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Parameter Dependence of Ballistic Velocity in Deterministic Diffusion in the Form of Devil's Staircase

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1 Introduction

The diffusion coefficient defined via conventional mean square displacements is not sufficient for describing non-Gaussian statistics of velocity caused by ballistic motion in diffusive processes. In order to remedy this deficiency, we introduce large-deviation statistics also known as thermo-dynamical formalism.

2 Large-deviation statistics of velocity in diffusive dynamics

Let us briefly describe large-deviation statistics following the series of studies by Fujisaka and Inoue [1]. Consider a stationary time series of velocity u . The average over time

interval T is given by this formula, $\bar{u}_T(t) = \frac{1}{T} \int_t^{t+T} u\{s\} ds$ which distributes when T is finite. When T is much larger than the correlation time of u , the distribution of coarse-grained u is assumed to be an exponential form $P_T(u) \propto e^{-S(u)T}$. Here we can

introduce the fluctuation spectrum $S(u)$ as $S(u) = -\lim_{T \rightarrow \infty} \frac{1}{T} \log P_T(u)$. When T is comparable to the correlation time, correlation cannot be ignored, so non-exponential or non-extensive statistics will be a problem, but here we do not discuss this point further. Let q be a real parameter. We introduce the generating function M_q of T by this

definition. $M_q(T) \equiv \langle e^{qT\bar{u}_T} \rangle = \int_{-\infty}^{\infty} P_T(u) e^{qTu} du$ We can here also assume the exponential distribution and introduce a characteristic function $\phi(q)$ as

$\phi(q) = \lim_{T \rightarrow \infty} \frac{1}{T} \log M_q(T)$. The Legendre transform holds between fluctuation spectrum $S(u)$ and characteristic function $\phi(q)$, which is obtained from saddle-point

calculations: $\frac{dS(u)}{du} = q$, $\phi(q) = -S(u(q)) + qu(q)$. In this transform a derivative

$d\phi/dq$ appears, and it is a weighted average of \bar{u}_T , $u(q) = \frac{d\phi(q)}{dq} = \lim_{T \rightarrow \infty} \frac{\bar{u}_T e^{qT\bar{u}_T}}{M_q(T)}$

so we find that q is a kind of weight index. We can also introduce susceptibility

$\chi(q) = \frac{du(q)}{dq}$ as a weighted variance. These statistical structure functions

$S(u), \phi(q), u(q), \chi(q)$ constitute the framework of statistical thermodynamics of temporal fluctuation, which characterize static properties of chaotic dynamics.

3 Conclusion

We deal with extracting the non-Gaussian characteristics of the phenomenon of diffusion. Then, we refer to the mapping system in which Klages and Dorfman discovered complex dependence of diffusion coefficients on the parameter [2]. A graph plotting the weighted mean velocity $u(q)$ of diffusive particles against parameter a is numerically obtained. This demonstrates that the graph $u(q \rightarrow \infty)$ corresponding to the velocity of the ballistic trajectory has a structure resembling the devil's staircase. It is found that the unstable periodic orbit corresponding to the ballistic motion with the largest velocity in this system changes in a complex manner depending on the value of parameter a . This seems to be one of the factors explaining why the graphs of $u(q)$ have complex structures.

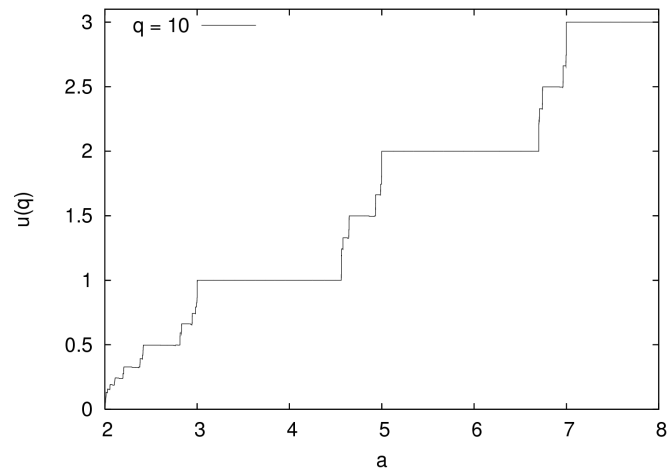


Fig. 1: Parameter dependence of ballistic velocity

References

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