] / \partial \omega$. Assuming "smoothness" between the first and second derivatives of $C_{1}\left[\omega_{2}\right]$ near the first point, the $C_{3}$ constant is chosen as a value that is sufficiently "close" to $C_{3}$ by Equation (3.104).

$$
\begin{equation*}
C_{3}=-\frac{\partial}{\partial \omega} C_{1}\left[\omega_{2}\right] \approx-\Delta \omega\left[\frac{\partial^{2}}{\partial \omega^{2}} C_{1}\left[\omega_{3}\right]\right] \tag{3.104}
\end{equation*}
$$

In Equation (3.104), $\Delta \omega$ is the frequency step set by the length of the FFT used to compute the STFT and $C_{1}\left[\omega_{3}\right]$ is the first point of the column.

Integrating $\gamma_{9}[t, \omega]$ over $\omega$ again,

$$
\begin{equation*}
\gamma_{10}[t, \omega]=\int_{0}^{\omega} \gamma_{9}\left[t, \omega^{\prime}\right] \partial \omega^{\prime}=C_{1}[\omega]+C_{2}+\omega C_{3} \tag{3.105}
\end{equation*}
$$

Since $C_{2}$ and $C_{3}$ have been estimated, both can be successively removed from Equation (3.105):

$$
\begin{gathered}
\gamma_{11}[t, \omega]=C_{1}[\omega]+\omega C_{3} \\
\gamma_{12}[t, \omega]=C_{1}[\omega]
\end{gathered}
$$

The $C_{1}[\omega]$ is removed from $\gamma_{4}[t, \omega]$ :

$$
\begin{equation*}
\gamma_{13}[t, \omega]=\gamma_{4}[t, \omega]-\gamma_{12}[t, \omega]=\frac{\partial}{\partial t} \log (A[t, \omega]) \tag{3.106}
\end{equation*}
$$

Integrating Equation (3.106) over $t$,

$$
\begin{equation*}
\gamma_{14}[t, \omega]=\int_{0}^{t} \gamma_{13}\left[t, \omega^{\prime}\right] \partial t^{\prime}=\log (A[t, \omega])+C_{4} \tag{3.107}
\end{equation*}
$$

The $C_{4} \in \mathbb{R}$ is a constant of integration. To find $C_{4}$, Equation (3.94) is integrated over $t$ and $C_{4}$ is calculated as Equation (3.109).

$$
\begin{gather*}
\gamma_{15}[t, \omega]=\int_{0}^{t} \gamma_{2}\left[t, \omega^{\prime}\right] \partial t^{\prime}=\log (A[t, \omega])-\frac{\omega t}{2 Q}+C_{4}  \tag{3.108}\\
C_{4}=\gamma_{15}[t, \omega]-\gamma_{1}[t, \omega] \tag{3.109}
\end{gather*}
$$

Removing the constant from $\gamma_{14}[t, \omega]$,

$$
\begin{equation*}
\gamma_{16}[t, \omega]=\log (A[t, \omega]) \tag{3.110}
\end{equation*}
$$

Equation (3.110) is an estimate of the un-attenuated signal in the log domain. An estimate of the kernel in the log domain is:

$$
\begin{equation*}
\gamma_{17}[t, \omega]=\gamma_{1}[t, \omega]-\gamma_{16}[t, \omega] \approx \frac{-\omega t}{2 Q} \tag{3.111}
\end{equation*}
$$

For acoustic signals that have passed through a snowpack, the cross-correlation operation (Section 2.8.2) will remove the negative sign in Equation (3.111). The negative sign is restored by a simple multiplication. Homomorphic filtering (Smith, 2007) is then applied to $\gamma_{17}[t, \omega]$ so that a minimum-phase approximation of the kernel $\Delta[t, \omega]$ is reconstructed. This is necessary
since the phase function $\Theta[t, \omega]$ has been removed and Equation (3.111) does not have a complex part. The un-attenuated time domain signal $h_{k}[t]$ is then reconstructed using the deconvolution given by Equation (3.88) and the inverse STFT.

To estimate $Q$, Equation (3.111) is normalized to the interval $[0,1]$. This is required since Equation (3.111) is an approximation of the kernel that attenuates the signal and numerical values slightly greater than 1.0 are not physically reasonable. A matrix $\hat{Q}[t, \omega]$ is calculated for each element of the $\gamma_{18}[t, \omega]$ matrix using the $-\omega t /(2 \hat{Q}[t, \omega])$ model. The values of $\hat{Q}[t, \omega]$ are then spatially averaged to obtain $Q$.

### 3.7.6. Differentiation and Integration Using FFTs

A high-accuracy method for computing the derivative and cumulative integral of a discrete signal is required for implementation of the Vanishing Constant Filter (VCF). Although the derivative can be computed using finite-differencing (Fletcher, 1991), this method will cause non-smooth discontinuities in the signals. Lanczos differentiators (Savitzky and Golay, 1964) are used to reduce noise when taking the derivative of a signal. However, the derivative computed using a Lanczos differentiator is only an approximation of a smoothed derivative.

To compute high-accuracy derivatives and cumulative intergrals, FFT methods were used. These methods are similar to computational procedures used for pseudospectral solutions of computational fluid mechanics problems (Fornberg, 1998) or for optical sensing applications (Yaroslavsky, 2003; Yaroslavsky et al., 2005).

Let $y[z]$ be a real-valued discrete signal that is a row or column of a matrix $M[t, \omega]$, where $z$ is an element number of the signal. Let $\Delta y$ be the spacing between discrete samples, so that $\Delta y=\Delta t$ for the timestep and $\Delta y=\Delta \omega$ for the step in angular frequency. Using linear differential operator theory from Oppenheim et al. (1997) and Hörmander (1983), the derivative of $y[z]$ is computed by the convolution:

$$
\begin{gather*}
\frac{\partial y[z]}{\partial \Delta y}=\frac{y[z] * u_{1}[z]}{\Delta y}  \tag{3.112}\\
u_{1}[z]=\left\{\begin{array}{cc}
1 & \text { for } z=\text { floor }\left(N_{k} / 2+1\right) \\
-1 & \text { for } z=\text { floor }\left(N_{k} / 2+1\right)+1 \\
0 & \text { otherwise }
\end{array}\right.
\end{gather*}
$$

In Equation (3.112), the number of elements in the kernel $u_{1}[z]$ is $N_{k}$ and $u_{1}[z]$ is a discrete version of the unit doublet function that is a discrete approximation of the Dirac delta function derivative. For the Vanishing Constant Filter (VCF), $N_{k}=1024$ was selected as an adequate kernel length. The kernel length $N_{k}$ is the same as commonly used for most signal processing applications (Smith, 2002). Computation of the derivative by a linear convolution using Equation (3.112) requires that one point is lost at the beginning of the signal. The output of the convolution is situated between the $N_{k} / 2+2$ and $N_{k} / 2+2+N-1$ points of the padded convolution, where $N$ is the length of the signal $y[z]$.

The cumulative integral is computed using a discrete version of the Heaviside function $U[z]$. Once again using theory from Oppenheim et al. (1997) and Hörmander (1983):

$$
\begin{gather*}
\int y\left[z^{\prime}\right] \partial z^{\prime}=(y[z] * U[z]) \Delta y  \tag{3.113}\\
U[z]=\left\{\begin{array}{lc}
1 & \text { for } z=\text { floor }\left(N_{k} / 2+1\right) \text { to } N \\
0 & \text { otherwise }
\end{array}\right.
\end{gather*}
$$

For the VCF, the length of the kernel $N_{k}$ used for integration was taken as $N_{k}=2 N$ to ensure that the integration filter operated over all frequencies in the input signal $y[z]$. Because no points are lost during integration, the output is situated between the $N_{k} / 2+1$ and $N_{k} / 2+2+N-1$ points of the convolution output. The convolution of Equation (3.113) is scaled by $\Delta y$.

### 3.7.7. Zero-offset Q

The numerical value of $Q$ found using the VCF is the $Q$ value for a particular microphone signal $r_{k}[t]$. This value of $Q$ is not the zero-offset value of $Q_{0}$ required for finding the loss tangent $\delta$ due to spherical spreading. Because the value of $Q$ will vary in a similar fashion to the slope of the spectral ratio with the square of the offset distance (Dasgupta and Clark, 1998), non-linear curve-fitting is used to determine $Q$.

Let $x$ be the offset distance of the microphone $k$ to the loudspeaker. These offsets are contained in the vector $\mathbf{x}_{\text {os }}$. The value of $Q_{0}$ is computed for each $r_{k}[t]$ over $1 \leq k \leq 6$, forming the vector $\hat{\mathbf{Q}}_{\mathrm{y}}$. Then the following model is used to find $Q_{0}$ by curve-fitting:

$$
\begin{equation*}
\hat{\mathbf{Q}}_{\mathbf{y}}=m_{s} \mathbf{x}_{\mathbf{o s}}^{2}+Q_{0} \tag{3.114}
\end{equation*}
$$

In Equation (3.114), $\hat{\mathbf{Q}}_{\mathrm{y}}$ is the dependent variable, the $\mathbf{x}_{\mathrm{os}}$ is the vector of microphone offsets, and $Q_{0}$ is the value of $Q$ at zero-offset that is determined from Equation (3.114).

### 3.8. FBS Analysis

To apply the UTAM model, $c_{s}[t, x]$ is required as a function of angular frequency $\omega$. To determine $c_{s}[t, \omega, x]$, the attenuation-corrected traces $h_{k}[t]$ for $1 \leq k \leq 6$ are taken into timefrequency space by the Gabor transform, yielding $H_{k}[t, \omega]$. The absolute value of each column $\left|H_{k}[t, \omega]\right|$ is treated as a filtered version of the signal, using the bandpass filter view of the STFT (Quatieri, 2001). The Gabor transform computed $m$ different frequencies with $1 \leq \ell \leq m$. This idea is similar to the group velocity analysis of Stoll (2002).

### 3.9. Speed Analysis

The speed $c_{s}$ of the P -wave in the pore spaces of the snowpack is found by creating a matrix $\Gamma\left[t, x, \omega_{\ell}\right]$ where each column of the matrix is a vector $\mid H_{k}\left[t, \omega_{\ell}\right]$ of filtered signals for a microphone $k$ at a frequency $\omega_{\ell}$. Each microphone signal is of length $n$, and since there are six microphones in total per row, the matrix $\Gamma\left[t, x, \omega_{\ell}\right]$ has $n$ rows and six columns. Velocity analysis algorithms from seismology are applied to obtain the speed $c_{s}\left[t, x, \omega_{\ell}\right]$ as a function of time $t$, offset $x$ and frequency $\omega_{\ell}$. The speed $c_{s}\left[t, x, \omega_{\ell}\right]$ is used as an input to the UTAM model and also for imaging of snow.

Toldi $(1985,1989)$ was perhaps the first investigator to automate the determination of velocity from a seismic dataset. The idea proposed by Toldi was to compute semblance plots of the seismic dataset and then use numerical optimization to fit a model of velocity to the dataset. Before the work of Toldi, velocity had to be "picked" by human interpretaton. This was accomplished by drawing a line on the semblance plot. Semblance is a 2D plot of velocity versus travel time, with the semblance plotted on the third axis. Higher values of semblance at a certain velocity imply that the wave is moving at that velocity through the seismic earth medium (Luo and Hale, 2012). Constrained versions of velocity estimation was later introduced by Clapp et al. (1998), and Fomel (2009) used a fast-marching eikonal equation solver (Alkhalifah and Fomel, 2001) to pick velocity. The difficulty with such methods is that the picking procedure can be inaccurate. Using plane-wave destructor filters (Claerbout, 1992; Fomel, 2002), Fomel (2007) showed that velocity could be estimated directly from a seismic dataset without the picking operation.

The plane-wave destructor filter used for finding the speed of sound in snow was implemented from Chen et al. (2013). Given a dataset of $\Gamma[t, x]$ with time $t$ and offset $x$, let $u[t, x]$ be the wavefield. Then given an appropriate choice of parameter $\Omega$, the plane wave destructor filter is the following partial differential equation that removes plane waves from the dataset (Claerbout, 1992):

$$
\begin{equation*}
\frac{\partial u[t, x]}{\partial x}+\Omega \frac{\partial u[t, x]}{\partial t}=0 \tag{3.115}
\end{equation*}
$$

The Z-transform of $u[t, x]$ is $U\left[Z_{t}, Z_{x}\right]$, and $\left\{Z_{t}, Z_{x}\right\}$ are the discrete time-shift and spatial shift operators. Let $C(\Omega)$ be an operator. Then using a three-point finite-difference approximation, the plane wave destructor filter is given by (Chen et al., 2013):

$$
\begin{gather*}
\left(1-Z_{x} Z_{t}^{\Omega}\right) U\left(Z_{t}, Z_{x}\right)=0 \\
C(\Omega)=B\left(Z_{t}\right)-Z_{x} B\left(1 / Z_{t}\right) \\
B\left(Z_{t}\right)=\frac{(1+\Omega)(2+\Omega)}{12} Z_{t}^{-1}+\frac{(2+\Omega)(2-\Omega)}{6}+\frac{(1-\Omega)(2-\Omega)}{12} Z_{t} \\
C\left(\Omega, Z_{x}, Z_{t}\right) U\left(Z_{x}, Z_{t}\right) \approx 0 \tag{3.116}
\end{gather*}
$$

Using Equation (3.116), Newton iteration (Chen et al., 2013) is used to find $\Omega$ as a constant value for the entire signal that has passed through the snowpack. The $\Omega$ parameter is found as a constant value to ensure stability of the Newton iteration when applied to SAS2 acoustic data. Smoothness of the solution is ensured by the use of shaping regularization (Fomel, 2007b), which is similar to a constraint when solving inverse problems (Hansen, 2010). Ill-posedness of the solution for the acoustic waves passed through snow did not allow for "layered" estimates of the speed of sound in snow. This means that the snowpack is assumed to be an effective "onelayer" model.

Assuming that $\Omega \neq 0$, the moveout velocity $v[t, x]$ is computed using the Ottolini equation (Fomel, 2007a; Ottolini, 1983):

$$
\begin{equation*}
v[t, x]=\left(\frac{x}{\Omega}\right)^{1 / 2} \tag{3.117}
\end{equation*}
$$

In Equation (3.117), the offset distance between the loudspeaker and a microphone is $x$. The moveout velocity is used in NMO (Normal Moveout). This is a geometrical correction of the acoustic data so that acoustic reflections are not curved due to spherical spreading (Claerbout, 1992). The NMO as applied by the Ottolini equation is (Fomel, 2007a):

$$
\begin{equation*}
t_{0}=\left(t^{2}-\Omega t x\right)^{1 / 2} \tag{3.118}
\end{equation*}
$$

In Equation (3.118), $t_{0}$ is the offset time that is remapped from the signal time at which the signals $\Gamma[t, x]$ are recorded. The time-remapping is accomplished by inverse B-spline interpolation (Fomel, 2000). The time-remapped $\Gamma[t, x]$ is $\Gamma_{M}[t, x]$.

The interval velocity $c_{s}$ was obtained from $\Omega$ by:

$$
\begin{gather*}
\vartheta=\Delta t \Omega+t \frac{\partial \Omega}{\partial t}=\Delta t \Omega, \text { since } \frac{\partial \Omega}{\partial t}=0 \text { and } \Omega \text { is constant } \\
c_{s}[t, x]=\frac{x^{2} \Delta x \Delta t \vartheta[t, x]-2 \Omega t^{2} \Delta x}{\Omega^{2} t(2 t \Delta x-x \vartheta[t, x])+\varepsilon_{s}} \tag{3.119}
\end{gather*}
$$

In Equation (3.119), $\Delta x$ is the offset distance step between the microphones, $\Delta t$ is the timestep, $\varepsilon_{s}$ is a small number that prevents division by zero, and all other variables are as previously defined. Once $c_{s}[t, x]$ is known, averaging over the entire matrix is used to obtain $c_{s}$ as a constant speed over the entire signal. The snow depth $\bar{y}$ is measured using a depth rod or SR50 ultrasonic depth gauge and simple kinematics is used to select the maximum cutoff time $t_{\max }$.

There is a need to select a maximum cutoff time $t_{\max }$ from a cutoff depth $\bar{y}$ to identify when the sound wave has reached the bottom of the snowpack. This is because attenuation of the sound wave in snow does not always permit the bottom of the snowpack to be clearly identified. This maximum cutoff time could be assumed. If the cutoff time $t_{\max }$ is assumed instead of measuring snow depth $\bar{y}$, late-time reflections from elements other than the snowpack will have a greater contribution to the error in determination of snow density, liquid water content and temperature using the UTAM model. Future work could use physics of reflections from the ice or soil at the base of the snowpack to identify a cutoff time using signal processing.

### 3.10. Signal Processing Flow

Figure 3.14 shows the signal processing flow. The diagram synthesizes all of the information given in this chapter, showing how the calculations proceed.

An MLS is generated using a LFSR (Section 3.6.3). The MLS is sent into the snowpack by a loudspeaker. As described in Section 3.5, six microphones at an offset distance from the
loudspeaker sense the reflections and output Pulse Density Modulated bitstreams $B_{k}^{*}[t]$. The bitstreams are scaled to produce $B_{k}[t]$. An IIR Butterworth lowpass filter with a cutoff frequency $f_{c}=44 \mathrm{kHz}$ is used to obtain $r_{k}[t]$. The kernel $\varphi_{k}[t]$ is computed by a crosscorrelation operation (Section 3.6.2). The cross-correlation operation was performed by the FPGA in the system (cf. Chapter 4). The Vanishing Constant Filter (VCF) is applied to $\varphi_{k}[t]$ so that an un-attenuated trace $h_{k}[t]$ is obtained (Section 3.7.5). Zero-offset $Q$ is obtained (Section 3.7.7). A Gabor transform is applied to calculate $\left|H_{k}[t, \omega]\right|$ for each microphone. Rows of $\left|H_{k}[t, \omega]\right|$ are extracted for velocity analysis. Each extracted row is $\left|H_{k}\left[t, \omega_{\ell}\right]\right|$. Velocity analysis for a number of $\omega_{\ell}$ is used to find $c_{s h}$ and the speeds at a number of different frequencies for input into the UTAM. A snowpack image is obtained from $\Gamma\left[t, x, \omega_{m}\right]$.

### 3.11. Computation of Parameters

The speed $c_{s}$ is found at a frequency $\omega_{\ell}$ and $\Omega$ is constant over time. There is only one value of $c_{s}$ over the entire time of the signal for a frequency $\omega_{\ell}$. This means that the UTAM model will output depth-averaged values of density $\bar{\rho}$, liquid water content $\bar{\theta}_{w}$, and temperature $\bar{T}$. The liquid water content is reported as a percentage. The measurements of $\bar{\rho}$ are similar to the depth-averaged values of density that are determined using an SR50 snow tube.


Figure 3.14. Signal processing flowchart. Details are given in the text.

## CHAPTER 4 <br> APPARATUS

### 4.1. Introduction

To collect acoustic data suitable for use with models of sound propagation through snow, prototype electronic systems were designed and constructed. The following sections describe the systems created and discuss engineering choices made during development. The apparatus is identified by the acronym SAS2 as a "System for the Acoustic Sensing of Snow." As a system, the apparatus is comprised of hardware and software components used to collect data as inputs for acoustic models. The penultimate development goal was to create a device suitable for collecting acoustic data to apply the Unified Thermoacoustic Model (UTAM) presented in Chapter 3. Although this development goal was the objective of the paper, three initial prototypes were created before the objective was achieved with a fourth prototype system. The sampling system design is related to the physics of sound propagation in porous media.

### 4.2. Initial Prototype Systems

Two prototypes of the SAS2 were initially created and deployed at field sites in the Wolf Creek Research Basin near Whitehorse, Yukon Territory, and also at field sites in the Rocky Mountains of Alberta. Figure 4.1 shows these two prototype systems. Both systems were portable and transported to each field site in a backpack.

Each system was comprised of a single loudspeaker and electret microphone situated on the front of an enclosure case. A semiconductor temperature sensor placed inside of a PVC Tee pipe superglued to the side of each case served as a shielded thermometer to measure air temperature. Inside of each enclosure case was an electronic circuit responsible for driving the loudspeaker and collecting acoustic data from the microphone. As shown in Figure 4.1a, the


Figure 4.1. Pictures of the two initial prototype devices. The first prototype is shown as (a), whereas the second prototype is shown as (b). A front view of the second prototype shows the positioning of the thermometer, loudspeaker and microphone.
system was positioned above the surface of the snowpack using a pistol-grip handle and the loudspeaker sent a Maximum Length Sequence (MLS) into snow. At the same time that the MLS was produced, the microphone detected the reflection response from the snowpack. An operational amplifier (op-amp) circuit and Analog-to-Digital Converter (ADC) was used to digitize the voltage output from the microphone. The voltage output was related to the instantaneous sound pressure incident at the microphone surface. A time series of sound pressure measurements from the microphone was used as an input for an acoustic model that determined depth-integrated SWE and density of snow. With only one microphone, the acoustic model was recursive, with successive reflections being used to estimate cumulative changes in porosity beneath the surface of the snowpack. The idea is similar to recursive seismic layerstripping models. Because the model is recursive, errors in previous measurements are
cumulative (Aki and Richards, 2002). Further details related to the sampling system and snow acoustic models are given by Kinar and Pomeroy (2008a, 2008b, 2009).

An initial prototype of a stationary version of the acoustic sensing system was deployed at the Upper Clearing site in the Marmot Creek Research Basin, situated in the Rocky Mountains. As shown in Figure 4.2, a metal mounting stand was used to suspend the acoustic gauge above the surface of the snowpack. For this prototype, six electret microphones were attached to the outside of an enclosure case at an offset distance from a loudspeaker situated inside of another enclosure case. Wires from the loudspeaker and microphones were passed back into a third enclosure case, on the side of which was a PVC Tee pipe set up in a similar configuration to the temperature sensor used for the two initial prototypes. The third enclosure case contained an electronic circuit that was triggered each hour by a datalogger. This electronic circuit was responsible for sending an MLS from the loudspeaker and sampling voltage data from each of the microphones. Three enclosure cases were required to provide space for the loudspeaker, microphones, and processing circuitry, demonstrating that the physical size of the sampling system will increase with a larger number of microphones. This was due to physical size of the microphones and the associated sampling circuitry.

A Campbell Scientific SR50 snow depth sensor was situated next to the prototype. A heat flux plate was buried under the snowpack near the ground surface to provide measurements of heat exchange at the snow-ground interface. An infrared thermometer was used to measure snow surface temperature, and a thermocouple array recorded temperatures throughout the snowpack.


Figure 4.2. Picture of the first stationary prototype device deployed at the Upper Clearing site in the Marmot Creek Research Basin, Rocky Mountains of Alberta.

### 4.3. Final Prototype Systems

Building on the ideas embodied in the three initial prototype systems, the fourth version of the SAS2 was designed to be used in both portable and stationary configurations. Two copies of the same system were created, one for each configuration. The data collected from this system was sufficient to provide inputs for the UTAM model so that measurements of snow thermal and physical parameters could be obtained. This is because the microphone array consisted of a larger number of sensitive microphones that were used to sample the reflected sound pressure wave; the microphones used for this system had a high sensitivity relative to the other sampling systems; and the sampling system geometry and electronic circuit permitted the detection of small-amplitude reflections from beneath the surface of the snowpack.

### 4.3.1. Hardware

The sensing system was comprised of a loudspeaker situated at an offset distance from a planar array of 24 microphones on a small printed circuit board (PCB), along with another PCB mounted in an enclosure case. The enclosure case was Bud Industries NBF-32424 with outside
dimensions of approximately 38.5 cm by 28 cm . Figure 4.3 shows a side view of the device, whereas Figure 4.4 shows the front sensing part. Due to the similarity of the two devices, pictures of the SAS2 used for the portable configuration are shown.

The loudspeaker was mounted in the enclosure case by first creating a hole with a circular saw, and then bolting the loudspeaker in place with flat-head machine screws inserted through holes created in the case with a drill. A countersink was used to ensure that the machine screws were flush with the front of the enclosure case. Rubber washers placed on the machine screw threads inside of the enclosure case prevented ingress from water and moisture. A design choice was made to not install a grille on the front of the loudspeaker since this would have changed the acoustic response of the loudspeaker (Newell and Holland, 2006). However, the grille would have protected the loudspeaker from mechanical damage.

The microphone circuit board was also mounted in a similar fashion to the loudspeaker assembly. Holes were created to mount the microphone PCB flush with the side of the enclosure case using flat-head machine screws. Once again, a countersink was used to ensure that the machine screws did not protrude from the front of the enclosure case. Creating a flat sensing surface helped to reduce spurious reflections received by the microphones.


Figure 4.3. Side view of SAS2 in an enclosure box.


Figure 4.4. Front view of SAS2. The microphone array and loudspeaker is identified on the diagram.

To allow the microphones to sense acoustic pressure waves, 24 holes with a diameter of $\sim 1 \mathrm{~mm}$ were created in the front of the enclosure case using a Dremmel tool and a drilling template. The holes acted as waveguides (Blackstock, 2000) for the received reflections, rejecting some sound pressure wave arrivals from angles away from the normal of each microphone.

When the microphone PCB was bolted to the inside wall of the enclosure case, each hole in the case was aligned with a microphone hole on the PCB. Adhesive acoustic vents (GORE part number GAW1020308 from Sealing Devices Inc., Lancaster NY) were applied to the outside of the enclosure case over the top of each microphone hole. The acoustic vent protected the microphones from water and moisture while permitting an acoustic pressure wave to be sensed by each microphone. Acoustic vents were applied to the outside of the enclosure case to prevent the possible formation of a Helmholtz resonator (Rossing and Fletcher, 1995). This may be created if a sealing gasket is placed inside of the case at the back of the microphone PCB (Khenkin and Lewis, 2012).

Holes were created on the sides of the enclosure case for waterproof connectors and switches. The switches were used to turn the system on/off, trigger the system, and turn on/off the onboard battery chargers. A USB input allowed for data to be offloaded from the system. An external trigger input allowed for the system to take a sample using an external switch. A 12 V input allowed for the system to be powered with a battery. Sealing gaskets were inserted between the enclosure case wall and each connector to protect the system from environmental ingress (Figure 4.3).

### 4.3.2. Mounting Platforms

For the portable system, a mounting attachment was created out of metal using a welding machine and an Oxy-Acetylene Torch. The mounting attachment was bolted on the top of the enclosure case (Figure 4.5). A child chariot carrier (Chariot Carriers Inc., Calgary, Alberta) was modified with a metal stand that mated with the mounting attachment (Figure 4.6). A hole was cut in the bottom of the child chariot carrier and a ski attachment kit replaced the child chariot carrier wheels. The mounting attachment held the SAS2 system above the surface of the snowpack at a distance of no more than 10 cm from the snowpack whilst an operator pushed the chariot carrier over the snow surface. A pushbutton connected via wires to a connector on the side of the enclosure case was used to trigger measurements when the portable mounting platform was not moving.

For the stationary system, a mounting plate was created out of stainless steel and bolted to the back of the enclosure case. The mounting attachment allowed for the acoustic device to be deployed on a boom arm above the snow surface. This deployment strategy is similar to the deployment of electronic devices on a met station (Figure 4.7).


Figure 4.5. Mounting attachment situated on the top side of the portable SAS2.


Figure 4.6. Portable version of the SAS2 mounted in a child chariot carrier.


Figure 4.7. Stationary version of the SAS2 mounted on a boom arm.

### 4.3.3. Electronic Circuits

Figure 4.8a is a picture of the two printed circuit boards (PCB) designed for the SAS2 system. A block diagram of the system is given as Figure 4.8 b. The microphone PCB was a $4-$ layer circuit board, whereas the main PCB was an 8-layer circuit board. Due to the similarity of the two circuit boards, Figure 4.8a only shows the version of PCB created for the portable configuration device. All electronic components used on the circuit board were specified for operation down to $-40^{\circ} \mathrm{C}$, the minimum temperature of the industrial operating range. Appendix A provides a complete schematic for the SAS2 system. Parts of the schematic are described below with the appendix page numbers given in brackets (i.e. A. 1 for the first page of the Appendix A).


Figure 4.8a. Two printed circuit boards (PCBs) designed for the SAS2 system. The main PCB and microphone PCBs are shown.


Figure 4.8b. Block diagram of the system, showing the power supply, main processing and analog frontend block.

As shown on pages A. 1 to A. 2 of Appendix A, and in Figure 4.8b, the SAS2 main circuit board had the ability to be powered by three different power supplies. A 12 V input allowed a lead-acid battery to provide power for the system. Three FETs and an ideal diode controller enabled automatic switching of system load between a regulated DC input from an external supply, the 12 V input, and a 3.7 V Lithium-ion battery with a $12.6 \mathrm{Amp} \cdot \mathrm{hr}$ capacity. The ideal diode controller automatically selected the input with the highest voltage. An overvoltage protection controller switched off either the 12 V or DC input if the applied voltage exceeded 19 V or if the inputs were of the wrong polarity. This protected the system from overvoltage due to improper connection to the wrong power supply. A battery charger (page A.9) with a maximum 1 A output current was powered via a connected solar panel input cable. The battery charger circuit was responsible for charging the connected 12 V lead-acid battery. This allowed the SAS2 system to dynamically manage power when used in a stationary configuration above the surface of the snowpack. ESD diodes on each power input protected the system from electrostatic discharge. Since the electric field strength in blowing snow has been recorded as high as $30 \mathrm{kV} \mathrm{m}^{-1}$ (Schmidt et al., 1999), ESD protection is necessary to ensure that the circuit will continue operation during the snow accumulation season.

An 8-bit microcontroller (pages A.4) powered with an always-on low-dropout 3.3 V linear regulator (page A.3) was used to control system operation. The signals from rocker switches on the side of the enclosure case were debounced using a hardware circuit before being input into microcontroller port pins. The rocker switches were responsible for signalling the microcontroller to either turn on/off the system, trigger a sample measurement, or to turn on a battery charger. The microcontroller program could selectively re-assign or disable the rocker switches. This allowed for two different configurations of the system, and also helped to prevent the system from being inadvertently switched on or triggered when deployed in a stationary configuration. A 5V-tolerant buffer and inverter implemented an auxiliary logic input to a port pin of the 8-bit microcontroller so an external device (such as a datalogger) could be used to turn on/off the system. A battery-backed real-time clock (page A.5) and I/O expander (page A.7) was connected via I2C bus to the 8 -bit microcontroller. This clock was used to obtain a timestamp for every acoustic measurement collected by the SAS2. Also interfaced with the I2C bus was a serial number IC, giving each SAS2 a unique identifier that could be determined by the 8 -bit microcontroller.

Port pins of the 8-bit microcontroller were connected to three bicolour LEDs to provide a visual indication of battery charger status and system operation (page A.5). The LEDs were bright enough to be semi-visible in sunlight. When the Lithium-ion battery charger (page A.10) or the Lead-Acid solar panel battery charger was switched on, the 8-bit microcontroller also turned on one of the LEDs. In a similar fashion to the rocker switches, the microcontroller software allowed for any of the LEDs to be disabled or re-assigned to another function. This saved power when the SAS2 system was configured in the stationary application.

The 8 -bit microcontroller software was written in the C programming language. The software implemented a user interface with a command-prompt over two serial ports ( 0 V to 3.3 V levels) at 115200 baud. One serial port was connected to a Bluetooth transceiver (page A.6), allowing the SAS2 system to be remotely controlled by a device such as a laptop. The other serial port was connected to a header for debugging. Removable jumpers on the circuit board also connected this serial port to a serial port of a 32-bit microcontroller by crossing RX/TX lines.

The 8 -bit microcontroller communicated with the 32 -bit microcontroller during system operation. The user interface implemented by the 8 -bit microcontroller enabled setting and reading of the system real-time clock, re-assignment of the LEDs and switches, and for checking voltage inputs and enabling battery chargers. The 8 -bit microcontroller also set up a real-time clock interrupt so that the system could be autonomously turned on once every hour to take a measurement. Once the measurement was complete, the 32 -bit microcontroller would send a command to the 8 -bit microcontroller, instructing that the system was to be turned off. A 15minute "kill switch" was programmed in the 8 -bit microcontroller software to turn the main system power off is the system malfunctioned.

A 5 V-tolerant buffer and inverter were used to sense a USB plug event when a 5 V powered USB cable was plugged into the SAS2. The output of the buffer and inverter pair was debounced using a hardware circuit. The debounced output was connected to a port pin of the 8bit microcontroller. The microcontroller software registered an interrupt for the port pin so that a change in voltage level associated with a USB plug event would turn on the SAS2 system. This enabled data to be transferred between the SAS2 and a computer.

To turn on the system, the 8 -bit microcontroller used a port pin connected to an FET gate driver IC that provided enough current to saturate an N-Channel FET acting as a switch for main
system power. For maximum power efficiency, three DC/DC buck-boost switcher circuits with inductors (pages A. 11 to A.13) were used to provide one 3.3 V and two 5 V power rails currentlimited to 2 A. The converters followed the design proposed by Ćuk and Middlebrook (1976), permitting the input voltage to be greater or less than the output voltage. The 3.3 V supply powered most of the ICs on the circuit board, including the microcontroller I/O ports. One 5 V supply powered the loudspeaker audio amplifier, whereas the other 5 V supply provided power for the microphone PCB. Power for the main microcontroller core and the FPGA core were provided by three step-down DC/DC switcher circuits with 1 V and 2.5 V outputs (pages A. 14 to A.15). All analog power circuits were designed using SPICE simulations to ensure proper operation under operating loads and environmental temperatures. A switched-capacitor inverter circuit (page A.15) provided a -5 V power supply rail for the Digital-to-Analog Converter (DAC) and op-amp signal chain use to produce the acoustic waveform.

The main microcontroller (page A.16) of the system was a 32-bit ARM9 processor (Atmel AT91SAM9G20B) running the GNU/Linux operating system. The operating frequency of the processor was 400 MHz . GNU/Linux was chosen as the OS for the SAS2 since the Free and Open Source (FOSS) kernel could be easily modified, and there were pre-existing device drivers already available for serial ports and the SD card. All other device drivers for the SAS2 hardware were custom-written for the SAS2 in the C programming language. The userspace program running on the embedded system was written in a mixture of $\mathrm{C} / \mathrm{C}++$, with low-level program code written exclusively in C for easy interfacing with the GNU/Linux system (file I/O, device driver accessing and sysfs support). Startup scripts were also written for the embedded system shell. Further information on device drivers and embedded Linux system development is given by Yaghmour et al. (2008) and Venkateswaran (2008).

When the system was turned on, the 8-bit microcontroller power-sequenced the processor core $\mathrm{DC} / \mathrm{DC}$ switchers. The processor core switchers were enabled 50 milliseconds after the 3.3 V power rail to ensure that transient inrush current at startup did not damage the processor. At startup, the AT91SAM9G20B embedded SAM-BA bootloader checked an SPI flash memory chip (page A.17) for a secondary bootloader (AT9Bootstrap), modified specifically for the SAS2 system. The secondary bootloader set up system clocks, enabled processor communication buses, and then loaded the U-Boot tertiary bootloader. As of the time of writing, the U-boot bootloader is widely used for booting Linux kernels on embedded systems, and it provides a user
interface with a command prompt over a serial port link (Yaghmour et al., 2008). The U-boot bootloader was responsible for loading the Linux kernel into volatile SDRAM memory from a UBIFS partition on the NAND flash IC. The total amount of SDRAM memory on the circuit board was 128 MB and the capacity of the NAND flash was 500 MB . The root file system of the NAND flash was stored on a separate UBIFS partition.

An unused I2C master connected to the main processor via I2C buffers allowed for external interfacing of add-on circuits via a male pin-head connector (page A.18). Moreover, a serial port level-shifter (page A.21) ensured that $-5 /+5 \mathrm{~V}$ levels were present on a header intended for interfacing either a datalogger or a modem. This allowed the SAS2 system to communicate with other embedded systems or for communications via a satellite or cellular phone modem. SDIO WiFi interfaces and radio modem interfaces were also built into the system. These features were intended for future application and were not used during deployment of the SAS2.

A full-speed ( $12 \mathrm{Mbit} / \mathrm{s}$ ) USB transceiver on the main processor was connected to a waterproof Mini-B connector on the side of the enclosure case via a header and an IC with an integrated USB bus switch and SD card switch (page A.20). The IC was capable of disconnecting the USB bus and the SD card bus from the main processor and emulating a mass storage device as the SD card. This allowed for a USB cable to be plugged into a laptop computer, and for data to be transferred from the embedded system at USB High Speed (480 $\mathrm{Mbit} / \mathrm{s}$ ) rates. A 5 V -tolerant buffer IC connected to a processor port pin enabled the userspace program to determine if a USB plug event occurred. This was accomplished by the main userspace program registering an interrupt with the Linux kernel using the sysfs kernel subsystem.

Data was sampled from each of the microphones on the microphone PCB by an FPGA (pages A. 25 to A.30), a reconfigurable logic device situated on the main PCB. The FPGA logic design was created using the Verilog hardware description language (HDL). Because FPGA devices cannot be debugged over JTAG, testbenches were created to model system operation before the design was synthesized in hardware.

An FPGA had to be used to sample the microphones due to the 2.1 MHz sampling rate. The high sampling rate did not permit a single microcontroller to be used due to latency involved in collecting and processing the data. The FPGA was also responsible for the cross-correlation operation required in MLS signal processing.

The digital data from the microphone PCB was streamed to the FPGA over a ribbon cable (page A.30). To ensure signal integrity, each digital bus line on the ribbon cable was paired with a ground wire acting as a shield. This entailed that data was transferred using every second wire on the ribbon cable. On the main PCB, a bus buffer with rail-to-rail switching further ensured signal integrity of the logic signals from the microphones.

The FPGA was responsible for producing a Maximum Length Sequence (MLS) signal using a Digital-to-Analog Converter (DAC) and at the same time sampling acoustic data from each of the microphones (page A.31). The analog design of the DAC and associated op-amp output buffer was conducted with reference to Terrell (1996), and the books edited by Jung (2005) and Kester (2005). The sampling rate of the DAC was 10 kHz , whereas the sampling rate of each microphone was 2.1 MHz . The acoustic data from the microphones was initially cached in a 64 MB SDRAM attached to the FPGA. Volatile SDRAM memory was chosen in lieu of simpler solutions such as SRAM due to the large memory density of the SDRAM. Although this increased the complexity of the logic design expressed in Verilog, it enabled larger amounts of acoustic data to be cached during sampling. Pages A. 32 to A. 34 are the schematics for the Microphone PCB.

The output of the audio DAC was connected to the input of a 3.5 W audio amplifier (page A.31). A loudspeaker with $8 \Omega$ impedance was connected to the output of the audio amplifier. The loudspeaker as an electromagnetic transducer was responsible for producing the audible MLS sequence. Because the diaphragm of the loudspeaker was exposed to the outside of the enclosure case, a waterproof loudspeaker was chosen.

The FPGA was interfaced with the main processor via an SRAM bus. The SRAM bus allowed for the microphone data to be quickly transferred to the main processor by having the FPGA act similar to an SRAM integrated circuit. The FPGA state machine also served as an I2C slave, allowing the main processor I2C master to send the FPGA state machine various commands over the I2C bus.

An acoustic measurement was triggered by the 8 -bit microcontroller communicating with the main processor over the serial port connection. The 8 -bit microcontroller software permitted acoustic measurements to be requested "on demand" using the Bluetooth interface, or on a set time schedule ranging from minutes to hours. On startup, the userspace program running on the main processor registered an interrupt with the serial port connected to the 8-bit microcontroller.

The userspace program then waited in a while loop until the 8 -bit microcontroller sent a command over the serial port. After a command was sent and a serial port interrupt occurred, the command string was then read from the serial port char device and matched to a list of possible commands.

After receiving a serial port command requesting an acoustic measurement, the main processor would send an I2C command to the FPGA state machine. The FPGA state machine logic would produce the MLS using the DAC and audio amplifier, and at the same time samples of digital data would be taken from the microphone bus. This is a well-suited operation for an FPGA because unlike most microcontrollers, there can be no sampling latency associated with FPGA logic devices (Maxfield, 2004). Moreover, this eliminates the latency inherent by design in GNU/Linux, despite the availability of various patches and techniques to make this OS a "real -time" operating system (Abbott, 2012; Yaghmour et al., 2008). Once the full MLS sequence had been produced, the FPGA logic would stop sampling microphone data.

During the sampling operation, the microphone data was stored in the SDRAM connected to the FPGA. The main processor polled an I2C slave register exposed by the FPGA state machine to determine when microphone data was available for reading from the SRAM bus. Once the sampling operation was complete and data was available for reading, the userspace program would send another command via sysfs and the I2C bus to switch the FPGA state machine over to SRAM emulation mode. The SRAM emulation mode permitted the main microcontroller to read acoustic data from the SDRAM attached to the FPGA as if the SDRAM was an SRAM.

Reading of acoustic data from the FPGA SRAM interface was accomplished using a custom char device driver written for the GNU/Linux OS. The userspace program would then store the acoustic data in an array. The array was cached in local SDRAM memory attached to the main processor so it could be passed to a function responsible for running an acoustic model to extract snowpack parameters.

Since the SD card was mounted as a directory on system startup, the GNU/Linux userspace program utilized ordinary file I/O operations to write the acoustic data to SD card along with a timestamp requested from the real-time clock interfaced with the 8-bit microcontroller. To provide visual indication of the end of the sampling process, the main processor instructed the 8 -bit microcontroller to toggle one of the bicolour LEDs. Alternately, if
the 8-bit microcontroller had turned the system on for acoustic measurements taken autonomously on a time schedule (i.e. once an hour), power to the main processor would be turned off after the SD card and other file systems had been un-mounted. Un-mounting of file systems is necessary before shut-down to make the SAS2 power off in a known state, and to ensure that the non-volatile UBIFS on the NAND flash is not inadvertently corrupted, despite the resilience of this file system to power cuts.

### 4.3.4. Microphone Selection

The microphones used for the SAS2 are classified as Microelectromechanical systems (MEMS) technology (Lyshevski, 2005), where the sensor is a mechanical device with physical dimensions measuring in the micrometer to millimeter range. The microphone selected was the omnidirectional ADMP421 from Analog Devices (Norwood, Massachusetts, USA). This microphone was initially selected for its small size ( 3 mm by 4 mm by 1 mm ), enabling an array of microphones to be created on a PCB. Since the microphone is small relative to the acoustic wavelengths of sound being measured by the SAS2 system, it is suitable for measuring lower frequencies of sound. When the wavelength of the incident sound pressure wave approaches the size of the microphone, acoustic measurements will be inaccurate due to reflections from the microphone itself (Raichel, 2006).

Moreover, since the microphone contains an Application-Specific Integrated Circuit (ASIC) (Barr, 2006) with an oversampling 1-bit Delta-Sigma ADC (Candy and Temes, 1991), there is no need to digitize voltages from each microphone. The digitized acoustic data is available as a Pulse-Density Modulated (PDM) bitstream sent over the ribbon cable to the FPGA on the main circuit board. This eliminates the need for an op-amp and ADC front-end for each microphone and simplifies signal processing, as discussed in Chapter 2.

The digital microphone sensitivity was specified in the datasheet as typically -26 dB fullscale, measured by the manufacturer using a nominal 1 kHz source at 94 dB SPL. The input noise level for the digital microphone is $L_{n}=32 \mathrm{~dB}$ SPL (Lewis, 2011). This represents the smallest SPL that can be measured by the microphone, since once a signal amplitude is less than the inherent noise level of the sensing system, it cannot be measured (Smith, 2002). Since the microphone is an electromechanical measuring device (Kinsler and Frey, 1962), there may be changes in the noise level with changes in temperature, air pressure, and relative humidity. A
measurement of changes in microphone noise level is outside of the scope of this thesis since it would require an anechoic chamber with adjustable environmental parameters.

For purposes of assessing the suitability of the ADMP421 for the SAS2 acoustic sensing application a sensitivity analysis was conducted. As shown in Figure 4.9, assume that the SAS2 is situated above a snowpack at a maximum distance of $y_{a}$. The microphone assembly and loudspeaker are situated in a bistatic sonar configuration above the surface of the snowpack. As an approximation, the snowpack is taken to be an effective acoustic medium. This is similar to the notion of lumped models in hydrology (Ponce, 1989). Furthermore, assume that under the snowpack is a frozen ground surface that reflects the sound wave.

Assume that an MLS is sent from the loudspeaker into the snow. The sound wave travels through the snowpack, is reflected by the ground surface, and is detected in the air medium by an ADMP421 microphone situated in the microphone assembly.


Figure 4.9. Physical system used for conducting a sensitivity analysis used for microphone selection. A loudspeaker and microphone assembly are situated at a distance of $y_{a}$ above the surface of the snowpack in a bistatic sonar arrangement. The snow depth is $y_{s}$, and the "lumped" attenuation coefficient of the snowpack is $a_{c}$. The RMS wave pressure at the
surface of the snowpack is $p_{1}$, whereas the RMS pressure of the reflected wave exiting the snowpack is $p_{2}$. The RMS pressure of the sound wave at the microphone assembly is $p_{3}$.

The attenuation coefficient of snow at the maximum MLS bandwidth frequency is given by $a_{c}$. Taking the snowpack as an effective acoustic medium $a_{c}$ is the mean attenuation coefficient for the snowpack. Since, in a similar fashion to most porous media (Allard and Atalla, 2009; Stoll, 1989), the attenuation of sound in snow is greater at higher frequencies, this $a_{c}$ value represents the maximum attenuation coefficient that can be expected for snow. The attenuation of sound in air is negligible for audible frequencies of sound less than 20 kHz (Rossing and Fletcher, 1995).

The loudspeaker is assumed to be a point source situated on a reflective surface halfspace that is the SAS2 enclosure case. As measured in the preceding section, the SPL of the loudspeaker source is taken to be $L_{s}=120 \mathrm{~dB}$ at a distance of $y=5 \mathrm{~cm}$ from the dust cap of the loudspeaker source.

Although sound pressure and sound power are not the same physical quantities, the sound power of the source is estimated in Watts as $P$ using Equations (4.1) and (4.2) below. Assuming hemispherical propagation of a point source, Equation (4.1) is given by Chadderton (2013), whereas Equation (4.2) is given by Crocker (1998), with a reference pressure $P_{0}$. Both equations are presented in re-arranged versions. The directionality factor $S_{0}=2 \mathrm{~m}^{2}$ is for a sound source on a reflective surface half-space.

$$
\begin{gather*}
L_{\mathrm{W}} \approx L_{s}-10 \log _{10}\left(\frac{S_{0}}{4 \pi y^{2}}\right) \approx 102 \mathrm{~dB}  \tag{4.1}\\
P=P_{0} 10^{L_{\mathrm{W}} / 10}=0.0158 \mathrm{~W}  \tag{4.2}\\
P_{0}=1.0 \times 10^{-12} \mathrm{~W}
\end{gather*}
$$

For hemispherical propagation of the wave in the air medium, the RMS pressure $p_{1}$ of the sound wave from the loudspeaker at the snow surface is (Raichel, 2006):

$$
\begin{gather*}
p_{1}=\left(\frac{\rho_{a} c_{a} P}{2 \pi y_{a}^{2}}\right)^{1 / 2}  \tag{4.3}\\
\rho_{a}=\frac{p_{a}}{R T}  \tag{4.4}\\
c_{a}=\left(\chi_{g} R_{g} T\right)^{1 / 2} \tag{4.5}
\end{gather*}
$$

In the above Equation (4.3) to Equation (4.5), $\rho_{a}$ is the density of the air in the medium above the surface of the snowpack, $c_{a}$ is the speed of sound in the air, $y_{a}$ is the distance from the SAS2 to the surface of the snowpack, $p_{a}$ is the barometric air pressure, $\gamma=1.4$ is the adiabatic index for air, the gas constant for air is $R_{g}=287.058 \mathrm{~J} \mathrm{~kg}^{-1} \mathrm{~K}^{-1}$, and the temperature $T$ (Kelvin). Equation (4.4) is the ideal gas law, and Equation (4.5) is given by Raichel (2006). After acoustic-to-seismic coupling of the sound wave to the snow medium (Albert and Hole, 2001; Albert, 2001; Boeckx et al., 2004), the round-trip attenuation of the sound wave through the snowpack as a porous medium is modelled as the incident pressure $p_{1}$ scaled by an exponential function (Meyer and Neumann, 1972). Sound pressure transmission coefficients (Blackstock, 2000; Ingard, 1988) represent the fractional amount of sound pressure energy transmitted through the air-snow and snow-air interfaces. The transmission coefficients are $\Lambda_{1}$ and $\Lambda_{2}$. The pressure reflection coefficient (Bjørndal and Frøysa, 2008; Blackstock, 2000) at the ground surface is taken to be the dimensionless value $\Gamma_{r}$. For the pressure wave exiting the snowpack medium, the RMS pressure is expressed as:

$$
\begin{gather*}
p_{2}=p_{1} \exp \left(-2 a_{c} y_{s}\right) \varsigma  \tag{4.6}\\
\varsigma=\left|\Lambda_{1}\right|\left|\Lambda_{2}\right|\left|\Gamma_{r}\right| \tag{4.7}
\end{gather*}
$$

In Equation (4.6), $a_{c}$ is the dimensionless attenuation coefficient at its maximum value and $y_{s}$ is the snow depth. Assuming that the wave exiting the snowpack has spherical spreading in the air medium, the RMS pressure of the sound wave arriving at the microphone is given by (Smith, 2010):

$$
\begin{equation*}
p_{3} \approx \frac{p_{2}}{\left(y_{a}\right)\left(m^{-1}\right)} \tag{4.8}
\end{equation*}
$$

The pressure $p_{3}$ is related to the SPL in decibels by (Raichel, 2006):

$$
\begin{gather*}
L_{3}=20 \log _{10}\left(\frac{p_{4}}{p_{\text {ref }}}\right)  \tag{4.9}\\
p_{r e f}=20 \times 10^{-6} \mathrm{~Pa}
\end{gather*}
$$

For the sensitivity analysis, assume that the model parameters are within the following ranges, with a baseline parameter being situated near the center of each range. The baseline parameter is the parameter that is held constant in one-at-a-time (OAT) sensitivity analysis designs (Saltelli et al., 2009). The parameter ranges used for the sensitivity analysis are:

- $-40^{\circ} \mathrm{C} \leq T \leq 0^{\circ} \mathrm{C}$ for the temperature range, with the lowest temperature coincident with the minimum operating temperature of the SAS2 device. The baseline temperature is $-20^{\circ} \mathrm{C}$.
- $30 \mathrm{kPa} \leq p_{a} \leq 120 \mathrm{kPa}$ as the normal range of barometric air pressures. This range includes most barometric air pressures at locations in Canada and on mountain tops. The baseline air pressure is 100 kPa .
- $\quad 0 \mathrm{~Np} \mathrm{~m}^{-1} \leq a_{c} \leq 2.0 \mathrm{~Np} \mathrm{~m}^{-1}$ as the range of maximum snow attenuation coefficients. This range was selected to show the threshold value of $L_{n}=32 \mathrm{~dB}$ SPL for the microphone sensitivity, and also to be coincident with the nominal range of snow attenuation coefficients measured by the experiments reported in this thesis. Similar snowpack attenuation coefficients have also been reported by Ishida (1965). The baseline attenuation coefficient is 1.0 , in the center of this range.
- $0 \leq \varsigma \leq 1.0$ as the range of transmission and reflection coefficients. This range includes the complete transfer of sound waves through the snowpack ( $\varsigma=1.0$ ) and the complete loss of reflected sound waves due to transmission and reflection $(\varsigma=0)$. The complete loss of reflected sound waves will occur if there is a large difference in magnitude between the effective acoustic impedances of the porous medium and the air. Although it is possible for transmission coefficients to be greater than unity, particularly for ocean waves breaking near the shore (Claerbout, 1985),
this phenomenon is not addressed within the context of this model. The baseline parameter is $\varsigma=0.5$.
- $5 \mathrm{~cm} \leq \mathrm{y}_{a} \leq 3 \mathrm{~m}$ as the distance from the loudspeaker and microphone array to the snow surface. The minimum value of the range is selected as being coincident with the closest distance that the microphones are placed to the snow surface using the portable configuration, whereas the maximum range is selected as being the maximum distance above the snow surface that the SAS2 would be mounted in a stationary configuration on a met station. The baseline value for this parameter is 1.5 m .
- $5 \mathrm{~cm} \leq y_{a} \leq 5 \mathrm{~m}$ as the depth of snow. This includes the range of snow depth found on the prairies (Pomeroy and Gray, 1995) and in some areas of the Rocky Mountains (Erxleben et al., 2002). Although snow depth in the mountains can greatly exceed 5 m (Hood and Hayashi, 2010), particularly for snow accumulation in glaciated areas (Martini et al., 2001), and for Arctic snowpacks that have accumulated in gullies (Woo and Sauriol, 1980), this sensitivity range is chosen to demonstrate the limitations of the acoustic sensing method relative to the microphone technology. The baseline value for this range is 2.5 m .

Figure 4.10 shows a sensitivity analysis of the model parameters. Changes in temperature (Figure 4.10a) cause an approximately linear drop in SPL received at the microphone. Since the change in temperature is associated with a reduction of SPL less than 1 dB , the ability for the microphone to resolve acoustic reflections is only weakly dependent on temperature, given that the noise level does not appreciably change over the temperature range. Alternately, changes in air pressure (Figure 4.10b) demonstrate an approximately linear increase in SPL as well, but the sensitivity of this parameter is similar to that of temperature.

The attenuation coefficient for the snowpack exhibits high sensitivity (Figure 4.10c), with an 80 dB drop in received SPL as $a_{c}$ varies over the range $0 \mathrm{~Np} \mathrm{~m}^{-1} \leq a_{c} \leq 2.0 \mathrm{~Np} \mathrm{~m}^{-1}$. The dashed line on the plot of Figure 4.10 c is the input noise level of the microphone at $L_{n}=32 \mathrm{~dB}$ SPL, showing that when $a_{c} \geq 1$, the SPL of the reflection at the microphone is under the input noise level and cannot be detected. The reduction in SPL is approximately linear with an increase in $a_{c}$.


Figure 4.10. Sensitivity analysis of model parameters. The dashed line on the plots represents the $L_{n}=32 \mathrm{~dB}$ SPL noise level of the microphone. If the SPL level of the reflection is smaller than the noise level it cannot be detected.

The reflection/transmission coefficient parameter $\varsigma$ exhibits an approximately hyperbolic increase over its range (Figure 4.10d), with the received SPL exceeding the input noise level for $\varsigma>0.25$. This suggests that at least $25 \%$ of the RMS sound pressure energy must be transmitted and reflected through the snowpack for the ADMP421 microphone to detect a pressure wave and make an acoustic measurement.

Figure 4.10 e shows that as the loudspeaker and microphone assembly moves further away from the snow surface, the SPL at the microphone exhibits an approximately hyperbolic decrease in SPL. When the distance to the snow surface exceeds 2 m , and the snowpack depth is 1.5 m with $\varsigma=0.5$ and $a_{c}=1.0$, the reflection from the snowpack is under the noise level of the microphone and cannot be detected.

Figure 4.10 f shows that as the snowpack depth increases, the received sound pressure at the microphone exhibits an approximately linear decrease. The received SPL is lower than the microphone noise level for snow depths approximately greater than 3 m , demonstrating that the snow depth is also a limiting factor for the detection of reflections from the snowpack.

This sensitivity analysis demonstrated that the ADMP421 microphone is a good choice for the SAS2 system due to the low noise level of $L_{n}=32 \mathrm{~dB}$ SPL. Given a range of parameters for snow, a microphone with this sensitivity level is able to detect reflections from the snowpack.

### 4.3.5. Microphone Sampling Rate

The sampling rate of each ADMP421 microphone was $f_{s}=2.1 \mathrm{MHz}$. Although this sampling rate is within the range of recommended sampling rates specified by the manufacturer, it is customary to use a Cascaded-Integrator-Comb filter (Lyons, 2004; Meyer-Baese, 2007) to downsample the microphone data to a professional audio rate of 96 kHz (Hegde, 2010).

Data was sampled from the microphones at a rate of $f_{s}=2.1 \mathrm{MHz}$ and not decimated until later in the signal processing chain, as discussed in Chapter 3. This was done for three reasons: (1) The MLS reference waveform was measured in an anechoic chamber at the same sampling rate, and the cross-correlation operation to obtain the snowpack filter kernel required that the signals be sampled at the same rate . (2) The sampling rate had to be high enough to resolve spatial differences in time of arrival of a reflected sound pressure wave in order to perform sound speed analysis using seismic algorithms (Claerbout, 1992; Fomel, 2007a). The $f_{s}=2.1 \mathrm{MHz}$ sampling rate was found to be sufficient for this application with the SAS2 system. (3) To prevent temporal aliasing, the sampling rate $f_{s}>2 f_{\max }$ must be greater than the
maximum frequency component $f_{\text {max }}$ in the received signal. This is the Nyquist criterion (Lyons, 2004; Smith, 2002). Since $f_{s}=2.1 \mathrm{MHz}$ is much greater than audible frequencies of sound, $f_{\text {max }}=20 \mathrm{kHz}$ and $f_{s} \gg 2 f_{\text {max }}$.

### 4.3.6. DAC Output and MLS Signal Amplitude

Using a DAC voltage of maximum 2.0 V to generate the MLS and a maximum total voltage gain of 36 dB at the audio amplifier results in an MLS being produced by an $8 \Omega$ loudspeaker at an A-weighted Sound Pressure Level (SPL) of $\sim 120 \mathrm{~dB}$, measured at a distance of $\sim 5 \mathrm{~cm}$ from the dust cap of the loudspeaker source in an anechoic chamber with an SPL meter.

The DAC voltage was selected to ensure that the DAC could produce an MLS without large amounts of distortion at the slew rate of the DAC op-amp output buffer. The slew rate is the maximum rate of change of the output voltage (Terrell, 1996). In addition, selection of a 2.0 V input to the audio amplifier ensured that "clipping" did not occur, where the audio amplifier is driven to produce a voltage output greater than its maximum rating (Cordell, 2010). The maximum DAC voltage was fixed and not changed over the entire time of operation of the SAS2 system. Some small changes in the output voltage level with temperature will occur due to tolerances of the electronic components (Kester, 2005). This was mitigated by sensitivity analysis of component changes with temperature during the design phase of the SAS2, and by using higher-precision tolerance of components (i.e $0.1 \%$ tolerances).

The input to the audio amplifier was band-limited by a built-in filter at the audio amplifier input. This helped to reduce the distortion (Vanderkooy, 1994) caused by the binary MLS. For future designs, an RC input filter could also be used as a low-pass input filter to the audio amplifier, or the MLS signal could be passed through a digital Butterworth filter before being produced by the ADC. The MLS signal could also be produced by the port pin of an FPGA.

### 4.3.7. Sound Pressure Level

Ideally, the higher the SPL of the sound wave near the source, the greater the penetration depth of snow and sound wave reflections will arrive back at the microphone array with SPL levels above the input noise level. Due to the fixed maximum voltage output of the DAC, the
maximum SPL of the loudspeaker was close to 120 dB and not changed for any of the experiments conducted with the SAS2 system.

The SPL near the loudspeaker cone is not high enough to cause permanent modification of the snowpack, as occurs when explosives are used for clearing minefields in snow-covered areas (Albert, 1983; Brown, 1979; Johnson, 1990). Since the RMS pressure of a 120 dB source signal is 20 Pa , this value is orders of magnitude lower than the Megapascal range of applied pressures that permanently deform the porous ice skeleton of snow (Johnson et al., 1993). However, additional testing could be done to determine the safety of acoustic system use for human beings with respect to daily noise doses (Raichel, 2006) or the effect of the noise on animals (West et al., 2007), particularly those living in the subnivian layer (Aitchison, 2001).

### 4.3.8. Length and Sampling Rate of MLS

For MLS testing applications, the duration of the MLS produced by the loudspeaker must exceed the response time of the physical system being tested (Schneider and Jamieson, 1993). Because of this criterion, the duration of the MLS for the SAS2 was selected to ensure that the temporal truncation of the snowpack response did not occur. Let the reflection from the snowpack have a duration $t_{m}$, and let $\left(L_{M}=2^{m}-1\right) \in \mathbb{N} \backslash\{0\}$ be the length of the MLS sequence, with $m \in \mathbb{N} \backslash\{0\}$. For a DAC sampling rate $f_{d s}$ the duration $t_{m l s}$ of the MLS sequence is:

$$
t_{m l s}=\frac{L}{f_{d s}}
$$

The MLS duration $t_{m l s}$ must exceed the maximum time taken for the sound wave to propagate through the snowpack. Moreover, the bandwidth $B$ of the MLS signal has to be low enough to permit acoustic wave penetration of the snowpack. The bandwidth of the MLS signal is given by (Rife and Vanderkooy, 1989):

$$
B=\left(\frac{f_{d s}}{L}\right)\left(\frac{L-1}{2}\right)
$$

To ensure that the DAC can produce an MLS sequence without large amounts of distortion, the DAC sampling rate was chosen as $f_{d s}=20 \mathrm{kHz}$. Then, the mathematical analysis below was used to select the MLS length $L$.

Using the same physical system as shown in Figure 4.9 above and Equations (4.5) and (3.75), simple kinematics was used to calculate the minimum value of $L_{\text {min }}$ required for a sound wave to travel from the loudspeaker through the air and snow mediums:

$$
\begin{equation*}
L_{\min } \geq\left\lceil f_{d s}\left(\frac{2 y_{a}}{\left(\chi_{g} R_{g} T\right)^{1 / 2}}+\frac{2 y_{s}}{c_{s}}\right)\right\rceil \tag{4.10}
\end{equation*}
$$

In the above Equation (4.10), $c_{s}$ is the speed of the Biot slow wave in the pore spaces of the effective snowpack medium, $\operatorname{ceil}(x)=\lceil x\rceil$ represents the ceiling function, and all other parameters are the same as those previously defined.

The air temperature $T$, distance to the snowpack surface $y_{a}$ and snow depth $y_{s}$ are taken as having the same ranges as parameters used for the sensitivity analysis in Chapter 3. The range of the slow wave speed in snow is taken as $10 \mathrm{~m} \mathrm{~s}^{-1} \leq c_{s} \leq 331 \mathrm{~m} \mathrm{~s}^{-1}$ with recourse to the observations made in this paper and the observations made by Ishida (1965) and Johnson (1982).

The value for $L_{M}$ is maximized when $y_{a}=3 \mathrm{~m}, y_{s}=5 \mathrm{~m}, \quad T=-40^{\circ} \mathrm{C}$, and $c_{s}=10 \mathrm{~m} \mathrm{~s}^{-1}$. When Equation (4.10) is evaluated using these parameters, $L_{M} \geq 20393$. Using Equation (3.75), this value for $L_{M}$ coincides with $t_{m l s}=1 \mathrm{~s}$. To increase the noise immunity of the MLS measurement, the length of the MLS sequence was increased to $L_{M}=32767$, corresponding to a total measurement time of $t_{m l s}=1.6 \mathrm{~s}$. Increasing the length of the MLS increases the signal-to-noise ratio (SNR) of the signal by spreading the signal energy over a longer time (Dunn and Hawksford, 1993). In addition, the choice of $L_{M}$ must also be made to ensure that sampled data from the microphones is small enough to be stored in SDRAM memory by the FPGA.

With $L_{M}=32767$ and $f_{d s}=20 \mathrm{kHz}$, the MLS bandwidth is $B=10 \mathrm{kHz}$. This is the maximum effective bandwidth of sound propagation through snow identified by Lang (1976). Previous experiments sending MLS sound waves through snow have also demonstrated the applicability of frequencies in the range of 20 Hz to 20 kHz for acoustic sensing of snow (Kinar and Pomeroy, 2009).

### 4.3.9. Loudspeaker

The loudspeaker used for the SAS2 system was the GF1004M from CUI Inc (Tualatin, OR). The frequency response curve of the loudspeaker had a low frequency limit of $\sim 20 \mathrm{~Hz}$. The response varied over 10 dB between 200 Hz and 10 kHz . This makes the loudspeaker suitable for sending the MLS with a maximum bandwidth of 10 kHz into snow. The cone of the loudspeaker was made of waterproof plastic, allowing it to be used in outdoor applications. The diameter of the loudspeaker was $d=9 \mathrm{~cm}$.

In a similar fashion to sonar transducers, the diameter of the loudspeaker relative to the size of the enclosure case was chosen to be as large as possible to maximize the acoustic power output of the source and maximize the emitted low frequencies (Wilson, 1989). Since lower frequencies have less attenuation in porous media than higher frequencies (Allard and Atalla, 2009), a large loudspeaker will ensure that low frequencies will be generated and sent into snow, thereby ensuring maximum propagation depth of the sound wave for frequencies in the MLS bandwidth.

### 4.3.10. Loudspeaker and Microphone Array Separation

The microphone array was situated at an offset distance of 10.8 cm from the loudspeaker. In a similar fashion to bistatic sonar (Cox, 1989) or radar (Willis, 1991), this offset distance was selected to ensure that the maximum input SPL of the microphones was not exceeded during production of the MLS by the loudspeaker. This also ensured that the direct wave (Burger et al., 2006) from the emitted source signal did not mask reflections from the snowpack. In the context of the SAS2 system, the direct wave is the acoustic wave that travels between the loudspeaker and microphones in the air medium and inside the enclosure case without propagating through the snowpack. This offset distance also separated the loudspeaker and the microphones so that the sampling system is similar to a common shot-gather in seismic data collection (Liner, 2004).

Increasing the distance between the loudspeaker and micropone array would increase the relative time of arrival of the sound wave between adjacent microphones, thereby allowing for a smaller microphone sampling rate to be used and potentially increasing the amount of information that can be used in the UTAM model. However, this would not permit the sampling system to be placed into a single enclosure case.

### 4.3.11. Array Design and Footprint

As shown in Figure 4.11, the microphones were situated in a rectangular grid comprised of 6 rows by 4 columns. For signal processing only using array elements along rows or columns, the planar array becomes a linear array of elements arranged along a line (Skolnik, 2001).

The planar rectangular array (Milligan, 2005) is normally used for beam-scanning, a digital signal processing procedure where the array can be effectively turned to any angle in a hemisphere to eliminate unwanted reflections from a particular direction. Wideband beamscanning (Liu and Weiss, 2010) is appropriate for use with a microphone array due to the large bandwidth of acoustic transducers.


Figure 4.11. Planar microphone array. The spacing between microphones is $x_{a}$ and $\theta$ is the incidence angle of the sound wave relative to the front of the array. There are 6 rows by 4 columns of microphones. Each horizontal row is a linear array "A" and each column is a linear array " $B$ " as marked on the figure.

However, because the microphone holes in the enclosure case acted similar to waveguides for the received acoustic pressure wave, thereby rejecting some spurious reflections from directions other than the snowpack beneath the sensor assembly, beam-scanning was not used for the experiments reported in this paper. This is suitable when the sensing array is situated close to the surface of the snowpack. Only linear arrays in the directions A and B shown in Figure 4.11 were used. This idea is justified below.

To ensure that spatial aliasing of waves received by the microphone array did not occur, the inter-element spacing $x_{a}$ of the array elements was chosen as (Barger, 1998):

$$
\begin{align*}
& x_{a}<\lambda / 2  \tag{4.11}\\
& \lambda=c_{a} / f_{a} \tag{4.12}
\end{align*}
$$

In Equation (4.11) and (4.12), $\lambda$ is the wavelength of the sound pressure wave incident on the array, $c_{a}$ is the speed of the sound wave, and $f_{a}$ is the frequency. For $-40^{\circ} \mathrm{C} \leq T \leq 0^{\circ} \mathrm{C}$, the speed of sound in air is $306 \mathrm{~m} \mathrm{~s}^{-1} \leq c_{a} \leq 331 \mathrm{~m} \mathrm{~s}^{-1}$. Taking the effective range of frequencies for the array as $20 \mathrm{~Hz} \leq f_{a} \leq 10 \mathrm{kHz}$, the minimum value of $x_{a}$ is such that $x_{a}<1.5 \mathrm{~cm}$. Given this constraint, the microphone separation was chosen as $x_{a}=7.5 \mathrm{~mm}$. This constraint was also affected by the spacing and layout of the microphones on the PCB.

Assuming an unshaded array, the magnitude of the beam pattern $D_{d B}(\theta)$ as a function of sound wave incidence angle $\theta$ of the "A" linear array marked on Figure 4.11 was calculated using Equations (4.13) to (4.15) from Barger (1998). Equation (4.16) is defined by Liu and Weiss (2010).

$$
\begin{gather*}
D(\theta)=\left(\frac{\sin (N X)}{\sin (X)}\right)^{2}  \tag{4.13}\\
X=\frac{1}{2} k x_{a} \sin (\theta)  \tag{4.14}\\
k=\frac{2 \pi f_{a}}{c_{a}}  \tag{4.15}\\
D_{d B}(\theta)=20 \log _{10} \frac{|D(\theta)|}{\max |D(\theta)|} \tag{4.16}
\end{gather*}
$$

In the above Equations (4.13) to (4.15), $N$ is the number of microphones in the linear array, and $k$ is the wavenumber. For speeds of sound in air ranging between $306 \mathrm{~m} \mathrm{~s}^{-1} \leq c_{a} \leq 331 \mathrm{~m} \mathrm{~s}^{-1}$ and for frequencies $20 \mathrm{~Hz} \leq f_{a} \leq 10 \mathrm{kHz}$, Figure 4.12 shows how the beam pattern of the array
changes for $N=6$ microphones over sound wave incidence angles of $\pm 90^{\circ}$. The $N=4$ case is similar to Figure 4.12, but with a smaller number of side lobes and a main lobe of slightly greater width. The main lobe is the maximum magnitude response of the array, whereas the side lobes represent smaller magnitude responses (Milligan, 2005).


Figure 4.12. Beam pattern of the array for $N=6$ microphones as a function of the angle of incidence $\theta$ of the sound wave. The dashed lines represent the width of the main lobe. For Figures 3.12 c and 3.12d, the main lobe width is situated close to $\theta= \pm 90^{\circ}$.

For an incident wave frequency of 10 kHz and a sound speed in the air medium of $306 \mathrm{~m} \mathrm{~s}^{-1}$ coinciding with a temperature of $-40^{\circ} \mathrm{C}$, the central beam lobe is between incidence angles of $\theta= \pm 43^{\circ}$ (Figure 4.12a). When the speed of sound in air rises with temperature to $c_{a}=331 \mathrm{~m} \mathrm{~s}^{-1}$ but the incident frequency of the sound wave remains the same, the main lobe widens to $\theta= \pm 47^{\circ}$, with notches of low magnitude (nulls) becoming smaller (Figure 4.12b). For an incident wave at frequency 20 Hz , the nulls are near the broadside extremes of the array at $\theta= \pm 90^{\circ}$, and the magnitude is mostly constant for sound waves incident to the array at all angles (Figure 4.12c and Figure 4.12d).

This analysis shows that the response of the linear microphone array is most directional for higher reflected frequencies of sound, with low reflected frequencies having the least
directional response. The smallest lobe width of $\theta= \pm 43^{\circ}$ establishes an upper limit to the effective footprint of the sensing device on the snowpack surface.

In a similar fashion to radar (Yankielun et al., 2004), the footprint in the context of the SAS2 design is the approximate area on the surface of the snowpack that is sensed by the microphone array. Assume that the minimum half-width of the central lobe given in radians is $\theta_{m}$. For a lobe width of $\theta= \pm 43^{\circ}$, the half-width is $\theta_{m}=43^{\circ}$. Then the effective sensing footprint $x_{s}$ of the array on the snow surface is approximated by:

$$
\begin{equation*}
x_{s} \approx 2 y_{a} \tan \left(\theta_{m}\right) \tag{4.17}
\end{equation*}
$$

For $5 \mathrm{~cm} \leq \mathrm{y}_{a} \leq 3 \mathrm{~m}$, evaluation of Equation (4.17) gives $9 \mathrm{~cm} \leq x_{s} \leq 5.6 \mathrm{~m}$ as the effective footprint on the snowpack surface. The 9 cm effective footprint on the snowpack surface is slightly larger than the theoretical inner diameter of 7.1 cm for the Canadian MSC snow sampler (Goodison et al., 1981), but 5.6 m is large enough for shrubs and vegetation buried beneath the snowpack to cause significant amounts of acoustic scattering in a similar fashion to objects on a seabed (Lurton, 2002). This suggests that wideband beamforming algorithms (Liu and Weiss, 2010) could be applied to reduce the effective footprint area of the imaging sensor. However, the microphone holes in the front of the enclosure case may function as waveguides for the received sound wave.

### 4.3.12. Snowpack Layer Resolution

Similar to seismic imaging, the smallest resolution of the acoustic measurement in the snowpack is related to the wavelength of the source signal. Assume that the source signal is comprised of a wavelet that propagates through the porous snowpack medium (Aki and Richards, 2002). This is true for an MLS after the cross-correlation signal processing operation has recovered the filter kernel (Borish and Angell, 1983). Then, destructive interference between wavelets reflected from successive acoustic "layers" in the snowpack establishes an imaging resolution (Widess, 1973):

$$
\begin{align*}
& \Delta y>\lambda_{s} / 4  \tag{4.18}\\
& \lambda_{s}=c_{s} / f_{a} \tag{4.19}
\end{align*}
$$

For $10 \mathrm{~m} \mathrm{~s}^{-1} \leq c_{s} \leq 331 \mathrm{~m} \mathrm{~s}^{-1}$ and $20 \mathrm{~Hz} \leq f_{a} \leq 10 \mathrm{kHz}$ for the MLS bandwidth, the minimum acoustic layer width $\Delta y$ that can be resolved is $\Delta y>2.5 \times 10^{-4} \mathrm{~m}$. This is a theoretical minimum and $\Delta y$ is dependent on the speed of the sound wave in the porous snowpack medium.

### 4.4 Anechoic Chamber Measurements

To empirically determine the MLS source signal sent from the loudspeaker as a system response $\mu_{k}[t]$, the SAS2 system was taken into an anechoic chamber at the University of Saskatchewan (Room 2C96.1, Department of Electrical and Computer Engineering). The anechoic chamber is a room designed specifically for acoustic measurements (Raichel, 2006). Sound-absorbing material and wedges mounted on the walls, ceiling and floor of the room reduce the ambient SPL to a minimum. The dimensions of the room are $6.9 \mathrm{~m} \times 4.5 \mathrm{~m} \times 4 \mathrm{~m}$. Figure 4.14 is a picture of the measurement being made in the anechoic chamber.

The SAS2 system was placed on a specially constructed metal table stand situated at a height of $73 \mathrm{~cm} \pm 5 \mathrm{~cm}$ above the metal grating floor. A loudspeaker was situated at a distance of 1.25 m from the microphone assembly on the end of a musician's microphone stand. A cable connected the loudspeaker to the SAS2 audio amplifier. The loudspeaker mounted on the front of the SAS2 enclosure case remained in place during the tests.

The room was closed before measurements commenced and the inside pressure was allowed to equilibrate for 15 minutes. The ambient temperature was $\sim 22{ }^{\circ} \mathrm{C}$. The Bluetooth interface was used to trigger the SAS2 from outside of the chamber, sending acoustic waves from the loudspeaker into the microphone assembly. Twenty measurements in total were taken and one measurement was selected for operational use in the SAS2 system.

Assuming that the room is a cuboid, $a_{s}=7 \mathrm{~cm}$ is the dimension of the sound source relative to the longest room dimension $d_{r}=6.9 \mathrm{~m}$, and $l_{w}=1 \mathrm{~m}$ is the length of the soundabsorbing wedges, the largest wavelength $\lambda_{s r}$ that can be measured in the room is given by (Ressl and Wundes, 2010):

$$
\begin{equation*}
\lambda_{s r}=2\left[d_{r}-\left(5 a_{s}+2 l_{w}\right)\right]=9.1 \mathrm{~m} \tag{4.20}
\end{equation*}
$$

At a temperature of $22^{\circ} \mathrm{C}$, this corresponds to a cutoff frequency of 37.8 Hz . This is the lowest assumed MLS cutoff frequency for the reference MLS measurement of system response.


Figure 4.13. Picture of the SAS2 measurement setup in the anechoic chamber.

Figure 4.14 shows a plot of the sound wave detected by one of the microphones in the anechoic chamber for the $5^{\text {th }}$ test, out of 22 tests in total. Signals collected from this test were used as the reference waveforms. Each microphone in the 24 microphone array had a reference signal similar to the one shown. The $5^{\text {th }}$ test was chosen because the amplitude of the signal envelope was more evenly distributed over the time that the signal was produced. Small movements of the loudspeaker on the end of the microphone stand did not always permit the amplitude to be as evenly distributed as the $5^{\text {th }}$ test signal.

Figure 4.15 shows the magnitude and phase of a signal (Figure 4.14) from the $5^{\text {th }}$ test, whereas Figure 4.16 shows the magnitude and phase from each of the 22 tests. The detected waveform frequency response is not flat (i.e. similar to Figure 3.10c) due to the frequency response of the loudspeaker convolved with the frequency response of the MLS. Signal averaging of all the test signals was not used due to small differences in the phase of the waveform resulting due to vibrations of the loudspeaker on the end of the microphone stand
(Figure 4.16b). This was a disadvantage of the measurement setup and necessitated the selection of a single reference waveform. Although a metal retaining stand for the loudspeaker was used in preliminary tests, this retaining stand modified the loudspeaker frequency response so that it could not be used as a reference signal in the signal processing inversion.


Figure 4.14. Plot of the sound wave detected by one of the microphones in the array for use as the reference signal. The sound wave was detected in the anechoic chamber.


Figure 4.15. Plots of the magnitude and phase of the reference signal. The phase is constrained to the interval $[-\pi, \pi]$.


Figure 4.16. Plots of the magnitude and phase of all signals collected in the anechoic chamber. Differences in color represent the different signals collected. This shows differences between the test signals collected in the anechoic chamber. The phase is constrained to the interval $[-\pi, \pi]$.

## CHAPTER 5 <br> FIELD LOCATIONS AND DATA COLLECTION

### 5.1. Introduction

To test the SAS2 on snow, ten field sites were selected in the Canadian Rocky Mountains of Alberta, in western Canada. The geographic area of the Canadian Rockies was used for these acoustic experiments for the following reasons.
(1) The Canadian Rockies are a natural laboratory with a wide range of vegetation types (Chinnappa and Hallworth, 1997; Habeck, 1987; Moss, 1955; Scotter and Zoltai, 1982), ecotones (Howell and Harris, 1978; Kernaghan and Harper, 2001; Macdonald, 1987) and mass and energy balances (Ellis et al., 2013, 2011; Helgason and Pomeroy, 2012a, 2012b; Pomeroy et al., 2012). Due to changes in elevation, uneven topography, forest type and canopy cover, snowpacks develop with large variability in depth, density, temperature and wetness. These factors provide a number of ideal locations for testing the SAS2 system developed in this thesis.
(2) Snowpacks in the Canadian Rockies provide water in the form of snowmelt to the Bow River, the City of Calgary, and the South Saskatchewan River (Pomeroy et al., 2007a). Moreover, mass and energy exchanges between the atmosphere and snowpacks in the Rockies influence local climatology (Helgason and Pomeroy, 2012b). Tests of the SAS2 in this environment validates the use of acoustic technologies for a future snow observation network that studies the availability and change in water resources to ensure adequate human use and consumption as well as a better understanding of the relationships between snow, water availability and climate.
(3) Locations with large changes in environmental conditions can be easily accessed by road or by a brief hike and sites with widely-varying changes in snowpack properties are situated close together, thereby simplifying the ability for data collection.
(4) The Marmot Creek Research Basin is an experimental site in the Rockies currently operated by the Centre for Hydrology. Because the watershed has been used extensively in the past and present for hydrological research (Forsythe, 1997; Golding and Swanson, 1986; Harlan, 1969; Storr, 1972, 1967), it provides a location where instrumentation can be set up and periodically checked by field personnel.
(5) The Centre for Hydrology has an office and laboratory facility situated at the University of Calgary's Barrier Lake Field Station located in the Kananaskis Valley. This simplifies the logistics of accessing the field sites and deploying instrumentation.

The geographic locations, site elevations and change in elevation at each field site are listed in Table 5.1. The sites span a large geographic area of the Canadian Rockies. The largest north-south line distance between sites is 160 km . The sites range in elevation from 1523.9 MASL to 2102.7 MASL. Descriptions of the sites are given in the next section.

### 5.2. Sites and Deployment

### 5.2.1. Upper Clearing Stationary (UCS) Site

The Upper Clearing (UCS) site was situated in the Marmot Creek Research Basin (MCRB). MCRB elevation ranges from 1600 MASL to 2825 MASL, placing the UCS site near the lower reach of the basin. The MCRB is near to the Nakiska Ski Resort, and the UCS site was accessed during the winter months by hiking up a snowmobile path that services this resort. The UCS site was situated 400 m from the snowmobile path.

| $\#$ | Name | Acrony <br> $\mathbf{m}$ | Latitude ( ${ }^{\circ} \mathbf{N}$ ) | Longitude ( ${ }^{\circ} \mathbf{W}$ ) | Minimum <br> Elevation <br> (MASL) | Maximum <br> Elevation <br> (MASL) | Max <br> Change <br> (MASL) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | Upper Clearing Stationary | UCS | 50.956524833000004 | 115.17526599999999 | 1846 | 1846 | 0 |
| 3 | Boulton Campground | BC | 50.637631167000002 | 115.107010333 | 1687.2 | 1722.8 | 35.6 |
| 6 | Bow Summit | BS | 51.715226999999994 | 116.4957945 | 2063.5 | 2102.7 | 39.2 |
| 2 | Upper Clearing Portable | UCP | 50.9564405 | 115.17563699999999 | 1837 | 1853.9 | 16.9 |
| 4 | Sawmill Creek | SC | 50.747676167000002 | 115.245452 | 1825.6 | 1857.1 | 31.5 |
| 7 | Bow Meadow | BM | 51.703477332999998 | 116.476415833 | 1995.7 | 2057 | 61.3 |
| 8 | Bow Lake | BL | 51.673902832999998 | 116.451093667 | 1934.4 | 1941.1 | 6.6 |
| 9 | Nakiska Ski Hills | NSH | 50.9427005 | 115.153063667 | 1523.9 | 1544.7 | 20.9 |
| 5 | Black Prince Cirque | BPC | 50.704814832999998 | 115.197102 | 1720.8 | 1751.2 | 30.4 |
| 10 | Icefields Parkway Forest | IPFS | 51.722083166999996 | 116.492946 |  | 2066.2 | 2089.7 |

Table 5.1. Names, acronyms, geographic locations and minimum and maximum elevations of acoustic sampling points at each site. The elevations are given in Meters Above Sea Level (MASL). The maximum change in elevation at each site is also given.

Figure 5.1 shows the stationary SAS2 setup at the UCS site, next to a meteorological tower near the centre of the clearing. Guy wires used to support the permanent meteorological tower are visible in the background. Surrounding the clearing is a forest of subalpine fir (Abies lasiocarpa) and Engelmann spruce (Picea engelmanni). Young spruce trees, small shrubs and sparse grass were buried under the snow in the clearing. Some of these trees are visible in the background of the picture.

The SAS2 was located on a boom arm suspended over the snow surface. An external solar panel provided auxiliary power to the SAS 2 so that a 12 V lead acid battery could be charged. Next to the SAS2 was a Campbell Scientific SR50 ultrasonic snow depth sensor. On the end of the boom arm was a Kipp \& Zonen CNR1 net radiometer to measure the incoming and outgoing shortwave and longwave radiation. A cup anemometer measured wind speed. A Vaisala HMP45CF hygrothermometer in a radiation shield measured air temperature and relative humidity near to the datalogger enclosure box. A shielded narrow beam Exergen IRTC infrared (IR) thermometer measured snow surface temperature near the met station. At the bottom of the snowpack was a heat flux plate (HFP01) to measure heat transport between the ground and snow surface.

A Type E thermocouple array developed by Helgason and Pomeroy (2012) and controlled by a multiplexer was buried under the snow surface at the position indicated in Figure 5.1. This is referred to as a "snow harp." The thermocouple array installation is shown as Figure 5.2, and dimensions of the thermocouple array are shown in Figure 5.3. The thermocouple array was composed of 11 vertical wires situated parallel to the ground surface. Each wire was suspended from a metal frame approximately 60 cm high. The distance between each wire was 3 cm . The array was located close to the datalogger box (Figure 5.2) since the wires connecting the thermocouples to the datalogger were short.


Figure 5.1. Stationary SAS2 setup at the UC site. Equipment deployed at the UC site is listed on the diagram as well as the distances between the ultrasonic SR50 and the SAS2.


Figure 5.2. Stationary SAS2 setup at the UC site showing the position of the snowharp thermocouple array relative to the met station setup.


Figure 5.3. Frame with thermocouples referred to as a "snowharp."

The SAS2 was programmed to take one acoustic sample per hour. The system automatically turned itself on, produced the sound wave, and then switched itself off. The one hour sampling rate was selected for two reasons. Snowpack evolution models and cold region hydrological process models are run using timesteps of at least one hour (Pomeroy et al., 2007b). These models have been validated using field observations of snow. This strongly implies that sampling of snowpack properties on a one hour time scale is likely suitable for acoustic observations. The second reason for the one hour sampling rate is due to battery capacity and system power consumption. To ensure that the system could be powered by a 12 V battery with finite capacity throughout the winter, the sampling rate had to be chosen to minimize power consumption.

From instruments deployed at the UCS site, a datalogger was used to sample: (1) snow depth; (2) incoming and outgoing solar radiation; (3) windspeed; (4) air temperature; (5) relative humidity; (6) snow surface temperature; (7) heat flux; and (8) snowharp temperatures once every 15 minutes. This is the same thermocouple sampling rate used by Brandt and Warren (1997), which was suitable for validation of an implicit finite-difference model of heat conduction in snow. Because the values sampled were logged to internal datalogger memory, the sampling rate was chosen to minimize the amount of time between samples and maximize the number of samples that could be stored. The thermocouple array was sampled once every minute and temperature measurements averaged by the datalogger to record one temperature per thermocouple once every 15 minutes. This sampling rate is four times greater than the sampling rate of the SAS2 to provide indication of changes in parameters between SAS2 samples.

The site was periodically visited at monthly intervals and a snowpit was dug near the stationary measurement device at a distance of $\sim 2 \mathrm{~m}$ from the SAS2. Because snowpit creation is an invasive process, the snowpit could not be dug directly under the SAS2 device. Snowpack stratigraphy was recorded and measurements of snow density were made with a triangular scoop (RIP) sampler. Measurements of snowpack wetness with a Denoth meter were made when the device was available. Because the Denoth meter electronics did not utilize a microcontroller, the twin-T bridge had to be balanced by turning a potentiometer. This added a very small human source of error to measurements of snow wetness made with the Denoth meter. The same Denoth meter was used for measurements taken at other field locations where data was collected.

### 5.2.2. Upper Clearing Portable (UCP) Site

The portable version of the SAS2 was taken to the UC site and used to collect acoustic measurements of snow in the clearing and nearby forest. For measurements taken in the clearing, the SAS2 was placed on the child chariot carrier and pushed along the snow surface (Figure 5.4). Acoustic measurements were made at sample points by pressing a button mounted on the handle-bars of the child chariot carrier. Acoustic "miss-firings" where the sensor was not pointed at the snow surface were excluded from the dataset. After the acoustic samples were taken, an ESC30 snow sampling tube (cf. Section 1.3.2) was used to measure depth-integrated SWE and density near each sampling point. Snow depth was measured using a rod. The procedure of taking the acoustic sample first and the invasive sample after was repeated at all other field sites. This was done to prevent refraction of the sound wave through the hole created in the snow surface by the extraction of the invasive sample.

The chariot carrier could not be pushed into this forest due to the presence of undergrowth. For all measurements made in the forest the SAS2 was taken off the child chariot carrier and placed on a mounting rod. The mounting rod was held in the hand of the investigator and the SAS2 was suspended over the snow surface when an acoustic sample was taken. When mounted on the child chariot carrier or held above the snow surface, the SAS2 was suspended at no more than a maximum of $\sim 15 \mathrm{~cm}$ from the snow surface. Figure 5.14 shows this method of sampling. At all other field sites, the SAS was also suspended at a distance of no more than $\sim 15 \mathrm{~cm}$ from the snow surface.

Two snowpits were created in the clearing. One of these snowpits (situated on the NE side of the clearing) is shown in Figure 5.5. The first snowpit was created near the stationary SAS2, and the second snowpit was created at the NE side of the clearing. Stratigraphy and density measurements were made in each snowpit using human observations and a scoop sampler. Snow wetness was also determined by inserting the Denoth meter into the snowpit wall. When the snowpit and SAS2 measurements were collected, the sky was overcast and light snow was falling.


Figure 5.4. Portable measurements being made at the UC with the SAS2 placed on the child chariot carrier. A solar panel is visible in the foreground.


Figure 5.5. Snowpit being sampled at UC site. The Denoth meter is visible in the foreground. A shovel was used to dig the snowpit, and gravimetric samples were extracted from the snowpit wall using a snow pit kit. A collapsible ruler was used to record snow depth, and a thermometer was inserted into the snowpack wall to measure temperature. The snow stratigraphy, density, Denoth meter measurements, and snow temperatures were recorded in a lab book. Visible in the background is the child chariot carrier on which the SAS2 was mounted.

### 4.2.3. Boulton Campground (BC) Site

The BCS was situated in Peter Lougheed Provincial Park within the Kananaskis Valley, 1 km east of the Lower Kananaskis Lake. The site was accessed via the Kananaskis Lakes Road and the Whiskey Jack Trail.

The portable version of the SAS2 was placed on the child chariot carrier and acoustic measurements were made in a clearing and nearby dense forest (Figure 5.6). The SAS2 remained mounted on the child chariot carrier for all sample points. An ESC snow sampler was used to measure depth-integrated SWE and density, and snow depth measurements were made
using a rod. Dielectric measurements of liquid water content were not made at this site because measurement equipment was not available.

Two snowpits were created at this site. One snowpit was dug in the clearing, whereas the other snowpit was dug in the forest underneath a tree. Light snow was falling when the snowpit and acoustic measurements were collected. This deposited a layer of low-density snow on top of the snow surface. For some acoustic measurements, the loudspeaker and microphones on the front of the enclosure case were inserted directly into the snowpack when the child chariot carrier inadvertently dipped into the snow. These measurements were later excluded from the dataset.


Figure 5.6. Picture of the BC site, showing the open clearing and nearby dense forest.

### 5.2.4 Sawmill Creek (SC) Site

The SC site was accessed via the Spray Lakes Road situated in the Peter Lougheed Provincial Park. The site was situated on a small plateau near Mount Inflexible.

Acoustic samples and gravimetric samples were made on a ridge situated next to the access road and in the nearby forest (Figure 5.7). The forest had undulating snow surface topography and uneven snow accumulation due to snow unloading (Figure 5.8). Detritus was observed on the snow surface within the forest. At some of the sampling points in the forest, the SAS2 was removed from the child chariot carrier and suspended above the snow surface via a mounting rod. This was done because the skis on the child chariot carrier were too big for maneuverability in the forest.

At the ecotone between the forest and the open ridge (Figure 5.9), small shrubs were buried beneath the snow surface whereas large shrubs were observed above the snow surface. At sampling points near the shrubs, water and detritus from the duff layer was observed at the bottom of the ESC tube. The sound of running water was heard near the shrubs. This implies that a small stream may have been situated under the snow near the shrubs. Five snowpits in total were made at the SC site, two on the ridge and two in the forest.


Figure 5.7. Picture of the SC site on the ridge looking toward Mount Inflexible.


Figure 5.8. Picture of the SC site taken in the forest. Undulating snow topography and detritus is visible on the snow surface.


Figure 5.9. Ecotone between the forest and the open ridge for the SC site. Water and detrius from the duff layer was found at the bottom of the ESC tube after a snow sample had been extracted, implying that a stream may have been located under the snowpack.

### 5.2.5 Black Prince Cirque (BPC)

The BPC site was accessed via the Spray Lakes Road in Peter Lougheed Provincial Park. The site was situated close to Smith-Dorrien Creek. The SAS2 was held in the hand of the investigator and suspended above the snow surface using the mounting rod. Once again, the SAS2 was suspended no more than $\sim 15 \mathrm{~cm}$ above the snow surface. Acoustic samples were taken along a transect starting in a clearing and ending in a forest 6 m from the bank of the Smith-Dorrien Creek (Figure 5.10). Measurements of snow depth were made along the transect, as well as depth-integrated SWE and density with an ESC30 sampler. Two snowpits were created, one in the clearing and the other in the forest. Buried shrubs approximately 60 cm high were found under the snow surface in the clearing.


Figure 5.10. Picture of the BPC site showing the clearing and the forest.

### 5.2.6 Bow Summit (BS)

The BS was situated in Banff National Park. The site was accessed via the Icefields Parkway (Highway 93 north). The site was situated 1 km east of Peyto Lake near the lower slopes of Mount Jimmy Simpson, and 4.4 km north of Bow Lake. The truck conveying the child chariot carrier and SAS2 was parked near the highway and the child chariot carrier was pulled up a scenic access road closed during the winter season. The site was situated 150 m from the access road. Fresh low-density snow had been deposited on the snowpack the previous night.

The SAS2 was placed on the child chariot carrier and acoustic samples were taken in a clearing and nearby forest (Figure 5.11). ESC snow sampler and snow depth measurements were made. Three snowpits were dug in the clearing. Measurements of snow stratigraphy, density and water content were made in the snowpits using gravimetric samplers and the Denoth meter.


Figure 5.11. Picture of the Bow Summit (BS) site, showing the clearing and nearby forest. Also shown is the child chariot carrier and a sled carrying shovels and gravimetric measurement instrumentation.

### 5.2.7 Bow Meadow (BM) Site

The BM site was situated in Banff National Park, 50 m to the west of the Icefields Parkway and 2 km south of the BSS. At lower elevations near the road, the site is characterized by an alluvial outwash plain with a meandering stream channel situated under the snowpack. At higher elevations, a forest is situated on the slopes of Mount Jimmy Simpson (Figure 5.12).

The SAS2 was placed on the child chariot carrier and acoustic samples were taken along a transect starting near the road and ending near the treeline of the forest. The elevation at the first point in the transect was 1855 MASL, and the elevation at the last point in the transect was 2057 MASL, with a total change in elevation of 202 m over the transect. The transect crossed the meandering stream, and acoustic samples of snow were taken on top of a snow bridge situated over the running water. Snow tube and snow depth measurements were made after acoustic measurements had been taken (Figure 5.13).

Four snowpits in total were created at this site. The first snowpit was created just west of the road, whereas the second snowpit was created west of the stream channel. The third snowpit was created on the mountain slope, and the fourth snowpit was created in the forest under a tree. The snow in this snowpit was observed to be unloaded snow. At higher elevations in the forest, the child chariot carrier could not be successfully maneuvered, and the SAS2 had to be held in the investigator's hand over the snow surface so that acoustic measurements could be taken. Figure 5.14 shows an example of this sampling configuration. The SAS2 was suspended no more than $\sim 15 \mathrm{~cm}$ above the snow surface.


Figure 5.12. Picture of the BM site, showing an alluvial outwash plain and a forest on the slopes of Mount Jimmy Simpson. A sampling transect crossed the plain and went up into the forest.


Figure 5.13. Acoustic and gravimetric samples being taken along the tranect. The child chariot carrier is visible in the foreground, and an ESC snow sampling tube is being weighed on a scale to determine depth-integrated SWE and density. The snow bridge situated over the stream is also visible.


Figure 5.14. Acoustic and gravimetric samples being taken along the transect with the SAS2 being held in the hand of the investigator. An acoustic wave is being sent into the snow. An ESC snow sampling tube and depth measurement rod is also visible.

### 4.2.8 Bow Lake (BL) Site

The BL site was situated 4 km south of the BMS site on the frozen ice of Bow Lake, a glacial lake originating from the Bow Glacier. The SAS2 was removed from the child chariot carrier and held over snow that had accumulated on top of the ice so that acoustic measurements could be made. Snow depth was measured using a rod, as well as depth-integrated SWE and density using an ESC snow sampler. The snow in the ESC snow sampler was observed to be wet due to water from the lake on top of the ice. No snowpits were dug at this site. The purpose of collecting data at the BLS was to observe the effects of water and ice on the acoustic sampling procedure.


Figure 5.15. Picture of the Bow Lake site situated on the frozen ice of Bow Lake.

### 5.2.9 Nakiska Ski Hill (NSH) Site

The NSH site was situated in the Nakiska Ski Resort Area close to the chairlift and administrative areas. The purpose of selecting this site was to determine if the acoustic sampling procedure could be used on late-season snow that was artificially-made. Acoustic samples were taken and sampling of snow with a snow tube and depth rod was used for comparison. In a similar fashion to the BLS, acoustic samples of snow were made with the SAS2 held over the snow surface. Sampling of snow also occurred on a steep ski hill. Snow at this site had undergone ablation. The snowcover was patchy and the surface had uneven melt topography. No snowpits were dug at this site due to the shallow snow depth.


Figure 5.16. Picture of the NSH site showing the slope with artificial old-season snow.

### 5.2.10 Icefields Parkway Forest (IFPF) Site

The IFPF was situated on a road and in a forest located near the Icefields Parkway (Figure 5.17). The snow at this site was late-season and had undergone heavy ablation and melt. To collect acoustic samples from the road and nearby forest, the child chariot carrier was not used and the acoustic gauge was held over the snow surface. Snow tube and depth rod samples were taken at each acoustic sampling point. Light rain and fog occurred when the data was collected. The snow was visibly wet. Once again, no snowpits were dug at this site due to the shallow snow depth.


Figure 5.17. Picture of the IPFS, showing the road and the nearby forest.

## CHAPTER 6

## RESULTS

### 6.1. Introduction

The following sections of this thesis present the results determined from acoustic data collected using stationary and portable versions of the SAS2 deployed at field sites described in Chapter 5. Observations made at the UCS site with the SAS2 and met station instrumentation is discussed. The portable version of the SAS2 was deployed and tested at different field sites.

### 6.2. Snowpack Measurements

### 6.2.1. Ultrasonic

At the UCS site, snow depth measurements were made over the time of observation using the ultrasonic SR50 sensor situated next to the stationary version of the SAS2. At all of the sites where the SAS2 was deployed, depth measurements were also made using a depth rod.

The ultrasonic SR50 snow depth measurements $\bar{y}$ (Figure 6.1a) corresponded reasonably well with nearby depth rod data measurements, exhibiting an RMSD of 9.0 cm and a MB of -8 cm for a maximum snow depth of 1 m . This indicated that the SR50 under-predicted measurements of snow depth, similar to the observations of Goodison (1985). The underprediction of snow depth $\bar{y}$ may have occurred due to partial penetration of the ultrasonic wave at the air-snow interface, as suggested by Goodison et al. (1984). Moreover, because snow depth measurements with the depth rod were not made directly under the SR50 sensor, variations in snow surface topography observed at the UC site may also be responsible for this error, given that the SR50 has a specified range resolution of 0.1 mm and a beam acceptance (half-width) of $22^{\circ}$. Some interception of snow on the mast arm and on the deployed SAS2 device may have caused less snow to accumulate under the SR50 and SAS2. This is due to the size of the SAS2 enclosure box.
(a)


N
(c)

(d)


Figure 6.1. Upper Clearing Stationary (UCS) site time-series data showing (a) SR50 snow depth $\bar{y}$, (b) average density $\bar{\rho}$, (c) SWE, and (d) liquid water content $\bar{\theta}_{w}$ as LWC.

Using Equation (4.17), and given that the SR50 was situated at a distance of $0.5 \mathrm{~m} \leq y_{a} \leq 1.5 \mathrm{~m}$ above the snow surface due to changes in snow depth, the theoretical SR50 beam footprint ranged between 0.40 m at a distance of 0.5 m from the snow surface, and a distance of 1.2 m when the SR50 was situated at a distance of 1.5 m . The error in depth measurement was greatest when the footprint was largest due to acoustic scattering of the sound wave by exposed vegetation at low snow depths and spurious reflections from vegetation and the met station mounting hardware situated at a distance of less than 1.2 m from the SR50. Moreover, because the horizontal separation distance between the SR50 and the SAS2 was 62 cm , this distance is greater than the SR50 footprint size of $x_{s} / 2=23 \mathrm{~cm}$ at a distance of 28 cm between the bottom of the SAS2 enclosure case and the bottom of the SR50 facing the snow surface. This implies that SR50 acoustic measurements were not adversely affected by the close proximity of the SAS2 enclosure case and reflections of the ultrasonic wave from the SAS2 enclosure case were of small magnitude.

Errors in snow depth measured using the SR50 occurred due to acoustic scattering of the sound wave during high wind and snowfall events. This effect is similar to scattering of sonar pulses in the ocean by suspended sediment (Richards et al., 1997). Large "spikes" corresponding to over-prediction errors in ultrasonic measurements of snow depth are visible in the time series plot of snow depth (Figure 6.1a). Before 15 April, the spikes occurred during snow deposition events when snow depth increased (Figure 6.5a). After 15 April, a general decrease in snow depth occurred due to ablation. The spikes of largest magnitude occurred during the time of ablation, suggesting that uneven surface topography or water on the snow surface during melt may have caused additional scattering of the acoustic pulse. Acoustic scattering from an uneven snow surface is similar to scattering of sonar from a rough seabed (Kinar and Pomeroy, 2008a, 2008b). After 15 May, the acoustic pulse from the SR50 contacted the ground surface. Residual amounts of snow on the ground may have been responsible for large over-prediction errors after 15 May, particularly on 23 May when $\sim 5.8 \mathrm{~cm}$ of snowfall occurred. The residual amounts of snow do not provide a horizontal surface for reflection of the ultrasonic wave. Snow ablation processes removed all of the snow from the UCS site by 26 May.

### 6.2.2. Ultrasonic and SAS2

The SR50 is an ultrasonic pulse-based measurement system that operates on principles similar to monopulse sonar and radar systems. Because the duration of the pulse is in the millisecond time range, the time-integrated energy of the sound wave produced by the SR50 is less than the time-integrated energy of the MLS sound wave sent from the SAS2 suspended over the surface of the snowpack. Unlike the SR50, time series of parameters measured by the SAS2 (Figure 6.1b,d; Figure 6.5a; Figure 6.9) do not show spurious measurement spikes of large magnitude, indicating the noise robustness of the MLS signal relative to pulse-based measurement systems. The time of the MLS signal was $t_{m l s}=1.6 \mathrm{~s}$. Moreover, unlike the ultrasonic SR50 sound waves that are reflected from the snow surface, the audible sound wave produced by the SAS2 will penetrate the snowpack, creating reflections from the snow and the ground surface.

Given the smallest half-width of the SAS2 sensor array as $\theta_{m}=43^{\circ}$ at 10 kHz (Section 4.3) and taking the vertical distance between the bottom of the SAS2 enclosure case and the bottom of the SR50 as 28 cm , evaluation of Equation (4.17) shows that the SAS2 footprint size of $x_{s} / 2=26 \mathrm{~cm}$ was less than 31 cm , implying that close proximity of the SR50 to the SAS2 did not interfere with the SAS2 acoustic measurement for higher frequencies received by the microphone array. However, design calculations given in Section 4.3 show that for low frequencies approaching 20 kHz , the main lobe width is $\theta= \pm 90^{\circ}$. Low-frequency reflections from the met station apparatus and vegetation at the UC site may have affected the SAS2 measurement for frequencies approaching 20 Hz . Because the SAS2 was situated at a distance of $0.22 \mathrm{~m} \leq y_{a} \leq 1.22 \mathrm{~m}$ from the snow surface due to changes in snow depth, the theoretical SAS2 device footprint ranged from 41 cm to 2.3 m for a half-width of $\theta_{m}=43^{\circ}$. This is larger than the device footprint of the SR50. Since the microphone holes on the SAS2 enclosure case acted as waveguides, these effects may have been considerably lessened.

### 6.2.3. Density

At the UCS site, the RMSD between stationary SAS2 and snowpit measurements of depth-averaged density $\bar{\rho}$ was $48.4 \mathrm{~kg} \mathrm{~m}^{-3}$, with a MB of $-4.1 \mathrm{~kg} \mathrm{~m}^{-3}$, indicating that the SAS2 installed at the UC site under-predicted density measurements (Figure 6.1b). The RMSD value
had the same order of magnitude as the RMSD values determined by comparing gravimetric and acoustic methods of snow density collected using the portable version of the SAS2 (Table 6.3). The SWE values at the UCS site were under-predicted as well (Figure 6.1c), with an RMSD of 48.5 mm and a MB of -28.1 mm . This is because SWE is the numerical product of snow depth and density and the SR50 was being used to measure snow depth at the UCS site. Figure 6.1b shows that a general rise in snow density was apparent over the time of the observation season due to snowpack metamorphism and compaction.

Because the portable version of the SAS2 deployed at the field sites listed in Table 6.3 was situated at distances of no more than $\sim 15 \mathrm{~cm}$ from the snow surface, the footprint at a halfwidth of $\theta_{m}=43^{\circ}$ for the portable device was $\sim 30 \mathrm{~cm}$ at this distance. Since the footprint of the stationary SAS2 device had a greater range at the UCS site relative to the maximum footprint range of the portable SAS2 device, there was an increased possibility for spurious reflections and acoustic scattering to affect the measurement during sampling of snow using the stationary version of the SAS2.

Although some variation in results can be explained by hardware differences between the stationary and portable versions of the SAS2, an observation made on 15 March by the portable version of the SAS2 situated close ( $\sim 2 \mathrm{~m}$ ) to the stationary version of the SAS2 had an RMSD of $15.6 \mathrm{~kg} \mathrm{~m}^{-3}$ and a MB of $14.3 \mathrm{~kg} \mathrm{~m}^{-3}$. This suggests that the stationary and portable versions of the acoustic gauge can obtain similar measurement results.

At the time of installation of the SAS2 on 1 February, the acoustic snow density $\bar{\rho}$ was measured as $145 \mathrm{~kg} \mathrm{~m}^{-3}$. The density $\bar{\rho}$ gradually increased between 1 February and 11 February to $225 \mathrm{~kg} \mathrm{~m}^{-3}$. The small amounts of snow deposited from 1 February and 11 February were not enough to offset larger increases in depth-integrated density $\bar{\rho}$ occurring due to mechanical compaction and metamorphic processes. This caused a corresponding rise in SWE as well due to small increases in snow depth $\bar{y}$ over this period. The rise in SWE occurred due to snowfall that increased snow depth.

During the first major snow deposition event lasting from 4 March to 6 March, the snow density $\bar{\rho}$ dropped from $205 \mathrm{~kg} \mathrm{~m}^{-3}$ to $160 \mathrm{~kg} \mathrm{~m}^{-3}$, showing that fresh low-density snow had been added to the snowpack, decreasing the depth-averaged density. From 6 March to 11 March, the snow density rose to $201 \mathrm{~kg} \mathrm{~m}^{-3}$. With a concomitant decrease in snow depth, this indicated
compaction of the snowpack and metamorphic processes that increased snow density. By 15 March, the snow density dropped to $170 \mathrm{~kg} \mathrm{~m}^{-3}$, once again indicating the addition of fresh lowdensity snow. The snow density continued to increase from 15 March to 30 March, due to slight compaction of the snowpack and metamorphic processes, reaching $256 \mathrm{~kg} \mathrm{~m}^{-3}$ by 30 March. Small drops in snow density were apparent during this time due to the addition of fresh snow to the snowpack.

On 31 March the snow density $\bar{\rho}$ was $221 \mathrm{~kg} \mathrm{~m}^{-3}$. On 3 April the snow density peaked at $323 \mathrm{~kg} \mathrm{~m}^{-3}$ showing that $\bar{\rho}$ increased a total of $102 \mathrm{~kg} \mathrm{~m}^{-3}$ over 4 days due to higher levels of liquid water content. A snow deposition event from 4 to 5 April caused an increase in snow depth of 10 cm from a depth of 88 cm to 99 cm and a decrease in snow density from $323 \mathrm{~kg} \mathrm{~m}^{-3}$ to $283 \mathrm{~kg} \mathrm{~m}^{-3}$. Decreases in density during snowfall events were also apparent during the 12 April and 14 April snowfall events.

A gradual increase in snow density from 17 March to 14 April due to compaction and metamorphic processes was apparent. On 17 March the snow density was $180 \mathrm{~kg} \mathrm{~m}^{-3}$, and this increased to $351 \mathrm{~kg} \mathrm{~m}^{-3}$ on 14 April. The last major snowfall before ablation occurred on 14 April, with a drop in snow density $\bar{\rho}$ to $294 \mathrm{~kg} \mathrm{~m}^{-3}$ indicating the addition of fresh snow to the snowpack.

The snow depth was 101 cm on 14 April, and all of the snow had undergone ablation by 14 May. The first major drop in snow depth occurred between 14 April and 21 April, when the snow depth decreased from 99 cm to 78 cm , a total of 21 cm . During this time rapid cycles in liquid water content occurred due to the production of meltwater during ablation. The density increased to $379 \mathrm{~kg} \mathrm{~m}^{-3}$ by 21 April. The second major drop in snow depth occurred between 21 April and 26 April, when the snow depth decreased from 76 cm to 48 cm , a total of 28 cm .

On 26 April, the snow depth was 48 cm , and the snow density $\bar{\rho}$ was $411 \mathrm{~kg} \mathrm{~m}^{-3}$, a measurement in the range of snow densities recorded at the NSH site (Table 6.4) for late-season old snow. The snow depth continued to decrease during ablation until 1 May, when a brief snowfall event increased snow depth to 50 cm . On 1 May, the snow density was $422 \mathrm{~kg} \mathrm{~m}^{-3}$, showing that despite the deposition of freshly-fallen snow at air temperatures exceeding $0{ }^{\circ} \mathrm{C}$, the density $\bar{\rho}$ did not decrease. The density continued to increase due to rapid metamorphism at higher air temperatures and snow surface temperatures near $0^{\circ} \mathrm{C}$ as the snowpack continued to warm.

Due to compaction and metamorphism of the snow during ablation, the snow depth reached 39 cm by 4 May, with the density increasing to $429 \mathrm{~kg} \mathrm{~m}^{-3}$. Another brief snowfall event on 5 May increased snow depth to 50 cm , but the snow density continued to increase to $429 \mathrm{~kg} \mathrm{~m}^{-3}$, indicating rapid metamorphism, compaction of the snowpack, and higher levels of liquid water content. By 11 May, the snow density $\bar{\rho}$ was within the range of mean snow densities observed at both the BL and NSH sites (Table 6.4), indicating that the snowpack was wet and comprised of old-season snow.

On 12 May, the acoustic snow density rose to $595 \mathrm{~kg} \mathrm{~m}^{-3}$ and remained at this density until the end of the observation season. The snow depth measured by the SR5 was 12.8 cm on 12 May, and all snow had ablated by 14 May. Despite a small drop in the acoustic measurement of density to $592 \mathrm{~kg} \mathrm{~m}^{-3}$ on 23 May when $\sim 5.8 \mathrm{~cm}$ of snowfall occurred, the nearly-constant acoustic snow density may represent a limitation of the acoustic method to measure wet, oldseason snow that had undergone extensive metamorphism over the snow accumulation and ablation season.

Figures 6.2 to 6.4 and Tables 6.1 and 6.2 show the RMSD and MB between ESC30 snowtube, snowpit and SAS2 acoustic measurements of density and SWE. The low MB magnitude between these quantities indicated that although the acoustic density $\bar{\rho}$ was slightly under-predicted, the SAS2 was able to obtain measurements similar to the ESC30 snowtube. The SAS2 can therefore be used as a replacement technology for non-invasively determining depth-averaged density $\bar{\rho}$ in lieu of a snowtube. The RMSD between ESC30 snowtube measurements of SWE and acoustic measurements of SWE showed slight over-prediction of SWE due to error in snow depth measurements collected using a rod. The maximum snow depth observed at all of the 10 observation sites was 2 m .

Table 6.1 shows that that acoustic measurements of $\bar{\rho}$ were under-predicted relative to gravimetric snowpit measurements due to a negative MB. The acoustic SWE measurements also showed this under-prediction error as well. The RMSD for comparisons between snowpit and acoustic measurements of $\bar{\rho}$ was less than the RMSD between snowpit and snowtube density, suggesting that the SAS2 is more accurate than ESC30 measurements and most similar to snowpit measurements of $\bar{\rho}$. Similar results were also found for comparisons between snowpit measurements and ESC30 measurements of SWE. Snowtube measurements were less


Figure 6.2. Comparison between depth-averaged density measured using an ESC30 snow tube and depth-averaged acoustic density measured using the portable version of the SAS2 at nine field sites. The black line on the plot is the $1: 1$ line. The Boulton Campground (BC), Bow Summit (BS), Upper Clearing Portable (UCP), Sawmill Creek (SC), Bow Meadows (BM), Bow Lake (BL), Nakiska Ski Hills (NSH), Black Prince Cirque (BPC), and Icefields Parkway Forest (IFPF) sites are listed in the legend. The number of samples is given as $N_{s}$.


Figure 6.3. Comparison between average density measured using an ESC30 snow tube and depth-averaged snowpit density measured at six field sites. The black line on the plot is the $1: 1$ line. The Boulton Campground (BC), Bow Summit (BS), Upper Clearing Portable (UCP), Sawmill Creek (SC), Bow Meadow (BM), and Black Prince Cirque (BPC) sites are listed in the legend. The number of samples is given as $N_{s}$.

| $\bar{\rho}$ comparison | RMSD $\left(\mathrm{kg} \mathrm{m}^{-3}\right)$ | MB $\left(\mathrm{kg} \mathrm{m}^{-3}\right)$ | $N_{\mathrm{s}}$ |
| :---: | :---: | :---: | :---: |
| ESC30 \& SAS2 | 54.0 | -8 | 452 |
| Snowpit \& Snowtube | 56.7 | -28.0 | 19 |
| Snowpit \& SAS2 | 24.2 | -4.9 | 19 |

Table 6.1. Overall comparisons between gravimetric and acoustic measurements of density $\bar{\rho}$.

| SWE comparison | RMSD (mm) | MB (mm) | $N_{\text {s }}$ |
| :---: | :---: | :---: | :---: |
| ESC30 \& SAS2 | 47.3 | 9.00 | 452 |
| Snowpit \& Snowtube | 112.4 | -10.11 | 19 |
| Snowpit \& SAS2 | 29.5 | -2.30 | 19 |

Table 6.2. Overall comparisons between gravimetric and acoustic measurements of SWE.


Figure 6.4. Comparison between depth-averaged gravimetric snowpit density and depthaveraged acoustic density measured using the portable version of the SAS2 at six field sites. The black line on the plot is the 1:1 line. The Boulton Campground (BC), Bow Summit (BS), Upper Clearing Portable (UCP), Sawmill Creek (SC), Bow Meadow (BM), and Black Prince Cirque (BPC) sites are listed in the legend.
accurate than snowpit measurements due to the error effects discussed in Section 1.3.2 of this thesis.

Table 6.3 aggregates data comparisons for the sites, showing that the RMSD values for site comparisons between snowpit and acoustic measurements of $\bar{\rho}$ were less than the RMSD values between snowpit and snowtube density. The Boulton Campground (BC) site data was the earliest dataset collected with the SAS2. The data points collected at the BS site had lower RMSD and MB magnitudes than the data points at the BC site, indicating that the BS site showed a better correspondence between acoustic and gravimetric density $\bar{\rho}$ due to smaller amounts of buried vegetation and less acoustic scattering. The Upper Clearing Portable (UCP) site data points showed more scatter than the previous BC and BS sites (Figure 6.2) and a higher range of acoustic densities (Table 6.4) due to error introduced by acoustic scattering caused by buried vegetation (small trees) in the clearing and in the forest.

The Sawmill Creek (SC) site data, collected later in the snow observation season, had higher densities than the previous sites due to snowpack metamorphism and higher levels of liquid water content (Table 6.4). This increased the scatter in the acoustic data points (Figure 6.2). The acoustic densities at the SC site showed greater error than the UCP and BS sites due to refraction of sound in the vicinity of tree wells, scattering of sound by buried vegetation in the clearing, forest and ecotone, as well as possible scattering and mode conversion of sound waves. Compared to all of the sites in Table 6.3, the Bow Meadows (BM) site had the lower RMSD values due to snowpacks with small amounts of buried vegetation that reduced acoustic scattering and snowpack stratigraphy with layer boundaries that best approximated horizontal lines. The horizontal line approximation was most similar to the acoustic model described in Section 3.5 of this thesis as a basis for operation of the SAS2.

The Bow Lake (BL) site dataset showed the highest error between gravimetric and acoustic measurements of $\bar{\rho}$, possibly owing to acoustic scattering and mode conversion of sound waves due to ponded water completely filling the pore spaces of the snow. This increased the scatter in the data points (Figure 6.2). The mean liquid water content determined using acoustics at the BL site was the second-highest of all of the field sites (Table 6.10), with higher levels of liquid water content only observed at the IFPF site on 1 June.

| Site | Dates | $\bar{\rho}$ comparison | $\operatorname{RMSD}\left(\mathrm{kg} \mathrm{m}^{-3}\right)$ | $\begin{gathered} \mathrm{MB} \\ \left(\mathrm{~kg} \mathrm{~m}^{-3}\right) \end{gathered}$ | $N_{\text {s }}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| UCS | 1 February to 31 May | ESC30 \& SAS2 | NA | NA | NA |
|  |  | Snowpit \& Snowtube | NA | NA | NA |
|  |  | Snowpit \& SAS2 | 48.4 | -4.10 | 6 |
| BC | 13 March | ESC30 \& SAS2 | 47.1 | 19.5 | 12 |
|  |  | Snowpit \& Snowtube | 67.4 | 64.5 | 2 |
|  |  | Snowpit \& SAS2 | 40.5 | -12.5 | 2 |
| BS | 14 March | ESC30 \& SAS2 | 29.0 | 3.2 | 117 |
|  |  | Snowpit \& Snowtube | 45.0 | -14.3 | 4 |
|  |  | Snowpit \& SAS2 | 18.1 | 7.3 | 4 |
| UCP | 15 March | ESC30 \& SAS2 | 25.2 | 3.8 | 35 |
|  |  | Snowpit \& Snowtube | 28.0 | 20.4 | 2 |
|  |  | Snowpit \& SAS2 | 15.5 | 15.0 | 2 |
| SC | 1 May | ESC30 \& SAS2 | 62.3 | 43.5 | 65 |
|  |  | Snowpit \& Snowtube | 82.6 | 75.2 | 5 |
|  |  | Snowpit \& SAS2 | 27.7 | -14.0 | 5 |
| BM | 3 May | ESC30 \& SAS2 | 17.9 | -2.8 | 63 |
|  |  | Snowpit \& Snowtube | 19.9 | -5.5 | 4 |
|  |  | Snowpit \& SAS2 | 8.2 | -6.2 | 4 |
| BL | 3 May | ESC30 \& SAS2 | 147.4 | -68.9 | 8 |
|  |  | Snowpit \& Snowtube | NA | NA | NA |
|  |  | Snowpit \& SAS2 | NA | NA | NA |
| NSH | 4 May | ESC30 \& SAS2 | 109.5 | -53.5 | 40 |
|  |  | Snowpit \& Snowtube | NA | NA | NA |
|  |  | Snowpit \& SAS2 | NA | NA | NA |
| BPC | 4 May | ESC30 \& SAS2 | 50.1 | 7.3 | 45 |
|  |  | Snowpit \& Snowtube | 62.6 | 33.7 | 2 |
|  |  | Snowpit \& SAS2 | 18.2 | -18.0 | 2 |
| IFPF | 1 June | ESC30 \& SAS2 | 36.7 | -13.5 | 9 |
|  |  | Snowpit \& Snowtube | NA | NA | NA |
|  |  | Snowpit \& SAS2 | NA | NA | NA |

Table 6.3. Comparisons between gravimetric and acoustic measurements of density $\bar{\rho}$ at individual field sites. Observations that are not available are marked with "NA." The RMSD, MB and number of samples $N_{s}$ are listed. The site locations are Upper Clearing Stationary (UCS), Boulton Campground (BC), Bow Summit (BS), Upper Clearing Portable (UCP), Sawmill Creek (SC), Bow Meadows (BM), Bow Lake (BL), Nakiska Ski Hills (NSH), Black Prince Cirque (BPC), and Icefields Parkway Forest Site (IFPF). The same site location names are used in the following tables. There were 10 sites in total.

| Site | Dates | Mean <br> $\left(\mathrm{kg} \mathrm{m}^{-3}\right)$ | Median <br> $\left(\mathrm{kg} \mathrm{m}^{-3}\right)$ | Min <br> $\left(\mathrm{kg} \mathrm{m}^{-3}\right)$ | Max <br> $\left(\mathrm{kg} \mathrm{m}^{-3}\right)$ | Range <br> $\left(\mathrm{kg} \mathrm{m}^{-3}\right)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| UCS | 1 Feb to 31 May | 291 | 241 | 145 | 597 | 452 |
| BC | 13 March | 202 | 195 | 154 | 261 | 107 |
| BS | 14 March | 242 | 239 | 190 | 312 | 122 |
| UCP | 15 March | 250 | 234 | 130 | 456 | 327 |
| SC | 1 May | 364 | 358 | 210 | 600 | 390 |
| BM | 3 May | 317 | 320 | 240 | 401 | 161 |
| BL | 3 May | 534 | 537 | 231 | 745 | 514 |
| NSH | 4 May | 420 | 459 | 38 | 635 | 597 |
| BPC | 4 May | 258 | 255 | 69 | 394 | 325 |
| IFPF | 1 June | 374 | 405 | 260 | 431 | 171 |

Table 6.4. Descriptive statistics of density $\bar{\rho}\left(\mathrm{kg} \mathrm{m}^{-3}\right)$ measured using the SAS2.

The artificial snow at the NSH site showed an under-prediction of acoustic snow density $\bar{\rho}$ but a higher accuracy than the BL site (Table 6.3) due to a lack of ponded water. The BPC site transect data, collected on same day as the NSH site transect data, showed lower RMSD error than the dataset collected at the NSH site (Table 6.3). Despite the presence of a few outliers (Figure 6.2), the IFPF site sample points had a lower RMSD error than the BL or NSH dataset. This suggested that although the snowpack was observed to be very wet at this field site, ponding of meltwater did not occur and water did not accumulate in the pore spaces of the snowpack. Because the snowpack was well-drained at the IFPF site, this reduced acoustic scattering and mode conversion occurring due to the ponded water. This reduced scatter in the data points shown in Figure 6.2.

Because time series data of snow density and SWE is not available at locations where the portable version of the SAS2 was deployed, more than one measurement cannot be used to show that error in the acoustically-determined data relative to gravimetric samples has a relationship with time. An increase in the scatter shown by Figure 6.2 may have occurred due to the inherent inaccuracies of the ESC30 snow sampler associated with loss of snow and water from the tube or due to mechanical modification of the snow sample during the invasive ESC30 sampling process. This suggests the inherent inaccuracies of the ESC30 sampler with respect to the SAS2.

### 6.2.4. Thermal

For most of the time of observation, the snowharp and depth-averaged snowharp temperature $\bar{T}$ observations at the UCS site (Figure 6.5 a , b) show results that are physically reasonable. The snowharp measurements were constrained to $\leq 0^{\circ} \mathrm{C}$, allowing for comparison with the acoustic SAS2 measurements that were constrained to $\leq 0^{\circ} \mathrm{C}$ by the UTAM model. The SR50 ultrasonic snow depth measurements are also shown on Figure 6.5b, indicating that there are four periods of observation. The averaged snowharp temperatures (Figure 6.5a) only use thermocouples that are completely covered by snow. These are the thermocouples situated at distances from the ground less than the snow depth measured by the SR50.

The first period coincided with the beginning of the observation season when the snowharp was not completely covered by snow; the second period coincided with the time when the snowharp was completely covered by snow; the third period coincided with the time of ablation, when there was a reduction in snow depth and uneven snow topography around the snowharp; and the fourth period coincided with the time when most of the snow had ablated. The RMSD and MB between snowharp and SAS2 measurements for each period is given by Table 6.5, whereas overall comparisons between snowpit and SAS2 and snowpit and snowharp measurements are given by Table 6.6.

| Measurement | First Period | Second Period | Third Period | Fourth Period |
| :---: | :---: | :---: | :---: | :---: |
| RMSD $\left({ }^{\circ} \mathrm{C}\right)$ | 3.0 | 1.1 | 1.2 | 0.32 |
| $\mathrm{MB}\left({ }^{\circ} \mathrm{C}\right)$ | -2.9 | -0.76 | -0.051 | -0.27 |

Table 6.5. Comparisons between snowharp and SAS2 measurements over observation periods.

| Measurement | Snowpit \& Acoustic |  <br> Snowharp |  <br> Acoustic |
| :---: | :---: | :---: | :---: |
| RMSD $\left({ }^{\circ} \mathrm{C}\right)$ | 3.9 | 2.6 | 1.8 |
| $\mathrm{MB}\left({ }^{\circ} \mathrm{C}\right)$ | -2.6 | 1.4 | -1.1 |

Table 6.6. Overall comparisons of snowharp temperature over the entire time of observation.
(a)

(b)

(c)


Figure 6.5. Upper Clearing Stationary (UCS) site time-series data showing (a) depth-averaged snowharp temperature and depthaveraged acoustic temperature $\bar{T}$, (b) spatial distribution of temperature observed using the snowharp, and (c) temperature at the snow surface measured using the IR thermometer.
(a)

(b)

(c)


Figure 6.6. Upper Clearing Stationary (UCS) site time-series data showing (a) ground temperature, (b) air temperature, and (c) heat flux at the bottom of the snowpack.

Tables 6.5 and 6.6 show that acoustic measurements of temperature $\bar{T}$ have the same order of magnitude as snowharp and snowpit measurements. The acoustic measurements made by the SAS2 are under-predicted relative to other measurements. Compared to the second and third observation periods (Table 6.5), the RMSD between snowharp and SAS2 measurements was highest during the first period of observation. Although thermocouples not covered with snow were excluded from the analysis, this may indicate that heat conduction along the thermocouple wires increased the temperature sensed by the thermocouples. The RMSD and MB magnitudes were lower during the second period of observation when the snowharp structure was completely covered with snow. By the third period when snow ablation occurred, the snow temperature $\bar{T}$ was close to $0^{\circ} \mathrm{C}$. Since both the snowharp and acoustic measurements are constrained to $\leq 0^{\circ} \mathrm{C}$, both the RMSD and MB magnitudes remained low during the third period. Constraining the measurements also caused the RMSD to remain low during the fourth period. However, the MB magnitude during the fourth period was higher than the other periods of observation.

Over the entire observation period (Table 6.6) the snowharp and acoustic measurements had the lowest RMSD and MB magnitudes. The snowpit and acoustic measurements had the highest RMSD and MB magnitudes due to spatial differences in snow temperature between the snowpit and SAS2 sampling locations. The snowpit and snowharp RMSD and MBs showed values that were intermediate between the RMSD and MB values used to compare the other measurements.

Despite a temperature anomaly apparent at the beginning of the observation period from 1 February to 4 February on the snowharp plot (Figure 6.5b) and possible inaccuracies of the invasive snowharp measurement method occurring due to heat conduction along the wires, the snowharp data showed that temperatures near the top of the snowpack were colder than temperatures near the ground surface. The difference in temperature between the top and bottom of the snowpack established a temperature gradient, resulting in depth hoar formation near the snowpack bottom as water vapour was transferred across pore spaces from warmer to colder surfaces. These processes are discussed in Section 2.2 of this thesis.

Diurnal heating and cooling cycles were apparent, with the snowpack gradually warming during the day due to a positive net shortwave radiation $K^{*}$ balance (Figure B.4a, Appendix B). The IR surface temperature of the snowpack (Figure 6.5c) was colder than the temperature at the
bottom of the snowpack (Figure 6.6a) due to longwave radiative cooling near the snow surface, evaporative cooling and a small but persistent negative heat flux ( $\sim-3 \mathrm{~W} \mathrm{~m}^{-2}$ ) from the ground into the snow during the snow accumulation season (Figure 6.6c). This observed heat flux had the same order of magnitude as the heat flux values listed in Table 2.1 of Chapter 2. Wet soil fed by groundwater exfiltration at the UC site where the ground was slow to freeze may have contributed to this heat flux.

After 16 April, the heat flux (Figure 6.6 c ) was small and positive $\left(\sim 0.2 \mathrm{~W} \mathrm{~m}^{-2}\right)$, indicating that the snowpack was transferring energy into the ground during the time of ablation due to rising temperatures and the production of meltwater after the snowpack reached $0{ }^{\circ} \mathrm{C}$. The ground temperature (Figure 6.6a) and heat flux plots (Figure 6.6c) were similar, with diurnal cycling of temperature most apparent on the plot of ground temperature (Figure 6.6a). The heat flux (Figure 6.6c) continued to increase until it exceeded $1.0 \mathrm{~W} \mathrm{~m}^{-2}$ on 13 May. After ablation of all snow at the UCS site by 14 May, the heat flux rose to $\sim 200 \mathrm{~W} \mathrm{~m}^{-2}$, indicating heating and cooling of the ground surface by direct radiative fluxes. A snow deposition event of short temporal duration lasting from 24-25 May depressed ground temperature and suppressed the heat flux associated with diurnal warming and cooling of the ground surface.

Comparisons between temperatures measured in snowpits using a thermometer and the acoustic temperature $\bar{T}$ measured using the SAS2 (Figure 6.7) showed an RMSD of $1.6^{\circ} \mathrm{C}$ and a MB of $-1.1^{\circ} \mathrm{C}$ for $N_{\mathrm{s}}=19$ samples. The RMSD and MB indicated that the acoustic temperature measurements are the same order of magnitude as the snowpit measurements of temperature. This strongly suggested that the SAS2 was able to measure snow temperature. Snow temperature $\bar{T}$ measured by the SAS2 increased over the winter season (Table 6.8 and Figure 6.5a), with higher mean temperatures apparent at the BM, BL, and NSH sites.


Figure 6.7. Comparison between depth-averaged temperature measured using snowpits and depth-averaged acoustic temperature measured using the portable and stationary versions of the SAS2 at ten field sites. The black line on the plot is the 1:1 line. The Upper Clearing Portable (UCP), Upper Clearing Stationary (UCS), Boulton Campground (BC), Sawmill Creek (SC), Black Prince Cirque (BPC), Bow Summit (BS), and Bow Meadows (BM) sites are listed in the legend.

| Site | Dates | $\bar{T} \mathrm{RMSD}\left({ }^{\circ} \mathrm{C}\right)$ | $\bar{T} \mathrm{MB}\left({ }^{\circ} \mathrm{C}\right)$ | $N_{\mathrm{s}}$ |
| :---: | :---: | :---: | :---: | :---: |
| UCS | 1 Feb to 31 May | 3.9 | -2.6 | 6 |
| BC | 13 March | 1.4 | -1.4 | 2 |
| BS | 14 March | 0.7 | 0.1 | 4 |
| UCP | 15 March | 0.7 | -0.4 | 2 |
| SC | 1 May | 2.5 | -2.2 | 5 |
| BM | 3 May | 1.0 | -0.7 | 4 |
| BL | 3 May | NA | NA | NA |
| NSH | 4 May | NA | NA | NA |
| BPC | 4 May | 0.2 | -1.5 | 2 |
| IFPF | 1 June | NA | NA | NA |

Table 6.7. Comparisons between snowpit and acoustic measurements of temperature $\bar{T}$.

| Site | Dates | Mean <br> $\left({ }^{\circ} \mathrm{C}\right)$ | Median <br> $\left({ }^{\circ} \mathrm{C}\right)$ | Min <br> $\left({ }^{\circ} \mathrm{C}\right)$ | Max <br> $\left({ }^{\circ} \mathrm{C}\right)$ | Range <br> $\left({ }^{\circ} \mathrm{C}\right)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| UCS | 1 Feb to 31 May | -3.4 | -1.5 | -15.0 | 0.0 | 15.0 |
| BC | 13 March | -4.9 | -4.5 | -9.0 | -2.0 | 7.0 |
| BS | 14 March | -6.0 | -5.6 | -8.6 | -3.1 | 5.5 |
| UCP | 15 March | -4.8 | -5.4 | -6.5 | -1.0 | 5.3 |
| SC | 1 May | -1.3 | -1.0 | -4.0 | 0.0 | 4.0 |
| BM | 3 May | -0.9 | -0.9 | -2.0 | -0.1 | 1.9 |
| BL | 3 May | -0.3 | -0.3 | -0.5 | -0.1 | 0.4 |
| NSH | 4 May | -1.4 | -1.0 | -4.0 | -0.5 | 3.5 |
| BPC | 4 May | -2.5 | -2.0 | -5.0 | -0.9 | 4.1 |
| IFPF | 1 June | -0.1 | -0.1 | -0.3 | 0.0 | 0.3 |

Table 6.8. Descriptive statistics of snow temperature $\bar{T}$ measured using the SAS2.

### 6.2.5. Liquid Water Content

Over the entire period of observation at the UCS site, there was an RMSD of 3.3\% and a MB of $2.5 \%$ between the stationary SAS2 and dielectric measurements of liquid water content $\bar{\theta}_{w}$, indicating that dielectric measurements were under-predicted relative to the acoustic measurements (Figure 6.1d). An RMSD of $3.2 \%$ and a MB of $-3.6 \%$ occurred between liquid water contents $\bar{\theta}_{w}$ measured using the stationary and portable versions of the SAS2, indicating spatial differences in liquid water content between measurement locations. The dielectric liquid water content and portable version of the SAS2 showed an RMSD of only $0.1 \%$ and a MB of $0.4 \%$, demonstrating the ability of the SAS2 to measure $\bar{\theta}_{w}$ within a $0.5 \%$ RMSD difference,
similar to the $\pm 0.5 \%$ liquid water content measurement error associated with dielectric measurements (Techel and Pielmeier, 2011).

Because the RMSD and MB magnitudes used to compare the dielectric and portable measurements of liquid water content was lower than the RMSD and MB magnitudes of the other $\bar{\theta}_{w}$ comparisons (Figure 6.1d), this suggested that $\bar{\theta}_{w}$ varied spatially between the stationary SAS2 sampling location and the portable sampling location situated close ( $\sim 2 \mathrm{~m}$ ) to the stationary device. The finding is supported by the literature reviewed in Section 2.2.9 of this thesis, showing that liquid water content is highly variable over distances spanning only 50 cm .

Some observations of liquid water content $\bar{\theta}_{w}$ are $<1.3 \%$ by volume within the dry region classified using the Techel and Pielmeier (2011) scheme, and most observations are < 8\% by volume as found by Kattelmann and Dozier (1999) for natural snowpacks. For wet snow during the time of ablation, the acoustic observations of liquid water content $\bar{\theta}_{w}$ approach $10 \%$. This is apparent for the stationary SAS2 observations made at the UCS site (Figure 6.1d) as well as for measurements of LWC made at other sites (Figure 6.8 and Table 6.10). The liquid water content values do not reach levels of $\sim 0 \%$ due to the presence of irreducible water content in the snowpack resulting due to the presence of a quasi-liquid layer surrounding the snow particles.

The liquid water content increased over the observation season as the snowpack warmed due to higher levels of net radiation and increasing air temperatures. This is apparent at the UCS site and at the portable sites visited later in the observation season (Table 6.10). All observations of liquid water content made using the SAS2 are within the range of liquid water contents given by the Martinec classification scheme and the Fierz et al. (1999) scheme. Because the acoustic values of water content observed at the UCS site are in the same range of liquid water contents observed by other investigators (Section 2.29), this strongly implies that the acoustic sensor measurements are physically reasonable, though it is not possible to exactly confirm the readings.

At the UCS site, the liquid water content $\bar{\theta}_{w}$ was lowest on 8 February at $1 \%$, indicating a very dry snowpack (Figure 6.1d). This is in agreement with the Fierz et al. (1999) classification of liquid water content being close to $0 \%$ for dry snow and irreducible liquid water contents $\theta_{e}$ of $1 \%$ to $2 \%$ (Marsh, 1991). The liquid water content never dropped as low as $0 \%$ due to the presence of irreducible liquid water held in the pore spaces of the snowpack.

After 8 February at the UCS site, higher levels of depth-averaged temperature $\bar{T}$ measured by the snowharp was apparent, and this is coincident with higher levels of liquid water content $\bar{\theta}_{w}$. Diurnal cycles of $\bar{\theta}_{w}$ were also present at this time, with a slight rise in liquid water content during the afternoon following peaks in shortwave radiation $K \downarrow$, indicating that energy was being transferred to the snowpack due to positive net shortwave radiation $K^{*}$.

Cycles of $\bar{\theta}_{w}$ were apparent, with the period of an average cycle (peak to trough of the waveform) lasting over the time period of $\sim 1$ day, indicating diurnal changes in $\bar{\theta}_{w}$ associated with warming of the snowpack during the day and cooling of the snowpack during the night. Because the cold content of the snowpack will increase due to a negative energy balance during the night, the cold content will only decrease when the net energy balance becomes positive and the snowpack warms. The cycles in depth-averaged water content $\bar{\theta}_{w}$ are created due to the heat capacity of the snowpack, which creates a time lag between the application of a positive energy balance and a rise in water content $\bar{\theta}_{w}$. Over the time of observation, the $\bar{\theta}_{w}$ values continued to increase due to increases in $K^{*}$ and air temperature. A gradual increase in snow average temperature $\bar{T}$ (Figure 6.5 a ) showed that the snowpack continued to warm over the observation season.

The water content once again reached a low level of $1 \%$ on 20 February (Figure 6.1d). This was the last time during the observation period when the liquid water content was as low as $1 \%$ due to gradually-increasing values of downward shortwave radiation $K \downarrow$, net shortwave radiation $K^{*}$, and air temperature occurring due to seasonal changes.

The first major drop in snow depth occurred between 14 April and 21 April, when the snow depth decreased from 99 cm to 78 cm due to ablation. The liquid water content $\bar{\theta}_{w}$ increased from $5.4 \%$ to $6 \%$ during this time (Figure 6.1d), continuing the upward trend apparent over the observation period. The second drop in snow depth occurred between 21 April and 26 April, when the snow depth decreased from 76 cm to 48 cm , a total of 28 cm . The water content was at $8.6 \%$ on 26 April, increasing a total of $2.6 \%$ from the observed $6 \%$ on 21 April.

After 10 May, a decrease in liquid water content $\bar{\theta}_{w}$ indicated that the snowpack was at the end of the ripening phase and had entered the output phase, as the snowpack was unable to
hold any more water. Beyond this point, water drained from the snowpack and the liquid water content did not increase.

The error between dielectric measurements and acoustic measurements of water content $\bar{\theta}_{w}$ collected at all of the field sites was small (Figure 6.8), as suggested by an RMSD of $0.5 \%$ and a MB of $-0.7 \%$ for $N_{s}=19$ samples. These results indicated that the RMSD was the same as the $0.5 \%$ measurement error associated with dielectric measurements, despite some small errors occurring due to manual balancing of the Denoth meter bridge. This demonstrated that the SAS2 was able to obtain measurements of liquid water content similar to dielectric measurements. The SAS2 can therefore be used in lieu of invasive dielectric measurements of liquid water content. Higher levels of liquid water content $\bar{\theta}_{w}$ were measured at sites visited later in the winter season than at sites visited at an earlier time (Table 6.10).


Figure 6.8. Comparison between depth-averaged liquid water content measured using dielectric methods and liquid water content measured using the SAS2 at field sites. The black line on the plot is the 1:1 line. The Boulton Campground (BC), Bow Summit (BS), Upper Clearing Portable (UCP), Sawmill Creek (SC), Bow Meadows (BM) and Black Prince Cirque (BPC) sites are listed in the legend.

| Site | Dates | $\bar{\theta}_{w}$ RMSD (\%) | $\bar{\theta}_{w}$ MB (\%) | $N_{s}$ |
| :---: | :---: | :---: | :---: | :---: |
| UCS | 1 Feb to 31 May | 3.3 | 2.5 | 6 |
| BC | 13 March | NA | NA | NA |
| BS | 14 March | 0.3 | 0.2 | 4 |
| UCP | 15 March | 0.2 | 0.1 | 2 |
| SC | 1 May | 0.8 | -0.6 | 5 |
| BM | 3 May | 0.4 | 0.2 | 4 |
| BL | 3 May | NA | NA | NA |
| NSH | 4 May | NA | NA | NA |
| BPC | 4 May | 0.2 | 0.2 | 2 |
| IFPF | 1 June | NA | NA | NA |

Table 6.9. Comparisons between dielectric and acoustic measurements of liquid water content $\bar{\theta}_{w}$.

| Site | Dates | Mean <br> $(\%)$ | Median <br> $(\%)$ | Min <br> $(\%)$ | Max <br> $(\%)$ | Range <br> $(\%)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| UCS | 1 Feb to 31 May | 4.2 | 4.1 | 1.0 | 10 | 9 |
| BC | 13 March | 1.8 | 1.7 | 1.1 | 2.5 | 1.4 |
| BS | 14 March | 2.0 | 2.1 | 1.1 | 3.1 | 2.0 |
| UCP | 15 March | 0.9 | 0.9 | 1.0 | 1.4 | 1.3 |
| SC | 1 May | 4.4 | 4.0 | 2.0 | 9.0 | 7.0 |
| BM | 3 May | 3.1 | 3.2 | 1.9 | 4.6 | 2.7 |
| BL | 3 May | 7.9 | 8.0 | 5.0 | 9.0 | 4.0 |
| NSH | 4 May | 6.8 | 7.0 | 4.0 | 9.0 | 5.0 |
| BPC | 4 May | 3.8 | 3.0 | 1.0 | 8.0 | 7.0 |
| IFPF | 1 June | 8.4 | 8.5 | 7.0 | 9.1 | 2.1 |

Table 6.10. Descriptive statistics of liquid water content $\bar{\theta}_{w}$ measured using the SAS2.

### 6.2.6. Acoustic

Figure 6.9a shows that at the UCS site, the speed of sound in snow, frequency-averaged from DC to the lowpass filter cutoff frequency ( $f_{c}=44 \mathrm{KHz}$ ), was less than the speed of sound in air. The speed of sound in air was computed using the air temperature (Figure 6.6b). Table 6.11 shows descriptive statistics of the speed of sound in snow at the field sites. The mean, minimum and maximum values of the speed of sound in snow at the field sites were less than the minimum speed of sound in air.

These findings suggest that the sound wave propagating through the snowpack is the Biot second P-wave that travels through the pore spaces of the snow medium. The physics of sound wave propagation in porous media shows that the Biot second P-wave may travel at a speed less than the speed of sound in air (Allard and Atalla, 2009; Johnson and Plona, 1980; Johnson et al., 1982).

Figure 6.9 b shows that the frequency-averaged attenuation $a_{c}$ was not constant over the snow accumulation and ablation season at the UCS site due to snowpack metamorphism and an increase in snow depth. Coincident with a rise in snow depth after the first large snow deposition event occurring on 4-5 March, the attenuation for the sound wave passing through the snowpack gradually increased during the time of snow accumulation and then decreased during the time of ablation after 16 April. For observations made at all of the field sites (Table 6.12), the mean, minimum and maximum attenuation values were in the range of $0 \mathrm{~Np} \mathrm{~m}^{-1} \leq a_{c} \leq 2.0 \mathrm{~Np} \mathrm{~m}^{-1}$. This range was used in the sensitivity analysis of Section 4.3.4, which showed that the theoretical maximum depth of snow over which an acoustic wave from the SAS2 could propagate was $\sim 3 \mathrm{~m}$.

The maximum snow depth at the UCS site was $\sim 1 \mathrm{~m}$, whereas the maximum snow depth at all of the sites (Table 6.13 ) was $\sim 2 \mathrm{~m}$. Because the SAS 2 was deployed at these field sites, this indicates that the SAS2 could obtain acoustic measurements of snow up to this maximum depth.

The $\Psi$ coefficient (Figure 6.9c) represents differences between the elastic constants of ice and the elastic constants of the snow particles (Section 3.2.3). For most of the snow accumulation and ablation season, Figure 6.16 c shows that $\Psi>1$. This suggests that the elastic constants of snow particles comprising the snowpack were less than the elastic constants of ice. During the time of ablation after 16 May, the $\Psi$ coefficient was $\Psi \leq 1$, thereby suggesting that the elastic constants of the snow particles were either the same or similar to the elastic constants of ice. Descriptive statistics (Table 6.14) showed that for most of the field sites, the mean, minimum and maximum values of $\Psi$ were $\Psi>1$. However, for the BL and IFPF field sites, the mean $\Psi$ coefficient was $\Psi \leq 1$, suggesting the presence of grain-coarsening of snow particles that occurred due to high liquid water contents and old, metamorphosed snow. Wet snow metamorphism that occurred due to the ponded water at the BL site may have been responsible for making the elastic constants of snow similar to the elastic constants of ice.

| Site | Dates | Min Air <br> $\left(\mathrm{m} \mathrm{s}^{-1}\right)$ | Mean <br> $\left(\mathrm{m} \mathrm{s}^{-1}\right)$ | Median <br> $\left(\mathrm{m} \mathrm{s}^{-1}\right)$ | Min <br> $\left(\mathrm{m} \mathrm{s}^{-1}\right)$ | Max <br> $\left(\mathrm{m} \mathrm{s}^{-1}\right)$ | Range <br> $\left(\mathrm{m} \mathrm{s}^{-1}\right)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| UCS | 1 Feb to 31 May | 319 | 257 | 258 | 190 | 318 | 128 |
| BC | 13 March | 329 | 307 | 308 | 292 | 311 | 19 |
| BS | 14 March | 329 | 302 | 302 | 292 | 309 | 17 |
| UCP | 15 March | 328 | 301 | 303 | 268 | 315 | 47 |
| SC | 1 May | 330 | 287 | 288 | 236 | 309 | 73 |
| BM | 3 May | 331 | 294 | 294 | 281 | 305 | 24 |
| BL | 3 May | 331 | 251 | 254 | 210 | 306 | 96 |
| NSH | 4 May | 335 | 274 | 270 | 227 | 327 | 100 |
| BPC | 4 May | 333 | 301 | 302 | 280 | 324 | 44 |
| IFPF | 1 June | 336 | 285 | 281 | 276 | 303 | 27 |

Table 6.11. Descriptive statistics of the speed of sound in snow $c_{s}\left(\mathrm{~m} \mathrm{~s}^{-1}\right)$ measured using the SAS2. The minimum speed of sound in air at each of the field sites is given in the column with the "Min Air" heading. The speed of sound in snow is less than the minimum speed of sound in air at each of the field sites.

| Site | Dates | Mean | Median | Min | Max | Range |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| UCS | 1 Feb to 31 May | 1.3 | 1.4 | 0.0 | 2.0 | 2.0 |
| BC | 13 March | 0.5 | 0.4 | 0.2 | 1.0 | 0.8 |
| BS | 14 March | 0.3 | 0.2 | 0.0 | 1.3 | 1.3 |
| UCP | 15 March | 1.2 | 1.2 | 0.5 | 1.8 | 1.3 |
| SC | 1 May | 0.6 | 0.5 | 0.1 | 1.9 | 1.8 |
| BM | 3 May | 1.2 | 1.2 | 0.1 | 1.7 | 1.6 |
| BL | 3 May | 1.9 | 1.9 | 1.8 | 2.0 | 0.2 |
| NSH | 4 May | 1.2 | 1.3 | 0.1 | 1.6 | 1.5 |
| BPC | 4 May | 0.5 | 0.4 | 0.1 | 1.4 | 1.3 |
| IFPF | 1 June | 1.7 | 1.7 | 1.5 | 1.8 | 0.3 |

Table 6.12. Descriptive statistics of sound attenuation $a_{c}$ measured using the SAS2.

| Site | Dates | Mean <br> $(\mathrm{cm})$ | Median <br> $(\mathrm{cm})$ | Min <br> $(\mathrm{cm})$ | Max <br> $(\mathrm{cm})$ | Range <br> $(\mathrm{cm})$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| UCS | 1 Feb to 31 May | 56 | 53 | 0 | 100 | 100 |
| BC | 13 March | 97 | 100 | 59 | 110 | 51 |
| BS | 14 March | 175 | 178 | 119 | 201 | 82 |
| UCP | 15 March | 78 | 75 | 26 | 123 | 97 |
| SC | 1 May | 98 | 97 | 18 | 160 | 142 |
| BM | 3 May | 125 | 125 | 65 | 162 | 97 |
| BL | 3 May | 58 | 53 | 51 | 70 | 19 |
| NSH | 4 May | 49 | 44 | 3 | 1.6 | 113 |
| BPC | 4 May | 57 | 56 | 22 | 92 | 70 |
| IFPF | 1 June | 43 | 42 | 30 | 55 | 25 |

Table 6.13. Descriptive statistics of snow depth $\bar{y}$ measured using a depth rod.

| Site | Dates | Mean | Median | Min | Max | Range |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| UCS | 1 Feb to 31 May | 1.4 | 1.5 | 0.0 | 2.0 | 2.0 |
| BC | 13 March | 1.2 | 1.3 | 0.5 | 1.6 | 1.1 |
| BS | 14 March | 1.3 | 1.3 | 1.0 | 1.9 | 0.9 |
| UCP | 15 March | 1.3 | 1.3 | 1.1 | 1.5 | 0.5 |
| SC | 1 May | 1.5 | 10.5 | 0.9 | 2.0 | 1.1 |
| BM | 3 May | 1.3 | 1.3 | 1.0 | 1.8 | 0.8 |
| BL | 3 May | 0.7 | 0.6 | 0.5 | 1.1 | 0.6 |
| NSH | 4 May | 1.1 | 1.2 | 0.1 | 1.8 | 1.7 |
| BPC | 4 May | 1.5 | 1.5 | 1.1 | 2.0 | 0.9 |
| IFPF | 1 June | 0.8 | 0.8 | 0.5 | 1.2 | 0.7 |

Table 6.14. Descriptive statistics of $\Psi$ coefficient measured using the SAS2.
(a)

(b)

(c)


Figure 6.9. Upper Clearing Stationary (UCS) site time-series data showing (a) frequency-averaged speed of sound in snow and in air, (b) frequency-averaged attenuation, and (c) $\Psi$ coefficient.

### 6.3. Acoustic Imaging Example

Figure 6.10 shows an example of an acoustic image created using the SAS2 at the UC stationary site on 15 March. The figure consists of processed signals arranged at cumulative microphone offset distances from the loudspeaker. The processing required to form this image is discussed in Chapter 3. The elements of the image matrix have been scaled to amplitudes of [ 0,1$]$ for visual display. The observation was made on 15 March shortly before a snowpit was created next to the acoustic sensor. The snowpit was created at a distance of $\sim 2 \mathrm{~m}$ from the acoustic sensor. The total snow depth was 91 cm . As shown by Figure 6.10, the snowpit stratigraphy was comprised of a wind-crusted surface layer with clumps and hard snow overlying a layer with rounded dendric particles; under this layer was a layer of snow particles with rounded columns; and at the bottom of the snowpack was a layer of depth hoar with facetted particles.

As shown by Figure 6.10, the snow density near the surface of the snowpack remained close to $130 \mathrm{~kg} \mathrm{~m}^{-3}$ with an increase in snow density to $220 \mathrm{~kg} \mathrm{~m}^{-3}$ and a slight drop in snow density to $214 \mathrm{~kg} \mathrm{~m}^{-3}$ in the bottom layers of the snowpack due to the presence of facetted depth hoar with unconsolidated crystals. The RMSD between the snowpit and acoustic measurements of $\bar{\rho}$ was $4.46 \mathrm{~kg} \mathrm{~m}^{-3}$, indicating that for this measurement, the SAS2 under-predicted depthaveraged snow density $\bar{\rho}$. The SWE was similarly under-predicted due to under-prediction of the density. The depth-averaged snowpit temperature $\bar{T}$ was $-1.6^{\circ} \mathrm{C}$, slightly greater than the acoustic measurement of temperature at $-2.5^{\circ} \mathrm{C}$. The RMSD between temperature measurements was $0.91^{\circ} \mathrm{C}$, indicating that there was $\sim 1^{\circ} \mathrm{C}$ difference between temperature measurements made from the side of a snowpit and acoustic measurements.
(a)

(b)

(c)

(d)


| Measurement | Gravimetric/Thermal/Dielectic | Acoustic | RMSD | MB |
| :---: | :---: | :---: | :---: | :---: |
| $\bar{\rho}\left(\mathrm{kg} \mathrm{m}^{-3}\right)$ | 173 | 168 | 4.46 | -4.46 |
| $\mathrm{SWE}(\mathrm{mm})$ | 157 | 142 | 15.6 | -15.6 |
| $\bar{T}\left({ }^{\circ} \mathrm{C}\right)$ | -1.60 | -2.50 | 0.910 | -0.910 |
| $\bar{\theta}_{w}$ | 0.960 | 4.70 | 3.70 | 3.70 |

Figure 6.10. Snowpit stratigraphy at Upper Clearing Stationary (UCS) site observed on 15 March. The snowpit was situated close to the SAS2 sensor; subplots are: (a) gravimetric density measurements; (b) temperature measured from the snowpit wall using a thermometer; (c) liquid water content; and (d) acoustic image of snowpit with cumulative offset. A table shows a comparison between the snowpit and acoustic measurements.

The acoustic image (Figure 6.10d) showed reflections from the snowpack. The resolution is qualitatively similar to a sonar image sensed using a sediment-profiling sonar system (Lurton, 2002) or a medical ultrasonography image (Ng and Swanevelder, 2011). Owing to the empirical measurement of the system response $\mu_{k}[t]$ (Section 4.4), there may be a loss of resolution and the creation of some spurious reflections in the SAS2 image. The resolution may be lower (i.e. the minimum resolvable width $\Delta y \gg 2.5 \times 10^{-4} \mathrm{~m}$ ) then the minimum resolution given by design calculations in Section 4.3.12.

Because the sound wave from the SAS2 is reflected due to changes in the acoustic impedance of snow, reflections shown by Figure 6.10d do not always coincide with snowpack layers observed by an experienced snow surveyor. This is a limitation of the acoustic method occurring due to the physics of sound propagation in snow and is similar to limitations of sonar and medical ultrasonography. The finding reported here is similar to the finding reported by Harper and Bradford (2003), who found that radar images do not always coincide with snowpit measurements due to small changes in the dielectric constant of snow. This implies that smallscale variability in snow acoustic properties may be a reason for differences between snowpits and the SAS2. Moreover, because time-to-depth conversion was accomplished using the average speeds of sound in snow, reflections may not exactly coincide with the depths of measured snowpack layers. The time-to-depth conversion in the air medium above the surface of the snowpack was accomplished by using the speed of the sound wave in air, calculated using air temperature.

Near the snow surface at 91 cm , the acoustic image from the SAS2 shows a highamplitude reflection associated with the sound wave crossing the air-snow interface. The highamplitude reflection occurred due to differences in acoustic impedance between the air and snow mediums. Above the air-snow interface, reflections of small amplitude are visible. These reflections may be multiple reflections from the snowpack, or could represent changes in the speed and attenuation of the sound wave in the boundary layer above the snow surface due to changes in temperature. A low-amplitude reflection was visible at 89 cm depth after the sound wave exited the wind-crusted layer and moved to a greater depth beneath the snow surface. Within the second snowpack layer from 89 cm to 71 cm , acoustic scattering of the sound wave and internal reflections caused a series of linear features to appear in the image. These linear features disappear after 71 cm , indicating the start of another layer in the snowpack. The linear
features are of higher amplitude than the features in the preceding layer from 71 cm to 52 cm . From 52 cm to the ground surface, there was a loss of resolution and an increase in attenuation as the sound wave propagated into the depth hoar layer. This attenuation occurs since the sound wave has propagated through a porous medium comprised of coarse particles (Stoll, 1989).

### 6.4. Seasonal Snowpack Changes

Figures 6.11 to 6.18 show snowpits and acoustic images observed at the UCS site. The first snowpit observed on 2 February showed a layer of fresh snow overlying a layer of looselypacked soft snow with extensive depth hoar formation at the bottom. An increase in snow density occurred with depth due to overburden compaction by overlying snow layers and depth hoar particles with large mass. The temperature continued to increase with depth beneath the snow surface, indicating the presence of a temperature gradient due to a ground heat flux. A temperature gradient was also apparent in the snowharp and heat flux plate data (Figure 6.5 and 6.6).

The layer of soft snow deposited on the top of the snowpack was visible on the acoustic image as a large amplitude reflection. Some internal reflections were caused due to acoustic scattering by the fresh snow. The layer of soft snow under the fresh snow layer was barely visible on the acoustic image. There was a loss of resolution and an increase in attenuation in the depth hoar layer. The RMSD and MB indicate good agreement between the measured and acoustic parameters.

By 16 February, the layer of fresh snow observed on 2 February had been compacted, and a small layer of fresh snow near the snow surface was also present. The density and temperature increased with depth. The acoustic image showed the top of the snowpack as a reflection with large amplitude. Some reduction in acoustic scattering was apparent in the second packed layer, with a change in the acoustic image apparent at less than 30 cm depth from the bottom of the snowpack, showing a change in density within the depth hoar layer.

The snowpit shown for 15 March has already been discussed in the previous section of this thesis (Figure 6.10). By 18 April the snowpit showed lower density at distances closer to the bottom of the snowpack due to the presence of wet layers near the snow surface. The snow temperature was near $0^{\circ} \mathrm{C}$, indicating that the snowpack was close to isothermal. The liquid
water content $\bar{\theta}_{w}$ decreased with depth due to warming of the snowpack near the snow surface. Acoustic imaging of the snowpack showed more spatial variability in the layers at this time.

By 25 April, the snowpack showed extensive metamorphism. A layer of wet snow on the top of the snowpack was observed above a slightly denser layer. Near the bottom of the snowpack was a layer of wet columns and wet depth hoar. The snowpack was isothermal at $0^{\circ} \mathrm{C}$, indicating that the snow was melting. The first two snow layers are visible on the acoustic image, with some internal reflections visible in the denser layer and a loss of resolution as well as an increase in attenuation for layers near the bottom of the snowpack.

On 2 May, the snowpack was once again isothermal at $0^{\circ} \mathrm{C}$. Wet snow metamorphism resulted in the destruction of snowpack layers observed earlier in the observation season. The water content was greatest near the snow surface due to warming by radiative fluxes. A decrease in water content was apparent for fresh snow, whereas the water content was slightly greater for the underlying layer of hard-packed crystals. The acoustic image shows the top layer as a highamplitude reflection, followed by internal reflections within the underlying snowpack layers.

By 10 May, the snowpack had a temperature near $0{ }^{\circ} \mathrm{C}$ shortly before the drop in water content observed after this date (Figure 6.1d). This is when the snowpack exited the ripening phase and the water content decreased during the output phase. The acoustic image showed a loss of resolution at a distance of less than 5 cm from the snow surface, indicating the presence of meltwater ponded between the bottom of the snowpack and the frozen ground surface. By 14 May, there was a rapid decrease in snow depth and water content. The snowpack was in the output phase where the production of meltwater occurred. Few reflections from the snowpack are visible on the acoustic image and the water content $\bar{\theta}_{w}$ was lower than the liquid water content $\bar{\theta}_{w}$ on 10 May.


| Measurement | Gravimetric/Thermal/Dielectric | Acoustic | RMSD | MB |
| :---: | :---: | :---: | :---: | :---: |
| $\bar{\rho}\left(\mathrm{kg} \mathrm{m}^{-3}\right)$ | 235 | 153 | 82.7 | -82.7 |
| SWE $(\mathrm{mm})$ | 129 | 67.5 | 61.9 | -61.9 |
| $\bar{T}\left({ }^{\circ} \mathrm{C}\right)$ | -1.30 | -7.70 | 6.40 | 6.40 |

Figure 6.11. Snowpit stratigraphy at Upper Clearing Stationary (UCS) site observed on 2 February. The snowpit was situated close to the SAS2 sensor; subplots are: (a) gravimetric density measurements; (b) temperature measured from the snowpit wall using a thermometer; and (c) acoustic image of snowpit with cumulative offset. A table shows a comparison between the snowpit and acoustic measurements.


| Measurement | Gravimetric/Thermal/Dielectric | Acoustic | RMSD | MB |
| :---: | :---: | :---: | :---: | :---: |
| $\bar{\rho}\left(\mathrm{kg} \mathrm{m}^{-3}\right)$ | 200 | 177 | 22.2 | -22.2 |
| SWE $(\mathrm{mm})$ | 110 | 89.8 | 20.2 | -20.2 |
| $\bar{T}\left({ }^{\circ} \mathrm{C}\right)$ | -2.80 | -9.90 | 7.10 | 7.10 |

Figure 6.12. Snowpit stratigraphy at Upper Clearing Stationary (UCS) site observed on 16 February. The snowpit was situated close to the SAS2 sensor; subplots are: (a) gravimetric density measurements; (b) temperature measured from the snowpit wall using a thermometer; and (c) acoustic image of snowpit with cumulative offset. A table shows a comparison between the snowpit and acoustic measurements.


| Measurement | Gravimetric/Thermal/Dielectric | Acoustic | RMSD | MB |
| :---: | :---: | :---: | :---: | :---: |
| $\bar{\rho}\left(\mathrm{kg} \mathrm{m}^{-3}\right)$ | 343 | 339 | 4.24 | -4.24 |
| $\mathrm{SWE}(\mathrm{mm})$ | 319 | 280 | 39.2 | -39.2 |
| $\bar{T}\left({ }^{\circ} \mathrm{C}\right)$ | -0.01 | -0.67 | 0.600 | 0.6 |
| $\bar{\theta}_{w}(\%)$ | 2.22 | 5.83 | 3.60 | 3.60 |

Figure 6.13. Snowpit stratigraphy at Upper Clearing Stationary (UCS) site observed on 18 April. The snowpit was situated close to the SAS2 sensor; subplots are: (a) gravimetric density measurements; (b) temperature measured from the snowpit wall using a thermometer; (c) liquid water content; and (d) acoustic image of snowpit with cumulative offset. A table shows a comparison between the snowpit and acoustic measurements.

（b）

（c）


| Measurement | Gravimetric／Thermal／Dielectric | Acoustic | RMSD | MB |
| :---: | :---: | :---: | :---: | :---: |
| $\bar{\rho}\left(\mathrm{kg} \mathrm{m}^{-3}\right)$ | 402 | 409 | 7.74 | 7.74 |
| $\mathrm{SWE}(\mathrm{mm})$ | 241 | 160 | 80.7 | -80.7 |
| $\bar{T}\left({ }^{\circ} \mathrm{C}\right)$ | 0.500 | -0.500 | 1.00 | 1.00 |
| $\bar{\theta}_{w}(\%)$ | 7.76 | 6.70 | 1.06 | 1.06 |

Figure 6．14．Snowpit stratigraphy at Upper Clearing Stationary（UCS）site observed on 25 April．The snowpit was situated close to the SAS2 sensor；subplots are：（a）gravimetric density measurements；（b）temperature measured from the snowpit wall using a thermometer；（c）liquid water content；and（d）acoustic image of snowpit with cumulative offset．A table shows a comparison between the snowpit and acoustic measurements．

(c)



| Measurement | Gravimetric/Thermal/Dielectric | Acoustic | RMSD | MB |
| :---: | :---: | :---: | :---: | :---: |
| $\bar{\rho}\left(\mathrm{kg} \mathrm{m}^{-3}\right)$ | 343 | 424 | 81.3 | 81.3 |
| $\mathrm{SWE}(\mathrm{mm})$ | 192 | 132 | 59.6 | -59.6 |
| $\bar{T}\left({ }^{\circ} \mathrm{C}\right)$ | 0.500 | -0.366 | 0.866 | -0.866 |
| $\bar{S} \bar{w}_{w}(\%)$ | 5.10 | 7.90 | 2.80 | 2.80 |

Figure 6.15. Snowpit stratigraphy at Upper Clearing Stationary (UCS) site observed on 2 May. The snowpit was situated close to the SAS2 sensor; subplots are: (a) gravimetric density measurements; (b) temperature measured from the snowpit wall using a thermometer; (c) liquid water content; and (d) acoustic image of snowpit with cumulative offset. A table shows a comparison between the snowpit and acoustic measurements.


| Measurement | Acoustic |
| :---: | :---: |
| $\bar{\rho}\left(\mathrm{kg} \mathrm{m}^{-3}\right)$ | 334 |
| $\mathrm{SWE}(\mathrm{mm})$ | 67.0 |
| $\bar{T}\left({ }^{\circ} \mathrm{C}\right)$ | -0.340 |
| $\bar{\theta}_{w}(\%)$ | 9.80 |

Figure 6.16. Acoustic image at Upper Clearing Stationary (UCS) site observed on 10 May at 12:00 hrs. The snowpit was situated close to the SAS2 sensor. A table shows the acoustic measurements observed by the SAS2. The snow depth was 20 cm (measured by the SR50)


| Measurement | Acoustic |
| :---: | :---: |
| $\bar{\rho}\left(\mathrm{kg} \mathrm{m}^{-3}\right)$ | 366 |
| $\mathrm{SWE}(\mathrm{mm})$ | 66.0 |
| $\bar{T}\left({ }^{\circ} \mathrm{C}\right)$ | 0.00 |
| $\bar{\theta}_{w}(\%)$ | 8.10 |

Figure 6.17. Acoustic image at Upper Clearing Stationary (UCS) site observed on 12 May at 12:00 hrs. The snowpit was situated close to the SAS2 sensor. A table shows the acoustic measurements observed by the SAS2. The snow depth was 18 cm (measured by the SR50).


| Measurement | Acoustic |
| :---: | :---: |
| $\bar{\rho}\left(\mathrm{kg} \mathrm{m}^{-3}\right)$ | 310 |
| $\mathrm{SWE}(\mathrm{mm})$ | 16.0 |
| $\bar{T}\left({ }^{\circ} \mathrm{C}\right)$ | 0.00 |
| $\bar{\theta}_{w}(\%)$ | 4.50 |

Figure 6.18. Acoustic image at Upper Clearing Stationary (UCS) site observed on 14 May at 12:00 hrs. The snowpit was situated close to the SAS2 sensor. A table shows the acoustic measurements observed by the SAS2. The snow depth was 5.1 cm (measured by the SR50)

### 6.5. Repeatability of Successive Measurements

Tables 6.15 to 6.17 and Figure 6.19 show the results of an experiment conducted on 2 February shortly before the digging of the first snowpit observation made at the UCS site. The stationary version of the SAS2 was repeatedly used to take 30 acoustic observations over a time period of 6 minutes. The duration of each acoustic sample was $t_{m l s}=1.6 \mathrm{~s}$. The purpose of this experiment was to test differences between repeated acoustic samples. The snow depth was 47 cm at this time and the SAS2 was situated at a distance of 75 cm above the snow surface. The experiment could not be conducted for every acoustic sample taken using the SAS2 due to the lack of capacity for storage of acoustic data on the SD card.

For depth-averaged density $\bar{\rho}$ the measures of central tendency (mean, median and mode) are in close agreement and the maximum and minimum values have a range of only $1.25 \mathrm{~kg} \mathrm{~m}^{-3}$, indicating that differences between 30 successive measurements are small. An RMSD of $0.34 \mathrm{~kg} \mathrm{~m}^{-3}$ and a MB of $0.10 \mathrm{~kg} \mathrm{~m}^{-3}$ also demonstrated the ability for the SAS2 to take successive measurements of density with values that are numerically similar. The negative skewness value of -0.03 indicated that the distribution was slightly skewed with more values greater than the mean. The kurtosis value of 2.16 showed that the distribution of $\bar{\rho}$ was platykurtic. This suggests that the histogram distribution is "flat" and the error between measurements was widely distributed over the range due to small variations in acoustic scattering between acoustic samples and small changes in wind speed under the SAS2 sensor assembly over the duration of the experiment.

The mean, median and mode were similar for depth-averaged temperature $\bar{T}$ with a range of only $0.19{ }^{\circ} \mathrm{C}$. Small values of RMSD and MB for the $\bar{T}$ parameter also demonstrate the ability for successive measurements of snow temperature to be numerically close to the measured values. The positive skewness value of 0.14 indicates that most values in the histogram distribution are less than the mean. The kurtosis value of 1.45 suggests that similar to successive measurements of $\bar{\rho}$, the distribution is platykurtic and the error between measurements is distributed over the range of $\bar{T}$ values.

Similar to the measurements of $\bar{\rho}$ and $\bar{\theta}_{w}$ the range of liquid water content values was small with low RMSD and MB values. The skewness of 0.61 for the dataset was higher than the
skewness values for the $\bar{\rho}$ and $\bar{\theta}_{w}$ datasets. The positive skewness value of 0.61 indicated that more values in the histogram distribution were less than the mean. The kurtosis value of 2.71 indicates that the distribution of depth-integrated liquid water content values $\bar{\theta}_{w}$ was not as "flat" as the histogram distributions of $\bar{\rho}$ and $\bar{T}$ parameters, with more values situated closer to the mean. These results strongly suggest that successive measurements taken by the SAS2 system are repeatable.


Figure 6.19. Experiment conducted on 2 February at the UCS site using the stationary version of the SAS2. The stationary version of the SAS2 was used to repeatedly take 30 acoustic observations over a time period of 10 minutes. (a) Histogram of repeated measurements for depth-integrated snow density $\bar{\rho}$. (b) Histogram of repeated measurements for depth-integrated temperature $\bar{T}$. (c) Histogram of repeated measurements for depth-integrated liquid water content $\bar{\theta}_{w}$.

| Statistic | $\bar{\rho}\left(\mathrm{kg} \mathrm{m}^{-3}\right)$ or $\%$ |
| :---: | :---: |
| Mean | 204.64 |
| median | 204.67 |
| mode | 204.01 |
| max | 205.26 |
| min | 204.01 |
| range | 1.25 |
| RMSD | 0.34 |
| MB | 0.10 |
| skewness | -0.03 |
| kurtosis | 2.16 |

Table 6.15. Descriptive statistics for the experiment of repeated measurements conducted on 2 February at the UCS site using the stationary version of the SAS2. The depth-integrated density is $\bar{\rho}$. The RMSD and MB are used to compare each of the 30 measurements to the mean of the 30 measurements. The mean of the 30 measurements is taken as the observed value.

| Statistic | $\bar{T}\left({ }^{\circ} \mathrm{C}\right)$ or $\%$ |
| :---: | :---: |
| mean | -7.60 |
| median | -7.60 |
| mode | -7.69 |
| max | -7.69 |
| min | -7.69 |
| range | 0.19 |
| RMSD | 0.06 |
| MB | -0.10 |
| skewness | 0.14 |
| kurtosis | 1.45 |

Table 6.16. Descriptive statistics for the experiment of repeated measurements conducted on 2 February at the UCS site using the stationary version of the SAS2. The depth-integrated temperature is $\bar{T}$. The RMSD and MB are calculated in the same manner as Table 5.1.

| Statistic | $\bar{\theta}_{w}$ (dimensionless) or $\%$ |
| :---: | :---: |
| mean | 3.19 |
| median | 3.18 |
| mode | 3.10 |
| max | 3.30 |
| min | 3.10 |
| range | 0.20 |
| RMSD | 0.05 |
| MB | 0.12 |
| skewness | 0.61 |
| kurtosis | 2.71 |

Table 6.17. Descriptive statistics for the experiment of repeated measurements conducted on 2 February at the UCS site using the stationary version of the SAS2. The depth-integrated liquid water content is $\bar{\theta}_{w}$. The RMSD and MB are calculated in the same manner as Table 5.1.

### 6.6. Influence of Wind on SAS2 Measurements

To assess the error introduced by wind on acoustic observations made using the SAS2, Figure 6.20 shows the anemometer-measured windspeed (Figure B.1a, Appendix B) plotted against the absolute temperature error $\left|\varepsilon_{\mathrm{T}}\right|$. The $\left|\varepsilon_{\mathrm{T}}\right|$ is the absolute difference between the depth-averaged measurements of temperature made using the snowharp and the depth-averaged measurements of $\bar{T}$ made using the SAS2. The temperature measurements were chosen because this is the most complete SAS2 dataset collected at the Upper Clearing Stationary (UCS) site over the snow accumulation and ablation season with $\mathrm{N}_{\mathrm{s}}=2390$ measurements.

A significant correlation between windspeed and $\left|\varepsilon_{\mathrm{T}}\right|$ will imply that wind movement of the SAS2 sensor above the surface of the snowpack or wind diffraction of sound waves influenced the acoustic measurements. The Spearman rank correlation coefficient was chosen as a measure of correlation because it is non-parametric and unlike the nominal Pearson rank correlation coefficient, the data does not have to be normally distributed. Moreover, the Spearman rank correlation coefficient tests for a possible non-linear relationship between the windspeed and the error $\left|\varepsilon_{\mathrm{T}}\right|$.

Choosing a $95 \%$ significance level and using a null hypothesis rejection test (Ebdon, 1991), the Spearman rank correlation coefficient of $r_{s}=0.03$ indicated that the null hypothesis cannot be rejected at the $95 \%$ level and there is no correlation between windspeed and $\left|\varepsilon_{\mathrm{T}}\right|$. This implies that small movements of the SAS2 on the met station boom and wind transport of sound waves did not influence acoustic measurements. This was likely due to the signal power and length of the Maximum Length Sequence (MLS). The errors in the acoustic measurements were therefore spread over the entire time that the MLS was produced, unlike the ultrasonic depth sensor with its monopulse source signal.


Figure 6.20. Correlations between the absolute temperature error $\left|\varepsilon_{\mathrm{T}}\right|$ and the anemometermeasured windspeed at the UCS site. The absolute temperature error $\left|\varepsilon_{\mathrm{T}}\right|$ is the absolute value of the difference between the depth-averaged snowharp and acoustic measurements of $\bar{T}$. The correlation $r_{s}$ is the non-parametric Spearman rank correlation coefficient and the number of samples is $N_{s}$. The dashed line is provided for visual reference.

### 6.7. Example of Portable SAS2 Observation Site

This section presents an example of observations made using the portable version of the SAS2. Observations made at the other field sites are given in Appendix B of this thesis.

The Sawmill Creek (SC) site was chosen because it is most demonstrative of acoustic observations made using the SAS2. The SC site transect started in a clearing on the ridge, headed into an avenue of trees in a gully, crossed an ecotone between the clearing and the forest, and then finished in the forest (Figure 6.21 and 6.22, Table 6.18). Similar to the other field sites (Table 6.3) the RMSD and MB magnitudes at the SC site were highest for the ESC30 and snowpit measurements and lowest for the snowpit and acoustic measurements, indicating that the acoustic device had a higher overall accuracy than the ESC30 sampler and could also be used in lieu of the ESC30 sampler.

The RMSD comparing dielectric to acoustic measurements of liquid water content $\bar{\theta}_{w}$ was $0.8 \%$ at the SC site, slightly higher than the $0.5 \%$ error in liquid water content nominally associated with dielectric methods. However, the error between acoustic and dielectric measurements still remained close to $0.5 \%$, similar to other site data shown in Table 6.9 and Appendix B.

In the same manner as the other sites, the error between gravimetric and acoustic measurements of $\bar{\rho}$ was greatest at sample points where vegetation was buried beneath the snowpack, demonstrating the effects of acoustic scattering. Larger differences in acoustic snow density relative to ESC30 density were apparent in the clearing situated on a ridge before a gully due to the presence of shrubs and small trees buried beneath the snow surface. These larger differences were not as apparent between snowpit and acoustic measurements, indicating that the ESC30 snowtube measurements were affected by the presence of vegetation.

The depth-averaged liquid water content $\bar{\theta}_{w}$ was lower on the ridge and in the avenue of trees than near the clearing and the ecotones, where a rise in liquid water content measured by the SAS2 was concomitant with observations of water at the bottom of the ESC30 tube used to extract snow samples. This also implied that a stream was situated under the snowpack, as suggested by the sound of running water near the shrubs. The liquid water content along the transect showed lower values in the forest than in the clearing or in the ecotone, indicating the presence of drier snow resulting due to lower levels of solar radiation occurring due to extinction of light by the forest canopy.

The acoustic SAS2 measurements showed a small drop in snow temperature on the ridge near the buried tree, suggesting the presence of the "chimney" effect identified by Sturm (1992), where temperatures are colder in the area surrounding the tree due to the rising of warm air through the void space surrounding the tree. The snow temperature remained approximately constant over the avenue of trees and clearing sections of the transect. The snow temperature $\bar{T}$ was higher in the clearing and approached $0{ }^{\circ} \mathrm{C}$, coincident with higher measurements of $\bar{\theta}_{w}$ and visual observations of liquid water within the snowpack. The drop in temperature in the forest indicated a reduction in shortwave radiation from the sun and lower values of $\bar{\theta}_{w}$. The temperature showed an increase in variability within the forest section of the transect due to transfer of heat from vegetation to the snowpack and spatial variability in canopy cover.

The speed of sound in snow (Figure 6.22) had a similar order of magnitude as the sound speeds observed at the UCS site (Figure 6.9a), once again suggesting that a P-wave propagated in the pore spaces of the medium. The air temperature at this site was $\sim 0^{\circ} \mathrm{C}$, corresponding with a speed of sound of $\sim 331 \mathrm{~m} \mathrm{~s}^{-1}$. The observed speed of sound in snow was $\ll 331 \mathrm{~m} \mathrm{~s}^{-1}$, indicating that the speed of sound in snow was less than the speed of sound in air.

The attenuation $a$ of sound in snow was spatially variable along the transect. The $\Psi$ coefficient showed that the elastic constants of the snow particles were less than the elastic constants of ice along the transect since $\Psi \geq 1$ (Figure 6.22).

Figures 6.23 to 6.27 show acoustic images corresponding with snowpits created at five locations along the transect. Most of the snowpits showed gradual increases in snow density with depth due to compression and metamorphic processes, with a slight decrease in density within layers near the bottom of the snowpack due to the presence of depth hoar. Reflections from the snow surface and a loss of resolution at the bottom of the snowpack due to the presence of depth hoar are visible on the image. Although buried shrubs were present at the bottom of the snowpack, the loss of resolution did not allow for visual identification of these elements.

Snowpit \#1 (Figure 6.23) in the clearing shows a reflection from an ice crust at the snow surface. The ice crust had a high enough permeability to permit propagation of the acoustic wave through the ice crust and into the snowpack. Compared to snowpit \#1, changes in snowpack stratigraphy are visible in the acoustic image of snowpit \#2 (Figure 6.24), which was sampled in the avenue of trees within the clearing. The acoustic image of snowpit \#2 showed that the snow surface did not generate as large as a reflection as the reflection from an ice crust
visible on the acoustic image of snowpit \#1. Fine layers are noticeable in the acoustic image of snowpit \#2 immediately below the surface showing changes in the acoustic properties of snow that do not immediately coincide with visually-observed snowpack stratigraphy.

The acoustic image of snowpit \#3 (Figure 6.25) shows a change in reflections near the top of the snowpack when the acoustic wave passed through a layer of loose and wet snow. Smaller reflections are apparent on both sides of the buried ice layer. A few multiple reflections appear in a dense layer near the bottom of the snowpack.

For snowpit \#4 (Figure 6.26) observed in the forest, a loose layer of unloaded snow from the canopy created multiple reflections at the top of the acoustic image. Some multiple reflections were also apparent in the loose layer of depth hoar near the bottom of the snowpack. The unloaded snow was also observed in the acoustic image of snowpit \#5 (Figure 6.27), where the fine structure of the unloaded snow also created multiple reflections. These reflections were not present after the sound wave passed through a hard layer of snow.

Figure 6.28 shows an example of an acoustic image with a buried conifer tree situated under the snowpack. This point is marked on the transect (Figure 6.21). Reflections occurring due to the buried tree are marked on the image. Unlike the other snowpits, where there is a loss of resolution at the bottom of the snowpack due to buried depth hoar, the void space created by the conifer tree is clearly visible. The additional reflections caused acoustic snow density $\bar{\rho}$ measured by the SAS2 to be over-predicted relative to measurements made by the ESC30 snowtube. However, despite the over-prediction of density, the acoustic snow density $\bar{\rho}$ is within the same order of magnitude as the densities collected using the ESC30 snowtube.

In the avenue of trees situated within the gully, acoustic scattering and diffraction of sound occurred for SAS2 samples taken near tree wells, resulting in over-prediction of density $\bar{\rho}$. For an acoustic sample taken near a tree well, propagation of the sound wave through the snow into the air within the tree well occurred, increasing the error in the acoustic measurement. Reflections from the tree trunk may have also contributed to this error.

Figure 6.29 shows an acoustic image taken near a tree well at a sample point marked on the transect (Figure 6.21). A loss of amplitude of the reflected sound wave at greater offset distances from the loudspeaker is indicative of diffraction of the sound wave by the tree well.


Figure 6.21. Sawmill Creek (SC) Site transects showing depth, average density $\bar{\rho}$, SWE, and depth-averaged liquid water content.


Figure 6.22. Sawmill Creek (SC) Site transects showing average temperature, frequency-averaged speed and attenuation, and $\Psi$ coefficient.

| $\bar{\rho}$ Comparison | Snowpit \& Snowtube | ESC30 \& SAS2 | Snowpit \& SAS2 |
| :---: | :---: | :---: | :---: |
| RMSD $\left(\mathrm{kg} \mathrm{m}^{-3}\right)$ | 82.6 | 62.3 | 27.7 |
| $\mathrm{MB}\left(\mathrm{kg} \mathrm{m}^{-3}\right)$ | 75.2 | 43.5 | -14.0 |


| SWE Comparison | Snowpit \& Snowtube | ESC30 \& SAS2 | Snowpit \& SAS2 |
| :---: | :---: | :---: | :---: |
| RMSD $(\mathrm{mm})$ | 182.4 | 79.7 | 24.9 |
| MB $(\mathrm{mm})$ | 84.4 | 44.9 | -7.6 |


| $\bar{\theta}_{w}$ Comparison | Dielectic \& SAS2 |
| :---: | :---: |
| RMSD (\%) | 0.8 |
| MB (\%) | -0.6 |


| $\bar{T}$ Comparison | Thermometer \& SAS |
| :---: | :---: |
| RMSD $\left({ }^{\circ} \mathrm{C}\right)$ | 2.5 |
| $\mathrm{MB}\left({ }^{\circ} \mathrm{C}\right)$ | -2.2 |

Table 6.18. Comparisons between snowpit, ESC30, and acoustic measurements of $\bar{\rho}, \mathrm{SWE}, \bar{\theta}_{w}$ and $\bar{T}$ for the SC site.


| Measurement | Gravimetric/Thermal/Dielectric | Acoustic | RMSD | MB |
| :---: | :---: | :---: | :---: | :---: |
| $\bar{\rho}\left(\mathrm{kg} \mathrm{m}^{-3}\right)$ | 371 | 340 | 31.0 | -31.0 |
| $\mathrm{SWE}(\mathrm{mm})$ | 274 | 252 | 22.0 | -22.0 |
| $T\left({ }^{\circ} \mathrm{C}\right)$ | 0.000 | -0.500 | 0.500 | -0.500 |
| $\bar{\theta}_{w}(\%)$ | 4.40 | 3.91 | 0.490 | -0.490 |

Figure 6.23. Snowpit \#1 stratigraphy at SC site. The snowpit was situated on the ridge; subplots are: (a) depth-averaged gravimetric density measurements; (b) depth-averaged temperature measured from the snowpit wall using a thermometer; (c) depth-averaged liquid water content (LWC) measured with Denoth meter; and (d) depth-averaged acoustic image of snowpit with cumulative offset. A table shows a comparison between the gravimetric, dielectric and acoustic measurements
(a)

(b)

(c)


| Measurement | Gravimetric/Thermal/Dielectric | Acoustic | RMSD | MB |
| :---: | :---: | :---: | :---: | :---: |
| $\bar{\rho}\left(\mathrm{kg} \mathrm{m}^{-3}\right)$ | 429 | 440 | 10.8 | 10.8 |
| SWE $(\mathrm{mm})$ | 494 | 506 | 12.4 | 12.4 |
| $\bar{T}\left({ }^{\circ} \mathrm{C}\right)$ | 0.000 | -2.000 | 1.900 | 1.900 |
| $\bar{\theta}_{w}(\%)$ | 4.86 | 3.90 | 0.96 | -0.96 |

Figure 6.24. Snowpit \#2 stratigraphy at SC site. The second snowpit was situated in a clearing on the ridge; subplots are: (a) snowpit measurements; (b) temperature measured from the snowpit wall using a thermometer; (c) depth-averaged liquid water content (LWC) measured with Denoth meter; and (d) acoustic image of snowpit with cumulative offset. A table shows a comparison between the gravimetric, dielectric and acoustic measurements.
(a)

(b)

(c)



| Measurement | Gravimetric/Thermal/Dielectric | Acoustic | RMSD | MB |
| :---: | :---: | :---: | :---: | :---: |
| $\bar{\rho}\left(\mathrm{kg} \mathrm{m}^{-3}\right)$ | 441 | 460 | 18.8 | 18.8 |
| SWE $(\mathrm{mm})$ | 684 | 713 | 29.2 | 29.2 |
| $\bar{T}\left({ }^{\circ} \mathrm{C}\right)$ | 0.00 | -1.50 | 1.50 | 1.50 |
| $\bar{\theta}_{w}(\%)$ | 4.40 | 4.70 | 0.280 | 0.280 |

Figure 6.25. Snowpit \#3 stratigraphy at SC site. The third snowpit was also situated in a clearing on the ridge; subplots are: (a) snowpit measurements; (b) temperature measured from the snowpit wall using a thermometer; (c) depth-averaged liquid water content (LWC) measured with Denoth meter; and (d) acoustic image of snowpit with cumulative offset. A table shows a comparison between the gravimetric, dielectric and acoustic measurements.
(a)

(b)

(c)



| Measurement | Gravimetric/Thermal/Dielectric | Acoustic | RMSD | MB |
| :---: | :---: | :---: | :---: | :---: |
| $\bar{\rho}\left(\mathrm{kg} \mathrm{m}^{-3}\right)$ | 395 | 360 | 35.1 | -35.1 |
| SWE $(\mathrm{mm})$ | 344 | 313 | 30.6 | -30.6 |
| $\bar{T}\left({ }^{\circ} \mathrm{C}\right)$ | 0.00 | -3.10 | 2.90 | 2.90 |
| $\bar{\theta}_{w}(\%)$ | 4.50 | 4.10 | 0.350 | -0.350 |

Figure 6.26. Snowpit \#4 stratigraphy at SC site. The fourth snowpit was situated in the forest nearby the ridge; subplots are: (a) gravimetric density measurements; (b) temperature measured from the snowpit wall using a thermometer; (c) depth-averaged liquid water content (LWC) measured with Denoth meter; and (d) acoustic image of snowpit with cumulative offset. A table shows a comparison between the gravimetric, dielectric and acoustic measurements.
(a)

(b)

(c)



| Measurement | Gravimetric/Thermal/Dielectric | Acoustic | RMSD | MB |
| :---: | :---: | :---: | :---: | :---: |
| $\bar{\rho}\left(\mathrm{kg} \mathrm{m}^{-3}\right)$ | 354 | 320 | 33.8 | -33.8 |
| SWE $(\mathrm{mm})$ | 276 | 250 | 26.3 | -26.3 |
| $\bar{T}\left({ }^{\circ} \mathrm{C}\right)$ | 0.00 | -4.00 | 3.900 | 3.900 |
| $\bar{S}_{w}(\%)$ | 4.44 | 3.10 | 1.33 | -1.33 |

Figure 6.27. Snowpit \#5 stratigraphy at SC site. The fifth snowpit was situated in the forest nearby the ridge; subplots are: (a) gravimetric density measurements; (b) temperature measured from the snowpit wall using a thermometer; (c) depth-averaged liquid water content (LWC) measured with Denoth meter, and (d) acoustic image of snowpit with cumulative offset. A table shows a comparison between the gravimetric, dielectric and acoustic measurements.


| Measurement | SR50 Gravimetric | Acoustic | RMSD | MB |
| :---: | :---: | :---: | :---: | :---: |
| $\bar{\rho}\left(\mathrm{kg} \mathrm{m}^{-3}\right)$ | 392 | 496 | 104 | 104 |
| SWE $(\mathrm{mm})$ | 470 | 595 | 125 | 125 |
| $\bar{T}\left({ }^{\circ} \mathrm{C}\right)$ | NA | -2.3 | NA | NA |
| $S_{w}(\%)$ | NA | 3 | NA | NA |

Figure 6.28. Example of acoustic scattering due to buried conifer tree at the SC site. The position of the buried tree is marked on the image above and the sampling point location is indicated on the transect (Figure 6.21). A table presents comparisons between measurements made with the SR50 snowtube and SAS2 acoustic gauge. Comparisons that cannot be made due to a lack of observations are marked as "NA" in the table. The snow depth at this observation point was measured using a depth rod as 120 cm .


Cumulative Offset (cm)

| Measurement | SR50 Gravimetric | Acoustic | RMSD | MB |
| :---: | :---: | :---: | :---: | :---: |
| $\bar{\rho}\left(\mathrm{kg} \mathrm{m}^{-3}\right)$ | 343 | 436 | 93 | 93 |
| $\mathrm{SWE}(\mathrm{mm})$ | 360 | 458 | 98 | 98 |
| $\bar{T}\left({ }^{\circ} \mathrm{C}\right)$ | NA | 0 | NA | NA |
| $S_{w}(\%)$ | NA | 9 | NA | NA |

Figure 6.29. Example of acoustic image taken at a sample point near a tree well at the SC site, showing a loss of amplitude with increasing offset distance due to diffraction of the sound wave by the tree well. The position of the tree well is marked on the image above and the sampling point location is indicated on the transect (Figure 6.21). A table presents comparisons between measurements made with the SR50 snowtube and SAS2 acoustic gauge. Comparisons that cannot be made due to a lack of observations are marked as "NA" in the table. The snow depth at this observation point was 105 cm .

### 6.8. Error Comparisons

Figures 6.30 and 6.31 were used to determine if a relationship existed between the magnitude and the absolute error between variables measured using acoustic and non-acoustic methods. The variable measured using a non-acoustic method was plotted on the x -axis of each plot, and the absolute error between the acoustic and non-acoustic method was plotted on the yaxis. In the same manner as Section 6.5 of this thesis, the Spearman rank correlation coefficient was used to check for a non-linear relationship. The dashed lines on the plots are provided for visual reference.

Figure 6.30a shows that there is a significant relationship between ESC30 snowtube measurements of density $\bar{\rho}$ and the error between acoustic and non-acoustic measurements of density. Points with higher SAS2 snowtube densities had a higher acoustic density error. However, despite the smaller number of snowpit samples relative to ESC30 samples, there was not a significant relationship between the depth-averaged snowpit density and the error in the determination of density using the SAS2 (Figure 6.30b). This suggests that the density error between ESC30 snowtube and acoustic measurements may have been influenced by error in ESC30 measurements such as a bias associated with snow samples of greater mass due to spring scale or snowtube inaccuracies. This suggestion is supported by the significant relationship shown by Figure 6.30c, which compares snowpit to snowtube measurement errors.

There is a significant relationship between snowpit and snowharp measurements and the error in acoustic measurement of temperature $\bar{T}$ (Figure 6.30d). The error is greatest for lower temperatures, indicating the presence of temperature shifts in the SAS2 measurement apparatus (circuit, loudspeaker and microphones) or error in the UTAM model at lower temperatures.

Despite larger errors in acoustic parameters at the BL field site due to ponded liquid water in the snowpack, Figure 6.30 e shows that there is not a significant relationship between dielectric measurements of liquid water content and the absolute error in acoustic measurement of liquid water content. Neither is there a significant relationship between liquid water content and the absolute error in acoustic density (Figure 6.30f). These results suggest that the SAS2 and UTAM model are able to determine liquid water content $\bar{\theta}_{w}$ and density $\bar{\rho}$ without being affected by the $\bar{\theta}_{w}$ magnitude, provided that a P-wave can propagate in the pore spaces of the snowpack.


Figure 6.30. Error comparisons between acoustic and non-acoustic parameters. The Spearman rank correlation coefficient is given by $r_{s}$ and the number of samples used in the analysis is $N_{s}$.

Figure 6.31 shows that there is no significant relationship between the measured snowpit density $\bar{\rho}$ and the error in determining acoustic temperature $\bar{T}$. Neither is there a relationship between dielectric measurements of LWC and acoustic measurements of temperature (Figure 6.31b). As the snow density $\bar{\rho}$ increases, the error in determination of acoustic liquid water content does not increase significantly (Figure 6.31c). There is also no significant relationship between snowpit temperature $\bar{T}$ and the error in acoustic determination of liquid water content (Figure 6.31d). These results show that the SAS2 measurements and UTAM model outputs are not biased by increases in magnitude of these measurement parameters.


Figure 6.31. Error comparisons between acoustic and non-acoustic parameters. The Spearman rank correlation coefficient is given by $r_{s}$ and the number of samples is $N_{s}$.

### 6.9. Effect of Signal Cutoff Time on Inverse UTAM Model Results

A maximum signal cutoff time $t_{\text {max }}$ was determined from measured snow depth $\bar{y}$ and the frequency-averaged speed of sound in snow. This signal cutoff time was used to truncate the signal so that the inverse UTAM model was only applied to part of the signal representative of the sound wave that had propagated through the snowpack. Reflections at time $t>t_{\max }$ were not used in the analysis. However, if snow depth measurements are not available then a reasonable assumption of the maximum cutoff time is required.

To assess the effect of $t_{\text {max }}$ on the error associated with measuring snowpack variables using the SAS2 and the inverse UTAM model, an error analysis (Figure 6.32) was performed using data collected at all of the field sites using the stationary and portable versions of the SAS2. The error analysis was conducted by progressively increasing the cutoff time up to $t_{\text {max }}=t_{\text {mis }}=1.6 \mathrm{~s}$. The error analysis series was treated similar to a time-domain signal and lowpass filtered using a Butterworth filter with a cutoff frequency of 50 Hz to suppress the effects of higher-frequency oscillations. A moving-average filter was then applied to further suppress noise.

Figure 6.32a shows that as the cutoff time increases, the mean absolute error between ESC30 density and acoustic density increases by $\sim 8 \mathrm{~kg} \mathrm{~m}^{-3}$, indicating that the error is relatively insensitive to cutoff time. Figure 6.32 b shows a similar increase and insensitivity to cutoff time for the mean absolute error between gravimetric snowpit density and acoustic density, with a density error range of $\sim 2 \mathrm{~kg} \mathrm{~m}^{-3}$. Figure 6.32 b shows a lower error range than Figure 6.32a due to the inherent error in samples collected using the ESC30 snow sampler. The error in temperature (Figure 6.32c) is also relatively insensitive to cutoff time. This to be expected since the UTAM model sensitivity analysis showed relative insensitivity to snow temperature. Furthermore, the mean absolute error between dielectric and acoustic measurements of LWC (Figure 6.32d) has a range of $<0.2 \%$, which is within the $0.5 \%$ range of errors associated with two different dielectric measurement techniques. The insensitivity of the inverse UTAM model to the cutoff time $t_{\text {max }}$ is due to reflections of relatively low amplitude that occurred at times after the sound wave propagated through the snowpack. These reflections contributed relatively little error to the inputs of the inverse UTAM model.


Figure 6.32. Error comparisons showing the effect of an assumed $t_{\max }$ on the mean absolute error between acoustic SAS2 measurements and measurements made using ESC30, gravimetric, thermometer and dielectric instruments.

## CHAPTER 7 <br> CONCLUSIONS

This thesis described the theory, design and testing results of the System for the Acoustic Sensing of Snow (SAS2), an electronic device used to obtain acoustic samples of sound waves passed into and reflected from snowpacks at field sites in the Rocky Mountains of Alberta, Canada. The research reported in this thesis suggested the following conclusions and opportunities for future investigation.

### 7.1. Non-invasive Measurement of Snowpack Variables

The SAS2 obtained non-invasive observations of depth-averaged density $\bar{\rho}$, temperature $\bar{T}$ and liquid water content $\bar{\theta}_{w}$ similar to measurements made with invasive ESC30 snowtube, snowpits, thermometers, thermocouples and dielectric measurement devices. The invasive methods of observation disrupted the snowpack, modifying it when an observation was made. The SAS2 used a sound wave that did not modify the snowpack. This permitted multiple measurements of snowpack variables to be made at the same location (Upper Clearing Stationary site) and measurements to be made at field sites without having to extract snow samples. Although snow depth or an assumption of a maximum signal cutoff time was required, it is generally easier to measure snow depth than snow density, temperature and liquid water content, particularly for stationary met station applications.

The measurement of snow density and liquid water content by the SAS2 could be used to provide an estimate of the dielectric constant (relative permittivity) and speed of propagation of electromagnetic waves in snow. Paired with a FMCW radar system, the SAS2 could provide information that would allow for the FMCW radar system to determine the depth to reflectors without the use of assumed or gravimetric snow densities.

Because the UTAM model is relatively insensitive to temperature, and given the differences between thermometer and acoustically-determined snow temperatures shown by the results, it is apparent that the SAS2 is a system with a primary purpose of determining snow density and liquid water content. An assumed ground temperature and a measured air temperature would obtain a reasonable estimate of snow temperature in lieu of acoustic measurements.

### 7.2. Unified Thermoacoustic Model (UTAM)

The Biot-Stoll constitutive equations modelling sound wave propagation through porous media were modified using a mixture theory approach to describe snow comprised of ice, air and water constituents. This was the UTAM model described in Chapter 2. A scaling coefficient modelled differences between the elastic constants of snow particles and ice.

The UTAM model allowed for the determination of the density, liquid water content and temperature of snow. Future updates of this model could restrict the solution for temperature or density based on the liquid water content.

Percolation theory (PT) was used to relate shear modulus and bulk modulus variables used in the Biot-Stoll model to snow porosity. The Consolidation Theory (CT) Model presented in this thesis provided an estimate of the porosity at which the snowpack ceased to act as a cellular solid and becomes an equivalent fluid. Mechanical parameters used as inputs for the Biot-Stoll constitutive equations were calculated using a mixture theory model and empirical equations were used to estimate mechanical and thermal parameters. Sensitivity analyses investigated changes in the model outputs to input parameters. Numerical methods were presented to evaluate these models.

Dielectric measurements of snowpack properties usually rely on empirical relationships between variables to make measurements. Unlike these dielectric measurements, the UTAM model did not require empirical calibration. Despite the use of empirical equations to estimate mechanical and thermal variables in the UTAM model, no relationships were utilized that directly related sound wave reflections to snowpack variables.

Because the Consolidation Theory (CT) model presented in this thesis predicts when the snowpack loses cohesion and becomes an equivalent fluid, the potential exists for future research to use this model for avalanche predictions.

Future work could be done to quantify uncertainty in the results of the inverse UTAM model. This would allow error bar estimates to be placed on the plots. A Monte Carlo method would be useful in this endeavor where the inputs are randomly varied and changes in the model outputs observed.

### 7.3. Signal Processing Techniques

A Maximum Length Sequence (MLS) was used as the source signal sent from the loudspeaker into the snowpack. The MLS was selected as a source signal due to its high noise immunity and time-integrated signal energy. The results presented in this thesis showed that an MLS source signal could be used for acoustic sampling of snow. Use of an MLS assumes that the snowpack is a Linear Time Invariant (LTI) system and does not change over the time of measurement ( $t_{m l s}=1.6 \mathrm{~s}$ ). Although another source signal without the LTI assumption such as the SineSweep (Farina, 2000) could be used in lieu of the MLS, further experimentation is required to determine if alternate source signals are usable for acoustic sampling of snow.

The SAS2 system response (source signal and microphones) was experimentally determined in an anechoic chamber. Future work could eliminate the need to measure this system response by a mathematical model of the SAS2 enclosure case and microphone assembly. Finite-difference or finite-element models of sound wave propagation through the enclosure case would be used to construct this model.

The Vanishing Constant Filter (VCF) was proposed to be able to measure and remove attenuation from a signal. This is a method of blind deconvolution. Future research could apply the VCF to seismic data, LIDAR data, or spectrometer data to measure attenuation of waves passing through rocks or sediment, the canopy of a forest, or light transmission through snow.

A maximum cutoff time $t_{\text {max }}$ was selected from a cutoff depth $\bar{y}$ to identify when the sound wave had reached the bottom of the snowpack due to attenuiation of the sound wave in snow. Attenuation did not always permit the bottom of the snowpack to be clearly identified using acoustics.

If the cutoff time $t_{\text {max }}$ is assumed instead of measuring snow depth $\bar{y}$, late-time reflections from elements other than the snowpack have small contributions to the error in determination of snow density, liquid water content and temperature using the inverse UTAM model. Because the inversion procedure is relatively insensitive to cutoff time $t_{\text {max }} \leq t_{\text {mis }}$, an average or maximum cutoff time could be used if snow depth is not known. This eliminates the need to measure snow depth.

### 7.4. System for the Acoustic Sensing of Snow (SAS2)

In a similar fashion to FMCW or bistatic radar and sonar, the SAS2 sensing system utilized a loudspeaker situated at an offset distance to a microphone assembly. The loudspeaker produced an audible sound wave in the frequency range from 20 Hz to 10 kHz at an SPL of 120 dB . Unlike the pulse from the ultrasonic SR50, the sound wave produced by the SAS2 entered the snowpack and was reflected from the snow due to changes in acoustic impedance. The sound wave was sensed in the air medium by the microphone assembly and processed using a custom-designed circuit.

The relative size of the SAS2 packaged in an enclosure case with dimensions of 38.5 cm by 28 cm reduced the portability of the device. Mounted on the chariot carrier, the portable SAS2 setup was difficult to transport to field sites with greater than $\sim 2 \mathrm{~m}$ of snow accumulation in the Rocky Mountains due to time and logistical constraints.

Further research could be conducted to reduce the size of the SAS2 and therefore increase the portability of the system. This entails the use of experimentation to find: an optimal separation distance between the loudspeaker and microphone assembly; the sampling rate of the microphones; and the maximum SPL of the loudspeaker required for acoustic sampling of snow. The possibility exists for an acoustic snow sampling system to be constructed that will fit in the pocket of a snow surveyor, similar to the size of a mobile phone device at the time of writing.

Further experimentation is also required to: validate whether the microphone holes created in the front of the enclosure case acted as waveguides, rejecting sound wave arrivals from angles away from the normal of each microphone hole; and to determine how error in the acoustic measurement is related to footprint size and response of the acoustic array.

An updated version of the sensing system could be created with a larger separation distance between the microphones and loudspeaker. This would still allow for the system to be portable, but would also permit a smaller microphone sampling rate and potentially allow for more information to be available for application of the UTAM or an updated model based on similar principles.

As shown in Chapter 4 and by the results of Chapter 6, the footprint size and system response may have ramifications associated with the accuracy and sampling size of each acoustic snow sample, particularly for stationary deployments of the SAS2 when the distance from the
snow surface to the SAS2 microphone assembly changes over time. This effect was most prevalent for the stationary version of the SAS2 system and also affected ultrasonic measurements made with the SR50 sensor. The stationary version of the SAS2 was situated at a distance of $0.22 \mathrm{~m} \leq y_{a} \leq 1.22 \mathrm{~m}$ from the snow surface, whereas the portable version of the SAS2 was situated at no more than a maximum distance of 15 cm from the snow surface. Nevertheless, despite the changing distance to the snow surface, both the SR50 and SAS2 were able to adequately obtain acoustic samples of snow depth (for the SR50) and acoustic samples of snow properties (for the SAS2).

Hardware modification of the SAS2 circuit board is required to add accelerometer and gyroscope integrated circuits so that accurate measurements of SAS2 gauge movement during the time of acoustic measurement can be determined. Movement of the SAS2 during sampling occurred when the SAS2 was placed on the child chariot carrier and when the SAS2 was held in the investigator's hand over the snow surface. The accelerometer and gyroscope measurements, as well as careful modelling of sound wave propagation through snow, would quantify the effects of movement on the acoustic sampling procedure.

Movement of the acoustic gauge during the time of acoustic sampling on the child chariot carrier or in the hand of the investigator may have contributed to error in acoustic measurements made at these site locations. Further experimentation is required to quantify this error. However, because the SAS2 could collect physically-reasonable data that corresponded well with gravimetric, dielectic and snow pit measurements, this implies that minuscule movements during sampling have a small effect on the operational use of the acoustic gauge. Placing the SAS2 at large (glancing) angles to the snow surface may induce mode conversion resulting in a P -wave changing into am S -wave and vice versa.

To measure snow depth so that a cutoff time for the signal can be provided, an ultrasonic depth gauge (i.e. SR50) could be incorporated into the SAS2 system. For stationary deployment of the SAS2 on a met station, the ultrasonic depth gauge could communicate with the SAS2 using the SDI-12 protocol. Communication would occur by placing a connector on the side of the SAS2 case so that the system can autonomously "talk with" and provide power for an ultrasonic depth gauge. Alternately, the connector could be used to provide a serial port with $\pm 5 \mathrm{~V}$ levels so that the SAS2 can communicate with a commercially-available datalogger. The
datalogger would provide snow depth information to the SAS2 obtained from an ultrasonic depth gauge attatched to the datalogger.

### 7.5. Deployment of SAS2

The results of experiments conducted using the SAS2 suggested that by using the apparatus described in this thesis the audible sound wave was able to make measurements of snow at 10 field sites in the Rocky Mountains.

Acoustic observations of snow were taken once every hour at the Upper Clearing Stationary (UCS) site. An experiment conducted at the UCS site showed that successive measurements taken by the acoustic system were repeatable. This experiment could not be conducted once every hour over the entire snow observation season due to the lack of storage capacity of data on the SD card. Repeating the experiment once every hour would allow for examination of footprint size changes on the repeatability of successive measurements as the snow depth changed. However, an updated hardware version of the SAS2 could be made in the future with more storage space (i.e. a greater number of SD card slots) for collecting a larger amount of acoustic data.

The wind speed and wind transport of sound waves did not appear to influence acoustic SAS2 measurements at the UCS site over the observation season. Measurements of blowing snow using the particle detector described by Brown and Pomeroy (1989) could also be used in future experiments to determine the influence of blowing snow events on the SAS2 signal. Because the SAS2 signal was longer in time ( $t_{m l s}=1.6 \mathrm{~s}$ ) and had greater time-integrated signal energy than the short pulse sent from the SR50, "spikes" in the acoustic SAS2 data were not present. These spikes occurred in the SR50 data during snow deposition events and ablation due to acoustic scattering of the ultrasonic SR50 pulse by snow particles passing under the sensor assembly and uneven snow surface topography during snowmelt.

### 7.6. Snowpack Measurements

Comparisons between snowpit and snowtube measurements of $\bar{\rho}$ were less accurate than comparisons between snowpit and SAS2 measurements, suggesting that the SAS2 has an accuracy most comparable to snowpit measurements of $\bar{\rho}$. The ESC30 snowtube was not as accurate as SAS2 and snowpit measurements. However, the SAS2 could be used in lieu of an ESC30 snowtube to determine depth-integrated measurements of density $\bar{\rho}$.

The Upper Clearing Stationary (UCS) site observations showed an increase in density over the time of the observation season, indicating compaction of the snowpack and metamorphic processes acting over the snow observation season. A decrease in density was observed to occur with the addition of fresh snow to the snowpack.

SAS2 measurements of liquid water content were similar to dielectric measurements and were within a nominal $0.5 \%$ error associated with dielectric measurements. This indicated that the SAS2 was able to function similar to a dielectric device used to measure liquid water content.

Measurements of depth-averaged liquid water content $\bar{\theta}_{w}$ at the Upper Clearing Stationary (UCS) site showed cycles coincident with inputs of net shortwave radiation that warmed the snowpack and reduced its cold content. Rising temperatures and an increase in the magnitude of the net shortwave radiation over the observation season increased the liquid water content $\bar{\theta}_{w}$ as the snowpack passed through the warming, ripening and output phases. The water output phase was observed as a reduction in liquid water content near the end of the ablation season when the snowpack could not hold any more melt water.

Observations at the UCS site showed that liquid water content $\bar{\theta}_{w}$ increased over the snow accumulation and ablation season. The $\bar{\theta}_{w}$ was higher later in the observation season than sites visited at an earlier time due to a greater magnitude of net shortwave radiation and higher air temperatures that warmed the snowpack and reduced its cold content.

For sites with higher levels of liquid water content and ponded water beneath the snowpack, acoustic scattering and mode conversion of sound waves may have occurred, increasing the error between SAS2 measurements relative to gravimetric measurements. This was most apparent at the Bow Lake (BL) site. Despite the presence of late-season snow at the Icefields Parkway Forest Site (IFPF), the RMSD and MB comparing gravimetric to acoustic densities $\bar{\rho}$ remained lower than the BL site, indicating that the snow may have been welldrained and ponding of water did not occur at the IFPF site. This indicated that acoustic measurement of snow using the SAS2 and UTAM required the propagation of an air-coupled sound wave through the pore spaces of the snowpack. The pore spaces of the snowpack could not be completely filled with ponded water.

The frequency-integrated speed of sound in snow was observed to be less than the speed of sound in air. This is similar to the observations made by other investigators of the speed of sound in snow (Section 2.5). The UTAM model and the Biot-Stoll model of sound propagation suggests that the air-coupled sound wave propagating in the pore spaces of the snowpack was a P-wave.

The $\Psi$ coefficient indicated that the elastic coefficients of the snow particles comprising the snowpack were different than the elastic coefficients of ice. There is a need to use experimental techniques to measure the elastic coefficients of snow particles to verify the values of the $\Psi$ coefficient determined using acoustics.

Although the sound wave produced by the ultrasonic SR50 gauge will be completely reflected from or may partially penetrate the snow surface (Goodison et al., 1984), complete attenuation measurements of sound in snow from frequencies approaching $\mathrm{DC}(\sim 0 \mathrm{~Hz})$ to the ultrasonic range (> $\sim 20 \mathrm{kHz}$ ) have not been reported in the literature. The experiments described in this paper showed that the frequency-averaged attenuation of sound in snow was in the range of $0 \mathrm{~Np} \mathrm{~m}^{-1} \leq a \leq 2 \mathrm{~Np} \mathrm{~m}^{-1}$ for a band-limited source signal in the range of 20 Hz to 10 kHz .

Over this range of attenuation coefficients, the sensitivity analysis given in Section 3.4 suggested that the maximum depth of snow that can be measured using the SAS2 system is 3 m . This is $\sim 1 \mathrm{~m}$ greater than the $\sim 2 \mathrm{~m}$ maximum depth of snow observed at the 10 field sites. The results showed that acoustic sampling of snow using the SAS2 was successful for snowpacks of $\sim 2 \mathrm{~m}$ depth. More sensitive microphones could be used to extend this range. However, microphones with a higher sensitivity may experience clipping due to close proximity of the loudspeaker to the microphone assembly.

Field sites with greater than $\sim 2 \mathrm{~m}$ of snow accumulation can be found in the Rocky Mountains and elsewhere in the world (i.e. the Alps). Although the logistics of visiting these sites and deploying instrumentation increases the complexity of field work, future research can be used to validate SAS2 measurements of the attenuation of sound in snow and the ability for the SAS2 system to measure snowpacks with greater than $\sim 2 \mathrm{~m}$ depth.

Infrasound (sound with frequencies less than 20 Hz ) could be used to ensure greater penetration of the snowpack for deep snowpacks. As predicted by the Biot theory, lower frequency sound waves will penetrate to greater depth beneath the snow surface due to lower levels of attenuation, whereas higher frequencies will have greater attenuation and will not travel
as far. However, the production of infrasound requires a sound source of greater dimension than the loudspeaker used in the SAS2 sensing system. Moreover, the physical size of the microphones and the microphone sensitivity will affect the ability to detect infrasound.

Reflections from the snowpack occurred due to changes in acoustic impedance. Images of these reflections did not necessarily coincide with snowpack layers as obtained by stratigraphy determination with a trained observer. The acoustic images were shown in this thesis. Further research is required to develop signal processing algorithms to extract snow stratigraphy information from the acoustic data. A possible way to do this would be to use neural networks (Smith, 2002) on the acoustic image data using a training dataset derived from snowpit observations of snow stratigraphy. The idea is similar to the use of algorithms to identify snowpack layers (Marshall et al., 2007; Tape et al., 2010).

Ice layers and crusts in the snowpack reflect sound waves. Ice layers that are semipermeable allow for sound waves to still propagate through the snowpack. Further research is required to quantify the permeability for which sound waves cannot propagate through ice layers in the snowpack.

For sampling points near tree wells, an error in the acoustic measurement occurred due to propagation of the sound wave through the snow into the air space within the tree well. Reflections from the tree trunk may have also contributed to this error.

Although a loss of resolution occurred at the bottom of the snowpack due to sound wave propagation through the depth hoar layer, an acoustic observation of the void space created due to a buried tree demonstrated that sound wave reflections from vegetation were sometimes visible. Further research could involve the mathematical modelling of sound waves in tree wells and the interaction of sound waves with vegetation using finite-difference or finite-element methods.

An increase in the error between gravimetric and acoustic measurements of density occurred when vegetation was buried beneath the snowpack. Because this vegetation could not always be detected due to a loss of resolution in the depth hoar layer, it was impossible to determine completely by the use of acoustics whether vegetation was buried beneath the snow surface.

Future work could be done to determine the effect of ground cover on the propagation of the acoustic wave through shallow snow. The vegetation under the shallow snowpack may cause attenuation or scattering of the acoustic wave.

The loss of resolution in the depth hoar layer occurred due to viscous damping caused by the sound wave travelling through a porous medium comprised of coarse particles (Stoll, 1989). In areas such as the Arctic, where extensive depth hoar formation occurs throughout the entire snowpack, it is possible that the SAS2 may not be able to perform acoustic sampling of snow. This may represent a limitation of the acoustic measurement technique presented in this thesis.

The SAS2 shows great promise for a number of potential applications. For continuous monitoring of snow, variables measured by the SAS2 could be used as internal state parameters in mathematical models of snowpack evolution. Rather than estimating the state parameters of the snowpack from mass and energy fluxes, the acoustic estimates could be used directly in the model. This suggests the way forward for a new generation of snowpack evolution models.

The SAS2 has the potential for reducing the time and effort taken to make observations of snow. Since the SAS2 is non-invasive when deployed in a stationary met station configuration, more than one sample can be taken at the same location, enabling continuous monitoring of snowpack parameters. Deployed at a location that is dangerous or impossible to visit during the winter season, the SAS2 enables the measurement of these parameters without human intervention. Deployed over a large geographic area, a network of SAS2 gauges may entail greater availability of data for global water security assessments or more accurate models for assessing the potential for drought or flooding.

The SAS2 could be used to non-invasively measure density and liquid water content on a slope or in an area where it is difficult or impossible to take samples. The SAS2 would be mounted on a small gantry crane or on a telescopic crane and moved over the snow surface under computer control. Such cranes are often used on movie sets for camera movement. Moreover, a small toy helicopter could be used to suspend the SAS2 over the snow surface.

There is a need to validate, test, and deploy a network of SAS2 gauges at a number of field sites situated at different locations in the Rocky Mountains, and in different mountain chains, countries and landscapes so that the models and instrumentation described in this paper can be verified, additionally tested in a number of cold environments, and potentially improved. A collaborative effort is required for this global undertaking.

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## APPENDIX A <br> CIRCUIT DIAGRAMS














## 5V Audio Power

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Main Microcontroller Core Power







2.4 GHz WiFi Module Option (Atheros ZCN-722M chipset in USB)








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1) Note that instead of using an audio ADC here, you could simply use a single-bit I/O port and use a low-pass filter (op-amp or passive) to create a
waveform that could be fed into the audio amp.
2) You could also use a high-speed switch to create a positive and negative voltage at a rate of 2.1 MHz , the same sampling rate as the mics. This would allow you to use the Fast Hadamard Transform as part of the deconvolution process. You would also have to pass the output through a low-pass filter as well.
3) You will have to test this by experimentation to see if there is another amplification stage required beyond low-pass filtering. Maybe create a small test circuit board with a CPLD? You don't want to exceed the slew rate of the audio amplifier!
4) Maybe use a passive filter (RLC) on the output of the ADC ?
5) Instead of using the passive filter, what you could do is digitally low-pass filter the MLS waveform before it is sent out from the system So you apply the digital filter in Matlab to the MLS waveform, and then the FPGA will only produce the low-pass filtered MLS waveform using the DAC! So you simply store the MLS waveform in the FPGA memory, or write code for a low-pass filter in Verilog and do the filtering on the FPGA!





## APPENDIX B <br> ADDITIONAL FIELD SITE OBSERVATIONS

(a)



Figure B.1. Upper Clearing Stationary (UCS) site time-series data showing (a) windspeed, and (b) Relative Humidity.


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Figure B.2. Upper Clearing Stationary (UCS) site time-series data showing (a) incoming shortwave radiation $K \downarrow$ and (b) outgoing shortwave radiation $K \uparrow$.



Figure B.3. Upper Clearing Stationary (UCS) site time-series data showing (a) incoming longwave radiation $L \downarrow$, and (b) outgoing longwave radiation $L \uparrow$.
(a)

(b)
u

(c)


Figure B.4. Upper Clearing Stationary (UCS) site time-series data showing (a) net shortwave radiation $K^{*}$, (b) net longwave radiation $L^{*}$, and (c) net all-wave radiation $Q^{*}$.


Figure B.5. Boulton Campground (BC) Site transects showing depth, average density $\bar{\rho}$, SWE , and depth-averaged liquid water content.


Figure B.6. Boulton Campground (BC) Site transects showing average temperature $\bar{T}$, frequency-averaged speed and attenuation, and $\Psi$ coefficient.

| $\bar{\rho}$ Comparison | Snowpit \& Snowtube | ESC30 \& SAS2 | Snowpit \& SAS2 |
| :---: | :---: | :---: | :---: |
| $\operatorname{RMSD}\left(\mathrm{kg} \mathrm{m}^{-3}\right)$ | 67.4 | 47.1 | 40.5 |
| $\mathrm{MB}\left(\mathrm{kg} \mathrm{m}^{-3}\right)$ | 64.5 | 19.5 | -12.5 |


| SWE Comparison | Snowpit \& Snowtube | ESC30 \& SAS2 | Snowpit \& SAS2 |
| :---: | :---: | :---: | :---: |
| RMSD $(\mathrm{mm})$ | 36.7 | 38.1 | 34.1 |
| MB $(\mathrm{mm})$ | 36.5 | 14.4 | -15.0 |


| $\bar{T}$ Comparison | Thermometer \& Acoustic |
| :---: | :---: |
| RMSD $\left({ }^{\circ} \mathrm{C}\right)$ | 1.4 |
| $\mathrm{MB}\left({ }^{\circ} \mathrm{C}\right)$ | -1.3 |

Table B.1. Comparisons between snowpit, ESC30, and acoustic measurements of $\bar{\rho}$, SWE and $\bar{T}$ for the Boulton Campground (BC) site.


Figure B.7. Bow Summit (BS) Site transects showing depth, average density $\bar{\rho}$, SWE, and depth-averaged liquid water content.


Figure B.8. Bow Summit (BS) Site transects showing average temperature, frequency-averaged speed and attenuation, and $\Psi$ coefficient.

| $\bar{\rho}$ Comparison | Snowpit \& Snowtube | ESC30 \& SAS2 | Snowpit \& SAS2 |
| :---: | :---: | :---: | :---: |
| RMSD $\left(\mathrm{kg} \mathrm{m}^{-3}\right)$ | 45.0 | 29.0 | 18.1 |
| $\mathrm{MB}\left(\mathrm{kg} \mathrm{m}^{-3}\right)$ | -14.3 | 3.2 | 7.3 |


| SWE Comparison | Snowpit \& Snowtube | ESC30 \& SAS2 | Snowpit \& SAS2 |
| :---: | :---: | :---: | :---: |
| RMSD $(\mathrm{mm})$ | 119.8 | 54.6 | 30.3 |
| MB $(\mathrm{mm})$ | -68.2 | -7.3 | 12.5 |


| $\bar{\theta}_{w}$ Comparison | Dielectric \& Acoustic |
| :---: | :---: |
| RMSD (\%) | 0.3 |
| MB (\%) | 0.2 |


| $\bar{T}$ Comparison | Thermometer \& Acoustic |
| :---: | :---: |
| $\operatorname{RMSD}\left({ }^{\circ} \mathrm{C}\right)$ | 0.7 |
| $\mathrm{MB}\left({ }^{\circ} \mathrm{C}\right)$ | 0.1 |

Table B.2. Comparisons between snowpit, ESC30, and acoustic measurements of $\bar{\rho}$, SWE, $\bar{\theta}_{w}$ and $\bar{T}$ for the Bow Summit (BS) site.


Figure B.9. Upper Clearing Portable (UCP) Site transects showing depth, average density $\bar{\rho}$, SWE, and depth-averaged liquid water content.


Figure B.10. Upper Clearing (UCP) Site transects showing average temperature $\bar{T}$, frequency-averaged speed and attenuation, and $\Psi$ coefficient.

| $\bar{\rho}$ Comparison | Snowpit \& Snowtube | ESC30 \& SAS2 | Snowpit \& SAS2 |
| :---: | :---: | :---: | :---: |
| RMSD $\left(\mathrm{kg} \mathrm{m}^{-3}\right)$ | 28.0 | 25.2 | 15.5 |
| $\mathrm{MB}\left(\mathrm{kg} \mathrm{m}^{-3}\right)$ | 20.4 | 3.8 | 15.0 |


| SWE Comparison | Snowpit \& Snowtube | ESC30 \& SAS2 | Snowpit \& SAS2 |
| :---: | :---: | :---: | :---: |
| RMSD $(\mathrm{mm})$ | 23.5 | 18.7 | 14.9 |
| MB $(\mathrm{mm})$ | 9.5 | 1.6 | 14.0 |


| $\bar{\theta}_{w}$ Comparison | Dielectic \& Acoustic |
| :---: | :---: |
| RMSD (\%) | 0.20 |
| $\mathrm{MB}(\%)$ | 0.10 |


| $\bar{T}$ Comparison | Thermometer \& Acoustic |
| :---: | :---: |
| RMSD $\left({ }^{\circ} \mathrm{C}\right)$ | 0.70 |
| $\mathrm{MB}\left({ }^{\circ} \mathrm{C}\right)$ | -0.40 |

Table B.3. Comparisons between snowpit, ESC30, and acoustic measurements of $\bar{\rho}, \mathrm{SWE}, \bar{\theta}_{w}$ and $\bar{T}$ for the UCP site.


Figure B.11. Bow Meadows (BM) Site transects showing depth, average density $\bar{\rho}$, SWE, and depth-averaged liquid water content.


Figure B.12. Bow Meadows (BM) Site transects showing average temperature, frequency-averaged speed and attenuation, and $\Psi$ coefficient.

| $\bar{\rho}$ Comparison | Snowpit \& Snowtube | ESC30 \& SAS2 | Snowpit \& SAS2 |
| :---: | :---: | :---: | :---: |
| $\operatorname{RMSD}\left(\mathrm{kg} \mathrm{m}^{-3}\right)$ | 19.9 | 17.9 | 8.2 |
| $\mathrm{MB}\left(\mathrm{kg} \mathrm{m}^{-3}\right)$ | -5.5 | -2.8 | -6.2 |


| SWE Comparison | Snowpit \& Snowtube | ESC30 \& SAS2 | Snowpit \& SAS2 |
| :---: | :---: | :---: | :---: |
| RMSD $(\mathrm{mm})$ | 42.4 | 24.3 | 24.7 |
| MB $(\mathrm{mm})$ | -30.0 | -5.6 | -7.8 |


| $\bar{\theta}_{w}$ Comparison | Dielectric \& Acoustic |
| :---: | :---: |
| RMSD (\%) | 0.4 |
| MB (\%) | 0.2 |


| $\bar{T}$ Comparison | Thermometer \& Acoustic |
| :---: | :---: |
| RMSD $\left({ }^{\circ} \mathrm{C}\right)$ | 1.0 |
| $\mathrm{MB}\left({ }^{\circ} \mathrm{C}\right)$ | -0.7 |

Table B.4. Comparisons between snowpit, ESC30, and acoustic measurements of $\bar{\rho}$, $\mathrm{SWE}, \bar{\theta}_{w}$ and $\bar{T}$ for the BM site.


Figure B.13. Bow Lake (BL) Site transects showing depth, average density, SWE, and depth-averaged liquid water content.


Figure B.14. Bow Lake (BL) Site transects showing average temperature, frequency-averaged speed and attenuation, and $\Psi$ coefficient.


Figure B.15. Nakiska Ski Hills (NSH) Site transects showing depth, average density, SWE, and depth-averated liquid water content.


Figure B.16. Nakiska Ski Hills (NSH) Site transects showing average temperature, frequency-averaged speed and attenuation, and $\Psi$ coefficient.


Figure B.17. Black Prince Cirque (BPC) Site transects showing depth, average density $\bar{\rho}$, SWE, and depth-averaged liquid water content.


Figure B.18. Black Prince Cirque (BPC) Site transects showing average temperature, frequency-averaged speed and attenuation, and $\Psi$ coefficient.

| $\bar{\rho}$ Comparison | Snowpit \& Snowtube | ESC30 \& SAS2 | Snowpit \& SAS2 |
| :---: | :---: | :---: | :---: |
| RMSD $\left(\mathrm{kg} \mathrm{m}^{-3}\right)$ | 62.6 | 50.1 | 18.2 |
| $\mathrm{MB}\left(\mathrm{kg} \mathrm{m}^{-3}\right)$ | 33.7 | 7.3 | -18.0 |


| SWE Comparison | Snowpit \& Snowtube | ESC30 \& SAS2 | Snowpit \& SAS2 |
| :---: | :---: | :---: | :---: |
| RMSD $(\mathrm{mm})$ | 51.6 | 21.9 | 11.0 |
| MB $(\mathrm{mm})$ | 35.5 | 3.6 | -10.1 |


| $\bar{\theta}_{w}$ Comparison | Dielectric \& Acoustic |
| :---: | :---: |
| RMSD (\%) | 0.2 |
| MB (\%) | 3.6 |


| $\bar{T}$ Comparison | Thermometer \& Acoustic |
| :---: | :---: |
| RMSD $\left({ }^{\circ} \mathrm{C}\right)$ | 1.6 |
| $\mathrm{MB}\left({ }^{\circ} \mathrm{C}\right)$ | -1.5 |

Table B.5. Comparisons between snowpit, ESC30, and acoustic measurements of $\bar{\rho}$, SWE, $\bar{\theta}_{w}$ and $\bar{T}$ for the BPC site.


Figure B.19. Icefields Parkway Forest (IPF) Site transects showing depth, average density, SWE, and depth-averaged liquid water content.


Figure B.20. Icefields Parkway Forest (IPF) Site transects showing average temperature, frequency-averaged speed and attenuation and $\Psi$ coefficient.


[^0]:    ${ }^{1}$ Steven G. Johnson, The NLopt nonlinear-optimization package, http://ab-initio.mit.edu/nlopt

[^1]:    ${ }^{2}$ http://www.mpir.org/
    ${ }^{3}$ http://www.mpfr.org/
    ${ }^{4}$ http://www.holoborodko.com/pavel/mpfr/

