Real and Nominal Effects of Monetary Policy Shocks

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Rokon Bhuiyan

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DEDICATION

To the Instructor of my Quantum Meditation Course *Gurugi Mahajatakh*

ABSTRACT

Using Canadian data we estimate the effects of monetary policy shocks on various real and nominal variables using a fully recursive VAR model. We decompose the nominal interest rate into an ex-ante real interest rate and inflationary expectations using the Blanchard-Quah structural VAR model with the identifying restriction that ex-ante real interest rate shocks have but a temporary impact on the nominal interest rate. The inflationary expectations are then employed to estimate a policy reaction function that identifies monetary policy shocks. We find that a positive shock introduced by raising the monetary aggregates raises inflationary expectations and temporarily lowers the ex-ante real interest rate. As well, it depreciates the Canadian dollar and generates other macro effects consistent with conventional monetary theory although these effects are not statistically significant. Using the overnight target rate as the monetary policy instrument we find that a contractionary monetary policy shock lowers inflationary expectations and raises the ex-ante real interest. Such a contractionary monetary policy shock also appreciates the Canadian currency, decreases industrial output and increases the unemployment rate. We obtain qualitatively better results using the overnight target rate rather than a monetary aggregate as the monetary policy instrument. Our estimated results are robust to various modifications of the basic VAR model and do not encounter empirical anomalies such as the liquidity and exchange rate puzzles found in some previous VAR studies of the effects of monetary policy shocks in an open economy.

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CONTENTS

1. INTRODUCTION

Although there has been much research in the past decade on the effects of monetary policy shocks in various macro-economic variables, most of them encountered puzzling dynamic responses[1]. For example, the liquidity puzzle is the finding that an increase in a monetary aggregate (such as M0, M1 and M2) is associated with an increase rather than a decrease in nominal interest rates (Leeper and Gordon, 1991). The price puzzle is the finding that, when monetary policy shocks are identified as innovations in an interest rate, the monetary tightening is associated with an increase rather than a decrease in the price level (Sims, 1992). The exchange rate puzzle is the finding that while a positive innovation in the interest rates in the United States is accompanied by an appreciation of U. S dollar relative to other G-7 countries (Eichenbaum and Evans, 1995), such monetary contraction in the other G-7 countries is often associated with depreciation in their currencies (Grilli and Roubini, 1995; Sims, 1992).

Empirical research involving both open and closed economies addressed these puzzles, and provided suggestions on how to explain those puzzles. According to Sims (1992), in the presence of money demand shocks, innovations in the monetary aggregates do not correctly represent exogenous changes in monetary policy. He, therefore, proposed innovations in the short-term interest rates as the indicator of a monetary policy change. Sims' solution, however, was not widely accepted as it leads to the price puzzle: a monetary contraction is accompanied by a persistent increase in the price level. Some other authors (Strongin, 1995; Eichenbaum and Evans, 1995) then suggested identifying monetary policy shocks with innovations in the narrow monetary aggregates, such as non-borrowed reserves.

One possible explanation of the price puzzle according to Sims is that interest rate innovations partly reflect inflationary pressures which in turn cause price increases. Grilli and Roubini (1995) provided evidence that this explanation of price puzzle also explains

[1] For an extensive review of these puzzles and early attempts to resolve them, see Kim and Roubini (2000).

the exchange rate puzzle. Later on, to test this explanation of price puzzle, Sims and Zha (1995) proposed a Structural VAR approach with contemporaneous restrictions that includes variables proxying for expected inflation. The results obtained in this way were consistent with the theory of monetary policy contraction: a monetary policy contraction was accompanied by an increase in interest rates, a reduction in the money supply, a transitory fall in output and a persistent reduction in the price level.

In a small open economy context, Cushman and Zha (1997) and Kim and Roubini (2000) argued to use the structural VAR method with contemporaneous restrictions on some variables to properly identify the policy reaction function. They believe that as the external shocks are also very important for domestic monetary policy in a small open economy, it is important to take those influences under consideration. By incorporating some foreign variables into the policy reaction function, they were able to solve the puzzles encountered by the previous studies.

In a different approach to the same problem, Kahn et al. (2002) argued that if inflationary expectations are not observable, one can not infer from an observed increase in nominal interest rates that a commensurate increase in the real interest rate occurred. It is, therefore, difficult in studies that examine nominal interest rates to distinguish between the interaction of central bank policy with real interest rates and its interaction with inflationary expectations. Also these studies cannot examine the extent to which monetary policy leads or reacts to changes in inflation and inflationary expectations as they consider realized inflation rates rather than inflationary expectations. To address these problems, Kahn et al. (2002) used the Israeli data of real interest rates and inflationary expectations, calculated from the market prices of indexed- and nominal-bonds, to measure the effects of monetary policy using the fully recursive VAR model. They found that monetary policy shocks, introduced by raising the overnight rate of the Bank of Israel, raises 1-year real interest rates, lowers inflationary expectations and appreciates the Israeli currency, effects which are consistent with economic theory. They also found that the monetary policy impacts are mainly concentrated on short-term real rates.

It can be, therefore, summarized that the puzzling responses of various macro-economic variables to monetary policy shocks originate either due to the lack of the consideration of inflationary expectations or due to the incorrect identification of monetary policy. In this thesis, to take into account of these facts, we proceed in two steps. First, we calculate inflationary expectations and ex-ante real interest rates using the Structural VAR method proposed by Blanchard and Quah (1989) with the identifying restrictions that real interest rate innovations have temporary effects while inflationary expectations innovations have permanent effects on nominal interest rates[2]. In the second step, using the data on real interest rates and inflationary expectations, we explicitly examine the separate reactions of both ex-ante real interest rates and inflationary expectations to monetary policy shocks. To do this, we use the fully recursive VAR model as used by Christiano et al. (1996), Edelberg and Marshall (1996), and Khan et al. (2002). We also examine the reaction of the central bank's monetary policy to changes in investor inflationary expectations and how the short-term end and the long-term end of the term structures of real interest rates react to monetary policy shocks. In addition, to have a diagnostic check of our model, we augment our basic model to include some non-financial variables that may also impact real interest rates and inflationary expectations. The exclusion of these variables may give us some misleading results if they are related to central bank monetary policy. The additional variables in the augmented model are industrial output, the unemployment rate and the US dollar exchange rate of Canadian currency.

Therefore, in sum, in our study, using a better set of data (inflationary expectations and ex-ante real interest rates) than was available in the previous studies, we are able answer the following questions: How do monetary policy shocks affect real interest rates and inflationary expectations? How differentially does the monetary policy impact on real rates of different maturities[3]? If the central bank's monetary policy shocks affect these variables, is there any lag in the policy's impact on these variables? How does the

[2] Using the same identifying restrictions, St-Amant (1995) and Gottschalk (2001) calculated the inflation expectations and ex-ante real interest rates from nominal interest rates. St-Amant calculated these rates using the U.S.A data and Gottschalk calculated these rates using the data of Euro area.

[3] In our model, we will use real interest rates of different maturities: one-year ex-ante real interest rate, ex-ante real forward rate of year two and ex-ante real forward rate of year three to see the effects of the central bank monetary policy on them.

monetary policy shock affect the exchange rates and other variables in the economy such as the unemployment rate, output level etc.? What is the magnitude of the policy's impact on these variables and how long does it last? Does the central bank's monetary policy respond to changes in inflationary expectations and other variables in the economy? To search for the answers of these questions, we used Canadian data in our analysis.

We find that a positive monetary policy shock introduced by increasing M1B (currency and all chequable deposits in chartered banks) temporarily lowers the ex-ante real interest rate and raises inflationary expectations. The effect of such a monetary policy shock on the nominal interest rate, which nets the effect of the shock on real interest rate and inflationary expectations, is a short-run decline in it which is smaller in magnitude than the ex-ante real rate. We find that the impact of a given monetary policy shock is smaller on long-term interest rate than on short-term interest rate. We also find that a positive monetary policy shock depreciates the Canadian currency and generates other macro effects consistent with the conventional monetary theory. To compare our results with previous studies, we also estimate our model using the overnight target rate as the monetary policy instrument. We find that a contractionary monetary policy shock introduced by raising the overnight target rate temporarily lowers inflationary expectations and increases the ex-ante real interest rate with statistically insignificant effect on the second and the third year ex-ante real forward rates. We also find that this type of monetary policy shock decreases output, increases the unemployment rate and appreciates the Canadian currency. Our results are qualitatively better using the overnight target rate instead of the monetary aggregate as the monetary policy instrument.

The remainder of the thesis is organized as follows. Chapters 2 and 3 provide estimate of the ex-ante real interest rate and inflationary expectations. Chapter 2 discusses the theory behind the decomposition of the nominal interest rate into the ex-ante real interest rate and the inflationary expectations using the Blanchard-Quah Structural Vector Auto Regression (VAR) methodology. In Chapter 3, we report the suitability of our data for the Blanchard-Quah model, the estimated variance decomposition of the nominal interest rate, the estimated impulse responses of nominal interest rates to ex-ante real interest rate and

the inflationary expectation shocks, and we present the estimated series of inflationary expectations and the ex-ante real interest rate. In Chapter 4, we describe the fully recursive VAR model used to estimate the effects of monetary policy shocks on various macroeconomic variables, and we empirically identify the feedback rule and the exogenous monetary policy shocks. Chapter 5 presents the estimated results including the impulse response of various macroeconomic variables to monetary policy shocks and the analysis of their implications. Chapter 6 concludes.

CHAPTER 2

2.1 The Theory behind the Decomposition of the Nominal Interest Rate into the Ex-ante Real Interest Rate and Inflationary Expectations

We apply the structural VAR methodology developed by Blanchard and Quah (1989) to decompose the Canadian one-year, two-year and three-year nominal interest rates into the expected inflation and the ex-ante real interest rate components following the approach by St-Amant (1996) and Gottschalk (2001). The starting point of St. Amant is the Fisher equation that states that the nominal interest rate is the sum of the expected inflation and the ex-ante real interest rate:

$$n_{t,k} = r_{t,k} + E(\pi_{t,k}) \qquad (1)$$

where $n_{t,k}$ is the nominal interest at time t on a bond with k periods till maturity, $r_{t,k}$ is the corresponding ex-ante real rate and $E(\pi_{t,k})$ denotes inflationary expectations for the time from t to t+k. The inflation forecast error $\varepsilon_{t,k}$ can be defined as the difference between the actual inflation $\pi_{t,k}$ and the expected inflation $E(\pi_{t,k})$:

$$\varepsilon_{t,k} = \pi_{t,k} - E(\pi_{t,k}) \qquad (2)$$

Now substituting (2) into (1), we get the following relation:

$$n_{t,k} - \pi_{t,k} = r_{t,k} - \varepsilon_{t,k} \qquad (3)$$

Therefore, the ex-post real rate ($n_{t,k} - \pi_{t,k}$) is the sum of the ex-ante real rate $r_{t,k}$ and the inflation forecast error $\varepsilon_{t,k}$. Under the assumptions that both the nominal interest rate and the inflation rate are integrated of order one and they are co-integrated, and that the inflation forecast error $\varepsilon_{t,k}$ is integrated of order zero, assumptions we test and confirm in Section III, then the ex-ante real rate $r_{t,k}$ must be stationary.

Gottschalk identifies three implications that flow from these assumptions. First, if the nominal interest rate is non-stationary, this variable can be decomposed into a non-stationary component comprised of changes in the nominal interest rate with a permanent character and a stationary component comprised of the transitory fluctuations in the interest rate. Second, if the nominal interest rate and the actual inflation rate are co-integrated, it implies that both variables share the common stochastic trend, and this stochastic trend is the source of the non-stationary of both variables. On the other hand, if the ex-ante real interest rate is stationary, the nominal trend has no long-run effect on this variable. Third, if the nominal interest rate and the actual inflation rate are co-integrated (1,1) and the inflation forecast error is integrated of order zero I(0), this implies that changes in inflationary expectations are the source of these permanent movements in the nominal interest rate.

Therefore, the permanent movements of the nominal interest rate obtained by using the Blanchard-Quah methodology will be the nothing other than those inflationary expectations. Since the permanent component of the nominal interest rate corresponds to inflationary expectations, the stationary component must be the ex-ante real interest rate. Therefore, using the identifying restrictions that shocks to the ex-ante real rate have only a transitory effect on the nominal interest rate while shocks to inflationary expectations induce a permanent change in the nominal interest rate, we can calculate inflationary expectations and the ex-ante real rate of interest.

2.2 The Blanchard-Quah Structural VAR Methodology[4]

Assuming our data satisfies the stationarity assumption (assumption that we test and confirm in Chapter 3), we turn to the structural VAR model developed by Blanchard and Quah (1989) to decompose the nominal interest rate into the ex-ante real interest rate and inflationary expectations. As mentioned earlier, our key assumption is that nominal interest rate fluctuations are a function of two non-autocorrelated and orthogonal types of

[4] The Blanchard-Quah VAR model of this chapter is based on Enders (2003).

shocks: inflationary expectations shocks (ε_p) and ex-ante real interest rate shocks (ε_r). Our objective is to identify these two shocks and thereafter compute the empirical measures of the ex-ante real interest rate and inflationary expectations components of the nominal interest rate. For this purpose, we use a bivariate model comprised of the first difference of the nominal interest rate (n_t) and real interest rate (r_t)[5].Define the first difference of nominal interest rate as y_t. Now assuming a lag-length of q, the simple bivariate Blanchard-Quah VAR model can be written as follows:

$$y_t = b_{10} - b_{12} r_t + \alpha_{11} y_{t-1} + \alpha_{12} r_{t-1} + \ldots\ldots\ldots\ldots \beta_{11} y_{t-q} + \beta_{12} r_{t-q} + \varepsilon_{pt} \qquad (4)$$

$$r_t = b_{20} - b_{21} y_t + \alpha_{21} y_{t-1} + \alpha_{22} r_{t-1} + \ldots\ldots\ldots\ldots \beta_{21} y_{t-q} + \beta r_{22} r_{t-q} + \varepsilon_{rt} \qquad (5)$$

where ε_{pt} and ε_{rt} are uncorrelated white-noise disturbances with standard deviations of σ_p and σ_r respectively.

These equations are in structural-form and not in reduced-form as both variables have contemporaneous effects on each other. As we will estimate the reduced-form VAR rather than the structural-form VAR, our next job is to transform the structural equations into the reduced-form equations. To do that, let's rewrite structural equations in matrix-form in the following way:

$$\begin{bmatrix} 1 & b_{12} \\ b_{21} & 1 \end{bmatrix} \begin{bmatrix} y_t \\ r_t \end{bmatrix} = \begin{bmatrix} b_{10} \\ b_{20} \end{bmatrix} + \begin{bmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{bmatrix} \begin{bmatrix} y_{t-1} \\ r_{t-1} \end{bmatrix} + \ldots\ldots\ldots \begin{bmatrix} \beta_{11} & \beta_{12} \\ \beta_{21} & \beta_{22} \end{bmatrix} \begin{bmatrix} y_{t-q} \\ r_{t-q} \end{bmatrix} + \begin{bmatrix} \varepsilon_{pt} \\ \varepsilon_{rt} \end{bmatrix}$$

and more compactly, we can write:

$$Bx_t = \Gamma_0 + \Gamma_1 x_{t-1} + \ldots\ldots\ldots\ldots + \Gamma_p x_{t-q} + \varepsilon_t \qquad (6)$$

[5] To use the Blanchard-Quah technique, both variables in the VAR model must be in a stationary form. Since the nominal interest rate is integrated of order one, we used it first differenced in our model. The second variable in the VAR model- the real interest rate- is already in a stationary form, and hence we don't need to take its first difference.

where $B = \begin{bmatrix} 1 & b_{12} \\ b_{21} & 1 \end{bmatrix}$

$$\Gamma_0 = \begin{bmatrix} b_{10} \\ b_{20} \end{bmatrix}$$

$$\Gamma_1 = \begin{bmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{bmatrix}$$

$$\Gamma_q = \begin{bmatrix} \beta_{11} & \beta_{12} \\ \beta_{21} & \beta_{22} \end{bmatrix}$$

$$x_t = \begin{bmatrix} y_t \\ r_t \end{bmatrix}$$

$$\varepsilon_t = \begin{bmatrix} \varepsilon_{pt} \\ \varepsilon_{rt} \end{bmatrix}$$

Therefore, pre multiplying both sides of (6) by B^{-1}, we get the VAR model in reduced-form or in standard-form as follows:

$$x_t = A_0 + A_1 x_{t-1} + \ldots\ldots\ldots\ldots + A_p x_{t-q} + e_t \qquad (7)$$

where,

$$A_0 = B^{-1}\Gamma_0$$
$$A_1 = B^{-1}\Gamma_1$$
$$A_p = B^{-1}\Gamma_p$$
$$e_t = B^{-1}\varepsilon_t$$

Defining a_{i0} as the element i of the vector A_0, a_{ij} as the element in row i and column j of the matrix A_1, d_{ij} as the element in row i and column j of the matrix A_q, and e_{it} as the element i of the vector e_t, we can rewrite (7) into the following reduced-form VAR model:

$$y_t = a_{10} + a_{11} y_{t-1} + a_{12} r_{t-1} + \ldots\ldots\ldots\ldots d_{11} y_{t-q} + d_{12} r_{t-q} + e_{1t} \qquad (8)$$

$$r_t = a_{20} + a_{21} y_{t-1} + a_{22} r_{t-1} + \ldots\ldots\ldots\ldots d_{21} y_{t-q} + d_{22} r_{t-q} + e_{2t} \qquad (9)$$

The error terms- e_{1t} and e_{2t} of the above reduced-form equations are composites of the structural shocks- ε_{pt} and ε_{rt}. Since $e_t = B^{-1}\varepsilon_t$ (defined above), we can express e_{1t} and e_{2t} in terms of ε_{pt} and ε_{rt} as follows:

$$e_{1t} = (\varepsilon_{pt} - b_{12}\varepsilon_{rt})/(1 - b_{12}b_{21}) \qquad (10)$$

$$e_{2t} = (\varepsilon_{rt} - b_{21}\varepsilon_{pt})/(1 - b_{12}b_{21}) \qquad (11)$$

According to the standard assumption of VAR, since ε_{pt} and ε_{rt} are white-noise process, e_{1t} and e_{2t} must have zero means, constant variance, and are individually serially uncorrelated. The important point to note here is that although each e_{1t} and e_{2t} have zero autocovariances, they are correlated with each other unless there is no contemporaneous effect of y_t on r_t and r_t on y_t, that is, unless the coefficients $b_{12} = b_{21} = 0$ [6].

Now if we ignore the intercept terms, following Enders (2003), the bivariate moving average (BMA) representation of $\{ y_t \}$ and $\{ r_t \}$ sequences can be written in the following form:

$$y_t = \sum_{k=0}^{\infty} c_{11}(k)\varepsilon_{pt-k} + \sum_{k=0}^{\infty} c_{12}(k)\varepsilon_{rt-k} \qquad (12)$$

$$r_t = \sum_{k=0}^{\infty} c_{21}(k)\varepsilon_{pt-k} + \sum_{k=0}^{\infty} c_{22}(k)\varepsilon_{rt-k} \qquad (13)$$

Using matrix notation, in a more compact form, we can rewrite the above equations as follows:

$$\begin{bmatrix} y_t \\ r_t \end{bmatrix} = \begin{bmatrix} C_{11}(L) & C_{12}(L) \\ C_{21}(L) & C_{22}(L) \end{bmatrix} \begin{bmatrix} \varepsilon_{pt} \\ \varepsilon_{rt} \end{bmatrix} \qquad (14)$$

where the $C_{ij}(L)$ are polynomials in lag operator L such that the individual coefficient of $C_{ij}(L)$ are denoted by $c_{ij}(k)$. For example, the second coefficient of $C_{12}(L)$ is $c_{12}(2)$ and the third coefficient of $C_{21}(L)$ is $c_{21}(3)$. Let's drop the time subscripts of the variance and the covariance terms and normalize the shocks for our convenience so that $\text{var}(\varepsilon_p) = 1$ and $\text{var}(\varepsilon_r) = 1$. If we name $\sum_{\varepsilon}$ the variance-covariance matrix of the innovations (structural shocks), we end up as follows:

[6] A detailed discussion of the properties of the structural shocks- ε_{pt} and ε_{rt} and the composite errors terms of the reduced-form equations- e_{1t} and e_{2t} are in Enders (2003).

$$\sum_{\varepsilon} = \begin{bmatrix} \text{var}(\varepsilon_p) & \text{cov}(\varepsilon_p, \varepsilon_r) \\ \text{cov}(\varepsilon_p \varepsilon_r) & \text{var}(\varepsilon_r) \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

As mentioned earlier, the key to decompose the nominal interest rate n_t into its trend and irregular component is to assume that ex-ante real interest rate shocks ε_{rt} have a temporary effect on the $\{n_t\}$ sequence. In the long run, therefore, if the nominal interest rate n_t is to be unaffected by the ex-ante real interest rate shock ε_{rt}, it must be the case that the cumulated effect of ε_{rt} shocks on the y_t sequence must be zero. So the coefficients in (12) must be such that

$$\sum_{k=0}^{\infty} c_{12}(k)\varepsilon_{rt-k} = 0 \qquad (15)$$

Our next job is then to recover ex-ante real interest rate shocks ε_{rt} and inflationary expectation shocks ε_{pt} from the VAR estimation. The reduced-form equations (8) and (9), in lag operator, can be written in the following matrix form:

$$\begin{bmatrix} y_t \\ r_t \end{bmatrix} = \begin{bmatrix} A_{11}(L) & A_{12}(L) \\ A_{21}(L) & A_{22}(L) \end{bmatrix} \begin{bmatrix} y_{t-1} \\ r_{t-1} \end{bmatrix} + \begin{bmatrix} e_{1t} \\ e_{2t} \end{bmatrix} \qquad (16)$$

In a more compact notation, we can rewrite the above equations as follows:

$$x_t = A(L)x_{t-1} + e_t \qquad (17)$$

where x_t = the column vector $(y_t, r_t)'$

e_t = the column vector $(e_{1t}, e_{2t})'$

A(L) = the 2X2 matrix with the elements equal to the polynomials $A_{ij}(L)$ and the

coefficients of $A_{ij}(L)$ are denoted by $a_{ij}(k)$.

As shown earlier, the VAR residuals in model (16) are composites of the pure innovations ε_{pt} and ε_{rt}. Therefore, we can relate the VAR residuals and the pure innovations as follows. We know e_t is the one-step ahead forecast error of y_t i.e., $e_{1t} = y_t - E_{t-1}y_t$. On the other hand, from the bivariate moving average (BMA) representation (equations (12) and (13)), one-step ahead forecast error can be defined as $c_{11}(0)\varepsilon_{1t} + c_{12}(0)\varepsilon_{2t}$. Therefore, we can write e_{1t} as follows:

$$e_{1t} = c_{11}(0)\varepsilon_{pt} + c_{12}(0)\varepsilon_{rt} \qquad (18)$$

Similarly, for e_{2t} we can write:

$$e_{2t} = c_{21}(0)\varepsilon_{pt} + c_{22}(0)\varepsilon_{rt} \qquad (19)$$

Combining (18) and (19), we get the following relationship in matrix notation:

$$\begin{bmatrix} e_{1t} \\ e_{2t} \end{bmatrix} = \begin{bmatrix} c_{11}(0) & c_{12}(0) \\ c_{21}(0) & c_{22}(0) \end{bmatrix} \begin{bmatrix} \varepsilon_{pt} \\ \varepsilon_{rt} \end{bmatrix} \qquad (20)$$

It is now evident that once we have the values of $c_{11}(0), c_{12}(0), c_{21}(0) and c_{22}(0)$, we can recover the pure innovations- ε_{pt} and ε_{rt} from the regression residuals- e_{1t} and e_{2t} of our estimated VAR model. To do this, we follow the Blanchard-Quah VAR technique. Following them, we use the relationship between (16) and the BMA model (14) plus the long run restriction that nominal interest is unaffected by the ex-ante real interest rate i.e., the cumulative effect of ε_{rt} shock on $\{ y_t \}$ sequence is zero (equation (15)). We, therefore, end up with the following four restrictions from which we calculate the numerical values of the coefficients: $c_{11}(0), c_{12}(0), c_{21}(0) and c_{22}(0)$ which, in turn, we use to recover the pure innovations- ε_{pt} and ε_{rt}.

Restriction 1:

Given (18) and using the assumption that the inflationary expectation shock ε_{pt} and the ex-ante real interest rate shock ε_{rt} are uncorrelated i.e., $E\varepsilon_{pt}\varepsilon_{rt} = 0$, we see that the normalization $Var(\varepsilon_p) = Var(\varepsilon_r) = 1$ means that the variance of e_{1t} is as follows[7]:

$$Var(e_1) = c_{11}(0)^2 + c_{12}(0)^2 \qquad (21)$$

Restriction 2:

Using the similar concept used in restriction 1, we get:

$$Var(e_2) = c_{21}(0)^2 + c_{22}(0)^2 \qquad (22)$$

Restriction 3:

The product of e_{1t} and e_{2t} is

$$e_{1t}e_{2t} = [c_{11}(0)\varepsilon_{pt} + c_{12}(0)\varepsilon_{rt}][c_{21}(0)\varepsilon_{pt} + c_{22}(0)\varepsilon_{rt}]$$

Taking the expectation, the covariance of the VAR residuals is:

$$Ee_{1t}e_{2t} = c_{11}(0)c_{21}(0) + c_{12}(0)c_{22}(0) \qquad (23)$$

Restriction 4:

The fourth restriction is the assumption that the ex-ante real interest rate shock ε_{rt} has no long-run effect on the nominal interest rate sequence n_t which is equation (15). Now our

[7] We can easily figure out restriction 1 and restriction 2 using the following matrix algebra. Dropping the time subscripts of the variables in (20), we can write it more compactly as follows:

$$e = c\varepsilon$$

$$i.e., ee' = c\varepsilon\varepsilon' c$$

$$i.e., Eee' = cIc$$

$$i.e., \begin{bmatrix} Var(e_1) & 0 \\ 0 & Var(e_2) \end{bmatrix} = \begin{bmatrix} c_{11}(0) & c_{12}(0) \\ c_{21}(0) & c_{22}(0) \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} c_{11}(0) & c_{12}(0) \\ c_{21}(0) & c_{22}(0) \end{bmatrix}$$

job is to transform this restriction into the VAR representation so that we can use this restriction to calculate the coefficients we need. We will proceed as follows.

We can rewrite the reduced form VAR, equation (17), as follows:

$$\begin{aligned} & x_t = A(L)Lx_t + e_t \\ & i.e., [I - A(L)L]x_t = e_t \\ & i.e., x_t = [I - A(L)L]^{-1} e_t \end{aligned} \tag{24}$$

For notational convenience, let's denote the determinant of [I – A(L)L] by D. Therefore, doing some algebra further equation (24) can be written as follows[8]:

$$\begin{bmatrix} y_t \\ r_t \end{bmatrix} = (1/D) \begin{bmatrix} 1 - A_{22}(L)L & A_{12}(L)L \\ A_{21}(L)L & 1 - A_{11}(L)L \end{bmatrix} \begin{bmatrix} e_{1t} \\ e_{2t} \end{bmatrix}$$

Using the definition of $A_{ij}(L)$, we get:

$$\begin{bmatrix} y_t \\ r_t \end{bmatrix} = (1/D) \begin{bmatrix} 1 - \Sigma a_{22}(L)L & \Sigma a_{12}(L)L \\ \Sigma a_{21}(L)L & 1 - \Sigma a_{11}(L)L \end{bmatrix} \begin{bmatrix} e_{1t} \\ e_{2t} \end{bmatrix}$$

Now the solution for y_t in terms of the current and lagged values of $\{e_{1t}\}$ and $\{e_{2t}\}$ is:

$$y_t = (1/D)\{[1 - \sum_{k=0}^{\infty} a_{22}(k)L^{k+1}]e_{1t} + [\sum_{k=0}^{\infty} a_{12}(k)L^{k+1}]e_{2t}\} \tag{25}$$

Replacing e_{1t} and e_{2t} in the (25) with ε_{pt} and ε_{rt} from equations (18) and (19), we get the following equation:

$$y_t = (1/D)\{[1 - \sum_{k=0}^{\infty} a_{22}(k)L^{k+1}](c_{11}(0)\varepsilon_{rt} + c_{12}(0)\varepsilon_{rt}) + [\sum_{k=0}^{\infty} a_{12}(k)L^{k+1}](c_{21}(0)\varepsilon_{pt} + c_{22}(0)\varepsilon_{rt}\}$$

[8] The details of the algebra are available in Enders (2003, p.334).

$$i.e., y_t = (1/D)\{[1-\sum_{k=0}^{\infty} a_{22}(k)L^{k+1}]c_{11}(0)\varepsilon_{pt} + [\sum_{k=0}^{\infty} a_{12}(k)L^{k+1}](c_{21}(0)\varepsilon_{pt} +$$

$$[1-\sum_{k=0}^{\infty} a_{22}(k)L^{k+1}]c_{12}(0)\varepsilon_{rt} + [\sum_{k=0}^{\infty} a_{12}(k)L^{k+1}]c_{22}(0)\varepsilon_{rt}\} \quad (26)$$

Therefore, using (26), the restriction that the ex-ante real interest rate shock $\{\varepsilon_{rt}\}$ has no long-run effect on the nominal interest rate n_t is:

$$[1-\sum_{k=0}^{\infty} a_{22}(k)L^{k+1}]c_{11}(0)\varepsilon_{rt} + [\sum_{k=0}^{\infty} a_{12}(k)L^{k+1}]c_{21}(0)\varepsilon_{rt} = 0$$

So our fourth restriction that for all possible realizations of the $\{\varepsilon_{rt}\}$ sequence, ex-ante real interest rate shocks $\{\varepsilon_{rt}\}$ will have only temporary effect on the y_t sequence (the first difference of nominal interest rate) and n_t itself (the nominal interest rate) is:

$$[1-\sum_{k=0}^{\infty} a_{22}(k)L^{k+1}]c_{11}(0) + [\sum_{k=0}^{\infty} a_{12}(k)L^{k+1}]c_{21}(0) \ = 0 \quad (27)$$

We now have four equations: (21), (22), (23) and (27) to get four unknown values: $c_{11}(0), c_{12}(0), c_{21}(0) and c_{22}(0)$. Once we have these values of $c_{ij}(0)$ and the residuals of the VAR $\{e_{1t}\}$ and $\{e_{2t}\}$, the entire $\{\varepsilon_{pt}\}$ and $\{\varepsilon_{rt}\}$ sequences can be identified using the following equations:

$$e_{1t-i} = c_{11}(0)\varepsilon_{pt-i} + c_{12}(0)\varepsilon_{rt-i} \quad (28)$$

and

$$e_{2t-i} = c_{21}(0)\varepsilon_{pt-i} + c_{22}(0)\varepsilon_{rt-i} \quad (29)$$

As our objective is to decompose the nominal interest into the ex-ante real interest rate and inflationary expectation, we will stop at this point. Since throughout the model we assume that the source of the change in the nominal interest rate is the inflationary

expectation shock $\{\varepsilon_{pt}\}$ and the ex-ante real interest rate shock $\{\varepsilon_{rt}\}$, the cumulation of these effects yields the level of the nominal interest rate as a function of inflationary expectations and ex-ante real interest rate shocks. This means that the cumulation of the effects of these structural shocks give the permanent and the stationary components of the nominal interest rates. Adding the calculated stationary components to the mean of the ex-post real interest rate (mean of the difference between the observed nominal interest rate and the contemporaneous inflation rate), therefore, gives us the ex-ante real rate. Once we have the ex-ante real interest rate, inflationary expectations estimates can be obtained by subtracting the estimated ex-ante real interest rate from the nominal interest rate as the nominal interest rate is the sum of these two rates.

2.3 Impulse Response Functions[9]

The impulse response functions give us the opportunity to visually observe the behavior of the nominal interest rate in response to the inflationary expectation shock ε_{pt} and the ex-ante real interest rate shock ε_{rt}. The practical way to derive the impulse response functions is to start with the reduced-form VAR model. Our two-variable VAR model in standard-form (reduced-form) with the nominal interest rate n_t and the real interest rate r_t in matrix notation can be written as follows:

$$\begin{bmatrix} n_t \\ r_t \end{bmatrix} = \begin{bmatrix} a_{10} \\ a_{20} \end{bmatrix} + \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} n_{t-1} \\ r_{t-1} \end{bmatrix} + \begin{bmatrix} e_{1t} \\ e_{2t} \end{bmatrix} \qquad (30)$$

Using the concept of Vector Moving Average Representation (VMA), (30) can be written as follows:

$$\begin{bmatrix} n_t \\ r_t \end{bmatrix} = \begin{bmatrix} \overline{n}_t \\ \overline{r}_t \end{bmatrix} + \sum_{i=0}^{\infty} \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}^i \begin{bmatrix} e_{1t-i} \\ e_{2t-1} \end{bmatrix} \qquad (31)$$

[9] For this econometric presentation, we heavily depend on Enders (2003).

Now for our purpose, we will rewrite (31) in terms of ε_{pt} and ε_{rt} sequences. From the relationship that $e_t = B^{-1}\varepsilon_t$, we find:

$$e_{1t} = (\varepsilon_{nt} - b_{12}\varepsilon_{rt})/(1 - b_{12}b_{21})$$

$$e_{2t} = (\varepsilon_{rt} - b_{21}\varepsilon_{nt})/(1 - b_{12}b_{21})$$

In matrix notation, we can rewrite the above equations as follows:

$$\begin{bmatrix} e_{1t} \\ e_{2t} \end{bmatrix} = [1/(1 - b_{12}b_{21})] \begin{bmatrix} 1 & -b_{12} \\ -b_{21} & 1 \end{bmatrix} \begin{bmatrix} \varepsilon_{nt} \\ \varepsilon_{rt} \end{bmatrix} \qquad (32)$$

Combining (31) and (32), we get:

$$\begin{bmatrix} n_t \\ r_t \end{bmatrix} = \begin{bmatrix} \overline{n}_t \\ \overline{r}_t \end{bmatrix} + [1/(1 - b_{12}b_{21})] \sum_{i=0}^{\infty} \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}^i \begin{bmatrix} 1 & -b_{12} \\ -b_{21} & 1 \end{bmatrix} \begin{bmatrix} \varepsilon_{nt} \\ \varepsilon_{rt} \end{bmatrix}$$

For notational convenience and simplification, as argued by Enders, (2003, p 305.), we can define the 2X2 matrix ϕ_i with elements $\phi_{jk}(i)$:

$$\phi_i = [A_1^i/(1 - b_{12}b_{21})] \begin{bmatrix} 1 & -b_{12} \\ -b_{21} & 1 \end{bmatrix}$$

Therefore, the moving average representation (31) can be written in terms ε_{pt} and ε_{rt} sequences as follows:

$$\begin{bmatrix} n_t \\ r_t \end{bmatrix} = \begin{bmatrix} \overline{n}_t \\ \overline{r}_t \end{bmatrix} + \sum_{i=0}^{\infty} \begin{bmatrix} \phi_{11}(i) & \phi_{12}(i) \\ \phi_{21}(i) & \phi_{22}(i) \end{bmatrix} \begin{bmatrix} \varepsilon_{nt-i} \\ \varepsilon_{rt-i} \end{bmatrix}$$

More compactly, we can write

$$x_t = \mu + \sum_{i=0}^{\infty} \phi_i \varepsilon_{t-i} \qquad (33)$$

The coefficients of ϕ_i shows the effects of inflationary expectations shocks, ε_{pt}, and ex-ante real interest rate shocks, ε_{rt}, on the entire time paths of nominal interest rate sequences $\{n_t\}$ and real interest rate sequences $\{r_t\}$. More precisely, the elements $\phi_{jk}(i)$ are the impact multipliers of the shocks of ε_{nt-i} and ε_{rt-i} on $\{n_t\}$ and $\{r_t\}$ sequences. For example, the coefficient $\phi_{12}(0)$ is the instantaneous impact of a one-unit change in the ex-ante real rate shock ε_{rt} on the nominal rate n_t. In the same way, updating by one period, the elements $\phi_{11}(1)$ and $\phi_{22}(1)$ represents the effects of one unit change in the inflationary expectation shock ε_{pt} on the nominal interest rate n_t and one unit change in the ex-ante real interest rate shock ε_{rt} on the real interest rate r_t respectively.

The cumulated effects of the impulses in ε_{pt} and ε_{rt} can be obtained by summing up the coefficients of the impulse response functions. For example, after m periods, the effect of the ex-ante real interest rate shock ε_{rt} on the nominal interest rate n_{t+m} is $\phi_{12}(m)$. Therefore, the cumulated sum of the effects of ε_{rt} on the $\{n_t\}$ sequence is:

$$\sum_{i=0}^{m} \phi_{12}(i)$$

As m approaches infinity, the above summation yields the long-run multiplier. Therefore, these four set of coefficients: $\phi_{11}(i), \phi_{12}(i), \phi_{21}(i) and \phi_{22}(i)$ are called the impulse response functions, and plotting these impulse response functions against time gives us the behavior of nominal interest rate series $\{n_t\}$ and real interest rate series $\{r_t\}$ in response to inflationary expectations and the ex-ante real interest rate shocks.

2.4 Variance Decomposition

Variance decomposition is another very practical way to take a closer look at the behavior of the variables we used in the VAR model. In our model, with the knowledge of the variance decomposition, we will be able to figure out the proportion of the

movements in the nominal interest rate sequence $\{n_t\}$ and the real interest rate sequence $\{r_t\}$ due to the inflationary expectation shocks ε_{pt} and the ex-ante real interest rate shocks ε_{rt}. To calculate the variance decomposition, we need to calculate the forecast errors of the VAR model in reduced-form. The standard-form VAR in (30) can be written more compactly as follows:

$$x_t = A_0 + A_1 x_{t-1} + e_t$$

Updating the above equation by one period and taking the conditional expectation of x_{t+1}, we get:

$$E_t x_{t+1} = A_0 + A_1 x_t$$

One-step ahead forecast error, therefore, can be defined as:

$$x_t - E_t x_{t+1} = e_{t+1}$$

Similarly, the two-step ahead forecast error is $e_{t+2} + A_1 e_{t+1}$, and in the same way, the m-step ahead forecast error is:

$$e_{t+m} + A_1 e_{t+m-1} + A_1^2 e_{t+m-2} + \ldots\ldots\ldots\ldots + A_1^{n-1} e_{t+1} \qquad (34)$$

Next we will describe these forecast errors in terms of $\{\varepsilon_t\}$ sequences rather than $\{e_t\}$ sequences. Using (33) to conditionally forecast x_{t+1}, the one-step ahead forecast error is $\phi_0 \varepsilon_{t+1}$. In the same way, m-step ahead forecast error $x_{t+m} - E_t x_{t+m}$ is:

$$x_{t+m} - E_t x_{t+m} = \sum_{i=0}^{m-1} \phi_i \varepsilon_{t+m-i}$$

Considering only on the $\{n_t\}$ sequence, the m-step ahead forecast error becomes:

$$n_{t+m} - E_t n_{t+m} = \phi_{11}(0)\varepsilon_{yt+m} + \phi_{11}(1)\varepsilon_{yt+m-1} + ... + \phi_{11}(m-1)\varepsilon_{yt+1} + \phi_{12}(0)\varepsilon_{zt+m} + \phi_{12}(1)\varepsilon_{zt+m-1} + ... + \phi_{12}(m-1)\varepsilon_{zt+1}$$

Therefore, denoting the m-step forecast error variance of y_{t+m} by $\sigma_n(m)^2$, we get:

$$\sigma_n(m)^2 = \sigma_n^2[\phi_{11}(0)^2 + \phi_{11}(1)^2 + ... + \phi_{11}(m-1)^2] + \sigma_r^2[\phi_{12}(0)^2 + \phi_{12}(1)^2 + ... + \phi_{12}(m-1)^2]$$

Since all the values of $\phi_{jk}(i)^2$ are necessarily nonnegative, it is evident from the above equation that the variance of the forecast error increases as the forecast horizon m increases. Now decomposing the m-step ahead total forecast error $\sigma_n(m)^2$ due to the inflationary expectation shocks ε_{pt} and the ex-ante real interest rate shocks ε_{rt} respectively, we get:

Proportion of forecast error due to shocks in ε_{pt} sequence:

$$\frac{\sigma_n^2[\phi_{11}(0)^2 + \phi_{11}(1)^2 + ... + \phi_{11}(m-1)^2]}{\sigma_n(m)^2}$$

and the proportion of forecast error due to shocks in ε_{rt} sequence:

$$\frac{\sigma_n^2[\phi_{12}(0)^2 + \phi_{12}(1)^2 + ... + \phi_{12}(m-1)^2]}{\sigma_n(m)^2}$$

If ex-ante real interest rate shocks ε_{rt} explain none of the forecast error variance of the nominal interest rate $\{n_t\}$ at al forecast horizons, we can say that the $\{n_t\}$ sequence is exogenous to the ex-ante real rate. In such a situation, the nominal interest rate $\{n_t\}$ sequence would evolve independently of the ex-ante real interest rate shock ε_{rt} and the real interest rate $\{r_t\}$ sequence. On the other hand, if ε_{rt} shocks explain all the forecast error variance in the $\{n_t\}$ sequence at all forecast horizons, the nominal interest rate $\{n_t\}$ would be entirely endogenous. In our bivariate VAR model, since we assume that the ex-

ante real interest rate shock ε_{rt} does not have a long-run effect on the nominal interest rate, it is expected that in later periods, the relative contribution of this shock on the nominal interest rate will be almost zero. On the other hand, the relative proportion of inflationary expectations shocks ε_{pt} in explaining the fluctuation of the nominal interest rate in subsequent periods will tend to one.

CHAPTER 3

3.1 The Stationarity Properties of the Data

We use Canadian monthly data for the nominal interest rate (n_t) with one-year, two-year and three-years to maturity, and the seasonally adjusted consumer price index (CPI) from 1980:1 to 2002:12. The inflation rate is calculated as the annualized monthly rate of change of the CPI. Our required assumptions are that the nominal interest rate and the inflation rate are both integrated of order one and that the two variables are co-integrated (1,-1). The stationary properties of the nominal interest rate, the real interest rate and the inflation rate are investigated using the Phillips-Perron test, the Augmented Dickey Fuller test (ADF), and the KPSS test. Both the ADF test and the Phillip-Perron tests have the null hypothesis of non-stationarity (unit-root) and the KPSS test has the null hypothesis of stationarity. The results of unit-root tests are reported in Table 1 and Table 2, and the results of co-integration test are reported in Table 3.

Table 1: Unit-root tests of the CPI and the Inflation Rate.

Variable	Unit Root Tests		
	ADF Test	**Phillips-Perron Test**	**KPSS Test**
ln(CPI)	-2.4049(c, t, 14)	-4.8725(c, t, 15)	1.7265(15)
Δ ln(CPI)	-3.0238(c, t, 13)	-16.0516(c, t, 14)	1.1674(14)
Δ^2 ln(CPI)	-6.8905(c,12)	-60.8905(c,13)	0.0344(13)

Δ is the first difference operator and Δ^2 is the second difference operator. The bracket indicate the inclusion of a constant, c, trend, t, and lag length. Lag lengths for the ADF test are chosen by the Ng-Perron(1995) recursive procedure and lag lengths for the Phillips-Perron and KPSS test are chosen by the Schwert (1989) formula .

Consider the CPI first. The Augmented Dickey Fuller (ADF) test cannot reject the null of unit root at 10 percent level but the Phillips-Perron (PP) test rejects the null hypothesis of unit root at 1% level of significance. The KPSS test rejects the null hypothesis of stationarity at 1% level of significance. Therefore, we conclude this variable is non-

stationary. For the inflation rate, the ADF test cannot reject the null hypothesis of unit root at 10% level of significance and the KPSS test rejects the null hypothesis of stationarity at 1% level of significance. However, once again the Phillip-Perron test rejects the null of unit root at the 1% level. The stationarity of the first difference of the inflation rate is supported by all three test procedures. Given these mixed results, we do not reject the maintained hypothesis that the inflation rate is integrated of order one. To explore the stationary characteristic of the inflation rate, we also check its auto correlation functions. We find that the auto correlation coefficient starts at a reasonably high value and it drops off as the lag length increases which suggests that this time series is non sationary.

Table 2: Unit-root tests of Nominal and Real Interest Rates

Variable	Unit Root Tests		
	ADF Test	Phillips-Perron Test	KPSS Test
One Year Rates			
Nominal Rate ($n_{t,1}$)	-3.2907 (c, t, 7)	3.4373 (c, t, 7)	2.7161 (7)
Δ Nominal Rate	-6.7362(c, 6)	-15.2798(c, 6)	0.0269 (6)
Real Rate ($n_{t,k} - \pi_t$)	-3.7108(c, 6)	-14.1801(c, 6)	1.2021 (6)
Two- Year Rates			
Nominal Rate ($n_{t,2}$)	-3.0030 (c, t, 7)	-4. 4754 (c, t, 7)	2.6805 (7)
Δ Nominal Rate	-6.7067(c, 6)	-13.4930(c, 6)	0.1040 (6)
($n_{t,k} - \pi_t$)	-4.1505(c, 4)	-13.2895(c, 4)	2.1101 (4)
Three- Year Rates			
Nominal Rate ($n_{t,3}$)	-3.1334 (c, t, 7)	-4.7362 (c, t, 7)	2.7207 (7)
Δ Nominal Rate	-6.9972(c, 6)	-13.3692(c, 6)	0.1062 (6)
Real Rate($n_{t,k} - \pi_t$)	-4.2688(c, 4)	-13.4652(c, 4)	1.8612 (4)

The brackets indicate the inclusion of a constant, c, trend, t, and lag length. The results are robust to the c and t assumptions. Lag lengths are chosen by the Ng-Perron(1995) recursive procedure.

Recall that it is assumed that there is a unit root in the nominal rate ($n_{t,k}$). Table 2 indicates that for the one- year nominal interest rate, neither the ADF test nor the Phillips-

Perron (PP) test can reject the null hypothesis of unit root at 5% level of significance, and the KPSS test rejects the null hypothesis of stationarity at 1% level of significance. Therefore, we conclude this variable is non-stationary. For the two-year nominal rate, the ADF cannot reject the null of unit root at 10 percent, but the PP rejects the null at 1 percent. The KPSS test rejects the null hypothesis of stationarity at 1% level of significance. For the three-year nominal rate, the ADF cannot reject the null hypothesis of unit root at 10% but the PP rejects at 1 percent while the KPSS rejects the null hypothesis of stationarity at 1 percent. The test procedures also support the hypothesis that the first difference of the nominal interest rates is stationary for nominal interest rates of all maturities. As with the inflation data, given the mixed results we do not reject the hypothesis that the nominal interest rates are integrated of order one

Finally consider the real rate of interest. We test this assumption by testing the equivalent assumption that $n_{t,k} - \pi_t$ is stationary. Both the ADF test and the Phillip-Perron test reject the null hypothesis of unit root at 1% level of significance for real interest rates of all maturities although the KPSS test does not support the null hypothesis of stationarity for any of these real interest rates. Since both the ADF test and the Phillip-Perron test strongly support the hypothesis of stationarity, we conclude that the real interest rate is stationary.

From the above findings we can conclude that the nominal interest rate and the inflation rate are co-integrated. To double check our conclusion, however, we also confirm that the nominal interest rates of all maturities and the inflation rate are co-integrated in Table-3. We use the Johansen co-integration test for this purpose. The first row presents the likelihood ratio test for which the null hypothesis is that these variables are not co-integrated. The second row presents the test that these variables share at most one co-integrating equation. Table-3 demonstrates that for all maturities, the likelihood ratio test statistic indicates the variables are co-integrated (1, -1).

Table 3: Cointegration tests of Nominal Interest Rates and Inflation rates

Variables	Eigen value	Likelihood Ratio	5 Percent Critical Value	1 Percent Critical Value
Inflation Rate	0.763	23.8655	15.41	20.04
1 Year Nominal Rate	0.0095	2.5781	3.76	6.65
Inflation Rate	0.1033	27.1494	15.41	20.04
2 Year Nominal Rate	0.0040	0.9695	3.76	6.65
Inflation Rate	0.0979	25.8440	15.41	20.04
3 Year Nominal Rate	0.0045	1.1051	3.76	6.65

3.2 Variance Decomposition and Impulse Responses

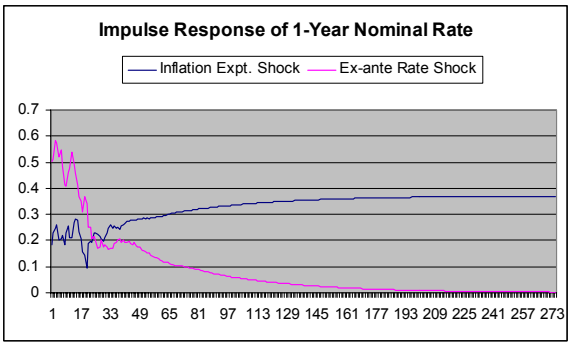
As we have confirmed the data satisfies all the required stationarity assumptions, our next step is to estimate the VAR model. We estimate three different reduced-form VAR models for 3 different nominal interest rates and corresponding real interest rates[10]. The two key outputs of VAR estimation that are of interest are the variance decompositions and impulse response functions. The decomposition of variance presented in Table 4 allows us to measure the relative importance of inflationary expectations and the ex-ante real interest rate shocks that underlie nominal interest rate fluctuations over different time horizons. It is evident from Table 4 that the proportion of the variance of nominal interest rates of all maturities explained by ex-ante real interest rate shocks gradually approaches zero in the long-run which is the result of the restriction that ex-ante real interest rate shocks have no permanent effect on the nominal interest rate. As in St. Amant (1996), both types of shocks have been important sources of nominal interest rate fluctuations.

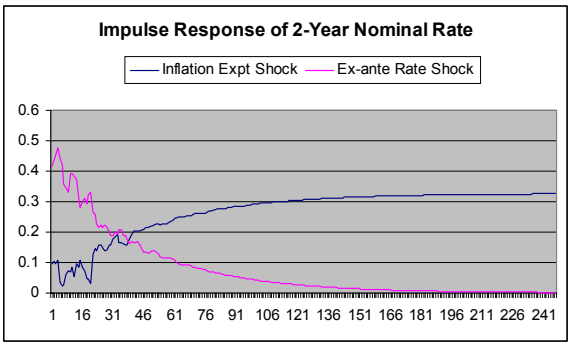
[10] We used the RATS program (Doan, 2000) to estimate the VAR models. In all the models we use a lag-length of 20 which was determined on the basis of Likelihood Ratio criterion and the Akaike Information criterion.

Table 4: Variance Decomposition of Nominal Interest Rates (in percent)

Horizons (Months)	**One-Year Rate**		**Two- Year Rate**		**Three- Year Rate**	
	Inflationary Expectation shock	Ex-ante Real Interest Rate shock	Inflationary Expectation shock	Ex-ante Real Interest Rate shock	Inflationary Expectation shock	Ex-ante Real Interest Rate shock
1	12	88	5	95	10	90
12	16	84	4	96	8	92
24	20	80	7	93	14	86
48	33	67	21	79	30	70
96	56	44	50	50	58	42
Long-term	100	0	100	0	100	0

Next we present the impulse responses of nominal interest rates to the structural shocks in Figure 1 wherein the horizontal axis measures the number of months. Figure 1 demonstrates that the effect of ex-ante real interest rate shocks disappear gradually while the effects of inflationary expectations shocks on nominal interest rates of all maturities are felt more dominantly in the later periods. This, as argued by St-Amant (1996, p.12) 'may reflect the dynamics of the adjustment of expectations to a change in the trend inflation'. Our impulse response functions are similar to those of Gottschalk (2001) and St-Amant (1996).





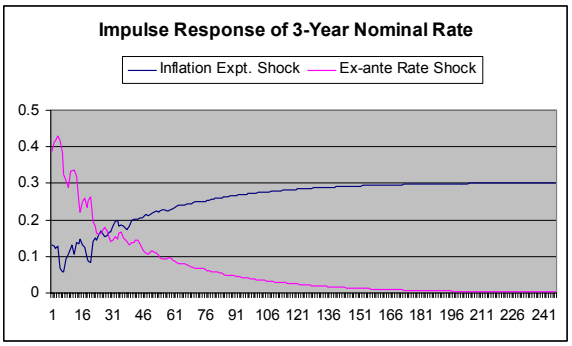


Figure 1: Impulse Responses of Nominal Interest Rates

3.3 The Ex-ante Real Interest Rate and Inflationary Expectations

To review, we estimate the ex-ante real interest rate and inflationary expectations by first computing the effects of ex-ante real rate shocks and inflationary expectations shocks on the nominal interest rate. The cumulation of these shocks provides the stationary and permanent components of nominal interest rates. An estimate of the ex-ante real interest rate is then obtained by adding the stationary components to the mean of the difference between the observed nominal interest rate and the contemporaneous rate of inflation i.e.,

the mean of the ex-post real interest rate. Then, the measure of inflationary expectations

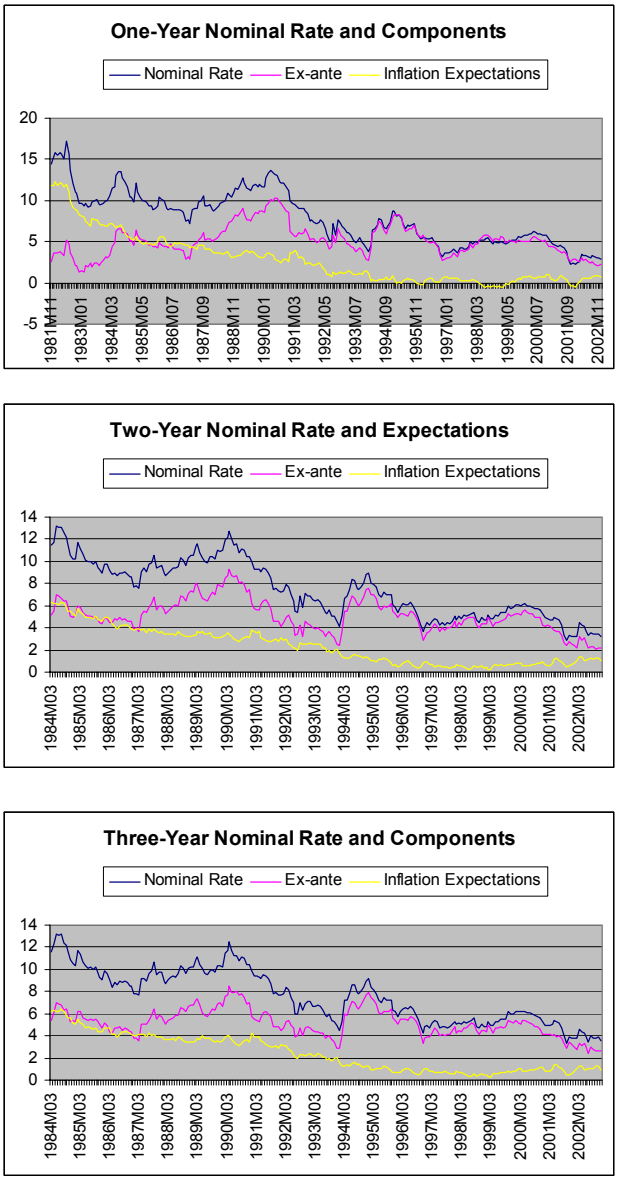


Figure 2: Nominal Interest Rate and Its Components

is calculated by subtracting the ex-ante real interest rate from the nominal interest rate. The estimated ex-ante real interest rate and the inflationary expectations of one-year, two-year and three-year along with the corresponding nominal interest rates are shown in Figure 2.

We also report the estimated series of the one-year inflationary expectation with the corresponding realized inflation rate in Figure 3. It is clear from the figure that the

estimated inflationary expectation series is less volatile than the realized inflation rate. It is also noticeable that expectations lag the turning points of actual inflation.

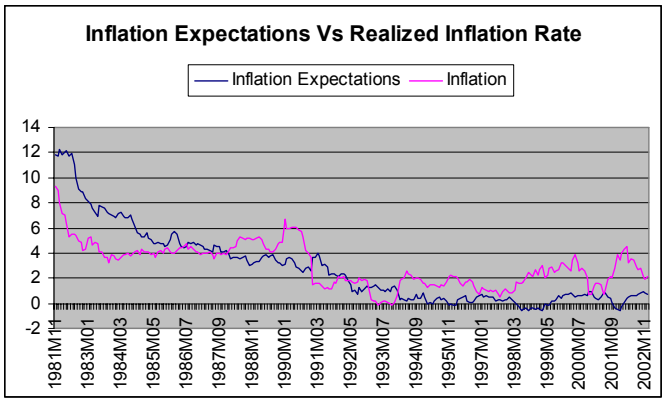


Figure 3: Inflationary Expectations and the Inflation Rate

Recall that we assume that the inflation forecast errors are integrated of order zero I(0). As reported in Table 5, the ADF test statistic support this hypothesis at the one percent level of significance while the Phillips-Perron test support this hypothesis at the five percent level of significance respectively[11].

Table 5: Unit Root Test of Inflation Forecast Errors

Variable	Unit Root Tests	
Inflation Forecast Error	ADF Test	Phillips-Perron Test
	-2.9148	-2.3582

[11] We use a lag-length of 3 for the ADF and the Phillips-Perron tests of the inflation forecast error which was determined on the basis of the Ng-Perron(1995) recursive procedure. We did not add any constant or trend the regression.

CHAPTER 4

4.1 A Framework for Analyzing the Effects of Monetary Policy Shocks

We use a fully recursive VAR model to estimate the effects of monetary policy shocks on various macroeconomic variables. The first step is to identify policy shocks that are orthogonal to the other shocks in the model. To do this, we follow the approach of Kahn et al. (2002) and Edelberg and Marshall (1996) to categorize all the variables in our model into three broad types.

The first type of variable (Type I variable) is the monetary policy instrument. We use both the monetary aggregate, M1B, and the overnight target rate (OT) as the monetary policy instruments. The second type of variable (Type II variable) is the contemporaneous inputs to the monetary policy rule, that is, the variables the central bank observes when setting its policy. In the basic model, we will include only one variable- the measure of inflationary expectations (EI) as the contemporaneous input to the policy process. In the diagnostic model, however, in addition to EI, we will include other variables, such as output (Y), the exchange rate (E) and unemployment rate (UNPR) as contemporaneous inputs to monetary policy. The third type of variable (Type III variable) in the basic model is a variable that responds to the change in policy. Since conventional theory treats the ex-ante real interest rate as the channel through which changes in policy are transmitted to policy targets, we use three alternative interest rates, R_1 , the one-year ex-ante real interest rate, F_2 , the two-year forward ex-ante real interest rate and F_3 , the three-year forward ex-ante real interest as our Type III variables[12].

[12] Assuming that R1 and R2 are the one-year and the two-year ex-ante real interest rates, and F2 is the ex-ante real forward rate of year two, the relationship between them (Bodie et. al., 2003) will be $(1+R2)^2 = (1+R1)(1+F2)$. From this equation, the ex-ante real forward of year two can be calculated as $F2 = \frac{(1+R2)^2}{(1+R1)} - 1$. Using the similar technique, the ex-ante real forward rate of year three can be calculated as $F3 = \frac{(1+R3)^2}{(1+R2)} - 1$, where R3 is the three-year ex-ante real interest.

Therefore our basic model includes three different variables: $[EI, M, R]$. We assume that the central bank's feedback rule is a linear function of contemporaneous values of Type II variables (inflationary expectations) and lagged values of all types of variables in the economy. That means that time t's change of monetary policy of the Bank of Canada is the sum of the following three things:

- the response of the Bank of Canada's policy to changes up to time t-1 in all variables in the model (i.e., lagged values of Type I, Type II and Type III variables),
- the response of the Bank of Canada's policy to time t changes in the non-policy Type II variable (inflationary expectations in the basic model), and
- the monetary policy shock.

Therefore, a monetary policy shock at time t is orthogonal to: changes in all variables in the model observed up to time t-1, and contemporaneous changes in the Type II non-policy variable (inflationary expectations in the basic model). So, by construction, a time t monetary policy shock of the Bank of Canada affects contemporaneous values of Type III variables (i.e., the real ex-ante interest rates of different maturities in the basic model) as well as all variables in the later periods[13].

The next two sections of this chapter describe the details of the fully recursive VAR model, the technique of how to identify the two portions of monetary policy- the feedback rule and exogenous monetary policy shocks, and how to get the impulse responses of monetary policy shocks.

4.2 The Recursive VAR Model to Estimate the Monetary Policy Shock

Our basic VAR system consists of three equations, and each equation in the system takes one of the three variables- inflationary expectations (EI), money supply (M) and the ex-ante real interest rate (R) to be its dependent variable. In the structural VAR system, for

[13] This framework assumes that the central concern of the Bank of Canada in the setting of policy is inflationary expectations because of the lag between changes in its instrument and the impact on its objective. Unless the bank targets inflationary expectations directly, it cannot hope to control inflation effectively.

each equation, the independent variables are lagged values of all three variables and the contemporaneous values of other variables. Suppose, our basic structural VAR model is the following:

$$EI_t = a_{10} - a_{12}M_t - a_{13}R_t + \gamma_{11}EI_{t-1} + \gamma_{12}M_{t-1} + \gamma_{13}R_{t-1} + .. + \delta_{11}EI_{t-q} + \delta_{12}M_{t-q} + \delta_{13}R_{t-q} + \varepsilon_{it} \quad (35)$$

$$M_t = a_{20} - a_{21}EI_t - a_{23}R_t + \gamma_{21}EI_{t-1} + \gamma_{22}M_{t-1} + \gamma_{23}R_{t-1} + .. + \delta_{21}EI_{t-q} + \delta_{22}M_{t-q} + \delta_{23}R_{t-q} + \varepsilon_{mt} \quad (36)$$

$$R_t = a_{30} - a_{31}EI_t - a_{32}M_t + \gamma_{31}EI_{t-1} + \gamma_{32}M_{t-1} + \gamma_{33}R_{t-1} + ..\delta_{31}EI_{t-q} + \delta_{32}M_{t-q} + \delta_{33}R_{t-q} + \varepsilon_{rt} \quad (37)$$

Here ε_{it} , ε_{mt} and ε_{rt} are white-noise disturbances with standard deviations of $\sigma_i, \sigma_m and \sigma_r$ respectively. In the above equations, the coefficients a_{ij} are the contemporaneous effects of an endogenous variable on two other endogenous variables. All other coefficients are effects on the lag variables.

To get the reduced-form version of the above structural equations, we transform them in matrix form as follows:

$$\begin{bmatrix} 1 & a_{12} & a_{13} \\ a_{21} & 1 & a_{23} \\ a_{31} & a_{32} & 1 \end{bmatrix} \begin{bmatrix} EI_t \\ M_t \\ R_t \end{bmatrix} = \begin{bmatrix} a_{10} \\ a_{20} \\ a_{30} \end{bmatrix} + \begin{bmatrix} \gamma_{11} & \gamma_{12} & \gamma_{13} \\ \gamma_{21} & \gamma_{22} & \gamma_{32} \\ \gamma_{31} & \gamma_{32} & \gamma_{33} \end{bmatrix} \begin{bmatrix} EI_{t-1} \\ M_{t-1} \\ R_{t-1} \end{bmatrix} + .. + \begin{bmatrix} \delta_{11} & \delta_{12} & \delta_{13} \\ \delta_{21} & \delta_{22} & \delta_{23} \\ \delta_{31} & \delta_{32} & \delta_{33} \end{bmatrix} \begin{bmatrix} EI_{t-q} \\ M_{t-q} \\ R_{t-q} \end{bmatrix} + \begin{bmatrix} \varepsilon_{it} \\ \varepsilon_{mt} \\ \varepsilon_{rt} \end{bmatrix}$$

i.e., $Ax_t = \Gamma_0 + \Gamma_1 x_{t-1} + ... + \Gamma_{t-q} + \varepsilon_t \quad (38)$

where $A = \begin{bmatrix} 1 & a_{12} & a_{13} \\ a_{21} & 1 & a_{23} \\ a_{31} & a_{32} & 1 \end{bmatrix}$

$$\Gamma_0 = \begin{bmatrix} a_{10} \\ a_{20} \\ a_{30} \end{bmatrix}$$

$$\Gamma_1 = \begin{bmatrix} \gamma_{11} & \gamma_{12} & \gamma_{13} \\ \gamma_{21} & \gamma_{22} & \gamma_{32} \\ \gamma_{31} & \gamma_{32} & \gamma_{33} \end{bmatrix}$$

$$\Gamma_q = \begin{bmatrix} \delta_{11} & \delta_{12} & \delta_{13} \\ \delta_{21} & \delta_{22} & \delta_{23} \\ \delta_{31} & \delta_{32} & \delta_{33} \end{bmatrix}$$

$$x_t = \begin{bmatrix} EI_t \\ M_t \\ R_t \end{bmatrix}$$

$$\varepsilon_t = \begin{bmatrix} \varepsilon_{it} \\ \varepsilon_{mt} \\ \varepsilon_{rt} \end{bmatrix}$$

Pre multiplying both sides of (38) by A^{-1}, we get the following reduced-form VAR model:

$$x_t = B_0 + B_1 x_{t-1} + ...B_q x_{t-q} + e_t \tag{39}$$

where
$$B_0 = A^{-1}\Gamma_0$$
$$B_1 = A^{-1}\Gamma_1$$
$$B_q = A^{-1}\Gamma_q$$
$$e_t = A^{-1}\varepsilon_t$$

The above reduced-form equation can be written into the following three reduced-form VAR model equations:

$$EI_t = b_{10} + b_{11}EI_{t-1} + b_{12}M_{t-1} + b_{13}R_{t-1} + ... + f_{11}EI_{t-q} + f_{12}M_{t-q} + f_{13}R_{t-q} + e_{1t} \tag{40}$$

$$M_t = b_{20} + b_{21}EI_{t-1} + b_{22}M_{t-1} + b_{23}R_{t-1} + ... + f_{21}EI_{t-q} + f_{22}M_{t-q} + f_{23}R_{t-q} + e_{2t} \tag{41}$$

$$R_t = b_{30} + b_{31}EI_{t-1} + b_{32}M_{t-1} + b_{33}R_{t-1} + ... + f_{31}EI_{t-q} + f_{32}M_{t-q} + f_{33}R_{t-q} + e_{3t} \tag{42}$$

It is quite straightforward that the structural equations (35) through (37) are not directly estimable as these equations violate the standard assumption that regressors can not be correlated with the error terms. In each equations of the structural VAR system, the contemporaneous variables are correlated with the error terms which prohibit the structural systerm to estimate directly. However, we don't have such problems in the reduced from VAR model as all the regressors in equations (40) through (42) are predetermined variables. Therefore, we can apply OLS to the reduced-form VAR system and can obtain the estimates of B_i where i ranges from zero to q and the variances of e_{it} and covarinces between them where i ranges from one to three.

Once we have the estimates of the reduced-from VAR equations our next job is to recover the parameters of the structural-form VAR equations from those of the reduced-form system. The problem we encounter in recovering the structural parameters from the reduced-from parameters is the number of estimated parameters of the reduced-from model is less than the number of recoverable parameters in the structural-form model. More precisely, in the reduced-from VAR model, we estimate three intercept terms (b_{10}, b_{20}, b_{30}) and $9q$ coefficients of the lag variables along with the calculated values of $\text{var}(e_{1t})$, $\text{var}(e_{2t})$, $\text{var}(e_{3t})$ and of covariances between them namely $\text{cov}(e_{1t}, e_{2t})$, $\text{cov}(e_{1t}, e_{2t})$, $\text{cov}(e_{1t}, e_{2t})$. Therefore, we have a total of $(9+9q)$ estimated parameters in the reduced-from VAR model. On the other hand, in the structural-form VAR system, we have a total of $(12+9q)$ parameters where there are three intercept terms (a_{10}, a_{20}, a_{30}), six contemporaneous coefficients ($a_{12}, a_{13}, a_{21}, a_{23}, a_{31}, a_{32}$), $9q$ lag coefficients and three variances- $\text{var}(\varepsilon_{it}), \text{var}(\varepsilon_{mt}), \text{var}(\varepsilon_{rt})$ or three standard deviations $\sigma_i, \sigma_m, \sigma_r$. Therefore, our structural system contains three more parameters than the reduced-form system that makes it under identified. Hence to make the structural system exactly identified we need three restrictions in it.

As mentioned in before, to identify the model, we follow the recursive system of Sims (1980). In our three variable basic VAR model $[EI_t, M_{1t}, R_{Tt}]$, we assume that money

supply (M_t) doesn't have a contemporaneous effect on inflationary expectations (EI_t) impying that the contemporaneous coefficient a_{12} is zero, and that the ex-ante real interest rate (R_t) doesn't have a contemporaneous effect on both inflationary expectations (EI_t) and money supply (M_t) implying that the contemporaneous coefficients a_{13} and a_{23} are zero. After imposing these restrictions on the contemporaneous coefficients, our structural VAR system becomes as follows:

$$EI_t = a_{10} + \gamma_{11} EI_{t-1} + \gamma_{12} M_{t-1} + \gamma_{13} R_{t-1} + .. + \delta_{11} EI_{t-q} + \delta_{12} M_{t-q} + \delta_{13} R_{t-q} + \varepsilon_{it} \quad (43)$$

$$M_t = a_{20} - a_{21} EI_t + \gamma_{21} EI_{t-1} + \gamma_{22} M_{t-1} + \gamma_{23} R_{t-1} + .. + \delta_{21} EI_{t-q} + \delta_{22} M_{t-q} + \delta_{23} R_{t-q} + \varepsilon_{mt} \quad (44)$$

$$R_t = a_{30} - a_{31} EI_t - a_{32} M_t + \gamma_{31} EI_{t-1} + \gamma_{32} M_{t-1} + \gamma_{33} R_{t-1} + .. \delta_{31} EI_{t-q} + \delta_{32} M_{t-q} + \delta_{33} R_{t-q} + \varepsilon_{rt} \quad (45)$$

Our restrictions, therefore, made the structural system exactly identified and we can recover all the parameters of this system that we will use in further analysis from the reduced-form VAR system. Since the structural shocks ε_t are white-noise process and the VAR disturbances e_t are composites of the structural shocks, it follows that the e_t also have zero means, constant variances, and are individually serially uncorrelated. If the reduced-from VAR disturbances vector e_t has a variance-covariance matrix V, we can write it as follows:

$$V = \begin{bmatrix} \text{var}(e_{1t}) & \text{cov}(e_{1t}, e_{2t}) & \text{cov}(e_{1t}, e_{3t}) \\ \text{cov}(e_{2t}, e_{1t}) & \text{var}(e_{2t}) & \text{cov}(e_{2t}, e_{3t}) \\ \text{cov}(e_{3t}, e_{1t}) & \text{cov}(e_{3t}, e_{2t}) & \text{var}(e_{3t}) \end{bmatrix}$$

Since the variances and the covariances in V are time independent, we can rewrite it as follows:

$$V = \begin{bmatrix} \sigma_1^2 & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_2^2 & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_3^2 \end{bmatrix}$$

On the other hand, we supposed that the structural disturbances $(\varepsilon_{it}, \varepsilon_{mt}, \varepsilon_{rt})$ are serially uncorrelated shocks with a covariance matrix equal to the identity matrix. That is, if we define the variance-covariance matrix of the structural disturbances as Λ then we find it as follows:

$$\Lambda = \begin{bmatrix} \text{var}(\varepsilon_{it}) & \text{cov}(\varepsilon_{it}, \varepsilon_{mt}) & \text{cov}(\varepsilon_{it}, \varepsilon_{rt}) \\ \text{cov}(\varepsilon_{mt}, \varepsilon_{it}) & \text{var}(\varepsilon_{mt}) & \text{cov}(\varepsilon_{mt}, \varepsilon_{rt}) \\ \text{cov}(\varepsilon_{rt}, \varepsilon_{it}) & \text{cov}(\varepsilon_{rt}, \varepsilon_{mt}) & \text{var}(\varepsilon_{rt}) \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

As defined earlier, the VAR disturbance vector e_t is a linear function of the underlying economic shocks ε_t as follows:

$$e_t = A^{-1} \varepsilon_t \qquad (46)$$

Since we assume that the coefficients a_{12}, a_{13} and a_{23} are all zero in matrix A, the structural disturbances (ε_t) and the estimated reduced-from VAR errors (e_t) are related as follows:

$$\begin{bmatrix} e_{1t} \\ e_{2t} \\ e_{3t} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ a_{21} & 1 & 0 \\ a_{31} & a_{32} & 1 \end{bmatrix}^{-1} \begin{bmatrix} \varepsilon_{it} \\ \varepsilon_{mt} \\ \varepsilon_{rt} \end{bmatrix}$$

Assuming $C = A^{-1}$, we can rewrite (46) as follows:

$$e_t = C \varepsilon_t \qquad (47)$$

Because of our restrictions that monetary policy doesn't have a contemporaneous effect on inflationary expectations and that the ex-ante real interest rate doesn't have a contemporaneous effect on both inflationary expectations and money supply ($a_{12} = a_{13}$

$= a_{23} = 0$), the C in (47) will be a unique lower triangular matrix. Equation (47), therefore, can be written more precisely as follows:

$$\begin{bmatrix} e_{1t} \\ e_{2t} \\ e_{3t} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ c_{21} & 1 & 0 \\ c_{31} & c_{31} & 1 \end{bmatrix} \begin{bmatrix} \varepsilon_{it} \\ \varepsilon_{mt} \\ \varepsilon_{rt} \end{bmatrix} \qquad (48)$$

This implies that the j th element of e_t is correlated with the first j elements of ε_t, but is orthogonal to the remaining elements of ε_t. In our three-variable basic model, we, therefore, have the following relationships between the reduced-form errors terms e_t and the ε_t.

$$e_{1t} = \varepsilon_{it}$$

$$e_{2t} = c_{21}\varepsilon_{it} + \varepsilon_{mt}$$

$$e_{3t} = c_{31}\varepsilon_{it} + c_{32}\varepsilon_{mt} + \varepsilon_{rt}$$

Since the variance-covariance matrix of ε_t is an identity matrix, it follows from (47) that C and is uniquely determined by the following relationship:

$$CC' = E[e_t e_t^{'}] = V \qquad (49)$$

4.3 The Feedback Rule, Exogenous Monetary Policy Shock and Impulse Response Function

Once the VAR model is estimated, our next job is estimate the responses of various macroeconomic variables due monetary policy shocks. Before estimating the impulse response functions we will first identify which portion of the monetary policy belongs to the feedback rule and which portion to the exogenous monetary policy shocks. We know while setting its monetary policy, the Bank of Canada both reacts to the economy as well as affects economic activity. It will be clear from the discussion of previous section and this section that we designed our VAR model to capture these cross-directional relationships between monetary policy and other macroeconomic variables.

Like previous research (for example, Khan et al., 2002, Edelberg and Marshall, 1996; Christiano et al. 1996 etc.), we define the feedback rule as a linear function ψ of a vector Ω_t of variables observed at or before date t. As mentioned earlier, the variable of time t that is used as a function of monetary policy is inflationary expectations in the basic model. The other variables that are used as function of monetary policy are the lagged values of all the variables used in the model. So the monetary policy can be completely described by the equation:

$$M_t = \Psi(\Omega_t) + c_{2,2}\ \varepsilon_{mt} \tag{50}$$

where ε_{mt} is the monetary policy shock and $c_{2,2}$ is the (2,2)th element of the matrix C. So in equation (50), $\Psi(\Omega_t)$ is the feedback rule component of monetary policy and $c_{2,2}\ \varepsilon_{mt}$ is the exogenous monetary policy shock component of monetary policy where Ω_t contains lagged values (dates t-1 and earlier) of all types of variables in the model, as well as the time t values of inflationary expectations. Therefore, in accordance with the assumption of the feedback rule, an exogenous shock ε_{mt} to the monetary policy cannot contemporaneously affect time t values of the elements in Ω_t although the lagged values of ε_{mt} can affect the variables in Ω_t.

Under the above assumptions and logical deductions, therefore, we can identify the first part of the right-hand side of equation (50) using the second equation of the reduced-form VAR model i.e., equation (41) as follows:

$$\Psi(\Omega_t) = b_{20} + b_{21}EI_{t-1} + b_{22}M_{t-1} + b_{23}R_{t-1} + \ldots + f_{21}EI_{t-q} + f_{22}M_{t-q} + f_{23}R_{t-q} + c_{21}\varepsilon_{it} \tag{51}$$

where c_{21} is the (2,1) th element of the matrix C and ε_{it} is the first element of ε_t. Here M_t is correlated with the first element of ε_t but is uncorrelated with the other element of ε_t[14]. Therefore, by construction, the shock $c_{2,2}\ \varepsilon_{mt}$ to monetary policy is uncorrelated

[14] AS we have only one variable ahead of the monetary policy variable in the basic model, we added $c_{21}\varepsilon_{it}$ with the lagged variables for the feedback rule equation $\Psi(\Omega)$. In the case of two variables ahead of the monetary policy variable we would add $c_{21}\varepsilon_{ir} + c_{22}\varepsilon_{it}$ and three variables we would add $c_{21}\varepsilon_{ir} + c_{22}\varepsilon_{it} + c_{23}\varepsilon_{it}$ and so on.

with Ω_t. Therefore, the feedback rule consists of the fitted equation of (M) plus a linear combination of the residual from the equation for inflationary expectations, EI[15]. The exogenous monetary policy shock is that portion of the residual in the (M) equation that is not correlated with this estimated feedback rule.

Our next job is to derive the impulse response functions. To derive the impulse response functions of our basic model we will start with transforming the reduced-from VAR model, taking only one lag for simplification, in the following matrix form:

$$\begin{bmatrix} EI_{tt} \\ M_i \\ R \end{bmatrix} = \begin{bmatrix} b_{10} \\ b_{20} \\ b_{30} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix} \begin{bmatrix} EI_{t-1} \\ M_{t-1} \\ R_{t-1} \end{bmatrix} + \begin{bmatrix} e_{1t} \\ e_{2t} \\ e_{3t} \end{bmatrix} \tag{52}$$

The vector moving average representation (VMA) of the above equation is as follows:

$$\begin{bmatrix} EI_t \\ M_t \\ R_t \end{bmatrix} = \begin{bmatrix} \overline{EI} \\ \overline{M} \\ \overline{R} \end{bmatrix} + \sum_{j=0}^{\infty} \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix}^j \begin{bmatrix} e_{1t-j} \\ e_{2t-j} \\ e_{3t-j} \end{bmatrix} \tag{53}$$

Replacing reduced-form errors e_t for the structural disturbances ε_t (using equation (46)) and doing the some more algebra, the moving average representation of (53) can be written as follows[16]:

[15] Since in basic model, only inflation expectations variable (EI) is ahead of monetary policy variable (M), i.e., only EI is contemporaneous input (type II variable) to the monetary policy, we add the linear combination of the residual from the equation for EI with the fitted equation of M. In the extended model, however, in addition to EI, we will add some more variables, like the unemployment rate (UNPR), the exchange rate (E) etc as type II variable, we will add their linear combination of the residuals also to the fitted equation of M.

[16] We are shortening the description here as we already discussed the impulse response function in section 2.3.

$$\begin{bmatrix} EI_t \\ M_t \\ R_t \end{bmatrix} = \begin{bmatrix} \overline{EI} \\ \overline{M} \\ \overline{R} \end{bmatrix} + \sum_{i=0}^{\infty} \begin{bmatrix} \varphi_{11}(i) & \varphi_{12}(i) & \varphi_{13}(i) \\ \varphi_{21}(i) & \varphi_{22}(i) & \varphi_{23}(i) \\ \varphi_{31}(i) & \varphi_{32}(i) & \varphi_{33}(i) \end{bmatrix} \begin{bmatrix} \varepsilon_{it-i} \\ \varepsilon_{mt-i} \\ \varepsilon_{rt-i} \end{bmatrix} \tag{54}$$

The sets of coefficients inside the summation matrix are the impulse response functions which are our main focus in estimating the effects of monetary policy shock. Since we have three variables in the basic VAR model and there are three impulse responses generated by each variable, there are a total of nine sets of impulse responses here. For example, in (54), $\varphi_{23}(0)$ shows the instantaneous impact of one unit change in exogenous monetary policy shock ε_{mt} on the ex-ante real interest rate(R). By the same token, $\varphi_{12}(1)$ is the effect on inflationary expectations (EI) after one period due to one unit change in monetary policy shock ε_{mt}. Similarly, $\varphi_{12}(0)$ is the instantaneous impact of one unit change in inflationary expectation shock ε_{it} on the money supply (M).

CHAPTER 5

In this chapter we present the estimated results. As mentioned in the introduction, we will estimate both the impacts of the central bank monetary policy on real and nominal variables as well as how the central bank reacts to changes in various macro-economic variables. All our data is monthly ranging from 1980 to 2002. The nominal interest rates used in the decomposition described in Section 2 are the one-year Government of Canada Treasury bill rate and the two-year and three-year Government of Canada benchmark bond yields. From the latter we calculate the two and three year forward rates. The Cansim series numbers of all variables are provided in Appendix 3.

5.1 The Impulse Responses of the Basic Model

First we report the impulse response of the Bank of Canada's monetary policy to a positive one standard deviation shock to inflationary expectations in Fig 4. With the increase in inflationary expectations under an inflation targeting regime, we anticipate the central bank's response is to tighten the money supply and we observe this response in Figure 4, although the response is insignificant. To see how the overnight rate responds in response to a positive inflationary expectations shock, we also report the impulse response of the overnight rate due to a positive one standard deviation inflationary expectations shock in Fig 5. As with Kahn et. al. (2002), the overnight rate response is more immediate than the monetary aggregate and is significant. This reinforces our view that our measure of inflationary expectations is a contemporaneous input to the policy process.

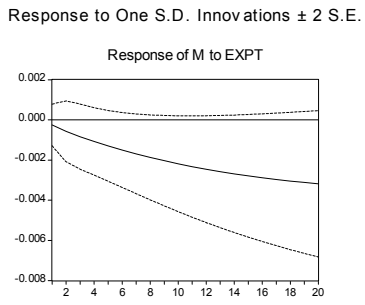


Figure 4: Impulse Response of M due to Inflationary Expectation Shocks

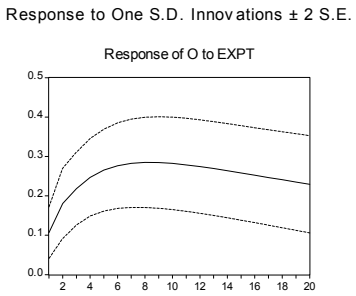


Figure 5: Impulse Response of the Overnight Rate to Inflationary Expectation Shocks

Second, we report the response of various macro-economic variables to a one standard deviation monetary policy shock from our VAR model[17]. We expect the innovation in the money supply to increase inflationary expectations and to reduce the real interest rate, although the degree of this impact on the real interest rate should vary depending on the maturity of the rate. In Figure 6 we report the reaction of inflationary expectations and the one-year ex-ante real interest rate to a positive monetary policy shock[18]. Observe from the impulse response functions that following a positive monetary policy shock, inflationary expectations increase (although the increase is not statistically significant) and the ex-ante real interest rate decreases. The effect on the ex-ante real interest rate remains statistically significant for eighteen months. Kahn et. al. (2002) also reported that for using monetary aggregate, M1, the effect on inflationary expectations is statistically insignificant while the effect on the one-year ex-ante real interest rate is statistically significant for about eight months. On the other hand, using the structural VAR model with contemporaneous restrictions and using a monetary aggregate (M1) as the monetary policy instrument, Cushman and Zha (1997) found that following a contractionary monetary policy shock the real interest rises and then declines, and the effect is significant for only the first two months.

[17] In the basic model, we use a lag-length of two which was determined on the basis of the Akaike Information Criterion. We keep using the same lag-length when we replace the ex-ante real rate with the ex-ante forward rate second year, the ex-ante forward rate of third year and the nominal interest rate which is also supported by the Akaike Information Criterion.

[18] The Solid line is the impulse response and the dashed lines contain the 95 percent confidence interval. These boundaries are calculated by 10,000 Monte Carlo repetitions.

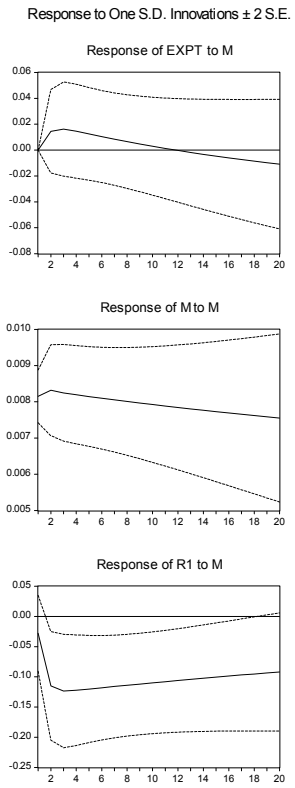


Figure 6: Impulse Response of Inflationary Expectations and the Ex-ante Real Rate due to Monetary Policy Shock

Although the Bank of Canada's policy impacts real interest rates at the short end of the maturity spectrum, we expect it may also impact real interest rates at longer horizons. Following Kahn et al. (2002), we use the forward ex-ante real interest rates of two- and three-years to estimate the longer-term impact of monetary shocks. We report the estimated impulse responses of the second- and third-year ex-ante forward rates in Figure 7. We find that the impact of a given monetary policy shock is smaller on the forward ex-ante real interest rate of the second year than on the ex-ante one-year real rate, and that the impact of this shock is smaller on the forward ex-ante real rate of the third year than on the ex-ante forward rate of second year. Therefore, we conclude that monetary policy shocks are more dominant on short-tem maturity interest rates. Edelberg and Marshall (1996) and Kahn et.al. (2002) also reported that monetary policy shocks have larger effects on short-term rates than on longer term rates.

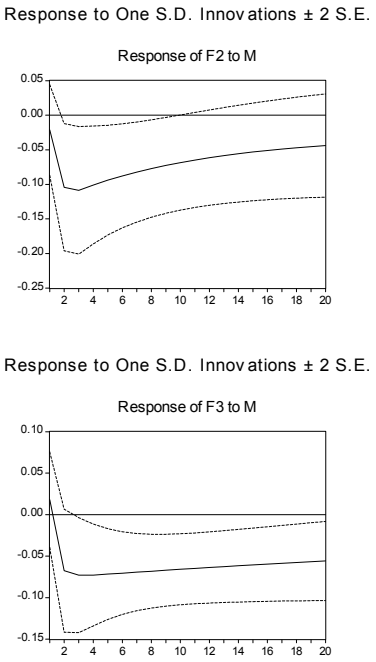


Figure 7: Impulse Response of the Second and Third Year Forward Rates

Most previous studies reported the response of nominal interest rates rather than ex-ante real interest rates to central bank monetary policy shocks[19]. To make our results comparable with these studies, we estimated the VAR model with nominal rates in place of real rates. It is important to note that the impact of monetary policy on nominal interest rates nets two opposite directional impacts-the impact on ex ante real interest rates and the impact on inflationary expectations. The shape and magnitude of the impact on nominal interest rates should, therefore, depend on the combined shape and magnitude of the impacts on the real interest rate and on inflationary expectations. We report the impact of a positive monetary policy shock on the nominal interest rate in Figure 8. As expected, the positive monetary policy shock lowers the one-year nominal interest rate, and it seems that this impact is a little smaller (0.10 percentage points) than the impact on the one year ex-ante real interest rate (0.13 percentage points). Kahn et al. (2002) also

[19] Both Edelberg and Marshall (1996) and Khan et al. (2002) examined the effects of monetary policy shocks on nominal interest rate. While Edelberg and Marshall used a proxy of inflation expectations in their model, Khan et al. used a market generated measure of the inflation expectations. Both studies reported relatively smaller effects on one-year nominal interest rate, and Edelberg and Marshall reported almost zero impact on longer-term nominal interest rates.

found a relatively smaller impact on the one-year nominal interest rate (0.30 percentage points) than on the one-year ex-ante real interest rate (0.40 percentage points). Using the overnight rate as the monetary policy instrument they found that a contractionary monetary policy shock raises the nominal interest rate which remains statistically significant for first four months. On the other hand, Cushman and Zha (1995), using the monetary aggregate as the monetary policy instrument, found that a contractionary monetary policy doesn't have any statistically significant effect on the nominal interest rate.

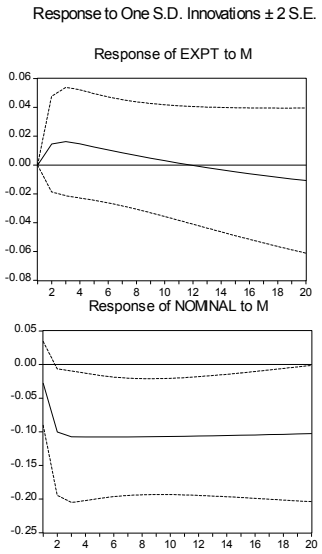


Figure 8: Impulse Responses of Inflationary Expectations and the Nominal Interest Rate

A criticism of the recursive VAR model is that its results crucially depend on the order of the variables in which they are estimated. We examine whether the change in the order of the variables impacts the estimates of the impulse response functions by re-estimating the model in the following order: M, EI and R. The estimated results are reported in Figure 9. It is clear from the figure that reversing the order of the variable doesn't have any significant impact on the impulse response functions.

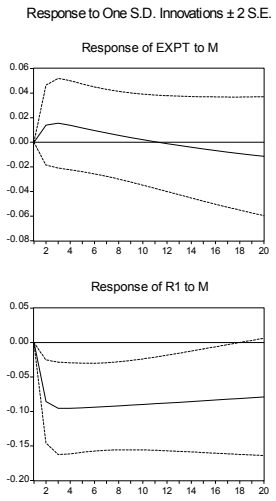


Figure 9: Impulse Responses for an Alternative Ordering of Variables

5.2 The Augmented Model

We augment the basic VAR model by incorporating some additional variables that may impact real interest rates and inflationary expectations. As suggested by prior research, if these variables are correlated with the monetary policy of the Bank of Canada, their omission may lead to erroneous conclusions about the impacts of monetary policy. The additional variables that we incorporate into the VAR model are the log of industrial production (Y), the unemployment rate (UNPR) and the log of Canadian/US dollar exchange rate (EXR) [20].

In this augmented model, we use inflationary expectations as Type II variable and specify the ex-ante real interest rate, the exchange rate, industrial output and the unemployment rate as Type III variables. The estimated impulse responses of this augmented model are reported in Figure 10. In the augmented model of Fig 10 with the ordering of the variables [EI, M, R1, EXR, Y, UNPR], inflationary expectations increase following a positive monetary policy shock for first five months but this increase is not significant. Following a positive monetary shock, the ex-ante real interest rate also decreases and the decrease remains significant for ten months. As is evident from Fig 10, the positive monetary policy shock temporarily depreciates the Canadian dollar (although it

[20] We use a lag-length of six in the augmented model which was determined on the basis of the Akaike Information criterion. The impulse responses do not change remarkably for using some other lags such as five, seven, eight or nine, and although we even get better impulse responses if we use lag-length of eight, we decided to use a lag-length of six as it was suggested by the Akaike Information Criterion.

appreciates in the first four months), increases industrial output (although it decreases in the first eight months) and lowers the unemployment rate (although it increases in the first four months) although these results are not statistically significant. Nevertheless,

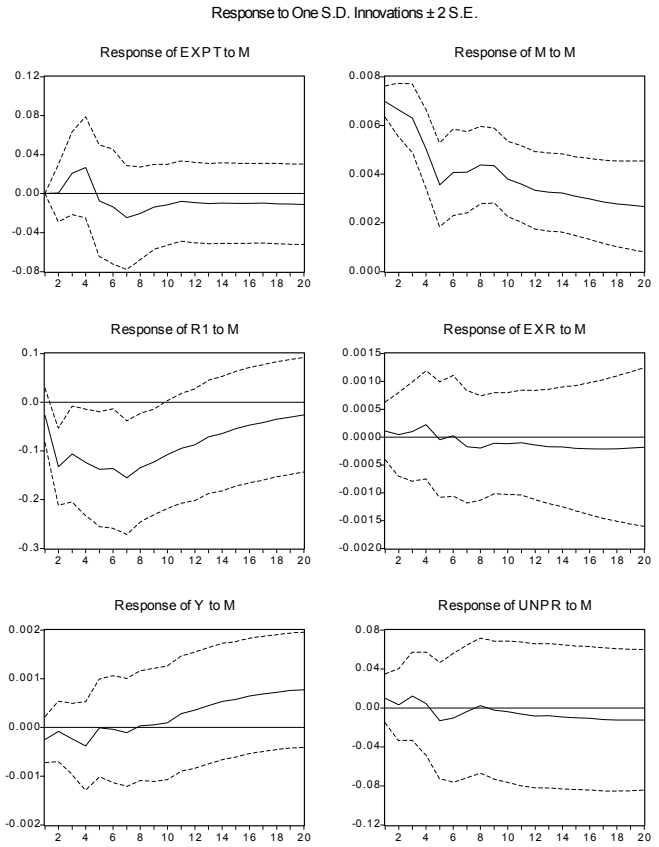


Figure 10: Impulse Responses of the Augmented Model.

the direction of movement is in accordance with conventional theory. Kahn et.al. (2002) also found insignificant effect on the unemployment rate although they obtained significant effects on the exchange rate from the second month to the third month. On the other hand, Cushman and Zha (1997) reported significant and consistent effects on the exchange rate and output. They found that a positive monetary policy shock increases output which remains significant for seven months and depreciates the Canadian currency which remains significant for thirteen months. They also noted that the monetary policy transmission mechanism occurs through the exchange rate because of the very short-run effect on the real interest rate.

To explore the possibility that the Bank of Canada might take the exchange rate as a contemporaneous input to the monetary policy reaction function along with inflationary expectations, we report the impulse response functions of the augmented model considering both inflationary expectations and the exchange rate as type II variable i.e., with the ordering of the variables [EI, EXR, M, R1, Y, UNPR] and [EXR, EI, M, R1, Y, UNPR] in Fig. 11 and Fig 12 respectively. As evident from Fig.11 and Fig.12, the impulse response functions don not change significantly for these new ordering of the variables.

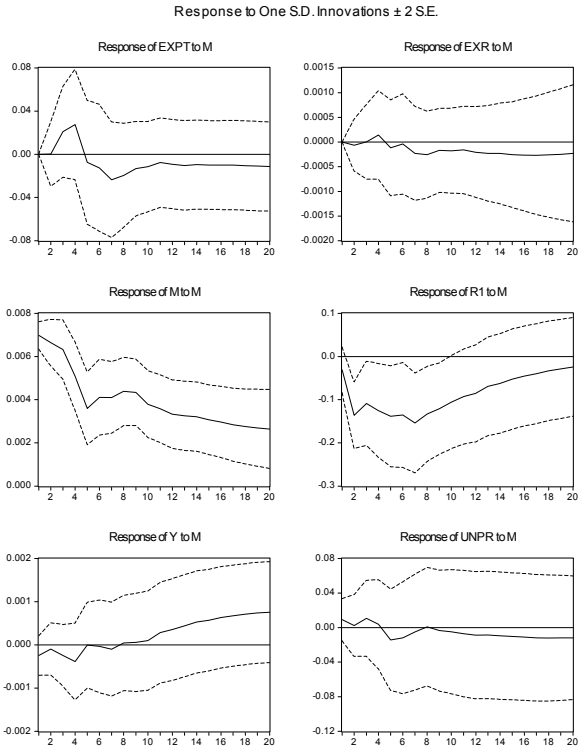


Figure 11: Impulse Responses with ordering [EI, EXR, M, Y, UNPR].

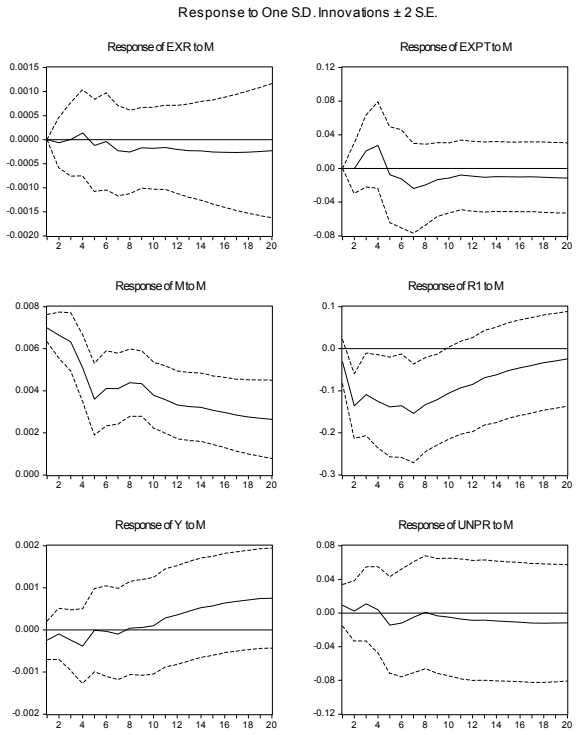


Figure 12: Impulse Responses with ordering [EXR, EI, M, Y, UNPR].

We also estimated the augmented model with the industrial output (Y) and the unemployment rate (UNPR) as Type II variables but the impulse responses do not change significantly to the change of this ordering. These impulse responses for these ordering are reported in Appendix 1. One thing to notice from the impulse responses of Fig A1 an Fig A2 in Appendix 1 is when we take the industrial output in addition to inflationary expectations as the contemporaneous input to the monetary policy reaction function, the effect on the industrial output marginally improves in the sense that the impulse response never goes above the zero line although it still remains statistically insignificant. Similarly when the unemployment is also taken as a contemporaneous input to the monetary policy reaction function its impulse response functions marginally improve meaning that it does not show any puzzle although the effect remains statistically insignificant as shown in Fig A3 and Fig A4.

Overall the impulse responses are not very satisfactory. This may reflect the fact that we use a monetary aggregate as the policy instrument and this variable may be affected by

other shocks in the model. Therefore, in the next section, we explore the overnight target rate as an alternative monetary policy instrument.

5.3 Impulse Responses Using the Overnight Target Rate

Many previous studies suggested that innovations in monetary aggregates might not truly represent the exogenous change in monetary policy. We, therefore, estimate our basic VAR model as well as the augmented VAR model using the overnight target rate as the monetary policy instrument[21].

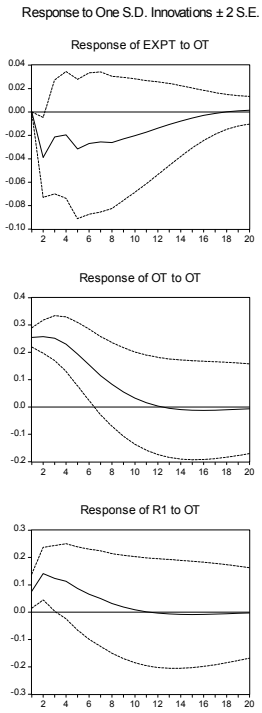
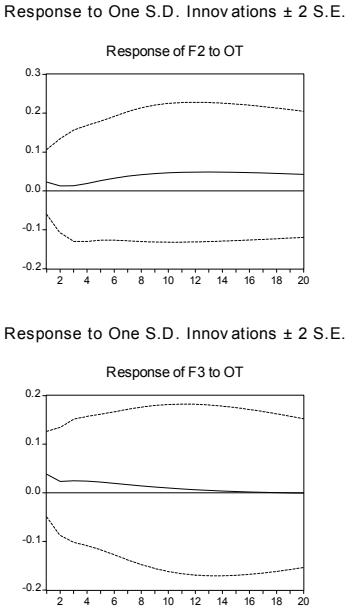


Figure 13: Impulse Responses of EI and R1 due to Overnight Target rate shocks.

[21] For Canada, we do not have the data on overnight target rate (OT) prior to 1994, and therefore, the sample period for the overnight target rate starts from 1994:1. To test our results, we also tried using the overnight rate (O) prior to 1994 and the overnight target rate from 1994 for the same model in which case we get statistically insignificant results for most of the variables. We also estimated our models using the overnight rate from 1980:1 to 2002:12 and get insignificant results for most of the variables. It would appear that innovations in the overnight rate do not measure exogenous monetary shocks either.

We report the impulse responses of the basic model using the overnight target rate in Fig 13[22]. It is evident from Fig 13 that a contractionary monetary policy shock introduced by increasing the overnight target rate (i.e., one standard deviation shock to the overnight target rate which is 0.2549 percentage points) lowers inflationary expectations by 0.04 percentage point and raises the ex-ante real interest rate by 0.15 percentage point. The shocks to inflationary expectations remain significant for three months and the shocks to the ex-ante real interest rate are significant for four months. It is important to recall that the impulse response functions of inflationary expectations in the basic model as well as in the augmented model using a monetary aggregate as the monetary policy instrument were statistically insignificant. This result suggests that innovations to the overnight target rate are a better measure of monetary policy shocks.

In Fig 14, we report the impulse responses of the second and the third year ex-ante real forward rates. As shown in Fig 14, the impulse responses of the second and the third year ex-ante real forward rates are insignificant using the overnight target rate while they were significant using the monetary aggregate. This result may reflect lags between changes in the overnight target rate and changes in the monetary aggregate.



[22] We used a lag-length of four in the basic model of the fully recursive VAR model which was determined on the basis of the Akaike Information Criterion. The impulse response functions are invariant for choosing

Figure 14: Impulse Responses of F2 and F3 due to Over Night Target Rate Shocks

Next we report the impulse response functions of the nominal interest rate due to a contractionary monetary policy shock introduced by a rise in the overnight target rate in Fig 15.

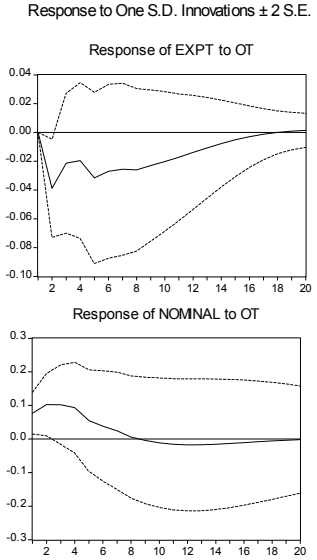


Figure 15: Impulse Responses of EI and Nominal due to Overnight Rate

As evident from Fig 15, the impact of the contractionary monetary policy shock (i.e., one standard deviation shock to the overnight target rate which is 0.2549 percentage points) on the one year nominal interest rate is smaller (0.10 percentage point) than its impact on the one year ex-ante real interest rate (0.15 percentage point). This result is quite consistent with our expectation as the effect on the nominal interest rate nets the opposite directional effects on inflationary expectations and on the ex-ante real interest rate. Kahn et.al. (2002) also found that the impact on the nominal interest rate (0.35 percentage point) is smaller than the impact on the ex-ante real interest rate (0.40 percentage point) using the overnight rate as the monetary policy instrument.

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other lag-lengths such as two, three or five. The solid line is the impulse response and the dashed lines contain the 95% confidence intervals which are calculated with 10,000 Monte Carlo repetitions.

Next we report the impulse response functions of the basic model using the ordering [OR, EI, R1] in Fig 16. We notice that the shape and the direction of the impulse response functions do not change for this ordering but the impulse response of inflationary expectations are not statistically significant whereas they are significant for the first three months using the ordering of the variables [EI, OT, R1]. This result suggests our identification scheme is valid.

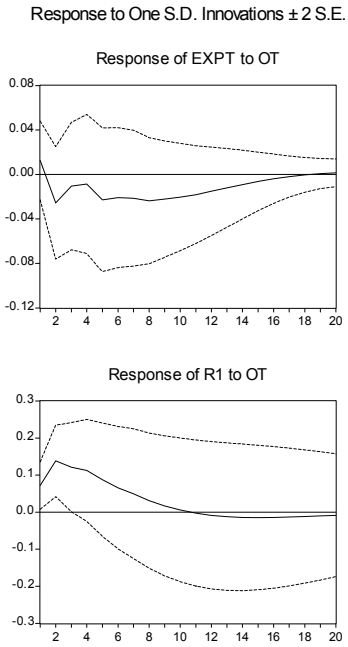

Figure 16: Impulse Responses of EI and R1 for ordering [OT, EI, R1]

The impulse responses of the augmented model with the overnight target rate are reported in Figure 17 with the ordering of [EI, OT, R1 EXR, Y, UNPR][23].

[23] We used a lag-length of three in the augmented model which was determined on the basis of the Akaike Information Criterion. The impulse response functions are invariant for choosing other lag-lengths such as two or four. The solid line is the impulse response and the dashed lines contain the 95% confidence intervals which are calculated with 10,000 Monte Carlo repetitions.

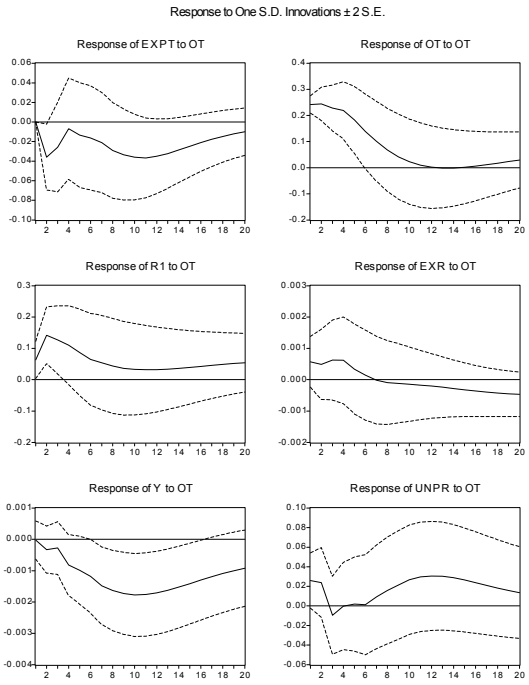

Figure 17: Impulse Responses with ordering [EXPT, OT, R1, EXR, Y, UNPR]

In the augmented model of Fig 17, we find that following a contractionary monetary policy shock (i.e., one standard deviation shock to the overnight target rate which is 0.2549 percentage points) inflationary expectations declines by 0.04 percentage point (statistically significant for three months), the ex-ante real interest rate increases by 0.15 percentage point (statistically significant for four months), the industrial output declines (statistically significant from month six to month sixteenth), exchange rate appreciates for first seven months (statistically insignificant) and the unemployment rate increases (statistically insignificant). We have two significant improvements in the augmented model using the overnight target rate as the monetary policy instrument over using a monetary aggregate. In the augmented model using the overnight target rate we have statistically significant effects both on inflationary expectations and on the industrial output while these effects were statistically insignificant using a monetary aggregate. Another thing that comes to attention in Fig 17 is when the shock on the industrial output reaches the peak in around month ten the shock on the unemployment rate also reaches its peak in the opposite direction. In other words, it implies that when the contractionary monetary policy reduces the industrial output it also increases the unemployment rate simultaneously.

Next we report the impulse response functions of the augmented model with the ordering of **[EI, EXR, OT, R1, Y, UNPR] and [EXR, EI, OT, R1, Y, UNPR] in** Fig 18 and Fig 19 respectively. It seems from these figures that the impulse responses do not improves for this ordering.

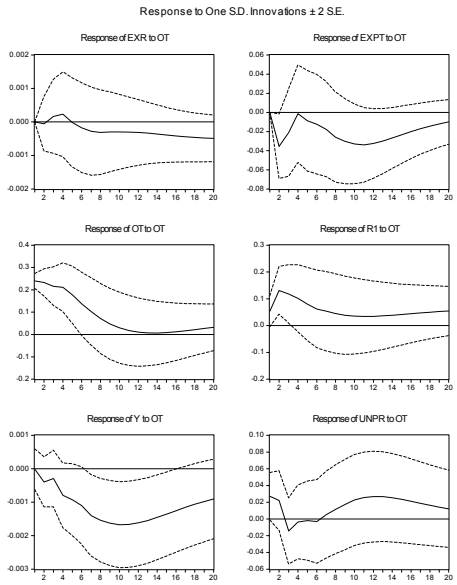

Figure 18: Impulse Responses with ordering [EI, Y, OT, R1, EXR, UNPR]

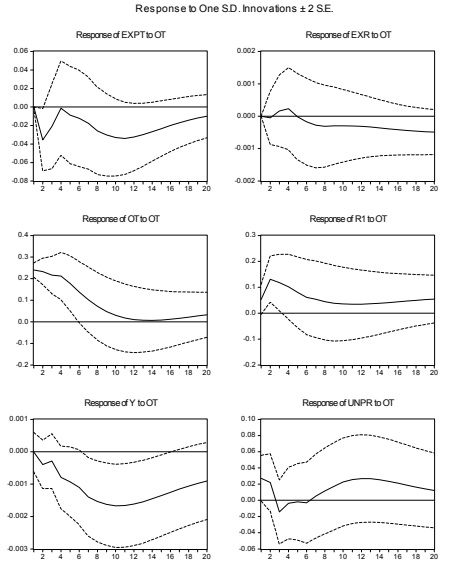

Figure 19: Impulse Responses with ordering [EI, Y, OT, R1, EXR, UNPR]

We also estimate the augmented model with all other possible ordering of the variables the impulse responses of which are reported in Appendix 2. It seems from the impulse responses in Appendix 2 that the estimated results are robust to the change of the ordering of the variables.

6. CONCLUSIONS

We estimated the impact of monetary policy on various real and nominal macroeconomic variables. Our approach of decomposing the nominal interest rate into the ex-ante real interest rate and inflationary expectations using the Blanchard-Quah VAR model made it possible to separately examine the reactions of these variables to monetary policy shocks. Using only inflationary expectations and/or inflationary expectations and other macroeconomic variables (industrial output, the exchange rate, and the unemployment rate) as contemporaneous input/inputs to the policy reaction function of the Bank of Canada, we don't encounter anomalies such as the liquidity or exchange rate puzzles that plagued early VAR studies of monetary policy shocks.

Our principal findings are that a positive one-standard-deviation monetary policy shock, identified as an innovation to M1B money, increases inflationary expectations by 0.02 percentage points and lowers the ex-ante real interest rate by 0.12 percentage points. The response of inflationary expectations and the ex-ante real interest rate reach their peak in about three months after the shock. We also find that the impact on the one-year rate ex-ante real interest rate is larger than the impact on the ex-ante forward rate of the second year which is 0.10 percentage points. On the other hand, the impact of this shock on the ex-ante forward rate of the third year 0.07 percentage points. The impact of a monetary policy shock on the one-year nominal interest rate which nets the impact on inflationary expectations and the ex-ante real interest rate is smaller (0.10 percentage point for a one-standard deviation shock in M1B) than its impact on one-year ex-ante real interest rate. Our estimated VAR model is robust to a change in the number of variables. We extended the model by including the exchange rate, industrial output and the unemployment rate and we find that a positive monetary policy shock temporarily depreciates the Canadian currency, increases real output and lower the unemployment rate although none of these effects are statistically significant.

When we use the overnight target rate (OT) as the monetary policy instrument rather than a monetary aggregate, we end up with significantly better results in many aspects. We

find that the impulse response functions of inflationary expectations and industrial output become statistically significant using the overnight target rate as the monetary policy instrument while these effects are insignificant when we use the monetary aggregate. In the model with the overnight target rate, we find that a contractionary monetary policy shock (i.e., one standard deviation shock to the overnight target rate which is 0.2549 percentage points) raises the one-year ex-ante real interest by 0.15 percentage point and lowers inflationary expectations by 0.04 percentage point. On the other hand, this monetary policy shock doesn't have any significant effect on the ex-ante real forward rate of year two and year three. Such a contractionary monetary policy also raises the one-year nominal interest rate which is smaller in magnitude (0.10 percentage point) than the one-year ex-ante real interest rate.

The augmented model with the overnight target rate as the monetary policy instrument gives even more attractive results. In the augmented model with output, the unemployment rate and the exchange rate as additional variables, we find that a contractionary monetary policy lowers industrial output which is statistically significant, increases the unemployment rate and appreciates the Canadian currency. Our estimated VAR model is robust to the changes in the ordering of the variables.

That we obtain significantly better results in recursive VAR model using the overnight target rate rather than a monetary aggregate likely follows because, unlike money, the overnight target rate cannot be influenced by private sector behavior except through the channel of an endogenous policy response of the central bank to changing economic conditions. Since our approach models this reaction function explicitly, we are able to estimate monetary policy shocks that are exogenous to other variables in the model. We conclude it is better to model monetary policy shocks in a recursive VAR model with the overnight target rate.

We believe that these results complement the work of Cushman and Zha (1997) in modeling the monetary policy in Canada. They demonstrate that to avoid anomalies that characterized previous attempts to estimate the effects of monetary policy shocks using a

monetary aggregate, it is necessary to impose identifying restrictions in a structural VAR model in order to separate the money demand function from the money supply (policy reaction) function and thereby estimate policy shocks that are exogenous. Since we are able to obtain qualitatively similar results with a more parsimonious specification using the overnight target rate as the policy instrument, we believe this approach could serve as a useful complement to that of Cushman and Zha (1997). Our results differ from Cushman and Zha (1997) in one important respect. They find that the transmission mechanism from monetary policy shocks to real output work primarily though an exchange rate effect. While we obtain this effect as well, we also obtain a significant role for the real interest rate channel. That is, we find that a positive one standard deviation shock to the overnight target rate (which is 0.2549 percentage points) raises the one-year ex-ante real interest rate by 0.15 percentage points.

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Appendix 1: Impulse Response of the Augmented Model for Using Monetary Aggregate

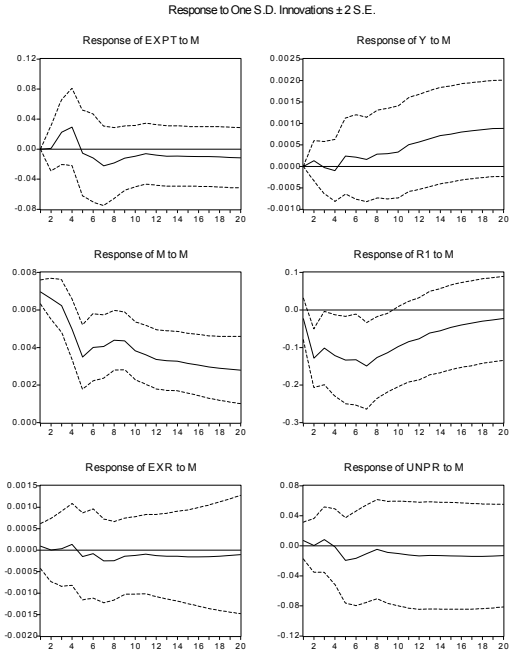

Figure A1: Impulse Responses with the ordering of [EI, Y, M, R1, EXR, UNPR]

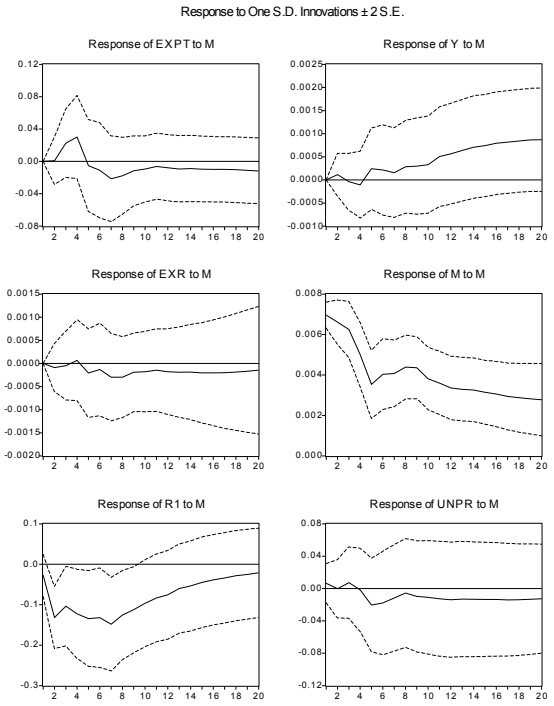

Figure A2: Impulse Responses with the ordering of [EI, Y, EXR, M, R1, UNPR]

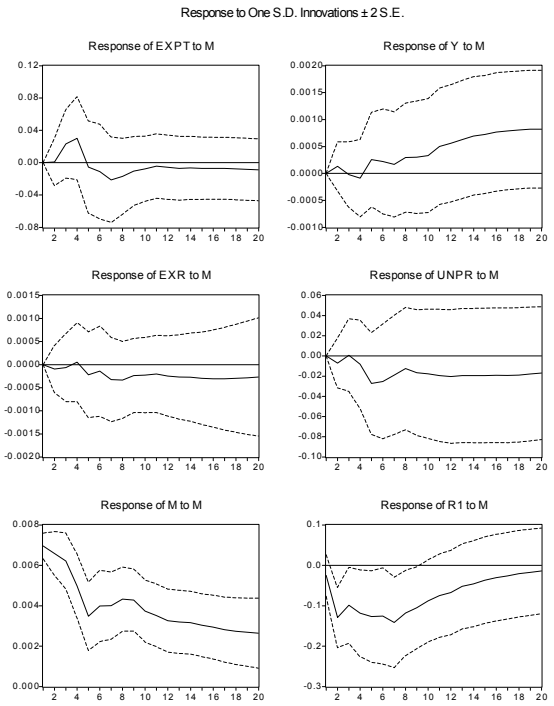

Figure A3: Impulse Responses with the ordering of [EI, Y, EXR, UNPR, M, R1]

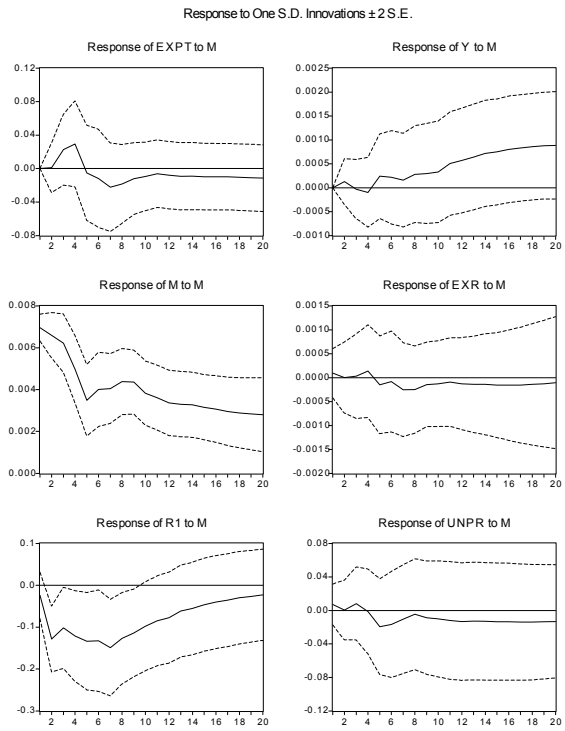

Figure A4: Impulse Responses with the ordering of [EI, Y, M EXR, R1, UNPR]

APPENDIX 2: Impulse Responses of the Augmented Model for using Overnight Target Rate

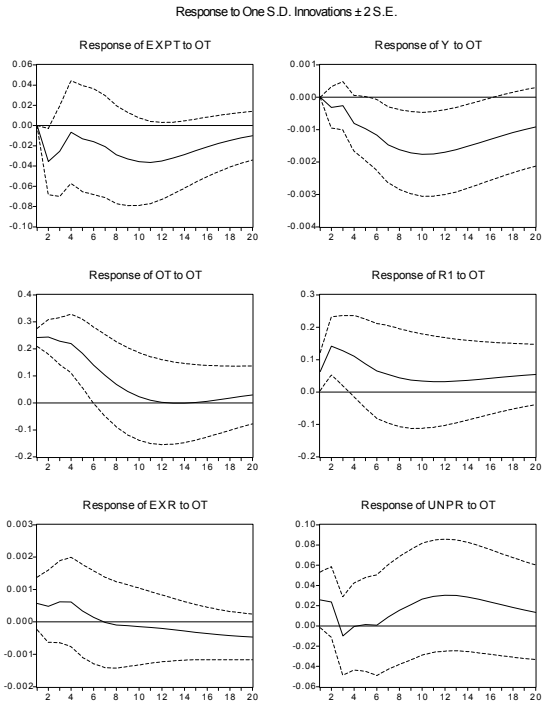

Figure A5: Impulse Responses with ordering [EI, Y, OT, R1, EXR, UNPR]

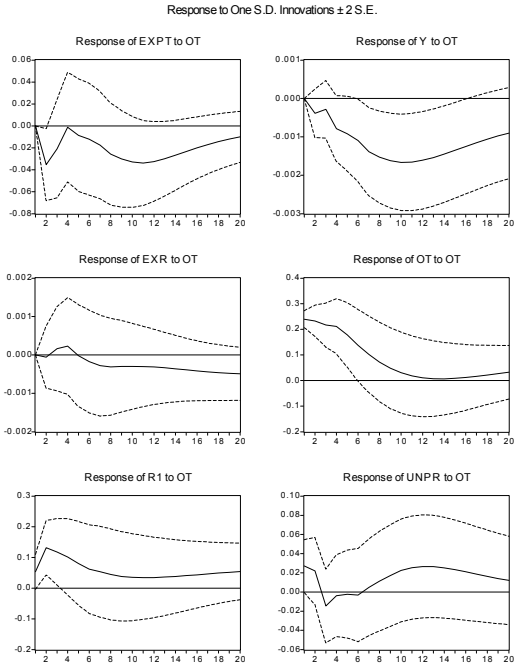

Figure A6: Impulse Responses with the ordering of [EI, Y, EXR, OT, R1, UNPR]

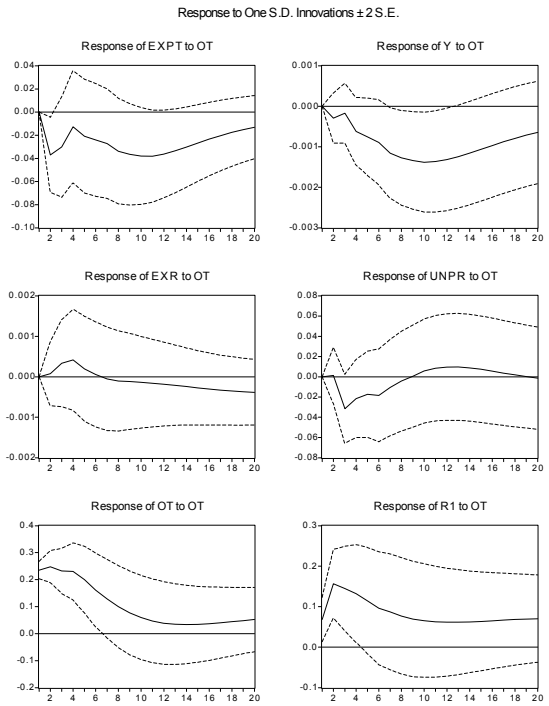

Figure A7: Impulse Responses with the ordering of [EI, Y, EXR, UNPR, OT, R1]

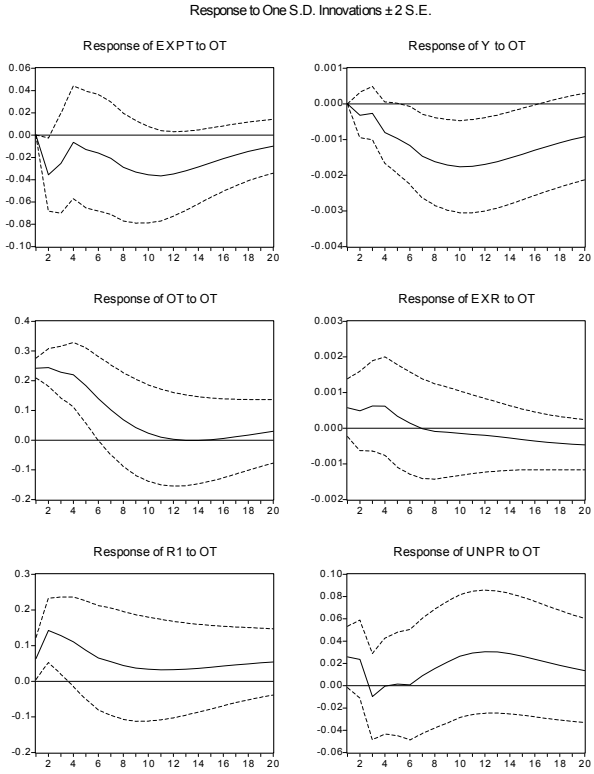

Figure A8: Impulse Responses with the ordering of [EI, Y, EXR, UNPR, OT, R1]

Appendix 3

Data Sources

Money Supply-M1B
Source: Cansim, Series Level-V37199

Overnight Rate
Source: Cansim, Series Level-V122514

One-year nominal interest rate- Government of Canada One-year Treasury Bills Rate
Source: Cansim, Series Level-V122533

Two-year nominal interest rate-Selected Government of Canada Benchmark Bond Yields
Source: Cansim, Series Level-V122538

Three-year nominal interest rate- Selected Government of Canada Benchmark Bond Yields
Source: Cansim, Series Level-V122539

Consumer Price Index (CPI)
Source: Cansim, Series Level-V737311

Exchange Rate-US dollar/Canadian dollar
Source: Cansim, Series Level-V37426

Industrial Production
Source: Cansim, Series Level-V2044332

Unemployment Rate
Source: Cansim, Series Level-V2062815

Overnight Target Rate
Source: Bank of Canada (Summary of Key Monetary Policy Variables: 1996-2002)
: Cansim Series Level-B114039 (1994-1996)